

# NetXPTO - LinkPlanner

20 de Outubro de 2017

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## **Capítulo 1**

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### **Introduction**

LinkPlanner is devoted to the simulation of point-to-point links.

## **Capítulo 2**

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### **Simulator Structure**

LinkPlanner is a signals open-source simulator.

The major entity is the system.

A system comprises a set of blocks.

The blocks interact with each other through signals.

#### **2.1 System**

#### **2.2 Blocks**

#### **2.3 Signals**

List of available signals:

- Signal

## **Capítulo 3**

## **Development Cycle**

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The NetXPTO-LinkPlanner has been developed by several people using git as a version control system. The NetXPTO-LinkPlanner repository is located in the GitHub site <http://github.com/netxpto/linkplanner>. The more updated functional version of the software is in the branch master. Master should be considered a functional beta version of the software. Periodically new releases are delivered from the master branch under the branch name Release<Year><Month><Day>. The integration of the work of all people is performed by Armando Nolasco Pinto in the branch Develop. Each developer has his/her own branch with his/her name.

## **Capítulo 4**

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## **Visualizer**

visualizer

## Capítulo 5

## Case Studies

### 5.1 QPSK Transmitter

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2017-08-25, Review, Armando Nolasco Pinto

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This system simulates a QPSK transmitter. A schematic representation of this system is shown in figure 5.1.



Figura 5.1: QPSK transmitter block diagram.

#### System Input Parameters

**Parameter:** *sourceMode*

**Description:** Specifies the operation mode of the binary source.

**Accepted Values:** PseudoRandom, Random, DeterministicAppendZeros, DeterministicCyclic.

**Parameter:** *patternLength*

**Description:** Specifies the pattern length used by the source in the PseudoRandom mode.

**Accepted Values:** Integer between 1 and 32.

**Parameter:** *bitStream*

**Description:** Specifies the bit stream generated by the source in the DeterministicCyclic and DeterministicAppendZeros mode.

**Accepted Values:** "XXX..", where X is 0 or 1.

**Parameter:** *bitPeriod*

**Description:** Specifies the bit period, i.e. the inverse of the bit-rate.

**Accepted Values:** Any positive real value.

**Parameter:** *iqAmplitudes*

**Description:** Specifies the IQ amplitudes.

**Accepted Values:** Any four pair of real values, for instance { { 1,1 },{ -1,1 },{ -1,-1 },{ 1,-1 } }, the first value correspond to the "00", the second to the "01", the third to the "10" and the forth to the "11".

**Parameter:** *numberOfBits*

**Description:** Specifies the number of bits generated by the binary source.

**Accepted Values:** Any positive integer value.

**Parameter:** *numberOfSamplesPerSymbol*

**Description:** Specifies the number of samples per symbol.

**Accepted Values:** Any positive integer value.

**Parameter:** *rollOffFactor*

**Description:** Specifies the roll off factor in the raised-cosine filter.

**Accepted Values:** A real value between 0 and 1.

**Parameter:** *impulseResponseTimeLength*

**Description:** Specifies the impulse response window time width in symbol periods.

**Accepted Values:** Any positive integer value.

>>> Romil

## 5.2 BPSK Transmission System

<b>Student Name</b>	:	Daniel Pereira
<b>Starting Date</b>	:	September 1, 2017
<b>Goal</b>	:	Estimate the BER in a Binary Phase Shift Keying optical transmission system with additive white Gaussian noise.
<b>Directory</b>	:	sdf/bpsk_system

Binary Phase Shift Keying (BPSK) is the simplest form of Phase Shift Keying (PSK), in which binary information is encoded into a two state constellation with the states being separated by a phase shift of  $\pi$ .

White noise is a random signal with equal intensity at all frequencies, having a constant power spectral density. White noise is said to be Gaussian (WGN) if its samples follow a normal distribution with zero mean and a certain variance  $\sigma^2$ . For WGN we know that its spectral density equals its variance. For the purpose of this work, additive WGN is used to model thermal noise typically observed in coherent receivers.

The purpose of this system is to simulate BPSK transmission in back-to-back configuration with additive WGN at the receiver, perform an accurate estimation of the BER and validate the estimation using theoretical values.

### 5.2.1 BER of BPSK with additive WGN (Theoretical Analysis)

The output of the system with added gaussian noise follows a normal distribution, whose first probabilistic moment can be readily obtained by knowledge of the optical power of the received signal and local oscillator,

$$m_i = 2\sqrt{P_L P_S} G_{ele} \cos(\Delta\theta_i), \quad (5.1)$$

where  $P_L$  and  $P_S$  are the optical powers, in watts, of the local oscillator and signal, respectively,  $G_{ele}$  is the gain of the transimpedance amplifier the coherent receiver and  $\Delta\theta_i$  is the phase difference between the local oscillator and the signal, for BPSK this takes the values  $\pi$  and 0, in which case (5.1) can be reduced to,

$$m_i = \pm 2\sqrt{P_L P_S} G_{ele}. \quad (5.2)$$

The second moment is directly chosen by inputting the spectral density of the noise  $\sigma^2$ , and thus is known *a priori*.

Both probabilist moments being known, the probability distribution of measurement results is given by a simple normal distribution,

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m_i)^2}{2\sigma^2}}. \quad (5.3)$$

The BER is calculated in the following manner,

$$BER = \frac{1}{2} \int_0^{+\infty} f(x|\Delta\theta = \pi) dx + \frac{1}{2} \int_{-\infty}^0 f(x|\Delta\theta = 0) dx, \quad (5.4)$$

given the symmetry of the system, this can be simplified to,

$$BER = \int_0^{+\infty} f(x|\Delta\theta = \pi)dx = \frac{1}{2}\operatorname{erfc}\left(\frac{-m_i}{\sqrt{2}\sigma}\right) \quad (5.5)$$

### 5.2.2 Simulation

A diagram of the system being simulated is presented in the Figure 5.2. A random binary string is generated and encoded in an optical signal using BPSK modulation. The decoding of the optical signal is accomplished by an homodyne receiver, which combines the signal with a local oscillator. The received binary signal is compared with the transmitted binary signal in order to estimate the Bit Error Rate (BER). The simulation is repeated for multiple signal power levels, each corresponding BER is recorded and plotted against the expectation value.

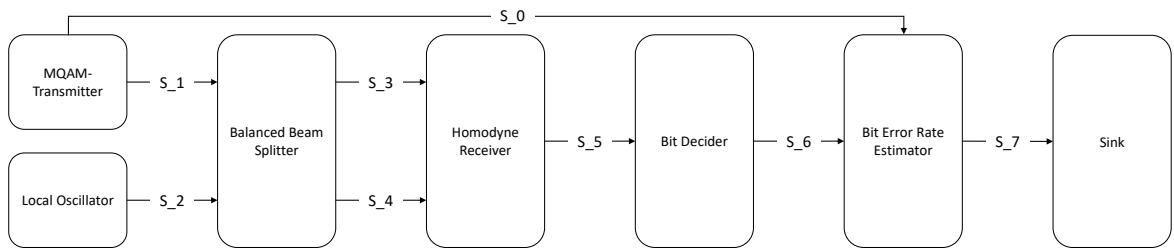


Figura 5.2: Overview of the BPSK system being simulated.

System Blocks	netxpto Blocks
BPSK Transmitter	MQamTransmitter
BPSK Receiver	HomodyneReceiver
BER Estimator	BitErrorRate

### Required files

#### Header Files

File	Description
netxpto.h	Generic purpose simulator definitions.
m_qam_transmitter.h	Generates the signal with coded constellation.
local_oscillator.h	Generates a continuous optical signal with set power and phase.
balanced_beam_splitter.h	Mixes the two optical signals set at it's input.
homodyne_reciever.h	Performs coherent detection on the input signal.
sampler.h	Samples the input signal at a user defined frequency.
bit_decider.h	Decodes the input signal into a binary string.
bit_error_rate.h	Calculates the bit error rate of the decoded string.
sink.h	Closes any unused signals.

### Source Files

File	Description
netxpto.cpp	Generic purpose simulator implementations.
m_qam_transmitter.cpp	Generates the signal with coded constellation.
local_oscillator.cpp	Generates a continuous optical signal with set power and phase
balanced_beam_splitter.h	Mixes the two optical signals set at it's input.
homodyne_reciever.cpp	Performs coherent detection on the input signal.
sampler.cpp	Samples the input signal at a user defined frequency.
bit_decider.cpp	Decodes the input signal into a binary string.
bit_error_rate.cpp	Calculates the bit error rate of the decoded string.
sink.cpp	Closes any unused signals.

### System Input Parameters

This system takes into account the following input parameters:

System Parameters	Description
numberOfBitsGenerated	Gives the number of bits to be simulated
bitPeriod	Sets the time between adjacent bits
samplesPerSymbol	Establishes the number of samples each bit in the string is given
pLength	PRBS pattern length
iqAmplitudesValues	Sets the state constellation
outOpticalPower_dBm	Sets the optical power, in units of dBm, at the transmitter output
loOutOpticalPower_dBm	Sets the optical power, in units of dBm, of the local oscillator used in the homodyne detector
localOscillatorPhase	Sets the initial phase of the local oscillator used in the homodyne detector
transferMatrix	Sets the transfer matrix of the beam splitter used in the homodyne detector
responsivity	Sets the responsivity of the photodiodes used in the homodyne detector
amplification	Sets the amplification of the trans-impedance amplifier used in the homodyne detector
noiseSpectralDensity	Sets the spectral density of the gaussian thermal noise added in the homodyne detector
confidence	Sets the confidence interval for the calculated QBER
midReportSize	Sets the number of bits between generated QBER mid-reports

### Inputs

This system takes no inputs.

## Outputs

This system outputs the following objects:

**Parameter:** Signals:

**Description:** Initial Binary String; ( $S_0$ )

**Description:** Optical Signal with coded Binary String; ( $S_1$ )

**Description:** Local Oscillator Optical Signal; ( $S_2$ )

**Description:** Beam Splitter Outputs; ( $S_3, S_4$ )

**Description:** Homodyne Detector Electrical Output; ( $S_5$ )

**Description:** Decoded Binary String; ( $S_6$ )

**Description:** BER result String; ( $S_7$ )

**Parameter:** Other:

**Description:** Bit Error Rate report in the form of a .txt file. (BER.txt)

## Simulation Results

The following results show the dependence of the error rate with the signal power assuming a constant Local Oscillator power of 0 dBm, the signal power was evaluated at levels between -70 and -25 dBm, in steps of 5 dBm between each. The simulation results are presented in orange with the computed lower and upper bounds, while the expected value, obtained from (5.5), is presented as a full blue line. A close agreement is observed between the simulation results and the expected value. The noise spectral density was set at  $\sqrt{2}0.0005 \text{ V}^2$ .

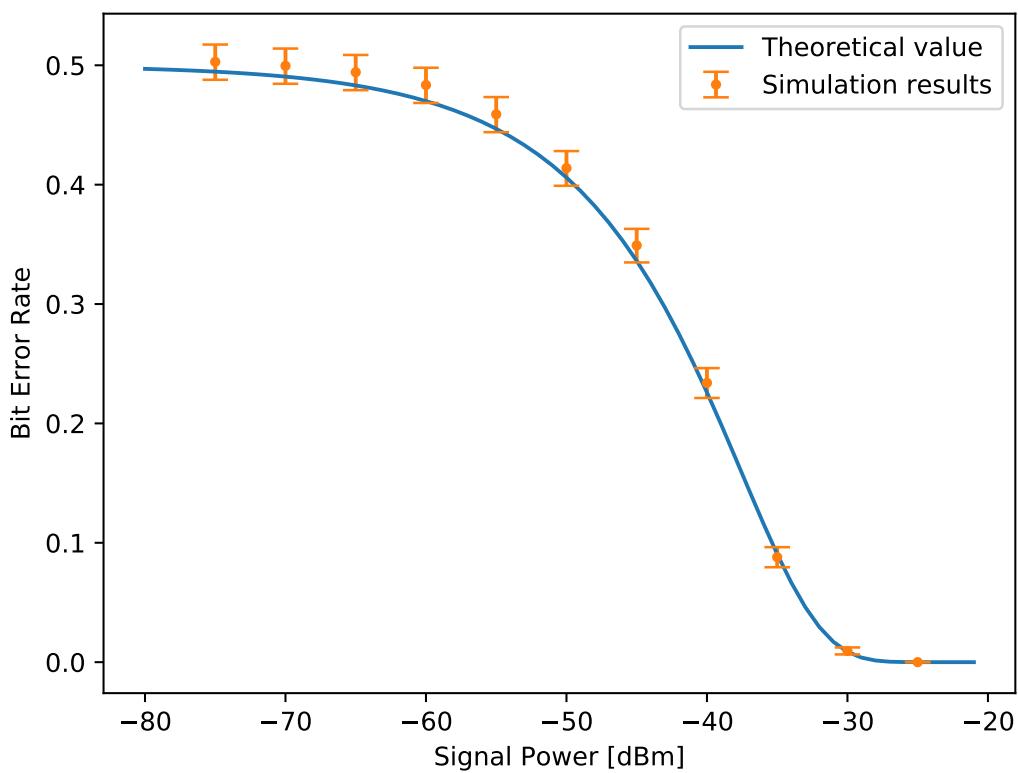


Figura 5.3: Bit Error Rate in function of the signal power in dBm at a constant local oscillator power level of 0 dBm. Theoretical values are presented as a full blue line while the simulated results are presented as a errorbar plot in orange, with the upper and lower bound computed in accordance with the method described in 6.2

## 5.3 M-QAM Transmission System

<b>Student Name</b>	:	Ana Luisa Carvalho
<b>Goal</b>	:	M-QAM system implementation with BER measurement and comparison with theoretical values.

### 5.3.1 Introduction

M-QAM, which stands for Quadrature Amplitude Modulation, is a modulation scheme that takes advantage of two carriers (usually sinusoidal waves) with a phase difference of  $\frac{\pi}{2}$ . The resultant output consists of a signal with both amplitude and phase variations. The two carriers, referred to as I (In-phase) and Q (Quadrature), can be represented as

$$I = A \cos(\phi) \quad (5.6)$$

$$Q = A \sin(\phi) \quad (5.7)$$

which means that any sinusoidal wave can be decomposed in its I and Q components:

$$A \cos(\omega t + \phi) = A (\cos(\omega t) \cos(\phi) - \sin(\omega t) \sin(\phi)) \quad (5.8)$$

$$= I \cos(\omega t) - Q \sin(\omega t), \quad (5.9)$$

where we have used the expression for the cosine of a sum and the definitions of I and Q.

For  $M=4$  the symbol constellation is shown figure ??.

$M$  can take several values: 2, 4, 16, 32, .... The first two correspond to BPSK and QPSK modulation, respectively.

### 5.3.2 Simulation setup

The M-QAM system transmission system is a complex block of code that simulates the modulation, transmission and demodulation of an optical signal using M-QAM modulation. It is composed of four blocks: a transmitter, a receiver, a sink and a block that performs a Bit Error Rate (BER) measurement.

**Current state:** The system currently being implemented is a QPSK system ( $M=4$ ).

**Future work:** Extend this block to include other values of  $M$ .

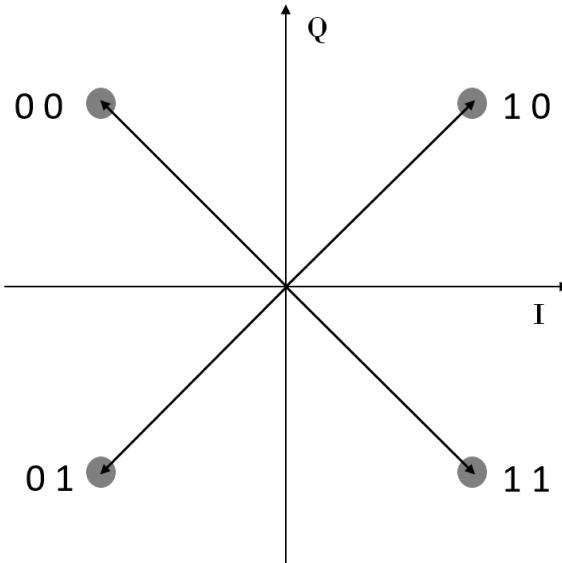


Figura 5.4: 4-QAM symbol constellation

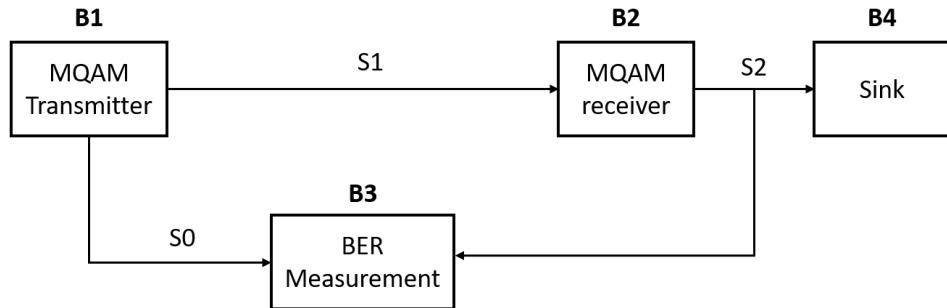


Figura 5.5: Schematic representation of the MQAM system.

### 5.3.3 Functional description

The schematic representation of the system is presented in figure 5.5.

A complete description of the M-QAM transmitter and M-QAM receiver blocks can be found in the *Library* chapter of this document as well as a detailed description of the independent blocks that compose these blocks.

The M-QAM transmitter is a complex block that generates one or two optical signals by encoding a binary string using M-QAM modulation. It also outputs a binary signal that is used to perform a BER measurement.

The M-QAM receiver is a homodyne receiver. It is a complex block that accepts one input optical signal and outputs a binary signal. It performs the M-QAM demodulation of the input signal by combining the optical signal with a local oscillator.

The demodulated optical signal is compared to the one produced by the transmitter in order to estimate the Bit Error Rate (BER).

The files corresponding to each of the system's blocks are summarized in table ???. Along

Tabela 5.1: Main system files

System blocks	cpp file	include file
Main	m_qam_system_sdf.cpp	—
M-QAM transmitter	m_qam_transmitter.cpp	m_qam_transmitter.h
M-QAM receiver	homodyne_receiver.cpp	homodyne_receiver.h
Sink	sink.cpp	sink.h
BER estimator	bit_error_rate.cpp	bit_error_rate.h

with the library and corresponding source files these allow for the full operation of the M-QAM system described here.

### 5.3.4 Input Parameters

### 5.3.5 Output Parameters

As output this block

### 5.3.6 BER measurement

Tabela 5.2: Input parameters

Parameter	Type	Description
numberOfBitsGenerated	t_integer	Determines the number of bits to be generated by the binary source
samplesPerSymbol	t_integer	
prbsPatternLength	int	Determines the length of the pseudorandom sequence pattern (used only when the binary source is operated in <i>PseudoRandom</i> mode)
bitPeriod	t_real	Temporal interval occupied by one bit
rollOffFactor	t_real	
signalOutputPower_dBm	t_real	Determines the power of the output optical signal in dBm
numberOfBitsReceived	int	
iqAmplitudeValues	vector<t_iqValues>	Determines the constellation used to encode the signal in IQ space
symbolPeriod	double	Given by bitPeriod / samplesPerSymbol
localOscillatorPower_dBm	t_real	Power of the local oscillator
responsivity	t_real	Responsivity of the photodiodes (1 corresponds to having all optical power transformed into electrical current)
amplification	t_real	??
noiseAmplitude	t_real	??
samplesToSkip	t_integer	Number of samples to be skipped by the <i>sampler</i> block
confidence	t_real	Determines the confidence limits for the BER estimation
midReportSize	t_integer	
bufferLength	t_integer	Corresponds to the number of samples that can be processed in each run of the system

## 5.4 Quantum Noise

<b>Student Name</b>	:	Diamantino Silva
<b>Starting Date</b>	:	October 19, 2017
<b>Goal</b>	:	Simulation of quantum noise in double homodyne detection.

### Introduction

This document describes an emission and detection system which is used to simulate the effects of quantum noise. The system is based on the transmission of coherent states as the support of information. The following section introduces some aspects about coherent states, with special focus on quantum noise.

We start by defining number states  $|n\rangle$  (or Fock states), which correspond to states with perfectly fixed number of photons. [1] Associated to those states are two operators, the creation  $\hat{a}^\dagger$  and annihilation  $\hat{a}$  operators, which in a simple way, remove or add one photon from a given number state. [2] Their action is defined as:

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \quad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \quad \hat{n}|n\rangle = n|n\rangle \quad (5.10)$$

in which  $\hat{n} = \hat{a}^\dagger\hat{a}$  is the number operator. Therefore, number states are eigenvectors of the number operator.

Coherent states have properties that closely resemble classical electromagnetic waves, and are generated by single-mode lasers well above the threshold. [1] We can define them, using number states in the following manner:

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad (5.11)$$

in which the complex number  $\alpha$  is the sole parameter that characterizes it. In fact, if we calculate the expected number of photons with  $\langle\alpha|\hat{n}|\alpha\rangle$  we will obtain  $|\alpha|^2$ . The coherent state is an eigenstate of the annihilation operator,  $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ .

Using the creation and annihilation operators, we can define two quadrature operators: [1]

$$\hat{X} = \frac{1}{2} (\hat{a}^\dagger + \hat{a}) \quad \hat{Y} = \frac{i}{2} (\hat{a}^\dagger - \hat{a}) \quad (5.12)$$

The expected value of these two operators, using a coherent state  $|\alpha\rangle$  are:

$$\langle\hat{X}\rangle = \text{Re}(\alpha) \quad \langle\hat{Y}\rangle = \text{Im}(\alpha) \quad (5.13)$$

We see that the expected value of these operators give us the real and imaginary part of  $\alpha$ . Now, we can obtain the uncertainty of these operators, using:

$$\text{Var}(\hat{X}) = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2 \quad (5.14)$$

For each of these quadrature operators the variance will be:

$$\text{Var}(\hat{X}) = \text{Var}(\hat{Y}) = \frac{1}{4} \quad (5.15)$$

This result show us that for both quadratures, the variance of measurement is the same and independent of the value of  $\alpha$ .

### Homodyne detection

The balanced homodyne technique is used to measure the phase of the input signal (S), relative to the phase of a local oscillator (LO), which has the same frequency as the input signal, but a much larger amplitude. The technique consists in combining the input signal and the local oscillator, using a 50:50 beam splitter, from whom two beams emerge, which are then converted to currents using photodides. Finally, the two currents are subtracted, resulting in an output current.

A phase of the local oscillator can be defined as the reference phase. A phase offset equal to 0 or  $\pi/2$  will give an output proportional to the signal's in-phase component or to the quadrature component, respectively. Therefore, the  $\hat{X}$  operator will correspond to the in-phase component and  $\hat{Y}$  operator correspond to quadrature component. [2]

In the lab and in our simulations, we will use a more complex system, the double balanced homodyne detection, which allows the simultaneous measurement of the in-phase and quadrature components. The signal is divided in two beam with half the power of the original. One of the beams will be used in a balanced homodyne detection with a local oscillator. The other beam will be used in another balanced homodyne detection, but using a local oscillator with a phase difference  $\pi/2$  relative to the first one.

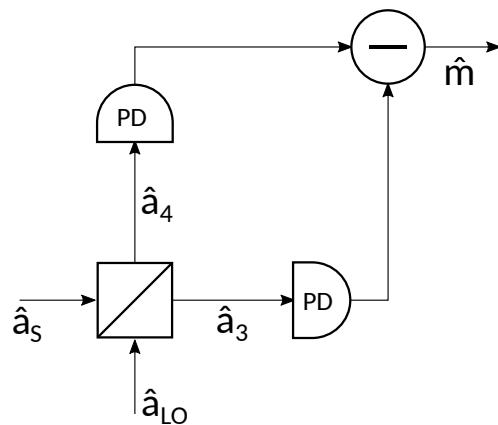


Figura 5.6: Balanced double homodyne detection.

### Noise sources in homodyne detection

The detection of light using photodiodes is subjected to various sources of noise. One of these sources is the electrical field itself. The interaction of the signal with the vacuum field adds quantum noise to the detection. Another source of noise comes from the detection system, such as photodiodes and other electrical circuits, originating various kinds of noise, such as thermal noise, dark noise and amplifier noise. [3] In the following sections, we will focus on two noise sources, quantum noise and thermal noise.

### Quantum Noise

In order to grasp this effect, the quantum mechanical description of balanced homodyne detection will be used, employing quantum operators to describe the effect of each component in the system (fig. ??). We start with the operators  $\hat{a}_S$  and  $\hat{a}_{LO}$  corresponding to the annihilation operator for the signal and local oscillator, which are the inputs in a beam divisor. The outputs will be  $\hat{a}_3$  and  $\hat{a}_4$ . Using a balanced beam splitter, we can write the output as:

$$\hat{a}_3 = \frac{1}{\sqrt{2}} (\hat{a}_S + \hat{a}_{LO}) \quad \hat{a}_4 = \frac{1}{\sqrt{2}} (\hat{a}_S - \hat{a}_{LO}) \quad (5.16)$$

The final output of a homodyne measurement will be proportional to the difference between the photocurrents in arm 3 and 4. Then

$$I_{34} = I_3 - I_4 \sim \langle \hat{n}_3 - \hat{n}_4 \rangle \quad (5.17)$$

We can define an operator that describes the difference of number of photons in arm 3 and arm 4:

$$\hat{m} = \hat{a}_3^\dagger \hat{a}_3 - \hat{a}_4^\dagger \hat{a}_4 \quad (5.18)$$

If we assume that the local oscillator produces the the coherent state  $|\beta\rangle$ , then the expected value of this measurement will be:

$$\langle m \rangle = 2|\alpha||\beta| \cos(\theta_\alpha - \theta_\beta) \quad \text{Var}(m) = |\alpha|^2 + |\beta|^2 \quad (5.19)$$

The local oscillator normally has a greater power than the signal , then  $|\alpha| \ll |\beta|$ . If we use as unit,  $2|\beta|$ , then these two quantities can be simplified to:

$$\langle m \rangle = |\alpha| \cos(\theta_\alpha - \theta_\beta) \quad \text{Var}(m) \approx \frac{1}{4} \quad (5.20)$$

[3]

Has we have seen previously, in order to measure two quadratures simultaneously, we can use double balanced homodyne detection. For each quadrature, the input signal now has half the power, so  $|\alpha| \rightarrow |\alpha/\sqrt{2}|$ . If we use a local oscillator that produces states  $|\beta\rangle$ , then we can divide it in two beams in state  $|\beta/\sqrt{2}\rangle$  and  $|i\beta/\sqrt{2}\rangle$  which will be used in each homodyne detection. In this setting, the expected values for each quadrature,  $X$  and  $Y$ , (in normalized values of  $\sqrt{2}|\beta|$ ) are:

$$\langle m_X \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \cos(\theta_\alpha - \theta_\beta) \quad \text{Var}(m_X) \approx \frac{1}{4} \quad (5.21)$$

$$\langle m_Y \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \sin(\theta_\alpha - \theta_\beta) \quad \text{Var}(m_Y) \approx \frac{1}{4} \quad (5.22)$$

Therefore the measurement of each quadrature will have half the amplitude, but the same variance.

### Thermal noise

Thermal noise is generated by electrons in response to temperature. It's contribution to the resulting current can be described by the following equation: [2]

$$\langle (\Delta i_T)^2 \rangle = 4K_B T_0 B / R_L \quad (5.23)$$

in which  $K_B$  it's Boltzmann's constant,  $T_0$  is the absolute temperature,  $B$  is the bandwidth and  $R_L$  is the receiver load impedance. The  $B$  value is imposed by default or chosen when the measurements are made, but the  $R_L$  value is dependent in the internal setup of the various components of the detection system. Nevertheless, for simulation purposes, we can just introduce an experimental value.

### Simulation

The simulation setup is represented in figure 5.7. The starting point is the MQAM, which generates random states from a given constelation. The output from the generator is received in the Optical Hybrid where it is mixed with a local oscillator, outputing two optical signal pairs. Each pair is converted to currents by two photodiodes, and then subtracted from each other, originating a current proportional to one of the quadratures of the input state. The other pair suffers the same process, but the resulting subtraction current will be proportional to another quadrature, dephased by  $\pi/2$  relative to the other quadrature.

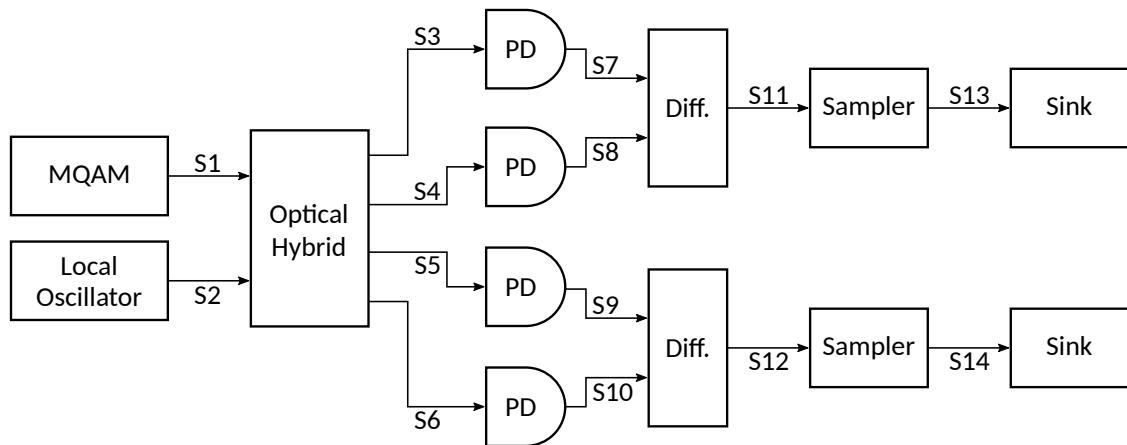


Figura 5.7: Overview of the simulated optical system.

System Blocks	netxpto Blocks
M - Quadrature Amplitude Modulator	MQamTransmitter
Local Oscillator	LocalOscillator
90deg Optical Hybrid	OpticalHybrid
Photodiode	Photodiode
Difference Circuit	Difference
Sampler	Sampler

## Required files

### Header Files

File	Description
netxpto.h	Generic purpose simulator definitions.
m_qam_transmitter.h	Outputs a QPSK modulated optical signal.
local_oscillator.h	Generates continuous coherent signal.
optical_hybrid.h	Mixes the two input signals into four outputs.
photodiode.h	Converts an optical signal to a current.
difference.h	Ouputs the difference between two input signals.
sampler.h	Samples the input signal.
sink.h	Closes any unused signals.

### Source Files

File	Description
netxpto.cpp	Generic purpose simulator definitions.
m_qam_transmitter.cpp	Outputs a QPSK modulated optical signal.
local_oscillator.cpp	Generates continuous coherent signal.
optical_hybrid.cpp	Mixes the two input signals into four outputs.
photodiode.h	Converts an optical signal to a current.
difference.h	Ouputs the difference between two input signals.
sampler.cpp	Samples the input signal.
sink.cpp	Empties the signal buffer.

## System Input Parameters

This system takes into account the following input parameters:

System Parameters	Description
numberOfBitsGenerated	Gives the number of bits to be simulated
bitPeriod	Sets the time between adjacent bits
wavelength	Sets the wavelength of the local oscillator in the MQAM
samplesPerSymbol	Establishes the number of samples each bit in the string is given
localOscillatorPower	Sets the optical power, in units of W, of the local oscillator inside the MQAM
localOscillatorPower	Sets the optical power, in units of W, of the local oscillator used for Bob's measurements
localOscillatorPhase	Sets the initial phase of the local oscillator used in the detection
transferMatrix	Sets the transfer matrix of the beam splitter used in the homodyne detector
responsivity	Sets the responsivity of the photodiodes used in the homodyne detectors
bufferLength	Sets the length of the buffer used in the signals
iqAmplitudeValues	Sets the amplitude of the states used in the MQAM
samplesToSkip	Sets the number of samples to skip when writing out some of the signal files

## Inputs

This system takes no inputs.

## Outputs

The system outputs the following objects:

**Parameter:** Signals:

**Description:** Binary Sequence used in the MQAM; ( $S_0$ )

**Description:** Local Oscillator used in the MQAM; ( $S_1$ )

**Description:** Local Oscillator used in the detection; ( $S_2$ )

**Description:** Optical Hybrid Outputs; ( $S_3, S_4, S_5, S_6$ )

**Description:** In phase Photodiodes output; ( $S_7, S_8$ )

**Description:** Quadrature Photodiode output; ( $S_9, S_{10}$ )

**Description:** In phase Difference output; ( $S_{11}$ )

**Description:** Quadrature Difference output; ( $S_{12}$ )

**Description:** In phase Sampler output; ( $S_{13}$ )

**Description:** Quadrature Sampler output; ( $S_{14}$ )

## Simulation Results

To test the simulated implementation, a series of states  $\{|\phi_i\rangle\}$  were generated and detected, resulting in a series of measurements  $\{(x_i, y_i)\}$ . The simulation result is presented in figure 5.8:

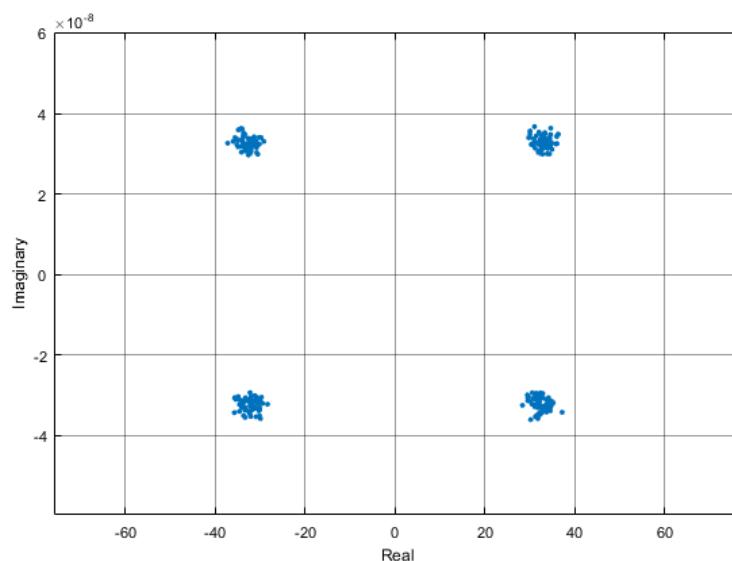


Figura 5.8: Simulation of a constelation of 4 states ( $n = 100$ )

We see that the measurements made groups in certain regions. Each of this groups is centered in the expected value ( $\langle X \rangle, \langle Y \rangle$ ) of one the generated states. Also, they show some

variance, which was tested for various expected number of photons,  $\langle n \rangle$ , resulting in figure 5.9:

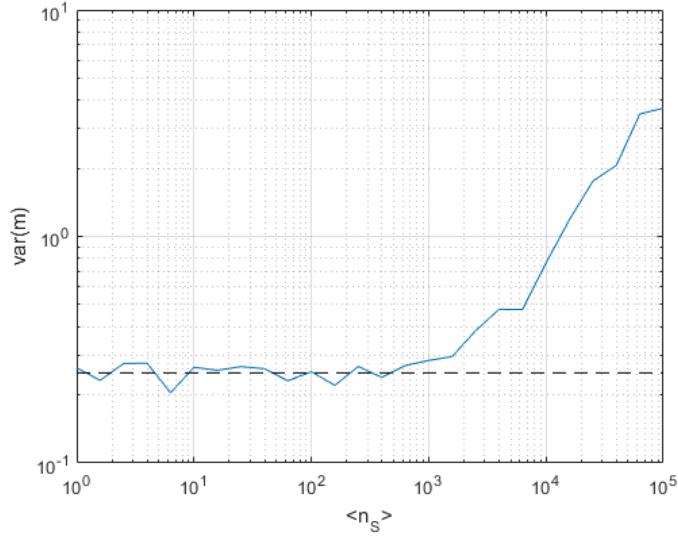


Figura 5.9: Simulation of the variance of  $m$ .  
Local oscillator expected number of photons:  $10^4$

It was expected that the variance should independent of the input's signal number of photons. Plot 5.9 shows that for low values of  $n_S$ , the simulation is in accordance with the theoretical prevision, with  $\text{Var}(X) = \text{Var}(Y) = \frac{1}{4}$ . For large values of  $n_S$ , when the number of photons is about the same has the local oscillator, the quantum noise variance starts to grow proportionally to  $n_S$ , in accordance with the non approximated calculation of quantum noise (eq. 5.19).

### Noise Variance with LO power Simulation

The following plot shows the behavior of current noise variance  $\langle (\Delta i)^2 \rangle$  with local oscilator power,  $P_{LO}$ :

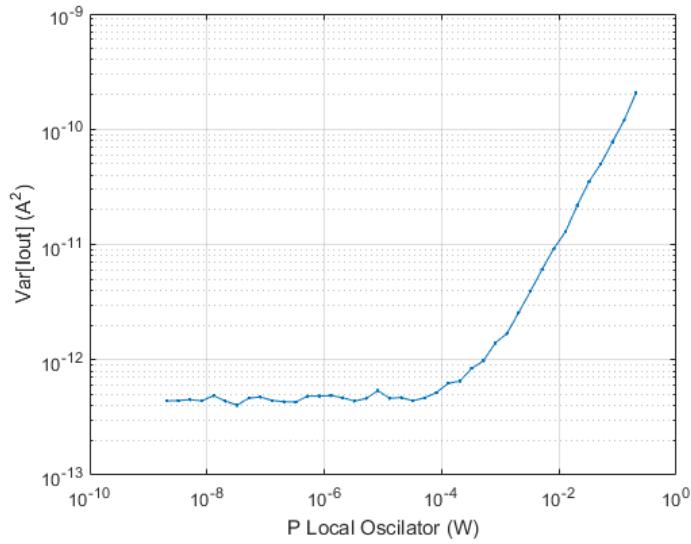


Figura 5.10: Output current variance in function of LO power.

We see that for low LO power, the dominant noise is the thermal contribution, but for higher power, quantum noise dominates, growing proportionally to  $P_{\text{LO}}$ . This in accordance with equation 5.20

### Comparison to experimental results

As we have seen earlier, the system will manifest noise even in the absence of input signal (eq 5.19). The experimental procedure for this measurement consisted in just inputting the local oscillator laser and no signal. For each value of  $P_{\text{LO}}$ , the local oscillator was pulsed, and for each pulse, the variance of the plateau of the maximum was calculated. The final variance was simply the mean of the variances of all pulses.

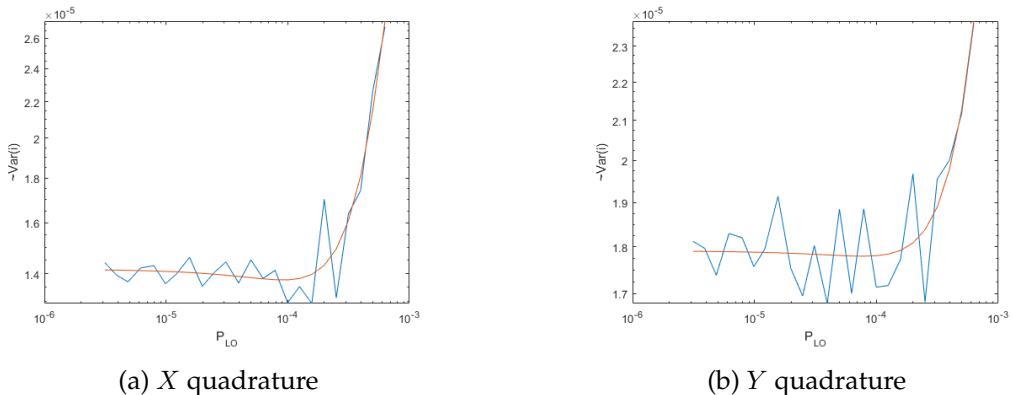


Figura 5.11: Noise variance dependency with local oscillator power for two different quadratures. Experimental vs fitted data.

Figures 5.11a and 5.11b show measures measurements of total noise for two different quadratures. For low power of LO, the noise variance flutuates around a constant value. For high power of LO, ( $P_{LO} > 10^{-4}W$ ), the variance of noise shows an increasing trend roughly proportional to  $P_{LO}^2$ . The expected growth should be proportional to  $P_{LO}$ , but the RIN noise, originated by the electric apparatus, grows quadratically with the power, dominating the noise amplitude for large  $P_{LO}$ .

So, we see that both the simulation and experimental data display a similar behaviour, but the quadratic growth of noise for large  $P_{LO}$  was not predicted in the simulations.

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## Bibliografia

- [1] Rodney Loudon. *The Quantum Theory of Light*. Oxford University Press, 2000.
- [2] Mark Fox. *Quantum Optics: An Introduction*. Oxford University Press, 2006.
- [3] Hans-A. Bachor and Timothy C. Ralph. *A Guide to Experiments in Quantum Optics*. Wiley-VCH, 2004.

## 5.5 Continuos Variable Quantum Transmission System

Student Name: Daniel Pereira

Starting Date: May 1, 2017

Goal: Simulation and experimental validation of a continuous variable transmission system.

Description of the Laboratorial and Simulation Setup:

Theoretical Description of the System:

Laboratorial and Simulation Results:

Discussion:

In this section a continuous variable quantum transmission system is analyzed. The results here presented follow closely the [?]. In [?], the security of a continuous variable quantum key distribution (CV-QKD) system is studied theoretically, here we complete that theoretical study with simulations results.

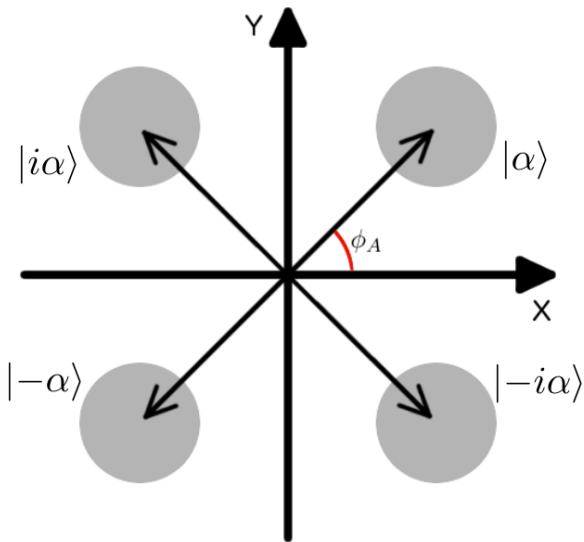


Figura 5.12: State constellation

The state constellation used in the system is presented in Figure 5.12. The emitter (usually named Alice) is going to use two basis, the  $45^\circ$  base and the  $-45^\circ$  base. In the  $45^\circ$  base, Alice sends one of two values, 1 and  $-1$ , which correspond to the states  $|\alpha\rangle$  and  $|- \alpha\rangle$ . In the  $-45^\circ$  base, Alice can also send one of two values, 1 and  $-1$ , which correspond to the states  $|i\alpha\rangle$  and  $|-i\alpha\rangle$ . At the end Alice is going to send one of the four states  $|\alpha\rangle$ ,  $|- \alpha\rangle$ ,  $| - i\alpha\rangle$ , and  $|i\alpha\rangle$ , with equal probability.

Because we don't know à prior which state is going be transmitted, neither which basis is going to be used, and to incorporate our "ignorance" in the system description, we can

work with the density operator. The density operator is a proper tool to describe "statistical mixtures". A "statistical mixtures" is one state, from a possible set, but we don't know which state it is. There is no state superposition.

Since all states have the same probability of occurring, the state density operator is given by:

$$\hat{\rho} = \frac{1}{4} (|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha| + |i\alpha\rangle\langle i\alpha| + |-i\alpha\rangle\langle -i\alpha|). \quad (5.24)$$

The probability to detect at the receiver the state  $|\alpha\rangle$  is given by

$$P(\alpha) = \langle\alpha|\hat{\rho}|\alpha\rangle = \frac{1}{4}. \quad (5.25)$$

Note that the density operator is equivalent to the wave function in terms of the system description.

From the receiver perspective, i.e. from the Bob perspective, and after knowing the base used by Alice. The density operator can be reduce to,

$$\hat{\rho}_1 = \frac{1}{2} (|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|), \quad (5.26)$$

$$\hat{\rho}_2 = \frac{1}{2} (|i\alpha\rangle\langle i\alpha| + |-i\alpha\rangle\langle -i\alpha|). \quad (5.27)$$

where 1 corresponds to the  $45^\circ$  base and  $-1$  corresponds to  $-45^\circ$ .

### Single Base Homodyne Detection

The probability of obtaining a quadrature  $\hat{X}_\phi = \hat{X}_1 \cos \phi + \hat{X}_2 \sin \phi$  when measuring the coherent state  $|\alpha\rangle$  is given by the following gaussian distribution:

$$|\langle X_\phi | \alpha \rangle|^2 = \sqrt{\frac{2}{\pi}} e^{-2(X_\phi - \alpha \cos \phi)^2}, \quad (5.28)$$

We can define the "correct" and "wrong" basis measurement probability density, respectively, as:

$$\langle X_i | \hat{\rho}_j | X_i \rangle = \begin{cases} \frac{1}{\sqrt{2\pi}} (e^{-2(X_i - \alpha)^2} + e^{-2(X_i + \alpha)^2}), & i = j \\ \sqrt{\frac{2}{\pi}} e^{-2X_i^2}, & i \neq j \end{cases}. \quad (5.29)$$

The post selection efficiency (PSE) can be defined as the probability of a measurement in the correct basis yields a result that satisfies the limit value  $X_0$ :

$$\begin{aligned} P(X_0, \alpha) &= \int_{-\infty}^{-X_0} \langle X_1 | \hat{\rho}_1 | X_1 \rangle dX_1 + \int_{X_0}^{\infty} \langle X_1 | \hat{\rho}_1 | X_1 \rangle dX_1 \\ &= \frac{1}{2} [\text{erfc}(\sqrt{2}(X_0 + \alpha)) + \text{erfc}(\sqrt{2}(X_0 - \alpha))]. \end{aligned} \quad (5.30)$$

The bit error rate (BER) is the normalized probability of, after choosing the correct basis, obtaining the wrong bit value:

$$Q(X_0, \alpha) = \frac{1}{P(X_0, \alpha)} \int_{-\infty}^{-X_0} |\langle X_i | \alpha \rangle| dX_i = \frac{\operatorname{erfc}(\sqrt{2}(X_0 + \alpha))}{2P(X_0, n)} \quad (5.31)$$

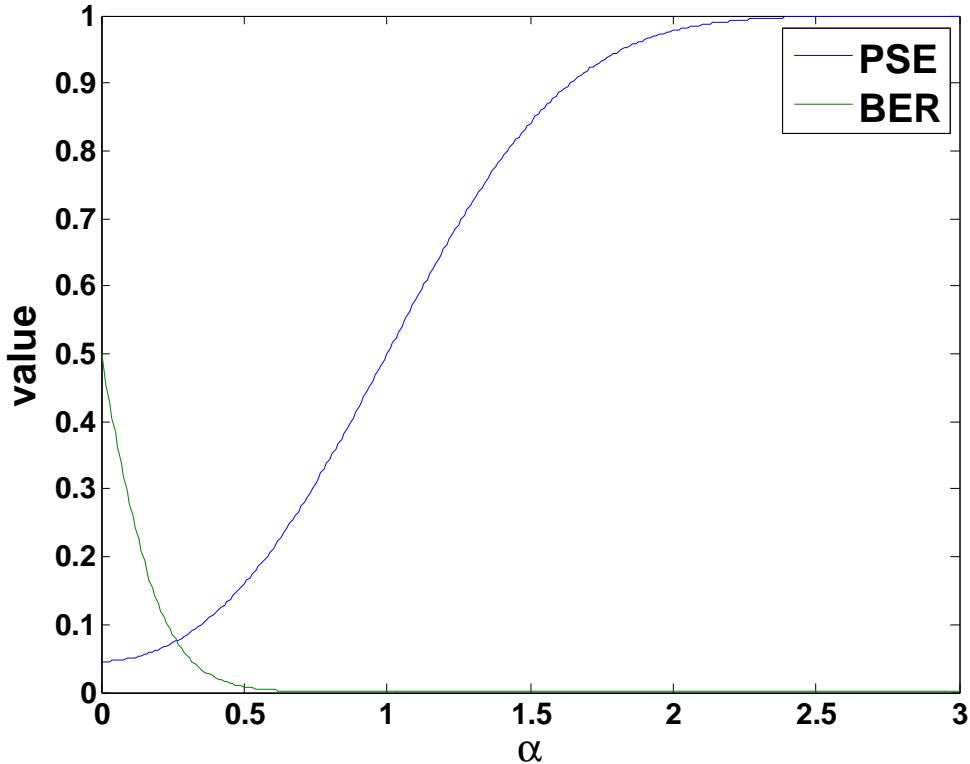


Figura 5.13: BER and PSE in function of  $\alpha$  for the single homodyne setup.  $X_0 = 1$  was used

### Double Homodyne setup

In our proposed double homodyne protocol both quadratures are measured simultaneously, as such the concept of correct and wrong basis measurements has no value. Our protocol also makes use of a locally generated Local Oscillator (LO), obtained from a different laser than the one used to generate the signal, thus we have to take into account the phase drift between both lasers. High intensity reference pulses are sent periodically to allow for an estimation of the phase drift. The double homodyne setup requires the signal to be divided into the two utilized detectors, so each measurement is made on a coherent state with half the amplitude of the incoming signal  $\alpha \rightarrow \frac{\alpha}{\sqrt{2}}$ .

For each incoming pulse we measure quadratures  $X_\phi$  and  $Y_\phi$ .  $\phi$  has contributions from both the encoded angle,  $\theta$ , and the phase difference between lasers,  $\epsilon$ , we assume  $\phi = \theta + \epsilon$ .

On the reference pulses no phase is encoded, that is  $\theta = 0$ , thus  $\epsilon$  can be estimated. Assuming  $\epsilon$  doesn't change between a reference pulse and the following signal pulse, the measured quadratures can be cast into the originally sent quadratures  $X_\theta$  and  $Y_\theta$  via:

$$\begin{aligned} X_\theta &= X_\phi \cos \epsilon - Y_\phi \sin \epsilon \\ Y_\theta &= X_\phi \sin \epsilon + Y_\phi \cos \epsilon \end{aligned} \quad (5.32)$$

Assuming an announcement of the coding basis, the density operators (5.26) and (5.27) still apply. We can now define the probability density of obtaining results  $X_\theta$  and  $Y_\theta$ , assuming a state in the  $X_1$  base was sent, as:

$$\langle X_\theta | \hat{\rho}_1 | X_\theta \rangle = \frac{\sqrt{\frac{2}{\pi}}}{4} \left( e^{-2(x_\theta - \frac{\alpha}{\sqrt{2}} \cos \theta)^2} + e^{-2(x_\theta + \frac{\alpha}{\sqrt{2}} \cos \theta)^2} \right), \quad (5.33)$$

$$\langle Y_\theta | \hat{\rho}_1 | Y_\theta \rangle = \frac{\sqrt{\frac{2}{\pi}}}{4} \left( e^{-2(y_\theta - \frac{\alpha}{\sqrt{2}} \sin \theta)^2} + e^{-2(y_\theta + \frac{\alpha}{\sqrt{2}} \sin \theta)^2} \right). \quad (5.34)$$

Now each state needs to satisfy two limit values,  $X_0$  and  $Y_0$ , to be accepted. Thus, the PSE is now defined as:

$$\begin{aligned} P_{DH}(X_0, Y_0, \alpha) &= \int_{-\infty}^{-X_0} \langle X_\theta | \hat{\rho}_1 | X_\theta \rangle dx_\theta \int_{-\infty}^{-Y_0} \langle Y_\theta | \hat{\rho}_1 | Y_\theta \rangle dy_\theta + \\ &\quad \int_{X_0}^{\infty} \langle X_\theta | \hat{\rho}_1 | X_\theta \rangle dx_\theta \int_{Y_0}^{\infty} \langle Y_\theta | \hat{\rho}_1 | Y_\theta \rangle dy_\theta \\ &= \frac{1}{4} \left\{ \operatorname{erfc} \left[ \sqrt{2} \left( X_0 - \frac{\alpha}{\sqrt{2}} \cos \theta \right) \right] + \operatorname{erfc} \left[ \sqrt{2} \left( X_0 + \frac{\alpha}{\sqrt{2}} \cos \theta \right) \right] \right\} \\ &\quad \left\{ \operatorname{erfc} \left[ \sqrt{2} \left( Y_0 - \frac{\alpha}{\sqrt{2}} \sin \theta \right) \right] + \operatorname{erfc} \left[ \sqrt{2} \left( Y_0 + \frac{\alpha}{\sqrt{2}} \sin \theta \right) \right] \right\}, \end{aligned} \quad (5.35)$$

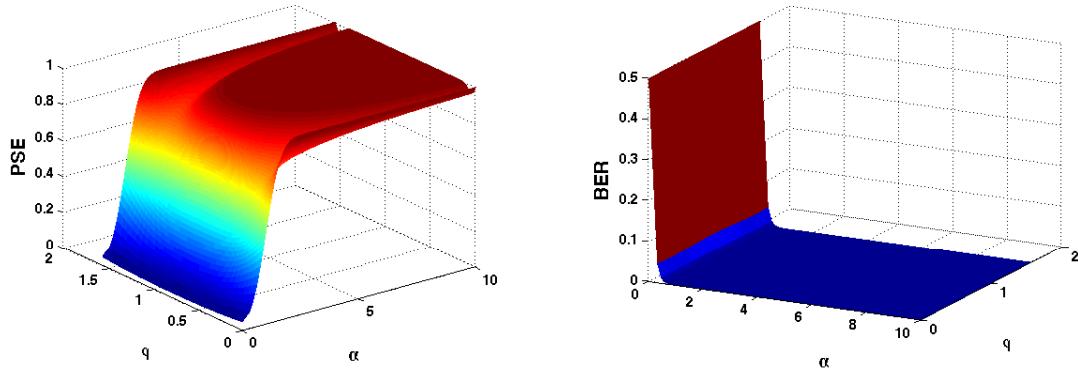
The DH subscript denotes Double Homodyne. In a somewhat similar manner, the BER is now defined as:

$$\begin{aligned} Q_{DH}(X_0, Y_0, \alpha) &= \frac{1}{P_{DH}} \left( \int_{-\infty}^{-X_0} \left| \langle X_\theta | \frac{\alpha}{\sqrt{2}} \rangle \right|^2 dx_\theta \int_{-\infty}^{-Y_0} \left| \langle Y_\theta | \frac{\alpha}{\sqrt{2}} \rangle \right|^2 dy_\theta + \right. \\ &\quad \left. \int_{X_0}^{\infty} \left| \langle X_\theta | -\frac{\alpha}{\sqrt{2}} \rangle \right|^2 dx_\theta \int_{Y_0}^{\infty} \left| \langle Y_\theta | -\frac{\alpha}{\sqrt{2}} \rangle \right|^2 dy_\theta \right) \\ &= \frac{1}{2P_{DH}} \operatorname{erfc} \left[ \sqrt{2} \left( X_0 + \frac{\alpha}{\sqrt{2}} \cos \theta \right) \right] \operatorname{erfc} \left[ \sqrt{2} \left( Y_0 + \frac{\alpha}{\sqrt{2}} \sin \theta \right) \right], \end{aligned} \quad (5.36)$$

note that, in this definition for BER, only values  $\theta \in [0, \frac{\pi}{2}]$  make sense (the sent state was  $\alpha$ ).

## Functional Description

Simplified diagrams of the systems being simulated are presented in Figures 5.15a. and 5.15b. Two optical signals are generated, one with a constant power level of 10 dBm and the other with power in multiples of the power corresponding to a single photon per



(a) PSE in function of  $\alpha$  and  $\theta$  for the double homodyne setup.  $X_0 = 1$  was used  
 (b) BER in function of  $\alpha$  and  $\theta$  for the double homodyne setup.  $X_0 = 1$  was used

Figura 5.14: Theoretical results for double homodyne setup.

sampling time ( $6.4078e \times 10^{-13}$  W for a sampling time of 200 ns). The two signals are mixed, with a Balanced Beam Splitter in the single homodyne case and with a  $90^\circ$  Optical Hybrid in the double homodyne one, and are subsequently evaluated with recourse to Homodyne Receivers.

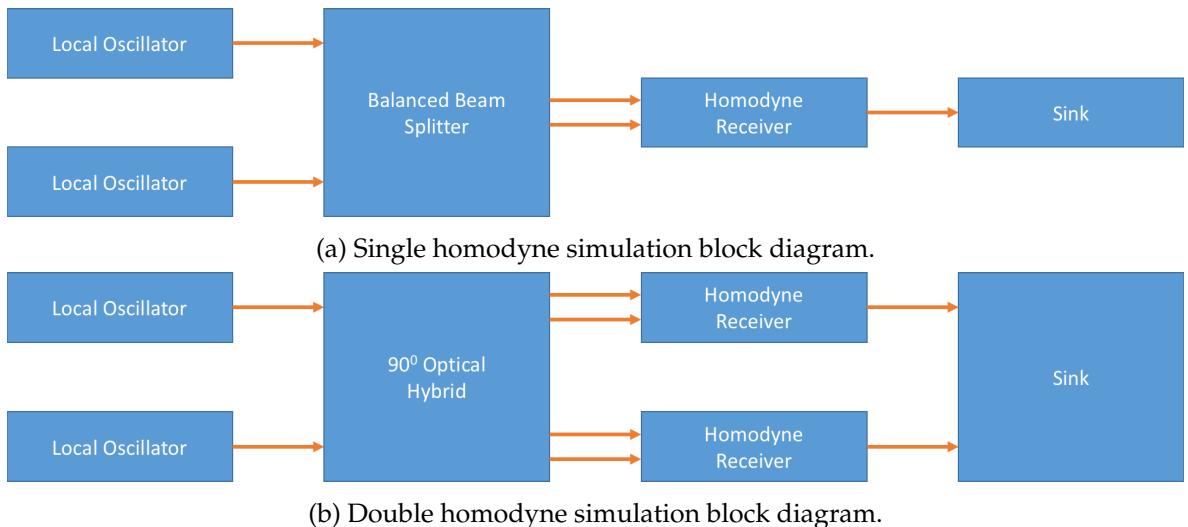


Figura 5.15: Block diagrams of both simulation results presented in this report.

System Blocks	netxpto Blocks
Local Oscillator	LocalOscillator
Homodyne Receiver	I_HomodyneReceiver
Balanced Beam Splitter	BalancedBeamSplitter
$90^\circ$ Optical Hybrid	OpticalHybrid

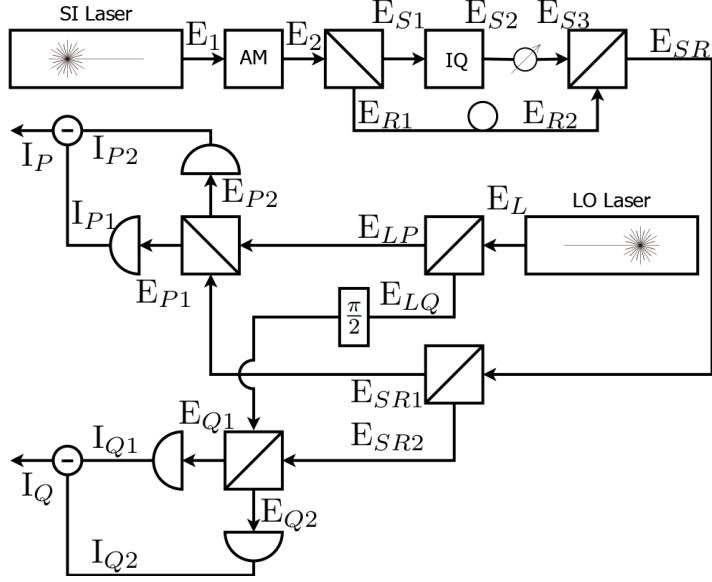


Figura 5.16: Simplified diagram of experimental setup.

### Theoretical study of the setup

A detailed diagram of the system studied is presented in Figure 5.16, with the respective mathematical treatment presented in equations (5.37) through (5.56).

$$E_1 = |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.37)$$

$$p(t) = \begin{cases} 1, & \frac{T}{8} \leq t < \frac{7T}{8} \\ 0, & \text{otherwise} \end{cases} \quad (5.38)$$

$$E_2 = p(t) |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.39)$$

$$E_{S1} = \frac{p(t)}{\sqrt{2}} |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.40)$$

$$E_{S2} = \frac{p(t)}{\sqrt{2}} |E_S| e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} \quad (5.41)$$

$$E_{S3} = \frac{p(t)}{\sqrt{2} \text{Att}} |E_S| e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} \quad (5.42)$$

$$E_{R1} = \frac{p(t)}{\sqrt{2}} |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.43)$$

$$E_{R2} = \frac{p(t - \frac{T}{2})}{\sqrt{2}} |E_S| e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \quad (5.44)$$

$$E_{SR} = \frac{|E_S|}{\sqrt{2}} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \quad (5.45)$$

$$E_{SR1} = \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \quad (5.46)$$

$$E_{SR2} = \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \quad (5.47)$$

$$E_L(t) = |E_L| e^{i(\omega_L t + \xi_L(t))} \quad (5.48)$$

$$E_{LP}(t) = \frac{1}{\sqrt{2}} |E_L| e^{i(\omega_L t + \xi_L(t))} \quad (5.49)$$

$$E_{LQ}(t) = \frac{1}{\sqrt{2}} |E_L| e^{i(\omega_L t + \xi_L(t) + \frac{\pi}{2})} \quad (5.50)$$

$$E_{P1}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t))} + \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.51)$$

$$E_{P2}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t))} - \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.52)$$

$$E_{Q1}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t) + \frac{\pi}{2})} + \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.53)$$

$$E_{Q2}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t) + \frac{\pi}{2})} - \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.54)$$

$$I_P(t) = \frac{|E_S| |E_L|}{\sqrt{2}} \left\{ \frac{p(t)}{\text{Att}} \cos((\omega_S - \omega_L)t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t) - \xi_L(t)) \right. \\ \left. + p \left( t - \frac{T}{2} \right) \cos \left( (\omega_S - \omega_L)t + \omega_S \frac{T}{2} + \xi_S \left( t - \frac{T}{2} \right) - \xi_L(t) \right) \right\} \quad (5.55)$$

$$I_Q(t) = \frac{|E_S| |E_L|}{\sqrt{2}} \left\{ \frac{p(t)}{\text{Att}} \sin((\omega_S - \omega_L)t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t) - \xi_L(t)) \right. \\ \left. + p \left( t - \frac{T}{2} \right) \sin \left( (\omega_S - \omega_L)t + \omega_S \frac{T}{2} + \xi_S \left( t - \frac{T}{2} \right) - \xi_L(t) \right) \right\} \quad (5.56)$$

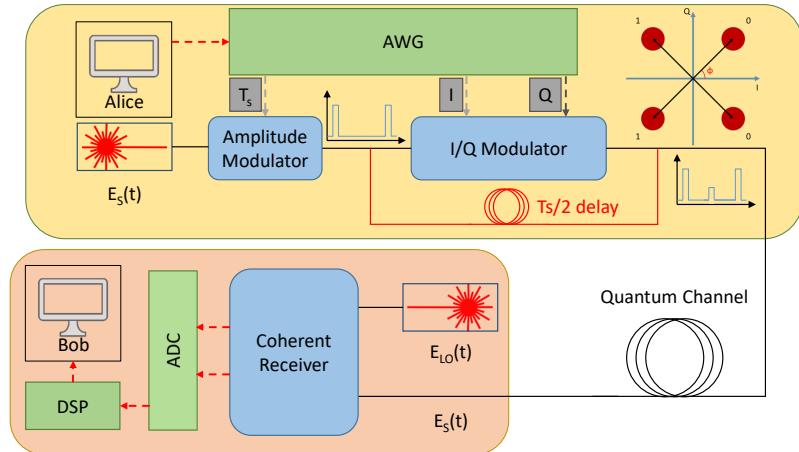


Figura 5.17: Diagram of full experimental setup.

### Experimental description

The main experimental setup utilized for the study presented in this document is presented in Figure 5.17. The setup contained two Yenista OSICS Band C/AG TLS lasers, tuned to a 1550 nm wavelength. A JDSU dual drive Mach-Zehnder Modulator with a Picosecond 5865 RF driver, employed at the output of one of the lasers, set the repetition rate at 1 GHz with a pulse time of 125 ps. The driving signal was generated by an Agilent Technologies BER Tester. A 50/50 beam-splitter was employed at the output of the Mach-Zehnder Modulator, one arm output is sent through an IQ Modulator while the other is sent through a fibre loop with length chosen such that the two arms have a relative delay of  $\sim 500$  ps. The employed IQ Modulator was a u2t photonics 32 GHz IQ Modulator with a SHF 807 RF driver, the driving signal being generated by a Tektronix AWG70002A Arbitrary Waveform Generator (AWG). A Variable Optical Attenuator (VOA) was set at the output of the IQ modulator to allow a fine tuning of the optical power to the desired level. The two arms output created by the first beam-splitter are combined by a 50/50 beam combiner. A fibre channel of  $\sim 10$  km was set between Alice's and Bob's setup. A Picometrix CR-100D 100G Integrated Balanced Receiver for Coherent Applications was employed to perform double homodyne measurements, recovering both the in-phase and in-quadrature components of the incoming light field. The response of the receiver was recorded by a Tektronix DPO77002SX-R3 oscilloscope, with an acquisition frequency of 100 GHz for a period of 400  $\mu$ s.

### Experimental results

#### Required files

Header Files

File	Description
netxpto.h	Generic purpose simulator definitions.
local_oscillator.h	Generates continuous coherent signal.
balanced_beam_splitter.h	Mixes the two input signals into two outputs.
optical_hybrid.h	Mixes the two input signals into four outputs.
homodyne_reciever.h	Performs coherent detection on the input signal.
sink.h	Closes any unused signals.

### Source Files

File	Description
netxpto.cpp	Generic purpose simulator definitions.
local_oscillator.cpp	Generates continuous coherent signal.
balanced_beam_splitter.cpp	Mixes the two input signals into two outputs.
optical_hybrid.cpp	Mixes the two input signals into four outputs.
homodyne_reciever.cpp	Performs coherent detection on the input signal.
sink.cpp	Closes any unused signals.

### System Input Parameters

This system takes into account the following input parameters:

System Parameters	Description
numberOfBitsGenerated	Gives the number of bits to be simulated
bitPeriod	Sets the time between adjacent bits
samplesPerSymbol	Establishes the number of samples each bit in the string is given
localOscillatorPower_dBm	Sets the optical power, in units of dBm, at the reference output
localOscillatorPower2	Sets the optical power, in units of W, of the signal
localOscillatorPhase1	Sets the initial phase of the local oscillator used for reference
localOscillatorPhase2	Sets the initial phase of the local oscillator used for signal
transferMatrix	Sets the transfer matrix of the beam splitter used in the homodyne detector
responsivity	Sets the responsivity of the photodiodes used in the homodyne detector
amplification	Sets the amplification of the trans-impedance amplifier used in the homodyne detector
electricalNoiseAmplitude	Sets the amplitude of the gaussian thermal noise added in the homodyne detector
shotNoise	Chooses if quantum shot noise is used in the simulation

### Inputs

This system takes no inputs.

## Outputs

The single homodyne system outputs the following objects:

**Parameter:** Signals:

**Description:** Local Oscillator Optical Reference; ( $S_1$ )

**Description:** Local Oscillator Optical Signal; ( $S_2$ )

**Description:** Beam Splitter Outputs; ( $S_3, S_4$ )

**Description:** Homodyne Detector Electrical Output; ( $S_5$ )

The double homodyne system outputs the following objects:

**Parameter:** Signals:

**Description:** Local Oscillator Optical Reference; ( $S_1$ )

**Description:** Local Oscillator Optical Signal; ( $S_2$ )

**Description:** 90° Optical Hybrid Outputs; ( $S_3, S_4, S_5, S_6$ )

**Description:** Homodyne Detector Electrical Output; ( $S_7$ )

## Simulation Results

### Single homodyne results

The numerical results presented in Figure 5.18 were obtained with the simulation described by the block diagram in Figure 5.15a. Theoretical results are a direct trace of (5.31). One can see that the numerical results adhere quite well to the expected curve.

### Double homodyne results

The numerical results presented in Figure 5.19 were obtained with the simulation described by the block diagram in Figure 5.15b. Theoretical results are a direct trace of (5.36) with  $\theta = \frac{\pi}{4}$ . One can see that the numerical results adhere quite well to the expected curve.

## Known Problems

1. Homodyne Super-Block not functioning

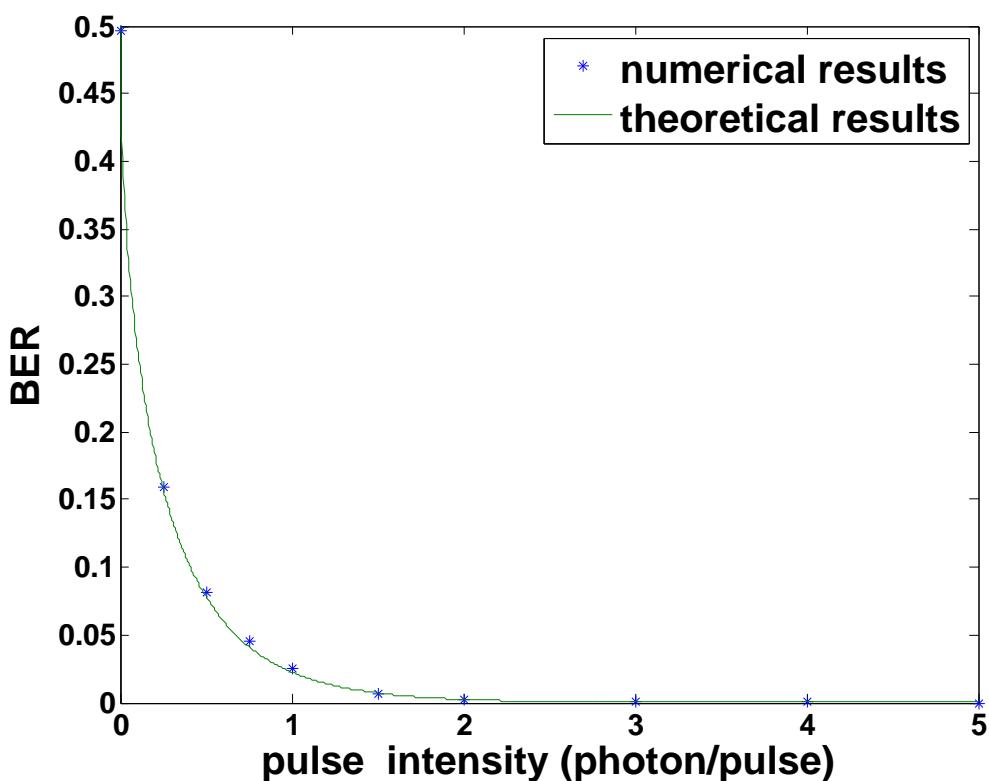


Figura 5.18: BER in function of  $\alpha$  for the single homodyne setup.  $X_0 = 0$  was used

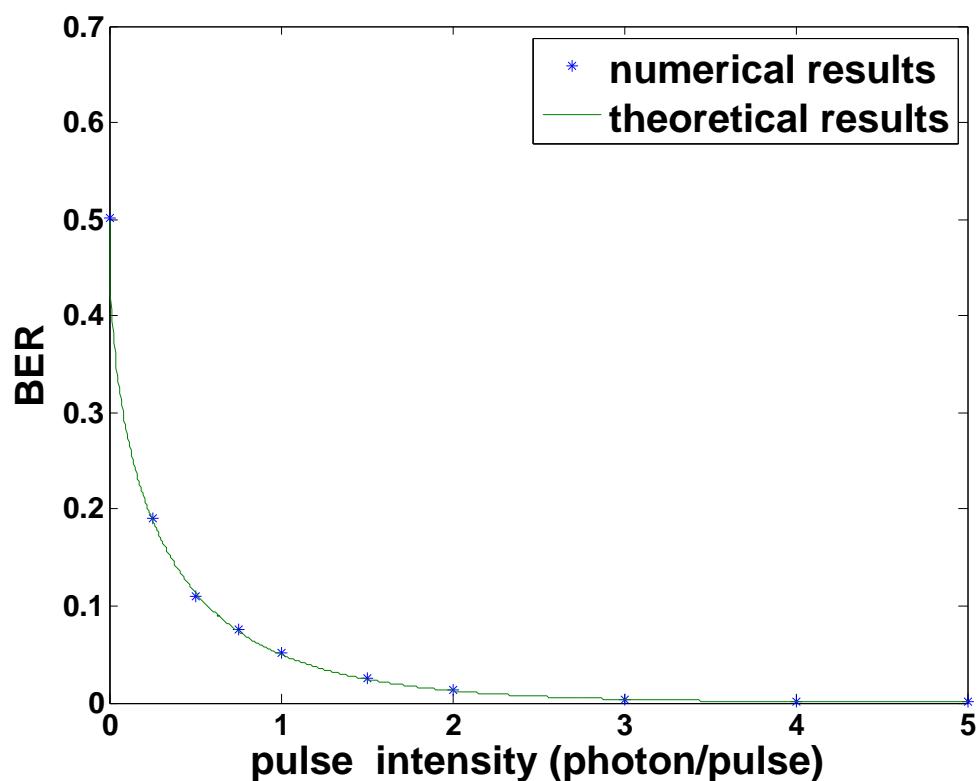


Figura 5.19: BER in function of  $\alpha$  for the double homodyne setup.  $X_0 = 0$  was used

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## Bibliografia

- [1] Rodney Loudon. *The Quantum Theory of Light*. Oxford University Press, 2000.
- [2] Mark Fox. *Quantum Optics: An Introduction*. Oxford University Press, 2006.
- [3] Hans-A. Bachor and Timothy C. Ralph. *A Guide to Experiments in Quantum Optics*. Wiley-VCH, 2004.

## 5.6 Kramers-Kronig Transceiver with Stokes PolDemux

<b>Student Name</b>	:	Romil Patel
<b>Starting Date</b>	:	August 16, 2017
<b>Goal</b>	:	Develop a simplified structure (low cost) for a coherent transceiver, that can be used in coherent PON, inter-data center connections, or metropolitan networks (optical path lengths < 100 km) We are going to assume a Kramers-Kronig transceiver with Stokes based PolDemux.

Coherent optical transmission schemes are spectrally efficient since they allow the encoding of information in both quadrature of sinusoid signal. However, the cost of coherent receiver becomes a major obstacle in the case of short-reach links applications like PON, inter-data-center communications and metropolitan network. In order to make the transceiver applicable in short-reach links, an architecture has been proposed which combines the advantages of coherent transmission and cost effectiveness of direct detection. The working principle of the proposed transceiver is based on the Kramers-Kronig (KK) relationship. The KK transceiver scheme allows digital compensation of propagation impairment because both amplitude and phase of the electrical field can be retrieved at the receiver.

### 5.6.1 Concept of Kramers-Kronig transceiver

The Kramers-Kronig relations are bidirectional mathematical relations, connecting the real and imaginary parts of any complex function that is analytic in the upper half-plane. For instance, if we consider a signal  $x(t) = x_r(t) + ix_i(t)$  then the following two terms describe the Kramers-Kronig relationship,

$$x_r(t) = -\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{x_i(t')}{t - t'} dt'$$

$$x_i(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{x_r(t')}{t - t'} dt'$$

The relationship displays that the real and the imaginary parts of the signal are related to each other though Hilbert transform. Therefore, if we have only real data then imaginary data can be calculated by its Hilbert transform. The proposed scheme allows the full reconstruction of the complex-valued electric-field signal from the amplitude data.

The following questions would give a comprehensive overview that how Kramers-Kroning transceiver functions and how phase can be extracted from the amplitude information.

### 1. What is Hilbert transform?

If we consider a filter  $H(\omega)$ , described in Figure 5.20, that has a unity magnitude response for all frequencies. Also, the phase response is  $-\pi/2$  for all positive frequencies and  $\pi/2$  for negative frequencies. The transfer function of this filter is,

$$H(i\omega) = -isgn(\omega) \quad (5.57)$$

Therefore, we can term it as a wideband phase shift network. The impulse response of this

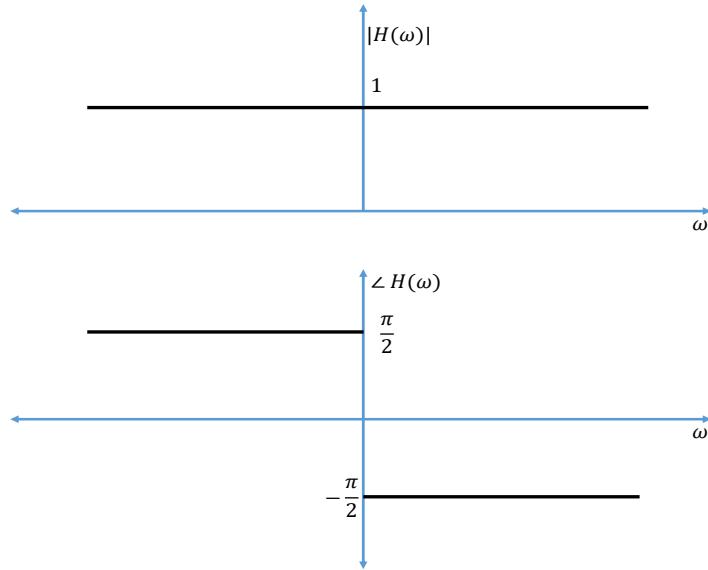


Figura 5.20: Magnitude and phase of Hilbert transformer

filter can be given as,

$$\begin{aligned} h(t) &= \mathcal{F}^{-1}[H(i\omega)] \\ &= -i\mathcal{F}^{-1}[sgn(\omega)] \\ &= -i\left(\frac{i}{\pi t}\right) \\ &= \frac{1}{\pi t} \end{aligned} \quad (5.58)$$

When this filter driven by an arbitrary signal  $s(t)$ , the filter produces the output as,

$$\begin{aligned} \hat{s}(t) &= s(t) * h(t) \\ &= \int_{-\infty}^{\infty} \frac{s(u)}{\pi(t-u)} du \end{aligned} \quad (5.59)$$

The function  $\hat{s}(t)$  is called the Hilbert transform if  $s(t)$ . Note that

$$\mathcal{F}[\hat{h}(t)] = H(\omega)S(\omega) = -isgn(\omega)S(\omega) \quad (5.60)$$

In conclusion, if we convolve any time domain signal with  $\frac{1}{\pi t}$  then it will give us Hilbert transformed signal in time domain. Similarly, from the convolution property of the Fourier transform, if we multiply  $-isgn(\omega)$  with any frequency domain signal  $S(\omega)$  then it'll give us Hilbert transformed signal in frequency domain.

## 2. What are the requirements for the KK receiver scheme?

The Kramers-Kronig receiver based communication scheme relies on identifying a condition which ensures that the received signal is **minimum phase**, in such a case its phase can be uniquely extracted from its intensity[2].

In order to achieve minimum phase condition at the receiver end, it is prerequisite that the transmitted signal should be **analytical signal** in which real and imaginary parts are related to each other by means of Hilbert transform of each other.

### Minimum phase signal:

A necessary and sufficient condition for a complex signal  $A(t)$  to be minimum phase is that the curve described in a complex plane by  $A(t)$  when  $t \rightarrow -\infty$  to  $t \rightarrow \infty$  **does not encircle the origin**. A minimum-phase signal, whether discrete-time or continuous-time, has an useful property that the natural logarithm of the magnitude of the frequency response is related to the phase angle of the frequency response by the Hilbert transform.

### Analytical signal:

An analytic signal is a complex-valued signal that has no negative frequency components, and its real and imaginary parts are related to each other by the Hilbert transform.

$$s_a(t) = s(t) + i\hat{s}(t) \quad (5.61)$$

where,  $s_a(t)$  is an analytical signal and  $\hat{s}(t)$  is the Hilbert transform of the signal  $s(t)$ . Such analytical signal can be used to generate Single Sideband Signal (SSB) signal.

### Single sideband (SSB) signal generation:

This section will represent the brief idea of generating SSB signal using Hilbert transform method. To understand this, we may express signal  $s(t)$  as a summation of the two complex-valued functions.

$$s(t) = \frac{1}{2}[s(t) + j\hat{s}(t)] + \frac{1}{2}[s(t) - j\hat{s}(t)] \quad (5.62)$$

where, the term  $\frac{1}{2}[s(t) + j\hat{s}(t)]$  is the analytical representation of the signal  $s(t)$  (from Equation 5.61). Another term represents the complex conjugate  $\frac{1}{2}[s(t) - j\hat{s}(t)]$  of this analytical signal. Such representation of the signal  $s_a(t)$  and  $s_a^*(t)$  divide the signal into non-negative frequency component and non-positive frequency component respectively. Alternatively, we can write it as,

$$\frac{1}{2}S_a(f) = \begin{cases} S(f) & \text{for } f > 0 \\ 0 & \text{for } f < 0 \end{cases} \quad (5.63)$$

where  $S_a(f)$  and  $S(f)$  are the Fourier transform of  $t_a(t)$  and  $s(t)$  respectively. The frequency translated version of  $S_a(f - f_0)$  contains only one side (positive) of  $S(f)$  and hence it is called single sideband signal  $s_{ssb}(t)$ ,

$$F^{-1}\{S_a(f - f_0)\} = s_a(t)e^{j2\pi f_0 t} = s_{ssb}(t) + j\hat{s}_{ssb}(t) \quad (5.64)$$

Therefore, from the Euler's formula,

$$\begin{aligned} s_{ssb}(t) &= Re\{s_a(t)e^{j2\pi f_0 t}\} \\ &= Re\{[s(t) + j\hat{s}(t)][\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)]\} \\ &= s(t)\cos(2\pi f_0 t) - \hat{s}(t)\sin(2\pi f_0 t) \end{aligned} \quad (5.65)$$

This Equation 5.65 displays the mathematical modeling of the upper sideband SSB signal. Similarly, we can generate lower sideband SSB signal by,

$$s_{ssb}(t) = s(t)\cos(2\pi f_0 t) + \hat{s}(t)\sin(2\pi f_0 t) \quad (5.66)$$

### 3. How we can use these signals and profit from them?

This section represents the justification that why we need to use these signals into our proposed transceiver system.

#### Analytical Signal:

If we denote an analytic signal  $A_s(t)$  as,

$$A_s(t) = A_{s,r}(t) + iA_{s,i}(t) \quad (5.67)$$

then in the equation 5.67, the real and imaginary parts  $A_{s,r}(t)$  and  $A_{s,i}(t)$  are related through the Kramers-Kronig relation with each other. An intuitive way to analyze the relation is based on expressing its Fourier transform  $A_s(\omega)$  as follows,

$$A_s(\omega) = \frac{1}{2}[1 + sgn(\omega)]A_s(\omega) \quad (5.68)$$

The equation 5.68 follows the SSB signal condition  $A_s(\omega) = 0$  for  $\omega < 0$ . Further, simplification of the signal can be summarized as follows:

$$\begin{aligned} A_s(\omega) &= \frac{1}{2}[1 + sgn(\omega)]A_s(\omega) \\ &= \frac{1}{2}A_s(\omega) + \frac{1}{2}sgn(\omega)A_s(\omega) \end{aligned} \quad (5.69)$$

Taking inverse Fourier transform of the equation 5.69,

$$\begin{aligned} A_s(t) &= IFT\{A_s(\omega)\} \\ &= \frac{1}{2}A_s(t) + \frac{1}{2}[IFT\{sgn(\omega)\} \otimes A_s(t)] \end{aligned} \quad (5.70)$$

The underlined term in Equation 5.70 displays that multiplication in frequency domain converted into the convolution in the time domain. Further, IFT of the function  $\text{sgn}(\omega)$  given as  $(-i/\pi t)$ . As a consequences, we can further simplify our equation as,

$$\begin{aligned} A_s(t) &= \frac{1}{2}A_s(t) + \frac{1}{2}\left[\frac{i}{\pi t} * A_s(t)\right] \\ \frac{A_s(t)}{2} &= \frac{1}{2}\left[\frac{i}{\pi t} * A_s(t)\right] \\ A_s(t) &= i\left[\frac{1}{\pi t} * A_s(t)\right] \\ A_s(t) &= \frac{i}{\pi}p.v.\int_{-\infty}^{\infty} \frac{A_s(t')}{t-t'}dt' \end{aligned} \quad (5.71)$$

Using Equation 5.67 into Equation 5.71,

$$A_{s,r}(t) + iA_{s,i}(t) = \frac{i}{\pi}p.v.\int_{-\infty}^{\infty} \frac{A_s(t')}{t-t'}dt' \quad (5.72)$$

Therefore,

$$\begin{aligned} A_{s,r}(t) + iA_{s,i}(t) &= \frac{i}{\pi}p.v.\int_{-\infty}^{\infty} \frac{A_{s,r}(t') + iA_{s,i}(t')}{t-t'}dt' \\ A_{s,r}(t) + iA_{s,i}(t) &= -\frac{1}{\pi}p.v.\int_{-\infty}^{\infty} \frac{A_{s,i}(t')}{t-t'}dt' + \frac{i}{\pi}p.v.\int_{-\infty}^{\infty} \frac{A_{s,r}(t')}{t-t'}dt' \end{aligned} \quad (5.73)$$

which leads to,

$$\begin{aligned} A_{s,r}(t) &= -\frac{1}{\pi}p.v.\int_{-\infty}^{\infty} \frac{A_{s,i}(t')}{t-t'}dt' \\ A_{s,i}(t) &= \frac{1}{\pi}p.v.\int_{-\infty}^{\infty} \frac{A_{s,r}(t')}{t-t'}dt' \end{aligned} \quad (5.74)$$

### Minimum Phase signal:

Given function  $A(t) = A_s(t) + \bar{E}$  never encircles the origin for  $t \in (-\infty, \infty)$ . if we define,

$$G(t) = \log\left[\frac{A(t)}{\bar{A}}\right] \quad (5.75)$$

then  $G(\omega)$ , the spectrum of  $G(t)$ , is such that  $G(\omega) = 0$  for  $\omega < 0$ . Under the hypothesis stated by Equation 5.72, we can write  $G(t)$  as,

$$G(t) = \frac{i}{\pi}p.v.\int_{-\infty}^{\infty} \frac{G(t')}{t-t'}dt' \quad (5.76)$$

From the equations 5.75 and 5.76,

$$\log|A(t)| - \log|\bar{A}| + i[\phi(t) - \bar{\phi}] = \frac{i}{\pi}p.v.\int_{-\infty}^{\infty} \frac{\log|A(t)| - \log|\bar{A}|}{t-t'}dt' + \frac{1}{\pi}p.v.\int_{-\infty}^{\infty} \frac{\phi(t) - \bar{\phi}}{t-t'}dt' \quad (5.77)$$

Comparing Imaginary part of Equation 5.77,

$$\phi(t) - \bar{\phi} = +\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|A(t)| - \log|\bar{A}|}{t - t'} dt' \quad (5.78)$$

In equation 5.78,  $\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|\bar{A}|}{t - t'} dt' = 0$  which leads to,

$$\begin{aligned} \phi(t) &= \bar{\phi} + \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|A(t)|}{t - t'} dt' \\ \phi(t) &= \bar{\phi} + \frac{1}{2\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|A(t)|^2}{t - t'} dt' \end{aligned} \quad (5.79)$$

### 5.6.2 KK scheme

If we consider the complex envelope of the incoming electric field by  $A_s(t)$  confined within the optical bandwidth denoted by B. The LO assumed to be a continuous wave (CW) signal whose amplitude is  $A_0$  whose frequency coincides with the left edge of the information-carrying signal spectrum. Here, we assumed that  $A_0$  is real-valued and positive, which is equivalent to referring all phase value to that of LO.

The complex envelope of the field striking upon the photo-diode can be given as,

$$A(t) = A_s(t) + A_0 \exp(i\pi Bt) \quad (5.80)$$

The photo current  $I$  produced by the photo-diode is proportional to the field intensity  $I = |A(t)|^2$ , here proportionality constant considered as 1 for the sake of simplicity. If  $A_0$  is large enough to ensure that the signal  $A(t)\exp(-i\pi Bt) = A_0 + A_s(t)\exp(-i\pi Bt)$  is minimum phase. The discussed hypothesis can be used to reconstruct the signal  $E_s(t)$  as follows[1]:

$$A_s(t) = \{\sqrt{I(t)} \exp[i\phi_E(t)] - A_0\} \exp(i\pi Bt) \quad (5.81)$$

$$\phi_A(t) = \frac{1}{2\pi} p.v. \int_{-\infty}^{\infty} dt' \frac{\log[I(t')]}{t - t'} \quad (5.82)$$

### 5.6.3 Simulation set-up

#### Transmitter set-up

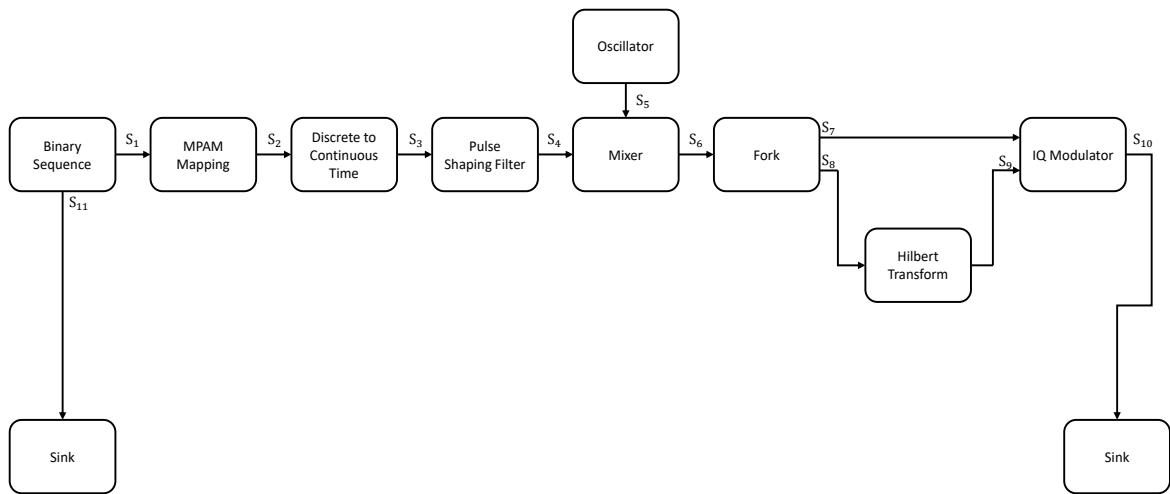


Figura 5.21: Transmitter simulation setup

#### Receiver set-up

### 5.6.4 Experimental set-up

If an optical signal received together with a CW tone at the band edge such that it satisfies the minimum phase condition, then its full complex signal can be recovered after direct-detection by using KK algorithm. The principle can be extended to the dual-polarization signals through a polarization diversity transceiver set-up. As shown in the Figure 5.23, the PDM received signal first spitted by a polarization beam splitter (PBS). Each polarization is combined with an LO tapped from the transmit laser. The laser's wavelength and power are set to ensure that the received signal should satisfy the minimum phase condition. After direct-detection, Kramers-Kronig algorithm is performed on each polarization separately to recover full complex signal. After compensation of the chromatic dispersion, stokes parameter based polarization demux. algorithm can be applied to recover PDM signals.

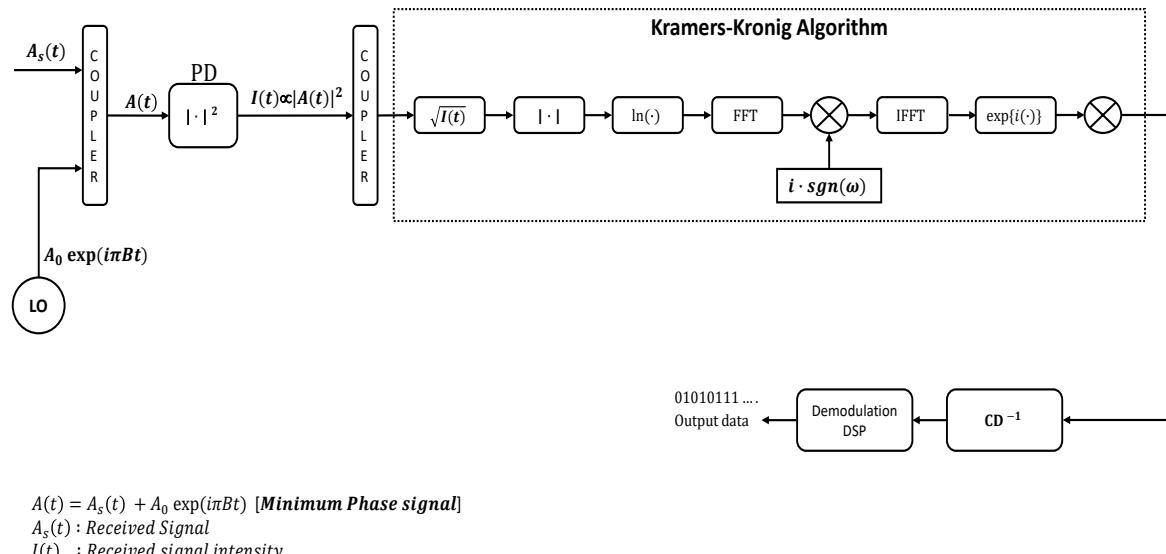


Figura 5.22: Receiver simulation setup

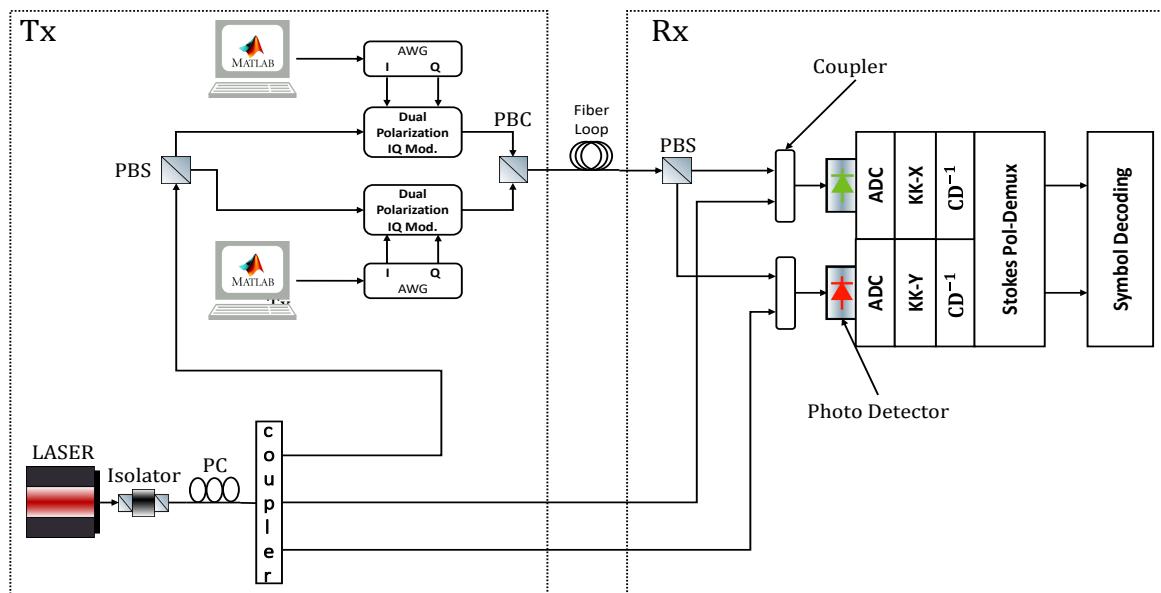


Figura 5.23: PDM Kramers-Kronig receiver experimental setup

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## Bibliografia

- [1] Antonio Mecozzi, Cristian Antonelli, and Mark Shtaif. *Kramers-Kronig Coherent Receiver*. Optica, vol.3, no.11, 2016, p.1220., doi:10.1364/optica.3.001220.
- [2] Antonio Mecozzi. *Retrieving the full optical response from amplitude data by Hilbert transform*. Opt. Comm. 282, 4183-4187.
- [3] Antonio Mecozzi. *A necessary and sufficient condition for minimum phase and implication of phase retrieval*. arXiv:1606.04861.

### APPENDICES

#### Appendix A : SSB with graphical explanation

This section describes the SSB signal generation using Hilbert transformation method (Phase Shift Method). Consider a message signal  $m(t)$  with its frequency domain spectrum  $M(F)$  as shown in Figure 5.24. From the Figure 5.24, we can see that both the side are scaled by factor '1' which means it represents the original signal.

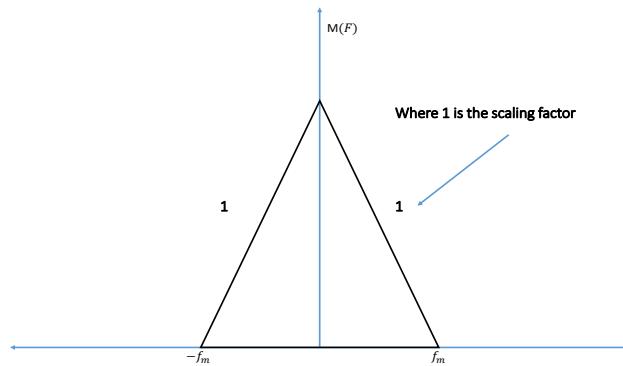


Figura 5.24: Original baseband signal

Now let's consider the modulated signal  $x(t)$  given as,

$$x(t) = m(t)\cos(2\pi f_c t) \quad (5.83)$$

Frequency domain representation of the equation 5.83 can be given as,

$$X(F) = \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c) \quad (5.84)$$

Here in equation 5.84, we can observe that each side band are scaled by  $\frac{1}{2}$  on the frequency spectrum. Figure displays the frequency domain representation of the modulated signal  $X(F)$ .

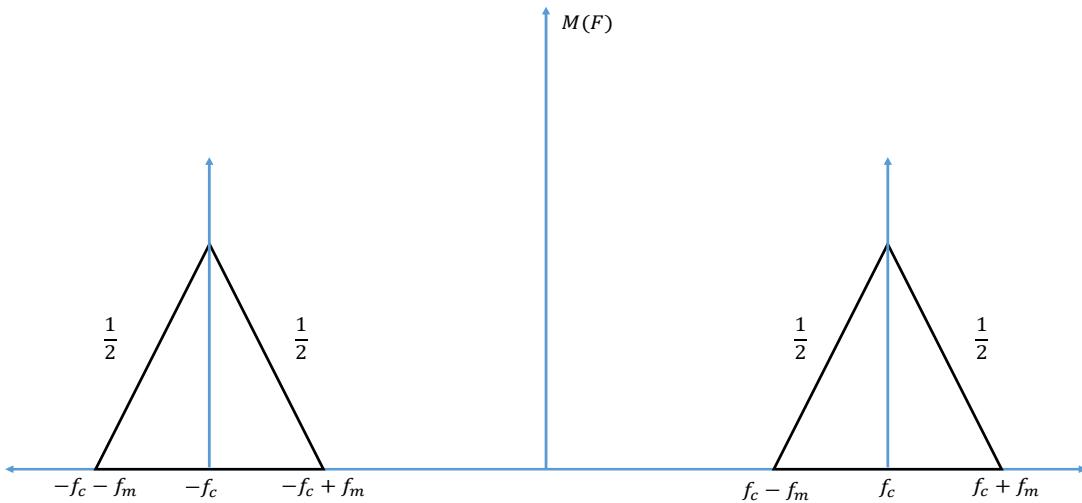


Figura 5.25: Original modulated signal

Next, we will discuss something more interesting which is called as Hilbert transform of the original message signal  $m(t)$ . As we discussed earlier, in the frequency domain, the Hilbert transformed signal  $\hat{M}(f)$  can be achieved by multiplying the Fourier transformed signal  $M(F)$  with  $[-isgn(F)]$ . Suppose we modulate the Hilbert transformed message signal  $\hat{m}(t)$

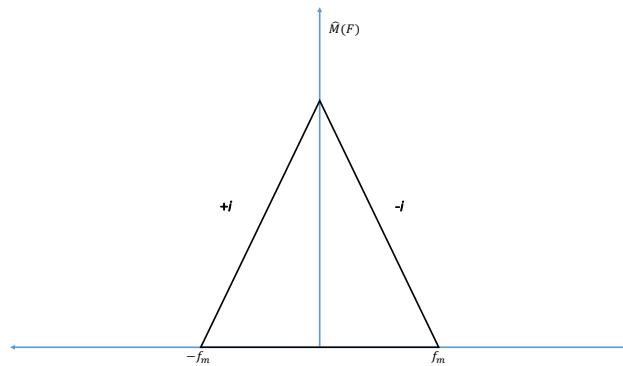


Figura 5.26: Hilbert transformed modulated signal

with the  $\sin(2\pi f_c t)$  (quadrature phase carrier), then we get the following results:

$$\begin{aligned}
\hat{m}(t)\sin(2\pi f_c t) &= \hat{m}(t) \frac{e^{j2\pi f_c t} - e^{-i2\pi f_c t}}{2} \\
&= \hat{m}(t) \frac{e^{j2\pi f_c t}}{2} - \hat{m}(t) \frac{e^{-i2\pi f_c t}}{2} \\
&= \frac{\hat{M}(f - f_c)}{2i} - \frac{\hat{M}(f + f_c)}{2i} \\
&= \frac{-i}{2} \hat{M}(f - f_c) + \frac{-i}{2} \hat{M}(f + f_c)
\end{aligned} \tag{5.85}$$

The detailed explanation of the equation 5.85 has been given in the Figure 5.27 and 5.28. Figure 5.27 displays the spectrum of the  $\hat{M}(f + f_c)$  and  $\hat{M}(f - f_c)$  for the positive and negative frequencies respectively. The final equation resolution of equation displays that both positive and negative side of the spectrum multiplied with  $\frac{i}{2}$  and  $\frac{-i}{2}$  respectively. Finally the spectrum of the signal  $\hat{m}(t)\sin(2\pi f_c t)$  can be given as Figure 5.28.

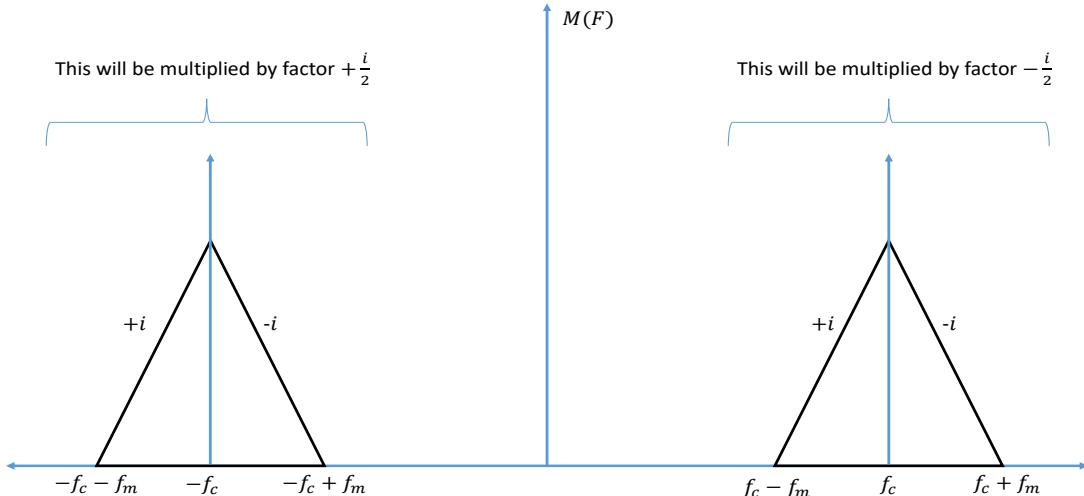


Figura 5.27: Hilbert transformed modulated signal

Further, summation of the two signals  $m(t)\cos(2\pi f_c t)$  and  $\hat{m}(t)\sin(2\pi f_c t)$  will generate the upper sideband SSB signal as follows,

$$u(t) = m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t) \tag{5.86}$$

From the above discussion, the spectrum of the Equation 5.86 can be given by the Figure 5.29. Similarly, for the lower sideband SSB can be generated by Equation,

$$u(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t) \tag{5.87}$$

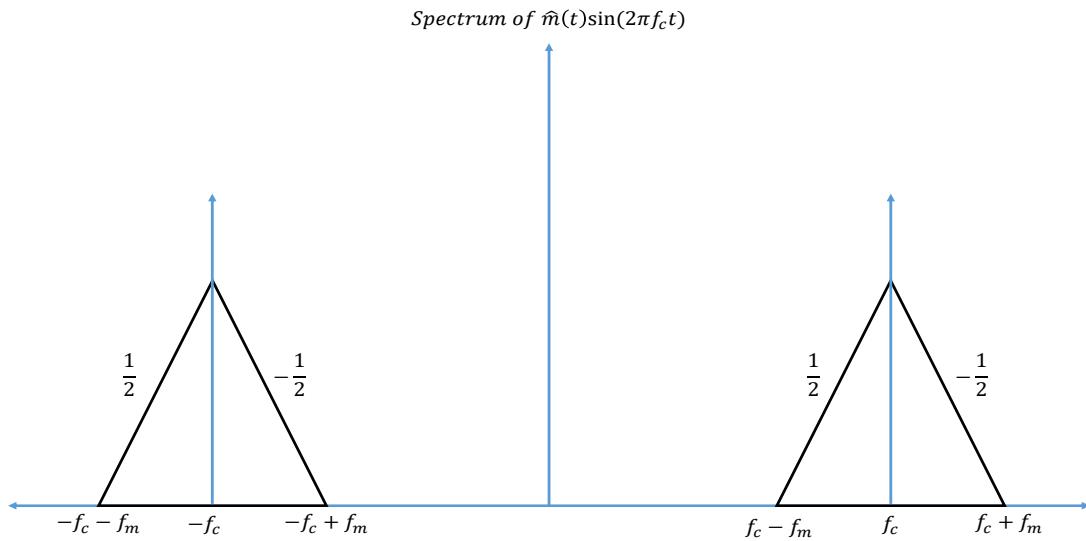


Figura 5.28: Hilbert transformed modulated signal

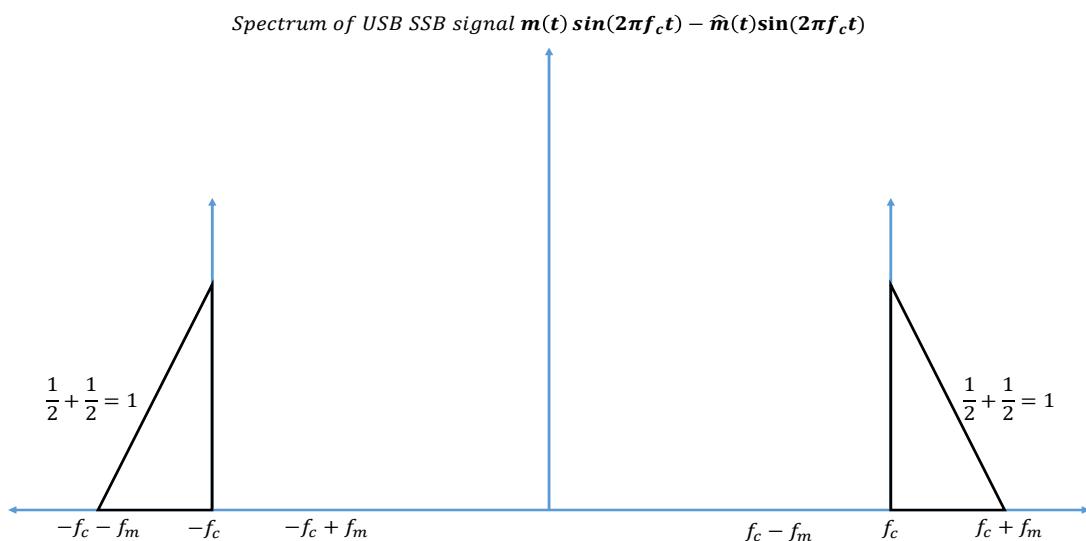


Figura 5.29: SSB signal spectrum

## 5.7 Quantum Oblivious Key Distribution with Discrete Variables

<b>Student Name</b>	:	Mariana Ramos
<b>Starting Date</b>	:	September 18, 2017
<b>Goal</b>	:	Quantum oblivious key distribution (QOKD) implementation with discrete variables.



Oblivious Transfer (OT) is a fundamental primitive in multi-party computation. The one-out-of-two OT consists in a communication protocol between Alice and Bob. At the beginning of the protocol Alice has two messages  $m_1$  and  $m_2$  and Bob wants to know one of them,  $m_b$ , without Alice knowing which one, i.e. without Alice knowing  $b$ , and Alice wants to keep the other message private, i.e. without Bob knowing  $m_{\bar{b}}$ . therefore two conditions must be fulfilled:

1. The protocol must be concealing, i.e at the beginning of the protocol Bob does not know nothing about Alice's messages, while at the end of the protocol Bob will learn the message  $m_b$  chosen by him.
2. The protocol is oblivious, i.e Alice cannot learn anything about Bob's choice, bit  $b$ , and Bob cannot learning nothing about the other message  $m_{\bar{b}}$ .

In order to implement OT between two parties (Alice and Bob) they must be able to exchange continuously oblivious keys, i.e a QOKD system must exist between them.

### 5.7.1 QO<sup>OKD</sup> system

In this section we are going to describe the quantum oblivious key distribution system.

Considering a discrete variables implementation, both Alice and Bob agree with the following correspondence, where + corresponds to *Rectilinear Basis* and  $\times$  corresponds to *Diagonal Basis*,

Basis	
0	+
1	$\times$

Alice and Bob also agree with the bit correspondence for each direction for each basis. For *Rectilinear basis*, "+",

	Basis "+"
0	$\rightarrow (0^\circ)$
1	$\uparrow (90^\circ)$

and for *Diagonal Basis*, " $\times$ ",

	Basis "x"
0	$\searrow (-45^\circ)$
1	$\nearrow (45^\circ)$

- The first step is to establish for both Alice and Bob the block length  $l$ . In this case, lets assume  $l = 16$ . Alice randomly generate a bit sequence with length  $l$ . Therefore, she must define two sets randomly:  $S_{A1}$  which contains the basis values; and  $S_{A2}$ , which contains the key values.

In that case, lets assume she generates the following sets  $S_{A1'}$  and  $S_{A2'}$ :

$$S_{A1'} = \{0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1\},$$

$$S_{A2'} = \{1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1\}.$$

- Next, Alice sends to Bob throughout a quantum channel  $l$  photons encoded using the basis defined in  $S_{A1'}$  and according to the key bits defined in  $S_{A2'}$ .

Therefore, in the current example, Alice sends the following photons,

$$\begin{aligned} S_{AB} &= \{\uparrow, \uparrow, \nearrow, \searrow, \rightarrow, \rightarrow, \searrow, \nearrow, \uparrow, \rightarrow, \searrow, \nearrow, \downarrow, \uparrow, \nearrow\} \\ &= \{90^\circ, 90^\circ, 45^\circ, -45^\circ, -45^\circ, 0^\circ, 0^\circ, -45^\circ, 45^\circ, 90^\circ, 0^\circ, -45^\circ, 45^\circ, -45^\circ, 90^\circ, 45^\circ\}. \end{aligned}$$



- Bob also randomly generates  $l = 16$  bits, which are going to define his measurement basis,  $S_{B1'}$ . Lets assume,

$$\begin{aligned} S_{B1'} &= \{0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1\} \\ &= \{+, \times, \times, +, +, \times, +, \times, \times, +, +, +, +, \times\}. \end{aligned}$$

Bob will get  $l$  results:

$$S_{B2'} = \{1, -, \underline{0}, 0, -, 1, \underline{1}, -, 1, -, 1, 0, 1, 1, \underline{0}, 1\}.$$

The ' $-$ ' corresponds to no clicks in Bob's detector. These no clicks happen due to attenuation. The underlined values are bits which were measured with a correct basis but an error has occurred.

- Bob is going to send a ' $\textcolor{red}{\cancel{-}}$ ' or a hash value to Alice for each measurement that he performed, thereby being ' $\textcolor{blue}{-1}$ ' the measurements which correspond to no clicks. In this case, we are going to assume that the hash value is calculated using the SHA-256 algorithm [?]. In detail, Bob has two sets  $S_{B1'}$  and  $S_{B2'}$  and he is going to generate the set  $S_{BH1}$  which have  $\textcolor{red}{l}$  hash values calculated for each position of  $S_{B1'}$  with the correspondent position of  $S_{B2'}$ . Therefore, Bob will send to Alice the following set:

$$S_{BH1} = \{\textcolor{blue}{sha}_1, -1, sha_2, sha_3, -1, sha_4, sha_5, -1, sha_6, -1, sha_7, sha_8, sha_9, sha_{10}, sha_{11}, sha_{12}\}.$$

5. Since Alice has received the confirmation of measurement from Bob, i.e after Alice has received  $S_{BH1}$ , she sends throughout a classical channel the basis which she has used to codify the photons, ~~which in this case we assumed~~  $S_{A1'} = \{0, -1, 1, 1, -1, 0, 0, -1, 1, -1, 0, 1, 1, 1, 0, 1\}$ .

Due to attenuation, the previous sets are reduced to the length  $l = 12$  and they shall be replaced by the following:

$$S_{A1} = \{0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1\},$$

$$S_{A2} = \{1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1\},$$

$$S_{B1} = \{0, 1, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1\},$$

$$S_{B2} = \{1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1\}$$



6. In order to know which photons were measured ~~correctly~~, Bob does the operation  $S_{B3} = S_{B1} \oplus S_{A1}$ . In the current example the operation will be:

$$\begin{array}{c|cccccccccccc} S_{B1} & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ \hline S_{A1} & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ \oplus & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}$$

In this way, Bob gets

$$S_{B3} = \{1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1\}.$$

When Bob uses the right basis he gets the values correctly, apart from possible errors in transmission, when he uses the wrong basis he just guess the value. The values "1" correspond to the values he measured correctly and "0" to the values he just guessed. Thus, Bob is building two sets of keys, one ' $r$ ' with correct ~~measurements~~ values and other ' $w$ ' with the ~~measurement~~ values that he just guessed.

Thus, Bob has two pair of sets, one for the ~~photons he measured correctly~~

$$S_{B_{rp}} = \{1, 2, 5, 6, 8, 11, 12\},$$

$$S_{B_{rb}} = \{1, 0, 1, 1, 0, 0, 1\},$$

where  $S_{B_{rp}}$  is the set of positions ~~he was right about~~ and  $S_{B_{rb}}$  is the set of bit values he measured for each position ~~he was right about~~. The other pair is for photons he just guessed the measurement,

$$S_{B_{wp}} = \{3, 4, 7, 9, 10\},$$

$$S_{B_{wb}} = \{0, 1, 1, 1, 1\},$$

where  $S_{B_{wp}}$  is the set of positions he just guessed the values and  $S_{B_{wb}}$  is the set of bit values he measured for each position he just guessed the values.

Nevertheless, due to errors in transmission, some of the bits in  $S_{B_{rb}}$  may be not right.

At this point, in order to test Bob's honesty, Alice is going to ask Bob to open some pairs of the right set and some pairs of the wrong set. These pairs are destroyed from them sets. Lets assume she asked to open two pairs of the right set and one pair of the wrong set. She verified these pairs using the hash function committed by Bob and concluded that he is being honest.

Now, Bob has the previous sets replaced by the following,

$$\begin{aligned} S_{B_{rp}} &= \{1, 2, 5, 6, 8\} \\ S_{B_{rb}} &= \{1, 0, 1, 1, 0\} \\ S_{B_{wp}} &= \{3, 4, 7, 9\} \\ S_{B_{wb}} &= \{0, 1, 1, 1\} \end{aligned}$$

Bob and Alice are going to use a modified version of *Cascade algorithm* to correct the errors due transmission.

### Modified version of Cascade Algorithm

The Cascade algorithm is often used with a key set where all values are supposed right. In this case, Bob has two pairs of sets, one with the position and bit values of photon he measured with the correct basis and other with position and bit values of photon he measured with the wrong basis. He only needs to apply the Cascade algorithm in the set that he measured the photons correctly [?]. However, he must apply a modified version of the Cascade in the other set in order to keep in secret from Alice which set corresponds to right and which set corresponds to wrong measurements.

- (a) Bob randomly generates a bit value. If he gets 0, he will send to Alice the set  $\{S_{B_{rp}}, S_{B_{wp}}\}$ . Otherwise, if he gets 1 he will send the set  $\{S_{B_{wp}}, S_{B_{rp}}\}$ . This guarantee that Alice does not know which is the right or wrong set. Lets assume this random bit is "1" and he sends  $\{S_{B_{wp}}, S_{B_{rp}}\}$ . Let assume Alice and Bob have agreed with  $N = 2$ , where  $N$  is the number of sub-blocks they will firstly divide the original set. After the first iteration, if there still be errors,  $N$  will be incremented by 1 for the next iteration. However, if there is some set which Bob thinks is already corrected, this block is discarded and the remaining blocks are divided again. If the set to be corrected has a odd number of bits, Bob and Alice must add an additional bit 0 at the end.
- (b) Now, Bob applies the Cascade algorithm in the two sets. However, to the wrong set he is going to apply a modified version, the *fake Cascade* in order to hiding that it is the set with wrong positions. In this case, he starts from the modified version of Cascade.
- (c) In this modified version Bob generates a random number to define  $i$  which is the number of iterations. Assuming  $s$  as the size of the set,  $i$  must be  $2 \geq i \leq s$ . As normal Cascade, in this modified version Bob divide the set in  $N$  blocks, where

$N$  is agreed with Alice at the beginning of the procedure and  $N \geq 1$ . Lets assume  $N = 1$ . After that, Alice sends to Bob the set with parity of the  $N$  blocks. He receives the set and do nothing with it. Bob says to Alice that an error has occurred and  $N$  is being incremented until  $N \leq \frac{s}{2}$  or until the number of iterations achieves  $i$ . Bob has been asking Alice for parities to keep her entertained during  $i$  iterations.

- (d) Bob acknowledge Alice that all values were fixed.
- (e) Next, Bob starts to apply the normal Cascade to the right set. Lets assumed they were agreed with first  $N = 2$ .

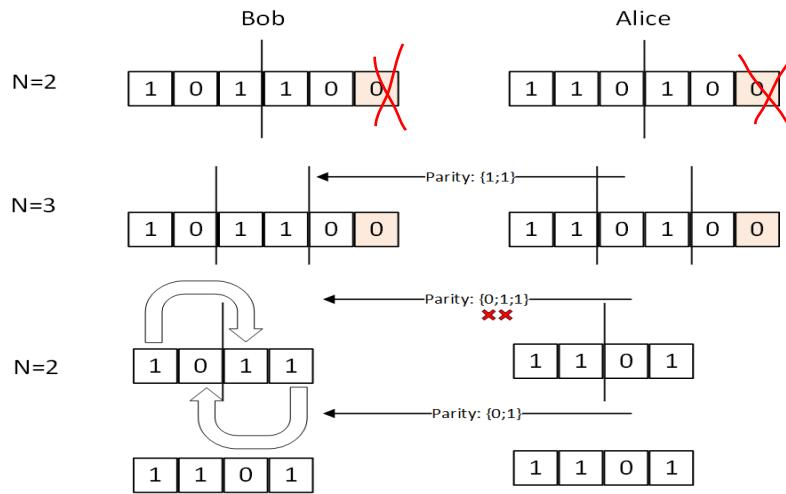


Figura 5.30: Cascade algorithm applied in the set of right values.

They start with  $N = 2$  and Alice sends the parity set. After that, as there is a possibility of error, Bob increments for  $N = 3$  and asks Alice to do the same. After that, Bob discard the last block since the parity matches with Alice's parity. With the other two blocks he change the bits position and combines the first bits and the second bits of each block and he asks Alice for the parity values again.

By analysing figure 5.30, the set of right positions is corrected and it will be replaced by the following:

$$S_{B_{rp}} = \{1, 2, 5, 6, 8\}$$

$$S_{B_{rb}} = \{1, 1, 0, 1, 0\}$$

~~7. Next, Bob calculates the minimum number between "ones" and "zeros" in  $S_{B3}$ , i.e~~

$$n = \min(\#0, \#1) = 4,$$

where  $\#0$  represents the number of zeros in  $S_{B2}$  and  $\#1$  the number of ones in  $S_{B2}$ . Bob sends to Alice, throughout a classical channel, the set  $4, w$ , where 4 is the  $n$  and

$w$  represents the nature of  $n$ , i.e in this case  $n$  corresponds to the number of wrong measured photons.

8. When Alice sends to Bob a photons set, they are building a set of pairs (array positions and bit values which correspond to measured photons at Bob's side and to the key bit with the photon was encoded at Alice's side). The main goal is to guarantee that Bob has the same number of right and wrong pairs. In addition, they must know information about  $t$  (represented in figure 5.31) which corresponds to the points where the previous condition is verified.

Since Bob has sent to Alice the information about the smallest set, in this example, Alice know that there are four pairs of wrong positions and five pairs of right positions. Alice must destroy one of the right pairs by asking Bob to open it. Therefore, at  $t = 8$  both know that there are the same number of right and wrong pairs thereby being the main goal guaranteed.

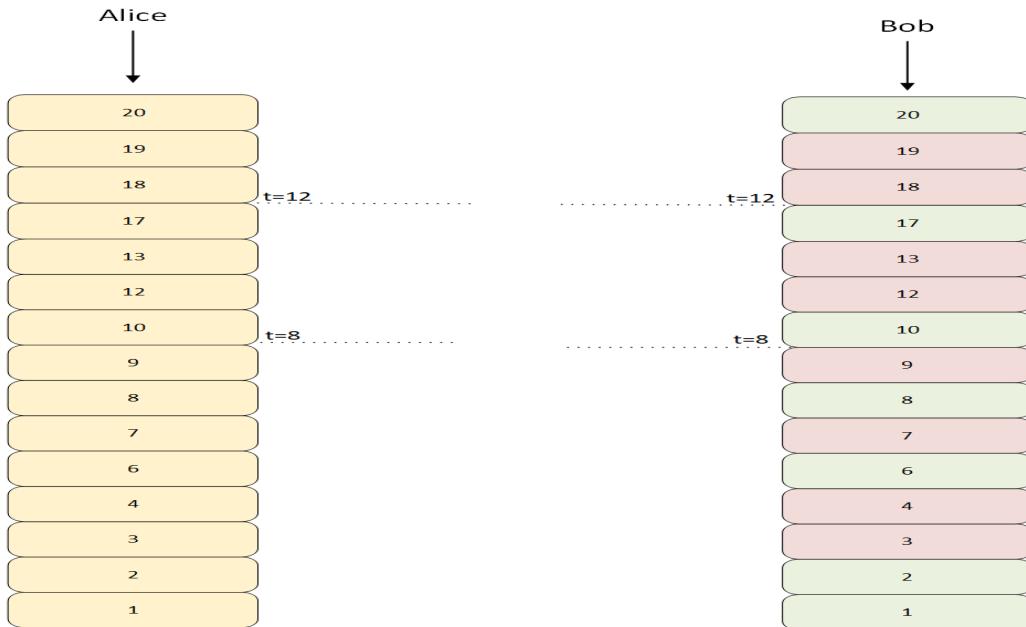


Figura 5.31: Alice and Bob key sets.

As we can see in figure 5.31, unlike Bob, Alice does not know which positions corresponds to right or wrong measurements performed by Bob. They have been building these sets during all protocol.

### 5.7.2 OT Protocol with QOKD system

At this time, we are going to describe the oblivious transfer protocol with detail. As it was referred at the beginning, Alice sends two messages to Bob and he wants to know one of them. Alice does not know which message Bob wants and Bob only know the message he

wants, i.e he does not know anything about the other message. Furthermore, only Alice knows information about messages  $m_0$  and  $m_1$ . In this case, lets assume the following two messages with size  $s = 4$ ,  $m_0 = \{0011\}$  and  $m_1 = \{0001\}$ . Alice must guarantee  $t = s \times 2$ . In order to do that, she must destroy the remaining pairs. In this case, there is no need to do that because they have a set for  $t = 8$  with the same number of wrong and right pairs.

1. Bob defines two new sub-sets,  $I_0$  and  $I_1$ .  $I_0$  is a set of values with photons array positions which Bob just guessed the measurement since he did not measure them with the same basis as Alice,  $I_1$  is a set of values with photons array positions which Bob measured correctly since he used the same basis as Alice used to encoded them. The position of the pairs of each right and wrong message are in the keys sets that they have been building during the protocol.

In this example, the message size is 4. Since, at this time  $t = 10$  and we have 5 right pairs and 5 wrong pairs, Alice ask to Bob to open one right pair and one wrong pair in order to both have exactly the message's size number of right and wrong pairs. Lets assume that Alice opened two pairs, position 15 which is a wrong measurement and position 10 which is a right measurement. We have now  $t = 8$ .

Next, Bob defines two sub-sets with size  $s = 4$ :

$$I_0 = \{3, 4, 7, 9\},$$

and

$$I_1 = \{1, 2, 6, 8\},$$

where  $I_0$  is the sequence of positions in which Bob was wrong about basis measurement and  $I_1$  is the sequence of positions in which Bob was right about basis measurement. Bob sends to Alice the set  $S_b$

Thus, if Bob wants to know  $m_0$  he must send to Alice throughout a classical channel the set  $S_0 = \{I_1, I_0\}$ , otherwise if he wants to know  $m_1$  he must send to Alice throughout a classical channel the set  $S_1 = \{I_0, I_1\}$ .

2. Alice is sure about Bob's honesty, since she knows he only has 4 right basis to measure the photons. In addition, Alice cannot know which message Bob chose because she did not know the order that he sent the sets.
3. Lets assume Bob sent  $S_0 = \{I_1, I_0\}$ . Alice defines two encryption keys  $K_0$  and  $K_1$  using the values in positions defined by Bob in the set sent by him. In this example, lets assume:

$$K_0 = \{1, 1, 1, 0\}$$

$$K_1 = \{0, 0, 0, 1\}.$$

Alice does the following operations:

$$m = \{m_0 \oplus K_0, m_1 \oplus K_1\}.$$

$$\begin{array}{c|cccc} m_0 & 0 & 0 & 1 & 1 \\ \hline K_0 & 1 & 1 & 1 & 0 \\ \oplus & 1 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{c|cccc} m_1 & 0 & 0 & 0 & 1 \\ \hline K_1 & 0 & 0 & 0 & 1 \\ \oplus & 0 & 0 & 0 & 0 \end{array}$$

Adding the two results,  $m$  will be:

$$m = \{1, 1, 0, 1, 0, 0, 0, 0\}.$$

After that, Alice sends to Bob the encrypted message  $m$  through a classical channel.

4. When Bob receives the message  $m$ , in the same way as Alice, Bob uses  $S_{B1}$ , values of positions given by  $I_1$  and  $I_0$  and does the decrypted operation. In this case, he does following operation:

$$\begin{array}{c|cccccccc} m & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ \oplus & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{array}$$

The first four bits corresponds to message 1 and he received  $\{0, 0, 1, 1\}$ , which is the right message  $m_0$  and  $\{0, 1, 1, 1\}$  which is a wrong message for  $m_1$ .

### 5.7.3 Simulation

First of all, the protocol will be simulated and then a experimental setup will be built in the laboratory.

The main goal of this simulation is to demonstrate that Bob was able to learn correctly message  $m_b$  and he does not know the message  $m_{\bar{b}}$ .

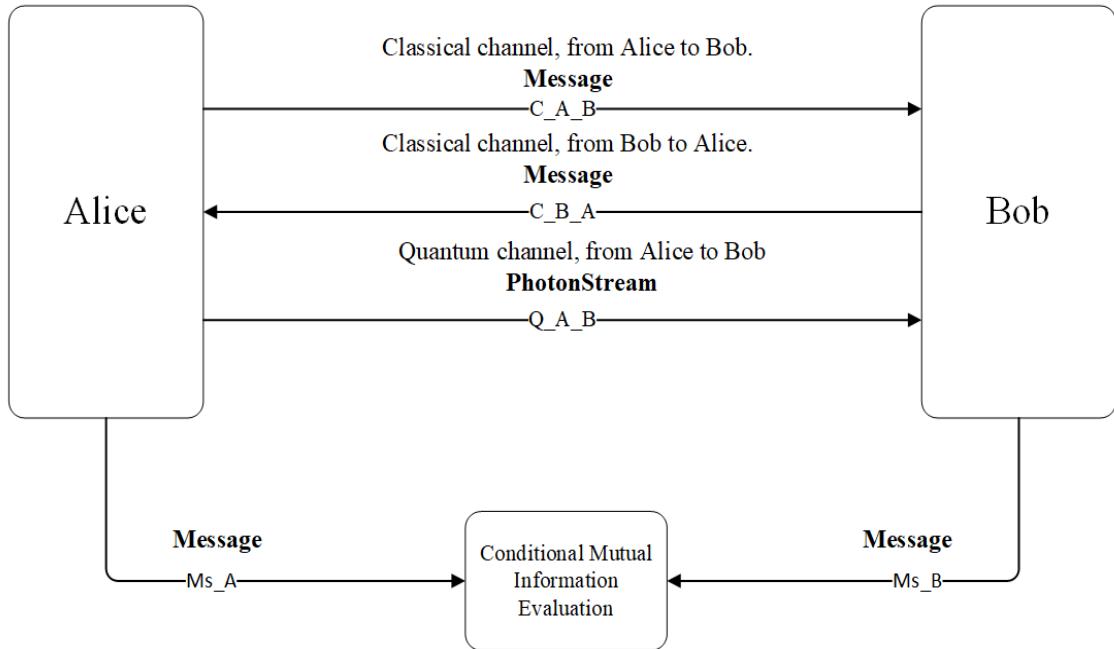


Figura 5.32: Simulation diagram at a top level

As one may see in figure 5.32 this simulation will have three top level blocks. Two of them are Alice and Bob and they are connected through two classical channels and one quantum channel. In addition, a third block will be performed in order to calculate the *Mutual Information*. The mutual information (MI) between Alice and Bob is defined in terms of their joint distribution.

1. In figure 5.33 one can observe a block diagram of the simulation at Alice's side. As it is shown in the figure, Alice must have two blocks for random number generation: one for basis generation in order to polarize the photons according this basis, and other for key random generation in order to have a random state to encode each photon. Furthermore, she has a Processor block for all logical operations: array analysis, hash function results validation, and others. This block also receives the start information, i.e. message size  $s$ , the expansion factor  $k$  and messages  $m_0$  and  $m_1$ , as well as information from Bob, i.e sets  $I_0$  and  $I_1$ , hash function results, and others. This block also must be responsible for send classical information to Bob. Finally, Processor block will also send a real continuous time signal to single photon generator, in order to generate photons according to this signal, and finally this block also sends to polarizer a real discrete signal in order to inform the polarizer which basis it should use. Therefore, she has two more blocks for quantum tasks: the single photon generator and the polarizer block which is responsible to encode the photons generated from the previous block and send them throughout a quantum channel from Alice to Bob.

Finally, Alice's processor has an output to Mutual Information top level block,  $Ms_A$ .

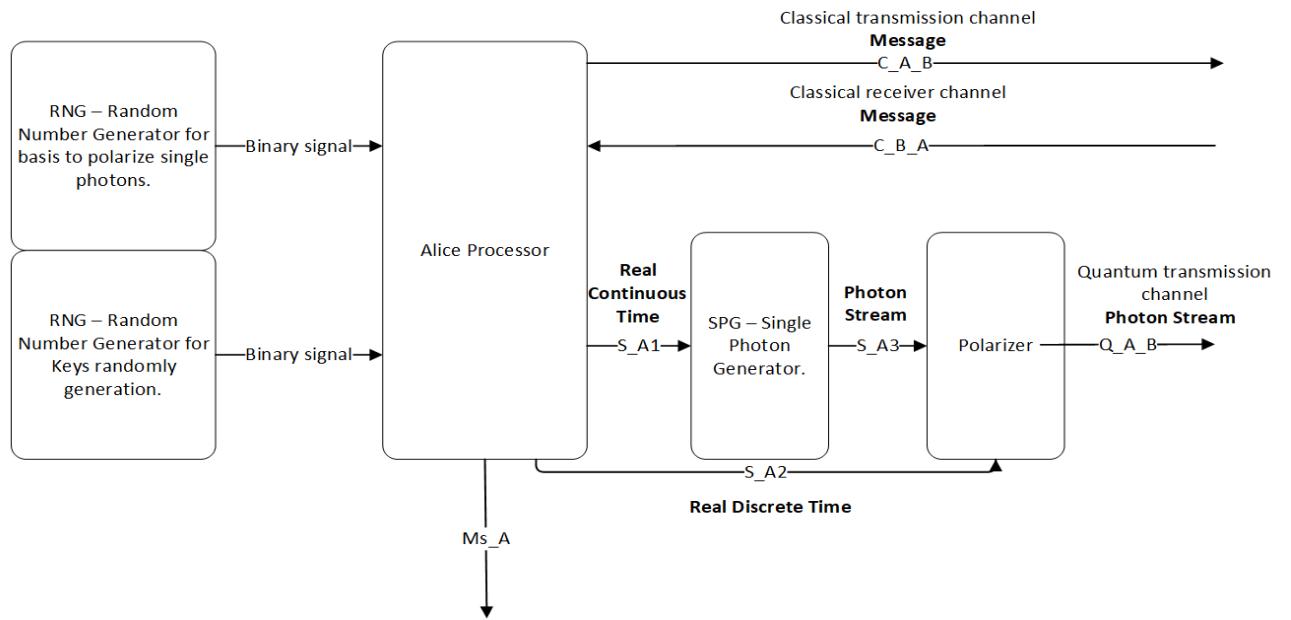


Figura 5.33: Simulation diagram - Alice's side

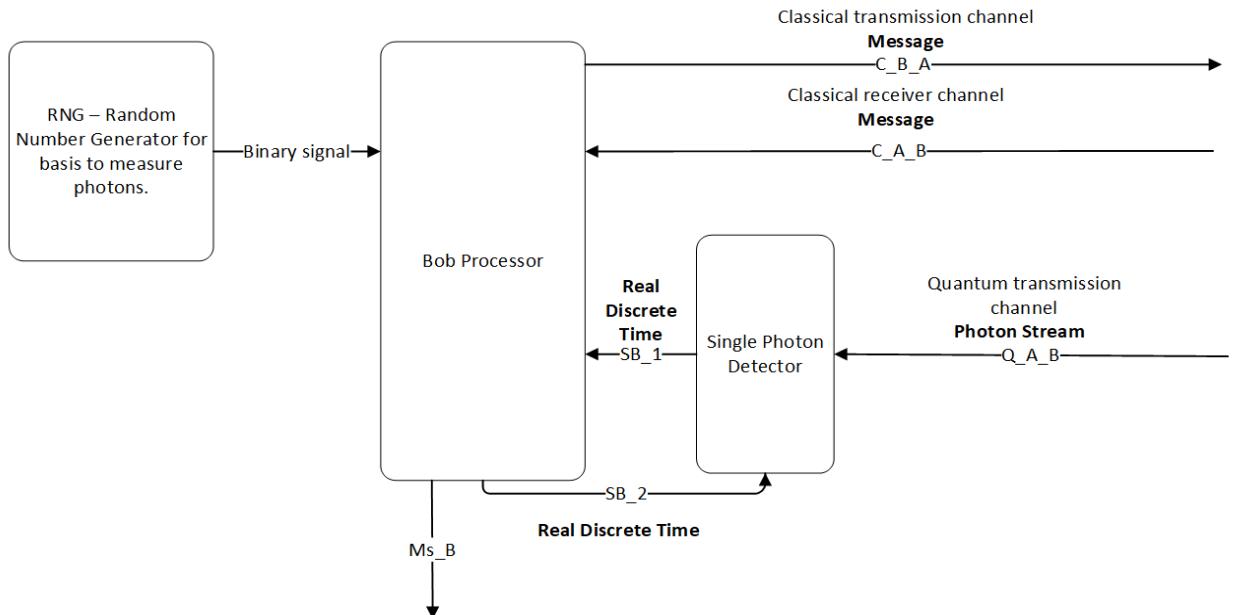


Figura 5.34: Simulation diagram - Bob's side

2. In figure 5.34 one can observe a block diagram of the simulation at Bob's side. From this side, Bob only has one block for Random Number Generation which is responsible for randomly generate basis values which Bob will use to measure the

photons sent by Alice throughout the quantum channel. Like Alice, Bob has a Processor block responsible for all logical tasks, i.e Hash function generation, analysing functions, etc. It receives information from Alice throughout a classical channel and a quantum channel but it sends information to Alice only throughout a classical channel. Furthermore, Bob has one more block for single photon detection which receives from processor block a real discrete time signal, in order to obtain the basis it should use to measure the photons.

Finally, Bob's processor has an output to Mutual Information top level block,  $M_{SB}$ .

3. Mutual Information calculation
- 4.
- 5.
- 6.

#### 5.7.4 Experimental

In figures 5.35 and 5.36 are presented the experimental setup to be performed in the lab. Starting with Alice's side and then Bob's side.

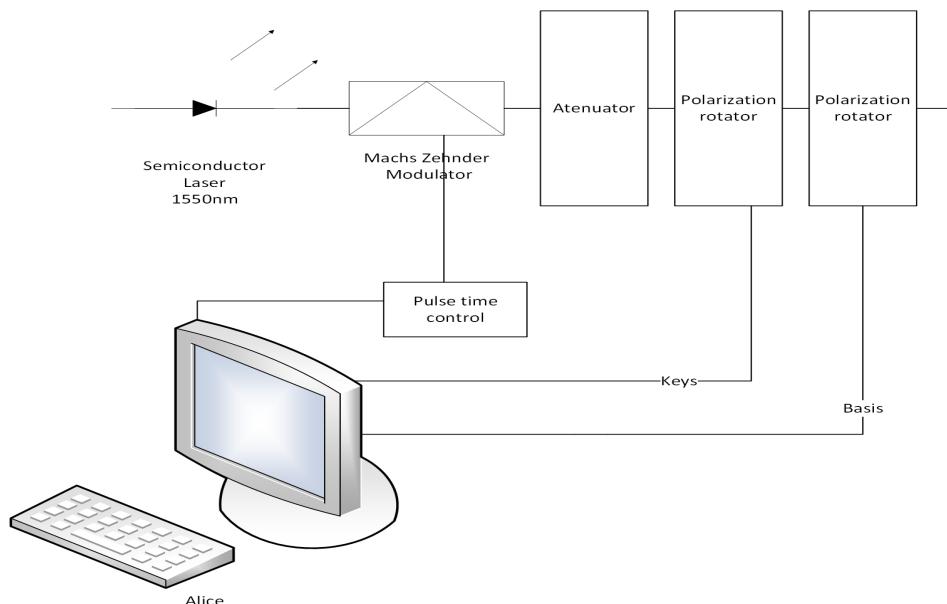


Figura 5.35: Quantum communication diagram - Alice's side

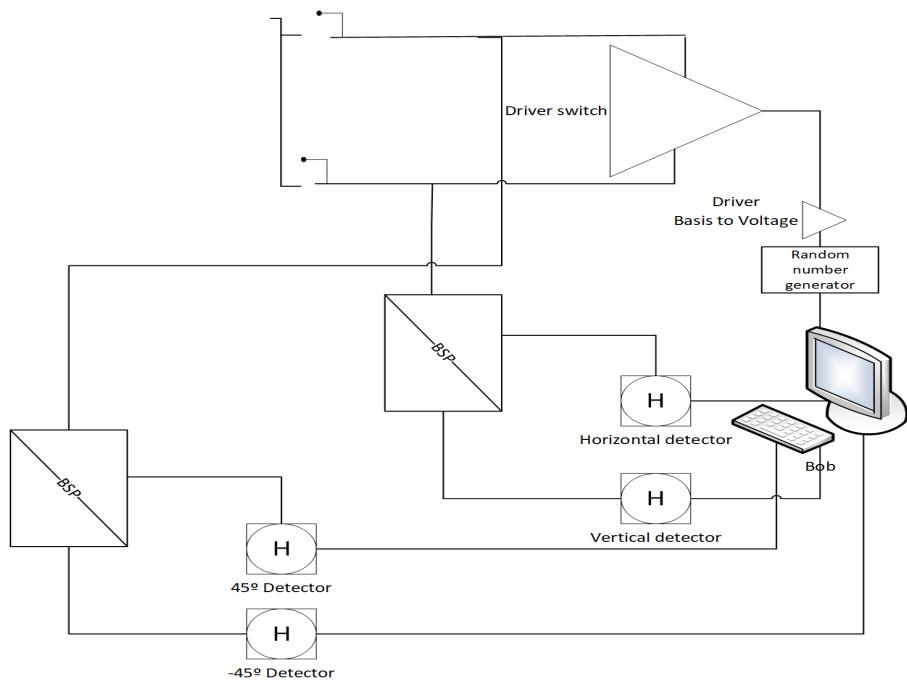


Figura 5.36: Quantum communication diagram - Bob's side

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## Bibliografia

- [1] Rodney Loudon. *The Quantum Theory of Light*. Oxford University Press, 2000.
- [2] Mark Fox. *Quantum Optics: An Introduction*. Oxford University Press, 2006.
- [3] Hans-A. Bachor and Timothy C. Ralph. *A Guide to Experiments in Quantum Optics*. Wiley-VCH, 2004.

## 5.8 Radio Over Fiber Transmission System

<b>Student Name</b>	:	Celestino Martins
<b>Starting Date</b>	:	September 25, 2017
<b>Goal</b>	:	Radio Over Fiber.

Radio over fiber (RoF) technology comprises the transmission over fiber technology, where light is modulated by a radio signal and transmitted over an optical fiber link to provide broadband wireless services. The connection is established between a base station (BS) and central processing units (CPUs) as shown in the Figure 5.37.

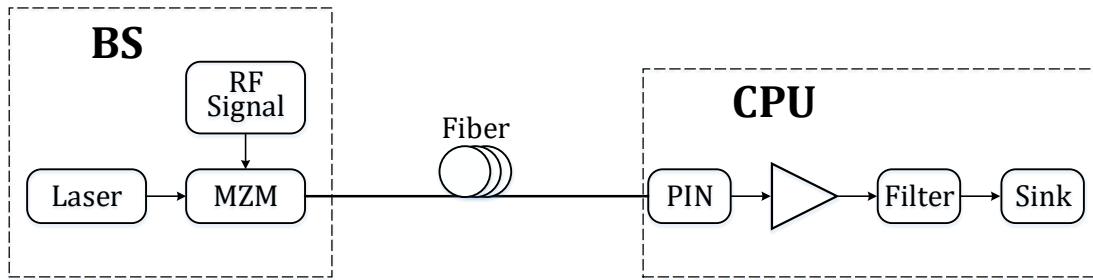


Figura 5.37: RoF transmission system block diagram.

In RoF systems, radio frequency (RF) signals are transported in optical form between a CPU and a set of BS before being radiated through the air. Each base station is adapted to communicate over a radio link with at least one user's mobile terminal located within the radio range of said BS. Considering an uplink connection in the Figure 5.37, the RF signals received from a mobile terminal are modulated using a laser and a mach-zehnder modulator (MZM) and then transmitted over optical link to CPU. In the CPU the optical signal is detected by a PIN, amplified and followed with an electrical filter. After these operations, digital signal processing techniques can be applied.

### 5.8.1 Simulation

### 5.8.2 Experimental

## **Capítulo 6**

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**Library**

## 6.1 Add

### Input Parameters

This block takes no parameters.

### Functional Description

This block accepts two signals and outputs one signal built from a sum of the two inputs. The input and output signals must be of the same type, if this is not the case the block returns an error.

### Input Signals

**Number:** 2

**Type:** Real, Complex or Complex\_XY signal (ContinuousTimeContinuousAmplitude)

### Output Signals

**Number:** 1

**Type:** Real, Complex or Complex\_XY signal (ContinuousTimeContinuousAmplitude)

## 6.2 Bit Error Rate

### Input Parameters

**Parameter:** setConfidence

**Parameter:** setMidReportSize

### Functional Description

This block accepts two binary strings and outputs a binary string, outputting a 1 if the two input samples are equal to each other and 0 if not. This block also outputs .txt files with a report of the estimated Bit Error Rate (BER),  $\widehat{\text{BER}}$  as well as the estimated confidence bounds for a given probability  $\alpha$ .

The  $\widehat{\text{BER}}$  is obtained by counting both the total number received bits,  $N_T$ , and the number of coincidences,  $K$ , and calculating their relative ratio:

$$\widehat{\text{BER}} = 1 - \frac{K}{N_T}. \quad (6.1)$$

The upper and lower bounds,  $\text{BER}_{\text{UB}}$  and  $\text{BER}_{\text{LB}}$  respectively, are calculated using the Clopper-Pearson confidence interval, which returns the following simplified expression for  $N_T > 40$  [?]:

$$\text{BER}_{\text{UB}} = \widehat{\text{BER}} + \frac{1}{\sqrt{N_T}} z_{\alpha/2} \sqrt{\widehat{\text{BER}}(1 - \widehat{\text{BER}})} + \frac{1}{3N_T} \left[ 2 \left( \frac{1}{2} - \widehat{\text{BER}} \right) z_{\alpha/2}^2 + (2 - \widehat{\text{BER}}) \right] \quad (6.2)$$

$$\text{BER}_{\text{LB}} = \widehat{\text{BER}} - \frac{1}{\sqrt{N_T}} z_{\alpha/2} \sqrt{\widehat{\text{BER}}(1 - \widehat{\text{BER}})} + \frac{1}{3N_T} \left[ 2 \left( \frac{1}{2} - \widehat{\text{BER}} \right) z_{\alpha/2}^2 - (1 + \widehat{\text{BER}}) \right], \quad (6.3)$$

where  $z_{\alpha/2}$  is the  $100(1 - \frac{\alpha}{2})$ th percentile of a standard normal distribution.

The block allows for mid-reports to be generated, the number of bits between reports is customizable, if it is set to 0 then the block will only output the final report.

### Input Signals

**Number:** 2

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number:** 1

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

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## Bibliografia

- [1] Rodney Loudon. *The Quantum Theory of Light*. Oxford University Press, 2000.
- [2] Mark Fox. *Quantum Optics: An Introduction*. Oxford University Press, 2006.
- [3] Hans-A. Bachor and Timothy C. Ralph. *A Guide to Experiments in Quantum Optics*. Wiley-VCH, 2004.

### 6.3 Binary source

This block generates a sequence of binary values (1 or 0) and it can work in four different modes:

- 1. Random
- 3. DeterministicCyclic
- 2. PseudoRandom
- 4. DeterministicAppendZeros

This blocks doesn't accept any input signal. It produces any number of output signals.

#### Input Parameters

**Parameter:** mode{PseudoRandom}  
 (Random, PseudoRandom, DeterministicCyclic, DeterministicAppendZeros)

**Parameter:** probabilityOfZero{0.5}  
 (real  $\in [0,1]$ )

**Parameter:** patternLength{7}  
 (integer  $\in [1,32]$ )

**Parameter:** bitStream{"0100011101010101"}  
 (string of 0's and 1's)

**Parameter:** numberOfWorks{-1}  
 (long int)

**Parameter:** bitPeriod{1.0/100e9}  
 (double)

#### Methods

```
BinarySource(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig,
OutputSig){};
```

```
void initialize(void);
```

```
bool runBlock(void);
```

```
void setMode(BinarySourceMode m) BinarySourceMode const getMode(void)
```

```
void setProbabilityOfZero(double pZero)
```

```
double const getProbabilityOfZero(void)
```

```
void setBitStream(string bStream)
```

```

string const getBitStream(void)

void setNumberOfBits(long int nOfBits)

long int const getNumberOfBits(void)

void setPatternLength(int pLength)

int const getPatternLength(void)

void setBitPeriod(double bPeriod)

double const getBitPeriod(void)

```

### Functional description

The *mode* parameter allows the user to select between one of the four operation modes of the binary source.

**Random Mode** Generates a 0 with probability *probabilityOfZero* and a 1 with probability  $1 - \text{probabilityOfZero}$ .

**Pseudorandom Mode** Generates a pseudorandom sequence with period  $2^{patternLength} - 1$ .

**DeterministicCyclic Mode** Generates the sequence of 0's and 1's specified by *bitStream* and then repeats it.

**DeterministicAppendZeros Mode** Generates the sequence of 0's and 1's specified by *bitStream* and then it fills the rest of the buffer space with zeros.

### Input Signals

**Number:** 0

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number:** 1 or more

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

## Examples

### Random Mode

**PseudoRandom Mode** As an example consider a pseudorandom sequence with *patternLength*=3 which contains a total of 7 ( $2^3 - 1$ ) bits. In this sequence it is possible to find every combination of 0's and 1's that compose a 3 bit long subsequence with the exception of 000. For this example the possible subsequences are 010, 110, 101, 100, 111, 001 and 100 (they appear in figure 6.1 numbered in this order). Some of these require wrap.

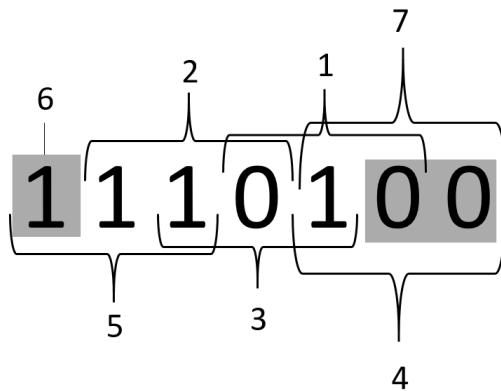


Figura 6.1: Example of a pseudorandom sequence with a pattern length equal to 3.

**DeterministicCyclic Mode** As an example take the *bit stream* '0100011101010101'. The generated binary signal is displayed in.

**DeterministicAppendZeros Mode** Take as an example the *bit stream* '0100011101010101'. The generated binary signal is displayed in 6.2.

### Sugestions for future improvement

Implement an input signal that can work as trigger.

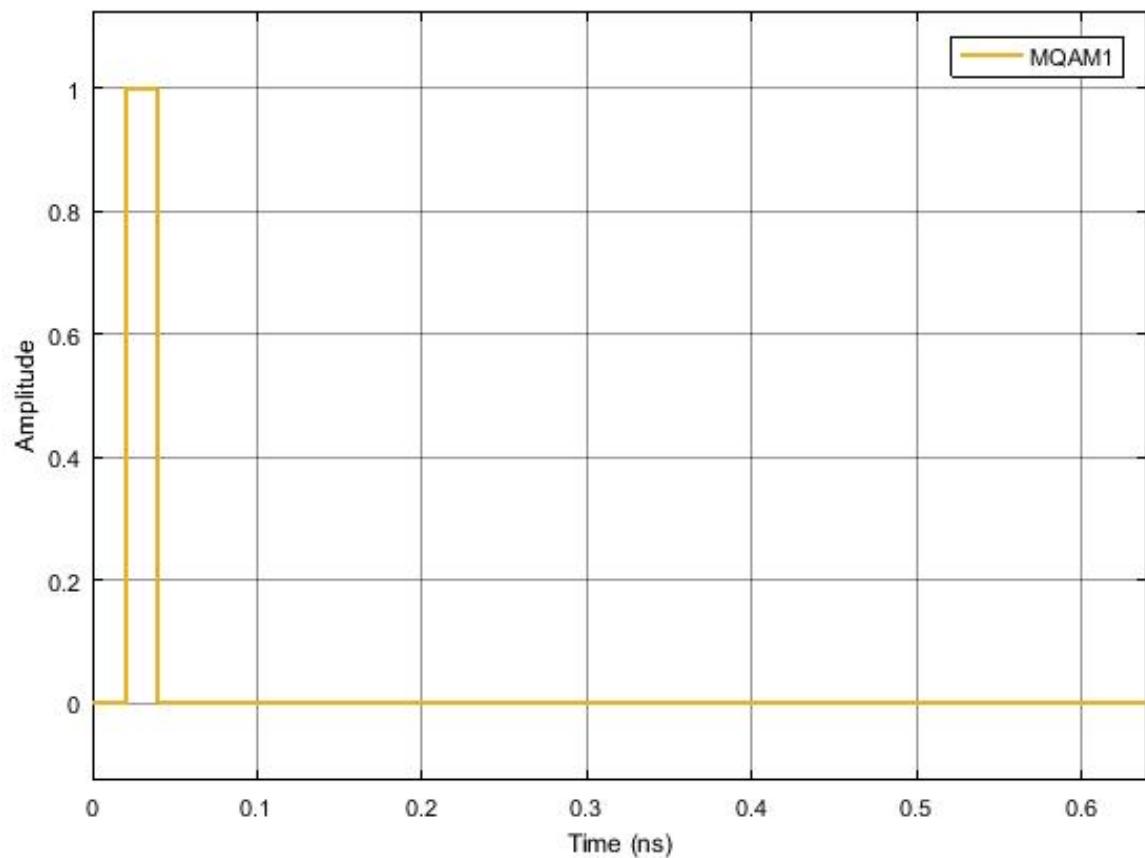


Figura 6.2: Binary signal generated by the block operating in the *Deterministic Append Zeros* mode with a binary sequence 01000...

## 6.4 Clock

This block doesn't accept any input signal. It outputs one signal that corresponds to a sequence of Dirac's delta functions with a user defined *period*.

### Input Parameters

**Parameter:** period{ 0.0 };

**Parameter:** samplingPeriod{ 0.0 };

### Methods

Clock()

Clock(vector<Signal \*> &InputSig, vector<Signal \*> &OutputSig) :Block(InputSig, OutputSig)

void initialize(void)

bool runBlock(void)

void setClockPeriod(double per)

void setSamplingPeriod(double sPeriod)

### Functional description

## Input Signals

**Number:** 0

## Output Signals

**Number:** 1

**Type:** Sequence of Dirac's delta functions.  
(TimeContinuousAmplitudeContinuousReal)

## Examples

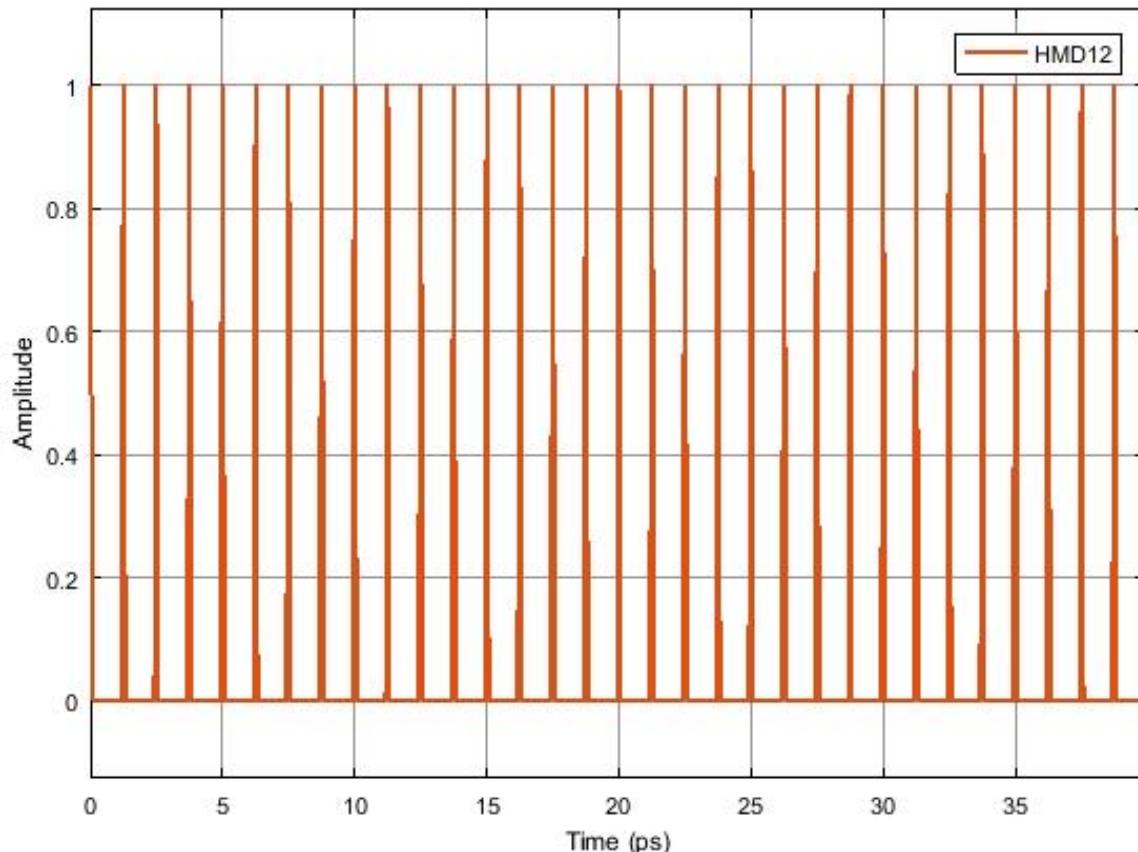


Figura 6.3: Example of the output signal of the clock

## Sugestions for future improvement

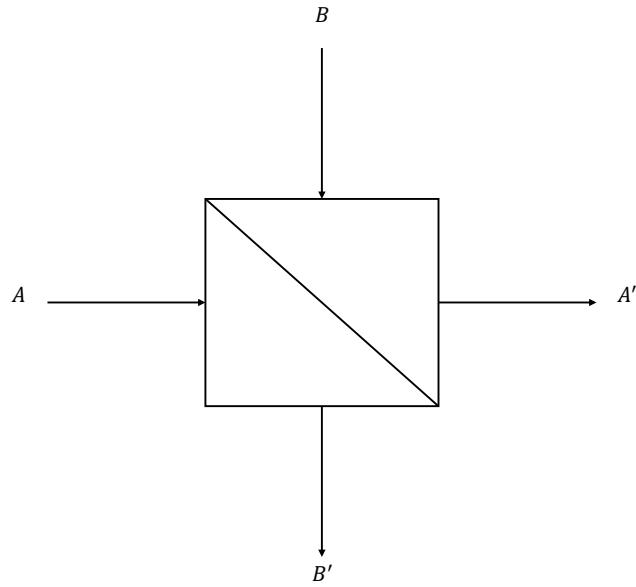


Figura 6.4: 2x2 coupler

## 6.5 Coupler 2 by 2

In general, the matrix representing 2x2 coupler can be summarized in the following way,

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} T & iR \\ iR & T \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} \quad (6.4)$$

Where, A and B represent inputs to the 2x2 coupler and A' and B' represent output of the 2x2 coupler. Parameters T and R represent transmitted and reflected part respectively which can be quantified in the following form,

$$T = \sqrt{1 - \eta_R} \quad (6.5)$$

$$R = \sqrt{\eta_R} \quad (6.6)$$

Where, value of the  $\sqrt{\eta_R}$  lies in the range of  $0 \leq \sqrt{\eta_R} \leq 1$ .

It is worth to mention that if we put  $\eta_R = 1/2$  then it leads to a special case of "Balanced Beam splitter" which equally distribute the input power into both output ports.

## 6.6 Decoder

This block accepts a complex electrical signal and outputs a sequence of binary values (0's and 1's). Each point of the input signal corresponds to a pair of bits.

### Input Parameters

**Parameter:** t\_integer m{ 4 }

**Parameter:** vector<t\_complex> iqAmplitudes{ { 1.0, 1.0 },{ -1.0, 1.0 },{ -1.0, -1.0 },{ 1.0, -1.0 } };

### Methods

Decoder()

Decoder(vector<Signal \*> &InputSig, vector<Signal \*> &OutputSig) :Block(InputSig, OutputSig)

void initialize(void)

bool runBlock(void)

void setM(int mValue)

void getM()

void setIqAmplitudes(vector<t\_iqValues> iqAmplitudesValues)

vector<t\_iqValues>getIqAmplitudes()

### Functional description

This block makes the correspondence between a complex electrical signal and pair of binary values using a predetermined constellation.

To do so it computes the distance in the complex plane between each value of the input signal and each value of the *iqAmplitudes* vector selecting only the shortest one. It then converts the point in the IQ plane to a pair of bits making the correspondence between the input signal and a pair of bits.

## Input Signals

**Number:** 1

**Type:** Electrical complex (TimeContinuousAmplitudeContinuousReal)

## Output Signals

**Number:** 1

**Type:** Binary

## Examples

As an example take an input signal with positive real and imaginary parts. It would correspond to the first point of the *iqAmplitudes* vector and therefore it would be associated to the pair of bits 00.

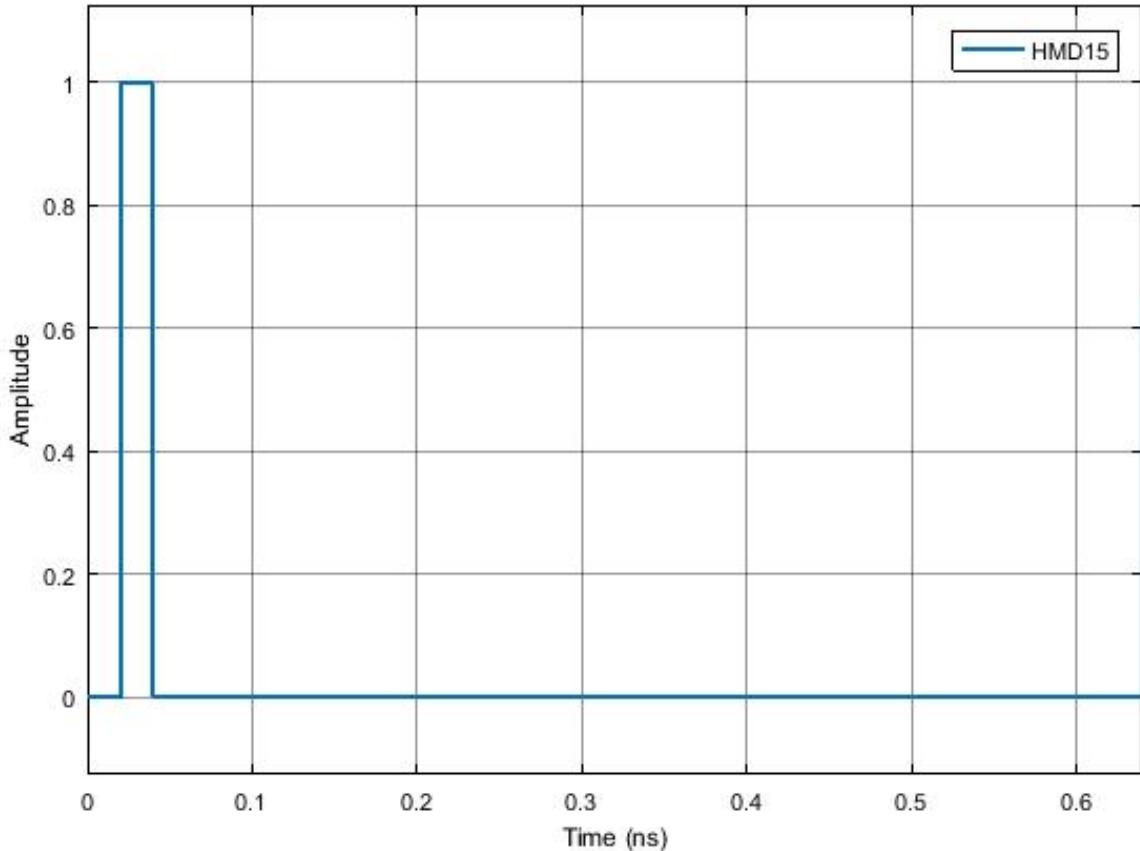


Figura 6.5: Example of the output signal of the decoder for a binary sequence 01. As expected it reproduces the initial bit stream

**Sugestions for future improvement**

## 6.7 Discrete to continuous time

This block converts a signal discrete in time to a signal continuous in time. It accepts one input signal that is a sequence of 1's and -1's and it produces one output signal that is a sequence of Dirac delta functions.

### Input Parameters

**Parameter:** `numberOfSamplesPerSymbol{8}`  
                   (int)

### Methods

```
DiscreteToContinuousTime(vector<Signal * > &inputSignals, vector<Signal * > &outputSignals) :Block(inputSignals, outputSignals){};
```

```
void initialize(void);
```

```
bool runBlock(void);
```

```
void setNumberOfSamplesPerSymbol(int nSamplesPerSymbol)
```

```
int const getNumberOfSamplesPerSymbol(void)
```

### Functional Description

This block reads the input signal buffer value, puts it in the output signal buffer and it fills the rest of the space available for that symbol with zeros. The space available in the buffer for each symbol is given by the parameter *numberOfSamplesPerSymbol*.

### Input Signals

**Number** : 1

**Type** : Sequence of 1's and -1's. (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number** : 1

**Type** : Sequence of Dirac delta functions (ContinuousTimeDiscreteAmplitude)

### Example

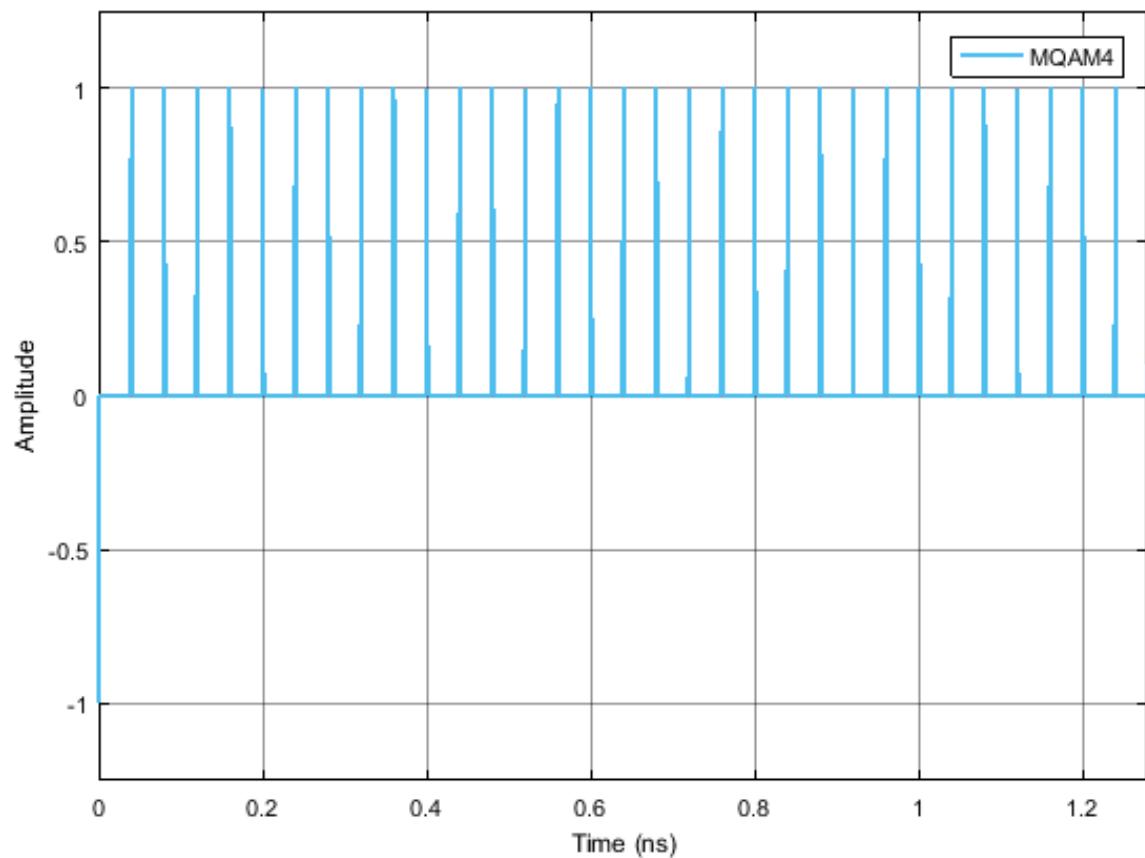


Figura 6.6: Example of the type of signal generated by this block for a binary sequence 0100...

## 6.8 Homodyne receiver

This block of code simulates the reception and demodulation of an optical signal (which is the input signal of the system) outputting a binary signal. A simplified schematic representation of this block is shown in figure 6.7.

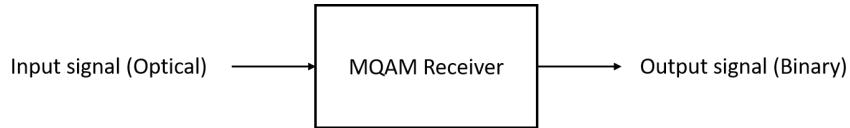


Figura 6.7: Basic configuration of the MQAM receiver

### Functional description

This block accepts one optical input signal and outputs one binary signal that corresponds to the M-QAM demodulation of the input signal. It is a complex block (as it can be seen from figure 6.8) of code made up of several simpler blocks whose description can be found in the *lib* repository.

In can also be seen from figure 6.8 that there's an extra internal (generated inside the homodyne receiver block) input signal generated by the *Clock*. This block is used to provide the sampling frequency to the *Sampler*.

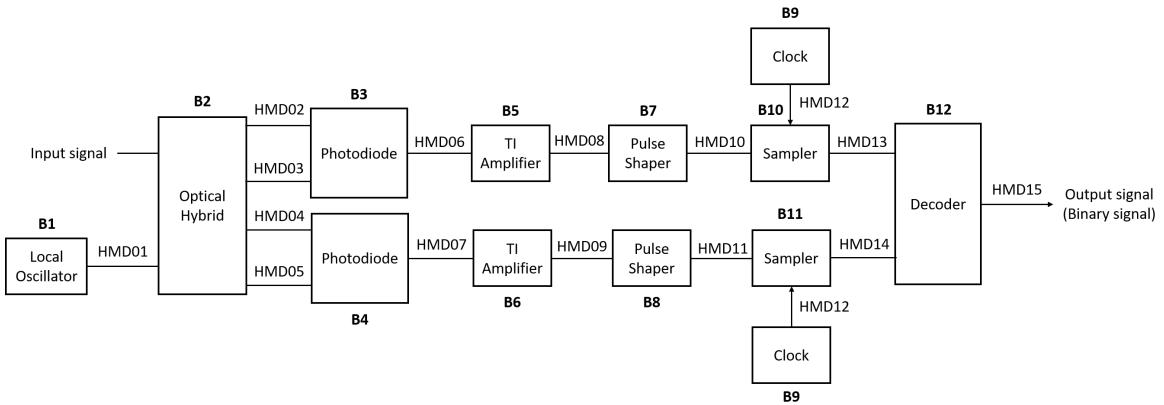


Figura 6.8: Schematic representation of the block homodyne receiver.

### Input parameters

This block has some input parameters that can be manipulated by the user in order to change the basic configuration of the receiver. Each parameter has associated a function that allows for its change. In the following table (table 6.2) the input parameters and corresponding functions are summarized.

Input parameters	Function	Type	Accepted values
IQ amplitudes	setIqAmplitudes	Vector of coordinate points in the I-Q plane	Example for a 4-qam mapping: { { 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 } }
Local oscillator power (in dBm)	setLocalOscillatorOpticalPower_dBm	double(t_real)	Any double greater than zero
Local oscillator phase	setLocalOscillatorPhase	double(t_real)	Any double greater than zero
Responsivity of the photodiodes	setResponsivity	double(t_real)	$\in [0,1]$
Amplification (of the TI amplifier)	setAmplification	double(t_real)	Positive real number
Noise amplitude (introduced by the TI amplifier)	setNoiseAmplitude	double(t_real)	Real number greater than zero
Samples to skip	setSamplesToSkip	int(t_integer)	
Save internal signals	setSaveInternalSignals	bool	True or False
Sampling period	setSamplingPeriod	double	Givem by symbolPeriod / samplesPerSymbol

Tabela 6.1: List of input parameters of the block MQAM receiver

## Methods

HomodyneReceiver(vector<Signal \*> &inputSignal, vector<Signal \*> &outputSignal)  
**(constructor)**

```

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues)
vector<t_iqValues> const getIqAmplitudes(void)
void setLocalOscillatorSamplingPeriod(double sPeriod)
void setLocalOscillatorOpticalPower(double opticalPower)
void setLocalOscillatorOpticalPower_dBm(double opticalPower_dBm)
void setLocalOscillatorPhase(double lOscillatorPhase)
void setLocalOscillatorOpticalWavelength(double lOscillatorWavelength)
void setSamplingPeriod(double sPeriod)

```

```
void setResponsivity(t_real Responsivity)  
void setAmplification(t_real Amplification)  
void setNoiseAmplitude(t_real NoiseAmplitude)  
void setImpulseResponseTimeLength(int impResponseTimeLength)  
void setFilterType(PulseShaperFilter fType)  
void setRollOffFactor(double rOffFactor)  
void setClockPeriod(double per)  
void setSamplesToSkip(int sToSkip)
```

**Input Signals**

**Number:** 1

**Type:** Optical signal

**Output Signals**

**Number:** 1

**Type:** Binary signal

**Example**

**Sugestions for future improvement**

## 6.9 IQ modulator

This blocks accepts one input signal continuous in both time and amplitude and it can produce either one or two output signals. It generates an optical signal and it can also generate a binary signal.

### Input Parameters

**Parameter:** outputOpticalPower{1e-3}  
(double)

**Parameter:** outputOpticalWavelength{1550e-9}  
(double)

**Parameter:** outputOpticalFrequency{speed\_of\_light/outputOpticalWavelength}  
(double)

### Methods

```
IqModulator(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig,
OutputSig){};
```

```
void initialize(void);
```

```
bool runBlock(void);
```

```
void setOutputOpticalPower(double outOpticalPower)
```

```
void setOutputOpticalPower_dBm(double outOpticalPower_dBm)
```

```
void setOutputOpticalWavelength(double outOpticalWavelength)
```

```
void setOutputOpticalFrequency(double outOpticalFrequency)
```

### Functional Description

This block takes the two parts of the signal: in phase and in amplitude and it combines them to produce a complex signal that contains information about the amplitude and the phase.

This complex signal is multiplied by  $\frac{1}{2}\sqrt{outputOpticalPower}$  in order to reintroduce the information about the energy (or power) of the signal. This signal corresponds to an optical signal and it can be a scalar or have two polarizations along perpendicular axis. It is the signal that is transmitted to the receptor.

The binary signal is sent to the Bit Error Rate (BER) measurement block.

### Input Signals

**Number :** 2

**Type :** Sequence of impulses modulated by the filter  
 (ContinuousTimeContiousAmplitude))

### Output Signals

**Number :** 1 or 2

**Type :** Complex signal (optical) (ContinuousTimeContinuousAmplitude) and binary signal (DiscreteTimeDiscreteAmplitude)

### Example

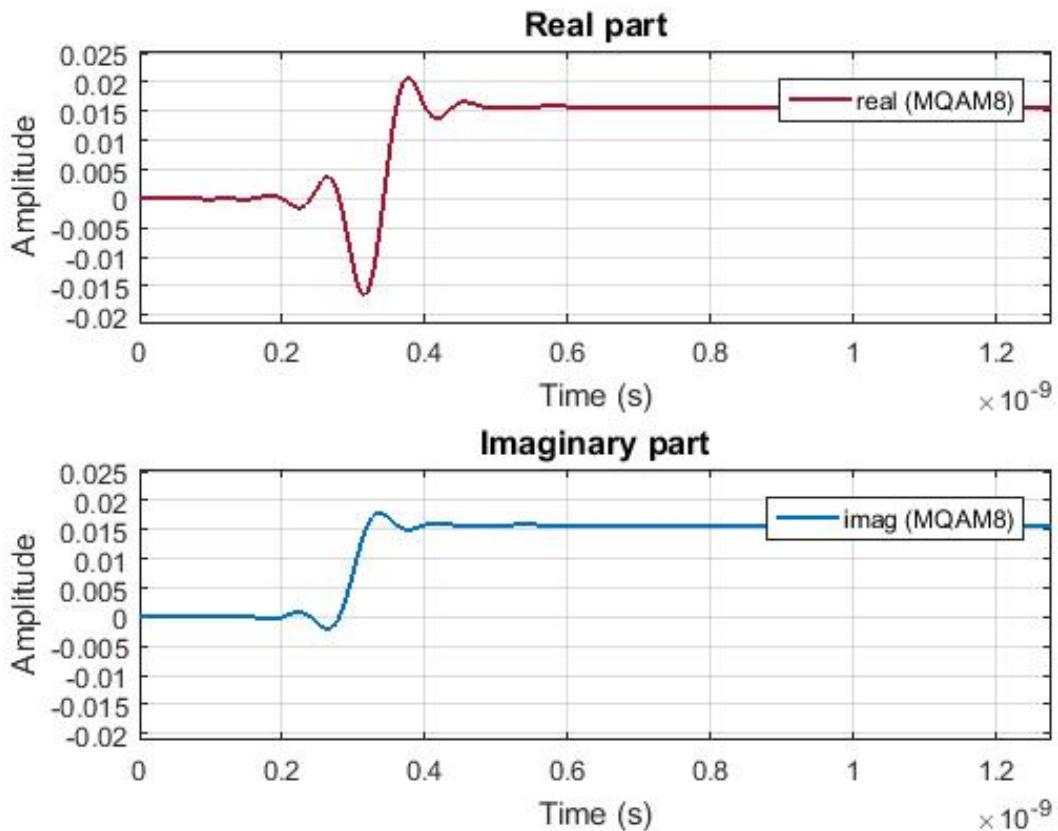


Figura 6.9: Example of a signal generated by this block for the initial binary signal 0100...

## 6.10 Local Oscillator

This block simulates a local oscillator with constant power and initial phase. It produces one output complex signal and it doesn't accept input signals.

### Input Parameters

**Parameter:** opticalPower{ 1e-3 }

**Parameter:** wavelength{ 1550e-9 }

**Parameter:** frequency{ SPEED\_OF\_LIGHT / wavelength }

**Parameter:** phase{ 0 }

**Parameter:** samplingPeriod{ 0.0 }

### Methods

LocalOscillator()

```
LocalOscillator(vector<Signal * > &InputSig, vector<Signal * > &OutputSig)
:Block(InputSig, OutputSig){};
```

```
void initialize(void);
```

```
bool runBlock(void);
```

```
void setSamplingPeriod(double sPeriod);
```

```
void setOpticalPower(double oPower);
```

```
void setOpticalPower_dBm(double oPower_dBm);
```

```
void setWavelength(double wlength);
```

```
void setPhase(double lOscillatorPhase);
```

### Functional description

This block generates a complex signal with a specified phase given by the input parameter *phase*.

**Input Signals**

**Number:** 0

**Output Signals**

**Number:** 1

**Type:** Optical signal

**Examples**

**Sugestions for future improvement**

## 6.11 MQAM mapper

This block does the mapping of the binary signal using a  $m$ -QAM modulation. It accepts one input signal of the binary type and it produces two output signals which are a sequence of 1's and -1's.

### Input Parameters

**Parameter:** m{4}  
 (m should be of the form  $2^n$  with n integer)

**Parameter:** iqAmplitudes{{ 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 }}

### Methods

```
MQamMapper(vector<Signal * > &InputSig, vector<Signal * > &OutputSig)
:Block(InputSig, OutputSig) {};

void initialize(void);

bool runBlock(void);

void setM(int mValue);

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues);
```

### Functional Description

In the case of  $m=4$  this block attributes to each pair of bits a point in the I-Q space. The constellation used is defined by the *iqAmplitudes* vector. The constellation used in this case is illustrated in figure 6.10.

### Input Signals

**Number :** 1

**Type :** Binary (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number :** 2

**Type :** Sequence of 1's and -1's (DiscreteTimeDiscreteAmplitude)

### Example

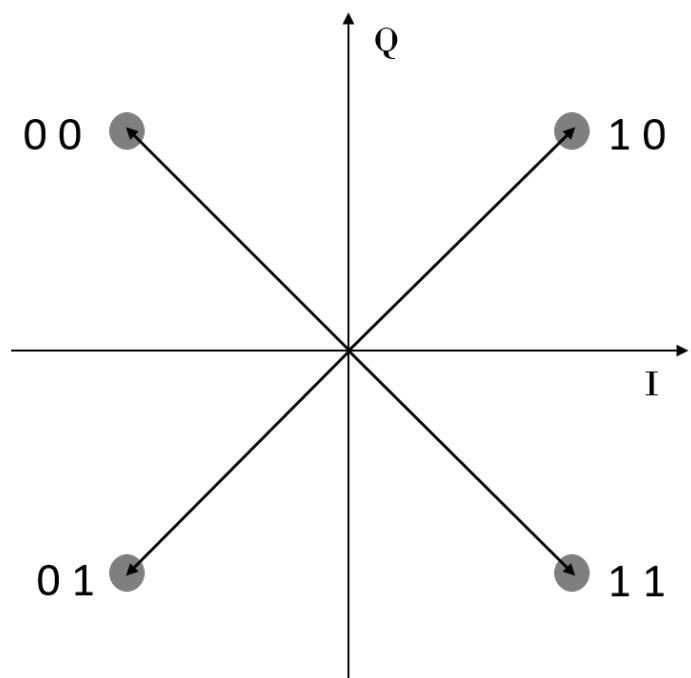


Figura 6.10: Constellation used to map the signal for  $m=4$

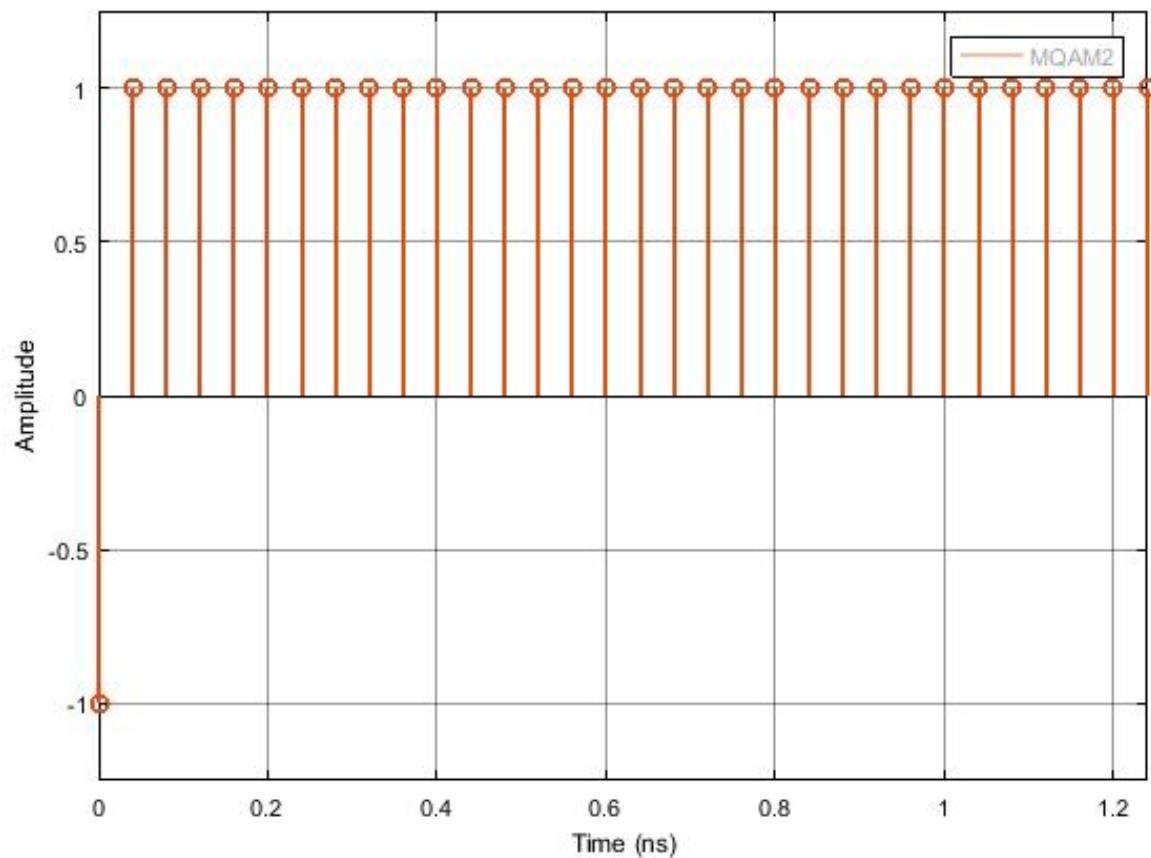


Figura 6.11: Example of the type of signal generated by this block for the initial binary signal 0100...

## 6.12 MQAM transmitter

This block generates a MQAM optical signal. It can also output the binary sequence. A schematic representation of this block is shown in figure 6.12.

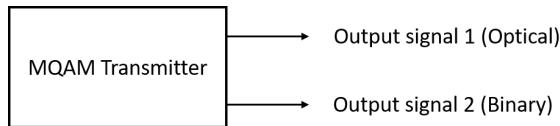


Figura 6.12: Basic configuration of the MQAM transmitter

### Functional description

This block generates an optical signal (output signal 1 in figure 6.13). The binary signal generated in the internal block Binary Source (block B1 in figure 6.13) can be used to perform a Bit Error Rate (BER) measurement and in that sense it works as an extra output signal (output signal 2 in figure 6.13).

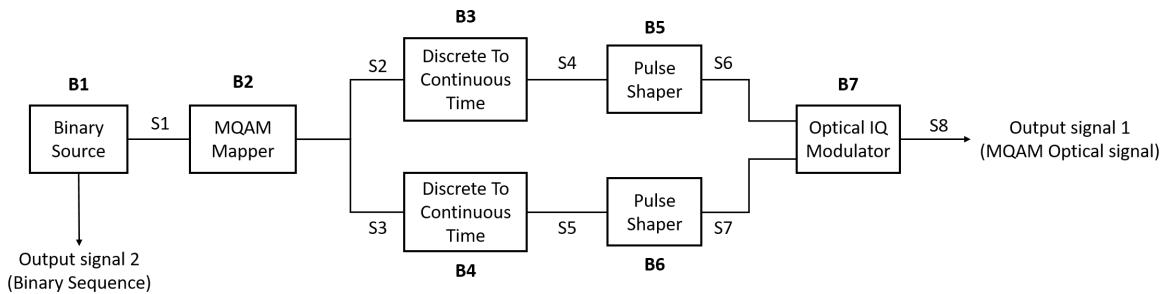


Figura 6.13: Schematic representation of the block MQAM transmitter.

### Input parameters

This block has a special set of functions that allow the user to change the basic configuration of the transmitter. The list of input parameters, functions used to change them and the values that each one can take are summarized in table 6.2.

Input parameters	Function	Type	Accepted values
Mode	setMode()	string	PseudoRandom Random DeterministicAppendZeros DeterministicCyclic
Number of bits generated	setNumberOfBits()	int	Any integer
Pattern length	setPatternLength()	int	Real number greater than zero
Number of bits	setNumberOfBits()	long	Integer number greater than zero
Number of samples per symbol	setNumberOfSamplesPerSymbol()	int	Integer number of the type $2^n$ with n also integer
Roll off factor	setRollOffFactor()	double	$\in [0,1]$
IQ amplitudes	setIqAmplitudes()	Vector of coordinate points in the I-Q plane	Example for a 4-qam mapping: { { 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 } }
Output optical power	setOutputOpticalPower()	int	Real number greater than zero
Save internal signals	setSaveInternalSignals()	bool	True or False

Tabela 6.2: List of input parameters of the block MQAM transmitter

## Methods

MQamTransmitter(vector<Signal \*> &inputSignal, vector<Signal \*> &outputSignal);  
**(constructor)**

```

void set(int opt);

void setMode(BinarySourceMode m)

BinarySourceMode const getMode(void)

void setProbabilityOfZero(double pZero)

double const getProbabilityOfZero(void)

void setBitStream(string bStream)

string const getBitStream(void)

```

```
void setNumberOfBits(long int nOfBits)

long int const getNumberOfBits(void)

void setPatternLength(int pLength)

int const getPatternLength(void)

void setBitPeriod(double bPeriod)

double const getBitPeriod(void)

void setM(int mValue) int const getM(void)

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues)

vector<t_iqValues> const getIqAmplitudes(void)

void setNumberOfSamplesPerSymbol(int n)

int const getNumberOfSamplesPerSymbol(void)

void setRollOffFactor(double rOffFactor)

double const getRollOffFactor(void)

void setSeeBeginningOfImpulseResponse(bool sBeginningOfImpulseResponse)

double const getSeeBeginningOfImpulseResponse(void)

void setOutputOpticalPower(t_real outOpticalPower)

t_real const getOutputOpticalPower(void)

void setOutputOpticalPower_dBm(t_real outOpticalPower_dBm)

t_real const getOutputOpticalPower_dBm(void)
```

## Output Signals

**Number:** 1 optical and 1 binary (optional)

**Type:** Optical signal

## Example

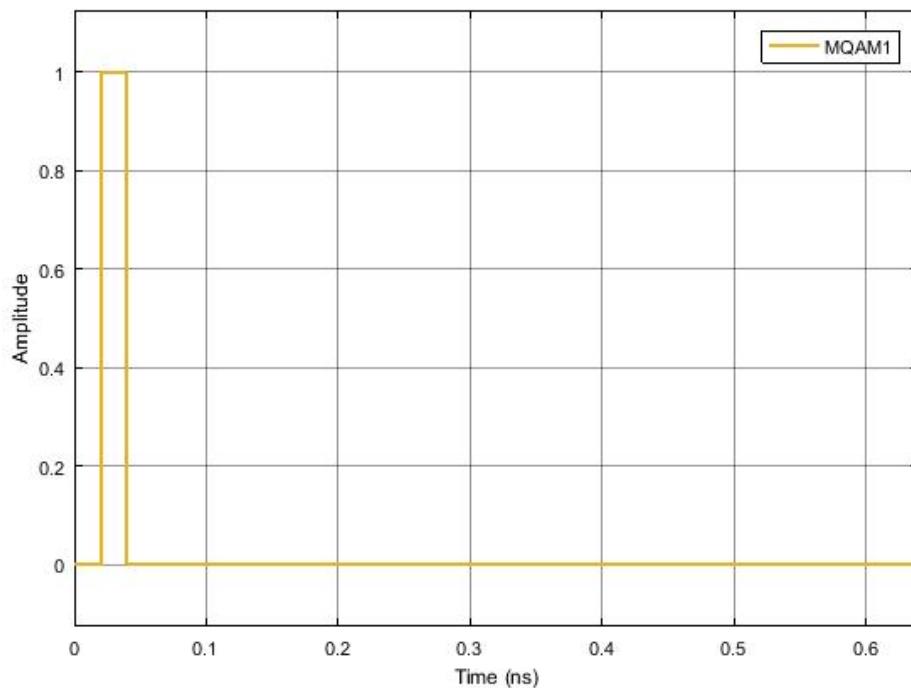


Figura 6.14: Example of the binary sequence generated by this block for a sequence 0100...

## Sugestions for future improvement

Add to the system another block similar to this one in order to generate two optical signals with perpendicular polarizations. This would allow to combine the two optical signals and generate an optical signal with any type of polarization.

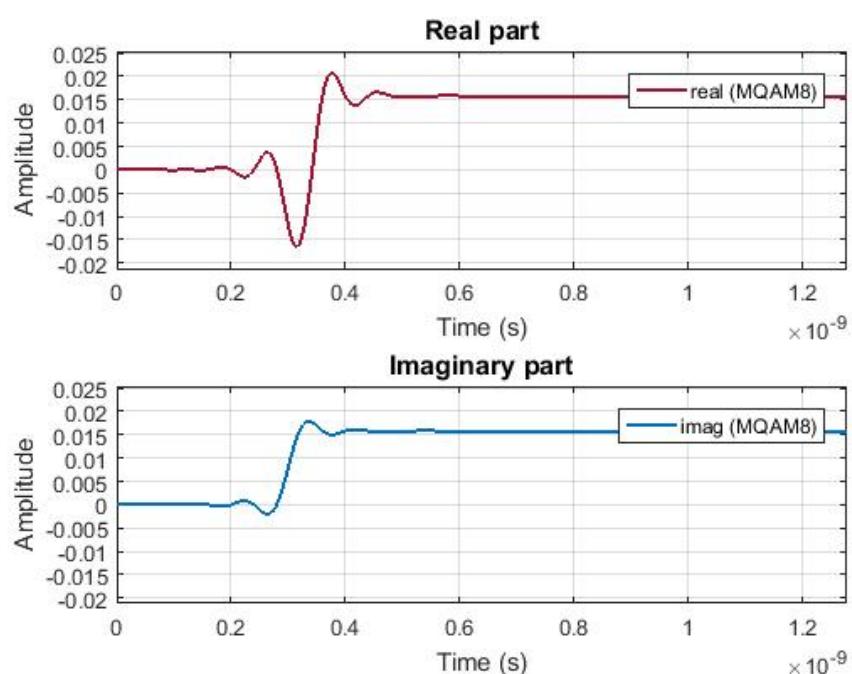


Figura 6.15: Example of the output optical signal generated by this block for a sequence 0100...

## **Capítulo 7**

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## **Mathlab Tools**

## 7.1 sgnToWfm

### Functional Description

This matlab function will build a WFM file given an input signal filename. The waveform file is compatible with the laboratory's AWG (Tektronix AWG70002A) and can be used to generate a real version of the chosen signal. However, in order to recreate it appropriately, the signal must be real, not exceed 8 G data points and have a sampling rate equal or bellow 16 GS/s.

```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] =  
sgnToWfm(fname_sgn, fname_wfm, nReadr);
```

### Example

#### Using one argument:

```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] =  
sgnToWfm('S6.sgn');
```

This creates a waveform file with the same name as the signal file name and uses all of the symbols it contains.

#### Using three arguments:

```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] =  
sgnToWfm('S6.sgn','myWaveform.wfm', 256);
```

This creates a waveform file with the name "myWaveform" and only extracts 256 symbols of the original signal.

### Inputs

**fname\_sgn:** Input filename of the signal you want to convert. It must be a real signal (Type: TimeContinuousAmplitudeContinuousReal).

**fname\_wfm:** Name that will be given to the waveform file.

**nReadr:** Number of symbols you want to extract from the signal.

### Outputs

A waveform file will be created in the Matlab current folder. This waveform files can be read, and later on, outputted by the Arbitrary Waveform Generator (AWG).

It will also return six variables in the workspace which are:

**data:** A vector with the signal data.

**symbolPeriod:** Equal to the symbol period of the corresponding signal.

**samplingPeriod:** Sampling period of the signal.

**type:** A string with the name of the signal type.

**numberOfSymbols:** Number of symbols retrieved from the signal.

**samplingRate:** Sampling rate of the signal.

The samplingRate variable should be noted as it will be important later on.

### Key performance specifications of the AWG

The AWG we will be using is the Tektronix AWG70002A which has the following key specifications:

**Sampling rate up to 16 GS/s:** This is the most important characteristic because it determines the maximum sampling rate that your signal can have. It must not be over 16 GS/s or else the AWG will not be able to recreate it appropriately.

**8 GSample waveform memory:** This determines how many data points your signal can have.

After making sure this specifications are respected you can create your waveform using the function. When you load your waveform, the AWG will output it and repeat it constantly until you stop playing it.

### Tutorial - Loading a signal to the AWG

**1. Using the function sgnToWfm:** Start up Matlab and change your current folder to mtools and add the signals folder that you want to convert to the Matlab search path. Use the function accordingly, putting as the input parameter the signal file name you want to convert.

**2. AWG sampling rate:** After calling the function there should be waveform file in the mtools folder, as well as a variable called samplingRate in the Matlab workspace. Make sure this is equal or bellow the maximum sampling frequency of the AWG (16 GS/s), or else the waveform can not be equal to the original signal. If it is higher you have to adjust the parameters in the simulation in order to decrease the sampling frequency of the signal(i.e. decreasing the bit period or reducing the samples per symbol).

**3. Loading the waveform file to the AWG:** Copy the waveform file to your pen drive and connect it to the AWG. With the software of the awg open,go to browse for waveform on the channel you want to use, and select the waveform file you created (Figure 7.1).

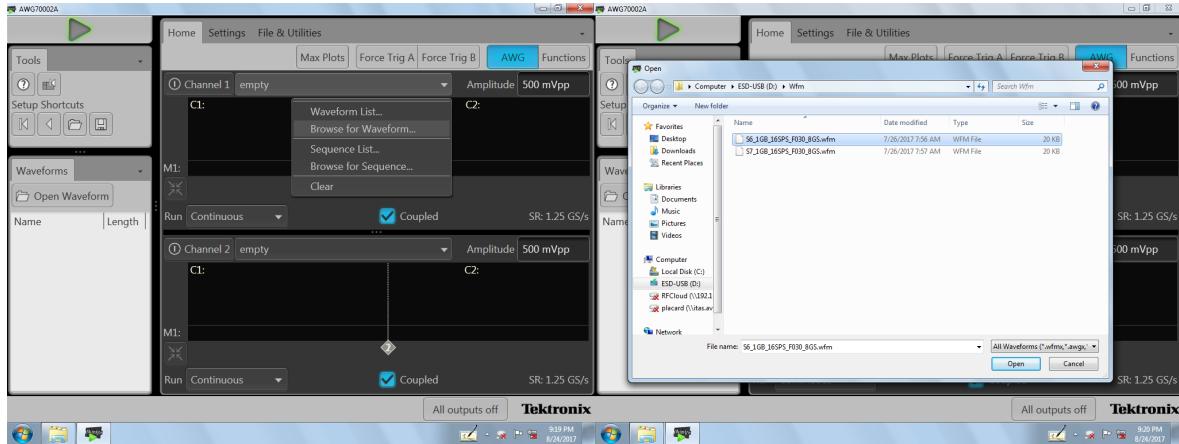


Figura 7.1: Selecting your waveform in the AWG

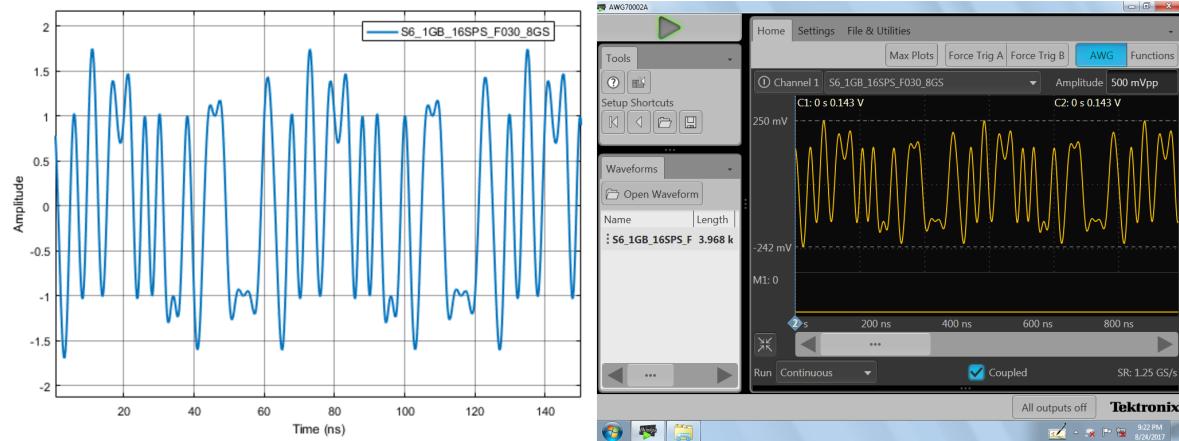


Figura 7.2: Comparison between the waveform in the AWG and the original signal before configuring the sampling rate

Now you should have the waveform displayed on the screen. Although it has the same shape, the waveform might not match the signal timing wise due to an incorrect sampling rate configured in the AWG. In this example (Figure 7.2), the original signal has a sample rate of 8 GS/s and the AWG is configured to 1.25 GS/s. Therefore it must be changed to the correct value. To do this go to the settings tab, clock settings, and change the sampling rate to be equal to the one of the original signal, 8 GS/s (Figure 7.3).

Compare the waveform in the AWG with the original signal, they should be identical (Figure 7.4).

**4. Generate the signal:** Output the wave by enabling the channel you want and clicking on the play button.

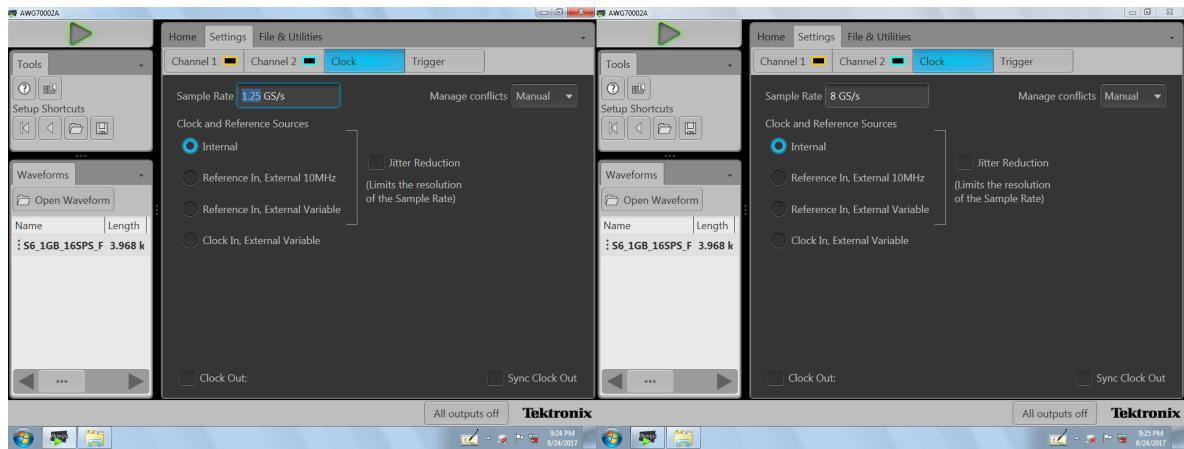


Figura 7.3: Configuring the right sampling rate

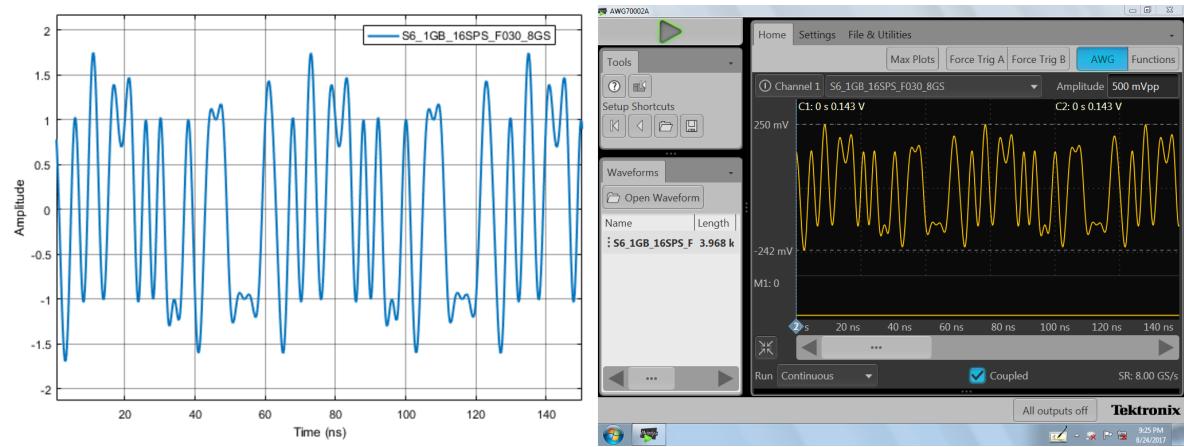


Figura 7.4: Comparison between the waveform in the AWG and the original signal after configuring the sampling rate

