

# NetXPTO - LinkPlanner

28 de Novembro de 2017

---

# Conteúdo

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>Simulator Structure</b>	<b>4</b>
2.1	System . . . . .	4
2.2	Blocks . . . . .	4
2.3	Signals . . . . .	4
<b>3</b>	<b>Development Cycle</b>	<b>5</b>
<b>4</b>	<b>Visualizer</b>	<b>6</b>
<b>5</b>	<b>Case Studies</b>	<b>7</b>
5.1	QPSK Transmitter . . . . .	7
5.2	BPSK Transmission System . . . . .	9
5.2.1	Theoretical Analysis . . . . .	9
5.2.2	Simulation Analysis . . . . .	10
5.2.3	Comparative Analysis . . . . .	14
5.3	M-QAM Transmission System . . . . .	17
5.3.1	Introduction . . . . .	17
5.3.2	Bit Error Rate for 4-QAM with Additive White Gaussian Noise (AWGN) . . . . .	17
5.3.3	Simulation setup . . . . .	19
5.3.4	Functional description . . . . .	19
5.3.5	Input Parameters . . . . .	20
5.3.6	Output Parameters . . . . .	20
5.3.7	BER measurement . . . . .	20
5.4	Quantum Noise . . . . .	22
5.4.1	Theoretical Description . . . . .	22
5.4.2	Simulation . . . . .	26
5.5	Continuos Variable Quantum Transmission System . . . . .	29
5.6	Kramers-Kronig Transceiver with Stokes PolDemux . . . . .	42

<b>Conteúdo</b>	<b>2</b>
5.6.1 Theoretical Analysis . . . . .	42
5.6.2 Simulation Analysis . . . . .	48
5.6.3 Experimental Analysis . . . . .	52
5.6.4 Comparative Analysis . . . . .	53
5.6.5 Know Problems . . . . .	53
5.7 Quantum Oblivious Key Distribution with Discrete Variables . . . . .	59
5.7.1 Quantum Oblivious Key Distribution System (QOKD) . . . . .	59
5.7.2 OT Protocol with QOKD system . . . . .	66
5.7.3 Simulation . . . . .	69
5.7.4 Experimental . . . . .	73
5.8 Radio Over Fiber Transmission System . . . . .	76
5.8.1 Simulation . . . . .	77
5.8.2 Experimental . . . . .	78
<b>6 Library</b>	<b>80</b>
6.1 Add . . . . .	81
6.2 Bit Error Rate . . . . .	82
6.3 Binary source . . . . .	85
6.4 Bit Decider . . . . .	89
6.5 Clock . . . . .	90
6.6 Coupler 2 by 2 . . . . .	92
6.7 Decoder . . . . .	93
6.8 Discrete to continuous time . . . . .	96
6.9 Homodyne receiver . . . . .	98
6.10 IQ modulator . . . . .	102
6.11 Local Oscillator . . . . .	104
6.12 MQAM mapper . . . . .	106
6.13 MQAM transmitter . . . . .	109
6.14 Fork . . . . .	114
<b>7 Matlab Tools</b>	<b>115</b>
7.1 Generation of AWG Compatible Signals . . . . .	116
7.1.1 sgnToWfm . . . . .	116
7.1.2 Loading a signal to the Tektronix AWG70002A . . . . .	117
7.2 Generation of AWG Compatible Signals . . . . .	120
7.2.1 sgnToWfm_20171121 . . . . .	120
<b>8 Algorithms</b>	<b>123</b>
8.1 Overlap-Save Method . . . . .	124
8.1.1 Frequency Response of Filter . . . . .	125
8.2 FFT . . . . .	129
8.3 IFFT . . . . .	134

## **Capítulo 1**

---

### **Introduction**

LinkPlanner is devoted to the simulation of point-to-point links.

## **Capítulo 2**

---

### **Simulator Structure**

LinkPlanner is a signals open-source simulator.

The major entity is the system.

A system comprises a set of blocks.

The blocks interact with each other through signals.

#### **2.1 System**

#### **2.2 Blocks**

#### **2.3 Signals**

List of available signals:

- Signal

## **Capítulo 3**

## **Development Cycle**

---

The NetXPTO-LinkPlanner has been developed by several people using git as a version control system. The NetXPTO-LinkPlanner repository is located in the GitHub site <http://github.com/netxpto/linkplanner>. The more updated functional version of the software is in the branch master. Master should be considered a functional beta version of the software. Periodically new releases are delivered from the master branch under the branch name Release<Year><Month><Day>. The integration of the work of all people is performed by Armando Nolasco Pinto in the branch Develop. Each developer has his/her own branch with his/her name.

## **Capítulo 4**

---

## **Visualizer**

visualizer

## Capítulo 5

## Case Studies

### 5.1 QPSK Transmitter

---

2017-08-25, Review, Armando Nolasco Pinto

---

This system simulates a QPSK transmitter. A schematic representation of this system is shown in figure 5.1.



Figura 5.1: QPSK transmitter block diagram.

#### System Input Parameters

**Parameter:** *sourceMode*

**Description:** Specifies the operation mode of the binary source.

**Accepted Values:** PseudoRandom, Random, DeterministicAppendZeros, DeterministicCyclic.

**Parameter:** *patternLength*

**Description:** Specifies the pattern length used by the source in the PseudoRandom mode.

**Accepted Values:** Integer between 1 and 32.

**Parameter:** *bitStream*

**Description:** Specifies the bit stream generated by the source in the DeterministicCyclic and DeterministicAppendZeros mode.

**Accepted Values:** "XXX..", where X is 0 or 1.

**Parameter:** *bitPeriod*

**Description:** Specifies the bit period, i.e. the inverse of the bit-rate.

**Accepted Values:** Any positive real value.

**Parameter:** *iqAmplitudes*

**Description:** Specifies the IQ amplitudes.

**Accepted Values:** Any four pair of real values, for instance { { 1,1 },{ -1,1 },{ -1,-1 },{ 1,-1 } }, the first value correspond to the "00", the second to the "01", the third to the "10" and the forth to the "11".

**Parameter:** *numberOfBits*

**Description:** Specifies the number of bits generated by the binary source.

**Accepted Values:** Any positive integer value.

**Parameter:** *numberOfSamplesPerSymbol*

**Description:** Specifies the number of samples per symbol.

**Accepted Values:** Any positive integer value.

**Parameter:** *rollOffFactor*

**Description:** Specifies the roll off factor in the raised-cosine filter.

**Accepted Values:** A real value between 0 and 1.

**Parameter:** *impulseResponseTimeLength*

**Description:** Specifies the impulse response window time width in symbol periods.

**Accepted Values:** Any positive integer value.

>>> Romil

## 5.2 BPSK Transmission System

<b>Student Name</b>	:	Daniel Pereira (2017/09/01 - )
<b>Goal</b>	:	Estimate the BER in a Binary Phase Shift Keying optical transmission system with additive white Gaussian noise. Comparison with theoretical results.
<b>Directory</b>	:	sdf/bpsk_system

Binary Phase Shift Keying (BPSK) is the simplest form of Phase Shift Keying (PSK), in which binary information is encoded into a two state constellation with the states being separated by a phase shift of  $\pi$  (see Figure 5.2).

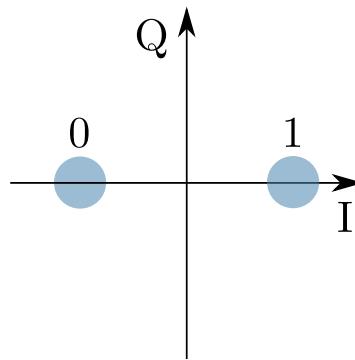


Figura 5.2: BPSK symbol constellation.

White noise is a random signal with equal intensity at all frequencies, having a constant power spectral density. White noise is said to be Gaussian (WGN) if its samples follow a normal distribution with zero mean and a certain variance  $\sigma^2$ . For WGN its spectral density equals its variance. For the purpose of this work, additive WGN is used to model thermal noise at the receivers.

The purpose of this system is to simulate BPSK transmission in back-to-back configuration with additive WGN at the receiver and to perform an accurate estimation of the BER and validate the estimation using theoretical values.

### 5.2.1 Theoretical Analysis

The output of the system with added gaussian noise follows a normal distribution, whose first probabilistic moment can be readily obtained by knowledge of the optical power of the received signal and local oscillator,

$$m_i = 2\sqrt{P_L P_S G_{ele}} \cos(\Delta\theta_i), \quad (5.1)$$

where  $P_L$  and  $P_S$  are the optical powers, in watts, of the local oscillator and signal, respectively,  $G_{ele}$  is the gain of the trans-impedance amplifier in the coherent receiver and

$\Delta\theta_i$  is the phase difference between the local oscillator and the signal, for BPSK this takes the values  $\pi$  and  $0$ , in which case (5.1) can be reduced to,

$$m_i = (-1)^{i+1} 2 \sqrt{P_L P_S} G_{ele}, \quad i = 0, 1. \quad (5.2)$$

The second moment is directly chosen by inputting the spectral density of the noise  $\sigma^2$ , and thus is known *a priori*.

Both probabilist moments being known, the probability distribution of measurement results is given by a simple normal distribution,

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m_i)^2}{2\sigma^2}}. \quad (5.3)$$

The BER is calculated in the following manner,

$$BER = \frac{1}{2} \int_0^{+\infty} f(x|\Delta\theta = \pi) dx + \frac{1}{2} \int_{-\infty}^0 f(x|\Delta\theta = 0) dx, \quad (5.4)$$

given the symmetry of the system, this can be simplified to,

$$BER = \int_0^{+\infty} f(x|\Delta\theta = \pi) dx = \frac{1}{2} \operatorname{erfc} \left( \frac{-m_0}{\sqrt{2}\sigma} \right) \quad (5.5)$$

### 5.2.2 Simulation Analysis

A diagram of the system being simulated is presented in the Figure 5.3. A random binary sequence is generated and encoded in an optical signal using BPSK modulation. The decoding of the optical signal is accomplished by an homodyne receiver, which combines the signal with a local oscillator. The received binary signal is compared with the transmitted binary signal in order to estimate the Bit Error Rate (BER). The simulation is repeated for multiple signal power levels, each corresponding BER is recorded and plotted against the expectation value.



Figura 5.3: Overview of the BPSK system being simulated.

### Required files

Header Files

<b>File</b>	<b>Description</b>
add.h	Adds the two input signals and outputs the result
balanced_beam_splitter.h	Mixes the two optical signals set at it's input.
binary_source.h	Generates a sequence of binary values (1 or 0)
bit_decider.h	Decodes the input signal into a binary string.
bit_error_rate.h	Calculates the bit error rate of the decoded string.
discrete_to_continuous_time.h	Converts a signal discrete in time to a signal continuous in time.
netxpto.h	Generic purpose simulator definitions.
m_qam_mapper.h	Maps the binary sequence into the coded constellation.
m_qam_transmitter.h	Generates the signal with coded constellation.
local_oscillator.h	Generates a continuous optical signal with set power and phase.
i_homodyne_reciever.h	Performs coherent detection on the input signal.
ideal_amplifier.h	Multiplies the input signal by a user defined gain factor and outputs the result.
iq_modulator.h	Maps two real valued signal into a complex optical bandpass signal.
photodiode.h	Converts complex optical bandpass signals into real value electrical signals.
pulse_shaper.h	Simulates the impulse response of a circuit.
sampler.h	Samples the input signal at a user defined frequency.
sink.h	Closes any unused signals.
super_block_interface.h	Allows superblocks to output multiple signals.
white_noise.h	Generates white gaussian noise with a user defined spectral density.

Source Files

<b>File</b>	<b>Description</b>
add.cpp	Adds the two input signals and outputs the result
balanced_beam_splitter.cpp	Mixes the two optical signals set at its input.
binary_source.cpp	Generates a sequence of binary values (1 or 0)
bit_decider.cpp	Decodes the input signal into a binary string.
bit_error_rate.cpp	Calculates the bit error rate of the decoded string.
discrete_to_continuous_time.cpp	Converts a signal discrete in time to a signal continuous in time.
netxpto.cpp	Generic purpose simulator definitions.
m_qam_mapper.cpp	Maps the binary sequence into the coded constellation.
m_qam_transmitter.cpp	Generates the signal with coded constellation.
local_oscillator.cpp	Generates a continuous optical signal with set power and phase.
i_homodyne_reciever.cpp	Performs coherent detection on the input signal.
ideal_amplifier.cpp	Multiplies the input signal by a user defined gain factor and outputs the result.
iq_modulator.cpp	Maps two real valued signal into a complex optical bandpass signal.
photodiode.cpp	Converts complex optical bandpass signals into real value electrical signals.
pulse_shaper.cpp	Simulates the impulse response of a circuit.
sampler.cpp	Samples the input signal at a user defined frequency.
sink.cpp	Closes any unused signals.
super_block_interface.cpp	Allows superblocks to output multiple signals.
white_noise.cpp	Generates white gaussian noise with a user defined spectral density.

### System Input Parameters

This system takes into account the following input parameters:

System Parameters	Description	Simulation Value
numberOfBitsGenerated	Gives the number of bits to be simulated	40000
bitPeriod	Sets the time between adjacent bits	$20 \times 10^{-12}$
samplesPerSymbol	Establishes the number of samples each bit in the string is given	16
pLength	PRBS pattern length	5
iqAmplitudesValues	Sets the state constellation	{ {−1, 0} , {1, 0} }
outOpticalPower_dBm	Sets the optical power, in units of dBm, at the transmitter output	Variable
loOutOpticalPower_dBm	Sets the optical power, in units of dBm, of the local oscillator used in the homodyne detector	0
localOscillatorPhase	Sets the initial phase of the local oscillator used in the homodyne detector	0
transferMatrix	Sets the transfer matrix of the beam splitter used in the homodyne detector	{ { $\frac{1}{\sqrt{2}}$ , $\frac{1}{\sqrt{2}}$ , $\frac{1}{\sqrt{2}}$ , $\frac{-1}{\sqrt{2}}$ } }
responsivity	Sets the responsivity of the photodiodes used in the homodyne detector	1
amplification	Sets the amplification of the trans-impedance amplifier used in the homodyne detector	$10^3$
noiseSpectralDensity	Sets the spectral density of the gaussian thermal noise added in the homodyne detector	$5 \times 10^{-4}\sqrt{2} \text{ V}^2$
confidence	Sets the confidence interval for the calculated QBER	0.95
midReportSize	Sets the number of bits between generated QBER mid-reports	0

## Inputs

This system takes no inputs.

## Outputs

This system outputs the following objects:

**Parameter:** Signals:

**Description:** Initial Binary String; ( $S_0$ )

**Description:** Optical Signal with coded Binary String; ( $S_1$ )

**Description:** Local Oscillator Optical Signal; ( $S_2$ )

**Description:** Beam Splitter Outputs; ( $S_3, S_4$ )

**Description:** Homodyne Detector Electrical Output; ( $S_5$ )

**Description:** Decoded Binary String; ( $S_6$ )

**Description:** BER result String; ( $S_7$ )

**Parameter:** Other:

**Description:** Bit Error Rate report in the form of a .txt file. (BER.txt)

### 5.2.3 Comparative Analysis

The following results show the dependence of the error rate with the signal power assuming a constant Local Oscillator power of 0 dBm, the signal power was evaluated at levels between -70 and -25 dBm, in steps of 5 dBm between each. The simulation results are presented in orange with the computed lower and upper bounds, while the expected value, obtained from (5.5), is presented as a full blue line. A close agreement is observed between the simulation results and the expected value. The noise spectral density was set at  $5 \times 10^{-4}\sqrt{2} \text{ V}^2$  [1].



Figura 5.4: Bit Error Rate in function of the signal power in dBm at a constant local oscillator power level of 0 dBm. Theoretical values are presented as a full blue line while the simulated results are presented as a errorbar plot in orange, with the upper and lower bound computed in accordance with the method described in 6.2

---

## Bibliografia

- [1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.
- [2] Ryo Namiki and Takuya Hirano. Security of quantum cryptography using balanced homodyne detection. *Physical Review A*, 67(2):022308, 2003.
- [3] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.
- [4] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.
- [5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

### 5.3 M-QAM Transmission System

<b>Student Name</b>	:	Ana Luisa Carvalho
<b>Goal</b>	:	M-QAM system implementation with BER measurement and comparison with theoretical values.

#### 5.3.1 Introduction

M-QAM, which stands for Quadrature Amplitude Modulation, is a modulation scheme that takes advantage of two carriers (usually sinusoidal waves) with a phase difference of  $\frac{\pi}{2}$ . The resultant output consists of a signal with both amplitude and phase variations. The two carriers, referred to as I (In-phase) and Q (Quadrature), can be represented as

$$I = A \cos(\phi) \quad (5.6)$$

$$Q = A \sin(\phi) \quad (5.7)$$

which means that any sinusoidal wave can be decomposed in its I and Q components:

$$A \cos(\omega t + \phi) = A (\cos(\omega t) \cos(\phi) - \sin(\omega t) \sin(\phi)) \quad (5.8)$$

$$= I \cos(\omega t) - Q \sin(\omega t), \quad (5.9)$$

where we have used the expression for the cosine of a sum and the definitions of I and Q.

For M= 4 the symbol constellation is shown figure ??.

M can take several values: 2, 4, 16, 32, .... The first two correspond to BPSK and QPSK modulation, respectively.

#### 5.3.2 Bit Error Rate for 4-QAM with Additive White Gaussian Noise (AWGN)

When demodulating a signal it is necessary to associate the received signal to the corresponding signal. The existence of noise in the channel means that we can only compute the probability that a given signal corresponds to a certain carrier and that's why we need to define the Bit Error Rate (BER). Using

$$P_i f(s|c_i) > P_j f(s|c_j), \quad i \neq j \quad (5.10)$$

where  $f(s|c_i)$  stands for the probability of detecting the signal  $s$  given that  $c_i$  was emitted. This inequality can be rewritten in the following way

$$P(c_i|s) > P(c_j|s) \quad (5.11)$$

where  $P(c_i|s)$  and  $P(c_j|s)$  are called *a posteriori* probabilities and represent the probability that  $c_i$  or  $c_j$  were transmitted given that  $s$  was received. In terms of the systems this simply means that we should select the signal most likely to have been transmitted.

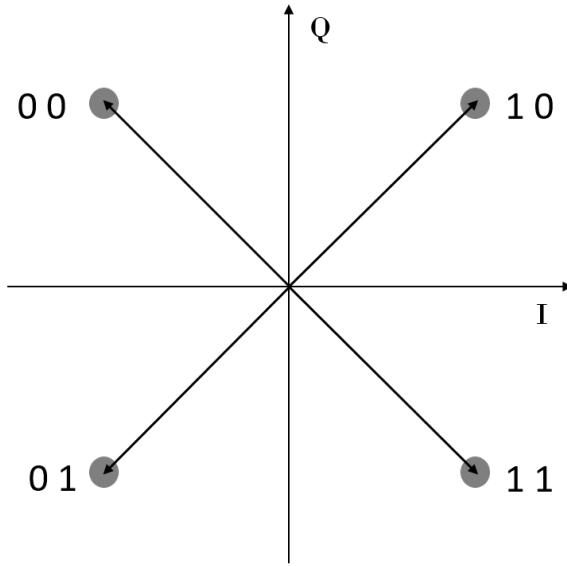


Figura 5.5: 4-QAM symbol constellation

In the case of additive white gaussian noise the  $f$  function is simply given by

$$f(s|c_i) = \frac{e^{-x^2/n_0}}{(\pi n_0)^{N/2}} \quad (5.12)$$

where  $x$  is the Euclidean distance in the I-Q plane between the signal received and carrier  $i$  and  $N$  is the number of noise samples.

When using 4-QAM modulation all points are at an equal distance from the origin (in the I-Q plane) so they all have the same energy given by

$$E = \frac{d^2}{2} \quad (5.13)$$

where  $d$  is the side of the square formed by the constellation points.

The probability that a given signal is identified correctly is given by

$$P_c = r^2 \quad (5.14)$$

where  $n_0/2$  is the noise variance for AWGN and

$$r = \int_{-d/2}^{\infty} \frac{e^{-x^2/n_0}}{\sqrt{\pi n_0}} dx. \quad (5.15)$$

The error probability,  $P_e$ , given by  $1 - P_c$  is given by

$$P_e = erfc \sqrt{\frac{E}{2n_0}}. \quad (5.16)$$

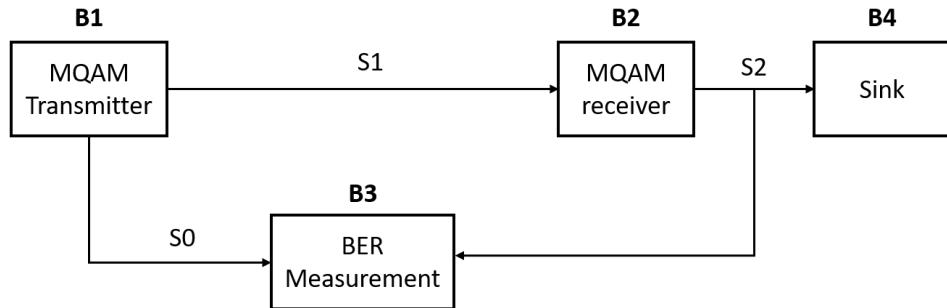


Figura 5.6: Schematic representation of the MQAM system.

### 5.3.3 Simulation setup

The M-QAM system transmission system is a complex block of code that simulates the modulation, transmission and demodulation of an optical signal using M-QAM modulation. It is composed of four blocks: a transmitter, a receiver, a sink and a block that performs a Bit Error Rate (BER) measurement.

**Current state:** The system currently being implemented is a QPSK system ( $M=4$ ).

**Future work:** Extend this block to include other values of  $M$ .

### 5.3.4 Functional description

The schematic representation of the system is presented in figure 5.6.

A complete description of the M-QAM transmitter and M-QAM receiver blocks can be found in the *Library* chapter of this document as well as a detailed description of the independent blocks that compose these blocks.

The M-QAM transmitter is a complex block that generates one or two optical signals by encoding a binary string using M-QAM modulation. It also outputs a binary signal that is used to perform a BER measurement.

The M-QAM receiver is a homodyne receiver. It is a complex block that accepts one input optical signal and outputs a binary signal. It performs the M-QAM demodulation of the input signal by combining the optical signal with a local oscillator.

The demodulated optical signal is compared to the one produced by the transmitter in order to estimate the Bit Error Rate (BER).

The files corresponding to each of the system's blocks are summarized in table ???. Along with the library and corresponding source files these allow for the full operation of the M-QAM system described here.

Tabela 5.1: Main system files

System blocks	cpp file	include file
Main	m_qam_system_sdf.cpp	—
M-QAM transmitter	m_qam_transmitter.cpp	m_qam_transmitter.h
M-QAM receiver	homodyne_receiver.cpp	homodyne_receiver.h
Sink	sink.cpp	sink.h
BER estimator	bit_error_rate.cpp	bit_error_rate.h

### 5.3.5 Input Parameters

### 5.3.6 Output Parameters

As output this block

### 5.3.7 BER measurement

Tabela 5.2: Input parameters

Parameter	Type	Description
numberOfBitsGenerated	t_integer	Determines the number of bits to be generated by the binary source
samplesPerSymbol	t_integer	
prbsPatternLength	int	Determines the length of the pseudorandom sequence pattern (used only when the binary source is operated in <i>PseudoRandom</i> mode)
bitPeriod	t_real	Temporal interval occupied by one bit
rollOffFactor	t_real	
signalOutputPower_dBm	t_real	Determines the power of the output optical signal in dBm
numberOfBitsReceived	int	
iqAmplitudeValues	vector<t_iqValues>	Determines the constellation used to encode the signal in IQ space
symbolPeriod	double	Given by bitPeriod / samplesPerSymbol
localOscillatorPower_dBm	t_real	Power of the local oscillator
responsivity	t_real	Responsivity of the photodiodes (1 corresponds to having all optical power transformed into electrical current)
amplification	t_real	??
noiseAmplitude	t_real	??
samplesToSkip	t_integer	Number of samples to be skipped by the <i>sampler</i> block
confidence	t_real	Determines the confidence limits for the BER estimation
midReportSize	t_integer	
bufferLength	t_integer	Corresponds to the number of samples that can be processed in each run of the system

## 5.4 Quantum Noise

<b>Student Name</b>	:	Diamantino Silva
<b>Starting Date</b>	:	October 19, 2017
<b>Goal</b>	:	Simulation of quantum noise in double homodyne detection.
<b>Directory</b>	:	sdf/quantum_noise

Quantum noise is an intrinsic property of light, a manifestation of the vacuum field fluctuations [?]. Contrarily to the majority of noise sources, that are overcomed by better equipment, quantum noise has a lower bound, given by the Heisenberg uncertainty principle, which cannot be broken. This propertie can be useful in some areas, such in quantum cryptography, were many protocols depend on it to ensure their security. The objective of this work is to develop a numerical model for the quantum noise in a double homodyne detection and to validate the numerical model with experimental results.

### 5.4.1 Theoretical Description

We start by defining number states  $|n\rangle$  (or Fock states), which correspond to states with perfectly fixed number of photons [?]. Associated to those states are two operators, the creation  $\hat{a}^\dagger$  and annihilation  $\hat{a}$  operators, which in a simple way, remove or add one photon from a given number state [?]. Their action is defined as

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \quad (5.17), \quad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \quad (5.18), \quad \hat{n}|n\rangle = n|n\rangle \quad (5.19)$$

in which  $\hat{n} = \hat{a}^\dagger\hat{a}$  is the number operator. Therefore, number states are eigenvectors of the number operator.

Coherent states have properties that closely resemble classical electromagnetic waves, and are generated by single-mode lasers well above the threshold. [?] We can defined them, using number states in the following manner

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad (5.20)$$

in which the complex number  $\alpha$  is the sole parameter that characterizes it. In fact, if we calculate the expected number of photons with  $\langle\alpha|\hat{n}|\alpha\rangle$  we will obtain  $|\alpha|^2$ . The coherent state is an eigenstate of the annihilation operator,  $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ .

Using the creation and annihilation operators, we can define two quadrature operators [?]

$$\hat{X} = \frac{1}{2} (\hat{a}^\dagger + \hat{a}) \quad (5.21), \quad \hat{Y} = \frac{i}{2} (\hat{a}^\dagger - \hat{a}) \quad (5.22)$$

The expected value of these two operators, using a coherent state  $|\alpha\rangle$  are

$$\langle \hat{X} \rangle = \text{Re}(\alpha) \quad (5.23), \quad \langle \hat{Y} \rangle = \text{Im}(\alpha) \quad (5.24)$$

We see that the expected value of these operators give us the real and imaginary part of  $\alpha$ . Now, we can obtain the uncertainty of these operators, using:

$$\text{Var}(\hat{X}) = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2 \quad (5.25)$$

For each of these quadrature operators the variance will be

$$\text{Var}(\hat{X}) = \text{Var}(\hat{Y}) = \frac{1}{4} \quad (5.26)$$

This result show us that for both quadratures, the variance of measurement is the same and independent of the value of  $\alpha$ .

### Homodyne detection

The measurent of a quadrature of an input signal (S) is made by using the balanced homodyne detection technique, which measures the phase difference between the input signal and a local oscillator (LO). The measurement of quadrature are made relative to a reference phase of the LO, such that if the measurement is made in-phase with this reference, the value will be proportional to the  $\hat{X}$  quadrature of the signal. If the phase of the LO is has an offset of  $\pi/2$  relative to the reference, the output will be proportional to the  $\hat{Y}$  quadrature of the signal.

Experimentally, the balanced homodyne detection requires a local oscillator with the same frequency as the input signal, but with a much larger amplitude. These two signals are combined using a 50:50 beam splitter, from were two beams emerge, which are then converted to currents using photodides. Finally, the two currents are subtracted, resulting in an output current proportional to a quadrature of the input signal [?].

In the lab and in our simulations, a more complex system is used, the double balanced homodyne detection, which allows the simultaneous measurement of the  $\hat{X}$  and  $\hat{Y}$  components. The signal is divided in two beam with half the power of the original. One of the beams is used in a balanced homodyne detection with a local oscillator. The other beam is used in another balanced homodyne detection, but using a local oscillator with a phase difference  $\pi/2$  relative to the first one.

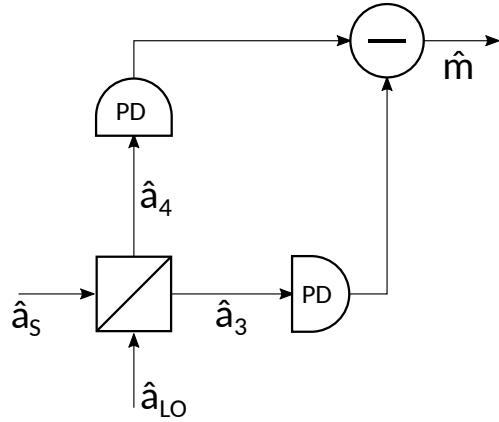


Figura 5.7: Balanced double homodyne detection.

### Noise sources in homodyne detection

The detection of light using photodiodes is subjected to various sources of noise. One of these sources is the electrical field itself. The interaction of the signal with the vacuum field adds quantum noise to the detection. Another source of noise comes from the detection system, such as photodiodes and other electrical circuits, originating various kinds of noise, such as thermal noise, dark noise and amplifier noise [?]. In the following sections, we will focus on two noise sources, quantum noise and thermal noise.

### Quantum Noise

In order to grasp this effect, the quantum mechanical description of balanced homodyne detection will be used, employing quantum operators to describe the effect of each component in the system (fig. ??). We start with the operators  $\hat{a}_S$  and  $\hat{a}_{LO}$  corresponding to the annihilation operator for the signal and local oscillator, which are the inputs in a beam divisor. The outputs will be  $\hat{a}_3$  and  $\hat{a}_4$ . Using a balanced beam splitter, we can write the output as

$$\hat{a}_3 = \frac{1}{\sqrt{2}} (\hat{a}_S + \hat{a}_{LO}) \quad (5.27), \quad \hat{a}_4 = \frac{1}{\sqrt{2}} (\hat{a}_S - \hat{a}_{LO}) \quad (5.28)$$

The final output of a homodyne measurement will be proportional to the difference between the photocurrents in arm 3 and 4. Then

$$I_{34} = I_3 - I_4 \sim \langle \hat{n}_3 - \hat{n}_4 \rangle \quad (5.29)$$

We can define an operator that describes the difference of number of photons in arm 3 and arm 4:

$$\hat{m} = \hat{a}_3^\dagger \hat{a}_3 - \hat{a}_4^\dagger \hat{a}_4 \quad (5.30)$$

If we assume that the local oscillator produces the the coherent state  $|\beta\rangle$ , then the expected value of this measurement will be

$$\langle m \rangle = 2|\alpha||\beta| \cos(\theta_\alpha - \theta_\beta) \quad (5.31), \quad \text{Var}(m) = |\alpha|^2 + |\beta|^2 \quad (5.32)$$

The local oscillator normally has a greater power than the signal, then  $|\alpha| \ll |\beta|$ . If we use as unit,  $2|\beta|$ , then these two quantities can be simplified to

$$\langle m \rangle = |\alpha| \cos(\theta_\alpha - \theta_\beta) \quad (5.33), \quad \text{Var}(m) \approx \frac{1}{4} \quad (5.34)$$

[?]

Has we have seen previously, in order to measure two quadratures simultaneously, we can use double balanced homodyne detection. For each quadrature, the input signal now has half the power, so  $|\alpha| \rightarrow |\alpha/\sqrt{2}|$ . If we use a local oscillator that produces states  $|\beta\rangle$ , then we can divide it in two beams in state  $|\beta/\sqrt{2}\rangle$  and  $|i\beta/\sqrt{2}\rangle$  which will be used in each homodyne detection. In this setting, the expected values for each quadrature,  $X$  and  $Y$ , (in normalized values of  $\sqrt{2}|\beta|$ ) are

$$\langle m_X \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \cos(\theta_\alpha - \theta_\beta) \quad (5.35), \quad \text{Var}(m_X) \approx \frac{1}{4} \quad (5.36)$$

$$\langle m_Y \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \sin(\theta_\alpha - \theta_\beta) \quad (5.37), \quad \text{Var}(m_Y) \approx \frac{1}{4} \quad (5.38)$$

Therefore the measurement of each quadrature will have half the amplitude, but the same variance.

### Thermal noise

Thermal noise is generated by electrons in response to temperature. It's contribution to the resulting current can be described by the following equation [?]

$$\langle (\Delta i_T)^2 \rangle = 4K_B T_0 B / R_L \quad (5.39)$$

in which  $K_B$  it's Boltzmann's constant,  $T_0$  is the absolute temperature,  $B$  is the bandwidth and  $R_L$  is the receiver load impedance. The  $B$  value is imposed by default or chosen when the measurements are made, but the  $R_L$  value is dependent in the internal setup of the various components of the detection system. Nevertheless, for simulation purposes, we can just introduce an experimental value.

### 5.4.2 Simulation

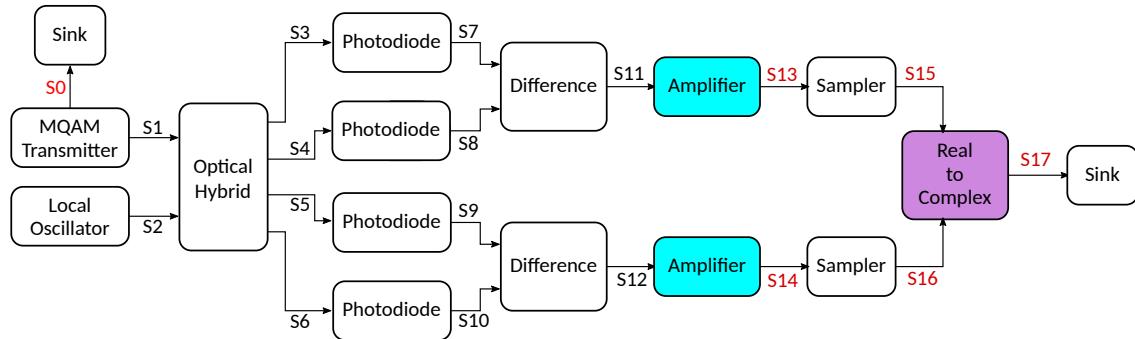


Figura 5.8: Overview of the simulated optical system.

List of signals used in the simulation:

Signal name	Signal type	Status
S0	Binary	check
S1	OpticalSignal	check
S2	OpticalSignal	check
S3	OpticalSignal	check
S4	OpticalSignal	check
S5	OpticalSignal	check
S6	OpticalSignal	check
S7	TimeContinuousAmplitudeContinuousReal	check
S8	TimeContinuousAmplitudeContinuousReal	check
S9	TimeContinuousAmplitudeContinuousReal	check
S10	TimeContinuousAmplitudeContinuousReal	check
S11	TimeContinuousAmplitudeContinuousReal	check
S12	TimeContinuousAmplitudeContinuousReal	check
S13	TimeContinuousAmplitudeContinuousReal	check
S14	TimeContinuousAmplitudeContinuousReal	check
S15	TimeDiscreteAmplitudeContinuousReal	check
S16	TimeDiscreteAmplitudeContinuousReal	check
S17	OpticalSignal	check

This system takes into account the following input parameters:

System Parameters	Default value	Description
localOscillatorPower1	$2.0505 \times 10^{-8} \text{W}$	Sets the optical power, in units of W, of the local oscillator inside the MQAM
localOscillatorPower2	$2.0505 \times 10^{-8} \text{W}$	Sets the optical power, in units of W, of the local oscillator used for Bob's measurements
localOscillatorPhase	0 rad	Sets the initial phase of the local oscillator used in the detection
responsivity	1 A/W	Sets the responsivity of the photodiodes used in the homodyne detectors
iqAmplitudeValues	$\{\{1, 1\}, \{-1, 1\}, \{-1, -1\}, \{1, -1\}\}$	Sets the amplitude of the states used in the MQAM

The simulation setup is represented in figure 5.8. The starting point is the MQAM, which generates random states from the constelation given by the variable `iqAmplitudeValues`. The output from the generator is received in the Optical Hybrid where it is mixed with a local oscillator, outputing two optical signal pairs. Each pair is converted to currents by two photodiodes, and the same currents are subtracted from each other, originating another current proportional to one of the quadratures of the input state. The other pair suffers the same process, but the resulting subtraction current will be proportional to another quadrature, dephased by  $\pi/2$  relative to the other quadrature.

### Required files

---

## Bibliografia

- [1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.
- [2] Ryo Namiki and Takuya Hirano. Security of quantum cryptography using balanced homodyne detection. *Physical Review A*, 67(2):022308, 2003.
- [3] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.
- [4] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.
- [5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

## 5.5 Continuos Variable Quantum Transmission System

Student Name: Daniel Pereira

Starting Date: May 1, 2017

Goal: Simulation and experimental validation of a continuous variable transmission system.

Description of the Laboratorial and Simulation Setup:

Theoretical Description of the System:

Laboratorial and Simulation Results:

Discussion:

In this section a continuous variable quantum transmission system is analyzed. The results here presented follow closely the [2]. In [2], the security of a continuous variable quantum key distribution (CV-QKD) system is studied theoretically, here we complete that theoretical study with simulations results.

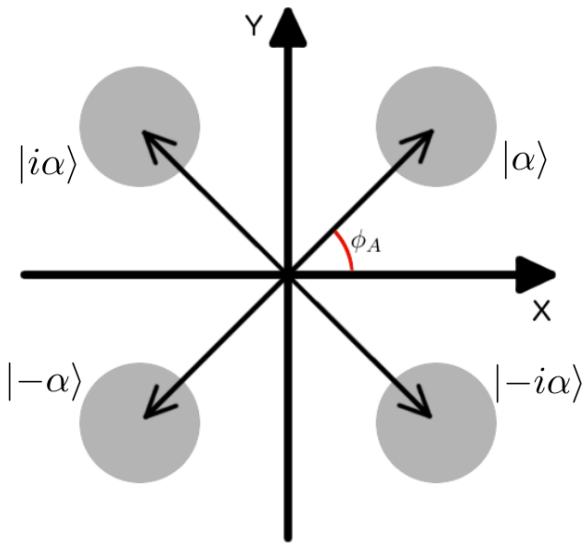


Figura 5.9: State constellation

The state constellation used in the system is presented in Figure 5.9. The emitter (usually named Alice) is going to use two basis, the  $45^\circ$  base and the  $-45^\circ$  base. In the  $45^\circ$  base, Alice sends one of two values, 1 and  $-1$ , which correspond to the states  $|\alpha\rangle$  and  $|- \alpha\rangle$ . In the  $-45^\circ$  base, Alice can also send one of two values, 1 and  $-1$ , which correspond to the states  $|i\alpha\rangle$  and  $|-i\alpha\rangle$ . At the end Alice is going to send one of the four states  $|\alpha\rangle$ ,  $|- \alpha\rangle$ ,  $| - i\alpha\rangle$ , and  $|i\alpha\rangle$ , with equal probability.

Because we don't know à prior which state is going be transmitted, neither which basis is going to be used, and to incorporate our "ignorance" in the system description, we can

work with the density operator. The density operator is a proper tool to describe "statistical mixtures". A "statistical mixtures" is one state, from a possible set, but we don't know which state it is. There is no state superposition.

Since all states have the same probability of occurring, the state density operator is given by:

$$\hat{\rho} = \frac{1}{4} (|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha| + |i\alpha\rangle\langle i\alpha| + |-i\alpha\rangle\langle -i\alpha|). \quad (5.40)$$

The probability to detect at the receiver the state  $|\alpha\rangle$  is given by

$$P(\alpha) = \langle\alpha|\hat{\rho}|\alpha\rangle = \frac{1}{4}. \quad (5.41)$$

Note that the density operator is equivalent to the wave function in terms of the system description.

From the receiver perspective, i.e. from the Bob perspective, and after knowing the base used by Alice. The density operator can be reduce to,

$$\hat{\rho}_1 = \frac{1}{2} (|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|), \quad (5.42)$$

$$\hat{\rho}_2 = \frac{1}{2} (|i\alpha\rangle\langle i\alpha| + |-i\alpha\rangle\langle -i\alpha|). \quad (5.43)$$

where 1 corresponds to the  $45^\circ$  base and  $-1$  corresponds to  $-45^\circ$ .

### Single Base Homodyne Detection

The probability of obtaining a quadrature  $\hat{X}_\phi = \hat{X}_1 \cos \phi + \hat{X}_2 \sin \phi$  when measuring the coherent state  $|\alpha\rangle$  is given by the following gaussian distribution:

$$|\langle X_\phi | \alpha \rangle|^2 = \sqrt{\frac{2}{\pi}} e^{-2(X_\phi - \alpha \cos \phi)^2}, \quad (5.44)$$

We can define the "correct" and "wrong" basis measurement probability density, respectively, as:

$$\langle X_i | \hat{\rho}_j | X_i \rangle = \begin{cases} \frac{1}{\sqrt{2\pi}} (e^{-2(X_i - \alpha)^2} + e^{-2(X_i + \alpha)^2}), & i = j \\ \sqrt{\frac{2}{\pi}} e^{-2X_i^2}, & i \neq j \end{cases}. \quad (5.45)$$

The post selection efficiency (PSE) can be defined as the probability of a measurement in the correct basis yields a result that satisfies the limit value  $X_0$ :

$$\begin{aligned} P(X_0, \alpha) &= \int_{-\infty}^{-X_0} \langle X_1 | \hat{\rho}_1 | X_1 \rangle dX_1 + \int_{X_0}^{\infty} \langle X_1 | \hat{\rho}_1 | X_1 \rangle dX_1 \\ &= \frac{1}{2} [\text{erfc}(\sqrt{2}(X_0 + \alpha)) + \text{erfc}(\sqrt{2}(X_0 - \alpha))]. \end{aligned} \quad (5.46)$$

The bit error rate (BER) is the normalized probability of, after choosing the correct basis, obtaining the wrong bit value:

$$Q(X_0, \alpha) = \frac{1}{P(X_0, \alpha)} \int_{-\infty}^{-X_0} |\langle X_i | \alpha \rangle| dX_i = \frac{\operatorname{erfc}(\sqrt{2}(X_0 + \alpha))}{2P(X_0, n)} \quad (5.47)$$

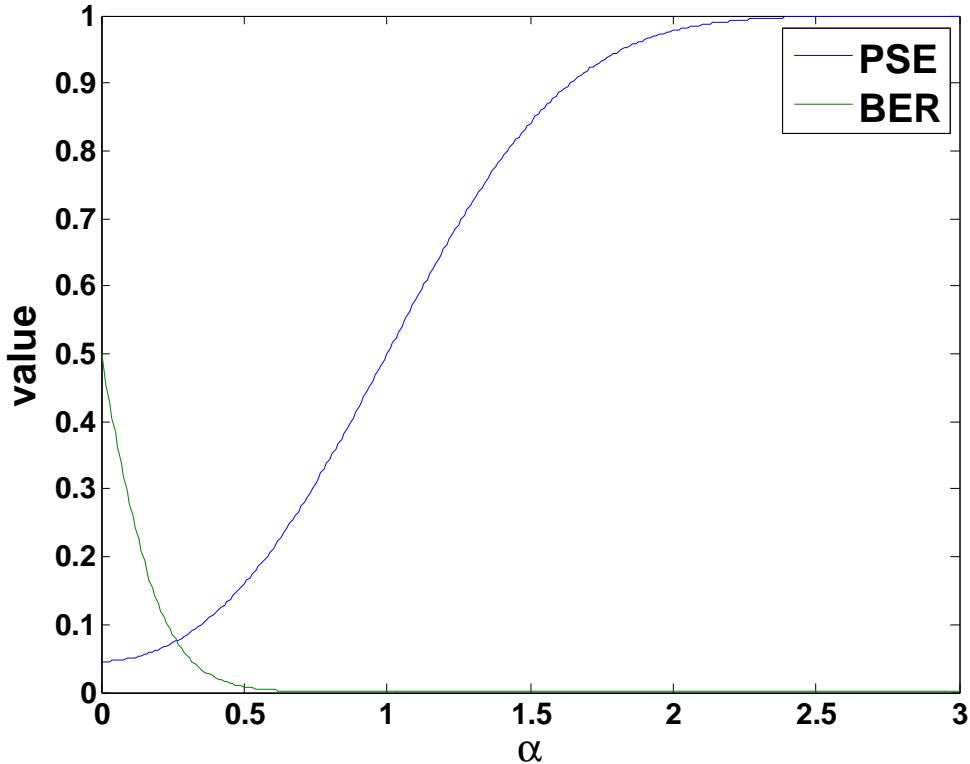


Figura 5.10: BER and PSE in function of  $\alpha$  for the single homodyne setup.  $X_0 = 1$  was used

### Double Homodyne setup

In our proposed double homodyne protocol both quadratures are measured simultaneously, as such the concept of correct and wrong basis measurements has no value. Our protocol also makes use of a locally generated Local Oscillator (LO), obtained from a different laser than the one used to generate the signal, thus we have to take into account the phase drift between both lasers. High intensity reference pulses are sent periodically to allow for an estimation of the phase drift. The double homodyne setup requires the signal to be divided into the two utilized detectors, so each measurement is made on a coherent state with half the amplitude of the incoming signal  $\alpha \rightarrow \frac{\alpha}{\sqrt{2}}$ .

For each incoming pulse we measure quadratures  $X_\phi$  and  $Y_\phi$ .  $\phi$  has contributions from both the encoded angle,  $\theta$ , and the phase difference between lasers,  $\epsilon$ , we assume  $\phi = \theta + \epsilon$ .

On the reference pulses no phase is encoded, that is  $\theta = 0$ , thus  $\epsilon$  can be estimated. Assuming  $\epsilon$  doesn't change between a reference pulse and the following signal pulse, the measured quadratures can be cast into the originally sent quadratures  $X_\theta$  and  $Y_\theta$  via:

$$\begin{aligned} X_\theta &= X_\phi \cos \epsilon - Y_\phi \sin \epsilon \\ Y_\theta &= X_\phi \sin \epsilon + Y_\phi \cos \epsilon \end{aligned} \quad (5.48)$$

Assuming an announcement of the coding basis, the density operators (5.42) and (5.43) still apply. We can now define the probability density of obtaining results  $X_\theta$  and  $Y_\theta$ , assuming a state in the  $X_1$  base was sent, as:

$$\langle X_\theta | \hat{\rho}_1 | X_\theta \rangle = \frac{\sqrt{\frac{2}{\pi}}}{4} \left( e^{-2(x_\theta - \frac{\alpha}{\sqrt{2}} \cos \theta)^2} + e^{-2(x_\theta + \frac{\alpha}{\sqrt{2}} \cos \theta)^2} \right), \quad (5.49)$$

$$\langle Y_\theta | \hat{\rho}_1 | Y_\theta \rangle = \frac{\sqrt{\frac{2}{\pi}}}{4} \left( e^{-2(y_\theta - \frac{\alpha}{\sqrt{2}} \sin \theta)^2} + e^{-2(y_\theta + \frac{\alpha}{\sqrt{2}} \sin \theta)^2} \right). \quad (5.50)$$

Now each state needs to satisfy two limit values,  $X_0$  and  $Y_0$ , to be accepted. Thus, the PSE is now defined as:

$$\begin{aligned} P_{DH}(X_0, Y_0, \alpha) &= \int_{-\infty}^{-X_0} \langle X_\theta | \hat{\rho}_1 | X_\theta \rangle dx_\theta \int_{-\infty}^{-Y_0} \langle Y_\theta | \hat{\rho}_1 | Y_\theta \rangle dy_\theta + \\ &\quad \int_{X_0}^{\infty} \langle X_\theta | \hat{\rho}_1 | X_\theta \rangle dx_\theta \int_{Y_0}^{\infty} \langle Y_\theta | \hat{\rho}_1 | Y_\theta \rangle dy_\theta \\ &= \frac{1}{4} \left\{ \operatorname{erfc} \left[ \sqrt{2} \left( X_0 - \frac{\alpha}{\sqrt{2}} \cos \theta \right) \right] + \operatorname{erfc} \left[ \sqrt{2} \left( X_0 + \frac{\alpha}{\sqrt{2}} \cos \theta \right) \right] \right\} \\ &\quad \left\{ \operatorname{erfc} \left[ \sqrt{2} \left( Y_0 - \frac{\alpha}{\sqrt{2}} \sin \theta \right) \right] + \operatorname{erfc} \left[ \sqrt{2} \left( Y_0 + \frac{\alpha}{\sqrt{2}} \sin \theta \right) \right] \right\}, \end{aligned} \quad (5.51)$$

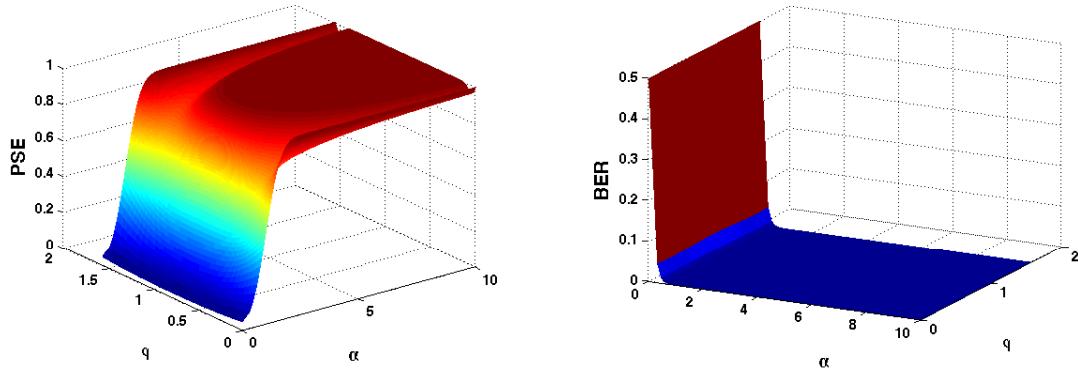
The DH subscript denotes Double Homodyne. In a somewhat similar manner, the BER is now defined as:

$$\begin{aligned} Q_{DH}(X_0, Y_0, \alpha) &= \frac{1}{P_{DH}} \left( \int_{-\infty}^{-X_0} \left| \langle X_\theta | \frac{\alpha}{\sqrt{2}} \rangle \right|^2 dx_\theta \int_{-\infty}^{-Y_0} \left| \langle Y_\theta | \frac{\alpha}{\sqrt{2}} \rangle \right|^2 dy_\theta + \right. \\ &\quad \left. \int_{X_0}^{\infty} \left| \langle X_\theta | -\frac{\alpha}{\sqrt{2}} \rangle \right|^2 dx_\theta \int_{Y_0}^{\infty} \left| \langle Y_\theta | -\frac{\alpha}{\sqrt{2}} \rangle \right|^2 dy_\theta \right) \\ &= \frac{1}{2P_{DH}} \operatorname{erfc} \left[ \sqrt{2} \left( X_0 + \frac{\alpha}{\sqrt{2}} \cos \theta \right) \right] \operatorname{erfc} \left[ \sqrt{2} \left( Y_0 + \frac{\alpha}{\sqrt{2}} \sin \theta \right) \right], \end{aligned} \quad (5.52)$$

note that, in this definition for BER, only values  $\theta \in [0, \frac{\pi}{2}]$  make sense (the sent state was  $\alpha$ ).

## Functional Description

Simplified diagrams of the systems being simulated are presented in Figures 5.12a. and 5.12b. Two optical signals are generated, one with a constant power level of 10 dBm and the other with power in multiples of the power corresponding to a single photon per



(a) PSE in function of  $\alpha$  and  $\theta$  for the double homodyne setup.  $X_0 = 1$  was used  
 (b) BER in function of  $\alpha$  and  $\theta$  for the double homodyne setup.  $X_0 = 1$  was used

Figura 5.11: Theoretical results for double homodyne setup.

sampling time ( $6.4078e \times 10^{-13}$  W for a sampling time of 200 ns). The two signals are mixed, with a Balanced Beam Splitter in the single homodyne case and with a  $90^\circ$  Optical Hybrid in the double homodyne one, and are subsequently evaluated with recourse to Homodyne Receivers.

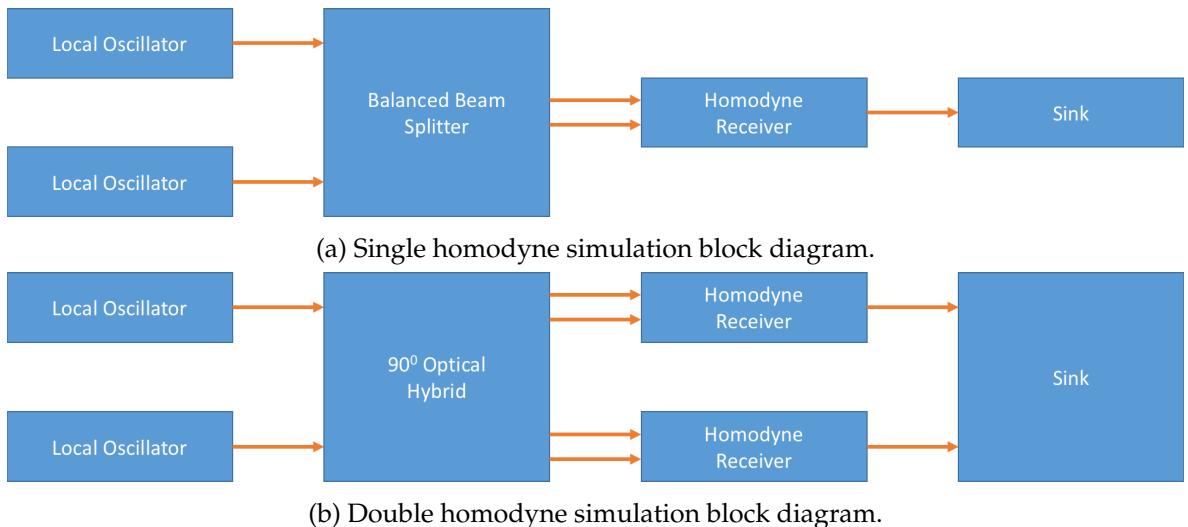


Figura 5.12: Block diagrams of both simulation results presented in this report.

System Blocks	netxpto Blocks
Local Oscillator	LocalOscillator
Homodyne Receiver	I_HomodyneReceiver
Balanced Beam Splitter	BalancedBeamSplitter
$90^\circ$ Optical Hybrid	OpticalHybrid

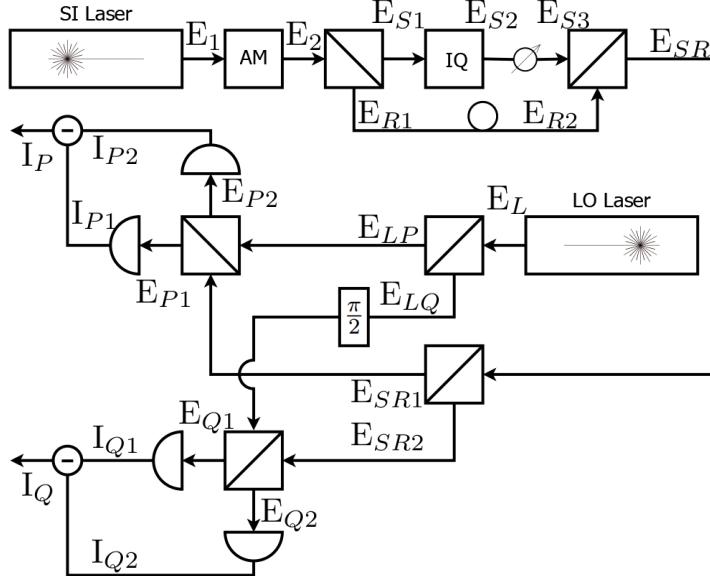


Figura 5.13: Simplified diagram of experimental setup.

### Theoretical study of the setup

A detailed diagram of the system studied is presented in Figure 5.13, with the respective mathematical treatment presented in equations (5.53) through (5.72).

$$E_1 = |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.53)$$

$$p(t) = \begin{cases} 1, & \frac{T}{8} \leq t < \frac{T}{8} \\ 0, & \text{otherwise} \end{cases} \quad (5.54)$$

$$E_2 = p(t) |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.55)$$

$$E_{S1} = \frac{p(t)}{\sqrt{2}} |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.56)$$

$$E_{S2} = \frac{p(t)}{\sqrt{2}} |E_S| e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} \quad (5.57)$$

$$E_{S3} = \frac{p(t)}{\sqrt{2} \text{Att}} |E_S| e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} \quad (5.58)$$

$$E_{R1} = \frac{p(t)}{\sqrt{2}} |E_S| e^{i(\omega_S t + \xi_S(t))} \quad (5.59)$$

$$E_{R2} = \frac{p(t - \frac{T}{2})}{\sqrt{2}} |E_S| e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \quad (5.60)$$

$$E_{SR} = \frac{|E_S|}{\sqrt{2}} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \quad (5.61)$$

$$E_{SR1} = \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \quad (5.62)$$

$$E_{SR2} = \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t))} + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \quad (5.63)$$

$$E_L(t) = |E_L| e^{i(\omega_L t + \xi_L(t))} \quad (5.64)$$

$$E_{LP}(t) = \frac{1}{\sqrt{2}} |E_L| e^{i(\omega_L t + \xi_L(t))} \quad (5.65)$$

$$E_{LQ}(t) = \frac{1}{\sqrt{2}} |E_L| e^{i(\omega_L t + \xi_L(t) + \frac{\pi}{2})} \quad (5.66)$$

$$E_{P1}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t))} + \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.67)$$

$$E_{P2}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t))} - \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.68)$$

$$E_{Q1}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t) + \frac{\pi}{2})} + \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.69)$$

$$E_{Q2}(t) = \frac{1}{\sqrt{2}} \left\{ \frac{|E_L|}{\sqrt{2}} e^{i(\omega_L t + \xi_L(t) + \frac{\pi}{2})} - \frac{|E_S|}{2} \left[ \frac{p(t)}{\text{Att}} e^{i(\omega_S t + \phi_{IQ} t + \xi_S(t) + \xi_{IQ}(t))} \right. \right. \\ \left. \left. + p \left( t - \frac{T}{2} \right) e^{i(\omega_S(t - \frac{T}{2}) + \xi_S(t - \frac{T}{2}))} \right] \right\} \quad (5.70)$$

$$I_P(t) = \frac{|E_S| |E_L|}{\sqrt{2}} \left\{ \frac{p(t)}{\text{Att}} \cos((\omega_S - \omega_L)t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t) - \xi_L(t)) \right. \\ \left. + p \left( t - \frac{T}{2} \right) \cos \left( (\omega_S - \omega_L)t + \omega_S \frac{T}{2} + \xi_S \left( t - \frac{T}{2} \right) - \xi_L(t) \right) \right\} \quad (5.71)$$

$$I_Q(t) = \frac{|E_S| |E_L|}{\sqrt{2}} \left\{ \frac{p(t)}{\text{Att}} \sin((\omega_S - \omega_L)t + \phi_{IQ}(t) + \xi_S(t) + \xi_{IQ}(t) - \xi_L(t)) \right. \\ \left. + p \left( t - \frac{T}{2} \right) \sin \left( (\omega_S - \omega_L)t + \omega_S \frac{T}{2} + \xi_S \left( t - \frac{T}{2} \right) - \xi_L(t) \right) \right\} \quad (5.72)$$

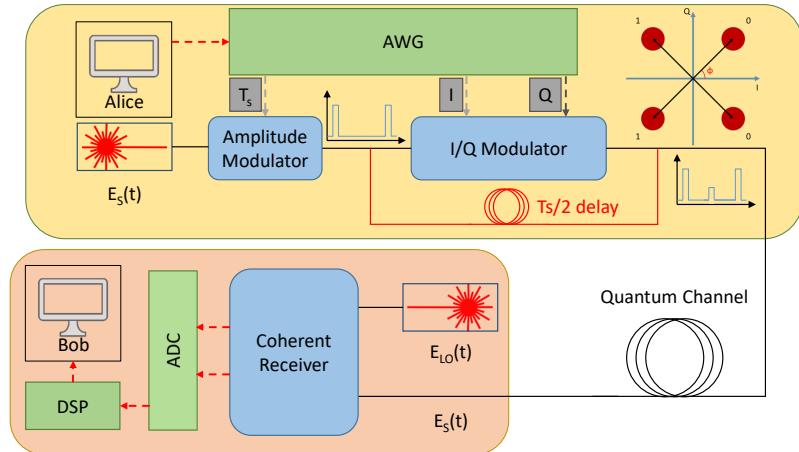


Figura 5.14: Diagram of full experimental setup.

### Experimental description

The main experimental setup utilized for the study presented in this document is presented in Figure 5.14. The setup contained two Yenista OSICS Band C/AG TLS lasers, tuned to a 1550 nm wavelength. A JDSU dual drive Mach-Zehnder Modulator with a Picosecond 5865 RF driver, employed at the output of one of the lasers, set the repetition rate at 1 GHz with a pulse time of 125 ps. The driving signal was generated by an Agilent Technologies BER Tester. A 50/50 beam-splitter was employed at the output of the Mach-Zehnder Modulator, one arm output is sent through an IQ Modulator while the other is sent through a fibre loop with length chosen such that the two arms have a relative delay of  $\sim 500$  ps. The employed IQ Modulator was a u2t photonics 32 GHz IQ Modulator with a SHF 807 RF driver, the driving signal being generated by a Tektronix AWG70002A Arbitrary Waveform Generator (AWG). A Variable Optical Attenuator (VOA) was set at the output of the IQ modulator to allow a fine tuning of the optical power to the desired level. The two arms output created by the first beam-splitter are combined by a 50/50 beam combiner. A fibre channel of  $\sim 10$  km was set between Alice's and Bob's setup. A Picometrix CR-100D 100G Integrated Balanced Receiver for Coherent Applications was employed to perform double homodyne measurements, recovering both the in-phase and in-quadrature components of the incoming light field. The response of the receiver was recorded by a Tektronix DPO77002SX-R3 oscilloscope, with an acquisition frequency of 100 GHz for a period of 400  $\mu$ s.

### Experimental results

#### Required files

Header Files

File	Description
netxpto.h	Generic purpose simulator definitions.
local_oscillator.h	Generates continuous coherent signal.
balanced_beam_splitter.h	Mixes the two input signals into two outputs.
optical_hybrid.h	Mixes the two input signals into four outputs.
homodyne_reciever.h	Performs coherent detection on the input signal.
sink.h	Closes any unused signals.

### Source Files

File	Description
netxpto.cpp	Generic purpose simulator definitions.
local_oscillator.cpp	Generates continuous coherent signal.
balanced_beam_splitter.cpp	Mixes the two input signals into two outputs.
optical_hybrid.cpp	Mixes the two input signals into four outputs.
homodyne_reciever.cpp	Performs coherent detection on the input signal.
sink.cpp	Closes any unused signals.

### System Input Parameters

This system takes into account the following input parameters:

System Parameters	Description
numberOfBitsGenerated	Gives the number of bits to be simulated
bitPeriod	Sets the time between adjacent bits
samplesPerSymbol	Establishes the number of samples each bit in the string is given
localOscillatorPower_dBm	Sets the optical power, in units of dBm, at the reference output
localOscillatorPower2	Sets the optical power, in units of W, of the signal
localOscillatorPhase1	Sets the initial phase of the local oscillator used for reference
localOscillatorPhase2	Sets the initial phase of the local oscillator used for signal
transferMatrix	Sets the transfer matrix of the beam splitter used in the homodyne detector
responsivity	Sets the responsivity of the photodiodes used in the homodyne detector
amplification	Sets the amplification of the trans-impedance amplifier used in the homodyne detector
electricalNoiseAmplitude	Sets the amplitude of the gaussian thermal noise added in the homodyne detector
shotNoise	Chooses if quantum shot noise is used in the simulation

### Inputs

This system takes no inputs.

## Outputs

The single homodyne system outputs the following objects:

**Parameter:** Signals:

**Description:** Local Oscillator Optical Reference; ( $S_1$ )

**Description:** Local Oscillator Optical Signal; ( $S_2$ )

**Description:** Beam Splitter Outputs; ( $S_3, S_4$ )

**Description:** Homodyne Detector Electrical Output; ( $S_5$ )

The double homodyne system outputs the following objects:

**Parameter:** Signals:

**Description:** Local Oscillator Optical Reference; ( $S_1$ )

**Description:** Local Oscillator Optical Signal; ( $S_2$ )

**Description:** 90° Optical Hybrid Outputs; ( $S_3, S_4, S_5, S_6$ )

**Description:** Homodyne Detector Electrical Output; ( $S_7$ )

## Simulation Results

### Single homodyne results

The numerical results presented in Figure 5.15 were obtained with the simulation described by the block diagram in Figure 5.12a. Theoretical results are a direct trace of (5.47). One can see that the numerical results adhere quite well to the expected curve.

### Double homodyne results

The numerical results presented in Figure 5.16 were obtained with the simulation described by the block diagram in Figure 5.12b. Theoretical results are a direct trace of (5.52) with  $\theta = \frac{\pi}{4}$ . One can see that the numerical results adhere quite well to the expected curve.

## Known Problems

1. Homodyne Super-Block not functioning

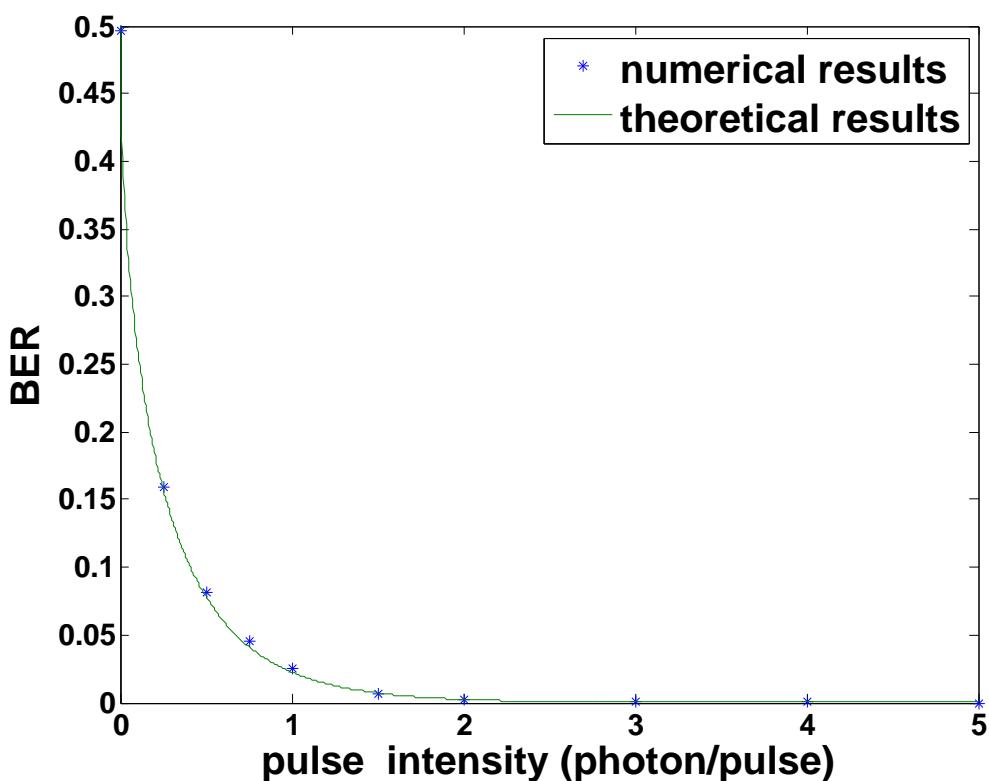


Figura 5.15: BER in function of  $\alpha$  for the single homodyne setup.  $X_0 = 0$  was used

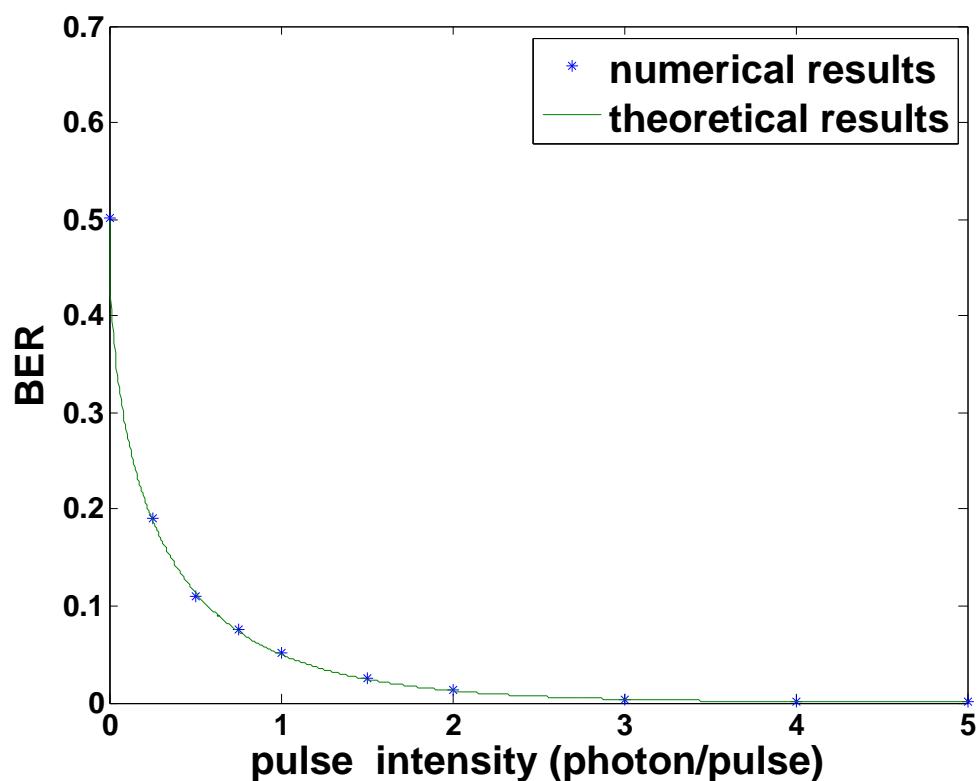


Figura 5.16: BER in function of  $\alpha$  for the double homodyne setup.  $X_0 = 0$  was used

---

## Bibliografia

- [1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.
- [2] Ryo Namiki and Takuya Hirano. Security of quantum cryptography using balanced homodyne detection. *Physical Review A*, 67(2):022308, 2003.
- [3] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.
- [4] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.
- [5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

## 5.6 Kramers-Kronig Transceiver with Stokes PolDemux

<b>Student Name</b>	:	Romil Patel
<b>Starting Date</b>	:	August 16, 2017
<b>Goal</b>	:	Develop a simplified structure (low cost) for a coherent transceiver, that can be used in coherent PON, inter-data center connections, or metropolitan networks (optical path lengths < 100 km). We are going to explore a Kramers-Kronig transceiver with Stokes based PolDemux.
<b>Directory</b>	:	LinkPlanner\doc\tex\sdf\simplified_coherent_receiver

Coherent optical transmission schemes are spectrally efficient since they allow the encoding of information in both quadrature of sinusoid signal. However, the cost of coherent receiver becomes a major obstacle in the case of short-reach links applications like PON, inter-data-center communications and metropolitan network. In order to make the transceiver applicable in short-reach links, an architecture has been proposed which combines the advantages of coherent transmission and cost effectiveness of direct detection. The working principle of the proposed transceiver is based on the Kramers-Kronig (KK) relationship. The KK transceiver scheme allows digital compensation of propagation impairment because both amplitude and phase of the electrical field can be retrieved at the receiver.

### 5.6.1 Theoretical Analysis

The Kramers-Kronig relations are bidirectional mathematical relations, connecting the real and imaginary parts of any complex function that is analytic in the upper half-plane. For instance, a signal  $x(t) = x_r(t) + ix_i(t)$  satisfies the Kramers-Kronig relationship if,

$$x_r(t) = -\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{x_i(t')}{t - t'} dt'$$

$$x_i(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{x_r(t')}{t - t'} dt'$$

This relationship imposes that the real and the imaginary parts of the signal are related to each other though Hilbert transform. Therefore, if we have the real part of the signal then the imaginary part can be calculated by its Hilbert transform.

For a signal that satisfies the Kramers-Kronig relationship, the real and imaginary part can be obtained only from the module. The following questions would give a comprehensive overview of Kramers-Kroning relation and the detailed mathematical calculation which depicts how phase can be extracted from the amplitude information.

### 1. What is Hilbert transform?

If we consider a filter  $H(\omega)$ , described in Figure 5.17, that has a unity magnitude response for all frequencies and the phase response is  $-\pi/2$  for all positive frequencies and  $\pi/2$  for negative frequencies. The transfer function of this filter is given by

$$H(\omega) = -isgn(\omega) \quad (5.73)$$

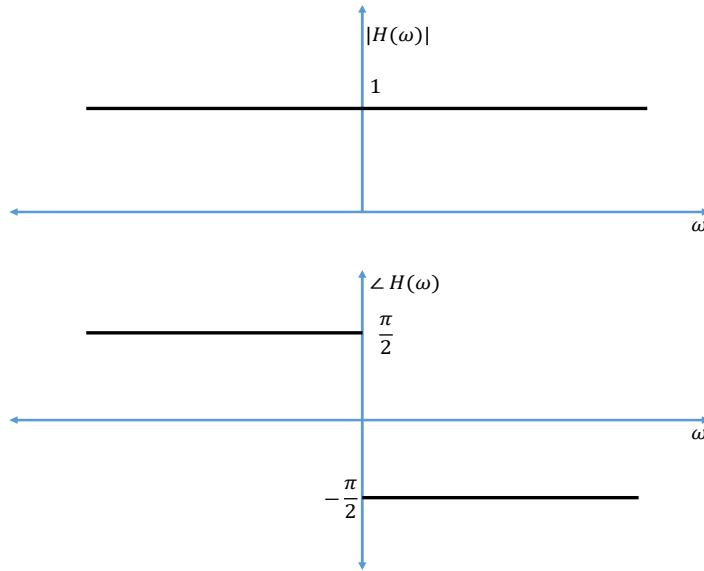


Figura 5.17: Magnitude and phase of Hilbert transform filter

The impulse response of this filter can be given as,

$$\begin{aligned} h(t) &= \mathcal{F}^{-1}[H(i\omega)] \\ &= -i\mathcal{F}^{-1}[sgn(\omega)] \\ &= -i\left(\frac{i}{\pi t}\right) \\ &= \frac{1}{\pi t} \end{aligned} \quad (5.74)$$

When this filter driven by an arbitrary signal  $s(t)$ , the filter produces the output as,

$$\begin{aligned} \hat{s}(t) &= s(t) * h(t) \\ &= \int_{-\infty}^{\infty} \frac{s(u)}{\pi(t-u)} du \end{aligned} \quad (5.75)$$

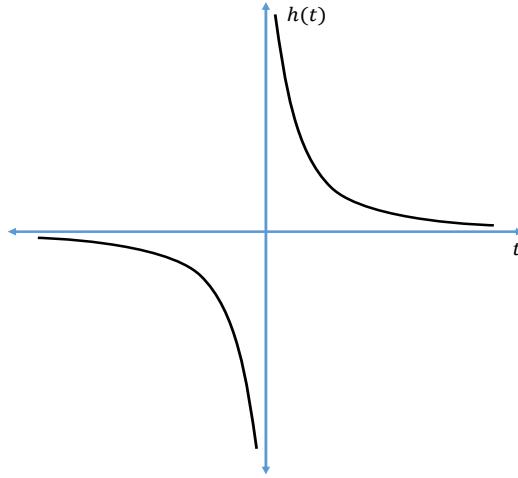


Figura 5.18: Impulse response  $h(t)$  of Hilbert transform filter

The function  $\hat{s}(t)$  is called the Hilbert transform if  $s(t)$ . Note that

$$\mathcal{F}[\hat{s}(t)] = H(\omega)S(\omega) = -isgn(\omega)S(\omega) \quad (5.76)$$

In conclusion, if we convolve any time domain signal with  $\frac{1}{\pi t}$  then it will give us Hilbert transformed signal in time domain. Similarly, from the convolution property of the Fourier transform, if we multiply  $-isgn(\omega)$  with any frequency domain signal  $S(\omega)$  then it'll give us Hilbert transformed signal in frequency domain.

## 2. What is analytical signal?

An analytic signal is a complex-valued signal that has no negative frequency components, and its real and imaginary parts are related to each other by the Hilbert transform.

$$s_a(t) = s(t) + i\hat{s}(t) \quad (5.77)$$

where,  $s_a(t)$  is an analytical signal and  $\hat{s}(t)$  is the Hilbert transform of the signal  $s(t)$ . Such analytical signal can be used to generate Single Sideband Signal (SSB) signal.

## 3. What is a SSB signal and how it can be generated?

By definition, the SSB is the signal which contains either upper sideband or lower sideband and hence it reduces the spectral occupancy by half.

This section will represent the brief idea of generating SSB signal using Hilbert transform method. To understand this, we may express signal  $s(t)$  as a summation of the two complex-valued functions.

$$s(t) = \frac{1}{2}[s(t) + i\hat{s}(t)] + \frac{1}{2}[s(t) - i\hat{s}(t)] \quad (5.78)$$

From Equation 5.77,

$$s(t) = s_a(t) + i s_a^*(t) \quad (5.79)$$

Such representation of  $s_a(t)$  and  $s_a^*(t)$  divide the signal into non-negative frequency component and non-positive frequency component respectively. Considering only non-negative frequency  $s_a(t)$  part, we can write it as

$$\frac{1}{2} S_a(f) = \begin{cases} S(f) & \text{for } f > 0 \\ 0 & \text{for } f < 0 \end{cases} \quad (5.80)$$

where  $S_a(f)$  and  $S(f)$  are the Fourier transform of  $t_a(t)$  and  $s(t)$  respectively. The frequency translated version of  $S_a(f - f_0)$  contains only one side (positive) of  $S(f)$  and hence it is called single sideband signal  $s_{ssb}(t)$ ,

$$F^{-1}\{S_a(f - f_0)\} = s_a(t)e^{i2\pi f_0 t} = s_{ssb}(t) + i\hat{s}_{ssb}(t) \quad (5.81)$$

Therefore, from the Euler's formula,

$$\begin{aligned} s_{ssb}(t) &= Re\{s_a(t)e^{i2\pi f_0 t}\} \\ &= Re\{[s(t) + i\hat{s}(t)][\cos(2\pi f_0 t) + i\sin(2\pi f_0 t)]\} \\ &= s(t)\cos(2\pi f_0 t) - \hat{s}(t)\sin(2\pi f_0 t) \end{aligned} \quad (5.82)$$

This Equation 5.82 displays the mathematical modeling of the upper sideband SSB signal. Similarly, we can generate lower sideband SSB signal by,

$$s_{ssb}(t) = s(t)\cos(2\pi f_0 t) + \hat{s}(t)\sin(2\pi f_0 t) \quad (5.83)$$

#### 4. What is minimum phase signal?

A necessary and sufficient condition for a complex signal  $A(t)$  to be minimum phase is that the curve described in a complex plane by  $A(t)$  when  $t \rightarrow -\infty$  to  $t \rightarrow \infty$  **does not encircle the origin**. A minimum-phase signal has an useful property that the natural logarithm of the magnitude of the frequency response is related to the phase angle of the frequency response by the Hilbert transform.

For instance, if we consider a complex data-carrying signal whose spectrum is contained between  $-B/2$  and  $B/2$ , and consider a SSB signal of the form,

$$x(t) = A + A_s(t)\exp(-i\pi Bt) \quad (5.84)$$

where  $A$  is a constant. Here,  $x(t)$  is minimum phase if and only if the winding number of its trajectory into complex plane is zero. The condition  $|A| > |A_s(t)|$  is sufficient for guaranteeing minimum phase property [3].

## 5. How we can use these signals and profit from them?

This section represents the justification that why we need to use these signals into our proposed transceiver system.

### Analytical Signal:

If we denote an analytic signal  $A_s(t)$  as,

$$A_s(t) = A_{s,r}(t) + iA_{s,i}(t) \quad (5.85)$$

then in the equation 5.85, the real and imaginary parts  $A_{s,r}(t)$  and  $A_{s,i}(t)$  are related through the Kramers-Kronig relation with each other. An intuitive way to analyze the relation is based on expressing its Fourier transform  $A_s(\omega)$  as follows,

$$A_s(\omega) = \frac{1}{2}[1 + sgn(\omega)]A_s(\omega) \quad (5.86)$$

The equation 5.86 follows the SSB signal condition  $A_s(\omega) = 0$  for  $\omega < 0$ . Further, simplification0n of the signal can be summarized as follows:

$$\begin{aligned} A_s(\omega) &= \frac{1}{2}[1 + sgn(\omega)]A_s(\omega) \\ &= \frac{1}{2}A_s(\omega) + \frac{1}{2}sgn(\omega)A_s(\omega) \end{aligned} \quad (5.87)$$

Taking inverse Fourier transform of the equation 5.87,

$$\begin{aligned} A_s(t) &= IFT\{A_s(\omega)\} \\ &= \underline{\frac{1}{2}A_s(t)} + \underline{\frac{1}{2}[IFT\{sgn(\omega)\} \otimes A_s(t)]} \end{aligned} \quad (5.88)$$

The underlined term in Equation 5.88 displays that multiplication in frequency domain converted into the convolution in the time domain. Further, IFT of the function  $sgn(\omega)$  given as  $(-i/\pi t)$ . As a consequences, we can further simplify our equation as,

$$\begin{aligned} A_s(t) &= \frac{1}{2}A_s(t) + \frac{1}{2}\left[\frac{i}{\pi t} \otimes A_s(t)\right] \\ \frac{A_s(t)}{2} &= \frac{1}{2}\left[\frac{i}{\pi t} \otimes A_s(t)\right] \\ A_s(t) &= i\left[\frac{1}{\pi t} \otimes A_s(t)\right] \\ A_s(t) &= \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_s(t')}{t - t'} dt' \end{aligned} \quad (5.89)$$

Using Equation 5.85 into Equation 5.89,

$$A_{s,r}(t) + iA_{s,i}(t) = \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_s(t')}{t - t'} dt' \quad (5.90)$$

Therefore,

$$\begin{aligned} A_{s,r}(t) + iA_{s,i}(t) &= \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,r}(t') + iA_{s,i}(t')}{t - t'} dt' \\ A_{s,r}(t) + iA_{s,i}(t) &= -\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,i}(t')}{t - t'} dt' + \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,r}(t')}{t - t'} dt' \end{aligned} \quad (5.91)$$

which leads to,

$$\begin{aligned} A_{s,r}(t) &= -\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,i}(t')}{t - t'} dt' \\ A_{s,i}(t) &= \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,r}(t')}{t - t'} dt' \end{aligned} \quad (5.92)$$

### Minimum Phase signal:

Given function  $A(t) = A_s(t) + \bar{A}$  never encircles the origin for  $t \in (-\infty, \infty)$ . if we define,

$$G(t) = \log \left[ \frac{A(t)}{\bar{A}} \right] \quad (5.93)$$

then  $G(\omega)$ , the spectrum of  $G(t)$ , is such that  $G(\omega) = 0$  for  $\omega < 0$ . Under the hypothesis stated by Equation 5.90, we can write  $G(t)$  as,

$$G(t) = \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{G(t')}{t - t'} dt' \quad (5.94)$$

From the equations 5.93 and 5.94,

$$\log|A(t)| - \log|\bar{A}| + i[\phi(t) - \bar{\phi}] = \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|A(t)| - \log|\bar{A}|}{t - t'} dt' + \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\phi(t) - \bar{\phi}}{t - t'} dt' \quad (5.95)$$

Comparing Imaginary part of Equation 5.95,

$$\phi(t) - \bar{\phi} = +\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|A(t)| - \log|\bar{A}|}{t - t'} dt' \quad (5.96)$$

In equation 5.96,  $\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|\bar{A}|}{t - t'} dt' = 0$  which leads to,

$$\begin{aligned} \phi(t) &= \bar{\phi} + \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|A(t)|}{t - t'} dt' \\ \phi(t) &= \bar{\phi} + \frac{1}{2\pi} p.v. \int_{-\infty}^{\infty} \frac{\log|A(t)|^2}{t - t'} dt' \end{aligned} \quad (5.97)$$

### 5.6.2 Simulation Analysis

#### Transmitter setup

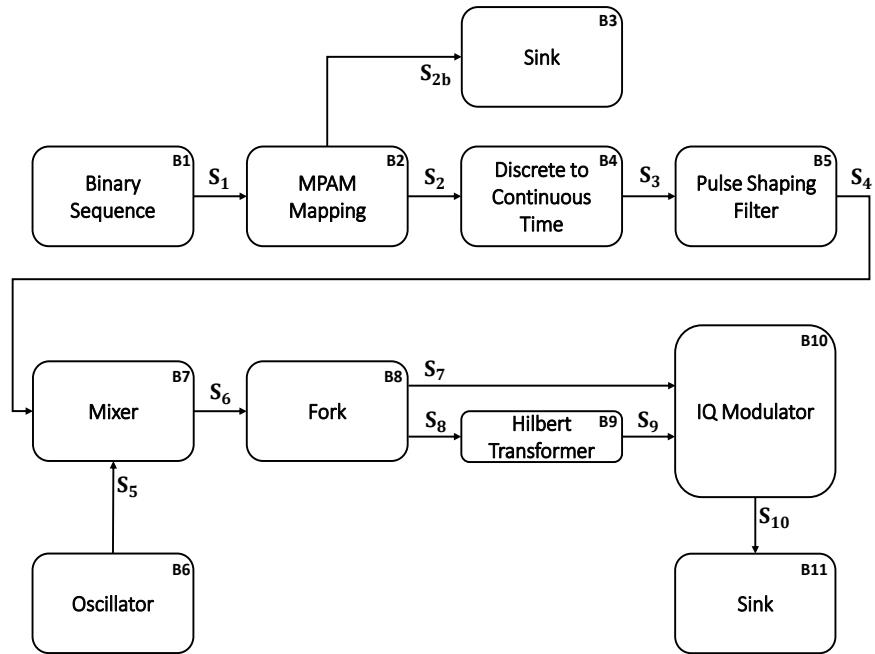


Figura 5.19: Transmitter simulation setup

#### System input parameters:

Parameter	Default Value	Description
sourceMode	PseudoRandom	
patternLength	5	
bitPeriod	1/1.25e9	
iqAmplitudes	{{0,0},{1,0},{2,0},{3,0}}	
numberOfBits	1000	
numberOfSamplesPerSymbol	8	
filterType	RaisedCosine	
rollOffFactor	0.3	
impulseResponseTimeLength	16	
outputOpticalPower	1e-3	

**Transmitter setup description:**

Header Files		
File name	Comments	Status
binary_source.h		✓
m_qam_mapper.h	DONE!	✓
discrete_to_continuous_time.h		✓
pulse_shaper.h		✓
RF_Oscillator.h	DONE!	✓
mixer.h	DONE!	✓
fork.h	DONE!	✓
hilbert_transform.h	DONE!	✓
iq_modulator.h		✓
sink.h		✓
netxpto.h		✓

Source Files		
File name	Comments	Status
binary_source.cpp		✓
m_qam_mapper.cpp	DONE!	✓
discrete_to_continuous_time.cpp		✓
pulse_shaper.cpp		✓
RF_Oscillator.cpp	DONE!	✓
mixer.cpp	DONE!	✓
fork.cpp	DONE!	✓
hilbert_transform.cpp		
iq_modulator.cpp		✓
sink.cpp		✓
netxpto.cpp		✓
kramers_kronig_transceiver_sdf.cpp		✓

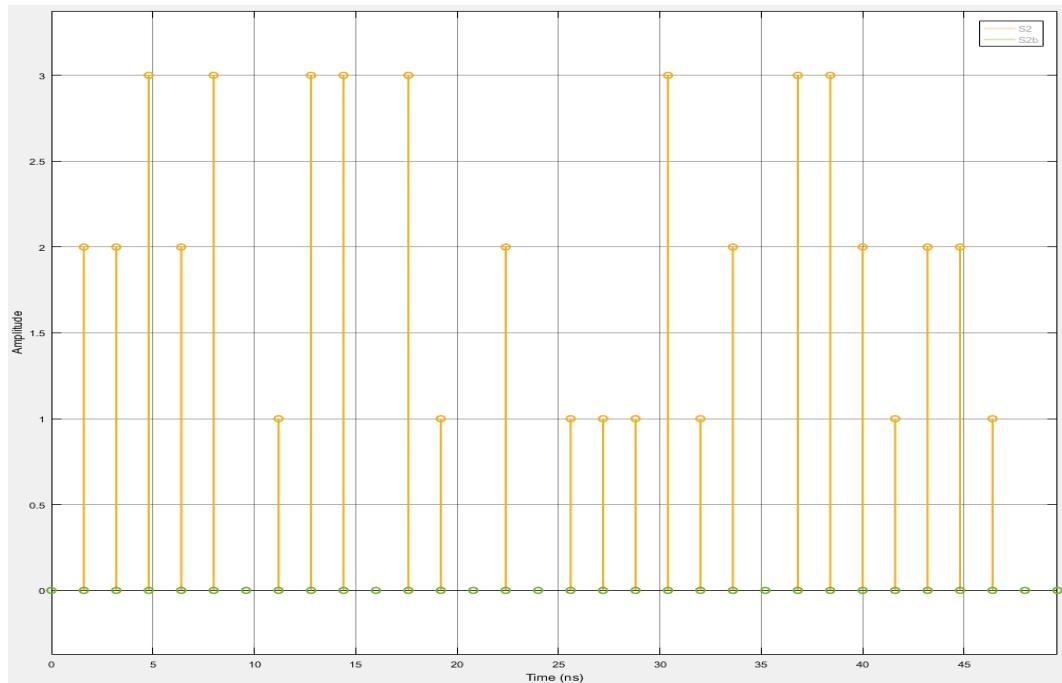
**Simulation results:**

Figura 5.20: S2 and S2b

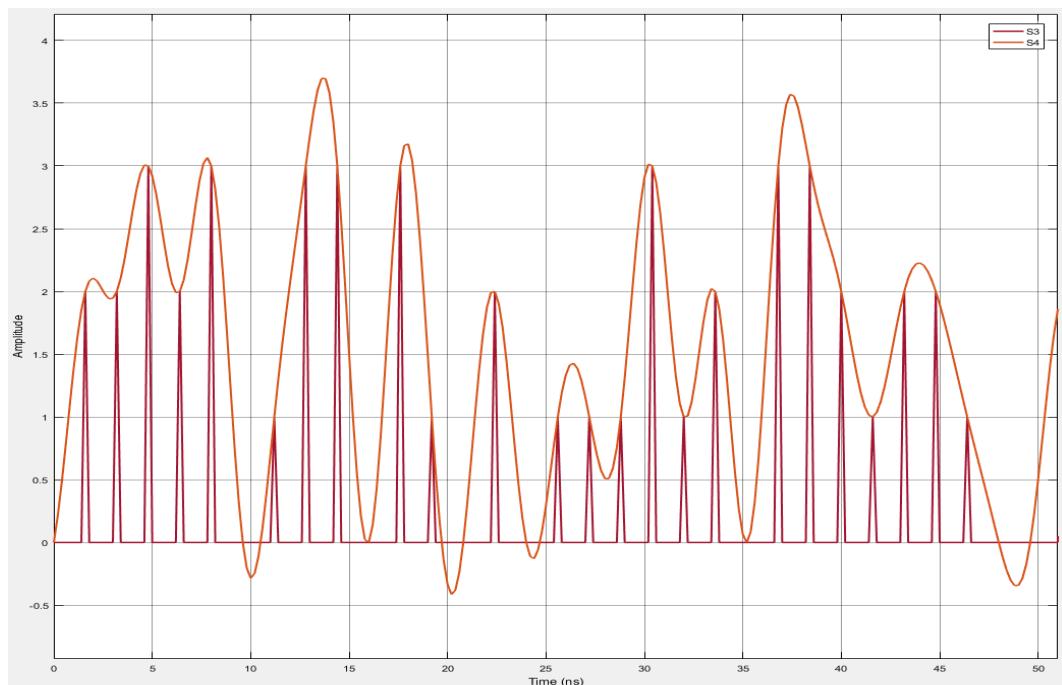


Figura 5.21: S3 and S4

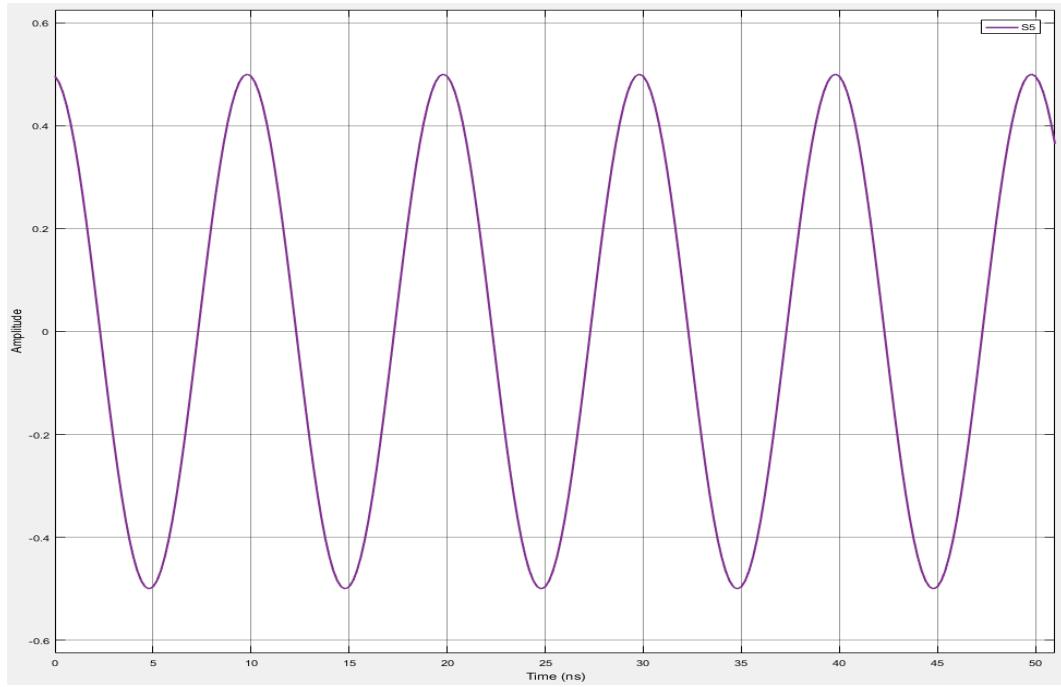


Figura 5.22: S5

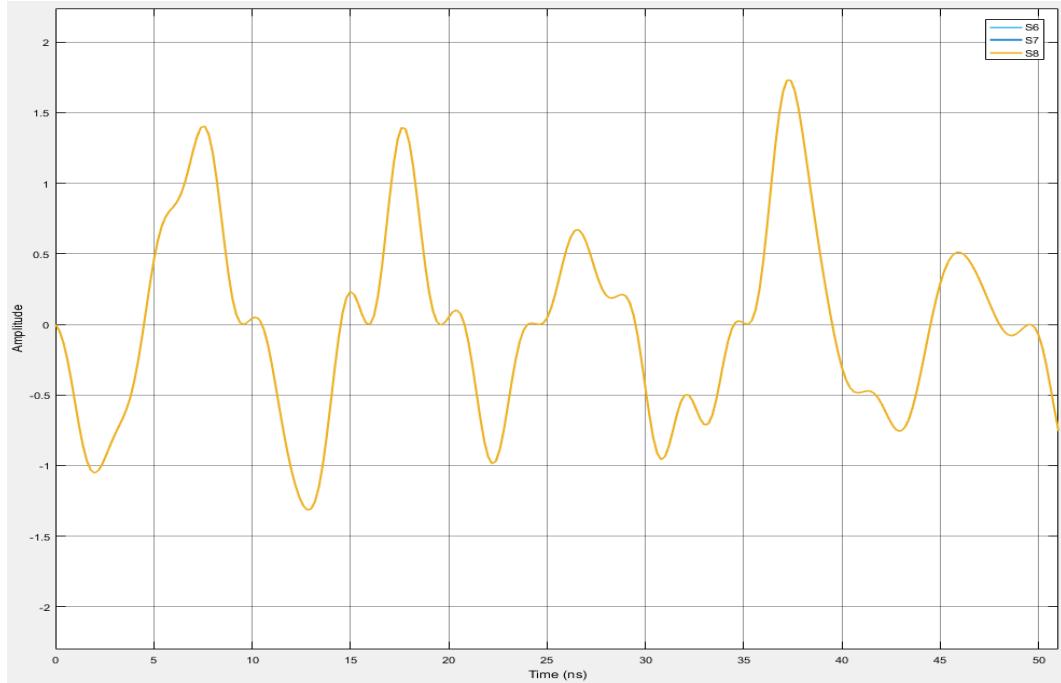


Figura 5.23: S6, S7 and S8

### Receiver setup

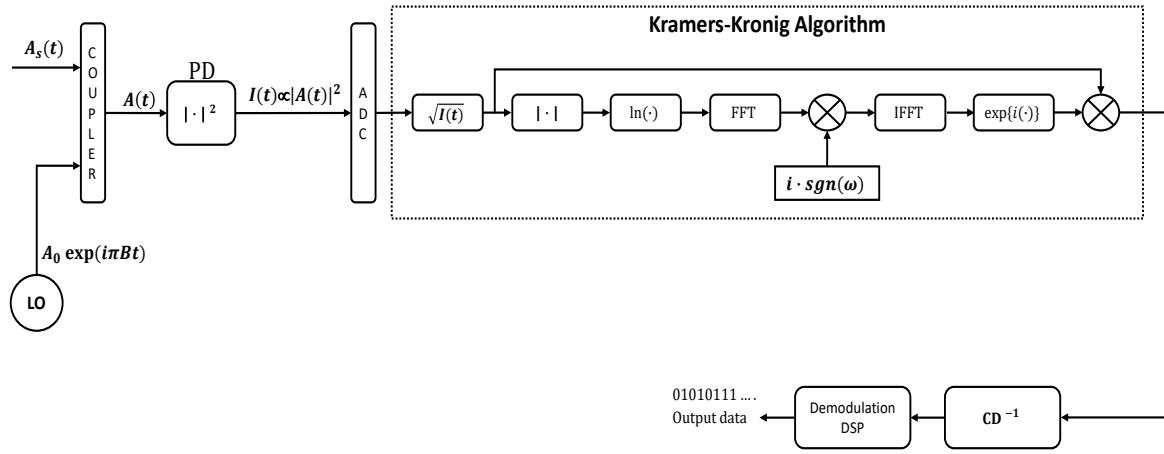


Figura 5.24: Receiver simulation setup

### 5.6.3 Experimental Analysis

As shown in the Figure 5.25, at the transmitter end, analytical signal generated with the help of netxpto and applied to the AWG. Waveform generated by AWG applied dual polarization IQ modulator which generates SSB signal in optical domain. SSB optical signal generated by both the IQ modulator is then combined using polarization beam combiner (PBC) and launched into the optical fiber.

At the receiver end, the PDM received signal first spitted by a polarization beam splitter (PBS). Each polarization is combined with an LO tapped from the transmit laser. The laser's wavelength and power are set to ensure that the received signal should satisfy the minimum phase condition. After direct-detection, Kramers-Kronig algorithm is performed on each polarization separately to recover full complex signal. After compensation of the chromatic dispersion, stokes parameter based poldemux algorithm can be applied to recover PDM signals.

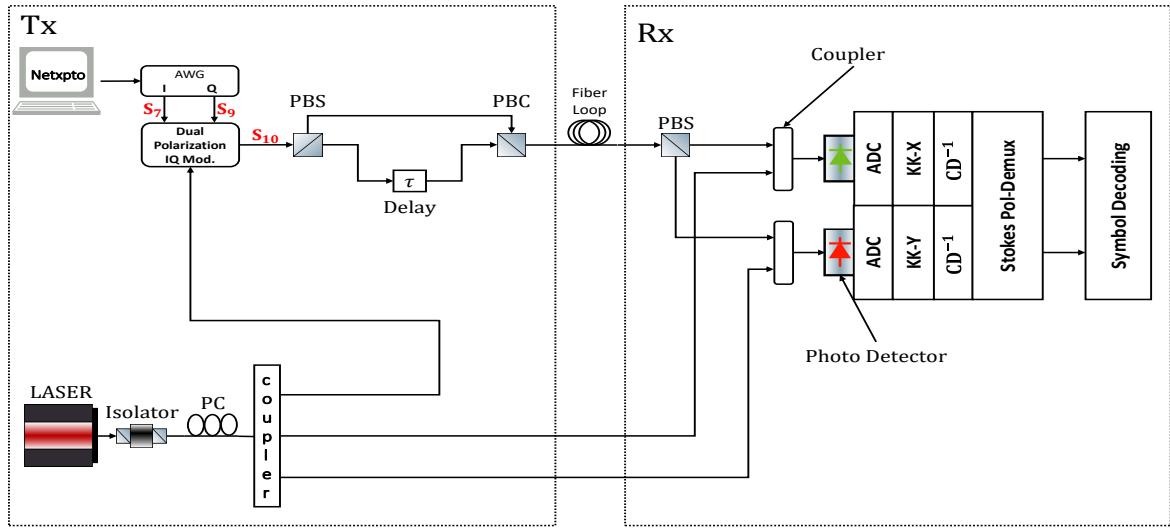


Figura 5.25: PDM Kramers-Kronig receiver experimental setup

### Status of equipment

Equipment name	Description	Status
LASER		✓
PC		✓
Coupler		✓
AWG		✓
Dual polarization IQ mod		✓
PBS		✓
PLC		✓
Delay		✓
Fiber loop		✓
Single photodetector	APD+TIA Optical Receiver: - Maximum bit rate: 10 Gb/s - Sensitivity: -26 dBm	✓

### 5.6.4 Comparative Analysis

### 5.6.5 Known Problems

Problem type	Description	Note
Simulator	Require a generalized block for FFT and IFFT	

---

## Bibliografia

- [1] Antonio Mecozzi, Cristian Antonelli, and Mark Shtaif. *Kramers-Kronig Coherent Receiver*. Optica, vol.3, no.11, 2016, p.1220., doi:10.1364/optica.3.001220.
- [2] Antonio Mecozzi. *Retrieving the full optical response from amplitude data by Hilbert transform*. Opt. Comm. 282, 4183-4187.
- [3] Antonio Mecozzi. *A necessary and sufficient condition for minimum phase and implication of phase retrieval*. arXiv:1606.04861.

### APPENDICES

#### Appendix A : SSB with graphical explanation

This section describes the SSB signal generation using Hilbert transformation method (Phase Shift Method). Consider a message signal  $m(t)$  with its frequency domain spectrum  $M(F)$  as shown in Figure 5.26. From the Figure 5.26, we can see that both the side are scaled by factor '1' which means it represents the original signal.

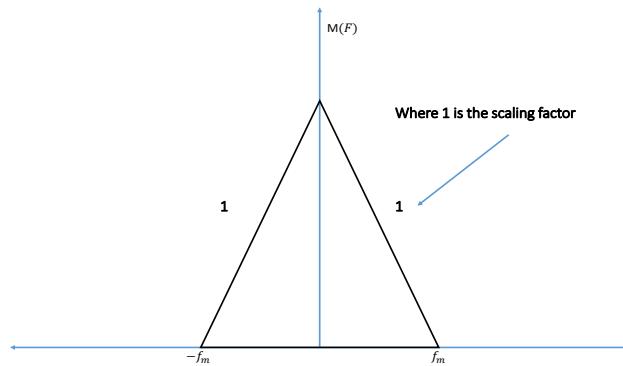


Figura 5.26: Original baseband signal

Now let's consider the modulated signal  $x(t)$  given as,

$$x(t) = m(t)\cos(2\pi f_c t) \quad (5.98)$$

Frequency domain representation of the equation 5.98 can be given as,

$$X(F) = \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c) \quad (5.99)$$

Here in equation 5.99, we can observe that each side band are scaled by  $\frac{1}{2}$  on the frequency spectrum. Figure displays the frequency domain representation of the modulated signal  $X(F)$ .

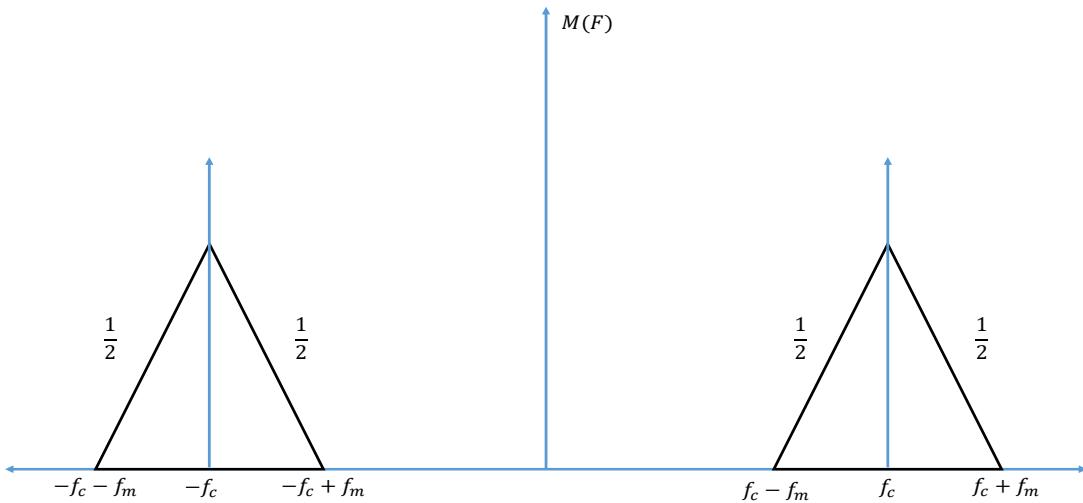


Figura 5.27: Original modulated signal

Next, we will discuss something more interesting which is called as Hilbert transform of the original message signal  $m(t)$ . As we discussed earlier, in the frequency domain, the Hilbert transformed signal  $\hat{M}(f)$  can be achieved by multiplying the Fourier transformed signal  $M(F)$  with  $[-isgn(F)]$ . Suppose we modulate the Hilbert transformed message signal  $\hat{m}(t)$

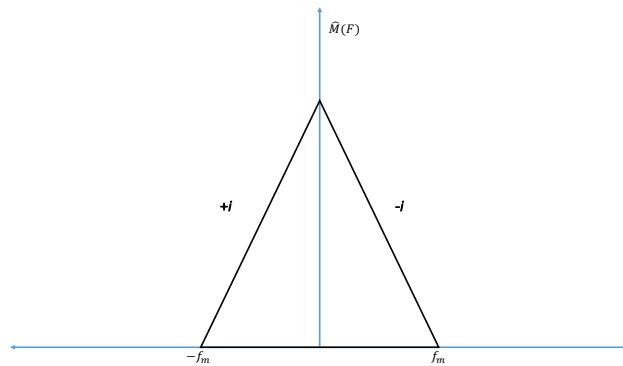


Figura 5.28: Hilbert transformed modulated signal

with the  $\sin(2\pi f_c t)$  (quadrature phase carrier), then we get the following results:

$$\begin{aligned}
 \hat{m}(t) \sin(2\pi f_c t) &= \hat{m}(t) \frac{e^{i2\pi f_c t} - e^{-i2\pi f_c t}}{2} \\
 &= \hat{m}(t) \frac{e^{i2\pi f_c t}}{2} - \hat{m}(t) \frac{e^{-i2\pi f_c t}}{2} \\
 &= \frac{\hat{M}(f - f_c)}{2i} - \frac{\hat{M}(f + f_c)}{2i} \\
 &= \frac{-i}{2} \hat{M}(f - f_c) + \frac{-i}{2} \hat{M}(f + f_c)
 \end{aligned} \tag{5.100}$$

The detailed explanation of the equation 5.100 has been given in the Figure 5.29 and 5.30. Figure 5.29 displays the spectrum of the  $\hat{M}(f + f_c)$  and  $\hat{M}(f - f_c)$  for the positive and negative frequencies respectively. The final equation resolution of equation displays that both positive and negative side of the spectrum multiplied with  $\frac{i}{2}$  and  $\frac{-i}{2}$  respectively. Finally the spectrum of the signal  $\hat{m}(t) \sin(2\pi f_c t)$  can be given as Figure 5.30.

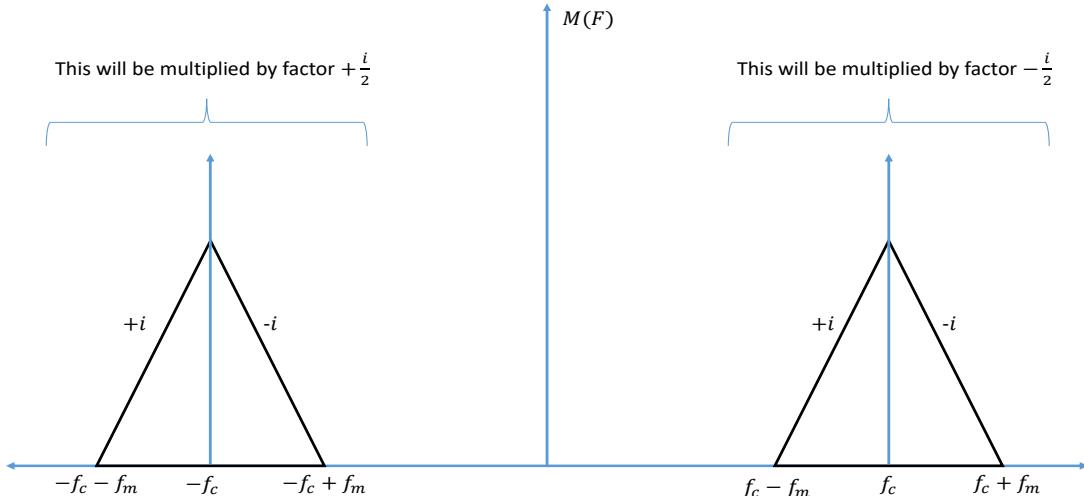


Figura 5.29: Hilbert transformed modulated signal

Further, summation of the two signals  $m(t) \cos(2\pi f_c t)$  and  $\hat{m}(t) \sin(2\pi f_c t)$  will generate the upper sideband SSB signal as follows,

$$u(t) = m(t) \cos(2\pi f_c t) - \hat{m}(t) \sin(2\pi f_c t) \tag{5.101}$$

From the above discussion, the spectrum of the Equation 5.101 can be given by the Figure 5.31. Similarly, for the lower sideband SSB can be generated by Equation,

$$u(t) = m(t) \cos(2\pi f_c t) + \hat{m}(t) \sin(2\pi f_c t) \tag{5.102}$$

#### Appendix B : Kramers-Kronig scheme

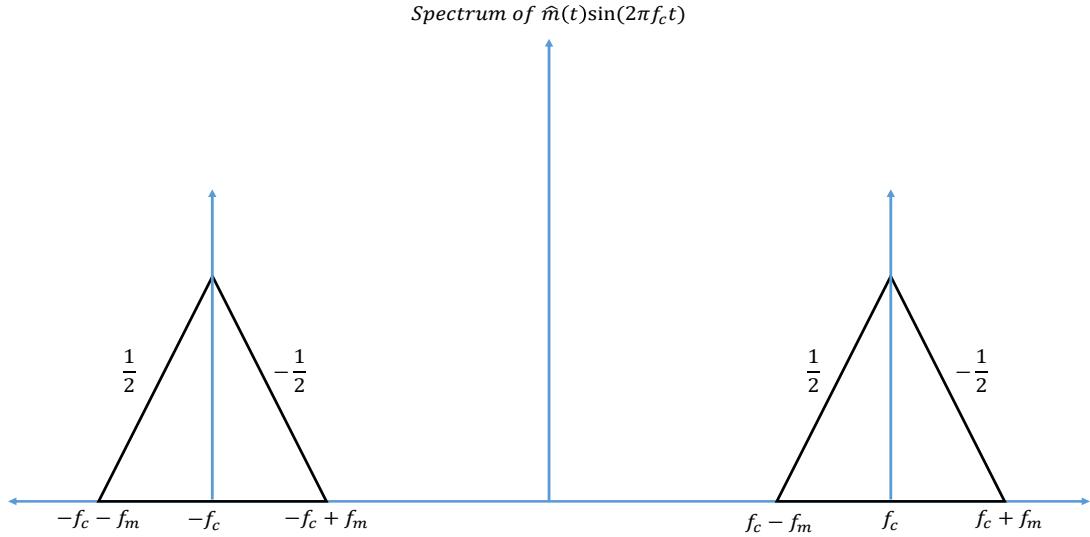


Figura 5.30: Hilbert transformed modulated signal

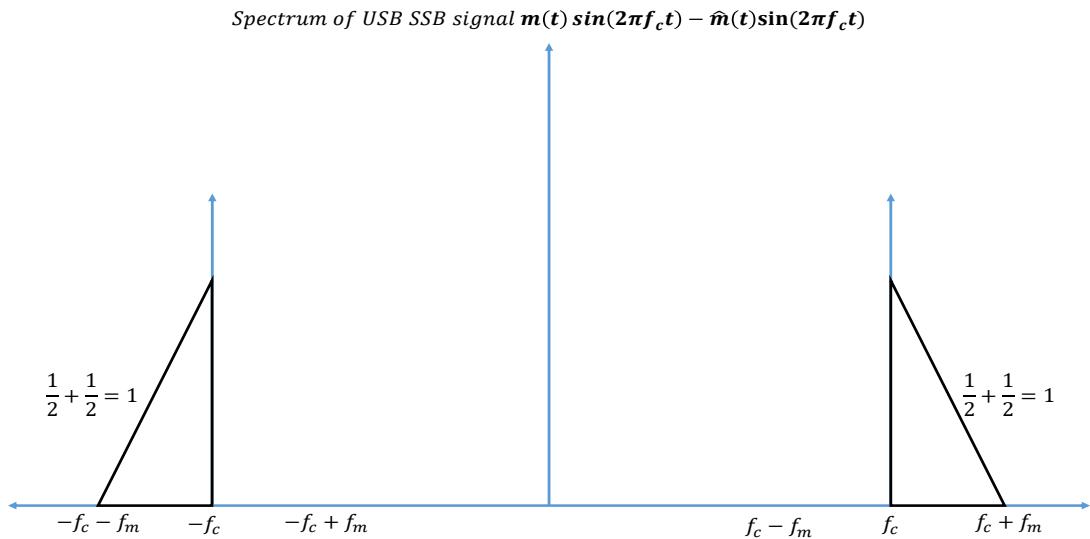


Figura 5.31: SSB signal spectrum

If we consider the complex envelope of the incoming electric field by  $A_s(t)$  confined within the optical bandwidth denoted by  $B$ . The LO assumed to be a continuous wave (CW) signal whose amplitude is  $A_0$  whose frequency coincides with the left edge of the information-carrying signal spectrum. Here, we assumed that  $A_0$  is real-valued and positive, which is equivalent to referring all phase value to that of LO.

The complex envelope of the field striking upon the photo-diode can be given as,

$$A(t) = A_s(t) + A_0 \exp(i\pi Bt) \quad (5.103)$$

The photo current  $I$  produced by the photo-diode is proportional to the field intensity  $I = |A(t)|^2$ , here proportionality constant considered as 1 for the sake of simplicity. If  $A_0$  is large enough to ensure that the signal  $A(t)\exp(-i\pi Bt) = A_0 + A_s(t)\exp(-i\pi Bt)$  is minimum phase. The discussed hypothesis can be used to reconstruct the signal  $E_s(t)$  as follows[1]:

$$A_s(t) = \{\sqrt{I(t)}\exp[i\phi_E(t)] - A_0\}\exp(i\pi Bt) \quad (5.104)$$

$$\phi_A(t) = \frac{1}{2\pi} p.v. \int_{-\infty}^{\infty} dt' \frac{\log[|I(t')|]}{t - t'} \quad (5.105)$$

## 5.7 Quantum Oblivious Key Distribution with Discrete Variables

<b>Student Name</b>	:	Mariana Ramos
<b>Starting Date</b>	:	September 18, 2017
<b>Goal</b>	:	Quantum oblivious key distribution (QOKD) implementation with discrete variables.
<b>Directory</b>	:	sdf/ot_with_discrete_variables.

Oblivious Transfer (OT) is a fundamental primitive in multi-party computation. The one-out-of-two OT consists in a communication protocol between Alice and Bob. At the beginning of the protocol Alice has two messages  $m_1$  and  $m_2$  and Bob wants to know one of them,  $m_b$ , without Alice knowing which one, i.e. without Alice knowing  $b$ , and Alice wants to keep the other message private, i.e. without Bob knowing  $m_{\bar{b}}$ . therefore two conditions must be fulfilled:

1. The protocol must be concealing, i.e at the beginning of the protocol Bob does not know nothing about Alice's messages, while at the end of the protocol Bob will learn the message  $m_b$  chosen by him.
2. The protocol is oblivious, i.e Alice cannot learn anything about Bob's choice, bit  $b$ , and Bob cannot learning nothing about the other message  $m_{\bar{b}}$ .

In order to implement OT between two parties (Alice and Bob) they must be able to exchange continuously oblivious keys, i.e a QOKD system must exist between them.

### 5.7.1 Quantum Oblivious Key Distribution System (QOKD)

In this section we are going to describe the Quantum Oblivious Key Distribution system (QOKD). The QOKD system enables two parties (Alice and Bob) to share a set of keys. These keys have the particularity of being half right and half wrong. Only Bob knows which are right and wrong keys.

Considering a discrete variables implementation, both Alice and Bob agree with the following correspondence, where + corresponds to *Rectilinear Basis* and  $\times$  corresponds to *Diagonal Basis*,

<i>Basis</i>	
0	+
1	$\times$

Alice and Bob also agree with the bit correspondence for each direction for each basis. For *Rectilinear basis*, "+",

	<i>Basis "+"</i>
0	$\rightarrow (0^\circ)$
1	$\uparrow (90^\circ)$

and for *Diagonal Basis*, "×",

	Basis "×"
0	↖ (−45°)
1	↗ (45°)

1. The first step is to establish for both Alice and Bob the block length  $l$ . In this case, lets assume  $l = 16$ . Alice randomly generate a bit sequence with length  $l$ . Therefore, she must define two sets randomly:  $S_{A1}$  which contains the basis values; and  $S_{A2}$ , which contains the key values.

In that case, lets assume she generates the following sets  $S_{A1'}$  and  $S_{A2'}$ :

$$S_{A1'} = \{0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1\},$$

$$S_{A2'} = \{1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1\}.$$

2. Next, Alice sends to Bob throughout a quantum channel  $l$  photons encoded using the basis defined in  $S_{A1'}$  and according to the key bits defined in  $S_{A2'}$ .

Therefore, in the current example, Alice sends the following photons,

$$\begin{aligned} S_{AB} &= \{\uparrow, \uparrow, \nearrow, \nwarrow, \rightarrow, \rightarrow, \downarrow, \nearrow, \uparrow, \rightarrow, \downarrow, \nearrow, \downarrow, \uparrow, \nearrow\} \\ &= \{90^\circ, 90^\circ, 45^\circ, -45^\circ, -45^\circ, 0^\circ, 0^\circ, -45^\circ, 45^\circ, 90^\circ, 0^\circ, -45^\circ, 45^\circ, -45^\circ, 90^\circ, 45^\circ\}. \end{aligned}$$

3. Bob also randomly generates  $l = 16$  bits, which are going to define his measurement basis,  $S_{B1'}$ . Lets assume,

$$\begin{aligned} S_{B1'} &= \{0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1\} \\ &= \{+, \times, \times, +, +, \times, +, \times, \times, +, \times, +, +, +, \times\}. \end{aligned}$$

Bob will get  $l$  results:

$$S_{B2'} = \{1, -, \underline{0}, 0, -, 1, \underline{1}, -, 1, -, 1, 0, 1, 1, \underline{0}, 1\}.$$

The "−" corresponds to no clicks in Bob's detector, due to attenuation. The underlined values are bits which were measured with a correct basis but an error has occurred due to imperfections in the quantum communication system.

4. Bob is going to send a "−1" or a hash value to Alice for each measurement that he performed, thereby being "−1" the measurements which correspond to no clicks. In this case, we are going to assume that the hash value is calculated using the SHA-256 algorithm [3]. In detail, Bob has two sets  $S_{B1'}$  and  $S_{B2'}$  and he is going to generate the set  $S_{BH1}$  with  $l$  values ("−1" or hash values calculated for each position of  $S_{B1'}$  with the correspondent position of  $S_{B2'}$ ). Therefore, Bob will send to Alice the following set:

$$S_{BH1} = \{S_1, -1, S_2, S_3, -1, S_4, S_5, -1, S_6, -1, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}\}.$$

5. Since Alice has received the confirmation of measurement from Bob, i.e after Alice has received  $S_{BH1}$ , she sends throughout a classical channel the basis which she has used to codify the photons updated with the information about the no received photons,

$$S_{A1'} = \{0, -1, 1, 1, -1, 0, 0, -1, 1, -1, 0, 1, 1, 1, 0, 1\}$$

Due to attenuation, the previous sets are reduced to the length 12 and they shall be replaced by the following:

$$S_{A1} = \{0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1\},$$

$$S_{A2} = \{1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1\},$$

$$S_{B1} = \{0, 1, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1\},$$

$$S_{B2} = \{1, \underline{0}, 0, 1, \underline{1}, 1, 1, 0, 1, 1, \underline{0}, 1\}$$

Note that  $S_{B2}$  still has errors.

6. In order to know which photons were measured correctly, Bob does the operation  $S_{B3} = S_{B1} \oplus S_{A1}$ . In the current example,

$$\begin{array}{c|cccccccccccc} S_{B1} & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ \hline S_{A1} & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ \oplus & \hline 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}$$

In this way, Bob gets

$$S_{B3} = \{1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1\}.$$

When Bob uses the right basis he gets the values correctly, apart from possible errors in transmission, when he uses the wrong basis he just guess the value. The values "1" correspond to the values he measured correctly and "0" to the values he just guessed. Thus, Bob is building two sets of keys, one with correct basis measurements values and other with the wrong basis measurement values that he just guessed.

Thus, Bob has two pair of sets, one for the right basis,

$$S_{B_{rp}} = \{1, 2, 5, 6, 8, 11, 12\},$$

$$S_{B_{rb}} = \{1, 0, 1, 1, 0, 0, 1\},$$

where  $S_{B_{rp}}$  is the set of positions and  $S_{B_{rb}}$  is the set of bit values he measured for each position. The other pair is for photons he measured with the wrong basis and then he just guessed the values,

$$S_{B_{wp}} = \{3, 4, 7, 9, 10\},$$

$$S_{B_{wb}} = \{0, 1, 1, 1, 1\},$$

where  $S_{B_{wp}}$  is the set of positions and  $S_{B_{wb}}$  is the set of bit values he measured for each position.

Nevertheless, due to errors in transmission, some bits in  $S_{B_{rb}}$  may be not right.

At this point, in order to test Bob's honesty and to estimate the QBER of the channel, Alice is going to ask Bob to open some pairs of the Bob's sets. The definition of the protocol to test Bob's honesty is still an open issue. However, depending on the QBER estimated by her, Alice must have a parameter to set the number of right position she wants to open, i.e she must open a minimum number of right position in order to guarantee a minimum QBER. This will increase the security of the protocol. Alice chooses some positions to open and tells Bob which positions she wants to open. Bob sends to Alice the pairs she chose and then these pairs are eliminated from them sets. Lets assume she asked to open the positions 10, 11 and 12. If she concludes Bob is not being honest, she stops the protocol and they must start it again. Otherwise, the protocol continues. Lets assume Alice has verified these pairs using the hash function committed by Bob and concluded that he is being honest. Therefore, she sends to Bob the QBER estimated by her.

Now, Bob has the previous sets replaced by the following,

$$\begin{aligned} S_{B_{rp}} &= \{1, 2, 5, 6, 8\} \\ S_{B_{rb}} &= \{1, 0, 1, 1, 0\} \\ S_{B_{wp}} &= \{3, 4, 7, 9\} \\ S_{B_{wb}} &= \{0, 1, 1, 1\} \end{aligned}$$

Bob is going to use a modified version of *Cascade algorithm* to correct the errors due transmission.

### Modified version of Cascade Algorithm

The Cascade algorithm is often used with a key set where all values are supposed right. In this case, Bob has two pairs of sets, one with the position and bit values of photon he measured with the correct basis and other with position and bit values of photon he measured with the wrong basis. He only needs to apply the Cascade algorithm in the set that he measured the photons correctly [4]. However, he must apply a modified version of the Cascade in the other set in order to keep in secret from Alice which set corresponds to right and which set corresponds to wrong measurements.

Bob randomly generates a bit value. If he gets 0, he will send to Alice the set  $\{S_{B_{rp}}, S_{B_{wp}}\}$ . Otherwise, if he gets 1 he will send the set  $\{S_{B_{wp}}, S_{B_{rp}}\}$ . This guarantee that Alice does not know which is the right or wrong set. Lets assume this random bit is "0"and he sends  $\{S_{B_{rp}}, S_{B_{wp}}\}$ .

- (a) Bob starts by applying the normal cascade to the set  $S_{B_{rb}}$ . After both know the error estimative Bob determine if the error rate is above the fail threshold. If it is

truth they must start the procedure again. Lets assume the estimated error rate is acceptable. Bob and Alice use a random permutation which is represented in figure 5.32 for a larger number of bits (agreed at the beginning) by applying it to the shifted keys, in order to guarantee the spread out of the error bits randomly and to separate consecutive errors from each other.

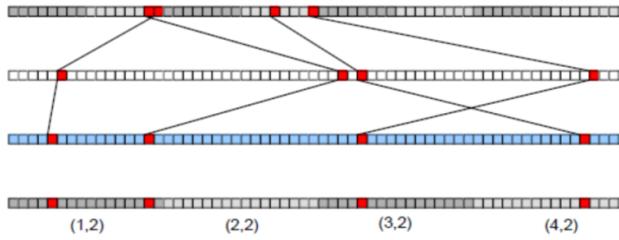


Figura 5.32: Cascade Algorithm - permutation

- (b) Bob and Alice divide all the shifted key bits into blocks of  $N$  bits depending on the estimated error rate in order to have one or no error per block. In general, the sets of keys are too large and it is easier to explain the algorithm based on a larger number of bits. Therefore, figure 5.33 represents the typical cascade initial steps. However, in this case, the set to be corrected only has five bits, therefore they divide the set in two sub-blocks, one with 3 bits and other with 2 bits.

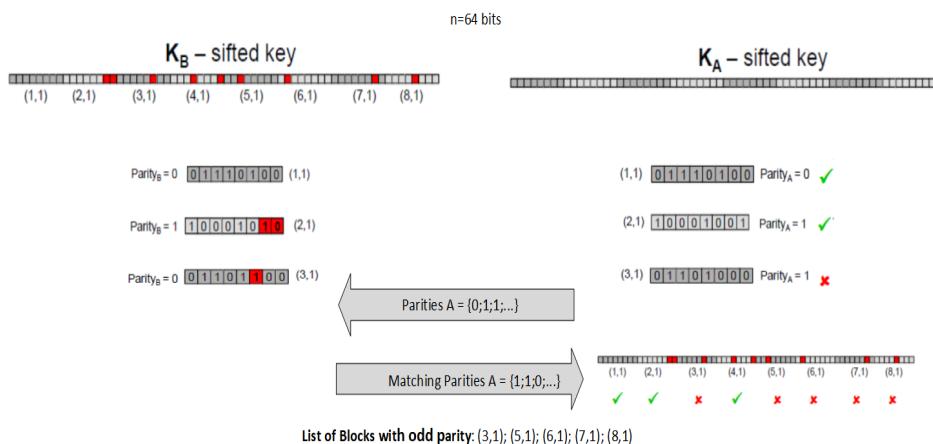


Figura 5.33: Cascade Algorithm

- (c) They use a classical channel to compare the block parities. For blocks with different parities, an odd number of errors must exist, otherwise an even number of errors would mask each other. Thus, the block in which the parities disagree is divided in half into two smaller blocks of length  $\frac{N}{2}$ , and another parity check is performed on the first sub-block, as one can see in figure 5.34. As it was referred above, there is at least one error in one sub-block being the error location revealed

by the parity of one sub-block. In other words, if the parity of the first sub-block passes, the error will be in the second sub-block. The sub-block with error will be sub-divided until the error is found.

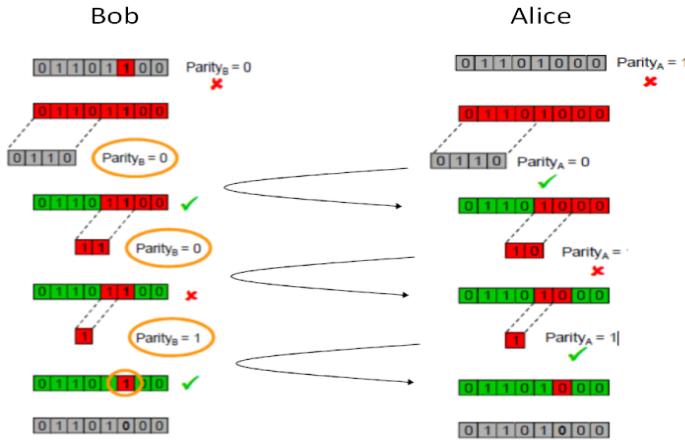


Figura 5.34: Cascade Algorithm - example of error correction

- (d) When the error is corrected, the last bit of the block is discard in order to prevent the gain of additional information by Bob.

In this case, lets assume the set of right positions was corrected with the algorithm described above and it will be replaced by the following:

$$\begin{aligned} S_{B_{rp}} &= \{1, 2, 5, 6, 8\} \\ S_{B_{rb}} &= \{1, 1, 0, 1, 0\} \end{aligned}$$

In order to test Alice's honesty, Bob must verify if the QBER sent by Alice is a realistic value. If it is not he stops the protocol and they must start again. Otherwise, the protocol continues.

After that, Bob needs to apply the Fake Cascade to the set  $S_{B_{wb}}$ . The main goal of this step is to convince Alice she is performing the real Cascade but she is not.

- (a) First of all, based on the positions contained in  $S_{B_{wb}}$ , Bob must build an array with the correspondent bits in a random order and informs Alice the order of positions. In order to best explain this version of the algorithm, lets assume a larger set of bits.

Bob sends to Alice throughout a classical channel the new positions order as if it were the permutation step represented in figure 5.32 in real Cascade algorithm.

- (b) Assuming each of them has a set with 32 bits randomly organized by Bob, they divide the supposed shifted keys in blocks with N bits according to the estimated

error rate. As the QBER is the same as for real cascade, Bob will assume the same number of errors, even if he starts for this modified version he can know the number of errors from QBER estimated by Alice.

- (c) Bob and Alice use a classical channel to compare the block parities. Alice sends to Bob her parity list. Based on Alice's parity list, Bob sends a block list with odd parities, i.e the blocks position in which parity supposed disagree. This list is randomly built based on the number of errors considered by Bob, i.e if he considered five errors from QBER estimative, he will distributed them randomly and after that he will fill the remaining spaces with even parities. Bob sends to Alice the set with the list of odd parities, i.e the list of sub-sets he has different parities than Alice.
- (d) The blocks with errors will be consecutively divided until they found the supposed errors. Since we have assumed there were five errors, this is the number of errors that Alice must supposedly correct.

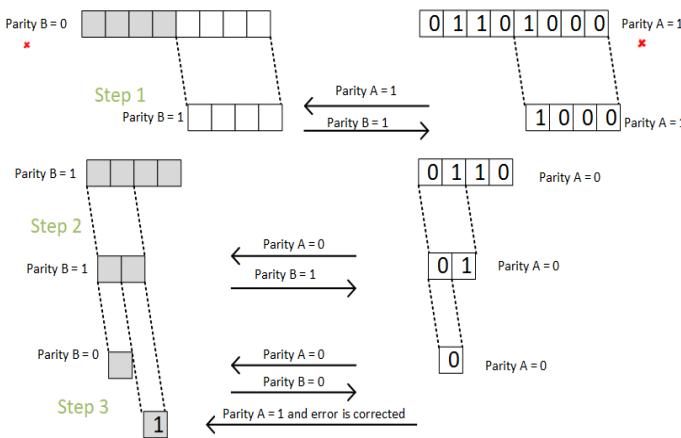


Figura 5.35: Fake cascade - example of error correction

Lets assume one of the blocks with error and analyse figure 5.35. Bob starts with a set filled with random bits, therefore we do not need to know which bits are. Alice starts by dividing her set in half with two blocks with  $N$  bits.

**Step 1:** Bob chooses one of the two blocks and informs Alice she must send the parity of this block. Lets assume he chose sub-block 2. She sends the parity and Bob is going to send his parity, which after know Alice's block parity he send the opposite parity. As referred in normal Cascade, there must be one error or no error in each block. Thus, since the parities disagree, the error must be in seconde block. They start the procedure to correct it.

**Step 2:** Bob divides again the sub-block in half with  $\frac{N}{2}$  bits and asks Alice for the parity of the first sub-block. She sends her parity equals to 0 and Bob sends to her the opposite parity again.

**Step 3:** They divide the sub-block in half again and Bob asks Alice for the parity

of the first bit. Alice sends to him the parity equals to her. As the error is not in the first bit, it must be in the second, therefore Bob is able to correct this bit with the information sent by Alice.

Note that Bob make his choice of which half analyse first using a random bit generator result. If he got "0" he starts with the first half of the sub-block, otherwise, if he got "1", he starts with the second half. In addition, they must discard the last bit of each block and sub-block in which fake Cascade were applied in order to guarantee that Bob does not gain additional information.

In this case, after apply the fake Cascade to  $S_{B_{wb}}$ , lets assume,

$$\begin{aligned} S_{B_{wp}} &= \{3, 4, 7, 9\} \\ S_{B_{wb}} &= \{0, 1, 1, 0\} \end{aligned}$$

If Bob starts by applying the fake Cascade, he must test Alice's honesty at the beginning of the real Cascade application, based on the number of errors he has. If he thinks that the QBER sent by Alice is unrealistic, he stops the protocol at this point.

7. When Alice sends to Bob a photons set, they are building a set of pairs (array positions and bit values which correspond to measured photons at Bob's side and to the key bit with the photon was encoded at Alice's side). The main goal is to guarantee that Bob has the same number of right and wrong pairs. In addition, they must know information about  $t$  (represented in figure 5.36) which corresponds to the points where the previous condition is verified.

Since Bob has sent to Alice the information about the smallest set, in this example, Alice know that there are four pairs of wrong positions and five pairs of right positions. Alice must destroy one of the right pairs by asking Bob to open it. Therefore, at  $t = 8$  both know that there are the same number of right and wrong pairs thereby being the main goal guaranteed.

As we can see in figure 5.36, unlike Bob, Alice does not know which positions corresponds to right or wrong measurements performed by Bob. They have been building these sets during all protocol.

### 5.7.2 OT Protocol with QOKD system

At this time, we are going to describe the oblivious transfer protocol with detail. As it was referred at the beginning, Alice sends two messages to Bob and he wants to know one of them. Alice does not know which message Bob wants and Bob only know the message he wants, i.e he does not know anything about the other message. Furthermore, only Alice knows information about messages  $m_0$  and  $m_1$ . In this case, lets assume the following two messages with size  $s = 4$ ,  $m_0 = \{0011\}$  and  $m_1 = \{0001\}$ . Alice must guarantee  $t = s \times 2$ . In order to do that, she must destroy the remaining pairs. In this case, there is no need to do that because they have a set for  $t = 8$  with the same number of wrong and right pairs.

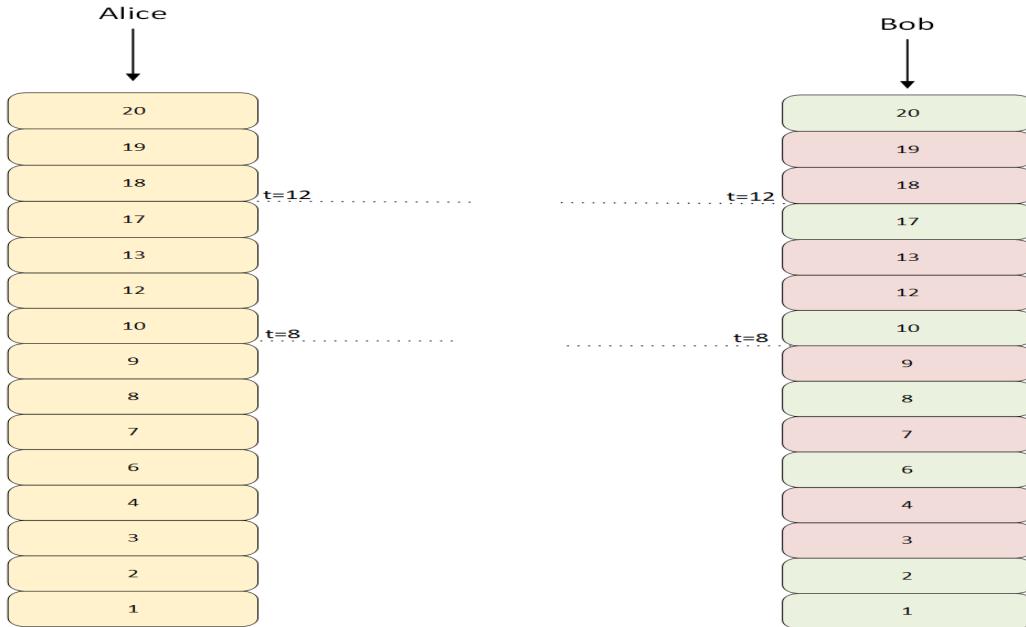


Figura 5.36: Alice and Bob key sets.

1. Bob defines two new sub-sets,  $I_0$  and  $I_1$ .  $I_0$  is a set of values with photons array positions which Bob just guessed the measurement since he did not measure them with the same basis as Alice,  $I_1$  is a set of values with photons array positions which Bob measured correctly since he used the same basis as Alice used to encoded them. The position of the pairs of each right and wrong message are in the keys sets that they have been building during the protocol.

In this example, the message size is 4. Since, at this time  $t = 10$  and we have 5 right pairs and 5 wrong pairs, Alice ask to Bob to open one right pair and one wrong pair in order to both have exactly the message's size number of right and wrong pairs. Lets assume that Alice opened two pairs, position 15 which is a wrong measurement and position 10 which is a right measurement. We have now  $t = 8$ .

Next, Bob defines two sub-sets with size  $s = 4$ :

$$I_0 = \{3, 4, 7, 9\},$$

and

$$I_1 = \{1, 2, 6, 8\},$$

where  $I_0$  is the sequence of positions in which Bob was wrong about basis measurement and  $I_1$  is the sequence of positions in which Bob was right about basis measurement. Bob sends to Alice the set  $S_b$

Thus, if Bob wants to know  $m_0$  he must send to Alice throughout a classical channel the set  $S_0 = \{I_1, I_0\}$ , otherwise if he wants to know  $m_1$  he must send to Alice throughout a classical channel the set  $S_1 = \{I_0, I_1\}$ .

2. Alice is sure about Bob's honesty, since she knows he only has 4 right basis to measure the photons. In addition, Alice cannot know which message Bob chose because she did not know the order that he sent the sets.
3. Lets assume Bob sent  $S_0 = \{I_1, I_0\}$ . Alice defines two encryption keys  $K_0$  and  $K_1$  using the values in positions defined by Bob in the set sent by him. In this example, lets assume:

$$K_0 = \{1, 1, 1, 0\}$$

$$K_1 = \{0, 0, 0, 1\}.$$

Alice does the following operations:

$$m = \{m_0 \oplus K_0, m_1 \oplus K_1\}.$$

$$\begin{array}{c|cccc} m_0 & 0 & 0 & 1 & 1 \\ \hline K_0 & 1 & 1 & 1 & 0 \\ \oplus & 1 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{c|cccc} m_1 & 0 & 0 & 0 & 1 \\ \hline K_1 & 0 & 0 & 0 & 1 \\ \oplus & 0 & 0 & 0 & 0 \end{array}$$

Adding the two results,  $m$  will be:

$$m = \{1, 1, 0, 1, 0, 0, 0, 0\}.$$

After that, Alice sends to Bob the encrypted message  $m$  through a classical channel.

4. When Bob receives the message  $m$ , in the same way as Alice, Bob uses  $S_{B1}$  values of positions given by  $I_1$  and  $I_0$  and does the decrypted operation. In this case, he does following operation:

$$\begin{array}{c|cccccccc} m & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ \oplus & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{array}$$

The first four bits corresponds to message 1 and he received  $\{0, 0, 1, 1\}$ , which is the right message  $m_0$  and  $\{0, 1, 1, 0\}$  which is a wrong message for  $m_1$ .

### 5.7.3 Simulation

First of all, the protocol will be simulated and then a experimental setup will be built in the laboratory.

The main goal of this simulation is to demonstrate that Bob was able to learn correctly message  $m_b$  and he does not know the message  $m_{\bar{b}}$ .

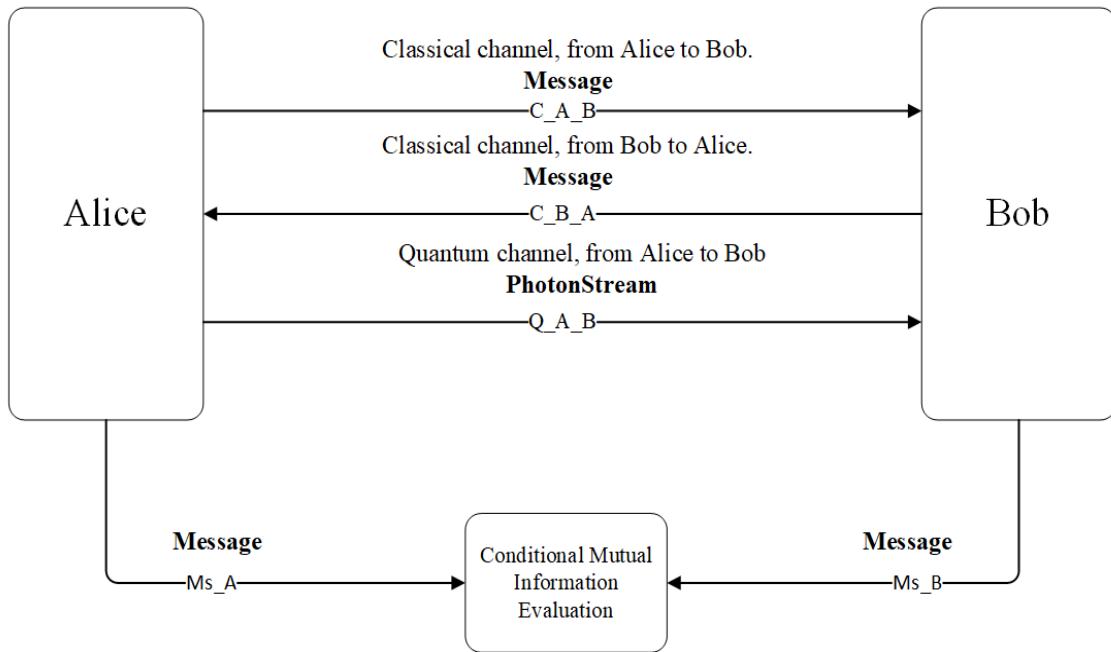


Figura 5.37: Simulation diagram at a top level

As one may see in figure 5.37 this simulation will have three top level blocks. Two of them are Alice and Bob and they are connected through two classical channels and one quantum channel. In addition, a third block will be performed in order to calculate the *Mutual Information*. The mutual information (MI) between Alice and Bob is defined in terms of their joint distribution.

1. In figure 5.38 one can observe a block diagram of the simulation at Alice's side. As it is shown in the figure, Alice must have one block for random number generation which is responsible for basis generation to polarize the photons, and for key random generation in order to have a random state to encode each photon. Furthermore, she has a Processor block for all logical operations: array analysis, hash function results validation, random number generation requests, and others. This block also receives the start information, i.e. message size  $s$  and messages  $m_0$  and  $m_1$ , as well as information from Bob, i.e sets  $I_0$  and  $I_1$ , hash function results, and others. In addition it is responsible for set the initial length  $l$  of the first array of photons which will send to Bob. This block also must be responsible for send classical information to Bob. Finally, Processor block will also send a real continuous time signal to single photon

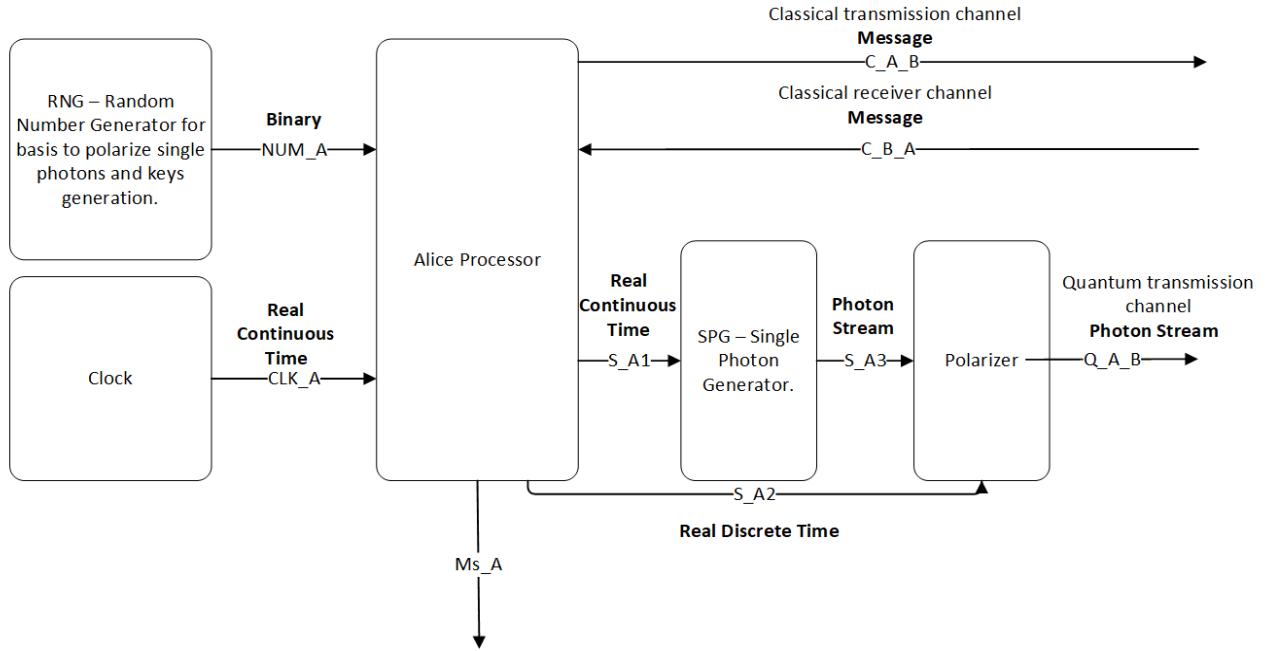


Figura 5.38: Simulation diagram - Alice's side

generator, in order to generate photons according to this signal, and finally this block also sends to polarizer a real discrete signal in order to inform the polarizer which basis it should use. Therefore, she has two more blocks for quantum tasks: the single photon generator and the polarizer block which is responsible to encode the photons generated from the previous block and send them throughout a quantum channel from Alice to Bob.

Finally, Alice's processor has an output to Mutual Information top level block,  $Ms_A$ .

2. In figure 5.39 one can observe a block diagram of the simulation at Bob's side. From this side, Bob has one block for Random Number Generation which is responsible for randomly generate basis values which Bob will use to measure the photons sent by Alice throughout the quantum channel. Furthermore, this Block will generate the random bits that Bob needs in Modified Version of Cascade Algorithm. Like Alice, Bob has a Processor block responsible for all logical tasks, i.e Hash function generation, analysing functions, requests for random number generator block, etc. Additionally, it receives information from Alice throughout a classical channel and a quantum channel but it sends information to Alice only throughout a classical channel. Furthermore, Bob has one more block for single photon detection which receives from processor block a real discrete time signal, in order to obtain the basis it should use to measure the photons.

Finally, Bob's processor has an output to Mutual Information top level block,  $Ms_B$ .

3. Mutual Information calculation

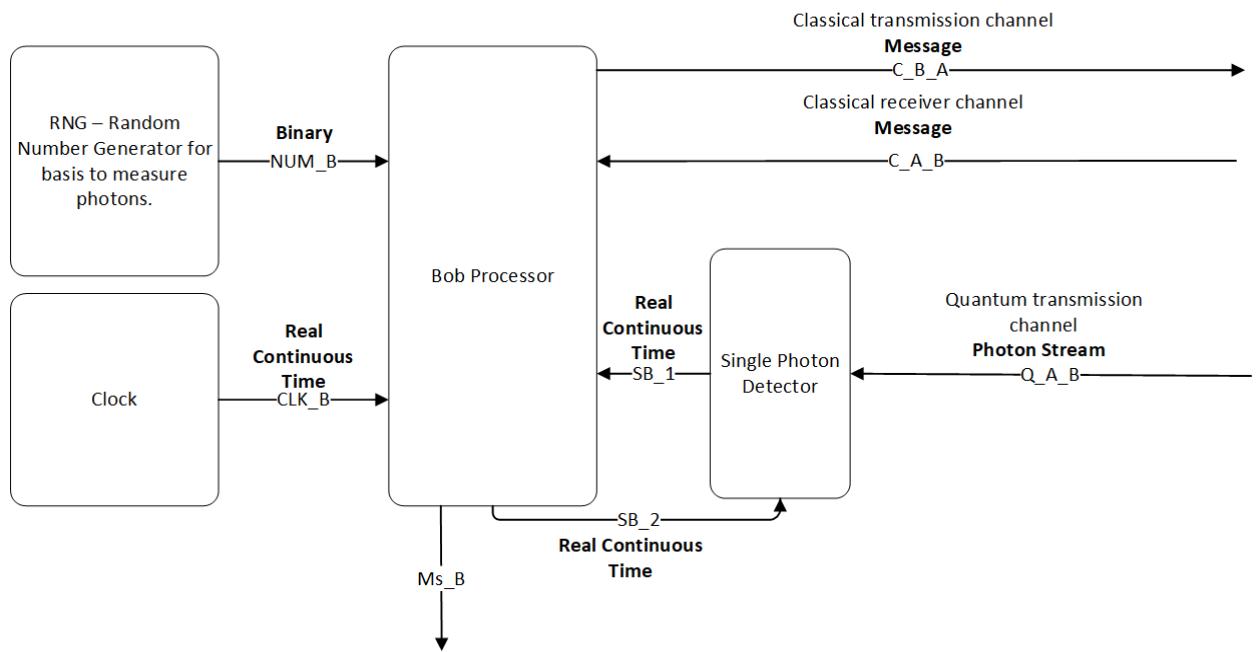


Figura 5.39: Simulation diagram - Bob's side

Tabela 5.3: System Signals

Signal name	Signal type	Status
NUM_A	Binary signal	
NUM_B	Binary signal	
CLK_A	Real continuous Time	
CLK_B	Real continuous Time	
C_A_B	Message	
C_B_A	Message	
S_A1	Real continuous Time	
S_A2	Real discrete Time	
S_A3	Photon Stream	
Q_A_B	Photon Stream	
Ms_A	Message	
Ms_B	Message	
S_B1	Real continuous Time	
S_B2	Real continuous Time	

Tabela 5.4: System input parameters

Parameter	Default Value	Description
messageSize	4	Size of the message Alice must send to Bob.
blockLength	16	Block length.
systemRate		

Tabela 5.5: Header Files

File name	Description	Status
random_number_generator.h		
real_continuous_time.h		
real_discrete_time.h		
single_photons_generator.h		
single_photons_detector.h		
encoder.h		
decoder.h		
messageToSend.h		
message_toReceive.h		
alice_tasks.h		
Bob_tasks.h		
mutual_information.h		
cascade_truth.h		
cascade_fake.h		
Sha256.h		

Tabela 5.6: Source Files

File name	Description	Status
random_number_generator.c		
real_continuous_time.c		
real_discrete_time.c		
single_photons_generator.c		
single_photons_detector.c		
encorder.c		
decoder.c		
messageToSend.c		
message_toReceive.c		
alice_tasks.c		
Bob_tasks.c		
mutual_information.c		
QOKD_main.c		
cascade_truth.c		
cascade_fake.c		
Sha256.c		

#### 5.7.4 Experimental

In figure 5.40 are presented the experimental setup to be performed in the lab. The main goal is to build an experimental setup in which Alice and Bob communicate through two classical channels and one quantum channel that will have only one direction (Alice to Bob).

We will use a experimental setup which has already been used in BB84 protocol implementation [?]. In figure 5.40 there are at Alice's side a Laser Semiconductor (**LD**), a Mach-Zenhder Modulator (**MZM**), three Faraday Mirrors (**FM1**, **FM2**, **FM3**), a four-port polarization beam splitter (**PBS1**), a polarization-insensitive phase modulator (**PM1**), a optical circulator (**OC1**) to allows the input and output optical pulses to be separated and finally a variable optical attenuator (**VOA**) in order to achieve the single-photon regime. At Bob's side there is first a polarizer controller (**PC**) to correct the birefringence effects introduced by the quantum channel, a optical circulator (**OC2**) to separate the input and output optical pulses, the photons received are decoded by using the same system which were used to encode them, and then the photon are recorded by two single photon detectors **D1**, **D2**) after a half-wave plate and a three-port polarization beam splitter (**PBS3**). Furthermore, the two processors (Alice and Bob) must be able to communicate in a bi-directional classical channel to exchange all information that they need.

In table 5.7 are described all components needed to build the experimental setup.

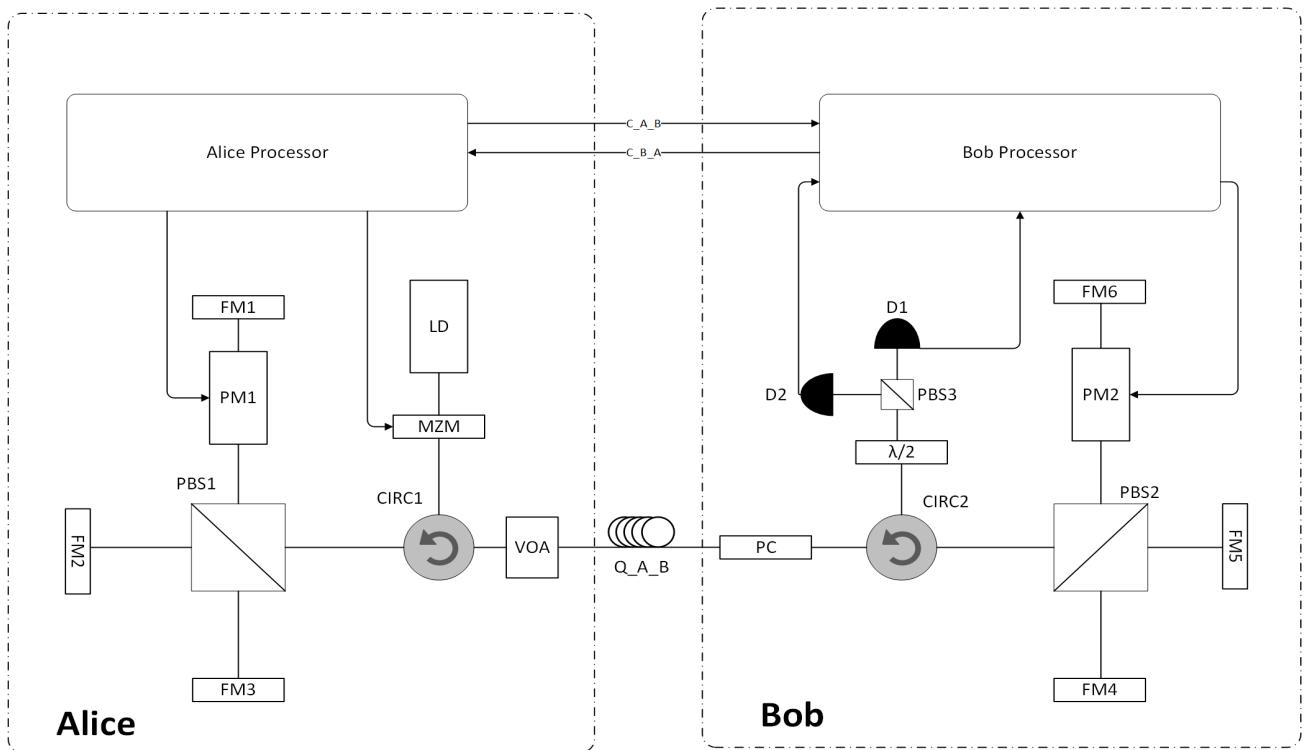


Figura 5.40: QOKD Experimental setup

Tabela 5.7: List of material

Material Name	Quantity	Status
Laser semiconductor 1550nm	1	✓
Manual polarization controller	5	
Faraday Mirror (FM)	6	
Mach-Zehnder Modulator	1	✓ 2.56GHz
Single Photon Detector	2	✓
Phase modulator	2	
Four-port polarization beam splitter	2	
Three-port polarization beam splitter	1	
Half-wave plate	1	
Optical circulator	2	
Variable Optical Attenuator	1	✓
Computer	1	

---

## Bibliografia

- [1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.
- [2] Ryo Namiki and Takuya Hirano. Security of quantum cryptography using balanced homodyne detection. *Physical Review A*, 67(2):022308, 2003.
- [3] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.
- [4] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.
- [5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

## 5.8 Radio Over Fiber Transmission System

<b>Student Name</b>	:	Celestino Martins
<b>Starting Date</b>	:	September 25, 2017
<b>Goal</b>	:	Simulation of Radio over fiber Transmission considering the uplink of base station cooperation systems.

Radio over fiber (RoF) technology comprises the transmission over fiber technology, where radio signal is modulated onto optical carrier and transmitted over an optical fiber link to provide a simple antenna front ends with increased capacity and broadband wireless services. In this network a central processing units (CPU) is connected to numerous base stations (BSs) via optic fibers. That means, RoF networks use optic fiber links to distribute radio frequency (RF) signals between the CPU and BSs. The downlink RF signals are distributed from a CPU to many BSs through the fibres, while the uplink signals received at BSs are sent back to the CPU for any signal processing. Figure 5.41 shows a general RoF architecture, where the wireless signals are transported over the optical fiber between a CPU and a set of base stations before being radiated through the air. RoF transmission systems are usually classified into two main categories, depending on the frequency range of the radio signal to be transported: i) RF-over-Fiber; ii) intermediate frequency (IF)-over-Fiber.

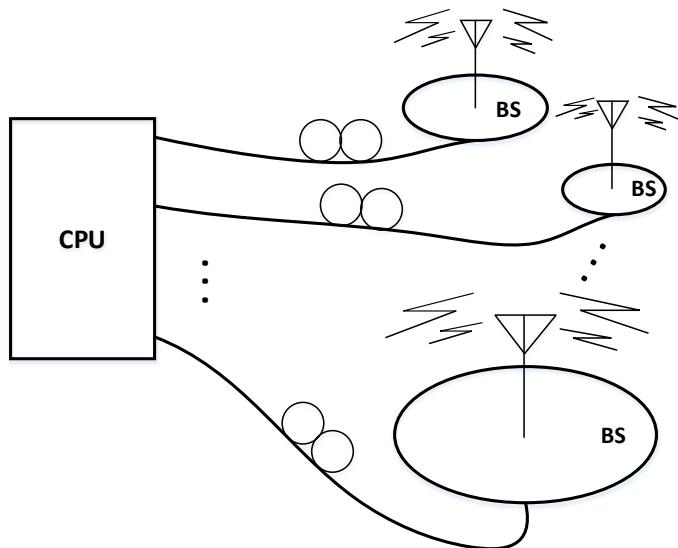


Figura 5.41: Schematic showing the concept of a centralized CPU architecture for future integrated optical wireless networks based on RoF.

- (i) In RF-over-Fiber architecture, a data-carrying RF (Radio Frequency) signal with a high frequency (usually greater than 10 GHz) is imposed on a lightwave signal before being transported over the optical link. Therefore, wireless signals are optically distributed to base stations directly at high frequencies and converted from the optical to electrical

domain at the base stations before being amplified and radiated by an antenna. As a result, no frequency up/down conversion is required at the various base stations, thereby resulting in simple and rather cost-effective implementation is enabled at the base stations.

- (ii) In IF-over-Fiber architecture, an IF (Intermediate Frequency) radio signal with a lower frequency (less than 10 GHz) is used for modulating light before being transported over the optical link. Therefore, before radiation through the air, the signal must be up-converted to RF at the base station.

In addition, the RoF technology can be implemented as analog RoF or digital RoF:

- (i) In analog RoF technology, the analog signal is transmitted over the optical fiber, being either RF signal, IF signal or baseband BB signal. In the optical transmitter, the RF/IF/BB signal is modulated onto the optical carrier by either using direct or external modulation of the laser. In this case, the signal distribution through RoF has the advantage of simplified BS design, however it is susceptible to fiber chromatic dispersion and nonlinearity generated by optical devices.
- (ii) In the digitized RoF the wireless carrier RF signal is first digitized prior to transport over the optical link. The digitalization of an RF signal produces a sampled digital signal in a serial form that can be directly modulated on an optical carrier, transmitted over the fiber optic link, and then detected like any other digital information. Modulation of the digital signal onto an optical carrier minimizes the nonlinear effects originating from the optical-to-electrical conversion function presented on analog RoF. In order to use not so high sample rates at the ADC/DAC components generally the bandpass sampling technique is applied to the RF signal.

### 5.8.1 Simulation

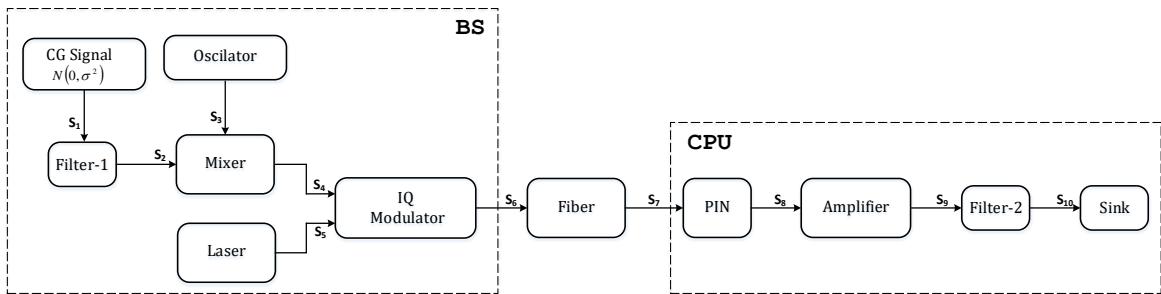


Figura 5.42: Simulation setup for the uplink of RoF transmission system. Complex Gaussian (CG);

Figure 5.42 depicts the simulation setup for an uplink of RoF, providing the connection between BS to CPU. At BS, we model the RF signal received from a mobile terminal, as zero

mean complex gaussian (CG) signal with a given bandwidth imposed by the low pass filter, Filter-1. The generated baseband signal is then up-converted to RF carrier frequency by utilizing an oscillator and a mixer. In this simulation we consider the RF carrier frequency between 2 to 5 GHz, according to the 5G technologies specifications. The generated RF passband signal modulates optical carrier by utilizing a laser and a mach-zehnder modulator (MZM). The optical signal is then transmitted to CPU using optical fiber. In CPU, the optical signal is detected by a PIN, amplified and followed with an electrical filter. After these operations, digital signal processing techniques can be applied to recover the transmitted signal.

Tabela 5.8: System Input Parameters.

Parameter	Default Value	Description
sourceMode		
symbolPeriod		
samplePeriod		
numberOfSamplesPerSymbol		
filterType1		
rollOffFactor		
filterType2		
outputOpticalPower		
outputOpticalWavelength		
rfCenterFrequency		
fiberAttenuation		

Tabela 5.9: Header Files for RoF Transmission System.

File Name	Description	Status
complex_gaussian_signal.h		
pulse_shaper.h		✓
local_oscillator.h		✓
mixer.h		
cw_laser.h		
iq_modulator.h		✓
fiber.h		
pin.h		
amplifier.h		
filter_rx.h		
sink.h		✓
netxpto.h		✓

### 5.8.2 Experimental

Tabela 5.10: Source Files for RoF Transmission System.

File Name	Description	Status
complex_gaussian_signal.cpp		
pulse_shaper.cpp		✓
local_oscillator.cpp		✓
mixer.cpp		
cw_laser.cpp		
iq_modulator.cpp		✓
fiber.cpp		
pin.cpp		
amplifier.cpp		
filter_rx.cpp		
sink.cpp		✓
netxpto.cpp		✓

## **Capítulo 6**

---

**Library**

## 6.1 Add

### Input Parameters

This block takes no parameters.

### Functional Description

This block accepts two signals and outputs one signal built from a sum of the two inputs. The input and output signals must be of the same type, if this is not the case the block returns an error.

### Input Signals

**Number:** 2

**Type:** Real, Complex or Complex\_XY signal (ContinuousTimeContinuousAmplitude)

### Output Signals

**Number:** 1

**Type:** Real, Complex or Complex\_XY signal (ContinuousTimeContinuousAmplitude)

## 6.2 Bit Error Rate

<b>Header File</b>	:	bit_error_rate.h
<b>Source File</b>	:	bit_error_rate.cpp
<b>Version</b>	:	20171810 (Responsible: Daniel Pereira)

### Input Parameters

Name	Type	Default Value
Confidence	double	0.95
MidReportSize	integer	0
LowestMinorant	double	$1 \times 10^{-10}$

### Input Signals

**Number:** 2

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number:** 1

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

### Functional Description

This block accepts two binary strings and outputs a binary string, outputting a 1 if the two input samples are equal to each other and 0 if not. This block also outputs .txt files with a report of the estimated Bit Error Rate (BER),  $\widehat{\text{BER}}$  as well as the estimated confidence bounds for a given probability  $\alpha$ .

The block allows for mid-reports to be generated, the number of bits between reports is customizable, if it is set to 0 then the block will only output the final report.

### Theoretical Description

The  $\widehat{\text{BER}}$  is obtained by counting both the total number received bits,  $N_T$ , and the number of coincidences,  $K$ , and calculating their relative ratio:

$$\widehat{\text{BER}} = 1 - \frac{K}{N_T}. \quad (6.1)$$

The upper and lower bounds,  $\text{BER}_{\text{UB}}$  and  $\text{BER}_{\text{LB}}$  respectively, are calculated using the Clopper-Pearson confidence interval, which returns the following simplified expression for

$N_T > 40$  [5]:

$$\text{BER}_{\text{UB}} = \widehat{\text{BER}} + \frac{1}{\sqrt{N_T}} z_{\alpha/2} \sqrt{\widehat{\text{BER}}(1 - \widehat{\text{BER}})} + \frac{1}{3N_T} \left[ 2 \left( \frac{1}{2} - \widehat{\text{BER}} \right) z_{\alpha/2}^2 + (2 - \widehat{\text{BER}}) \right] \quad (6.2)$$

$$\text{BER}_{\text{LB}} = \widehat{\text{BER}} - \frac{1}{\sqrt{N_T}} z_{\alpha/2} \sqrt{\widehat{\text{BER}}(1 - \widehat{\text{BER}})} + \frac{1}{3N_T} \left[ 2 \left( \frac{1}{2} - \widehat{\text{BER}} \right) z_{\alpha/2}^2 - (1 + \widehat{\text{BER}}) \right], \quad (6.3)$$

where  $z_{\alpha/2}$  is the  $100(1 - \frac{\alpha}{2})$ th percentile of a standard normal distribution.

---

## Bibliografia

- [1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.
- [2] Ryo Namiki and Takuya Hirano. Security of quantum cryptography using balanced homodyne detection. *Physical Review A*, 67(2):022308, 2003.
- [3] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.
- [4] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.
- [5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

### 6.3 Binary source

This block generates a sequence of binary values (1 or 0) and it can work in four different modes:

- 1. Random
- 3. DeterministicCyclic
- 2. PseudoRandom
- 4. DeterministicAppendZeros

This blocks doesn't accept any input signal. It produces any number of output signals.

#### Input Parameters

**Parameter:** mode{PseudoRandom}  
 (Random, PseudoRandom, DeterministicCyclic, DeterministicAppendZeros)

**Parameter:** probabilityOfZero{0.5}  
 (real  $\in [0,1]$ )

**Parameter:** patternLength{7}  
 (integer  $\in [1,32]$ )

**Parameter:** bitStream{"0100011101010101"}  
 (string of 0's and 1's)

**Parameter:** numberOfWorks{-1}  
 (long int)

**Parameter:** bitPeriod{1.0/100e9}  
 (double)

#### Methods

```
BinarySource(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig,
OutputSig){};
```

```
void initialize(void);
```

```
bool runBlock(void);
```

```
void setMode(BinarySourceMode m) BinarySourceMode const getMode(void)
```

```
void setProbabilityOfZero(double pZero)
```

```
double const getProbabilityOfZero(void)
```

```
void setBitStream(string bStream)
```

```

string const getBitStream(void)

void setNumberOfBits(long int nOfBits)

long int const getNumberOfBits(void)

void setPatternLength(int pLength)

int const getPatternLength(void)

void setBitPeriod(double bPeriod)

double const getBitPeriod(void)

```

### Functional description

The *mode* parameter allows the user to select between one of the four operation modes of the binary source.

**Random Mode** Generates a 0 with probability *probabilityOfZero* and a 1 with probability  $1 - \text{probabilityOfZero}$ .

**Pseudorandom Mode** Generates a pseudorandom sequence with period  $2^{patternLength} - 1$ .

**DeterministicCyclic Mode** Generates the sequence of 0's and 1's specified by *bitStream* and then repeats it.

**DeterministicAppendZeros Mode** Generates the sequence of 0's and 1's specified by *bitStream* and then it fills the rest of the buffer space with zeros.

### Input Signals

**Number:** 0

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number:** 1 or more

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

## Examples

### Random Mode

**PseudoRandom Mode** As an example consider a pseudorandom sequence with *patternLength*=3 which contains a total of 7 ( $2^3 - 1$ ) bits. In this sequence it is possible to find every combination of 0's and 1's that compose a 3 bit long subsequence with the exception of 000. For this example the possible subsequences are 010, 110, 101, 100, 111, 001 and 100 (they appear in figure 6.1 numbered in this order). Some of these require wrap.

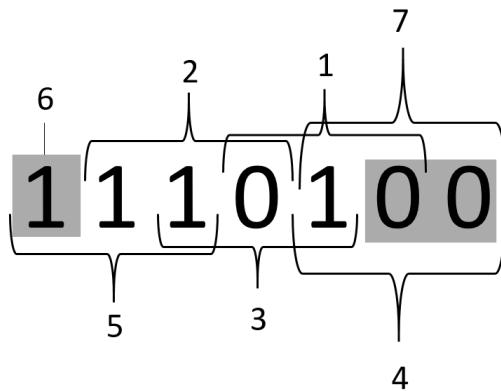


Figura 6.1: Example of a pseudorandom sequence with a pattern length equal to 3.

**DeterministicCyclic Mode** As an example take the *bit stream* '0100011101010101'. The generated binary signal is displayed in.

**DeterministicAppendZeros Mode** Take as an example the *bit stream* '0100011101010101'. The generated binary signal is displayed in 6.2.

### Sugestions for future improvement

Implement an input signal that can work as trigger.

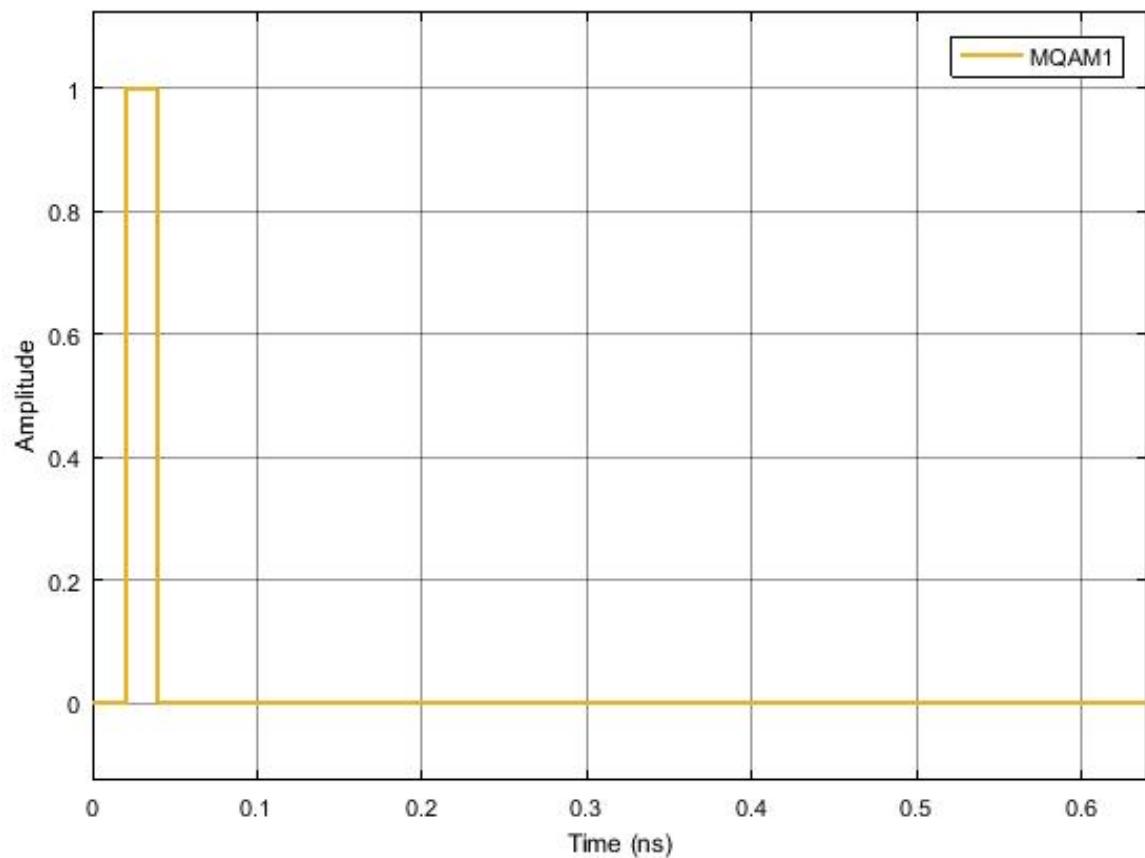


Figura 6.2: Binary signal generated by the block operating in the *Deterministic Append Zeros* mode with a binary sequence 01000...

## 6.4 Bit Decider

### Input Parameters

**Parameter:** setPosReferenceValue

**Parameter:** setNegReferenceValue

### Functional Description

This block accepts one real discrete signal and outputs a binary string, outputting a 1 if the input sample is above the predetermined reference level and 0 if it is below another reference value. The reference values are defined by the values of *PosReferenceValue* and *NegReferenceValue*.

### Input Signals

**Number:** 1

**Type:** Real signal (DiscreteTimeContinuousAmplitude)

### Output Signals

**Number:** 1

**Type:** Binary (DiscreteTimeDiscreteAmplitude)

## 6.5 Clock

This block doesn't accept any input signal. It outputs one signal that corresponds to a sequence of Dirac's delta functions with a user defined *period*.

### Input Parameters

**Parameter:** period{ 0.0 };

**Parameter:** samplingPeriod{ 0.0 };

### Methods

Clock()

Clock(vector<Signal \*> &InputSig, vector<Signal \*> &OutputSig) :Block(InputSig, OutputSig)

void initialize(void)

bool runBlock(void)

void setClockPeriod(double per)

void setSamplingPeriod(double sPeriod)

### Functional description

## Input Signals

**Number:** 0

## Output Signals

**Number:** 1

**Type:** Sequence of Dirac's delta functions.  
(TimeContinuousAmplitudeContinuousReal)

## Examples

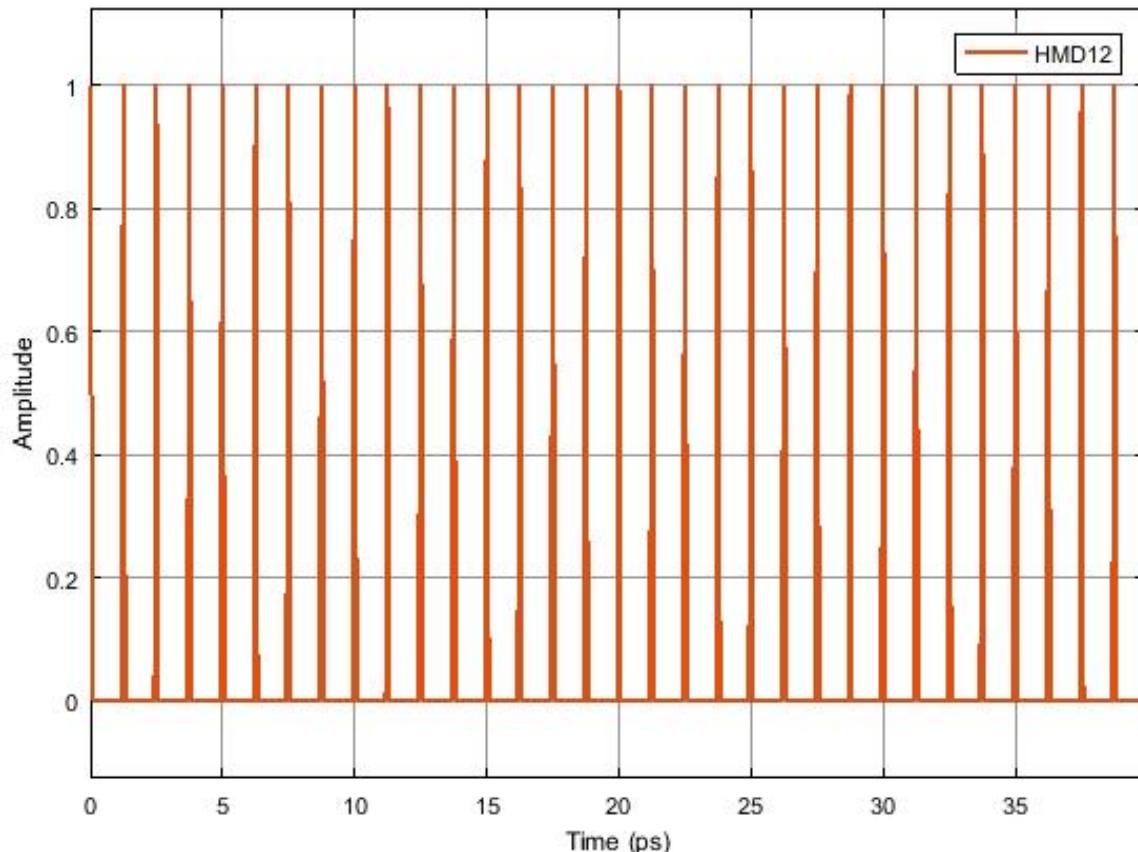


Figura 6.3: Example of the output signal of the clock

## Sugestions for future improvement

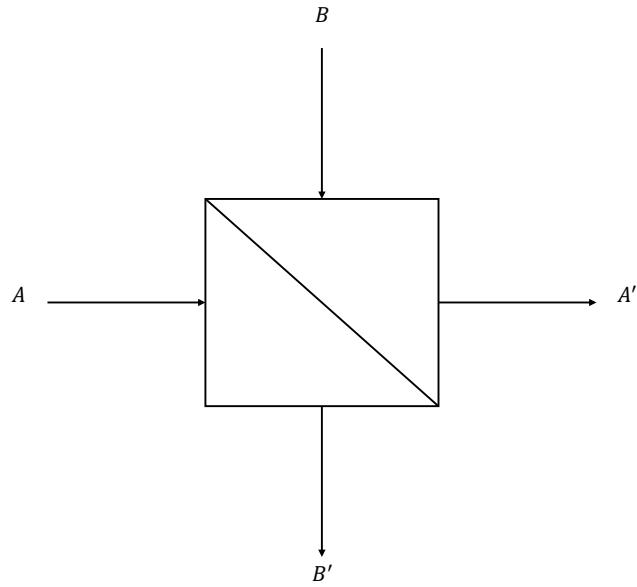


Figura 6.4: 2x2 coupler

## 6.6 Coupler 2 by 2

In general, the matrix representing 2x2 coupler can be summarized in the following way,

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} T & iR \\ iR & T \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} \quad (6.4)$$

Where, A and B represent inputs to the 2x2 coupler and A' and B' represent output of the 2x2 coupler. Parameters T and R represent transmitted and reflected part respectively which can be quantified in the following form,

$$T = \sqrt{1 - \eta_R} \quad (6.5)$$

$$R = \sqrt{\eta_R} \quad (6.6)$$

Where, value of the  $\sqrt{\eta_R}$  lies in the range of  $0 \leq \sqrt{\eta_R} \leq 1$ .

It is worth to mention that if we put  $\eta_R = 1/2$  then it leads to a special case of "Balanced Beam splitter" which equally distribute the input power into both output ports.

## 6.7 Decoder

This block accepts a complex electrical signal and outputs a sequence of binary values (0's and 1's). Each point of the input signal corresponds to a pair of bits.

### Input Parameters

**Parameter:** t\_integer m{ 4 }

**Parameter:** vector<t\_complex> iqAmplitudes{ { 1.0, 1.0 },{ -1.0, 1.0 },{ -1.0, -1.0 },{ 1.0, -1.0 } };

### Methods

Decoder()

```
Decoder(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig,
OutputSig)
```

void initialize(void)

bool runBlock(void)

void setM(int mValue)

void getM()

void setIqAmplitudes(vector<t\_iqValues> iqAmplitudesValues)

vector<t\_iqValues>getIqAmplitudes()

### Functional description

This block makes the correspondence between a complex electrical signal and pair of binary values using a predetermined constellation.

To do so it computes the distance in the complex plane between each value of the input signal and each value of the *iqAmplitudes* vector selecting only the shortest one. It then converts the point in the IQ plane to a pair of bits making the correspondence between the input signal and a pair of bits.

## Input Signals

**Number:** 1

**Type:** Electrical complex (TimeContinuousAmplitudeContinuousReal)

## Output Signals

**Number:** 1

**Type:** Binary

## Examples

As an example take an input signal with positive real and imaginary parts. It would correspond to the first point of the *iqAmplitudes* vector and therefore it would be associated to the pair of bits 00.

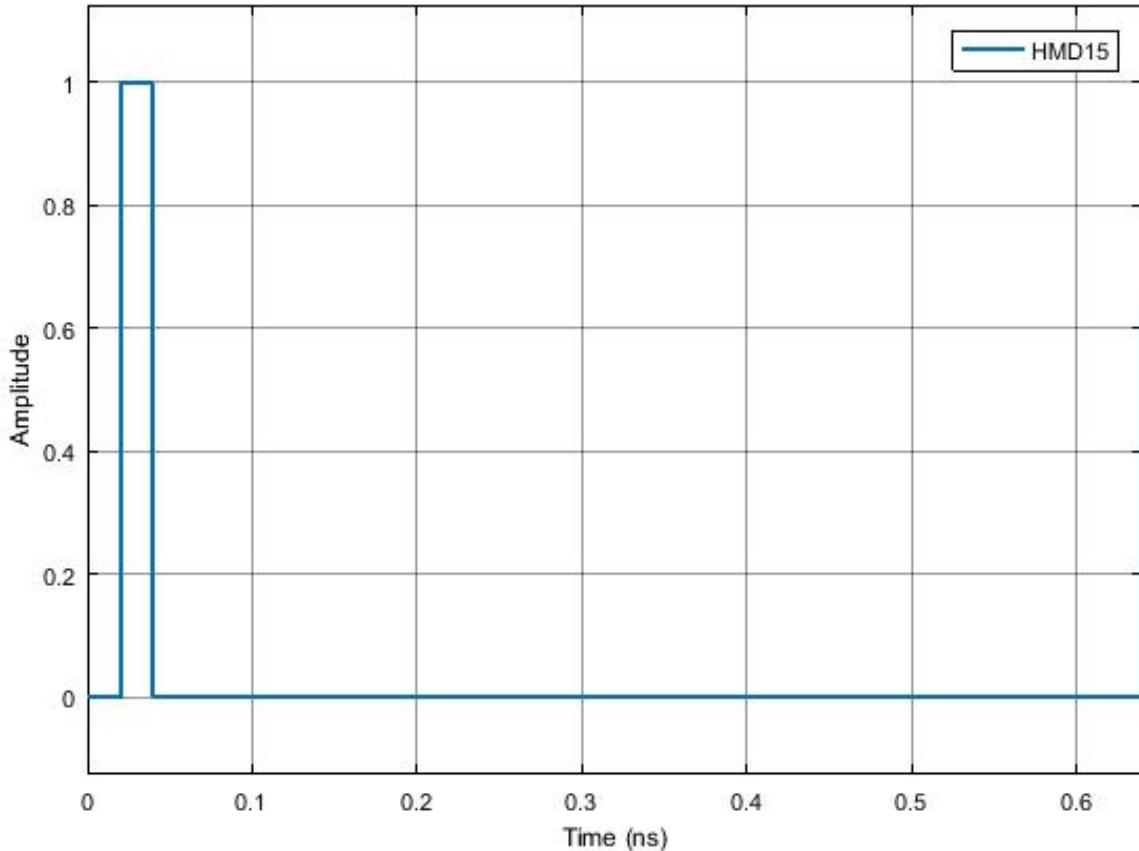


Figura 6.5: Example of the output signal of the decoder for a binary sequence 01. As expected it reproduces the initial bit stream

**Sugestions for future improvement**

## 6.8 Discrete to continuous time

This block converts a signal discrete in time to a signal continuous in time. It accepts one input signal that is a sequence of 1's and -1's and it produces one output signal that is a sequence of Dirac delta functions.

### Input Parameters

**Parameter:** `numberOfSamplesPerSymbol{8}`  
 (int)

### Methods

```
DiscreteToContinuousTime(vector<Signal * > &inputSignals, vector<Signal * > &outputSignals) :Block(inputSignals, outputSignals){};
```

```
void initialize(void);
```

```
bool runBlock(void);
```

```
void setNumberOfSamplesPerSymbol(int nSamplesPerSymbol)
```

```
int const getNumberOfSamplesPerSymbol(void)
```

### Functional Description

This block reads the input signal buffer value, puts it in the output signal buffer and it fills the rest of the space available for that symbol with zeros. The space available in the buffer for each symbol is given by the parameter *numberOfSamplesPerSymbol*.

### Input Signals

**Number :** 1

**Type :** Sequence of 1's and -1's. (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number :** 1

**Type :** Sequence of Dirac delta functions (ContinuousTimeDiscreteAmplitude)

### Example

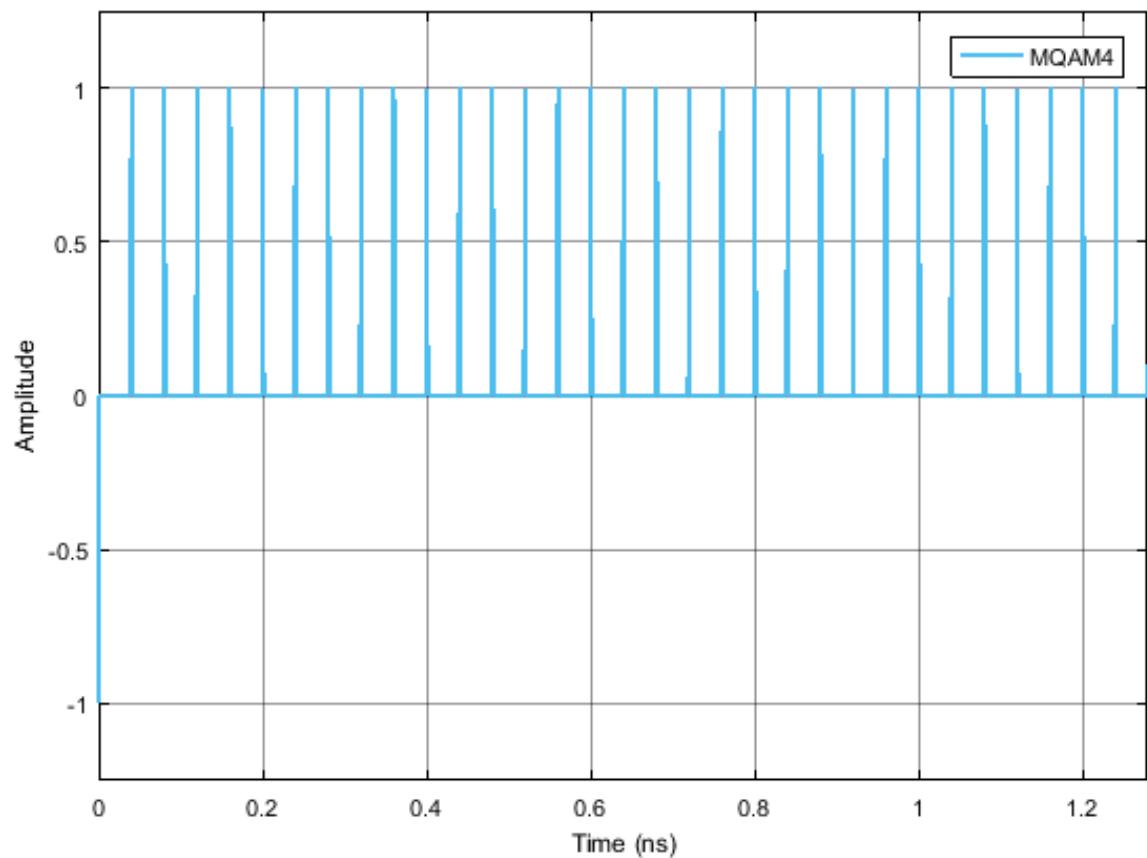


Figura 6.6: Example of the type of signal generated by this block for a binary sequence 0100...

## 6.9 Homodyne receiver

This block of code simulates the reception and demodulation of an optical signal (which is the input signal of the system) outputting a binary signal. A simplified schematic representation of this block is shown in figure 6.7.



Figura 6.7: Basic configuration of the MQAM receiver

### Functional description

This block accepts one optical input signal and outputs one binary signal that corresponds to the M-QAM demodulation of the input signal. It is a complex block (as it can be seen from figure 6.8) of code made up of several simpler blocks whose description can be found in the *lib* repository.

In can also be seen from figure 6.8 that there's an extra internal (generated inside the homodyne receiver block) input signal generated by the *Clock*. This block is used to provide the sampling frequency to the *Sampler*.

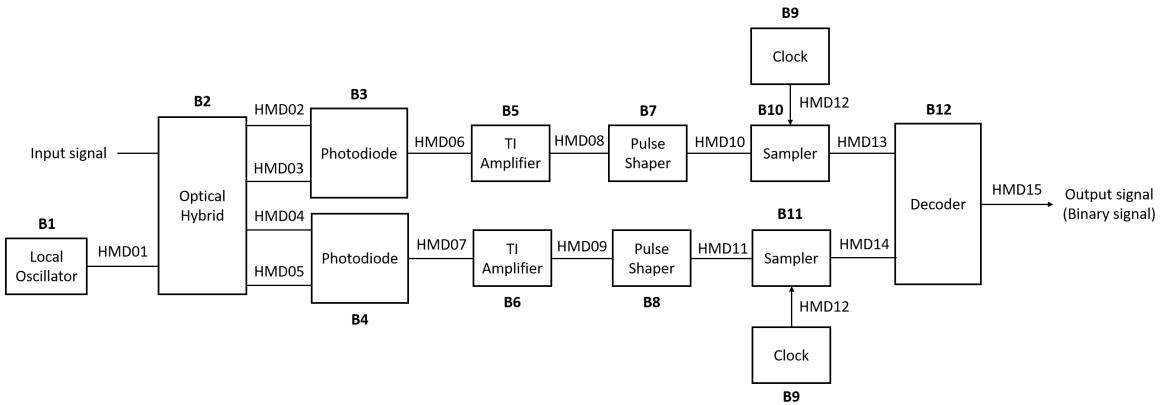


Figura 6.8: Schematic representation of the block homodyne receiver.

### Input parameters

This block has some input parameters that can be manipulated by the user in order to change the basic configuration of the receiver. Each parameter has associated a function that allows for its change. In the following table (table 6.2) the input parameters and corresponding functions are summarized.

Input parameters	Function	Type	Accepted values
IQ amplitudes	setIqAmplitudes	Vector of coordinate points in the I-Q plane	Example for a 4-qam mapping: { { 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 } }
Local oscillator power (in dBm)	setLocalOscillatorOpticalPower_dBm	double(t_real)	Any double greater than zero
Local oscillator phase	setLocalOscillatorPhase	double(t_real)	Any double greater than zero
Responsivity of the photodiodes	setResponsivity	double(t_real)	$\in [0,1]$
Amplification (of the TI amplifier)	setAmplification	double(t_real)	Positive real number
Noise amplitude (introduced by the TI amplifier)	setNoiseAmplitude	double(t_real)	Real number greater than zero
Samples to skip	setSamplesToSkip	int(t_integer)	
Save internal signals	setSaveInternalSignals	bool	True or False
Sampling period	setSamplingPeriod	double	Givem by symbolPeriod / samplesPerSymbol

Tabela 6.1: List of input parameters of the block MQAM receiver

## Methods

HomodyneReceiver(vector<Signal \*> &inputSignal, vector<Signal \*> &outputSignal)  
**(constructor)**

```

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues)
vector<t_iqValues> const getIqAmplitudes(void)
void setLocalOscillatorSamplingPeriod(double sPeriod)
void setLocalOscillatorOpticalPower(double opticalPower)
void setLocalOscillatorOpticalPower_dBm(double opticalPower_dBm)
void setLocalOscillatorPhase(double lOscillatorPhase)
void setLocalOscillatorOpticalWavelength(double lOscillatorWavelength)
void setSamplingPeriod(double sPeriod)

```

```
void setResponsivity(t_real Responsivity)  
void setAmplification(t_real Amplification)  
void setNoiseAmplitude(t_real NoiseAmplitude)  
void setImpulseResponseTimeLength(int impResponseTimeLength)  
void setFilterType(PulseShaperFilter fType)  
void setRollOffFactor(double rOffFactor)  
void setClockPeriod(double per)  
void setSamplesToSkip(int sToSkip)
```

**Input Signals**

**Number:** 1

**Type:** Optical signal

**Output Signals**

**Number:** 1

**Type:** Binary signal

**Example**

**Sugestions for future improvement**

## 6.10 IQ modulator

This blocks accepts one input signal continuous in both time and amplitude and it can produce either one or two output signals. It generates an optical signal and it can also generate a binary signal.

### Input Parameters

**Parameter:** outputOpticalPower{1e-3}  
(double)

**Parameter:** outputOpticalWavelength{1550e-9}  
(double)

**Parameter:** outputOpticalFrequency{speed\_of\_light/outputOpticalWavelength}  
(double)

### Methods

```
IqModulator(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig,
OutputSig){};
```

```
void initialize(void);
```

```
bool runBlock(void);
```

```
void setOutputOpticalPower(double outOpticalPower)
```

```
void setOutputOpticalPower_dBm(double outOpticalPower_dBm)
```

```
void setOutputOpticalWavelength(double outOpticalWavelength)
```

```
void setOutputOpticalFrequency(double outOpticalFrequency)
```

### Functional Description

This block takes the two parts of the signal: in phase and in amplitude and it combines them to produce a complex signal that contains information about the amplitude and the phase.

This complex signal is multiplied by  $\frac{1}{2}\sqrt{outputOpticalPower}$  in order to reintroduce the information about the energy (or power) of the signal. This signal corresponds to an optical signal and it can be a scalar or have two polarizations along perpendicular axis. It is the signal that is transmitted to the receptor.

The binary signal is sent to the Bit Error Rate (BER) measurement block.

### Input Signals

**Number :** 2

**Type :** Sequence of impulses modulated by the filter  
 (ContinuousTimeContiousAmplitude))

### Output Signals

**Number :** 1 or 2

**Type :** Complex signal (optical) (ContinuousTimeContinuousAmplitude) and binary signal (DiscreteTimeDiscreteAmplitude)

### Example

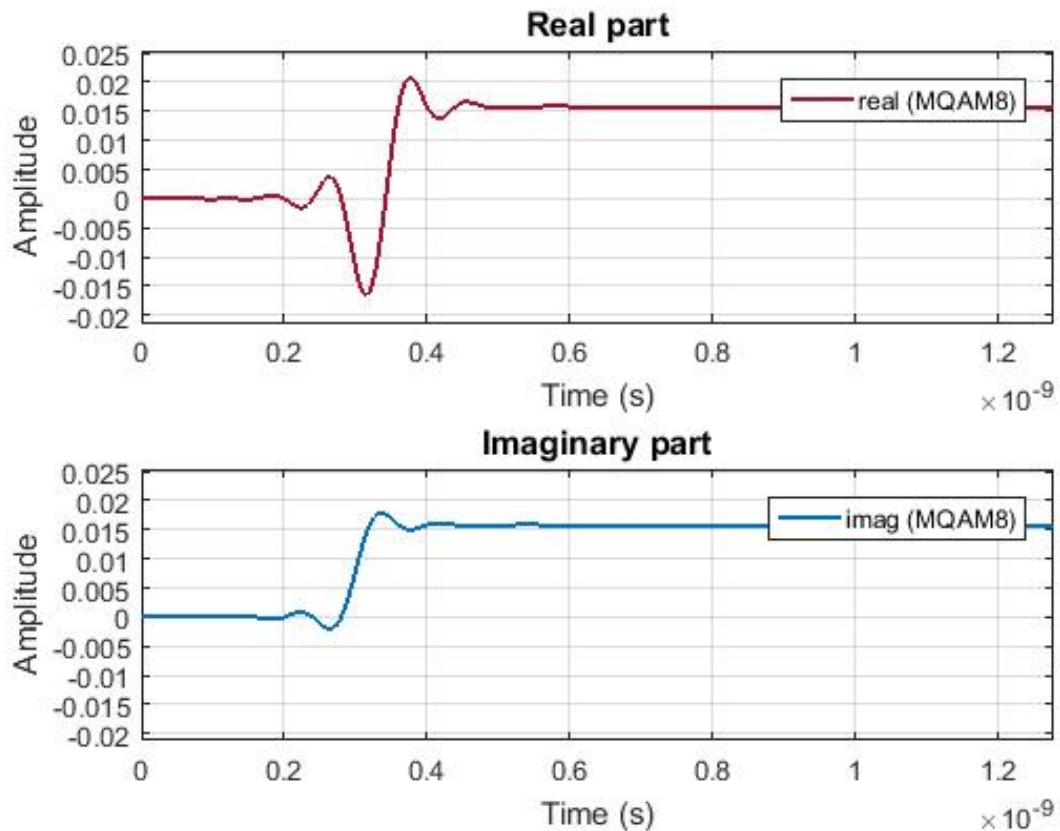


Figura 6.9: Example of a signal generated by this block for the initial binary signal 0100...

## 6.11 Local Oscillator

This block simulates a local oscillator with constant power and initial phase. It produces one output complex signal and it doesn't accept input signals.

### Input Parameters

**Parameter:** opticalPower{ 1e-3 }

**Parameter:** wavelength{ 1550e-9 }

**Parameter:** frequency{ SPEED\_OF\_LIGHT / wavelength }

**Parameter:** phase{ 0 }

**Parameter:** samplingPeriod{ 0.0 }

### Methods

LocalOscillator()

```
LocalOscillator(vector<Signal * > &InputSig, vector<Signal * > &OutputSig)
:Block(InputSig, OutputSig){};
```

void initialize(void);

bool runBlock(void);

void setSamplingPeriod(double sPeriod);

void setOpticalPower(double oPower);

void setOpticalPower\_dBm(double oPower\_dBm);

void setWavelength(double wlength);

void setPhase(double lOscillatorPhase);

### Functional description

This block generates a complex signal with a specified phase given by the input parameter *phase*.

**Input Signals**

**Number:** 0

**Output Signals**

**Number:** 1

**Type:** Optical signal

**Examples**

**Sugestions for future improvement**

## 6.12 MQAM mapper

This block does the mapping of the binary signal using a  $m$ -QAM modulation. It accepts one input signal of the binary type and it produces two output signals which are a sequence of 1's and -1's.

### Input Parameters

**Parameter:** m{4}  
(m should be of the form  $2^n$  with n integer)

**Parameter:** iqAmplitudes{{ 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 }}

### Methods

```
MQamMapper(vector<Signal * > &InputSig, vector<Signal * > &OutputSig)
:Block(InputSig, OutputSig) {};
void initialize(void);
bool runBlock(void);
void setM(int mValue);
void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues);
```

### Functional Description

In the case of  $m=4$  this block attributes to each pair of bits a point in the I-Q space. The constellation used is defined by the *iqAmplitudes* vector. The constellation used in this case is illustrated in figure 6.10.

### Input Signals

**Number :** 1

**Type :** Binary (DiscreteTimeDiscreteAmplitude)

### Output Signals

**Number :** 2

**Type :** Sequence of 1's and -1's (DiscreteTimeDiscreteAmplitude)

### Example

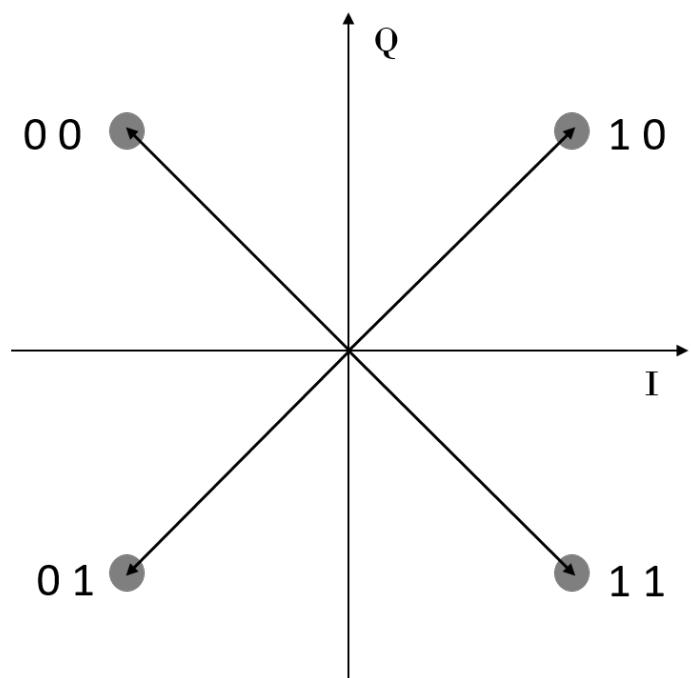


Figura 6.10: Constellation used to map the signal for  $m=4$

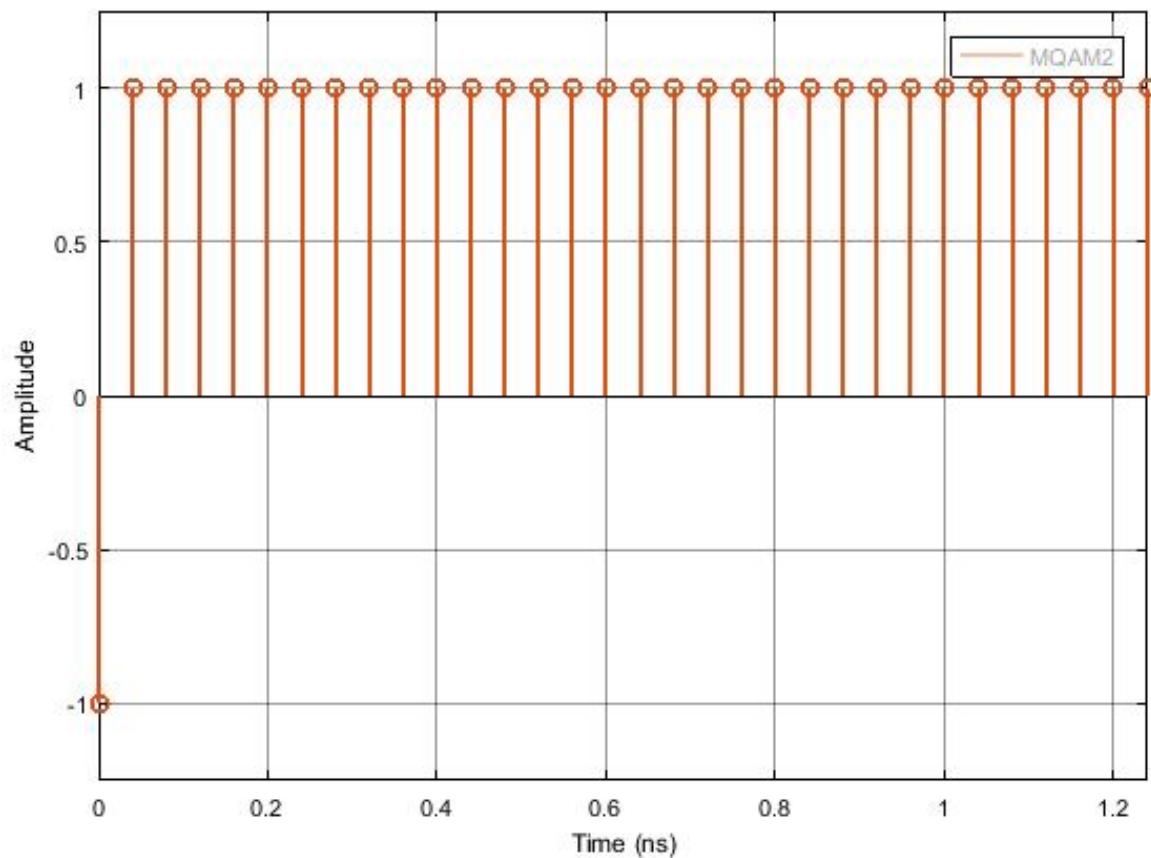


Figura 6.11: Example of the type of signal generated by this block for the initial binary signal 0100...

### 6.13 MQAM transmitter

This block generates a MQAM optical signal. It can also output the binary sequence. A schematic representation of this block is shown in figure 6.12.

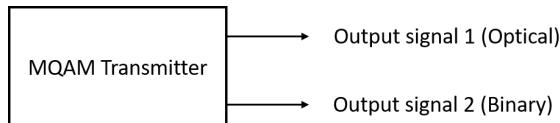


Figura 6.12: Basic configuration of the MQAM transmitter

#### Functional description

This block generates an optical signal (output signal 1 in figure 6.13). The binary signal generated in the internal block Binary Source (block B1 in figure 6.13) can be used to perform a Bit Error Rate (BER) measurement and in that sense it works as an extra output signal (output signal 2 in figure 6.13).

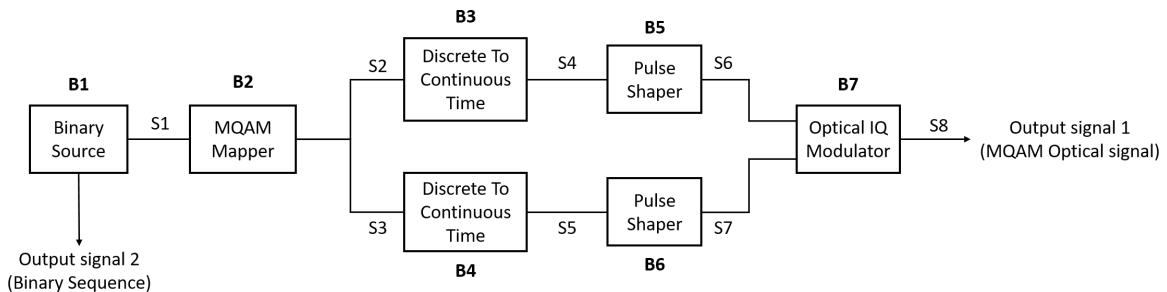


Figura 6.13: Schematic representation of the block MQAM transmitter.

#### Input parameters

This block has a special set of functions that allow the user to change the basic configuration of the transmitter. The list of input parameters, functions used to change them and the values that each one can take are summarized in table 6.2.

Input parameters	Function	Type	Accepted values
Mode	setMode()	string	PseudoRandom Random DeterministicAppendZeros DeterministicCyclic
Number of bits generated	setNumberOfBits()	int	Any integer
Pattern length	setPatternLength()	int	Real number greater than zero
Number of bits	setNumberOfBits()	long	Integer number greater than zero
Number of samples per symbol	setNumberOfSamplesPerSymbol()	int	Integer number of the type $2^n$ with n also integer
Roll off factor	setRollOffFactor()	double	$\in [0,1]$
IQ amplitudes	setIqAmplitudes()	Vector of coordinate points in the I-Q plane	Example for a 4-qam mapping: { { 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 } }
Output optical power	setOutputOpticalPower()	int	Real number greater than zero
Save internal signals	setSaveInternalSignals()	bool	True or False

Tabela 6.2: List of input parameters of the block MQAM transmitter

## Methods

MQamTransmitter(vector<Signal \*> &inputSignal, vector<Signal \*> &outputSignal);  
**(constructor)**

```

void set(int opt);

void setMode(BinarySourceMode m)

BinarySourceMode const getMode(void)

void setProbabilityOfZero(double pZero)

double const getProbabilityOfZero(void)

void setBitStream(string bStream)

string const getBitStream(void)

```

```
void setNumberOfBits(long int nOfBits)

long int const getNumberOfBits(void)

void setPatternLength(int pLength)

int const getPatternLength(void)

void setBitPeriod(double bPeriod)

double const getBitPeriod(void)

void setM(int mValue) int const getM(void)

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues)

vector<t_iqValues> const getIqAmplitudes(void)

void setNumberOfSamplesPerSymbol(int n)

int const getNumberOfSamplesPerSymbol(void)

void setRollOffFactor(double rOffFactor)

double const getRollOffFactor(void)

void setSeeBeginningOfImpulseResponse(bool sBeginningOfImpulseResponse)

double const getSeeBeginningOfImpulseResponse(void)

void setOutputOpticalPower(t_real outOpticalPower)

t_real const getOutputOpticalPower(void)

void setOutputOpticalPower_dBm(t_real outOpticalPower_dBm)

t_real const getOutputOpticalPower_dBm(void)
```

## Output Signals

**Number:** 1 optical and 1 binary (optional)

**Type:** Optical signal

## Example

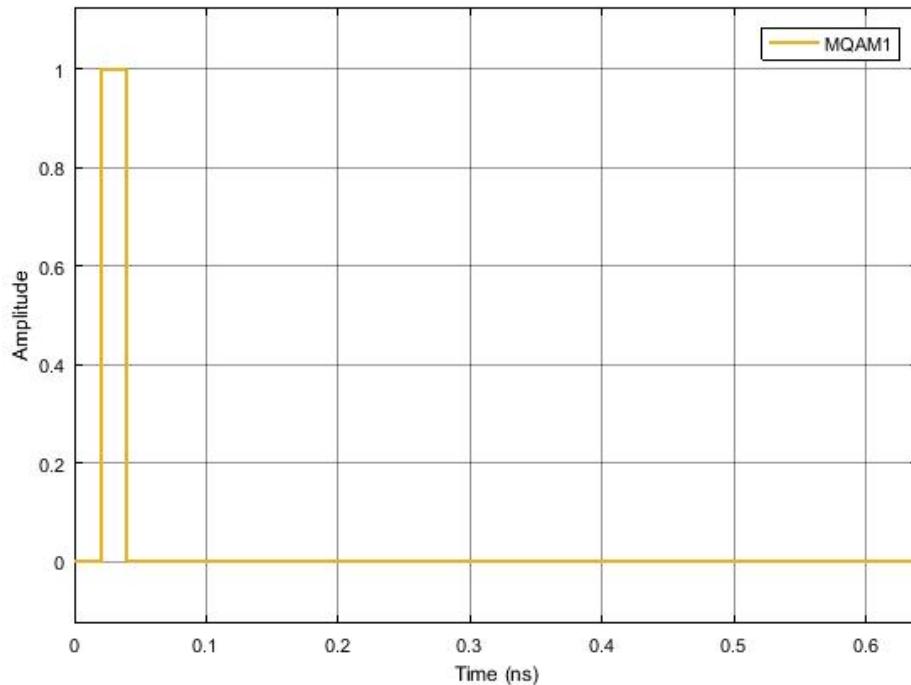


Figura 6.14: Example of the binary sequence generated by this block for a sequence 0100...

## Sugestions for future improvement

Add to the system another block similar to this one in order to generate two optical signals with perpendicular polarizations. This would allow to combine the two optical signals and generate an optical signal with any type of polarization.

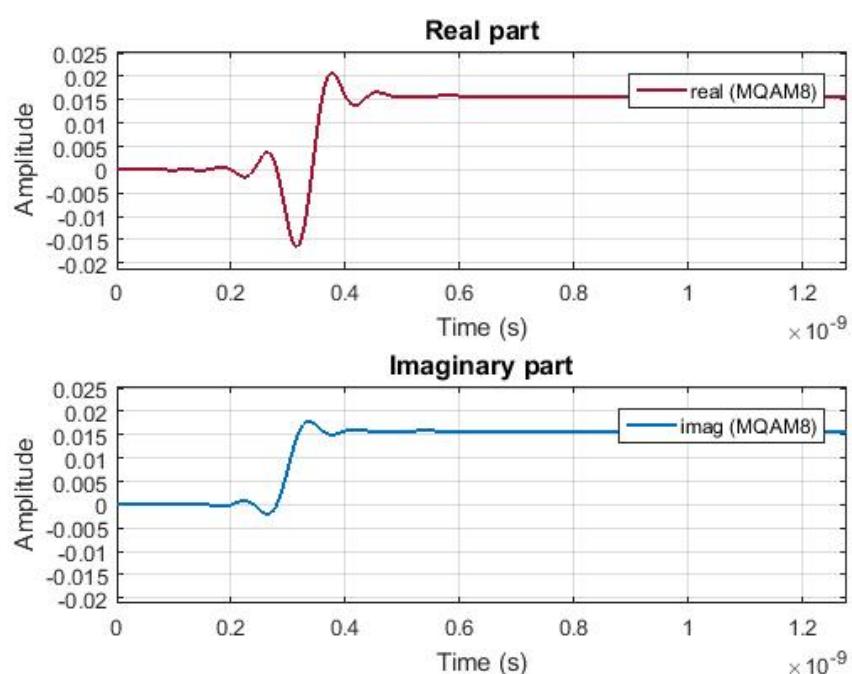


Figura 6.15: Example of the output optical signal generated by this block for a sequence 0100...

## 6.14 Fork

<b>Header File</b>	:	for.h
<b>Source File</b>	:	fork_20171119.cpp
<b>Version</b>	:	20171119 ( <b>Student Name:</b> Romil Patel)

### Input Parameters

— NA —

### Input Signals

**Number:** 1

**Type:** Any type (BinaryValue, IntegerValue, RealValue, ComplexValue, ComplexValueXY, PhotonValue, PhotonValueMP)

### Output Signals

**Number:** 2

**Type:** Same as applied to the input.

**Number:** 3

**Type:** Same as applied to the input.

### Functional Description

This block accepts any type signal and outputs two replicas of the input signal.

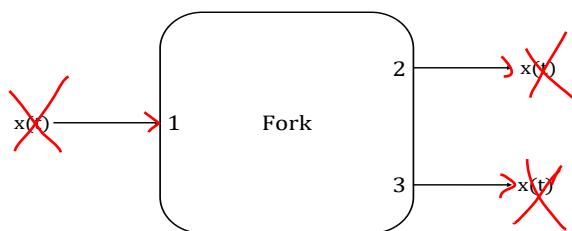


Figura 6.16: Fork

## **Capítulo 7**

---

### **Mathlab Tools**

## 7.1 Generation of AWG Compatible Signals

<b>Student Name</b>	:	Francisco Marques dos Santos
<b>Starting Date</b>	:	September 1, 2017
<b>Goal</b>	:	Convert simulation signals into waveform files compatible with the laboratory's Arbitrary Waveform Generator
<b>Directory</b>	:	mtools

This section shows how to convert a simulation signal into an AWG compatible waveform file through the use of a matlab function called sgnToWfm. This allows the application of simulated signals into real world systems.



### 7.1.1 sgnToWfm



```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm(fname_sgn, nReadr, fname_wfm);
```

#### Inputs

**fname\_sgn:** Input filename of the signal (\*.sgn) you want to convert. It must be a real signal (Type: TimeContinuousAmplitudeContinuousReal).

**nReadr:** Number of symbols you want to extract from the signal.

**fname\_wfm:** Name that will be given to the waveform file.

#### Outputs

A waveform file will be created in the Matlab current folder. It will also return six variables in the workspace which are:

**data:** A vector with the signal data.

**symbolPeriod:** Equal to the symbol period of the corresponding signal.

**samplingPeriod:** Sampling period of the signal.

**type:** A string with the name of the signal type.

**numberOfSymbols:** Number of symbols retrieved from the signal.

**samplingRate:** Sampling rate of the signal.

## Functional Description

This matlab function generates a \*.wfm file given an input signal file (\*.sgn). The waveform file is compatible with the laboratory's Arbitrary Waveform Generator (Tektronix AWG70002A). In order to recreate it appropriately, the signal must be real, not exceed  $8 \times 10^9$  samples and have a sampling rate equal or bellow 16 GS/s.

### **This function can be called with one, two or three arguments:**

Using one argument:

```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm('S6.sgn');
```

This creates a waveform file with the same name as the \*.sgn file and uses all of the samples it contains.

Using two arguments:

```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm('S6.sgn',256);
```

This creates a waveform file with the same name as the signal file name and the number of samples used equals nReadr x samplesPerSymbol. The samplesPerSymbol constant is defined in the \*.sgn file.

Using three arguments:

```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm('S6.sgn',256,'myWaveform.wfm');
```

This creates a waveform file with the name "myWaveform"and the number of samples used equals nReadr x samplesPerSymbol. The samplesPerSymbol constant is defined in the \*.sgn file.

### **7.1.2 Loading a signal to the Tektronix AWG70002A**

The AWG we will be using is the Tektronix AWG70002A which has the following key specifications:

**Sampling rate up to 16 GS/s:** This is the most important characteristic because it determines the maximum sampling rate that your signal can have. It must not be over 16 GS/s or else the AWG will not be able to recreate it appropriately.

**8 GSample waveform memory:** This determines how many data points your signal can have.

After making sure this specifications are respected you can create your waveform using the function. When you load your waveform, the AWG will output it and repeat it constantly until you stop playing it.

**1. Using the function sgnToWfm:** Start up Matlab and change your current folder to mtools and add the signals folder that you want to convert to the Matlab search path. Use the function accordingly, putting as the input parameter the signal file name you want to convert.

**2. AWG sampling rate:** After calling the function there should be waveform file in the mtools folder, as well as a variable called samplingRate in the Matlab workspace. Make sure this is equal or bellow the maximum sampling frequency of the AWG (16 GS/s), or else the waveform can not be equal to the original signal. If it is higher you have to adjust the parameters in the simulation in order to decrease the sampling frequency of the signal(i.e. decreasing the bit period or reducing the samples per symbol).

**3. Loading the waveform file to the AWG:** Copy the waveform file to your pen drive and connect it to the AWG. With the software of the awg open, go to browse for waveform on the channel you want to use, and select the waveform file you created (Figure 7.1).

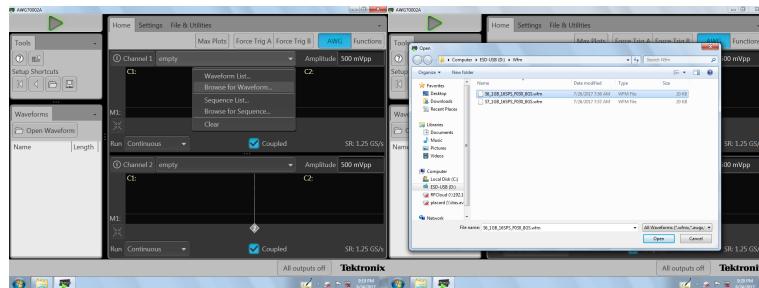


Figura 7.1: Selecting your waveform in the AWG

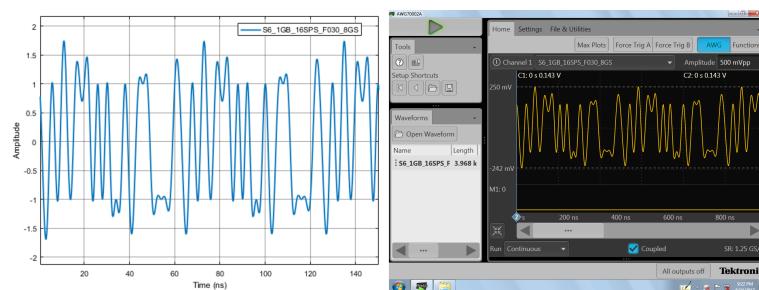


Figura 7.2: Comparison between the waveform in the AWG and the original signal before configuring the sampling rate

Now you should have the waveform displayed on the screen. Although it has the same shape, the waveform might not match the signal timing wise due to an incorrect sampling rate configured in the AWG. In this example (Figure 7.2), the original signal has a sample

rate of 8 GS/s and the AWG is configured to 1.25 GS/s. Therefore it must be changed to the correct value. To do this go to the settings tab, clock settings, and change the sampling rate to be equal to the one of the original signal, 8 GS/s (Figure 7.3). Compare the waveform in

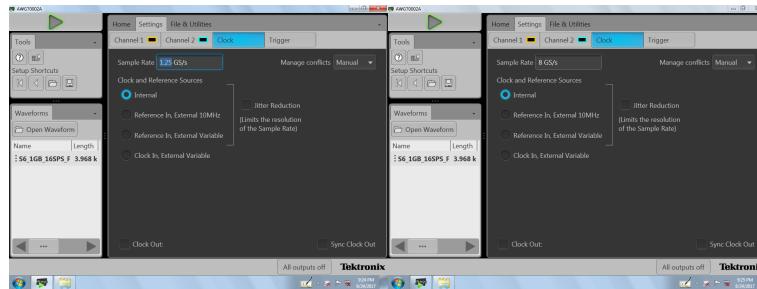


Figura 7.3: Configuring the right sampling rate

the AWG with the original signal, they should be identical (Figure 7.4).

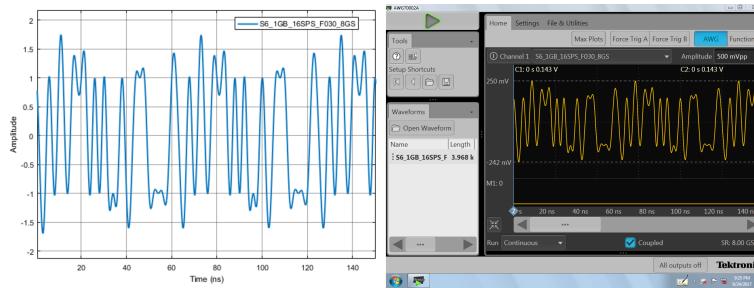


Figura 7.4: Comparison between the waveform in the AWG and the original signal after configuring the sampling rate

**4. Generate the signal:** Output the wave by enabling the channel you want and clicking on the play button.

## 7.2 Generation of AWG Compatible Signals

<b>Student Name</b>	:	Romil Patel
<b>Starting Date</b>	:	November 21, 2017
<b>Goal</b>	:	Convert simulation signals into waveform files compatible with the laboratory's Arbitrary Waveform Generator. (Updated version considering maximum sampling frequency and maximum length of the signal. )
<b>Directory</b>	:	\doc\tex\mtools\sgnToWfm_20171121

This section explains how to convert a simulation signal into an AWG compatible waveform file by using of a matlab function namely sgnToWfm. This allows the application of simulated signals into real world systems.

### 7.2.1 sgnToWfm\_20171121

#### Structure of a function

[dataDecimate, data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRateDecimate] = sgnToWfm\_20171121(fname\_sgn, nReadr, fname\_wfm)

#### Inputs

Name	Description
<b>fname_sgn</b>	Input the filename of the signal (*.sgn) you want to convert. It must be a real signal (type: TimeContinuousAmplitudeContinuousReal).
<b>nReadr</b>	Number of symbols you want to extract from the signal.
<b>fname_wfm</b>	Name that will be given to the waveform file.

## Output

Name	Description
<b>dataDecimate</b>	A vector which contains decimated signal data by an appropriate decimation factor to make it compatible with the AWG.
<b>data</b>	A vector which contains signal data without decimation.
<b>symbolPeriod</b>	Symbol period of the signal.
<b>samplingPeriod</b>	Sampling period of the signal.
<b>type</b>	A string which contains the name of the signal data type.
<b>numberOfSymbols</b>	Number of symbols retrieved from the signal.
<b>samplingRateDecimate</b>	Reduced sampling rate which is compatible with AWG. (i.e. less than 16 GSa/s)
<b>r</b>	Recommended decimation factor

## Functional Description

This matlab function generates a \*.wfm file given an input signal file (\*.sgn). The waveform file is compatible with the laboratory's Arbitrary Waveform Generator (Tektronix AWG70002A). In order to recreate it appropriately, the signal must be real, not exceed  $8 \times 10^9$  samples and have a sampling rate equal or bellow 16 GS/s.

**This function can be called with one, two or three arguments:**

**Using one argument:**

```
[dataDecimate,
data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRateDecimate] =
sgnToWfm_20171121sgnToWfm('S6.sgn')
```

This creates a waveform file with the same name as the \*.sgn file and uses all of the samples it contains.

**Using two arguments:**

```
[dataDecimate,
data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRateDecimate] =
sgnToWfm_20171121sgnToWfm('S6.sgn',256);
```

This creates a waveform file with the same name as the signal file name and the number of samples used equals nReadr x samplesPerSymbol. The samplesPerSymbol constant is defined in the \*.sgn file.

**Using three arguments:**

```
[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] =  
sgnToWfm('S6.sgn',256,'myWaveform.wfm');
```

This creates a waveform file with the name "myWaveform" and the number of samples used equals nReadr x samplesPerSymbol. The samplesPerSymbol constant is defined in the \*.sgn file.

## **Capítulo 8**

---

## **Algorithms**

## 8.1 Overlap-Save Method

Linear filtering can be easily implemented in time-domain resorting to the use of finite impulse response (FIR) digital filters and convolution property as,

$$y(n) = \sum_{k=0}^{R-1} x(n-k)h(k), \quad (8.1)$$

where  $x(n)$  is the input signal,  $h(k)$  is the FIR filter coefficients,  $R$  is the length of FIR filter coefficients and  $y(n)$  represents the filtered output signal. Analysing this equation we can note that, for a block signal of length  $R$ , the required number of operations for the direct implementation of equation (8.2) evolves with  $R^2$ ,  $\mathcal{O}(R^2)$ . This limitation imposed the emergence of algorithm, where the linear convolution is calculated faster than the directly implementing of (8.2). In this sense, it is used the computation of linear filtering in frequency domain resorting to the use of fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) algorithms as However, for long input sequence, the direct implementation of frequency domain filtering in real-time is limited by the limited memory of the digital processors. Hence, the filtering in frequency-domain is implemented by sectioning or block the input signal, such that the practical implementation of FFT and IFFT is feasible. In order to implement the non-cyclic convolution with the finite-length of cyclic convolution that the FFT provides, overlap-save and overlap-add method are use, enabling that the complexity evolves in log scale  $\mathcal{O}(N \log_2 N)$ . The general method is to split the input signal into manageable blocks, then apply the FFT to to perform the linear convolution and at the end recombine the output blocks such that it is avoided the wrap-around errors due to the circular convolution imposed by FFT.

The overlap-save method can be computed in the following steps:

1. Step 1:

**Parameter:** Determine the length  $R$  of impulse response,  $h(n)$ ;

2. Step 2:

**Parameter:** Define the size of FFT and IFFT operation,  $N$ ;

3. Step 3:

**Parameter:** Determine the length of block  $L$  to section the input sequence  $x(n)$ , considering that  $N = L + R - 1$ ;

4. Step 4:

**Parameter:** Pad  $L - 1$  zeros to the impulse response  $h(n)$  to obtain the length  $N$ ;

5. Step 5:

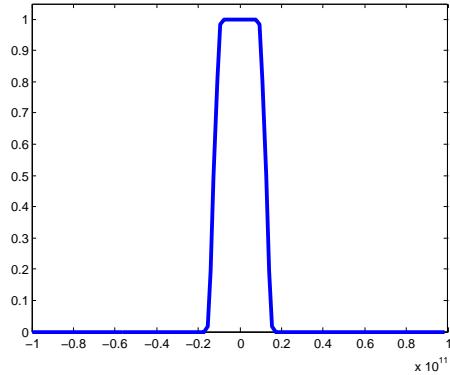


Figura 8.1: Frequency response of raised-cosine filter.

**Parameter:** Make the segments of the input sequences of length  $L$ ,  $x_i(n)$ , where index  $i$  correspond to the  $i^{th}$  block. Overlap  $R - 1$  samples of the previous block at the beginning of the segmented block to obtain a block of length  $N$ . In the first block, it is added  $R - 1$  null samples;

6. Step 6:

**Parameter:** Compute the circular convolution of segmented input sequence  $x_i(n)$  and  $h(n)$  described as,

$$y_i(n) = x_i(n) \circledast h(n). \quad (8.2)$$

This is obtained in the following steps:

**Description:** Compute the FFT of  $x_i$  and  $h$  both with length  $N$ ;

**Description:** Compute the multiplication of  $X_i(f)$  and the transfer function  $H(f)$ ;

**Description:** Compute the IFFT of the multiplication result to obtain the time-domain block signal,  $y_i$ ;

7. Step 7:

**Parameter:** Discarded  $R - 1$  initial samples from the  $y_i$ , and save only the error-free  $N - R - 1$  samples in the output record.

In the Figure 8.4 it is illustrated an example of overlap-save method.

### 8.1.1 Frequency Response of Filter

The frequency response of filter can be directly defined by using the frequency-domain formula, or it can be equivalently calculated from the FFT of impulse response of the filter. In this sense, we present an example of FIR filter (*raised-cosine filter*) to illustrate these two cases.

### Frequency-domain Formula

The frequency-domain description of raised-cosine filter can be given as,

$$H(f) = \begin{cases} 1, & |f| \leq \frac{1-\beta}{2T} \\ \frac{1}{2} \left[ 1 + \cos \left( \frac{\pi T}{\beta} \left[ |f| - \frac{1-\beta}{2T} \right] \right) \right], & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$

where,  $f$  is the frequency,  $0 \leq \beta \leq 1$  corresponds to the roll-off factor and  $T$  is the reciprocal of the symbol-rate. A plot of direct implementation of frequency response of raised-cosine filter is presented in Figure 8.1, for a roll-off factor of 0.3. The frequency response,  $H(f)$ , calculated for  $N$  frequency bins, which can be defined as,

$$f = -\frac{f_{windowTF}}{2} : \frac{f_{windowTF}}{N} : \left( \frac{f_{windowTF}}{2} - \frac{f_{windowTF}}{N} \right), \quad (8.3)$$

where  $f_{windowTF}$  is the sampling frequency. This, imposes that the length of  $H(f)$  is  $N$  as expected for the  $N$ -point FFT and the transfer function multiplication.

### FFT of Impulse Response

Alternatively to the frequency-domain formula, we can obtain the frequency response of filter by calculating the FFT of its impulse response. In the case of raised-cosine filter, the impulse response is given as,

$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\pi t\beta/T)}{1 - (2\beta t/T)^2}, \quad (8.4)$$

where  $t$  is the time. Figure 8.2 shows a plot of impulse response of raised-cosine filter for a roll-off factor of 0.3. Before calculating the FFT of the impulse response we must zero-pad the

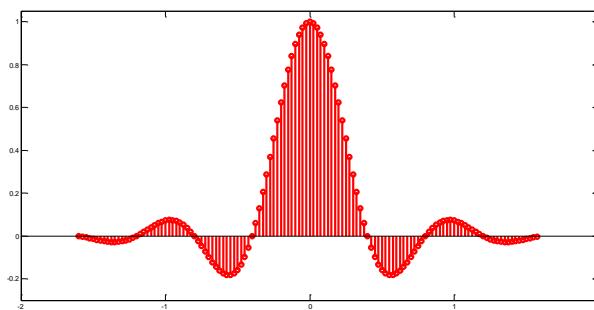


Figura 8.2: Impulse response of raised-cosine filter.

impulse response, which has the length  $R$ , to the length  $N$ . In this case  $N$  is the FFT length, which is efficiently defined as power of 2. The zero-padding process can be performed by appending  $L - 1$  zeros at the end of impulse response, as shown in the Figure 8.3.

In both cases, the frequency response of the filter will be limited to the frequency interval  $f_{windowTF}, [-\frac{f_{windowTF}}{2}, \frac{f_{windowTF}}{2}]$ , and this range show us the  $N$  frequency components

Figura 8.3: Zero-padding of impulse response  $h(n)$ .

obtained from FFT. The minimum frequency bin is  $-\frac{f_{windowTF}}{2}$  and the maximum bin is  $\frac{f_{windowTF}}{2}$ , in which  $f_{windowTF}$  corresponds to the sampling frequency. We can note that the spectral width of  $H(f)$  is  $f_{windowTF}$ , which is the inverse of sampling period,  $dt$ . It is also important to note that, for a given sampling frequency, the frequency resolution,  $\Delta f$ , of  $H(f)$  depends on the parameter  $N$  and it increases with  $N$ ,  $\Delta f = \frac{f_{windowTF}}{N}$ .

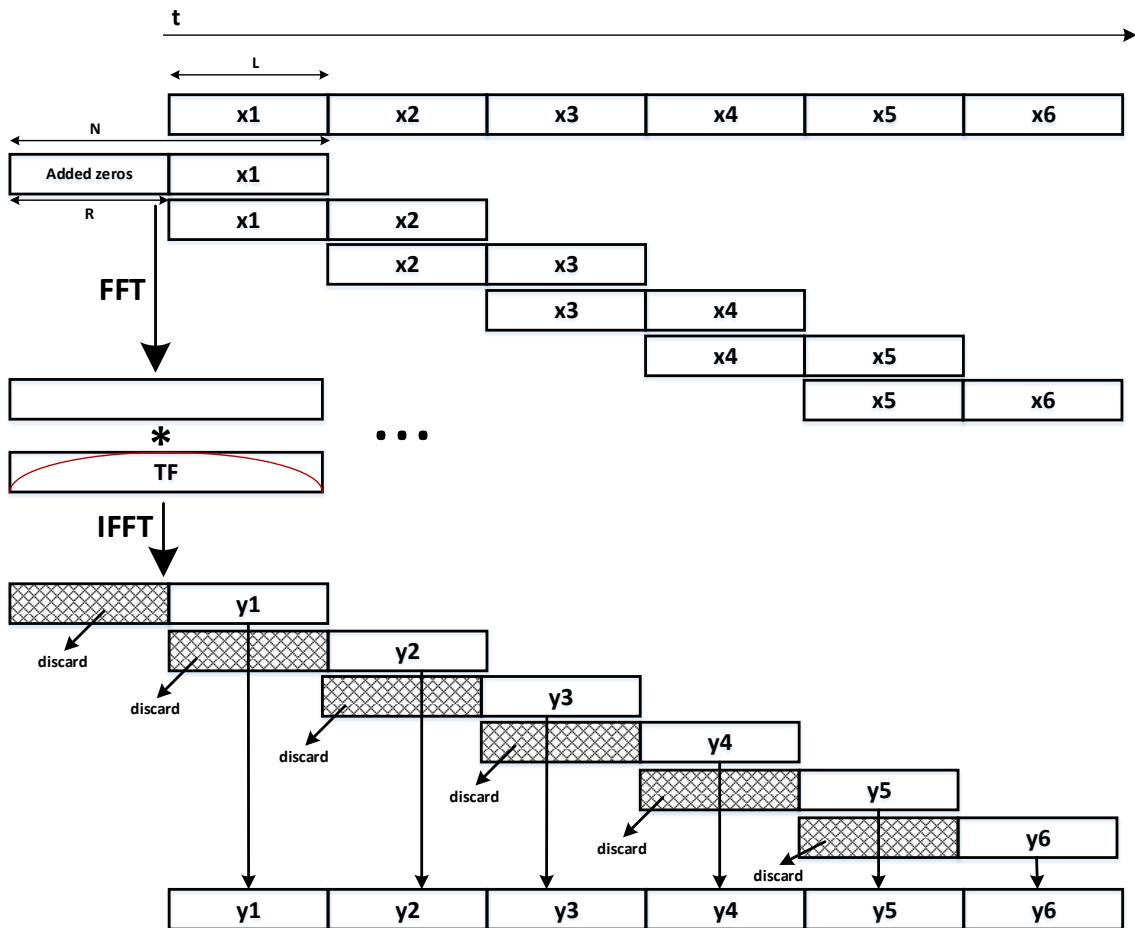


Figura 8.4: Illustration of Overlap-save method.

---

## Bibliografia

- [1] Blahut, R.E. *Fast Algorithms for Digital Signal Processing*, Addison-Wesley, Reading, MA, 1985.
- [2] Steven W. Smith. *The Scientist and Engineer's Guide to Digital Signal Processing*. California Technical Publishing, San Diego, CA, USA, 1997.

## 8.2 FFT

### Definitions of DFT and IDFT

The discrete Fourier transform of a sequence of N complex number  $x_0, x_1, x_2, \dots, x_{N-1}$  into another sequence of complex number  $X_0, X_1, X_2, \dots, X_{N-1}$  defined by,

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi kn/N} \quad 0 \leq k \leq N - 1 \quad (8.5)$$

Similarly, inverse discrete Fourier transform can be given as,

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \cdot e^{i2\pi kn/N} \quad 0 \leq k \leq N - 1 \quad (8.6)$$

Using these equation, it requires  $N^2$  calculation for the transformation from time domain to frequency domain and vice-versa. Therefore, it is not efficient for the practical implementation purpose. Radix-2 based FFT calculation reduces the number of computation to  $N \log(n)$

*explain*

### Algorithm

An alternate representation of the Fourier transform can give a flexibility to utilize the same code for the FFT and IFFT with appropriate input arguments. The algorithm for the FFT will follow the following formula,

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi kn/N} \quad 0 \leq k \leq N - 1 \quad (8.7)$$

Similarly, for IFFT,

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \cdot e^{i2\pi kn/N} \quad 0 \leq k \leq N - 1 \quad (8.8)$$

*n = 1 for iFFT*

From the above manipulation discussed in equations 8.7 and 8.8, we can write only one script for the implementations Fourier algorithm and manipulate its functionality as a FFT or IFFT by applying an appropriate input arguments.

The generalized form for the algorithm can be given as,

$$\underbrace{\text{OUTPUT}}_{\text{OUTPUT}} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \underbrace{\text{INPUT}}_{\text{INPUT}} \cdot \underbrace{e^{sign \cdot i2\pi kn/N}}_{\text{sign}}$$

$$0 \leq k \leq N - 1 \quad (8.9)$$

where,

$\text{INPUT} \rightarrow$  Input complex signal

$\text{OUTPUT} \rightarrow$  Output complex signal &  $sign \rightarrow$  '-1' for FFT and '1' for IFFT

**Function description**

To perform FFT operation, the input argument to the function can be given as follows,

$$\underbrace{OUTPUT}_{\text{out}} = \underbrace{\text{transform}}_{\text{in}}(INPUT, -1)$$

in a similar way, IFFT can be manipulated as,

$$OUTPUT = \text{transform}(INPUT, 1)$$

## Flowchart

This algorithm converts time domain signal to frequency domain for both real and complex signal type. In case of real signal, we have to manipulate imaginary part to be zero for the execution of algorithm. The figure 8.8 displays top level architecture of the FFT algorithm. If the length of the input signal is  $2^N$ , then it'll execute Radix-2 algorithm otherwise it'll execute Bluestein algorithm.

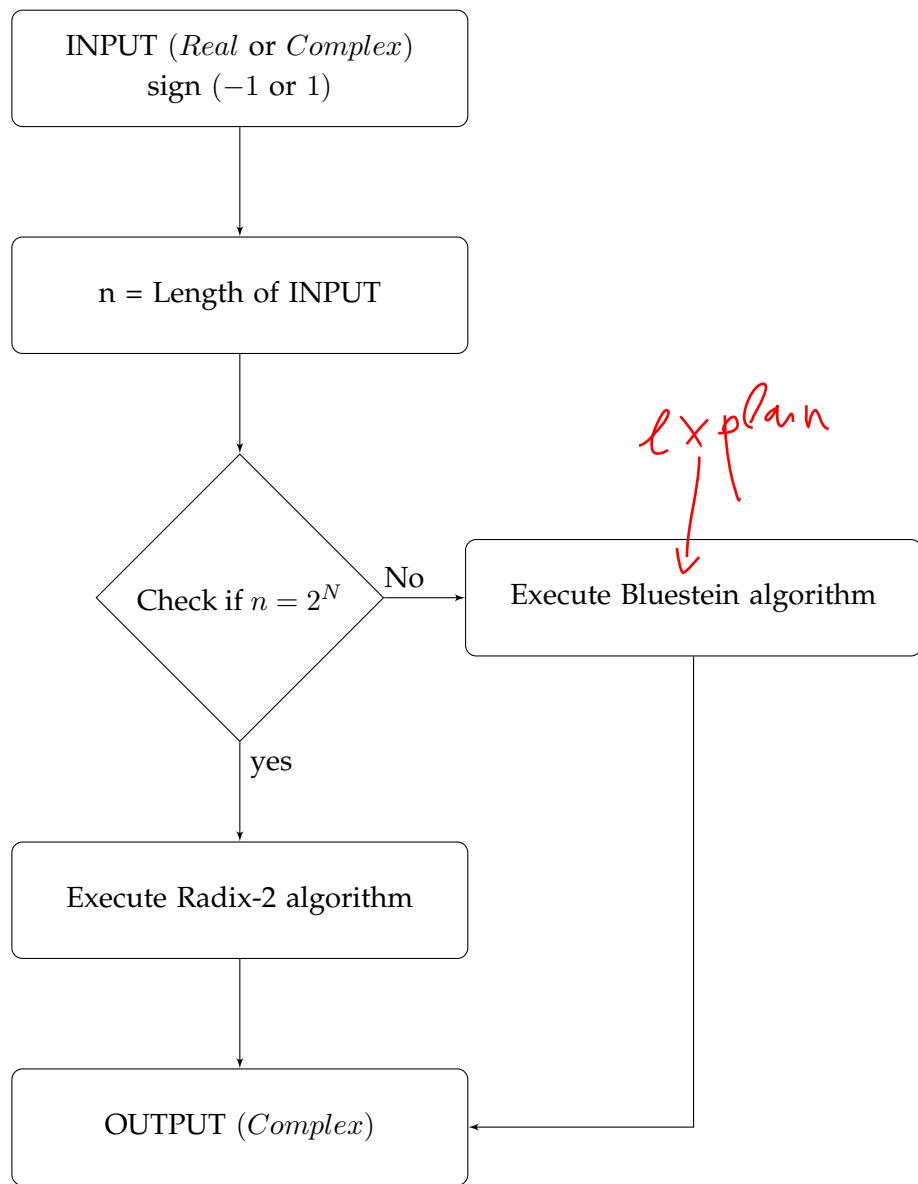


Figura 8.5: Top level architecture of the FFT algorithm

### Radix-2 algorithm

The architecture of the algorithm is to accept time domain complex signal and generate frequency domain complex output signal. In the case of real input signal, first we have to convert it into the complex form by adding zero to the imaginary part.

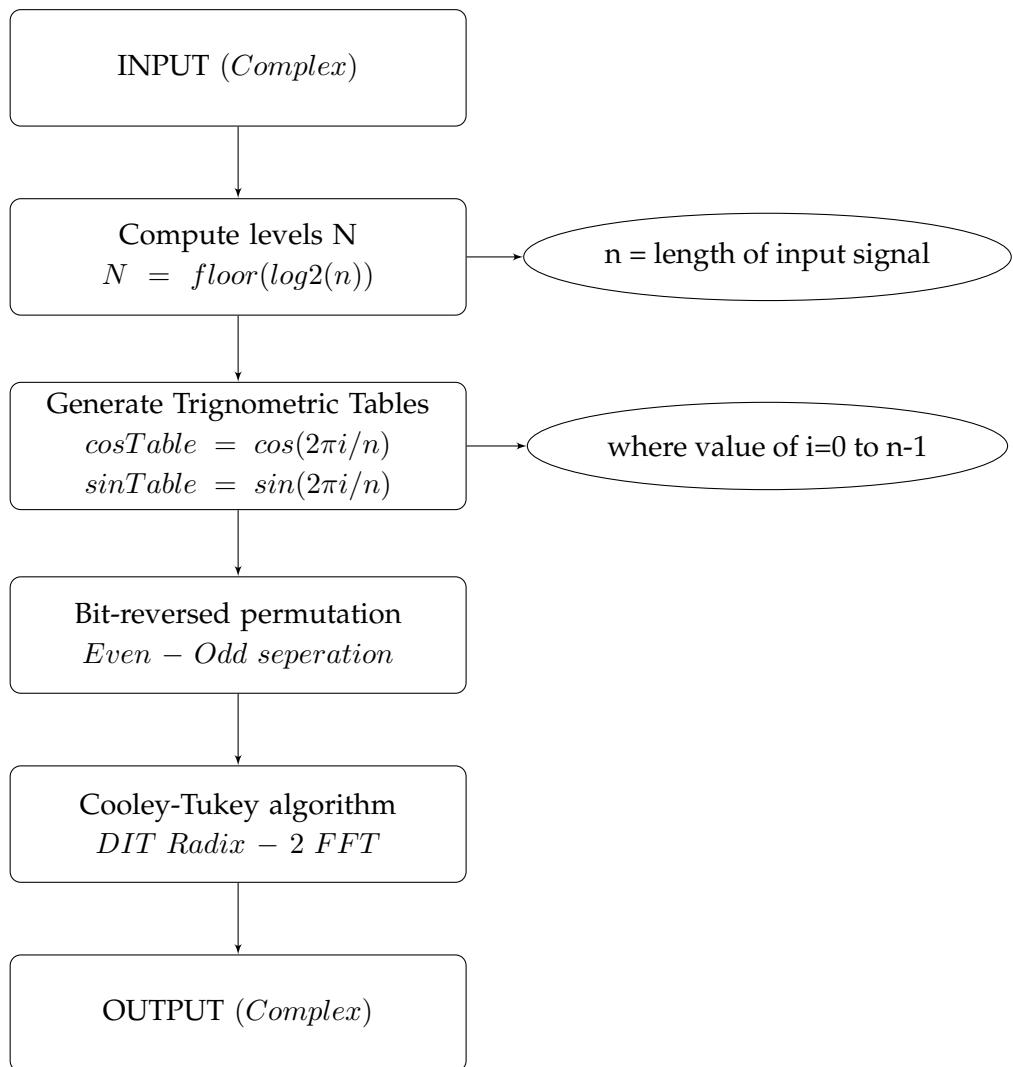


Figura 8.6: Flowchart of Cooley-Tukey DIT Radix-2 algorithm

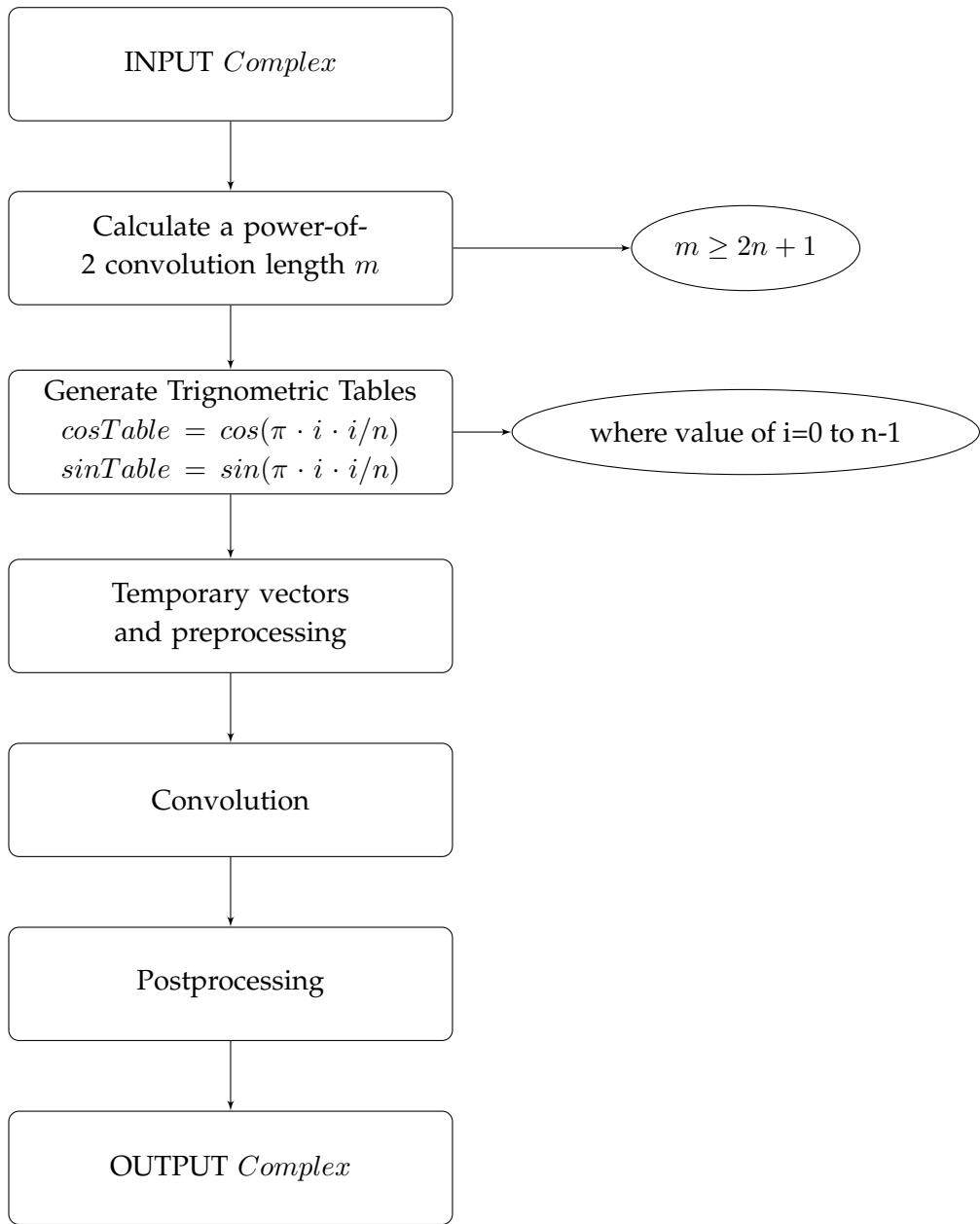
**Bluestein algorithm**

Figura 8.7: Flowchart of Bluestein algorithm

### 8.3 IFFT

This is the generalized IFFT algorithm which facilitates frequency domain to time domain conversion. First, the frequency domain input signal separated into real and imaginary part. The implementation of the IFFT can be carried out by swaping the position of the real and imaginary part of the FFT functions. In other words, the function  $FFT(imaginary, real)$  calculates the IFFT of the frequency domain signal applied at the input. Following that, the output of the IFFT function normalized to get exact time domain signal.

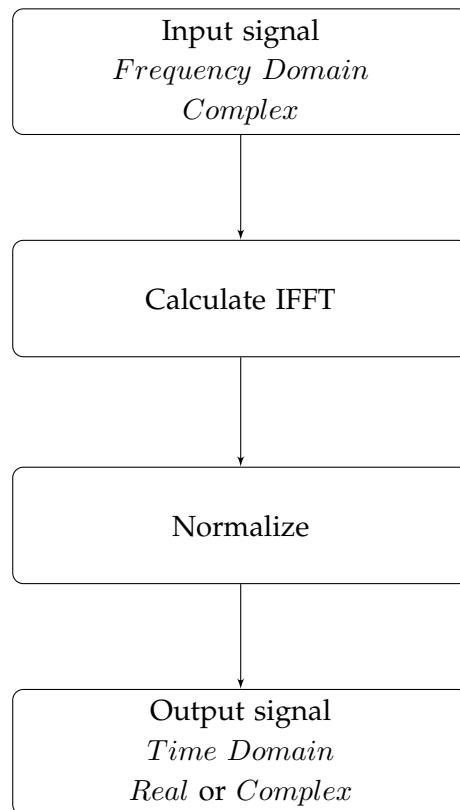


Figura 8.8: Top level architecture of the IFFT algorithm

