

Final Assessment Test - November 2018

Course: MAT3004 - Applied Linear Algebra

Class NBR(s): 2146 / 2164 / 2165 / 2166 / 2167 / 4786

Slot: C2+TC2+TCC2+V5

Max. Marks: 100

Time: Three Hours

General Instructions (if any): R=Set of all real numbers.

Answer any FIVE Questions $(5 \times 20 = 100 \text{ Marks})$



- (i) Find three elementary matrices E_1 , E_2 , E_3 such that $E_3E_2E_1A = U$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{bmatrix}$. [10]
 - (ii) Under what conditions on b, Ax = C, $C = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is solvable?
- [10] b) (i) Find all possible values of a, b, c so that $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ Such that $A^{-1} = A$. (ii) Find left and right inverse of $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ and determine A is invertible or not.



(i) Show that set of non-singular 2×2 matrices is not a vector space.

- [10]
- (ii) Prove or disprove set of all $n \times n$ symmetric matrices forms subspace of $M_{n \times n}(R)$.
- Find a basis for the subspace $V = \{x \in \mathbb{R}^3 \mid Ax = 0\}$, where $A = \begin{bmatrix} 2 & 0 & 5 \\ 1 & 1 & -1/2 \end{bmatrix}$. [10] Also, write down the dimension of V.



Find the bases for four fundamental subspaces of $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \end{bmatrix}$.

[10]

Find the polynomial which passes through (0, -5), (1, -3), (-1, -15), (2,39), (-2,9).

[10]

- Let $L: P_2 \to P_2$ defined by $L(a + bx + cx^2) = ax + bx^2$

[10]

Show that L is linear transformation.

- (ii) Determine Ker (L) and Range (L).
- Find the basis change matrix P from the basis α to β where

[10]

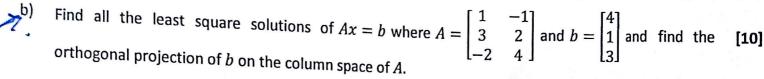
$$\alpha = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right\} \; ; \; \beta = \left\{ \begin{pmatrix} -3 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 9 \end{pmatrix} \right\} \text{ for } R^3. \text{ Use this result to find the basis change matrix from } \beta \text{ to } \alpha.$$

Let <, $>: P_2 \times P_2 \rightarrow R$ be defined by

- $< a_0 + a_1 x + a_2 x^2, b_0 + b_1 x + b_2 x^2 > = a_0 b_0 + a_1 b_1 + a_2 b_2.$
 - Prove that <, > is an inner product on $P_2(R)$.
- Find the angle between the vectors f(x) = 1 x and $g(x) = x^2$. (i) Show that the vectors $w_1 = (0,2,0)$, $w_2 = (3,0,3)$, $w_3 = (-4,0,4)$ form an orthogonal basis for b) R^3 with Euclidean inner product and use that basis to an orthonormal basis by normalizing each [10]
 - (ii) Express the vector u=(1,2,4) as a linear combination of the orthonormal basis vector obtained



Find the matrix of the orthogonal projection $Proj_W: \mathbb{R}^3 \to \mathbb{R}^3$ where $W = \text{Span}\{(1,1,1), (1,-1,0)\}.$ [10]



Encrypt the message "MATHEMATICS" by using the matrix
$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$
 (A-1, B-2, ..., Z-26). [15]

b) Find the standard matrix for an orthogonal projection on the y-axis, followed by a contraction with factor $k = \frac{1}{2}$. [5]