Final Assessment Test – November 2018

VIT Vellore Institute of Technology

Course: **MAT3004 - Applied Linear Algebra**

Class NBR(s): **2146 / 2164 / 2165 / 2166 / 2167 / 4786**

Slot: **C2+TC2+TCC2+V5**

Time: **Three Hours**

Max. Marks: **100**

General Instructions (if any): R=Set of all real numbers.

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) (i) Find three elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1A = U$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{bmatrix}$. **[10]**

   (ii) Under what conditions on b, $Ax = C, C = \begin{pmatrix} 2 \\ -1 \\ b \end{pmatrix}$ is solvable?

   b) (i) Find all possible values of a, b, c so that $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ Such that $A^{-1} = A$. **[10]**

   (ii) Find left and right inverse of A= $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ and determine A is invertible or not.

2. a) (i) Show that set of non-singular 2×2 matrices is not a vector space. **[10]**

   (ii) Prove or disprove set of all $n \times n$ symmetric matrices forms subspace of $M_{n\times n}(R)$.

   b) Find a basis for the subspace $V = \{x \in R^3 \mid Ax = 0\}$, where $A = \begin{bmatrix} 2 & 0 & 5 \\ 1 & 1 & -1/2 \end{bmatrix}$. **[10]**

   Also, write down the dimension of V.

3. a) Find the bases for four fundamental subspaces of $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \end{bmatrix}$. **[10]**

   b) Find the polynomial which passes through $(0,-5), (1,-3), (-1,-15), (2,39), (-2,9)$. **[10]**

4. a) Let $L: P_2 \to P_2$ defined by $L(a + bx + cx^2) = ax + bx^2$ **[10]**

   (i) Show that L is linear transformation.

   (ii) Determine Ker (L) and Range (L).

   b) Find the basis change matrix P from the basis α to β where **[10]**

$$\alpha = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right\} \; ; \beta = \left\{ \begin{pmatrix} -3 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 9 \end{pmatrix} \right\}$$

for R^3. Use this result to find the basis change matrix from β to α.

5\. a) Let $<,>: P_2 \times P_2 \to R$ be defined by
$< a_0 + a_1x + a_2x^2, b_0 + b_1x + b_2x^2 > = a_0b_0 + a_1b_1 + a_2b_2.$ **[10]**
(i) Prove that $<,>$ is an inner product on $P_2\ (R)$. 1 (
(ii) Find the angle between the vectors $f(x) = 1 - x$ and $g(x) = x^2$.

b) (i) Show that the vectors $w_1 = (0,2,0), w_2 = (3,0,3), w_3 = (-4,0,4)$ form an orthogonal basis for R^3 with Euclidean inner product and use that basis to an orthonormal basis by normalizing each vector. **[10]**
(ii) Express the vector $u = (1,2,4)$ as a linear combination of the orthonormal basis vector obtained in part (i).

6\. a) Find the matrix of the orthogonal projection $Proj_W: R^3 \to R^3$ where $W = \text{Span}\{(1,1,1), (1,-1,0)\}$. **[10]**

b) Find all the least square solutions of $Ax = b$ where $A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$ and find the orthogonal projection of b on the column space of A. **[10]**

7\. a) Encrypt the message "MATHEMATICS" by using the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ (A-1, B-2, ..., Z-26). **[15]**

b) Find the standard matrix for an orthogonal projection on the y-axis, followed by a contraction with factor $k = \frac{1}{2}$. **[5]**

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