Zewail City for Science and Technology
Nanotechnology and Nanoelectronics Program
Midterm #1 - FALL 2020

NANENG 512
Applied Digital Control and Drives
Time: 60 Minutes

Name: Model Answer ID: ..........

**This examination paper has 5 questions in 7 pages, including a formula sheet**

# Question 1

[2.5 Points]

**Choose the suitable answer then write it in the table.**

| Question 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Your Answer | c | a | d | c | d | c | b | d | b | a |

1. Transfer function of a system is used to calculate which of the following?
   (a) The order of the system
   (b) The time constant
   (c) The output for any given input
   (d) The steady state gain

2. An increase in gain, in most systems, leads to
   (a) Smaller damping ratio
   (b) Larger damping ratio
   (c) Constant damping ratio
   (d) None of the above

3. Static error coefficients are used as a measure of the effectiveness of closed loop systems for specified input signal like:
   (a) Acceleration
   (b) Velocity
   (c) Position
   (d) All of the above

4. A control system in which the control action is somehow dependent on the output is
   (a) Open-loop system
   (b) Semiclosed loop system
   (c) Closed-loop system

(d) None

5. The position and velocity errors of a type-2 system are:

   (a) Constant, Constant
   (b) Constant, Infinity
   (c) Zero, Constant
   (d) Zero, Zero

6. Let $L(s) = \frac{(s+5)(s+3)}{(s+2)^2}$ be an open loop transfer function in a standard feedback architecture. Let the input to the system be a unit step input. What is the static error $e_\infty$ of the system?

   (a) $\frac{1}{2}$
   (b) $\frac{15}{19}$
   (c) $\frac{4}{19}$
   (d) 1

$R(s) = \frac{1}{s} \Rightarrow e_\infty = \lim_{s\to 0} \frac{sR(s)}{1+L(s)} = \frac{1}{1+\frac{(s+5)(s+3)}{(s+2)^2}}$

$e_\infty = \lim_{s\to 0} \frac{(s+2)^2}{(s+2)^2+(s+5)(s+3)} = \frac{4}{19}$

7. Which of the following is a characteristic of a first order system?

   (a) Its step response has a non-zero slope at the origin
   (b) It can be modelled using a first-order differential equation
   (c) It does not exhibit oscillatory behavior when excited
   (d) All of the above

8. Steady state error is always zero in response to the displacement input for

   (a) Type 0 system
   (b) Type 1 system
   (c) Type 2 system
   (d) Type ($N > 1$) system for $N = 0, 1, 2 \ldots .N$

9. When modelling a second-order underdamped system from step response data, what are the key parameters that we need to determine?

   (a) Time-constant and steady-state gain
   (b) Damping ratio, undamped natural frequency, and steady-state gain
   (c) Cut-off frequency and steady-state gain

10. Consider the system of figure 1, the steady state error is less than 2% to step inputs for

system type is $>0 \Rightarrow e_\infty = 0 < 2\%$

   (a) any value of $K_1$
   (b) no values of $K_1$
   (c) $K_1 > 50$
   (d) None of the above

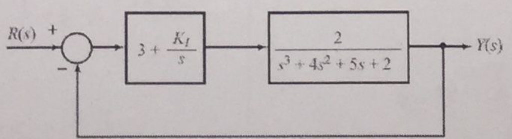



Figure 1

Proof:

$\frac{3s+K_1}{s}$

$G(s) = \left(3+\frac{K_1}{s}\right)\left(\frac{2}{s^3+4s^2+5s+2}\right)$

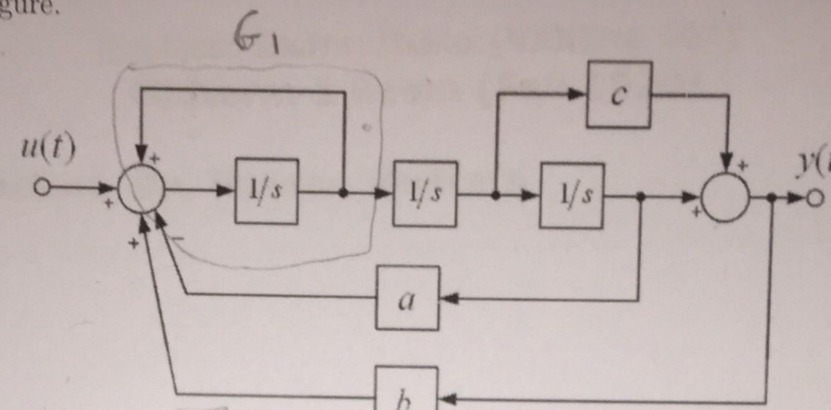
$= \frac{6s+2K_1}{s^4+4s^3+5s^2+2s}$

$e_\infty = \lim_{s\to 0} \frac{s \times \frac{1}{s}}{1+\frac{6s+2K_1}{s^4+4s^3+5s^2+2s}} = 0$

# Question 2

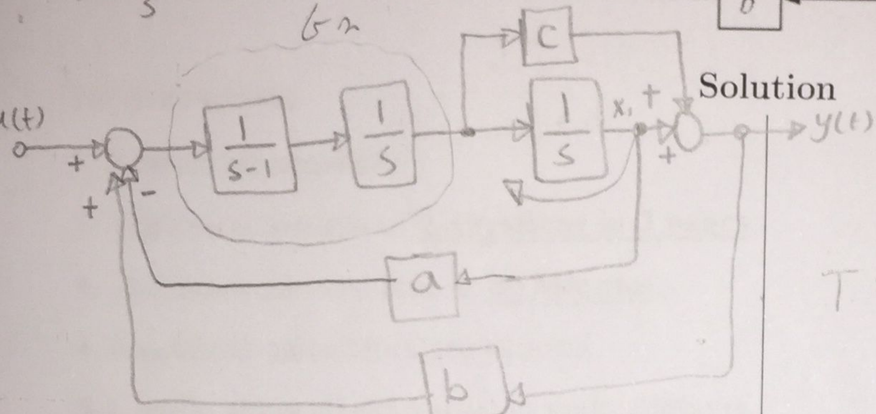
[4 Points]

Use block-diagram reduction or Mason's rule to find the transfer function for the system shown in the following Figure.




$$G_1 = \frac{\frac{1}{s}}{1-\frac{1}{s}} = \frac{1}{s-1}$$

## Solution



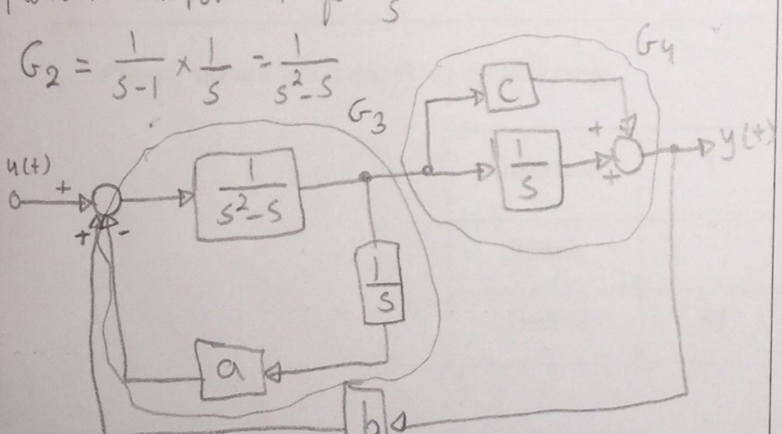

$$T = \frac{\frac{\cancel{s}}{s^3-s^2+a} \times \frac{1+sc}{\cancel{s}}}{1 - b \times \frac{\cancel{s}}{s^3-s^2+a} \times \frac{1+sc}{\cancel{s}}}$$

$$T = \frac{1+sc}{s^3-s^2+a-b(1+sc)}$$

$$= \frac{1+sc}{s^3-s^2-bcs+a-b} \quad \#$$

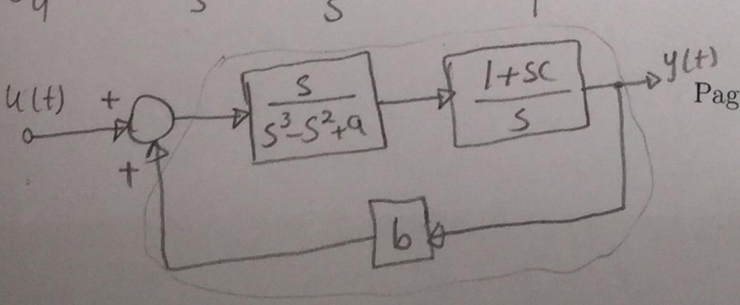
Move Branch point $x_1$ before $\frac{1}{s}$ and get $G_2$:

$$G_2 = \frac{1}{s-1} \times \frac{1}{s} = \frac{1}{s^2-s}$$




$$G_3 = \frac{\frac{1}{s^2-s}}{1 + a \times \frac{1}{s} \times \frac{1}{s^2-s}} = \frac{s}{s^3-s^2+a}$$

$$G_4 = c + \frac{1}{s} = \frac{1+sc}{s}$$

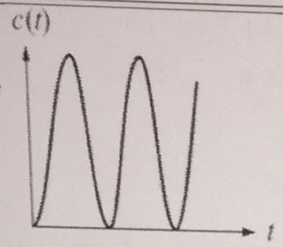
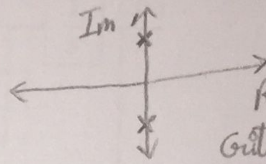
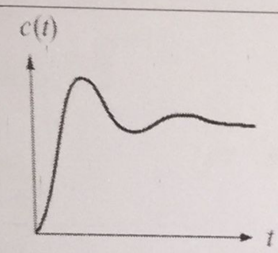
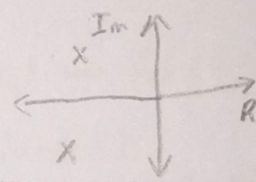
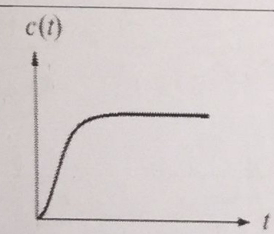
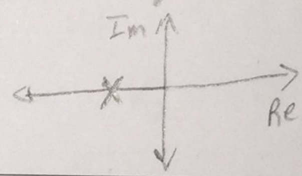
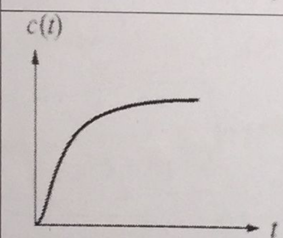
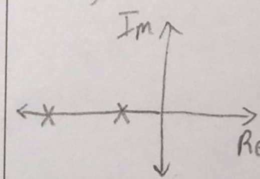


Poles: $p = -\omega_n \zeta \pm \omega_n \sqrt{\zeta^2 - 1}$

## Question 3

[4 Points]

Given the expected unit step response of a second order system, define the type of the response and the range of the corresponding damping ratio for each case, and sketch the expected locations of the poles of such a system.

| Step Response | Step Response Type | $\zeta$ | Poles |
|---|---|---|---|
| <br> | undamped | $\zeta = 0$ | Imaginary : $\pm \omega_n j$<br><br><br>Critically stable |
| <br> | underdamped | $0 < \zeta < 1$ | Complex conjugates: $-\omega_n \zeta \pm j \omega_n \sqrt{1-\zeta^2}$<br><br><br>stable |
| <br> | critically damped | $\zeta = 1$ | Real, Equal : $-\omega_n$<br><br><br>stable |
| <br> | overdamped | $\zeta > 1$ | Real, distinct : $-\omega_n \zeta \pm \omega_n \sqrt{\zeta^2 - 1}$<br><br><br>stable |

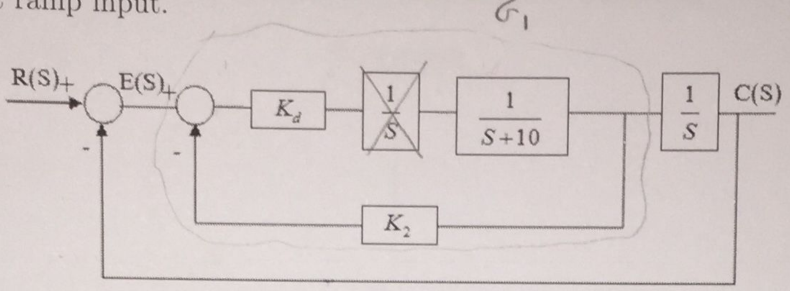
# Question 4

[4 Points]

The automatic control of an airplane is one example that requires multiple-variable feedback methods. A simplified model where the rolling motion can be considered independent of other motions is shown in figure. The step response desired has an overshoot less than or equal 10% and a rise time less than or equal 4 second.

- Select the parameters $K_d$ and $K_2$ that achieve the desired response.
- Find the minimum steady state error that can be obtained by varying the values of $K_d$ and $K_2$, for a unit ramp input.

$M_p = 0.1 \quad t_r = 4$ sec.




$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.1$$

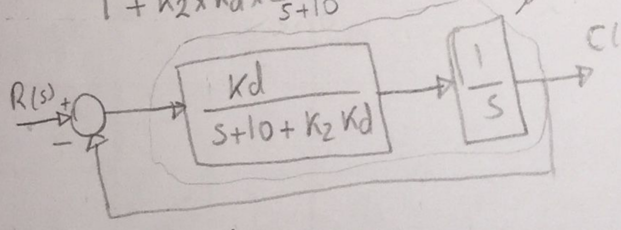
$$\zeta = 0.59$$

## Solution

$$t_r = \frac{\pi - \theta}{\omega_n\sqrt{1-\zeta^2}} \qquad \theta = \cos^{-1}(\zeta) = 0.94 \text{ rad}$$

$$\omega_n = \frac{\pi - \theta}{t_r\sqrt{1-\zeta^2}} = \frac{\pi - 0.94}{4\times\sqrt{1-(0.59)^2}} = 0.682 \text{ rad/sec.}$$

$$G_1 = \frac{Kd \times \frac{1}{s+10}}{1 + K_2 \times Kd \times \frac{1}{s+10}} = \frac{Kd}{s+10+K_2 Kd}$$




$$\frac{C(s)}{R(s)} = \frac{\frac{Kd}{(s+10+K_2Kd)} \times \frac{1}{s}}{1 + \frac{Kd}{s+10+K_2Kd} \times \frac{1}{s}}$$

$$= \frac{Kd}{(s+10+K_2Kd)(s) + Kd} = \frac{Kd}{s^2 + (10+K_2Kd)s + Kd}$$

char. Eq. $= s^2 + (10+K_2Kd)s + Kd$

Compare with $\to s^2 + 2\omega_n\zeta s + \omega_n^2$

$\therefore \omega_n^2 = Kd \Rightarrow Kd = 0.682^2 = 0.465$

$\therefore 2\omega_n\zeta = 10 + K_2 Kd$

$$K_2 = \frac{2\omega_n\zeta - 10}{Kd} = \frac{2\times0.682\times0.59 - 10}{0.465}$$

$$K_2 = -19.775$$

for a unit Ramp: $R(s) = \frac{1}{s^2}$

$$e_\infty = \lim_{s\to0} \frac{sR(s)}{1+G_2(s)} = \lim_{s\to0} \frac{1}{s} \times \frac{1}{1+G_2(s)}$$

Assume $sG_2(s) \gg s \Rightarrow e_\infty = \lim_{s\to0} \frac{1}{sG_2(s)}$

$$e_\infty = \lim_{s\to0} \frac{1}{s \times \frac{Kd}{s+10+K_2Kd} \times \frac{1}{s}} = \lim_{s\to0} \frac{s+10+KdK_2}{Kd}$$

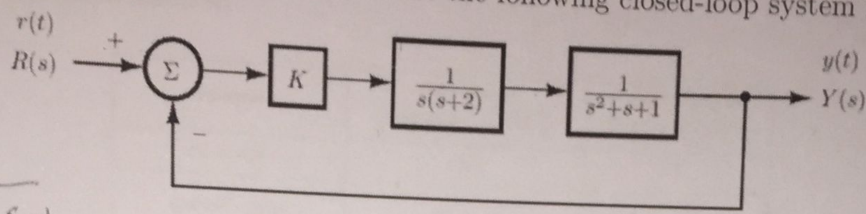
$$e_\infty = \frac{10}{Kd} + K_2$$

Kd should be very Large: $Kd \to \infty$

$K_2$ should be very small: $K_2 \to 0$

# Question 5

Determine the range of values of $K$ such that the following closed-loop system is **stable**. [3 Points]




## Solution

$$\frac{Y(s)}{R(s)} = \frac{K \times \frac{1}{S(S+2)} \times \frac{1}{S^2+S+1}}{1 + K \times \frac{1}{S(S+2)} \times \frac{1}{S^2+S+1}}$$

$$= \frac{K}{S(S+2)\times(S^2+S+1) + K} = \frac{K}{S^4+2S^3+S^3+2S^2+S^2+2S+K}$$

$$\frac{Y(s)}{R(s)} = \frac{K}{S^4+3S^3+3S^2+2S+K}$$

char. Eq: $S^4+3S^3+3S^2+2S+K$ → use Routh

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 3 | K |
| $S^3$ | 3 | 2 | 0 |
| $S^2$ | $\frac{7}{3}$ | K | 0 |
| $S^1$ | $\frac{\frac{14}{3}-3K}{\frac{7}{3}}$ | 0 | 0 |
| $S^0$ | K | 0 | 0 |
| | 0 | 0 | 0 |

$\therefore \frac{\frac{14}{3}-3K}{\frac{7}{3}} > 0$ and $K > 0$

$\frac{14}{3} > 3K$

$K < \frac{14}{9}$

$\therefore 0 < K < \frac{14}{9}$ #

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Good Luck!!!