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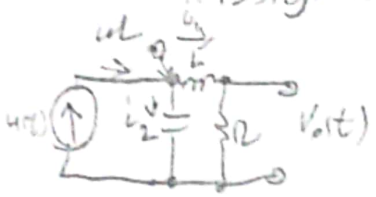
Nanong 512
Assignment 4.

[1] Derive T.F $\frac{V_o(s)}{U_o(s)}$



For electrical system :-

Resistor: $V_R(t) = i_R(t) R$, $i_R(t) = \frac{V_R(t)}{R}$

Inductor: $V_L(t) = L\frac{di_L(t)}{dt}$, $i_L(t) = \frac{1}{L}\int V_L(t)dt$

Capacitor: $V_C(t) = \frac{1}{C}\int i_C(t)dt$, $i_C(t) = C\frac{dV_C(t)}{dt}$

KCL at node 0 :- $u = i_1 + i_2 = i_c + i_R$

$i_2 = C\frac{d\left(V_o + L\frac{di}{dt}\right)}{dt} = C\frac{dV_o}{dt} + CL\frac{d^2 i}{dt^2}$; $i_1 = i_R = \frac{V_o}{R}$

$\therefore u = C\frac{dV_o}{dt} + CL\frac{d^2 i}{dt^2} + \frac{V_o}{R}$; Let $x_1 = V_o$, $\dot{x}_1 = \frac{dV_o}{dt}$, $i_1 = x_1/R$

$x_2 = i_1$, $\dot{x}_2 = di_1/dt$

$x_3 = \dot{x}_2$, $\dot{x}_3 = d^2 i_1/dt^2$

$\therefore u = \frac{x_1}{R} + C\dot{x}_1 + CL\dot{x}_3$

$\therefore \dot{x}_1 = \frac{u}{LC} - \frac{x_1}{RLC} - R\frac{x_3}{L}$; $\dot{x}_2 = x_3$; $\dot{x}_1 = Rx_3$

matrix form:
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 0 & R\\ 0 & 0 & 1\\ -1/RLC & 0 & -R/L\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + u\underbrace{\begin{bmatrix}0\\ 0\\ 1/LC\end{bmatrix}}_{B}$$

$y = \underbrace{[1\ 0\ 0]}_{C}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$; $\therefore \frac{y}{u} = C[SI-A]^{-1}B + D$, $\because D = 0$

$\therefore \frac{y}{u} = C[SI-A]^{-1}B$

$$[SI-A] = \begin{bmatrix}S & 0 & -R\\ 0 & S & -1\\ 1/RLC & 0 & S+R/L\end{bmatrix} \therefore [SI-A]^{-1} = \frac{adj}{det}$$

$$adj = \begin{bmatrix}\frac{S^2L+SR}{L}+1 & -1/RLC & -S/RLC\\ 0 & \frac{S^2L+SR}{L}+1/LC & S/RLC\\ RS & S & S^2\end{bmatrix}^T = \begin{bmatrix}\frac{S^2L+SR}{L}+1 & 0 & RS\\ 0 & \frac{S^2L+SR}{L}+\frac{1}{LC} & S\\ RS & S/RLC & S^2\end{bmatrix}$$

$det = |[\,]| = \frac{S[S^2LC+SCR+1]}{LC}$; $[SI-A]^{-1} = \begin{bmatrix}\frac{C(S^2L+SR+L)}{S(S^2LC+SCR+1)} & 0 & \frac{RLC}{S^2LC+SCR+1}\\ \frac{-1}{S(S^2LC+SCR+1)} & 1/S & \frac{LC}{S^2LC+SCR+1}\\ \frac{-1}{R(S^2LC+SCR+1)} & \frac{1}{R(S^2LC+SCR+1)} & \frac{SLC}{S^2LC+SCR+1}\end{bmatrix}$

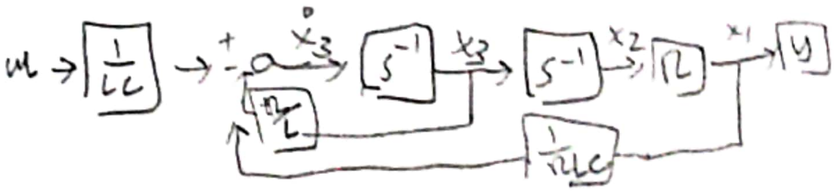
$\therefore \frac{y}{u} = C[SI-A]^{-1}B$

$\rightarrow C[SI-A]^{-1} = \left[\frac{C(S^2L+SR+L)}{S(S^2LC+SCR+1)}\quad 0 \quad \frac{RLC}{S^2LC+SCR+1}\right]$

$C[SI-A]^{-1}B = \left[0 + 0 + \frac{R}{S^2LC+SCR+1}\right]$

$$\boxed{\frac{y}{u} = \frac{R}{S^2LC+SCR+1}}$$

state diagram:



7 controllable, observable, diagonal $\frac{Y(z)}{U(z)} = \frac{z^2-2z+3}{z^3+7z^2+14z+8}$

$\frac{Y}{U} \cdot \frac{Q}{Q} \rightarrow \frac{Y}{Q} = z^2-2z+3$

$\frac{Q}{U} = \frac{1}{z^3+7z^2+14z+8}$; $U = z^3Q+7z^2Q+14zQ+8Q$

$x_1(k) = Q \quad x_1(k+1) = zQ$

$x_2(k) = zQ = x_1(k+1) \quad , x_2(k+1) = z^2Q$

$x_3(k) = z^2Q \quad , x_3(k+1) = z^3Q$

$u(k) = x_3(k+1)+7x_3k+14x_2k+8x_1k$

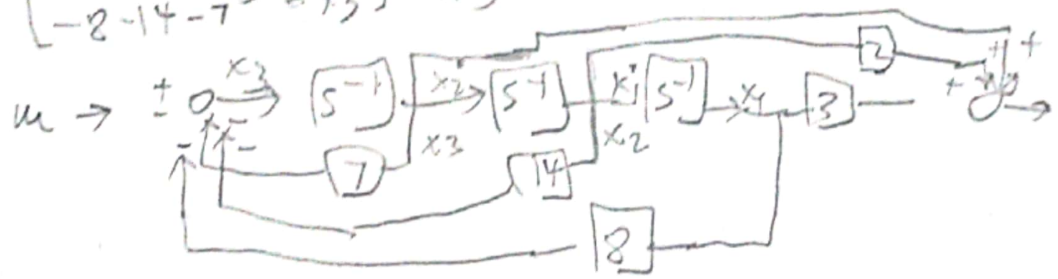
$x_3(k+1) = u(k)-7x_3(k)-14x_2(k)-8x_1(k)$

$x_2(k+1) = x_3(k) \qquad x_1(k+1) = x_2(k)$

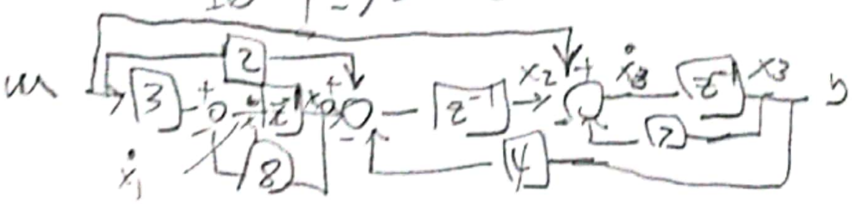
$Y = z^2Q - 2zQ + 3Q$

$y(k) = x_3(k) + 2x_2(k) + 3x_1(k)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad , y = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



Observable $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -14 \\ 0 & 1 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \\ 2 \\ 7 \end{bmatrix} u \quad y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$



Diagonal $\frac{Y(z)}{U(z)} = \frac{z^2-2z+3}{(z+4)(z+1)(z+2)}$, Partial Fractions $\frac{Y}{U} = \frac{4.5}{z+4} + \frac{2}{z+1} + \frac{-5.5}{z+2}$

$x_1 = \frac{U(z)}{z+4} \rightarrow zx_1+4x_1 = U(z)$

$x_1(k+1) = u(k) - 4x_1(k)$

$x_2 = \frac{U(z)}{z+1} \rightarrow zx_2+x_2 = U(z)$

$x_2(k+1) = u(k) - x_2(k)$

$x_3 = \frac{U(z)}{z+2} \quad x_3(k+1) = u(k) - 2x_3(k) \quad \therefore y(k) = 4.5x_1(k)+2x_2(k)+3x_3(k)$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u \quad , y(k) = \begin{bmatrix} 4.5 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$