Zewail City for Science and Technology
Nanotechnology and Nanoelectronics Program
Midterm #2 - FALL 2020

NANENG 512
Applied Digital Control and Drives
Time: 60 Minutes

Name: Model Answer ID: ....

This examination paper has 4 questions in 5 pages, including a formula sheet

Question 1

[3 Points]

Consider a unity feedback control system with the forward transfer function:

$$G(S) = \frac{K(S+1)}{S(S+2)(S+4)^2}$$

PID controller is used to control the system, apply the second method of Ziegler-Nichols tunning rule to determine the values of the PID parameters.

Solution

$$G_c(s) = K_p\left(1 + T_d S + \frac{1}{T_i S}\right)$$

$$G_c(s) = 0.6K_{cr}\left(1 + 0.125 P_{cr} S + \frac{1}{0.5 P_{cr} S}\right)$$

$\therefore$ C/Ch eqn:

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1)}{s(s+2)(s+4)^2} = 0$$

$$\therefore s^4 + 10s^3 + 32s^2 + (32+K)s + K = 0$$

Put $s = j\omega$

$$\omega^4 + (-10j\omega^3) - 32\omega^2 + (32+K)j\omega + K = 0$$

Real Part = 0

$$\omega^4 - 32\omega^2 + K = 0 \quad \text{—} \quad (1)$$

imag. Part = 0

$$-10\omega^3 + (32+K)\omega = 0 \quad \text{—} \quad (2)$$

Solve (1) and (2)

$$\therefore \omega_{cr} = 4.83$$

$$K_{cr} = 201.7$$

$$\therefore P_{cr} = \frac{2\pi}{\omega_{cr}} = 1.2997$$

$$\therefore K_p = 0.6 K_{cr} = 121$$

$$T_i = 0.5 P_{cr} = 0.64985$$

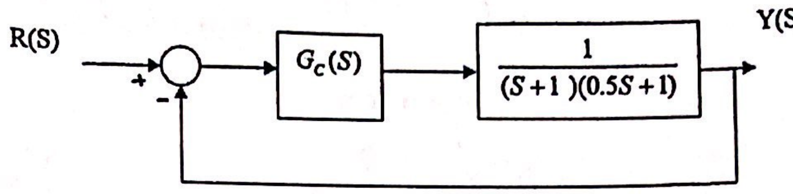
$$T_d = 0.125 P_{cr} = 0.1625$$

Question 2

[7 Points]

A mobile robot using a vision system as the measurement device can be represented by the block diagram in the below figure. Design a series controller $G_C(S)$ such that:

- The steady state error due to a unit step input is equal to zero.
- The relative damping ratio (ζ) of the complex dominant poles equal 0.7.
- The velocity error constant (K_V) is 0.9.



Find the imporvement in the steady state response due to a unit step reference input. Sketch the resulting time reponse to a unit step input.

Solution

We need Gc(S) to improve transient and steady state responses then we will choose PID.

$$G(S) = K_p + K_d S + \frac{K_i}{S}$$

C/ch eqn ; $1 + G_c(S)\, G(S) = 0$

$$\therefore S^3 + (1.5 + K_d)S^2 + (1 + K_p)S + K_i = 0$$

3rd order eqn:

$$(S+\alpha)(S^2 + 2\zeta\omega_n S + \omega_n^2) = 0$$

$$S^3 + (\alpha + 2\zeta\omega_n)S^2 + (\omega_n^2 + 2\zeta\omega_n\alpha)S + \alpha\omega_n^2 = 0$$

$\therefore 1.5 + K_d = \alpha + 2\zeta\omega_n$ —— (1)

$1 + K_p = 2\zeta\omega_n\alpha + \omega_n^2$ —— (2)

$K_i = \alpha\omega_n^2$ —— (3)

we have $\zeta = 0.7$

& $K_v = \lim_{s\to 0} S\,G(S) = 0.9$

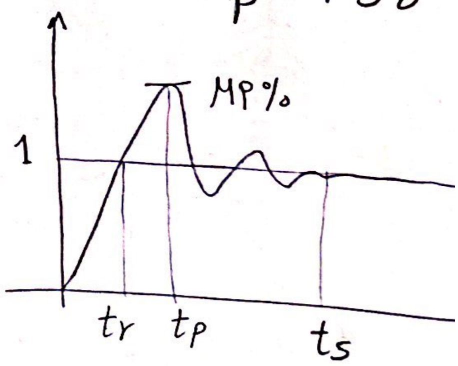
$\therefore K_i = 0.9$

$\alpha = 5\zeta\omega_n$

$\therefore \omega_n = 0.6359$

$\alpha = 2.23$

$K_d = 1.62$ & $K_p = 1.38$



Improvements:

$e_{ss.\,before} = 0.5$

$e_{ss\,after} = \text{zero}$

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Question 3

[3 Points]

For the following system:

$$\frac{Y(Z)}{U(Z)} = \frac{Z^2+1}{Z^2(Z^2+0.2)}$$

- Deduce the observable canonical form.
- Draw the corresponding state diagram.

Solution

$$\frac{Y(Z)}{U(Z)} = \frac{Z^2+1}{Z^2(Z^2+0.2)} * \frac{Z^{-2}}{Z^{-2}}$$

$$= \frac{Z^{-2}+Z^{-4}}{1+0.2Z^{-2}}$$

$$y(Z) = \left(U(Z) - 0.2y(Z)\right)Z^{-2} + U(Z)Z^{-4}$$

$$= \underbrace{Z^{-1}\underbrace{\left[\underbrace{Z^{-1}\left(U(Z) - 0.2y(Z) + \underbrace{Z^{-1}\underbrace{(Z^{-1}U(Z))}_{X_1(Z)}}_{X_2(Z)}\right)}_{X_3(Z)}\right]}_{X_4(Z)}}$$

$\therefore X_1(K+1) = U(K)$

$X_2(K+1) = X_1(K)$

$X_3(K+1) = U(K) - 0.2y(K) + X_2(K)$

$X_4(K+1) = X_3(K)$

$y(K) = X_4(K)$

$$\underline{X}(K+1) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.2 \\ 0 & 0 & 1 & 0 \end{bmatrix}\underline{X}(K) + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$y(K) = [0\ 0\ 0\ 1]\ \underline{X}(K)$$

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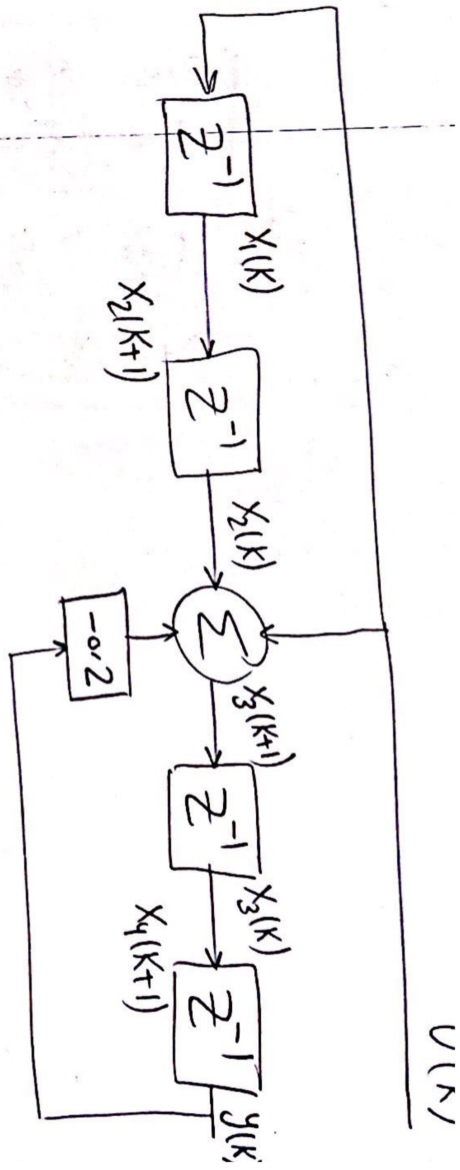

Question 4

[3 Points]

Given the following system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+2) \end{bmatrix} = \begin{bmatrix} a & 0.2 \\ 0.3 & b \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

1. Find the ranges of values of a and b for which the system is completely state controllable and completely state observable.
2. For $a = 0.3$ and $b = 0.4$. Find the system transfer function and identifiy the uncontrollable pole.

Solution

① $M = [B \;\; AB]$

$= \begin{bmatrix} 1 & a-0.2 \\ -1 & 0.3-b \end{bmatrix}$

$|M| = 0$

$0.3 - b = -a + 0.2$

$\therefore a - b = -0.1$

& $N = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2a+0.3 & 0.4+b \end{bmatrix}$

$|N| = 0$

$0.8 + 2b = 2a + 0.3$

$\therefore a - b = \frac{1}{4}$

$\therefore$ for Controllability

$\rightarrow a - b \neq -0.1$

& for observability

$\rightarrow a - b \neq \frac{1}{4}$

② $G = \begin{bmatrix} 0.3 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}$

$PTF = C(ZI - G)^{-1} H$

$= \dfrac{\cancel{Z - 0.6}}{\cancel{(Z-0.6)}(Z-0.1)}$

$\therefore$ uncontrollable Pole is

$\boxed{Z = 0.6}$

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Good Luck!!!