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Assignment 3

1 - $F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$; $e^{j\omega} = \cos(\omega) + j\sin(\omega)$ $j = \sqrt{-1}$

1- $f(t) = e^{-3|t|} \sin 2t = e^{-3t} \sin 2t\, u(t) + e^{3t} \sin 2t\, u(-t)$

Using Table identity: $F[e^{-at} u(t)] \rightarrow \frac{1}{j\omega + a}$

$i^2 = -1$ $\therefore i = \frac{-1}{i}$ $\therefore \frac{1}{i} = -i$

$x_1(t) = e^{-3t} \sin 2t\, u(t)$ $\therefore X_1(t) = \int_0^{\infty} e^{-3t} \frac{(e^{2ti} - e^{-2ti})}{2i} e^{-j\omega t}$

$\therefore X_1(t) = \frac{-i}{2} \int_0^{\infty} e^{-t(3+j\omega-2j)} - e^{-t(3+j\omega+2j)}$

$\therefore X_1(j\omega) = \frac{-i}{2} \left[\frac{1}{3+j\omega-2j} - \frac{1}{3+j\omega+2j} \right] = \frac{1/2 i}{3+j\omega-2j} - \frac{1/2 i}{3+j\omega+2j}$

$x_2(t) = e^{3t} \sin 2t\, u(-t) = -x_1(-t) \overset{FT}{\longleftrightarrow} X_2(j\omega) = -X_1(-j\omega)$

$X_2(j\omega) = \frac{1/2 i}{3-j\omega-2j} - \frac{1/2 j}{3-j\omega+2j}$

$\therefore X(j\omega) = X_1(j\omega) + X_2(j\omega) = \boxed{\frac{3j}{9+(\omega+2)^2} - \frac{3j}{9+(\omega-2)^2}}$

2 - $\delta(t+1) + \delta(t-1)$

$X(j\omega) = \int_{-\infty}^{\infty} [\delta(t+1) + \delta(t-1)] e^{-j\omega t} dt$

$\boxed{X(j\omega) = e^{j\omega} + e^{-j\omega} \equiv 2\cos\omega}$

$\because \cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$

$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

2 1- $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

1- $X(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [2\pi\delta(\omega) + \pi\delta(\omega-4\pi) + \pi\delta(\omega+4\pi)] e^{j\omega t} d\omega$

$= \frac{1}{2\pi}[2\pi e^{0jt} + \pi e^{4j\pi t} + \pi e^{-4j\pi t}]$

$\boxed{x(t) = 1 + \frac{1}{2}e^{4j\pi t} + \frac{1}{2}e^{-4j\pi t} \equiv 1 + \cos(4\pi t)}$

2- $x(t) = \frac{1}{2\pi}\int_0^2 2e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{-2}^{0}(-2)e^{j\omega t} d\omega$

$= \frac{e^{j2t} - 1}{\pi j t} - \frac{1 - e^{-j2t}}{\pi j t} = \frac{1}{\pi t j}\left\{ e^{j2t} + e^{-j2t} - 2 \right\}$

$\boxed{x(t) = \frac{-4j\sin^2 t}{\pi t}}$

3 a) $y[n] = x[n-2] - 2x[n-8]$

Linearity check:- $x_1[n] \rightarrow y_1[n] = x_1[n-2] - 2x_1[n-8]$

$x_2[n] \rightarrow y_2[n] = x_2[n-2] - 2x_2[n-8]$

$ax_1[n] + bx_2[n] = x_3[n] \rightarrow y_3[n] = [x_1 + x_2][n-2] - 2[x_1 + x_2][n-8]$

$y_3[n] = x_3[n][n-2] - 2x_3[n-8]$

$= ay_1[n] + by_2[n]$

$\therefore$ $\boxed{\text{Linear}}$

Time-invariance:- $x_1[n] \rightarrow y_1[n] = x_1[n-2] - 2x_1[n-8]$

$x_1[n-n_0] = x_2[n] \rightarrow y_2[n] = x_2[n-2] - 2x_2[n-8]$

$= x_1[n-2-n_0] - 2x[n-8-n_0]$

$= y_1[n-n_0]$

$\therefore$ $\boxed{\text{Time-invariant}}$

$\boxed{\text{Causal}}$

$\boxed{\text{Stable}}$

$\boxed{\text{not memoryless}}$

$\boxed{3}$ b) $y[n]=nx[n]$ $x_1[n] \rightarrow y_1[n]=nx_1[n]$

$x_2[n] \rightarrow y_2[n]=nx_2[n]$

$ax_1[n]+bx_2[n]=x_3[n] \rightarrow y_3[n]=nx_3[n]$

$= n(ax_1[n]+bx_2[n]) = ay_1[n]+by_2[n]$

$\therefore$ $\boxed{\text{Linear}}$

Time invariance: $x_1[n] \rightarrow y_1[n]=nx_1[n]$

$x_1[n-n_0]=x_2[n] \rightarrow y_2[n]=nx_2[n]$

$= nx_1[n-n_0] \neq y_1[n-n_0]$

$\because y_1[n-n_0]=(n-n_0)\,x_1[n-n_0]$ $\therefore$ $\boxed{\text{time-variant}}$

$\boxed{\text{Not stable}}$ bec. if $x[n]=1$ for all n $\therefore$ $y[n] \rightarrow \infty$ as $n \rightarrow \infty$

$\boxed{\text{memoryless}}$, $y[n]$ depends only on $x[n]$

$\therefore$ it is also $\boxed{\text{causal}}$

c) $y(t)=x(t^3)$ $ax_1(t^3)+bx_2(t^3) = x_3(t^3) = ay_1(t)+by_2(t)$

$\therefore$ $\boxed{\text{Linear}}$

continous & depends on t $\therefore$ $\boxed{\text{not memoryless}}$

$\boxed{\text{not causal}}$