Problem 1

Compute the Fourier transform of each of the following signals:

1- $e^{-3|t|}\sin(2t)$

$$X(t) = \underbrace{e^{-3|t|}}_{x_1(t)} \sin(2t) \qquad x_1(t) = \begin{cases} e^{3t} & t<0 \\ e^{-3t} & t>0 \end{cases}$$

$$X_1(j\omega) = \int x_1(t)\, e^{-j\omega t}\, dt$$

$$X_1(j\omega) = \int_{-\infty}^{0} e^{3t} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-3t} e^{-j\omega t}\, dt$$

$$= \left.\frac{e^{t(3-j\omega)}}{(3-j\omega)}\right|_{-\infty}^{0} + \left.\frac{e^{-t(3+j\omega)}}{-(3+j\omega)}\right|_{0}^{\infty}$$

$$X_1(j\omega) = \frac{1}{3-j\omega} + \frac{1}{3+j\omega}$$

$$x_1(t)\sin(2t) \longleftrightarrow \frac{j}{2}\left[X_1(\omega+2) - X(\omega-2)\right]$$

$$X(j\omega) = \frac{j}{2}\left[\frac{1}{3-j(\omega+2)} + \frac{1}{3+j(\omega+2)} - \frac{1}{3-j(\omega-2)} - \frac{1}{3+j(\omega-2)}\right]$$

2- $\delta(t+1) + \delta(t-1)$

$$x(t) = \delta(t+1) + \delta(t-1)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\therefore X(j\omega) = \int_{-\infty}^{\infty} \delta(t+1)\, e^{-j\omega t}\, dt + \int_{-\infty} \delta(t-1)\, e^{-j\omega t}\, dt$$

$$= \left. e^{-j\omega t}\right|_{t=-1} + \left. e^{-j\omega t}\right|_{t=1}$$

$$X(j\omega) = e^{j\omega} + e^{-j\omega} = 2\cos\omega$$

Problem 2

Compute the Inverse Fourier transform of each of the following signals:

1- $X(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

$$X(j\omega) = 2\pi\,\delta(\omega) + \pi\,\delta(\omega - 4\pi) + \pi\,\delta(\omega + 4\pi)$$

$$x(t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$\therefore x(t) = \frac{1}{2\pi}\int 2\pi\,\delta(\omega)\, e^{j\omega t}\, d\omega + \frac{1}{2\pi}\int \pi\,\delta(\omega - 4\pi)\, e^{j\omega t}\, d\omega$$

$$+ \frac{1}{2\pi}\int \pi\,\delta(\omega + 4\pi)\, e^{j\omega t}\, d\omega$$

$$x(t) = e^{j\omega t}\Big|_{\omega=0} + \frac{1}{2}\, e^{j\omega t}\Big|_{\omega=4\pi} + \frac{1}{2}\, e^{j\omega t}\Big|_{\omega=-4\pi}$$

$$x(t) = 1 + \cos 4\pi t$$

2- $x(j\omega) = \begin{cases} 2 & 0 \le \omega \le 2 \\ -2 & -2 \le \omega \le 0 \\ 0 & |\omega| > 2 \end{cases}$

$$\therefore x(t) = \frac{1}{2\pi}\int_0^2 2\, e^{j\omega t}\, d\omega + \frac{1}{2\pi}\int^0 -2\, e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{\pi}\, \frac{e^{j\omega t}}{jt}\Big|_0^2 + \frac{-1}{\pi}\, \frac{e^{j\omega t}}{jt}\Big|_{-2}^0$$

$$x(t) = \frac{1}{j\pi t}\left[e^{j2t} - 1\right] - \frac{1}{j\pi t}\left[1 - e^{-j2t}\right]$$

Problem 3

- Determine which of the following properties hold and which do not hold for each of the following continues time systems. Justify your answer. $X(t)$ is the system input while $y(t)$ is the system output?
- Memoryless
- Causality
- Linearity

a. y[n] = x[n-2]-2x[n-8]

$$y[n] = x[n-2] - 2x[n-8]$$

$$x[n] \longrightarrow \boxed{Sys.} \longrightarrow x[n-2]-2x[n-8]$$

$$\therefore x_1[n] \longrightarrow y_1[n] = x_1[n-2] - 2x_1[n-8]$$

$$x_2[n] \longrightarrow y_2[n] = x_2[n-2] - 2x_2[n-8]$$

$$(a x_1[n] + b x_2[n]) \longrightarrow \boxed{Sys.} \longrightarrow [a x_1[n-2] + b x_2[n-2]] - 2[a x_1[n-8] + b x_2[n-8]]$$

$x_3[n]$

$$y_3[n] = a[x_1[n-2] - 2x_1[n-8]] + b[x_2[n-2] - 2x_2[n-8]]$$

$$y_3[n] = a y_1[n] + b y_2[n] \quad \therefore \text{Sys. is linear}$$

→ System is Causal (Depend on Past i/p)

→ System needs Memory.

b. y[n] = nx[n]

$$y[n] = n\,x[n]$$

→ System is Memoryless / Causal as it depend on present i/p only.

$$x_1[n] \longrightarrow y_1[n] = n\,x_1[n]$$

$$x_2[n] \longrightarrow y_2[n] = n\,x_2[n]$$

$$x_3[n] \longrightarrow y_3[n] = n\,x_3[n] \quad \text{where } x_3[n] = a x_1[n] + b x_2[n]$$

$$= n\,[a x_1[n] + b x_2[n]]$$

$$y_3[n] = a y_1[n] + b y_2[n] \quad \text{Sys is linear.}$$

c. $y(t) = x(t^3)$

$y(t) = x(t^3)$

System needs memory , Non Causal as o/p depend on future i/p

$y(0) = x(0)$

$y(-1) = x(-1)$

$y(2) = x(8)$ future

$x(t) \rightarrow \boxed{sys.} \rightarrow x(t^3)$

$x_1(t) \rightarrow y_1(t) = x_1(t^3)$

$x_2(t) \rightarrow y_2(t) = x_2(t^3)$

$a\,x_1(t) + b\,x_2(t) \longrightarrow a\,x_1(t^3) + b\,x_2(t^3)$

$a\,y_1(t) + b\,y_2(t)$ sys is linear