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Naneng 307
Assignment 1

[1] Given: Ar, $\alpha_e = 1.7\times10^{-40}\ Fm^2$, Density $D = 1.8\,g\,cm^{-3}$, Solid Ar ($T<84K$)
$N_A = Avag = 6.02\times10^{23}$, $M_{at} = 39.95\ gmol^{-1}$, $\epsilon_0 = 8.85\times10^{-12}$, $\epsilon_r = ?$

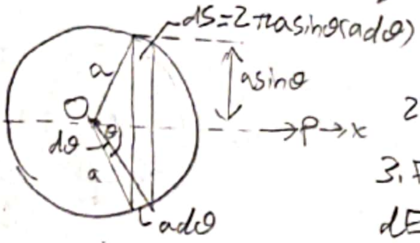
1. No. atoms per unit volume $N = \dfrac{N_A D}{M_{at}} = \dfrac{(6.02\times10^{23})(1.8)}{39.95} = 2.7124\times10^{22}\ cm^{-3}$

2. $\because \epsilon_r = 1 + \chi_e = 1 + \dfrac{N\alpha_e}{\epsilon_0} = 1 + \dfrac{(2.7124\times10^{23})(1.7\times10^{-40})\cdot(10^{-2})^{-3}}{8.85\times10^{-12}}$ $\boxed{= 1.5208 \simeq 1.521}$

$N = 2.7124\times10^{22}(10^{-2})^{-3}\ m^{-3}$

3. Using Clausius-Mossotti eq:
$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0}$$; solving for ϵ_r:- $\boxed{\epsilon_r = 1.6302}$

[2] a) Derive Lorentz Eq: 1. Assume $E_{dipoles} = 0$ $\therefore E_{loc} = E + E_s$ -(1)



For this spherical shell: $dS = (2\pi a\sin\theta)(a\,d\theta)$

2. Polarization charge: $dQ_p = P_n dS = (P\cos\theta)(2\pi a\sin\theta)(a\,d\theta)$

3. Field at O from dQ_p in x-direction:-
$$dE_s = \frac{dQ_p}{4\pi\epsilon_0 a^2}\cos\theta = \frac{(P\cos\theta)(2\pi a\sin\theta)(a\,d\theta)}{4\pi\epsilon_0 a^2}\cos\theta$$

4. Total Field from S: $E_s = \int_0^{\pi} dE_s = \int_0^{\pi} \dfrac{(P\cos\theta)(\sin\theta)}{2\epsilon_0}\cos\theta\, d\theta$

5. Substitution, using
$x = \cos\theta$, $dx = -\sin\theta\, d\theta$ $\therefore E_s = \displaystyle\int_{\cos(0)}^{\cos(\pi)} -\frac{P}{2\epsilon_0}x^2dx = -\frac{P}{2\epsilon_0}\int_1^{-1} x^2 dx$

$$\therefore E_s = \frac{P}{2\epsilon_0}\frac{x^3}{3}\Big|_{-1}^{1} = \frac{P}{2\epsilon_0}\left[\frac{1}{3}+\frac{1}{3}\right] = \frac{P}{3\epsilon_0} \quad -(2)$$

6. From (1) & (2); $\boxed{E_{loc} = E + \frac{1}{3\epsilon_0}P \text{ - Lorentz eq.}}$

b) 1. Polarization $P = (\epsilon_r - 1)\epsilon_0 E$ -(1), $E_{loc} = E + \dfrac{1}{3\epsilon_0}P$ -(2)

2. Substitute (1) in (2): $E_{loc} = E + \dfrac{(\epsilon_r - 1)\epsilon_0 E}{3\epsilon_0} = E\left[1 + \dfrac{\epsilon_r - 1}{3}\right] = E\left[\dfrac{3+\epsilon_r-1}{3}\right]$

3. $\boxed{\therefore E_{loc} = E\left[\frac{\epsilon_r + 2}{3}\right]}$

[3] a) $\mu_r = 1 + \chi_m = 1 + 100 = \boxed{(d)\ 101}$

b) $\because C = \dfrac{\epsilon_0\epsilon_r A}{d}$, $d_{new} = 2d$ & ϵ_r inserted $\therefore \dfrac{C_{new}}{C_{old}} = \dfrac{2}{1} = \dfrac{\left[\frac{\epsilon_0\epsilon_r A}{2d}\right]}{\frac{\epsilon_0 A}{d}} = \dfrac{1}{2}\epsilon_r$

$\therefore \boxed{\epsilon_r = (d)\ 4.0}$