Q1) $n = \frac{d N_A}{M_{at}} = 8.5 * 20^{22}\ cm^{-3}$

$\mu = \frac{\sigma}{en} = 43.4\ cm^2 V^{-1} s^{-1}$

$\tau = \frac{\mu m_e}{e} = 2.5 * 10^{-14}\ s$

$\ell = u \tau = 1.5 * 10^6 * 2.5 * 10^{-14}$

$= 39\ nm$

Q2) $E_x = \frac{V_{dx}}{\mu_d} = \frac{10^3}{43.4 * 10^{-4}} = 2.3 * 10^5\ V m^{-1}$

$J_x = \sigma E_x = 1.4 * 10^7\ A mm^2$

Q3) $\frac{1}{2} KT = \frac{1}{4} M a^2 w^2$ $\quad a^2 = \frac{2KT}{M w^2}$ $\longrightarrow \frac{63.56 * 10^{-3}}{N_a}$

$J = \sigma E$

$\sigma = e m \mu$

$\mu = \frac{e\tau}{m_e}$

$s = \pi a^2$

$N_s = \frac{d N_A}{M_{at}}$ $\quad \therefore s = 3.9 * 10^{-22} m^2$

$\tau = \frac{1}{s \mu N_s} = \frac{1}{3.9 * 10^{-22} * 1.6 * 10^6 * 8.5 * 10^{28}} = 1.9 * 10^{-14} s$

$\mu_d = \frac{e\tau}{m_e} = \frac{1.6 * 10^{-19} * 1.9 * 10^{-14}\ s}{9.1 * 10^{-31}} = 33\ cm^2 V^{-1} s^{-1}$

$= 3.3 * 10^{-3}\ m^2 V^{-1} s^{-1}$

$\sigma = e n \mu_d =$

$1.6 * 10^{-19} * 8.5 * 10^{22} * 33 = 4.5 * 10^5\ \Omega^{-1} cm^{-1}$

Q4) Atomic Concentration is

(a) $n_{at} = \frac{DN_A}{M_{at}} = \frac{971.2 * 6.022 \times 10^{23}}{22.99 * 10^{-3}} = 2.544 * 10^{28} \ m^{-3}$

also same as number of electrons (1 atom 1 e^-)

$d = \frac{1}{\sqrt[3]{V}} = \frac{1}{n_{at}^{1/3}} = \frac{1}{(2.544 \times 10^{28})^{1/3}} = 3.4 * 10^{-10} \ m$

(b) Density $= D = \frac{(\text{atoms in unit cell})(\text{mass of 1 atom})}{\text{Volume unit cell}}$

$= \frac{2 * \frac{M_{at}}{N_A}}{a^3}$

$\therefore a = \left[\frac{2M_{at}}{DN_A}\right]^{1/3} = \left[\frac{2 * 2.99 * 10^{-3}}{9712 * 10^3 * 6.022 * 10^{23}}\right]^{1/3}$

$a = 4.2 * 10^{-10} \ m \longrightarrow ,0.4284 \, nm$

$(4R)^2 = 3a^2 \longrightarrow R = \frac{1}{4}\sqrt{3a^2} = 1.85 \times 10^{-10} \ m$

$PE = \frac{e^2}{4\pi \varepsilon R} = \frac{-1.6 * 10^{-19} * 1.6 * 10^{-b}}{4 \pi \varepsilon_0 R}$

$\hookrightarrow FR$

$= -1.24 * 10^{18} \ J$

$\simeq -7.76 \ eV$

(c) $E_{thermal} = \frac{3}{2} KT \rightarrow .639 \ eV$ $\boxed{300T}$

$KT \simeq PE = \frac{1}{2} m_e u^2 \rightarrow u = 1.65 * 10^6 \ m/s$

(d) $\sigma = en\mu$, $n = n_{at}$

$\therefore \sigma = en * \mu = 1.6 * 10^{-19} * 2.544 * 10^{28} * 53$

$= 2.16 * 10^7 \ \Omega^{-1} m^{-1}$

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Q5) (a) $\rho(T) = \rho_0[1+\alpha_0(T-T_0)]$

$\rho(-40°C) = \rho_0[1+\alpha_0(-40°C - 0°C)]$ 0°C as reference

$\rho(25°C) = \rho_0[1+\alpha_0(25-0)]$

$\cancel{\alpha_0 \text{ at } T_0 = 0}$

α_0 at $T_0 = 0$

$$\frac{\rho(-40°C)}{\rho(25°C)} = \frac{1+\alpha_0(-40°C)}{1+\alpha_0(25°C)}$$

$$\therefore \rho(-40°C) = 2.72\times10^{-8}\ \frac{1+4.29\times10^{-3}\times40}{1+4.29\times10^{-3}\times25}$$

$$= 2.035\times10^{-8}\ \Omega m$$

(b) $\rho = \rho_0[1+\alpha_0(T-T_0)]$

$\rho_2 = \rho_0(1+\alpha_0(T_2-T_0))]$

$\rho_0 = \rho_2(1+\alpha_2(T_0-T_2))]$

$\rightarrow \alpha_2 = \alpha_0/[1-(T_2 T_0)\alpha_0]$

$\alpha_2(-40) = 4.29\times10^{-3}/[1+(-40-0)(4.29\times10^{-3})]$

$= 5.18\times10^{-3}\ 1/°C$

or $\alpha_0 = \frac{1}{\alpha_0}\left[\frac{d\rho}{dT}\right]_{T_0}$

(c) $\frac{1}{\rho} = \sigma = en\mu \rightarrow \mu = \frac{e\tau}{m_e}$

$\therefore \tau = \frac{m_e}{\rho e^2 n}$

$n_{AT} = \frac{N_A d}{M_{at}} = \frac{6.022\times10^{23}\times2700}{.027} = 6.022\times10^{28}\ m^{-3}$

$= n = 3n_{AT} = 1.807\times10^{29}\ m^{-3}$

$$\therefore \tau = \frac{9.1\times10^{-31}}{2.72\times10^{-8}\times(1.602\times10^{-19})^2\times1.807\times10^{29}}$$

$= 7.22\times10^{-15}\ s$

$\therefore \mu_d = \frac{e\tau}{m_e} = 1.27\times10^{-3}\ m^2V^{-1}s^{-1}$

$= 12.7\ cm^2V^{-1}s^{-1}$

(d) $l = v\tau = 1.65 * 10^{6} * 7.22 \times 10^{-15}$

$= 1.44 * 10^{-8} = 14.4\,mm.$

(e) $P = I^2 R$ if I is Constant

$$\frac{P(-40) - P(25^\circ)}{P(25^\circ)} = \frac{P_{40^\circ} - P_{25}}{(P_{25})}$$

$= -25.4\%$

(Q6) $\sigma = en\mu_d$

From Table at $0^\circ C$, $\rho_0 = 20.5 \ n\Omega m$

$\rho = \rho_0 [1 + \alpha_0 (T - T_0)]$ α_0 From Table $= 1/242 \ K^{-1}$

$\therefore \rho(22) = 20.5[1 + \frac{1}{242}(293 - 273)]$

$= 22.36 \ n\Omega m$

$\because$ Au atom give one electron:

$n = \frac{dN_A}{M_{at}}$ $d \rightarrow$ density

$N_A \rightarrow$ Avogadro

$M_{at} \rightarrow$ atomic mass

$n = \frac{19300 * 6.022 \times 10^{23}}{.169} = 5.91 * 10^{28} \ m^{-3}$

or $5.91 * 10^{22} \ cm^{-3}$

$\therefore \mu_\sigma = \frac{\sigma}{en} = \frac{22.36 * 10^{-9}}{1.6 * 10^{-19} * 5.91 * 10^{28}}$

$= 4.72 * 10^{-3} m^2 V^{-1} s^{-1} = 47.2 \ cm^2 V^{-1} S^{-1}$

$\therefore$ when mean speed $= 1.4 * 10^6 m/s$

$\therefore \ell = \tau * u = \frac{\mu_d m_e u}{e}$

$= \frac{4.72 * 10^{-3} * 9.1 * 10^{-31} * 1.4 * 10^6}{1.6 * 10^{-19}}$

$\therefore \ell = 3.76 * 10^{-8} m$

$= 37.6 \ nm$

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Q7) From nordheim rule , $\rho_{alloy} = \rho_0 + C\,x(1-x)$

$$\alpha_{alloy} = \frac{1}{\rho_{alloy}} \frac{d\rho_{alloy}}{dT} \quad \text{from TCR definition}$$

$$= \frac{1}{\rho_{alloy}} \frac{d}{dT}\left[\rho_0 + Cx(1-x)\right]$$

$\because$ C is Temp independant

$$\therefore \alpha_{alloy} = \frac{1}{\rho_{alloy}} \frac{d\rho_0}{dT} \rightarrow \alpha_0 \rho_0$$

$$\alpha_{alloy} = \frac{1}{\rho_{alloy}} \rho_0 \alpha_0$$

$$\therefore \alpha_{alloy}\, \rho_{alloy} = \rho_0 \alpha_0 \rightarrow (1)$$

at room Temp. From table $\rho_{cu} = 17.1\, n\Omega m$

$\alpha_{cu} = 4 * 10^{-3}\, K^{-1}$, $C = 1310\, n\Omega m$

$$\therefore \text{from } (1) \quad \rho_{alloy} = \frac{\alpha_{cu}\, \rho_{cu}}{\alpha_{alloy}} = \frac{.044 * 117.1}{.0004} = 171\, n\Omega m \text{ much larger than Cu}$$

$$\rho_{alloy} = \rho_{cu} + Cx(1-x)$$

$$171 = 17.1 + (1310)\, x\, (1-x)$$

$$x^2 - x + .1175 = 0$$

$\therefore x = .136$ (not .866 because Ni is the one dissolved in Cu, so smaller percentage).

$\therefore$ Cu = 86.4% , Ni = 13.6%

Q8) $n = \frac{\sigma}{e n \mu_d} = \frac{8.37 * 10^{-8}}{1.6 * 10^{-19} * 6 * 10^{-4}}$

$= 1.24 * 10^{29}\ Cm^{-3}$

atomic Concentration

$n_{at} = \frac{d N_A}{M_{at}} = \frac{7.31 * 10^{3}\ 6.022 \times 10^{23}}{114.82 * 10^{-3}}$

$= 3.834 * 10^{26}\ m^{-3}$

$\therefore$ number of Conduction e^- per atom

$= \frac{n}{n_{at}} = 3.24 \simeq 3e^-$

$\mu_d = \frac{e\tau}{m_e}$ $\therefore \tau = \frac{\mu_d m_e}{e} = \frac{6 * 10^{-4} * 9.1 * 10^{-31}}{1.6 * 10^{-19}}$

$= 3.4 * 10^{-15}\ s$

$v = 1.74 * 10^{6}$ $\therefore$ mean free path ℓ:

$\therefore \ell = v\tau = 1.74 * 10^{6} * 3.4 \times 10^{-15}$

$= 5.9 \times 10^{-9}\ m = 5.9\ nm$

Interatomic Separation $d \simeq \frac{1}{\sqrt[3]{n_{at}}} = \frac{1}{3.8 * 10^{23\ 1/3}}$

$= 0.3\ nm$

e^- pass 20 atoms before Scattering again.

(c) $K = \sigma T C_{WFL} = 8.77 * 10^{-8}\ (27 + 273)\ (2.44 * 10^{-8})$

$\therefore K = 85.4\ W m^{-1} K^{-1}$

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Q9) 15% air pores are the dispersed, $X_d = 0.15$

phase From mixture rule:

$$\rho_{eff} = \rho \frac{1+\frac{1}{2}X_d}{1-X_d} = 62\,n\Omega m\left(\frac{1+\frac{0.15}{2}}{1-0.15}\right)$$

$$\rho_{eff} = 78.41\,n\Omega m$$

Now using Reynolds and Hough rule:

$$\frac{\sigma - \sigma_{alloy}}{\sigma + 2\sigma_{alloy}} = X\frac{\sigma_{air} - \sigma_{alloy}}{\sigma_{air} + 2\sigma_{alloy}}$$

$$\sigma_{air} = 0 \quad , \quad \sigma_{alloy} = \frac{1}{62\,n\Omega m}$$

$$\therefore \frac{\sigma - \frac{1}{62}}{\sigma + \frac{2}{62}} = 0.15\,\frac{0-\frac{1}{62}}{0+\frac{2}{62}} \quad \therefore \sigma = 1.27\times10^{7}\,\Omega^{-1}m^{-1}$$

$$\therefore \rho_{eff} = \frac{1}{\sigma} = 78.41\,n\Omega m$$

→ Same as first calculation with mixture rule

(mixture rule is a special case of Reynolds).

(Q10)

(a) $$\frac{\sigma - \sigma_c}{\sigma + 2\sigma_c} = \chi \frac{\sigma_d - \sigma_c}{\sigma_d + 2\sigma_c}$$

$\because$ Porus $\therefore$ 47 % air

$\because$ $\sigma_{air} = 0$

$$\sigma_{graphic} = \frac{1}{9.1\,\mu\Omega m}$$

$$\therefore \frac{\sigma - \frac{1}{9.1 * 10^{-6}}}{\sigma + \frac{2}{9.1 * 10^{-6}}} = 0.47 \frac{0 - \frac{1}{9.1 * 10^{-6}}}{0 + \frac{2}{9.1 * 10^{-6}}}$$

$$\sigma = \frac{1}{21.2 * 10^{-6}} \qquad \therefore \rho_{eff} = 21.2 * 10^{-6}\,\Omega m$$

(b) For silver $\alpha_0 = \frac{1}{242}$ at $0\,^\circ C$

$\rho_0 = 14.7$ nm at $0\,^\circ C$

$$\therefore \rho_{silver} = \rho_0 [1 + \alpha_0 (T - T_0)]$$

$$= 14.7\,n\Omega m \left[1 + \frac{1}{242}(293 - 273)\right]$$

$$= 15.91\,n\Omega m$$

$\because$ $\rho_d < 0.1\,\rho_c$

$\therefore$ Use mixture rule with $X_d = 0.3$

$$\therefore \rho_{eff} = \rho_c \frac{1 - X_d}{1 + 2X_d} = 9.1\,\mu\Omega \times \frac{1 - 0.3}{1 + 2 * 0.3}$$

$\rho_{eff} = 3.98\,\mu\Omega m$, much smaller than (a)

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(Q11) (a1) Thermal resistence is $\Theta = \frac{L}{KA}$

so we find K first

$K = \sigma_{brass}\, C_{WL}\, T$

$\sigma_{brass} = \frac{1}{\rho_{brass}}$

$\rho_{brass} = \rho_{Cu} + C_{Zn-in-Cu}\, X(1-X)$

$= 17.1\, n\Omega m + (300)(0.2)(1-0.2)$

$= 65.1\, n\Omega m = 65.1 * 10^{-9}\, \Omega m$

$\therefore K = \frac{2.44 * 10^{-8} * 293}{65.1 \times 10^{-9}\, \Omega m}$

$\therefore K(20^\circ c) = 109.8\; W K^{-1} m^{-1}$

$\therefore \Theta = \frac{L}{KA} = \frac{5 \times 10^{-3}}{109.8 * \pi \left(\frac{40 \times 10^{-3}}{2}\right)^2} = 0.0362\; K W^{-1}$

(a2) $\frac{dQ}{dT} = 100 W \quad \because \frac{dQ}{dT} = \frac{AK\Delta T}{\Delta X} = \frac{\Delta T}{\Theta} = P$

$\therefore \Delta T = P\Theta = 100 * 3.62 * 10^{-2} = 3.62\, K \text{ or } ^\circ c$

(b) $\Delta T = P\Theta$

if $\Delta T_1 \longrightarrow \frac{\Delta T}{2}$

$\therefore \Theta \longrightarrow \frac{\Theta}{2}$

$\Theta = \frac{L}{KA}$ $\therefore$ K should be doubled

$\because K \propto \frac{1}{\rho_{brass}}$

$\therefore \rho_{brass}$ is half $\therefore \frac{\rho_{brass}}{2} = \rho_{Cu} + C\, X_{new}(1-X_{new})$

$\frac{65.1}{2} = 17 + 300\, X_{new}(1-X_{new})$

$\therefore X_{new} = 5.5\%$

$\therefore$ ~~97~~.5% Cu

5.5% Zn