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Naneng 430
Assignment I

[1] Given: Lossless dielectric medium ∴ $\sigma \simeq 0$, $\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$, $\sigma \ll \omega\epsilon$; $E_x = E_0 \cos(\omega t - \beta z)$
∴ propagation in the a_z direction, ; $f = 5\,GHz$ ∴ $\omega = 2\pi f = 2\pi(5\times10^9) = 3.14\times10^{10}\,Hz$
$\lambda = 3cm = 3^{-2}\,m$ ∴ $\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda} = \frac{2\pi}{3\times10^{-2}} = \frac{200}{3}\pi\ rad/m \simeq 209.4$

⊙ Propagation constant $k = \frac{2\pi}{\lambda} = \frac{200}{3}\pi = 209.4\ rad/m$

⊙ Phase velocity $v_p = \frac{\omega}{k} = \frac{10\pi(10^9)}{\frac{200}{3}\pi} = 1.5\times10^8\,m/s$

⊙ relative permittivity: ∵ $c = \frac{1}{\sqrt{\epsilon_0\mu}}$ & $v_p = \frac{1}{\sqrt{\epsilon_r\epsilon_0\mu}}$ ∴ $c^2 = \frac{1}{\epsilon_0\mu}$ & $v_p^2 = \frac{1}{\epsilon_r\epsilon_0\mu}$

∴ $\epsilon_r = \frac{1}{v_p^2}\frac{1}{\epsilon_0\mu} = \frac{c^2}{v_p^2} = \left(\frac{c}{v_p}\right)^2 = 2^2 = 4$

⊙ wave impedance $n = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{\mu_r}{4}}$, assuming $\mu_r = 1$ (lossless dielectric)

∴ $n = \frac{377}{\sqrt{4}} = 188.5\,\Omega$

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[2] Given: $f = 10\,GHz = 10\times10^9\,Hz$; polystyrene ($\epsilon_r = 2.5$ $\tan\delta = 0.001$)

attenuation constant: ∴ Lossy medium ∴ $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\epsilon}\right]^2} - 1\right]}$

∵ $\mu_r = 1$, $\epsilon_r = 2.5$, $\tan\theta = \frac{\sigma}{\omega\epsilon} = \tan\delta = 0.001$, $\omega = 2\pi f = 2\pi[1\times10^{10}]$

∴ $\alpha = (2\times10^{10})(\pi)\sqrt{\frac{2.5\mu_0\epsilon_0}{2}\left[\sqrt{1+0.001^2} - 1\right]} = \boxed{0.1657\ Np/m}$

∴ $\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\epsilon}\right]^2} + 1\right]} = (2\times10^{10})(\pi)\sqrt{\frac{2.5\mu_0\epsilon_0}{2}\left[\sqrt{1+0.001^2} + 1\right]} = 331.4\ rad/m$

∵ $\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda}$ ∴ $\lambda = \frac{2\pi}{\beta} = 18.96\,mm$

⊙ phase velocity: Two methods: 1. ∵ $\beta = \frac{\omega}{u}$ ∴ $u = \frac{\omega}{\beta} = \frac{(2\times10^{10})(\pi)}{\beta} = 1.896\times10^8\ m/s$

or 2. $u = f\lambda = 10^{10}\lambda = \boxed{1.896\times10^8\,m/s}$

∴ $\sigma = \omega\epsilon\tan\delta = [2.5\epsilon_0][2\times10^{10}\pi][0.001] = 1.39\times10^{-3}\ S/m$

⊙ Intrinsic Impedance ∴ $|n| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} = \frac{\sqrt{\frac{\mu_0}{2.5\epsilon_0}}}{\left[1+0.001^2\right]^{1/4}} = 238.3\,\Omega$

phase: ∵ $\tan 2\theta_n = \frac{\sigma}{\omega\epsilon}$ ∴ $\theta_n = \frac{\tan^{-1}(0.001)}{2} = 0.0286°$ ∴ $\boxed{n = 238.3\,\Omega\angle 0.0286°}$

⊙ Comparing with air: air is considered lossless $\sigma \simeq 0$ and $\alpha = 0$
while here is a lossy medium with $\sigma = 1.39\times10^{-3}\,S/m$ and $\alpha = 0.1657\,Np/m$
Accordingly phase velocity u here is less than c as expected for lossy medium
while $u = c$ for air (approximation).

③ Given $\sigma = 4.5\times10^{-3}$ S/m , $\epsilon_r = 3-j$, $f = 350MHz = 350\times10^6 Hz$, $\mu_r = 2-3j$

Lossy medium $\therefore \omega = 2\pi f = 700\times10^6 \pi$ Hz

For intrinsic impedence (η is complex) $\therefore \eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$$\therefore \eta = \sqrt{\frac{[700\times10^6 \pi \mu_0 j][2-3j]}{4.5\times10^{-3} + [700\times10^6 \pi \epsilon_0][3-j][j]}} = \sqrt{130827.3998 - 88238.15557i}$$

$$\therefore \eta = \sqrt{157802\angle -33.498^\circ} = \boxed{397.244\angle -16.499^\circ = 379.9 - 116.1i}$$

Distance required $\because \gamma = j\omega\sqrt{\mu\epsilon} = 700\times10^6 \pi j \sqrt{\mu_0\epsilon_0 [2-3j][3-j]}$ & $c = \dfrac{1}{\sqrt{\mu_0\epsilon_0}}$

$$\therefore \gamma = \frac{700\times10^6 \pi j \sqrt{3-11j}}{3\times10^8} = 15.024 + 19.67j$$

$\because 20dB = 20\log(10)$ $\because \gamma = \alpha + j\beta \therefore \alpha = 15.024$ Np/m , $\beta = 19.67$ rad/m

$$\therefore \frac{E}{E_0} = \frac{1}{10} = e^{-\alpha z} \Rightarrow \ln(10) = \alpha z \quad \therefore z = \frac{\ln(10)}{15.024} = 0.15326$$

$\therefore \boxed{z = 15.326cm}$ depth.