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Naneng 430
midterm

Q1 EQ, sketches. Ampl, phase, velocity EM plane wave each medium.
+ relation between E & H

I. Perfect dielectric:
Here it is a lossless dielectric ($\sigma = 0$, $\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$ or $\sigma \ll \omega\epsilon$)
for lossless: $\alpha = 0$, $\beta = \omega\sqrt{\mu\epsilon}$

$\because \gamma = j\omega\sqrt{\mu\epsilon} = \alpha + j\beta$

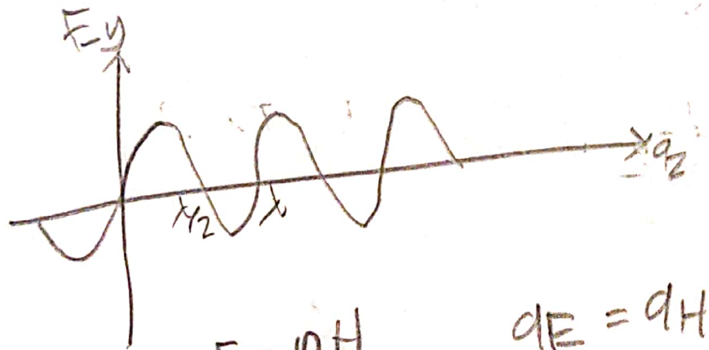
$\because u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$ ∴ wave velocity $= u = \frac{1}{\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}}$

$\lambda = \frac{2\pi}{\beta}$; $\eta = \sqrt{\frac{\mu}{\epsilon}} \angle 0°$

$E = \cancel{Re[E_c \exp j(\omega t - \beta z)]} = E_0 \cos(\omega t - \beta z + \phi_0)$

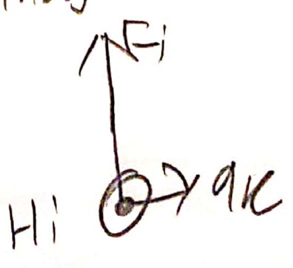
$= Re[\underbrace{E_0 \exp(j\phi_0)}_{E_c} \exp j(\omega t - \beta z)]$

$= E_c \cos(\omega t - \beta z)\, a_y$ +az propagation
oscillation in +ay dir.



$E = \eta H$ $\quad a_E = a_H \times a_k$

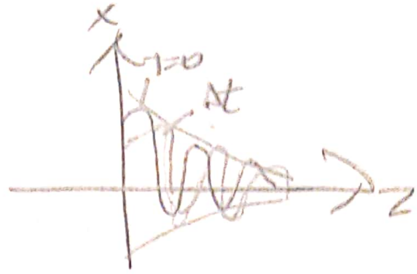
E, H, k are orthogonal to each other



2- Lossy: $(\sigma \neq 0, \epsilon = \epsilon_0 \epsilon_r, \mu = \mu_0 \mu_r)$ $\therefore \alpha \neq 0$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega \epsilon}\right]^2} - 1\right]} \qquad \beta = \omega \sqrt{\frac{\mu \epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega \epsilon}\right]^2} + 1\right]}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{u} \qquad H_0 = \frac{E_0}{\eta}$$



$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| \angle \phi_\eta$$

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

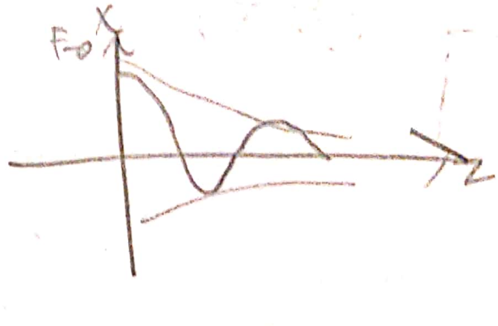
$$H(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \phi_\eta)\, a_y$$

3- Good conductor $(\sigma \simeq \infty, \epsilon = \epsilon_0 \epsilon_r, \mu = \mu_0 \mu_r, \sigma \gg \omega\epsilon)$

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\pi f \mu \sigma} \qquad u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu \sigma}}, \quad \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \qquad \therefore E \text{ leads } H \text{ by } 45^\circ$$

$$\therefore E = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x \quad \therefore H = \frac{E_0}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\, a_y$$



4- Perfect conductor $\sigma = \infty$ or $\cancel{R}$ $\rho = 0$

does not exist in real life

However follows same equations as good conductor and same behaviour.

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Q2 $E = 8\cos(\omega t - 4x - 3z)a_y$ V/m $\quad z>0\ \mu_r=1\ \epsilon_r=2.5\ \sigma=0$

~~$\beta = 3 = \frac{\omega}{c}\ \therefore \omega = 3c = 9\times10^8$ Hz~~

~~$\omega = 2\pi f\ \therefore f = \frac{\omega}{2\pi} = \frac{3c}{2\pi}$~~

air freespace | lossless dielectric $\mu_r=1\ \epsilon_r=2.5$

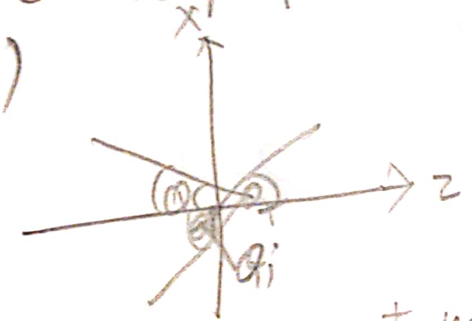
$z=0$

a) $k_i = 4a_x + 3a_z \rightarrow k_i = 5 = \omega\sqrt{\mu_0\epsilon_0} = \frac{\omega}{c} \rightarrow \omega = 5c = 15\times10^8$ rad/s

unit vector normal to interface ($z=0$) is a_z.
the plane containing k and a_z is y=constant wich is xz plane
The plane of incidence. since E_i is normal to this plane
we have perpendicular polarization

$\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{4}{3} \rightarrow \theta_i = 53.13^\circ$

b)



c) let $E_r = E_{ro}\cos(\omega t - k_r \cdot r)a_y$

$k_r = k_{rx}a_x - k_{rz}a_z \quad \theta_r = \theta_i \quad k_r = k_i = 5$

$k_{rx} = k_r\sin\theta_r \quad k_{rz} = k_r\cos\theta_r$

$\because$ both k_r and k_i are in same meidum

$\therefore k_r = 4a_x - 3a_z \quad ;$ snell's law

$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\epsilon_1}}{c\sqrt{\mu_2\epsilon_2}}\sin\theta_i = \frac{\sin 53.13^\circ}{\sqrt{2.5}} \therefore \theta_t = 30.39^\circ$

~~So~~ from ~~#~~ Γ_r eq: $\eta_1 = \eta_0 = 377$

$\therefore \eta_2 = \sqrt{\frac{\mu_0\mu_{r2}}{\epsilon_0\epsilon_{r2}}} = \frac{377}{\sqrt{2.5}} = 238.4$

in Γ_r eq: $\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{238.4\cos 53.13^\circ - 377\cos 30.39^\circ}{238.4\cos 53.13^\circ + 377\cos 30.39^\circ} = -0.389$

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$\therefore E_{r0} = \Gamma_r E_{i0} = -0.389(8) = -3.112$

$E_r = -3.112\cos(15\times10^8 t - 4x + 3z)a_y$ V/m

d) $E_t = E_{t0}\cos(\omega t - k_t \cdot r)a_y$

$k_t = \beta_2 = \omega\sqrt{\mu_2\epsilon_2} = \frac{\omega}{c}\sqrt{\mu_{r2}\epsilon_{r2}} = \frac{15\times10^8}{3\times10^8}\sqrt{1\times2.5}$

$= 7.906$

as $k_{tx} = k_t\sin\theta_t = 4$

$k_{tz} = k_t\cos\theta_t = 6.819$

$k_t = 4a_x + 6.819a_z$

$\because k_{ix} = k_{rx} = k_{tx}$ (boundary conditions)

$\therefore \tau_t = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = 0.611$

$\because \tau = 1 + \Gamma$ $\therefore E_{t0} = \tau_t E_{i0} = 0.611\times8 = 4.888$

$E_t = 4.88\cos(15\times10^8 t - 4x - 6.819z)a_y$

$H_t = \frac{a_{k_t}\times E_t}{\eta_2} = (-17.69a_x + 10.37a_z)\cos(15\times10^8 t - 4x - 6.819z)$ mA/m

4 Q2 $E = 8\cos(\omega t - 4x - 3z)\,a_y$ V/m $z \geq 0$ $\mu_r = 1$ $\epsilon_r = 2.5$ $\sigma = 0$

~~$\beta = 3 = \frac{\omega}{c}$ ∴ $\omega = 3c = 9\times10^{8}$ rad/s~~

~~$\omega = 2\pi f$ ∴ $f = \frac{\omega}{2\pi} = \frac{3c}{2\pi}$~~

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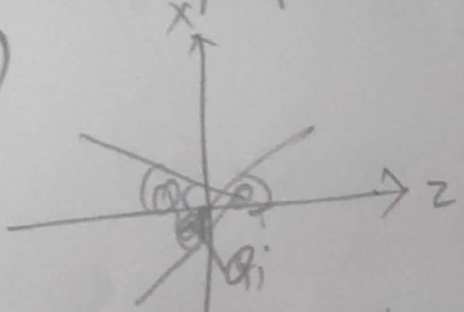

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$k_r = k_{rx}a_x - k_{rz}a_z$ $\theta_r = \theta_i$ $k_r = k_i = 5$

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∴ $k_r = 4a_x - 3a_z$; ∴ snell's law

$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\epsilon_1}}{c\sqrt{\mu_2\epsilon_2}}\sin\theta_i = \frac{\sin 53.13^\circ}{\sqrt{2.5}}$ ∴ $\theta_t = 30.39^\circ$

from Γ_r eq: $\eta_1 = \eta_0 = 377$

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∴ Γ_r eq: $\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{238.4\cos 53.13^\circ - 377\cos 30.39^\circ}{238.4\cos 53.13^\circ + 377\cos 30.39^\circ} = -0.389$

Q3 $f_c = 5GHz$, TE_{01} @ $f_c = 2GHz$ $\omega_c = 2\pi f$

a) TE_{10} mn air-filled

$\omega_c = 10\pi \times 10^9 Hz$ $\omega_c = 4\pi \times 10^9 Hz$ $\mu = \mu_0$, $\epsilon = \epsilon_0$

$$\omega_c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\therefore \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \left[\frac{\omega}{c}\right]^2$$

case 1 $m=1, n=0$

$$\left(\frac{100}{3}\pi\right)^2 = \left(\frac{\pi}{a}\right)^2 \therefore a =$$

$$f_c = c\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

case 2 $m=0, n=1$

$$\frac{2f}{c} = \frac{1}{b} \therefore b = \frac{1}{20/3} = 0.15m$$

$$b = 15cm$$

$$\left(\frac{5f}{c}\right)^2 = \left(\frac{1}{a}\right)^2 \therefore a = \frac{1}{50/3} = 0.06m = 6cm$$

b) For TE_{02} mode ($m=0, n=2$)

$$f_c = 3\times10^8 \sqrt{\frac{m^2}{0.06^2} + \frac{n^2}{0.15^2}} = 4 \quad 3GHz$$

TE_{03} mode $\therefore f_c = 6GHz$

TE_{04} mode $\therefore f_c = 8GHz$.

c) $\epsilon_r = 2.25$ $\mu_r = 1$ $\therefore$ $f_{c_{mn}} = \frac{1}{\sqrt{2.25}} f_{c\,original}$

$$TM_{11} \therefore f_c = \frac{c}{\sqrt{2.25}} \sqrt{\frac{1}{0.06^2} + \frac{1}{0.15^2}} = 3.59GHz$$

$m=1, n=1$