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Naneng 430
Assignment 2

Q) Air-filled rectangular wg. $a = 3cm$ & $b = 2cm$

a) Considering TM modes, what is the cuttoff freq. of wg.? (no propagation)

For TM_n mode, with $n = 0, 1, 2, \dots$:- $\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$

For air : $\mu_r = 1$ & $\epsilon_r = 1$ $\therefore \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c = 3\times10^8 m/s$

$\because$ for TM mode, if n or m is zero, all fields are zero.

$\therefore$ TM_{11} is lowest order mode of all TM_{mn} modes $\therefore m = n = 1$

$\therefore \omega_c = c\sqrt{\left[\frac{\pi}{0.03}\right]^2 + \left[\frac{\pi}{0.02}\right]^2} = 5.66\times10^{10} Hz.$ $\because \omega = 2\pi f$

$\therefore f_c = \frac{1}{2\pi}\omega_c = 9.0076\times10^9 Hz = 9.0076 GHz.$

b) Plots are done in excel: governing eq's:-

$\beta = \beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$, $\because \beta' = \frac{\omega}{u} = \omega\sqrt{\mu\epsilon}$ $\therefore \beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$\therefore \beta = \frac{2\pi f}{c}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$V_{ph} = \frac{c}{n}$ $\therefore n = \frac{c}{V_{ph}}$ & $V_{ph} = \frac{\omega}{\beta}$ $\therefore n_{eff} = \frac{c\beta}{\omega} = \frac{c\beta}{2\pi f}$

c) $V_{ph} = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$, For TM_{mn} T_{11} : $f_c \doteq 9.0076 GHz$

T_{21} : $f_c = 12.49 GHz$ T_{12} ; $f_c = 15.8 GHz$

d) Even for combined modes, speed doesn't exceed 'c'

$\therefore V_g < c$