ZEWAIL CITY: UNIVERSITY OF SCIENCE AND TECHNOLOGY

Principles of Microwave and Waveguides: Final Assessment Report

Naneng 430– Spring 2020

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I. Contents

II. Introduction

As most electrical circuits can be modeled as a “black box” with a linear network with equivalent resistance (R), Inductance (L), capacitance (C) and dependent sources, it is essential to have a visual model that represents this black box and it’s RLC network. Considering the case of a two-port network, which is completely described in terms of voltage (V) and currents (I) as shown in Fig. 1:

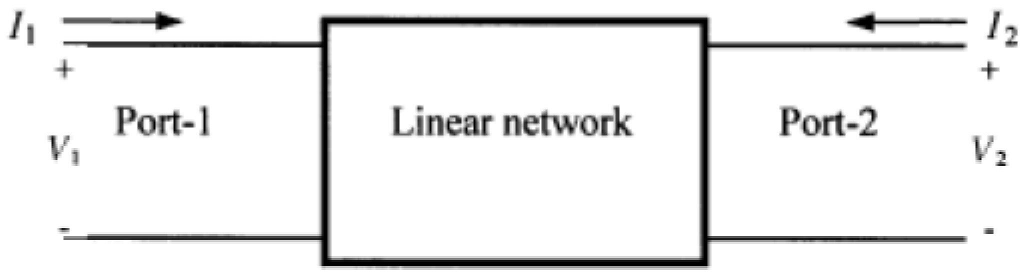

Fig. 1 two-port network

Accordingly, characterizing this network is essential for describing the electrical and/or optical properties which can be carried by one of the following sets of parameters (out of many others as well):

1. Impedance (Z) parameters
2. Admittance (Y) parameters
3. Hybrid (H) parameters
4. Transmission (ABCD) parameters
5. Scattering (S) parameters

On the other hand, to characterize high-frequency and microwave circuits, the that defines normal or real parameters such as Z or Y cannot be achieved as this requires opening or short circuiting the network. However, at these operating frequencies it is not possible to carry out these open/short circuit conditions due to parasitic inductance or capacitance. Not to mention, these states would lead to standing wave which would cause oscillation and destruction of the device. Moreover, for non-TEM propagation mode, direct measurement of voltage and current is not possible as only power form E and H fields are accessible and measurable. Conclusively, a different set of parameters other than the conventional Z, Y or H parameters models systems that operate at the high-frequency or microwave region, these are known as scattering (S) parameters.

III. Scattering Parameters

A. Definition

S-parameters relate incident to reflected power waves, instead of voltage and current. Therefore, the open/short circuit condition required for defining Z, Y or H parameters is not utilized which would cause standing wave oscillations for high-frequency or microwave circuits that would destruct the device. Additionally, as S-parameters relates reflected and incident voltage waves (derived from power waves), there is an advantage of calculating important optical properties such as reflection coefficient, gain, loss etc. from the information provided. It is worth mentioning, analogous to ABCD parameters, S-parameters can be cascaded for multiple devises to predict total system performance. Finally, Z, Y, or H parameters may be obtained from S-parameters through simple conversions.

As mentioned in the introduction, in order to fully comprehend a circuit model, a visual representation is required. Hence, considering a linear n-port network (Fig. 2) that represents any physical device that operates a microwave circuit (i.e. transmission lines or waveguides) such as those shown in Fig. 3.

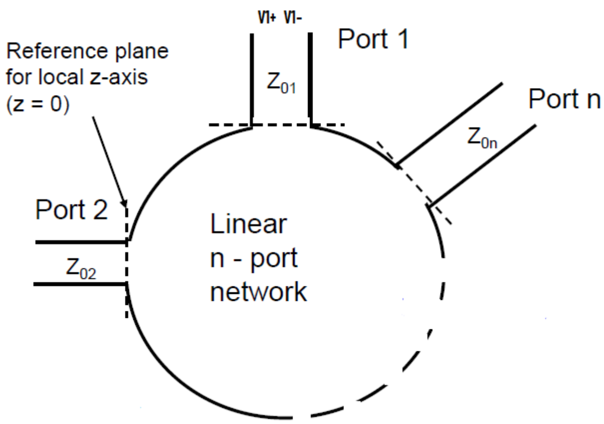

Fig. 2 Linear n-port network

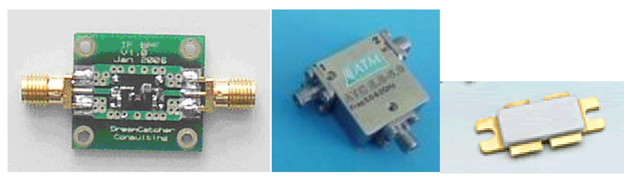

Fig. 3 Physical device models

Each port in the model is attached to a transmission line (T-line) in which the electromagnetic (EM) wave propagates. Accordingly, all ports are referenced to a T-line with known Z_0. Notwithstanding, at the port/network interface, the EM wave is composed of incident (+) and reflected (-) components. Thus, the derived voltage or current waves at the port takes the general from of an EM wave. Therefore, the S-matrix relates reflected voltage wave at port n (V_n^-) to incident voltage wave at port n (V_n^+), generally as follows:

$$
\begin{aligned}
V_1^- &= s_{11}V_1^+ + s_{12}V_2^+ + s_{13}V_3^+ + \cdots + s_{1n}V_n^+ \\
V_2^- &= s_{21}V_1^+ + s_{22}V_2^+ + s_{23}V_3^+ + \cdots + s_{2n}V_n^+ \\
&\vdots \\
V_n^- &= s_{n1}V_1^+ + s_{n2}V_2^+ + s_{n3}V_3^+ + \cdots + s_{nn}V_n^+
\end{aligned}
$$

Or in matrix form:

$$
\begin{bmatrix} V_1^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ \vdots \\ V_N^+ \end{bmatrix}
$$

To find an element of the S-matrix:

$$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0 \text{ for } k\neq j}$$

S_{ij} is measured by driving port j with an incident wave V_j^+, and measuring the reflected wave amplitude, V_i^-, coming out of port i. It is essential that incident waves on all ports except j_{th} port are set to zero. Hence, all ports should be terminated in matched load to avoid reflections. Accordingly, S_{ii} is the reflection coefficient seen looking into port i when all other ports are terminated in matched loads, and S_{ij} is the transmission coefficient from port j to port i when all other ports are terminated in matched loads. Matched load ensures zero reflection and eliminates standing wave.

B. Network conversion relations

From their definitions, impedance and admittance are inverse of each other or:

$$[Z] = [Y]^{-1}$$

From mathematical derivations utilizing the unity matrix [U], a relation between S, Z, Y, and U matrices each with their respective parameters is obtained:

$$[S] = ([Z] + [U])^{-1}([Z] - [U])$$

IV. Network Properties

A. Reciprocity

"A network is reciprocal if a zero impedance source and a zero impedance ammeter can be placed at any locations in a network and their positions interchanged without changing the ammeter reading" Lorentz. Mathematically, reciprocity is defined as when:

$$[S] = [S]^T$$

Where $[S]^T$ is the transpose of the $[S]$ matrix or for a square matrix:

$$[S] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \xrightarrow{\text{Transpose of [S], written as } [S]^t} \quad [S]^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

B. Symmetric

Whereas, symmetry of a matrix is achieved when a = d for the same [S] matrix or:

$$[S] = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$$

C. Lossless

A lossless network occurs when:

$$[S][S]^* = [U]$$

A lossless and reciprocal network: $[S][S]^* = [S]^T\,[S]^* = [U]$

D. Matched

As per the definition of matched load presented in the definition of scattering parameters, it is worthwhile to mention, that if all ports of a network are matched then the S-matrix takes the following form as S_{ii} = 0 throughout diagonal:

$$[S] = \begin{bmatrix} 0 & \cdots & S_{1N} \\ \vdots & 0 & \vdots \\ S_{N1} & \cdots & 0 \end{bmatrix}$$

Ultimately, these properties assist in analyzing the differences and similarities between distinct multiport networks.

V. Power Dividers and Directional Couplers

A. Definition

Power Dividers and directional couplers are passive microwave elements employed for power division or combination. Power dividers provide output signals in phase equal power division ratio, but with unequal power division ratios. Directional couplers arbitrarily divide power.

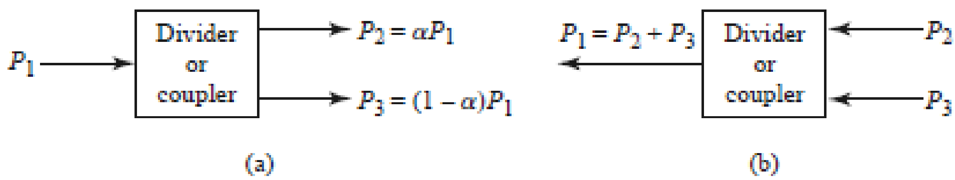

Fig. 4 Power division and combining. (a) Power division. (b) Power combining.

B. Three-Port Network

Utilizing the simplest power divider type, which is a T-junction, a three-port network with two inputs and one output. It has nine independent elements:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

If the device is passive and contains no anisotropic materials, then it must be reciprocal and symmetric so $S_{ij} = S_{ji}$. Also making the junction lossless and matched at all ports would lead to avoiding power loss. Using matched and reciprocal properties covered, [S] takes the form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Accordingly, after applying the lossless condition and satisfying corresponding unitary conditions, the following set of equations are obtained:

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{23}|^2 = 1,$$
$$|S_{13}|^2 + |S_{23}|^2 = 1,$$
$$S_{13}^* S_{23} = 0,$$
$$S_{23}^* S_{12} = 0$$
$$S_{12}^* S_{13} = 0$$

Unfortunately, these equations are inconsistent. Conclusively, a three-port network cannot be simultaneously lossless, reciprocal, and matched at all ports. At least one of these conditions must be violated for a real device to theoretically exist.

C. Circulator

A real example of a three-port network, alas one that is not reciprocal $S_{ij} \neq S_{ji}$ ensures the possibility that all ports are matched and lossless. This device is a circulator and its S-matrix takes the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Taking into consideration that this network is not reciprocal, the same analysis is done as before and results in a solution set.

D. Resisitive divider

Alternatively, a three-port network may be lossless and reciprocal. However, for physical realization, only two of its ports are matched. Such is a resistive divider. The S-matrix when ports 1 and 2 are matched takes the form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}.$$

With only 2 out of 3 ports matched, there is a solution set when applying the unitary condition to obtain a lossless matrix.

VI. References

1. Pozar, D., 2012. *Microwave Engineering*. 4th ed.
2. Farhat, M., 2020. *MICROWAVE NETWORK ANALYSIS*.