NANENG 430

Formula Sheet

TABLE 1.1 Summary of Results for Plane Wave Propagation in Various Media

	Type of Medium		
Quantity	Lossless ($\epsilon'' = \sigma = 0$)	General Lossy	Good Conductor ($\epsilon'' \gg \epsilon'$ or $\sigma \gg \omega\epsilon'$)
Complex propagation constant	$\gamma = j\omega\sqrt{\mu\epsilon}$	$\gamma = j\omega\sqrt{\mu\epsilon}$ $= j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\frac{\sigma}{\omega\epsilon'}}$	$\gamma = (1+j)\sqrt{\omega\mu\sigma/2}$
Phase constant (wave number)	$\beta = k = \omega\sqrt{\mu\epsilon}$	$\beta = \mathrm{Im}\{\gamma\}$	$\beta = \mathrm{Im}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Attenuation constant	$\alpha = 0$	$\alpha = \mathrm{Re}\{\gamma\}$	$\alpha = \mathrm{Re}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Impedance	$\eta = \sqrt{\mu/\epsilon} = \omega\mu/k$	$\eta = j\omega\mu/\gamma$	$\eta = (1+j)\sqrt{\omega\mu/2\sigma}$
Skin depth	$\delta_s = \infty$	$\delta_s = 1/\alpha$	$\delta_s = \sqrt{2/\omega\mu\sigma}$
Wavelength	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$
Phase velocity	$v_p = \omega/\beta$	$v_p = \omega/\beta$	$v_p = \omega/\beta$

$$\Gamma = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i},$$

$$T = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}.$$

TE/TM modes

Wave Propagation in Dielectric Medium	Wave Propagation in a Waveguide
$\beta' = \omega / u' = \omega\sqrt{\mu\varepsilon}$	$\beta = \beta'\sqrt{1-\left[\frac{f_c}{f}\right]^2}$
$\eta' = \sqrt{\mu / \varepsilon}$	$\eta_{TE} = \dfrac{\eta'}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$, $\eta_{TM} = \eta'\sqrt{1-\left[\frac{f_c}{f}\right]^2}$
$u' = \omega / \beta' = f\lambda = 1/\sqrt{\mu\varepsilon}$	$u_p = \dfrac{\omega}{\beta'\sqrt{1-\left[\frac{f_c}{f}\right]^2}} = \omega / \beta$
$\lambda' = u' / f$	$\lambda = \dfrac{\lambda'}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$

		$n = 0,1,2,3,\ldots$	$n = 1,2,3,\ldots$
Quantity	TEM Mode	TM_n Mode	TE_n Mode
k	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
k_c	0	$n\pi/d$	$n\pi/d$
β	$k = \omega\sqrt{\mu\epsilon}$	$\sqrt{k^2 - k_c^2}$	$\sqrt{k^2 - k_c^2}$
λ_c	∞	$2\pi/k_c = 2d/n$	$2\pi/k_c = 2d/n$
λ_g	$2\pi/k$	$2\pi/\beta$	$2\pi/\beta$
v_p	$\omega/k = 1/\sqrt{\mu\epsilon}$	ω/β	ω/β
α_d	$(k\tan\delta)/2$	$(k^2\tan\delta)/2\beta$	$(k^2\tan\delta)/2\beta$
α_c	$R_s/\eta d$	$2kR_s/\beta\eta d$	$2k_c^2 R_s/k\beta\eta d$
E_z	0	$A\sin(n\pi y/d)e^{-j\beta z}$	0
H_z	0	0	$B\cos(n\pi y/d)e^{-j\beta z}$
E_x	0	0	$(j\omega\mu/k_c)B\sin(n\pi y/d)e^{-j\beta z}$
E_y	$(-V_o/d)e^{-j\beta z}$	$(-j\beta/k_c)A\cos(n\pi y/d)e^{-j\beta z}$	0
H_x	$(V_o/\eta d)e^{-j\beta z}$	$(j\omega\epsilon/k_c)A\cos(n\pi y/d)e^{-j\beta z}$	0
H_y	0	0	$(j\beta/k_c)B_n\sin(n\pi y/d)e^{-j\beta z}$
Z	$Z_{\mathrm{TEM}} = \eta d/W$	$Z_{\mathrm{TM}} = \beta\eta/k$	$Z_{\mathrm{TE}} = k\eta/\beta$