Assignment 1

Q.(1)

$x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(10$

– Given

(a)

$F_s = 200 Hz$

$$x(n) = 3\cos\left(\frac{50\pi n}{200}\right) + 10\sin\left(\frac{300\pi n}{200}\right) - \cos\left(\frac{100\pi n}{200}\right)$$

$$x(n) = 3\cos\left(\frac{\pi n}{4}\right) + 10\sin\left(\frac{3\pi n}{2}\right) - \cos\left(\frac{\pi n}{2}\right)$$

$$x(n) = 3\cos(\pi/4 n) - 10\sin(\pi/2 n) - \cos(\pi/2 n)$$

(b)

$$F_1 = \frac{50\pi}{2\pi} = 25 Hz$$

$$F_2 = \frac{300\pi}{2\pi} = 150 Hz$$

$$F_3 = \frac{100\pi}{2\pi} = 50 Hz$$

$Max(25, 150, 50) = 150 Hz$

$F_m = 150 Hz$

∴ For succesfull reconstruction

$F_s \geq 2F_m$

∴ $200 \geq 2 \times 150$

$200 \geq 300$ (Not true)

∴ Reconstruction is not possible

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Reconstructed signal

$$x(t) = 3\cos\left(\frac{\pi}{4} \times 200t\right) - 10\sin\left(\frac{\pi}{2} \times 200t\right)$$
$$- \cos\left(\frac{\pi}{2} \times 200t\right)$$

$$x(t) = 3\cos(50\pi t) - 10\sin(100\pi t) - \cos(100\pi t)$$

Q.2

$$x(t) = \sin(480\pi t) + 3\sin(720\pi t)$$

$F_s = 600\ Hz$

a)

$$F_1 = \frac{480\pi}{2\pi} = 240\ Hz$$

$$F_2 = \frac{720\pi}{2\pi} = 360\ Hz$$

$\therefore F_m = Max(240, 360) = 360\ Hz$

~~$\therefore$ Folding frequency $= 2 \times 360 = 720\ Hz$~~

$\therefore$ Nyquist rate $= 2 \times 360 = 720\ Hz$

Folding frequency $= \frac{600}{2} = 300\ Hz$

b)

$$x(n) = \sin\left(\frac{480\pi n}{600}\right) + 3\sin\left(\frac{720\pi n}{600}\right)$$

$$x[n] = \sin\left(\frac{4}{5}\pi n\right) + 3\sin\left(\frac{6}{5}\pi n\right)$$

The frequencies are $\frac{4}{5}\pi$ rad and $\frac{6}{5}\pi$ rad

Q.3

(a)

$$x[n] = \cos(0.3\pi n)\,u[n]$$

$$\omega = 0.3\pi$$

$$\frac{2\pi}{\omega} = \frac{N}{K}$$

$$\frac{2\pi}{0.3\pi} = \frac{N}{K}$$

$$\therefore \frac{20}{3} = \frac{N}{K}$$

$\therefore N = 20$

$\therefore x[n]$ is periodic with period 20

(b) $x[n] = \cos(0.3\pi n + 0.5\pi)\,u[n]$

$\Rightarrow$ The phase shift does't affect freq

$\therefore \omega = 0.3\pi$

$$\frac{2\pi}{\omega} = \frac{N}{K} \Rightarrow \frac{2\pi}{0.3\pi} = \frac{N}{K} \Rightarrow \frac{20}{3} = \frac{N}{K}$$

$\therefore N = 20$

$\therefore x[n]$ is periodic with period 20

(c) $x[n] = \cos(0.3\pi n) + \cos(0.5\pi n)$

$\omega_1 = 0.3\pi$	$\omega_2 = 0.5\pi$
$\frac{N_1}{k_1} = \frac{2\pi}{0.3\pi}$	$\frac{N_2}{k_2} = \frac{2\pi}{0.5\pi}$
$N_1 = 20$	$N_2 = 4$

$\therefore$ Period $= LCM(20, 4) = 20$

$x[n]$ is periodic with period 20

(d) $x[n] = \cos(0.3n)u[n]$

$\Rightarrow$ $w = 0.3$

$2\pi f = 0.3$

$f = \frac{0.3}{2\pi}$

$\frac{2\pi}{0.3} = \frac{N}{K}$ N is not an integer

For period $x[n] = x[n+N]$

N is not an integer.

$\therefore x[n]$ is not periodic.

Q-4

a) $x[n] = (0.5)^n u[n] + (2)^n u[-n-1]$

$$E = \sum_{-\infty}^{\infty} |x[n]|^2$$

$$E = \sum_{n=-0}^{\infty} |(0.5)^n u[n]|^2 + \sum_{-\infty}^{-1} |(2)^n u[n]|^2$$

$$E = \sum_{n=0}^{\infty} (0.5)^{2n} + \sum_{-\infty}^{-1} 2^{2n}$$

$$E = \left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right) + \left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \right)$$

$$E = \frac{1}{1-1/4} + \frac{1/4}{1-1/4}$$

$$= \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

$x[n]$ is energy signal

(b)

$$x[n] = \cos(0.5\pi n)u[n]$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=0}^{\infty} |\cos(0.5\pi n)|^2$$

when $n = \text{even} \to 1$; $n = \text{odd} \to 0$

$$E = 1 + 0 + 1 + 0 + \cdots$$

$$E \to \infty$$

$$P = \lim_{N\to\infty} \frac{1}{N+1} \sum_{n=0}^{N} |x[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{N+1} \left[1 + 0 + \cdots + (N+1)\right]$$

$$P = \lim_{N\to\infty} \frac{1}{\cancel{N+1}} \frac{\cancel{(N+1)}}{2}$$

$$P = \frac{1}{2}$$

$\therefore$ $x[n]$ is Power Signal

$x[n] = (0.5)^n u[n] + \delta[n] + 2^n u[-n-1]$

$x[-n] = (0.5)^{-n} u[-n] + \delta[-n] + 2^{-n} u[n-1]$
$= 2^n u[-n] + \delta[-n] + (0.5) u[n-1]$

$x[n] \neq x[-n]$ and $x[n] \neq -x[-n]$

$\therefore x[n]$ is neither even nor odd.

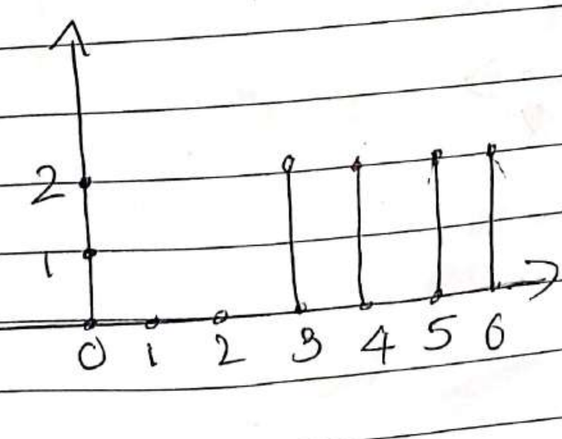
$x[n] = \cos(0.3\pi n) u[n]$

$x[-n] = \cos(-0.3\pi n) u[-n]$
$x[-n] = \cos(0.3\pi n) u[-n]$

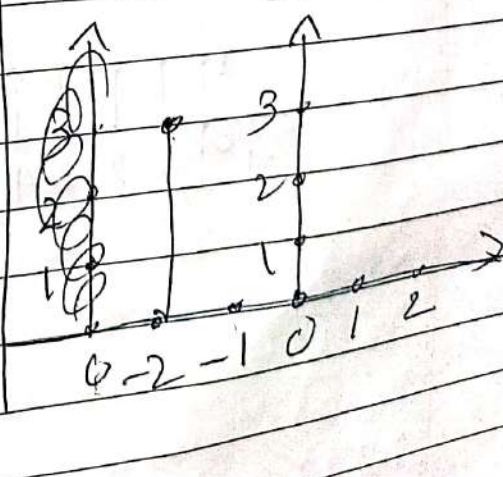
$x[n] = x[-n]$
$\therefore x[n]$ is even

$x[n] = 2u[n-3] - 3\delta[n+2]$

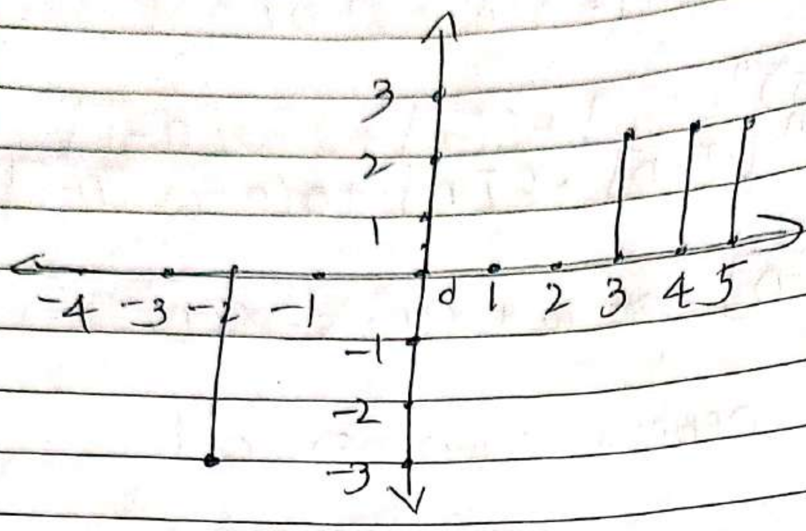
$2u[n-3]$



$3\delta[n+2]$

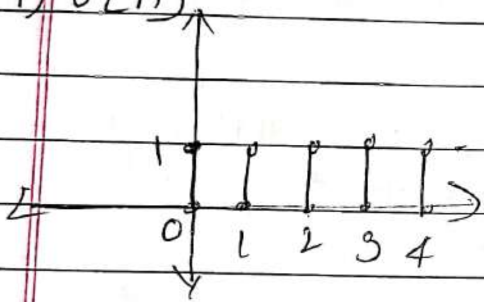


$2u[n-3] - 3\delta[n+2]$

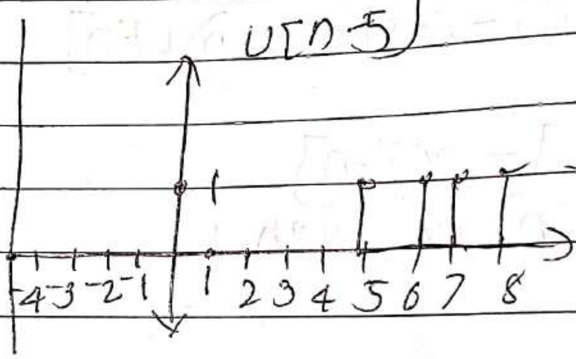


(b) $x[n] = u[n] - u[n-5]$

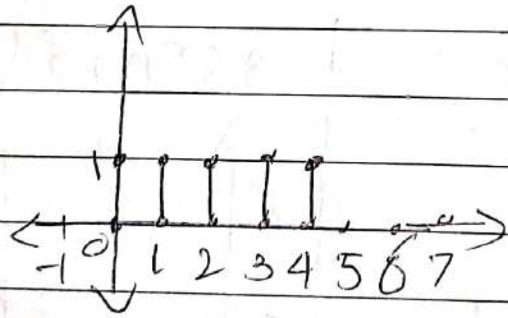
i) $u[n]$



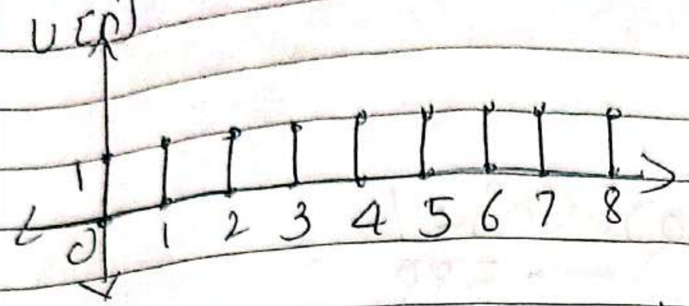
$u[n-5]$

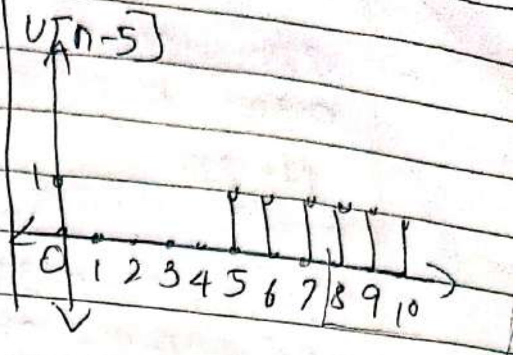


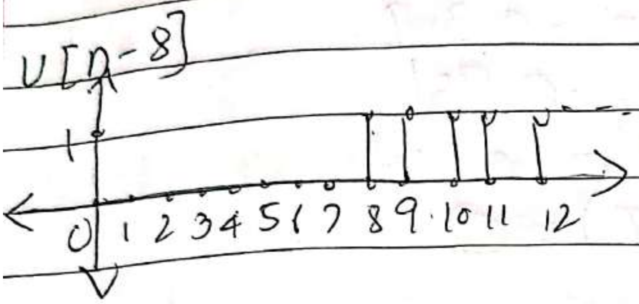
$u[n] - u[n-5]$

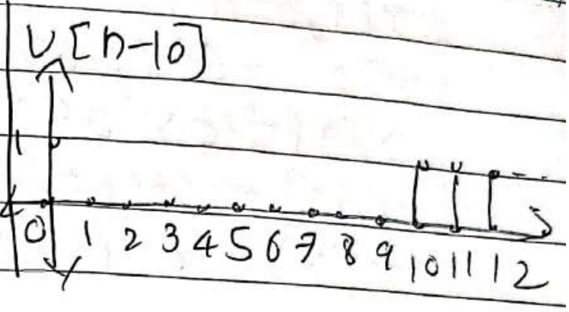


$x[n] = u[n] + u[n-5] - u[n-8] - u[n-10]$

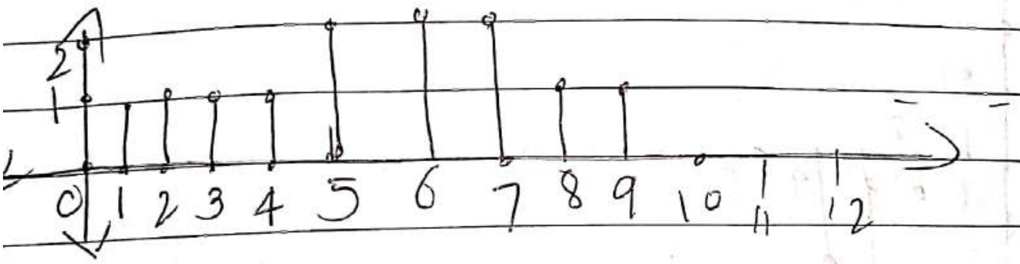




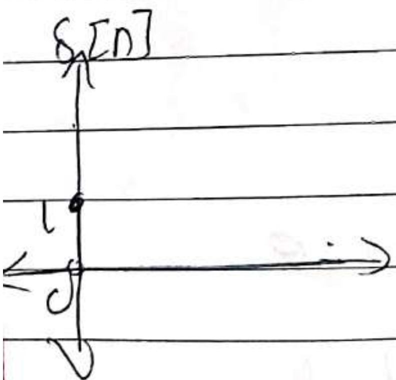


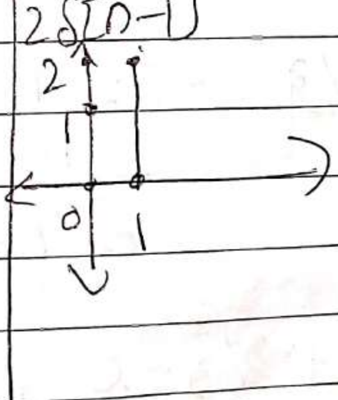


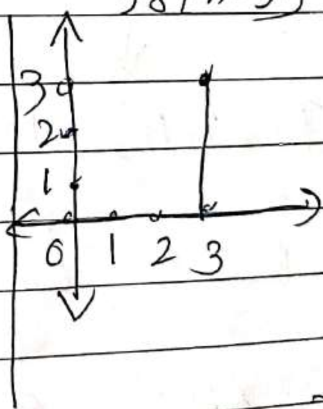
$u[n] + u[n-5] - u[n-8] - u[n-10]$



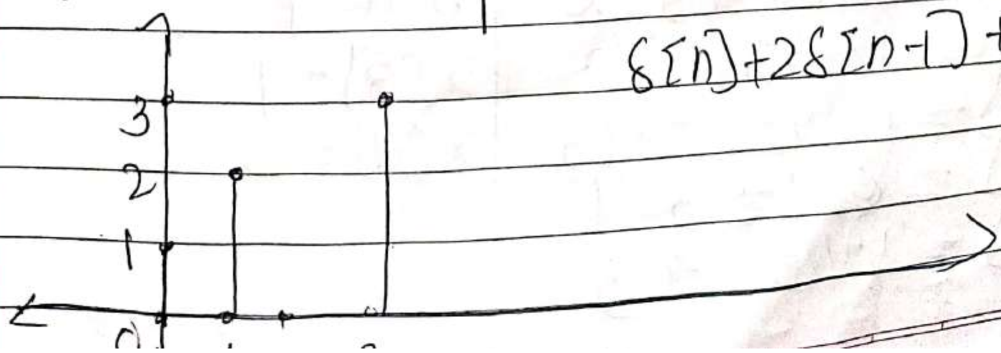
$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-3]$







$\delta[n] + 2\delta[n-1] + 3\delta[n-3]$



(c) $x[n] = \cos(0.3\pi n)u[n]$

$\Rightarrow \frac{2\pi}{0.3\pi} = \frac{N}{K}$

$N = 20$

$x[0] = \cos(0.3\pi 0)u[n] = 1$

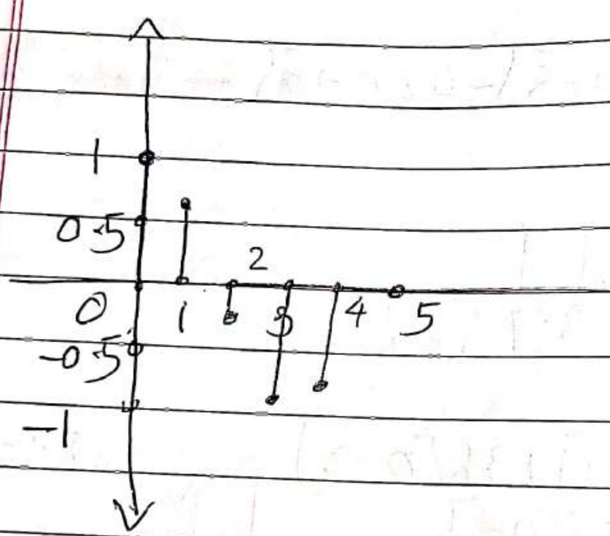
$x[1] = \cos(0.3\pi 1) = 0.588$

$x[2] = \cos(0.6\pi) = -0.309$

$x[3] = \cos(0.9\pi) = -0.951$

$x[4] = \cos(1.2\pi) = -0.8.4$

$x[5] = \cos(1.5\pi) = 0$



Q.7 $x[n] = 1 + n/3 \quad -3 \leq n \leq 1$

$\quad\quad 1 \quad\quad 2 \leq n \leq 3$

$\quad\quad 0 \quad\quad$ otherwise

$x[-3] = 1 + -3/3 = 0$

$x[-2] = 1 - 2/3 = 1/3$

$x[-1] = 1 - 1/3 = 2/3$

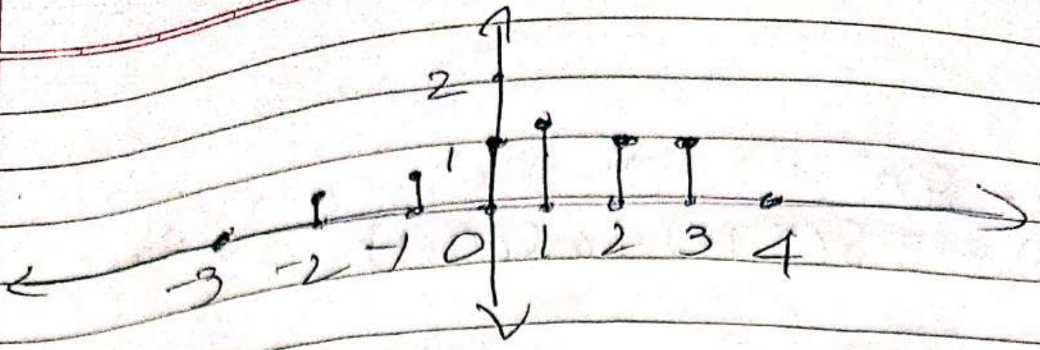
$x[0] = 1 - 0/3 = 1$

$x[1] = 1 + 1/3 = 4/3$

$x[2] = 1$

$x[3] = 1$

$x[4] = 0$



$$x[n] = \{0, 1/3, 2/3, \underset{\uparrow}{1}, 4/3, 1, 1\}$$

$$x[-n] = \{1, 1, 4/3, \underset{\uparrow}{1}, 2/3, 1/3, 0\}$$

$$x_e[n] = \frac{x[n] + x[-n]}{2} = \frac{1}{2}\{1, 4/3, 2, \underset{\uparrow}{2}, 2, 4/3, 1\}$$

$$= \left\{\frac{1}{2}, 2/3, 1, \underset{\uparrow}{1}, 1, 2/3, 1/2\right\}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2} = \frac{1}{2}\{-1, -2/3, -2/3, 0, 2/3, 2/3,$$

$1 - \frac{1}{3}$

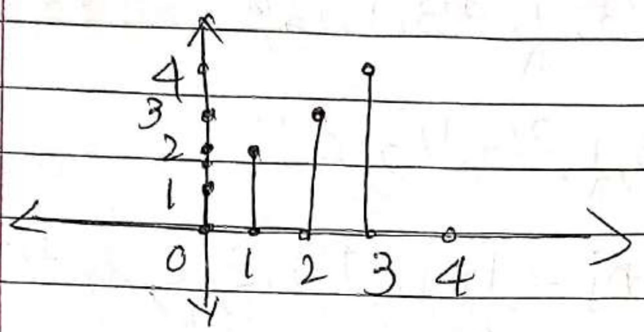
$$= \left\{-\frac{1}{2}, -1/3, -\frac{1}{3}, 0, 1/3, 1/3, 1\right\}$$

1.2

Q.3

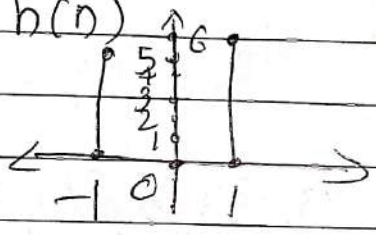
$$x(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)$$
$$h(n) = 5\delta(n+1) + 6\delta(n-1)$$

$x(n)$



$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

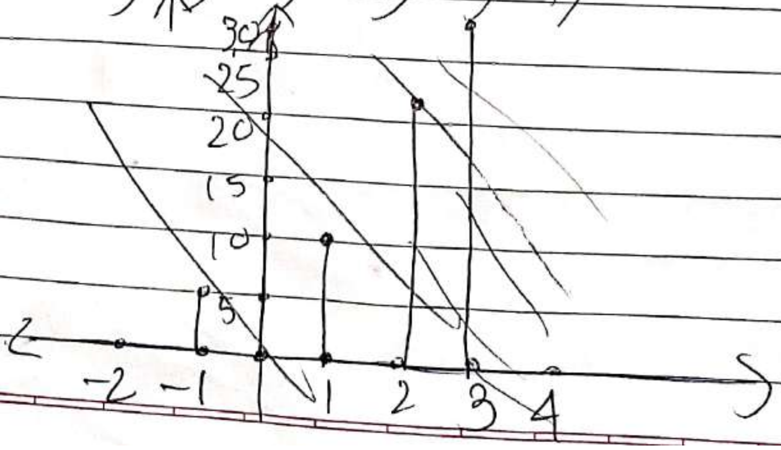
$h(n)$

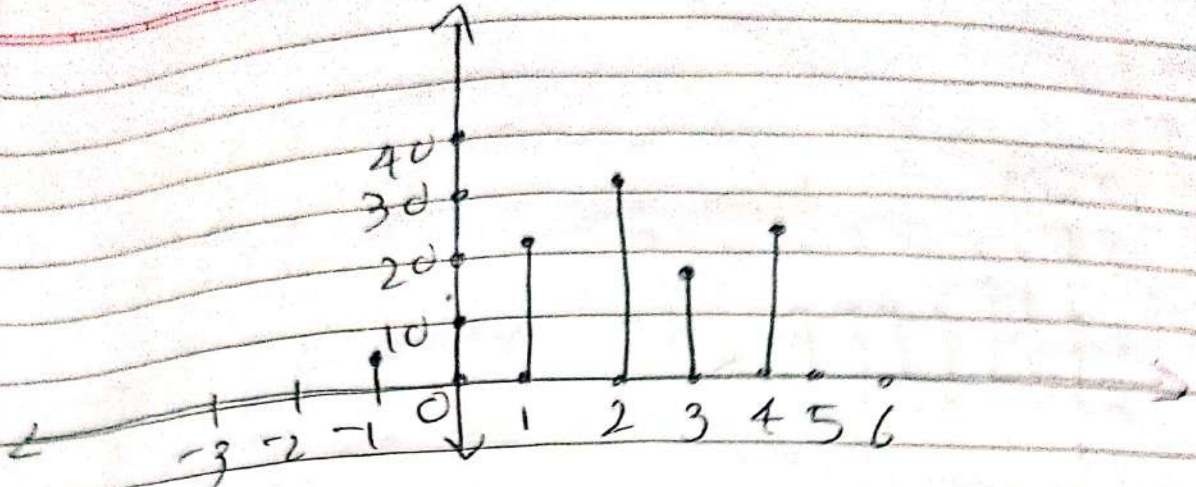


$h(n) = \{5, \underset{\uparrow}{0}, 6\}$

$h(n)$ \ $x(n)$	1	2	3	4
5	5	10	15	20
0	0	0	0	0
6	6	12	18	24

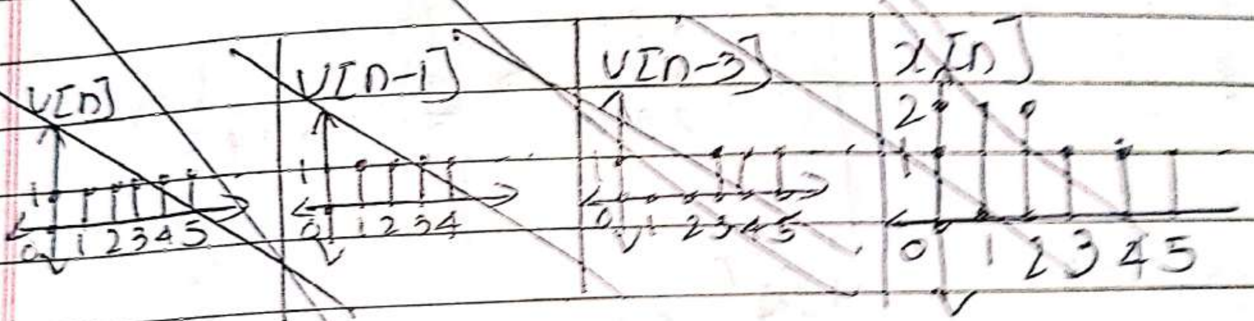
$y[n] = \{5, \underset{\uparrow}{10}, 21, 32, 18, 24\}$





Q.4 $x(n) = u[n] + u(n-1) - u(n-3)$

$h(n) = u(n-1) + u(n-2) - u(n-4) - u(n-5)$

=> 

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$$y[n] = \sum_{k=-\infty}^{\infty} (u[k] + u[k-1] - u[k-3]) \cdot (u(n-k-1) + u(n-k-2) - u(n-k-$$

Q.8

$x[n] = \{1, 4, 6, 3\}$
$h[n] = \{7, 5, 8, 9\}$

$$y(n) = \begin{bmatrix} 1 & 3 & 6 & 4 \\ 4 & 1 & 3 & 6 \\ 6 & 4 & 1 & 3 \\ 3 & 6 & 4 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 106 \\ 111 \\ 97 \\ 113 \end{bmatrix}$$

Q.10

$x[n] = \{1, 4, 6, 3\}$
$h[n] = \{7, 5, 8, 9\}$

h(-n)	7	9	8	5
x[-n]	7	9	8	5
1	28	36	32	20
4	21	27	24	15
3	42	54	48	30
6				

$$y[n] = \{\underset{\uparrow}{7}, 37, 65, 106, 98, 63, 30\}$$