Null Space

Null Space :-

Subspace of vector space consisting of vectors that under a given linear transformation & mapped on to 0 vector

- called Kernel of matrix.

$m \times n$ matrix A has its nullspace as the collection of ~~row~~ those vectors in R^n that A maps to the zero vector.

$$N(A) = \{ x \in R^n \mid Ax = 0 \}$$

Method

1. Find the sol^n set of $Ax = 0$.
2. Perform row echelon reduction.
3. Get the eq^n & set the free variables α & β as req^d.
4. Write the nullspace in terms of α & β.

5.

Q. Find ~~nullspace~~ :- sol^n set of $AX = 0$.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

$R_1 \to R_1 - R_2$
$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

Then we write the free variables as:

$x_1 - x_3 - 2x_4 = 0$ —— ①

$x_2 + 2x_3 + 3x_4 = 0$ —— ②

Let $x_3 = \alpha$, $x_4 = \beta$

$x_1 = \alpha + 2\beta$ — (Put $x_3 = \alpha$, $x_4 = \beta$ in ①)

$x_2 = -2\alpha - 3\beta$ — (Put $x_3 = \alpha$, $x_4 = \beta$ in ②)

Sol^n ~~set~~ Set

$$= \alpha \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore N(A) = \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Q. $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 4 & 2 & 0 & 0 & 3 \\ 1 & 1 & 1 & -2 & 1 \\ 2 & 2 & 0 & 0 & 2 \\ 1 & 1 & 2 & -4 & 1 \end{bmatrix}$

$AX = 0$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 4 & 2 & 0 & 0 & 3 \\ 1 & 1 & 1 & -2 & 1 \\ 2 & 2 & 0 & 0 & 2 \\ 1 & 1 & 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

~~$R_2 \to R_2 + R_1$~~ $R_3 \to R_3 - 4R_1$

$R_4 \to R_4 - R_1$

$R_5 \to R_5 - 2R_1$

$R_6 \to R_6 - R_1$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

$R_2 \leftrightarrow R_3$:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 0 \end{bmatrix}$$

$R_6 \leftrightarrow R_3$:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 2 & -4 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{bmatrix}$$

$R_4 \to R_4 - \frac{1}{2} R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 2 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{bmatrix}$$

$R_4 \to R_4 - \frac{1}{2} R_3$

$R_6 \to R_6 - \frac{1}{2} R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 2 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

$x_1 + x_2 + x_5 = 0$

$-2x_2 - x_5 = 0$

$2x_3 - 4x_4 = 0$

Put $x_4 = \alpha$, $x_5 = \cancel{\beta} - 2\beta$

$x_3 = 2\alpha$, $x_2 = \beta$

$x_1 = +\beta$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \beta \\ \beta \\ 2\alpha \\ \alpha \\ -2\beta \end{bmatrix}$$

$$= \alpha \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

~~Nullsp~~ : Dimension of null space = 2

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Dimension of vector space

Defⁿ If V is a vector space.
If $\{v_1, v_2, \ldots, v_t\}$ is a basis of V
then dimension of V is defined by
$\dim(m) = t$.
If V has no finite bases, then V has infinite dimension.
~~If~~

$\dim(\{0\}) = 0$
$\dim(P_n) = n+1$ — (P_n :- Polynomial of degree n)
$\dim(R^n) = n$
$\dim(M_{mn}) = mn$ → ?

Q. What is the dimension of the vector space of polynomial in x with real coefficients having degree at most 3.

⟹ $ax^3 + bx^2 + cx + d = P(x)$
$n = 3$ — (Degree)
$\dim(P_3) = 3+1 = 4$
$\{x^3, x^2, x, 1\}$ as a basis.

Q. Find the dimension of the solⁿ space ~~eigenspace~~ of the following homogenous system

→ $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \\ 1 & -4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$R_2 \leftrightarrow R_3$ $\begin{bmatrix} 1 & -1 & 2 \\ 1 & -4 & 6 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & -1 & 2 \\ -2 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x - y + 2z = 0$ —— ①

$-2x - y = 0$ —— ②

If we put $x = t$,

$y = -2t$, $z = -1.5t$

Solⁿ space $= (x, y, z) = (t, -2t, -1.5t)$

$$= t \begin{bmatrix} 1 \\ -2 \\ -1.5 \end{bmatrix}$$

Dimension of solⁿ space $= 1$

Q. Find the dimension of plane $x + 2z = 0$ in R^3.

Let ~~$y = t$~~, $z = s$, then general solⁿ of $x + 2z = 0$ is

$x = -2s$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2s \\ t \\ s \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Dimension of plane $= 2$