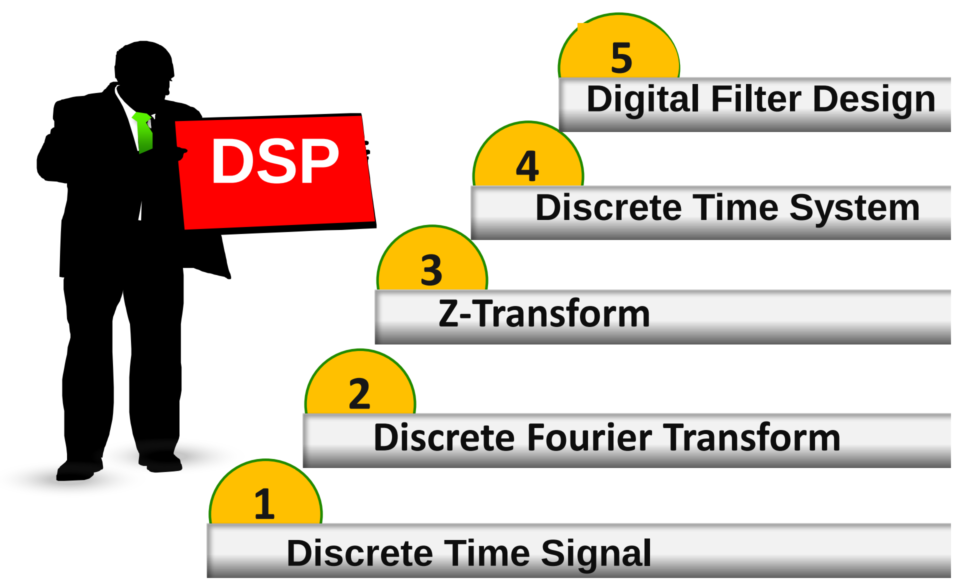
Digital Signal Processing

	TOPIC
1	Discrete Time signals
2	Discrete Fourier Transform
3	Z-Transform
4	Discrete Time System
5	Digital Filter Design



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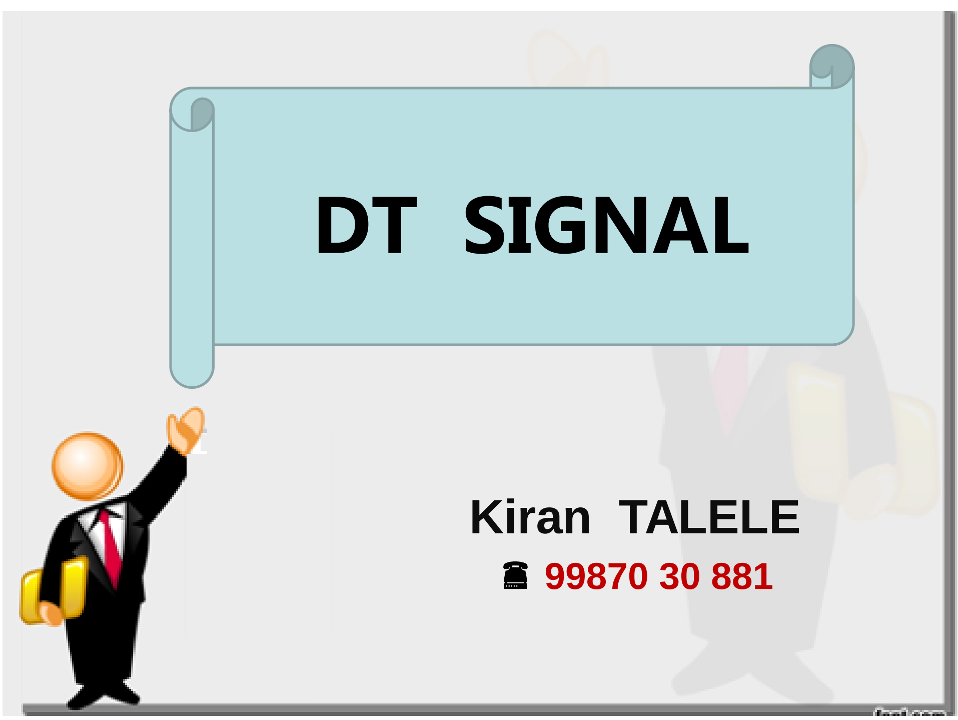

Chapter-1 : Discrete Time Signal

Objective : To learn the fundamental concepts of Digital Signal Processing

Outcome : At the end of module students will be able to,

- **Understand** the concept of DT Signal
- **Evaluate** Sampling and Reconstruction
- **Perform** signal manipulation
- **Represent** DT Signal using standard signals
- **Classify** DT signals
- **Perform** Convolution and Correlation operation

Contents

Standard Discrete Time Signals

(1) Delta Signal OR Impulse Signal

OR Unit Sample Signal

(2) Unit Step Signal

(3) Sinusoidal Signal

Classification of DT Signals

(1) Deterministic / Non-deterministic

(2) Causal / Anti-Causal / Bothsided

(3) Periodic / Non-Periodic

(4) Energy / Power / Neither Energy Nor Power

(5) Even / Odd / Neither Even Nor Odd

(I) Deterministic / Non Deterministic Signal

If Discrete Time Signal x[n] can be represented mathematically,
Then x[n] is called Deterministic Signal

Otherwise x[n] is called Non Deterministic Signal.

e.g. x[n] = Sinusoidal Signal : cos (0.6 π n)
Noise Signal ➔ Non Deterministic
ECG Signal ➔ Non Deterministic

(II) Causal / Anti-Causal / Bothsided

1. Causal Signal :

If **x[n] = 0** for all **n < 0**
Then x[n] is **causal** Signal.

Examples :

(i) $x[n] = \{ \underset{\uparrow}{9}, 9, 8, 7, 0, 3, 0, 8, 8, 1 \}$

(ii) x[n] = u[n]

2. Anti-causal Signal :

If **$x[n] = 0$** for all **$n \geq 0$**
Then $x[n]$ is **Anti-causal** signal.

Examples :

$x[n] = \{ 9, 9, 8, 7, 0, 3, 0, 8, 8, 1, \underset{\uparrow}{0} \}$

$x[n] = u[-n-1]$

3. Both Sided Signal :

If $x[n]$ is Neither Causal
Nor Anti-causal
Then $x[n]$ is **Both-sided** signal.

Examples :

$x[n] = \{ 1, 2, \underset{\uparrow}{3}, 4, 5, 6 \}$

$x[n] = \{ 1, 2, 3, 4, 5, \underset{\uparrow}{6} \}$

Example of Causal / Anti-causal & Both Sided (infinite length) Signals :

(i) Causal signal : $x[n] = u[n]$

(ii) Anti-causal signal : $x[n] = u[-n-1]$

(iii) Both sided signal : $x[n] = u[n] + u[-n-1]$

(III) Periodic / Non-periodic Signal

If Digital frequency of signal x[n] is a rational number then x[n] is periodic.
Otherwise x[n] is Non-periodic.

Examples:

(1) $x[n] = \cos(0.3\pi n)$

(2) $x[n] = \cos(0.3\pi n + 0.5\pi)$

(3) $x[n] = \cos(0.3 n)$

(IV) Energy / Power / Neither Energy Nor Power

1. Energy of signal is defined as,

$$E = \sum_{n=-\infty}^{\infty} | x[n] |^2$$

If Energy of x[n] is finite Then x[n] is an Energy Signal.

Examples :

Energy : 30 (Finite)

$x[n] = (½)^n u[n]$ Energy : 2 (Finite)

2. Average Power of signal is defined as,

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} | x[n] |^2$$

If Power of x[n] is finite and nonzero Then x[n] is a Power Signal.

Example :

$x[n] = u[n]$ Energy : ∞

Power : 0.5 (Finite)

Ex-1. x[n]=u[n] ← Unit Step Signal,

(i) To find Energy :

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$\mathbf{E} = \sum_{n=0}^{\infty} |\mathbf{u[n]}|^2$$

$$E = 1^2+1^2+1^2+1^2 \ \ldots$$

$$E = \infty$$

Since Energy is infinite,
u[n] is NOT an Energy Signal.

(ii) To find Average Power :

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum |u[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |u[n]|^2$$

$$P = \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$P = \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}} \qquad P = \frac{1}{2}$$

Since Average Power is finite and non zero, u[n] is a Power Signal.

Ex-2 x[n] = r[n] ← Unit Ramp Signal,

(i) To find Energy :

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$E = \sum_{n=0}^{\infty} |r[n]|^2$$

$$E = (0)^2 + (1)^2 + (2)^2 + \ldots$$

$$E = \infty$$

Since Energy is infinite,
x[n]=r[n] is NOT an Energy Signal.

(ii) To find Average Power :

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |r[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \left\{(0)^2 + (1)^2 + (2)^2 + \ldots\right\}$$

$$P = \infty$$

Since Average Power is infinite ,
x[n] is NOT a Power Signal.

Ex-3 $x[n]= e^{2n}\, u[n]$,

(i) To find Energy :

$$E = \sum_{n=-\infty}^{\infty} |\, x[n]\,|^2$$

$$E = \sum_{n=0}^{\infty} |\, e^{2n}\,|^2$$

$$E = \sum_{n=0}^{\infty} (e^4)^n$$

$$E = \sum_{n=0}^{\infty} (54.59)^n$$

$$E = \infty$$

Since Energy is infinite, x[n] is NOT an Energy Signal.

(ii) To find Average Power :

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |\, x[n]\,|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |\, e^{2n}\,|^2$$

$$P = \infty$$

Since Average Power is infinite , x[n] is NOT a Power Signal.

Ex-4 $x[n] = (0.5)^n u[n]$,

(i) To find Energy : $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

$$E = \sum_{n=0}^{\infty} (r)^n$$
$$= \frac{1}{1-r} \quad \text{iff } r < 1$$
$$= \infty \quad \text{Otherwise}$$

$$E = \sum_{n=0}^{\infty} |(0.5)^n u[n]|^2$$

$$E = \sum_{n=0}^{\infty} (0.25)^n$$

$$E = \frac{1}{1 - 0.25} \qquad E = \frac{4}{3} \text{ (Finite)}$$

Since Energy is finite, x[n] is an Energy Signal.

(V) Even / Odd / Neither Even Nor Odd

1. EVEN Signal

If $x[n] = x[-n]$
Then $x[n]$ is Even Signal.

2. ODD Signal

If $x[n] = -x[-n]$
Then $x[n]$ is Odd Signal.

Examples :

(i) Even signal : $x[n] = \{ -1 \quad -2 \quad \underset{\uparrow}{3} \quad -2 \quad -1 \}$

(ii) Odd Signal $x[n] = \{ 1 \quad 2 \quad \underset{\uparrow}{0} \quad -2 \quad -1 \}$

(iii) Neither Even nor Odd signal : $x[n] = \{ \underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \}$

NOTE : (1) When digital frequencies are separated by multiples of $\pm 2\pi$ Then Discrete Time Signals are exactly Same..(i.e. their Samp values are identical)

(2) Range of Digital frequency ω is $(-\pi, \pi]$ for mathematical analysis.

(3) Range of Digital frequency f is $\left(\frac{-1}{2}, \frac{1}{2} \right]$ for mathematical analysi

(1) What is DSP ?

Ans : **Digital Signal Processing is a technique that converts signals from real world sources (usually in analog form) into digital data that can then be analyzed.**

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(2) What do you mean by real time signal ?

Ans :
- **Signal is processed with the same speed it is captured.**
- **Signal is not stored before processing.**
- **Entire input signal is never available before processing.**

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- **For example, in digital telephone system, Signal is captured, Sampled, Processed , Transmitted and Made it available to the end user.**

Real Time Processing is Online Processing

(3) What is Analog Signal, Digital Signal, CT signal, DT Signal ?

➢ **Analog Signal : Signal value can be anything. NO fixed signal level.**

Eg $x(t) = \cos(100\pi\, t)$. ← Continuous Sinusoidal signal

$x[n] = \{ 10.5, 4.7, 3.5, 5.7, 3.8 \}$ ← Sampled signal

Digital Signal :

Only two levels +5v and 0 V.
ie. Logically High and Low.

Eg. Binary data

0	1	0	1	1	0	1	0

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- **Continuous Time Signal : Signal is defined for every value of time. Signal value can be anything.**

  Eg. $x(t) = \cos(100\pi\, t)$. 🡐 Continuous Sinusoidal Signal

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Discrete Time Signal :

Signal is defined for Discrete instant of Time.

NOT for every value of time. Signal value can be anything.

Eg. x[n] = { 10.5, 4.7, 3.5, 5.7, 3.8 }
🡐 sampled signal

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CONCEPT of Signal Processing

Signals commonly need to be processed in a variety of ways. For example, the output signal from a transducer may well be contaminated with unwanted electrical "noise".

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The electrodes attached to a patient's chest when an ECG is taken measure tiny electrical voltage changes due to the activity of the heart and other muscles.

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Analog and Digital signals

In many cases, the signal is initially in the form of an analog electrical voltage or current, produced for example by a microphone or some other type of transducer.

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In some situations the data is already in digital form - such as the output from the readout system of a CD (compact disc) player.

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An analog signal must be converted into digital (i.e. numerical) form before DSP techniques can be applied.

There are two main kinds of filter, *analog* and *digital*. They are quite different in their physical makeup and in how they work.

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An analog filter uses analog electronic circuits made up from components such as resistors, capacitors and op amps to produce the required filtering effect. Such filter circuits are widely used in such applications as noise reduction, video signal enhancement, graphic equalizers in hi-fi systems, and many other areas.

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There are well-established standard techniques for designing an analog filter circuit for a given requirement. At all stages, the signal being filtered is an electrical voltage or current which is the direct analogue of the physical quantity (e.g. a sound or video signal or transducer output) involved

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Analog and digital filters

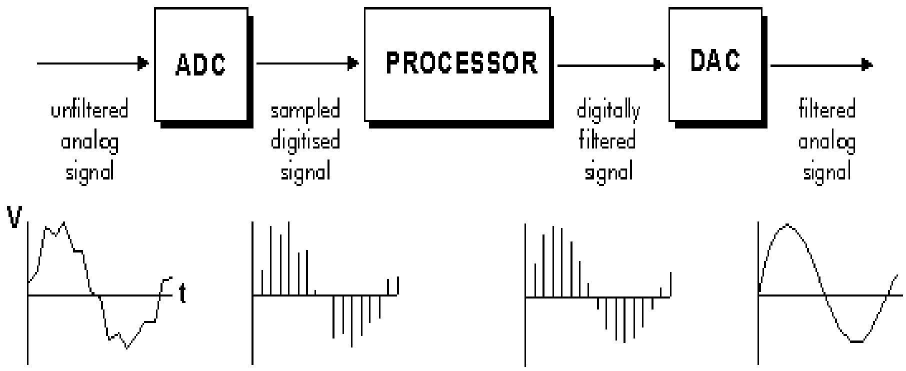
In signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful parts of the signal, such as the components lying within a certain frequency range.

The following block diagram illustrates the basic idea.

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialized DSP (Digital Signal Processor) chip.

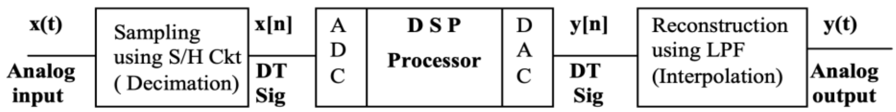
- **Note that in a digital filter, the signal is represented by a sequence of numbers, rather than a voltage or current**.

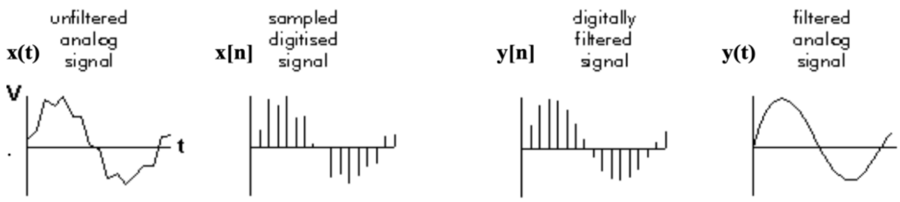
The following diagram shows the basic setup of such a system.



Fast DSP processors can handle complex combinations of filters in parallel or cascade (series), making the hardware requirements relatively *simple* and *compact* in comparison with the equivalent analog-circuitry

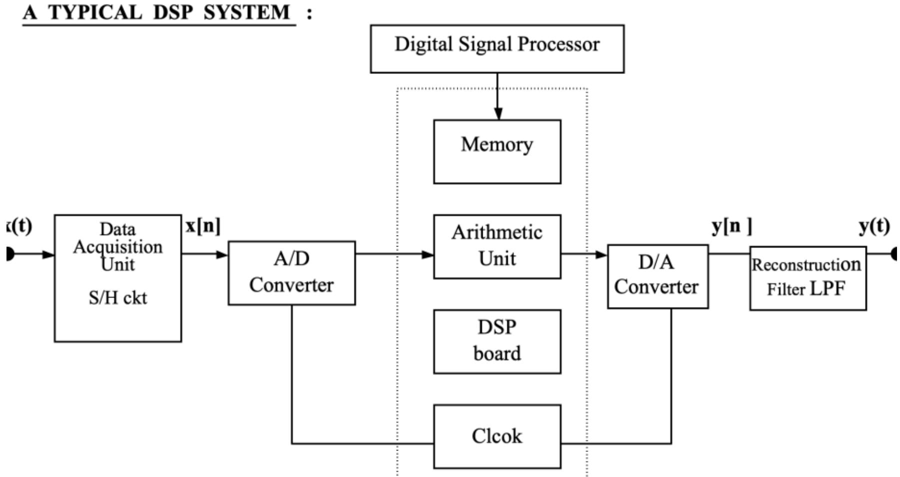
What is DSP SYSTEM?



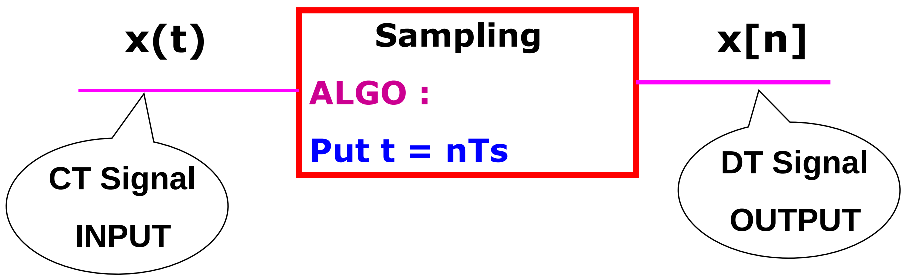


A Typical DSP System….

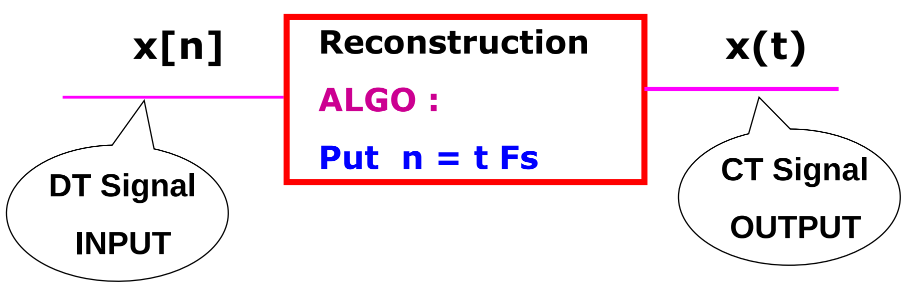
A TYPICAL DSP SYSTEM :



Sampling



Reconstruction



H.W. Let **$x[t] = 10 \cos(100\pi t) + 20 \cos(120\pi t) - 5 \sin(50\pi t)$.** If x(t) is sampled with sampling frequency Fs = 200 Hz. What will be Discrete Time Signal x[n] at n=0 ?

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