

# Image Enhancement Using Spatial Filtering



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## Image Enhancement by Neighbourhood Processing : **Spatial Filtering**

- (i) In Neighbourhood operation, Tx. function works on a **group of input pixel values**
- (ii) The Output pixel value at (x,y) position is obtained by **masking operation**
- (iii) The use of spatial mask for image processing is called spatial filtering and the mask is called spatial filter

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**Masking Operation:** Consider an image F and filter mask W

$$\begin{array}{c} \mathbf{F} \\ = \end{array} \begin{array}{|c|c|c|} \hline Z_1 & Z_2 & Z_3 \\ \hline Z_4 & Z_5 & Z_6 \\ \hline Z_7 & Z_8 & Z_9 \\ \hline \end{array} * \begin{array}{c} w = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} \end{array} \quad \mathbf{A} = \begin{array}{|c|c|c|} \hline & & \\ \hline & \mathbf{R} & \\ \hline & & \\ \hline \end{array}$$

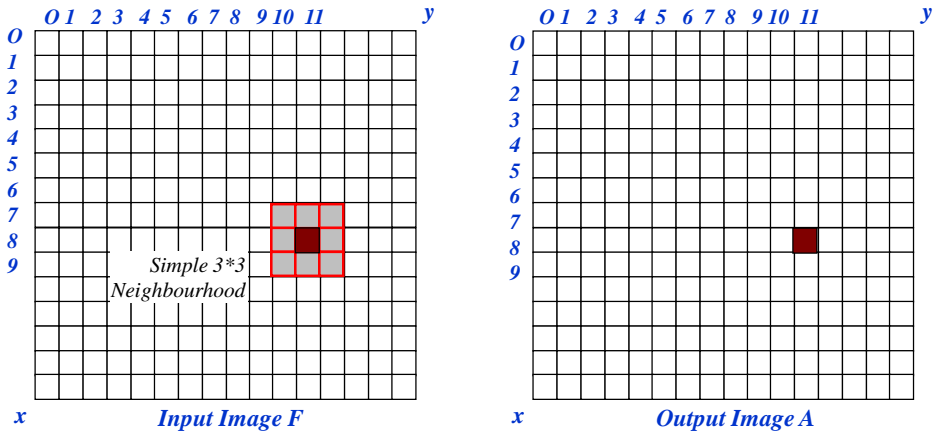
The response of mask is given by,

$$\mathbf{R} = \mathbf{z}_1 w_1 + \mathbf{z}_2 w_2 + \mathbf{z}_3 w_3 + \dots + \mathbf{z}_9 w_9$$

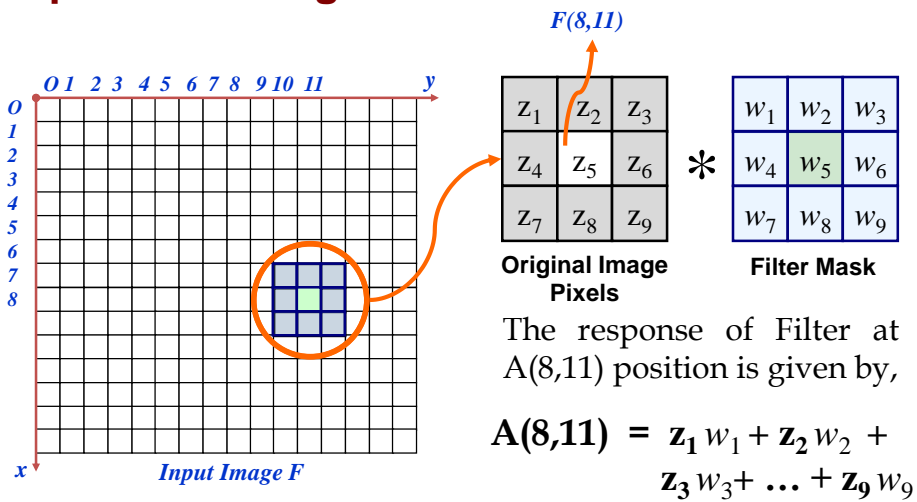
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## Spatial Filtering



## Spatial Filtering



The above is repeated for every pixel in the original image to generate the smoothed image

## List of Spatial Filters

### [I] Smoothing Linear Filters (3)

Eg. Low Pass Averaging Filter, Weighted Average Filter, Trimmed Averaging Filter and Geometric Mean Filter

### [II] Smoothing Non-Linear Filters (3)

Eg. Median, Max and Min Filter.

### [III] Sharpening : First Order Derivative Filters (4)

Eg. Robert, Prewit, Sobel and Fri-Chen filter

### [IV] Sharpening Second Order Derivative Filters (3)

Eg. Laplacian Filter, HPF and High Boost Filter

## [ I ] Smoothing Linear Filters

### (1) Low Pass Averaging Filter

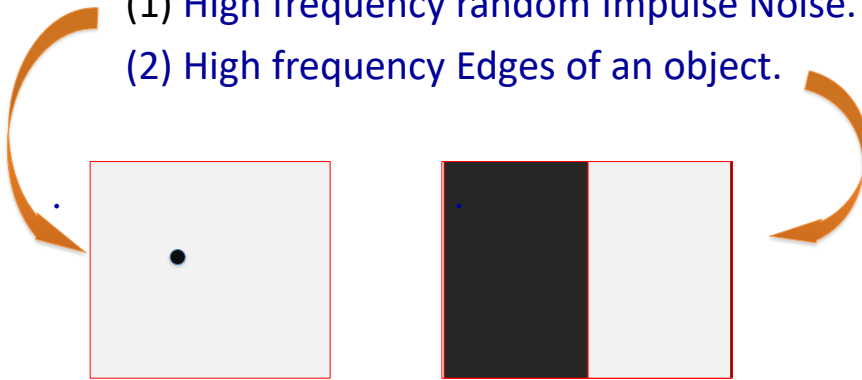
- (i) LPF attenuates High frequency Components and allows to pass LOW frequency components of the image.
- (ii) LPF reduces the difference between neighbouring pixels by distributing its' energy using averaging technique.

(iii) LPF removes sharp transitions in the image

- Sharp transitions are due to :

(1) High frequency random Impulse Noise.

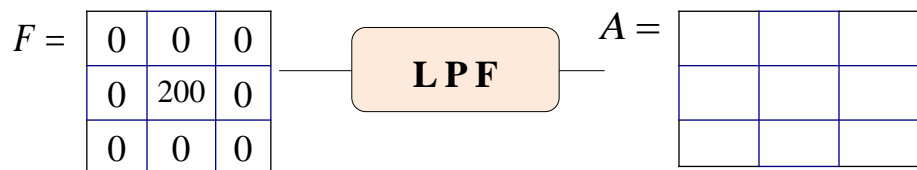
(2) High frequency Edges of an object.



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Eg. Consider a Digital Image  $F$  with impulse noise



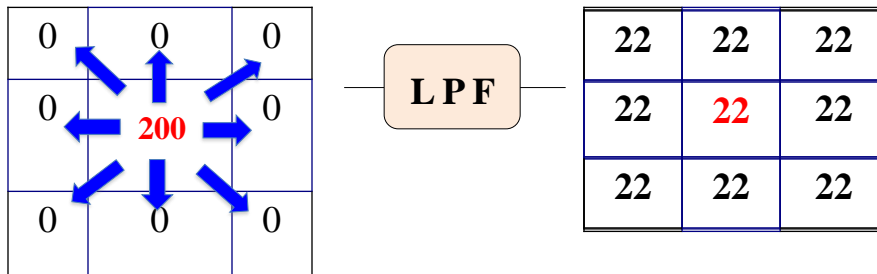
(IV) LPF reduces the difference between the neighbouring pixels by distributing its energy in all directions by averaging techniques

(v) The net effect of LP filtering is Image Blurring

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Eg. Digital Image with impulse noise

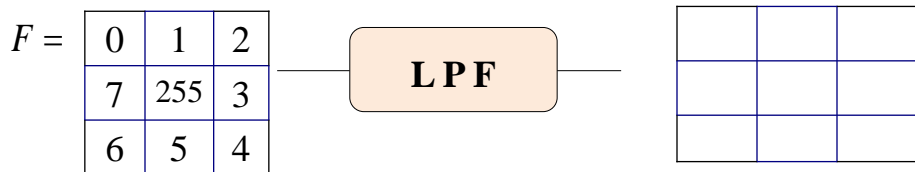


LPF reduces the difference between the neighbouring pixels by distributing its energy in all directions by averaging techniques

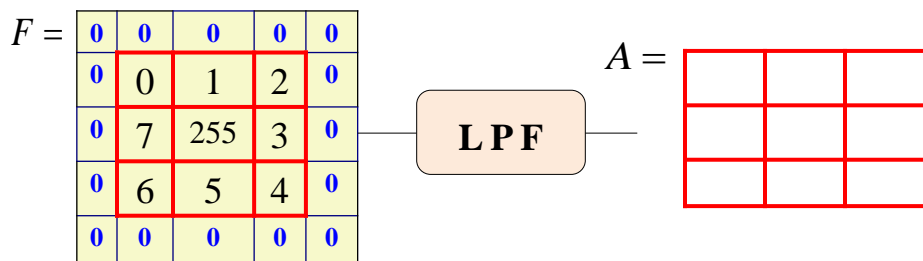
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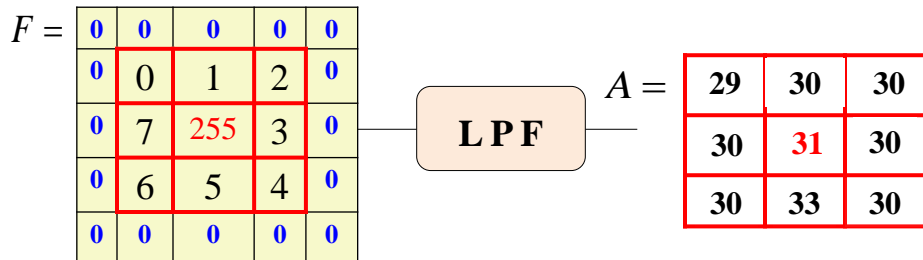
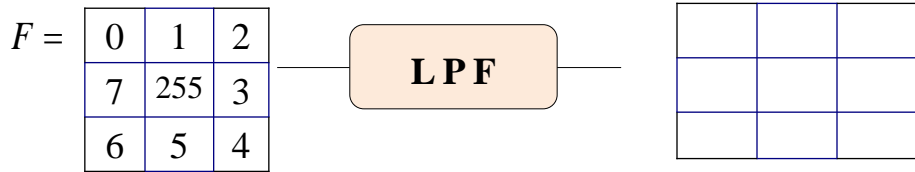
Eg-1 : Digital Image with impulse noise



Case-1 : Assume virtual Rows and Columns with Zero pixel Values

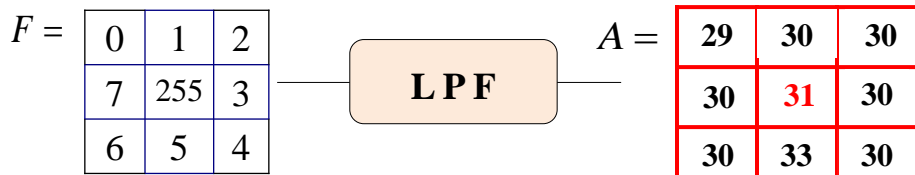


Eg-2 : Consider a Digital Image with impulse noise



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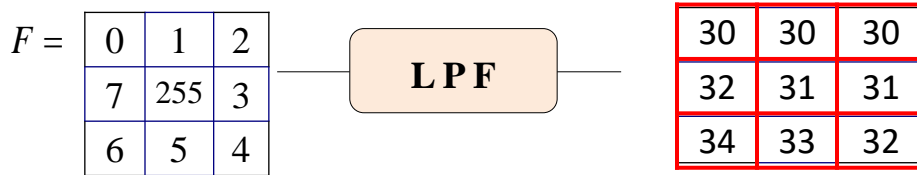
All the pixel in the neighbourhood of impulse noise have almost same value.

Thus high frequency impulse noise is completely suppressed in the output image.

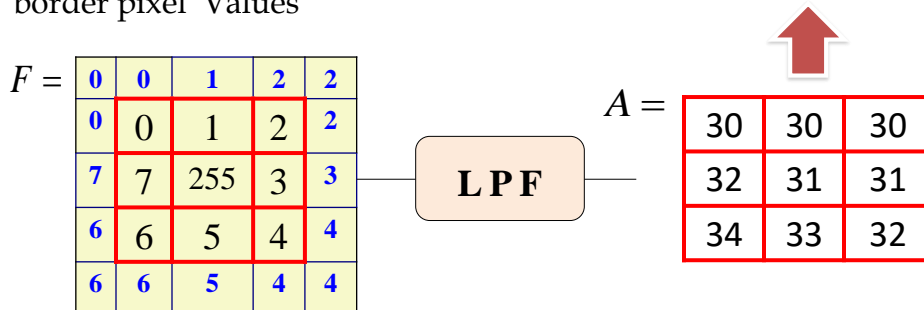
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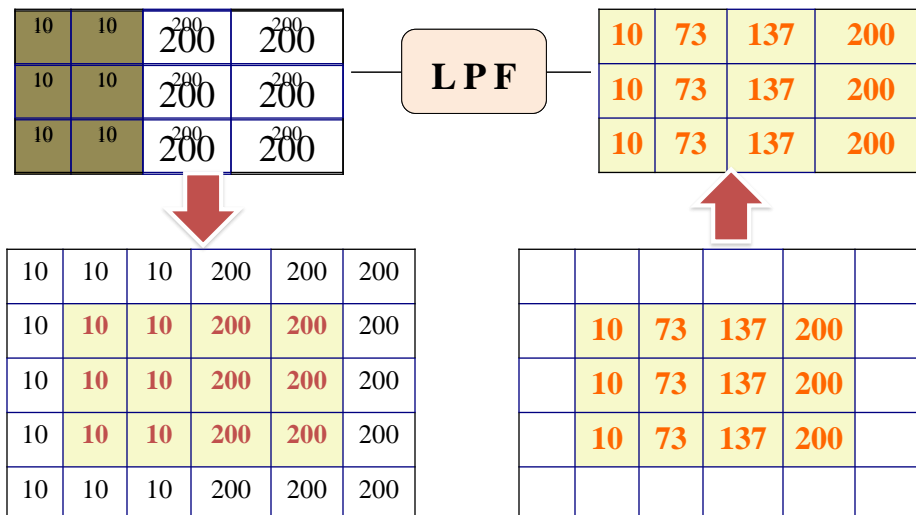
## Digital Image with impulse noise



Case-2 : Assume Virtual Rows and Columns with repeated border pixel Values



## Eg-2. Digital Image with High Frequency Edge



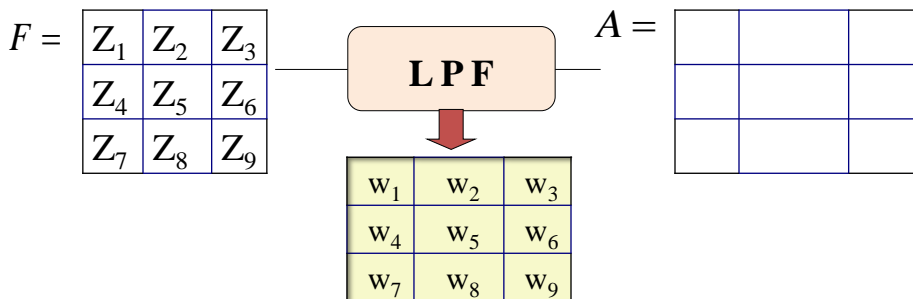


## Summary :

- All the pixels in the output image have almost same value.
- Averaging of gray values in the neighbourhood reduces the sharp transitions in the images.
- Thus High frequency Impulse noise is completely suppressed. This is desirable effect.
- However high frequency edges are blurred.  
This is undesired effect.

## Derivation of LPF Mask

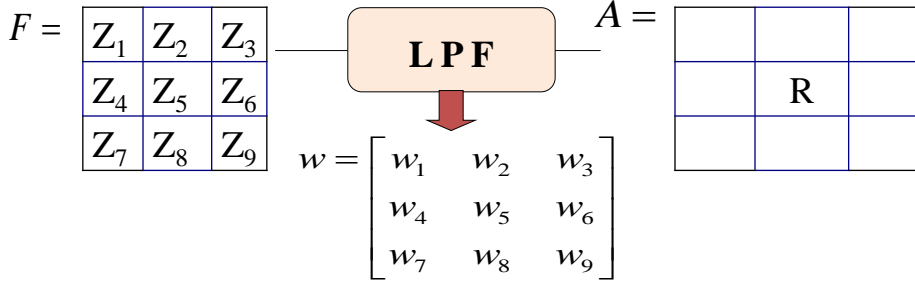
Consider a Digital Image  $F$  and LPF Mask  $w$  as shown in figure below,



The response of LPF mask at  $A(x,y)$  position is given by,

$$R = z_1 w_1 + z_2 w_2 + z_3 w_3 + \dots + z_9 w_9 \dots (I)$$

Consider a Digital Image  $F$  and LPF Mask  $w$  as shown in figure below,

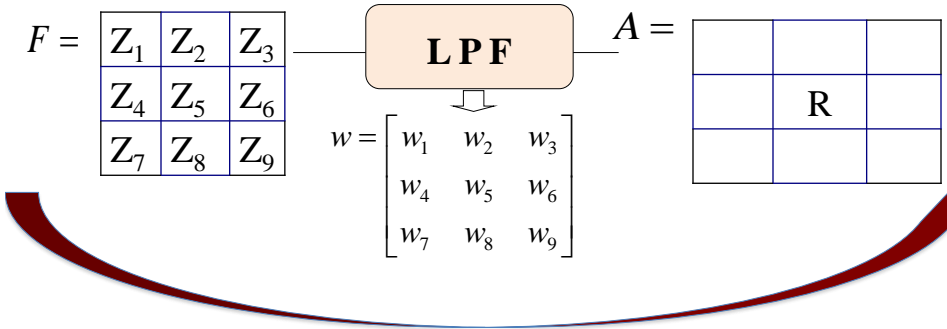


The response of LPF mask at  $A(x,y)$  position is given by,

$$\mathbf{R} = \mathbf{z}_1 w_1 + \mathbf{z}_2 w_2 + \mathbf{z}_3 w_3 + \dots + \mathbf{z}_9 w_9 \dots\dots(\mathbf{I})$$

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By averaging output pixel value at  $A(x,y)$  position is given by,

$$\mathbf{R} = \frac{1}{9} ( \mathbf{z}_1 + \mathbf{z}_2 + \mathbf{z}_3 + \dots + \mathbf{z}_9 )$$

$$\mathbf{R} = \frac{1}{9} \mathbf{z}_1 + \frac{1}{9} \mathbf{z}_2 + \dots + \frac{1}{9} \mathbf{z}_9 ) \dots\dots (\mathbf{II})$$

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$$R = z_1 w_1 + z_2 w_2 + z_3 w_3 + \dots + z_9 w_9 \dots\dots(I)$$

$$R = \frac{1}{9} z_1 z_1 + \frac{1}{9} z_2 + \dots + \frac{1}{9} z_9 \dots\dots (II)$$

By equating equation (I) and (II) we get,

$$W = \begin{bmatrix} w_1 = \frac{1}{9} & w_2 = \frac{1}{9} & w_3 = \frac{1}{9} \\ w_4 = \frac{1}{9} & w_5 = \frac{1}{9} & w_6 = \frac{1}{9} \\ w_7 = \frac{1}{9} & w_8 = \frac{1}{9} & w_9 = \frac{1}{9} \end{bmatrix} \rightarrow W = \frac{1}{9} \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

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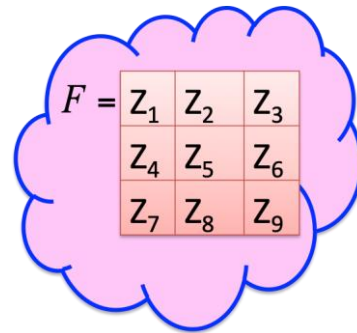
## (2) Low Pass Weighted Averaging Filters

- (i) In this filter, output pixel value at each (x,y) position is obtained by averaging the weighted neighbouring pixel values.
- (ii) The **Weight** (i.e. Scaling factor) depends on the position of the neighbouring pixel with reference to the center pixel.

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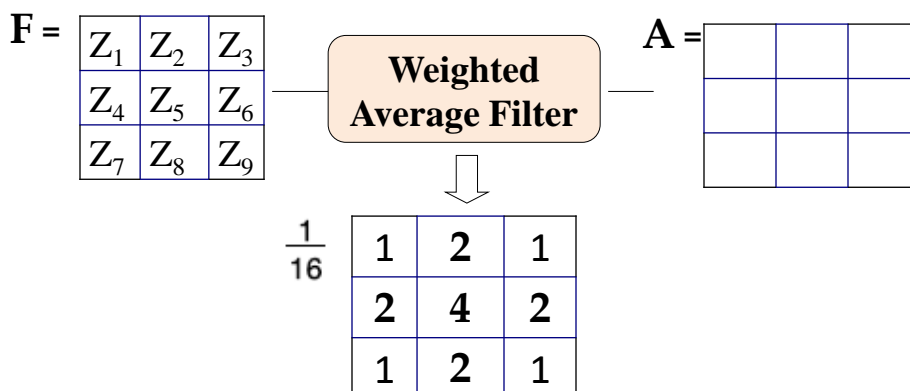
|   | Pixel Position               | Weight |
|---|------------------------------|--------|
| 1 | Center Pixel                 | 4      |
| 2 | 4 Directional Neighbour      | 2      |
| 3 | Diagonal Direction Neighbour | 1      |



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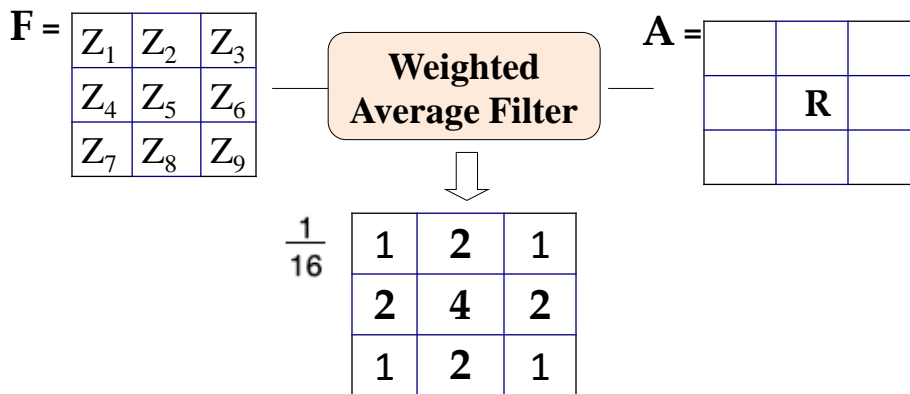
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Consider a Digital Image  $F$  and filter mask  $w$  as shown in fig below,



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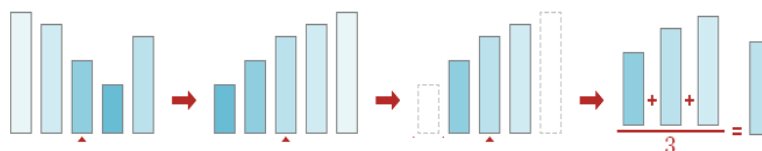
The response of mask at  $A(x,y)$  position is given by,

$$R = \frac{1}{16} [(z_1 + 2z_2 + z_3) + (2z_4 + 4z_5 + 2z_6) + (z_7 + 2z_8 + z_9)]$$

### (3) Trimmed Averaging Filter

A non-linear smoothing filter.

This filter deals well with images containing spiky noise, but it is less effective at attenuating random noise



## Trimmed Mean Filter ALGORITHM :

1. Place a window over element;
2. Pick up elements;
3. Order elements;
4. Discard elements at the beginning and at the end of the got
5. ordered set;
5. Take an average — sum up the remaining elements and divide the sum by their number.

Since the filter applies a mean to the untrimmed data, it is not edge preserving

## (4) Alpha ( $\alpha$ )-Trimmed Averaging Filter

A non-linear smoothing filter.

For ex. Consider below window size

$F =$

|    |    |    |
|----|----|----|
| 20 | 20 | 18 |
| 21 | 19 | 12 |
| 19 | 22 | 10 |

$A =$

|  |          |  |
|--|----------|--|
|  |          |  |
|  | $A(x,y)$ |  |
|  |          |  |

The ordered list is :

$F = \{ 8, 10, 12, 19, 19, 20, 20, 21, 22 \}$

### To find $A(x,y)$

For different values of  $\alpha$ , the output is as follows:

**Case-1 : When  $\alpha = 0$**

$$F = \{ 8, 10, 12, 19, 19, 20, 20, 21, 22 \}$$

$$A(x,y) = 18.8$$

**Case-2 : When  $\alpha = 1$**

$$F = \{ \cancel{8}, 10, 12, 19, 19, 20, 20, 21, \cancel{22} \}$$

$$A(x,y) = 17.3$$

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**Case-3 : When  $\alpha = 2$**

$$F = \{ \cancel{8}, \cancel{10}, 12, 19, 19, 20, 20, \cancel{21}, \cancel{22} \}$$

$$A(x,y) = 18.0$$

**Case-4 : When  $\alpha = 3$**

$$F = \{ \cancel{8}, \cancel{10}, \cancel{12}, 19, 19, 20, \cancel{20}, \cancel{21}, \cancel{22} \}$$

$$A(x,y) = 19.3$$

**Case-5 : When  $\alpha = 4$**

$$F = \{ \cancel{8}, \cancel{10}, \cancel{12}, \cancel{19}, 19, \cancel{20}, \cancel{20}, \cancel{21}, \cancel{22} \}$$

$$A(x,y) = 19.0$$

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This filter is a combination of mean filter and median filter as you said.

This is an algorithmic approach that tries to combine properties of the mean filter with properties of the median filter.

## (5) Geometric Mean Filter

The geometric mean filter is an image filtering process meant to smooth and reduce noise of an image.

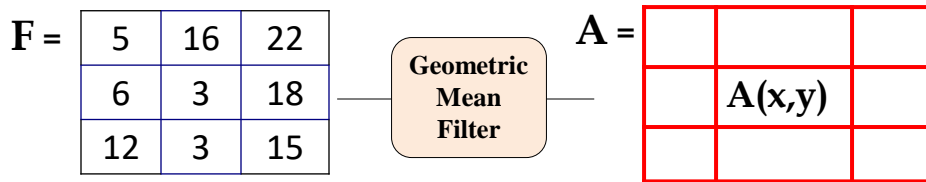
It is based on the mathematic geometric mean.

The output image  $G(x,y)$  of a geometric mean is given by

$$G(x,y) = \left[ \prod F(i,j) \right]^{\frac{1}{mn}}$$



Eg. Consider a digital subimage F



The output pixel value at  $A(x,y)$  position is given by,

$$A(x,y) = (5 * 16 * 22 * 6 * 3 * 18 * 12 * 3 * 15)^{(1/9)}$$

$$A(x,y) = 8.77$$

## [II] Smoothing Non-Linear Filters

[ also called as ordered statistic filters ]

### (1) Median Filter

- (i) Median filter is a Non-Linear ordered statistic filter.
- (ii) Output pixel value at  $(x,y)$  position is obtained by selecting the median of neighbourhood of input pixel value.

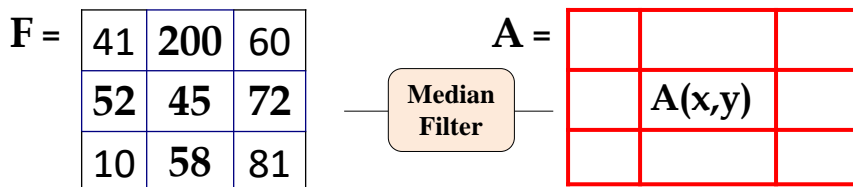
## \* Median Filter ALGORITHM

- (I) Arrange the pixels in the window either in Increasing order OR in Decreasing order.
- (II) Select the middle value  
If number of pixels in the window are even,  
Then find average of the middle two values.
- (III) Replace the input pixel value by the selected median value in the output image.

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Eg. Consider a Digital Image F



1. Arrange All the pixel values in ascending order

|    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|-----|
| 10 | 41 | 45 | 52 | 58 | 60 | 72 | 81 | 200 |
|----|----|----|----|----|----|----|----|-----|

2. Select the middle value

$A(x,y) = 58$

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## Ex of Digital Image with Salt and Pepper Noise :

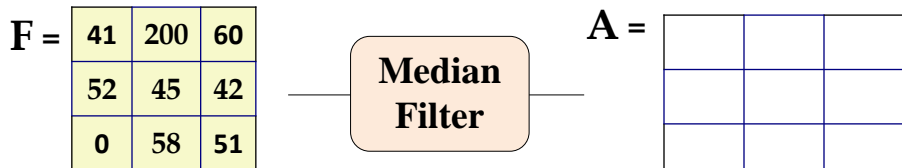
|     |    |     |    |                                |
|-----|----|-----|----|--------------------------------|
| F = | 41 | 200 | 60 | Salt Noise Value = 200 (White) |
|     | 52 | 45  | 42 |                                |
|     | 0  | 58  | 51 |                                |
|     |    |     |    | Pepper Noise Value = 0 (Black) |

Most of the pixels in the image are in the range  
[ 40 to 60 ]

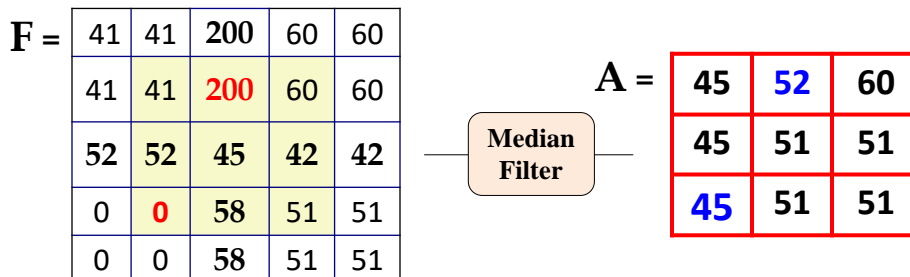
There are two extreme values in the image :  
i.e. 200 (Salt Noise) and 0 (Pepper Noise)

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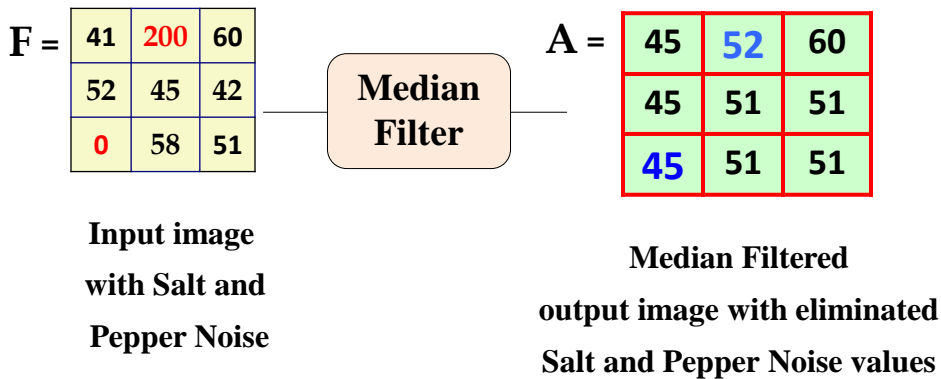


To remove noisy pixels, assume virtual rows and columns with repeated border pixel values



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$F = \begin{bmatrix} 41 & 200 & 60 \\ 52 & 45 & 42 \\ 0 & 58 & 51 \end{bmatrix}$

$A = \begin{bmatrix} 45 & 52 & 60 \\ 45 & 51 & 51 \\ 45 & 51 & 51 \end{bmatrix}$

- Median Filter forces the points with distinct gray levels to be more like their neighbours
- When all the pixel values are arranged in order, distinct values occupy extreme positions and Median filter selects median i.e. middle vales of ordered pixel values.
- Therefore, noisy pixel values are eliminated median filter

**F =**

|    |     |    |
|----|-----|----|
| 41 | 200 | 60 |
| 52 | 45  | 42 |
| 0  | 58  | 51 |

**A =**

|    |    |    |
|----|----|----|
| 45 | 52 | 60 |
| 45 | 51 | 51 |
| 45 | 51 | 51 |

- **Eliminated noisy pixel values are replaced by one of the neighbouring pixel values.**
- **Therefore, Median Filter is the most suitable to remove Salt and Pepper Noise in the image.**

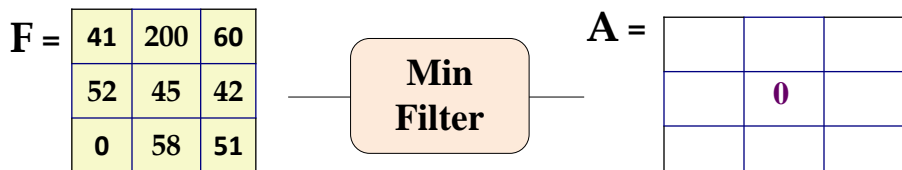
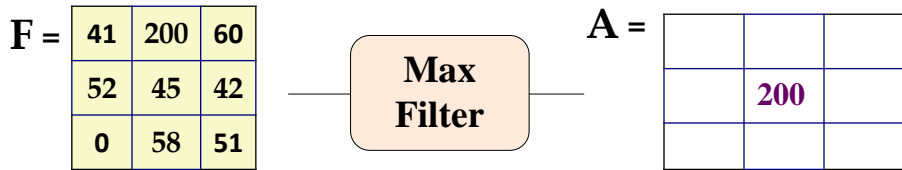
## **(2) Max Filter**

- Max filter is a Non-Linear ordered statistic filter.
- Output pixel value at (x,y) position is obtained by selecting the Maximum of neighbourhood of input pixel value.

## **(3) Min Filter**

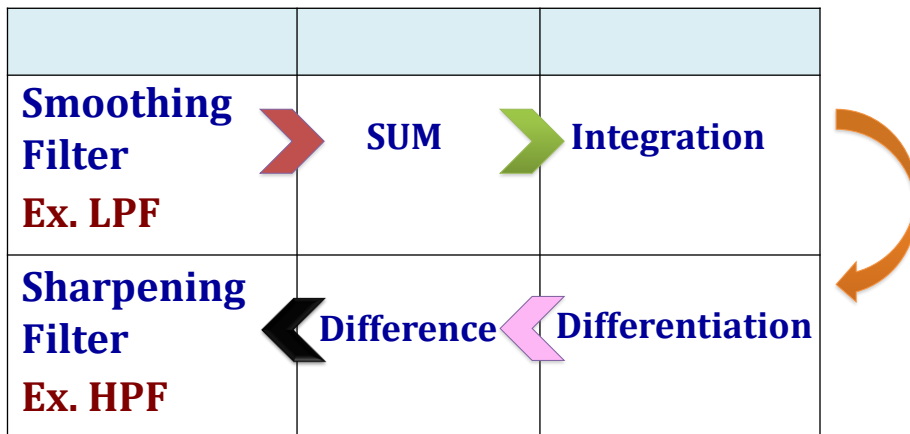
- Min filter is a Non-Linear ordered statistic filter.
- Output pixel value at (x,y) position is obtained by selecting the Minimum of neighbourhood of input pixel value.

Eg.



## Sharpening Derivative Filters

**Concept :**



### [ III ] Sharpening First Order Derivative filters [also called as Gradient Filters]

Eg: Robert, Prewit, Sobel and Fri-chen filters

- Method of differentiation is Gradient
- The Gradient of F at (x,y) position is defined as,

$$\nabla f = \left[ \frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right] = [G_x, G_y]$$

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Where,

$$G_x = \frac{\partial f}{\partial x} = \lim_{h \rightarrow 1} \frac{f(x+h, y) - f(x, y)}{h}$$

$$G_x = f(x+1, y) - f(x, y)$$

$$G_y = \frac{\partial f}{\partial y} = \lim_{h \rightarrow 1} \frac{f(x, y+h) - f(x, y)}{h}$$

$$G_y = f(x, y+1) - f(x, y)$$

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$$G_x = f(x+1, y) - f(x, y)$$

$$G_y = f(x, y+1) - f(x, y)$$

|  |  |  |  |  |
|--|--|--|--|--|
|  |  | F = \begin{matrix} & y-1 & y & y+1 \\ x-1 & & f(x-1, y) & \\ x & f(x, y-1) & f(x, y) & f(x, y+1) \\ x+1 & & f(x+1, y) & \end{matrix} |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

## 1. Robert Filter

(1<sup>st</sup> Order Derivative Sharpening filter)

- To find  $G_x$

$$G_x = f(x+1, y) - f(x, y)$$

$$G_x = z_8 - z_5$$

|         |  |    |   |
|---------|--|----|---|
| $G_x =$ |  |    |   |
|         |  |    |   |
|         |  | -1 | 0 |
|         |  | 1  | 0 |

|  |  |  |  |  |
|--|--|--|--|--|
|  |  | F = \begin{matrix} & y \\ x & \begin{matrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{matrix} \end{matrix} |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

- To find  $G_y$

$$G_y = f(x, y+1) - f(x, y)$$

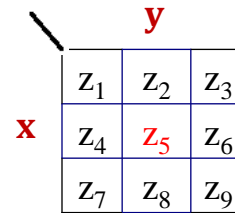
$$G_y = z_6 - z_5$$

|         |  |    |   |
|---------|--|----|---|
| $G_y =$ |  |    |   |
|         |  |    |   |
|         |  | -1 | 1 |
|         |  | 0  | 0 |



## 2. Prewitt Filter

( 1<sup>st</sup> Order Derivative - Sharpening filter )



|     |       |       |       |
|-----|-------|-------|-------|
|     | $z_1$ | $z_2$ | $z_3$ |
| $x$ | $z_4$ | $z_5$ | $z_6$ |
|     | $z_7$ | $z_8$ | $z_9$ |

### ▪ To find $G_x$

$$G_x = (z_7 - z_1) + (z_8 - z_2) + (z_9 - z_3)$$

$$G_x =$$

|      |      |      |
|------|------|------|
| $-1$ | $-1$ | $-1$ |
| $0$  | $0$  | $0$  |
| $1$  | $1$  | $1$  |

### ▪ To find $G_y$

$$G_y = (z_3 - z_1) + (z_6 - z_4) + (z_9 - z_7)$$

$$G_y =$$

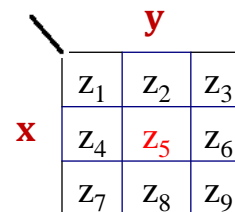
|      |     |     |
|------|-----|-----|
| $-1$ | $0$ | $1$ |
| $-1$ | $0$ | $1$ |
| $-1$ | $0$ | $1$ |

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## 3. Sobel Filter

( 1<sup>st</sup> Order Derivative - Sharpening filter )



|     |       |       |       |
|-----|-------|-------|-------|
|     | $z_1$ | $z_2$ | $z_3$ |
| $x$ | $z_4$ | $z_5$ | $z_6$ |
|     | $z_7$ | $z_8$ | $z_9$ |

### ▪ To find $G_x$

$$G_x = (z_7 - z_1) + 2(z_8 - z_2) + (z_9 - z_3)$$

$$G_x =$$

|      |      |      |
|------|------|------|
| $-1$ | $-2$ | $-1$ |
| $0$  | $0$  | $0$  |
| $1$  | $2$  | $1$  |

### ▪ To find $G_y$

$$G_y = (z_3 - z_1) + 2(z_6 - z_4) + (z_9 - z_7)$$

$$G_y =$$

|      |     |     |
|------|-----|-----|
| $-1$ | $0$ | $1$ |
| $-2$ | $0$ | $2$ |
| $-1$ | $0$ | $1$ |

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#### 4. Fri-Chen Filter

( 1<sup>st</sup> Order Derivative - Sharpening filter )

|          |                |                      |                |
|----------|----------------|----------------------|----------------|
|          | <b>y</b>       |                      |                |
|          | z <sub>1</sub> | z <sub>2</sub>       | z <sub>3</sub> |
| <b>x</b> | z <sub>4</sub> | <b>z<sub>5</sub></b> | z <sub>6</sub> |
|          | z <sub>7</sub> | z <sub>8</sub>       | z <sub>9</sub> |

▪ To find G<sub>x</sub>

$$G_x = (z_7 - z_1) + \sqrt{2}(z_8 - z_2) + (z_9 - z_3)$$

$$G_x = \begin{bmatrix} -1 & -\sqrt{2} & 1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$$

▪ To find G<sub>y</sub>

$$G_y = (z_3 - z_1) + \sqrt{2}(z_6 - z_4) + (z_9 - z_7)$$

$$G_y = \begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix}$$

1. The Magnitude of Gradient at (x,y) is given by,

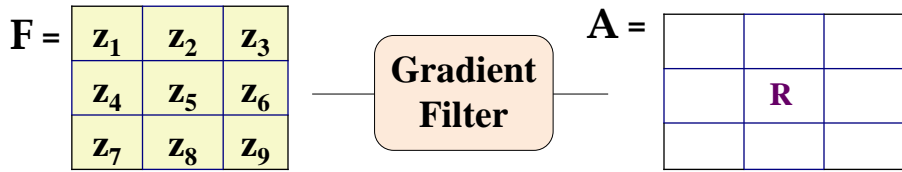
$$|\nabla f| = \sqrt{G_x^2 + G_y^2}$$

$$|\nabla f| = |G_x| + |G_y|$$

2. Orientation at (x,y) is given by,

$$\alpha = \tan^{-1} \left( \frac{G_y}{G_x} \right)$$

### 3. To find response of the Gradient filter,



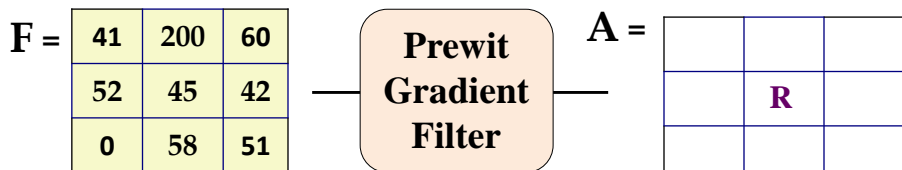
where,

$$R = |G_x| + |G_y|$$

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### Example-1 :



Prewitt Vertical Mask is given by,

$$G_x = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

#### (i) To find $G_x$

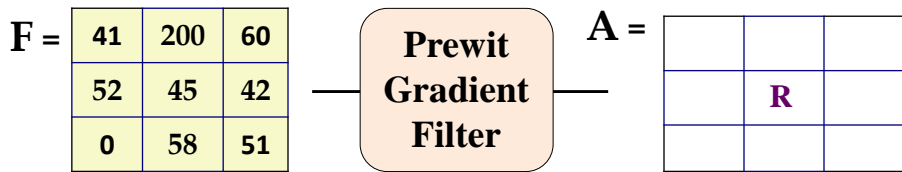
$$G_x = (-41) + (-142) + (-9)$$

$$G_x = -192$$

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Similarly,



Prewit Horizontal Mask is given by,

|    |   |   |
|----|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

(ii) To find  $G_y$

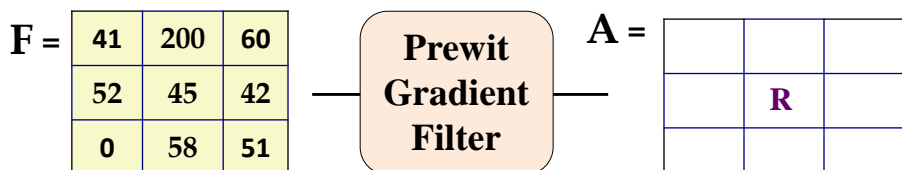
$$G_y = (19) + (-10) + (51)$$

$$G_x = 60$$

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Example...



(iii) To find  $R$

$$R = |G_x| + |G_y|$$

$$R = (192) + (60)$$

$$R = (252)$$

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## [IV] Second Order Derivative filters

Eg. Laplacian Filter, HPF and High Boost Filter

### (2) Laplacian Filter

The Laplacian filter is Second Order Derivative filter

The Laplacian of Digital image  $f$  at  $(x,y)$  position is given by,

$$\nabla^2 f = G_x^2 + G_y^2$$

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#### (i) Find $G^2_x$

$$G_x = f(x+1,y) - f(x,y)$$

|     | y-1        | y          | y+1        |
|-----|------------|------------|------------|
| x-1 |            | $f(x-1,y)$ |            |
| X   | $f(x,y-1)$ | $f(x,y)$   | $f(x,y+1)$ |
| x+1 |            | $f(x+1,y)$ |            |

$$G^2_x = [f(x+1,y) - f(x,y)] - [f(x,y) - f(x-1,y)]$$

$$G^2_x = f(x+1,y) - 2f(x,y) + f(x-1,y)$$

$$G^2_x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{Scale by } (-1)} G^2_x = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

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(i) Find  $G^2y$

$$Gy = f(x, y+1) - f(x, y)$$

$$G^2y = [f(x, y+1) - f(x, y)] - [f(x, y) - f(x, y-1)]$$

$$G^2y = f(x, y+1) - 2f(x, y) + f(x, y-1)$$

$$G^2y = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Scale by } (-1)} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

(iii) Find  $\nabla^2 f$

$$\nabla^2 f = G_x^2 + G_y^2$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \leftarrow \text{Laplacian 4 directional Mask}$$

**(iv) Find  $\nabla^2 f$**

$$\nabla^2 f = \underset{\text{( 4 Directional)}}{\nabla^2 f} + \underset{\text{( Diagonal Directional)}}{\nabla^2 f}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \leftarrow \text{Laplacian 8 directional Mask}$$

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## (2) High Pass Filter

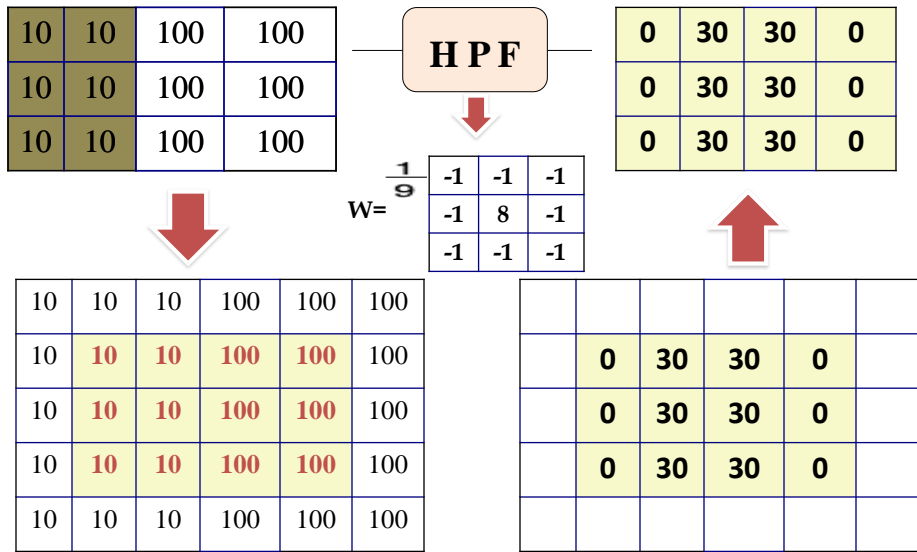
- (i) HPF attenuates LOW frequency Components and allows to pass HIGH frequency components of the image.
- (ii) HPF image can be obtained by subtracting LPF image from the original image.
- (iii) HPF Mask is given by,

$$W = \frac{1}{9} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

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Eg. Consider a Digital Image F as given below

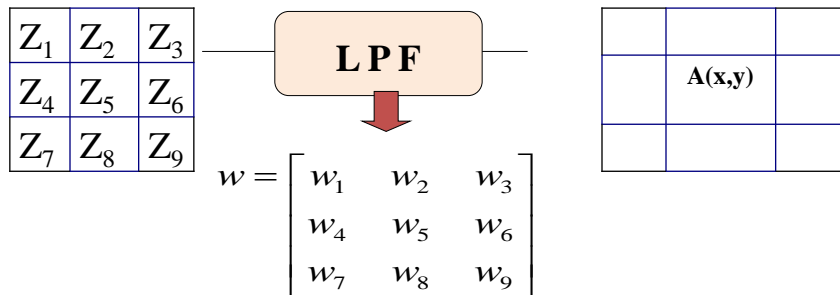


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## Derivation of HPF Mask

Consider a Digital Image F and HPF Mask w as shown in figure below,



The response of HPF mask at A(x,y) position is given by,  

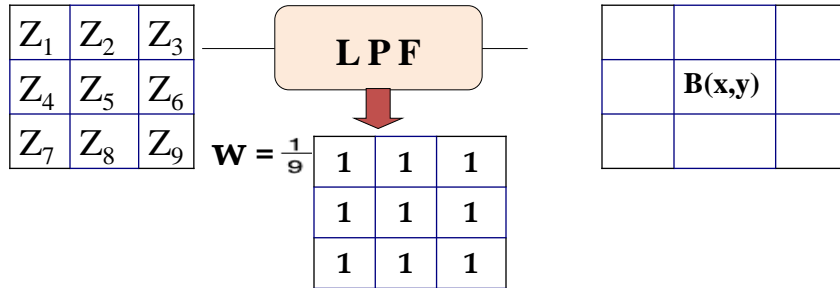
$$A(x,y) = Z_1 w_1 + Z_2 w_2 + Z_3 w_3 + \dots + Z_9 w_9$$
.....(I)

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Consider a Digital Image  $F$  and LPF Mask  $w$  as shown in figure below,



The response of LPF mask at  $B(x,y)$  position is given by,

$$B(x,y) = \frac{1}{9} z_1 + \frac{1}{9} z_2 + \dots + \frac{1}{9} z_9 \dots\dots (II)$$

Now, **Original Image = LPF Image + HPF Image**

$$\text{HPF Image} = \text{Original Image} - \text{LPF Image}$$

$$\begin{bmatrix} & & \\ & A(x,y) & \\ & & \end{bmatrix} = \begin{bmatrix} Z_1 & Z_2 & Z_3 \\ Z_4 & Z_5 & Z_6 \\ Z_7 & Z_8 & Z_9 \end{bmatrix} - \begin{bmatrix} & & \\ & B(x,y) & \\ & & \end{bmatrix}$$

At  $(x,y)$  position,

$$A(x,y) = Z_5 - B(x,y)$$

Now,  $A(x,y) = Z_5 - B(x,y)$

Where,  $B(x,y) = \frac{1}{9}z_1 - \frac{1}{9}z_2 + + \dots \frac{1}{9}z_9$

$$A(x,y) = Z_5 - \left( \frac{1}{9}z_1 + \frac{1}{9}z_2 + + \dots + \frac{1}{9}z_9 \right)$$

$$= Z_5 - \frac{1}{9}z_1 - \frac{1}{9}z_2 - \frac{1}{9}z_3 - \frac{1}{9}z_4 - \frac{1}{9}z_5 \dots \frac{1}{9}z_9$$

$$A(x,y) = -\frac{1}{9}z_1 - \frac{1}{9}z_2 - \frac{1}{9}z_3 - \frac{1}{9}z_4 + \frac{8}{9}z_5 - \frac{1}{9}z_6 - \frac{1}{9}z_7 - \frac{1}{9}z_8 - \frac{1}{9}z_9 \text{ ----(III)}$$

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$$A(x,y) = z_1 w_1 + z_2 w_2 + z_3 w_3 + \dots + z_9 w_9 \text{ .....(I)}$$

$$A(x,y) = -\frac{1}{9}z_1 - \frac{1}{9}z_2 - \frac{1}{9}z_3 - \frac{1}{9}z_4 + \frac{8}{9}z_5 - \frac{1}{9}z_6 - \frac{1}{9}z_7 - \frac{1}{9}z_8 - \frac{1}{9}z_9 \text{ ----(III)}$$

By equating equation (I) and (III) we get,

$$W = \begin{bmatrix} W_1 = \frac{-1}{9} & W_2 = \frac{-1}{9} & W_3 = \frac{-1}{9} \\ W_4 = \frac{-1}{9} & W_5 = \frac{8}{9} & W_6 = \frac{-1}{9} \\ W_7 = \frac{-1}{9} & W_8 = \frac{-1}{9} & W_9 = \frac{-1}{9} \end{bmatrix} \Rightarrow W = \frac{1}{9} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

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### (3) High Boost Filter

High Boost Filter (HBF) Image is obtained by adding HPF image with the original Image.

$$\text{HBF Image} = \text{Original Image} + \text{HPF Image}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline & A(x,y) & \\ \hline & & \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline Z_1 & Z_2 & Z_3 \\ \hline Z_4 & Z_5 & Z_6 \\ \hline Z_7 & Z_8 & Z_9 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline & & \\ \hline & B(x,y) & \\ \hline & & \\ \hline \end{array}$$

At (x,y) position,

$$A(x,y) = Z_5 + B(x,y)$$

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$$\text{Now, } A(x,y) = Z_5 + B(x,y)$$

$$\text{Where, } B(x,y) = -\frac{1}{9}z_1 - \frac{1}{9}z_2 - \frac{1}{9}z_3 - \frac{1}{9}z_4 - \frac{1}{9}z_5 \dots - \frac{1}{9}z_9$$

By Substituting,

$$A(x,y) = Z_5 - \frac{1}{9}z_1 - \frac{1}{9}z_2 - \frac{1}{9}z_3 - \frac{1}{9}z_4 + \frac{8}{9}z_5 \dots - \frac{1}{9}z_9$$

$$A(x,y) = -\frac{1}{9}z_1 - \frac{1}{9}z_2 - \frac{1}{9}z_3 - \frac{1}{9}z_4 + \frac{17}{9}z_5 \dots - \frac{1}{9}z_9$$

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$$A(x,y) = -\frac{1}{9} z_1 - \frac{1}{9} z_2 - \frac{1}{9} z_3 - \frac{1}{9} z_4 + \frac{17}{9} z_5 \dots - \frac{1}{9} z_9$$

$$\text{Let } A(x,y) = z_1 w_1 + z_2 w_2 + z_3 w_3 + \dots + z_9 w_9$$

By equating, HBF Mask is then given by,

$$W = \begin{bmatrix} w_1 = -\frac{1}{9} & w_2 = -\frac{1}{9} & w_3 = -\frac{1}{9} \\ w_4 = -\frac{1}{9} & w_5 = \frac{17}{9} & w_6 = -\frac{1}{9} \\ w_7 = -\frac{1}{9} & w_8 = -\frac{1}{9} & w_9 = -\frac{1}{9} \end{bmatrix} \rightarrow W = \frac{1}{9} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 17 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

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