Lec 5

* Impulse Response of Digital Signal :-

$x(n)$ Input → [DT System eg:- Digital Filter] → $y(n)$ output

$\Downarrow$ $\delta(n)$ Impulse Signal

$\Downarrow$ $h(n)$ Impulse Response of the System.

⇒ Impulse Signal gives the $h(n)$ as Output.

When $h(n)$ is given :-

$x(n)$ → [$h(n)$] → $y(n) = \underbrace{x(n) * h(n)}_{\text{linear convolution}}$

⇒ $h(n)$ is representation of Signal.

* Linear Convolution :-

$$\boxed{y(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m)}$$ IMP

⇒ linear convolution application is to find the output of a filter.

* Types of Filter:-

$x(n)$ I/P → [DT Signal System / Eg:- Digital Filter] → $y(n)$ O/P

IIR Filter | FIR Filter

Eg:- $h(n) = (0.5)^n u(n)$
Infinite Impulse Response

Eg:- $h(n) = \{5, \underset{\uparrow}{6}, 7\}$
Finite Impulse Response.

Eg:- Given $h(n) = \{5, \underset{\uparrow}{6}, 7\}$, Find the response of filter to the $x(n)$ input $= \{\underset{\uparrow}{1}, 2, 3, 4\}$.

Ans. To find response :- O/P:- $(y(n))$

$x(n)$ I/P → [$h(n)$] → $y(n)$ O/P

Step 1:-

Find $y(n)$ for $n \geq 0$

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\,h(n-m)$$

$n=0$; $y(0) = \sum x(m)\,h(-m)$

$y(1) = \sum x(m)\,h(1-m)$

$y(2) = \sum x(m)\,h(2-m)$

$\vdots$

$x(m) = x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$h(m) = \{5, \underset{\uparrow}{6}, 7\}$

$h(-m) = \{7, \underset{\uparrow}{6}, 5\}$

$h(1-m) = \{\_, 7, 6, 5\}$

$\vdots$

$x(m)$		1	2	3	4
$h(-m)$	7	6	5		
$h(1-m)$		7	6	5	
$h(2-m)$			7	6	5
$h(3-m)$				7	6
$h(4-m)$					7

~~$y(n) = x(0) \times h(-m) = 42$~~

$y(0) = \Sigma x(m) h(-m)$

$y(0) = [7\times0 + 1\times6 + 2\times5]$

$= 16$

$y(1) = 1\times7 + 2\times6 + 3\times5 = \boxed{34}$

$y(2) = 7\times2 + 6\times3 + 5\times4 = \boxed{52}$

$y(3) = 7\times3 + 6\times4 = \boxed{45}$

$y(4) = 7\times4 = \boxed{28}$

Step 2:-

Find $y(n)$ for $n < 0$

$\therefore \quad y(-1) = x(m) (h(-1-m)$

$y(-2) = x(m) h(-2-m)$

$\vdots$

$x(m) = x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$x(m)$			1	2	3	4	
$h(0-m)$		0	6	5	5		$y(0) = 34$
$h(-1-m)$		7	6	5			$y(-1) =$
$h(-2-m)$	7	6	5				

$x(m)$				1	2	3	4	
$h(-m)$			7	6	5			$y(0) = 16$
$h(-1-m)$		7	6	5				$y(-1) = 5$
$h(-2-m)$	7	6	5					$y(-2) = 0$

$$\therefore \boxed{y(n) = \{5, \underset{\uparrow}{16}, 34, 52, 45, 28\}}$$

Q2)- Given $h(n) = \{\underset{\uparrow}{5}, 6, 7\}$. Find the response of the filter to the input values $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Ans

$$\underset{I/P}{x(n)} \longrightarrow \boxed{h(n)} \longrightarrow \underset{O/P}{y(n)}$$

Step 1:-

Find y(n) for $n \geq 0$

$$\boxed{y(n) = \sum_{M=-\infty}^{M=\infty} x(M)\, h(n-M)}$$

$y(0) = \Sigma\, x(M)\, h(-M)$

$y(1) = \Sigma\, x(M)\, h(1-M)$

$y(2) = \Sigma\, x(M)\, h(2-M)$

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x(M)	1	2	3	4			
h(-M) 7, 6	5						y(0) = 5
h(1-M) 7	6	5					y(1) = 16
h(2-M)	7	6	5				y(2) = 34
h(3-M)		7	6	5			y(3) = 52
h(4-M)			7	6	5		y(4) = 45
h(5-M)				7	6	5	y(5) = 28

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Step 2:-

For $n < 0$

$y(-1) = x(m)\, h(-1-m)$

$y(-2) = x(m)\, h(-2-m)$

$x(m)$	1	2	3	4	
$h(-m)$ 7,6	5				$y(0) = 5$
$h(-1-m)$ 7,6,5					$y(-1) = 0$
	-	-	-	-	$y(-2) = 0$
					$\vdots$

$\therefore y(n) = \{ \underset{\uparrow}{5}, 16, 34, 52, 45, 28 \}$

* NOTE :-

$\boxed{\text{Length of linear convolution} = \text{Length of 1st signal} + \text{length of 2nd signal} - 1.}$