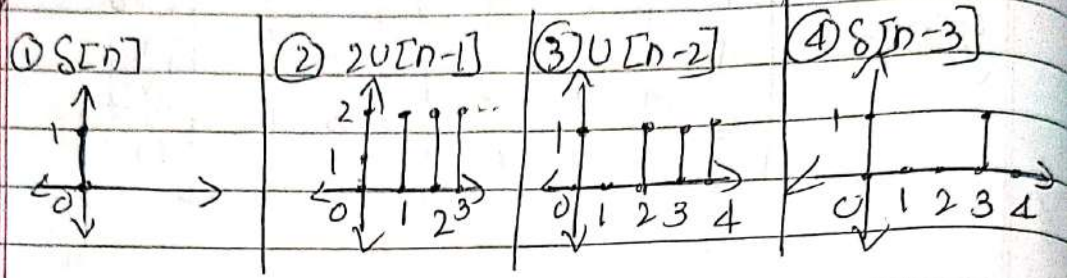
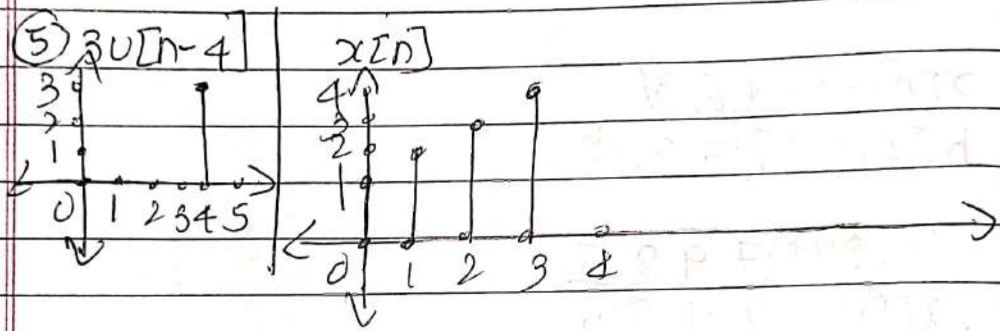
Q. 1

a)

$$x[n] = \delta[n] + 2u[n-1] + u[n-2] + \delta[n-3] - 3u[n-$$

$\Rightarrow$





$x[n] = \{1, 2, 3, 4\}$

$$X(k) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$W_4 = e^{-j\frac{2\pi}{N}} = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$$

$$W = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X(k) = W_N x(n)$$

$1-2j-j+4$

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$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$1+2j-3-4j$

$$X(k) = \{ \underset{\uparrow}{10}, -2+2j, -2, -2-2j \}$$

Q.2

$x[n] = \{1, 2, 3, 4\}$

(a) $P[k] = 8X[k]$

$p(n) = 8x[n] = \{8, 16, 24, 32\}$

(b) $q[k] = 8 + X[k]$

$q(n) = 8\delta(n) + x(n) = \{9, 2, 3, 4\}$

Q. 3

$X[k] = \{1, 2, 3, 4\}$

a) $p[n] = x[n-1]$

If $DFT\{x(n)\} = X(k)$

then $DFT\{x(n-n_0)\} = X(k) e^{-j\frac{2\pi k}{N} n_0}$

$P[0] = X(0) e^{-j\frac{2\pi(0)(1)}{4}} = X(0) = 1$

$P[1] = X(1) e^{-j\frac{2\pi(1)(1)}{4}} = 2e^{-j\pi/2} = -2j$

$P[2] = X(2) e^{-j\frac{2\pi(2)(1)}{4}} = 3e^{-j\pi} = -3$

$P[3] = X(3) e^{-j\frac{2\pi(3)(1)}{4}} = 4e^{-j3\pi/2} = 4j$

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b) $q[n] = x[n+1]$

Here $n_0 = -1$

$$Q(k) = X(k)e^{-j2\pi k n_0/N} = X(k)e^{j\pi/2 k}$$

$$Q(0) = X(0)e^{j\pi/2(0)} = (1)(1) = 1$$
$$Q(1) = X(1)e^{j\pi/2(1)} = 2j$$
$$Q(2) = X(2)e^{j\pi/2(2)} = -3$$
$$Q(3) = X(3)e^{j\pi/2(3)} = -4j$$

$Q(k) = \{1, 2j, -3, -4j\}$

Q.4

$a[n] = \{1, 2, 3, 4\}$

a) $A[k] = W \cdot a[n]$

$$\begin{bmatrix} A(0) \\ A(1) \\ A(2) \\ A(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$A[k] = \{10, -2+2j, -2, -2-2j\}$

c) $b[n] = \{3, 4, 1, 2\}$

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$\therefore b[n] = a[n-2]$

$\therefore B[k] = A(k) \cdot e^{-\frac{2\pi j (2) k}{4}} = A(k) \cdot e^{-\pi j k}$

$= A(k) (e^{-\pi j})^k = A(k) (\cos(\pi) - j\sin(\pi))^k$

$B(k) = (-1)^k A(k)$

(c) $c[n] = \{4, 6, 4, 6\}$

$\cancel{c[n] = x[n] + x[n-}$

$c[n] = a[n] + a[n-2]$

$\therefore C(k) = A(k) + (-1)^k A(k) = (1 + (-1)^k) A(k)$

(d) $d[n] = \{-2, -2, 2, 2\}$

$d[n] = a[n] - a[n-2]$

$D(k) = A(k) - (-1)^k A[k] = [1 - (-1)^k] A[k]$

(e)

$e[n] = \{5, 3, 5, 7\}$

$e[n] = a[n] + a[n-1]$

$E(k) = A(k) + A(k) e^{-\frac{2\pi j (1) k}{4}}$

$= A[k] + \cancel{A[k] W_N^k} W_N^k A[k]$

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Q.5 $x(n) = \{1,1,1,1,0,0,0,0\}$

$X(k) = DFT\{x(n)\}$

a) $a[n] = \{1,1,1,1,1,1,1,1\}$

$a[n] = x(n) + x(n-4)$

$A(k) = X(k) + X(k) e^{\frac{2\pi j \cancel{4} k}{\cancel{4}}}$

$= X(k) + X(k) e^{-2\pi j k}$

$= X(k) + X(k) (e^{-2\pi j})^{k}$

$= X(k) + X(k) [\cos(2\pi) - j\sin(2\pi)]^{k}$

$= X(k) + X(k) [1 - 0]^{k}$

$= [1 + (1)^{k}] X[k]$

(b) $b[n] = \{1,1,1,1,-1,-1,-1,-1\}$

$b[n] = x[n] - x[n-4]$

$B(k) = X(k) - X(k) e^{-2\pi j k}$

$B(k) = [1 - (1)^{k}] X(k)$

(c)

$c[n] = x[n] - x[n-1]$

$C(k) = X(k) - W_N^{k} X(k)$

(d) $d[n] = \{2,0,0,0,0,2,2,2\}$

$d[n] = 2x[-n] = 2x[8-n]$

$D(k) = 2X(8-n)$

Q. 6

$X[k] = \{1,2,3,4\}$

(a) $p[n] = (-1)^n x[n]$

$p[n] = x[n]\, e^{\frac{j2\pi n l}{N}}$

$(-1)^n \cancel{x[n]} = e^{\frac{j2\pi n l}{4}} \cancel{x[n]}$

$(-1)^n = \left(e^{j\frac{\pi}{2}}\right)^{nl}$

$-1 = \left(e^{j\pi/2}\right)^{l}$

$-1 = (j)^l$

$\therefore l = 2$

$\therefore P(k) = X(k-2)_N = \{3,4,1,2\}$

(b) $q[n] = x[n]\cos\left(\frac{n\pi}{2}\right)$

$\cancel{x[n]}\, e^{\frac{j2\pi n l}{4}} = \cancel{x[n]} \cos\left(\frac{n\pi}{2}\right)$

$e^{j\frac{\pi}{2} n l} = \cos\left(\frac{n\pi}{2}\right)$

$\cos\left(\frac{nl\pi}{2}\right) + j\sin\left(\frac{nl\pi}{2}\right) = \cos\left(\frac{n\pi}{2}\right)$

$\frac{nl\pi}{2} = \frac{n\pi}{2} \Rightarrow l = 1$

$\frac{nl\pi}{2} = n\pi \Rightarrow l = 2$

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(b)

$$q[n] = x[n]\cos\left(\frac{\pi n}{2}\right)$$

$$q[n] = x[n]\left(\frac{e^{j\frac{n\pi}{2}} + e^{-j\frac{n\pi}{2}}}{2}\right)$$

$$q[n] = \frac{1}{2}x[n]e^{j\frac{n\pi}{2}} + \frac{1}{2}x[n]e^{-j\frac{n\pi}{2}}$$

$$\text{DFT}\{q[n]\} = \text{DFT}\left\{\frac{1}{2}x[n]e^{j\frac{n\pi}{2}}\right\} + \text{DFT}\left\{\frac{1}{2}x[n]e^{-j\frac{n\pi}{2}}\right\}$$

$$Q(k) = \frac{1}{2}X(k-1) + \frac{1}{2}X(k+1)$$

$$e^{j\frac{2\pi}{4}nl} = e^{j\frac{n\pi}{2}}$$

$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$a[n] = \{1, 0, 0, 0, 0, 1, 1, 1\}$

$x(-n) \Rightarrow a[n] = x[-n]$

$\therefore A(k) = X(-k)$

$b[n] = \{2, 1, 1, 1, 0, 1, 1, 1\}$

$b[n] = x[n] + x[-n]$

Taking DFT on both sides

$B[k] = X[k] + X[-k]$

~~$x[n] = \{1, 2, 3, 4\}$~~

$\left.\begin{array}{l} DFT\{x[n]\} = X[k] \\ X[k] = \{1, 2, 3, 4\} \end{array}\right\}$ Given.

a) $p[n] = x[-n]$

$\therefore P(k) = X[-k] = \{1, 4, 3, 2\}$

b) $q[n] = x[-n+1]$

let's Find

$DFT\{x[-n]\} = X[-k] = \{1, 4, 3, 2\}$

$\{x[-n+1]\} = X[-k]\, e^{\frac{-2\pi j(-1)k}{4}} = X[-k]\, e^{\frac{\pi j k}{2}}$

$= X[-k]\,(j)^k$

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(b)

$q[n] = x[-n+1]$

$DFT\{x[-n]\} = X(-k)$

$DFT\{x[-n+1]\} = X[-k]\, e^{-j\frac{2\pi(-k)(-1)}{4}}$

$= X[-k]\, e^{-\frac{j\pi k}{2}}$

$= X[-k]\,(-j)^k$

$= \{1, 4, 3, 2\}(-j)^k$

$= \{1, -4j, -3, 2j\}$

$\therefore Q(k) = \{1, -4j, -3, 2j\}$

(c) $r[n] = x[-n-1]$

$R(k) = DFT\{x[-n-1]\}$

$DFT\{x[-n-1]\} = DFT\{x[-n]\}\, e^{-j\frac{2\pi(1)(-k)}{N}}$

$= X[-k]\, e^{j\frac{2\pi}{4}k}$

$= X[-k]\, e^{j\pi/2\, k}$

$= \{1, 4, 3, 2\}(j)^k$

$= \{1, 4j, -3, -2j\}$

$\therefore R[k] = \{1, 4j, -3, -2j\}$

9.9. $x[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & 4 \leq n \leq 7 \end{cases}$

$x[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}$

a) $p[n] = \{1, 0, 0, 0, 0, 1, 1, 1\}$

$p[n] = x[-n]$

$\therefore P[k] = DFT\{x[-n]\} = X[-k]$

b) $q[n] = \{0, 0, 1, 1, 1, 1, 0, 0\}$

$q[n] = x[n-2]$

$Q[k] = DFT\{x[n-2]\} = X[k]e^{\frac{-j2\pi k 2}{4}}$

$= X[k]e^{-j\pi k} = \cancel{X[k]\,W_N^{2}} \; W_N^{2k} X[k]$

9.10
a) $P[k] = \{0, \boxed{\ }, 2+j, -1, \boxed{\ }, j\}$

$N = 6$

By symetry property

$P[k] = P^*[N-k]$

$P[k] = P^*[6-k]$

$P[1] = P^*[6-1]$

$P[1] = P^*[5] = -j$

Similary

$P[4] = P^*[6-4]$

$P[4] = P^*[2]$

$P[4] = 2-j$

$P[k] = \{0, -j, 2+j, -1, 2-j, j\}$

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(b) $e[k] = \{1, 2, \square, \square, 0, 1-j, -2, \square\}$

$N = 8$

$\Rightarrow$ By symmetry property,

$e[k] = e^*[N-k]$
$e[k] = e^*[8-k]$

$\therefore e[2] = e^*[8-2] = e^*[6] = -2$
$\therefore e[3] = e^*[8-3] = e^*[5] = 1+j$
$\therefore e[7] = e^*[8-7] = e^*[1] = 2$

$\therefore e[k] = \{1, 2, -2, 1+j, 0, 1-j, -2, 2\}$

Q.12

$P[n] = \frac{1}{2}\left\{\frac{x[n] + x[-n]}{2}\right\}$

$\therefore P[n] = x_e[n]$

$\therefore P[k] = DFT\{P[n]\} = X_e[k]$

$= \frac{1}{2}\left[\frac{X[k] + X[-k]}{2}\right]$

$X[k] = \{1, 4+2j, 6+4j, 2j, 6, -2j, 6-4j, 4-2j\}$
$X[-k] = \{1, 4-2j, 6-4j, -2j, 6, 2j, 6+4j, 4+2j\}$

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$$= \frac{1}{2}\{2, 8, 12, 0, 12, 0, 12, 8\}$$

$$= \{1, 4, 6, 0, 6, 0, 6, 4\}$$

Q.3 $x[n] = \{(1+j), (2+2j), (3+3j), (4+2j)\}$

a) $X[k]$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix} = \begin{bmatrix} 10+8j \\ 3j \\ -2 \\ -2-4i \end{bmatrix}$$

$X[k] = [10+8j, 3j, -2, -2-4i]$

b)

$DFT\{x^*[n]\} = X^*[-k]$

$\therefore DFT\{x^*[n]\} = \{10-8j, -2+4j, -2, -3j\}$

c)

$P[n] = \{1, 2, 3, 4\}$

$$P[n] = \frac{x[n] + x^*[n]}{2}$$

$$P(k) = \frac{1}{2}X(k) + \frac{1}{2}X^*[-k]$$

$P(k) = \frac{1}{2}[20, -2+7j, -4, -2-7j]$

Q.13

$x[n] = \{1+j,\ 2+2j,\ 3+3j,\ 4+2j\}$

$N = 4$

a) $\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix}$

$X(0) = 1+j+2+2j+3+3j+4+2j = 10+8j$
$X(1) = 1+j-2j+2-3-3j+4j-2 = -2$
$X(2) = 1+j-2-2j+3+3j-4-2j = -2$
$X(3) = 1+j+2j-2-3-3j-4j+2 = -2-4j$

$X[k] = [10+8j,\ -2,\ -2,\ -2-4j]$

b) $DFT\{x^*[n]\} = X^*[N-k]$

$\therefore DFT\{x^*[n]\} = \{10-8j,\ -2+4j,\ -2,\ -2\}$

c) $P[n] = \{1, 2, 3, 4\}$

$P[n] = \frac{1}{2}\{x[n] + x^*[n]\}$

$\therefore P(k) = \frac{1}{2}\{X[k] + X^*[N-k]\}$

$= \frac{1}{2}\{20,\ -4+4j,\ -4,\ -4-4j\} = \{10,\ -2+2j,\ -2$

$= \{10,\ -2+2j,\ -2,\ -2-2j\}$

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d) $q[n] = \{1, 2, 3, 2\}$

$q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$

$\therefore Q(k) = \frac{1}{2j}\{X(k) - X^*[N-k]\}$

$= \frac{1}{2j}\{16j, -4j, 0, -4j\}$

$= \{8, -2, 0, -2\}$

Q.14

If

$g[n] = x_1[n] \odot x_2[n]$

$\therefore Y[k] = X_1[k] \cdot X_2[k]$

Put $X_1[k] = X_2[k]$

$\therefore Y[k] = X_1^2[k]$

$\therefore y[n] = x_1[n] \odot x_1[n]$

$$\therefore \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 26 \\ 28 \\ 26 \\ 20 \end{bmatrix}$$

$\therefore IDFT[P[k]] = \{26, 28, 26, 20\}$

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b) $q[n] = x[n] \otimes x[n]$

$Q[k] = X[k] \cdot X[k]$

Question is wrong

Q.15

$x[n] = \{1, 2, 3, 2\}$ $\quad h[n] = \{1, 2, 3, 4\}$

a)

$$r_{xy} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 22 \\ 18 \\ 18 \\ 22 \end{bmatrix}$$

$r_{xy} = \{22, 18, 18, 22\}$

b)

$X[k] = DFT\{x[n]\} = \{8, -2, 0, -2\}$

$H[k] = DFT\{h[n]\} = \{10, -2-2j, 2, -2+2j\}$

$DFT\{r_{xy}\} = X[k] \cdot H^*[k]$

$= \{8, -2, 0, -2\} \cdot \{10, -2+2j, 2, -2-2j\}$

$= \{80, 4-4j, 0, 4+4j\}$

$r_{xy} = IDFT\{80, 4-4j, 0, 4+4j\}$
$= \{22, 18, 18, 22\}$

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Q.16. $x[n] = \{1, 2, 3, 2\}$

a) $X[K] = \{8, -2, 0, -2\}$

$$E = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

$$= \frac{1}{4} \sum_{k=0}^{3} |X[k]|^2 = \frac{1}{4}[64+4+0+4]$$

$$= 18J$$

b) $$E = \sum_{n=0}^{N-1} |x[n]|^2 = [1+4+9+4] = 18J$$

Q.17.

a) $X[0]$

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

Put $N=1$

$$1^2 = 1\,(X[0])^2$$

$X[0] = 1$

b)

$X[0]^2 + X[1]^2 + X[2]^2$

b) $$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

Put

$N=2$

$$1^2 + (-2)^2 = \frac{1}{2}\left[1 + X[1]^2\right]$$

$10 = 1 + X[1]^2$

$X[1]^2 = 9$

$X[1] = 3$

Put

$N=3$

$$1^2 + (-2)^2 + 3^2 = \frac{1}{3}\left[1 + 9 + X[2]^2\right]$$

$42 = 10 + X[2]^2$

$X[2]^2 = 32$

$X[2] = \sqrt{32} = 4\sqrt{2}$

Put $N=4$

$$1^2 + (-2)^2 + 3^2 + (-4)^2 = \frac{1}{4}\left[1 + 9 + 32 + X[3]^2\right]$$

$120 = 42 + X[3]^2$

$X[3]^2 = 120 - 42$

$X[3] = \sqrt{78}$

(c) $\sum_{k=0}^{5} |X[k]|^2$

$$N \sum_{n=0}^{5} |x(n)|^2 = [1^2 + 2^2 + 3^2 + 4^2 + 5^2 + -6^2] \times 6$$
$$= 546$$

Q.18