* Circular Convolution :- (Periodic Signal)

$$\Rightarrow \boxed{y(n) = \sum_{m=0}^{N-1} x(m)\, \underbrace{h((n-m))}_{\text{circular shifting}}}$$

Q] Eg :- $x[n] = \{1, 2, 3, 4\}$ $L = 4pt$
$h[n] = \{5, 6, 7\}$ $M = 3pt$

$\Rightarrow$ Find $y(n) = x(n) \circledast h(n)$

Notation for circular convolution.

Ans :- Step 1 :-

We need a common period N.

$\therefore N = \max(L, M)$
$= \max(4, 3)$
$N = 4$

Step 2 :- Zero padding :-

$x(n) = \{1, 2, 3, 4\}$
$h(n) = \{\underset{\uparrow}{5}, 6, 7, 0\}$
} Periodic with period $N = 4$

$h(-n) = \{\underset{\uparrow}{5}, 0, 7, 6\}$ (For periodic signals)

Step 3:-

$$y(n) = \sum_{m=0}^{N-1} x(m)\, h(n-m)$$

$n = 0$

$y(0) = \sum x(m)\, h(-m)$

$y(1) = \sum x(m)\, h(1-m)$

$y(2) = \sum x(m)\, h(2-m)$

$\vdots$

$x(m)$	1	2	3	4	$y(n)$
$h(-m)$	5	0	7	6	$y(0) = 50$
$h(1-m)$	6	5	0	7	$y(1) = 44$
$h(2-m)$	7	6	5	0	$y(2) = 34$
$h(3-m)$	0	7	6	5	$y(3) = 52$
$h(4-m)$	5	0	7	6	$y(4) = 50$

⇒ Hence we know circular convolution gives a periodic output.

Another Way:- (Practical Implementation (code))

$$\underset{h(-m),\, h(1-m), \ldots}{\begin{bmatrix} 5 & 0 & 7 & 6 \\ 6 & 5 & 0 & 7 \\ 7 & 6 & 5 & 0 \\ 0 & 7 & 6 & 5 \end{bmatrix}} \underset{x(n)}{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}} = \begin{bmatrix} 50 \\ 44 \\ 34 \\ 52 \end{bmatrix}$$

Q2) $x(n) = \{1, 2, 3, 4\}$
$h(n) = \{5, 6, 7\}$

Find $y(n) = x(n) \circledS h(n)$

$\therefore N = 5.$

Ans **Step 1:-**

$x(n) = \{1, 2, 3, 4, 0\}$
$h(n) = \{5, 6, 7, 0, 0\}$
$h(-M) = \{5, 0, 0, 7, 6\}$

Step 2(:-)

$$y(n) = \sum_{M=0}^{N-1} x(M)\, h(n-M)$$

$y(0) = x(M)\, h(-M)$
$y(1) = x(M)\, h(1-M)$
$y(2) = x(M)\, h(2-M)$
$y(3) = x(M)\, h(3-M)$
$\vdots$

Step 3:-

$x(M)$	1	2	3	4	0	$y(n)$
$h(-M)$	5	0	0	7	6	$y(0) = 33$
$h(1-M)$	6	5	0	0	7	$y(1) = 16$
$h(2-M)$	7	6	5	0	0	$y(2) = 34$
$h(3-M)$	0	7	6	5	0	$y(3) = 52$
$h(4-M)$	0	0	7	6	5	$y(4) = 45$

$\therefore y(n) = \{33, 16, 34, 52, 45\} \Rightarrow$ ans.

Q3)- $x(n) = \{1, 2, 3, 4\}$
$h(n) = \{5, 6, 7\}$
$N = 6$
Find $y(n) = x(n) \; ⑥ \; h(n)$

Ans ~~$h(-m)$~~ =
step 1:- $x(n)\{1, 2, 3, 4, 0, 0\}$
$h(n) = \{5, 6, 7, 0, 0, 0\}$

step 2:-
$$y(n) = \sum_{m=0}^{N-1} x(m) h(n-m)$$

$y(0) = x(m) h(-m)$
$y(1) = x(m) h(1-m)$
$y(2) = x(m) h(2-m)$
$\vdots$

$h(-m) = \{5, 0, 0, 0, 7, 6\}$

step 3:-

$$\begin{bmatrix} 5 & 0 & 0 & 0 & 7 & 6 \\ 6 & 5 & 0 & 0 & 0 & 7 \\ 7 & 6 & 5 & 0 & 0 & 0 \\ 0 & 7 & 6 & 5 & 0 & 0 \\ 0 & 0 & 7 & 6 & 5 & 0 \\ 0 & 0 & 0 & 7 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 6+10 \\ 7+12+15 \\ 14+18+\cancel{15}\,20 \\ 21+24 \\ 28+0 \end{bmatrix}$$

$\therefore y(n) = \{5, 16, 34, 52, 45, 28\}$ //

Q4)- $x(n) = \{1, 2, 3, 4\}$ $L = 4pt$
$h(n) = \{5, 6, 7\}$ $M = 3pt$

Find $y(n) = x(n) * h(n)$

Here circle is not there hence it is not circular convolution, but it is linear convolution. Signal is non-periodic

Ans

∴ Step 1:-

$$y(n) = \sum_{-\infty}^{\infty} x(m) h(n-m)$$

$\therefore y(0) = x(m) h(-m)$
$y(1) = x(m) h(1-m)$
$y(2) = x(m) h(2-m)$
⋮

$h(-m) = \{7, 6, \underset{\uparrow}{5}\}$

∴

$x(m)$			1	2	3	4	$y(n)$
$h(-m)$	7	6	5				$y(0) = 5$
$h(1-m)$		7	6	5			$y(1) = 16$
$h(2-m)$			7	6	5		$y(2) = 34$
$h(3-m)$				7	6	5	$y(3) = 52$
$h(4-m)$					7	6	$y(4) = 45$
$h(5-m)$						7	$y(5) = 28$

$y(n) = \{\underset{\uparrow}{5}, 16, 34, 52, 45, 28\}$

IMP ✱ NOTE :-

1) If $N \geq \underset{(L)}{x(n)} + \underset{(M)}{h(n)} - 1$

Then :-

Result of circular convolution = Result of linear convolution

2) To find linear convolution using circular convolution :-
Select N such that :-

$$\boxed{N \geq L + M - 1}$$

HW Q) Given :- $x(n) = [1, 2, 3, 4]$
$h(n) = [5, 6, 7]$

Find $y(n) = h(n) * x(n)$

IMP.

Ans $\boxed{\text{Linear convolution is commutative, i.e :- } x(n) * h(n) = h(n) * x(n)}$

$\therefore$ $y(n) = h(n) * x(n)$ will be same as $y(n) = x(n) * h(n)$.

Ans :- $y(n) = \{5, 16, 34, 52, 45, 28\}$

~~Ans :-~~

* Correlation :-

Types of correlation :-

1) **Auto correlation :-**

⇒ For same signals it is auto correlation

$$y(n) = x(n) \underset{\substack{\uparrow \\ \text{Notation}}}{\underline{o}} x(n)$$

$$\boxed{y(n) = \sum_{M=-\infty}^{\infty} x(M)\, x(M-n)}$$

2) **Cross Correlation :-**

$$y(n) = x(n) \circ h(n)$$

$$\boxed{y(n) = \sum_{M=-\infty}^{\infty} x(M)\, h(M-n)}$$

Q) $x(n) = \{1, 2, 3, 4\}$
$h(n) = \{5, 6, 7\}$
Find correlation of $x(n)$ and $h(n)$

Ans **Step 1** :- Find $y(n)$ for $n \geq 0$

$$y(n) = \sum_{M=-\infty}^{\infty} x(M)\, h(M-n)$$

$$y(0) = \sum_{M=-\infty}^{\infty} x(M)\, h(M)$$

$$y(1) = \sum_{-\infty}^{\infty} x(m)\, h(m-1)$$

⋮

$x(m)$	1	2	3	4	$y(n)$
$h(m)$	5	6	7		$y(0) = 38$
$h(m-1)$		5	6	7	$y(1) = 56$
$h(m-2)$			5	6	$y(2) = 39$
$h(m-3)$				5	$y(3) = 20$

<u>Step 2:-</u> For $n \leq 0$

$$\therefore y(n) = \sum_{m=\infty}^{\infty} x(m)\, h(m-n)$$

$$y(0) = \sum_{m=\infty}^{\infty} x(m)\, h(m)$$

$$y(-1) = \sum_{m=\infty}^{\infty} x(m)\, h(m+1)$$

$$y(-2) = \sum_{m=\infty}^{\infty} x(m)\, h(m+2)$$

$x(m)$	1	2	3	4	$y(n)$
$h(m)$	5	6	7		$y(0) = 38$
$h(m+1)$ 5	6	7			$y(-1) = 20$
$h(m+2)$ 5 6	7				$y(-2) = 7$

$$\therefore y(n) = \{7, 20, \underset{\uparrow}{38}, 56, 39, 20\}$$

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* DSP System:-

← DT system →

$x(t)$ I/P:- Continuous Signal	Sampling using Sample & Hold Circuit (Decimation)	$x(n)$ Discrete Signal	A D C	DSP Processor eg:- TMS 320	D A C	$y(n)$ Output DT Signal	Reconstruction using LPF (interpolation)

- DSP Algorithm:- Convolution
  Correlation
  Linear Filtering

- ADC:- Analog to digital Converter
- DAC:- Digital to Analog Converter

* Example of DSP Systems is Digital Telephone System:-

⇒ Audio Signal Frequency is (20Hz - 20kHz) but:-

- Bandlimited (Band limit of frequency) of Audio signal range is (20 - 3.4kHz).

⇒ According to Sampling Theorem:-

$\boxed{F_s \geq 2 F_{max}}$ (F_s:- Sampling Frequency)

- Normally F_s for Telephone Communication is taken as 8kHz
- No of Sample/sec = F_s in Hz
  ∴ No of Sample/sec = 8000 Hz
  ADC = 8 bit

∴ Data Rate = 64 Kbps.

* Sampling :-

$x(t)$	Sampling Algo	$x(n)$
CT Signal (I/P)	Put $t = nTs$	DT Signal o/P

Q) $x(t) = 12\cos(20\pi t) + 6\sin(200\pi t)$

$Fs = 400Hz$

Find $x(n)$.

Ans $x(t)$:- Continuous Time Signal.

Now $x(t) = 12\cos(20\pi t) + 6\sin(200\pi t)$

Put $t = nTs = \frac{n}{Fs} = \frac{n}{400}$

$$x[nTs] = 12\cos\left(20\pi\left(\frac{n}{400}\right)\right) + 6\sin\left(200\pi \times \frac{n}{400}\right)$$

$$x[n] = 12\cos\left(\frac{\pi n}{20}\right) + 6\sin\left(\frac{\pi n}{2}\right)$$

$$\boxed{\therefore x[n] = 12\cos\left(\frac{\pi}{20}n\right) + 6\sin\left(\frac{\pi}{2}n\right)}$$

★ To check if this correct or not we need to check if ω is range of $(-\pi, \pi$

∵ Digital Frequency ω range is $(-\pi, \pi)$

⇒ If not in range make / convert it in range.

$$\left.\begin{array}{l} \therefore \omega_1 = \frac{\pi}{20} \\ \\ \omega_2 = \frac{\pi}{2} \end{array}\right\} \rightarrow \text{Both are in range of } -\pi \text{ to } \pi\text{, hence correct.}$$

FS = 8kHz

* Reconstruction :-

$x(n)$	Reconstruction Algo	$x(t)$
I/P	Put $n = tFs$	O/P
DT signal		continuous signal

eg :- $x(n) = 12\cos\left(\frac{\pi}{20} n\right) + 6\sin\left(\frac{\pi}{2} n\right)$

$Fs = 400\,Hz$

Calculate $x(t)$.

Ans Put $n = tFs$

$$\therefore x(tFs) = 12\cos\left(\frac{\pi}{20} \times t \times 400\right) + 6\sin\left(\frac{\pi}{2} \times t \times 400\right)$$

$$x(tFs) = 12\cos(20\pi t) + 6\sin(200\pi t)$$

$\because$ Fs is constant

$\Downarrow$

$$\boxed{x(t) = 12\cos(20\pi t) + 6\sin(200\pi t)}$$

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Sampling :-

Q) $x(t)\ 12\cos(20\pi t)+6\sin(200\pi t)$

$F_s = 80Hz$

calculate $x(n)$

Ans Put $t = nT_s = \frac{n}{F_s} = \frac{n}{80}$

$$\therefore x(nT_s) = 12\cos\left(20\pi \times \frac{n}{80}\right) + 6\sin\left(200\pi \cdot \frac{n}{80}\right)$$

$$= 12\cos\left(\frac{\pi n}{4}\right) + 6\sin\left(\frac{5\pi n}{2}\right)$$

Here ω is not in Range $(-\pi, \pi)$

$\therefore$ We convert ~~it~~ in the form. to make it in Range

$$\therefore \omega = \frac{5\pi}{2} = 2\pi - \frac{5\pi}{2}$$

$$= \frac{\pi}{2}.$$

$$\therefore x(nT_s) = 12\cos\left(\frac{\pi}{4}n\right) + 6\sin\left(\frac{\pi}{2}n\right)$$

$$\therefore x(n) = 12\cos\left(\frac{\pi}{4}n\right) + 6\sin\left(\frac{\pi}{2}n\right) //.$$

Reconstruction :-

$$x(n) = 12\cos\left(\frac{\pi}{4}n\right) + 6\sin\left(\frac{\pi}{2}n\right)$$

Find $x(T)$

$\therefore$ put $= n = tF_s$

$$\therefore x(tF_s) = 12\cos\left(\frac{\pi}{4} t \times 80\right) + 6\sin\left(\frac{\pi}{2} \times t \times 80\right)$$

$x(t) = 12\cos(20\pi t) + 6\sin(40\pi t)$.

Now Original $x(t) \neq$ Reconstructed $x(t)$.

* Sampling Theorem:-

⇒ According to Sampling theorem, inorder to reconstruct the Original continuous Time Signal, Sampling frequency Fs should be greater than twice of Maximum frequency Component present in the signal.

$\boxed{\text{i.e:- } F_s \geq 2F_{max}}$

Eg:- $x(t) = 12\cos(20\pi t) + 6\sin(200\pi t)$

$\omega_1 = 20\pi \qquad \omega_2 = 200\pi$

$2\pi f_1 = 20\pi \qquad 2\pi f_2 = 200\pi$

$f_1 = 10Hz \qquad f_2 = 100Hz$

$f_{max} = 100Hz$

$\therefore f_{max} \geq 2(100)$

$\boxed{f_{max} \geq 200Hz}$

* Classification of DT Signals:-

1. Deterministic / Non-Deterministic
2. Causal / Anti-Causal / Bothside
3. Periodic / Non-Periodic
4. Energy / Power / Neither Energy nor Power
5. Even / Odd / Neither even nor odd.

1) Deterministic / Non-Deterministic :-

⇒ The signals which can be represented mathematically are deterministic signals.

⇒ Eg:- Sigma, Delta, Cos ~~etc~~, Sin, etc

⇒ The signals which cannot be represented mathematically are non-deterministic signals.

Eg:- Biomedical signals, Noise signals etc.

2) Causal / Anti-causal / Bothside :-

Causal signal :-

⇒ $x[n] = 0$ for $n < 0$

$x[n]$ is causal signal.

Eg:- $x[n] = \{ \underset{\uparrow}{1}, 2, 3, 4 \}$

Anti-Causal Signal :-

⇒ $x[n] = 0$ for $n \geq 0$

$x[n]$ = Anti-causal Signal

Eg:- $x[n] = \{ 1, 2, 3, 4, \underset{\uparrow}{0}, 0, 0, 0 \}$

Bothside :-

Both -ve & +ve side are not zero.

⇒ $x[n] = \{ 1, 2, 3, 4, \underset{\uparrow}{5}, 6, 7, 8 \}$

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3] Periodic / Non-Periodic Signal :-

→ If Digital frequency of signal $x[n]$ is a rational number then $x[n]$ is periodic
otherwise it is Non-Periodic.

eg :- $x[n] = \cos(0.3\pi n)$

$\omega = 0.3\pi$

$2\pi f = 0.3\pi$

$f = \frac{0.3}{2}$

$f = \frac{3}{20}$ (Ratio of Integers / Ratio of Integers)

∴ Periodic

eg :- $x[n] = \cos(0.3n)$

$\omega = 0.3$

$2\pi f = 0.3$

$f = \frac{0.3}{2\pi}$ (Not a Integer / Integer ratio)

∴ This is Non-Periodic Signal.

IMP 4] Energy / Power / Neither Energy nor Power :-

#] Energy of signal is defined as :-

→ $\boxed{E = \sum_{n=-\infty}^{\infty} |x[n]|^2}$

[Mod is because if $x[n] = 2+3i$ then we take its mod :- $\sqrt{2^2+3^2} = \sqrt{13}$ → so only real one is generated]

⇒ While transmitting a signal, there should not be loss of information, energy loss can be there.

⇒ If Energy of $x[n]$ is finite then $x[n]$ is an energy signal.

Eg:- $x[n] = \{1, 2, 3, 4\}$

$$\text{Energy} = 1^2 + 2^2 + 3^2 + 4^2 = 30$$

Eg:- $x[n] = (1/2)^n u[n]$

$$\sum_{n=0}^{\infty} a^n = \begin{cases} \frac{1}{1-a} & \text{if } a < 1 \\ \infty & \text{otherwise} \end{cases}$$

Average Power of Signal is defined as:-

⇒ If Power of $x[n]$ is finite and non-zero, Then $x[n]$ is a Power Signal.

⇒ $$\boxed{P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2}$$

⇒ If it is an energy signal, then it is not a Power Signal.

* NOTE:-

→ Delta Signal:- Energy Signal
→ Unit U[n] :- Power Signal
→ Ramp R[n] :- Neither Energy nor Power Signal

5) Even / Odd / Neither even nor Odd :-

\# Even Signal :-

⇒ If $x(n) = x(-n)$

\# Odd Signal :-

⇒ If $x[n] = -x[-n]$

eg:- $x(n) = \{1, 2, \underset{\uparrow}{3}, -2, -1\}$

$x(-n) = \{-1, -2, \underset{\uparrow}{3}, 2, 1\}$

This not odd signal :- ∵ ~~3≠-3~~

For $x(n)$ the value of $x(-n)$ should be negative of $x(n)$ value.

⇒ $x(n) = \{1, 2, \underset{\uparrow}{0}, -2, -1\}$

$x(-n) = \{-1, -2, 0, 2, 1\}$

yess, Odd signal since $-0 = 0$

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Chapter 2 :- Discrete Fourier Transform (DTFT)

1) DTFT of DT signal $x(n)$ is defined as:-

$$\boxed{X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jn\omega}}$$

Here it is plotted against frequency.

2) Inverse ~~DTFT~~ DTFT of $X(\omega)$ is defined as:-

$$\boxed{x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega}$$

Eg:- $x(n) = \{1, 2, 3, 4\}$

Find $X(\omega)$

Ans To find $X(\omega)$ i.e.:- DTFT

$$\boxed{X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jn\omega}}$$ ($j \Rightarrow$ imaginary part)

$$X(\omega) = \sum_{n=0}^{n=3} x(n)\, e^{-jn\omega}$$

$$= x(0) = x(0)\, e^{-j0\omega} + x(1)\, e^{-j1\omega} + x(2)\, e^{-j2\omega} + x(3)\, e^{-j3\omega}$$

$$= 1(1) + 2e^{-j\omega} + 3e^{-2j\omega} + 4e^{-3j\omega}$$

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DTFT :-

⇒ Not practical for (real-time) computation on a digital computer

⇒ Solution :- Limit the extent of the summation to N points & evaluate the continuous function of frequency at N ~~equidist~~ equispaced points.

Derivation of DFT Equation :-

By DTFT :-

$$\Rightarrow X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega}$$

for :-
Range of $\omega = [-\pi, \pi]$ $(-\pi < \omega \leq \pi)$

Put $\omega = \frac{2\pi k}{N}$

$$X\left[\frac{2\pi k}{N}\right] = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-jn\left(\frac{2\pi k}{N}\right)}$$

Note :- In frequency domain we need to change ω.

$(2\pi/N$:- Constant)

$$\therefore X(k) = \sum x(n) e^{-j\frac{2\pi}{N}(nk)}$$

Put $\boxed{W_N = e^{-j\frac{2\pi}{N}}}$

$$\boxed{\therefore X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}}$$

(W_N :- ~~Tridale~~ Twiddle Factor)

eg :- $x(n) = \{1, 2, 3, 4\}$
Find $X[k]$

Ans To find $X(k)$:-

By ~~DTFT~~ DFT :-   (DFT :- Gives Finite Values
DTFT :- Gives ∞ Values)

$\Rightarrow$ $X[k] = \sum_{n=0}^{N-1} x(n) W_N^{nk}$

where $N = 4$ (Length of $x(n)$)

$\Rightarrow$ $\boxed{W_N^1 = e^{-j\frac{2\pi}{N}}}$

$X(k) = \sum_{n=0}^{3} x(n) W_N^{nk}$

$X(k) = x(0) W_N^0 + x(1) W_N^k + x(2) W_N^{2k} + x(3) W_N^{3k}$

$= W_N^0 + 2W_N^k + 3W_N^{2k} + 4W_N^{3k}$

$= 1 + 2W_N^k + 3W_N^{2k} + 4W_N^{3k}$

$k=0$:- $X(0) = 1+2+3+4 = 10$

$k=1$:- $X(1) = 1 + 2W_N^1 + 3W_N^2 + 4W_N^3$

$k=2$:- $X(2) = 1 + 2W_N^2 + 3W_N^4 + 4W_N^6$

$k=3$:- $X(3) = 1 + 2W_N^3 + 3W_N^6 + 4W_N^9$

$W_N^1 = +e^{-j\frac{2\pi}{N}} = +e^{-j\frac{2\pi}{4}} = e^{-j\pi/2}$

$e^{-j\pi/2} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)$

$= 0 - j(1)$

$W_N^1 = -j$

$\therefore W_N^2 = W_N^1 W_N^1$

$= (-j)(-j) = j^2 = -1$ $(i^2 = -1)$

$$W_N^3 = (-j)(-j)(-j)$$
$$= (j^2)(-j)$$
$$= (-1)(-j)$$
$$= j$$

$\cancel{W_N^4 =}$

$$\therefore \; x(1) = 1 + 2(-j) + 3(-1) + 4(j)$$
$$= 1 - 2j - 3 + 4j$$
$$= -2 + 2j$$

$$x(2) = 1 + 2W_N^2 + 3W_N^4 + 4W_N^6$$
$$= 1 + 2(-1) + 3(1) + 4(-1)$$
$$= 1 - 2 + 3 - 4$$
$$= -2$$

$$x(3) = 1 + 2W_N^3 + 3W_N^6 + 4W_N^9$$
$$= 1 + 2(j) + 3(-1) + 4(j^3)$$
$$= 1 + 2j - 3 - 4j$$
$$= \cancel{-2+6j} \quad -2 - 2j$$

Ans:- $$x[k] = \begin{bmatrix} 10 & k=0 \\ -2+2j & k=1 \\ -2 & k=2 \\ -2-2j & k=3 \end{bmatrix}$$