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Module 1
Matrices

A set of mn numbers real or complex arranged in a rectangular array of m rows & n columns like

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ - & - & - & - \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

each called a matrix of order $m \times n$ & it is read as m by n.

The elements $a_{11}, a_{12} \ldots a_{mn}$ constituting $m \times n$ matrix are called as elements or constituents.

Matrix is usually denoted by capital letters from A, B, C & it is shortly denoted by a_{ij} where a_{ij} is the element in ith row & jth column.

$\therefore$ we get $A = [a_{ij}]_{m \times n}$

where $i = 1, 2 \ldots m$ & $j = 1, 2 \ldots n$

Types of Matrices:-

1] Square matrix:- A matrix of order $m \times n$ is called as a square matrix iff $m = n$ i.e no. of row is same as the no. of columns.

eg:- $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}_{2 \times 2}$

2] Row Matrix:- A matrix which contains only 1 row is called as row matrix

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3] Column Matrix:- A matrix having only 1 column is called as a column matrix.

4] Diagonal Matrix:- A square matrix in which all non-diagonal elements are zero.

eg:- $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5] Scalar Matrix:- It is a diagonal matrix in which all diagonal elements are same.

eg:- $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

6] Unit matrix:- It is a scalar matrix with diagonal ~~matrix~~ elements as 1.

eg:- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7] Upper Triangular Matrix:- It is a square matrix in which all the elements below the principle diagonal are zero.

$A = \begin{bmatrix} 2 & 1 & 7 \\ 0 & -7 & 5 \\ 0 & 0 & 7 \end{bmatrix}$

8] Lower Δ Matrix:- It is a square matrix in which all the elements above the principle diagonal are zero.

eg:- $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -7 & 0 \\ 4 & 5 & 7 \end{bmatrix}$

9] Null matrix:- A square matrix having every element 0 is called as null matrix. It is denoted by O

10] Transpose of a matrix:- A matrix obtained from a given matrix by changing its rows to column & vice-versa.

eg:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $A^T = A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Idempotent Matrix:- A square matrix such that $A^2 = A$ is called as Idempotent matrix:-

Involuntary Matrix:- A □ matrix such that $A^2 = I$ is called as involuntary matrix where I is unit matrix.

Nilpotent matrix:- A □ matrix is called as a Nilpotent matrix if there exists a +ve integer x such that $A^x = 0$, where x is known as index of the Nilpotent matrix.

Hermitian Matrix:- A □ matrix is said to be hermitian in $A^\theta = A$, θ is complex conjugate

Skew Hermitian Matrix:- A □ matrix is said to be skew hermitian in $A^\theta = -A$ & $A = -A^\theta$

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Rectangular Matrix:- $m \neq n$

A matrix is said to be skew-symmetric

A matrix is said to be non-singular if & only if $|A| \neq 0$.
if $|A| = 0$ singular

A matrix is said to be orthogonal matrix iff $AA' = A'A = I$

Minor of an element:-

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\text{Minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

The minor of an element of $|A|$ is a determinant obtained from $|A|$ row & column containing that element.

$$\therefore \text{Minor of } a_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

Cofactor of an element:-

$$\text{Cofactor of } a_{11} = (-1)^{i+j} \cdot \text{Minor of } a_{11}$$
$$= 1 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

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Cofactor of $a_{32} = (-1)^{3+2}$
$= -1 \cdot \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

If we multiply the minor of an element in the ith row & jth col. of the determinant by $(-1)^{i+j}$, the product is called as cofactor of an element.

Adjoint of a Matrix:-

$Adj(A) = [\text{matrix of cofactors}]^T$

Let A be a square matrix of order n. Adjoint of matrix is denoted by $adj(A)$ & is defined by transpose matrix of its cofactor.

Row Echelon form:-

Q) Convert the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & 8 \end{bmatrix}$ into row equivalent form:-

→ Given $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & 8 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

Interchange Row 1 & Row 2 because the first non-zero entry in first column is in row 2

$\therefore A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 2 & 7 & 8 \end{bmatrix}$

$R_3 \rightarrow R_3 - 2R_1$

$\therefore A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix}$

$R_2 \rightarrow R_3 - 3R_2$

$\therefore A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

This is the required row Echelon form for the given matrix

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Q) Reduce the following matrix to row echelon form & find its rank:-

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$\to$ $R_3 \to R_3 - R_1$
$R_2 \to R_2 + 2R_1$

$$\therefore A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$\therefore R_2 \to R_2 - 3R_4$
$\therefore R_3 \to R_3 + 2R_4$

$$\therefore A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

By $R_2 \leftrightarrow R_4$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is the required row ~~equivalent~~ Echelon form

Here the no. of non-zero rows is 2 $\therefore$
Rank = 2

Q) Reduce the following matrix to row echelon form

$$A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$$

→ $R_2 \to R_2 - R_1,\ R_3 \to R_3 - 2R_1,\ R_4 \to R_4 - 3R_1$

$$\therefore A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & -2 & -2 & 2 \end{bmatrix}$$

By $R_3 \to R_3 - 2R_2,\ R_4 \to R_4 - 2R_2$

$$\therefore A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 3

Q] Reduce the following matrix in row echelon form:-

$$\to A = \begin{bmatrix} 0 & 2 & -6 & -2 \\ 0 & 2 & 6 & 10 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_4$

$$A = \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 2 & 6 & 10 \\ 3 & 1 & 0 & 2 \\ 0 & 2 & -6 & -2 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_2$

$$\therefore A = \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 2 & 6 & 10 \\ 0 & & & \\ 0 & & & \end{bmatrix}$$

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6/02/23 Module 2:- Soln of System of linear eqns

Gauss Elimination method:-

① In this method 1st we express the system of eqn as AX = B

② Find the Augmented matrix for the given system.

③ Find the transform of the augmented matrix & convert it into row equivalent form.

④ Find eqns corresponding to an upper Δ matrix

⑤ Using back substitution method find soln for the given system of eqn.

Q] Apply gauss eliminate method to solve the following system of eqn:-

$2x - 2y + 3z = 2$

$x + 2y - z = 3$

$3x - y + 2z = 1$

→ Augmented matrix of the above eqns are

$$[A|B] = [A:B] = \left[\begin{array}{ccc|c} 2 & -2 & 3 & 2 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \end{array}\right]$$

Transforming augmented matrix to row Echelon form

$R_1 \leftrightarrow R_2$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -2 & 3 & 2 \\ 3 & -1 & 2 & 1 \end{array}\right]$$

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$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & -7 & 5 & -8 \end{array}\right]$$

By $R_3 \to R_3 - R_2$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & -1 & 0 & -4 \end{array}\right]$$

~~Using~~ $\therefore x + 2y - z = 3$ — ①

$\therefore -6y + 5z = -4$ — ②

$\therefore -y = -4$ — ③

$\therefore$ Using $y = 4$ in ②

$z = 4$

$\therefore x = -1$

Q] Apply gauss eliminate method to solve the following system of eqn:-

$x + 2y + 3z - t = 10$

$2x + 3y - 3z - t = 1$

$3x + 2y - 4z + 3t = 2$

$2x - y + 2z + 3t = 7$

→ We 1st write the eqn in matrix form & then reduce the coefficient matrix to upper △ matrix by elementry row transformate

~~[A:B] =~~

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 3 & -3 & -1 \\ 3 & 2 & -4 & 3 \\ 2 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 2 \\ 7 \end{bmatrix}$$

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By $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -1 & -9 & 1 \\ 0 & -4 & -13 & 6 \\ 0 & -5 & -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 10 \\ -19 \\ -28 \\ -13 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$, $R_4 \to R_4 - 5R_2$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -1 & -9 & 1 \\ 0 & 0 & 23 & 2 \\ 0 & 0 & 41 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 10 \\ -19 \\ 48 \\ 82 \end{bmatrix}$$

$\therefore x + 2y + 3z - t = 10$ —— ①

$-y - 9z + t = -19$ —— ②

$23z + 2t = 48$ —— ③

$\therefore 41z = 82$

$\therefore z = 2$

$\therefore$ from ③

$2t = 2 \quad \therefore t = 1$

$\therefore y = 2$ — from ②

$\therefore x = 10 - 4 - 6 + 1 = 1$

This is the reqd soln of the given system.

Q] Apply gauss eliminate method to solve the following system of eqns :-

$x + 2y + 3z = 14$

$3x + 3y + 4z = 21$

$4x + 5y + 7z = 35$

$\to$

$$\begin{bmatrix}1&2&3\\3&8&4\\4&5&7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}14\\21\\35\end{bmatrix}$$

$R_2 \to R_2 - 3R_1 \,,\ R_3 \to R_3 - 4R_1$

$$\begin{bmatrix}1&2&3\\0&-3&-5\\0&-3&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}14\\-21\\-21\end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix}1&2&3\\0&-3&-5\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}14\\-21\\0\end{bmatrix}$$

$\therefore\ x+2y+3z=14$ —— ①

$-3y-5z=-21$ —— ②

Put $z=t$

$\therefore\ -3y-5t=-21$

$\therefore\ y=\left(\dfrac{21-5t}{3}\right)$

$x+\dfrac{42-10t}{3}+3t=14$

$\therefore\ 3x=\dfrac{42-42+10t-9t}{3}$

$=\dfrac{t}{3}$

Q] $x+2y+3z+4w=30$

$2x+3y+6z+5w=46$

$3x+4y+8z-6w=9$

$4x-y-z+2w=7$

P.T.O

→

Gauss Jacobi Method:-

Consider the following system of eqns

$$\left.\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3 \end{aligned}\right\} \text{— (1)}$$

$AX=B \Rightarrow$ General eqn of linear system of eqn

Where a_1, b_2 & c_3 are large as compared to the remaining coefficients
we rewrite the equations from (1)

$$\left.\begin{aligned} x&=\frac{1}{a_1}\left[d_1-b_1y-c_1z\right]\\ y&=\frac{1}{b_2}\left[d_2-a_2x-c_2z\right] \text{ \&}\\ z&=\frac{1}{c_3}\left[d_3-a_3x-b_3y\right] \end{aligned}\right\} \text{— (2)}$$

Procedure:- Now we start with the assumption that the roots of the eqns are
$x=x_0$, $y=y_0$ & $z=z_0$

Putting these values in eqn (2)
The first approximate is

$$x_1=\frac{1}{a_1}\left[d_1-b_1y_0-c_1z_0\right]$$

$$y_1=\frac{1}{b_2}\left[d_2-a_2x_0-c_2z_0\right],\quad z_1=\frac{1}{c_3}\left[d_3-a_3x_0-b_3y_0\right]$$

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Also we start with the assumptn that the roots of the eqns are
$x = x_1, y = y_1, z = z_1$

putting these values in eqn (2) we get a better approximate

$$x_2 = \frac{1}{a_1}\left[d_1 - b_1 y_1 - c_1 z_1\right]$$

$$y_2 = \frac{1}{b_2}\left[d_2 - a_2 x_1 - c_2 z_1\right]$$

$$z_2 = \frac{1}{c_3}\left[d_3 - a_3 x_1 - b_3 y_1\right]$$

Now iterate (repeat) the procedure as many times as we want till we arrive at design accuracy.

Q] Apply gauss Jacobi Method to solve the following system of eqns.

$15x + 2y + z = 18$
$2x + 20y - 3z = 19$ upto 6th iterate
$3x - 6y + 25z = 22$

$$\left.\begin{aligned} x &= \frac{1}{15}\left[18 - 2y - z\right] \\ y &= \frac{1}{20}\left[19 - 2x + 3z\right] \\ z &= \frac{1}{25}\left[22 - 3x + 6y\right] \end{aligned}\right\} \longrightarrow (1)$$

1st ⇒ $x = 0, y = 0, z = 0$ compulsory

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$x_1 = \frac{1}{15}\left[18 - 2(0) - 0\right]$

$x_1 = \frac{18}{15} = 1.2$

$y_1 = \frac{1}{20}\left[19 - 20(0) + 3(0)\right] = 0.95$

$z_1 = \frac{1}{25}\left[22 - 3(0) + 6(0)\right] = 0.88$

2^{nd} $x_1 = 1.2$, $y_1 = 0.95$, $z_1 = 0.88$

$\therefore x_2 = \frac{1}{15}\left[18 - 2(0.95) - 0.88\right]$

$\therefore x_2 = 1.0147$

$\therefore y_2 = \frac{1}{20}\left[19 - 20(1.2) + 3(0.88)\right]$

$= 0.962$

$z_2 = 0.964$

3^{rd} iterate

$x_3 = 1.0075$, $y_3 = 0.9931$, $z_3 = 0.9891$

4^{th} "

$x_4 = 1.0016$, $y_4 = 0.9976$, $z_4 = 0.9974$

5^{th} "

$x_5 = 1.00025$, $y_5 = 0.9994$, $z_5 = 0.9980$

6^{th} ".

$x_6 = 1.0002$, $y_6 = 0.996$, $z_6 = 0.9998$

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6] $4x+y+3z=17$
$x+5y+z=14$ upto 5 iterates
$2x-y+8z=12$

$\rightarrow x=\frac{1}{4}[17-y-3z]$

$y=\frac{1}{5}[14-x-z]$

$z=\frac{1}{8}[12-2x+y]$

1st put $x=0, y=0, z=0$

$\therefore x_1=\frac{1}{4}[17]=4.25$

$\therefore y_1=2.8, z_1=1.5$

2nd put $x_1=4.25, y_1=2.8, z_1=1.5$

$x_2=2.425, y_2=1.65, z_2=0.7875$

3rd $x_3=3.2468, y_3=2.1575, z_3=1.1$

4th $x_3=2.8856$

Reduced Row Echelon form:-

Q] Convert the following matrix into reduced (Identity) row echelon form:- where

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 9 & 2 & 3 \end{bmatrix}$$

→ By $R_1 \to \frac{R_1}{2}$, $R_2 \to R_2 - \frac{9}{2} R_1$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & -15 \end{bmatrix}$$

By $R_2 \to \frac{R_2}{2}$ $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -15/2 \end{bmatrix}$

This is the reqd reduced row echelon form.

Q] $A = \begin{bmatrix} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{bmatrix}$

→ $R_2 \to R_2 + \frac{3}{2} R_1$, $R_3 \to R_3 + R_1$

$$A = \begin{bmatrix} 2 & 1 & -1 & 8 \\ 0 & 1/2 & 1/2 & 1 \\ 0 & 2 & 1 & 5 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$

$$A = \begin{bmatrix} 2 & 1 & -1 & 8 \\ 0 & 1/2 & 1/2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

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$R_2 \to R_2 + \frac{1}{2}R_3$, $R_2 \to R_2 + \cancel{3/2 R_3} - R_3$

$$A = \begin{bmatrix} 2 & 1 & 0 & 7 \\ 0 & 1/2 & 0 & 3/2 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$R_3 \to (-1)R_3$

$$A = \begin{bmatrix} 2 & 1 & 0 & 7 \\ 0 & 1/2 & 0 & 3/2 \\ 0 & 0 & +1 & -1 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$, $R_2 \to R_2 \times 2$

$$A = \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & +1 & -1 \end{bmatrix}$$

$R_1 \to R_1/2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

This is the reqd. reduced row echelon form

Q] $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 7 \end{bmatrix}$

$\to R_1 \to R_1 - 3R_2$

$$A = \begin{bmatrix} 1 & 0 & -22 \\ 0 & 1 & 7 \end{bmatrix}$$

Gauss Jordan Method:-

1] In this method we first express the system of equation as $Ax = B$
2] Find the augmented matrix for the given system.
3] Converts the augmented matrix into row echelon form.

Q] Apply Gauss Jordan Method to solve the following eqns $x + 3y = 7$
$3x + 4y = 11$

→ Augmented matrix for the given system is

$$[A|B] = [A:B] = \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 3 & 4 & 11 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$$[A:B] = \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & -5 & -10 \end{array}\right]$$

$R_2 \to \frac{R_2}{-5}$

$$[A:B] = \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & 1 & 2 \end{array}\right]$$

$R_1 \to R_1 - 3R_2$

$$[A:B] = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$ This is the red. row echelon form.

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$\therefore x=1$ & $y=2$

Q] $x+y+z=5$
$2x+3y+5z=8$
$4x+0y+5z=2$

→ Augmented matrix is

$$[A|B]=\left[\begin{array}{ccc|c}1&1&1&5\\2&3&5&8\\4&0&5&2\end{array}\right]$$

$R_2\to R_2-2R_1,\ R_3\to R_3-4R_1$

$$[A|B]=\left[\begin{array}{ccc|c}1&1&1&5\\0&1&3&-2\\0&-4&1&-18\end{array}\right]$$

$R_1\to R_1-R_2,\ R_3\to R_3+4R_2$

$$[A|B]=\left[\begin{array}{ccc|c}1&0&-2&7\\0&1&3&-2\\0&0&13&-26\end{array}\right]$$

$R_3\to R_3/13$

$$[A|B]=\left[\begin{array}{ccc|c}1&0&-2&7\\0&1&3&-2\\0&0&1&-2\end{array}\right]$$

$R_1\to R_1+2R_3,\ R_2\to R_2-3R_3$

$$[A|B]=\left[\begin{array}{ccc|c}1&0&0&3\\0&1&0&4\\0&0&1&-2\end{array}\right]$$

$x=3,\ y=4,\ z=-2$

9] $x + 2y = 4$
$x - 2y = 6$

→ Augmented Matrix

$$[A|B] = \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & -2 & 6 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$[A|B] = \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -4 & 2 \end{array}\right]$$

$R_2 \to R_2 / -4$

$$[A|B] = \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & -1/2 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$

$$[A|B] = \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -1/2 \end{array}\right]$$

$x = 5,\ y = -1/2$

Gauss Seidel Method:-

This is the modificate of Gauss Jacobi Method.
Consider the system of eqns are

$$\left.\begin{aligned} a_1x+b_1y+c_1z &= d_1 \\ a_2x+b_2y+c_2z &= d_2 \\ a_3x+b_3y+c_3z &= d_3 \end{aligned}\right\} \text{---} (1)$$

we write these eqns as

$$\left.\begin{aligned} \therefore x &= \frac{1}{a_1}[d_1-b_1y-c_1z] \\ \therefore y &= \frac{1}{b_2}[d_2-\cancel{b_2y}\,a_2y-c_2z] \\ \therefore z &= \frac{1}{c_3}[d_3-a_3x-b_3y] \end{aligned}\right\} \text{---} (2)$$

We start with initial approximate x_0, y_0, z_0 for x, y, z resp.
Then substituting $y=y_0$ & $z=z_0$ in the first of eqn (2)

$$\therefore x_1 = \frac{1}{a_1}[d_1-b_1y_0-c_1z_0]$$

Then for getting y_1 substituting $x=x_1$, $z=z_0$ in the second of eqn (2)

$$\therefore y_1 = \frac{1}{b_2}[d_2-a_2x_1-c_2z_0].$$

Then for getting z_1 substitute $x=x_1$ & $y=y_1$ in the 3rd of eqn (2)

$$\therefore z_1 = \frac{1}{c_3}[d_3-a_3x_1-b_3y_1]$$ & so on

i.e as soon as the new approximate of an unknown is found it is immediately

used in the next step.
This process is repeated till the values of x, y, z are obtained till desired degree of accuracy.

Q] Apply Gauss Seidel method to solve the following eqns.

$5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$ → leading diagonal dominant form

→ $x = \frac{1}{5}[12 - 2y - z]$

$y = \frac{1}{4}[15 - x - 2z]$

$z = \frac{1}{5}[20 - x - 2y]$

for 1^{st} approximate
for x_1 put $y_0 = z_0 = 0$

$x_1 = 2.4$
for y_1, put $x = x_1$ & $z = z_0$
∴ $y_1 = 3.15$

for z_1, put $x = x_1$ & $y = y_1$
∴ $z_1 = 2.26$

for 2^{nd} iterate
$x_2 = 0.688$ $z_2 = 2.8832$

$y_2 = 2.448$

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for 3rd iterate

$x_3 = \cancel{0.90032}\ 0.84416 \quad z_3 = 2.9922$
$y_3 = 2.09736$

for 4th iterate

$x_4 = 0.962616, \ y_4 = 2.018246, \ z_4 = 3.00217$

for 5th iterate

$x_5 = 0.9942676, \ y_5 = 2.0003481$
$z_5 = 3.001007$

for 6th iterate

$x_6 = 0.99966, \ y_6 = 1.99958$
$z_6 = 3.001748$

$\therefore$ after 6th iterate the values are
$x = 1, \ y = 2 \ \& \ z = 3$

Q] In terms of x_1, x_2, x_3, x_4,
$13x_1 + 5x_2 - 3x_3 + x_4 = 18$
$2x_1 + 12x_2 + x_3 - 4x_4 = 13$
$3x_1 - 4x_2 + 10x_3 + x_4 = 29$
$2x_1 + x_2 - 2x_3 + 9x_4 = 31$

$\rightarrow x_1 = \frac{1}{13}\left[18 - 5x_2 + 3x_3 - x_4\right]$

$x_2 = \frac{1}{12}\left[13 - 2x_1 - x_3 + 4x_4\right]$

$x_3 = \frac{1}{10}\left[29 - 3x_1 + 4x_2 - x_4\right]$

$$x_4 = \frac{1}{9}\left[31 - 2x_1 - x_2 + 3x_3\right]$$

1st ~~iter~~

	x_1	x_2	x_3	x_4
1st	1.3846	0.85256	2.8256	3.98389
2nd	1.4023	1.94211	2.8577	3.86959
3rd	0.999	1.4686	3.000781	4.00397
4th	1.0119	1.99927	2.99609	~~3.33011~~ 3.9961
5th	0.99967	1.9991	3.00012	4.00021
6th	1.0003	~~2.15005~~ 2	3	4

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Module 5:
Eigen value & Eigen vector

Characteristic Matrix:-

For a given matrix A,
$A-\lambda I$ is called as a characteristic Matrix.
where λ is a scalar & I is the unit matrix.
for ex:-

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\therefore A-\lambda I = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A-\lambda I = \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

Characteristic polynomial:-

The determinant $|A-\lambda I|$ when expended will give a polynomial which we called as characteristic polynomial.

Repeat the previous example

$\therefore \cancel{|A-\lambda I| = (2-\lambda)(}$

Take $|A-\lambda I|$

$\therefore$ after simplificate we get

$|A-\lambda I| = \lambda^3 - 7\lambda^2 + 11\lambda - 5$

Characteristic equation:-

The $|A-\lambda I| =$ zero is called characteristic eqn

$$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

Characteristic root or Eigen value:-

~~Charact~~ Notes:-

1] If λ is a eigen value of a matrix A & X is corresponding eigen vector then $AX = \lambda X$

Properties of eigen value or eigen vector

or

Certain relations between eigen value & eigen vector.

1] Show that the eigen values of a Hermition matrix are all real.

→ Given that A is Hermition matrix.

i.e $A^\theta = A$

WKT if λ is eigen value & X is corresponding eigen vector

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$\therefore AX = \lambda X$ ——— ①

Taking complex conjugate to eqⁿ ①

$\therefore (AX)^{\theta} = (\lambda X)^{\theta}$

$\therefore X^{\theta} A^{\theta} = \bar{\lambda} X^{\theta}$

$\therefore$

But A is Hermitian

$\therefore X^{\theta} A = \bar{\lambda} X^{\theta}$.

Post multiply by X to both sides

$\therefore X^{\theta}(AX) = \bar{\lambda}(X^{\theta}X)$ ——— ③

$\therefore X^{\theta} \lambda X = \bar{\lambda}(X^{\theta}X)$ ——— from ①

$\therefore \lambda(X^{\theta}X) - \bar{\lambda}(X^{\theta}X) = 0$

$\therefore (\lambda - \bar{\lambda})(X^{\theta}X) = 0$

$\therefore \lambda - \bar{\lambda} = 0$

$\therefore \lambda = \bar{\lambda}$ ——— ③

Hence proved

$\because X^{\theta}X$ is not a zero vector

$\therefore$ from eqⁿ ② & ③

we get $\lambda = \bar{\lambda}$ showing that λ is real

This shows that the eigen values of Hermitian matrix are all real.

2] Show that the matrices A & $P^{-1}(AP)$ Have the same eigen values.

→ WKT if λ is the eigen value of a matrix A & X is the corresponding eigen vector then

$AX = \lambda X$

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Premultiply by P^{-1} both sides

$P^{-1}(AX) = P^{-1}(\lambda X)$

$\therefore P^{-1}AP(P^{-1}X) = \cancel{P}\lambda(P^{-1}X)$

$\therefore (P^{-1}AP)(P^{-1}X) = \lambda(P^{-1}X)$

$\therefore [(P^{-1}AP) - \lambda]P^{-1}X = 0$

$\therefore P^{-1}X$ is not a zero term as X is not zero vector

$\therefore P^{-1}AP - \lambda = 0$

$\therefore P^{-1}AP = \lambda$

This shows that λ is an eigen value of $P^{-1}AP$. Hence in general the two matrices A & $P^{-1}AP$ have the same eigen values.

Q] Show that the eigen value of an unitary matrix are of unit modulus (have absolute value 1).

→ given that the matrix A is unitary matrix

i.e $A^{\theta}A = AA^{\theta} = I$ —— (*)

we know that if λ is an eigen value of matrix A & X is the corresponding eigen vector then $AX = \lambda X$ —— (1)

Taking complex conjugate to both sides of eq^n (1)

$\therefore (AX)^{\theta} = (\lambda X)^{\theta}$

$\therefore X^{\theta}A^{\theta} = \bar{\lambda}X^{\theta}$ —— (2)

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$\therefore$ from ① & ② by multiplicate

$$(X^\theta A^\theta)(AX) = \bar{\lambda}\lambda(X^\theta X)$$

$$\therefore X^\theta(A^\theta A)X = \lambda\bar{\lambda}(X^\theta X)$$

$$\therefore X^\theta I X = \lambda\bar{\lambda}(X^\theta X) \text{ — from eq}^n \text{ ⊛}$$

$$\therefore X^\theta X = \lambda\bar{\lambda}(X^\theta X)$$

$$\therefore (1-\lambda\bar{\lambda})(X^\theta X) = 0$$

$$X^\theta X \neq 0$$

$$\therefore \lambda\bar{\lambda} = 1$$

$$\therefore |\lambda| = 1$$

Diagonal & Transferring Matrix:-

a] Find Eigen value & Eigen vector of Matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

b] Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is a diagonalisable. ~~matrix~~ Also find the the diagonal matrix & transforming matrix.

Algebraic

+ By using characteristic eq^n

i.e $|A-\lambda I| = 0$

$$\therefore \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

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$\therefore \begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ -3 & -4 & 1-\lambda \end{vmatrix} = 0$

$\therefore (8-\lambda)(-(3+\lambda)(1-\lambda)-8)$
$+8(4-4\lambda-6)-2(-16-9-3\lambda) = 0$

$\therefore -(8-\lambda)(3-3\lambda-\lambda+\lambda^2+8)+8(-2-4\lambda)$
$-2(-25-3\lambda) = 0$

$\therefore -(8-\lambda)(11-4\lambda+\lambda^2)-16-32\lambda$
$+50+6\lambda = 0$
$\therefore -88+32\lambda-8\lambda^2+11\lambda-4\lambda^2+\lambda^3$
$-16-32\lambda+50+6\lambda = 0$

$\therefore \lambda^3-12\lambda^2+17\lambda-54 = 0$

$\therefore \lambda = 1, 2, 3$

let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $[A-I\lambda]X = 0$

$\therefore \begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

put $\lambda = 1$

$\therefore \begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ —— (2)

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Thus $\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$ is the first eigen vector

corresponding to $\lambda = 1$

Also put $\lambda = 2$ in (2)

Thus $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ is the second eigen vector

corresponding to $\lambda = 2$

put $\lambda = 3$ in (2)

Thus $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ is the third eigen vector

Here all eigen values are diff. & algebric & geometric matrices are same i.e A is diagonalisable.

$\therefore M^{-1}AM = D$ the given matrix A will be diagonalised to the diagonal matrix

$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ by the transforming matrix $M = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

Cayley-Hamiltons Theorem or CH theorem:-

Statement:- Every $\square$ matrix satisfies its characteristic eqn is called as Cayley's Hamilton's theorem.

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Q) verify the CH theorem for the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$ Also find A^{-1}

→ Using characteristic eqn

$|A - \lambda I| = 0$

i.e $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$

$\therefore \begin{vmatrix} 1-\lambda & 0 & 2 \\ 1 & 1-\lambda & 2 \\ 2 & 1 & -1-\lambda \end{vmatrix} = 0$

$\therefore \cancel{(1-\lambda)(1-\lambda+1+\lambda) + 2(1-2+2\lambda) = 0}$

$\therefore \cancel{(1-\lambda)(-(1)^2}$

$\therefore (1-\lambda)\left[-(1^2-\lambda^2) - 2\right] + 2(1-2+2\lambda) = 0$

$\therefore (1-\lambda)(+\lambda^2-3) + (-2) + 4\lambda = 0$

$\therefore \lambda^2 - \lambda^3 - 3 + 3\lambda - 2 + 4\lambda = 0$

$\cancel{\therefore -\lambda^3 + 8\lambda - 5 = 0}$

$\therefore -\lambda^3 + \lambda^2 + 7\lambda - 5 = 0$

$\therefore \lambda^3 - \lambda^2 - 7\lambda + 5 = 0$ —— (1)

But as per CH theorem which states that every □ matrix satisfies its own characteristic eqn

i.e replace λ by A in (1)

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Here we have to show that

$$A^3 - A^2 - 7A - 5 = 0 \quad \text{---} \quad (*)$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 2 \\ 1 & 0 & 7 \end{bmatrix}$$

$$A^3 = A^2 A$$

$$= \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 2 \\ 1 & 0 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 & 14 \\ 13 & 5 & 16 \\ 15 & 7 & -5 \end{bmatrix}$$

$$\therefore LHS = \begin{bmatrix} 7 & 2 & 14 \\ 13 & 5 & 16 \\ 15 & 7 & -5 \end{bmatrix} - \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 2 \\ 1 & 0 & 7 \end{bmatrix} - 7 \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$+ 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore 5 \times 1$ so I as $I = 1$

$$\therefore LHS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore LHS = 0 = RHS$

$\therefore$ A satisfies its characteristic eqn

hence CH theorem is verified.

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~~Pre~~ multiply by A^{-1} in eq^n (*)

$\therefore A^{-1}(A^3 - A^2 - 7A + 5) = A^{-1} \cdot O$

$\therefore A^2 - A - 7 + 5A^{-1} = O$

$\therefore 5A^{-1} = A + 7I - A^2$

$$\therefore A^{-1} = \frac{1}{5}\left(\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 2 \\ 1 & 0 & 7 \end{bmatrix}\right)$$

$$= \frac{1}{5}\begin{bmatrix} 3 & -2 & 2 \\ -5 & 5 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3/5 & -2/5 & 2/5 \\ -1 & 1 & 0 \\ 1/5 & 1/5 & -1 \end{bmatrix}$$

6] Find the characteristic eq^n of matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ & verify that ~~is~~ it is satisfied by A & obtain A^{-1} & A^4.

→ By using characteristic eq^n
$|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$\therefore (2-\lambda)((2-\lambda)^2 - 1) + 1(-2+\lambda+1) + 1(1-2+\lambda) = 0$

$\therefore (2-\lambda)^3 - 2 + \lambda + \lambda - 1 + \lambda - 1 = 0$

$\therefore 8 - 12\lambda + 6\lambda^2 - \lambda^3 - 2 + 3\lambda - 2 = 0$

$\therefore -\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$

$\therefore \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

By CH theorem

Put $\lambda = A$

$\therefore A^3 - 6A^2 + 9A - 4 = 0$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 9A - 4 = 0$

$$\therefore LHS = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix}$$

$$+ \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

LHS = 0 = RHS

$\therefore$ CH theorem verified

$$\therefore A^4 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$= \begin{bmatrix} 86 & -85 & 85 \\ -85 & 86 & -85 \\ 85 & -85 & 86 \end{bmatrix}$$

$\therefore$ ~~A~~ for A^{-1}

Multiply throughout by A^{-1}

$\therefore$ $\cancel{A^2 - A =}$ $A^2 - 6A + 9 - 4A^{-1} = 0$

$$\therefore A^{-1} = \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \right)$$

$$= \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

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Q] Find A^{-1} of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ using CH theorem

functn of a square matrix is denoted by $\phi(A)$
∴ the matrix is of ~~other~~ order 2×2
∴ we take
$\phi(A) = \alpha_1 A + \alpha_0 I$
where α_0 & α_1 are arbitary constants
∴ the matrix is of order 3×3
then we take the function of square matrix as
$\phi(A) = \alpha_2 A^2 + \alpha_1 A + \alpha_0 I$
where α_0, α_1 & α_2 are the arbitary constants

Q if $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ then prove that $A^{50} = \begin{bmatrix} -149 & -150 \\ 150 & 151 \end{bmatrix}$

→ By using characteristic eqⁿ
i.e $|A - \lambda I| = 0$

∴ $\begin{vmatrix} 2-\lambda & 3 \\ -3 & -4-\lambda \end{vmatrix} = 0$

∴ $-(2-\lambda)(4+\lambda) + 9 = 0$

∴ $-8 - 2\lambda + 4\lambda + \lambda^2 + 9 = 0$

∴ $\lambda^2 + 2\lambda + 1 = 0$

∴ $(\lambda+1)^2 = 0$

i.e $\lambda + 1 = 0$ & $\lambda + 1 = 0$

∴ $\lambda = -1$ & $\lambda = -1$

Here eigen values are repeated

∴ Let $\phi(A) = A^{50} = \alpha_1 A + \alpha_0 I$ —— (*)

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where α_0 & α_1 are arbitrary constant
we assume that the above equality is satisfied by characteristic root of A

i.e replace A by λ in eq^n (*)

$\therefore \lambda^{50} = \alpha_1 \lambda + \alpha_0 I$ — (I)

put $\lambda = -1$

$\therefore (-1)^{50} = \alpha_1(-1) + \alpha_0$

$\therefore 1 = -\alpha_1 + \alpha_0$ — (1)

Diff. eq^n (I) w.r.t to λ

$\therefore 50\lambda^{49} = \alpha_1 + 0$

$\therefore \alpha_1 = 50\lambda^{49}$

put $\lambda = -1$

$\therefore \alpha_1 = 50(-1)^{49}$

$\therefore \alpha_1 = -50$ — (2)

$\therefore$ from (1) & (2)

$1 = -(-50) + \alpha_0$

$\therefore \alpha_0 = -49$

then substituting the values of α_0 & α_1 in eq^n (*)

$$\therefore A^{50} = -50\begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix} - 49\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -149 & -150 \\ 150 & 151 \end{bmatrix}$$

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6] If $A = \begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix}$ find $\cos A$

→ By using characteristic eqⁿ
i.e $|A - \lambda I| = 0$

$\therefore \begin{vmatrix} \pi-\lambda & \pi/4 \\ 0 & \pi/2-\lambda \end{vmatrix} = 0$

$\therefore \lambda = \frac{\pi}{2}$ & $\lambda = \pi$ Here eigen values are distinct

$\therefore$ the functⁿ is $\phi(A) = \cos A = \alpha_1 A + \alpha_0 I$ — (1)
we assume that the above equality is satisfied by characteristic root of A
$\therefore$ replace A by λ in (1)
$\therefore \cos\lambda = \alpha_1 \lambda + \alpha_0$ — (2)
Put $\lambda = \pi/2$ in eq (2)

$\therefore \cos\pi/2 = \alpha_1 \frac{\pi}{2} + \alpha_0$

$\therefore 0 = \alpha_1 \frac{\pi}{2} + \alpha_0$ — (3)

put $\lambda = \pi$ in eq (2)

$\therefore -1 = \alpha_1 \pi + \alpha_0$ — (4)

$\therefore$ from (3) & (4)

$\therefore -1 = \alpha_1 \frac{\pi}{2}$

$\therefore \alpha_1 = -\frac{2}{\pi}$

$\therefore \alpha_0 = 1$

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$\therefore$ Substituting α_0 & α_1 in eqn (1)

$$\therefore \cos A = -\frac{2}{\pi}\begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -1/2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1/2 \\ 0 & 0 \end{bmatrix}$$

Q] $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ find A^{50}

$\to$ Using $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 1-\lambda & 0 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = 0$$

$\therefore (1-\lambda)(\lambda^2 - 1) = 0$

$\therefore \cancel{\lambda^2} - 1 \cancel{-\lambda^3} + \lambda = 0$

$\therefore -\lambda^3 + \lambda^2 + \lambda - 1 = 0$

$\therefore \lambda = 1, 1, -1$

$\therefore$ Here eigen values 2 eigen values are repeated & 3rd value is different

Let $\phi(A) = A^{50} = \alpha_2 A^2 + \alpha_1 A + \alpha_0 I$ —— (*)

We assume that the above equality is satisfied by characteristic root of A

$\therefore$ replace A by λ, in eqn (*)

P.T.O

How to find Eigen value & Eigen vector by elementary row transformation:-

Q] Find the eigen values & ~~eigen~~ bases for the eigen spaces(vector), of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ by row transformate

→ By using characteristic eqⁿ

$|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\therefore (6-\lambda)((3-\lambda)^2-1) + 2(-6+2\lambda+2) + 2(2-6+2\lambda) = 0$$

$$\therefore (6-\lambda)(9-6\lambda+\lambda^2-1) + 4\lambda - 8 - 8 + 4\lambda = 0$$

$$\therefore 48 - 36\lambda + 6\lambda^2 - \cancel{8\lambda} + 6\lambda^2 - \lambda^3 + \lambda + \cancel{8\lambda} - 16 = 0$$

$$\therefore \lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$\therefore \lambda = 2, 2, 8$$

i) for $\lambda_1 = 2$ in ① $\therefore [A - \lambda I]X = 0$ gives

$$\therefore \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

By $R_2 \leftrightarrow R_1$

$$\therefore \begin{bmatrix} -2 & 1 & -1 \\ 4 & -2 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

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By $R_2 \to R_2 + 2R_1$, $R_3 \to R_3 + R_1$

$$\begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore -2x_1 + x_2 - x_3 = 0$

$\therefore 2x_1 - x_2 + x_3 = 0$ —— (2)

Here we see that rank of matrix is 1 & the no. of variables are 3 hence there are $3-1=2$ linearly independent solns

so that put $x_2 = 2s$

$x_3 = -2t$ in (2)

$\therefore 2x_1 - 2s - 2t = 0$

$\therefore x = s+t$

$$\therefore X = \begin{bmatrix} s+t \\ 2s+0 \\ 0-2t \end{bmatrix} = s\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

Thus the vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ are linearly independent

Also the above eigenvectors form the bases for the eigen spaces corresponding to $\lambda = 2$

$\therefore$ the dimension of eigen space is 2.

ii) for $\lambda_2 = 8$ in (1)

$\therefore [A - \lambda_2 I]X = 0$ gives

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

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By $R_2 \to R_2 - R_1$ & $R_3 \to R_3 + R_1$

$$\begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

By $R_2 \to R_3 - R_2$ $$\begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

The rank of the matrix is 2 & there are 3 variables hence there ~~are~~ is 1 linearly independent sol^ns

$\therefore -2x_1 - 2x_2 + 2x_3 = 0$; $-3x_2 - 3x_3 = 0$

i.e. $2x_1 + 2x_2 - 2x_3 = 0$; $\therefore x_2 = -x_3$ — (4)

$\therefore x_1 + x_2 - x_3 = 0$

put $x_2 = -t$ $\therefore x_3 = t$

$\therefore x_1 + (-t) - t = 0$

$\therefore x_1 = 2t$

$$\therefore X = \begin{bmatrix} 2t \\ -t \\ t \end{bmatrix} = t \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence the above vector is the bases for eigen space corresponding to $\lambda = 8$

Q] $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

→ By using characteristic eq^n

$|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0 \quad \text{--- (1)}$$

$$\therefore (2-\lambda)\big((3-\lambda)(2-\lambda)-2\big) - 2(2-\lambda-1) + 1(2-3+\lambda) = 0$$

$$\therefore (2-\lambda)^2(3-\lambda) - 2 + 2\lambda - 2 + 2\lambda - 1 + \lambda = 0$$

$\therefore (4$

$\lambda = 1, 1, 5$

$\therefore$ (i) for $\lambda_1 = 1$ in (1)

$\therefore [A - \lambda_1 I]X = 0$ gives

$$\therefore \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

By $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore x_1 + 2x_2 + x_3 = 0$ --- (2)

$\therefore$ Rank of above matrix is 1 & no. of variables is 3 $\therefore$ we have $3-1=2$ linearly independent sol^ns

$\therefore$ put $x_2 = -s$ in (2)

$x_3 = -t$

$\therefore x_1 - 2s - t = 0$

$\therefore x_1 = 2s + t$

$$\therefore X = \begin{bmatrix} 2s+t \\ -s+0 \\ 0-t \end{bmatrix} = s\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Hence the above vector is bases ~~of~~ for eigen spaces corresponding to $\lambda = 1$

ii] for $\lambda_2 = 5$ in ① $\therefore [A - \lambda_2 I]X = 0$ gives

$$\therefore \begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

By $R_1 \leftrightarrow R_3$

$$\therefore \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore$ By $R_2 \to R_2 - R_1$, $R_3 \to R_3 + 3R_1$

$$\therefore \begin{bmatrix} 1 & 2 & -3 \\ 0 & -4 & 2 \\ 0 & 8 & -8 \end{bmatrix}$$

Module 2:-

Condition of Consistency & Inconsistency for Homogenous & Non Homogeneous eqⁿˢ

A set of linear eqⁿˢ:

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$

$\cdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$

Such eqⁿˢ are called as non-homogenous eqⁿˢ

On the other hand all $b_1, b_2, \ldots, b_m$ are zero's then the eqⁿˢ are called as homogeneous eqⁿ

Note:-

augmented matrix

1] If rank $A <$ rank$[A:B]$ then the eqⁿˢ are inconsistent, i.e. they have no solⁿ

2] If rank $A =$ rank$[A:B]$ then the eqⁿˢ are consistent, i.e. they poses a solⁿ.

3] Further if $r = n$ i.e rank$(A) =$ no. of unknowns then the system has unique solⁿˢ, also note that the system has unique solⁿ iff the coefficient matrix is non-singular.

5] If $r < n$ i.e rank$(A) <$ no. of unknowns, the system has a infinite solⁿ

In this case $(n-r)$ unknowns called parameters can be assigned arbitary values. The remaining unknowns the can be expressed in terms of those parameter.

P.T.O

Q] $x + y + z = 3$
$x + 2y + 3z = 4$
$x + 4y + 9z = 6$ soln the eqns

→ we have $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

By $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

By $R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \quad \text{——} \ (1)$$

This

$A \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ & $[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{array}\right]$

the rank of A = rank[A:B] which is 3
∴ the eqns are consistent
further the rank(A) = no. of unknowns
Hence the system as unique soln

from (1)

$x + y + z = 3$ ∴ $x = 2$
$y + 2z = 1$ ∴ $y = 1$
$2z = 0$ ∴ $z = 0$

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Q) Test for consistency & if possible to solve the eqn
$2x - 3y + 7z = 5$
$3x + y - 3z = 13$
$2x + 19y - 47z = 32$

$$\rightarrow \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

By $R_1 \rightarrow R_1 - R_2$
$$\begin{bmatrix} -1 & -4 & 10 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 13 \\ 32 \end{bmatrix}$$

By $-R_1 \rightarrow R_1$
$$\begin{bmatrix} 1 & 4 & -10 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 32 \end{bmatrix}$$

By $R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - 2R_1$
$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 11 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \\ 16 \end{bmatrix}$$

By $R_3 \rightarrow R_3 + R_2$
$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \\ 5 \end{bmatrix} \quad \text{—— (1)}$$

$$A = \begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 0 & 0 \end{bmatrix} \qquad [A:B] = \left[\begin{array}{ccc|c} 1 & 4 & -10 & 8 \\ 0 & -11 & 27 & -11 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

Rank(A) = 2, Rank[A:B] = 3

$\therefore$ the system of eqns are ~~con~~ inconsistent

This is ~~cor~~ clear even otherwise because from eqn (1) gives $0x + 0y + 0z = 5$ which is absurd.

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∴ the eqns are inconsistent.

Q] Investigate for what values of λ & μ the eqns $x+y+z=6$
$x+2y+3z=10$
$x+2y+\lambda z=\mu$ have ① No soln
② Unique soln
③ Infinite solns

→ We have
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ \mu-10 \end{bmatrix}$$

② the system have unique soln if the coefficient matrix is non singular. rank(A) i.e $r =$ no. of unknowns which is $n=3$
this requires $\lambda-3 \neq 0 \Rightarrow \lambda \neq 3$
if $\lambda \neq 3$, the system has unique soln

③ if $\lambda=3$, coefficient matrix & augmented matrix becomes

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \& \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & \mu-10 \end{array}\right]$$

∴ Rank(A) $= \cancel{3}2$ & Rank$[A:B] = 2$ if $\mu=10$

Thus if $\lambda = 3$ & $\mu = 10$ then the system is consistent
but rank(A) < no. of unknowns, hence the eqns have infinite soln

① if $\lambda = 3$ & $\mu \neq 10$
Rank(A) = 2 & Rank([A : B]) = 3
i.e they are not equal.
$\therefore$ the eqns are inconsistent & they posses no soln

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Mod 2:-

Crout's Method:-

In LA Crout's Matrix decomposite is an LU decomposite which decomposes a matrix into lower & upper Δ matrix

Steps to Remember:-

1] get Matrix A & B from the given set of eqns

2] First we take $L \times U = A$

where $L = \begin{bmatrix} 1 & 0 & 0 \\ r_{21} & 1 & 0 \\ r_{31} & r_{32} & 1 \end{bmatrix}$ & $U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$

Lower Δ matrix with diagonal elements $= 1$

Upper Δ matrix

3] Then calculate the elements of lower & upper Δ matrix

4] Find V by solving $LV = B$ by forward substitution where $V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

then finally we take the original system of eqn ~~where~~ $UX = V$.

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

find X by back substitution.

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Q] Apply Crout's method to solve the eqn

$3x+2y+7z=4$
$2x+3y+z=5$
$3x+4y+z=7$

$\rightarrow A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$

~~for~~ 1st we take $LU = A$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ r_{21} & 1 & 0 \\ r_{31} & r_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \quad \text{---} (1)$$

$$\therefore \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ r_{21}u_{11} & r_{21}u_{12}+u_{22} & r_{21}u_{13}+u_{23} \\ r_{31}u_{11} & r_{31}u_{12}+r_{32}u_{22} & r_{31}u_{13}+r_{32}u_{23}+u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

$\therefore u_{11} = 3, \; u_{12} = 2, \; u_{13} = 7$

$\therefore r_{21} = \frac{2}{3}, \; r_{31} = \frac{3}{3} = 1$

$u_{22} = 3 - \frac{2}{3} \times 2 = \frac{5}{3}, \quad u_{23} = 1 - \frac{2}{3} \times 7 = \frac{-11}{3}$

$r_{32} = \frac{1}{u_{22}} (4 - 1 \times 2) = \frac{3}{5} \times 2 = \frac{6}{5}$

$u_{33} = 1 - 1 \times 7 - \frac{6}{5} \times \left(\frac{-11}{3}\right)$

$\cancel{= \frac{-6+66}{15} = \frac{150}{15} = 10} \quad \frac{-8}{5}$

Substituting all the above values in eqn (1)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \quad \text{--- (2)}$$

$LV = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ \frac{2}{3}V_1 + V_2 \\ V_1 + \frac{6}{5}V_2 + V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$\therefore V_1 = 4$

$\therefore V_2 = 5 - \frac{8}{3} = \frac{7}{3}$

$V_3 = 7 - 4 - \frac{14}{5} = \frac{1}{5}$

Hence the original system of eqns is

$UX = V$

$$\therefore \begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7/3 \\ 1/5 \end{bmatrix}$$

$\therefore z = \frac{-1}{8} \times \frac{8}{8} = \frac{-1}{8}$

$\frac{5}{3}y - \frac{11}{3}z = \frac{7}{3}$

$\therefore \frac{5}{8}\,5y + \frac{11}{8} = \frac{7}{\cancel{10}}$

$\therefore y = \frac{1}{5}\left(\frac{45}{8}\right) = \frac{9}{8}$

$\therefore 3x + 2y + 7z = 4$

$\therefore 3x = 4 - \frac{18}{8} + \frac{7}{8}$

$= 4 - \frac{11}{8} = \frac{21}{8}$

$\cancel{\therefore x = 13}$

$\therefore x = \frac{7}{8}$

Q] $x_1 + x_2 + 4x_3 = 43$

$2x_1 + 10x_2 + x_3 = 63$

$25x_1 + 2x_2 + x_3 = 69$