Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

(Autonomous College Affiliated to University of Mumbai)

End Semester Examination

August 2021

Max. Marks: 60 **Duration: 2 hours**

Class: FE **Semester: II**

Course Code: MA201 **Branch: ETRX/EXTC/IT/COMPS**

Name of the Course: Differential Equations and Complex Analysis

Instructions:

(1) All Questions are Compulsory
(2) Assume suitable data if necessary
(3) Full working to be shown
(4) Start each section on a fresh page.

Question No.	Section1	Max. Marks	CO
Q.1			1
(A)	Solve: $2\left(1+x^2\sqrt{y}\right)ydx+\left(x^2\sqrt{y}+2\right)xdy=0$ **OR** Solve: $tany\frac{dy}{dx}+tanx(1-cosy)=0$	5	
(B)	Using Euler's method find the approximate value of y at x= 0.1 (exact up to 4 decimal places) for the differential equation $\frac{dy}{dx}=x-y^2$, with initial conditions y(0)=1 in 5 steps.	5	
Q.2			2
(A)	Solve the system of differential equations $\frac{dx}{dt}=4x-3y-4t^2+5t$ $\frac{dy}{dt}=6x-7y-6t^2+7t+1$	7	
(B)	Solve: $\left(\frac{d^2}{dx^2}-\frac{2}{x^2}\right)\left(\frac{d}{dx}+\frac{1}{x}\right)y=\cos(logx)$	8	
Q.3			3
(A)	The current 'i' in a circuit, containing a resistance R, and a condenser of capacity C, and connected to a constant emf E is given by, $Ri+\frac{q}{C}=\frac{E}{R}$. Find the charge 'q' if given that q=0 when t=0.	5	

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Q.4	**Section2**		4
(A)	List down all the roots of $x^6 - 1 = 0$. Hence, find the roots of $x^4 + x^2 + 1 = 0$ that are common to $x^6 - 1 = 0$.	6	
(B)	If $\tanh(\alpha + i\beta) = x + iy$ prove that, (i) $x^2 + y^2 + 1 = 2x\coth 2\alpha$ (ii) $x^2 + y^2 + 2y\cot 2\beta = 1$ **OR** If $\cos(\theta + i\emptyset) = re^{i\alpha}$ prove that, (i) $r^2 = \frac{1}{2}(\cosh 2\emptyset + \cos 2\theta)$ (ii) $\tan\alpha = -\tan\theta\tanh\emptyset$	6	
(C)	If $\sqrt{i}^{\sqrt{i}^{\cdot^{\cdot^{\infty}}}} = \alpha + i\beta$ prove considering the principal value, (i) $\alpha^2 + \beta^2 = e^{\frac{-\pi\beta}{2}}$ (ii) $\tan\left(\frac{\alpha\pi}{4}\right) = \frac{\beta}{\alpha}$	6	
Q.5			5
(A)	Using Milne Thompson's method, construct an analytic function f(z) = u+iv if $u = \frac{\sin x + \sinh y}{\cosh y - \cos x}$	6	
Q.6			6
(A)	Evaluate $\oint(z^3 + z + 1 - 2\bar{z})dz$ over the curve C, where C is the circle $x^2 + y^2 = 2$. **OR** Evaluate $\oint \frac{z^2+1}{(z-1)^2(z-2)} dz$ over the curve $\lvert z - 2\rvert = 2$.	6	
	ALL THE BEST 😊		