classmate

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* Range of Digital Frequency 'ω'

⇒ Consider DT Signal :-

- $x_1(n) = \cos(0.3\pi n)$ where $\omega_1 = 0.3\pi$ rad
- $x_2(n) = \cos(2.3\pi n)$ where $\omega_2 = 2.3\pi$ rad

Now, $x_2(n) = \cos(2.3\pi n)$

$= \cos[(0.3\pi n) + (2\pi n)]$

$(\cos(A+B) = \cos A \cos B - \sin A \sin B)$

$\therefore = \cos(0.3\pi n)\cos(2\pi n) - \sin(0.3\pi n)\sin(2\pi n)$

$= \cos(0.3\pi n)(1) - 0$ $\left[\because \sin(2\pi n) = 0, \ \cos(2\pi n) = 1\right]$

$= \cos(0.3\pi n)$

$\therefore \boxed{x_2(n) = x_1(n)}$

Similarly, if $x_3(n) = 4.3\pi n$, $x_4(n) = 8.3\pi n$ } Multiples of $2\pi n$, it will be same as $x_1(n)$

$x_5(n) = 1.7\pi n$ is also $= x_1(n)$.

* NOTE :-

1] Range of Digital frequency (ω) is ~~[-π, π]~~ $(-\pi, \pi] \rightarrow$

i.e $\boxed{-\pi < \omega \le \pi}$

[(: Not included
] : Included]

2] Range of Digital frequency (f) is $(-1/2, 1/2] \rightarrow$

i.e :- $\left[-\frac{1}{2} < f \le \frac{1}{2}\right]$

$\because \omega = 2\pi f$

$\frac{-\pi}{2\pi} = f$ & $\frac{\pi}{2\pi} = f$

$\therefore \boxed{-\frac{1}{2} < f \leq \frac{1}{2}}$

* Linear Shifting of Non Periodic Digital Signal

Eg:- $x[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Find $x[n-1]$, $x[n+1]$, $x[-n]$
$x[-n+1]$, $x[-n-1]$

To Find:- $x[n-1]$

$\Rightarrow$ Let $y[n] = x[n-1]$

Steps:-

I) Find $y[n]$ for $n \geq 0$

$\therefore y(0) = x(-1) = 0$
$y(1) = x(0) = 1$
$y(2) = x(1) = 2$
$y(3) = x(2) = 3$
$y(4) = x(3) = 4$
$y(5) = x(4) = 0$ (continue till we again get a repeating value, here we get a 0 so stop)
$\vdots$

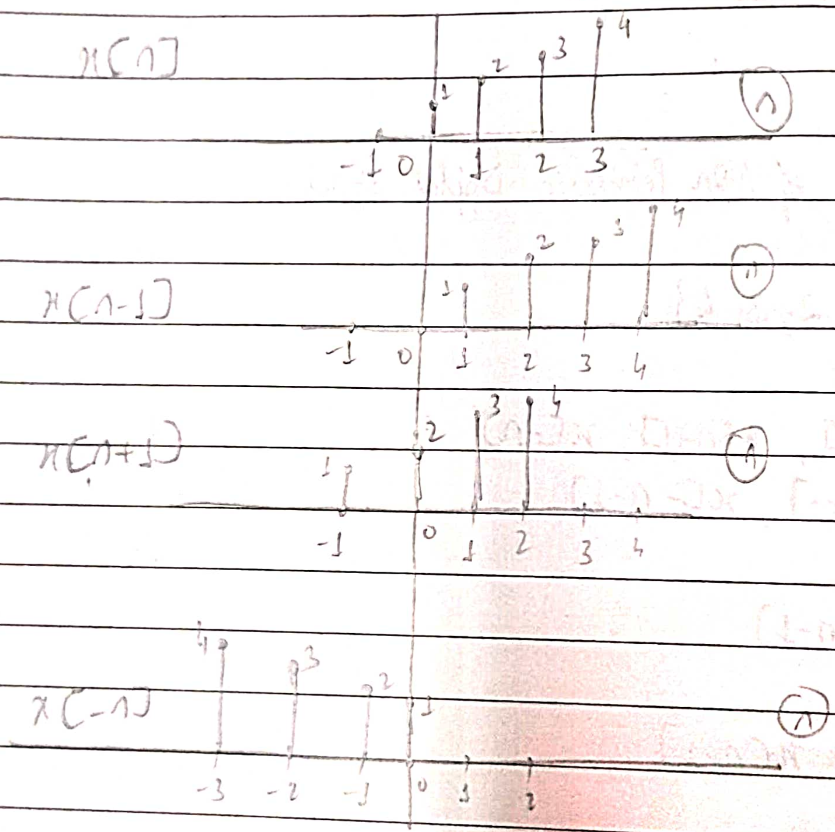
II) Find $y[n]$ for $n < 0$

$\therefore y(-1) = x(-2) = 0$
$y(-2) = x(-3) = 0$
$\vdots$

Ans:-

$\therefore \; x[n-1] = [\underset{\uparrow}{0}, 1, 2, 3, 4]$

$\therefore$



Note:-

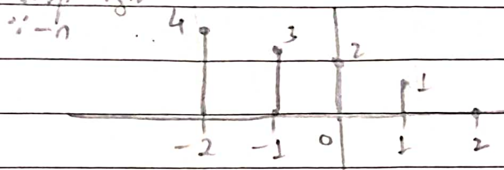
$x[n-1]$ = $x[n]$ shifted right by 1

$x[n+1]$ = $x[n]$ shifted left by 1

$x[-n]$ = $x[n]$'s mirror image.

opp sign, shift right

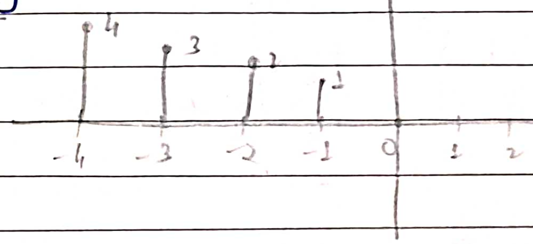
$x[-n+1]$ ∴ $-n$



(Here shift right of $x(-n)$ by 1)

$x[-n-1]$

same sign: shift left of $x(-n)$



(Here shift left of $x(-n)$ by 1)

* Trick to Remember:-

→ $-ve$ & $+ve$ sign (i.e. opp signs): shift Right of $x[n]$ / $x[-n]$

→ $-ve$ & $-ve$ / $+ve$ & $+ve$ (i.e. same sign): shift left of $x[n]$ / $x[-n]$

* Circular Shifting of Periodic Discrete Time Signal:-

eg:- $x[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Find:-

$x[n-1]$

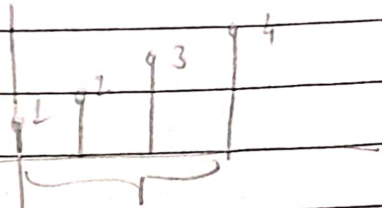
$x[n+1]$

$x[-n]$

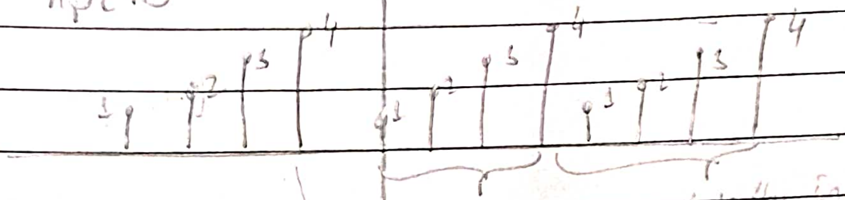
$x[-n+1]$

$x[-n-1]$

$x[n]$



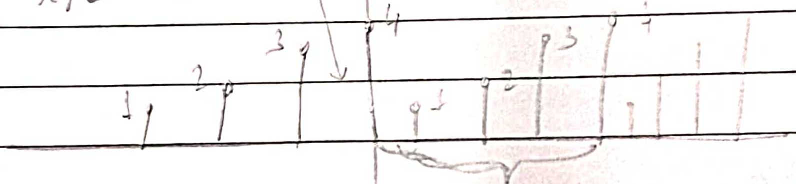
$x_p(n)$



Actually is periodic

$x_p(n-1)$

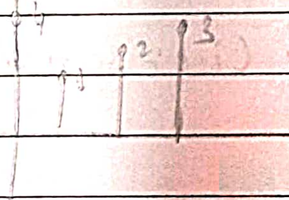
($x_p(n) \rightarrow$ right shifted by 1)



Repeating

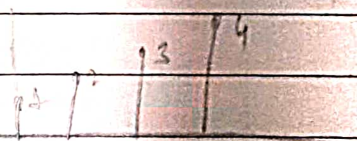
$x[n-1]$

$x[n-1] = 1$ cycle of $x_p(n-1)$

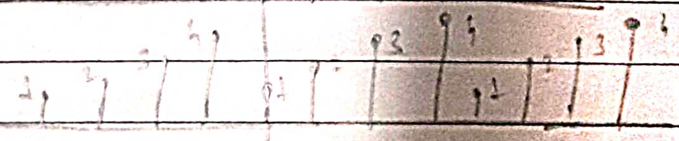


∴ To find $x[-n]$:-

$x[n]$

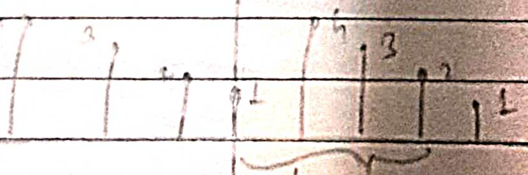


Then plot $x_p(n)$

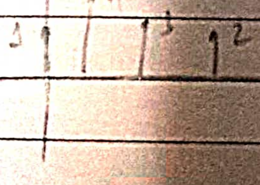


$x_p(-n)$ = mirror image of $x_p(n)$

Then plot $x_p(-n)$



Then plot $x[-n]$



$x[-n] = 1$ cycle of $x_p(-n)$

$\therefore \quad x[n] = \{1, 2, 3, 4\}$

1] $x[n-1] = \{4, 1, 2, 3\}$ — Keep 1st index same then reverse the rest.

2] $x[n+1] = \{2, 3, 4, 1\}$

3] $x[-n] = \{1, 4, 3, 2\}$

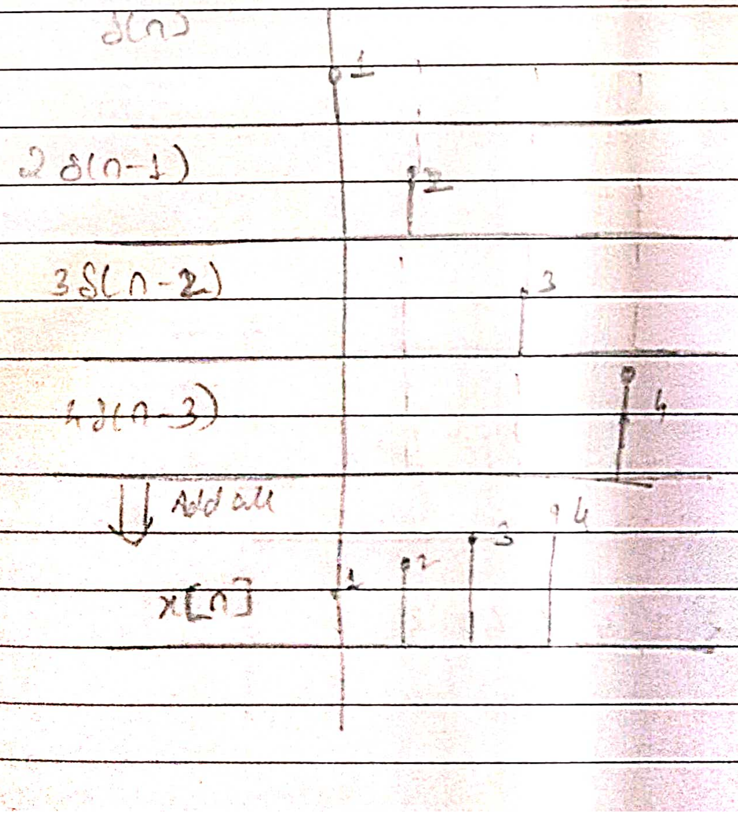
4] $x[-n+1] = \{2, 1, 4, 3\}$

5] $x[-n-1] = \{4, 3, 2, 1\}$

* <u>Representation of DT signal $x[n]$ in terms of shifted $\delta(n)$ signals.</u>

Eg:- $x[n] = \{1, 2, 3, 4\}$

$$x[n] = \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)$$



Q) $x[n] = \delta(n) + 2\delta(n-2) + 3\delta(n-3) + 4\delta(n-4)$

Ans $\therefore x[n] = \{\underset{\uparrow}{1}, 0, 2, 3, 4\}$

($\because \delta(n-1)$ is missing it will be 0 at index 1)

IMP Q) $x[n] = \delta(n) + 2u[n-1] + \delta[n-2] + 2u[n-3] - 4\delta[n-4]$

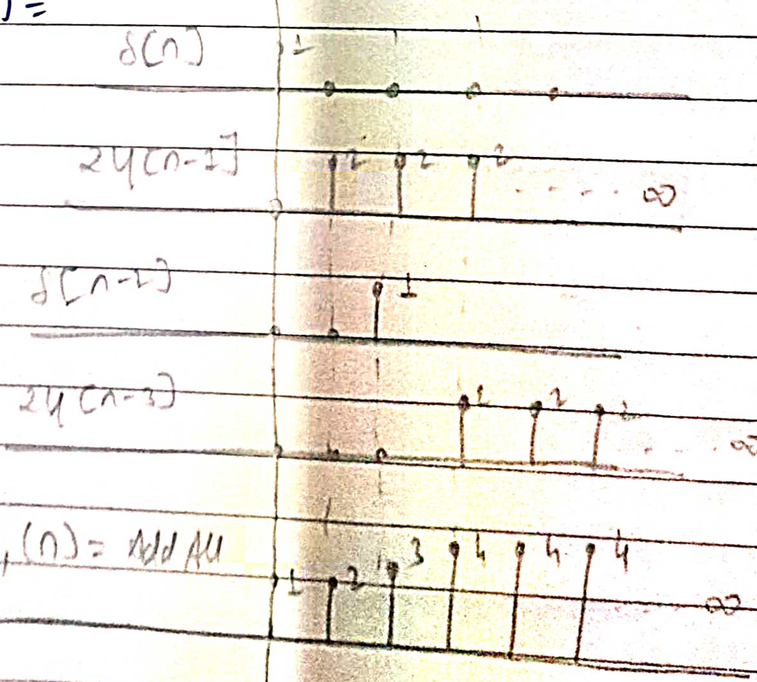
$\cancel{-4[n-4]}$ $-4u[n-5]$

Ans <u>Now:-</u> $x[n] = (\delta(n) + 2u[n-1] + \delta[n-2] + 2u[n-3])$

$-(4\delta[n-4] + 4[n-5])$

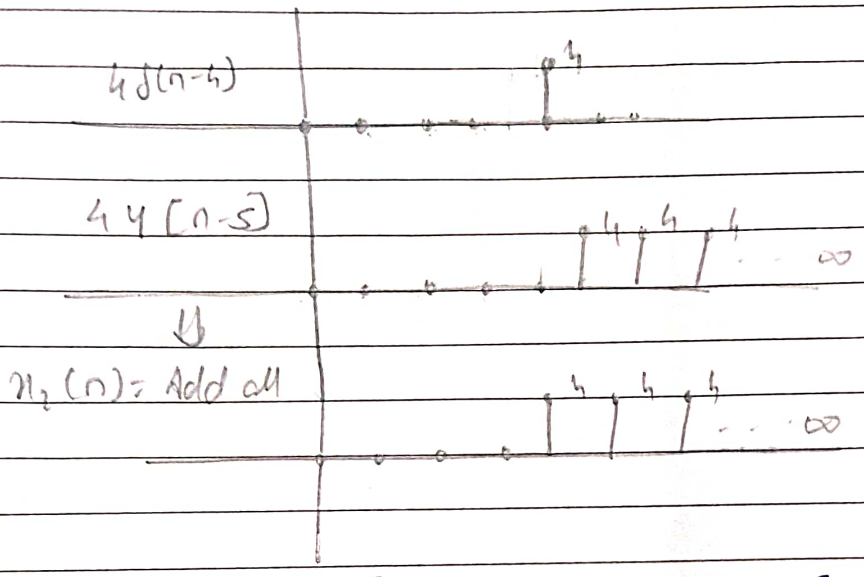
$\therefore \boxed{x[n] = x_1[n] - x_2[n]}$

<u>Step 1</u> $x_1[n] =$



$x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 4, 4 \ldots\}$

Step 2 $x_2(n) =$



$$\therefore x_2(n) = \{\underset{\uparrow}{0}, 0, 0, 0, 4, 4, 4 \ldots\}$$

Step 3 $\therefore x(n) = x_1(n) - x_2(n)$

$$\therefore \boxed{x(n) = \{1, 2, 3, 4\}}$$