16/03/23 Mod 5

Derogatary & Non Derogatary Matrix:-

Monic polynomial:-

A polynomial in x in which highest power of x has coefficient 1 (unitary)

eg:- 1] $x^3 - 2x^2 + x - 1 = 0$ is a Monic polynomial.

2] $2x^4 + x - 7 = 0$ is not a Monic polynomial.

Q $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ Show that matrix A is Non - Derogatary matrix.

$-1 \cancel{-2} + \cancel{4} - 3$

$= 0$

→ Using Characteristic eqn

$|A - \lambda I| = 0$

$\therefore \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 3-\lambda & 4 \\ 3 & 4 & 5-\lambda \end{vmatrix} = 0$

$\therefore \lambda^3 - 9\lambda^2 + (-1 + (-4) + (-1))\lambda - |A| = 0$

$|A| = 0$

$\therefore \lambda^3 - 9\lambda^2 - 6\lambda = 0$

$\therefore \lambda_1 = 9.623 \quad \lambda_3 = 0$

$\therefore \lambda_2 = -0.623$

$\therefore$ All eigen values are distinct & characteristic eqn is satisfied by A &

Hence A is non derogatary

Q] Show that $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is a Non Derogatary Matrix

$2(-4)+(-2)+3(2)$
$=-8-4+6$

→ By using $|A-I\lambda|=0$

$$\therefore \begin{vmatrix} 2-\lambda & -2 & 3 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -1 \end{vmatrix} = 0$$

$\therefore \lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$

$\therefore \lambda_1 = -2,\ \lambda_2 = 3,\ \lambda_3 = 1$

$\therefore$ A is Non derogatary Matrix

Q] $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is a Derogatary matrix

→ By using $|A-\lambda I|=0$

$$\begin{vmatrix} 5-\lambda & -6 & -6 \\ -1 & 4-\lambda & 2 \\ 3 & -6 & -4-\lambda \end{vmatrix} = 0$$

$\therefore \lambda^3 - 5\lambda^2 + [(-4)+(-2)+(14)]\lambda - (5(-4)+6(-2)$
$-6(-6)) = 0$

$\therefore \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$

$\therefore \lambda = 1, 2$

1	1	-5	8	-4
		1	-4	4
	1	-4	4	0

$\therefore \lambda = 1, 2, 2$

$\therefore (\lambda^2 - 4\lambda + \lambda) = 0$

$\therefore \lambda = 2$

Here two eigen values are repeated & 3rd values is different. So first we find the minimal polynomial of A. But we know that each characteristic root of A is also a root

$\therefore (x-1)(x-2) = x^2 - 3x + 2$

Let us see annihilates the matrix A or not

Replace x by A.

$\therefore A^2 - 3A + 2I = A \times A - 3A + 2I$

$$A^2 = \begin{bmatrix} 13 & -18 & -18 \\ -3 & 10 & 6 \\ 9 & -18 & -14 \end{bmatrix}$$

$$\therefore A^2 - 3A + 2I = \begin{bmatrix} 13 & -18 & -18 \\ -3 & 10 & 6 \\ 9 & -18 & -14 \end{bmatrix} - \begin{bmatrix} 15 & -18 & -18 \\ -3 & 12 & 6 \\ 9 & -18 & -12 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $f(x) = x^3 - 3x + 2$ annihilates the matrix A

Hence $f(x)$ is a minimal polynomial & Monic polynomial

Hence A is derogatory

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Q] Check whether $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is a derogatory matrix or not.

→ Using by $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 7-\lambda & 4 & -1 \\ 4 & 7-\lambda & -1 \\ 4 & -4 & 4-\lambda \end{vmatrix} = 0$$

$\therefore \lambda^3 - 18\lambda^2 + (24+24+33)\lambda - (7(24) - 4(12)$
$-1(12)) = 0$

$\therefore \lambda^3 - 18\lambda^2 + 81\lambda - 108 = 0$

$\therefore \lambda = 12, 3, 3$

Repeat the lines

$\therefore (x-12)(x-3) = 0$

$\therefore x^2 - 15x + 36 = 0$

Let us see annihilates the matrix A or not

$\therefore A^2 - 15A + 36 \cancel{I} = ?$

$$A^2 = \begin{bmatrix} 69 & 60 & -15 \\ 60 & 69 & -15 \\ -60 & -60 & 24 \end{bmatrix}$$

$$\therefore A^2 - 15A + 36I = \begin{bmatrix} 69 & 60 & -15 \\ 60 & 69 & -15 \\ -60 & -60 & 24 \end{bmatrix} - 15\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix} + \begin{bmatrix} 36 & 0 & 0 \\ 0 & 36 & 0 \\ 0 & 0 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $f(x) = x^3 - 3x + 2$ annihilates matrix A
Hence $f(x)$ is a Minimal polynomial & Monic polynomial.
Hence A is derogatory.

Singular Value Decomposition (S.V.D)

Theorem:- A rectangular Matrix A_{mn} can be decomposed into product of 3 matrices, an orthogonal matrix U_{mm}, a diagonal Matrix D_{mn} & transpose of another orthogonal Matrix V'_{nn}

$\therefore A_{mn} = U_{mm} \cdot D_{mn} \cdot V'_{nn}$
$\because$ U & V are orthogonal matrix
$\therefore UU' = I$ & $VV' = I$
Further the columns of U are orthogonal eigen vectors of AA' & columns of V are orthonormal eigen vectors of $A'A$
The diagonal values of D are called as singular values & hence the name "Singular Value Decomposition" or "SVD"
The normalized eigen ~~vec~~ vectors of AA' are the columns of V & the normalized eigen vectors of AA' are the columns of U

Steps to find SVD:-

I] Find eigen values of $A'A$
II] Obtain square ~~r~~ root of eigen values obtained in step (I) & write the diagonal matrix D whose diagonal elements are D's square roots arranged in a descending order

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Let $D = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$

where σ_1, σ_2 & σ_3 are the sq. roots of the diagonal elements.

(3) Obtain eigen vectors of $A'A$
These Eigen vectors when normalized are the columns of V, say ~~v_1, v_2~~ v_1, v_2, v_3.

(4) The columns of U are given by

$U_1 = \frac{1}{\sigma_1} A v_1$

$U_2 = \frac{1}{\sigma_2} A v_2$

$U_3 = \frac{1}{\sigma_3} A v_3$

5] Thus we get $A = UDV'$

Q] Find SVD of $A = \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$

→ Step I: We first obtain the eigen values of

$A'A = \begin{bmatrix} 4 & -3 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$

$= \begin{bmatrix} 25 & 7 \\ 7 & 25 \end{bmatrix}$

∴ Characteristic eqn of $A'A$ is

~~$A'A$~~ ∴ $(25-\lambda)^2 - \overset{49}{\cancel{7}} = 0$ ——— (1)

∴ $625 - 50\lambda + \lambda^2 - \cancel{7}49 = 0$

∴ $\lambda^2 - 50\lambda + \underset{576}{\cancel{618}} = 0$ ∴ $\lambda = 32, 18$

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step 2
Now we take sq root of Eigen values to get D & arrange them in descending order

for $\lambda = 32$, $\sigma_1 = \sqrt{32} = 4\sqrt{2}$
$\lambda = 18$, $\sigma_2 = \sqrt{18} = 3\sqrt{2}$

$\therefore D = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} = \begin{bmatrix} 4\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{bmatrix}$

Step 3:- Now we obtain E.vector of A'A
for $\lambda_1 = 32$

$\therefore [A - \lambda_1 I]X = 0$

$\therefore \begin{bmatrix} -7 & 7 \\ 7 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

By $R_2 + R_1$ & $R_1/7$

$\therefore \begin{bmatrix} -1 & +1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\therefore -x_1 + x_2 = 0$ put $x_2 = t$

$\therefore x_1 - x_2 = 0$

$\therefore x_1 = t$ $\therefore V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda_2 = 18$

$\begin{bmatrix} 7 & 7 \\ 7 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$R_2 \to R_2 - R_1$ & $R_1/7$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\therefore x_1 + x_2 = 0$ put $x_2 = -t$

$\therefore x_1 = t$ $\therefore V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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dot product

$\therefore V_1 \cdot V_2 = (1,1)\cdot(1,-1) = 1\cdot 1 + 1\cdot(-1) = 1-1 = 0$

SVD hai

$\therefore V_1$ & V_2 are orthogonal
we now normalise them by dividing them by their norms

$\|V_1\| = \sqrt{x_1^2 + x_2^2} = \sqrt{1^2+1^2} = \sqrt{2}$

norm of V_1

$\|V_2\| = \sqrt{x_1^2+x_2^2} = \sqrt{1^2+(-1)^2} = \sqrt{2}$

$\therefore$ The normalised vector

$$V_1 = \frac{V_1}{\|V_1\|} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$V_2 = \frac{V_2}{\|V_2\|} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

Thus $V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

Step 4:- the columns of U are given by

$$U_1 = \frac{1}{\sigma_1} A V_1$$

$$= \frac{1}{4\sqrt{2}} \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$U_2 = \frac{1}{\sigma_2} AV_2$

$= \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

Thus $U = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Step 5:-

∴ We get $A = UDV'$

$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

Q] Find SVD of $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$

→ $\cancel{(2-\lambda)^2 = 0}$ $A'A = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 6 & 13 \end{bmatrix}$

∴ $(4-\lambda)(13-\lambda) - 36 = 0$

∴ $52 - 17\lambda + \lambda^2 - 36 = 0$

∴ $\lambda_1 = 16, \lambda_2 = 1$

∴ for $\lambda_1 = 16, \sigma_1 = 4$

$\lambda_2 = 1, \sigma_2 = 1$

$D = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

∴ $[A'A - \lambda I]X = 0$

∴ for $\lambda_1 = 16$

∴ $\begin{bmatrix} -12 & 6 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

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$R_2 \to R_2 + R_1/2$, $R_1/2$

$$\therefore \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\therefore -2x_1 + x_2 = 0$

$\therefore 2x_1 - x_2 = 0$

put $x_2 = 2t$

$\therefore x_1 = t$

$$\therefore V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\lambda_2 = 1$

$$\therefore \begin{bmatrix} 3 & 6 \\ 6 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 3 & 6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\therefore x_1 + 2x_2 = 0$

~~put $x_1 = t$~~ put $x_2 = -t$

~~$\therefore x_2 = t/2$~~ $x_1 = 2t$

$$\therefore V_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Application of Diagonalisation to solving system of Differential eqn :-

Q] Solve the system of Differential eqn

$$y' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} y$$

→ Soln :- we can write the system as

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad \text{——} \quad (*)$$

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Now Diagonalising A

$\therefore$ It's charactorsitic eqn is $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$\therefore 4 - 4\lambda + \lambda^2 - 1 = 0$

$\therefore \lambda^2 - 4\lambda + 3 = 0$

$\therefore \lambda = 1$ or $\lambda = 3$

$\therefore$ for eigenvectors $\lambda_1 = 1$

$[A - \lambda_1 I][X] = 0$

$$\therefore \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\therefore x_1 + x_2 = 0$

put $x_2 = -t$ $\quad \therefore v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ or $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\therefore x_1 = t$

for eigen vectors $\lambda_2 = 3$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\therefore -x_1 + x_2 = 0$

$\therefore x_1 - x_2 = 0$

$\therefore$ put $x_2 = t$ $\quad \therefore x = t$ $\quad \therefore v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

P.T.O

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thus $P = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$

Now we know $P^{-1} = \frac{\text{adj} P}{|P|}$ —— ①

$|P| = -1 - 1 = -2$

$\therefore \text{adj} P = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

$\therefore P^{-1} = \frac{1}{-2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

& $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$\therefore A = PDP^{-1}$

$\therefore$ from eqn ⊛

$y' = PDP^{-1} \times y$

premultiply by P^{-1}

$\therefore (P^{-1}y)' = DP^{-1}y$ —— ②

Let $P^{-1}y = z$

$\therefore z' = Dz$

$\therefore \begin{bmatrix} z_1' \\ z_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$\therefore z_1' = z_1$

& $z_2' = 3z_2$

On solving these eqns we get

$z_1 = C_1e^{t}, \; z_2 = C_2e^{3t}$ ←—— ③

$\therefore z = P^{-1}y$

$\therefore$ Multiply by P

$\therefore Pz = y$

P.T.O

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 e^{t} \\ c_2 e^{3t} \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -c_1 e^{t} + c_2 e^{3t} \\ c_1 e^{t} + c_2 e^{3t} \end{bmatrix}$$

$$\therefore \vec{y} = \begin{bmatrix} -c_1 e^{t} + c_2 e^{3t} \\ c_1 e^{t} + c_2 e^{3t} \end{bmatrix}$$

y vector

Q] with initial condition $y_1(0) = 1$ & $y_2(0) = 6$

$$y' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} y$$

→ We write the system as

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad \text{---} \; (*)$$

Let $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$

Now Diagonalising A

$$\therefore |A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 4 & -2-\lambda \end{vmatrix} = 0$$

$$\therefore (1-\lambda)(-2-\lambda) - 4 = 0$$

$$\therefore -2 + \lambda + \lambda^2 - 4 = 0$$

$$\therefore \lambda^2 + \lambda - 6 = 0$$

$$\therefore \lambda = 2, -3$$

for $\lambda = 2$, $\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$$\cancel{\therefore \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0}$$

$$\therefore \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \quad \therefore V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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$\lambda_2 = -3$, $\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{bmatrix} 4 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\therefore 4x_1 + x_2 = 0$

put $x_2 = -4t$

$\therefore x = t$

$\therefore V_2 \begin{bmatrix} 1 \\ -4 \end{bmatrix}$

$\therefore P = \begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$

Now $P^{-1} = \dfrac{\operatorname{adj} P}{|P|}$, $\operatorname{adj} P = \begin{bmatrix} -4 & -1 \\ -1 & 1 \end{bmatrix}$

$|P| = -4 - 1 = -5$

$\therefore P^{-1} = \dfrac{1}{5}\begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}$ —— (1)

$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$

$\therefore A = PDP^{-1}$, from (*) we write $y' = PDP^{-1}y$

premultiply by P^{-1}

$\therefore (P^{-1}y)' = DP^{-1}y$ —— (2)

Let $z = P^{-1}y$

$\therefore z' = Dz$

$\therefore \begin{bmatrix} z_1' \\ z_2' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$\therefore z_1' = 2z_1$ & $z_2' = -3z_2$

$\therefore$ On solving the eq^ns we get

$z_1 = c_1 e^{2t}$ & $z_2 = c_2 e^{-3t}$ —— (3)

$\therefore z = P^{-1}y$

Multiply by P

$\therefore Pz = y$

$\therefore \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$\therefore \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} c_1 e^{2t} + c_2 e^{-3t} \\ c_1 e^{2t} - 4c_2 e^{-3t} \end{bmatrix}$ —— (I)

when $t=0$ $y_1(0)=1$ & $y_2(0)=6$ in (I)

$\therefore \begin{bmatrix} 1 \\ 6 \end{bmatrix} = \begin{bmatrix} c_1 + c_2 \\ c_1 - 4c_2 \end{bmatrix}$

$\therefore c_1 = 2, \ c_2 = -1$

$\therefore \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2e^{2t} - 1e^{-3t} \\ 2e^{2t} + 4e^{-3t} \end{bmatrix}$

Q) $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ with $\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\rightarrow \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Now Diagonize A

$|A - \lambda I| = 0 \quad \therefore \begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = 0$

$\therefore 16 - 8\lambda + \lambda^2 - 1 = 0$

$\therefore \lambda^2 - 8\lambda + 15 = 0$

$\therefore (\lambda - 3)(\lambda - 5) = 0$

$\therefore \lambda_1 = 3 \ \& \ \lambda_2 = 5$

for $\lambda_1 = 3$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$

$\therefore \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$

$\therefore y_1 + y_2 = 0$

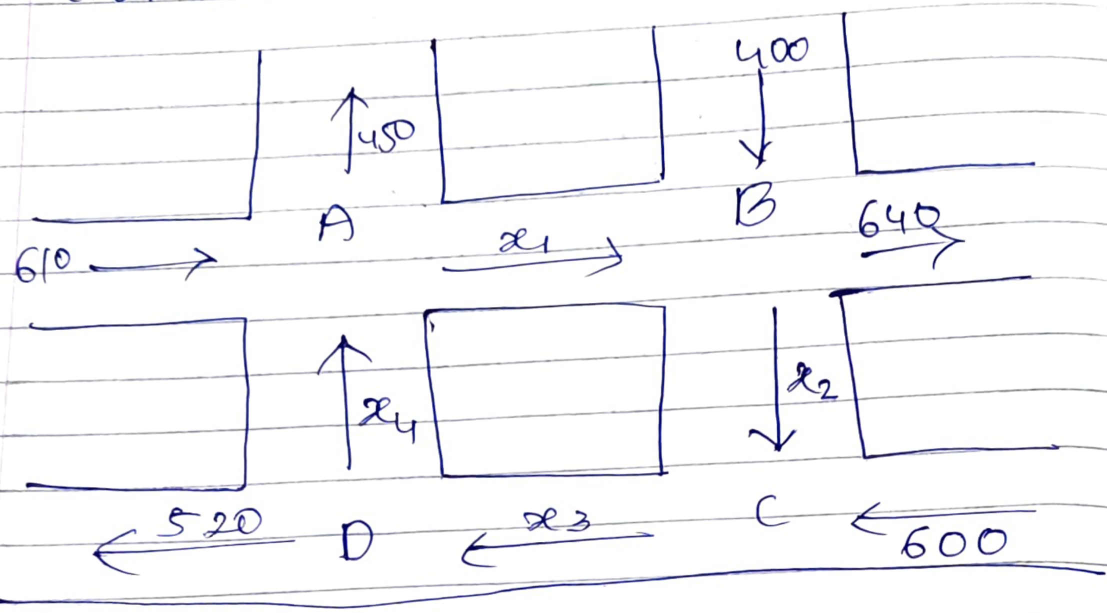
put $y_2 = -t$

$\therefore y_1 = t$

$\therefore V_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ ~~for~~.

Module 2

Applicatn of solving system of eqns in traffic control:-



Q) Determine the amount of traffic between each of the 4 intersections:-

→ At each intersection, the incoming traffic has to match the outgoing traffic then we write the nodes A, B, C & D as

At node A: $610 + x_4 = 450 + x_1$

" " B: $x_1 + 400 = 640 + x_2$

" " C: $x_2 + 600 = x_3$

" " D: $x_3 = x_4 + 520$

∴ we have $-x_1 + x_4 = -160$

$x_1 - x_2 = 240$

$x_3 - x_2 = 600$

$x_3 - x_4 = 520$

On simplificatn we get

$x_2 = x_1 - 240$

$x_3 = x_1 + 360$

$x_4 = x_1 + 160$

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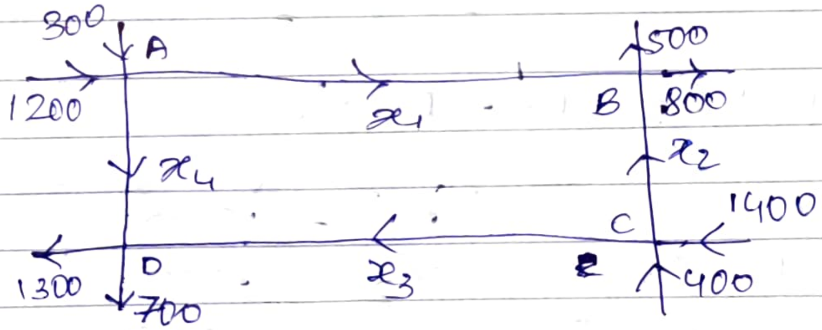
~~we need~~ from the above eqns
we needed to install only one sensor x_1 to count the no. of cars passing through lane A. Thus we can fully determine the traffic flow through this 4 branches. If we can count the no. of cars passing from A to B in a given time

Q] If x_1, x_2, x_3 & x_4 are the no. of vehicles travelling through each road per hour. find x_1, x_2, x_3 & x_4 from the traffic digream given below.



→ At each intersection, the incoming traffic has to match the outgoing traffic then we write the nodes A, B, C & D as

node A: $1200 + 300 = x_1 + x_4$

$x_1 + x_4 = 1500$

node C: $x_2 + x_3 = 1800$

node B = $x_1 + x_2 = 1300$

node D = $x_3 + x_4 = 2000$

On simplificate we get

$\therefore x_4 = 1500 - x_1$

$\therefore x_2 = 1300 - x_1$

$\therefore x_3 = x_1 + 500$

Using ~~xx~~ row Echelon form,

$AX = B$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1800 \\ 1300 \\ 2000 \end{bmatrix}$$

$R_3 \to R_3 - R_1$ & $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1800 \\ -2000 \\ 2000 \end{bmatrix}$$

$R_4 \to R_4 + R_3$; $R_2 \to R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1500 \\ -200 \\ -2000 \\ 0 \end{bmatrix}$$

~~000 x_1~~ + $R_4 \to R_4 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1800 \\ -2000 \\ 0 \end{bmatrix}$$

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Module 3

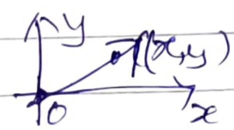
Vector Space.

One dimensional Space:-

r^1 Set of all points in space, on a line is called 1 dimensional Space

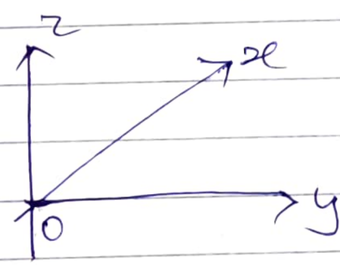
2 dimensional Space:-

r^2 The set of all ordered pair (x, y) where $-\infty < x < \infty$ & $-\infty < y < \infty$ is called 2 dimensional space.



3 dimensional Space:-

r^3 Set of all ordered ~~pair~~ triplets (x, y, z) where $-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$



Ordered n tuple:-

If n is a +ve integer then the sequence

r^n $x_1, x_2 \ldots x_n$ is called an ordered n tuple represents a vector or a point in n dimensional space denoted by r^n.

Zero space:-

The set of a single point zero is called as zero space.