Fast Fourier Transform

	TOPIC
1	Radix-2 Cooley & Tuckey's DIT-FFT Algorithm,
3	DIT-FFT Flowgraph for N=4 & 8,
3	Comparison of Complex and Real, Multiplication and Additions of DFT and FFT
4	Inverse FFT algorithm

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Chapter-2B : Fast Fourier Transform

Objective : To illustrate FFT calculations mathematically

Outcomes :

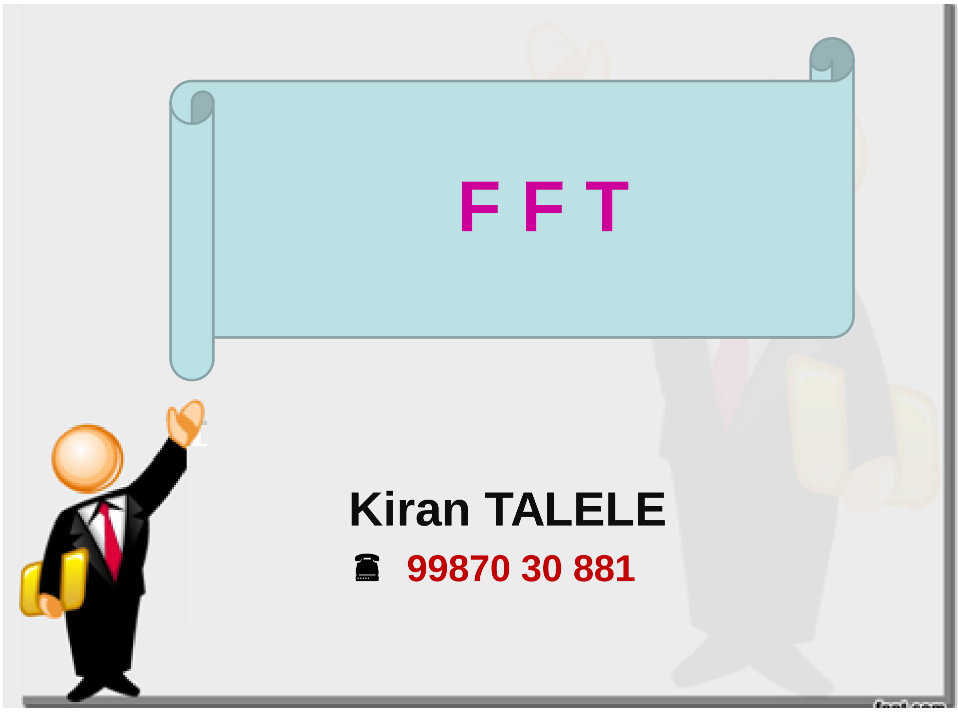
At the end of module, students will be able to ,

- **Develop** FFT flow-graph
- **Compare** DFT and FFT computationally
- **Perform** forward and Inverse FFT
- **Plot** signal spectrum in frequency domain

- In 1965, James W. Cooley and John W. Tukey (IEEE 1982 Medal of Honor recipient) published a paper describing the Fast Fourier Transform (FFT) algorithm, which led to an explosion in Digital Signal Processing.
- Their landmark research offered enormous improvements in processing speeds and played an essential role in the digital revolution.

James COOLEY

John TUKEY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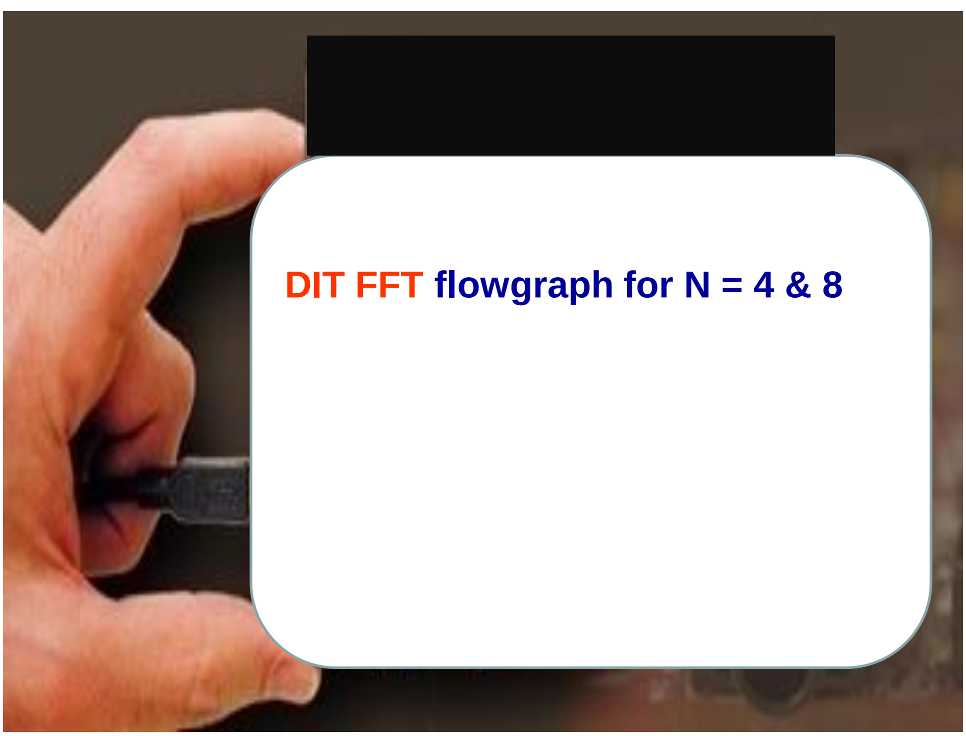

DIT FFT flowgraph for N = 4 & 8

DIT FFT flowgraph for N = 4

Step-1 : Derive DIT-FFT equation

- By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

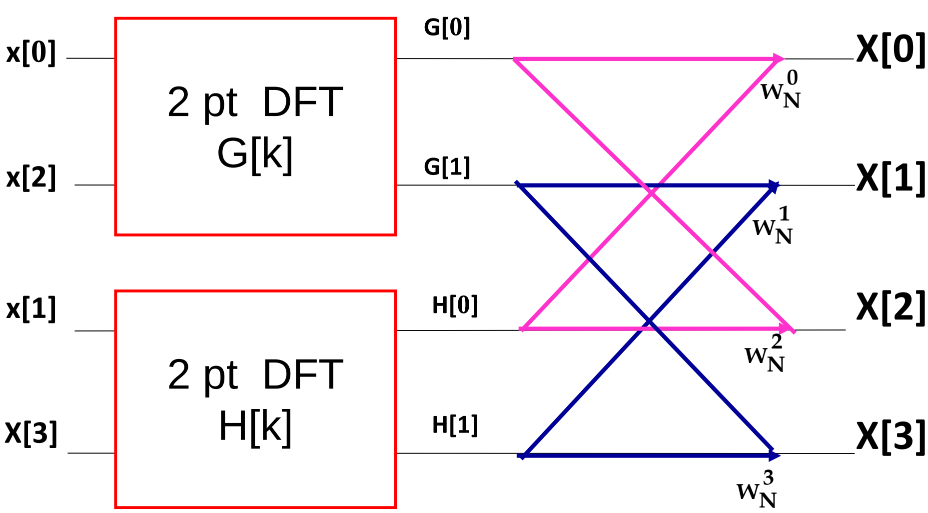
$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] \; W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] \; W_N^{(2r+1)k}$$

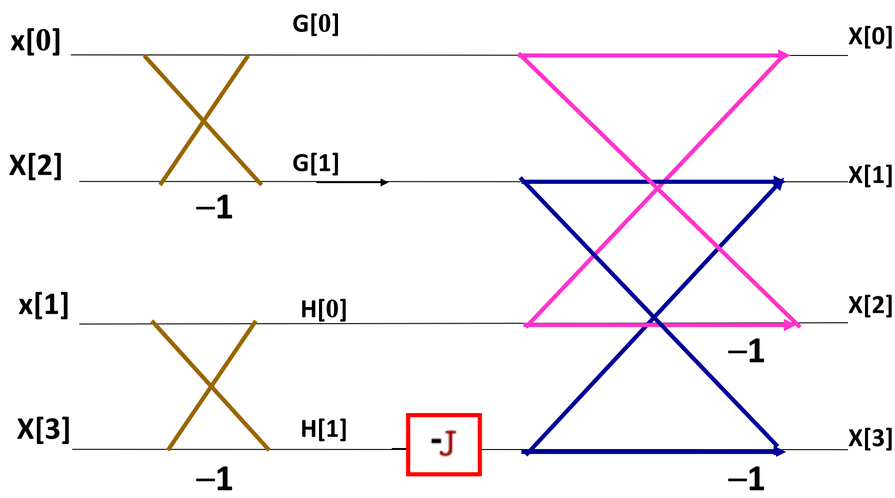
$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] \; W_{\frac{N}{2}}^{rk} + W_N^k \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] \; W_{\frac{N}{2}}^{rk}$$

$$\mathbf{X[k] = G[k] + W_N^k \; H[k]}$$

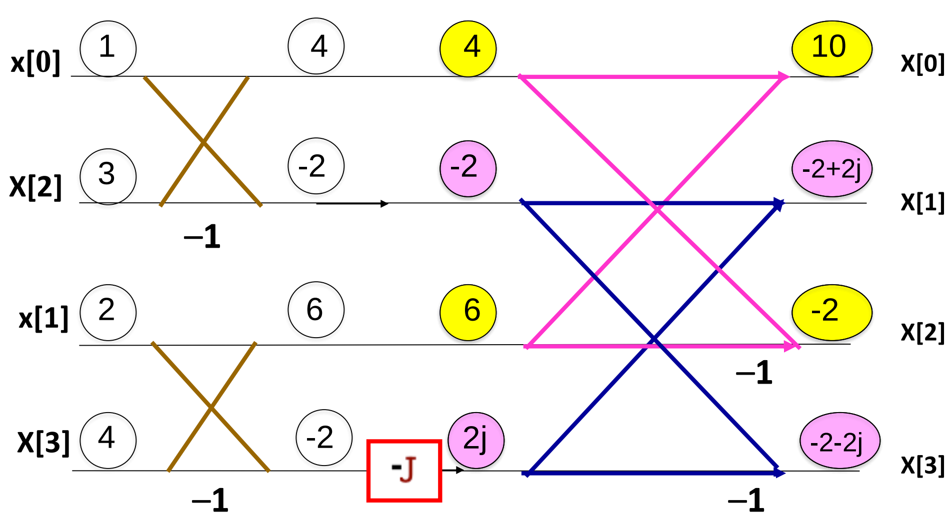
N pt $\quad \frac{N}{2}$ **pt** $\quad \frac{N}{2}$ **pt**

Step-2 : Derive DIT-FFT flowgraph



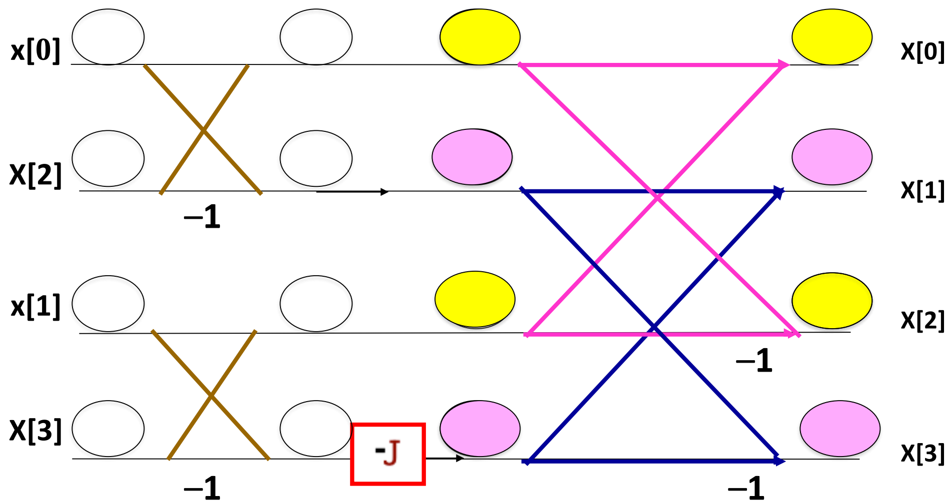


Ex-1 : Given x[n] = { 1, 2, 3, 4 }. Find X[k] using DITFFT



Ex-2 : Given x[n] = { 1, 4, 3, 2 }. Find X[k] using DITFFT

Solution : To Find X[k] using FFT



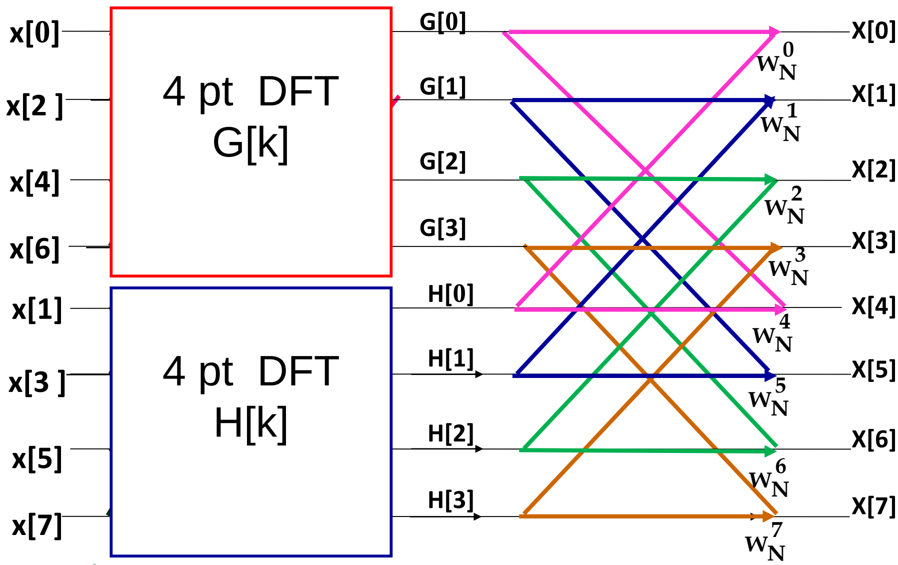
DIT FFT flowgraph for N = 8

Step-1 : Derive DIT-FFT equation

$$X[k] = G[k] + W_N^k \; H[k]$$

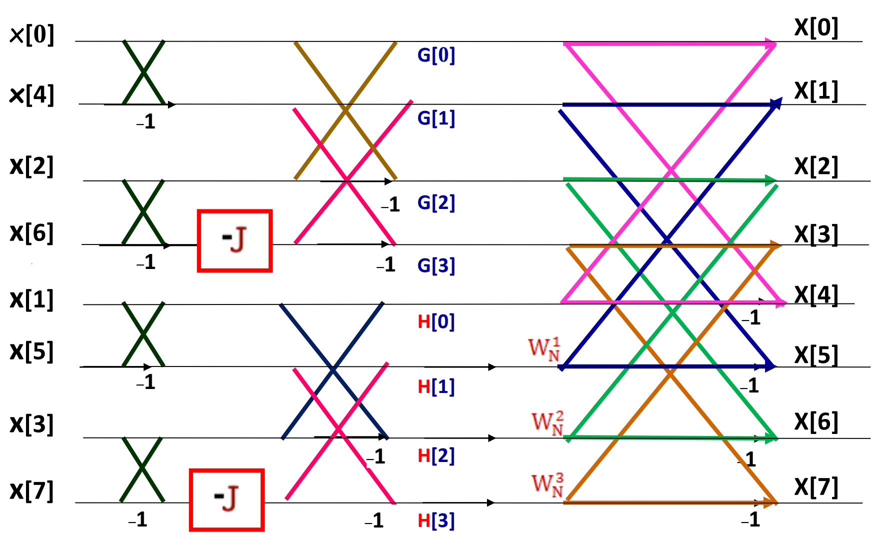
N pt $\quad \frac{N}{2}$ pt $\quad \frac{N}{2}$ pt

Step-2 : Derive DIT-FFT flowgraph



$$X[k] = G[k] + W_N^k \ H[k]$$

8 pt **4 pt** **4 pt**



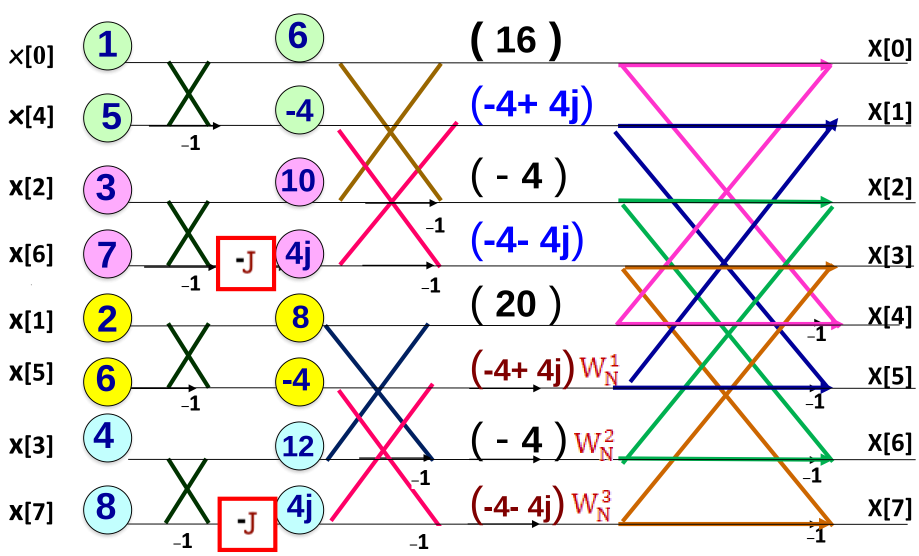
DIT-FFT Flowgraph for N = 8

Ex-1 : Given x[n] = { 1, 2, 3, 4, 5, 6, 7, 8 }.

Find X[k] using DIT-FFT.

Solution : To Find X[k] using DIT-FFT

Given x[n] = { 1, 2, 3, 4, 5, 6, 7, 8 }



Find X[k] :

(1) $X[0] = (16) + (20)$

$X(0) = 36$

(2) $X[1] = (-4+4j) + (-4+4j)\, W_N^1$

$= (-4+4j) + (-4+4j)(0.707 - j\,0.707)$

$X(1) = -4 + j\,9.656$

(3) $X[2] = (-4) + (-4)\, W_N^2$

$= (-4) + (-4)(-j)$

$X(2) = -4 + 4j$

(4) $X[3] = (-4-4j) + (-4-4j)\, W_N^3$

$= (-4-4j) + (-4-4j)(-0.707 - j\,0.707)$

$X[3] = -4 + j\,1.656$

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(5) $X[4] = (16) - (20)$

$X[4] = -4$

(6) $X[5] = (-4-4j) - (-4-4j)\, W_N^1$

$= (-4-4j) - (-4-4j)[0.707 - j\,0.707]$

$X[5] = -4 - j\,1.656$

(7) $X[6] = (-4) - (-4)\, W_N^2$

$= (-4) - (-4)[-j]$

$X[6] = -4 - 4j$

(8) X[7] = (-4-4j) − (-4-4j) W_N^3

X[7] = (-4-4j) − (-4-4j) [-0.707-j 0.707]

X[7] = − 4 − j 9.656

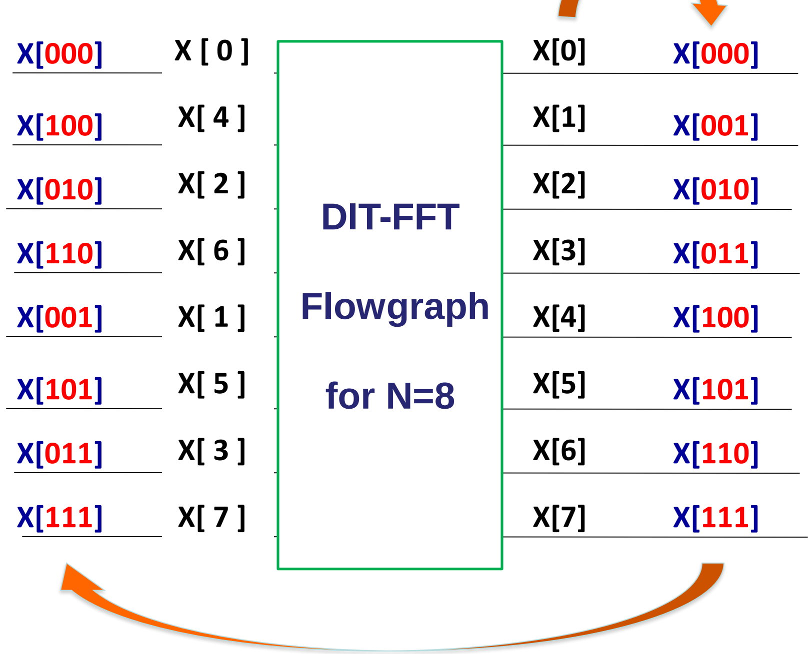
ANS :

$$X[k]=\begin{bmatrix} 36 & k=0 \\ -4+9.656j & \\ -4+4j & \\ -4+1.656j & \\ -4 & \\ -4-1.656j & \\ -4-4j & \\ -4-9.656j & \end{bmatrix}$$

Computational Efficiency of FFT

N	Complex Multiplications		Speed Improvement factor.
	By DFT	By FFT.	
4	16	4	4.0
8	64	12	5.3
16	256	32	8.0
32	1024	80	12.8
64	4096	192	21.3
128	16384	448	36.6
256	65536	1024	64.0

Bit Reversal Technique :



Bit Reversal Technique :

Input with Index in Binary	Input sequence		Output Sequence	Output with Index in Binary
x[0 0 0]	x[0]		X[0]	X[0 0 0]
x[1 0 0]	x[4]		X[1]	X[0 0 1]
x[0 1 0]	x[2]	**DIT-FFT**	X[2]	X[0 1 0]
x[1 1 0]	x[6]	Flowgraph	X[3]	X[0 1 1]
x[0 0 1]	x[1]	For	X[4]	X[1 0 0]
x[1 0 1]	x[5]	N = 8	X[5]	X[1 0 1]
x[0 1 1]	x[3]		X[6]	X[1 1 0]
X[1 1 1]	x[7]		X[7]	X[1 1 1]

Inverse FFT Algorithm

Using **Forward** FFT Flowgraph

Using **Inverse** FFT Flowgraph

Inverse FFT using Forward FFT Flowgraph

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-nk}$$

By Complex Conjugate on Both Sides,

$$x^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} X^*[k]\, W_N^{nk}$$

$$x^*[n] = \frac{1}{N}\,\text{FFT}\{X^*[k]\}$$

IFFT ALGORITHM

I. Find $X^*[k]$
II. Find FFT ($X^*[k]$)
III. Find x[n] using IFFT equation

By Complex Conjugate on Both Sides,

$$\boxed{x[n] = \frac{1}{N}\left(\text{FFT}\{X^*[k]\}\right)^*}$$

This is an IFFT equation

Ex-1. Given $X[k] = \begin{bmatrix} 66 & k=0 \\ -22+2j & \\ -2 & \\ -22-2j & \end{bmatrix}$

Find x[n] using Forward FFT.

Solution : To find x[n]

By IFFT equation:

$$x[n] = \frac{1}{N}\left(FFT\{X^*[k]\}\right)^*$$

IFFT ALGORITHM

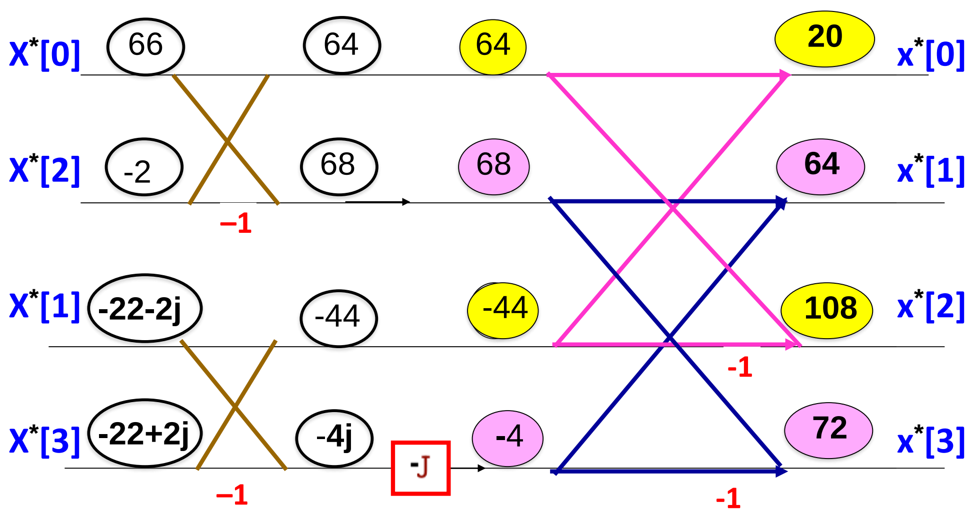
I. Find $X^*[k]$
II. Find FFT ($X^*[k]$)
III. Find x[n] using IFFT equation

I. Find $X^*[k]$

Now, $X[k] = \begin{bmatrix} 66 & k=0 \\ -22 + 2j & \\ -2 & \\ -22 - 2j & \end{bmatrix}$

$X^*[k] = \begin{bmatrix} 66 & k=0 \\ -22 - 2j & \\ -2 & \\ -22 + 2j & \end{bmatrix}$

II. Find FFT (X*[k])



III. Find x[n]

By IFFT :

$$x[n] = \frac{1}{N}\left(\text{FFT}\{X^*[k]\}\right)^*$$

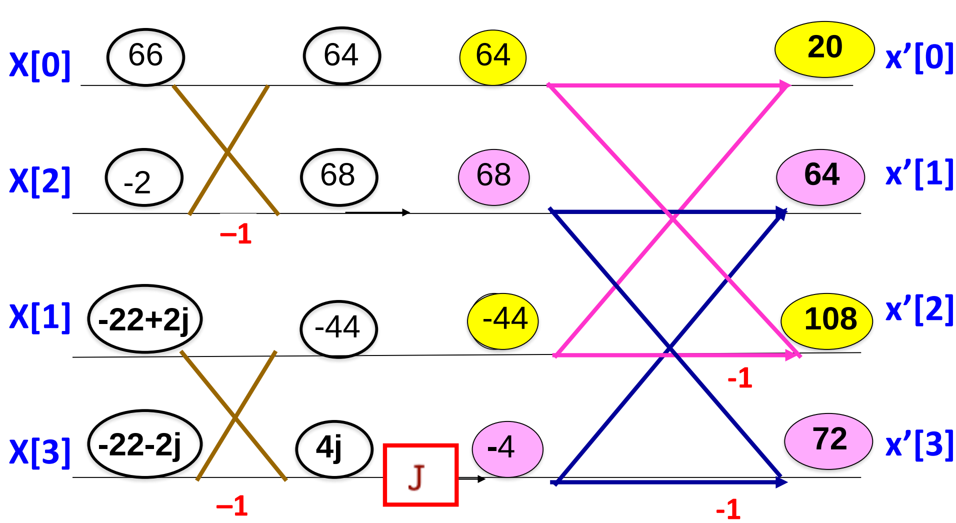
$$x[n] = \frac{1}{4}\left(\begin{bmatrix} 20 & n=0 \\ 64 & \\ 108 & \\ 72 & \end{bmatrix}\right)^*$$

$$x[n] = \begin{bmatrix} 5 & n=0 \\ 16 & \\ 27 & \\ 18 & \end{bmatrix}$$

Ex-2. Given $X[k] = \begin{bmatrix} 66 & k=0 \\ -22+2j & \\ -2 & \\ -22-2j & \end{bmatrix}$

Find x[n] using Inverse FFT.

To Find x[n] using Inverse FFT



III. Find x[n]

By IFFT : $x[n] = \frac{1}{N} x'[n]$

$$x[n] = \frac{1}{4} \left(\begin{bmatrix} 20 & n=0 \\ 64 & \\ 108 & \\ 72 & \end{bmatrix} \right)$$

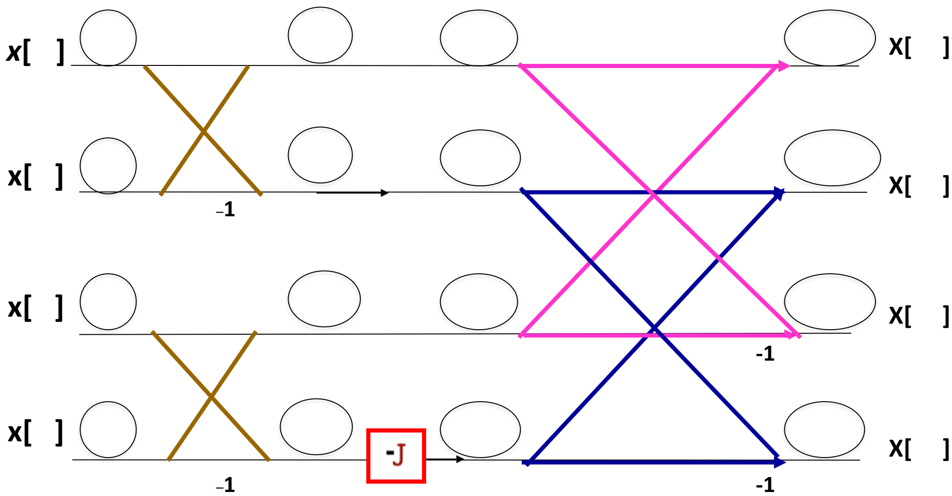
$$x[n] = \begin{bmatrix} 5 & n=0 \\ 16 & \\ 27 & \\ 18 & \end{bmatrix}$$

Ex : Let x[n] = {1, 2, 3, 4}

(a) Find X[k] using DIT-FFT.
(b) Let p[n] = { 1, 0, 2, 0, 3, 0, 4, 0 }.
Find P[k] using X[k].

Solution : (a) To finds X[k] using DITFFT

Given x[n] = { 1, 2, 3, 4 }.



(b) Let p[n] = { 1, 0, 2, 0, 3, 0, 4, 0 }.
Find P[k] using X[k].

- **To find P[k]**

Let $$\underset{8\text{ pt}}{P[k]} = \underset{4\text{ pt}}{G[k]} + W_N^k \underset{4\text{ pt}}{H[k]}$$ **---Eqn (1)**

Where G[k] = DFT{ p(2r) } and H[k] = DFT { p(2r+1) }

$$G[k] = \text{DFT}\begin{bmatrix} p[0] \\ p[2] \\ p[4] \\ p[6] \end{bmatrix} \qquad H[k] = \text{DFT}\begin{bmatrix} p[1] \\ p[3] \\ p[5] \\ p[7] \end{bmatrix}$$

$$G[k] = \text{DFT}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad H[k] = \text{DFT}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

G[k] = X[k] **H[k] = 0**

By Substituting G[k] = X[k] and H[k] = 0 in Eqn (1) we get,

P[k] = X[k]

P[0] = X[0] = = 10

P[1] = X[1] = = -2+2J

P[2] = X[2] = = -2

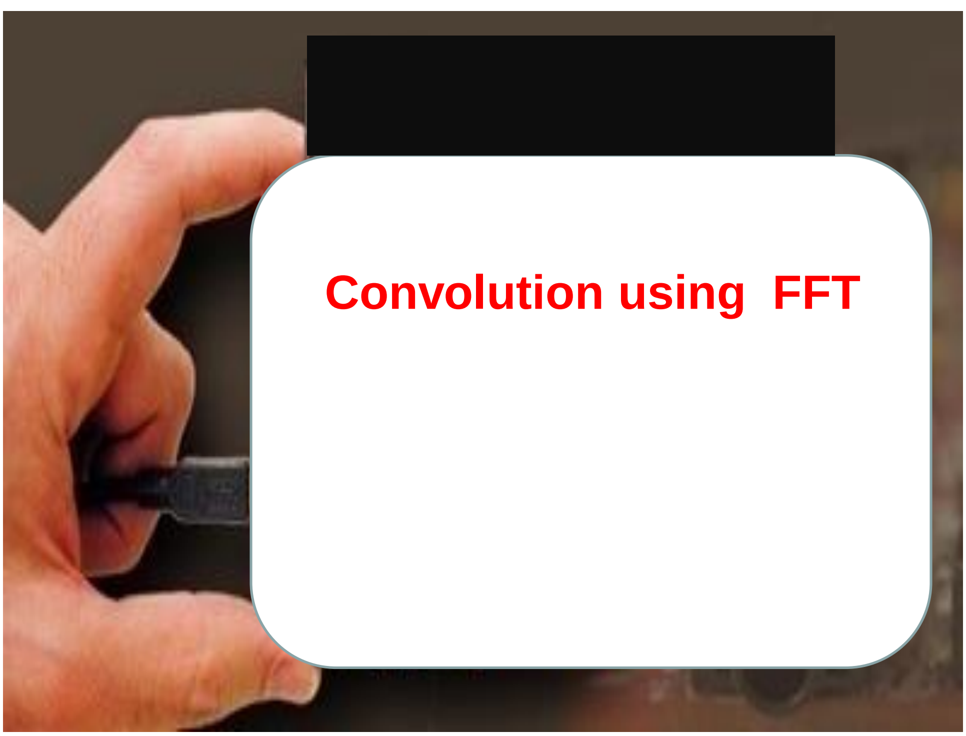
P[3] = X[3] = = -2-2J

P[4] = X[4] = = 10

P[5] = X[5] = = -2+2J

P[6] = X[6] = = -2

P[7] = X[7] = = -2-2J



Ex-1 **Let x[n] = [1, 2, 3, 4 } and h[n] = { 5, 6, 7 }**
Find Circular Convolution using FFT

Solution :

Here x[n] is L=4 point and h[n] is M = 3 point

I. Select N

N = Max (L,M)

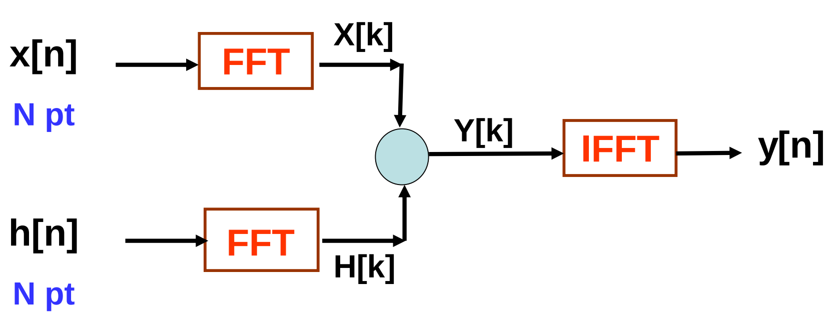
N = Max (4,3) == **4**

II. **Zero Padding**

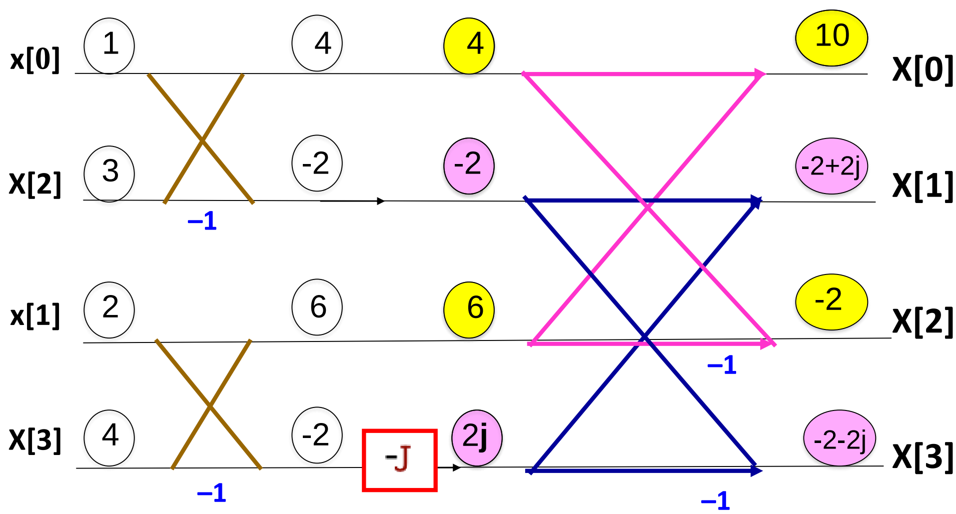
x[n] = [1, 2, 3, 4 }

h[n] = { 5, 6, 7, 0 }

III. **Find y[n] = x[n] ⊗ h[n] using FFT**



(1). **Find X[k] using DIT-FFT**



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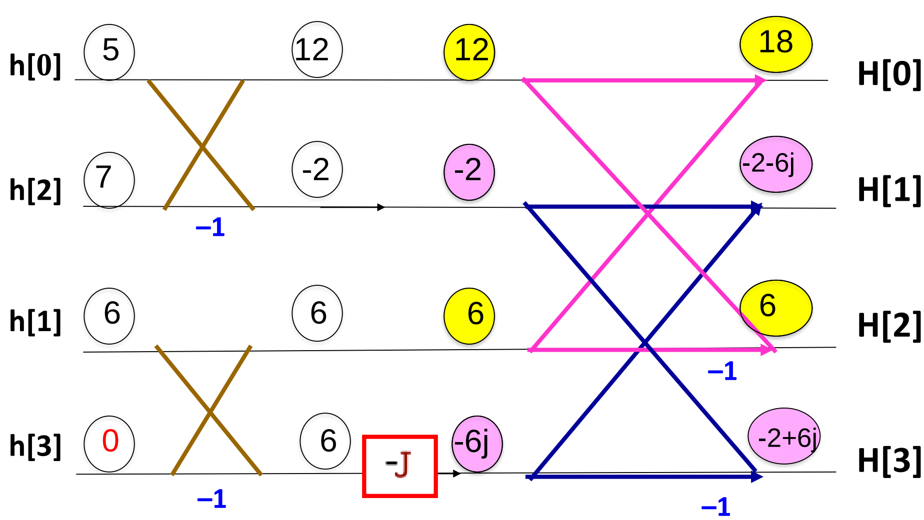
(2). **Find H[k] using DIT-FFT**



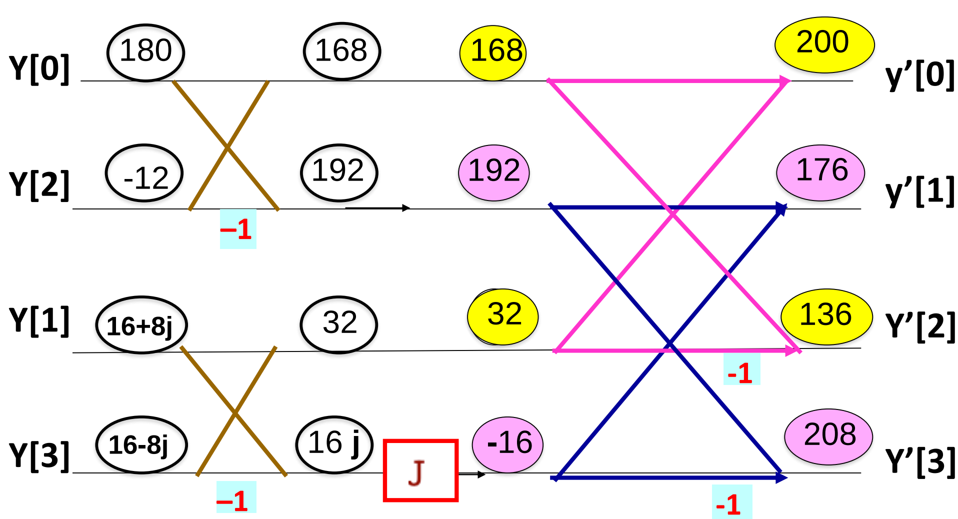
(3). **Find Y[k]**

$$Y[k] = X[k]\ H[k]$$

$$Y[k] = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}\begin{array}{l} k=0 \\ \\ \\ \\ \end{array} \begin{bmatrix} 18 \\ -2-6j \\ 6 \\ -2+6j \end{bmatrix}\begin{array}{l} k=0 \\ \\ \\ \\ \end{array}$$

$$Y[k] = \begin{bmatrix} 180 \\ 16+8j \\ -12 \\ 16-8j \end{bmatrix}\begin{array}{l} k=0 \\ \\ \\ \\ \end{array}$$

III. **Find y[n] By Inverse FFT**



Now $y[n] = \frac{1}{N} y'[n]$

$$y[n] = \frac{1}{4}\begin{bmatrix} 200 \\ 176 \\ 136 \\ 208 \end{bmatrix} \; n=0$$

$$y[n] = \begin{bmatrix} 50 \\ 44 \\ 34 \\ 52 \end{bmatrix} \; n=0$$

To verify :

x[n] = [1, 2, 3, 4 }

h[n] = { 5, 6, 7, 0 }

$$y[n] = \begin{bmatrix} 5 & 0 & 7 & 6 \\ 6 & 5 & 0 & 7 \\ 7 & 6 & 5 & 0 \\ 0 & 7 & 6 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$y[n] = \begin{bmatrix} 50 \\ 44 \\ 34 \\ 52 \end{bmatrix}$$

Ex-1 Let x[n] = [1, 2, 3 } and h[n] = { 5, 6 }

Find Linear Convolution using FFT

Solution :

Here x[n] is L= 3 point and h[n] is M = 2 point

I. **Select N**

$N \geq L+M-1$

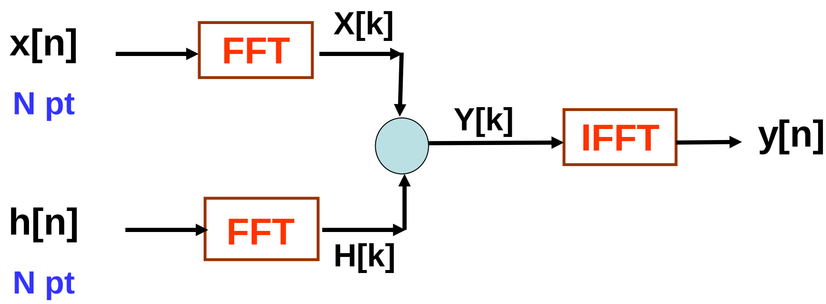
$N \geq 3+2-1 == 4$

II. **Zero Padding**

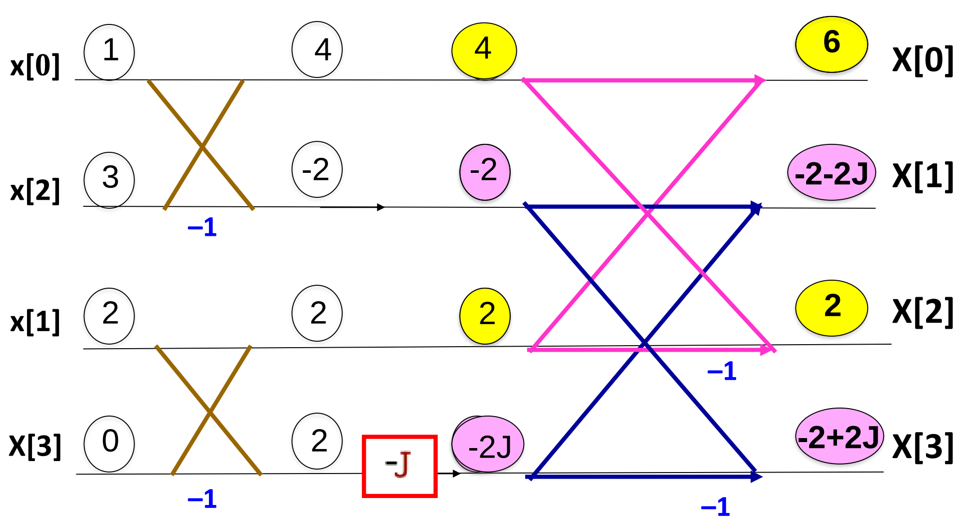
x[n] = [1, 2, 3, 0 }

h[n] = { 5, 6, 0, 0 }

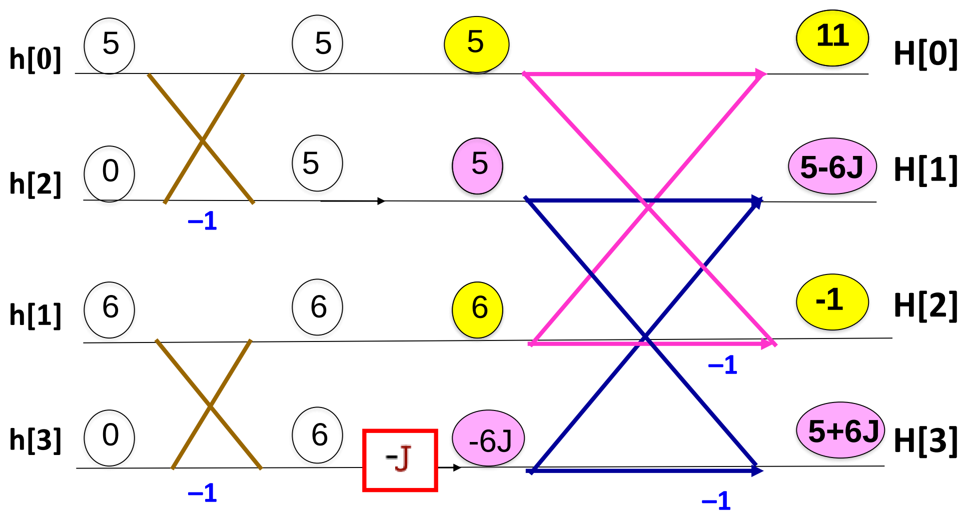
III. Find y[n] = x[n] ⊗ h[n] using FFT



(1). Find X[k] using DIT-FFT



(2). Find H[k] using DIT-FFT



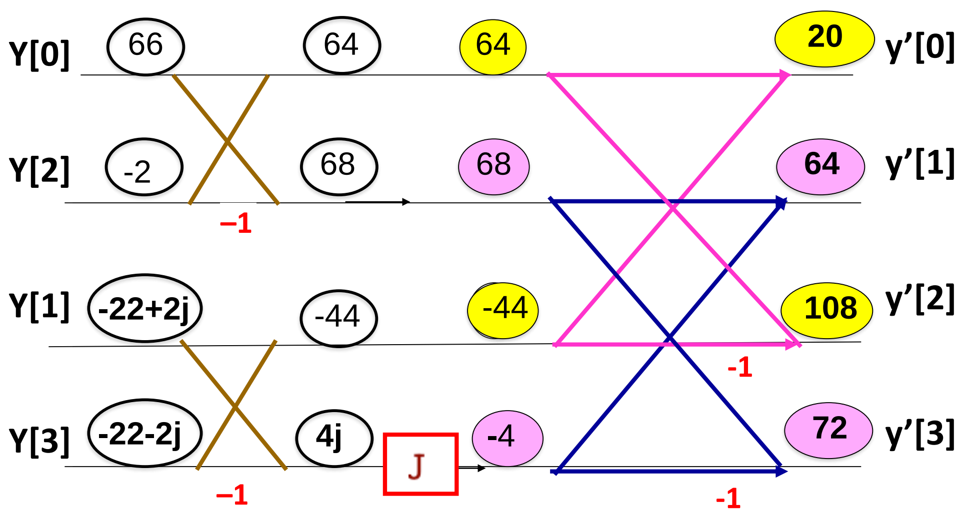
(3). Find Y[k]

$$Y[k] = X[k]\ H[k]$$

$$Y[k] = \begin{bmatrix} 6 & k=0 \\ -2-2j & \\ 2 & \\ -2+2j & \end{bmatrix} \begin{bmatrix} 11 & k=0 \\ 5-6j & \\ -1 & \\ 5+6j & \end{bmatrix}$$

$$Y[k] = \begin{bmatrix} 66 & k=0 \\ -22+2j & \\ -2 & \\ -22-2j & \end{bmatrix}$$

III. Find y[n] using Inverse FFT



Now $y[n] = \frac{1}{N}\, y'[n]$

$$y[n] = \frac{1}{4}\begin{bmatrix}20\\64\\108\\72\end{bmatrix}\begin{matrix}n=0\\ \\ \\ \\ \end{matrix}$$

$$y[n] = \begin{bmatrix}5\\16\\27\\18\end{bmatrix}\begin{matrix}n=0\\ \\ \\ \\ \end{matrix}$$

To verify :

x[n]		1	2	3	y[n]
h[-n]	6	5			5
		6	5		16
			6	5	27
				6	18

APLICATIONS OF D F T / F F T

(i) Linear Filtering : To find output of a digital FIR filter for any given input say x[n].

(ii) Spectral Analysis : To find magnitude spectrum and phase spectrum of signal.

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Linear Filtering : To find output of a digital FIR filter for any given input say x[n]

x[n] —— **Digital FIR Filter h[n]** —— **y[n]=x[n]* h[n]**

LC by LC Formula Method

LC by CC Formula Method

LC by DFT Formula Method

LC by FFT Algorithm

None of above mehod is suitable for Real Time Application

Limitations of LC by FFT Algorithms

(1) Algorithm is NOT suitable for Real Time Applications where entire input signal is not available.

Examples include

(i) ECG Monitoring system

(ii) Digital Telephone System

(iii) Weather Monitoring System

Limitations of LC by FFT Algorithms...

(2) Algorithm is NOT suitable for Long Data Sequence.

Examples include
i) Digital Song in the form of wave file (Fs = 44.1 KHz)
ii) ECG/Weather Monitoring Systems.

In most of the real Time applications data is Long sequence.

If N is too large as for long data sequences,
then there is a significant delay in processing, that will make processing, almost impossible for real-time applications

Linear FIR filtering using FAST Algorithm

x[n] —— **Digital FIR Filter h[n]** —— **y[n]=x[n]* h[n]**

Overlap Add Method

Overlap Save Method

Overlap Add fast Convolution Algorithm

I. Decompose x[n] into L point sequences

II. Select N

III. Append h[n] with (N-M) zeros and find H [k] using N pt FFT flowgraph

IV. Append each input signal data block by (N-L) zeros and find DFT of each block using N pt FFT algorithm

V. Let Yi [k] = Xi [k] . H [k]   for i = 0,1…..

VI. Obtain $y_i[n]$ by N pt iFFT Algorithm

VII. Find y[n] i.e. $y[n] = \sum_{i=0}^{N-1} y_i[n - iL]$

Ex. Given

$h[n] = \delta[n] + 2\,\delta[n-1] + 3\,\delta[n-2] + 4\,\delta[n-3]$

Find the response of the filter to the input x[n]={ 2, 0, 2, 0, 2, 1, 0, 2, 1 } using Overlap Add Method.

Solution : To Output of the filter i.e. y[n]

Given h[n]= { 1, 2, 3, 4 }. Length M = 4
and x[n] = { 2, 0, 2, 0, 2, 1, 0, 2, 1 }

(I) Select L

Assume N = 8 for Radix 2 FFT Algorithm

Now **N = L + M -1**

8 = L + 4 – 1

L = 5

Given h[n]= { 1, 2, 3, 4 }. Length M = 4
and x[n] = { 2, 0, 2, 0, 2 , 1, 0, 2, 1 }

(II) Decompose x[n]

By decomposing x[n] into L=5, we get,

$x_1[n]$ ={ 2, 0, 2, 0, 2 }

$x_2[n]$ ={ 1, 0, 2, 1, 0 }

(III) Zero Padding

$x_1[n] = \{\ 2,\ 0,\ 2,\ 0,\ 2,\ \underbrace{0,\ 0, 0}\ \}$

$x_2[n] = \{\ 1,\ 0,\ 2,\ 1,\ 0,\ \underbrace{0,\ 0, 0}\ \}$

h[n] = { 1, 2, 3, 4, **0, 0**, **0, 0** }.

h[-n] = { 1, **0, 0, 0, 0**, 4, 3, 2 }

(IV) Find $y_1[n]$

$$y_1[n] = \sum_{m=0}^{N-1} x_1[m]\ h[((n-m))]$$

$$y_1[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} == \begin{bmatrix} 2 \\ 4 \\ 8 \\ 12 \\ 8 \\ 12 \\ 6 \\ 8 \end{bmatrix}$$

(V) Find $y_2[n]$

$$y_2[n] = \sum_{m=0}^{N-1} x_2[m] \; h[((n-m))]$$

$$y_2[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} == \begin{bmatrix} 1 \\ 2 \\ 5 \\ 9 \\ 8 \\ 11 \\ 4 \\ 0 \end{bmatrix}$$

(VI) Find y[n]

Now $y[n] = y_1[n] + y_2[n-L]$

Put L = 5

$y[n] = y_1[n] + y_2[n-5]$

To find y[n] :

$y_1[n]$ = { 2 4 8 12 8 12 6 8 }

\+

$y_2[n-5]$= { 0 0 0 0 0 1 2 5 9 8 11 4 0 }

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y[n] = { 2, 4, 8,12, 8, 13, 8, 13, 9, 8, 11, 4, 0 }

- **Overlap Save fast Convolution Algorithm**

1. Decompose x[n] into L point sequences
2. Select $N \geq L + M + 1$
3. Begin each decomposed input sequence with (N–L) values of previous sequence
4. Append h[n] with (N–M) zeros and find H[k] using N point FFT.
5. Perform N – point FFT on the selected data block X_i [n]
6. Let Yi[k] = Xi[k] . H[k]
7. Perform N point iFFT of Yi[k]
8. Discard the first (N–L) values of y_i [n] and save the remaining values of y_i [n]
9. y[n] is obtained by concatenating all the saved values of y_i[n]

Ex. Given

$h[n] = \delta[n] + 2\,\delta[n-1] + 3\,\delta[n-2] + 4\,\delta[n-3]$

Find the response of the filter to the input x[n]={ 2, 0, 2, 0, 2, 1, 0, 2, 1 } using Overlap Save Method.

Solution : To Output of the filter i.e. y[n]

Given h[n]= { 1, 2, 3, 4 }. Length M = 4
and   x[n] = { 2, 0, 2, 0, 2, 1, 0, 2, 1 }

(I) Select L

Assume N = 8 for Radix 2 FFT Algorithm

Now N = L + M -1

8 = L + 4 – 1

L = 5

Given h[n]= { 1, 2, 3, 4 }. Length M = 4
and   x[n] = { 2, 0, 2, 0, 2 , 1, 0, 2, 1 }

(II) Decompose x[n]

By decomposing x[n] into L=5, we get,

$x_1[n]$ ={ 2, 0, 2, 0, 2 }

$x_2[n]$ ={ 1, 0, 2, 1, 0 }

$x_3[n]$ ={ 0, 0, 0, 0, 0 }

(III) Modify input Sequence

$x_1[n] = \{\ 0,\ 0,\ 0,\ 2,\ 0,\ \underbrace{2,\ 0,\ 2}\ \}$

$x_2[n] = \{\ \overbrace{2,\ 0,\ 2,}\ 1,\ 0,\ \underbrace{2,\ 1,\ 0}\ \}$

$x_3[n] = \{\ \overbrace{2, 1, 0,}\ 0,\ 0,\ 0,\ 0,\ 0\ \}$

h[n] = { 1, 2, 3, 4, **0, 0**, **0, 0** }.

h[-n] = {1, **0, 0, 0, 0**, 4, 3, 2 }

(IV) Find $y_1[n]$

$$y_1[n] = \sum_{m=0}^{N-1} x_1[m]\ h[((n-m))]$$

$$y_1[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 2 \\ 0 \\ 2 \end{bmatrix} == \begin{pmatrix} 12 \\ 6 \\ 8 \\ 2 \\ 4 \\ 8 \\ 12 \\ 8 \end{pmatrix}$$

(V) Find $y_2[n]$

$$y_2[n] = \sum_{m=0}^{N-1} x_2[m]\ h[((n-m))]$$

$$y_2[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} == \begin{bmatrix} 13 \\ 8 \\ 8 \\ 13 \\ 8 \\ 13 \\ 9 \\ 8 \end{bmatrix}$$

(VI) Find $y_3[n]$

$$y_3[n] = \sum_{m=0}^{N-1} x_3[m]\ h[((n-m))]$$

$$y_3[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} == \begin{bmatrix} 2 \\ 5 \\ 8 \\ 11 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(VII) Find y[n]

$$y_1[n] = \{ \cancel{12, 6, 8}, \underbrace{2, 4, 8, 12, 8} \}$$

$$y_2[n] = \{ \cancel{13, 8, 8}, \underbrace{13, 8, 13, 9, 8} \}$$

$$y_3[n] = \{ \cancel{2, 5, 8}, \underbrace{11, 4, 0, 0, 0} \}$$

By discarding first (M-1) values from each output sequence we get,

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$$y[n] = \{ \underset{\uparrow}{2}, 4, 8, 12, 8, \underbrace{13, 8, 13, 9, 8}, \underbrace{11, 4, 0, 0, 0} \}$$

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