b)

* Plot Magnitude spectrum :-

⇒ For $x(n) = \{1, 2, 3, 4\}$ ⟵ Time Domain Sequence

$$X(k) = \left[\begin{array}{ll|l} 10 & k=0 & \omega = 0 \\ -2+2j & k=1 & \omega = \pi/2 \\ -2 & k=2 & \omega = \pi \\ -2-2j & k=3 & \omega = 3\pi/2 \end{array}\right.$$

↓
Frequency Domain

$\because \omega = \frac{2\pi k}{N}$ $(N=4)$

$\therefore \omega = \frac{2\pi k}{4} = \frac{\pi k}{2}$

★ Sampling Frequency depends on Maximum Frequency.
★ In Discrete Time signal, Frequency and Time are not related.

⇒ We write for k and not do not write frequency because $\omega = \frac{2\pi k}{N}$

$\frac{2\pi}{N}$ is constant, k is something which is changing.

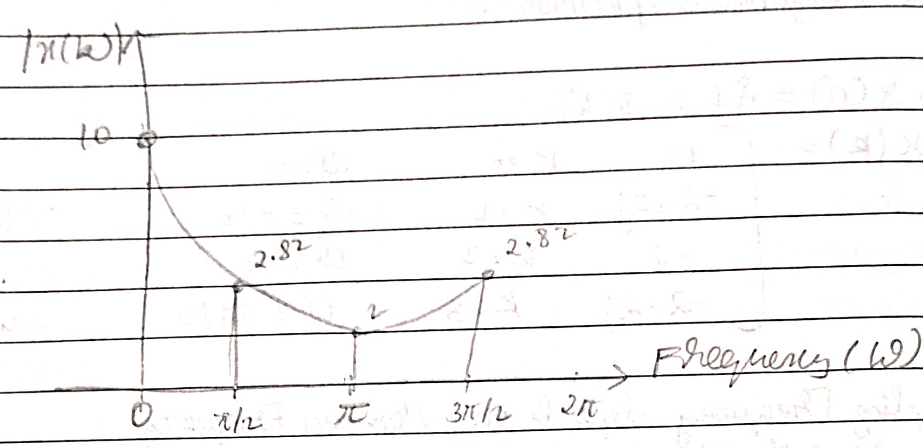
⇒ $$X(k) = \left[\begin{array}{ll|l} 10 & k=0 & \omega = 0 \\ 2\sqrt{2} & k=1 & \omega = \pi/2 \\ 2 & k=2 & \omega = \pi \\ 2\sqrt{2} & k=3 & \omega = 3\pi/2 \end{array}\right]$$

We need to calculate real part (magnitude) = $|a+ib| = \sqrt{a^2+b^2}$

Magnitude Spectrum :-

⇒ Gives various frequency component present in the signal.
⇒ The strength of signal depends on Magnitude
⇒ Magnitude spectrum is a continuous spectrum, i.e :- ω for every point.

Magnitude Spectrum.



∵ Magnitude spectrum is continuous, we connect the points freehand.

Magnitude Representation of DFT & IDFS

IMP ⇒ DFT (Discrete Fourier Theorm) :- . gives Discrete spectrum
· gives Approx Spectrum ; · gives Periodic Signal.

⇒ IDFT (Inverse Discrete Fourier Theorm)

eg:- $x(n) = [1, 2, 3, 4]$
Find $x(k)$

Ans To find $x(k)$:-

⇒ $$\boxed{x(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}}$$

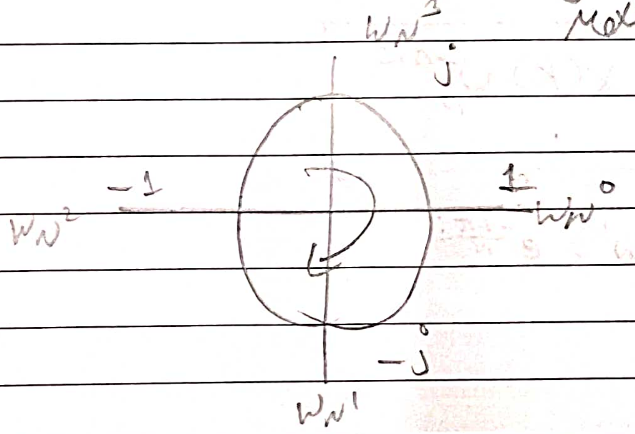
where $N = 4$ or
$W_N' = e^{-j 2\pi/N}$

$$X(k) = x(0)W_N^0 + x(1)W_N^k + x(2)W_N^{2k} + x(3)W_N^{3k}$$

In Matrix Form:-

$$\begin{array}{c} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array}\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \underbrace{\begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix}}_{\text{Matrix Form}} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$X(k) \qquad\qquad\qquad\qquad x(n)$$



$$\therefore \begin{array}{c} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array}\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$X(k) = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3+4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2-2j \\ -2 \\ -2-2j \end{bmatrix} \begin{array}{c} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array}$$

//

Q2) Eg:- Given $x(k) = \begin{bmatrix} -10 \\ -2-2j \\ -2 \\ -2-2j \end{bmatrix}, \begin{bmatrix} k=0 \\ k=1 \\ k=2 \\ k=3 \end{bmatrix}$

Calculate $x(n)$:-

Ans To find $x(n)$:-

By Inverse DFT :- (IDFT) :-

IMP

$$\boxed{x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-nk}}$$

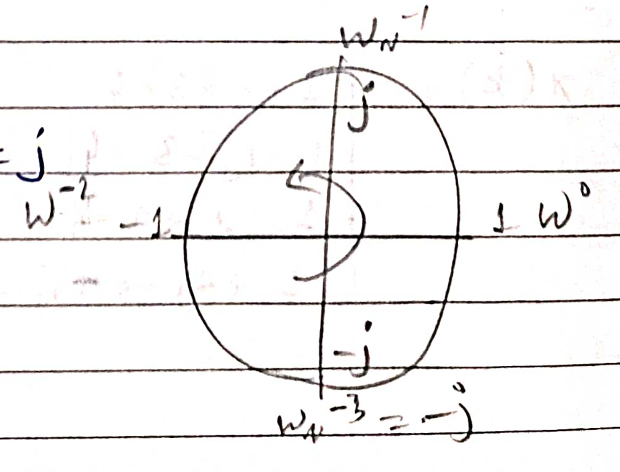
where $N=4$; $W_N^1 = e^{\frac{-2\pi}{N} j}$

In Matrix Form :-

$$x(n) = \frac{1}{N} \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} & W_N^{-3} \\ W_N^0 & W_N^{-2} & W_N^{-4} & W_N^{-6} \\ W_N^0 & W_N^{-3} & W_N^{-6} & W_N^{-9} \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}$$

$W_N^1 = -j$

$W_N^{-1} = \frac{1}{W_N^1} = \frac{1}{-j} = j$



$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} -10 \\ -2-2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -10 -2-2j -2 -2-2j \\ -10 + j(-2-2j) + 2 - j(-2-2j) \\ -10 -1(-2-2j) - 2 - 1(-2-2j) \\ -10 - j(-2-2j) + 2 + j(-2-2j) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -16-4j \\ -10+2 + \cancel{2} - \cancel{2} - 2j + 2j \\ -10 + \cancel{2} + \cancel{2} + 2 + 2j + 2j \\ -10 - \cancel{2} + \cancel{2} + 2 - 2j - 2j \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -4(4+j) \\ -8 \\ -8+4j \\ -8-4j \end{bmatrix}$$

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* NOTE:-

⇒ $x[n] = \{1, \underset{\uparrow}{2}, 3, 4\}$

- To calculate DFT for this :-

First find DTFT :-

then ~~#~~ convert DTFT to DFT by replacing ω by :-

* **Properties :-**

1) **Linear & Scaling Property :-**

$x_1[n] = X_1[k]$
$x_2[n] = X_2[k]$

$p[n] = x_1[n] + x_2[n]$

⇒ $\boxed{\therefore P[k] = X_1[k] + X_2[k]}$ IMP

eg:- $q[n] = 2 + x[n]$, Find $q[k]$ using $x[k]$
⇒ Pg 9.

2) **Periodicity Property :-**

⇒ DFT & IDFT produces periodic signals
Pg 10.

3) Time Shift Property
4) Frequency Shift Property
5) Time Reversal Property.
6) Symmetry Property
7) Even Signal Property
8) Odd Signal Property
9) Complex Conjugate Sequence Property