maths

The purpose here is to derive mathematically the expected performance of a decoder on pre-stimulus data in a ordered sequence of stimuli such as implemented in Demarchi et al. 2019.

Under null hypothesis

We assume the null hypothesis is true: there is only sensory but no predictive/anticipatory activity.

Let p be the probability the decoder finds the correct input. We assume $p_A = p_B = p_C = p_D$, i.e. the decoder performs equally on all stimuli types. p is the *sensitivity* of the classifier.

Based on the transition matrix of the ordered sequenece in Demarchi et al. 2019, we consider the following pairs of consecutive stimuli and their occurrence rates :

S_{-1}	S_0	occurrence rate
A	A	0.25
B	A	0
C	A	0
D	A	0.75

Equiprobable errors

We further assume that when the classifier fails, it might return any other class with equal probability, i.e.

$$\forall Y \neq X, p(Y|X) = \frac{1-p}{3}$$

Testing on S_{-1}

Then for any S_0, the probability that a decoder tested on S_{-1} finds the identity of S_0 in the ordered sequence is:

$$p(Y=S_0|X=S_0)=\frac{1}{4}p(S_0|S_{-1}=S_0)+\frac{3}{4}p(S_0|S_{-1}\neq S_0)=\frac{1}{4}p+\frac{3}{4}\frac{1-p}{3}=\frac{1}{4}$$

In conclusion, under the assumption of equiprobable errors, the performance of the decoder tested on the previous trial is **at chance level**.

Testing on S_{-2}

We can extend the same reasoning to testing on S_{-2}:

$$p(Y=S_0|X=S_0)=\frac{1}{16}p(S_0|S_{-2}=S_0)+\frac{15}{16}p(S_0|S_{-2}\neq S_0)=\frac{1}{16}p+\frac{5}{16}\frac{1-p}{3}=\frac{5-4p}{16}$$

Considering that $p>\frac{1}{4}$, the latter quantity is necessarily smaller than $\frac{1}{4}$, i.e. the decoder performs **below chance level**.

Non-equiprobable misses

In practice, we might expect pairs of stimuli to be more often confounded if they are physically close:

$$p(B|A)>p(C|A)>p(D|A)$$

A complete confounding matrix can be obtained at any latency from training and testing on random sequences. Here, for convenience, we parametrize the 3 possible error rates as having a geometric relationship:

$$p(B|A)=kp(C|A)=k^2p(D|A)$$

where $k>1$

We can extend this logic to all others X and $p(Y\neq X|X)$, with the additional constraint that:

$$\sum_{Y\neq X}p(Y|X)=1-p$$

An example of a plausible confounding matrix with $p = 0.35$ (based on fig.2a in Demarchi et al. 2019) and $k = 2$:

$$\begin{matrix} .35 & .372 & .186 & .093 \\ .26 & .35 & .26 & .13 \\ .13 & .26 & .35 & .26 \\ .093 & .186 & .372 & .35 \end{matrix}$$

However, this confusion matrix is not plausible as we end up with some error rates being larger than the true positive rate (.372 vs. .35). Let us try with $k = 1.5$:

$$\begin{matrix} .35 & .308 & .205 & .137 \\ .244 & .35 & .244 & .163 \\ .163 & .244 & .35 & .244 \\ .137 & .205 & .308 & .35 \end{matrix}$$

Using this.35+3.08 confusion matrix and the transition matrix, we can now calculate the probability that a decoder tested on S_{-1} finds the identity of S_0 in the ordered sequence:

$$p(Y = S_0|X = S_0) = \sum_{x \in X} p(Y = x|x) = \frac{1}{4}((\frac{1}{4}p + \frac{3}{4}.137) + 3(\frac{1}{4}p + \frac{3}{4}.244)) = \frac{1}{4}p + \frac{3 * .137 + 9 * .244}{16} = .2504$$

While the accuracy is now **above chance level**, it remains largely under the accuracy reported by Demarchi et al. 2019 (fig. 3) which is between .255 and .26.

The unbalance can be made a little more dramatic with $k = 1.7$: Let us try with $k = 1.5$:

$$\begin{matrix} .35 & .336 & .198 & .116 \\ .251 & .35 & .251 & .148 \\ .148 & .251 & .35 & .251 \\ .116 & .198 & .336 & .35 \end{matrix}$$

But even under the unfavourable conditions, the accuracy is still only .204.