Homework 3: Oscillation

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การบ้านครั้งที่ 3

จงหาจุดตรึงและตรวจสอบพฤติกรรมของจุดตรึง(มีเสถียรภาพหรือไม่)ของแบบจำลองต่อไปนี้ว่ามีการกวัดแกว่งหรือไม่ พร้อมทั้งแสดงกราฟของ P_n สำหรับ $n = 0,1,2,3,\ldots,10$ เมื่อกำหนดค่าเริ่มต้น $P_0 = -2,$ และ $P_0 = 3$

1. $P_{n+1} = 2P_n - 3$
2. $P_{n+1} = 0.5P_n + 1.5$
3. $P_{n+1} = -0.5P_n - 3$
4. $P_{n+1} = -1.4P_n - 3$

Tasks in each model

1. Find fixed point of the equation
2. Test stability of the model

Equation 1: $P_{n+1} = 2P_n - 3$

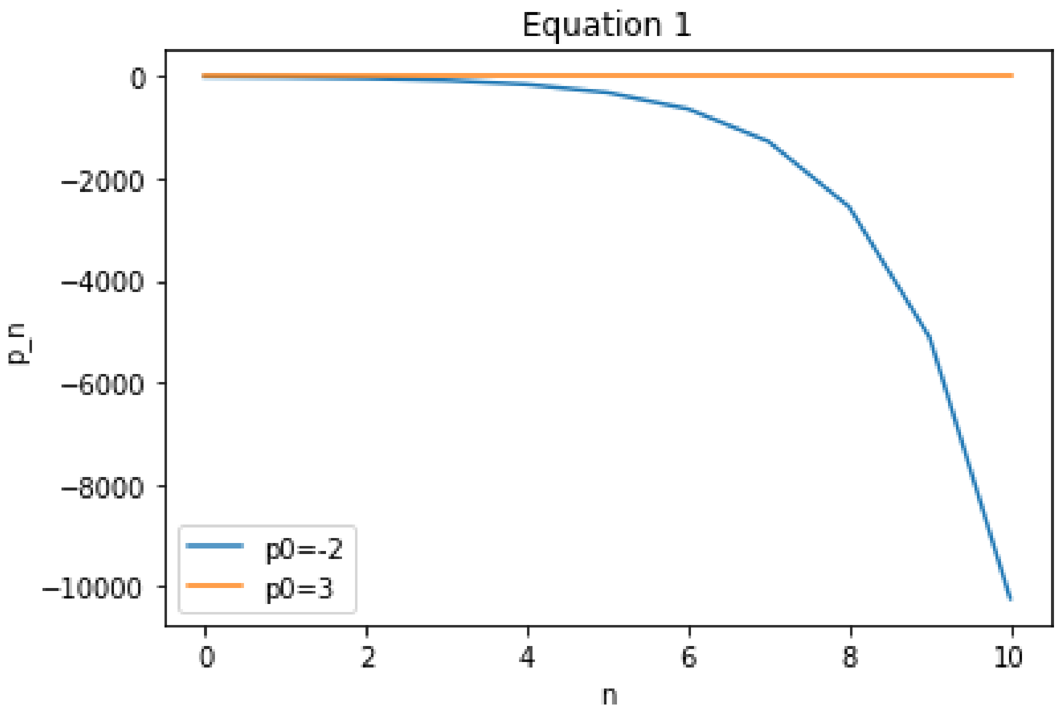
From equation,

$a = 2$

$b = -3$

Plot graph and show values

```
n:  [ 0  1  2  3  4  5  6  7  8  9 10]
p_n (p_0=-2):  [-7, -17, -37, -77, -157, -317, -637, -1277, -2557, -5117,
-10237]
p_n (p_0=3):  [3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3]
```



Since the coefficient in front of $P_n > 0$, both line results are the **monotonic solutions.**

- **Fixed point** is equal to $p = \frac{b}{1-a} = \frac{-3}{1-2} = 3$
  Where $p_0 = 3$, the equation is at equilibrium.
- **Stability test**
  From the equation coefficient $a = 2$ and $|a| = |2| > 1$, model is **unstable**. As we can see from model at original $p_0 = -2$, model diverges out of fixed point 3.

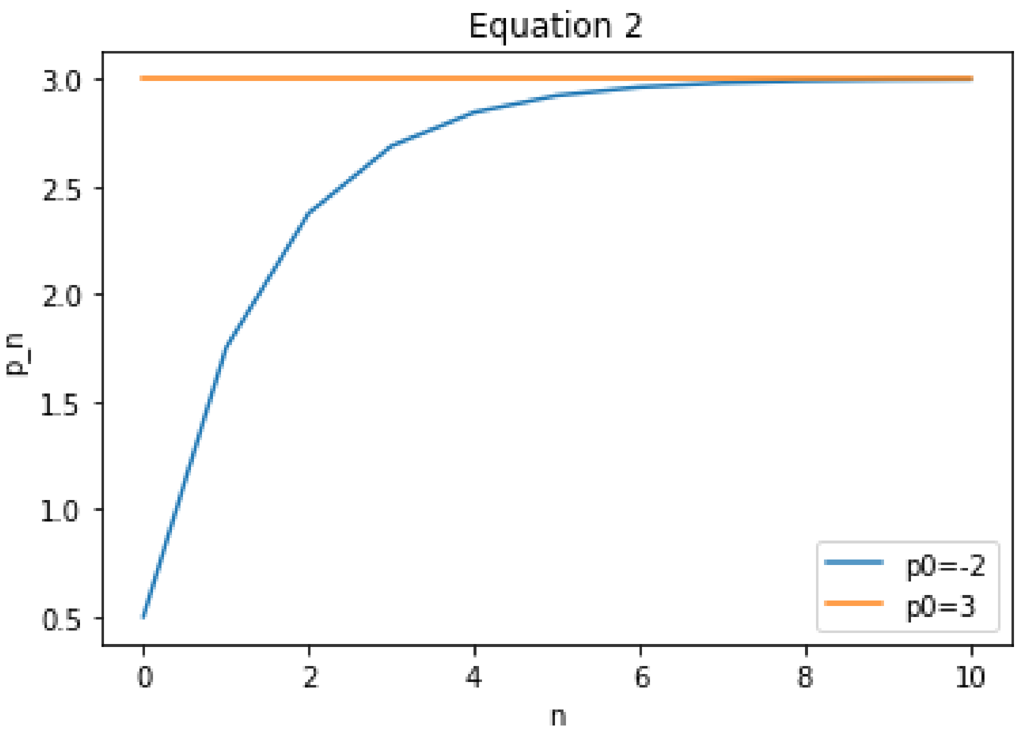
Equation 2: $P_{n+1} = 0.5P_n + 1.5$

From equation,

$a = 0.5$
$b = 1.5$

Plot graph and show values

```
n:  [ 0  1  2  3  4  5  6  7  8  9 10]
p_n (p_0=-2):  [0.5, 1.75, 2.375, 2.6875, 2.84375, 2.921875, 2.9609375,
2.98046875, 2.990234375, 2.9951171875, 2.99755859375]
p_n (p_0=3):  [3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0]
```



Since this equation $a > 0$, the model results in a **monotonic solution.**

- **Fixed point** is equal to $p = \frac{b}{1-a} = \frac{1.5}{1-0.5} = 3$
  Where $p_0 = 3$, the equation is at the equilibrium.
  And $p_0 = -2$, the equation converges to 3, which is close to fixed point 3.
- **Stability test**
  Since $a = 0.5$ and $|a| = |0.5| < 1$, the model is **stable** as the original $p_0 = -2$ model converges close to 3 while n goes on.

Equation 3: $P_{n+1} = -0.5P_n - 3$

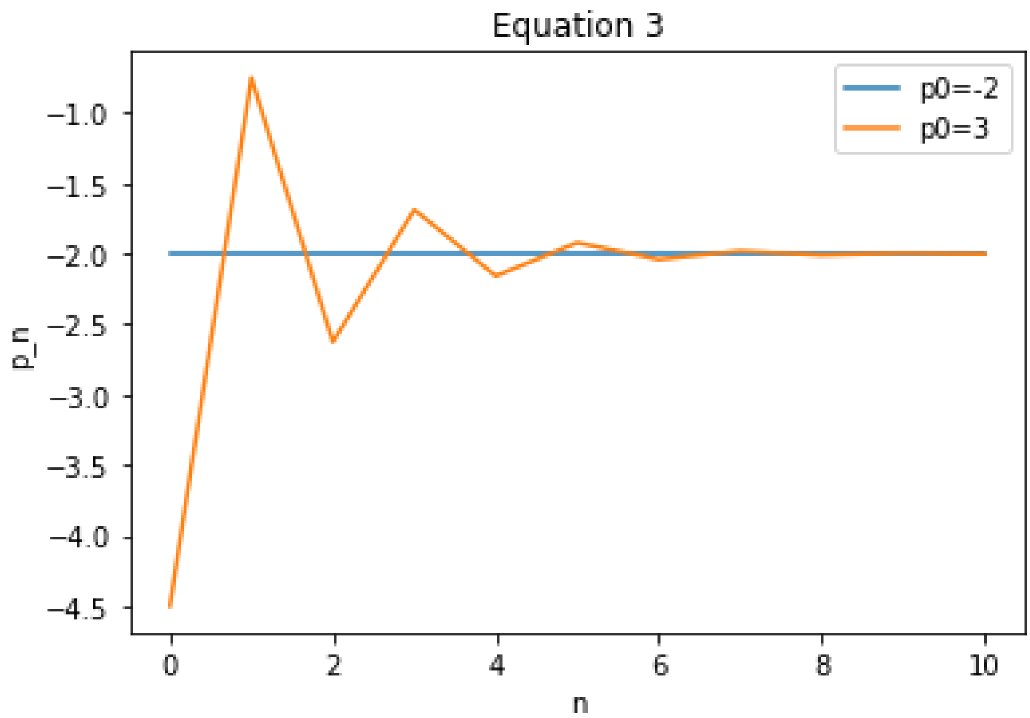
From equation,

$a = -0.5$

$b = -3$

Plot graph and show values

```
n:  [ 0  1  2  3  4  5  6  7  8  9 10]
p_n (p_0=-2):  [-2.0, -2.0, -2.0, -2.0, -2.0, -2.0, -2.0, -2.0, -2.0, -2.0, -2.0]
p_n (p_0=3):  [-4.5, -0.75, -2.625, -1.6875, -2.15625, -1.921875, -2.0390625, -1.98046875, -2.009765625, -1.9951171875, -2.00244140625]
```



Since this equation $a < 0$, the model causes an **oscillation.**

- **Fixed point** is equal to $p = \frac{b}{1-a} = \frac{-3}{1+0.5} = -2$
  Where $p_n = -2$, the equation is at equilibrium.
- **Stability test**
  Since $a = 0.5$ and $|a| = |-0.5| < 1$, the model is **stable** as the $p_0 = 3$ model converges close to -2 while n goes on.

Equation 4: $P_{n+1} = -1.4P_n - 3$

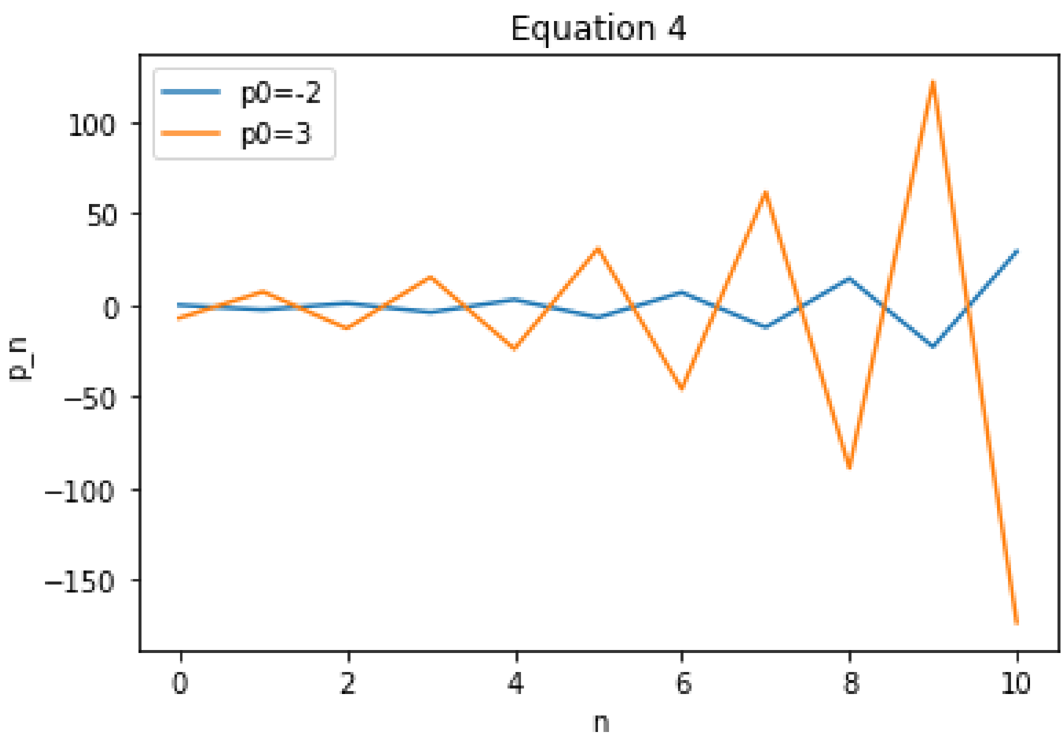
From equation,

$a = -1.4$

$b = -3$

Plot graph and shows values

```
n:  [ 0  1  2  3  4  5  6  7  8  9 10]
p_n (p_0=-2):  [-0.20000000000000018, -2.7199999999999998,
0.8079999999999994, -4.131199999999999, 2.7836799999999977,
-6.897151999999997, 6.656012799999994, -12.318417919999991,
14.245785087999987, -22.94409912319998, 29.121738772479972]
p_n (p_0=3):  [-7.199999999999999, 7.079999999999998, -12.911999999999997,
15.076799999999995, -24.10751999999999, 30.750527999999998,
-46.050739199999974, 61.47103487999996, -89.05944883199994,
121.68322836479992, -173.35651971071988]
```



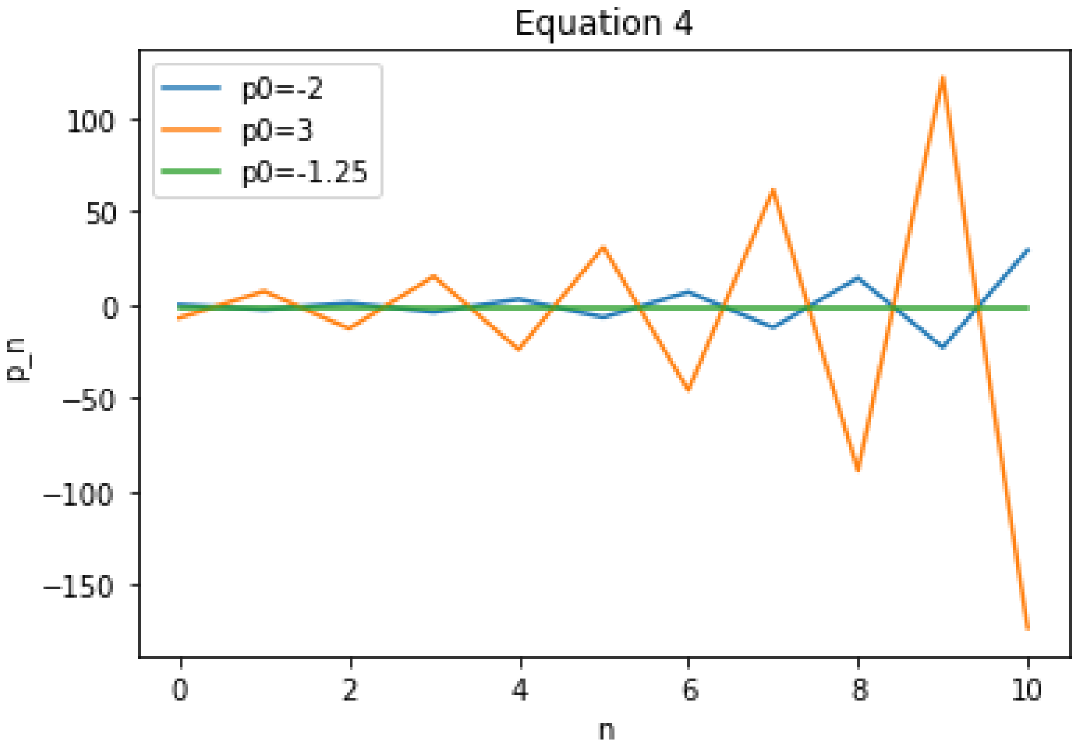
Since this equation $a < 0$, the model causes a huge **oscillation** more than equation 3.

- **Fixed point** is equal to $p = \frac{b}{1-a} = \frac{-3}{1+1.4} = -1.25$
- **Stability test**
  Since $a = 0.5$ and $|a| = |-1.4| > 1$, the model is **unstable** as both models from the original points $p_0 = 3$ and $p_0 = -2$ diverge out of a certain point.

Extra test from equation 4, add more original point $p_0 = -1.25$ to see whether it is the fixed point or not

Plot graph and shows values

```
n:  [ 0  1  2  3  4  5  6  7  8  9 10]
p_n (p_0=-2):  [-0.20000000000000018, -2.7199999999999998,
0.8079999999999994, -4.131199999999999, 2.7836799999999977,
-6.897151999999997, 6.656012799999994, -12.318417919999991,
14.245785087999987, -22.94409912319998, 29.121738772479972]
p_n (p_0=3):  [-7.199999999999999, 7.079999999999998, -12.911999999999997,
15.076799999999995, -24.10751999999999, 30.75052799999998,
-46.050739199999974, 61.47103487999996, -89.05944883199994,
121.68322836479992, -173.35651971071988]
p_n (p_0=-1.25):  [-1.25, -1.25, -1.25, -1.25, -1.25, -1.25, -1.25, -1.25,
-1.25, -1.25, -1.25]
```



This graph shows that at $p = -1.25$ is the fixed point of equation 4.