Romain Santiard

In 2015 in Saudi Arabia, more than 2400 peoples died near the city of Mecca because of a crowd movement. That is not an isolated case. Indeed, over the last few decades, a lot of different events involving big crowds led to catastrophic scenarios. Since 1990, around 330 peoples have died every year as a result, which is 30 times more than from shark attacks.
In a world approaching 10 billion inhabitants, with geopolitical tensions growing, the study of crowds and the planning of events that bring together thousands of people has become a burning topic.
The purpose of this project is to build simple crowds model. First we will show the existence of a critic mass theorized in 1978 by Mark Granovetter. Then we will study the influence of the stress level on a crowd that wants to escape a room, and finally the effect of stress in the same scenario.

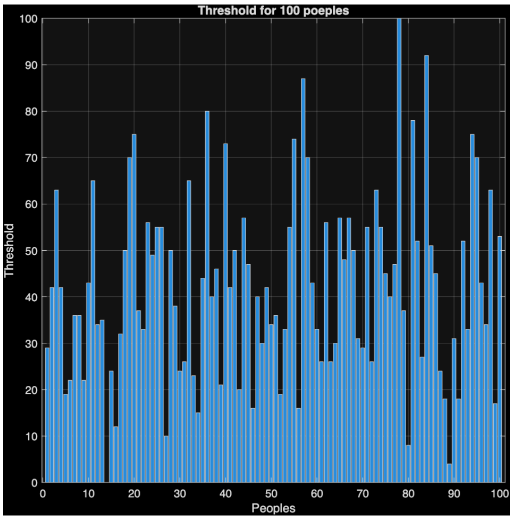
1. Build of a simple crowd model

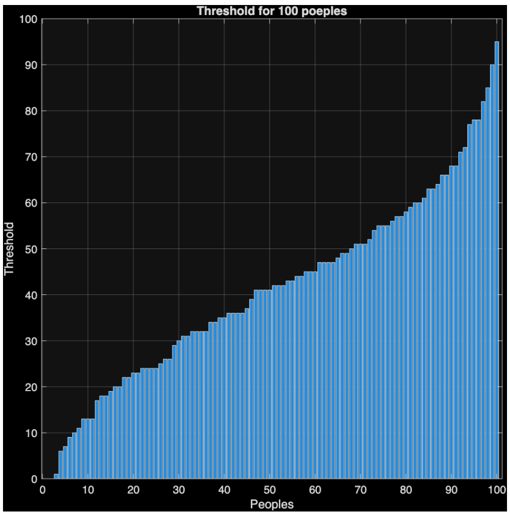
The idea of this project come from a youtube video of the channel *Fouloscopie* run by Mehdi Moussaïd . He presents a simple discret model where he explains the theory of Mark Granovetter. He takes the example of a hola in a soccer stadium and finds that bellow 33 peoples, the wave doesn't start but with 33 peoples, it works.

1.1. Threshold of a person

We can distinguish two main contagions model. Simple contagions only require one contact to spread : virus, basic information, viral content on social media… And complex contagions : joining a protest movement, adopting a new diet…
The critic mass notion was therozied by Mark Granovetter in 1978. He discovered that unlike simple contagions, complex one needed a social reinforcment. For those kind of contagion, we need a certain proportion of our neighbourhood to be contaminated to follow the habit in turn. One person is not enough most of the time.
We define the threshold as the number of person in our circle who need to do something for us to did it too.

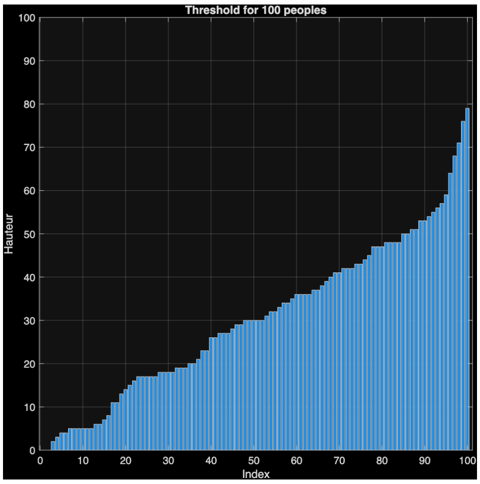
Here is an example of 100 different peoples (x axis) and there threshold (y axis). In bulk and sorted by increasing value. People with a high threshold tend to be more conservative, while people with lower one are motivated activists.

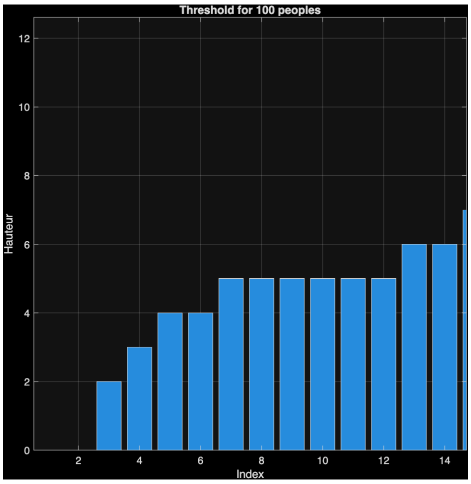




On the sorted graph, we see that the 2 peoples with a zero threshold will be the first to do something. The next one need at least 1 person to do it too, so he will do it. But the propagation will stop then, because the fourth person need 6 peoples to do it in turn while there are only 3 for now.

But we can imagine another distribution of threshold as the following one :





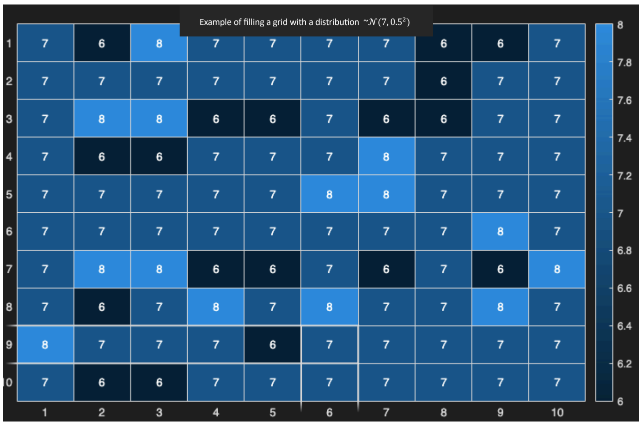
Two peoples spontaneously initiated the movement, the third one follows because he needs 2 persons, the third one too, he needs 3 persons and so on. Every people respecting the rule : for x their position on the increasing graph and y their threshold, x < y, the wave propagate all over the 100 peoples. But it would have been enough for the third person to have threshold of 3 for the movement to stop right from the start.

1.2. Rules of stress propagation

I use the same rule as Mehdi did. I assigned to each cell of my grid an individual stress threshold between 1 and 24. Indeed, for each people, this will determine, in a 5*5 square centered on a define person, the number of people needed to make this person become stressed in turn.

The distribution of this individual stress thershold will follow a gaussian law ~$\mathcal{N}(\mu, \sigma^2)$. It aims to represent a crowd with a few very stressed temperaments people and a few very relaxed people but a large majority of people between these two natures.

Based on the different studies I found related to the modeling of crowds with thresholds, I found that $\mu = 7$ and $\sigma = 0.5$ should be appropriate. Indeed, these values correspond to approximately 30% of a local neighborhood of 24 individuals, the empirically observed threshold for triggering phase transitions in collective systems. Moreover, a low sigma value allows for a gradual transition.



1.3. Initial stressed peoples distribution

To start a panic movement, there need to be something to triggers it : an explosion, a suspicious comportement by someone… I only wanted to start my simulation with the conscequence of a such event : initial stressed peoples.
At the beginning of the project, I was very confused because the curve results I had didn't show the threshold I was looking for. And then, I realized that I chose to randomly distribute a given number of initial stressed people. It was not realistic pricesely because the trigger had small chances to stressed people randomly all over the crowd. This random distribution was creating lots of different hotbed, which was not the case I wanted to study.
Then, I made my program chose a random place around which people had a probability to be stressed expenontially decreasing with the distance between them and the trigger cell. It works well and led to the apparition of this long-awaited threshold.

1.4. Which curve plotting to find the threshold ?

In this video Mehdi show the existence of the threshold with a given crowd, and a commun individual stess threshold for every one. He starts the movement at the same place each time, and keep by keeping the same crowd. He repeats the simulation by increasing the initial stressed people number by one each time and find the number 33.

I didn't wanted to have the same approach. I wanted to use my model to make a huge number of simulations. I wanted to plot the number of time the crowd ended up completely panicked on a given number of simulation as a function of the number of initial stressed people.

For a M*N grid, with a number of initial stressed people going from 1 to p, with n simulation each time, M*n*p*n numbers between 1 and 24 had to be randomly chosen for the individual stressed thresholds, and the same number of 0 or 1 for the initial state of each person following a particular exponential distribution. For M = N = 100, p = 20, n = 300, it gives 2*100*100*20*300 = 120 millions d'entiers, codés chacun sur 4 octets, soit 480 millions d'octects, donc environ 0.447 Go.
Initially, I tried to only use *Matlab* to obtain my curve, but it takes about 2 hours with my first script. I then moved to another one using *C* to compute all the grid I needed in two big files :

- *thresholds_matrix.c* for the distribution of the stressed thresholds :

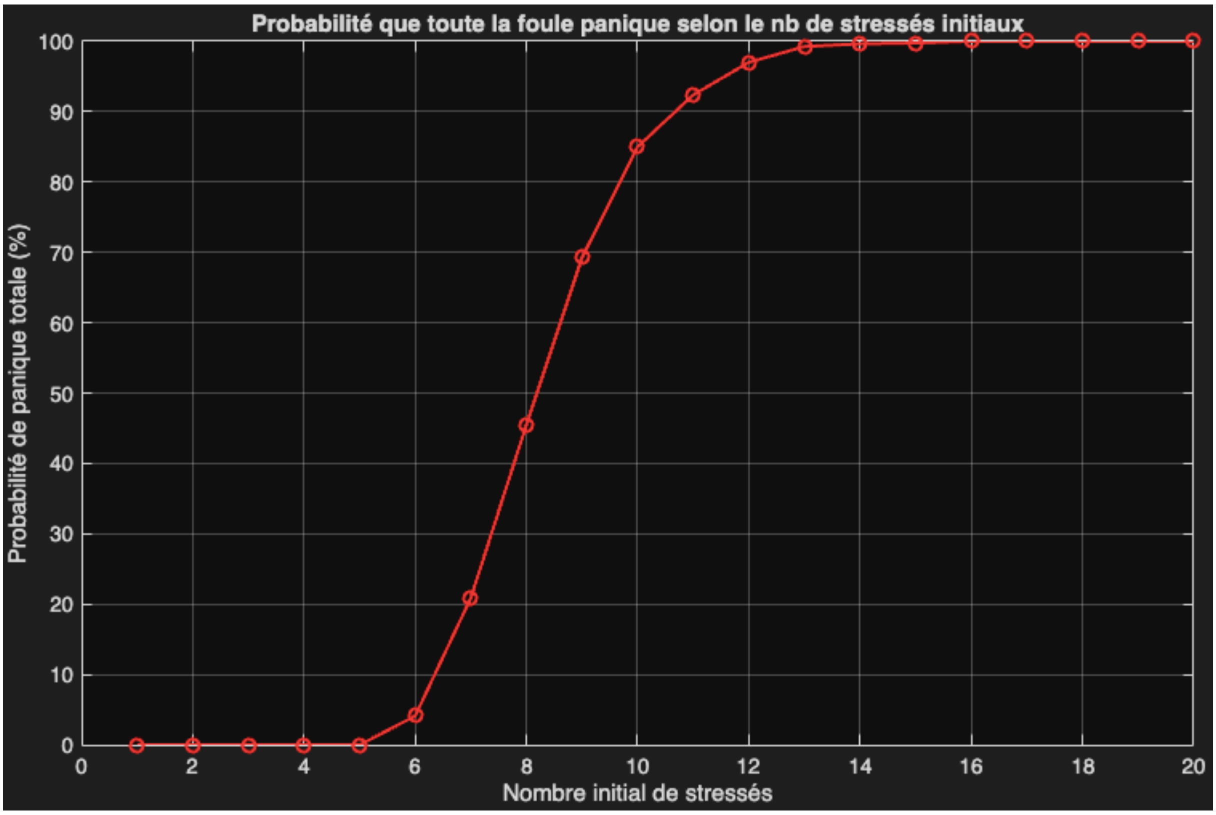
$$\begin{bmatrix} \mathrm{matrix}(1,1) & \mathrm{matrix}(1,2) & \cdots & \mathrm{matrix}(1,n) \\ \mathrm{matrix}(2,1) & \mathrm{matrix}(2,2) & \cdots & \mathrm{matrix}(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{matrix}(r,1) & \mathrm{matrix}(r,2) & \cdots & \mathrm{matrix}(r,n) \end{bmatrix}$$

where
r = max number of initial stressed people for the curve plotting (*max_k in the scripts)*
n = number of simulation for a given number of initial stressed people (*n_sim)*

matrix(i,j) is the distribution of the M*N integrer ~$\mathcal{N}(\mu, \sigma^2)$ clamped in [1;24] for the jth matrix with i peoples initially stressed.

- *state_only.c* for the initial state of each people (same construction as the previous matrix)

I executed those two script which both created a file of around 0.22 Go. I then needed to open all the values with matlab and stock them in clean matrix following the previous form (files created with C were just a long sequence of binary number). I managed to obtain the curve bellow.

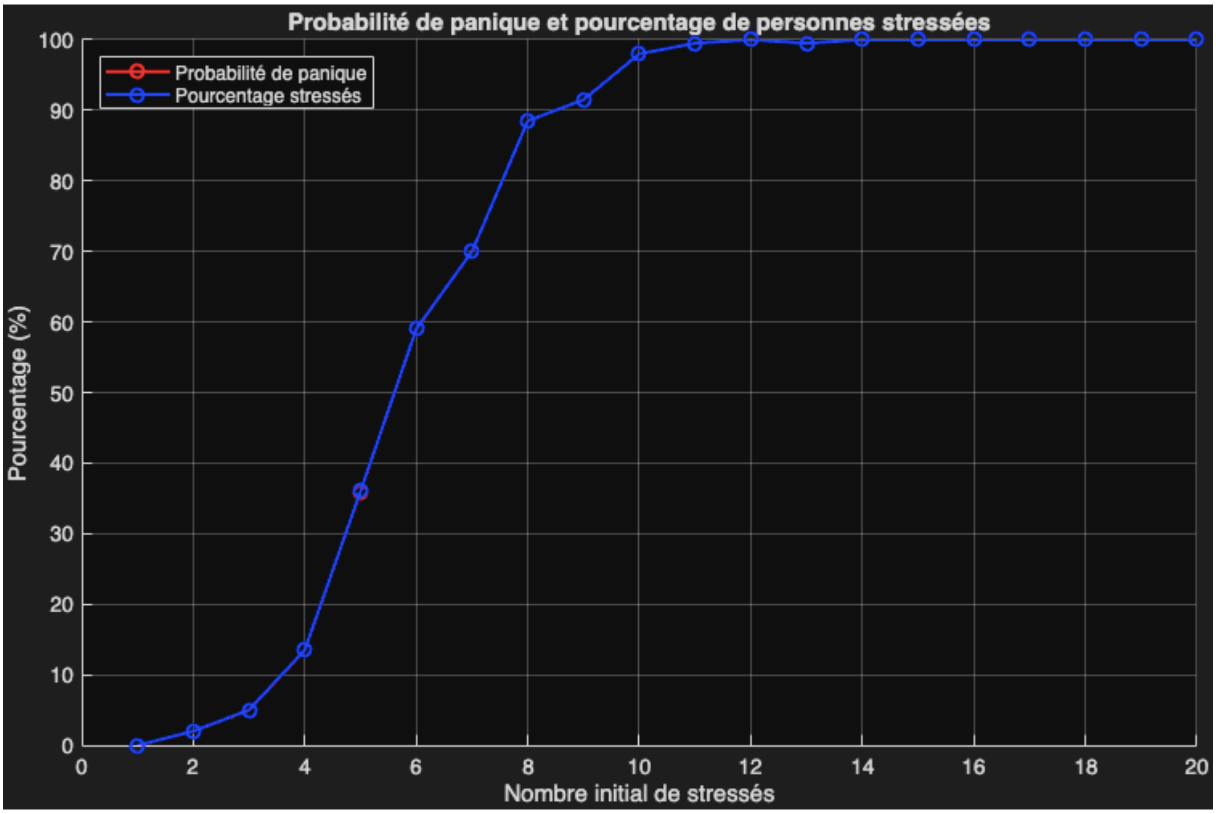


Note : I encountered a particular problem when constructing the graph: even with a very high number of people initially stressed, it almost never happened that the crowd ended up completely stressed. Indeed, some cells at the edge of the grid remain relaxed, even though they are surrounded only by tense people. This is because the squares at the edge have only 14 neighbors, instead of 24, and those at the corners have only 8. With a normal distribution with a mean of 7-8 and a standard deviation of 2-3, a significant number of squares (out of 10,000) have a stress index strictly greater than 14 and 8.
I therefore decided not to count the cells on the edge with a thickness of 1 when classifying the entire grid as stressed. This amounts to neglecting edge effects.

1.5. Study of the curve and parameters influence

At first sight, we can define an interval for the threshold value : ~[6;10]. But this assomption is not very rigorous. Let's see how we can mathematically find a threshold on a curve like this. The simpliest approach is to find the point where the tangent is the stieppest which means the point where the derivate is the highest. However, the curve does not correspond to a function that is differentiable at every point (it is a succession of segment). So we calculated the slope of each segment using their coordonates.
X = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
Y = [0, 0, 0, 0, 0, 4.125, 20.875, 45.5, 69.25, 85, 92.375, 97, 99.25, 99.625, 99.75, 100, 100, 100, 100]
pente = [0, 0, 0, 0, 4.125, 16.75, 24.625, 23.75, 15.75, 7.375, 4.625, 2.25, 0.6, 0.125, 0.025, 0, 0, 0, 0]

So the threshold is 7. But the slope of the segment after 7 is very close to the one after 8. To distinguish those two values, we could plot the pourcentage of the number of people stressed at the end of the wave propagation over the number of people that could be stressed if all simulations ended up with a total panicked crowd.



But we see this curve is so similar to the one we previously plotted, that they are overlapping. Indeed, we can distinguinsh two main scenarios. In the first one, the stress wave start to propagate within the crowd, and it finished all stressed. The second one, the wave does not start to propagate and only initial stressed people are affected (and sometimes a very few other people). So there is no case where the stress wave stopped after having affected a significant fraction of the crowd. That is why the two curves are overlapping. We can not define one clear and well-defined threshold, but we can notice that the crowd is the most feeble and unpredictable for 7-8 initial stressed people.

Here is a summary of the influence of different parameters on stress propagation :

<table>
<tr><th>High value</th><th>Low value</th></tr>
<tr><td colspan="2">R : Appears in t <code>p[idx] = exp(-(dist/R)*(dist/R));</code> <code>(only_state.c)</code></td></tr>
<tr><td colspan="2">It is the scale factor for the distance in the core that determines the probability of placing the initial triggers around the center <code>cx, cy</code>.</td></tr>
<tr><td>The inital triggers are more dispersed, so stress can appear over a wide area from the outset.</td><td>The initial triggers are concentrated, stress starts locally and spreads more slowly.</td></tr>
<tr><td colspan="2">Mu : appears in <code>int val = (int)lround(mu + sigma * z);</code> <code>(big_matrice.c)</code></td></tr>
<tr><td colspan="2">defines the stress threshold for each cell. It is the average of the distribution of the thersholds.</td></tr>
<tr><td>Each cell requires many stressed neighbors to become stressed. The spread is slower and more difficult.</td><td>Each people only need a few people to become contaminated in turn. The spread is easy and quick.</td></tr>
<tr><td colspan="2">Sigma : appears in <code>int val = (int)lround(mu + sigma * z);</code> <code>(big_matrice.c)</code></td></tr>
<tr><td colspan="2">controls the dispersion of the threshold around the mean mu.</td></tr>
<tr><td>Very varied thresholds. Some areas becomes stressed easily, others not. The spread is irregular, with “pockets” of resistance.</td><td>Homogeneous distribution of the thresholds. The propagation is predictable and evenly distributed.</td></tr>
</table>

---

2. Impact of the stress level of a crowd on its escape time

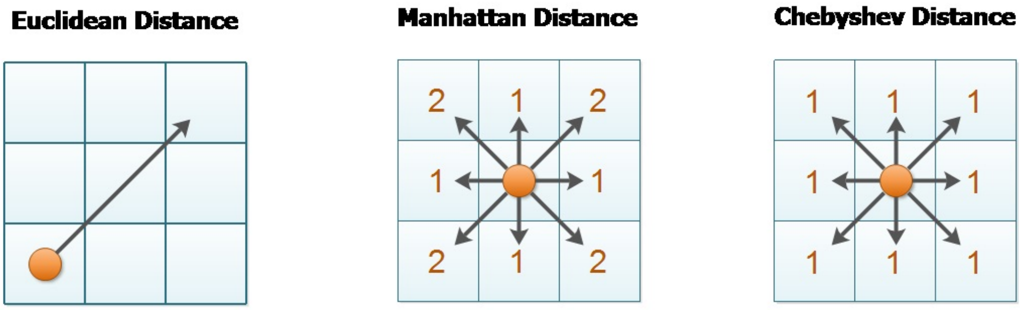
Now we developped a consistent stress propagation model. We are going to use it in order to verify a commun assumption : A crowd can exit a room faster by staying calm in a tense situation.

2.1 Continuous or discret approach ?

I started this project even before the previous one, I just wanted to have a little simulation of people trying to escape a room. Every people was represented by a circle and at each time step, circles moves towards the exit, following a unit vector pointing it with a defined velocity. But the model was to heavy for a high number of people (over ~20) and it was hard to define a rule so that the cricles could not overlap. That is why a chose a grid as I did after for the stress propagation model.

2.1 Distance grid

With a discret approach, I had do find something to replace the unit vectors that were pointing towards the exit. I chose to assign a value to each cell depending on its distance from the exit (a predefine cell of the grid). I had different option to calculate those distances :

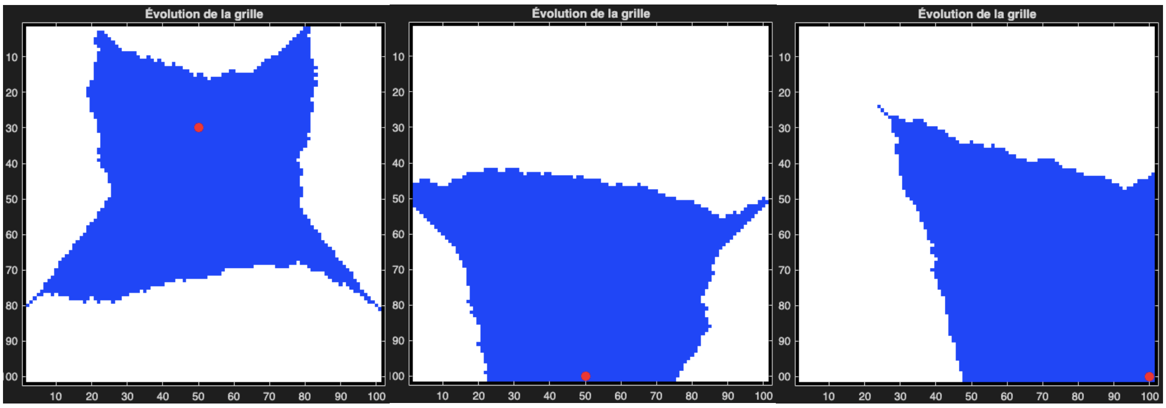


$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $|x_1 - x_2| + |y_1 - y_2|$ $\max(|x_1 - x_2|, |y_1 - y_2|)$

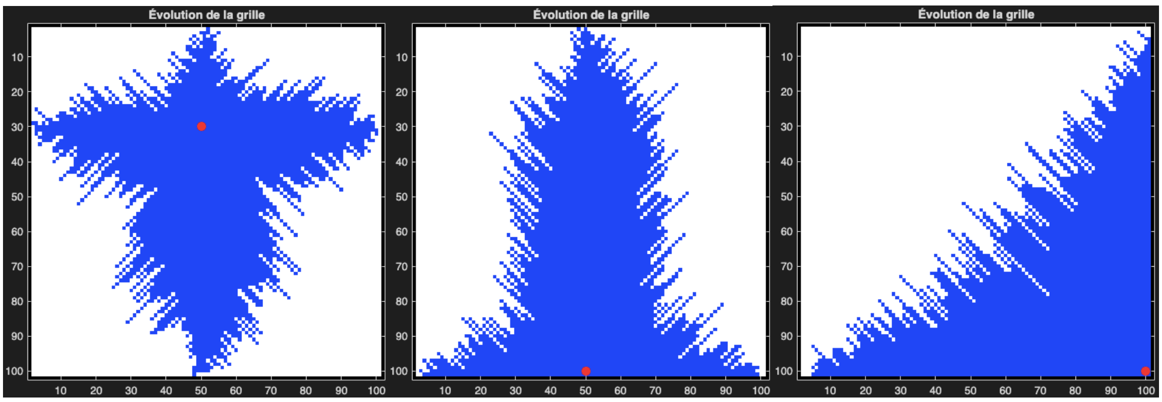
opengenus.org

The difference is very different depending on the distance chosen :

Shape of the crowd around the exit with Tchebytchev distance :



Shape with Manhattan distance (no visible differences with Euclidian distance) :



We can agree that the shapes obtained are not natural. Moreover, independently of the distance, at each time step, a person is looking for the cell around him (in a 3*3 square). But when several cells have the minimal score, the person chose the first one. And the order in which cells are read in the square is :

1	4	7
2	5	8
3	6	9

So the left-upper direction is favored.

The solution used is using a function : imgaussfit to smooth the euclidian distance and avoid unatural movement of the crowd. We define a gaussian kernel. It assign for a given cell, the weight each value around it will have. We fix the mean and the standard deviation then Matlab will adjust the size of the kernel.

$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$\varphi_{\text{lisse}}(i,j) = \sum_{dx=-k}^{k} \sum_{dy=-k}^{k} G(dx,dy) \cdot \varphi(i+dx, j+dy)$$

Here is an example for a small grid (5*5 where the cell (2,4) is the exit).

$$\begin{bmatrix} 4.47 & 3.61 & 2.83 & 2.24 & 2 \\ 3.61 & 2.83 & 2.24 & 1.41 & 1 \\ 3 & 2.24 & 1.41 & 1 & 0 \\ 3.61 & 2.83 & 2.24 & 1.41 & 1 \\ 4.47 & 3.61 & 2.83 & 2.24 & 2 \end{bmatrix}$$

and after the gaussian filter :

$$\begin{bmatrix} 4.045 & 3.442 & 2.703 & 2.124 & 1.821 \\ 3.474 & 2.902 & 2.188 & 1.553 & 1.129 \\ 3.112 & 2.250 & 1.848 & 1.184 & 0.676 \\ 3.610 & 2.830 & 2.240 & 1.410 & 1.129 \\ 4.045 & 3.442 & 2.703 & 2.124 & 1.821 \end{bmatrix}$$

For the cell of coordinates (3,4) for example, with the kernel define on the left ($\sigma \cong 1$), with $\odot$ the matrix convolution product :

$$\varphi_{\text{lisse}}(3,4) = \begin{bmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{bmatrix} \odot \begin{bmatrix} 2.188 & 1.553 & 1.129 \\ 1.848 & 1.184 & 0.676 \\ 2.24 & 1.41 & 1.129 \end{bmatrix}$$

$$= \frac{1}{16} \begin{pmatrix} 1 \times 2.188 + 2 \times 1.553 + 1 \times 1.129 + \\ 2 \times 1.848 + 4 \times 1.184 + 2 \times 0.676 + \\ 1 \times 2.24 + 2 \times 1.41 + 1 \times 1.129 \end{pmatrix}$$

$$= \frac{1}{16} \times 18.944$$

$$= 1.184$$

At the end of the application of the gaussian filter, we assign a negative value to the exit cell to be sure that it is still attractive.

2.3 Stress level system

Now each cell has a score that enable to have a natural escape, we want to complexify a bit our model to take in count the stress level of the crowd. We use the approach proposed by Dirk Helbing and Péter Molnár in *Social force model for pedestrian dynamics (1995).* The idea is to combine two “forces” in a linear combination. The value of the coefficient will change depending on the stress state of the crowd.
The first “force” is the will to escape, it is symbolize by the number we assign to each cell in the previous part. The new one is the “repulsion force”. It consists on the idea that in a calm crowd, people avoid crowding together too much when going out and proritize less crowded area. At a given cell A and a given time step, we calculate for each cell around A the density in a 3*3 square. It gives a density score.
Then each cell has a new score define by :

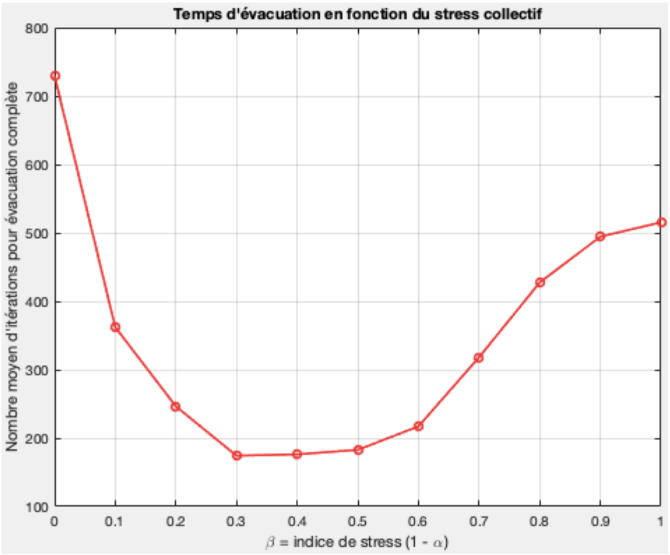
$$S(i,j) = \varphi_{\text{lisse}}(i,j) + \alpha \cdot D(i,j)$$

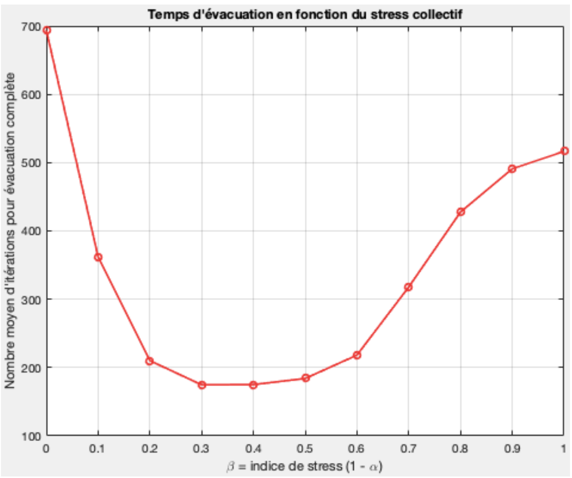
And at each time step, every person will go in the empty cell around him with the lowest score. (We assume that each score is different for each cell so there can not be conflict between two candidates cells.)

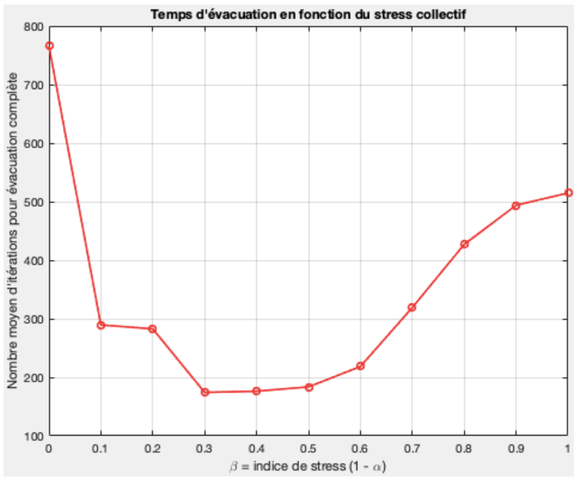
A calm crowd will prioritise separsely populated area rather than going out at all costs. The stress level of a crowd decrease with α. So we can define a stress index given by $\beta = 1-\alpha$.

2.4 Plot and study of the curve

Now we have a model that can simulate the escape of a crowd with a stress level we can control. We can now plot the curve of the number of time step a crowd need to escape a room as a function of its stress level (β).







After plotting this graph several times, we notice that the values vary greatly. This is particulary true for a crowd with β = 0. However, the overall trend is always similar. Indeed, as expected, a stress crowd takes more time to escape a room than a calm one. But, a too calm crowd is so relaxed and peaceful that it takes even more time to escape than very stressed people. Each agent avoids dense areas too much creating :

- excessive dispersion : agent scatter
- localized blockages : mutual avoidance creates dead ends
- loss of coordination : no collective movement toward the exit

We also observe that the part that varies from one curve to another is the one with $\beta < 0.3$. Below this value, the crowd is difficult to predict, while above it, the values remain close.

3. Conclusion and outllook

Conclusion

This project succesfully developed and analyzed two interconnected crowds models that capture essential aspects of collective human comportments in stressfull situations.
First we implemented Granovetter's threshold model to demonstrate the existence of a critical mass phenomenon in stress propagation. The simulations revealed that :

- Stress spread non-linearly through crowds, with a transition around 7-8 initially stressed agents.
- The system shows critical behavior where small changes in initial conditions lead to completely different outcomes.
- Edge effects significantly influence propagation, requiring careful boundary treatment in descrete models.

Then, we asked how stress levels affect evacuation efficiency using a social-force inspired model. We found that :

- An optimal stress level ($\beta \cong 0.3$-0.7) minimizes evacuation time.
- Excessively calm crowds ($\beta < 0.3$) become inefficient due to over-avoidance of dense areas.
- Highly stressed crowds ($\beta > 0.7$) exhibit chaotic behavior that hinders coordinated movement.
- The U-shpaed curve confirms that moderate stress facilitates better coordination than extreme calmness or panic.

Outlook

- Rooms with several exits
- Heterogeneous agents : varying speed (children, elderly, disabled people…)
- Comparison with real-world data from security cameras or controlled experiments
- Influence of the architechture on the critic mass and the escape time (pillars, obstacles…)

4. References

1. **Granovetter, M. (1978).** *Threshold models of collective behavior.* *American Journal of Sociology*, 83(6), 1420-1443.

2. **Helbing, D., & Molnár, P. (1995).** *Social force model for pedestrian dynamics.* *Physical Review E*, 51(5), 4282-4286.

3. **Moussaïd, M., et al. (2011).** *How simple rules determine pedestrian behavior and crowd disasters.* *Proceedings of the National Academy of Sciences*, 108(17), 6884-6888.