1.1 THE SCHRÖDINGER EQUATION

Imagine a particle of mass m, constrained to move along the x-axis, subject to some specified force $F(x, t)$ (Figure 1.1). The program of *classical* mechanics is to determine the position of the particle at any given time: $x(t)$. Once we know that, we can figure out the velocity ($v = dx/dt$), the momentum ($p = mv$), the kinetic energy ($T = (1/2)mv^2$), or any other dynamical variable of interest. And how do we go about determining $x(t)$? We apply Newton's second law: $F = ma$. (For conservative systems—the only kind we shall consider, and, fortunately, the only kind that *occur* at the microscopic level—the force can be expressed as the derivative of a potential energy function,[1] $F = -\partial V/\partial x$, and Newton's law reads $m\, d^2x/dt^2 = -\partial V/\partial x$.) This, together with appropriate initial conditions (typically the position and velocity at $t = 0$), determines $x(t)$.

Quantum mechanics approaches this same problem quite differently. In this case what we're looking for is the **wave function**, $\Psi(x, t)$, of the particle, and we get it by solving the **Schrödinger equation**:

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi.} \qquad [1.1]$$

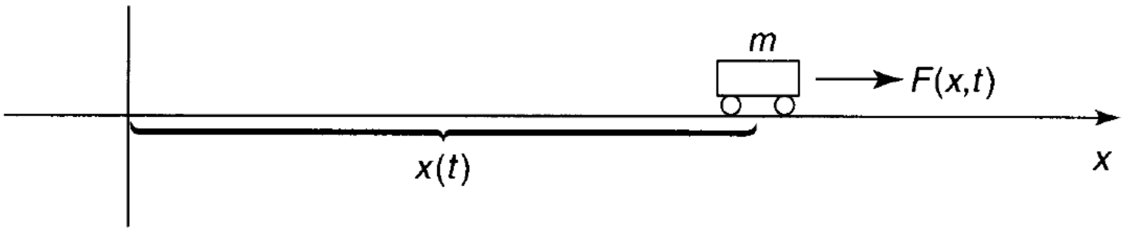

Figure 1.1: A "particle" constrained to move in one dimension under the influence of a specified force.

Here i is the square root of -1, and $\hbar$ is Planck's constant—or rather, his original constant (h) divided by 2π:

$$\hbar = \frac{h}{2\pi} = 1.054573 \times 10^{-34} \text{J s}. \qquad [1.2]$$

La **constante de Planck** es una constante física que desempeña un papel central en la teoría de la mecánica cuántica y recibe su nombre de su descubridor, Max Planck, uno de los padres de dicha teoría. Denotada como h, es la constante que frecuentemente se define como el cuanto elemental de acción. Planck la denominaría precisamente «cuanto de acción» (en alemán, *Wirkungsquantum*), debido a que la cantidad denominada acción de un proceso físico (el producto de la energía implicada y el tiempo empleado) solo podía tomar valores discretos, es decir, múltiplos enteros de h.

En aplicaciones donde la frecuencia viene expresada en términos de radianes por segundo o frecuencia angular, es útil incluir el factor $1/2\pi$ dentro de la constante de Planck. La constante resultante, «constante de Planck reducida» o «constante de Dirac», se expresa como $\hbar$ ("*h barra*"):

$$\hbar = \frac{h}{2\pi}$$

The Schrödinger equation plays a role logically analogous to Newton's second law: Given suitable initial conditions [typically, $\Psi(x, 0)$], the Schrödinger equation determines $\Psi(x, t)$ for all future time, just as, in classical mechanics, Newton's law determines $x(t)$ for all future time.