Review: **Basic Trigonometric Identities**

104003 Differential and Integral Calculus I
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Tutorial Handout – January 30, 2011 – Kayla Jacobs

Pythagorean Identity:

$$\sin^2 x + \cos^2 x = 1.$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Angle Sum & Differences:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double-Angle:

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta \\ &= \frac{2\tan\theta}{1+\tan^2\theta}\end{aligned}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \\ &= \frac{1-\tan^2\theta}{1+\tan^2\theta}\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half-Angle:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\begin{aligned}\tan\frac{\theta}{2} &= \csc\theta - \cot\theta \\ &= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \\ &= \frac{\sin\theta}{1+\cos\theta} \\ &= \frac{1-\cos\theta}{\sin\theta}\end{aligned}$$

Product-to-Sum:

$$\sin\theta\sin\varphi = \frac{\cos(\theta-\varphi) - \cos(\theta+\varphi)}{2}$$

$$\sin\theta\cos\varphi = \frac{\sin(\theta+\varphi) + \sin(\theta-\varphi)}{2}$$

$$\cos\theta\sin\varphi = \frac{\sin(\theta+\varphi) - \sin(\theta-\varphi)}{2}$$

$$\cos\theta\cos\varphi = \frac{\cos(\theta-\varphi) + \cos(\theta+\varphi)}{2}$$

Sum-to-Product:

$$\sin\theta \pm \sin\varphi = 2\sin\left(\frac{\theta \pm \varphi}{2}\right)\cos\left(\frac{\theta \mp \varphi}{2}\right)$$

$$\cos\theta + \cos\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta - \cos\varphi = -2\sin\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

Review: **Logarithms and Exponentials**

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$$\log_b x = n \quad \leftrightarrow \quad b^n = x$$

Exponent Rule	***Logarithm Rule***
$b^0 = 1$	$\log_b 1 = 0$
$b^1 = b$	$\log_b b = 1$
$b^{(\log_b x)} = x$	$\log_b(b^x) = x$
$b^x \cdot b^y = b^{x+y}$	$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
$b^x / b^y = b^{x-y}$	$\log_b(x / y) = \log_b x - \log_b y$
$(b^x)^y = b^{x \cdot y}$	$\log_b(x \cdot y) = y \cdot \log_b x$

More Helpful Logarithm Rules:

$$(\log_a b) \cdot (\log_b x) = \log_a x$$

$$\log_b x = \log_a x / \log_a b$$

$$\log_b a = 1 / \log_a b$$

(All rules are for any positive a, b, x, and y.)