1.3 PROBABILITY

∗Problem 1.1 For the distribution of ages in the example in Section 1.3,

(a) Compute $\langle j^2 \rangle$ and $\langle j \rangle^2$.

(b) Determine Δj for each j, and use Equation 1.11 to compute the standard deviation.

(c) Use your results in (a) and (b) to check Equation 1.12.

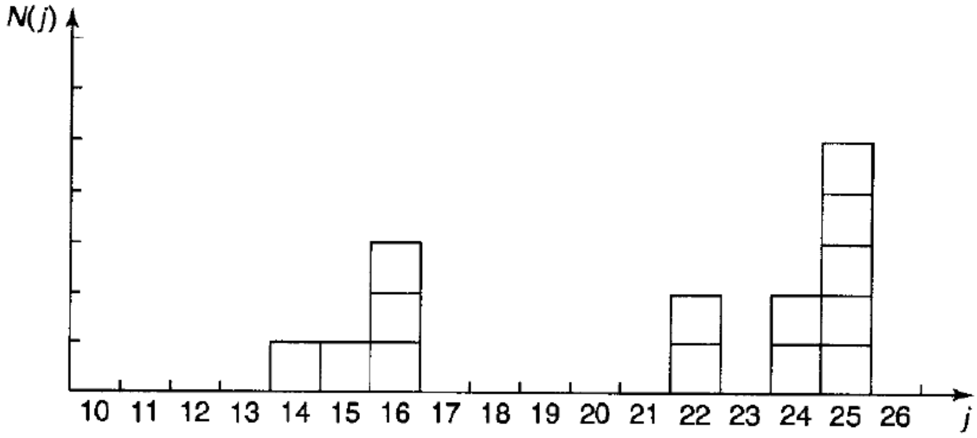


Figure 1.4: Histogram showing the number of people, $N(j)$, with age j, for the example in Section 1.3.

SOLVING A

Primero computamos la probabilidad de cada edad:

$$P(x = 14) = \frac{1}{14}$$

$$P(x = 15) = \frac{1}{14}$$

$$P(x = 16) = \frac{3}{14}$$

$$P(x = 22) = \frac{2}{14}$$

$$P(x = 24) = \frac{2}{14}$$

$$P(x = 25) = \frac{5}{14}$$

Luego calculamos el promedio de los cuadrados de las edades:

$$< j^2 > = \sum j^2 P(x)$$

$$< j^2 > = 14^2\left(\frac{1}{14}\right) + 15^2\left(\frac{1}{14}\right) + 16^2\left(\frac{3}{14}\right) + 22^2\left(\frac{2}{14}\right) + 24^2\left(\frac{2}{14}\right) + 25^2\left(\frac{5}{14}\right)$$

$$< j^2 > = \left(\frac{196}{14}\right) + \left(\frac{225}{14}\right) + \left(\frac{256 * 3}{14}\right) + \left(\frac{484 * 2}{14}\right) + \left(\frac{576 * 2}{14}\right) + \left(\frac{625 * 5}{14}\right)$$

$$< j^2 > = \left(\frac{196}{14}\right) + \left(\frac{225}{14}\right) + \left(\frac{256 * 3}{14}\right) + \left(\frac{484 * 2}{14}\right) + \left(\frac{576 * 2}{14}\right) + \left(\frac{625 * 5}{14}\right)$$

$$< j^2 > = \left(\frac{196}{14}\right) + \left(\frac{225}{14}\right) + \left(\frac{768}{14}\right) + \left(\frac{968}{14}\right) + \left(\frac{1152}{14}\right) + \left(\frac{3125}{14}\right)$$

$$< j^2 > = \frac{3217}{7} = 459.57$$

El siguiente script de javascipt puede ser usado para validar los resultados.

```
function avg_squared_of_j(data, N)
{
    var avg = 0;
    for (var j = 0; j < data.length; j += 2)
    {
        avg += (data[j]*data[j]) * (data[j+1]/N);
    }
    return avg;
}
```

Usage:

```
avg_squared_of_j([j1, j1count, j2, j2count...], N=sampleQuantity);
```

Example:

```
avg_squared_of_j([14, 1, 15, 1, 16, 3, 22, 2, 24, 2, 25, 5], 14);
```

Al computar la función anterior se puede determinar que efectivamente $< j^2 > = \frac{3217}{7} = 459.57$

SOLVING B

Primero calculamos el avg

$$< j > = \sum \frac{jN(j)}{N} = \sum jP(j)$$

$$< j > = 14\left(\frac{1}{14}\right) + 15\left(\frac{1}{14}\right) + 16\left(\frac{3}{14}\right) + 22\left(\frac{2}{14}\right) + 24\left(\frac{2}{14}\right) + 25\left(\frac{5}{14}\right)$$

$$< j > = \left(\frac{14}{14}\right) + \left(\frac{15}{14}\right) + \left(\frac{16 * 3}{14}\right) + \left(\frac{22 * 2}{14}\right) + \left(\frac{24 * 2}{14}\right) + \left(\frac{25 * 5}{14}\right)$$

$$< j > = \left(\frac{14}{14}\right) + \left(\frac{15}{14}\right) + \left(\frac{48}{14}\right) + \left(\frac{44}{14}\right) + \left(\frac{48}{14}\right) + \left(\frac{125}{14}\right)$$

$$< j > = 21$$

```
function avg_of_j(data, N)
{
    var avg = 0;
    for (var j = 0; j < data.length; j += 2)
    {
        avg += (data[j]) * (data[j+1]/N);
    }
    return avg;
}
```

Usage:

```
avg_of_j([j1, j1count, j2, j2count...], N=sampleQuantity);
```

Example:

```
avg_of_j([14, 1, 15, 1, 16, 3, 22, 2, 24, 2, 25, 5], 14);
```

Al computar la función anterior se puede determinar que efectivamente $< j > = 21$

Ahora calcularemos la distancia entre $< j >$ y j

$$\Delta j = j - < j >$$

$$\Delta j_{14} = 14 - 21 = -7$$

$$\Delta j_{15} = 15 - 21 = -6$$

$$\Delta j_{16} = 16 - 21 = -5$$

$$\Delta j_{22} = 22 - 21 = 1$$

$$\Delta j_{24} = 24 - 21 = 3$$

$$\Delta j_{25} = 25 - 21 = 4$$

Now let`s use equation [1.11] for calculate the standard deviation

$$\sigma^2 \equiv \langle (\Delta j)^2 \rangle . \qquad [1.11]$$

First we square each distance.

$$\Delta j_{14}^2 = (14 - 21)^2 = (-7)^2 = 49$$

$$\Delta j_{15}^2 = (15 - 21)^2 = (-6)^2 = 36$$

$$\Delta j_{16}^2 = (16 - 21)^2 = (-5)^2 = 25$$

$$\Delta j_{22}^2 = (22 - 21)^2 = 1^2 = 1$$

$$\Delta j_{24}^2 = (24 - 21)^2 = 3^2 = 9$$

$$\Delta j_{25}^2 = (25 - 21)^2 = 4^2 = 16$$

Now let`s calculate varince, note that $j$ was squared in last step so we won`t doet again in the formula of average of j squared.

$$\sigma^2 = <(\Delta j)^2> = \sum \frac{\Delta j N(j)}{N} = \sum \Delta j P(j)$$

$$\sigma^2 = 49\left(\frac{1}{14}\right) + 36\left(\frac{1}{14}\right) + 25\left(\frac{3}{14}\right) + 1\left(\frac{2}{14}\right) + 9\left(\frac{2}{14}\right) + 16\left(\frac{5}{14}\right)$$

$$\sigma^2 = \frac{130}{7} = 18.57$$

You can vlidate the result using a variance calculator and indrocind the respective values, next i post and screendshot of my validation process.

The previous calculator can be found here.

SOLVING C

$$\sigma^2 = \langle j^2 \rangle - \langle j \rangle^2. \qquad [1.12]$$

$$\sigma^2 = <j^2> - <j>^2 = \frac{3217}{7} - 21^2 = \frac{3217}{7} - 441$$

$$\sigma^2 = \frac{130}{7} = 18.57$$

Problem 1.2 Consider the first 25 digits in the decimal expansion of π (3, 1, 4, 1, 5, 9, . . .).

(a) If you selected one number at random from this set, what are the probabilities of getting each of the 10 digits?

(b) What is the most probable digit? What is the median digit? What is the average value?

(c) Find the standard deviation for this distribution.

The 25 first numbers of pi are:

3.141592653589793238462643

3.141592653589793238462643

And

$$N(j=1)=2$$

$$N(j=2)=3$$

$$N(j=3)=5$$

$$N(j=4)=3$$

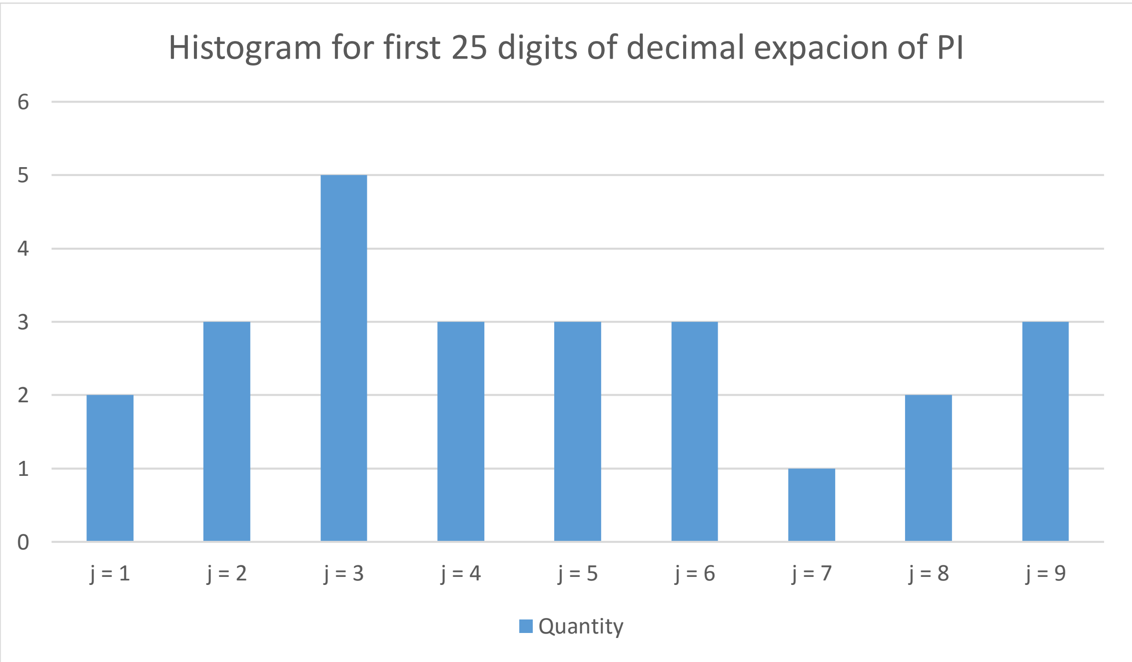
$$N(j=5)=3$$

$$N(j=6)=3$$

$$N(j=7)=1$$

$$N(j=8)=2$$

$$N(j=9)=3$$



SOLVING A

$$P(j=1)=\frac{N(j=1)}{N}=\frac{2}{25}$$

$$P(j=2)=\frac{N(j=2)}{N}=\frac{3}{25}$$

$$P(j=3)=\frac{N(j=3)}{N}=\frac{5}{25}$$

$$P(j=4)=\frac{N(j=4)}{N}=\frac{3}{25}$$

$$P(j=5)=\frac{N(j=5)}{N}=\frac{3}{25}$$

$$P(j=6)=\frac{N(j=6)}{N}=\frac{3}{25}$$

$$P(j=7)=\frac{N(j=7)}{N}=\frac{1}{25}$$

$$P(j=8)=\frac{N(j=8)}{N}=\frac{2}{25}$$

$$P(j=9)=\frac{N(j=9)}{N}=\frac{3}{25}$$

SOLVING B

The most probable digit is 3. While the median is:

Population size:25

Median: 4

Mean Geometric mean Harmonic mean Mode Standard Deviation Variance Mean absolute deviation |
All dispersion data Box plot

Data is from: Population Sample

Enter comma separated data (numbers only):

1,1,2,2,2,3,3,3,3,3,3,4,4,4,5,5,5,6,6,6,7,
8,8,9,9,9

SUBMIT DATA RESET

Median formulas

This calculator uses two different formulas for calculating the median, depending on whether the number of observations is odd, or it is even:

When the number of observations is odd the formula is:

$$x_m = x_{\frac{n+1}{2}}$$

When the number of observations is even the formula is:

$$x_m = \frac{x_{\frac{n}{2}} + x_{\frac{n+2}{2}}}{2}$$

where n is the number of observations.

And the average

Mean (Average):	4.72

SOLVING C

The standart deviation

Variance (Sample Standard), s^2	6.3766666666667
Population Standard Deviation, σ	2.4741867350707

Problem 1.3 The needle on a broken car speedometer is free to swing, and bounces perfectly off the pins at either end, so that if you give it a flick it is equally likely to come to rest at any angle between 0 and π.

(a) What is the probability density, $\rho(\theta)$? [$\rho(\theta)\, d\theta$ is the probability that the needle will come to rest between θ and $(\theta + d\theta)$.] Graph $\rho(\theta)$ as a function of θ, from $-\pi/2$ to $3\pi/2$. (Of course, part of this interval is excluded, so ρ is zero there.) Make sure that the total probability is 1.

(b) Compute $\langle\theta\rangle$, $\langle\theta^2\rangle$, and σ for this distribution.

(c) Compute $\langle\sin\theta\rangle$, $\langle\cos\theta\rangle$, and $\langle\cos^2\theta\rangle$.

Problem 1.4 We consider the same device as the previous problem, but this time we are interested in the x-coordinate of the needle point—that is, the "shadow", or "projection", of the needle on the horizontal line.

(a) What is the probability density $\rho(x)$? [$\rho(x)\, dx$ is the probability that the projection lies between x and $(x + dx)$.] Graph $\rho(x)$ as a function of x, from $-2r$ to $+2r$, where r is the length of the needle. Make sure the total probability is 1. [*Hint*: You know (from Problem 1.3) the probability that θ is in a given range; the question is, what interval dx corresponds to the interval $d\theta$?]

(b) Compute $\langle x\rangle$, $\langle x^2\rangle$, and σ for this distribution. Explain how you could have obtained these results from part (c) of Problem 1.3.

****Problem 1.5** A needle of length l is dropped at random onto a sheet of paper ruled with parallel lines a distance l apart. What is the probability that the needle will cross a line? [*Hint*: Refer to Problem 1.4.]

***Problem 1.6** Consider the **Gaussian** distribution

$$\rho(x) = Ae^{-\lambda(x-a)^2},$$

where A, a, and λ are constants. (Look up any integrals you need.)

(a) Use Equation 1.16 to determine A.

(b) Find $\langle x \rangle$, $\langle x^2 \rangle$, and σ.

(c) Sketch the graph of $\rho(x)$.