Next: About this document ...

INTEGRATION OF TRIGONOMETRIC INTEGRALS

Recall the definitions of the trigonometric functions.

- $\tan x = \dfrac{\sin x}{\cos x}$
- $\sec x = \dfrac{1}{\cos x}$
- $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{\tan x}$
- $\csc x = \dfrac{1}{\sin x}$

The following indefinite integrals involve all of these well-known trigonometric functions. Some of the following trigonometry identities may be needed.

- A.) $\cos^2 x + \sin^2 x = 1$
- B.) $\sin 2x = 2 \sin x \cos x$
- C.) $\cos 2x = 2\cos^2 x - 1$ so that $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- D.) $\cos 2x = 1 - 2\sin^2 x$ so that $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
- E.) $\cos 2x = \cos^2 x - \sin^2 x$
- F.) $1 + \tan^2 x = \sec^2 x$ so that $\tan^2 x = \sec^2 x - 1$
- G.) $1 + \cot^2 x = \csc^2 x$ so that $\cot^2 x = \csc^2 x - 1$

It is assumed that you are familiar with the following rules of differentiation.

- $D(\sin x) = \cos x$
- $D(\cos x) = -\sin x$
- $D(\tan x) = \sec^2 x$
- $D(\cot x) = -\csc^2 x$
- $D(\sec x) = \sec x \tan x$
- $D(\csc x) = -\csc x \cot x$

These lead directly to the following indefinite integrals.

- 1.) $\displaystyle\int \cos x \ dx \ = \ \sin x + C$
- 2.) $\displaystyle\int \sin x \ dx \ = \ -\cos x + C$
- 3.) $\displaystyle\int \sec^2 x \ dx \ = \ \tan x + C$
- 4.) $\displaystyle\int \csc^2 x \ dx \ = \ -\cot x + C$
- 5.) $\displaystyle\int \sec x \tan x \ dx \ = \ \sec x + C$
- 6.) $\displaystyle\int \csc x \cot x \ dx \ = \ -\csc x + C$

The next four indefinite integrals result from trig identities and u-substitution.

- 7.) $\displaystyle\int \tan x \ dx \ = \ \ln|\sec x| + C$
- 8.) $\displaystyle\int \cot x \ dx \ = \ \ln|\sin x| + C$
- 9.) $\displaystyle\int \sec x \ dx \ = \ \ln|\sec x + \tan x| + C$
- 10.) $\displaystyle\int \csc x \ dx \ = \ \ln|\csc x - \cot x| + C$

We will assume knowledge of the following well-known, basic indefinite integral formulas :

- $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C$, where n is a constant $(n \neq -1)$
- $\int \dfrac{1}{x} \, dx = \ln|x| + C$
- $\int e^x \, dx = e^x + C$
- $\int k f(x) \, dx = k \int f(x) \, dx$, where k is a constant
- $\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$

Most of the following problems are average. A few are challenging. Many use the method of u-substitution. Make careful and precise use of the differential notation dx and du and be careful when arithmetically and algebraically simplifying expressions.

- *PROBLEM 1 :* Integrate $\int \sin 3x \, dx$.

  Click HERE to see a detailed solution to problem 1.

- *PROBLEM 2 :* Integrate $\int \tan 5x \, dx$.

  Click HERE to see a detailed solution to problem 2.

- *PROBLEM 3 :* Integrate $\int 5 \sec 4x \tan 4x \, dx$.

  Click HERE to see a detailed solution to problem 3.