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Generation of an Anatomically Based Three-Dimensional Model of the Conducting Airways

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Abstract—An anatomically accurate model of the conducting airways is essential for adequately simulating gas mixing, particle deposition, heat and water transfer, and fluid distribution. We have extended a two-dimensional tree-growing algorithm to three dimensions for generation of a host-shape dependent three-dimensional conducting airway model. Terminal branches in the model are both length limited and volume-supplied limited. A limit is imposed on the maximum possible branch angle between a daughter and parent branch. Comparison of the resulting model with morphometric data shows that the algorithm produces branching and length ratios, path lengths, numbers of branches, and branching angles very close to those from the experimental data. The correlation between statistics from the generated model and those from morphometric studies suggests that the conducting airway structure can be described adequately using a ''supply and demand'' algorithm. The resulting model is a computational mesh that can be used for simulating transport phenomena. © *2000 Biomedical Engineering Society.* [S0090-6964(00)00207-1]

Keywords—Respiratory, Computational model, Lungs.

INTRODUCTION

Mathematical models of the lungs have been used extensively to investigate gas mixing,[2,4,13,14,19,24] aerosol deposition,[3,21,17] and flow distribution.[5] Mathematical models can provide useful insights into gas and particulate concentrations, pressure, and flow in the smaller airways, unobtainable by direct measurement.

The airways in the human lungs can be broadly classified into two main groups: the conducting airways (from the trachea to the terminal bronchioles) whose function is to transport gas to and from the more peripheral gas exchange surfaces and the respiratory airways where gas exchange with the blood takes place. The complex of respiratory branches arising from a single terminal bronchiole is a pulmonary acinus.

The conducting airways in the human lungs are a complex asymmetric branching structure. However, most computational models of the conducting airways have used simplified models of airway anatomy to keep the amount of computation required to solve the governing equations relatively small. The most commonly used conducting airway model has been Weibel's symmetric model A.[26] This model has provided a good basis for the development of a range of increasingly sophisticated gas transport and mixing models. Because every pathway from the trachea to the terminal bronchioles is identical, the governing equations have usually been solved down only one pathway. Although the symmetric model can be computationally inexpensive, it obviously cannot be used to investigate the effect of anatomical asymmetry on transport problems. Asymmetry of the lung structure becomes important when considering gravity effects in aerosol deposition, or when different regions of the lungs have different mechanical properties, or experience different pressures during breathing. The importance of branching asymmetry for accurately simulating gas mixing has also been recognized.

The position of an airway can be described relative to the stem branch (usually the trachea) by assigning the stem branch to be ''generation'' one, then branches arising from this are generation two, and so on. This is classification by Weibel generation.[26] Airway position can also be described by Horsfield order,[9] where the terminal branches are assigned order one, and parent branches are assigned one order higher than their ''daughter'' branch of highest order. Strahler ordering[9] is similar to Horsfield ordering in that the terminal branches are assigned order one, and ordering proceeds up to the stem branch. When the daughter branches are of the same order, the parent branch is assigned one order higher; when the daughter branches have different orders, the parent branch is assigned the same order as the daughter branch with highest order. Parent and daughter branches of the same order are then replaced by a single branch.

Horsfield and Cumming[9] took detailed measurements from a cast of the conducting airways. From this Horsfield *et al.*[10] derived a conducting airway model with regular asymmetry based on delta, the difference in

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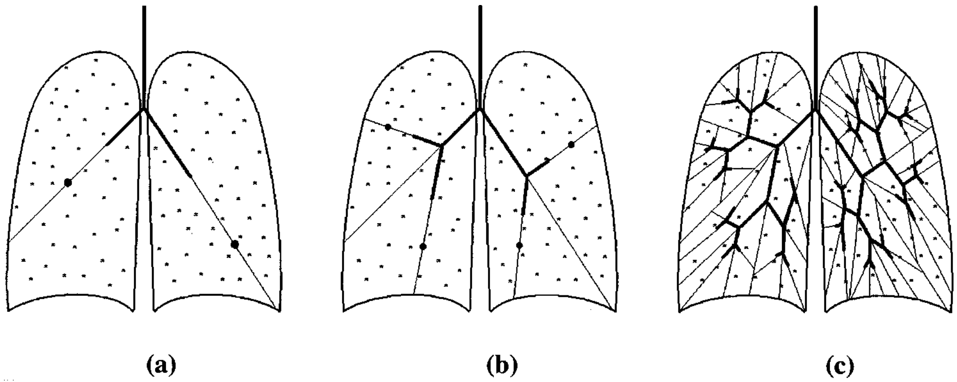

FIGURE 1. Two-dimensional tree generation using the method of Wang *et al.* (see Ref. 25). (a) Host spaces filled with random points, centers of mass calculated for each half lung, dividing line projected to boundary, and branch constructed on the line. (b) Random points split into four sets, and branching repeated. (c) Branching continues until only one point left in each set.

Horsfield order between daughter branches. The delta model can be used to describe the numbers of branches in each order, and their connectivity. Gillis and Lutchen[5] have used the delta model to predict flow distributions among the acini during heterogeneous bronchoconstriction. Although the model has proved useful for this type of calculation, it does not incorporate spatial positions for the branches, and therefore is not directly useful for investigating gravity-dependent or structure-dependent effects.

Perzl *et al.*[15] have developed an approach for reconstructing conducting airway models from high-resolution computed tomography. Their procedure has the potential to produce a realistic conducting airway model based directly on anatomy, but the process requires extensive imaging of an individual lung to produce a single model.

The deterministic algorithm proposed by Kitaoka *et al.*[12] to generate a three-dimensional conducting airway model is based on a power law relationship between airway diameter and flow. The algorithm uses nine ''basic'' rules to assign branch diameters, angles, and lengths in a bifurcating system that aims to supply uniformly distributed acini. Four ''supplementary'' rules were found necessary to ensure that the model approached a realistic branching pattern. The algorithm successfully generated a visually lung-like tree in an idealized thoracic cavity. However, the resulting model was markedly more asymmetric than the human lungs, and the algorithm was shown to be very sensitive to the assumed geometry of the thoracic cavity, and to a number of parameters fundamental to the method. Not enough information was provided with the model to support the author's claim that it compared well with morphometric data.

The purpose of this paper is to present a method for generating conducting airway models that ''grow'' into any given lung space, that correspond statistically to published data for average dimensions and order distributions, and that can be used as a computational mesh.

THE GENERATED MODEL

Basis for the Model

The lungs are a space-filling structure: the conducting airways bifurcate (divide into two daughter branches) repeatedly to supply around 30,000 respiratory units[6] distributed within two irregularly shaped lung cavities. An algorithm to generate a conducting airway model should therefore consider the supply of respiratory units as well as physical constraints on branch dimensions and orientation.

A Monte Carlo method for growth of bifurcating systems in two-dimensional space in response to host geometry has been described by Wang *et al.*[25] The first step in the method is to calculate a large collection of random points on a plane, then the following algorithm is used to generate a bifurcating system to service the points:

(1) The center of mass of the points is found by averaging the individual coordinate positions [see Fig. 1(a)].
(2) An imaginary dividing line is drawn from a given starting point through the center of mass to the boundary. A branch is generated from the starting point, lying on the dividing line, and extending a defined fractional distance towards the center of mass [see Fig. 1(b)].
(3) The dividing line is used to define two new subcollections of random points.
(4) The end of the generated branch is used as the start point for branching into the subsets of points [see Fig. 1(c)].

(5) Branching ends when the subcollection of points contains only one point.

The two-dimensional bifurcation algorithm is extended to three dimensions for generation of an anatomically based lung model. The generated model extends from the trachea to the terminal conducting airways, which are the transitional/terminal bronchioles as defined by Haefeli-Bleuer and Weibel.[6]

The Host Mesh

A host volume is defined into which the model is generated. The lung surface used for the host volume in this study was originally a surface mesh derived from magnetic resonance imaging (MRI) data.[1] The surface mesh describes only the surface of the lungs, as more detailed information was not visible on the MRI slices. The horizontal and oblique fissures were determined by examination of slices through the torso from the Visual Man project.[23] These torso slices are 1 mm sections through a frozen cadaver, on which the lung fissures are identifiable. The horizontal and oblique fissures were tracked through the lung, and their positions transferred to the surface lung mesh. The description of the fissures and a refined version of the surface mesh were combined to make a three-dimensional volume mesh. Cubic Hermite basis functions were used in the mathematical description of the surfaces, enabling a good description of the curved surfaces with a relatively small number of elements. The host space for the conducting airway mesh generation is in the form of five subspaces: one for each typical human lobe.

The central airways (from the trachea to the lobar bronchi) are based mainly upon the study of Horsfield and Cumming,[9] but have been adjusted to fit the MRI data and model from Bradley *et al.*[1] The end of each lobar bronchus provides a starting point for model generation into each corresponding lobe.

Adaptations for Three Dimensions

The two-dimensional bifurcation algorithm begins with generation of a large number of random points in the host space. Instead of the random point approach, a fine uniform grid of points was fit inside the three-dimensional host spaces. A uniform grid was computationally less expensive than generating random points. The irregular shape of the host spaces means that meshes generated using the uniform grid will be ('grown') in response to host geometry rather than according to a defined distribution of the random points. Wang *et al.*[25] have stated that the two-dimensional bifurcation algorithm would be equivalent to an area-halving algorithm if the points were numerous and evenly spaced, therefore it is expected that for a fine uniform grid in the host volumes a three-dimensional bifurcation algorithm should approach a volume-halving algorithm.

An obvious difference between a two-dimensional and a three-dimensional bifurcation algorithm is that, whereas plane branching uses a line to divide the collection of points, volume branching will require a plane. In the two-dimensional algorithm using the line from the starting point through the center of mass to split the points is an obvious choice. For consistency between the three-dimensional and two-dimensional algorithms, the splitting plane in the three-dimensional algorithm should include the line from the starting point through the center of mass. However, the choice of a splitting plane that includes this line is infinite. The vector in the direction of the parent branch and the coordinates of the center of mass were used to define the splitting plane.

The two-dimensional algorithm aims to supply each random point in the host space with a terminal branch, hence termination of an airway path occurs when a subcollection of random points contains only one point. In contrast, it is not appropriate to have a terminal branch supplying each uniform grid point in the three-dimensional host volumes, as the terminal airways are not necessarily uniformly distributed nor are they as numerous as the number of grid points. Terminal branches occur in the three-dimensional model when their length is less than a defined limit, or the subcollection of points the branch supplies is less than a defined number. The limit on the number of points in a subcollection ensures that a space exists at the end of each terminal branch for an acinus of a realistic size.

Branch lengths decrease with order in the conducting airway tree. A limit on the length is imposed such that generated branches with a length less than or equal to the length limit are considered to be terminal bronchioles. The terminal bronchiole is on the order of 1–1.5 mm,[6,26] therefore a limit of 1.2 mm was imposed to allow some variation in the terminal bronchiole length. That is, some terminal bronchioles will be longer than the length limit as they will have arisen from the limit on the number of points in a subcollection, and some terminal bronchioles may be far shorter than the length limit.

When a terminal branch arises in the generated conducting airway tree, the grid points assigned to that branch constitute a space that cannot be filled by neighboring branches. If the terminal branch is of a relatively low generation, then the unfilled volume in the host mesh may be far larger than the size of a single acinus. To enable neighboring branches to partially fill the space below a terminal branch, the closest branch to the terminal branch is found, and the grid points are reassigned to it. A limit is set on the highest generation for which point reassignment occurs.

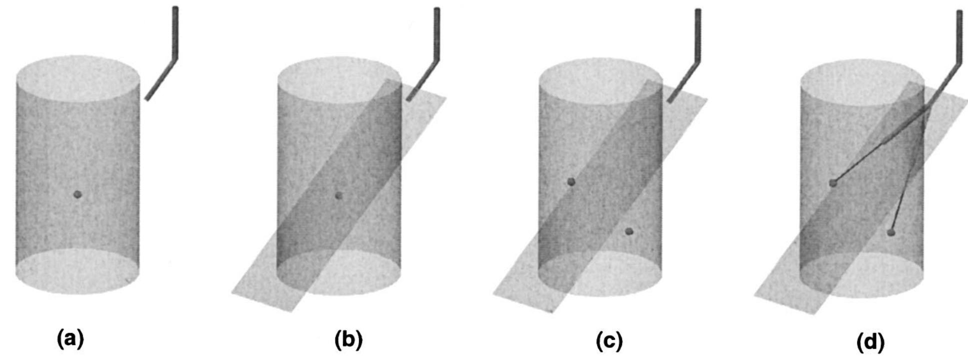

FIGURE 2. Three-dimensional tree generation in a simple volume: (a) calculated center of mass of uniform points; (b) uniform points split into two sets using the plane containing the center of mass and the lobar bronchus vector; (c) center of mass of subsets calculated; (d) branches generated on lines from end of lobar bronchus to centers of mass.

Because of the irregular shape of the host volumes, the center of mass of a subcollection of grid points could be positioned such that the resulting branch has a large branching angle between itself and its parent branch. In reality, the angle of branching is usually below 60°,[7,22] therefore a branch angle limit of 60° is included in the model generation. When a new branch has a branching angle larger than the limit, its angle is reduced to be equal to the angle limit. The branch is altered such that it still lies in the plane defined by its parent branch vector and the line from the end of the parent branch through the center of mass.

As each branch is produced, a check is performed to ensure that the end of the new branch lies inside the host volume. This is done by checking that the end of the new branch lies close enough to one of the grid points to qualify as being inside the host. If the end of the branch is outside of the host volume, then the branch length is reduced until the end is within the host. If this makes the branch shorter than the length limit, then the branch becomes a terminal airway.

The Three-Dimensional Host-Filling Algorithm

After filling the host volume with a fine uniform grid of points, the algorithm used for generating a volume-filling tree is as follows (illustrated in a simple cylinder in Fig. 2):

(1) The center of mass of the points contained by a single host lobe is found by averaging the individual coordinate positions [see Fig. 2(a)].

(2) The vector in the direction of the corresponding lobar bronchus and the coordinates of the center of mass are used to define a splitting plane. The splitting plane is extended to the host boundaries, and points on either side of the plane are assigned into two subcollections of points [see Fig. 2(b)].

(3) The center of mass of each subcollection of points is calculated [see Fig. 2(c)].

(4) An imaginary line is constructed from the end of the lobar bronchus to each center of mass [see Fig. 2(d)].

(5) For each subcollection of points a branch is generated from the end of the lobar bronchus, lying on the imaginary line, and extending a defined fractional distance (the ''branching fraction'') toward the center of mass [see Fig. 2(d)].

(6) The angle between the projection of the parent branch and the new generated branch is calculated. This is the ''branch angle.'' If the branch angle is greater than an angle limit then the angle is set equal to the limit, such that the resulting branch continues to lie in the plane of its parent branch and the imaginary dividing line.

(7) The length of the branch is calculated. If the length is less than or equal to a length limit, then the branch is a terminal airway.

(8) The position of the branch end is checked to make sure it is inside the host space. If the branch end is outside the host space, then its length is reduced until the end point lies within the host.

(9) The number of grid points in the subcollection is compared with the point number limit. If the number of points is smaller than the limit, then the branch is a terminal airway.

(10) When all branching is completed for a single generation, grid points from terminal branches (up to a generation limit) are reassigned to the closest neighboring branches.

(11) The process continues until all pathways are terminated by a terminal airway.

Diameters are randomly assigned to branches in each Horsfield order using data from Horsfield[7] as mean values, and a coefficient of variation of 0.1.

RESULTS

Trees were generated in the right middle lobe using a length limit of 1.2 mm, angle limit of 60°, and a variable

TABLE 1. Effect of branching fraction on the number of terminal branches N_{TB}, the mean generation for terminal branches TBgen, and the mean branch angle in the generated orders θ for the right middle lobe.

Fraction	N_{TB}	TB_{gen}	θ
0.30	3152	16.705	31.13
0.35	3111	16.387	35.56
0.37	3035	16.124	36.41
0.39	3025	16.020	37.97
0.40	2998	15.957	38.45
0.41	2999	15.979	39.21
0.43	2992	15.957	40.73
0.45	2972	15.902	41.80
0.50	2939	15.806	44.69

branching fraction. The branching fraction affects the branch angles, numbers of branches in each order, and the average generation for the terminal branches. Table 1 summarizes the trees generated using different values for the branching fraction. It can be seen that decreasing the branching fraction increases the number of terminal branches and the average terminal generation, and decreases the average branch angle. With a branching fraction of 0.4 the average branch angle is close to the theoretical ideal angle,[8] and the average terminal generation is less than 16, as observed by Raabe *et al.*[18] A branching fraction of 0.4 was therefore adopted for generating all of the conducting airway trees.

A tree was generated in each of the five host volumes using a length limit of 1.2 mm, angle limit of 60°, and a branching fraction of 0.4. The five trees were then combined to produce a full conducting airway model. The full model is shown in Fig. 3, with each lobe tree highlighted in a different color: right upper lobe (green), right middle lobe (red), right lower lobe (blue), left upper lobe (yellow), and left lower lobe (purple). Generation of the full conducting airway model took approximately 1 h of computing time on a Silicon Graphics Origin 2000 computer using a single processor (250 MHz).

It is necessary to compare the generated model with morphometric studies and comparable mathematical models. The studies by Weibel,[26] Horsfield and Cumming,[9] and Phalen *et al.*[16] (based on the data from Raabe *et al.*[18]) and the regular asymmetric model presented by Horsfield *et al.*[10] provides useful information for comparison with statistics from the generated model. The diameter-flow model of Kitaoka *et al.*[12] also provides some statistics for comparison.

Weibel[26] measured dimensions of branches from a cast of the human lungs. Measurements were performed on all branches down to generation 4, then measurements were less complete down to generation 10 where only 10% of the branches were measured. This study was used to develop the widely used symmetric model A.

Horsfield and Cumming[9] performed detailed measurements on a resin cast of the human lungs. Measurements

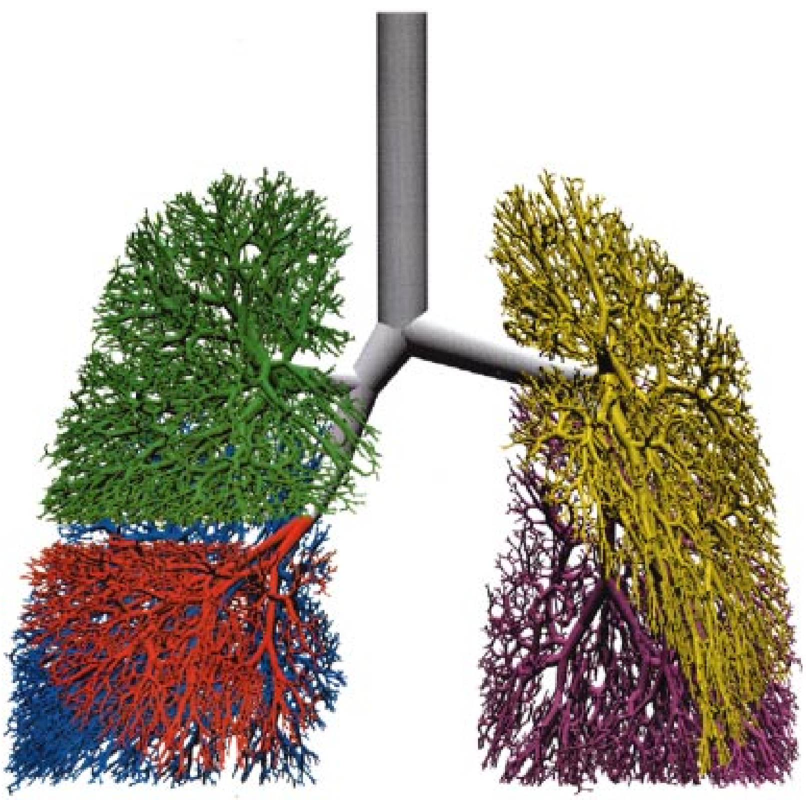

FIGURE 3. The generated conducting airway model, viewed from the front. Right upper lobe (green), right middle lobe (red), right lower lobe (blue), left upper lobe (yellow), left lower lobe (purple).

were taken from the trachea down to airways 0.7 mm in diameter (defined as the ''lobular branches''). These data were complete down to airways of 2.2 mm diameter, and 90% complete at the lobular branches. The lobular branches were assumed to be two or three generations proximal to the terminal bronchioles. Therefore, the Horsfield data are incomplete for the last few orders of the conducting airway tree. However, measurements of individual ''lobules'' confirmed that branching in the terminal region is nearly symmetrical. Horsfield *et al.*[10] developed a conducting airway model with regular asymmetry based on the study of Horsfield and Cumming.[9]

Raabe *et al.*[18] made silicon rubber casts of two human lungs. They made measurements of diameter, length, branching angle, and the angle the branch made with the direction of gravity. For one of the casts, measurements were complete down to branches of 3 mm diameter. The other cast was measured down to a range of diameters, but for three of its lobes these data were 10% complete at the terminal bronchioles.

Horsfield *et al.*[10] presented conducting airway models derived from Horsfield and Cumming[9] using delta, the difference in Horsfield order of daughter branches. Horsfield's delta prescribes an average asymmetry within the lobes. Each daughter pair has a difference in order of 3 down to branches of 0.7 mm diameter.[9]

The diameter-flow algorithm presented by Kitaoka *et al.*[12] generates a lung-like tree into an idealized thoracic cavity by using a relationship between diameter and flow to constrain model branch diameter, angle, and length. The approximate number of terminal branches is specified by setting a flow limit. That is, flow decreases with each branch division down to the flow limit, at which point the branch is declared to be terminal. Branches are produced into ''branching planes,'' where the branching plane of the daughter branches is generally constrained to be perpendicular to the branching plane of the parent branch. Branches that project outside of the host boundary are either forced back inside the host, or are terminal branches.

Table 2 shows the average number of generations from the trachea to the terminal bronchioles for the generated model and the comparison studies. Weibel[26] found an average of 16 generations from the trachea to the terminal bronchiole, whereas Horsfield and Cumming[9] measured an average of 14.6 generations from the trachea to the lobular branches. Assuming that there are two or three further generations with symmetric branching, this gives an average of 16.6–17.6 generations to the terminal bronchioles. Phalen *et al.*[16] give the average number of generations from the trachea to the terminal branches for each lobe. Kitaoka *et al.*[12] give an average for their whole model of 17.6. The average values from the generated model are around 16, which is similar to the comparison studies, except for the right lower lobe which has an average of 17.42 generations. This is higher than the Weibel study, but closer to the Horsfield and Phalen studies.

TABLE 2. Average number of generations from the trachea to the terminal bronachioles for Weibel,[a] Horsfield and Cumming,[b] Phalen *et al.*,[c] and the generated model, for each of the five lobes.

Lobe	Weibel	Horsfield	Phalen	Generated
R.U.L.	16	17.6	15	16.37
R.M.L.	16	17.6	15	15.99
R.L.L.	16	17.6	17	17.42
L.U.L.	16	17.6	15	16.34
L.L.L.	16	17.6	16	16.16

[a]See Ref. 26.
[b]See Ref. 9.
[c]See Ref. 16

The shortest path length from the carina (the first division of the trachea) to a lobular branch measured by Horsfield and Cumming[9] was 75 mm, and the longest was 215 mm. The shortest path from the carina to the end of the terminal bronchiole in the generated model was 102 mm and the longest 239 mm. The path lengths are therefore slightly longer in the generated model, which is partly accounted for by the extra few generations from the lobular branch to the terminal bronchiole in the generated tree. The range of bifurcations to reach a terminal branch in the generated model is 10–26, compared with 8–25 from Horsfield and Cumming,[9] 11–28 in the delta model, and 8–32 in the diameter-flow model.

The number of terminal bronchioles in the generated model is 29,445. Weibel[26] estimated 65,536 terminal bronchioles, whereas Horsfield and Cumming[9] estimated 27,992. A more recent study of the respiratory airways by Haefeli-Bleuer and Weibel[6] estimates the number of terminal bronchioles to be in the range of 26,000–32,000. The generated model therefore lies in the middle of this range.

For the Horsfield delta model, a fixed delta of 3 was prescribed for the airways from the lobar bronchi to order 7 branches (if we assume that terminal bronchioles are order 1). The Horsfield orders and delta values were calculated for the generated model. Figure 4 shows the number of branches in an order (N_w) divided by the number of branches in the previous order (N_{w-1}) for the generated orders (that is, not for the orders where the numbers of branches were predefined). For the delta model this is usually 1.38. For the generated model, the curve varies around 1.38 in orders 6–20. For orders 1–5, the branching pattern becomes more symmetric, and this is reflected in the rise of N_w/N_{w-1} for both models. As can be seen in Fig. 5, the mean delta value from the generated model is similar to that of the Horsfield delta model.

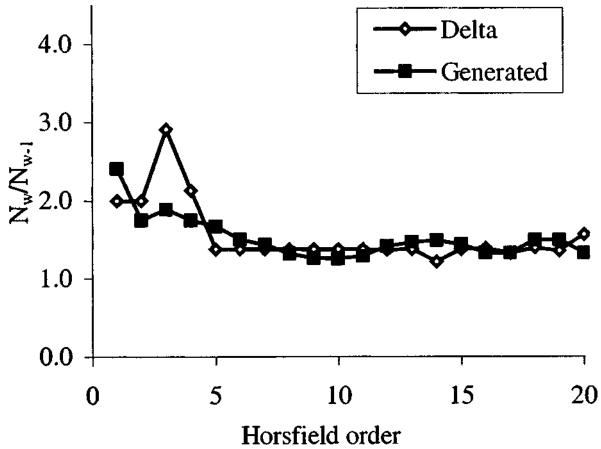

FIGURE 4. Comparison of the number of branches in an order divided by the number of branches in the previous order, for the delta and generated models.

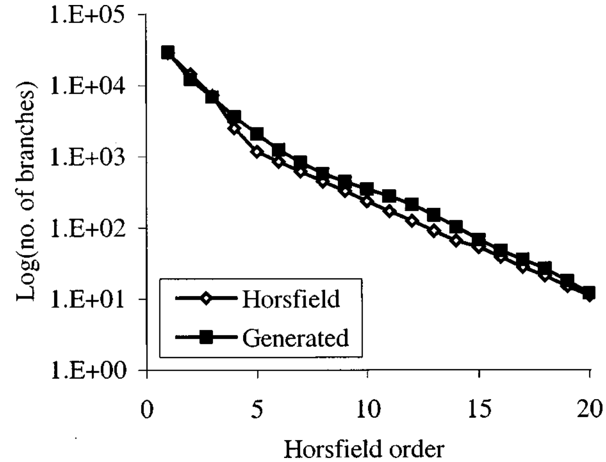

FIGURE 6. Comparison of log(number of branches) against order, for the delta and generated models.

Figure 6 plots Horsfield order against log(number of branches) for the generated and delta models. The curves are similar for the two models, as are the slopes. The slope of the plot of order against log(number of branches) is the ''branching ratio,'' R_b. For the generated model R_b is 1.387, compared with 1.38 for the delta model.

The length ratio R_l and the diameter ratio R_d are calculated from the slopes of order against log (length) and order against log (diameter), respectively. R_l and R_d are calculated for the generated model from the curves in Figs. 7 and 8. Their values are 1.114 for R_l and 1.109 for R_d.

The branching, length, and diameter ratios can also be calculated using Strahler ordering. Table 3 compares branching, length, and diameter ratios calculated from Strahler ordering in the generated model, from Horsfield and Cumming,[9] and from Phalen *et al.*[16]

The branching angles in the generated model are plotted in Fig. 9. The mean angle is fairly constant around 37° for orders 1–22, comprising the sublobar bronchi and bronchioles.

Table 4 compares the proportion of terminal branches in each lobe for the generated model with the lobar volumes given by Horsfield and Cumming[10] for the delta model and for the lung.

DISCUSSION

The host space into which the generated model was produced is an anatomically accurate description of a thoracic cavity, taken from MRI data and anatomical slices. The resulting model is space filling into this host, in contrast to the symmetric model of Weibel and the Horsfield delta model which do not specify spatial positioning of branches. The diameter-flow model is also

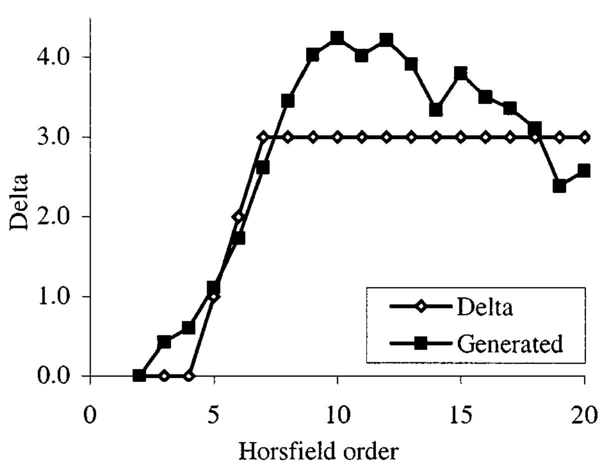

FIGURE 5. Comparison of the delta value per order, for the delta and generated models.

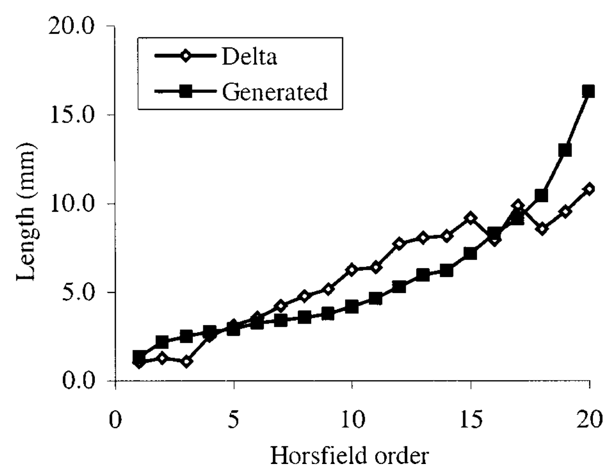

FIGURE 7. Comparison of branch length against order, for the delta and generated models.

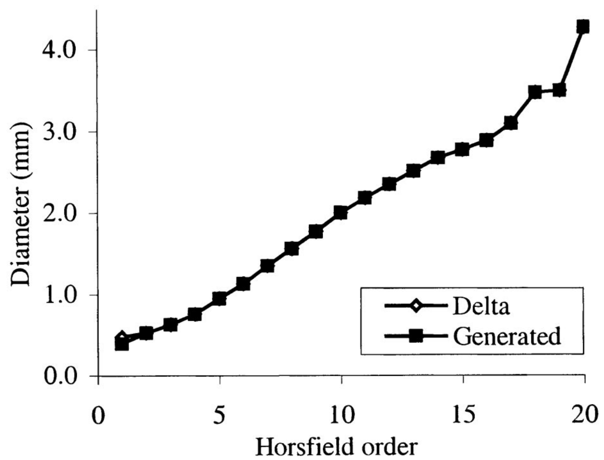

FIGURE 8. Comparison of the branch diameter against order, for the delta and generated models.

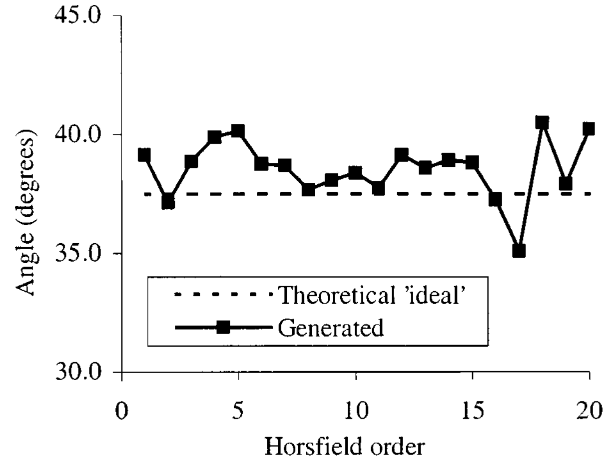

FIGURE 9. Comparison of the average branch angle from the generated model with the theoretical ideal angle.

space filling. However, it was generated into an idealized geometry. When more accurate descriptions of host geometry were used, the number of branches terminated because they projected outside the host grew larger.

The generation algorithm presented in this paper produces a strictly bifurcating tree; that is, there are no divisions into three or more branches. Trifurcations have been observed in the lung, but on close inspection have usually been found to be two bifurcations connected by a very short branch.[6,9,26] The generation algorithm is therefore capable of producing an apparent trichotomy. In terms of function, a trichotomy will behave similarly to two bifurcations joined by a short branch.

The number of branches in an order, branch length, and branch diameter are generally logarithmic functions of order. From calculations based on minimum flow resistance and minimum entropy production the relationship between R_b, R_d, and R_l has been shown to approximate $R_b^{1/3}=R_d=R_l$.[11] Values for R_b, R_d, and R_l have been calculated for the generated model using both Horsfield ordering and Strahler ordering. The Horsfield R_b from the generated and delta models are very close in value, but the Strahler R_b from the generated model is slightly smaller than values calculated from morphometric studies. The Strahler R_b for the delta model is for branches down to a diameter of 0.7 mm; for branches with smaller diameter R_b is 2.320,[7] therefore R_b from the generated model (which is for all of the conducting airway branches, regardless of diameter) lies within the measured range. Both the Horsfield and Strahler ratios for branch numbers, length, and diameter in the generated model exhibit the cube-root relationship. In contrast, the diameter-flow algorithm produced a tree with higher asymmetry than the human lungs. Constraining the algorithm to produce a more symmetric tree resulted in more branches terminating because they projected outside the host boundary.

Although the numbers of branches and the branch lengths are generated using the three-dimensional algorithm, the diameters have been assigned randomly, based on order with an arbitrarily chosen coefficient of variation of 0.1. The diameter ratio will therefore be very close to that for the delta model, regardless of the generated tree. The coefficient of variation was chosen because of the absence of information on diameter variations in the literature. Diameter could also be assigned based on the expected relationship between length and diameter (that is, length is approximately three times the diameter). This raises difficulties when the length is significantly shorter than the mean length for the branch

TABLE 3. Branching ratios calculated using Strahler ordering for Horsfield and Cumming,[a] Phalen *et al.*,[b] and the generated model.

Ratio	Horsfield	Phalen	Generated
R_b	2.805	2.508	2.358
R_d	1.427	1.351	1.323
R_l	1.402	1.333	1.344

[a]See Ref. 9.
[b]See Ref. 16.

TABLE 4. Percentage volumes of lobes for Horsfield and Cumming,[a] the delta model, and the generated model.

Lobe	Horsfield	Delta	Host	Generated
R.U.L.	21	19	22.9	22.8
R.M.L.	9	10	10.3	10.2
R.L.L.	25	26	22.5	22.5
L.U.L.	20	19	23.5	23.7
L.L.L.	25	26	20.8	20.8

[a]See Ref. 9.

order. In this case, the diameter could be significantly smaller than the diameters of its daughter branches. It is assumed that for computational purposes, either the relationship between diameter and order or between diameter and length would be deemed the most appropriate method for assigning diameter values. With a more detailed assignment of diameter dimensions it would be possible to investigate major/minor daughter relationships.

The average terminal generation in the delta model is 17.6, which is an overestimate according to Phalen *et al.*[16] This is the same average value resulting from the diameter-flow algorithm. The generated model has average terminal generations within the range observed by the comparison studies of Weibel,[26] Phalen,[16] and Horsfield and Cumming.[9] This is important from a functional point of view, as the number of bifurcations down an airway path will have a significant influence on the distribution and behavior of gas flow.

The delta model has a fixed difference between daughter branch orders from the lobar bronchi down to order 6. The generated model has a range of delta values, but the mean value of delta varies between 4 and 2 in the lobar bronchi to order 6 range. For lower orders, delta decreases rapidly in both models, reflecting more symmetric branching. This is consistent with observation.[9]

The fine grid of uniform points in the host volume is split by the plane that contains the parent branch and the center of mass (the ''splitting plane''). The plane is therefore not constrained to a set angle as in the diameter-flow algorithm, where the splitting plane is generally at 90° to the branching plane of the parent branch. The branching planes in the generated model have an average angle between them of 90.1° with a standard deviation of 39.0°. In reality, successive bifurcations often lie in planes at right angles to each other.[7] This arrangement reduces unequal distribution of flow that can be caused by having bifurcations occurring in the same plane.[20]

Branch angles have been observed, on average, to increase toward the periphery of the tree.[9,16] In contrast, the generated model shows a fairly constant mean branch angle for the generated branches, and a steadily decreasing minimum angle with order. Although the branching angles in the generated model do not exhibit the observed trend, their mean value is close to the theoretical ''ideal'' value proposed by Horsfield and Cumming[8] of 37°28′.

Horsfield and Cumming[9] found that alveoli situated near the hilum of the lung were supplied by recurrent bronchi which turned back from the larger intrasegmental branches through 180° or more. The nature of the generation algorithm means that this is also the mechanism by which alveoli in this area are supplied.

The volume proportions of the right upper, right middle, and left upper lobes for the host lung are higher than for the lung studied by Horsfield and Cumming.[9] The proportions for the lower lobes are lower, in particular the left lower lobe. However, the number of terminal branches generated in each lobe is in proportion to the percentage volume of lung.

Although the generation algorithm is used to ''grow'' a model into a defined volume, it does not attempt to mimic the actual growth process of a lung; the algorithm is only concerned with simulating a single adult conducting airway tree. Real growth would require extension of the algorithm to include a deforming host geometry and a more complex set of rules to govern branch splitting.

The conducting airway model has been generated as a computational finite element mesh, with information stored about branch length, diameter, and spatial position. By coupling appropriate respiratory airway models to the terminal branches in the conducting airway model, gas transport phenomena can be investigated.

CONCLUSION

We have shown that the generation algorithm produces a conducting airway model with branching characteristics close to those measured. The model has branching, length, and diameter ratios consistent with morphometric studies, and the average terminal generation lies within the observed range. The branch angles are consistently close to the ideal angle proposed by Horsfield and Cumming.[8]

Weibel's symmetric model[26] and Horsfield's delta model[10] have proved very useful for specific lung function investigations, but Weibel's model is limited to studies in which asymmetry is not important, and both models are only useful for studies in which the spatial distribution of airway paths can be neglected. To thoroughly investigate gas mixing, aerosol deposition, and water/heat transfers, a more complete model of the conducting airways is required. The diameter-flow algorithm presented by Kitaoka *et al.*[12] produces a three-dimensional airway model but the algorithm is relatively complicated and is very sensitive to changes in the assumed host geometry and fundamental model parameters.

The three-dimensional algorithm described in this paper provides an effective method for generating conducting airway models specific to a given host geometry. This will be useful for ''normal'' lungs, but may be of particular use for lungs with structural abnormalities.

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