Reinforcement learning

Episode 6

Policy gradient methods

Yandex
Data Factory

LAMBDA

British Hedgehog
Preservation Society

Small experiment

The next slide contains a question

Please respond as fast as you can!

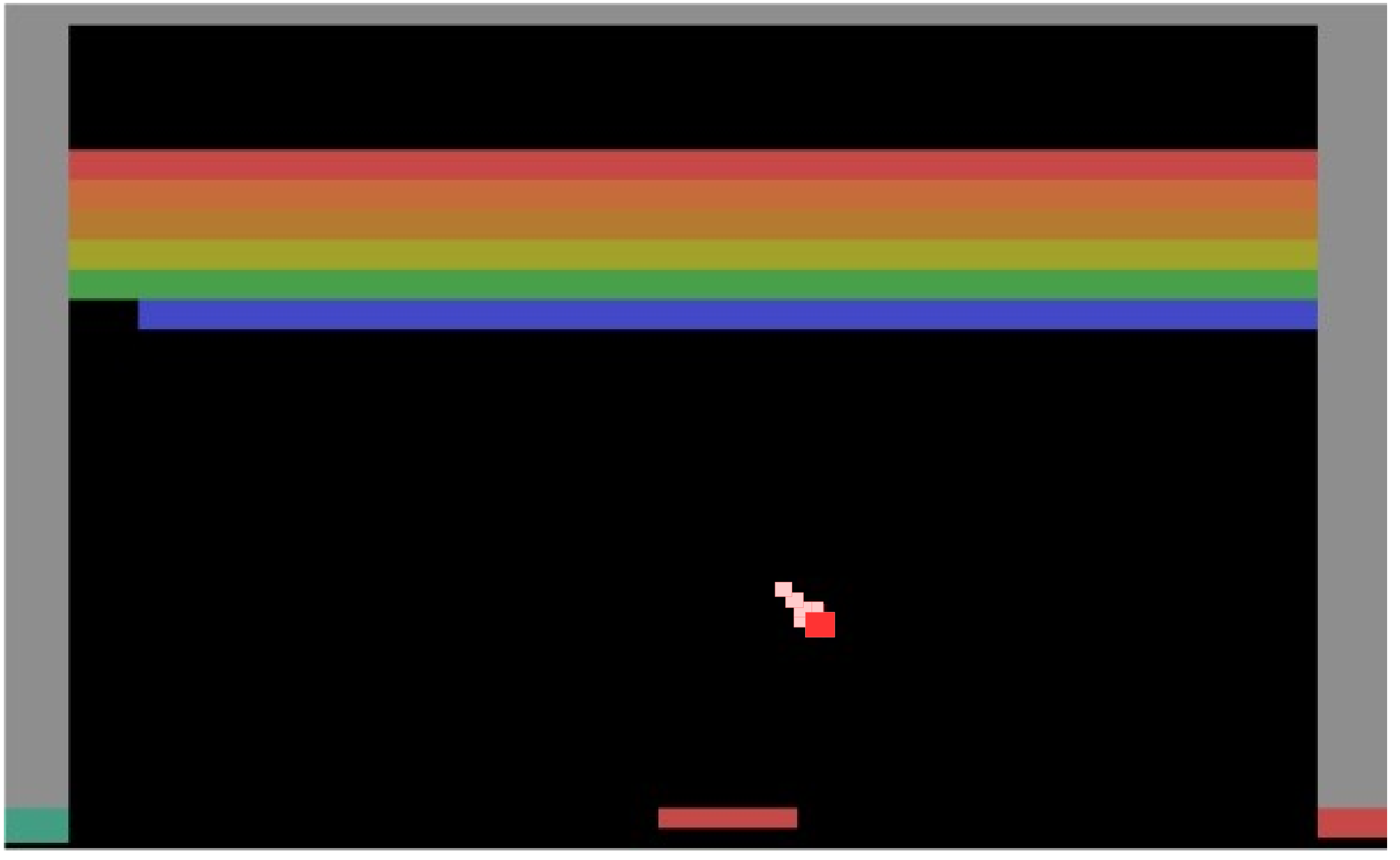
Small experiment

left or **right**?

Small experiment

Right! Ready for next one?

Small experiment



What's **Q(s,right)** under gamma=0.99?

Small experiment

What's **Q(s,right)** under gamma=0.99?

Conclusion

- Often computing q-values is harder than picking optimal actions!
- We could avoid learning value functions by directly learning agent's policy $\pi_\theta(a|s)$

Trivia: what algorithm works that way?

(of those we studied)

Conclusion

- Often computing q-values is harder than picking optimal actions!
- We could avoid learning value functions by directly learning agent's policy $\pi_\theta(a|s)$

Trivia: what algorithm works that way?

e.g. crossentropy method

NOT how humans survived

argmax[
Q(s,pet the tiger)
Q(s,run from tiger)
Q(s,provoke tiger)
Q(s,ignore tiger)
]

how humans survived

$$\pi(run|s)=1$$

Policies

In general, two kinds

- Deterministic policy

$$a = \pi_\theta(s)$$

- Stochastic policy

$$a \sim \pi_\theta(a|s)$$

Trivia: Any reason why stochastic is better?

Policies

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Trivia: Any reason why stochastic can be better?

Policies

In general, two kinds

- Deterministic policy

same action each time

Genetic algos (week 0)
Deterministic policy gradient

$$a = \pi_\theta(s)$$

- Stochastic policy

sampling takes care of exploration

Crossentropy method
Policy gradient

$$a \sim \pi_\theta(a|s)$$

Policies

In general, two kinds

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Deterministic policy gradient

$$a = \pi_\theta(s)$$

- Stochastic policy

sampling takes care of exploration

Crossentropy method
Policy gradient

$$a \sim \pi_\theta(a|s)$$

categorical, normal, mixture of normal, whatever

Two approaches

- **Value based:**

  Learn value function $Q_\theta(s,a)$ or $V_\theta(s)$

  Infer policy $\pi(a|s)=[a=\underset{a}{argmax}\,Q_\theta(s,a)]$

- **Policy based:**

  Explicitly learn policy $\pi_\theta(a|s)$ or $\pi_\theta(s)\rightarrow a$

  Implicitly maximize reward over policy

Recap: crossentropy method

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions
  - elite = take M best sessions and concatenate

$$\theta_{i+1} = \theta_i + \alpha \nabla \sum_i \log \pi_{\theta_i}(a_i | s_i) \cdot [s_i, a_i \in Elite]$$

Trivia: Can we adapt it for discounted rewards? (with **γ**)

Recap: crossentropy method

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TD version: elite (s,a) that have highest R(s,a)

(select elites independently from each state)

Policy gradient main idea

Why so complicated?

We'd rather simply maximize R over pi!

Objective

Expected reward:

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a) \\ \dots}}{E} R(s, a, s', a', \dots)$$

Expected discounted reward:

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} Q(s, a)$$

Objective

Expected reward: R(z) setting

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a) \\ \ldots}}{E} R(s, a, s', a', \ldots)$$

Expected discounted reward: R(s,a) = r + y*R(s',a')

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} Q(s, a)$$

“true” Q-function

Objective

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} Q(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s,a)\, da\, ds$$

Objective

Agent's policy

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} Q(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s,a)\, da\, ds$$

True action value

state visitation frequency
(may depend on policy)

Trivia: how do we compute that?

Objective

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} Q(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s,a)\, da\, ds$$

Empirical action value
a.k.a. R(s,a)

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} Q(s,a)$$

sample N sessions

Objective

$$J = \mathop{E}_{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}} Q(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s,a)\, da\, ds$$

Empirical action value
a.k.a. R(s,a)

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} Q(s,a)$$

sample N sessions

Can we optimize policy now?

Objective

$$J = \mathop{E}_{\substack{s\sim p(s) \\ a\sim\pi_\theta(s|a)}} Q(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s,a)\, da\, ds$$

parameters “sit” here

Empirical action value
a.k.a. R(s,a)

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a\in z_i} Q(s,a)$$

We don't know how to compute dJ/dtheta

Optimization

Finite differences

- Change policy a little, evaluate

$$\nabla J \approx \frac{J_{\theta+\epsilon} - J_{\theta}}{\epsilon}$$

Stochastic optimization

- Good old crossentropy method
- Maximize probability of “elite” actions

Optimization

Finite differences

– Change policy a little, evaluate

$$\nabla J \approx \frac{J_{\theta+\epsilon} - J_{\theta}}{\epsilon}$$

Stochastic optimization

– Good old crossentropy method

– Maximize probability of “elite” actions

Trivia: any problems with those two?

Optimization

Finite differences

- Change policy a little, evaluate

$$\nabla J \approx \frac{J_{\theta+\epsilon} - J_{\theta}}{\epsilon}$$

VERY noizy, especially if both J are sampled

Stochastic optimization

“quantile convergence” problems with stochastic MDPs

- Good old crossentropy method
- Maximize probability of “elite” actions

Objective

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} Q(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s,a)\, da\, ds$$

Wish list:

- Analytical derivative
- Easy/stable approximations

Logderivative trick

Simple math

$$\nabla \log \pi(z) = ???$$

(try chain rule)

Logderivative trick

Simple math

$$\nabla \log \pi(z) = \frac{1}{\pi(z)} \cdot \nabla \pi(z)$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) Q(s,a)\, da\, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) Q(s,a) \, da \, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

$$\nabla J = \int_s p(s) \int_a \pi_\theta(a|s) \nabla \log \pi_\theta(a|s) Q(s,a) \, da \, ds$$

Trivia: anything curious about that formula?

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) Q(s,a)\, da\, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

$$\nabla J = \underbrace{\int_s p(s) \int_a \pi_\theta(a|s)}_{\text{that's expectation :)}} \nabla \log \pi_\theta(a|s) Q(s,a)\, da\, ds$$

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) Q(s,a) \, da \, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

$$\nabla J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

Policy gradient (REINFORCE)

- Policy gradient

$$\nabla J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

- Approximate with sampling

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

  - Ascend $\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \nabla J$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$

Trivia: is it off- or on-policy?

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

  - Ascend $\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \nabla J$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$
- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$

    actions under current policy = on-policy
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

  - Ascend $\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \nabla J$

value-based Vs policy-based

Value-based

- Q-learning, SARSA, MCTS value-iteration

- Solves harder problem
- Artificial exploration
- Learns from partial experience (temporal difference)
- Evaluates strategy for free :)

Policy-based

- REINFORCE, CEM

- Solves easier problem
- Innate exploration
- Innate stochasticity
- Support continuous action space
- Learns from full session only

value-based Vs policy-based

Value-based

- Q-learning, SARSA, MCTS value-iteration

- Solves harder problem
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Policy-based

- REINFORCE, CEM

We'll learn much more soon!

- Solves easier problem
- Innate exploration
- Innate stochasticity
- Support continuous action space
- ~~Learns from full session only~~

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

What is better for learning:
random action in good state
or
great action in bad state?

REINFORCE baseline

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

Q(s,a) = V(s) + A(s,a)

Actions influence A(s,a) only, so V(s) is irrelevant

REINFORCE baseline

- Initialize NN weights $\theta_0 \leftarrow random$
- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot (Q(s,a) - b(s))$$

Anything that doesn't depend on action
ideally, b(s) = V(s)

Actor-critic

- Learn both V(s) and $\pi_\theta(a|s)$
- Hope for best of both worlds :)

Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

Non-**trivia:** how can we estimate **A(s,a)**
from (s,a,r,s') and V-function?

Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

$$A(s,a)=Q(s,a)-V(s)$$

$$Q(s,a)=r+\gamma\cdot V(s')$$

$$A(s,a)=r+\gamma\cdot V(s')-V(s)$$

Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

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$$A(s,a)=Q(s,a)-V(s)$$

$$Q(s,a)=r+\gamma\cdot V(s')$$

$$A(s,a)=r+\gamma\cdot V(s')-V(s)$$

Also: n-step version

Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

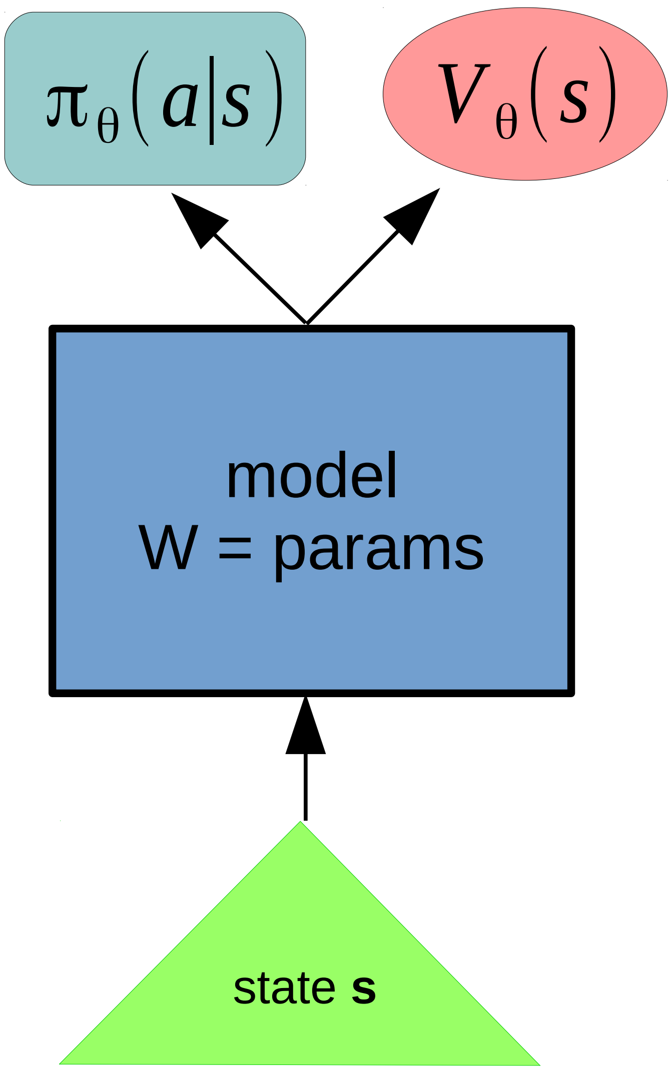
Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

$$A(s,a)=r+\gamma\cdot V(s')-V(s)$$

$$\nabla J_{actor}\approx\frac{1}{N}\sum_{i=0}^{N}\sum_{s,a\in z_i}\nabla\log\pi_\theta(a|s)\cdot\underbrace{A(s,a)}_{\text{consider const}}$$

Trivia: how do we train V then?

Advantage actor-critic



Improve policy:

$$\nabla J_{actor} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot A(s,a)$$

Improve value:

$$L_{critic} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} (V_\theta(s) - [r + \gamma \cdot V(s')])^2$$

Continuous action spaces

What if there's continuously many actions?

- Robot control: control motor voltage
- Trading: assign money to equity

How does the algorithm change?

Continuous action spaces

What if there's continuously many actions?

- Robot control: control motor voltage
- Trading: assign money to equity

How does the algorithm change?

it doesn't :)
Just plug in a different formula for pi(a|s), e.g. normal distribution

Duct tape zone

- V(s) errors less important than in Q-learning
  - actor still learns even if critic is random, just slower
- Regularize with entropy
  - to prevent premature convergence
- Learn on parallel sessions
  - Or super-small experience replay
- Use logsoftmax for numerical stability

Let's code!