Reinforcement learning

Episode 4B

Deep reinforcement learning

What we already know:

- Q-learning

$$L = E_{s \sim S, a \sim A}[(Q(s,a) - (r + \gamma \max_{a'} Q(s', a')))^2]$$

- Approximation of q-values with respect to state $Q(s, a, \Theta)$, where Θ is the vector of weights

- Experience replay

This is not enough!

Autocorrelation

- Target is based on prediction

$$r + \gamma \max_{a'} Q(s', a', \Theta)$$

- Since we use function approximation, when we update $Q(s, a, \Theta)$ we also update $Q(s', a, \Theta)$ towards that direction
- In worst case network may diverge, but usually it becomes unstable.

- **How to stabilize weights?**

Target network

Idea: use network with frozen weights to compute the target

$$L(\Theta) = E_{s\sim S, a\sim A}[(Q(s,a,\Theta) - (r + \gamma \boxed{\max_{a'} Q(s',a',\Theta^-)}))^2]$$

where Θ^- is the frozen weights

Const

Hard target network:

Update Θ^- every **n** steps and set its weights as Θ

Target network

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$$L(\Theta) = E_{s\sim S,a\sim A}[(Q(s,a,\Theta) - (r + \gamma \underbrace{\boxed{\max_{a'} Q(s',a',\Theta^-)}}_{\textbf{Const}}))^2]$$

where Θ^- is the frozen weights

Hard target network:

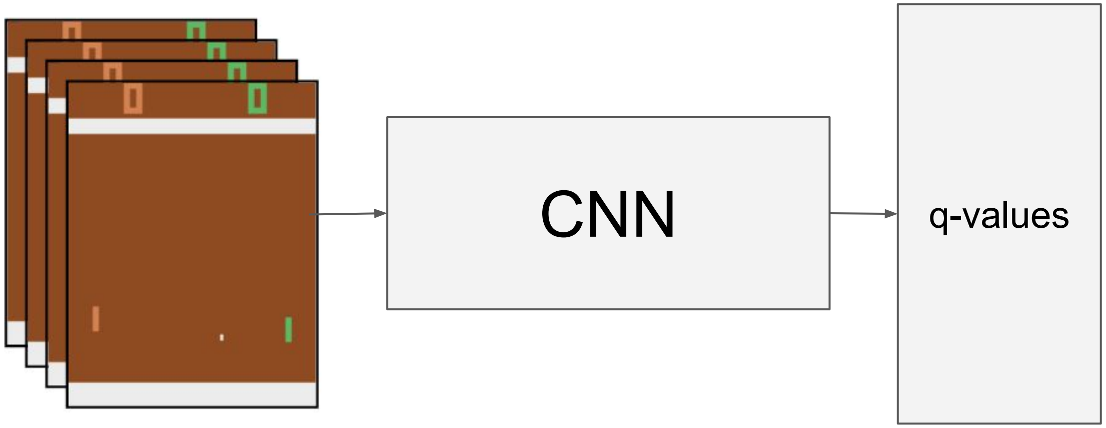
Update Θ^- every **n** steps and set its weights as Θ

Soft target network:

Update Θ^- every step:

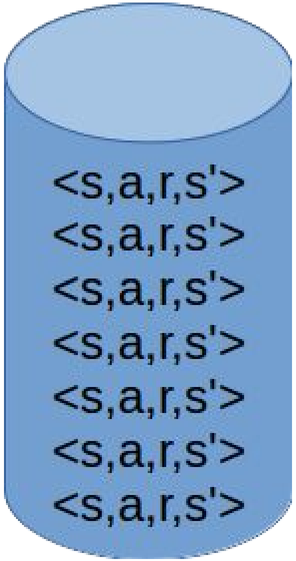
$$\Theta^- = (1-\alpha)\Theta^- + \alpha\Theta$$

Playing Atari with Deep Reinforcement Learning (2013, Deepmind)



4 last frames as input

Experience replay



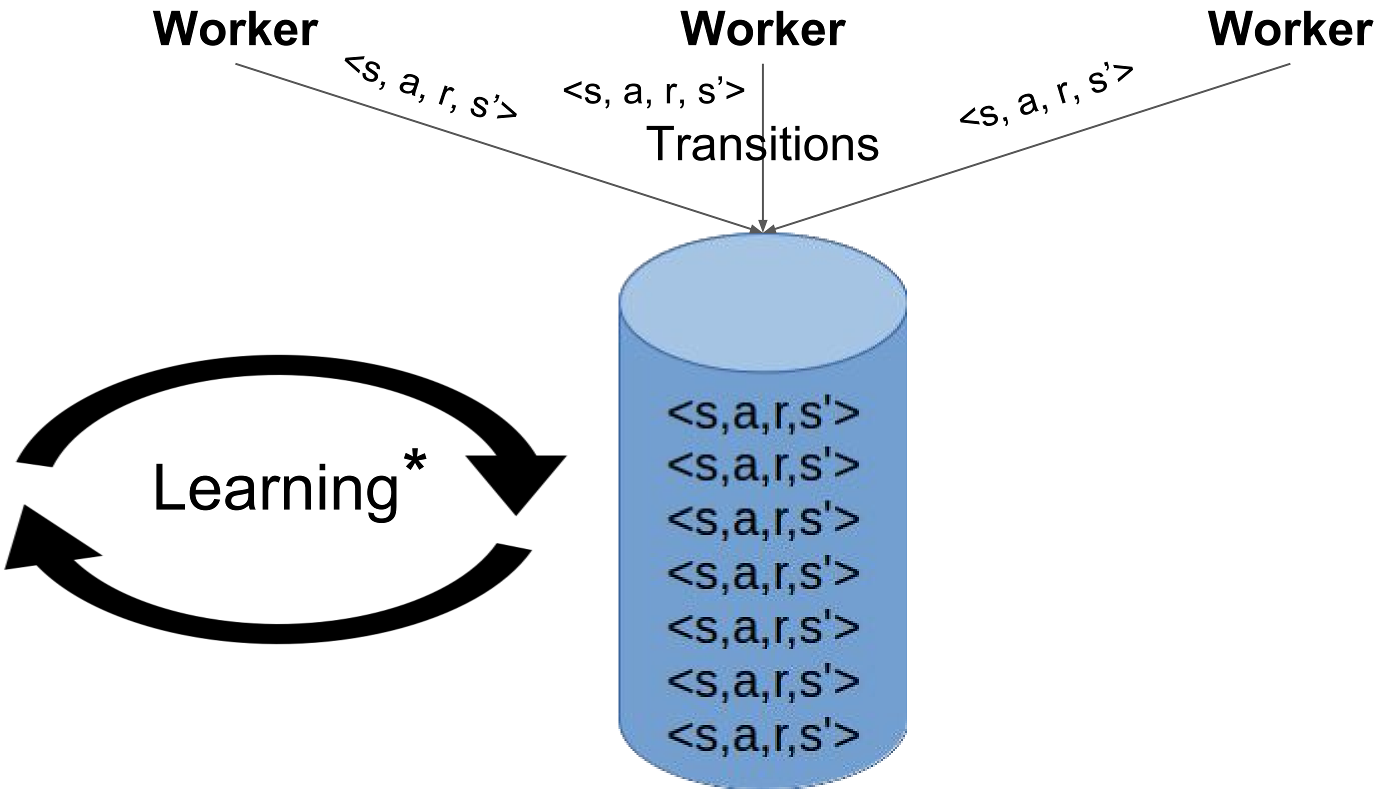
10^6 last transitions

Update weights using:

$$L(\Theta) = E_{s\sim S,a\sim A}[(Q(s,a,\Theta) - (r + \gamma \max_{a'} Q(s',a',\Theta^-)))^2]$$

Update Θ^- every 5000 train steps

Asynchronous Methods for Deep Reinforcement Learning (2016, Deepmind)



***:** $L(\Theta) = E_{s \sim S, a \sim A}[(Q(s,a,\Theta) - (r + \gamma \max_{a'} Q(s',a',\Theta^-)))^2]$

Problem of overestimation

We use “max” operator to compute the target

$$L(s,a) = (Q(s,a) - (r + \gamma \max_{a'} Q(s',a')))^2$$

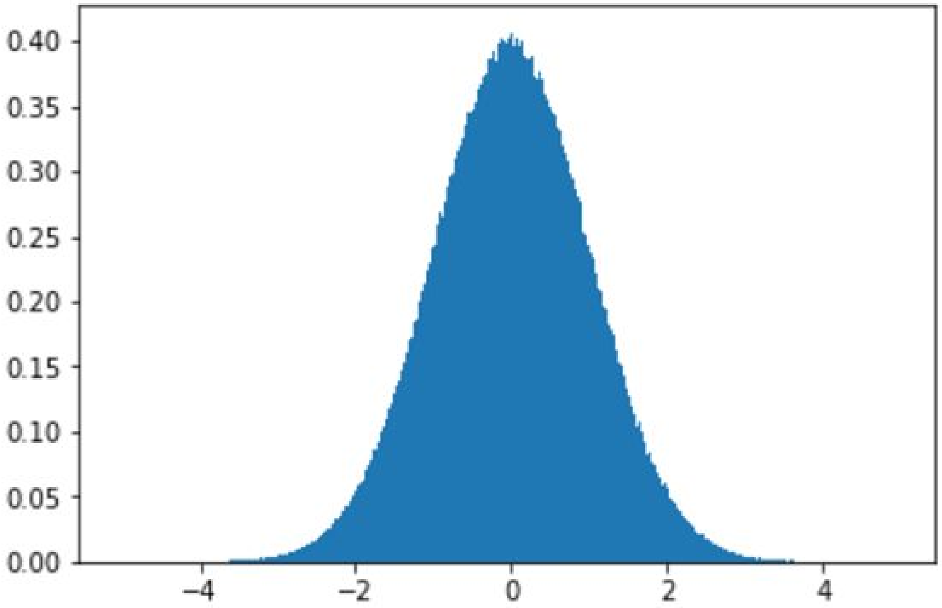
Surprisingly here is a problem

(although we want $\mathrm{E}_{s\sim S, a\sim A}[L(s,a)]$ to be equal zero)

Problem of overestimation

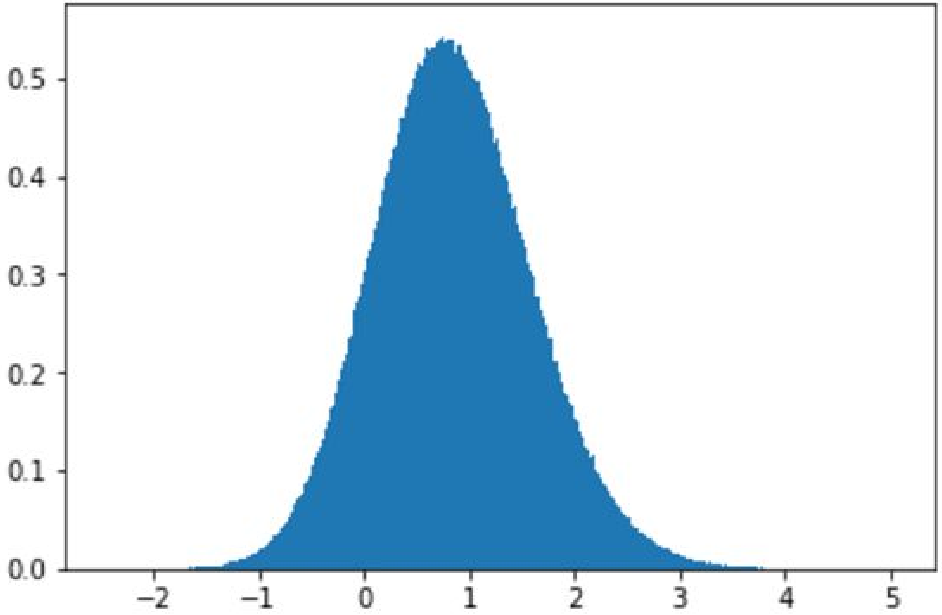
Normal distribution
3*10^6 samples

mean: ~0.0004



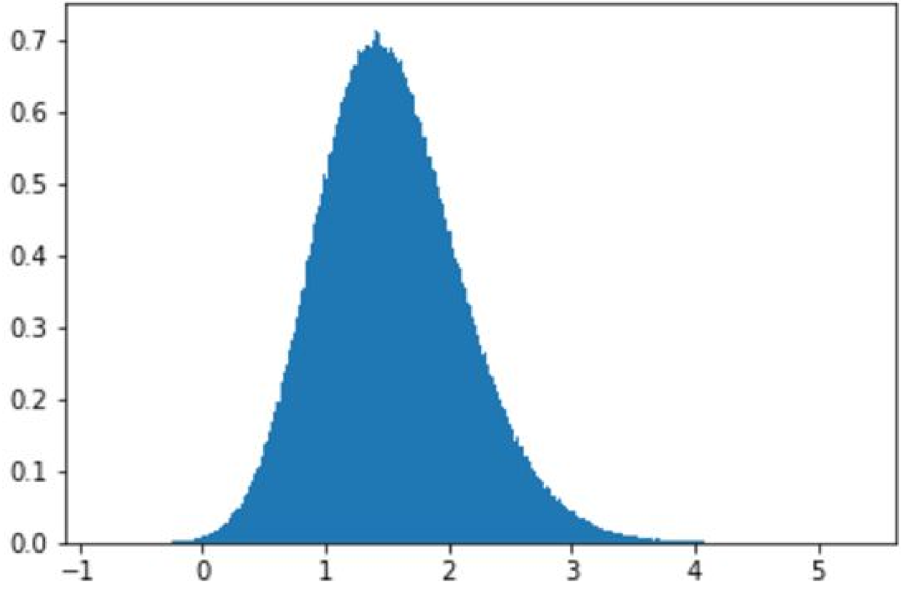
Problem of overestimation

Normal distribution
$3*10^6$ x 3 samples
Then take maximum of every tuple
mean: ~0.8467

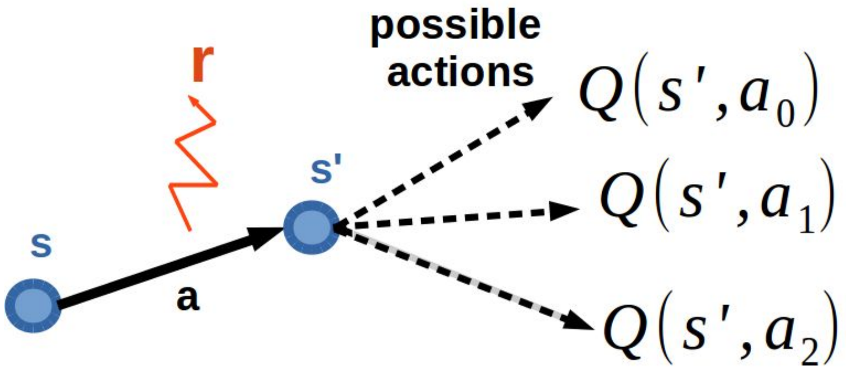


Problem of overestimation

Normal distribution
3*10^6 x 10 samples
Then take maximum of every tuple
mean: ~1.538



Problem of overestimation



Suppose true $Q(s', a')$ are equal **0** for all a'

But we have an approximation (or other kind) error $\sim N(0, \sigma^2)$

So $Q(s, a)$ should be equal r

But if we update $Q(s, a)$ towards $r + \gamma \max_{a'} Q(s', a')$

we will have overestimated $Q(s, a)$ > r because

$$\mathrm{E}[\max_{a'} Q(s', a')] >= \max_{a'} E[Q(s', a')]$$

Double Q-learning (NIPS 2010)

Idea: use two estimators of q-values: Q^A, Q^B
They should compensate mistakes of each other because they will be independent
Let's get argmax from another estimator!

$$y = r + \gamma \max_{a'} Q(s', a')$$

- Q-learning target

$$y = r + \gamma Q(s', \operatorname{argmax}_{a'} Q(s', a'))$$

- Rewrited Q-learning target

$$y = r + \gamma Q^A(s', \operatorname{argmax}_{a} Q^B(s', a'))$$

- Double Q-learning target

Double Q-learning (NIPS 2010)

Algorithm 1 Double Q-learning

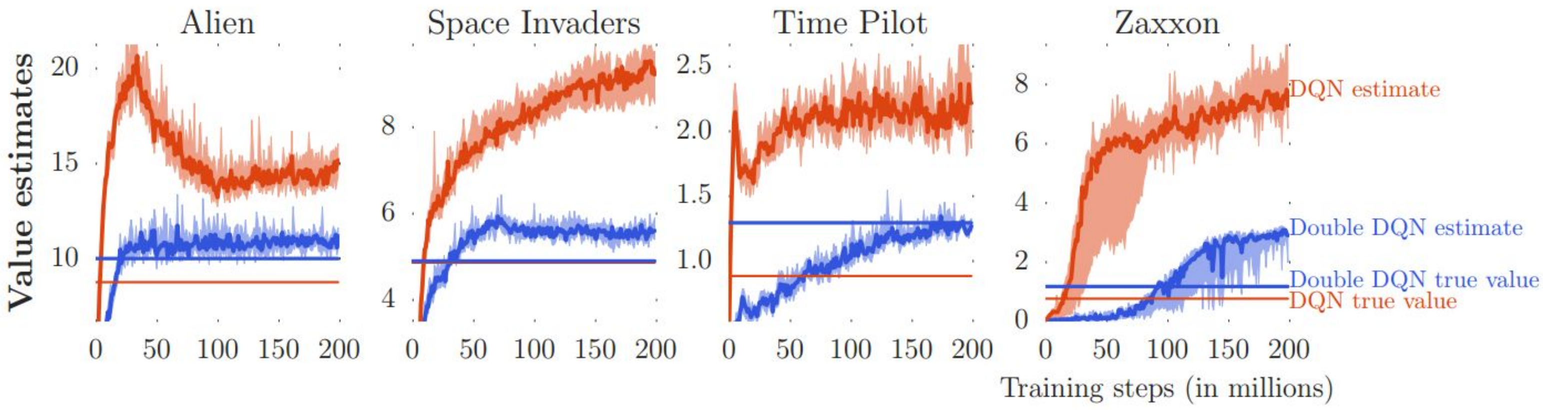
```
Initialize Q^A, Q^B, s
repeat
    Choose a, based on Q^A(s,·) and Q^B(s,·), observe r, s'
    Choose (e.g. random) either UPDATE(A) or UPDATE(B)
    if UPDATE(A) then
        Define a* = arg max_a Q^A(s', a)
        Q^A(s,a) ← Q^A(s,a) + α(s,a) (r + γQ^B(s', a*) − Q^A(s,a))
    else if UPDATE(B) then
        Define b* = arg max_a Q^B(s', a)
        Q^B(s,a) ← Q^B(s,a) + α(s,a)(r + γQ^A(s', b*) − Q^B(s,a))
    end if
    s ← s'
until end
```

How to apply this algorithm in deep reinforcement learning?

Deep Reinforcement Learning with Double Q-learning (Deepmind, 2015)

Idea: use main network to choose action!

$$y_{dqn} = r + \gamma \max_{a'} Q(s', a', \Theta^-)$$
$$y_{ddqn} = r + \gamma Q(s', \mathrm{argmax}_{a'} Q(s', a', \Theta), \Theta^-)$$



	DQN	Double DQN	Double DQN (tuned)
Median	47.5%	88.4%	116.7%
Mean	122.0%	273.1%	475.2%

Prioritized Experience Replay (2016, Deepmind)

Idea: sample transitions from xp-replay more clever

We want to set probability for every transition. Let's use the absolute value of TD-error of transition as a probability!

$$\text{TD-error } \delta = Q(s,a) - (r + \gamma Q(s', \text{argmax}_{a'} Q(s', a', \Theta), \Theta^-))$$

$$p = |\delta|$$

$$P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha}$$ where α is the priority parameter (when α is 0 it's the uniform case)

Do you see the problem?

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Do you see the problem?

Transitions become non i.i.d. and therefore we introduce the bias.

Prioritized Experience Replay (2016, Deepmind)

Solution: we can correct the bias by using importance-sampling weights

$$w_i = \left(\frac{1}{N} \cdot \frac{1}{P(i)}\right)^\beta$$ where β is the parameter

We also normalize weights by $1/\max_i w_i$ (here is not mathematical reason)

When we put transition into experience replay, we set maximal priority $p_t = \max_{i<t} p_i$

Prioritized Experience Replay (2016, Deepmind)

Algorithm 1 Double DQN with proportional prioritization

Input: minibatch k, step-size η, replay period K and size N, exponents α and β, budget T.
Initialize replay memory $\mathcal{H} = \emptyset$, $\Delta = 0$, $p_1 = 1$
Observe S_0 and choose $A_0 \sim \pi_\theta(S_0)$
for $t = 1$ **to** T **do**
    Observe S_t, R_t, γ_t
    Store transition $(S_{t-1}, A_{t-1}, R_t, \gamma_t, S_t)$ in $\mathcal{H}$ with maximal priority $p_t = \max_{i<t} p_i$
    if $t \equiv 0 \mod K$ **then**
        for $j = 1$ **to** k **do**
            Sample transition $j \sim P(j) = p_j^\alpha / \sum_i p_i^\alpha$
            Compute importance-sampling weight $w_j = (N \cdot P(j))^{-\beta} / \max_i w_i$
            Compute TD-error $\delta_j = R_j + \gamma_j Q_{\text{target}}\left(S_j, \arg\max_a Q(S_j, a)\right) - Q(S_{j-1}, A_{j-1})$
            Update transition priority $p_j \leftarrow |\delta_j|$
            Accumulate weight-change $\Delta \leftarrow \Delta + w_j \cdot \delta_j \cdot \nabla_\theta Q(S_{j-1}, A_{j-1})$
        end for
        Update weights $\theta \leftarrow \theta + \eta \cdot \Delta$, reset $\Delta = 0$
        From time to time copy weights into target network $\theta_{\text{target}} \leftarrow \theta$
    end if
    Choose action $A_t \sim \pi_\theta(S_t)$
end for

Prioritized Experience Replay (2016, Deepmind)

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      Update transition priority $p_j \leftarrow |\delta_j|$
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    end for
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    From time to time copy weights into target network $\theta_{\text{target}} \leftarrow \theta$
  end if
  Choose action $A_t \sim \pi_\theta(S_t)$
end for

It is the bonus homework!

Let’s watch a video…

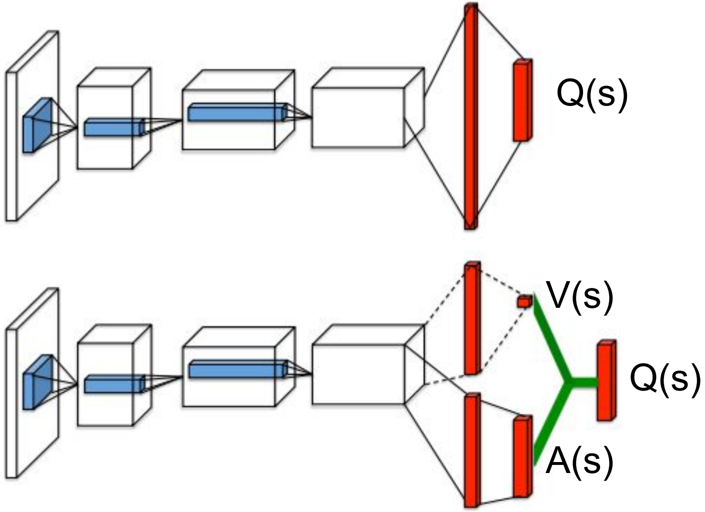
https://www.youtube.com/watch?v=UXurvvDY93o

Dueling Network Architectures for Deep Reinforcement Learning (2016, Deepmind)

Idea: change the network's architecture.

Recall:
Advantage Function A(s,a) = Q(s,a) - V(s)

So, Q(s,a) = A(s,a) + V(s)



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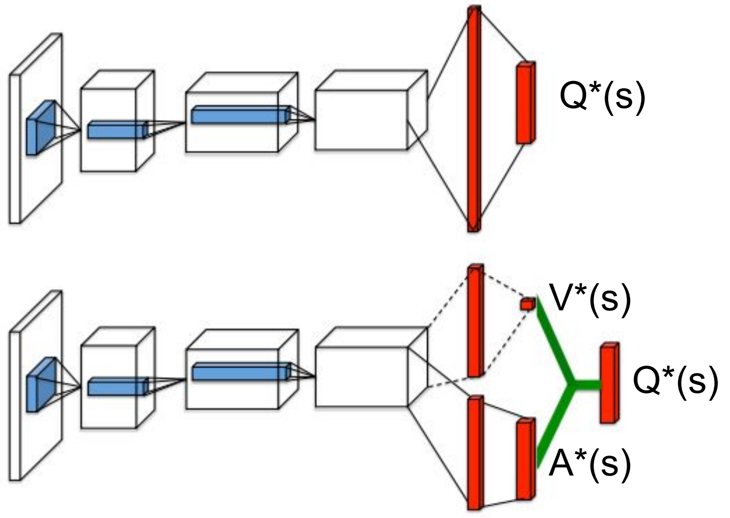

Recall:
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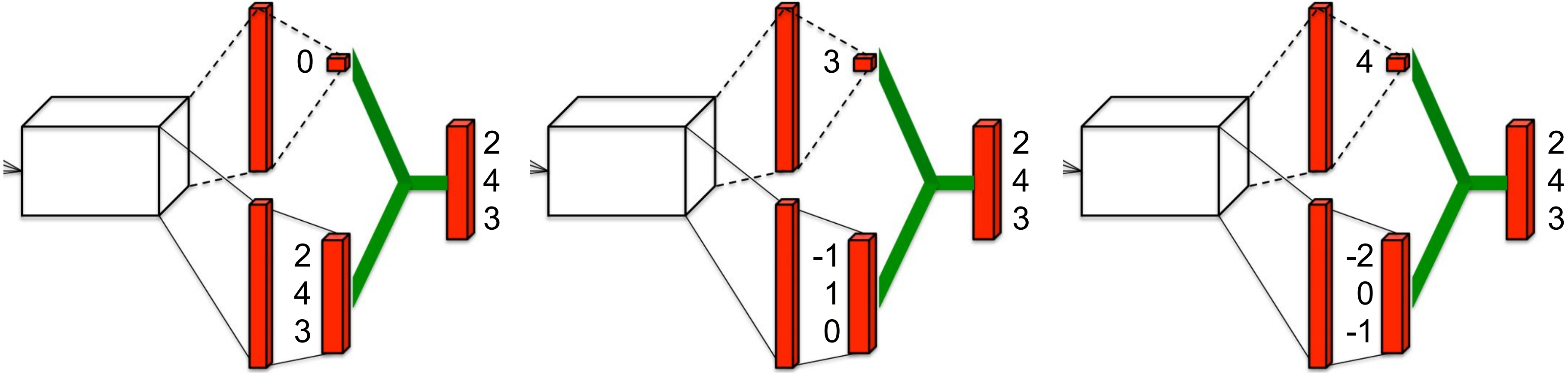
So, Q(s,a) = A(s,a) + V(s)

Here is a problem!

Dueling Network Architectures for Deep Reinforcement Learning (2016, Deepmind)

Here is one extra freedom degree!

Example:



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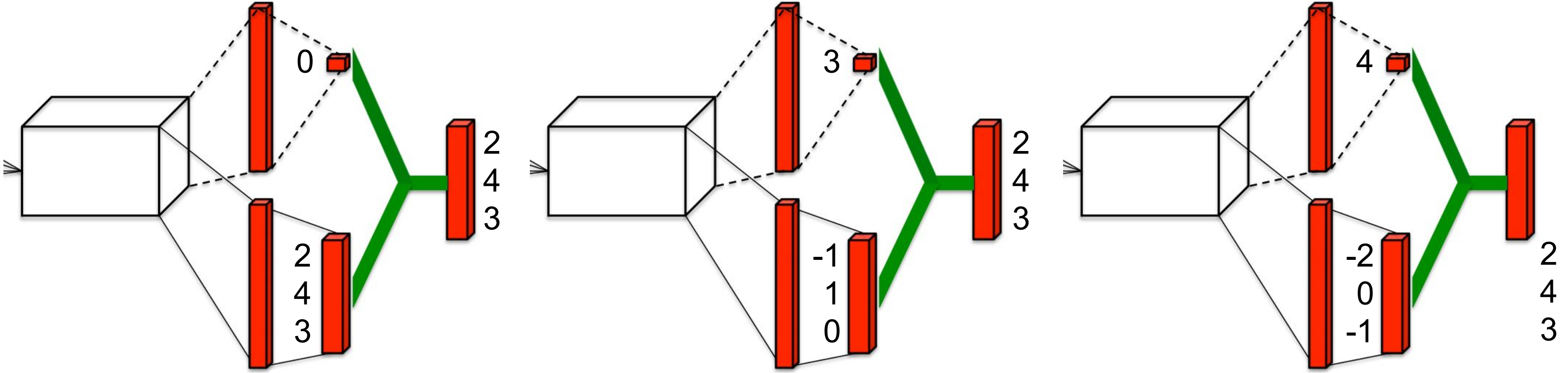

What is correct?

Hint 1:

$$a^*(s) = \operatorname{argmax}_{a'} Q^*(s, a')$$

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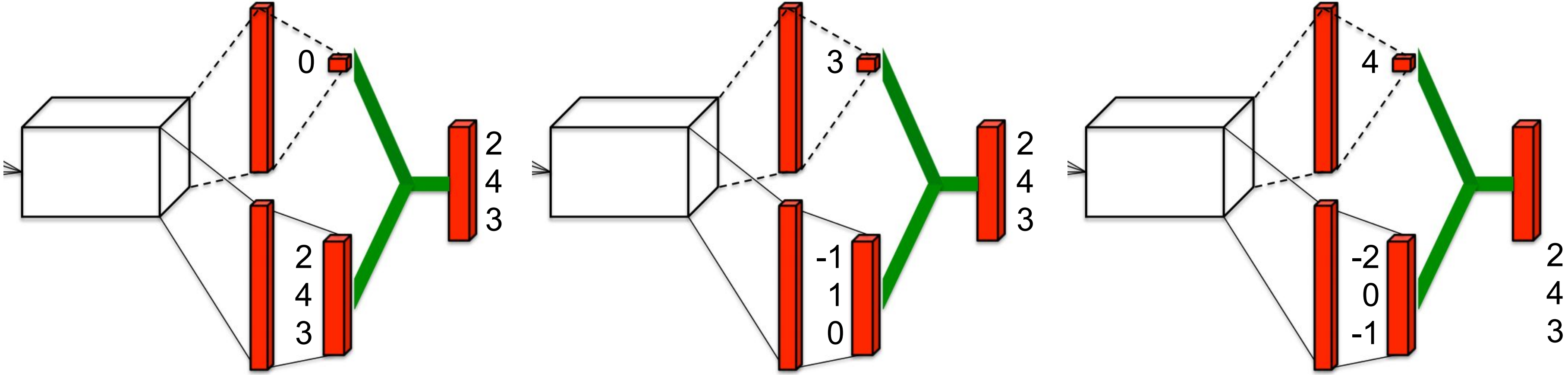

What is correct?

Hint 1:

$$a^*(s) = \text{argmax}_{a'} Q^*(s, a')$$

Hint 2:

$$V^*(s) = \sum_a \pi^*(s, a) Q^*(s, a)$$

Dueling Network Architectures for Deep Reinforcement Learning (2016, Deepmind)

Solution: require $\max_{a' \in |\mathcal{A}|} A(s, a'; \theta, \alpha)$ to be equal to zero!

So the **Q-function** computes as:

$$Q(s, a; \theta, \alpha, \beta) = V(s; \theta, \beta) + \left(A(s, a; \theta, \alpha) - \max_{a' \in |\mathcal{A}|} A(s, a'; \theta, \alpha) \right)$$

Authors of this papers also introduced this way to compute Q-values:

$$Q(s, a; \theta, \alpha, \beta) = V(s; \theta, \beta) + \left(A(s, a; \theta, \alpha) - \frac{1}{|\mathcal{A}|} \sum_{a'} A(s, a'; \theta, \alpha) \right)$$

They wrote that this variant increases stability of the optimization
(The fact that this loses the original semantics of Q doesn’t matter)

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Questions?