Reinforcement learning

Episode 3

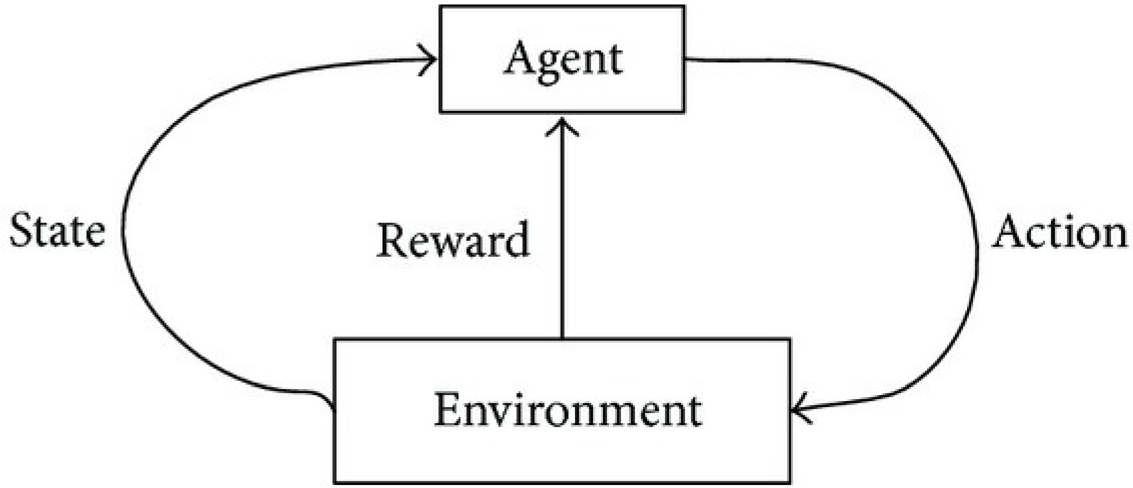
Value-based methods

Yandex Data Factory

LAMBDA

British Hedgehog Preservation Society

Recap: Discounted reward MDP



Classic MDP(Markov Decision Process)
Agent interacts with environment

- Environment states: $s \in S$
- Agent actions: $a \in A$
- State transition: $P(s_{t+1} | s_t, a_t)$
- Reward: $r_t = r(s_t, a_t)$

Recap: Discounted reward MDP

Objective:
Total action value

$$R_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \ldots + \gamma^n \cdot r_{t+n}$$

$$R_t = \sum_i \gamma^i \cdot r_{t+i} \qquad \gamma \in (0,1)\, const$$

γ ~ patience

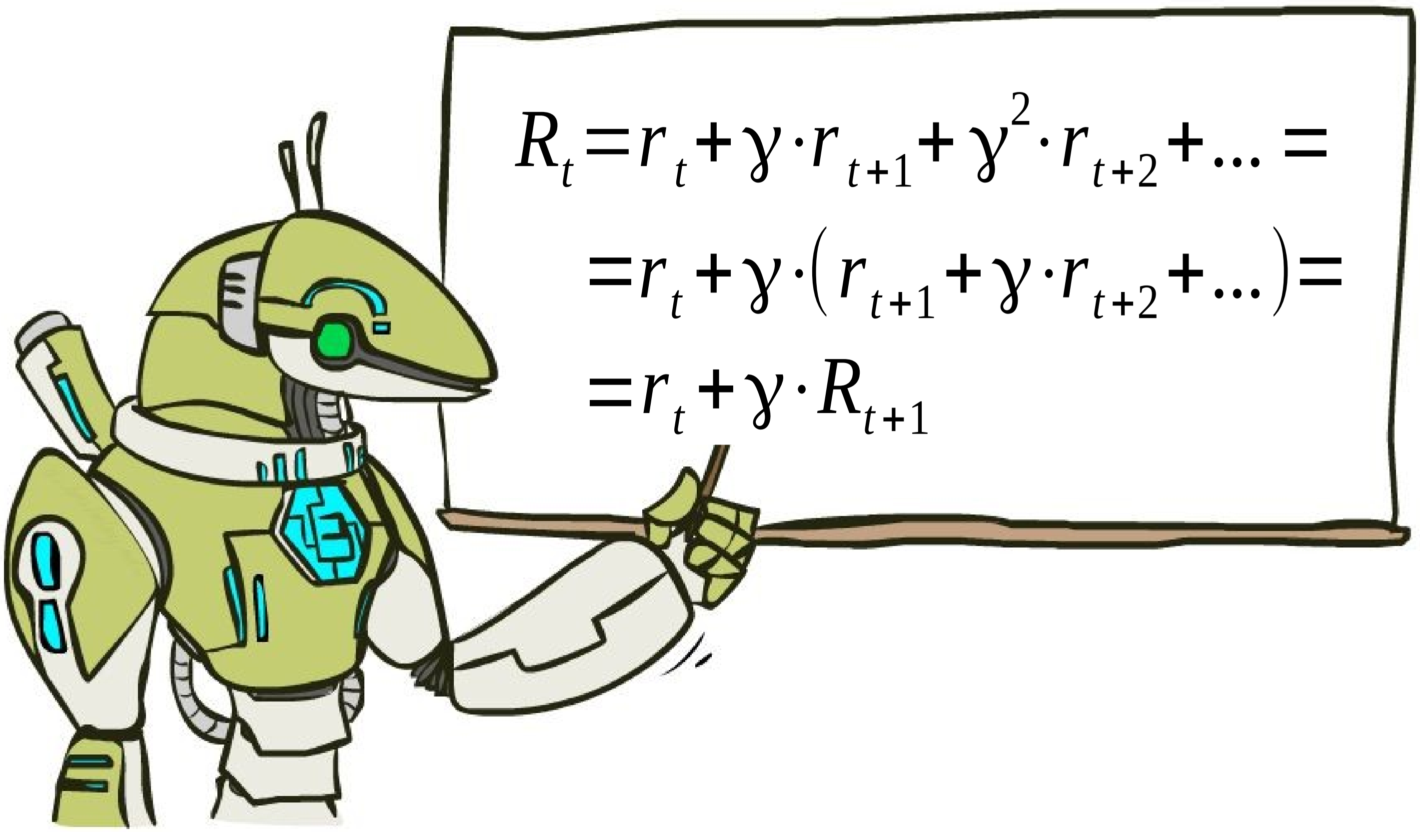
Cake tomorrow is γ as good as now

Reinforcement learning:

- Find policy that maximizes the expected reward

$$\pi = P(a|s) : E[R] \rightarrow max$$

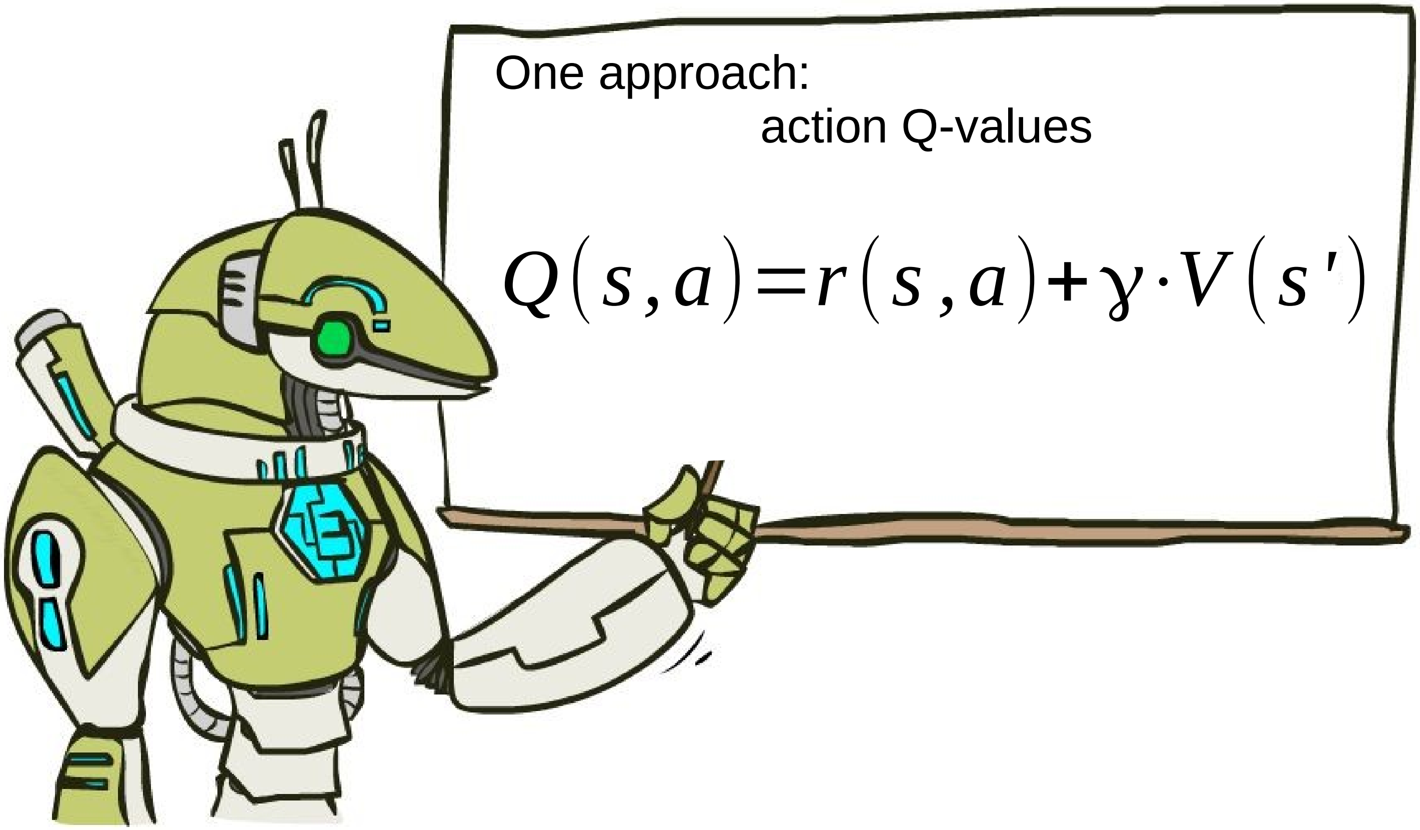
Recap: Optimal policy



$$R_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \ldots =$$
$$= r_t + \gamma \cdot \left(r_{t+1} + \gamma \cdot r_{t+2} + \ldots \right) =$$
$$= r_t + \gamma \cdot R_{t+1}$$

We rewrite R with sheer power of math!

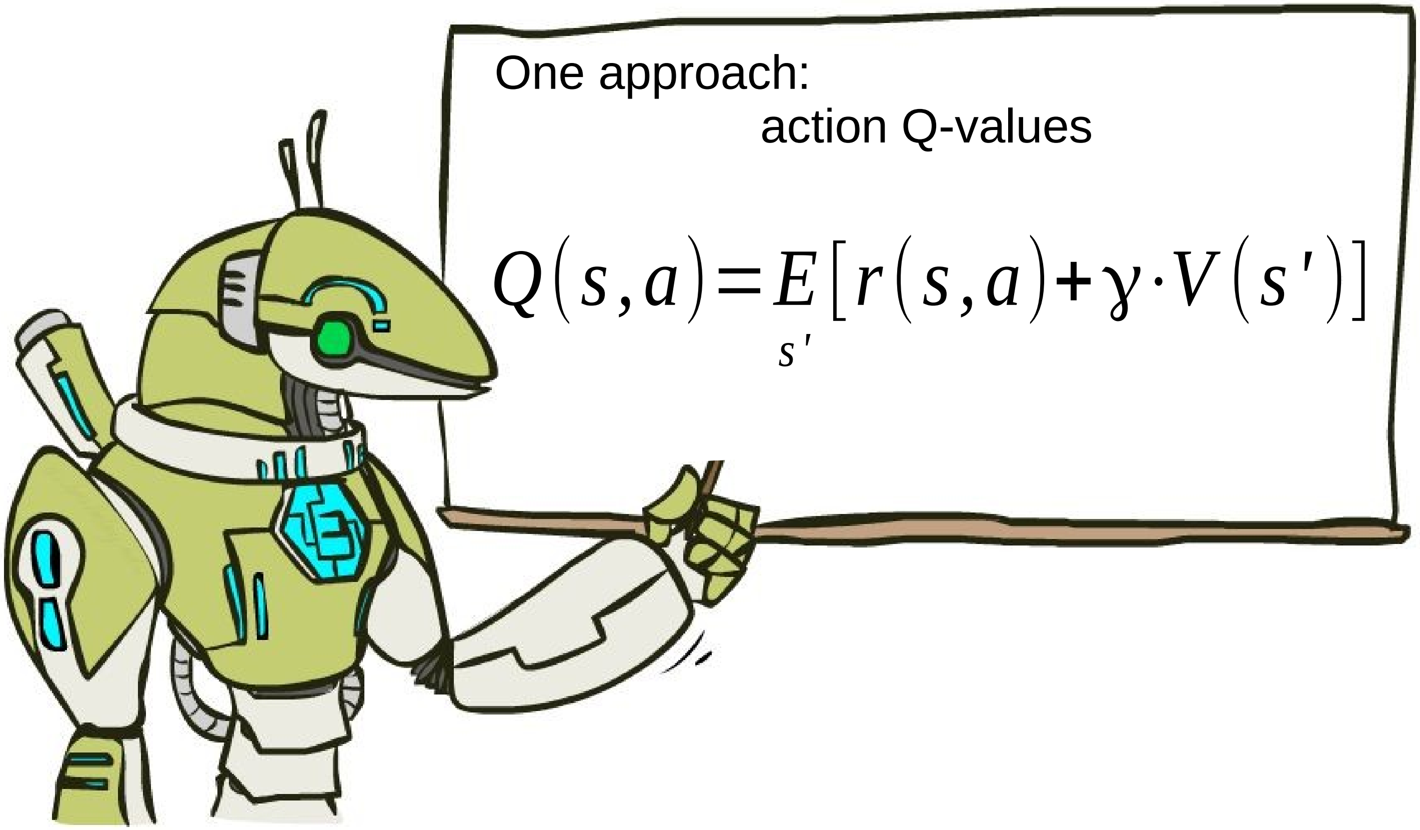
Recap: Q-learning



Action value Q(s,a) is the expected total reward **R** agent gets from state **s** by taking action **a** and following policy **π** from next state.

Trivia: how do we get policy **π**(a|s) given Q(s,a)?

Recap: Q-learning



Action value Q(s,a) is the expected total reward **R** agent gets from state **s** by taking action **a** and following policy **π** from next state.

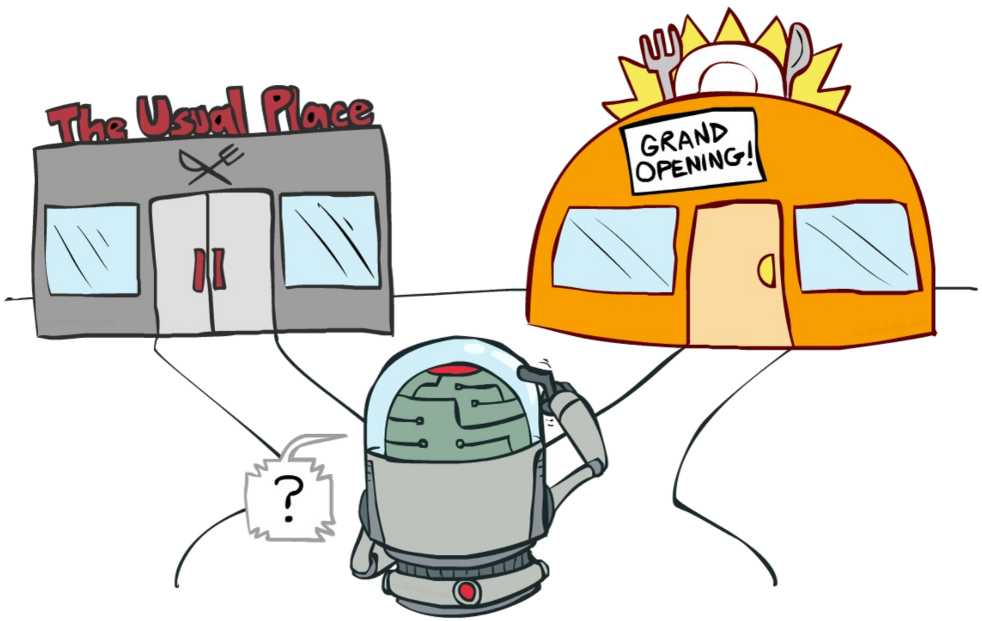
$$\pi(s) : argmax_a Q(s,a)$$

Recap: Exploration Vs Exploitation

Balance between using what you learned and trying to find something even better

ε-greedy

- With probability ε take random action;
- Otherwise take optimal action.



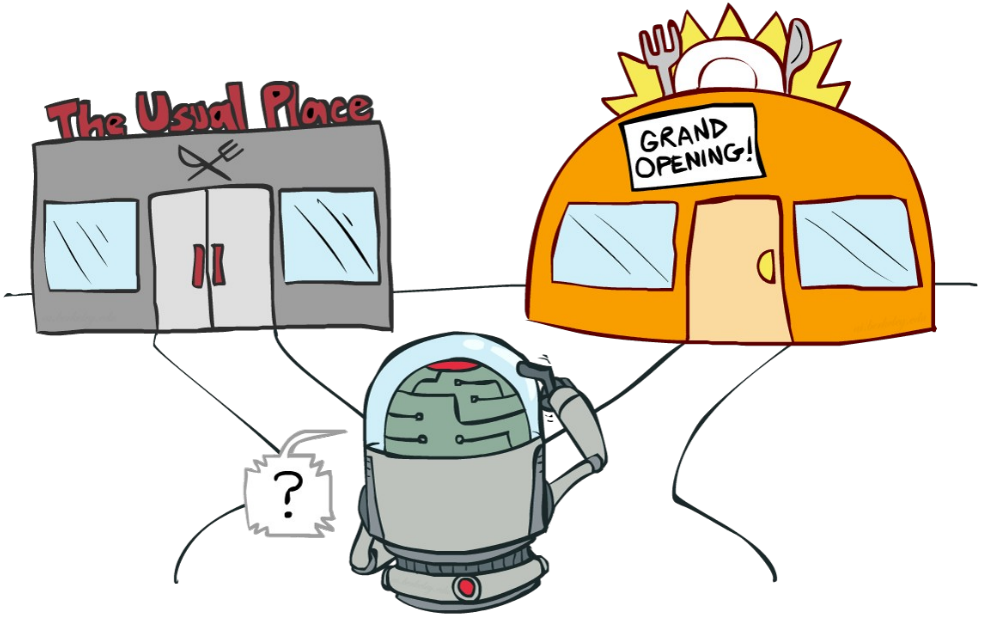
Trivia: how to define $\pi(a|s)$ now?

Exploration Vs Exploitation

Balance between using what you learned and trying to find something even better

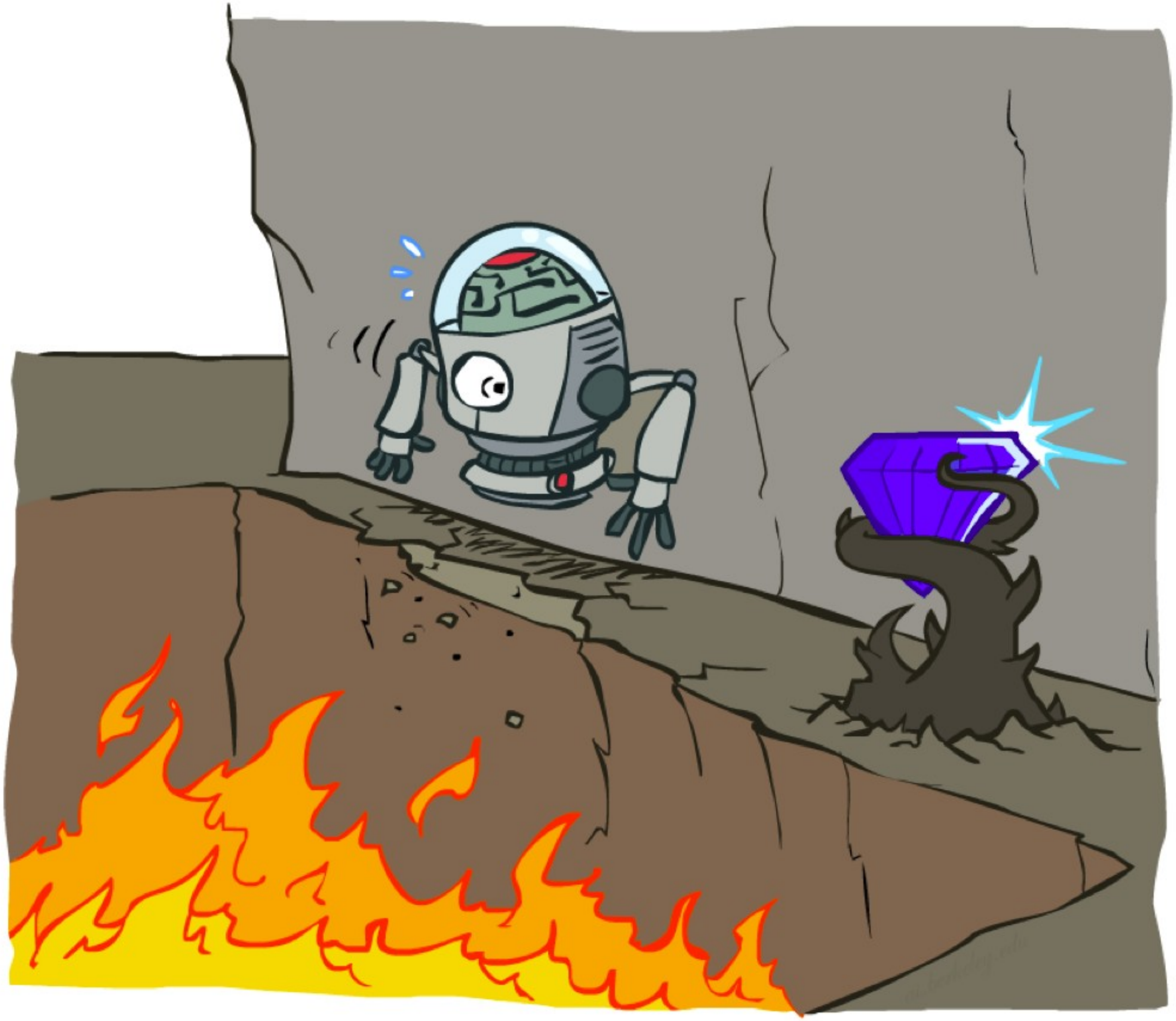
ε-greedy

- With probability ε take random action;
- Otherwise take optimal action.



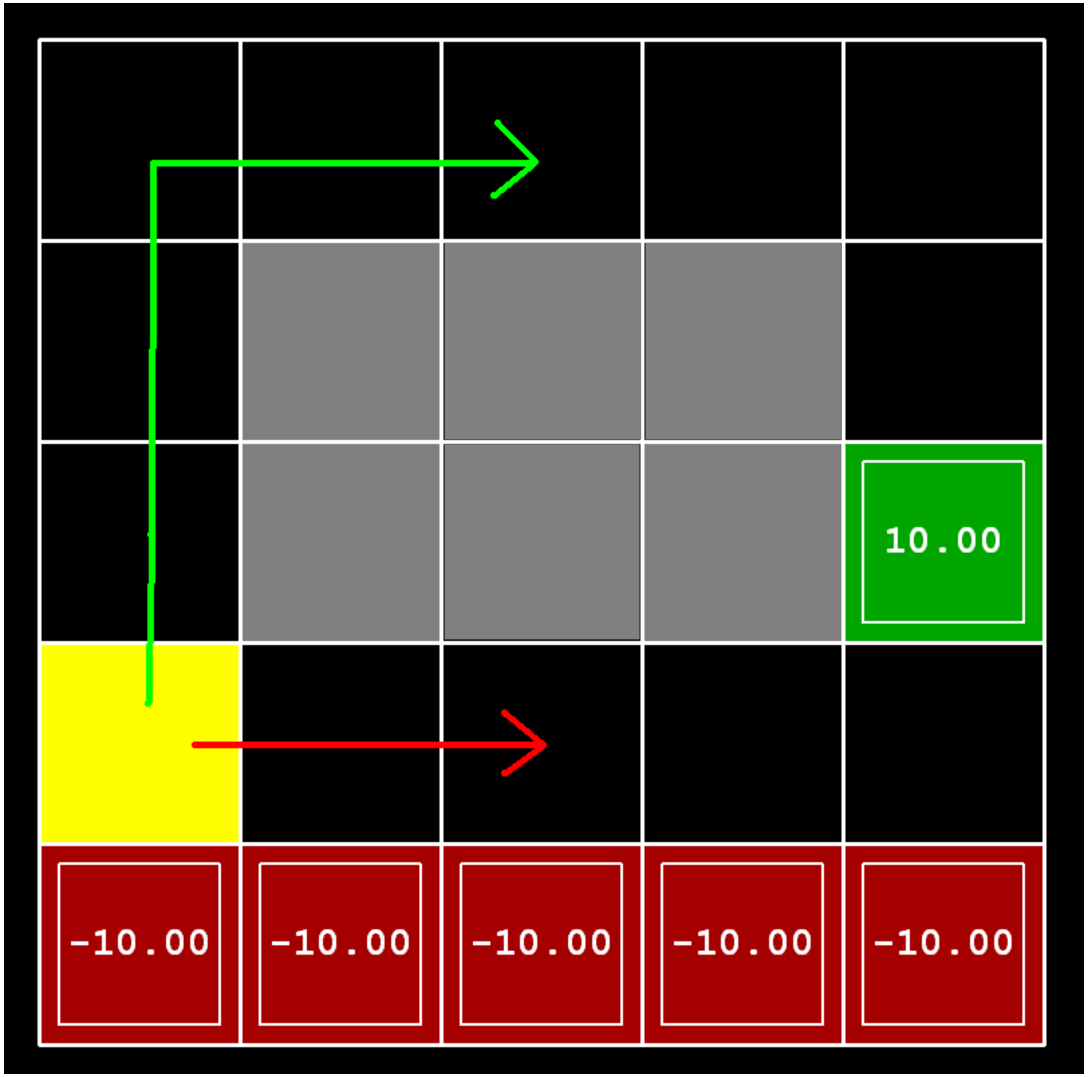
$$\pi(a|s):(1-\epsilon)[a=argmax_a Q(s,a)]+\frac{\epsilon}{|A|}$$

Cliff world



Picture from Berkeley CS188x

Cliff world



Conditions

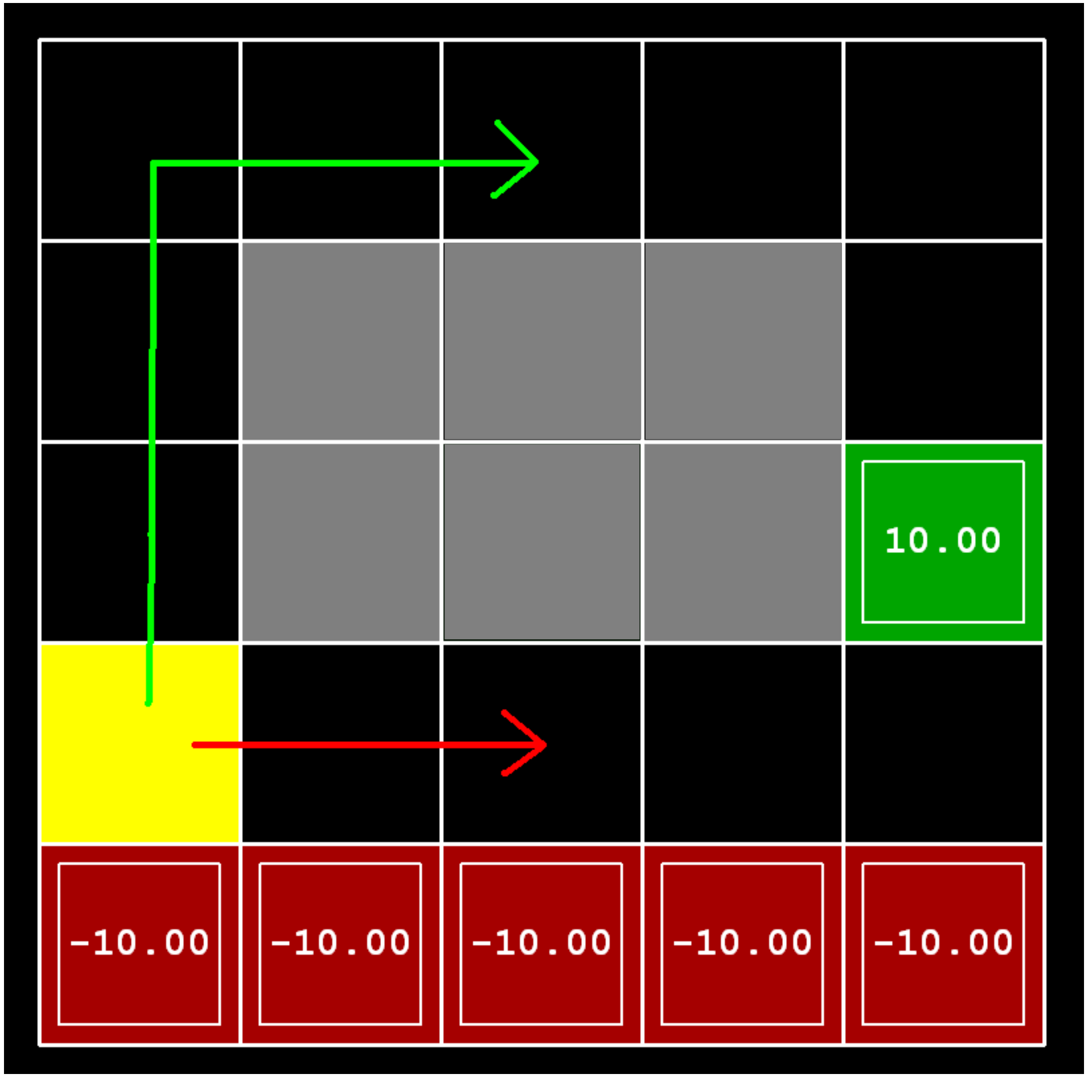
- Q-learning

  $\gamma = 0.99 \quad \epsilon = 0.1$

- no slipping

Trivia:
What will q-learning learn?

Cliff world



Conditions

- Q-learning

$$\gamma = 0.99 \quad \epsilon = 0.1$$

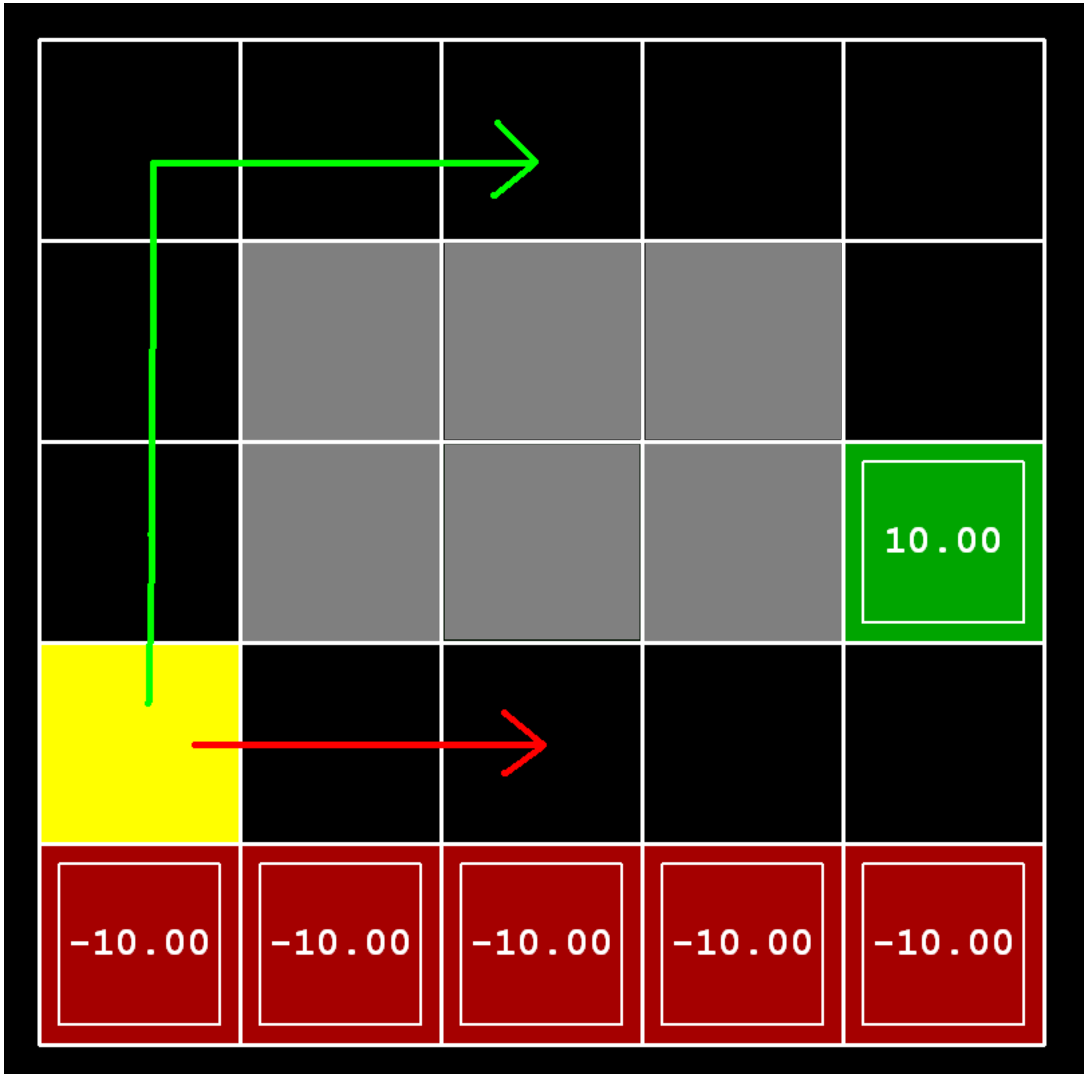
- no slipping

Trivia:
What will q-learning learn?

follow the short path

Will it maximize reward?

Cliff world



Conditions

- Q-learning

$\gamma = 0.99 \quad \epsilon = 0.1$

- no slipping

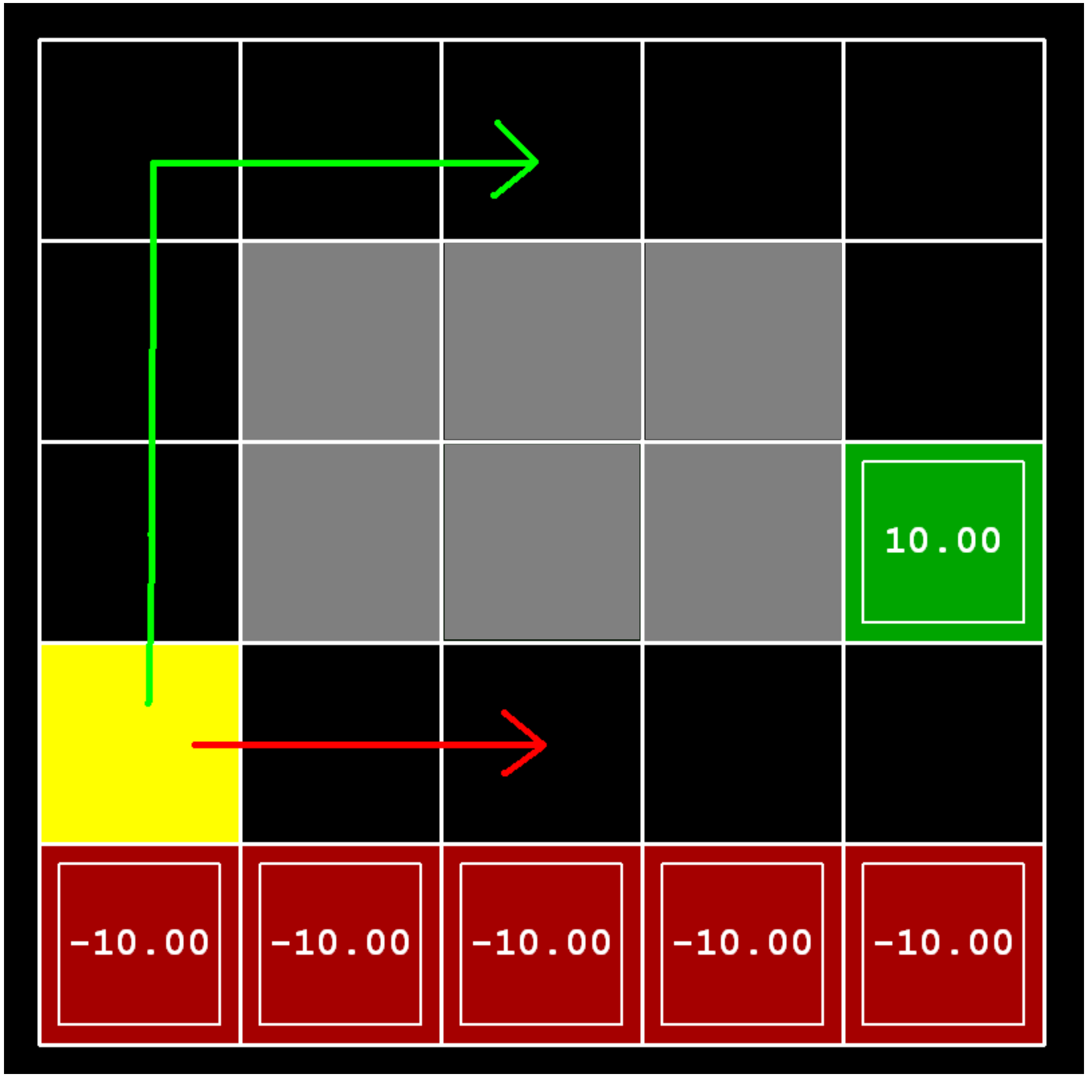
Trivia:
What will q-learning learn?

follow the short path

Will it maximize reward?

no, robot will fall due to epsilon-greedy "exploration"

Cliff world



Conditions

- Q-learning

$$\gamma = 0.99 \quad \epsilon = 0.1$$

- no slipping

Decisions must account for actual policy!
e.g. ε-greedy policy

Generalized value iteration

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

“better Q(s,a)”

Q-learning VS SARSA

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

"better Q(s,a)"

Q-learning

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(s', a')$$

Q-learning VS SARSA

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

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Q-learning

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(s',a')$$

SARSA

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \underset{a' \sim \pi(a'|s')}{E} Q(s',a')$$

Q-learning VS SARSA

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

“better Q(s,a)”

Q-learning

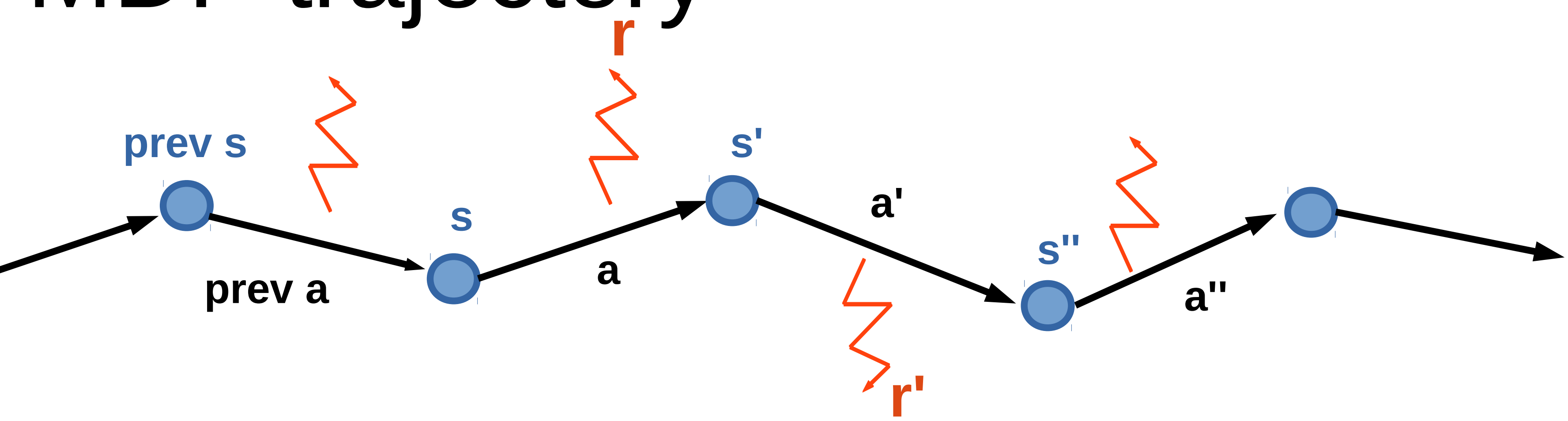
$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(\underline{s'}, a')$$

Expected from s' ~ P(s'|s,a)

SARSA

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \mathop{E}_{a' \sim \pi(a'|s')} Q(\underline{s'}, a')$$

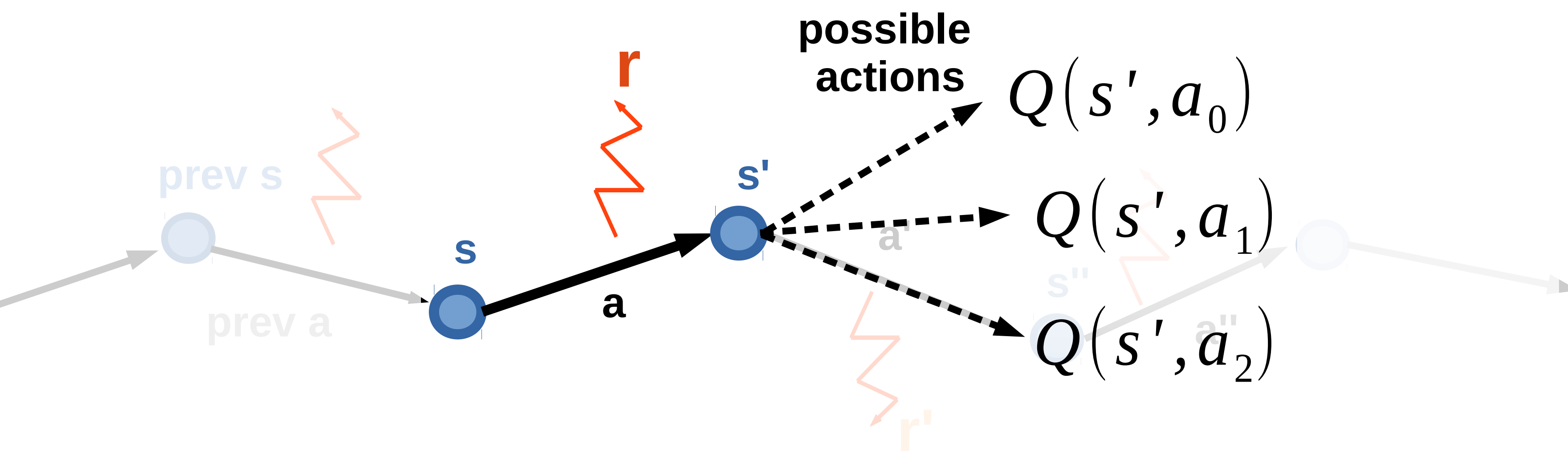
MDP trajectory



- sample sequence of
  - states (s)
  - actions (a)
  - rewards (r)

- Can be infinite, we can't wait that long

Recap: Q-learning

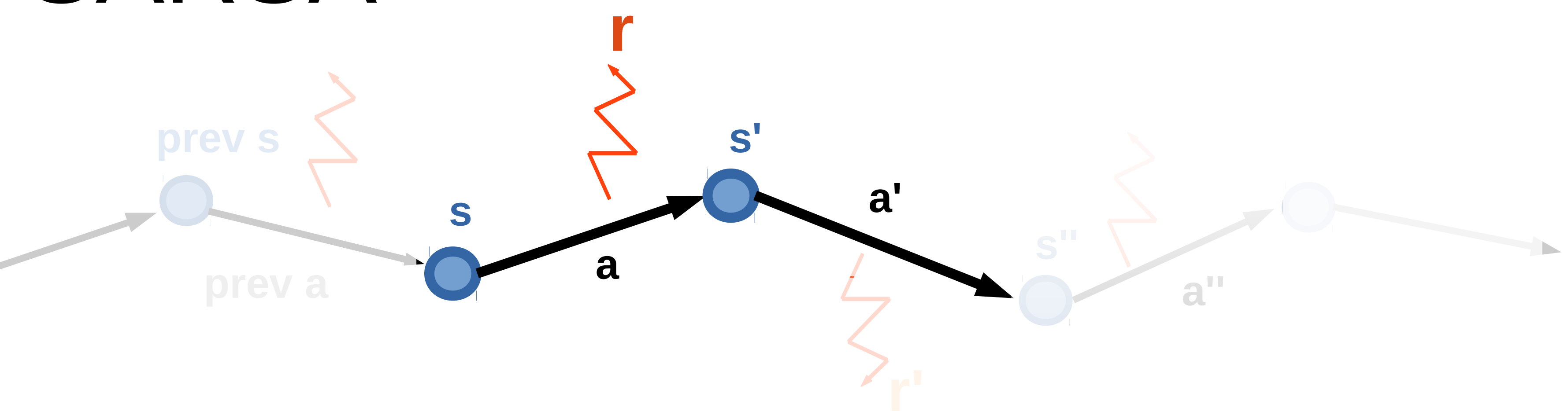


$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

- Sample <**s**,**a**,**r**,**s'**> from env
- Compute $\hat{Q}(s,a) = r(s,a) + \gamma \max_{a_i} Q(s',a_i)$
- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha) Q(s,a)$

SARSA

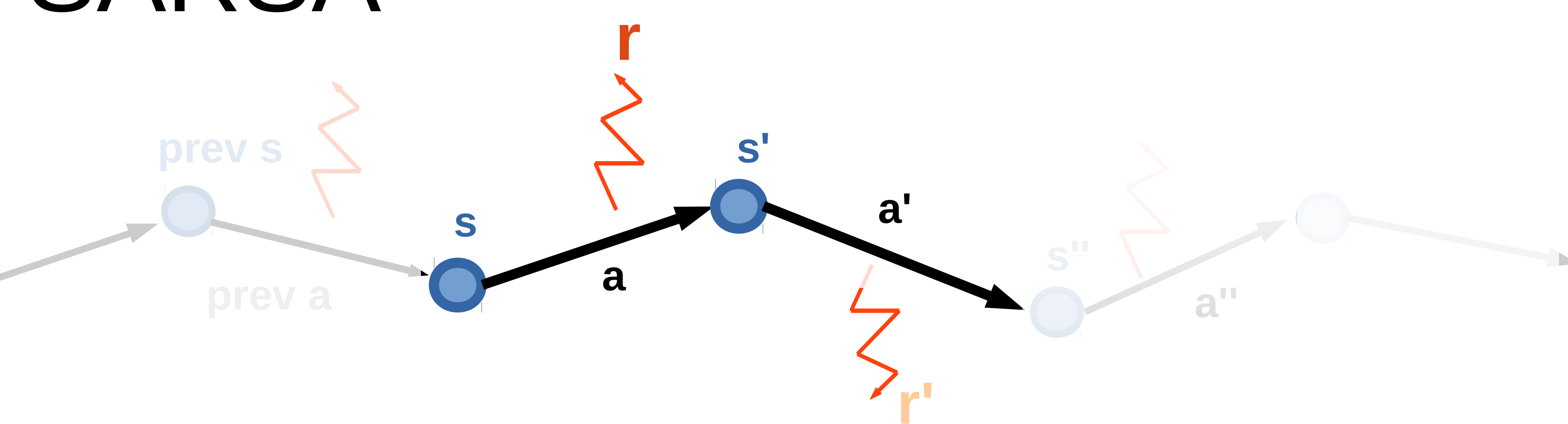


$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

- Sample <**s,a,r,s',a'**> from env
- Compute $\hat{Q}(s,a) = r(s,a) + \gamma Q(s',a')$
- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha) Q(s,a)$

SARSA



$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

hence “SARSA”

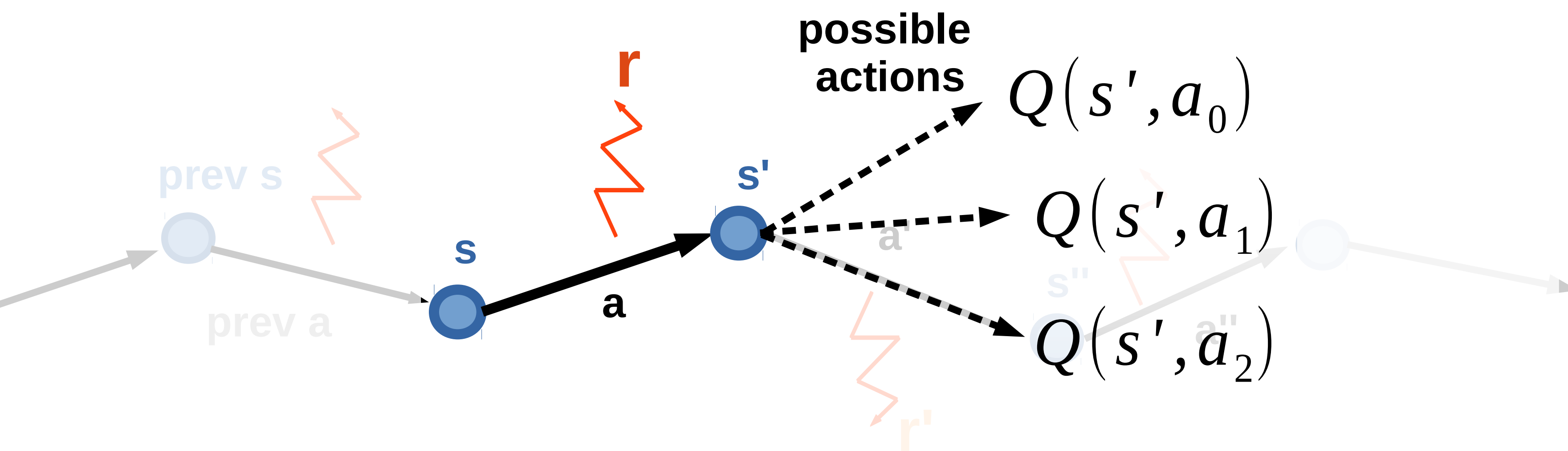
- Sample <**s**,**a**,**r**,**s'**,**a'**> from env

- Compute $\hat{Q}(s,a) = r(s,a) + \gamma Q(\underline{s',a'})$

next action (not max)

- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha) Q(s,a)$

Expected value SARSA

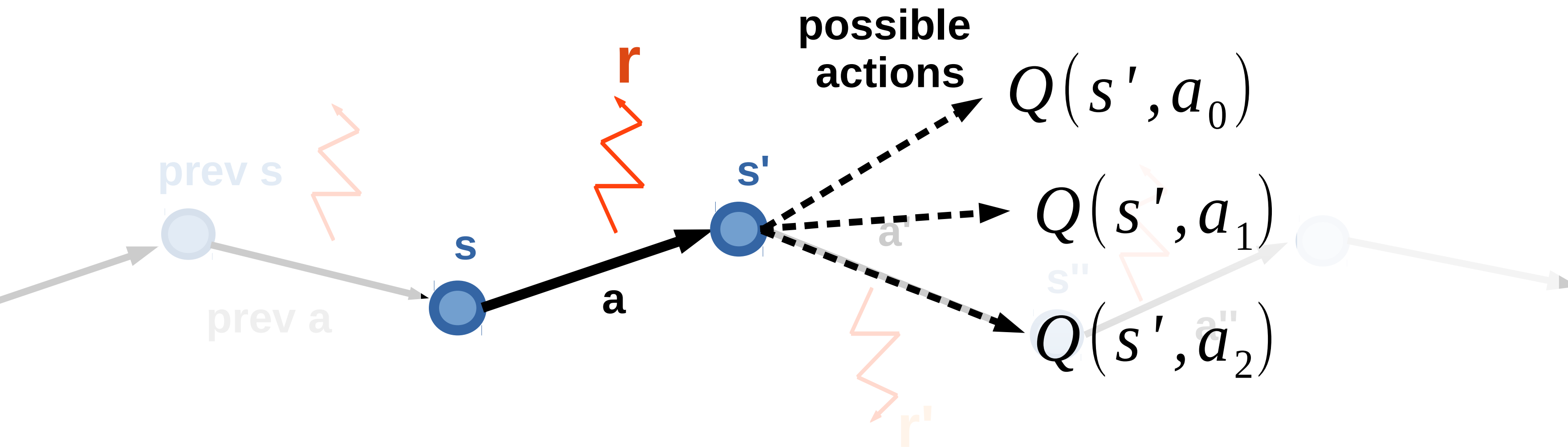


$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

- Sample <**s**,**a**,**r**,**s'**> from env
- Compute $\hat{Q}(s,a) = r(s,a) + \gamma \underset{a_i \sim \pi(a|s')}{E} Q(s',a_i)$
- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha) Q(s,a)$

Expected value SARSA



$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

- Sample <**s,a,r,s'**> from env

Expected value

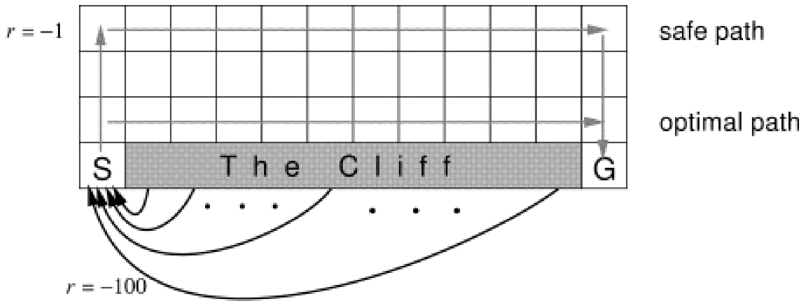
- Compute $\hat{Q}(s,a) = r(s,a) + \gamma \underset{a_i \sim \pi(a|s')}{E} Q(s',a_i)$

- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha) Q(s,a)$

Difference

- SARSA converges to optimal policy

- Q-learning policy **would be** optimal without exploration



On-policy vs Off-policy

Two problem setups

on-policy

Agent **can** pick actions

- Most obvious setup :)
- Agent always follows his **own** policy

off-policy

Agent **can't** pick actions

- Learning with exploration, playing without exploration
- Learning from expert (expert is imperfect)
- Learning from sessions (recorded data)

On-policy vs Off-policy

Two problem setups

on-policy

Agent **can** pick actions

- On-policy algorithms **can't** learn off-policy

(but they be faster/better)

off-policy

Agent **can't** pick actions

- Off-policy algorithms **can** learn on-policy

learn optimal policy even if agent takes random actions

Trivia: which of Q-learning, SARSA and exp. val. SARSA will **only** work on-policy?

On-policy vs Off-policy

Two problem setups

on-policy	**off-policy**
Agent **can** pick actions	Agent **can't** pick actions
– On-policy algorithms **can't** learn off-policy	– Off-policy algorithms **can** learn on-policy
– SARSA	– Q-learning
– more coming soon	– Expected Value SARSA

Trivia: will Crossentropy Method converge if it learns off-policy from agent that takes random actions?

On-policy vs Off-policy

Two problem setups

on-policy

Agent **can** pick actions

- On-policy algorithms **can't** learn off-policy
- SARSA
- more coming soon

off-policy

Agent **can't** pick actions

- Off-policy algorithms **can** learn on-policy
- Q-learning
- Expected Value SARSA

Trivia: will Crossentropy Method converge if it learns off-policy from agent that takes random actions? **Well, no :)**

N-step algorithms

Recall R?

$$R_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \ldots =$$

$$= r_t + \gamma \cdot \left(r_{t+1} + \gamma \cdot r_{t+2} + \ldots \right) =$$

$$= r_t + \gamma \cdot R_{t+1} =$$

$$= r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot R_{t+2}$$

N-step SARSA

- General formula

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma Q(s_{t+1}, a_{t+1})$$

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma r(s_{t+1}, a_{t+1}) + \gamma^2 Q(s_{t+2}, a_{t+2})$$

N-step SARSA

- General formula

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha) Q(s_t, a_t)$$

1-step SARSA

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma Q(s_{t+1}, a_{t+1})$$

2-step SARSA

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma r(s_{t+1}, a_{t+1}) + \gamma^2 Q(s_{t+2}, a_{t+2})$$

3-step SARSA

$\hat{Q}(s_t, a_t) =$ **???**

N-step SARSA

- General formula

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

1-step SARSA

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma Q(s_{t+1}, a_{t+1})$$

2-step SARSA

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma r(s_{t+1}, a_{t+1}) + \gamma^2 Q(s_{t+2}, a_{t+2})$$

3-step SARSA

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma r(s_{t+1}, a_{t+1}) + \gamma^2 r(s_{t+2}, a_{t+2}) + \gamma^3 Q(s_{t+3}, a_{t+3})$$

N-step SARSA

- General formula

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

1-step SARSA

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma Q(s_{t+1}, a_{t+1})$$

N-step SARSA

$$\hat{Q}(s_t, a_t) = [\sum_{\tau=t}^{\tau<t+n} \gamma^{\tau} r(s_{t+\tau}, a_{t+\tau})] + \gamma^{n} Q(s_{t+n}, a_{t+n})$$

N-step algorithms

- General formula

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

N-step SARSA

$$\hat{Q}(s_t, a_t) = [\sum_{\tau=t}^{\tau<t+n} \gamma^{\tau} r(s_{t+\tau}, a_{t+\tau})] + \gamma^{n} Q(s_{t+n}, a_{t+n})$$

N-step Q-learning

$$\hat{Q}(s_t, a_t) = [\sum_{\tau=t}^{\tau<t+n} \gamma^{\tau} r(s_{t+\tau}, a_{t+\tau})] + \gamma^{n} \cdot \max_{a} Q(s_{t+n}, a)$$

Trivia: which of these methods work off-policy?

N-step algorithms

- General formula

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1-\alpha) Q(s_t, a_t)$$

N-step SARSA

$$\hat{Q}(s_t, a_t) = [\sum_{\tau=t}^{\tau<t+n} \gamma^{\tau} r(s_{t+\tau}, a_{t+\tau})] + \gamma^{n} Q(s_{t+n}, a_{t+n})$$

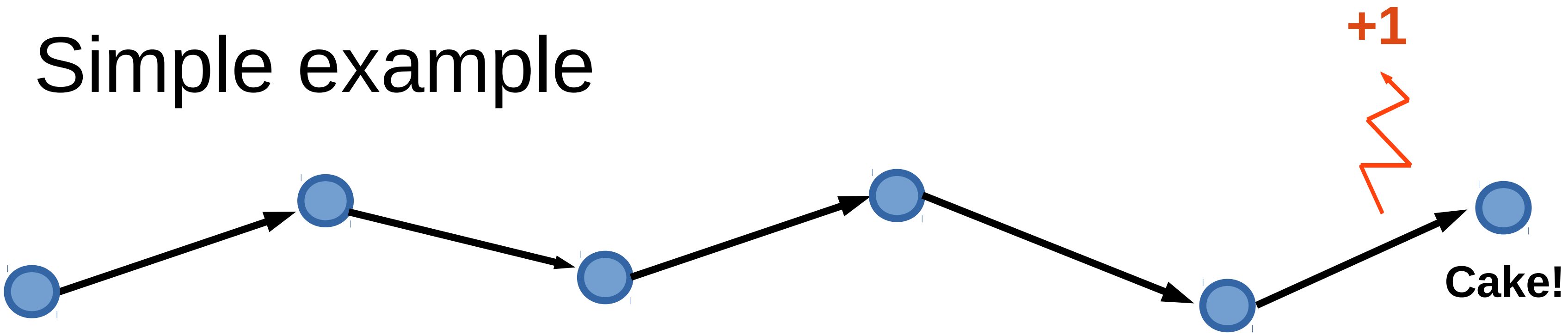
N-step Q-learning

$$\hat{Q}(s_t, a_t) = [\sum_{\tau=t}^{\tau<t+n} \gamma^{\tau} r(s_{t+\tau}, a_{t+\tau})] + \gamma^{n} \cdot \max_{a} Q(s_{t+n}, a)$$

Trivia: which of these methods work off-policy? **None of them!**

1-step Vs n-step

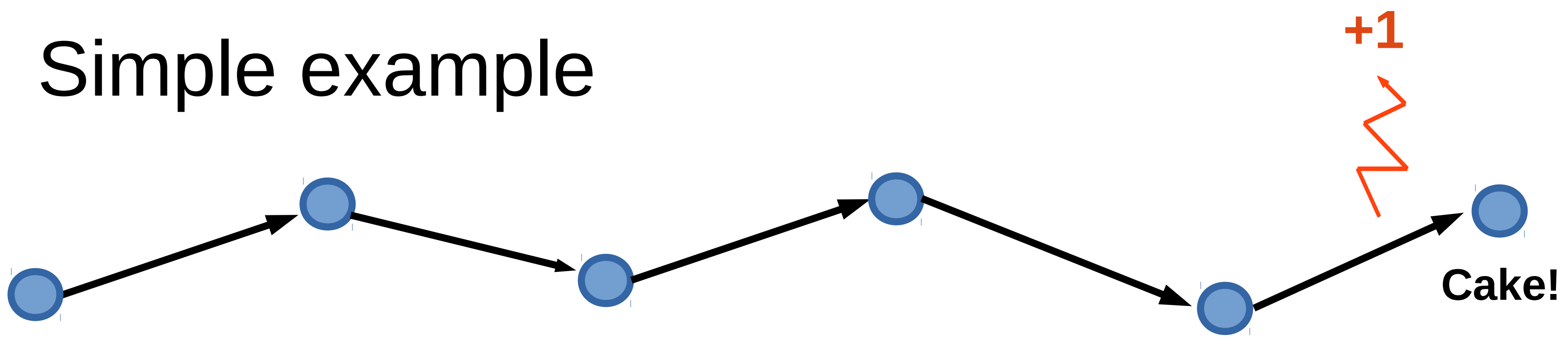
- Simple example



How many games does it take for **SARSA** to learn the optimal policy?

1-step Vs n-step

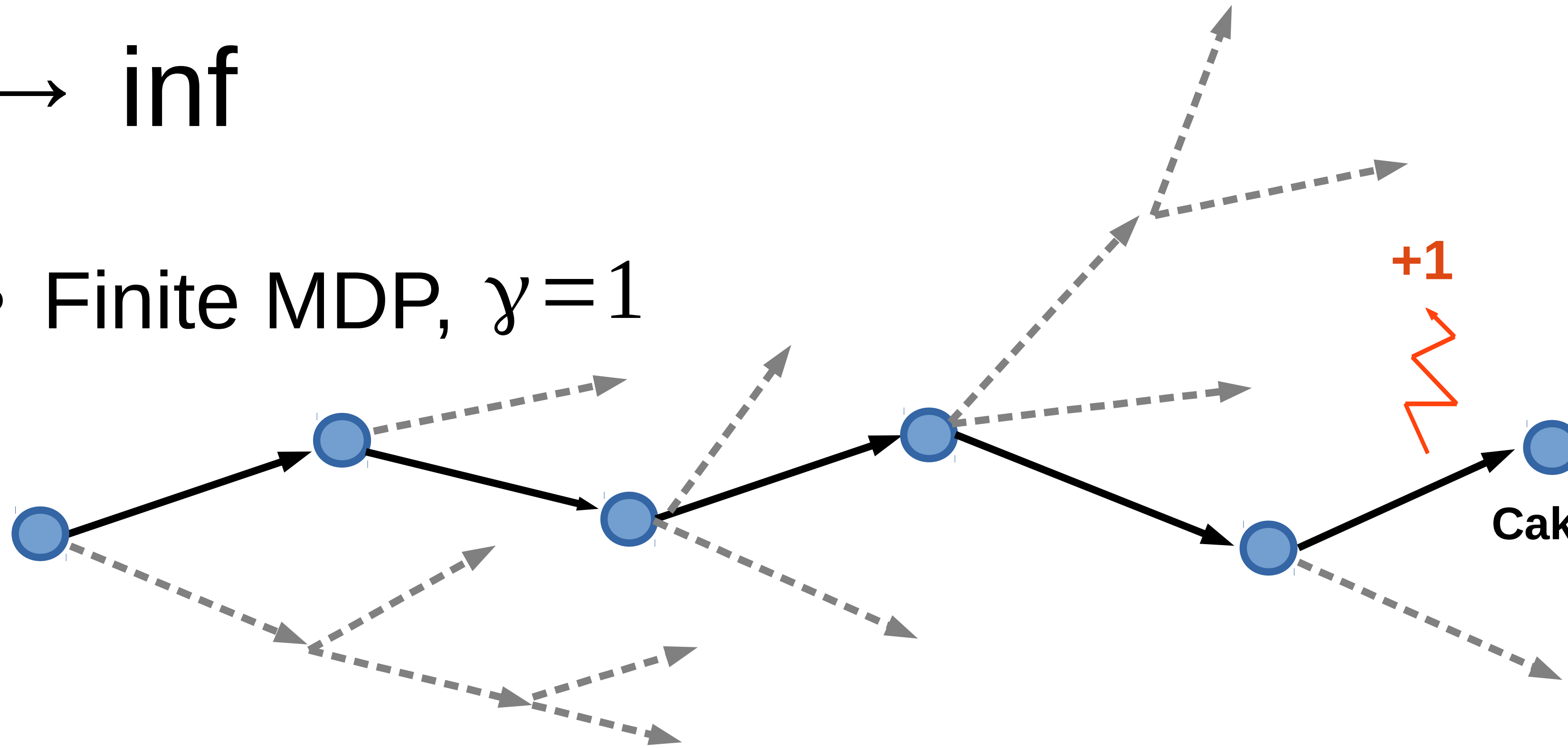
- Simple example



- SARSA needs 5 steps, n-step SARSA needs 1
- Nuts and bolts
  - Nonlinear approximations learn much faster!
  - Play for N steps, than learn (batched)

n → inf

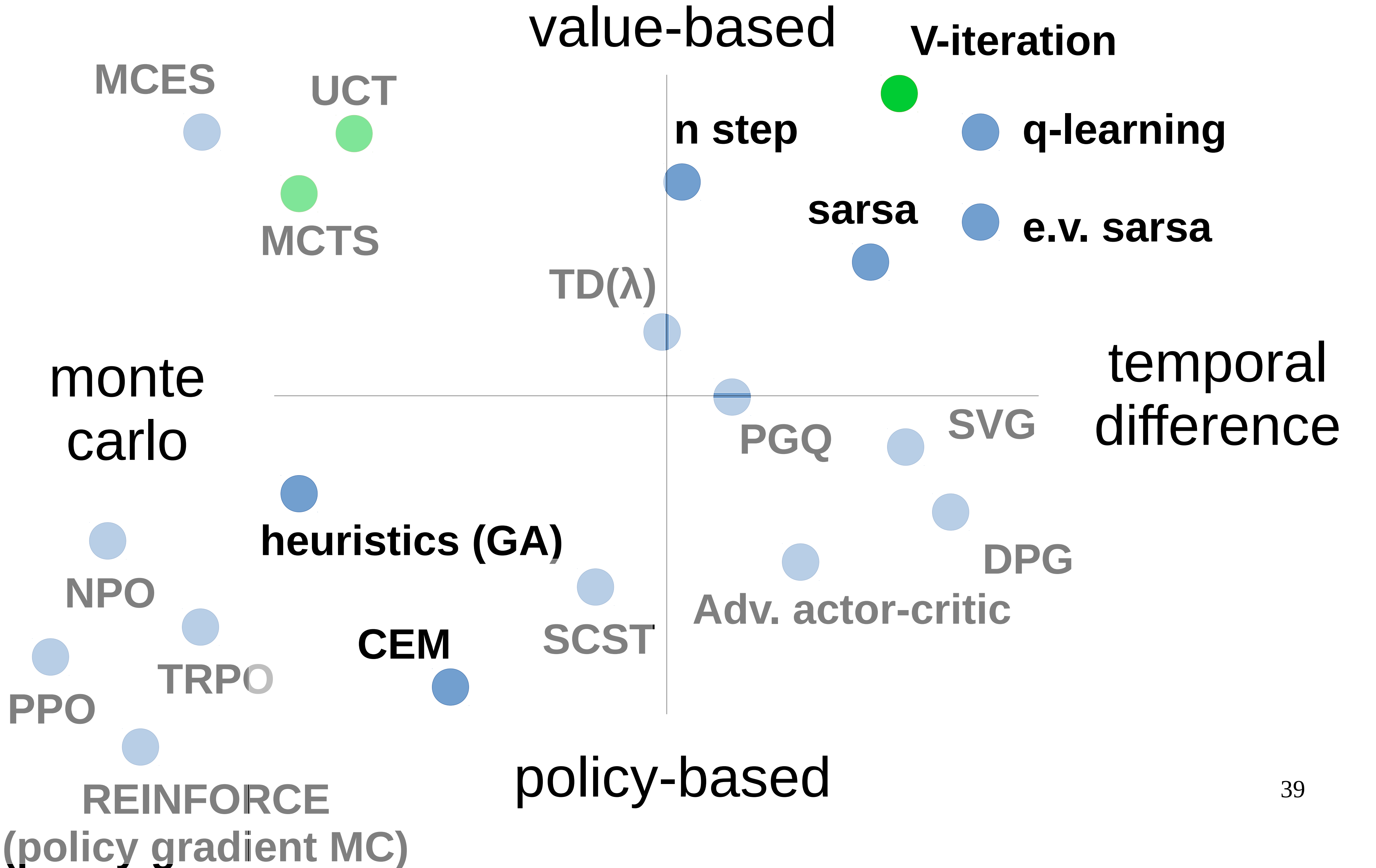
- Finite MDP, $\gamma=1$



- Sample many trajectories (or tree search)
- Compute expected $Q(s,a)=\underset{\substack{s'\sim p(s'|s,a),\\ a'\sim\pi(a'|s')\\ s''\sim p(s''|s',a')\\ \dots}}{E} R(s,a)$

minimal assumptions, unbiased, large variance

Long road ahead



Let's write some code!