

ENHANCING LASER-DRIVEN ION ACCELERATION THROUGH COMPUTATIONAL METHODS

DISSERTATION

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ABSTRACT

An abstract goes here. It should be less than **500 words**.

To you, my love.

ACKNOWLEDGMENTS

I want to thank your mom...

VITA

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Publications

Applying Machine Learning Methods to Laser Acceleration of Protons: Lessons Learned from Synthetic Data

Towards Automated Learning with Ultra-Intense Laser Systems Operating in the kHz Repetition Rate Regime

Intelligent Control of MeV Electrons and Protons

Can Two Pulses Enhance Proton and Electron Acceleration?

Fields of Study

Major Field: Physics

Table of Contents

	Page
Abstract	ii
Dedication	iii
Acknowledgments	iv
Vita	v
List of Figures	viii
List of Tables	x
List of Abbreviations	xi

Chapters

1 Introduction	1
1.1 Lasers	1
1.1.1 Chirped Pulse Amplification	1
1.1.2	1
1.2 High Energy Density Science	1
1.2.1 Fusion	1
1.2.2 Particle Acceleration	1
1.3 This Work	1
2 Laser-Plasma Interactions and Ion-Acceleration	2
2.1 Plasma Physics	2
2.1.1 Gaussian Laser	2
2.1.2 Single Particle Motions	3
2.1.3 Properties of a Plasma	5
2.1.4 Absorption of Energy	8
2.2 Ion Acceleration	13
2.2.1 Target Normal Sheath Acceleration	14
2.2.2 Other Acceleration Mechanisms	17
3 Computational Methods	19
3.1 The Particle-In-Cell Method	19
3.1.1 Densities and Shape Factors	20
3.1.2 Field Solver and Particle Push	21
3.1.3 EPOCH Code	23
3.2 Machine Learning	23

3.2.1	Simple Models	23
3.2.2	Advanced Models	23
3.2.3	Neural Networks	23
4	Particle-in-Cell Simulations of Enhanced Target Normal Sheath Acceleration	24
4.1	Theory	24
4.1.1	Spatially Aligned Pulses	24
4.1.2	Spatially and Temporally Aligned Pulses	26
4.2	Results	30
4.3	Discussion	30
5	Machine Learning Methods Applied to Synthetic Ion Acceleration Data	31
5.1	Modified Fuchs et. al. Model	31
5.1.1	Plasma Expansion into a Vacuum	31
5.1.2	Modified Fuchs Model	35
5.1.3	Further Model Modifications	36
5.2	Results	39
5.2.1	First Analysis	39
5.2.2	Second Analysis	39
5.3	Discussion	39
6	Optimization and Control of a kHz Laser System	40
6.1	Background	40
6.2	Methods	40
6.3	Discussion	40
7	Conclusion	41
7.1	Summary	41
7.2	Future Work	41
	Bibliography	42
 Appendices		
A	Energy Conservation in EPOCH Particle-in-Cell Simulations Due to Finite Numbers of Particles	49
A.1	Background	49

List of Figures

Figure		Page
2.1	An initially charge-neutral plasma is depicted on the left. On the right, the electrons are displaced by a distance x creating a charge separation and electric field akin to a parallel-plate capacitor directed towards the right. Adapted from Smith [1].	5
2.2	Visualization of the electric potential as a function of radial distance away from a positive point charge at the origin in three scenarios: vacuum (left), plasma (center), ideal conductor (right). Brighter colors show a higher value of ϕ . In the center panel, the debye length λ_D is shown.	8
2.3	Absorption fraction as a function of incidence angle θ_i . For resonance absorption, the density scale length L_p is varied in terms of the laser wavelength $\lambda = 0.8 \mu\text{m}$. For the Brunel mechanism, fractions are plotted for two regimes $a_0 \ll 1$ (where a value of $a_0 = 0.1$ was chosen) and for $a_0 \gg 1$ (which has no dependence on a_0).	11
2.4	Experimentally recorded hot electron temperatures as a function of irradiance $I\lambda^2$ are plotted as red squares. The empirical scaling models are given by Wilks [2](pink, solid), Gibbon and Bell [3](blue, ashed), Forslund et. al. [4](green, dash-dot), and Brunel [5](black, dotted). Figure is taken from Gibbon [6]	14
2.5	The Target Normal Sheath Acceleration (TNSA) process is depicted. First, an intense laser pulse irradiates the front side of a target foil of few μm thickness. This generates hot electrons that stream through the foil and re-emerge in a cloud on the rear side. The charge separation of the hot electrons and positively charged target creates intense longitudinal fields ($\sim \text{TV/m}$) that accelerate light ions in the mostly target normal direction. This figure was taken from Roth [7]	16
2.6	The regimes of various three different acceleration mechanisms are displayed in terms eq. (2.34). This figure was taken from Roth [8]	18
3.1	The top hat ($S_0(x)$), triangle ($S_1(x)$) and 3 rd order spline ($S_3(x)$) are plotted in 1D.	21

3.2	The “Yee” grid is depicted (left) where the electric and magnetic field components are staggered by half a cell. The fields, currents, position, and velocity make use of the staggered grid by leapfrog time integration (right). This picture was taken from the WarpX documentation	22
4.1	Proton energy spectrum from 1D PIC simulations depicted for three different temporal delays (including $\Delta t = 0$) from FIG. 3 in Markey et. al. [9]. . . .	25
4.2	Schematic of the incident laser on the half-cavity target (left) in Scott et. al. [10]. The radius of the cavity foil determines the delay ($\tau = 2r/c$) between the main pulse and the post-pulse (reflected laser light of $\sim 40\%$) energy of the main pulse).	25
4.3	Geometry of the two pulse scheme as shown in FIG. 1 of Ferri et. al. (2019) [11]. The single pulse has a total energy of 1.1J at an incidence angle of $\phi = 45^\circ$ and is shown through several time snapshots (a-c). In (d-f), the double pulse is shown through those same time snapshots with energies of 0.55 J in each pulse (the 1.55 J is a typo from the original figure). Other parameters include $\tau_{\text{fwhm}} = 38$ fs, thickness = 3 μm , material = aluminum, $w_{0,\text{fwhm}} = 5$ μm , $I_0 = 7 \times 10^{19} \text{ W cm}^{-2}$	26
4.4	Double pulse effectiveness in terms of changing preplasma scale length (a) and total laser energy (b,c) from Ferri et. al. (2019) [11].	28
4.5	FIG. 2 from Yao et al. [12]. Case 0 shows electron signals from a single pulse. Case 1 shows electron signals from a double pulse with spatial separation 120 μm . Case 2 shows electron signals from a double pulse with no spatial separation.	29
5.1	The net charge density (left) as a function of position $x/c_s t$ and normalized electric field E/E_0 (right) for $\omega_{pi} t = 50$ taken from Fig 1 and 2 in Mora’s Paper [13]. On the right, the self-similar electric field from eq. (5.9) is plotted with a dashed line.	34
5.2	The electron density profile of the pre-expanded target is depicted for various times t_0 . In this figure, $n(0) \equiv n_{\text{max}}$. Taken from Desai et al. [14] where z was used as the distance along the laser axis instead of x as done in this work.	37
5.3	The dotted black line shows the maximum proton energy predicted by eq. (5.19) with the pump depletion considerations in section 5.1.3 assuming $t_0 = 60$ ps, $I_0 = 10^{19} \text{ W cm}^{-2}$, $\kappa = 10^{-7}$, $d = 0.5$ μm . The red stars indicate the predicted positions of maximum proton energy ~ 12 μm . This plot is overlayed on top of an experimental maximum proton energy distribution from Morrison et. al. [15]. This figure is taken from Desai et. al. [14]. . . .	38

List of Tables

Table

Page

Chapter 1

INTRODUCTION

1.1 Lasers

Explain what a laser stands for, stimulated emission, lasing medium, what the Ti:sapphire crystal does and why it is chosen, CPA

1.1.1 Chirped Pulse Amplification

1.1.2 ...

1.2 High Energy Density Science

1.2.1 Fusion

1.2.2 Particle Acceleration

1.3 This Work

Talk about prior work

- Talk about motivating question

- Talk about how work is organized.

Chapter 2

LASER-PLASMA INTERACTIONS AND ION-ACCELERATION

2.1 Plasma Physics

Chen [16] describes a plasma as

a *quasineutral* gas of charged and neutral particles which exhibits *collective behavior*

In this section, the quasineutrality and collective behavior of plasmas will be discussed. One way to create a plasma is through the interaction of matter with an energetic laser. The laser has enough energy to ionize atoms to a hot dense soup that satisfy the definition of Chen. Plasmas do exist in other contexts, like those found in stars and various astrophysical systems, but these will not be explained in detail in this section. Here, we'll overview the physics of gaussian laser beams, particle motions in electric fields, and the properties of plasmas.

2.1.1 Gaussian Laser

In order to heat up a material with a laser efficiently, the energy would ideally be concentrated to a small point. For example, the typical incandescent household light bulb radiates energy equally in all directions and does not do a good job of focusing energy to a point. But, using a magnifying glass can effectively concentrate the energy. For this reason, focusing optics play an important part in the field of plasma physics.

However, to obtain a focused light source, we can use lasers that have the capability of producing narrow beams. The intended output of many lasers have an electric field described by the fundantal transverse electromagnetic mode [17] (TEM₀₀) described by the following electric field

$$E(r, x) = E_0 \hat{y} \frac{w_0}{w(x)} \exp\left(-\frac{r^2}{w(x)^2}\right) \cos\left(kx - \arctan(x/x_R) + \frac{kr^2}{2R(x)}\right) \quad (2.1)$$

where \hat{x} is the propagation direction, \hat{y} is the polarization direction, and $r = \sqrt{y^2 + z^2}$ is the radial distance away from the laser axis. In the cosine function, $R(z) = z[1 + (z_R/z)^2]$ is the radius of curvature and $\arctan(z/z_R)$ is the guoy phase. For our purposes, these won't have too much relevance. Additionally, the beam radius w is expressed as

$$w(z) = w_0 \sqrt{1 + (z/z_R)^2} \quad (2.2)$$

and has a minimum value at the *beam waist* $w(0) \equiv w_0$ at the focal position of the laser. The length scale over which the beam can propagate without diverging significantly is the *Rayleigh range* $x_R \equiv \frac{\pi w_0^2}{\lambda}$. The peak intensity is related to the electric field by $I_0 = \frac{1}{2} \epsilon_0 c E_0^2$ and eq. (2.1) shows that the intensity decays as $I(x, r) = I_0 (w_0/w(x))^2 \exp(-2r^2/w(x)^2)$ with increasing r and x . If we integrate this intensity distribution over the entire $y - z$ plane, we obtain the peak power $P_0 = \frac{\pi \omega_0^2}{2} I_0$. Furthermore, we can integrate the power over the pulse duration (assuming the pulse has a $\sin^2(t)$ envelope) to obtain the total energy in the pulse

$$E = \frac{\pi \omega_0^2}{2} I_0 \tau_{\text{fwhm}} \quad (2.3)$$

where τ_{fwhm} is the full-width half-max and is equal to half the pulse duration for a $\sin^2(t)$ pulse envelope.

2.1.2 Single Particle Motions

Electrostatics is governed by the Maxwell's equations which describe the allowed wave-like solutions for electric and magnetic fields in both matter and vacuum. The most relevant equation in this section is Gauss' Law or Poisson's Equation which can be expressed as

$$-\nabla^2 \phi = \nabla \cdot \vec{E} \equiv \frac{\rho}{\epsilon_0} \quad (2.4)$$

which relates the electrostatic potential ϕ or electric field E to the charge density ρ . This equation highlights how electric fields are directed radially outward from positive charges and inward towards negative charges. The motion of an electron in the influence of an electric field E or magnetic field B is given by the *Lorentz force* F_L

$$F_L \equiv -e(\vec{E} + \vec{v} \times \vec{B}) \quad (2.5)$$

Quiver Energy

To gain intuition about some quantities of interest for laser-matter interactions, let's consider a simple problem of an electron of charge $-e$ governed by eq. (2.5) with a negligible magnetic field B . Additionally, only consider 1D motion in the oscillating field

$E = E_0 \cos(\omega t)$ for a laser field of frequency ω . Then, the equation of motion is

$$\frac{dv}{dt} = -\frac{eE_0}{m} \cos(\omega t) \quad (2.6)$$

We can integrate this equation to obtain the velocity and position as a function of time (assuming $x_0 = v_0 = 0$)

$$v(t) = -v_{\text{osc}} \sin(\omega t) \quad (2.7)$$

$$x(t) = \frac{v_{\text{osc}}}{\omega} [\cos(\omega t) - 1] \quad (2.8)$$

where $v_{\text{osc}} \equiv (eE_0)/(m\omega)$ is defined as the *quiver velocity*. From Equation 2.7, we can calculate the kinetic energy gained by an electron as $U_p \equiv \frac{1}{2}m\langle v^2 \rangle = \frac{1}{4}mv_{\text{osc}}^2$ which is known as the *ponderomotive potential (energy)*. This energy represents the cycle-averaged quiver energy of an electron in an electromagnetic field. A more commonly used term is the dimensionless *normalized vector potential* a_0 which is closely related to the quiver velocity

$$a_0 \equiv v_{\text{osc}}/c = \frac{eE_0}{m\omega c} \quad (2.9)$$

Ultra-intense laser-matter interactions involve relativistic electrons which are produced when $a_0 \gtrsim 1$. In terms of the field, $a_0 \sim 1$ corresponds to a peak electric field $E_0 = \frac{2\pi mc^2}{e\lambda} \simeq 4 \text{ TV m}^{-1}$. In terms of the peak intensity, $I_0 = \frac{1}{2}c\epsilon_0 E_0^2 \simeq 2 \times 10^{18} \text{ W cm}^{-2}$ for this electric field. Consequently, the threshold for relativistic interactions is commonly understood as $I_0 \gtrsim 1 \times 10^{18} \text{ W cm}^{-2}$.

Ponderomotive Force

The above approach yields some important scales for laser-matter interactions, but only describes the interaction of a plane wave that is spatially homogeneous. A real laser field would be spatially inhomogeneous and we can express $E(x) \approx E_0 + xE'_0(x)$ to first order. This modifies the equation of motion as

$$\frac{dv}{dt} = -\frac{eE_0}{m} \cos(\omega t) - \frac{eE'_0}{m} \cos(\omega t) \left[\frac{v_{\text{osc}}}{\omega} (\cos(\omega t) - 1) \right] \quad (2.10)$$

where we've inserted the expression for x from eq. (2.8) which should be approximately true for small x . This equation can be simplified and separated into oscillating and non-oscillating components as

$$\frac{dv}{dt} = -\frac{eE_0}{m} [\cos(\omega t)(1 - \frac{E'_0 v_{\text{osc}}}{e_0 \omega}) + \frac{E'_0 v_{\text{osc}}}{2\omega E_0} \cos(2\omega t)] - \frac{eE'_0 v_{\text{osc}}}{2m\omega} \quad (2.11)$$

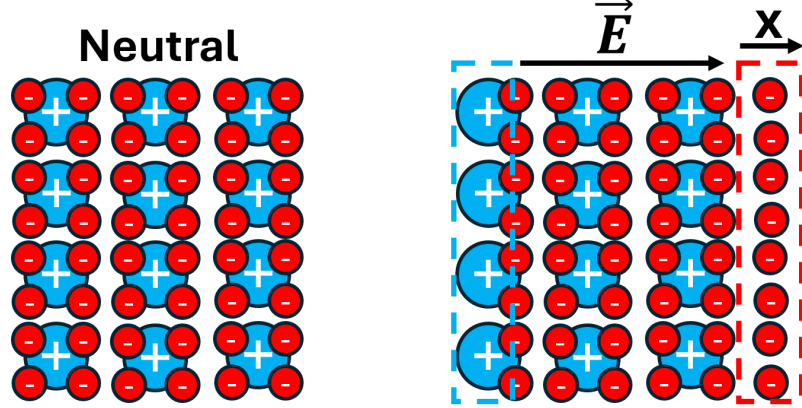


Figure 2.1: An initially charge-neutral plasma is depicted on the left. On the right, the electrons are displaced by a distance x creating a charge separation and electric field akin to a parallel-plate capacitor directed towards the right. Adapted from Smith [1].

Over many cycles, the oscillating components will average out to zero and the remaining term is given by $\langle F_p \rangle = m \frac{dv}{dt} = -\frac{eE'_0 v_{\text{osc}}}{2m\omega}$ and is called the *ponderomotive force*. We can generalize this to 3D and express this time-averaged force in several different, equivalent ways

$$\langle F_p \rangle = -\frac{e^2}{2m\omega^2} |E_0| \nabla E_0 = -\frac{mc^2}{4} \nabla (a_0^2) = -\nabla U_p \quad (2.12)$$

where

$$U_p = \frac{e^2 E_0^2}{4m\omega^2} = \frac{1}{4} m v_{\text{osc}}^2 \quad (2.13)$$

is the ponderomotive potential energy introduced earlier.

The ponderomotive force is an important mechanism in the absorption of laser energy by electrons which will be expanded upon in section 2.1.4.

2.1.3 Properties of a Plasma

The quasi-neutrality condition reflects the fact that a plasma is charge neutral throughout its volume in a similar way to an ideal conductor: mobile electrons will reorganize themselves in the presence of an external electric field to maintain zero field (or constant potential). The simplest plasma description will assume the ions are immobile (due to being much heavier than the electrons) and can be treated as a constant neutralizing background density n_i for the electrons of density $n_{e,0} = Zn_i$ (for a plasma with atomic number Z).

Plasma Electron Oscillations

A simple example can be illustrated by fig. 2.1 which shows a sheet of negative charge density $-\sigma = -en_e x$ displaced to the right a small distance x . The region in the bulk of the plasma will experience a force from the parallel plate “capacitor” fields directed to the left.

$$F = m \frac{d^2 x}{dt^2} = -e \frac{en_e x}{\epsilon_0} \quad (2.14)$$

which has the form of a restoring force that brings the charge imbalance back to the center of the plasma. This oscillatory motion has an associated frequency

$$\omega_{p,e} = \sqrt{\frac{n_e e^2}{m_e \epsilon_0}} \quad (2.15)$$

that gives the timescale for electron motion in the plasma. This characteristic frequency shows why plasmas support collective motion (in opposition to a neutral gas in which collisions between individual particles only happen). To get a feeling for this timescale, let’s assume a somewhat typical electron density $1 \times 10^{29} \text{ m}^{-3}$ in laser-plasma interactions to yield a timescale of $\omega_{p,e}^{-1} \simeq 0.1 \text{ fs}$.

Naturally (without externally imposed forces), these fluctuations in charge would be caused by thermal motions of electrons with a characteristic speed v_{th}

$$v_{th} = \sqrt{\frac{k_B T_e}{m}} \quad (2.16)$$

Due to the strong restoring force from the charge separation, the electrons can only move a short distance λ_D , called the Debye length, out of equilibrium in this timescale. We can estimate this length by equating $v_{th} = \lambda_D / t \simeq \lambda_D \omega_{p,e}$ and solve for λ_D .

$$\lambda_D = \frac{v_{th}}{\omega_{p,e}} = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}} \quad (2.17)$$

Physically, λ_D gives a length scale over which the electrostatic force persists in a plasma. Within a distance λ_D from some perturbation, charges will feel a force, and outside this distance, the charges will be completely shielded like that of an ideal conductor.

Fluid Model

This description of a plasma as a sea of electrons with collective motion that allows wave-like motions naturally lends itself toward a fluid model. The first component of this model stems from eq. (2.5) whose explicit time and space dependence can be expressed through $\frac{dp}{dt} = m(\frac{\partial v}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial v}{\partial t}) = m(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x})$ (just considering one spatial dimension for simplicity).

The second component of this model is the effect of the pressure gradient from thermal motions. Particles will tend to migrate from areas of higher pressure to lower pressure, where the thermal pressure is typically given by the familiar ideal gas law equation of state $p = n_e k_B T_e$. Consequently, the equation of motion should have a term that is opposite to the pressure gradient direction (i.e. $-\nabla p$). Combining these two components together in a generalized 3D equation results in

$$mn_e \left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right) = -en_e (\vec{E} + \vec{u} \times \vec{B}) - \nabla p \quad (2.18)$$

where we've changed the single particle velocity \vec{v} to the fluid velocity \vec{u} and multiplied by the electron density n_e to ensure correct units with the pressure gradient term. Then, let's look for a simple radially symmetric solution where the fluid velocity $u = 0$, magnetic field B is negligible, and the temperature is constant (isothermal). Then,

$$n_e e E = -k_B T_e \frac{\partial n_e}{\partial r} \quad (2.19)$$

and by relating the electric field to the potential $E = -\frac{dV}{dx}$, this equation can be integrated from $n_{e,0} \rightarrow n_e$ and $0 \rightarrow \phi$ to obtain

$$n_e = n_{e,0} \exp\left(\frac{e\phi}{k_B T_e}\right) \quad (2.20)$$

which is referred to as the *Boltzmann relation* for electrons [16]. We can get an approximate solution to this equation when the potential ϕ is only slightly larger than the equilibrium $\phi = 0$, which can be found when $e\phi \ll k_B T_e$. We can Taylor expand the density to obtain

$$n_e \approx n_{e,0} \left(1 + \frac{e\phi}{k_B T_e} \right) \quad (2.21)$$

and if we assume a fully ionized plasma with immobile ions of charge Z , the density of ions satisfies $n_{e,0} = Z n_i$ and eq. (2.4) becomes

$$\epsilon_0 \nabla^2 \phi = -en_{e,0} + en_e = en_{e,0} \left[1 + \frac{e\phi}{k_B T_e} - 1 \right] = \frac{e^2 n_{e,0} \phi}{k_B T_e} \quad (2.22)$$

This equation admits solutions of an exponentially decaying potential

$$\phi(r) = \frac{Q}{r} \exp\left(-\frac{r}{\lambda_D}\right) \quad (2.23)$$

where $\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n_{e,0} e^2}}$ is the Debye length from eq. (2.17). A visualization of the decaying potential from eq. (2.23) is shown in fig. 2.2. Looking at the center panel, we can see the fields drop off quickly within a distance λ_D in contrast to the left panel's potential that extends much further out in distance r . The exponentially decaying potential is a feature of plasmas and highlights the ability of plasma electrons to shield fields in a distance λ_D .

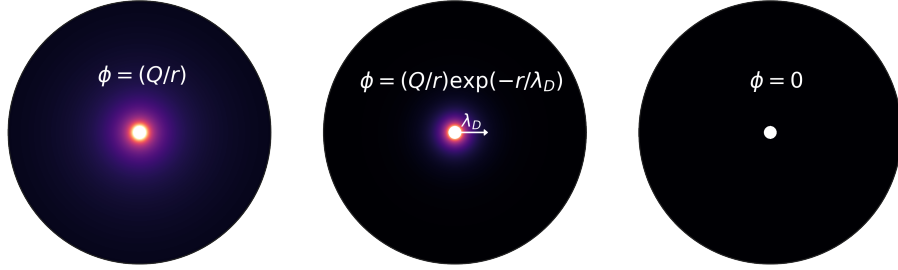


Figure 2.2: Visualization of the electric potential as a function of radial distance away from a positive point charge at the origin in three scenarios: vacuum (left), plasma (center), ideal conductor (right). Brighter colors show a higher value of ϕ . In the center panel, the debye length λ_D is shown.

Plasma Conditions

Putting all this together, we can define several conditions that must be satisfied for a plasma [16]. Quasi-neutrality dictates that the plasma should be largely charge neutral. The only regions that aren't charge neutral are those that fall within λ_D of some charge imbalance. Therefore, if L is the length scale of the system in which the plasma resides, we require that $\lambda_D \ll L$. However, this condition is not sufficient because an ideal conductor has $\lambda_D = 0$ but is not a plasma due to the absence of collective behavior. Collective behavior can be enforced by asserting that there are enough electrons N_D within a spherical volume of radius λ_D . The corresponding equation is $N_D = n_e(\frac{4}{3}\pi\lambda_D^3) \gg 1$. The final condition is that electrostatic interactions should dominate over collisions because the collective behavior (e.g. plasma oscillations) originates from the electrostatic forces. This means that the period of oscillations ($\omega_{p,e}^{-1}$) should be less than the mean time between collisions.

2.1.4 Absorption of Energy

In order for a laser to couple energy to the plasma electrons, some absorption mechanism needs to take place. The most obvious way that electrons can gain energy is through collisions with other energetic electrons and ions. However, the collision frequency is known to get smaller as the temperature goes up [6], so much so that plasmas can be treated as collisionless for ultra-intense laser experiments. Below, some of the most common known heating mechanisms are summarized.

Critical Density

First, we will look at how the electric field from an oscillating electric field penetrates a plasma. Using eqs. (3.2) and (3.4) combined with the vector identity $\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$ [17], we can solve for the vector wave equation in terms of only \vec{E} .

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \vec{E} = \mu_0 \frac{\partial \vec{J}}{\partial t} + \nabla(\nabla \cdot \vec{E}) \quad (2.24)$$

We can look for solutions of $\vec{E} = E(x) \cos(\omega t) \hat{x}$ that vary spatially only in the x direction. We are assuming $E(0)$ is the amplitude of the electric field at the boundary between vacuum $x < 0$ and matter $x > 0$ and wish to understand the form of $E(x)$ when $x > 0$. To proceed, we can assume the current density can be related to the drift velocity [18] by $\vec{J} = -n_e e u$ where u is the electron fluid velocity that satisfies eq. (2.18). Ignoring B and thermal pressure, this relationship becomes

$$\frac{\partial \vec{J}}{\partial t} = \frac{n_e e^2}{m} E = \omega_p^2 \epsilon_0 \vec{E} \quad (2.25)$$

using eq. (2.15) which can be combined with eq. (2.24) to obtain a differential equation for the electric field

$$[\nabla^2 + \frac{\omega^2}{c^2} (1 - \frac{\omega_p^2}{\omega^2})] \vec{E} = 0 \quad (2.26)$$

By just focusing on the x-dependence, we can simplify this equation to

$$\frac{d^2 E}{dx^2} = \frac{1}{l_s^2} E \quad (2.27)$$

where $l_s^2 \equiv \frac{c^2}{\omega^2 - \omega_p^2}$ defines the *skin depth* l_s . In the case where $\omega < \omega_p$, l_s^2 is negative and the solution has a sinusoidal dependence

$$E(x) = E(0) \cos(x/l_s) \quad (2.28)$$

On the other hand, when $\omega > \omega_p$, l_s^2 is positive and the solution has an exponential dependence

$$E(x) = E(0) \exp(-x/l_s) \quad (2.29)$$

where $l_s \equiv c/\omega_p$ is defined as the *skin depth*. This “evanescent” behavior when $\omega > \omega_p$ occurs because the electrons cannot respond fast enough to the higher frequency ω . Since the field cannot propagate effectively for $x > 0$, the plasma ends up reflecting a significant portion of the light. Since wavelength (and frequency) is fixed from the laser, we can reformulate this finding in terms of electron density. The critical density n_c is defined as the electron

density where $\omega = \omega_{p,e}$. Using eq. (2.15), this can be expressed as

$$n_c \equiv \frac{m\epsilon_0}{e^2}\omega^2 \quad (2.30)$$

When $n_e > n_c$, the plasma is said to be *overdense* and most of the laser light gets reflected. When $n_e < n_c$, the plasma is said to be *underdense*, and the laser light can propagate through the plasma.

A typical Ti:Sapphire laser has a wavelength of $0.8\text{ }\mu\text{m}$ which corresponds to a critical density of $n_c \simeq 1.7 \times 10^{27} \text{ m}^{-3}$. In this work, two materials are of interest: gold and ethylene glycol which have densities of 19.3 g cm^{-3} and 1.11 g cm^{-3} respectively. These mass densities correspond to a number density of electrons $5.9 \times 10^{28} \text{ m}^{-3}$ and $1.1 \times 10^{28} \text{ m}^{-3}$ respectively assuming a singly ionized plasma. If the plasmas were multiply ionized, these densities would be even higher. Even though these “solid density” plasmas are clearly overdense, experiments show energy is able to efficiently couple to the electrons. Consequently, there must exist mechanisms of absorption that are consistent with the fact that most of the laser energy can only be deposited in a small depth l_s into the plasma.

Resonance Absorption

The previous discussion applied to an electric field directed in the x-direction. For gaussian laser beams, the electric field is always perpendicular to the direction of propagation. So, if the laser beam is to be directed toward a target at $x = 0$, normal incidence would imply that the electric field $E_x = 0$. Additionally, an s-polarized beam would have an electric field only in the z direction. As a result, the typical model of a laser beam depositing energy into a plasma involves a p-polarized laser beam traveling obliquely in the $x - y$ plane with some angle of incidence θ_i measured with respect to the normal direction of the target. Physically, plasma oscillations occur through fluctuations in density which are going to be the strongest in the x direction due to the interface between vacuum and matter.

Furthermore, Kruer [19] argues that the reflection of light at oblique incidence occurs at a density $n_e = n_c \cos^2(\theta_i)$ by enforcing momentum conservation of the electric field component in the y direction. Even though we’ve discussed density profile as an abrupt step: from 0 to n_e from crossing $x = 0$, an actual experiment would see some pre-heating of the target before the target becomes ionized and behaves as a mirror at the critical density. This can be characterized by some scale length $L_p \equiv n_e(\frac{\partial n_e}{\partial x})^{-1}$ which is smaller for steeper density profiles. As a result, at higher incidence angles, the evanescent portion of the electric field has to travel further into the underdense region of the target to reach the critical density.

When the frequency of the laser $\omega \simeq \omega_p$, the laser light is in resonance with the plasma oscillations and the energy can be most efficiently absorbed. To maximize the amount of

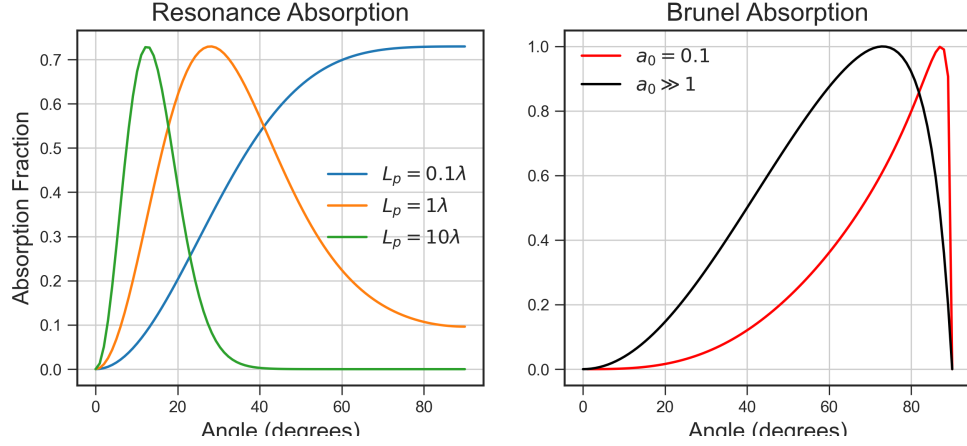


Figure 2.3: Absorption fraction as a function of incidence angle θ_i . For resonance absorption, the density scale length L_p is varied in terms of the laser wavelength $\lambda = 0.8 \mu\text{m}$. For the Brunel mechanism, fractions are plotted for two regimes $a_0 \ll 1$ (where a value of $a_0 = 0.1$ was chosen) and for $a_0 \gg 1$ (which has no dependence on a_0).

energy reaching the critical density surface in resonance, we would want θ_i to be large so that the electric field has a significant component in the x direction, but also small enough so that the field doesn't diminish too much by traveling in the evanescent underdense region. Denisov [20] and others [21–23] addresses this question and develops a model for this so-called *resonance absorption*. An approximate version of this formula is given by Kruer [19]

$$\phi(\tau) \simeq 2.3\tau \exp(-2\tau^3/3) \quad (2.31)$$

where $\tau \equiv (kL_p)^{1/3} \sin(\theta_i)$ takes into account both the scale length and incidence angle. fig. 2.3 shows the fractional absorption $\phi(\tau)^2/2$ of the incident light from this model as a function of θ_i for various scale lengths. There is an optimal angle $\theta_{max} \approx \arcsin(0.8(kL_p)^{-1/3})$ that maximizes the absorption fraction (although Kruer notes that his simple model overestimates the peak absorption – the fraction should peak at around 0.5 [19]).

Brunel Heating

Resonance absorption only makes sense when the amplitude of the plasma oscillations $x_{osc} = v_{osc}/\omega = \frac{a_0\lambda}{2\pi}$ is less than the scale length L_p [6], otherwise there is not enough available space for the oscillations to take place. For $\lambda = 0.8 \mu\text{m}$ and $a_0 = 1$, $x_{osc} = 127 \text{ nm}$. However, for extremely small scale-lengths, efficient electron heating can still be observed [24]. Consequently, a different type of heating mechanism is responsible in this case (somewhat confusingly) called “not so resonant, resonant absorption” [5]. This model, developed by Brunel, is also known as *vacuum heating* and will be explained below.

Before explaining the Brunel mechanism, I should the basics of the 3-step model used for high-harmonic generation [25]. Readers may find this model more familiar due to the recent 2023 Nobel Physics Prize won by Ohio State’s Pierre Agostini [26] which utilized the ideas of this model to produce attosecond pulses. This model involves a strong oscillating electric field $E(x) = E_0 \cos(\omega t)$ incident on an atom. Now, assume an electron is ionized at $t = t^*$. Under the influence of the oscillating field, the (initially stationary) electron will gain and lose energy by moving away from the atom and returning back toward the atom. When $t^* \neq n\pi$ for integer n , it is actually possible for the electron to return back to the atom with non-zero energy. In fact, when $\omega t^* \approx 17^\circ + n(180^\circ)$, the electron returns with an energy peaking at $3.17U_p$ where U_p is the ponderomotive potential given by eq. (2.13). Furthermore, modeling the ionization rate through quantum-mechanical tunneling of an electron through a Coulomb potential warped by the oscillating laser field, we also determine that the most probable energy for an electron is strongly peaked at the $3.17U_p$ cutoff. In short, this model shows how a laser field can produce electrons with energy on the scale of the ponderomotive potential with high probability at a frequency of twice per optical cycle.

In the Brunel mechanism [5], we are considering a laser field incident on a planar target at $x > 0$ and vacuum at $x < 0$. In order for the electrons to escape the target, there needs to be some component of the electric field in the x direction. Thus, we need to consider oblique incidence and p-polarization just like with resonance absorption. When the plasma scale length is small, the electrons will be able to travel far enough in the $x < 0$ to escape the plasma entirely and gain energy on the order of U_p in a similar fashion to Corkum’s model [25]. Electrons arriving back to the target at just the right time will penetrate deeper than the skin depth $l_s \approx c/\omega_p$ and be inaccessible to the laser field [6]. These *hot electrons*, generated primarily on the front surface of the target, will provide the energy to heat the remainder of the overdense region of the target that the laser field cannot directly access.

As mentioned, the optimal angle would appear to be for grazing incidence ($\theta_i = 90^\circ$), but Gibbon notes that accounting for imperfect reflection of the laser field and relativistic energies of the electrons, the efficiency no longer diverges at $\theta = 90^\circ$ [6]. In fig. 2.3, some estimates for the absorption efficiency are plotted according to a simplified model developed by Gibbon [6] based on Brunel [5].

Other Mechanisms

When the laser field penetrates a distance $l_s \approx c/\omega_p$ into the overdense region of the target, the electrons can heat up through collisions at an absorption rate $\eta \propto \frac{\nu_{ei}}{\omega_{p,i}}$ [6] where ν_{ei} is the electron-ion collision frequency. This type of absorption is called the *skin effect*. In this case, we see Fresnel-like reflection and absorption (see [27]) that is effective for large incidence angles. Even when the collision frequency is low, we can still get efficient absorption as long as the thermal electron motions are large compared to the skin depth (i.e. $v_{th}/\omega > l_s$)

[6]. This phenomena is called the *anomalous skin effect* that is also most effective at large incidence angles.

All of the mentioned phenomena work best at oblique incidence in p-polarization. But, for relativistic intensities, additional heating mechanisms arise. When $a_0 \gtrsim 1$, the magnetic portion of eq. (2.5) becomes significant. At normal incidence, the electric and magnetic field components both fall in the $y - z$ plane. The electric fields move the electrons strongly causing a current \vec{J} which in turn will interact with the laser magnetic field \vec{B} in the direction perpendicular to both. As a result, this type of heating is known as $\vec{J} \times \vec{B}$ heating [6, 28]. Because $\vec{J} \times \vec{B}$ is in the direction of propagation, this effect is most pronounced at normal incidence. (TALK ABOUT HOW CIRCULAR POLARIZATION NEGATES JXB HEATING)

At even higher intensities, the laser can directly impart energy to the electrons through radiation pressure [29] because photons themselves carry momentum. These mechanisms are explored further in section 2.2. In reality, all experiments involve a combination of several different electron heating mechanisms. Consequently, many experiments and simulations have been devoted to parametric studies that show how parameters (L_p , θ_i , polarization, a_0 , etc.) affect electron absorption (CITATION NEEDED)

2.2 Ion Acceleration

The previous section gave an overview of the laser-plasma interactions and how they can efficiently couple energy into hot electrons. Regardless of heating mechanism, one theme is common to all – the energy gained by an electron’s quiver motion in an oscillatory field, known as the ponderomotive potential (eq. (2.13)) sets the scale for the hot electron temperature T_h . That equation was only valid for non-relativistic electrons, so we must replace $U_p = \frac{1}{4}mv_{\text{osc}}^2$ with the relativistic kinetic energy $U_p \equiv (\gamma - 1)mc^2$ where $\gamma = 1/\sqrt{1 - \frac{v_{\text{osc}}^2}{c^2}}$ defines the lorentz factor. We can combine this with the relativistic momentum $p = \gamma mv_{\text{osc}}$ and eq. (2.9) to determine an approximate expression of γ in terms of a_0

$$\gamma \simeq \sqrt{1 + a_0^2} = \sqrt{1 + \frac{I_{18}\lambda_{\mu m}^2}{1.37}} \quad (2.32)$$

where I_{18} is the peak intensity of the laser pulse in $1 \times 10^{18} \text{ W cm}^{-2}$ and $\lambda_{\mu m}$ is the wavelength in μm . In 1992, Wilks [2] conducted simulations to show that T_h is on the order of U_p .

$$k_B T_h = mc^2(\gamma - 1) \quad (2.33)$$

In fig. 2.4, we can see that the *Wilks scaling* (pink) closely matches ultra-intense laser experiments. The other scalings in the figure are similarly validated by computational simulations and are all proportional to $(I\lambda^2)^\alpha$, where $0 < \alpha \leq 1$. As a result, the product

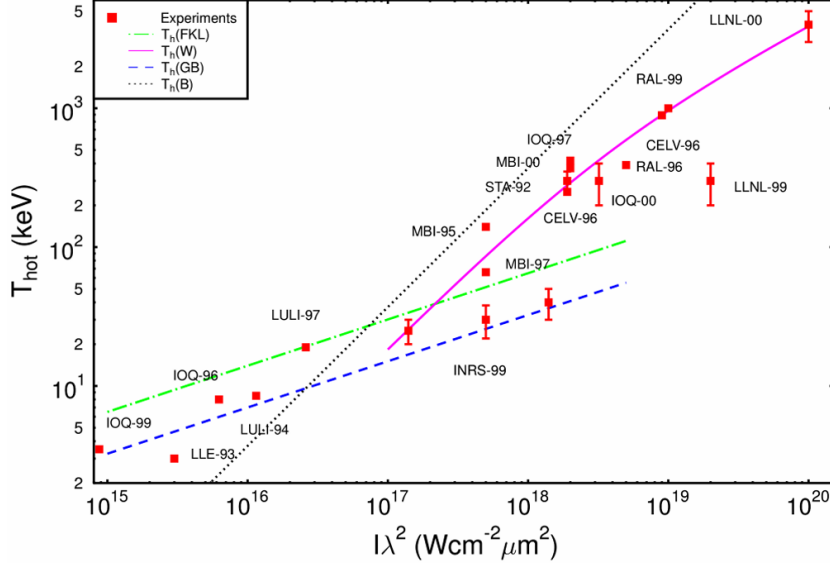


Figure 2.4: Experimentally recorded hot electron temperatures as a function of irradiance $I\lambda^2$ are plotted as red squares. The empirical scaling models are given by Wilks [2](pink, solid), Gibbon and Bell [3](blue, dashed), Forslund et. al. [4](green, dash-dot), and Brunel [5](black, dotted). Figure is taken from Gibbon [6]

$I\lambda^2$ is an important quantity in laser-plasma experiments and is called the *irradiance*. Wilks also outlines a way to measure the hot electron temperature in his simulations [2] by taking the slope of $\frac{dN_e}{dE}$ in the *MeV* regime and is something I do in (CITE DUB PULSE SIM FIGUR HOT ELECTRON)

Since protons are 1836 times as massive as electrons, they are much harder to accelerate and on the scale of femtosecond pulse interactions, they are essentially immobile. (CITATION) Despite this, ultra-intense laser experiments have demonstrated proton acceleration is possible. This section will explain the Target Normal Sheath Acceleration (TNSA) mechanism for accelerating protons and light ions which is heavily dependent on T_h . Then, we will discuss alternative acceleration mechanisms. Finally, we'll overview some important applications.

2.2.1 Target Normal Sheath Acceleration

The observation of energetic protons off the rear side of thin plastic and gold targets has been documented throughout a variety of experiments since the 80s [30]. It might sound unintuitive that we would even see protons in the first place; after all, when shooting a target like aluminum, one would expect aluminum ions. It turns out that there is always an important and measurable surface contamination layer, primarily composed of hydrogen

and light hydrocarbons [31]. Allen [32] showed that when removing the surface contaminant from the backside, we see a strong suppression in ion acceleration. This points to the contaminant layer being the crux of what is accelerated.

TNSA Models

Expansion models have been long known since the 70s and 80s (e.g. Crowe [33] and Kishimoto [34]) that describe the acceleration of protons with experiments (e.g. Tan [30]) as well. However, the advent of Chirped Pulse Amplification [35] in 1985 by Donna Strickland and Gerard Mourou allowed the intensities of the laser light to increase to relativistic levels ($a_0 > 1$) with sub-ps pulse durations. This technology dramatically impacted the field of laser-plasma interactions because it allowed new relativistic regimes of ion acceleration to be explored – for this work they won the Nobel Prize [36].

The field of ultra-intense proton acceleration kicked off in the year of 2000 with a group at Michigan [37] finding 1.6 MeV protons from a thin aluminum foil with a $3 \times 10^{18} \text{ W cm}^{-2}$ class laser at normal incidence. Then, Rutherford Appleton Laboratory found 30 MeV protons [38] from a $5 \times 10^{19} \text{ W cm}^{-2}$ class laser incident on a lead target at 45° incidence. Shortly after, LLNL found energies up to 58 MeV [39] from a $3 \times 10^{20} \text{ W cm}^{-2}$ class laser on a gold target at 45° incidence.

Now that efficient MeV proton acceleration had been achieved through multiple studies, a more thorough comprehensive picture of the physical process was desired. In 2001, Wilks [40] summarized much of the existing literature including the isothermal expansion model [33], existence of a maximum cutoff energy [34], and dependence on hot temperature [2]. Then, he described the Target Normal Sheath Acceleration process in the following way [40]

... the prepulse creates large plasma in front of a solid target. Once the main pulse hits the target, a cloud of energetic electrons (1-10 MeV in effective temperature) is generated, which extends past the ions on both the front and back of the target. Since the protons on the back are in a sharp, flat density gradient, they are accelerated quickly (in the first few μm off the target) to high energies in the forward direction ... On the front, the outermost ions are in a sphere, in a long scale length plasma (due to prepulse) and therefore are accelerated to lower energies, and are spread out into 2π steradians.

A visual of the TNSA process can be seen in fig. 2.5. Although Wilks [40] provided a physical picture of the TNSA process, existing models didn't always match up to experiments. For this reason, the ensuing decade saw much progress in the development of models to describe the spectrum of TNSA accelerated protons. Perego [41] gives a good review of some of the leading models developed and tested against experiments in the 2000s and these models will be summarized below.

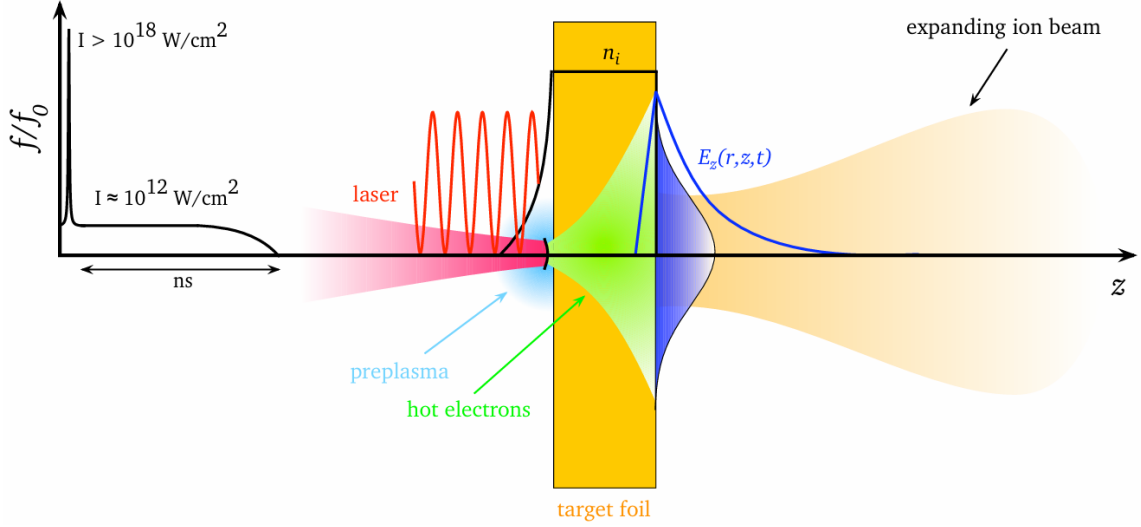


Figure 2.5: The Target Normal Sheath Acceleration (TNSA) process is depicted. First, an intense laser pulse irradiates the front side of a target foil of few μm thickness. This generates hot electrons that stream through the foil and re-emerge in a cloud on the rear side. The charge separation of the hot electrons and positively charged target creates intense longitudinal fields ($\sim TV/m$) that accelerate light ions in the mostly target normal direction. This figure was taken from Roth [7]

First, are the isothermal expansion (fluid) models which include Mora’s “Plasma Expansion into a Vacuum” [13] (2003) that combine eqs. (2.4) and (2.20) with fluid eqs. (5.6a) and (5.6b). This model underlies the work done in chapter 5, where it is explained in more detail, and has the issue of predicting proton energies that can go up to arbitrarily high values. As a remedy, Mora introduces a finite acceleration time τ which is of the order of the pulse duration. Mora [42] addresses this in a different way (2005) by instead assuming an adiabatic model and limiting the target to be a thin foil (instead of a semi-infinite target).

Alternatively, Passoni and Lontano [43] introduces an upper limit to the integration range of the electric potential instead of using the fluid equations. In this approach, the electrostatic fields determined from the potential are considered static, and the ensuing ion dynamics is determined by placing a test ion in the field. Further iterations incorporate some distribution of speeds for the electrons (non-relativistic Maxwell-Boltzmann [44] or relativistic Maxwell-Jüttner [45]) and use an empirically determined scaling for the peak energy of electrons (as a function of laser energy) that do not escape the system [46].

Furthermore, some hybrid models include elements of both fluid and quasistatic models like Robinson [47] and Albright [48].

Optimization of TNSA process

Since the TNSA process is intimately related to the hot electron process at the front of the target and the flat density gradient at the back, many efforts have been taken to design targets that optimize ion acceleration. Patel [49] used spherically shaped targets to act as a lens that can focused the proton beam. MacKinnon [50] showed lower target thickness leads to higher proton energy due to a higher mean density of hot electrons at the surface. More recent experiments have even used nanowires [51] and microtubes [52]. Many experiments generally find that there is an optimal level of pre-expansion of the target that enhances hot electron generation and ion acceleration (e.g. McKenna [53]).

Another way to increase the peak proton energy of the emitted spectrum is to use two spatially aligned pulses. If one pulse has a delay with respect to the other, the first pulse could pre-expand the target to provide an optimal electron density at the front surface [54]. If the pulses are also temporally aligned, the constructive interfere at the target front surface may prove beneficial [11]. The second approach is called *Enhanced Target Normal Sheath Acceleration* (eTNSA) and chapter 4 is devoted to this phenomenon.

See the review article by Roth [7] for a more comprehensive list of the different approaches to enhance TNSA.

2.2.2 Other Acceleration Mechanisms

TNSA is not the only method in which protons can be accelerated. For intensities of greater than $10^{21} \text{ W cm}^{-2}$, laser-induced ion shocks can start to play a significant role [55]. For even higher intensities $\sim 10^{23} \text{ W cm}^{-2}$, the radiation pressure of the electromagnetic wave can efficiently transfer momentum to the ions [55]. See Macchi [29] for a more in depth discussion on these topics. One way to differentiate the TNSA regime from other regimes is through the following equation relating a_0 to various properties of the laser and target

$$a_0 = n_e \lambda r_e l_0 = 224 \left(\frac{n_e}{1 \times 10^{29} \text{ m}^{-3}} \right) \left(\frac{l_0}{1 \mu\text{m}} \right) \quad (2.34)$$

Lezhnin [8] uses this equation to differentiate TNSA from two other mechanisms: Radiation Pressure Acceleration (RPA) and Coulomb Explosions; this can be seen in fig. 2.6. If the laser intensity is sufficiently high and density is low enough to be transparent, the laser can quickly sweep away most electrons to leave behind a strongly positive target. The repelling coulomb force will cause the protons to expand outwards in all directions.

When the radiation pressure $P_{\text{rad}} \approx 2I_0/c$ is significant enough to overcome the thermal expanding pressure $n_e k_B T_e$, ions can accelerate directly through the transfer of momentum. [29]. In this regime, laser absorption into hot electrons by traditional mechanisms would be detrimental. By shooting the laser at normal incidence with circular polarization, resonance absorption and $\vec{J} \times \vec{B}$ heating can be minimized as seen in section 2.1.4.

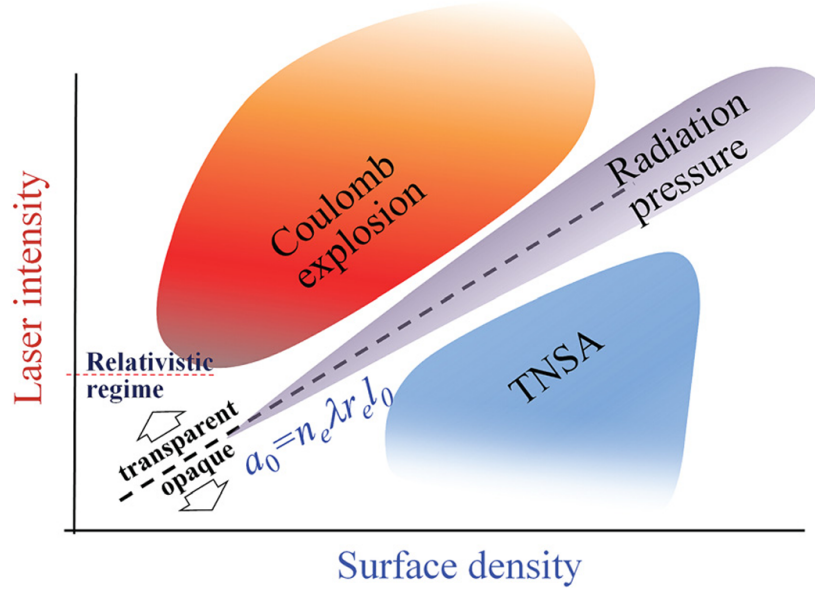


Figure 2.6: The regimes of various three different acceleration mechanisms are displayed in terms eq. (2.34). This figure was taken from Roth [8]

For thick targets, this immense pressure can impart a parabolic deformation that allows the laser to penetrate further. This is the regime of *hole boring*. Targets thin enough where the hole boring process reaches the target rear in a time less than the pulse duration are in the *light sail* regime. [29].

Wakefield Acceleration

All the aforementioned acceleration processes only make sense for overdense targets whose electron density is greater than the critical density ($n_e > n_c$). This is because the critical density surface is the primary area where the laser deposits energy into hot electrons. If the target plasma has $n_e < n_c$, the target is said to be underdense and there is no critical density surface where the laser interacts with. Tajima and Dawson [56] first proposed the idea of a “Laser Electron Accelerator” in 1979 that is capable of accelerating electrons to high energies through the non-linear ponderomotive force. If the conditions are just right, the electrons can “surf” a plasma wave in the wake of the pulse and pull along positive ions in a process now known as *Laser Wakefield Acceleration*. A comprehensive review of the subject can be found here [57].

Chapter 3

COMPUTATIONAL METHODS

3.1 The Particle-In-Cell Method

The Particle-In-Cell (PIC) method involves solving Maxwell's Equations on a grid

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (3.1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3.2)$$

$$\nabla \cdot \vec{B} = 0 \quad (3.3)$$

$$\nabla \times \vec{B} = \mu_0(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}) \quad (3.4)$$

This is combined with the lorentz force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (3.5)$$

which determines the motions (i.e. \vec{r} and \vec{v}) of charged particles by integration. It is impossible to keep track of the true numbers of particles in this type of simulation which would be roughly on the order of Avogadro's number $\sim 10^{23}$. Instead, we lump many particles together into what is called a *macro particle*. For example, one “macro electron” could contain 1 trillion “real electrons”. Also, we cannot hope to have infinite precision in calculating quantities of interest. Spatially, we must separate the simulation volume into a grid where each cell has length Δx , Δy , and Δz in the x, y, and z direction respectively. Temporally, we introduce a time step Δt which allows us to propagate Maxwell's Equations forward in time by Δt for every iteration.

For simplicity, non-relativistic equations will be introduced in this section, but they can easily be generalized to the relativistic versions which are implemented in modern PIC codes. Additionally, some of the equations will assume a 2D grid, but a 3D grid is similarly straightforward to generalize.

3.1.1 Densities and Shape Factors

When a simulation is initialized, all the particles will have a defined position and velocity. The charge density $\rho_{i,j}$ (for the cell at the i^{th} and j^{th} grid point in the x and y directions) is easy to compute – it is simply the sum of all the charges q_α closest to grid point (i, j) divided by the cell area: $\rho_{i,j} \equiv \frac{\sum_\alpha q_\alpha}{\Delta x \Delta y}$ (in 2D symmetry we additionally divide by 1 meter in the z direction to get the units right). The current density $\vec{J}_{i,j}$ can be obtained similarly – $\vec{J}_{i,j} \equiv \frac{\sum_\alpha q_\alpha \vec{v}_\alpha}{\Delta x \Delta y}$. Assigning the densities to the nearest grid point in this manner is sensibly called Nearest Grid Point (NGP) by Birdsall and Langdon [58].

Since the PIC approach contains many real particles in each macro particle, it is desired to smooth the macro particle densities throughout the cell(s). We can modify the individual density contributions of particles by a shape factor $S(\vec{r}_\alpha - \vec{r})$ that depends on a particle's location \vec{r}_α in relation to a grid point located at \vec{r} . This shape factor is normalized so that integrating it over the area of the simulation yields 1 to ensure the particle number is properly being conserved. The simplest improvement over NGP would be the *top hat* shape factor (also called Cloud in Cell [58]) which assigns density contributions proportional to proximity of the nearest cells within $(\Delta x, \Delta y)$. This has the shape of a uniform distribution and thus looks like a “top hat” in 1D. It is a 0th order shape factor and can be represented by the following equation

$$S_0(x) \equiv \begin{cases} 1 & \text{if } |x| \leq 0.5\Delta x \\ 0 & \text{otherwise} \end{cases} \quad (3.6)$$

A further improvement will weigh the particles closer to a particular grid point higher than a particle further away. If this weighting is linear over an area $(2\Delta x, 2\Delta y)$, it is called the *triangle* shape factor and represented by the following equation in 1D

$$S_1(x) \equiv \begin{cases} 1 - \frac{|x|}{\Delta x} & \text{if } |x| \leq \Delta x \\ 0 & \text{otherwise} \end{cases} \quad (3.7)$$

It turns out that the higher order shape factors $S_n(x)$ can be represented by convolutions of $S_0(x)$

$$S_n(x) \equiv \int_{-\infty}^{\infty} S_{n-1}(x') S_0(x - x') dx' \quad (3.8)$$

and the shape factors for $n \geq 2$ are commonly called n-splines. The third order spline is used in this work and weights particles over an area $(4\Delta x, 4\Delta y)$ and is represented in 1D by

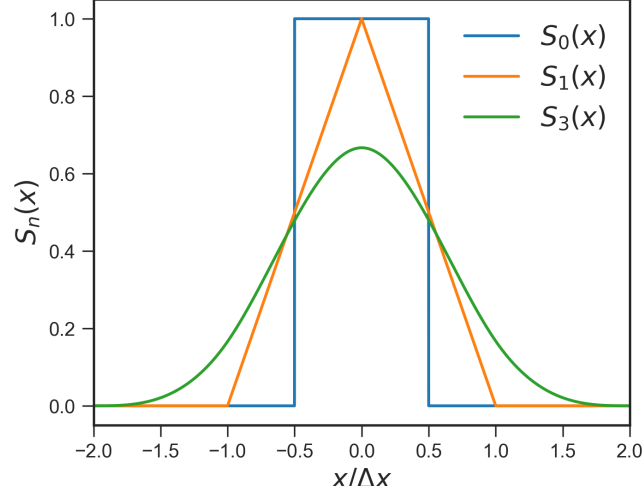


Figure 3.1: The top hat ($S_0(x)$), triangle ($S_1(x)$) and 3rd order spline ($S_3(x)$) are plotted in 1D.

$$b_3(x) = \begin{cases} \frac{1}{6}(8 - 12|\tilde{x}| + 6\tilde{x}^2 - \tilde{x}^3) & \text{if } 1 \leq |\tilde{x}| \leq 2 \\ \frac{1}{6}(4 - 6\tilde{x}^2 + 3\tilde{x}^3) & \text{if } |\tilde{x}| \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (3.9)$$

where $\tilde{x} \equiv x/\Delta x$ normalizes the position x . See fig. 3.1 for a comparison of the three shape factors.

These shape factors not only apply to the calculation of densities, but also to the electric and magnetic fields. In this way, the fields used to update particle positions and velocities are averaged over neighboring cells.

3.1.2 Field Solver and Particle Push

The PIC method is able to make efficient use of the second order accurate central difference approximation to compute derivatives. A simpler method like Euler integration is only first order accurate and will suffer in terms of accuracy. Higher order methods like 4th order Runge-Kutta have much higher computational costs in terms of operations per time step and memory consumption. The central difference scheme is accomplished by alternately calculating electric and magnetic fields, staggered by half a time step, in an approach called *leapfrog integration* [58]. This can be seen in the right half of fig. 3.2 where the calculations of E and J, B alternate in a “leapfrog” fashion. It turns out that this staggering also comes with some nice properties like automatically satisfying eq. (3.3). By rearranging eq. (3.2),

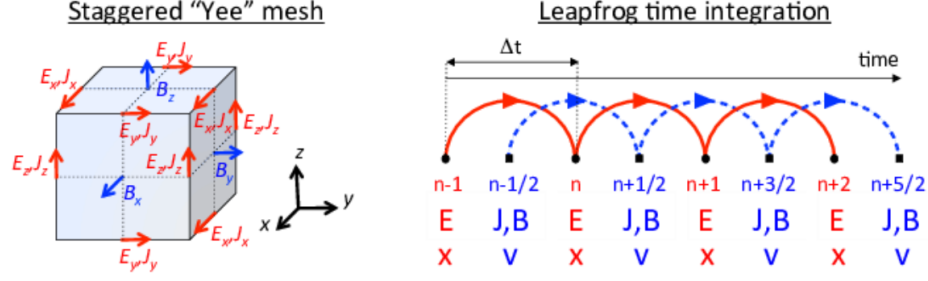


Figure 3.2: The “Yee” grid is depicted (left) where the electric and magnetic field components are staggered by half a cell. The fields, currents, position, and velocity make use of the staggered grid by leapfrog time integration (right). This picture was taken from the WarpX documentation

we can update the electric and magnetic fields through the following equations [59]

$$\vec{E}^{n+1} = \vec{E}^n + \Delta t (c^2 \nabla \times \vec{B}^{n+\frac{1}{2}} - \frac{1}{\epsilon_0} \vec{J}^{n+\frac{1}{2}}) \quad (3.10)$$

$$\vec{B}^{n+\frac{1}{2}} = \vec{B}^{n-\frac{1}{2}} - \Delta t (\nabla \times \vec{E}^n) \quad (3.11)$$

where $\vec{J}^{n+\frac{1}{2}} \equiv \frac{\sum_{\alpha} q_{\alpha} \vec{v}_{\alpha}^{n+\frac{1}{2}}}{\Delta x \Delta y}$ depends on the velocity. The updated velocity for each particle is calculated through the force from eq. (3.5).

$$\frac{v_{\alpha}^{n+\frac{1}{2}} - v_{\alpha}^{n-\frac{1}{2}}}{\Delta t} = \frac{q}{m} [E_{\alpha}^n + \frac{v_{\alpha}^{n+\frac{1}{2}} + v_{\alpha}^{n-\frac{1}{2}}}{2} \times B_{\alpha}^n] \quad (3.12)$$

The α subscript indicates the quantities are calculated for each particle; thus, the fields are smoothed out by the shape factor (e.g. $E_{\alpha}^n \equiv \int E^n S_3(x - x_{\alpha}, y - y_{\alpha}, z - z_{\alpha}) dx dy dz$). In practice, eqs. (3.10) and (3.11) are broken up into half-steps so that the electric and magnetic field are known for all half-steps. At first glance, eq. (3.12) does not appear to have an explicit solution for $v^{n+\frac{1}{2}}$. There are implicit methods that can solve this equation which are used in codes like LSP (CITE LSP). It turns out that there is an explicit solution given by the *Boris Rotation Algorithm*. If we define

$$v^{n+\frac{1}{2}} = v^+ + \frac{qE^n}{2m} \Delta t \quad (3.13)$$

$$v^{n-\frac{1}{2}} = v^- - \frac{qE^n}{2m} \Delta t \quad (3.14)$$

we can separate out the electric field dependence to get

$$\frac{v^+ - v^-}{\Delta t} = \frac{q}{m} [\frac{v^+ + v^-}{2} \times B] \quad (3.15)$$

which can conveniently be calculated through a rotation [58] through the following steps:

1. Compute v^- from eq. (3.14).
2. Compute $\vec{t} \equiv \frac{q\Delta t}{2m}\vec{B}^n$ (equivalent in magnitude to $\tan(\theta/2)$ where θ is the rotation angle)
3. Compute $\vec{s} = \frac{2\vec{t}}{1+t^2}$ (equivalent in magnitude to $\sin(\theta/2)$)
4. Compute $\vec{v}' = \vec{v}^- + \vec{v}^- \times \vec{t}$.
5. Compute $\vec{v}^{n+\frac{1}{2}}$ from eq. (3.13).

Now, the particles can be advanced or “pushed” from

$$x^{n+1} = x^n + v^{n+\frac{1}{2}}\Delta t \quad (3.16)$$

For completeness, the velocity initially needs to be pushed backwards from $v^0 \rightarrow v^{-\frac{1}{2}}$. This is not done in a time-centered way, but is only needed at the start of the simulation.

3.1.3 EPOCH Code

Clearly Top-hat shape functions should never be used for laser-solid simulations.

Background

Methods

Results

3.2 Machine Learning

3.2.1 Simple Models

Polynomial Regression

Support Vector Regression

3.2.2 Advanced Models

Gaussian Process

3.2.3 Neural Networks

Chapter 4

PARTICLE-IN-CELL SIMULATIONS OF ENHANCED TARGET NORMAL SHEATH ACCELERATION

This chapter details the work I did in conducting PIC simulations to better understand the Enhanced Target Normal Sheath Acceleration (eTNSA) mechanism that our group tried to demonstrate using LLNL’s Titan Laser in March of 2024. As a result, I will mostly focus on the simulation aspect, but include some relevant comparisons to the experiment.

4.1 Theory

4.1.1 Spatially Aligned Pulses

As explained in section 2.2.1, there is generally an optimal level of pre-expansion of the target to maximize the TNSA proton energies [53, 60]. While this could be attributed to a low contrast pulse, this could also be an artificially injected pre-pulse whose intensity and temporal delay are tunable. Robinson et. al. [61] first addresses the idea of using multiple high intensity 40 fs laser pulses with the first being one-tenth to one-quarter of the intensity of the second. They termed this novel two-stage process “multiple pulse sheath acceleration”. This study actually found a reduction in peak proton energy through numerical simulations, but did find the existence of “spectral peaks” – spikes in the energy spectrum at specific energies. A few years later, Markey et. al. [9] examined a similar setup experimentally, varying the temporal separation by 0.75-2.5 ps. Interestingly, they found an enhancement in the peak proton energy with a 0.75 ps separation and pulse energy ratio of 0.4:1. They conducted 1D PIC simulations shown in fig. 4.1 that verify this as well.

In 2018, Ferri et. al. [54] re-examines the same question, but uses the same intensity for both pulses. He finds that ultimately, little to no delay is optimal and as the delay gets larger, the enhancement reduces to the single pulse result. He finds the acceleration process can be affected by the second pulse for time delays as long as 0.6 ps for 3 μm targets and

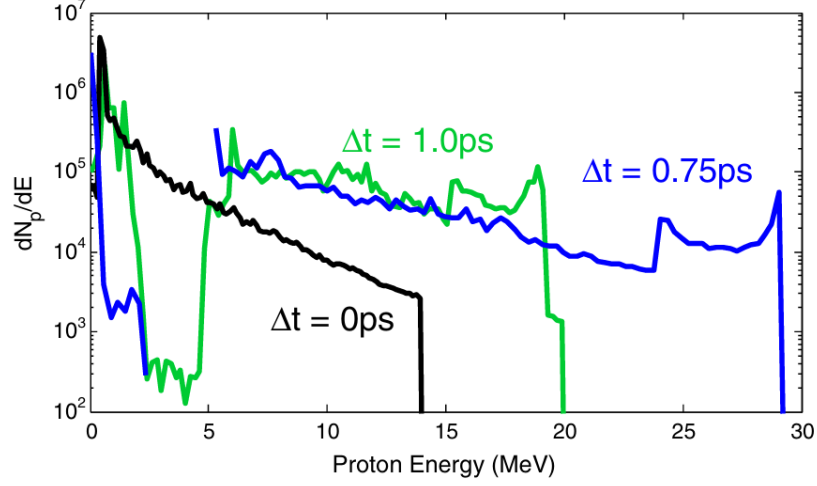


Figure 4.1: Proton energy spectrum from 1D PIC simulations depicted for three different temporal delays (including $\Delta t = 0$) from FIG. 3 in Markey et. al. [9].

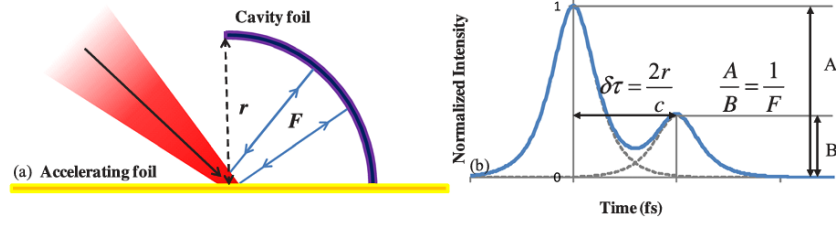


Figure 4.2: Schematic of the incident laser on the half-cavity target (left) in Scott et. al. [10]. The radius of the cavity foil determines the delay ($\tau = 2r/c$) between the main pulse and the post-pulse (reflected laser light of $\sim 40\%$) energy of the main pulse).

1 ps for $6\mu\text{m}$ targets.

In a different approach Scott et. al. [10] realized that much of the laser light that is reflected is wasted, so a mechanism to re-direct this light back into the target would be desired. They designed a target with a half-cavity foil as seen in fig. 4.2 that will take light reflected off the accelerating foil and re-reflect it back towards the accelerating foil. They find that this can increase the laser to proton energy efficiency by up to $\sim 55\%$.

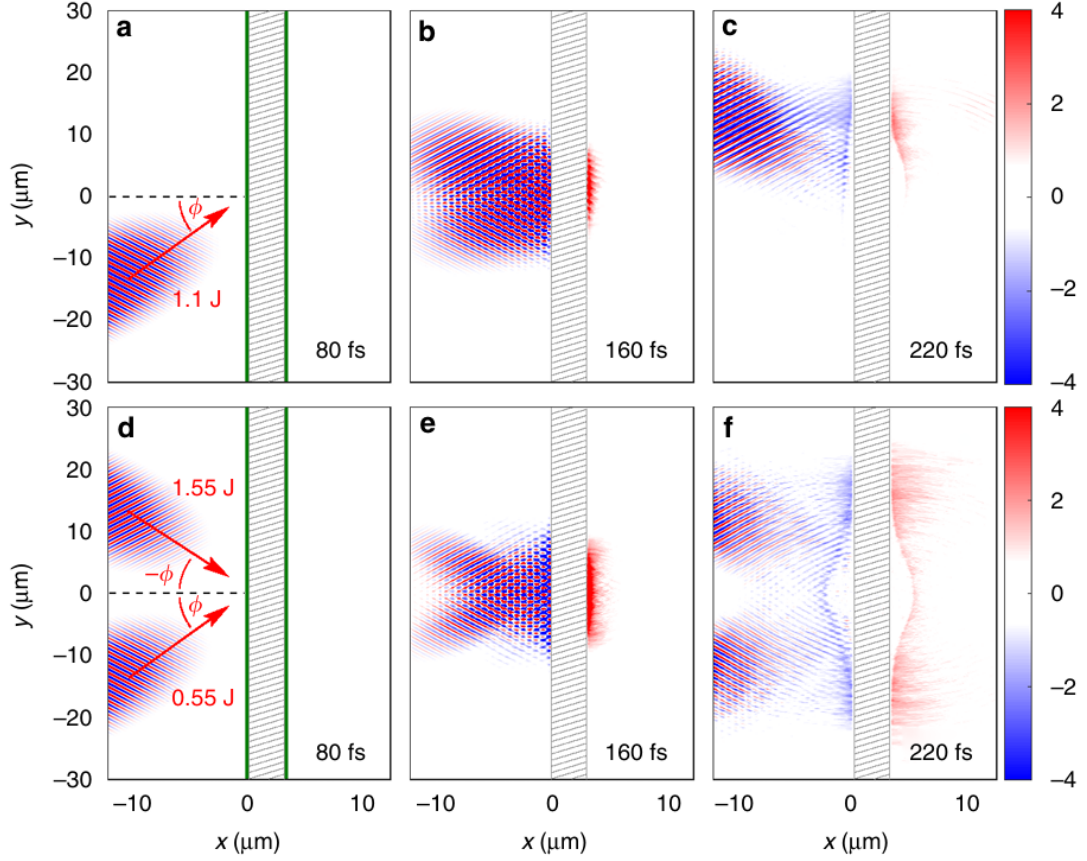


Figure 4.3: Geometry of the two pulse scheme as shown in FIG. 1 of Ferri et. al. (2019) [11]. The single pulse has a total energy of 1.1J at an incidence angle of $\phi = 45^\circ$ and is shown through several time snapshots (a-c). In (d-f), the double pulse is shown through those same time snapshots with energies of 0.55 J in each pulse (the 1.55 J is a typo from the original figure). Other parameters include $\tau_{\text{fwhm}} = 38$ fs, thickness = $3 \mu\text{m}$, material = aluminum, $w_{0,\text{fwhm}} = 5 \mu\text{m}$, $I_0 = 7 \times 10^{19} \text{ W cm}^{-2}$.

4.1.2 Spatially and Temporally Aligned Pulses

Ferri

Given the results of the previous section [9, 10, 54] that use short-delay pre or post pulses, Ferri decided to study how a spatially *and* temporally aligned pulse can enhance proton acceleration. Ferri, Siminos, and Fulop showed that the double pulse setup can result in an almost doubling of the proton energy and five-fold enhancement in the number of protons [11] through PIC simulations. This phenomena, referred to as eTNSA, is depicted in fig. 4.3 which shows the constructive interference of the fields at the front of the target in the center panel (e) and enhanced TNSA fields at the target rear (f).

The constructive interference of the electric fields can be understood by considering the electric field of two p-polarized pulses coming in at angles of incidence ϕ and $-\phi$

$$E_{x,1} = -E_0 \sin(\phi) \sin(ky \sin(\phi) - \omega t + kx \cos(\phi)) \quad (4.1)$$

$$E_{x,2} = -E_0 \sin(\phi) \sin(ky \sin(\phi) - \omega t - kx \cos(\phi)) \quad (4.2)$$

Adding these two fields together results in

$$E_{\text{double}} = -2E_0 \sin(\phi) \sin(ky \sin(\phi) - \omega t) \cos(kx \cos(\phi)) \quad (4.3)$$

which has an amplitude of $|E_{\text{double}}| = 2E_0 \sin(\phi)$. In contrast, a single pulse with twice the energy (intensity) would only have a $\sqrt{2}$ larger electric field ($|E_{\text{single}}| = \sqrt{2}E_0 \sin(\phi)$) which explains the enhanced electric fields seen in the bottom row of fig. 4.3. In an ideal situation where 100 % of the laser pulse is reflected, the angles of the two pulses do not need to be equal and opposite – we will get a standing wave pattern from the constructive interference of the incident wave with its reflection and the same $\sqrt{2}$ field enhancement.

Unlike the methods described in section 4.1.1 (which see enhanced proton acceleration from a pre-expanded target), eTNSA relies on the presence of an undisturbed target from the vacuum heating mechanism [5] explained in section 2.1.4. Vacuum heating relies on the dominance of the electron's quiver motion in the oscillatory electric field in the x-direction, but the magnetic field (which is relevant when $a_0 \gtrsim 1$) may impart a $\vec{v} \times \vec{B}$ Lorentz force (eq. (2.5)) that negatively affects the electron acceleration in the x-direction. Brunel even assumes in his original 1987 paper [5] that we can ignore $\vec{v} \times \vec{B}$ interactions if the laser is split into two equal and opposite angle pulses. Ferri confirms [11], through simulations, that the magnetic field is indeed suppressed in the double pulse case which results in hot electrons accelerated to higher energies.

Furthermore, Ferri performed some simulations that explored the effect of changing the preplasma scale length L at a fixed energy, and changing the total laser energy for the flat target [11] which can be seen in fig. 4.4. He finds that a higher preplasma scale length generally increases the max proton energy up to a certain scale length of around $L = 0.6 \mu\text{m}$. The enhancement is seen for all scale lengths, but the gap between single and double pulse diminishes for larger scale lengths. In the double pulse case, a more favorable scaling with laser energy is seen for both the hot electron temperature and maximum proton energy ($E_{0,\text{tot}}$ as opposed to $\sqrt{E_{0,\text{tot}}}$) when comparing double pulse against single. In addition, Ferri comments that, for pulses with total energy greater than 10 J, the proton layer on the rear side starts to entirely disconnect from the bulk of the target during the acceleration, causing proton energies to saturate and become lower than the expected linear scaling.

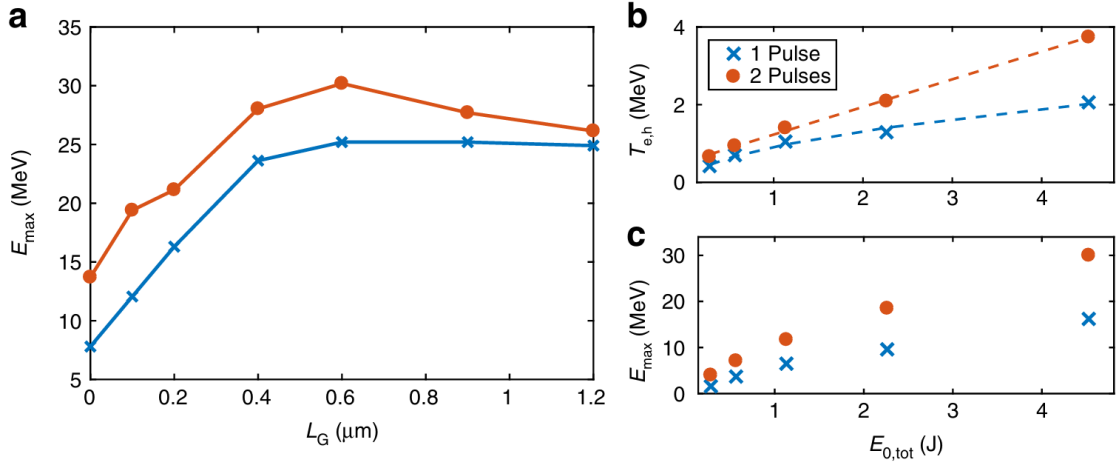


Figure 4.4: Double pulse effectiveness in terms of changing preplasma scale length (a) and total laser energy (b,c) from Ferri et. al. (2019) [11].

Other Simulations

In 2021, eTNSA was again demonstrated through simulations in Rahman et. al. [62] for a mJ class laser, around 2 orders of magnitude lower energy than the setup in Ferri’s work. This mJ class laser is based on the experimental facility at the Wright Patterson Air Force Base (see chapter 6 for more details) which utilizes a thinner target of $\sim 460 \text{ nm}$. Despite the major difference in laser and target parameters, the findings are similar – increased hot electron temperature and proton energies with double pulse compared to single pulse. However, Rahman finds that the presence of a preplasma actually *reduces* the maximum proton energy. But, like Ferri et. al., the gap between single and double pulse becomes smaller in the presence of a preplasma.

In 2024, Khan and Saxena [63] revisit the double pulse scheme but with an applied longitudinal kilo-Tesla level magnetic field. The purpose of the magnetic field is to reduce the divergence of the hot electron beam. This guides the electrons and enhances the TNSA process to see a higher maximum cutoff energy in the proton spectrum [64]. In this study, the laser was based off of the experimental set-up at the Rutherford Appleton Lab with a peak intensity of $5.5 \times 10^{20} \text{ W cm}^{-2}$ incident on a $7 \mu\text{m}$ polyethylene target.

Experiments

Due to multiple studies demonstrating eTNSA [11, 62, 63], experiments are needed to validate the PIC simulations. Morace et al. [65] showed that splitting a 270 J beam ($2.5 \times 10^{18} \text{ W cm}^{-2}$ peak intensity) into multiple “beamlets” on Al foil targets enhances

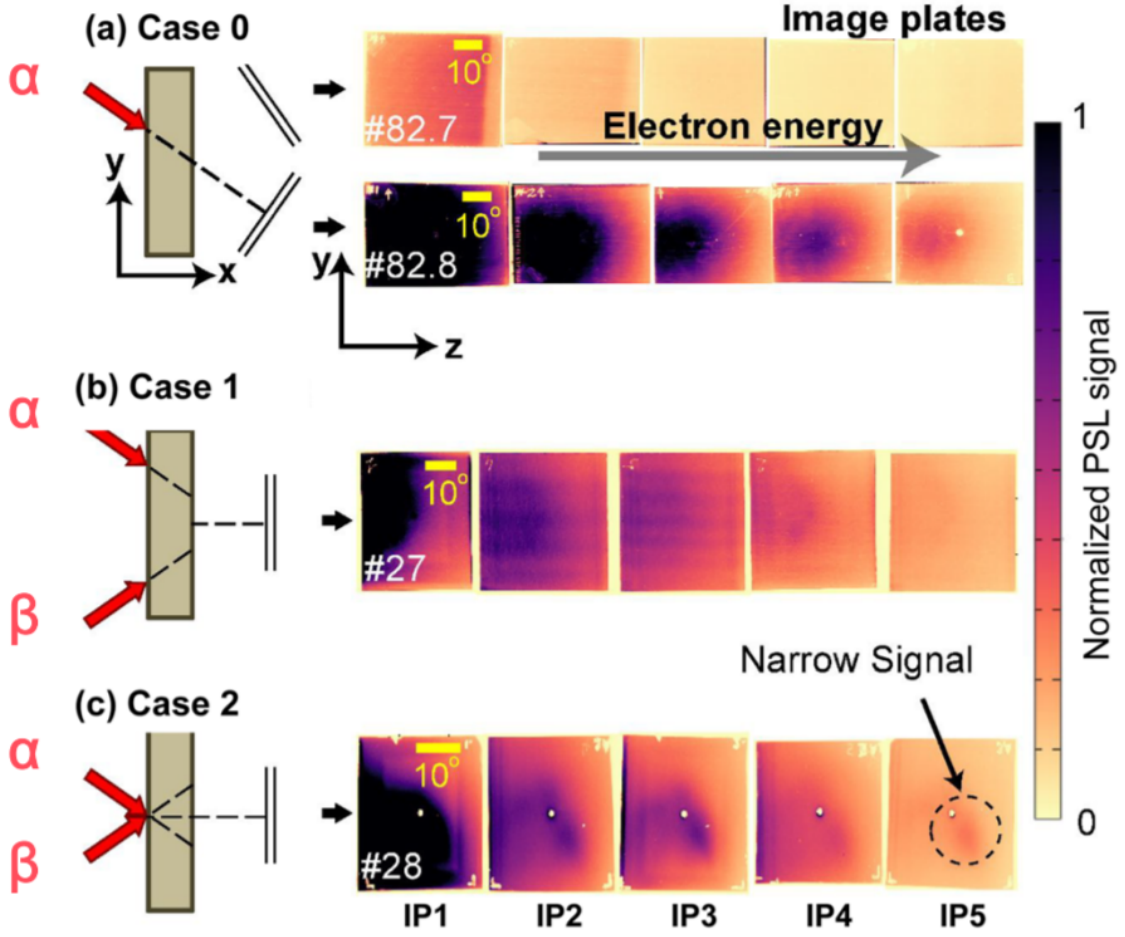


Figure 4.5: FIG. 2 from Yao et al. [12]. Case 0 shows electron signals from a single pulse. Case 1 shows electron signals from a double pulse with spatial separation $120\text{ }\mu\text{m}$. Case 2 shows electron signals from a double pulse with no spatial separation.

the proton energy spectrum. These beamlets (dependent on the incidence angle) induced critical surface density modulations that strongly improved absorption into hot electrons.

Yao et al. [12] investigated the double pulse effect by changing the transverse spatial separation between two temporally aligned pulses as shown in fig. 4.5. This figure shows two possible beams α and β which come in at equal and opposite incidence angles with α angled in the $-y$ direction. Comparing (b) and (c) shows an enhancement of the double pulse electron energy when the pulses are spatially overlapped. In (a), the electron energy is recorded along the laser beam axis in contrast to (b) and (c) and this shows that most the electrons are getting accelerated along the laser beam axis. This suggests that the $\vec{J} \times \vec{B}$ mechanism is taking place (in contrast to vacuum heating or resonance absorption which drives electrons in the target normal direction). In terms of the protons, case 2 (spatially

aligned double pulses) shows a more collimated central small beam which is absent in case 1 (spatially offset double pulses).

4.2 Results

Morace et. al. [65] was able to provide evidence ... Yao et. al. [12] was supported double pulse but didn't do a direct comparison with single pulse. Not enough experimental results out there, so we did an experiment ...

4.3 Discussion

Chapter 5

MACHINE LEARNING METHODS APPLIED TO SYNTHETIC ION ACCELERATION DATA

This chapter details the work I did developing and investigating a synthetic dataset based on a model by Fuchs [55].

5.1 Modified Fuchs et. al. Model

In this section, I will describe the model from which we generated the synthetic datasets [14, 66]. First, the expansion of a plasma into a vacuum [13] is used to determine the maximum proton energy and the number of accelerated protons per unit energy $\frac{dN}{dE}$. Following Fuchs [55], we introduce define the acceleration time in proportion to the pulse duration of the laser and adopt a scaling (e.g. eq. (2.33)) to relate hot electron temperature to the ponderomotive potential. This, in combination with other empirical estimates, allows calculating a proton energy spectrum from up to 7 parameters: main pulse intensity, contrast, wavelength, pulse duration, target thickness, target focal position, and laser spot size.

5.1.1 Plasma Expansion into a Vacuum

This model was developed by Mora [13] in 2003 who built off of earlier efforts [33, 34] in examining an isothermal expansion model. The model begins with the assumption that ions are contained in the semi-infinite interval $n_i = n_{i0}$ for $x < 0$ and no ions initially in the vacuum region for $x > 0$. The electrons are distributed according to the boltzmann relation given by eq. (2.20) where $n_{e0} = n_e(x = -\infty)$ is the electron density in the unperturbed plasma. Through this relation, $\phi(-\infty) = 0$. The initial electron density is related to the ion density $n_{e0} = Zn_{i0}$ where Z is the ion charge number for a fully ionized plasma. The potential also satisfies the Poisson equation eq. (2.4) where $\rho/m = -e(n_e - Zn_i)$ is the

mass density of the electrons. The solution of eq. (2.4) at $t = 0$ is found by integration [33] (where $E \equiv -\frac{d\phi}{dx}$) as

$$\frac{1}{2}\epsilon_0 E^2 = n_{e0}k_B T_e \begin{cases} \exp(\frac{e\phi}{k_B T_e} - 1 - \frac{e\phi}{k_B T_e}) & \text{if } x < 0 \\ \exp(\frac{e\phi}{k_B T_e}) & \text{if } x > 0 \end{cases} \quad (5.1)$$

From enforcing continuity of eq. (5.1) at $x = 0$ (the location of the ion front initially) we determine $\phi = -k_B T_e / e$ to arrive at

$$E_{front,0} = \sqrt{\frac{2}{\exp(1)}} E_0 \quad (5.2)$$

where $E_0 \equiv \sqrt{n_{e0}k_B T_e / \epsilon_0}$. To get an estimate of the electric field at the ion front when $t > 0$ we need to consider what the characteristic time scale for ion motion is: the plasma ion frequency $\omega_{p,i}$

$$\omega_{p,i} \equiv \sqrt{\frac{Z n_{e0} e^2}{m_i \epsilon_0}} \quad (5.3)$$

which is analogous to eq. (2.15). So, in relation to the time-scale of plasma ion oscillations, a long time would refer to $\omega_{p,i} t \gg 1$. The ion fluid sound speed c_s is given by

$$c_s = \sqrt{\frac{Z k_B T_e}{m_i}} \quad (5.4)$$

and which is very similar to eq. (2.16). Using the definition of the Debye length (eq. (2.17)) and sound speed c_s we can re-express eq. (5.3) as

$$\omega_{p,i} t = \sqrt{\frac{Z k_B T_e}{m_i}} \sqrt{\frac{n_{e0} e^2}{\epsilon_0 k_B T_e}} t = (c_s t) (\lambda_{D0}) \quad (5.5)$$

where λ_{D0} is the initial Debye length and c_s is the ion sound speed. As we know from chapter 2, when λ_D is smaller than the characteristic length scale of a system, the quasi-neutrality condition for a plasma is satisfied. In this case, the length scale would be $c_s t$ and we can show that asserting the condition $\omega_{p,i} t > 1$ is equivalent to $\lambda_D < c_s t$. We can continue by incorporating equations of continuity and the Lorentz force (eq. (2.5)) which can be expressed as

$$\frac{\partial n_i}{\partial t} + v_i \frac{\partial n_i}{\partial x} = -n_i \frac{\partial v_i}{\partial x} \quad (5.6a)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{Ze}{m_i} \frac{\partial V}{\partial x} \quad (5.6b)$$

This set of fluid equations can be solved numerically with the initial conditions for n_i , E , and $v_i = 0$, but it is more instructive to consider a “self-similar solution” that describes the ions moving with speed

$$v_i = c_s + x/t \quad (5.7)$$

for $x + c_s t > 0$. It is self-similar in the sense that the specific length and time scales are not important, only their ratio x/t . In this self-similar region, quasi-neutrality is maintained and the expanding electron density can be expressed as

$$n_e = Z n_i = n_{e0} \exp\left(-\frac{x}{c_s t} - 1\right) \quad (5.8)$$

By combining eqs. (5.6a), (5.6b), (5.7) and (5.8), we can arrive at a solution for the self-similar electric field in this quasi-neutral region

$$E_{SS} = \frac{m_i c_s}{Z e t} = \frac{k_B T_e}{e c_s t} = \frac{E_0}{\omega_{p,i} t} \quad (5.9)$$

Physically, we can interpret this as a sheet of positive charge $\sigma = \epsilon_0 E_{SS}$ at $x = -c_s t$ and a sheet of negative charge $-\sigma$ at the plasma edge. The location of this plasma edge (i.e. the location of the ion front) can be roughly obtained by equating the local Debye length $\lambda_D = \lambda_{D0} \sqrt{n_{e0}/n_e}$ to the scale length $c_s t$.

$$x_{i,front} = c_s t [2 \ln(\omega_{p,i} t) - 1] \quad (5.10)$$

and the ion velocity at the front can also be obtained

$$v_{i,front} = 2 c_s \ln(\omega_{p,i} t) \quad (5.11)$$

The ion velocity can plug back into eq. (5.6b) to find out that $E_{front,SS} = 2E_{SS}$. Mora found an approximate solution to E_{front} that matches $E_{front,0}$ and $E_{front,SS}$ in their respective cases ($t = 0$ and $\omega_{p,i} t \gg 1$) as

$$E_{front} \simeq \frac{2E_0}{\sqrt{2 \exp(1) + (\omega_{p,i} t)^2}} \quad (5.12)$$

This formula not only reaches the correct values in the limiting cases, but also effectively interpolates in the intermediary regions (i.e. $\omega_{p,i} t \sim 1$) when compared to a numerical code that solves eqs. (5.6a) and (5.6b) without assuming a self-similar solution. In fig. 5.1a, we see the net charge density at some time $\omega_{p,i} t = 50$ after the start of a 1D plasma expansion simulation. We can identify the -2σ with the fastest expanding electrons and the $+\sigma$ region next to it as the positive ions getting pulled along. In fig. 5.1b, we can see the electric field between these two charged regions peaks $\simeq 2E_{ss}$. Then, using this formula with eq. (5.6b), we can determine the ion front velocity as

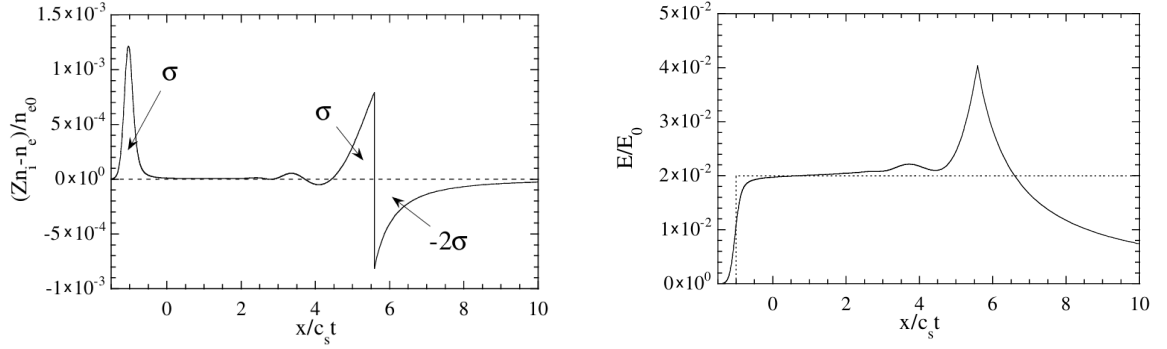


Figure 5.1: The net charge density (left) as a function of position $x/c_s t$ and normalized electric field E/E_0 (right) for $\omega_{pi}t = 50$ taken from Fig 1 and 2 in Mora's Paper [13]. On the right, the self-similar electric field from eq. (5.9) is plotted with a dashed line.

$$v_{i,front} = 2c_s \ln(\tau + \sqrt{\tau^2 + 1}) \quad (5.13)$$

where we've defined a normalized acceleration time $\tau \equiv \omega_{p,i}t/\sqrt{2\exp(1)}$. Additionally, in the limit $\omega_{p,i}t \gg 1$, eq. (5.10) becomes

$$x_{i,front} \simeq c_s t [2 \ln(\omega_{p,i}t) + \ln(2) - 3] \quad (5.14)$$

The per-ion kinetic energy can now be calculated as

$$\begin{aligned} \mathcal{E} &\equiv \frac{1}{2} m_i v_{i,front}^2 = 2m_i c_s^2 \ln(\tau + \sqrt{\tau^2 + 1})^2 \\ &= 2Zk_B T_e \ln(\tau + \sqrt{\tau^2 + 1})^2 \end{aligned} \quad (5.15)$$

Using eq. (5.8), we can determine the number of accelerated ions between $x = -c_s t$ and $x = x$ as

$$N_i \equiv \int_{-c_s t}^x n_i(x') dx' = n_{i0} c_s t [1 - \exp(-\frac{x}{c_s t} - 1)] \quad (5.16)$$

and using eq. (5.7), we can show that this is equivalent to

$$N_i(x) = n_{i0} c_s t [1 - \exp(-\sqrt{\frac{2\mathcal{E}}{\mathcal{E}_0}})] \quad (5.17)$$

where $\mathcal{E}_0 \equiv Zk_B T_e$. Now that the number of ions is expressed in terms of the energy \mathcal{E} , we can determine the number of accelerated ions per unit energy (per unit surface) as

$$\frac{dN}{d\mathcal{E}} = \frac{n_{i0}c_s t}{\sqrt{2\mathcal{E}\mathcal{E}_0}} \exp\left(-\sqrt{\frac{2\mathcal{E}}{\mathcal{E}_0}}\right) \quad (5.18)$$

5.1.2 Modified Fuchs Model

When $\tau \rightarrow \infty$, eq. (5.15) diverges to ∞ . This is an inherent limitation of the isothermal fluid model, and different models are able to avoid this issue [42, 67, 68]. However, a simple fix to this model involves assuming that this acceleration time is finite and proportional to the pulse duration. Physically, it makes sense that the protons are only getting accelerated on the timescale of laser-target interactions. This is the approach taken by Fuchs [55] and he expresses eq. (5.15) as

$$E_{\max} = 2k_B T_h [\ln(t_p + \sqrt{t_p^2 + 1})]^2 \quad (5.19)$$

where $\tau \equiv \omega_{p,i} t_{\text{acc}} / \sqrt{2 \exp(1)}$ just like the Mora model. We've also set $Z = 1$ to signify that we are looking for hydrogen ions (i.e. protons) The crucial difference is that we express the acceleration time as

$$t_{\text{acc}} \approx 1.3 \tau_{\text{FWHM}} \quad (5.20)$$

One can assume that the absorption fraction of hot electrons η (with respect to the total laser energy E_L) is given by $\eta_e = 1.2 \times 10^{-15} I^{0.74} \text{ W cm}^{-2}$ with a maximum of 0.5, determined from empirical scalings (e.g. see fig. 3 from Key [69]). Additionally, the average energy of the hot electrons is set by the Wilks scaling eq. (2.33). Putting this together,

$$N_e = \eta_e \frac{E_L}{T_h} \quad (5.21)$$

would be the total number of hot electrons accelerated into the target. These electrons spread out in a roughly cylindrical volume of area S_{sheath} and length $c\tau_{\text{fwhm}}$ where the circular sheath cross section can be estimated by $S_{\text{sheath}} = \pi(r_0 + d \tan(\theta))^2$. Here, $r_0 = w(x) \frac{\sqrt{2 \ln(2)}}{2}$ is half of the (spatial) full width at half maximum of the intensity distribution at position x . The effective radius of the sheath has an additional factor of $d \tan(\theta)$ where d is the initial target thickness and θ is the half-angle divergence of the hot electron within the target (taken as $\theta = 25^\circ$). As a result, the hot electron number density can be expressed as

$$n_{e0} = \frac{N_e}{c\tau_{\text{fwhm}} S_{\text{sheath}}} \quad (5.22)$$

With an estimate of the hot electron density, the proton spectrum can now be computed from eq. (5.18) as

$$\frac{dN}{dE} = N_0 \frac{\exp(-\sqrt{2E/k_B T_h})}{\sqrt{2E k_B T_h}} \quad (5.23)$$

where $N_0 \equiv n_{e0} c_s t_{\text{acc}} S_{\text{sheath}}$ is defined for convenience. Using a dimensionless scale for energy $\varepsilon \equiv \sqrt{2E/k_B T_h}$, we can calculate the number of protons and total energy in protons through integrating eq. (5.23)

$$N = N_0 (\exp(-\varepsilon_{\min}) - \exp(-\varepsilon_{\max})) \quad (5.24)$$

$$E_{\text{tot}} = N_0 \frac{k_B T_h}{2} [\exp(-\varepsilon_{\min})(2 + \varepsilon_{\min}(2 + \varepsilon_{\min})) - \exp(-\varepsilon_{\max})(2 + \varepsilon_{\max}(2 + \varepsilon_{\max}))] \quad (5.25)$$

where $\varepsilon_{\min} = \sqrt{2E_{\min}/k_B T_h}$ defines a minimum energy cutoff (ε_{\max} is analogous and chosen by eq. (5.19)). Furthermore, we can calculate the average proton energy by dividing eq. (5.24) from eq. (5.25)

$$E_{\text{avg}} \equiv \frac{E_{\text{tot}}}{N} \quad (5.26)$$

The combination of eqs. (5.19) and (5.23) have been tested across many of the early TNSA experiments of the early 2000s for a wide range of laser intensities and pulse durations with good accuracy (see fig. 4 from Fuchs [55]).

5.1.3 Further Model Modifications

When restricted to a particular laser system, the wavelength, pulse duration, and spot size are fixed. Considering the model in section 5.1.2, only three adjustable parameters would be of interest – target thickness d , peak intensity I_0 and target focal position x . To introduce complexity into our model, we wanted to consider the effect that a pre-expanded target would have on the proton acceleration. The pre-expansion may enhance the hot electron generation, but expansion on the rear side of the target would reduce the effectiveness of the TNSA process. We incorporate this effect by allowing the laser to have a finite contrast κ which relates the intensity of the main laser pulse I_0 to the intensity of a secondary laser pulse I_{pre} as $I_{\text{pre}} = \kappa I_0$. This pre-pulse is treated as a spike in intensity that occurs t_0 before the arrival of the main pulse. The pre-expanded target would have a new effective thickness given by

$$d_{\text{eff}} = d + 2c_s t_0 \quad (5.27)$$

where c_s is the ion sound speed from eq. (5.4) in which the target is expanding outwards from both sides. Here T_e is the temperature due to the pre-pulse and can be calculated by

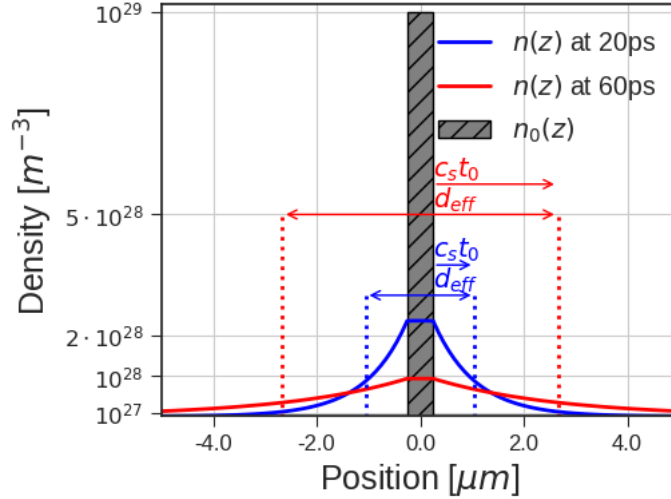


Figure 5.2: The electron density profile of the pre-expanded target is depicted for various times t_0 . In this figure, $n(0) \equiv n_{\max}$. Taken from Desai et al. [14] where z was used as the distance along the laser axis instead of x as done in this work.

assuming that $T_e \propto I$ and that an intensity of $10^{12} \text{ W cm}^{-2}$ produces electron temperatures of $T_{\text{pre},0} = 50 \text{ eV}$. Since n_e decreases as d gets larger and $\omega_{p,i} \propto \sqrt{n_e}$, eq. (5.19) is inversely proportional to the target thickness. So, a larger prepulse with a longer time to expand t_0 will see a higher effective target thickness. Furthermore, when the target is off focus, the effective pre-pulse intensity on target is less which results in less expansion.

In addition, some of the main pulse energy can be depleted by traveling through the underdense region of this new pre-expanded target. These effects will be referred to as *pump depletion* and are inspired by arguments from Decker [70]. Decker describes pump depletion as an “etching” process where traveling through the plasma causes wavefront edge to recede at a speed given by the “etching velocity”

$$v_{\text{etch}} = (\omega_{p,e}/\omega)^2 c \quad (5.28)$$

Note that this speed continuously changes throughout the exponential-scale electron density which falls off like $n \sim \exp(-x/c_s t_0)$ on both sides of the target (see fig. 5.2 for a visual). Due to conservation of particle number, if the target expands, the maximum density n_{\max} will also lower and is given by $n_{\max} = \frac{n_{e0} d}{d_{\text{eff}}}$. We can integrate v_{etch} with respect to time, but it is more convenient in terms of the position since we know the range over which the under-dense plasma exists. The plasma edge x_f is given by eq. (5.14) and we will integrate up to the location of the critical density $x_0 = c_s t_0 (\ln(n_{\max}) - \ln(n_c))$. Utilizing the change of variables $dx = c dt$ (due to the pulse traveling at the speed of light c), the “etching distance” can be calculated as [14]

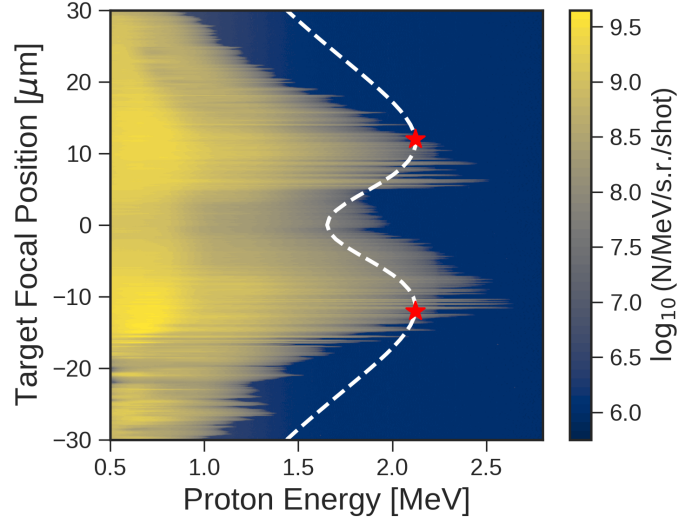


Figure 5.3: The dotted black line shows the maximum proton energy predicted by eq. (5.19) with the pump depletion considerations in section 5.1.3 assuming $t_0 = 60$ ps, $I_0 = 10^{19}$ W cm $^{-2}$, $\kappa = 10^{-7}$, $d = 0.5$ μ m. The red stars indicate the predicted positions of maximum proton energy ~ 12 μ m. This plot is overlayed on top of an experimental maximum proton energy distribution from Morrison et. al. [15]. This figure is taken from Desai et. al. [14].

$$L_{\text{etch}} \equiv \int_{x_0}^{x_f} v_{\text{etch}} \frac{1}{c} dx = \frac{e^2 n_{\text{max}} c_s t_0}{\epsilon_0 m_e \omega^2} \left(\exp\left(-\frac{x_0}{c_s t_0}\right) - \exp\left(-\frac{x_f}{c_s t_0}\right) \right) \quad (5.29)$$

Finally, this etching reduces the effective pulse duration by

$$\tau_{\text{fwhm,eff}} = \tau_{\text{fwhm}} \left(1 - \frac{L_{\text{etch}}}{c \tau_{\text{fwhm}}} \right) \quad (5.30)$$

This model, however, is not without its flaws. First, our calculations assume a critical density of a tenth of the amount of the actual critical density. Second, instead of defining $d_{\text{eff}} = d_0 + 2x_0$ (which would be the true effective density that remains above critical density), we substitute it with eq. (5.27). Third, we modify the multiplier seen in eq. (5.20) from 1.3 to 25 which is a significant departure. Finally, the proportionality $T_e \propto I$ with $T_{\text{pre},0} = 50$ eV is chosen arbitrarily. Despite these drawbacks, we obtain model predictions similar to what is seen in fig. 5.3 to account for the maximum proton energy dip at peak focus. The goal of creating this model modification is to add complexity to the underlying physics for the purposes of evaluating the effectiveness of machine learning models, not to invent new physics.

5.2 Results

5.2.1 First Analysis

5.2.2 Second Analysis

5.3 Discussion

Chapter 6

OPTIMIZATION AND CONTROL OF A kHz LASER SYSTEM

Experimental Layout: CCD covered with aluminum which stops low energy protons and ions of lower charge to mass ratio (need to confirm this is true)

6.1 Background

6.2 Methods

6.3 Discussion

Chapter 7

CONCLUSION

7.1 Summary

7.2 Future Work

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Appendix A

ENERGY CONSERVATION IN EPOCH PARTICLE-IN-CELL SIMULATIONS DUE TO FINITE NUMBERS OF PARTICLES

This appendix focuses on unpublished work that was done jointly with Ricky Oropeza and Joseph Smith. My contributions to this project were primarily done as a pre-candidacy student.

A.1 Background

Particle-in-Cell (PIC) simulations provide a useful but imperfect model of various plasma phenomena. In this work, the impact of the finite number of particles in a PIC simulation on the energy conservation is considered and explored through ultra-intense laser interactions with a thin, near solid density target in the Target Normal Sheath Acceleration (TNSA) regime.

One

I have tried to consider the Hockney calculations regarding the E-Field Fluctuations, but I did not fully examine how this would turn into a heating model. Here, I will finish the calculation. In the Hockney 1971 paper, they state a formula for the time-averaged squared electric field fluctuations (which comes from Plasma Kinetic Theory by Montgomery and Tidman Ch. 8.2)

$$\frac{\langle E^2 \rangle}{8\pi} = \frac{k_B T}{2} \int \int_{-\infty}^{+\infty} \frac{dk_x dk_y}{(2\pi)^2} \frac{1}{(1 + (k_x^2 + k_y^2)\lambda_D^2)} \quad (\text{A.1})$$

which is in units of energy per unit volume. We can convert this integral to polar coordinates by letting $k = \sqrt{k_x^2 + k_y^2}$ and integrating with respect to $k \cdot dk \cdot d\phi$. Also, we can convert the expression from CGS to SI Units

$$\frac{\epsilon_0 \langle E^2 \rangle}{2} = \frac{k_B T}{4\pi} \int_0^\infty \frac{k dk}{1 + k^2 \lambda_D^2} \quad (\text{A.2})$$

To solve this integral, we let $u \equiv k \lambda_D$ to get

$$\langle E^2 \rangle = \frac{k_B T}{2\pi \epsilon_0 \lambda_D^2} \int_0^\infty \frac{u du}{1 + u^2} \quad (\text{A.3})$$

which diverges at infinity, so we set limits to the integral 0 and u_{max} which correspond to the maximum relevant wavenumber in our simulation. This can be solved analytically as

$$\langle E^2 \rangle = \frac{k_B T}{4\pi \epsilon_0 \lambda_D^2} \text{Log}(1 + u_{max}^2) \quad (\text{A.4})$$

Here, $u_{max} = 2\pi \frac{\lambda_D}{\Delta x} R$ where R is some multiplicative factor that tells us where to implement our cutoff. $R = 1$ corresponds to saying that the maximum wavenumber considered is limited the the cell size: $k_{max} = \frac{2\pi}{\Delta x}$.

Also, from the Hockney 1971 Paper, they give an estimate of how the squared electric field fluctuations contribute to the kinetic energy increase of one macro particle

$$\Delta KE = \frac{1}{2} m \langle \Delta v^2 \rangle = \frac{q^2}{m} \langle E^2 \rangle \tau_{corr} t \quad (\text{A.5})$$

where we can identify $\tau_{corr} = \frac{2\pi}{\omega_{pe}}$ with the plasma period and we already have $\langle E^2 \rangle$. Additionally, we can express the charge of one macroparticle as $q = e \frac{N}{N_{mac}} = \frac{en}{n_{mac}} = \frac{en \Delta x^2}{n_{ppc}}$. Combining this all together, we will get

$$\Delta KE = \left(\frac{e}{m_e} \right) \frac{en \Delta x^2}{n_{ppc}} \frac{k_B T}{4\pi \epsilon_0 \lambda_D^2} \text{Log}(1 + u_{max}^2) \frac{2\pi}{\omega_{pe}} t \quad (\text{A.6})$$

now, we can substitute the expressions for the plasma oscillation frequency and debye length to arrive at

$$\Delta KE = \frac{n_e^{3/2} \Delta x^2}{n_{ppc}} \text{Log}(1 + u_{max}^2) \sqrt{\frac{e^6}{4m_e \epsilon_0^3}} \quad (\text{A.7})$$

From here, I can do the same procedure I did with the arber formula and let $n \rightarrow 10^{29} n_{23}$, $\Delta x \rightarrow 10^{-9} \Delta x_{nm}$, and $t \rightarrow 10^{-12} t_{ps}$ to get

$$\frac{\Delta T_{eV}}{\Delta t_{ps}} = (1.07 \times 10^7 eV) \frac{n_{23}^{3/2} \Delta x_{nm}^2}{n_{ppc}} \text{Log}(1 + u_{max}^2) \quad (\text{A.8})$$

which, to me, does not line up with the heating constant we see in the Arber formula, so I will just call the constant out front C_H . Additionally, I will add another hyper-parameter that multiplies the debye ratio called R and when $R = 1$, we would arrive at the final formula:

$$\frac{\Delta T_{eV}}{\Delta t_{ps}} = C_H \frac{n_{23}^{3/2} \Delta x_{nm}^2}{n_{ppc}} \text{Log}(1 + 4\pi^2 (R \frac{\lambda_D}{\Delta x})^2) \quad (\text{A.9})$$

which is identical to the Arber-Model, with the exception of the Log Term. When the cell size resolves the debye length ($\lambda_D > \Delta x$), the log term is very slowly growing as Δx gets smaller, but this is overshadowed by the Δx^2 term outside of the Log. As a result, this would not behave much differently than the Arber formula when the cell size is small, which is in the regime where there is not much numerical heating anyway.

When the Debye Length is very poorly under-resolved ($\lambda_D \ll \Delta x$), we can simplify the expression as

$$\frac{\Delta T_{eV}}{\Delta t_{ps}} = C_H \frac{n_{23}^{3/2} \Delta x_{nm}^2}{n_{ppc}} 4\pi^2 (R \frac{\lambda_D}{\Delta x})^2 = 4\pi^2 R^2 C_H \frac{n_{23}^{3/2}}{n_{ppc}} \lambda_{D,nm}^2 \quad (\text{A.10})$$

which is now no longer dependent on the cell size Δx . This is consistent with our observation that after a certain cell size is reached, the amount of heating becomes relatively constant.

Two

I read the Hockney papers to understand where the heating time formulas come from which are used by Arber for his formula. I realized that there is no particular reason why $(\lambda_D/\Delta x)$ has to be squared, so we should allow the possibility that it may not be squared. Maybe, this relationship breaks down when the simulation gets sufficiently under-resolved. Regardless, I decided to run some simulations that would allow me to explore how effective such a model to be. Let us assume, from the Arber Paper, that the heating time is only dependent on the dimensionless debye ratio and the inverse of the plasma frequency.

$$\tau_H \sim \frac{n_{ppc}}{\omega_{pe}} (\lambda_D/\Delta x)^\alpha \quad (\text{A.11})$$

The two other dimensionless parameters are $\omega_{pe}\Delta t$ and $v_{th}\Delta t/\Delta x$ which we are going to ignore for now, but if necessary could be included in a more advanced model. From my previous sims, I am convinced that the dependence on n_{ppc} is correct, so I will be leaving that as a constant. This would correspond to an Arber-like formula of

$$\frac{dT_{eV}}{dt_{ps}} = C_H T_{0,eV}^{1-\alpha/2} \Delta x_{nm}^\alpha n_{23}^{(\alpha+1)/2} \quad (\text{A.12})$$

where for a spline shape function, $C_H = \alpha_H/n_{ppc} = 2/25 = 0.08$. Here, I assumed that $n_{ppc} = 25$ which is what I used for the sims in the next section. I want to test this model for a variety of different ratios.

Three

In our PIC simulation, we compute the velocities at the next timestep v_{i+1} by taking the velocity at the previous timestep v_i and modifying it by some correction term that is proportional to the timestep length Δt . However, due to the fact that we use finite grid, approximate real particles with macro particles, use a finite time-step, errors in rounding, use of finite-difference, etc. we will develop some errors in our calculation of the (let's just consider electric) field δE . This would correspond to miscalculation in the force by $\delta F = e\delta E\Delta t$ which would deliver an impulse $m\delta v$. As a result, our error in calculating the velocity will be

$$\delta v = \frac{e}{m}\Delta t\delta E \quad (\text{A.13})$$

Now, we can make an assumption that these errors in the calculation of fields will be distributed in a random fashion, so that we can treat each velocity deviation as a random walk in velocity space. If we consider Δv as the total deviation of the calculated velocity from the true value, we should expect $\langle \Delta v \rangle = 0$ due to the symmetry of this random walk. However, the squared deviations on average will increase over time (cite wikipedia). If we are considering n time-steps (each with the same random error δE), we would have

$$\langle \Delta v^2 \rangle = n\delta v^2 = n\frac{e^2}{m^2}\Delta t^2\delta E^2 \quad (\text{A.14})$$

Additionally, the average change in kinetic energy per particle would be given by

$$\langle h \rangle = \frac{1}{2}m\langle (v_0 + \Delta v)^2 \rangle - \frac{1}{2}m\langle v_0^2 \rangle = \frac{1}{2}m(2v_0\langle \Delta v \rangle) + \frac{1}{2}m\langle \Delta v^2 \rangle = \frac{1}{2}m\langle \Delta v^2 \rangle \quad (\text{A.15})$$

due to $\langle \Delta v \rangle = 0$. We can clearly see that this means that the average kinetic energy per particle will increase linearly with the number of time-steps n . And, in fact, this is exactly what we see in PIC sims.