

MA7008 – Financial Mathematics

Coursework 2023/24

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The goal of this case study is to statistically analyze a portfolio consisting of randomly selected stocks based on financial mathematics concepts so that a potential investor can be advised to choose the most efficient portfolio.

1. Initially, any 5 random stocks were selected from the UK stock market and their historical prices were downloaded from yahoo finance. It is important that these prices date back sufficiently in order to yield positive average returns and also, they should be of a suitable length of at least a year and hence, these prices were downloaded from 11/06/2015 to 18/11/2016.

The 5 companies included in the portfolio are:

AstraZeneca: It is a worldwide pharma giant and does business on a large scale in UK. It employs around 8700 people of UK and supports 66000 jobs in UK.

Unilever: A company which basically manufactures household items has a strong presence in the UK market. It is spread across 400 brands and 190 countries in the world.

Rio Tinto: It is company that provides innovative solutions to find the materials that people need in daily lives so that there is minimal damage to the environment. It is large company employing over 52000 employees across the world.

HSBC: It is a banking heavy-weight which provides banking facilities to people of over 50 countries and helps them achieve their monetary objectives.

Diageo: It is a leading beverage company of UK and has a strong hold on the UK beverage market. Founded in 1997, it is a relatively young company compared to the other four but it is a global giant in premium drinks.

The adjusted close prices of all these stocks are placed in one excel worksheet along with the date. The next step is to calculate the daily returns of each stock which is done in the following manner:

H3	✓	:	✕	✓	f_x	=(B3-B4)/B4						
	A	B	C	D	E	F	G	H	I	J	K	L
1			Prices							Returns		
2	Date	AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo		AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo
3	18/11/2016	3466.907	2460.067	2006.724365	437.506958	1723.00049		-0.0038018	0	-0.0288	0.002848	0.000987
4	17/11/2016	3480.1377	2460.067	2066.229004	436.264404	1721.30188		-0.0031009	-0.00272	0.019403	0.005409	0.012235
5	16/11/2016	3490.9629	2466.77	2026.901245	433.917389	1700.49683		-0.0018345	-0.01449	-0.01348	-0.0152	0.007547
6	15/11/2016	3497.3787	2503.039	2054.601318	440.613342	1687.75903		-0.000916	0.013731	-0.04499	0.005514	0.008883
7	14/11/2016	3500.5852	2469.135	2151.381348	438.197266	1672.89844		0.0091308	-0.01011	0.024927	0.024036	-0.00631
8	11/11/2016	3468.9114	2494.366	2099.058594	427.911926	1683.51331		-0.0178228	-0.0055	-0.02494	-0.00864	-0.00502
9	10/11/2016	3531.8589	2508.165	2152.749023	431.639465	1692.00488		-0.0373729	-0.03926	0.025746	0.004821	-0.03604
10	09/11/2016	3668.9792	2610.667	2098.716309	429.568634	1755.26929		0.0280867	-0.02099	0.068419	0.00371	-0.01148
11	08/11/2016	3568.7451	2666.649	1964.319214	427.980957	1775.6499		0.0045143	0.006398	0.016637	-0.0037	0.00917
12	07/11/2016	3552.7073	2649.698	1932.17334	429.568634	1759.5155		0.0143087	0.012809	0.035747	0.046234	0.006558
13	04/11/2016	3502.5898	2616.187	1865.488037	410.58551	1748.05139		-0.0149962	-0.01103	-0.02188	-0.00435	-0.0058
14	03/11/2016	3555.915	2645.361	1907.209229	412.380402	1758.2417		-0.0201083	-0.01728	-0.01466	-0.01125	-0.02907
15	02/11/2016	3628.8857	2691.881	1935.593384	417.074432	1810.89111		0.002992	-0.00857	-0.00492	-0.01355	-0.0125
16	01/11/2016	3618.0605	2715.141	1945.168213	422.803772	1833.81934		-0.0165647	0.00746	0.001585	-0.006	-0.00918
17	31/10/2016	3679.0022	2695.035	1942.090698	425.357758	1850.80273		-0.0062812	-0.00712	0.002825	-0.01329	-0.00366
18	28/10/2016	3702.2568	2714.353	1936.619019	431.087158	1857.59644		0.0005416	0.001163	0.012516	-0.00478	-0.00069
19	27/10/2016	3700.2527	2711.199	1912.680664	433.158112	1858.87024		-0.0340171	0.004206	0.00197	0.008032	-0.00046
20	26/10/2016	3830.5571	2699.845	1908.919189	429.706726	1859.7196		-0.0071703	-0.00705	-0.00179	-0.00607	-0.00613
21	25/10/2016	3858.2217	2719	1912.339111	432.329834	1871.18311		-0.0081427	0.001728	0.045039	0.003204	0.011708
22	24/10/2016	3889.8958	2714.31	1829.922119	430.949249	1849.52905		-0.0203958	-0.0043	0.002623	-0.00335	-0.00023
23	21/10/2016	3970.8853	2726.038	1825.134399	432.398743	1849.95398		-0.0051231	-0.00072	0.019095	0.007884	-0.00525
24	20/10/2016	3991.3333	2727.993	1790.936523	429.016357	1859.7196		0.0074891	0.006201	-0.00191	0.007851	0.004357
25	19/10/2016	3961.6638	2711.182	1794.356323	425.674255	1851.6521		-0.012295	-0.00801	0.003059	0.00289	0.003451
26	18/10/2016	4010.9788	2733.075	1788.884888	424.447662	1845.28333		0.0055281	-0.00257	0.010431	0.015157	-0.00046
27	17/10/2016	3988.9275	2740.111	1770.417969	418.110565	1846.1322		0.0055592	-0.01212	-0.00289	-0.00968	-0.00731

After that, the average daily return, daily variance, average annual return and annual variance are calculated using the appropriate formulae as shown below:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1			Prices							Returns			
2	Date	AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo		AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo	
3	18/11/2016	3466.907	2460.067	2006.724	437.507	1723		-0.0038018	0	-0.0288	0.002848	0.000987	
4	17/11/2016	3480.1377	2460.067	2066.229	436.2644	1721.302		-0.0031009	-0.00272	0.019403	0.005409	0.012235	
5	16/11/2016	3490.9629	2466.77	2026.901	433.9174	1700.497		-0.0018345	-0.01449	-0.01348	-0.0152	0.007547	
6	15/11/2016	3497.3787	2503.039	2054.601	440.6133	1687.759		-0.000916	0.013731	-0.04499	0.005514	0.008883	
7	14/11/2016	3500.5852	2469.135	2151.381	438.1973	1672.898		0.0091308	-0.01011	0.024927	0.024036	-0.00631	
8	11/11/2016	3468.9114	2494.366	2099.059	427.9119	1683.513		-0.0178228	-0.0055	-0.02494	-0.00864	-0.00502	
9	10/11/2016	3531.8589	2508.165	2152.749	431.6395	1692.005		-0.0373729	-0.03926	0.025746	0.004821	-0.03604	
10	09/11/2016	3668.9792	2610.667	2098.716	429.5686	1755.269		0.0280867	-0.02099	0.068419	0.00371	-0.01148	
11	08/11/2016	3568.7451	2666.649	1964.319	427.981	1775.65		0.0045143	0.006398	0.016637	-0.0037	0.00917	
12	07/11/2016	3552.7073	2649.698	1932.173	429.5686	1759.516		0.0143087	0.012809	0.035747	0.046234	0.006558	
13	04/11/2016	3502.5898	2616.187	1865.488	410.5855	1748.051		-0.0149962	-0.01103	-0.02188	-0.00435	-0.0058	
14	03/11/2016	3555.915	2645.361	1907.209	412.3804	1758.242		-0.0201083	-0.01728	-0.01466	-0.01125	-0.02907	
15	02/11/2016	3628.8857	2691.881	1935.593	417.0744	1810.891		0.002992	-0.00857	-0.00492	-0.01355	-0.0125	
16	01/11/2016	3618.0605	2715.141	1945.168	422.8038	1833.819		-0.0165647	0.00746	0.001585	-0.006	-0.00918	
17	31/10/2016	3679.0022	2695.035	1942.091	425.3578	1850.803		-0.0062812	-0.00712	0.002825	-0.01329	-0.00366	
18	28/10/2016	3702.2568	2714.353	1936.619	431.0872	1857.596		0.0005416	0.001163	0.012516	-0.00478	-0.00069	
19	27/10/2016	3700.2527	2711.199	1912.681	433.1581	1858.87		-0.0340171	0.004206	0.00197	0.008032	-0.00046	
20	26/10/2016	3830.5571	2699.845	1908.919	429.7067	1859.72		-0.0071703	-0.00705	-0.00179	-0.00607	-0.00613	
21	25/10/2016	3858.2217	2719	1912.339	432.3298	1871.183		-0.0081427	0.001728	0.045039	0.003204	0.011708	
22	24/10/2016	3889.8958	2714.31	1829.922	430.9492	1849.529		-0.0203958	-0.0043	0.002623	-0.00335	-0.00023	
23	21/10/2016	3970.8853	2726.038	1825.134	432.3987	1849.954		-0.0051231	-0.00072	0.019095	0.007884	-0.00525	
24	20/10/2016	3991.3333	2727.993	1790.937	429.0164	1859.72		0.0074891	0.006201	-0.00191	0.007851	0.004357	
25	19/10/2016	3961.6638	2711.182	1794.356	425.6743	1851.652		-0.012295	-0.00801	0.003059	0.00289	0.003451	
26	18/10/2016	4010.9788	2733.075	1788.885	424.4477	1845.283		0.0055281	-0.00257	0.010431	0.015157	-0.00046	
27	17/10/2016	3988.9275	2740.111	1770.418	418.1106	1846.132		0.0055592	-0.01212	-0.00289	-0.00968	-0.00731	

			AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo				
			Average Daily Re=AVERAGE(H3:H369)								
			AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo				
			Average Daily Return	0.0003066	0.000461	0.000654	0.000499	0.0003696			
			Variance(daily=VAR(H3:H369))								

372						AstraZeneca	Unilever
373			Average Daily Return			0.0003066	0.000461
374			Variance(daily)			0.0002326	0.000172
375			Average Annual Return			=D373*252	

	A	B	C	D
357	30/06/2015	3041.1609	2054.092	1648.1548
358	29/06/2015	3099.8042	2073.655	1697.0195
359	26/06/2015	3173.2041	2130.839	1707.7384
360	25/06/2015	3165.2588	2139.115	1723.1855
361	24/06/2015	3215.958	2145.135	1753.1346
362	23/06/2015	3233.7397	2151.154	1739.2642
363	22/06/2015	3209.9036	2148.144	1746.5145
364	19/06/2015	3169.7998	2085.694	1722.5553
365	18/06/2015	3172.8259	2087.199	1733.5887
366	17/06/2015	3146.7197	2078.17	1716.5651
367	16/06/2015	3155.4214	2099.99	1731.067
368	15/06/2015	3139.5313	2089.456	1769.5278
369	12/06/2015	3186.4465	2097.732	1785.2906
370	11/06/2015	3232.6047	2143.629	1790.02
371				
372				AstraZeneca
373	Average Daily Return			0.0003066
374	Variance(daily)			0.0002326
375	Average Annual Return			0.0772585
376	Annual Variance			=D374*252

By using the above formulae, a table was obtained as seen below:

	A	B	C	D	E	F	G	H
371								
372				AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo
373	Average Daily Return			0.000306581	0.00046084	0.00065365	0.000499	0.0003696
374	Variance(daily)			0.000232605	0.00017183	0.00068784	0.000251	0.0001398
375	Average Annual Return			0.07725853	0.11613247	0.16471878	0.125649	0.0931515
376	Annual Variance			0.058616382	0.0433017	0.17333638	0.063256	0.0352223

From this table, the expected returns of each stock are the same as the Average Annual Return and the standard deviation or volatility of each stock is the square root of the Annual variance as shown below:

N	O	P
	Returns	Standard Deviation
AstraZeneca	0.077259	=SQRT(D376)
Unilever	0.116132	
Rio Tinto	0.164719	
HSBC	0.125649	
Diageo	0.093152	

Hence, the expected returns and volatility of each stock are obtained as seen below:

N	O	P
	Returns	Standard Deviation
AstraZeneca	0.077259	0.242108204
Unilever	0.116132	0.20809061
Rio Tinto	0.164719	0.416336863
HSBC	0.125649	0.251507118
Diageo	0.093152	0.187676094

Now, the variance-covariance matrix is obtained using the daily returns of the stocks as seen below:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1			Prices							Returns			
2	Date	AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo		AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo	
3	18/11/2016	3466.907	2460.067	2006.724	437.507	1723		-0.0038018	0	-0.0288	0.002848	0.000987	
4	17/11/2016	3480.1377	2460.067	2066.229	436.2644	1721.302		-0.0031009	-0.00272	0.019403	0.005409	0.012235	
5	16/11/2016	3490.9629	2466.77	2026.901	433.9174	1700.497		-0.0018345	-0.01449	-0.01348	-0.0152	0.007547	
6	15/11/2016	3497.3787	2503.039	2054.601	440.6133	1687.759		-0.000916	0.013731	-0.04499	0.005514	0.008883	
7	14/11/2016	3500.5852	2469.135	2151.381	438.1973	1672.898		0.0091308	-0.01011	0.024927	0.024036	-0.00631	
8	11/11/2016	3468.9114	2494.366	2099.059	427.9119	1683.513		-0.0178228	-0.0055	-0.02494	-0.00864	-0.00502	
9	10/11/2016	3531.8589	2508.165	2152.749	431.6395	1692.005		-0.0373729	-0.03926	0.025746	0.004821	-0.03604	
10	09/11/2016	3668.9792	2610.667	2098.716	429.5686	1755.269		0.0280867	-0.02099	0.068419	0.00371	-0.01148	
11	08/11/2016	3568.7451	2666.649	1964.319	427.981	1775.65		0.0045143	0.006398	0.016637	-0.0037	0.00917	
12	07/11/2016	3552.7073	2649.698	1932.173	429.5686	1759.516		0.0143087	0.012809	0.035747	0.046234	0.006558	
13	04/11/2016	3502.5898	2616.187	1865.488	410.5855	1748.051		-0.0149962	-0.01103	-0.02188	-0.00435	-0.0058	
14	03/11/2016	3555.915	2645.361	1907.209	412.3804	1758.242		-0.0201083	-0.01728	-0.01466	-0.01125	-0.02907	
15	02/11/2016	3628.8857	2691.881	1935.593	417.0744	1810.891		0.002992	-0.00857	-0.00492	-0.01355	-0.0125	
16	01/11/2016	3618.0605	2715.141	1945.168	422.8038	1833.819		-0.0165647	0.00746	0.001585	-0.006	-0.00918	
17	31/10/2016	3679.0022	2695.035	1942.091	425.3578	1850.803		-0.0062812	-0.00712	0.002825	-0.01329	-0.00366	
18	28/10/2016	3702.2568	2714.353	1936.619	431.0872	1857.596		0.0005416	0.001163	0.012516	-0.00478	-0.00069	
19	27/10/2016	3700.2527	2711.199	1912.681	433.1581	1858.87		-0.0340171	0.004206	0.00197	0.008032	-0.00046	
20	26/10/2016	3830.5571	2699.845	1908.919	429.7067	1859.72		-0.0071703	-0.00705	-0.00179	-0.00607	-0.00613	
21	25/10/2016	3858.2217	2719	1912.339	432.3298	1871.183		-0.0081427	0.001728	0.045039	0.003204	0.011708	
22	24/10/2016	3889.8958	2714.31	1829.922	430.9492	1849.529		-0.0203958	-0.0043	0.002623	-0.00335	-0.00023	
23	21/10/2016	3970.8853	2726.038	1825.134	432.3987	1849.954		-0.0051231	-0.00072	0.019095	0.007884	-0.00525	
24	20/10/2016	3991.3333	2727.993	1790.937	429.0164	1859.72		0.0074891	0.006201	-0.00191	0.007851	0.004357	
25	19/10/2016	3961.6638	2711.182	1794.356	425.6743	1851.652		-0.012295	-0.00801	0.003059	0.00289	0.003451	
26	18/10/2016	4010.9788	2733.075	1788.885	424.4477	1845.283		0.0055281	-0.00257	0.010431	0.015157	-0.00046	
27	17/10/2016	3988.9275	2740.111	1770.418	418.1106	1846.132		0.0055592	-0.01212	-0.00289	-0.00968	-0.00731	

T3							=VARP(Sheet1!\$H\$3:\$H\$369)*252
	S	T	U	V	W	X	
1		Variance-Covariance Matrix					
2		AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo	
3	AstraZeneca	0.05845666	0.0314717	0.020504	0.026162	0.029402	
4	Unilever	0.03147171	0.0431837	0.023698	0.027611	0.031325	
5	Rio Tinto	0.02050431	0.0236983	0.172864	0.0563	0.02402	
6	HSBC	0.02616243	0.0276112	0.0563	0.063083	0.023116	
7	Diageo	0.02940224	0.0313249	0.02402	0.023116	0.035126	
8							

T4							=0.000124887719642945*252
	S	T	U	V	W	X	
1		Variance-Covariance Matrix					
2		AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo	
3	AstraZeneca	0.05845666	0.0314717	0.020504	0.026162	0.029402	
4	Unilever	0.03147171	0.0431837	0.023698	0.027611	0.031325	
5	Rio Tinto	0.02050431	0.0236983	0.172864	0.0563	0.02402	
6	HSBC	0.02616243	0.0276112	0.0563	0.063083	0.023116	
7	Diageo	0.02940224	0.0313249	0.02402	0.023116	0.035126	
8							

U3	✖	✓	f_x	=TRANSPPOSE(T4:T7)		
	S	T	U	V	W	X
1		Variance-Covariance Matrix				
2		<i>AstraZeneca</i>	<i>Unilever</i>	<i>Rio Tinto</i>	<i>HSBC</i>	<i>Diageo</i>
3	<i>AstraZeneca</i>	0.05845666	0.0314717	0.020504	0.026162	0.029402
4	<i>Unilever</i>	0.03147171	0.0431837	0.023698	0.027611	0.031325
5	<i>Rio Tinto</i>	0.02050431	0.0236983	0.172864	0.0563	0.02402
6	<i>HSBC</i>	0.02616243	0.0276112	0.0563	0.063083	0.023116
7	<i>Diageo</i>	0.02940224	0.0313249	0.02402	0.023116	0.035126
8						

Hence, the Variance-Covariance matrix is obtained as shown below:

	Q	R	S	T	U	V
	Variance-Covariance Matrix					
	<i>AstraZeneca</i>	<i>Unilever</i>	<i>Rio Tinto</i>	<i>HSBC</i>	<i>Diageo</i>	
<i>AstraZeneca</i>	0.0584567	0.031472	0.020504	0.026162	0.029402	
<i>Unilever</i>	0.0314717	0.043184	0.023698	0.027611	0.031325	
<i>Rio Tinto</i>	0.0205043	0.023698	0.172864	0.0563	0.02402	
<i>HSBC</i>	0.0261624	0.027611	0.0563	0.063083	0.023116	
<i>Diageo</i>	0.0294022	0.031325	0.02402	0.023116	0.035126	

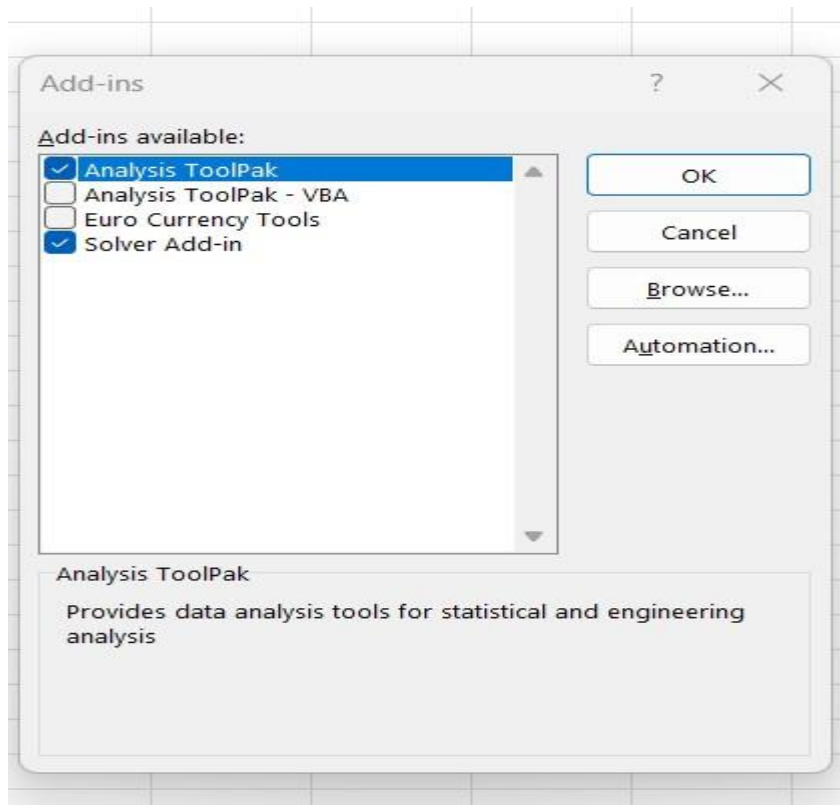
It is noteworthy that all values in the above matrix are positive and thus, the returns of all the stocks can be said to be moving in a positive direction along with the market.

2. The equally weighted portfolio is obtained as shown below:

Q19		X		✓fx		=MMULT(TRANPOSE(Q12:Q16),Q2:Q6)		
	K	L	M	N	C	P	Q	R
1							Returns	Standard Deviation
2	HSBC	Diageo	FTSE 100	FTSE 100 Returns		AstraZeneca	0.077259	0.242108204
3	0.002848	0.000987	6,775.77	0.00		Unilever	0.116132	0.20809061
4	0.005409	0.012235	6,794.71	0.01		Rio Tinto	0.164719	0.416336863
5	-0.0152	0.007547	6,749.72	-0.01		HSBC	0.125649	0.251507118
6	0.005514	0.008883	6,792.74	0.01		Diageo	0.093152	0.187676094
7	0.024036	-0.00631	6,753.18	0.00				
8	-0.00864	-0.00502	6,730.43	-0.01				
9	0.004821	-0.03604	6,827.98	-0.01		Equally Weighted Portfolio		
10	0.00371	-0.01148	6,911.84	0.01				
11	-0.0037	0.00917	6,843.13	0.01			Weights	
12	0.046234	0.006558	6,806.90	0.02		AstraZeneca	0.2	
13	-0.00435	-0.0058	6,693.26	-0.01		Unilever	0.2	
14	-0.01125	-0.02907	6,790.51	-0.01		Rio Tinto	0.2	
15	-0.01355	-0.0125	6,845.42	-0.01		HSBC	0.2	
16	-0.006	-0.00918	6,917.14	-0.01		Diageo	0.2	
17	-0.01329	-0.00366	6,954.22	-0.01		Sum	1	
18	-0.00478	-0.00069	6,996.26	0.00				
19	0.008032	-0.00046	6,986.57	0.00		Expected Return	0.115382	
20	-0.00607	-0.00613	6,958.09	-0.01		Standard Deviation	0.195953	

Q20															=SQRT(MMULT(MMULT(TRANPOSE(Q12:Q16),T3:X7),Q12:Q16))														
	K	L	M	N	C	P	Q	R	S	T	U	V	W	X															
1							Returns	Standard Deviation		Variance-Covariance Matrix																			
2	HSBC	Diageo	FTSE 100	FTSE 100 Returns		AstraZeneca	0.077259	0.242108204		AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo															
3	0.002848	0.000987	6,775.77	0.00		Unilever	0.116132	0.20809061	AstraZeneca	0.05845666	0.0314717	0.020504	0.026162	0.029402															
4	0.005409	0.012235	6,794.71	0.01		Rio Tinto	0.164719	0.416336863	Unilever	0.03147171	0.0431837	0.023698	0.027611	0.031325															
5	-0.0152	0.007547	6,749.72	-0.01		HSBC	0.125649	0.251507118	Rio Tinto	0.02050431	0.0236983	0.172864	0.0563	0.02402															
6	0.005514	0.008883	6,792.74	0.01		Diageo	0.093152	0.187676094	HSBC	0.02616243	0.0276112	0.0563	0.063083	0.023116															
7	0.024036	-0.00631	6,753.18	0.00					Diageo	0.02940224	0.0313249	0.02402	0.023116	0.035126															
8	-0.00864	-0.00502	6,730.43	-0.01																									
9	0.004821	-0.03604	6,827.98	-0.01		Equally Weighted Portfolio			Risk free rate	0.015																			
10	0.00371	-0.01148	6,911.84	0.01																									
11	-0.0037	0.00917	6,843.13	0.01			Weights		AstraZeneca_w	Unilever_w	Rio Tinto_w	HSBC_w	Diageo_w	Volatility															
12	0.046234	0.006558	6,806.90	0.02		AstraZeneca	0.2		0	0	1	0	0	0.42															
13	-0.00435	-0.0058	6,693.26	-0.01		Unilever	0.2		0.024775723	0.01376352	0.9301195	0.011068	0.020274	0.39															
14	-0.01125	-0.02907	6,790.51	-0.01		Rio Tinto	0.2		0.060879793	0.01995283	0.8648264	0.014286	0.040055	0.37															
15	-0.01355	-0.0125	6,845.42	-0.01		HSBC	0.2		0.103735296	0.02251008	0.8031792	0.01534	0.055236	0.35															
16	-0.006	-0.00918	6,917.14	-0.01		Diageo	0.2		0.151033068	0.02374729	0.7432091	0.015802	0.066209	0.33															
17	-0.01329	-0.00366	6,954.22	-0.01		Sum	1		0.201246257	0.0244369	0.6841648	0.016048	0.074105	0.31															
18	-0.00478	-0.00069	6,996.26	0.00					0.253380044	0.02486393	0.6256715	0.016196	0.079889	0.29															
19	0.008032	-0.00046	6,986.57	0.00		Expected Return	0.115382		0.306789872	0.02514964	0.5675338	0.016293	0.084234	0.27															
20	-0.00607	-0.00613	6,958.09	-0.01		Standard Deviation	0.195953		0.361075381	0.0253523	0.5096286	0.016361	0.087582	0.26															
21	0.003204	0.011708	7,017.64	0.00					0.415975391	0.02550265	0.4518848	0.016417	0.090275	0.25															

Now, the solver function was used to obtain the optimal risky portfolio. To do this, add-ins need to be included from File>Options>Add-ins in Excel. The two add-ins required in this case study were added as shown below:



These add-ins can be found under the Data tab in Excel. The maximum expected return of the portfolio was calculated using the solver function initially as seen below:

	K	L	M	N	C	P	Q
9	0.004821	-0.03604	6,827.98	-0.01		Equally Weighted Portfolio	
10	0.00371	-0.01148	6,911.84	0.01			
11	-0.0037	0.00917	6,843.13	0.01			
12	0.046234	0.006558	6,806.90	0.02			
13	-0.00435	-0.0058	6,693.26	-0.01			
14	-0.01125	-0.02907	6,790.51	-0.01			
15	-0.01355	-0.0125	6,845.42	-0.01			
16	-0.006	-0.00918	6,917.14	-0.01			
17	-0.01329	-0.00366	6,954.22	-0.01			
18	-0.00478	-0.00069	6,996.26	0.00			
19	0.008032	-0.00046	6,986.57	0.00		Expected Return	0.115382
20	-0.00607	-0.00613	6,958.09	-0.01		Standard Deviation	0.195953
21	0.003204	0.011708	7,017.64	0.00			
22	-0.00335	-0.00023	6,986.40	0.00		Optimal Risky Portfolio	
23	0.007884	-0.00525	7,020.47	0.00			
24	0.007851	0.004357	7,026.90	0.00			
25	0.00289	0.003451	7,021.92	0.00			
26	0.015157	-0.00046	7,000.06	0.01			
27	-0.00968	-0.00731	6,947.55	-0.01			
28	0.017572	-0.00387	7,013.55	0.01			
29	-0.01233	-0.00091	6,977.74	-0.01			
30	-0.00323	-0.01256	7,024.01	-0.01			
31	-0.01008	-0.00425	7,070.88	0.00			
32	0.008555	0.002913	7,097.50	0.01		Expected Return	0.164719
33	0.028045	0.00518	7,044.39	0.01		Standard Deviation	0.42
34	0.002662	-0.00023	6,999.96	0.00			
35	0.008051	-0.02094	7,033.25	-0.01			
36	0.014636	0.010345	7,074.34	0.01			

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Help, Solve, Close

It is important to note that the expected returns were maximised by changing the weights of the stocks in the portfolio and by implementing the constraint that the sum of the weights of all stocks in the portfolio should be equal to 1.

Now, the solver function is used to generate different weights of each stock for expected returns less than the maximum value obtained earlier as seen below:

	K	L	M	N	C	P	Q
9	0.004821	-0.03604	6,827.98	-0.01	Equally Weighted Portfolio		
10	0.00371	-0.01148	6,911.84	0.01			
11	-0.0037	0.00917	6,843.13	0.01		Weights	
12	0.046234	0.006558	6,806.90	0.02	AstraZeneca	0.2	
13	-0.00435	-0.0058	6,693.26	-0.01	Unilever	0.2	
14	-0.01125	-0.02907	6,790.51	-0.01	Rio Tinto	0.2	
15	-0.01355	-0.0125	6,845.42	-0.01	HSBC	0.2	
16	-0.006	-0.00918	6,917.14	-0.01	Diageo	0.2	
17	-0.01329	-0.00366	6,954.22	-0.01	Sum	1	
18	-0.00478	-0.00069	6,996.26	0.00			
19	0.008032	-0.00046	6,986.57	0.00	Expected Return	0.115382	
20	-0.00607	-0.00613	6,958.09	-0.01	Standard Deviation	0.195953	
21	0.003204	0.011708	7,017.64	0.00			
22	-0.00335	-0.00023	6,986.40	0.00	Optimal Risky Portfolio		
23	0.007884	-0.00525	7,020.47	0.00			
24	0.007851	0.004357	7,026.90	0.00		Weights	
25	0.00289	0.003451	7,021.92	0.00	AstraZeneca	0.159517	
26	0.015157	-0.00046	7,000.06	0.01	Unilever	0.43	
27	-0.00968	-0.00731	6,947.55	-0.01	Rio Tinto	0.16	
28	0.017572	-0.00387	7,013.55	0.01	HSBC	0.26	
29	-0.01233	-0.00091	6,977.74	-0.01	Diageo	0.00	
30	-0.00323	-0.01256	7,024.01	-0.01	Sum	1	
31	-0.01008	-0.00425	7,070.88	0.00			
32	0.008555	0.002913	7,097.50	0.01	Expected Return	0.12	
33	0.028045	0.00518	7,044.39	0.01	Standard Deviation	0.20	
34	0.002662	-0.00023	6,999.96	0.00			

Solver Parameters

Set Objective:

To: ☐ Max ☐ Min ☒ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$Q\$30 = 1

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method

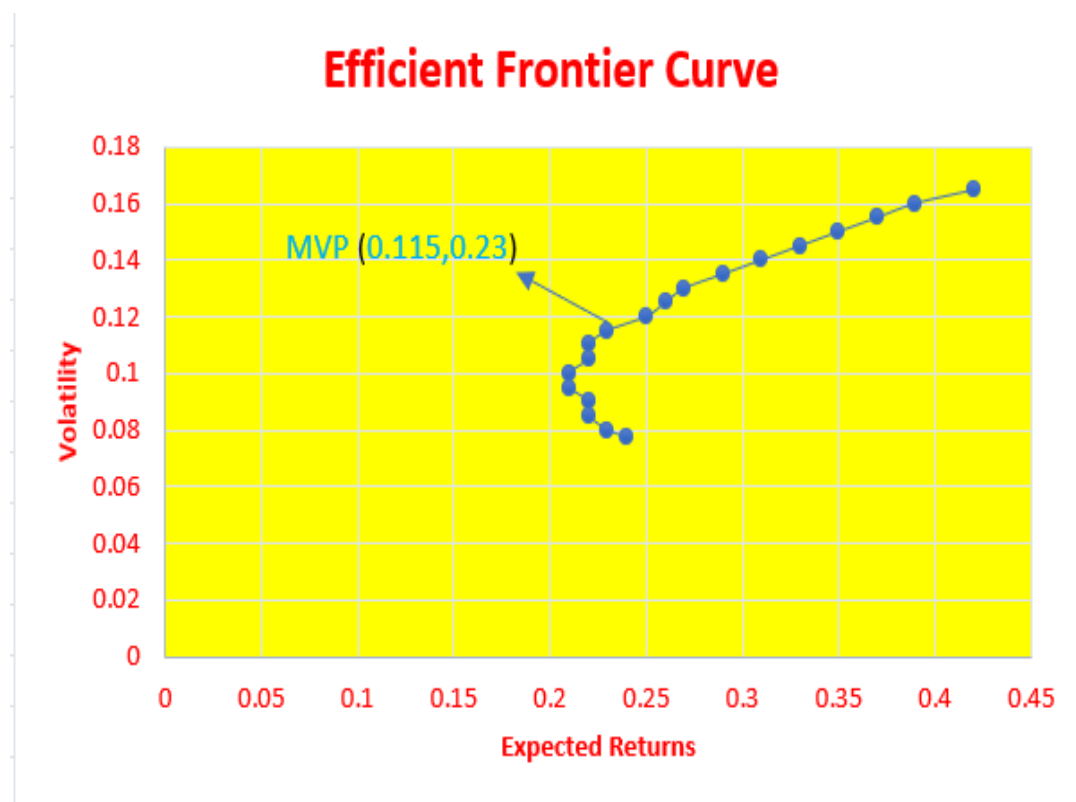
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons:

Repeating the above process, till the solver function finds a feasible solution, the below shown table was obtained:

AstraZeneca_w	Unilever_w	Rio Tinto_w	HSBC_w	Diageo_w	Volatility	Expected Returns
0	0	1	0	0	0.42	0.164719
0.024775723	0.01376352	0.9301195	0.011068	0.020274	0.39	0.16
0.060879793	0.01995283	0.8648264	0.014286	0.040055	0.37	0.155
0.103735296	0.02251008	0.8031792	0.01534	0.055236	0.35	0.15
0.151033068	0.02374729	0.7432091	0.015802	0.066209	0.33	0.145
0.201246257	0.0244369	0.6841648	0.016048	0.074105	0.31	0.14
0.253380044	0.02486393	0.6256715	0.016196	0.079889	0.29	0.135
0.306789872	0.02514964	0.5675338	0.016293	0.084234	0.27	0.13
0.361075381	0.0253523	0.5096286	0.016361	0.087582	0.26	0.125
0.415975391	0.02550265	0.4518848	0.016412	0.090225	0.25	0.12
0.471320746	0.02561822	0.3942547	0.01645	0.092356	0.23	0.115
0.529037129	0.02551127	0.3372684	0.016395	0.091788	0.22	0.11
0.586946547	0.02536651	0.2803444	0.016321	0.091022	0.22	0.105
0.64516849	0.02515974	0.2235242	0.016214	0.089933	0.21	0.1
0.703951343	0.02484082	0.166891	0.01605	0.088267	0.21	0.095
0.763896957	0.02428696	0.1106461	0.015762	0.085408	0.22	0.09
0.826890256	0.02310457	0.0554332	0.015142	0.07943	0.22	0.085
0.902692417	0.01918053	0.0046138	0.013044	0.060469	0.23	0.08
0.999991134	0.00	0	0	0	0.24	0.077259

By plotting the expected returns v/s the volatility, the efficient frontier curve is obtained as seen below:



3. The next step is to calculate the Sharpe ratio using the above results and the given risk-free rate of 1.5%. This was achieved using the formula as shown below:

Z12									
	R	S	T	U	V	W	X	Y	Z
8									
9		Risk free rate	0.015						
10									
11		AstraZeneca_w	Unilever_w	Rio Tinto_w	HSBC_w	Diageo_w	Volatility	Expected Returns	Sharpe ratio
12		0	0	1	0	0	0.42	0.164719	0.3564738
13		0.024775723	0.01376352	0.9301195	0.011068	0.020274	0.39	0.16	0.3717949
14		0.060879793	0.01995283	0.8648264	0.014286	0.040055	0.37	0.155	0.3783784
15		0.103735296	0.02251008	0.8031792	0.01534	0.055236	0.35	0.15	0.3857143
16		0.151033068	0.02374729	0.7432091	0.015802	0.066209	0.33	0.145	0.3939394
17		0.201246257	0.0244369	0.6841648	0.016048	0.074105	0.31	0.14	0.4032258
18		0.253380044	0.02486393	0.6256715	0.016196	0.079889	0.29	0.135	0.4137931
19		0.306789872	0.02514964	0.5675338	0.016293	0.084234	0.27	0.13	0.4259259
20		0.361075381	0.0253523	0.5096286	0.016361	0.087582	0.26	0.125	0.4230769
21		0.415975391	0.02550265	0.4518848	0.016412	0.090225	0.25	0.12	0.42
22		0.471320746	0.02561822	0.3942547	0.01645	0.092356	0.23	0.115	0.4347826
23		0.529037129	0.02551127	0.3372684	0.016395	0.091788	0.22	0.11	0.4318182
24		0.586946547	0.02536651	0.2803444	0.016321	0.091022	0.22	0.105	0.4090909
25		0.64516849	0.02515974	0.2235242	0.016214	0.089933	0.21	0.1	0.4047619
26		0.703951343	0.02484082	0.166891	0.01605	0.088267	0.21	0.095	0.3809524
27		0.763896957	0.02428696	0.1106461	0.015762	0.085408	0.22	0.09	0.3409091
28		0.826890256	0.02310457	0.0554332	0.015142	0.07943	0.22	0.085	0.3181818
29		0.902692417	0.01918053	0.0046138	0.013044	0.060469	0.23	0.08	0.2826087
30		0.999991134	0.00	0	0	0	0.24	0.077259	0.2594125
31									

The values for which the Sharpe ratio is the maximum is the optimal portfolio as seen below:

AstraZeneca_w	Unilever_w	Rio Tinto_w	HSBC_w	Diageo_w	Volatility	Expected Returns	Sharpe ratio		
0	0	1	0	0	0.42	0.164719	0.3564738		
0.024775723	0.01376352	0.9301195	0.011068	0.020274	0.39	0.16	0.3717949		
0.060879793	0.01995283	0.8648264	0.014286	0.040055	0.37	0.155	0.3783784		
0.103735296	0.02251008	0.8031792	0.01534	0.055236	0.35	0.15	0.3857143		
0.151033068	0.02374729	0.7432091	0.015802	0.066209	0.33	0.145	0.3939394		
0.201246257	0.0244369	0.6841648	0.016048	0.074105	0.31	0.14	0.4032258		
0.253380044	0.02486393	0.6256715	0.016196	0.079889	0.29	0.135	0.4137931		
0.306789872	0.02514964	0.5675338	0.016293	0.084234	0.27	0.13	0.4259259		
0.361075381	0.0253523	0.5096286	0.016361	0.087582	0.26	0.125	0.4230769		
0.415975391	0.02550265	0.4518848	0.016412	0.090225	0.25	0.12	0.42		
0.471320746	0.02561822	0.3942547	0.01645	0.092356	0.23	0.115	0.4347826	Optimal Portfolio	
0.529037129	0.02551127	0.3372684	0.016395	0.091788	0.22	0.11	0.4318182		
0.586946547	0.02536651	0.2803444	0.016321	0.091022	0.22	0.105	0.4090909		
0.64516849	0.02515974	0.2235242	0.016214	0.089933	0.21	0.1	0.4047619		
0.703951343	0.02484082	0.166891	0.01605	0.088267	0.21	0.095	0.3809524		
0.763896957	0.02428696	0.1106461	0.015762	0.085408	0.22	0.09	0.3409091		
0.826890256	0.02310457	0.0554332	0.015142	0.07943	0.22	0.085	0.3181818		
0.902692417	0.01918053	0.0046138	0.013044	0.060469	0.23	0.08	0.2826087		
0.999991134	0.00	0	0	0	0.24	0.077259	0.2594125		

Now, the volatility and the returns are calculated based on the optimal portfolio values and assumed weights as shown below:

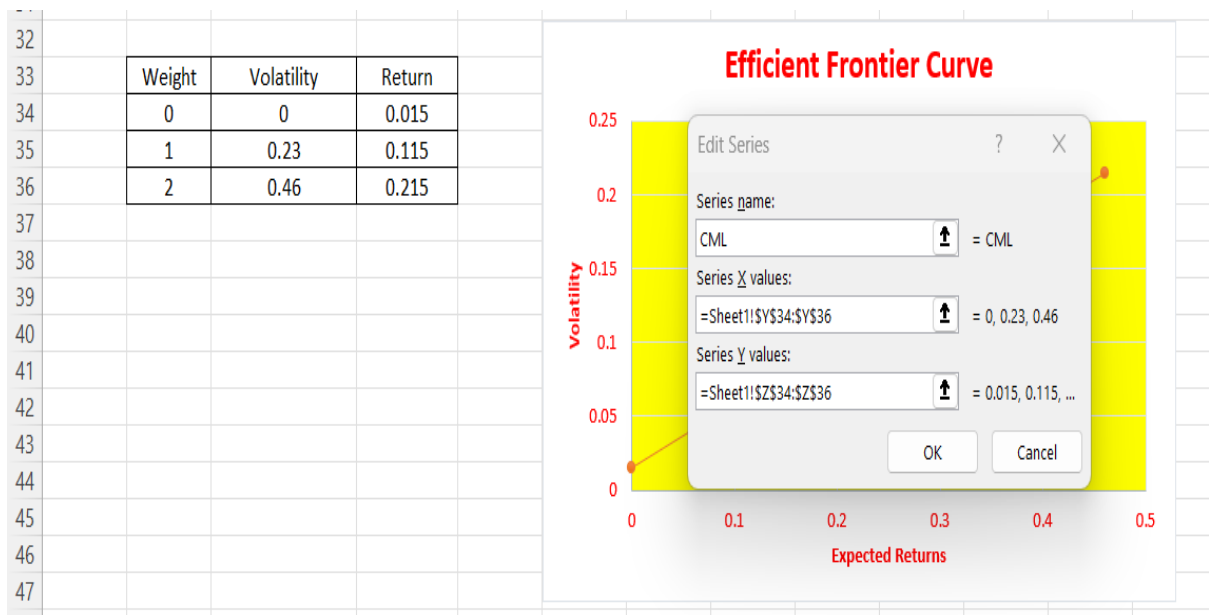
Y34											
	R	S	T	U	V	W	X	Y	Z	AA	AB
9		Risk free rate	0.015								
10											
11		AstraZeneca_w	Unilever_w	Rio Tinto_w	HSBC_w	Diageo_w	Volatility	Expected Returns	Sharpe ratio		
12		0	0	1	0	0	0.42	0.164719	0.3564738		
13		0.024775723	0.01376352	0.9301195	0.011068	0.020274	0.39	0.16	0.3717949		
14		0.060879793	0.01995283	0.8648264	0.014286	0.040055	0.37	0.155	0.3783784		
15		0.103735296	0.02251008	0.8031792	0.01534	0.055236	0.35	0.15	0.3857143		
16		0.151033068	0.02374729	0.7432091	0.015802	0.066209	0.33	0.145	0.3939394		
17		0.201246257	0.0244369	0.6841648	0.016048	0.074105	0.31	0.14	0.4032258		
18		0.253380044	0.02486393	0.6256715	0.016196	0.079889	0.29	0.135	0.4137931		
19		0.306789872	0.02514964	0.5675338	0.016293	0.084234	0.27	0.13	0.4259259		
20		0.361075381	0.0253523	0.5096286	0.016361	0.087582	0.26	0.125	0.4230769		
21		0.415975391	0.02550265	0.4518848	0.016412	0.090225	0.25	0.12	0.42		
22		0.471320746	0.02561822	0.3942547	0.01645	0.092356	0.23	0.115	0.4347826	Optimal Portfolio	
23		0.529037129	0.02551127	0.3372684	0.016395	0.091788	0.22	0.11	0.4318182		
24		0.586946547	0.02536651	0.2803444	0.016321	0.091022	0.22	0.105	0.4090909		
25		0.64516849	0.02515974	0.2235242	0.016214	0.089933	0.21	0.1	0.4047619		
26		0.703951343	0.02484082	0.166891	0.01605	0.088267	0.21	0.095	0.3809524		
27		0.763896957	0.02428696	0.1106461	0.015762	0.085408	0.22	0.09	0.3409091		
28		0.826890256	0.02310457	0.0554332	0.015142	0.07943	0.22	0.085	0.3181818		
29		0.902692417	0.01918053	0.0046138	0.013044	0.060469	0.23	0.08	0.2826087		
30		0.999991134	0.00	0	0	0	0.24	0.077259	0.2594125		
31											
32											
33							Weight	Volatility	Return		
34							0	0	0.015		
35							1	0.23	0.115		

Z34											
	R	S	T	U	V	W	X	Y	Z	AA	AB
9		Risk free rate	0.015								
10											
11		AstraZeneca_w	Unilever_w	Rio Tinto_w	HSBC_w	Diageo_w	Volatility	Expected Returns	Sharpe ratio		
12		0	0	1	0	0	0.42	0.164719	0.3564738		
13		0.024775723	0.01376352	0.9301195	0.011068	0.020274	0.39	0.16	0.3717949		
14		0.060879793	0.01995283	0.8648264	0.014286	0.040055	0.37	0.155	0.3783784		
15		0.103735296	0.02251008	0.8031792	0.01534	0.055236	0.35	0.15	0.3857143		
16		0.151033068	0.02374729	0.7432091	0.015802	0.066209	0.33	0.145	0.3939394		
17		0.201246257	0.0244369	0.6841648	0.016048	0.074105	0.31	0.14	0.4032258		
18		0.253380044	0.02486393	0.6256715	0.016196	0.079889	0.29	0.135	0.4137931		
19		0.306789872	0.02514964	0.5675338	0.016293	0.084234	0.27	0.13	0.4259259		
20		0.361075381	0.0253523	0.5096286	0.016361	0.087582	0.26	0.125	0.4230769		
21		0.415975391	0.02550265	0.4518848	0.016412	0.090225	0.25	0.12	0.42		
22		0.471320746	0.02561822	0.3942547	0.01645	0.092356	0.23	0.115	0.4347826	Optimal Portfolio	
23		0.529037129	0.02551127	0.3372684	0.016395	0.091788	0.22	0.11	0.4318182		
24		0.586946547	0.02536651	0.2803444	0.016321	0.091022	0.22	0.105	0.4090909		
25		0.64516849	0.02515974	0.2235242	0.016214	0.089933	0.21	0.1	0.4047619		
26		0.703951343	0.02484082	0.166891	0.01605	0.088267	0.21	0.095	0.3809524		
27		0.763896957	0.02428696	0.1106461	0.015762	0.085408	0.22	0.09	0.3409091		
28		0.826890256	0.02310457	0.0554332	0.015142	0.07943	0.22	0.085	0.3181818		
29		0.902692417	0.01918053	0.0046138	0.013044	0.060469	0.23	0.08	0.2826087		
30		0.999991134	0.00	0	0	0	0.24	0.077259	0.2594125		
31											
32											
33							Weight	Volatility	Return		
34							0	0	0.015		
35							1	0.23	0.115		

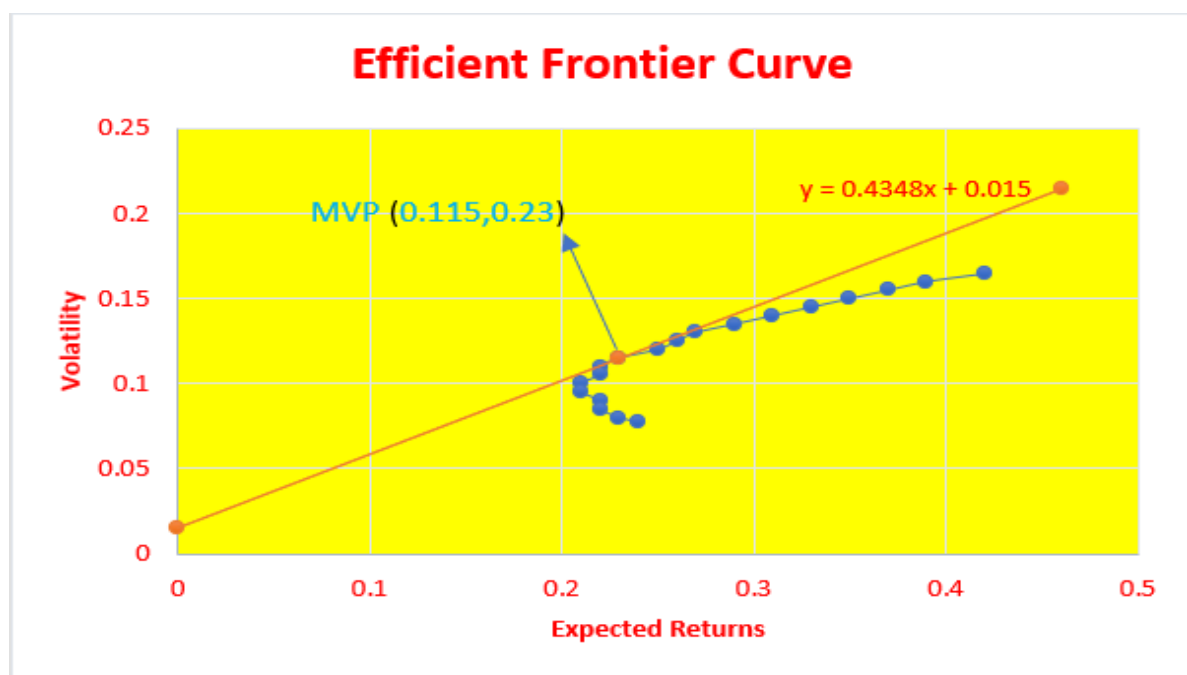
It is important to note that the optimal portfolio values need to be locked in the formulae so that the correct result is obtained for all values. We now have the following table:

Weight	Volatility	Return
0	0	0.015
1	0.23	0.115
2	0.46	0.215

The next step is to obtain the Capital Market Line which is done using the following method:



The Capital Market Line and its equation are thus, obtained as shown below:



Economic Significance of CML:

As seen above, the CML provides the optimal portfolio values which give the highest possible returns for a specified amount of risk. It thus becomes a vital cog in today's modern financial market for investors who want to achieve maximum returns by taking a certain amount of risk. It becomes an essential tool for achieving the correct balance one needs to obtain between risk and return. Portfolio managers and investors utilize the CML to adjust their portfolios so that the optimal returns can be gained. The point at which the CML and the efficient frontier curve coincide is the MVP or the most valuable portfolio point as seen in the above figure.

4. Using regression analysis, the *beta* for each stock was calculated which is the measure of risk of a stock with respect to the stock market. For obtaining this, FTSE 100 historical prices which is the benchmark of the UK stock market were downloaded and the returns were calculated as it was earlier done for each of the stocks.

N3 =(M3-M4)/M4								
	G	H	I	J	K	L	M	N
1				Returns				
2		AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo	FTSE 100	FTSE 100 Returns
3		-0.0038018	0	-0.0288	0.002848	0.000987	6,775.77	0.00
4		-0.0031009	-0.00272	0.019403	0.005409	0.012235	6,794.71	0.01
5		-0.0018345	-0.01449	-0.01348	-0.0152	0.007547	6,749.72	-0.01
6		-0.000916	0.013731	-0.04499	0.005514	0.008883	6,792.74	0.01
7		0.0091308	-0.01011	0.024927	0.024036	-0.00631	6,753.18	0.00
8		-0.0178228	-0.0055	-0.02494	-0.00864	-0.00502	6,730.43	-0.01
9		-0.0373729	-0.03926	0.025746	0.004821	-0.03604	6,827.98	-0.01
10		0.0280867	-0.02099	0.068419	0.00371	-0.01148	6,911.84	0.01
11		0.0045143	0.006398	0.016637	-0.0037	0.00917	6,843.13	0.01
12		0.0143087	0.012809	0.035747	0.046234	0.006558	6,806.90	0.02
13		-0.0149962	-0.01103	-0.02188	-0.00435	-0.0058	6,693.26	-0.01
14		-0.0201083	-0.01728	-0.01466	-0.01125	-0.02907	6,790.51	-0.01
15		0.002992	-0.00857	-0.00492	-0.01355	-0.0125	6,845.42	-0.01
16		-0.0165647	0.00746	0.001585	-0.006	-0.00918	6,917.14	-0.01
17		-0.0062812	-0.00712	0.002825	-0.01329	-0.00366	6,954.22	-0.01
18		0.0005416	0.001163	0.012516	-0.00478	-0.00069	6,996.26	0.00
19		-0.0340171	0.004206	0.00197	0.008032	-0.00046	6,986.57	0.00
20		-0.0071703	-0.00705	-0.00179	-0.00607	-0.00613	6,958.09	-0.01
21		-0.0081427	0.001728	0.045039	0.003204	0.011708	7,017.64	0.00
22		-0.0203958	-0.0043	0.002623	-0.00335	-0.00023	6,986.40	0.00
23		-0.0051231	-0.00072	0.019095	0.007884	-0.00525	7,020.47	0.00
24		0.0074891	0.006201	-0.00191	0.007851	0.004357	7,026.90	0.00
25		-0.012295	-0.00801	0.003059	0.00289	0.003451	7,021.92	0.00
26		0.0055281	-0.00257	0.010431	0.015157	-0.00046	7,000.06	0.01
27		0.0055592	-0.01212	-0.00289	-0.00968	-0.00731	6,947.55	-0.01

AstraZeneca *beta*:

[illegible]

The *beta* value for AstraZeneca is highlighted in yellow and is 0.110202 which is <1 . Hence, it can be safely concluded that **AstraZeneca is less volatile or less risky than the market as a whole.**

Unilever *beta*:

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	<i>Regression Statistics</i>								
4	Multiple R	0.052534							
5	R Square	0.00276							
6	Adjusted R	2.77E-05							
7	Standard E	0.013108							
8	Observatic	367							
9									
10	ANOVA								
11		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
12	Regression	1	0.000174	0.000174	1.010141	0.315535			
13	Residual	365	0.062717	0.000172					
14	Total	366	0.062891						
15									
16		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
17	Intercept	0.000459	0.000684	0.670141	0.503192	-0.00089	0.001804	-0.00089	0.001804
18	X Variable	0.059534	0.059234	1.005058	0.315535	-0.05695	0.176018	-0.05695	0.176018
19									

The *beta* value for Unilever is highlighted in yellow and is 0.059534 which is <1 . Hence, it can be safely concluded that **Unilever is less volatile or less risky than the market as a whole.**

The *beta* value for HSBC is highlighted in yellow and is 0.110181 which is <1 . Hence, it can be safely concluded that **HSBC is less volatile or less risky than the market as a whole.**

Diageo *beta*:

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	<i>Regression Statistics</i>								
4	Multiple R	0.056314							
5	R Square	0.003171							
6	Adjusted R	0.00044							
7	Standard E	0.01182							
8	Observatic	367							
9									
10	ANOVA								
11		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
12	Regressior	1	0.000162	0.000162	1.1612	0.281928			
13	Residual	365	0.050994	0.00014					
14	Total	366	0.051156						
15									
16		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
17	Intercept	0.000367	0.000617	0.595509	0.551872	-0.00085	0.001581	-0.00085	0.001581
18	X Variable	0.057556	0.053412	1.07759	0.281928	-0.04748	0.162591	-0.04748	0.162591
19									

The *beta* value for Diageo is highlighted in yellow and is 0.057556 which is <1 . Hence, it can be safely concluded that **Diageo is less volatile or less risky than the market as a whole.**

The next step is to estimate the Value at Risk for the portfolio. This is achieved using the following formulae:

B383								
372				AstraZeneca	Unilever	Rio Tinto	HSBC	Diageo
373		Average Daily Return		0.000306581	0.00046084	0.00065365	0.000499	0.0003696
374		Variance(daily)		0.000232605	0.00017183	0.00068784	0.000251	0.0001398
375		Average Annual Return		0.07725853	0.11613247	0.16471878	0.125649	0.0931515
376		Annual Variance		0.058616382	0.0433017	0.17333638	0.063256	0.0352223
377								
378		Calculating VaR						
379	Expected Return	11.50%		Asset Value	100000			
380	Expected Volatility	23%						
381	Time(days)	5						
382	Confidence Level	0.95						
383	Stress Event(Z(1-alpha))	-1.6448536						

The expected return and volatility are taken from the optimal portfolio and confidence level of 95% is given. The time(days) and Asset value are assumed.

	A	B	C	D
372				
373		Average Daily Return		
374		Variance(daily)		
375		Average Annual Return		
376		Annual Variance		
377				
378		Calculating VaR		
379		Expected Return	11.50%	
380		Expected Volatility	23%	
381		Time(days)	5	
382		Confidence Level	0.95	
383		Stress Event($Z(1-\alpha)$)	-1.6448536	
384				
385		VaR	0.06171072	
386		VaR(£)	6171.0723	

Thus, the Value at Risk (5%) of the portfolio is 6171.0723 GBP for an asset value of 100000. The contribution of each stock to this value was obtained using the individual weight contribution of each stock to the optimal portfolio as seen below:

[illegible]

Thus, the contribution of all stocks to Value at Risk of the portfolio was obtained as follows:

388	Contribution to VaR	£	
389	AstraZeneca	2908.5544	
390	Unilever	158.09188	
391	Rio Tinto	2432.9744	
392	HSBC	101.51616	
393	Diageo	569.93544	

It is noteworthy that the majority of the contribution to the VaR is coming from two stocks namely, AstraZeneca and Rio Tinto while HSBC contributes the least to VaR.

- Now, finally a single asset namely Rio Tinto was chosen to estimate its volatility using ARCH/GARCH models and their extensions in R. **To decide whether to use ARCH or GARCH, the values of annualized returns and standard deviation are squared and checked for equality. If they are equal, ARCH is preferred and if they are not equal GARCH is the go-to model. In this case, the values were found to be unequal and hence, GARCH model was chosen.**

Initially, the returns of Rio Tinto were copied into a separate csv file and that file was read into R.

E9			
	A	B	
1	StockReturns		
2	-0.028798666		
3	0.019402898		
4	-0.01348197		
5	-0.044985065		
6	0.024926772		
7	-0.024940403		
8	0.025745602		
9	0.068419172		
10	0.016637158		
11	0.03574684		
12	-0.021875519		
13	-0.014664317		
14	-0.004922366		
15	0.00158464		
16	0.002825377		
17	0.012515605		
18	0.001970474		
19	-0.001788345		
20	0.045038524		
21	0.002623215		
22	0.019094968		
23	-0.001905864		
24	0.003058573		
25	0.010430824		
26	-0.002889046		
27	0.007763821		
28	0.048568762		

The following R packages and their respective libraries were installed to implement the GARCH models and their extensions:

- tseries
- fGarch
- rugarch

Initially, the garch models under tseries library were implemented and the results were compared using AIC as shown below:

```
> AIC(g11,g20,g21,g22,g02,g12)
      df      AIC
g11   3 -2400.077
g20   3 -2350.052
g21   4 -2394.138
g22   5 -2395.336
g02   3 -2369.347
g12   4 -2397.457
>
> AIC(g11,g20,g21,g22,g02,g12,k=log(3586))
      df      AIC
g11   3 -2381.522
g20   3 -2331.497
g21   4 -2369.399
g22   5 -2364.412
g02   3 -2350.793
g12   4 -2372.718
```

It can be noted that the model g11 or GARCH (1,1) has the least AIC and degrees of freedom and thus is the best model time series model. Its summary was then obtained as follows:

```
> summary(g11)

Call:
garch(x = ts(Return), order = c(1, 1), trace = FALSE)

Model:
GARCH(1,1)

Residuals:
    Min       1Q   Median       3Q      Max
-3.6847 -0.4769  0.0235  0.5729  3.7428

Coefficient(s):
      Estimate Std. Error t value Pr(>|t|)
a0 1.816e-05   1.182e-05   1.537  0.1243
a1 6.056e-02   2.674e-02   2.265  0.0235 *
b1 9.115e-01   3.836e-02  23.759 <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
  Jarque Bera Test

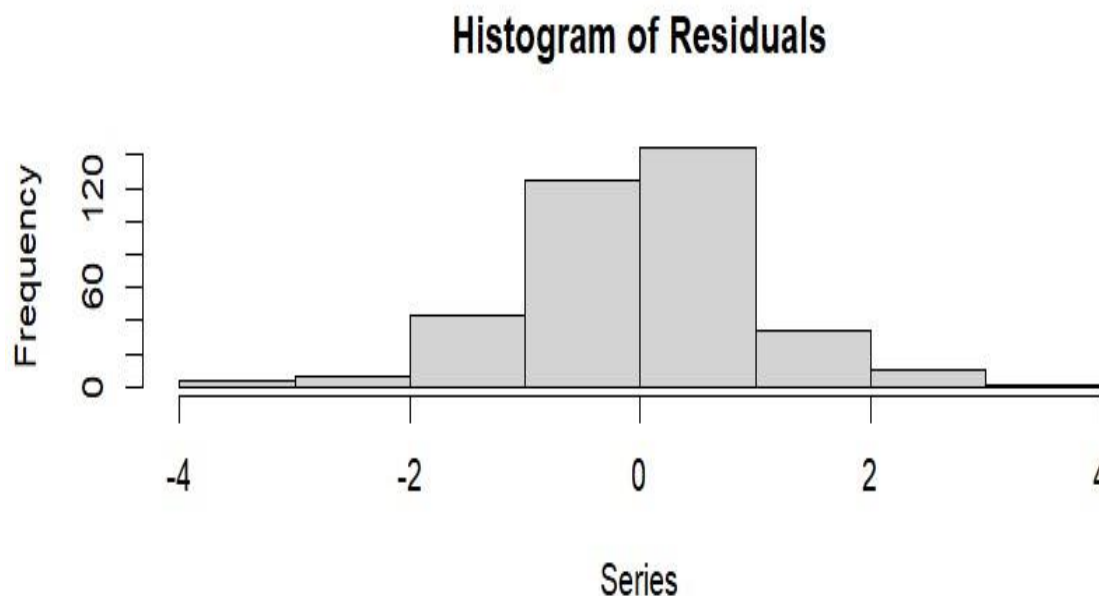
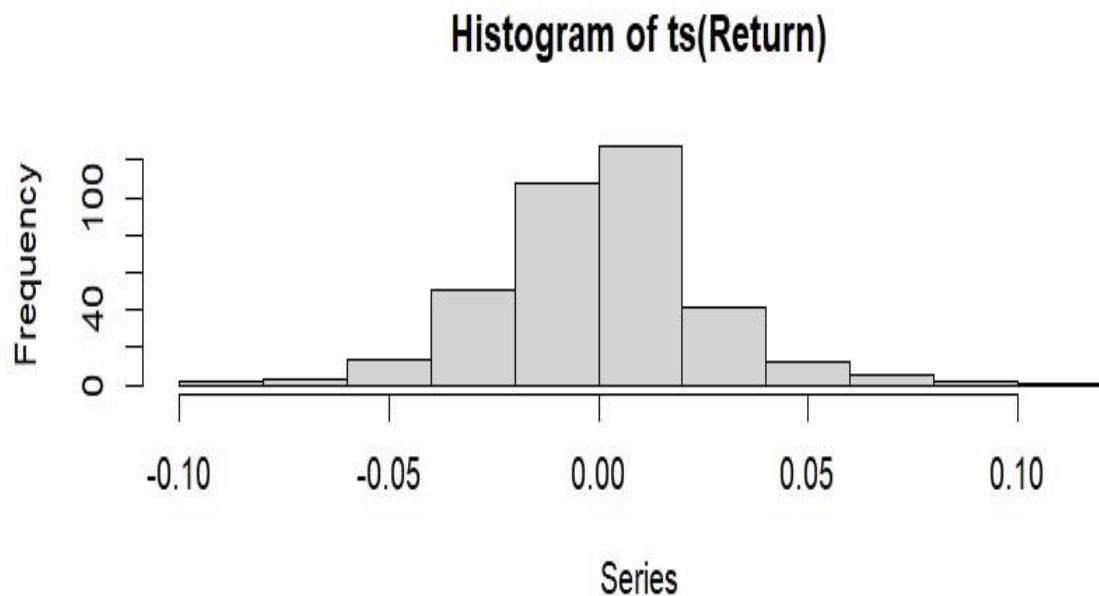
data: Residuals
X-squared = 27.984, df = 2, p-value = 8.382e-07

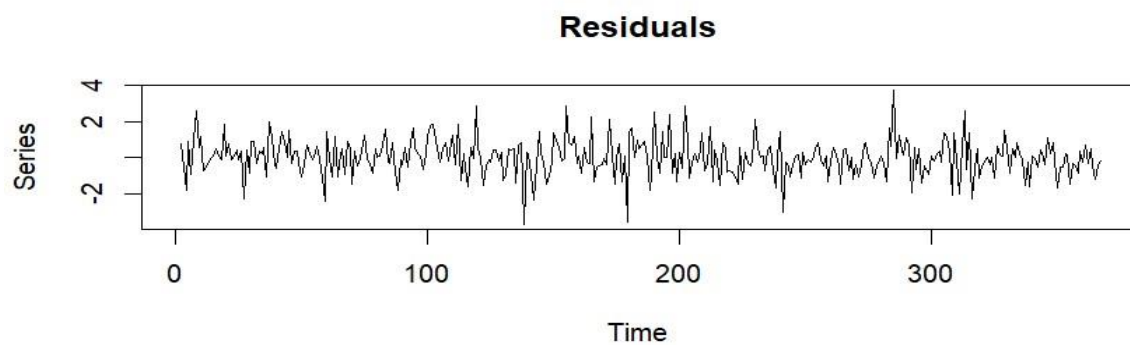
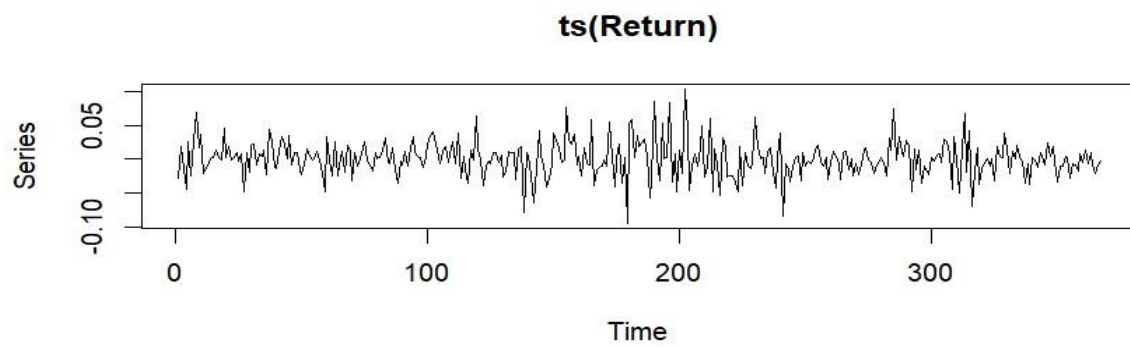
Box-Ljung test

data: Squared.Residuals
X-squared = 1.2223, df = 1, p-value = 0.2689
```

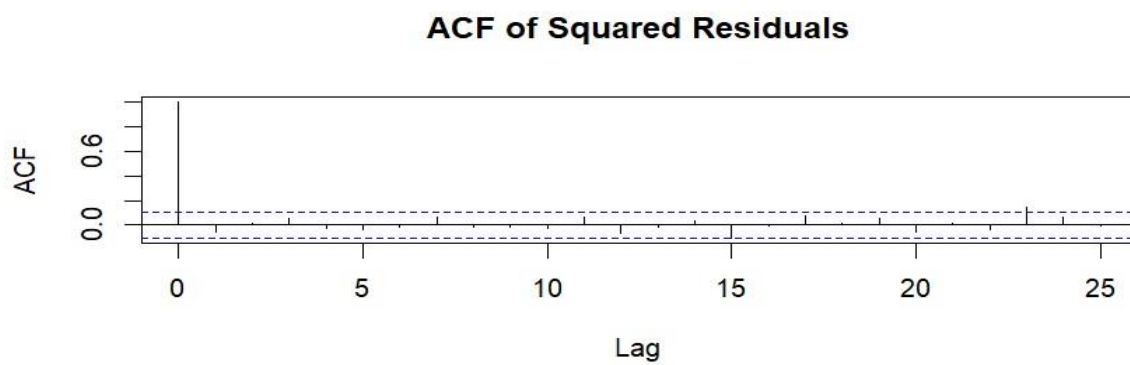
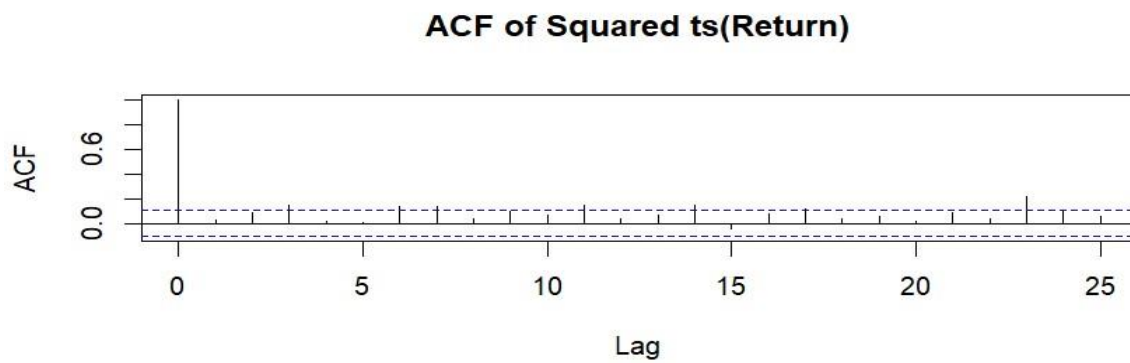
The above model has three fitted parameters namely, a_0 which is very small and negligible, a_1 which is 0.00605 and b_1 which is 0.9115. The t-values are very high and p-values are pretty small. It has two diagnostic tests. The Jarque Bera test has very small p-value, hence the null hypothesis that the skewness and kurtosis are zero is rejected. The Box-Ljung test has a p-value of 0.2689. Hence, the null hypothesis that the squared residuals of the fitted model have no auto correlation was accepted.

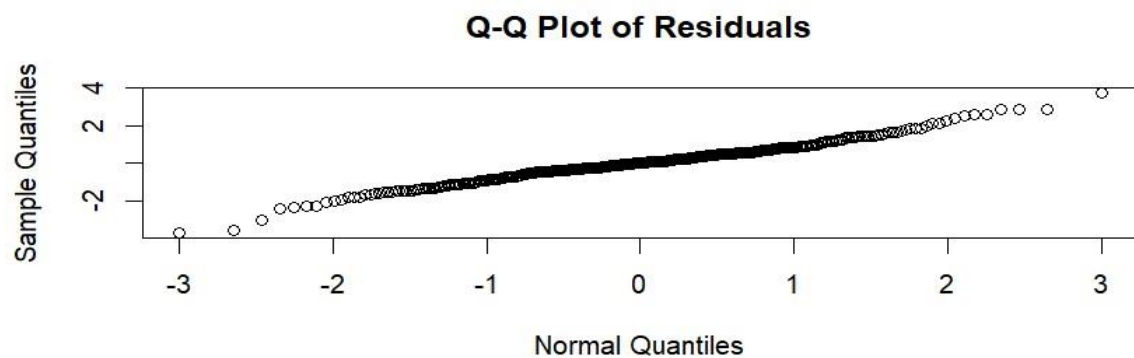
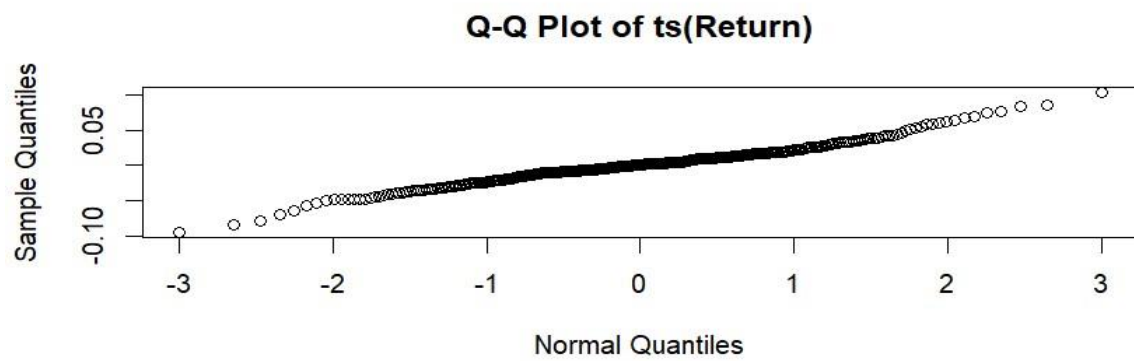
After this, the `plot(g11)` R command gives the following plots:



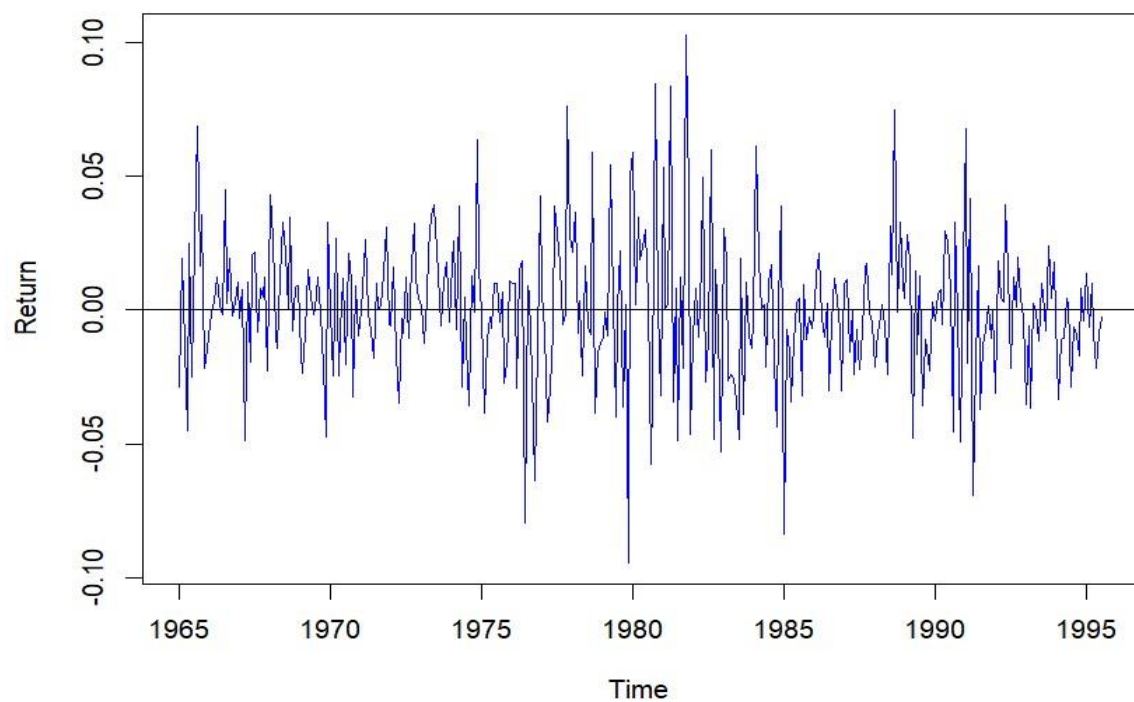


The R command `plot(predict(g11))` gave the following results:





Now, the function `garchFit()` in package `fGarch` was used to fit a model. Initially, the command `predict(g1)` gave the following output:



And then the summary of the model was obtained as seen below:

```
> summary(g1)

Title:
GARCH Modelling

Call:
garchFit(formula = ~garch(1, 1), data = Return, trace = FALSE)

Mean and Variance Equation:
data ~ garch(1, 1)
<environment: 0x0000026c8bf0d358>
[data = Return]

Conditional Distribution:
norm

Coefficient(s):
      mu      omega    alpha1    beta1
2.4080e-04 1.7966e-05 6.0012e-02 9.1236e-01

Std. Errors:
based on Hessian

Error Analysis:
      Estimate Std. Error t value Pr(>|t|)
mu      2.408e-04 1.272e-03   0.189 0.84986
omega   1.797e-05 1.114e-05   1.613 0.10677
alpha1  6.001e-02 2.229e-02   2.693 0.00708 **
beta1   9.124e-01 2.972e-02  30.696 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
829.9225    normalized: 2.261369

Standardised Residuals Tests:

      Statistic      p-Value
Jarque-Bera Test  R    Chi^2 27.5854105 1.023067e-06
Shapiro-Wilk Test R     W      0.9841097 4.580606e-04
Ljung-Box Test   R    Q(10)  4.9059448 8.973717e-01
Ljung-Box Test   R    Q(15) 12.2539632 6.597132e-01
Ljung-Box Test   R    Q(20) 17.5921783 6.142537e-01
Ljung-Box Test   R^2  Q(10)  5.6295817 8.453653e-01
Ljung-Box Test   R^2  Q(15) 13.5342037 5.611082e-01
Ljung-Box Test   R^2  Q(20) 17.8985872 5.940888e-01
LM Arch Test     R     TR^2   8.5162474 7.435991e-01

Information Criterion Statistics:
      AIC      BIC      SIC      HQIC
-4.500940 -4.458375 -4.501174 -4.484027
```

This model has four fitted parameters with high t-values and small p-values. This package provides four tests for residuals testing whether all the parameters are simultaneously equal to zero or not.

Finally, the function `ugarchfit()` in package `rugarch` was used to fit a model. It gave the following outputs:

```
> m1
```

```
*-----*
*          GARCH Model Fit          *
*-----*
```

Conditional Variance Dynamics

```
-----
GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(1,0,1)
Distribution      : norm
```

Optimal Parameters

```
-----
      Estimate Std. Error  t value Pr(>|t|)
mu      0.000244   0.001259   0.193472 0.846589
ar1     -0.013950   1.300081  -0.010730 0.991439
ma1      0.002319   1.299651   0.001785 0.998576
omega    0.000018   0.000011   1.593992 0.110938
alpha1   0.059351   0.022374   2.652709 0.007985
beta1    0.913161   0.030019  30.419118 0.000000
```

Robust Standard Errors:

```
      Estimate Std. Error  t value Pr(>|t|)
mu      0.000244   0.001450   0.168009 0.866576
ar1     -0.013950   0.350629  -0.039787 0.968263
ma1      0.002319   0.362634   0.006396 0.994897
omega    0.000018   0.000011   1.656924 0.097535
alpha1   0.059351   0.019802   2.997189 0.002725
beta1    0.913161   0.023224  39.319259 0.000000
```

LogLikelihood : 829.9452

Information Criteria

```
-----
Akaike      -4.4902
Bayes       -4.4263
Shibata     -4.4907
Hannan-Quinn -4.4648
```

Weighted Ljung-Box Test on Standardized Residuals

```

-----
                statistic p-value
Lag[1]          0.02505  0.8742
Lag[2*(p+q)+(p+q)-1][5]  1.89075  0.9750
Lag[4*(p+q)+(p+q)-1][9]  2.86925  0.9095
d.o.f=2
H0 : No serial correlation

```

Weighted Ljung-Box Test on Standardized Squared Residuals

```

-----
                statistic p-value
Lag[1]          1.288  0.2564
Lag[2*(p+q)+(p+q)-1][5]  2.222  0.5669
Lag[4*(p+q)+(p+q)-1][9]  3.406  0.6904
d.o.f=2

```

Weighted ARCH LM Tests

```

-----
Statistic Shape Scale P-Value
ARCH Lag[3]    1.094 0.500 2.000 0.2957
ARCH Lag[5]    1.599 1.440 1.667 0.5668
ARCH Lag[7]    2.558 2.315 1.543 0.6005

```

Nyblom stability test

Joint Statistic: 1.7063

Individual Statistics:

```

mu      0.60091
ar1     0.07460
ma1     0.07426
omega   0.15131
alpha1  0.15058
beta1   0.19317

```

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.49 1.68 2.12

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

```

-----
                t-value  prob sig
Sign Bias      0.3423 0.7323
Negative Sign Bias 1.4271 0.1544
Positive Sign Bias 0.3579 0.7206
Joint Effect    2.3213 0.5084

```

Adjusted Pearson Goodness-of-Fit Test:

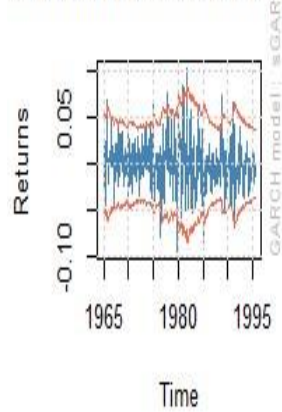
```

-----
group statistic p-value(g-1)
1  20  32.95  0.02439
2  30  39.68  0.08938
3  40  56.43  0.03501
4  50  54.39  0.27676

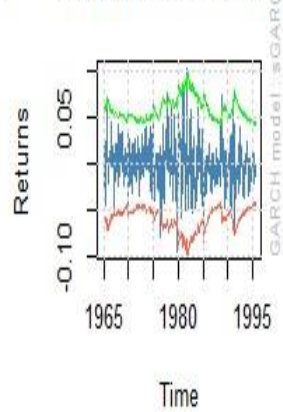
```

Elapsed time : 0.09292912

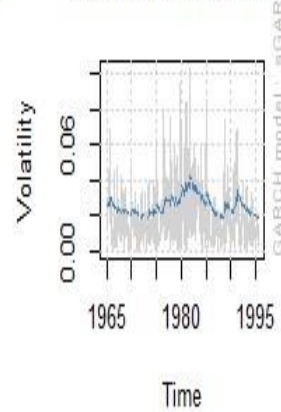
Series with 2 Conditional SD Superimp



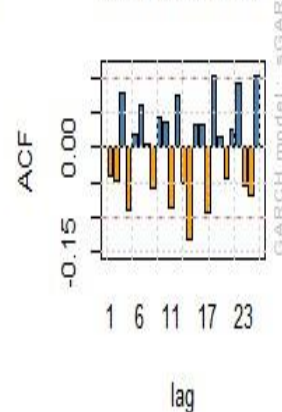
Series with with 1% VaR Limits



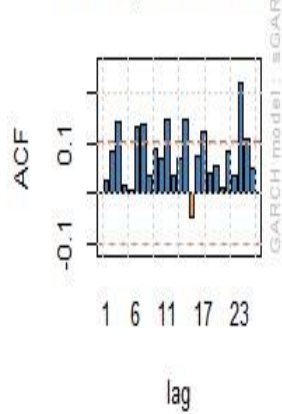
Conditional SD (vs |returns|)



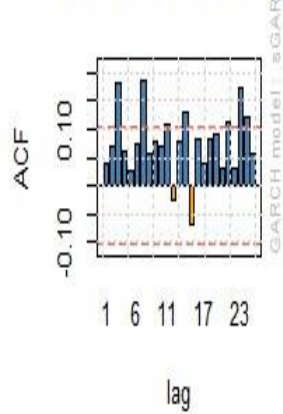
ACF of Observations



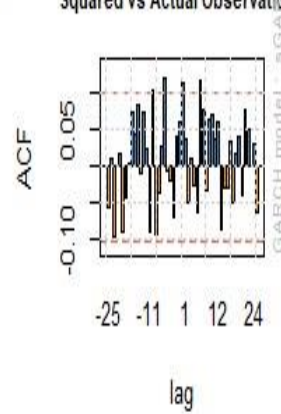
ACF of Squared Observations



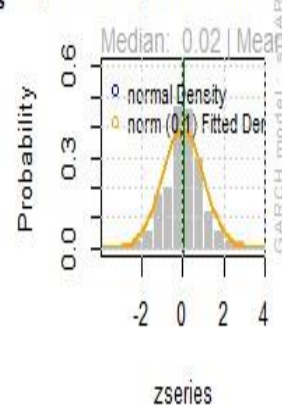
ACF of Absolute Observations



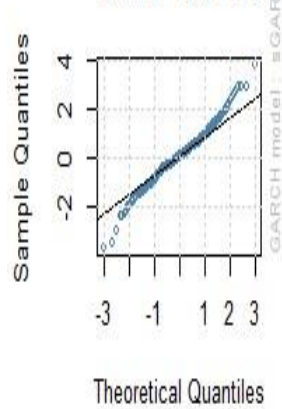
Cross-Correlations of Squared vs Actual Observations



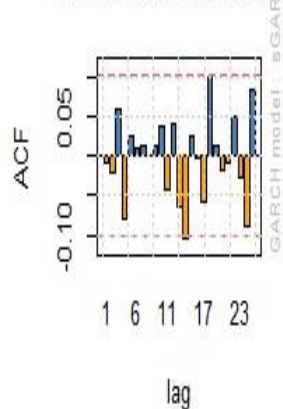
Empirical Density of Standardized Resi



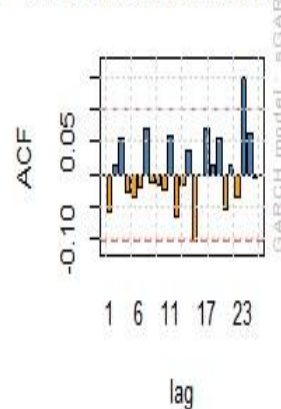
norm - QQ Plot



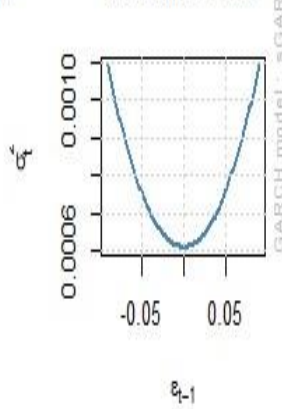
ACF of Standardized Residuals



ACF of Squared Standardized Residu



News Impact Curve



6. Conclusion:

What are the stocks that I should include in my portfolio to get the best possible returns? This is the billion-dollar question every potential investor has in mind while buying stocks. This case study provides useful insights to help investors maximize their returns in the stock market for a certain amount of risk.

Initially, the investor can select any random stocks of choice to be included in the portfolio. Then, the returns and volatility of each stock compared to the market over a certain period of time (typically a year) should be calculated. If these values are positive, then the portfolio is bound to give positive returns as compared to the market. If any stock has negative returns, it can be replaced with a stock having positive average returns. The correlation between the asset returns can also be calculated using the Variance-Covariance matrix as done in section 1 of this case study. This helps the investor understand which stock gives better returns compared to the portfolio itself. The Capital Market Line should be obtained with the aid of the efficient frontier curve for the portfolio which is an accurate measure of the risk one needs to take to get the maximum possible returns. The CML is a pivotal measure used nowadays for portfolio optimization.

The *beta* value of a stock identifies the amount of risk the investor has to take to invest in the stock as compared to the market. If the *beta* value is greater than 1, the stock is riskier than the market and should be replaced with a stock having *beta* value less than 1.

The software R Studio can also be utilized to find the best efficient portfolio by implementing the ARCH/GARCH models and its extensions which are used to find the volatility of an asset.

All in all, the stocks included in the portfolio in this case study can be considered good for investment as all the stocks are less risky than the market as a whole and have decent expected returns.

Any potential investor should take into account all the above performance measurements to determine the best efficient portfolio.