Before Taking an Undergraduate Math Course

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1 Introduction

The jump from high school to college level mathematics is often difficult. The focus shifts away from the rote computations in algebra and calculus toward conceptual overhead (multivariable calculus, linear algebra, differential equations) and proof-writing (discrete math, real analysis). Consequently, best practices for studying change, toughening the adjustment period for engineering students. This note contains tips that I wish I had before taking my first undergraduate mathematics courses. To be fair, many of them are "obvious" in hindsight, but it is easy to let go of discipline when you are faced with the stress of assignments and exams.

2 General Tips

1. **Treat definitions as you would in a memorization course.** You will be exposed to much more conceptual material, particularly "definitions" and "theorems", rather than methods of computing things. Memorize these as if they were dates in a history course or reactions in a chemistry course. Using flash cards in a math course was definitely foreign to me, but the bottom line is that you will be tested on manipulating these definitions and theorems. You will struggle if the statement of these definitions are not already loaded in your brain.

2. **Completely separate the "learning" phase and the "practice" stage of studying.** Many students that I have tutored try to learn a concept by doing problems. It might seem sensible at first; the exam is going to contain problems, and if I build up an arsenal of problem solving strategies, I will do well. However, many professors like to test the "limit" of understanding rather than the "core". A significant portion of exams can be dedicated to material that you have never seen before, but should be able to figure out if you apply the concepts from the course. This motivates having a phase 1 were you do *not* focus on problems at all, but focus only on concepts. Why is this theorem true? How was this formula derived? This usually involves reading a chapter in the textbook, or slowly going through course slides. After that, you start problems, and the problem-solving techniques will come naturally. Starting by memorizing problem-solving strategies will give you an inflexible grasp of the subject, and paralyze you on unseen problems.

3. **Distinguish between formulas, definitions, and intuitions.** Why is 0! (0 factorial) equal to 1? Using the formula this is not very helpful, but think about the following (intuitive) definition:

   $$n! = \text{The number of ways to order a list of } n \text{ items}$$

   How many ways are there to form an empty (0 item) list? One - a blank page. The definitions of mathematical objects and operations are not always equivalent to the way that you calculate them. The question to as is: what is this object trying to model?

4. **Be prepared to answer the following question.**

   What is a { *insert mathematical object here* }?

   This might seem particularly obvious, but for many students that I've worked with, this simple question will yield vague, possibly in the ballpark answers behind a wall of stuttering. This is actually not about reciting the definition, but a somewhat "extended' definition that I outline in the next section.

3 What is a ____?

I will ask this question very, very often to make sure everyone is studying (not just practicing), and tightening their conceptual understanding. There are 3 necessary parts to answering this question, and we will use four examples to illustrate this point.

1. Probability: What is an event?
2. Linear Algebra: What is a kernel/nullspace?
3. Calculus: What is a derivative?

3.1 What type of mathematical object is it?

Is it a set, a number, a vector, a function, a matrix, an integer, or something else?

1. An event is a set.
2. A kernel is a set.
3. A derivative is a function.

Sometimes, we might want to specify the type further (set of what?). Similarly, when defining functions, we tend to say what the input type and output type of the function is. (Ex: the function f takes in any real number x in $(-\infty, \infty)$ and outputs another real number y in the interval $[0, 1]$.

3.2 Who/what does the object belong to?

This is the part that is often forgotten. Objects can be properties, derivations, or extensions of other objects. Here, you might want to use variables to name the objects to not get confused.

1. An event E *of an experiment* $\mathcal{E}$ is a set that is...
2. A kernel $\ker A$ *of a matrix* A is a set of vectors such that...
3. A derivative $f'(x)$ *of a function* $f(x)$ is also a function, defined by...

Some objects do not belong to any other object, and in those cases you can skip this part.

3.3 What restrictions are on this object?

Finally, this is the more familiar part of the definition. Students generally skip to this, without saying what the object is or who it belongs to. If the definition is written, it is useful to add the notation.

1. An event E of an experiment $\mathcal{E}$ is a set that is a subset of $\mathcal{E}$'s sample space.
2. A kernel of a matrix A (or $\ker A$) is a set of vectors such that each vector x solves $Ax = 0$.
3. A derivative $f'(x)$ of a function $f(x)$ is also a function, equal to the following limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

While these sentences might sound awkward grammatically, and can be condensed, they are written to explicitly contain the three parts of the answer. If you can answer this question about every object/concept you run into in a math course, as far as studying goes, you will be golden. Good luck!