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MA Assignment-4

I have attached the screenshots for my written work

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(A1) Consider the Cobb-douglas equation & the production function in the Solow Model

a) i) We know that the equation describing the evolution of capital stock i.e k with the per unit of effective labour is of the form

$k = sf(k) - (n+g+\delta)k$ — (1)

From Cobb-douglas equation, we have $f(k) = k^{\alpha}$

Putting, the value of $f(k)$ in (1),

$\Rightarrow k = sk^{\alpha} - (n+g+\delta)k$

Since, the growth rate is balanced here, k turns out to be zero. It means the investment being done per unit effective labour equates to the break-even investment per unit of effective labour. Therefore, $sk^{*\alpha} = (n+g+\delta)k^{*}$

$\Rightarrow \cancel{k^{*} = sk^{\alpha} = (n+g+\delta)k}$

$\Rightarrow k^{*} = \left[\frac{s}{n+g+\delta}\right]^{\frac{1}{1-\alpha}}$ → Answer by rearranging the above equation terms

ii) For getting the balanced growth rate output per unit of effective labour, we can substitute into $y = k^{\alpha}$

$$\therefore \quad y^* = \left[\frac{s}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}} \longrightarrow \text{Answer}$$

iii) We know that consumption per unit of effective labour on the balanced growth path is given by $c^* = y^*(1-s)$

Since, we know y^* value from above equation, we have

$$c^* = \left[\frac{s}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}}(1-s)$$

$$= (1-s)\left[\frac{s}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

b) The golden-rule value of the capital stock is that level at which the level at which the consumption per unit of effective labour gets maximised.

From previous parts, we know that

$$k^* = \left[\frac{s}{(n+g+\delta)}\right]^{\frac{1}{1-\alpha}}$$

$$\Rightarrow \quad s = (n+g+\delta)\,k^{*(1-\alpha)} \quad \text{—} \quad 1$$

Also, $c^* = (1-s)\left[\dfrac{s}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}}$

Putting 1, in c^*, we have

$$c^* = \left[1 - (n+g+\delta)\, k^{*(1-\alpha)}\right] \cdot \left[\frac{(n+g+\delta)\, k^{*(1-\alpha)}}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

solving this, we have

$$c^* = k^{*\alpha} - (n+g+\delta)\, k^* \quad \text{—} \quad (2)$$

Now, we would maximize the equation (2) w.r.t to k^*. Differentiating w.r.t k^*

$$\therefore \frac{\partial c^*}{\partial k^*} = \alpha k^{*\alpha-1} - (n+g+\delta)$$

For getting the maxima, we equate this value to zero

$$\Rightarrow \alpha k^{*\alpha-1} - (n+g+\delta) = 0$$

$$\Rightarrow \alpha k^{*\alpha-1} = n+g+\delta \quad \text{—} \quad (3)$$

This equation (3) can be thought like a function $f'(k)$, a differential function which defines the golden-rule level of capital w.r.t effective labour.

solving for k^* from eq 3,

$$k^* = k^*_{\text{Golden Ratio}} = \left[\frac{\alpha}{n+g+\delta}\right]^{\frac{1}{1-\alpha}}$$

$\rightarrow$ Answer

c) We know that

$$s = (n+g+\delta)\, k^{*(1-\alpha)} \quad \& \quad k^* = \left[\frac{\alpha}{n+g+\delta}\right]^{\frac{1}{1-\alpha}}$$ from

previous answer

Putting k^* in equation;

$$s = (n+g+\delta)\left[\frac{\alpha}{n+g+\delta}\right]^{\frac{1-\alpha}{1-\alpha}}$$

$$= \left[\frac{\alpha}{\cancel{n+g+\delta}}\right]^{\cancel{\frac{1-\alpha}{1-\alpha}}}\left[\cancel{n+g+\delta}\right] = \alpha$$

$\therefore$ So, we can say that with Cobb-Douglas function the saving rate s for reaching the value of golden ratio is equal to output elasticity w.r.t capital.

(A2)

a) When δ decreases i.e the rate of depreciation falls, it affects the Break-Even-Investment lines. The Break-Even-Investment lines become less steep as their value decreases. This can be understood as for any value of k_t, less investment is required to sustain its initial value. Therefore, the BGP value is increased when rate of depreciation i.e δ falls

b) When the rate of technological process increases, the lines become steeper. The effectiveness from an increase in a lower k_t & hence at initial values more investment is required for ~~k~~ k^t ~~to~~ to be constant. However, the k_{BGP} values decrease.

c) When α increases in the Cobb-douglas equation the actual investment tends to increase. If the share to the capital size, more investment is needed. This makes the value of capital at BGP to increase.

d) When workers put in more effort, the actual investment lines move upwards. However, this does not effect the labour effectiveness, therefore BEI remains to be constant. It can be visualized from CD Equation as in previous question

(A3) a) When the jump happens, the output per unit of effective labour falls immediately. This also makes the capital per unit of effective labour fall. These makes the values of $\bar{k} = \frac{K}{AL}$ to decrease. This also makes $\bar{y} = \frac{A_t}{AL}$ to fall.

This is majorly because the technological processes & the value of capital stock does not change instantly. From differentiating we can also see that $\frac{\partial \bar{k}}{\partial L}$ makes $\frac{\partial \bar{y}}{\partial L} < 0$

where $\bar{Y}$ = Output / Effective Labour (AL)

b) The one-time jump is only interpreted as an initial situation which does not effect the economy later. When the value of $\bar{k}$ decreases, the actual investment exceeds the break-even investment. The economy tends to be new saving & investment exceeds the negative causing effects like depreciation etc. This k^* rise's & eventually this leads to an increase in the output per unit of effective labour i.e Y.

c) The output per unit of effective labour will rise at the same rate until its reaches the original level before the jump at the critical point k^*, the investment per unit of effective labour (AL) just exceeds the technological process & the depreciation involved. When, k reaches to its original level of k^*, the output per unit of effective labour also would come to its original value & thus $Y = Y^*$ at this point.

Graphs for all parts

(From Book)