MATH 164, Optimization
Assignment 5

- **Due**: June 8 (Friday). Late homeworks will not be accepted.

1. Consider the linear program

$$\begin{aligned} \text{maximize} \quad & 2x_1 + x_2 \\ \text{subject to} \quad & 0 \leq x_1 \leq 5 \\ & 0 \leq x_2 \leq 7 \\ & x_1 + x_2 \leq 9. \end{aligned}$$

   Convert the problem to standard form and solve it using the simplex method.

2. Problem 20.4 from textbook.

3. Consider the problem

$$\begin{aligned} \text{minimize} \quad & \|\mathbf{x} - \mathbf{x}_0\|^2 \\ \text{subject to} \quad & \|\mathbf{x}\|^2 = 9, \end{aligned}$$

   where $\mathbf{x}_0 = [1, \sqrt{3}]^\top$. Find all points satisfying the Lagrange condition for the problem.

4. Consider the problem

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2}\|\mathbf{x}\|^2 \\ \text{subject to} \quad & \mathbf{a}^\top \mathbf{x} = b \\ & \mathbf{x} \geq \mathbf{0}, \end{aligned}$$

   where $\mathbf{a} \in \mathbb{R}^n$, $\mathbf{a} \geq \mathbf{0}$, and $b \in \mathbb{R}$, $b > 0$. Show that if a solution (or global minimizer) to the problem exists, then it is unique, and find an expression for it in terms of $\mathbf{a}$ and b. (Hint: show that the KKT point is unique.)

5. Problem 23.4 from textbook.