# MATH 164 Optimization
# Assignment 3

- **Due**: Monday, May 14. Late homeworks will not be accepted.

1. Suppose $Q \succ 0$ is of $n \times n$ size. Given a set of linearly independent vectors $\{\mathbf{p}^0, \ldots, \mathbf{p}^{n-1}\}$ in $\mathbb{R}^n$, perform the Gram-Schmidt procedure as follows:

$$\begin{aligned} \mathbf{d}^0 &= \mathbf{p}^0, \\ \mathbf{d}^{k+1} &= \mathbf{p}^{k+1} - \sum_{i=0}^{k} \frac{(\mathbf{p}^{k+1})^\top Q \mathbf{d}^i}{(\mathbf{d}^i)^\top Q \mathbf{d}^i} \mathbf{d}^i, \quad \forall k = 0, \ldots, n-2. \end{aligned}$$

   Show that $\mathbf{d}^0, \ldots, \mathbf{d}^{n-1}$ are $Q$-conjugate.

2. Problem 11.1 from textbook.

3. Let $Q \succ 0$. Suppose the nonzero vectors $\mathbf{d}^0, \ldots, \mathbf{d}^{n-1}$ are $Q$-conjugate, show that they are linearly independent.

4. Consider the algorithm

$$\mathbf{x}^{k+1} = \mathbf{x}^k - \alpha_k P_k \nabla f(\mathbf{x}^k),$$

   where $f : \mathbb{R}^2 \to \mathbb{R}$, $P_k = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$ with $a \in \mathbb{R}$, and

$$\alpha_k = \arg\min_{\alpha \geq 0} f(\mathbf{x}^k - \alpha P_k \nabla f(\mathbf{x}^k)).$$

   Suppose that at some iteration $k$, we have $\nabla f(\mathbf{x}^k) = [1, 2]^\top$. Find the largest range of values of $a$ that guarantees that $\alpha_k > 0$ for any $f$.