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OPTIMIZATION OF PRELIMINARY DESIGN OF AXIAL FLOW TURBINES FOR TURBOPUMP APPLICATIONS BASED ON REDUCED ORDER METHODOLOGY

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OPTIMIZATION OF PRELIMINARY DESIGN OF AXIAL FLOW TURBINES FOR TURBOPUMP APPLICATIONS BASED ON REDUCED ORDER METHODOLOGY

Ronaldo Chaves Reis

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ITA

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Ah, já estava esquecendo... agradeço também, mais uma vez ao TEX, por ele não possuir vírus de macro :-)

"Foguete do tipo NASA saindo da atmosfera,
Tem turbina nessa raba e agora ninguém me pega"
— Pablo Vittar

Resumo

Aqui começa o resumo do referido trabalho. Não tenho a menor idéia do que colocar aqui. Sendo assim, vou inventar. Lá vai: Este trabalho apresenta uma metodologia de controle de posição das juntas passivas de um manipulador subatuado de uma maneira subótima. O termo subatuado se refere ao fato de que nem todas as juntas ou graus de liberdade do sistema são equipados com atuadores, o que ocorre na prática devido a falhas ou como resultado de projeto. As juntas passivas de manipuladores desse tipo são indiretamente controladas pelo movimento das juntas ativas usando as características de acoplamento da dinâmica de manipuladores. A utilização de redundância de atuação das juntas ativas permite a minimização de alguns critérios, como consumo de energia, por exemplo. Apesar da estrutura cinemática de manipuladores subatuados ser idêntica a do totalmente atuado, em geral suas caraterísticas dinâmicas diferem devido a presença de juntas passivas. Assim, apresentamos a modelagem dinâmica de um manipulador subatuado e o conceito de índice de acoplamento. Este índice é utilizado na sequência de controle ótimo do manipulador. A hipótese de que o número de juntas ativas seja maior que o número de passivas ($n_a > n_p$) permite o controle ótimo das juntas passivas, uma vez que na etapa de controle destas há mais entradas (torques nos atuadores das juntas ativas), que elementos a controlar (posição das juntas passivas).

Abstract

Well, the book is on the table. This work presents a control methodologie for the position of the passive joints of an underactuated manipulator in a suboptimal way. The term underactuated refers to the fact that not all the joints or degrees of freedom of the system are equipped with actuators, which occurs in practice due to failures or as design result. The passive joints of manipulators like this are indirectly controlled by the motion of the active joints using the dynamic coupling characteristics. The utilization of actuation redundancy of the active joints allows the minimization of some criteria, like energy consumption, for example. Although the kinematic structure of an underactuated manipulator is identical to that of a similar fully actuated one, in general their dynamic characteristics are different due to the presence of passive joints. Thus, we present the dynamic modelling of an underactuated manipulator and the concept of coulpling index. This index is used in the sequence of the optimal control of the manipulator.

List of Figures

List of Tables

List of Abbreviations and Acronyms

CTq	computed torque
DC	direct current
EAR	Equação Algébrica de Riccati
GDL	graus de liberdade
ISR	interrupção de serviço e rotina
LMI	linear matrices inequalities
MIMO	multiple input multiple output
PD	proporcional derivativo
PID	proporcional integrativo derivativo
PTP	point to point
UARMII	Underactuated Robot Manipulator II
VSC	variable structure control

List of Symbols

a	Distância
$\mathbf{a}$	Vetor de distâncias
$\mathbf{e}_j$	Vetor unitário de dimensão n e com o j-ésimo componente igual a 1
$\mathbf{K}$	Matriz de rigidez
m_1	Massa do cumpim
δ_{k-k_f}	Delta de Kronecker no instante k_f

Contents

1

Introduction

1.1 Motivation

Gas turbines play an important role when it comes to propulsion systems for both aeronautical and space applications. Their role is also increasingly critical in delivering energy security, dynamically evening out the peaks and troughs in production from intermittent power sources such as wind, solar and hydroelectricity systems. Therefore, several countries including their thousands of companies, scientists and engineers have already dedicated lots of financial, temporal and computational resources to the development of turbines, seeking above all to improve their efficiency.

It is know that any gain in the efficiency of a gas turbine has direct effects on the reduction of fuel consumptioon and consequent financial savings, mainly when comes to axial flow turbines as they are present in several applications and have an exceptional high scalability. Such benefit can also be pursued and achieved if turbine preliminary design tools were more efficient and speeded up the conceptual phase of the its development. Then, there is a demand and need for tools of this type.

1.2 Thesis Structure

The chapter 1 aims to introduce the project, where are exposed a brief identification of the problem, overall scene and the motivation to solve it, in order to support and justify the need for this work.

In the following, chapter 2 puts in the general and specific objectives, theme identification and contributions of the present work.

Chapter 3 presents the literature review on two fronts, along the presentation of the nomenclature used. The first is the physics that describes the axial flow turbines and its design, which basically includes concepts of fluid and thermodynamics, based on a reduced order methodology. Then, the second presents a description of the genetic algorithms and their characteristics that try to mimic the concepts of natural and genetic evolution, as a tool to find optimal solutions of the problem.

Chapter 4 brings the methodology used for the development of the computational tool, with the definition of the optimization process.

Chapter 5 provides the results obtained from simulations, considering a turbine XX as a validation case. Then, an axial flow turbine is designed as beeing the optimal solution of the problem descripted in the previous chapter. Discussion and literature comparison also compoud this chapter, along the critical evaluation of the tool performance capability.

Chapter 6 and last delivers final consideration and proposes future work of the studied theme.

2

Objective

2.1 General Objective

The main objective of the study is to design a computational tool for optimization of preliminary design of axial flow turbines with focus on turbopump usage. Project aims to use a reduced order methodology on turbine physics and genetic algorithm to handle the optimization process.

The process to find the optimal solution will be guided by maximizing the pressure ratio through turbine stage and also maximizing the overall efficiency in the operational envelope of the axial flow turbine.

2.2 Theme Identification

2.3 Contribution of the Present Work

This computer-aided engineering software aimed to be developed in this work has already been developed by other companies and countries. However, as this theme has strategical and state sovereignty characteristics due to the multidisciplinary techonoligical dependencies, either this tool is restrictly available to be used in foreign countries were it was developed, either they are very expensive.

That stated, this tool has great potential to bring benefits to governamental organizations as Brazilian Air Force (FAB) and to companies and groups of the Defense Industrial

Base (BID) of Brazil.

3

Literature Review

3.1 Turbines

A meanline and simplified approach for turbomachine design can produce qualitative information of great value to the designers and engineers in the first phase of development with small computational cost as drawback. For instance, specific trends about turbine performance and operation ease the definition of machine basic geometry (dimensions and shapes), with low usage of temporal and computational resources.

The first phase consists of a detailed analysis of main requirements of the project. The engine cycle analysis provides information as inlet conditions, pressure ratio, mass flow and rotation. These variables are good candidates to be part of the inlet vector of a **black box** scheme.

The following relation of previous work are intended to be studied in this part of the dissertation:

1. (SARAVANAMUTTOO *et al.*, 2017)
2. ME-211 class notes
3. (OVSYANNIKOV; BOROVSKIY, 1971)
4. (BELYAEV E. N.; TCHERVAKOV, 1999)
5. (OLIVEIRA *et al.*, 2020)
6. (SCHOBEIRI, 2018)

7. (DENTON, 1993)

8. (MAIA *et al.*, 2019)

9. (DENTON; XU, 1998)

10. (CRAIG; COX, 1970)

11. (LEACH, 1983)

12. (KADHIM, 2018)

13. (WALLIS; DENTON, 1998)

14. (LEE *et al.*, 2018)

15. (NOH *et al.*, 2004)

16. (OHLSSON, 1962)

17. (CHO *et al.*, 2008)

18. (VARMA; SOUNDRANAYAGAM, 2012)

3.1.1 Loss Models

3.2 Optimization

The procedure of problem formulation starts on creating a mathematical model of the optimal design problem, in order to solve it by an optimization algorithm.

According to (DEB, 2012) about problem formulation, it is necessary to realize firstly the need of using an optimization in this specific problem. The author also illustrate that optimization algorithms are routinely used in aerospace design activities to minimize the overall weight, since every element or component adds to the overall weight of the aircraft or spacecraft. Thus, the first step of this problem formulation is already satisfied.

Then, the next step is to choose the important design variables associated with the turbine problem. As said before, these variables can be fluid inlet thermodynamics properties conditions, pressure ratio through turbine stage, mass flow and rotation, since they came from requirements of high order components after engine cycle analysis.

In addition, the formulation of optimal design problems involves other considerations, such as constraints, objective function, and variable bounds, as suggested by (DEB, 2012).

The figure 3.1 shows a sumarizing flowchart about the procedure illustrated previously. Once declared and defined the optimal design problem, it is chosen the suitable optimization algorithm.

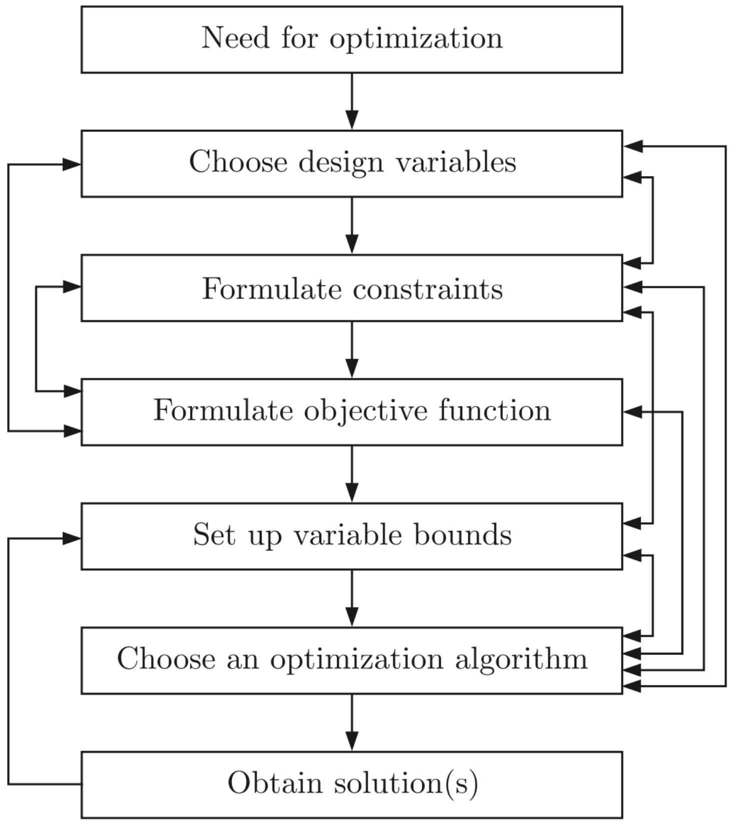

Figure 1.1 **A flowchart of the optimal design procedure.**

FIGURE 3.1 – Flowchart of the optimal design procedure. Source (DEB, 2012).

3.2.1 Root-Finding

Finding the roots of an equation is common to many engineering activities and many root-finding problems are parts of the optimization process. This problem can be solved using an optimization technique by suitably choosing an objective function, as suggested by (DEB, 2012). The problem of finding $f(x)$ roots could be converted to, for instance, minimizing $|f(x)|$ or minimizing $[f(x)]^2$. The former has an advantage about using derivative-based optimization methods.

Once the optimization problem is formed, a bracketing technique can be used to first bracket the root and then a region-elimination method or a gradient-based search method can be used to find the root with the desired accuracy (DEB, 2012).

3.2.2 Design Variables

3.2.2.1 Sensibility of design variables

How impactant can be a variable in relation to an objective function. It is possible that a variable is more sensitive to an aspect than others.

3.2.3 Objective Function

It is suggested by (DEB, 2012) the avoidance of several objective functions. Usually, it is selected the main of them and set it as unique objective function. Modification of the other functions comes along in order to force restrictions to the problem, bounding them into specific range, that was *a priori* analised.

3.2.4 Constraints

3.2.5 Design Variables Bounds

It is set the bounds for design variables based on the previous analysis, and considering that the optimal solution is inside this range. Then, it is analised the firsts results and reset the interval for more precise results.

3.2.6 Modeling

Normally, mathematical modeling of the optimization problem is not easily reachable. An alternative is modify the governing equations, mainly when exist experimental and observed values of the system in study. Thus, if E and/or any differential equations of E

describes a particular process, it is possible to add modification parameters and then a new equation as function of these parameters. The equation 3.1 describes a possible function of search space β (post-added parameters space), related to E observed and simulated. Such process has similarities with a linear regression techniques, where f represents the error or residue to be minimized.

$$f(\beta) = (E_{observed} - E_{simulated})^2 \tag{3.1}$$

As demonstred by (DEB, 2012), the addition of these artificial parameters can bring advantages in the modeling and understanding of the problem, in terms of insights.

The following relation of previous work are intended to be studied in this part of the dissertation:

1. (MENGISTU; GHALY, 2004)
2. (ASGARSHAMSI *et al.*, 2014)
3. (THORN; HARTFIELD, 2016)
4. (ARABNIA; GHALY, 2009)
5. (CERVANTES; HARTFIELD, 2018)
6. (JUANGPHANICH *et al.*, 2019)
7. (JUANGPHANICH *et al.*, 2017)
8. (AGROMAYOR; NORD, 2019)
9. (ABADI *et al.*, 2017)
10. (AMINYAVARI *et al.*, 2016)
11. (BA *et al.*, 2019)
12. (LÉONARD; ADAM, 2008)
13. (ÖKSüZ; AKMANDOR, 2010)
14. (ARABNIA, 2012)

15. (JENKINS, 1982)

16. (JENKINS, 1983)

17. (CERVANTES, 2018)

18. (SIVASHANMUGAM, 2011)

19. (MENGISTU, 2005)

20. (AMANO; XU, 2003)

21. (RAO; SAVSANI, 2012)

22. (DEB, 2012)

3.3 Problem Declaration

The optimization task will be accomplished with the use of genetic algorithms, while the preliminary design of the turbine will be simplified and one-dimensional.

It is intended to use a simpler case study in order to understand the operation of the optimization and familiarization with the method. Such a base example can be done considering only one design variable. After this phase, the process formulated as a whole should be resolved. It is intended to limit the number of design variables to X, which are easily to make them dimensionless for comparison purposes.

3.4 Genetic Algorithms

According to (DEB, 2012), genetic algorithms allow a easy way to find multiples optimal solutions simultaneously in a single simulation run. In the case of multiobjective optimization problems, a set of optimal solutions emerges known as Pareto Optimal Solutios or Pareto Frontier.

3.4.1 NSGA-II

The following relation of previous work are intended to be studied in this part of the dissertation:

1. (DEB *et al.*, 2002)

4

Methodology

Methodology

5

Discussion

Discussion

6

Conclusion

Conclusion

Bibliography

ABADI, S. N. R.; AHMADPOUR, A.; ABADI, S.; MEYER, J. CFD-based shape optimization of steam turbine blade cascade in transonic two phase flows. **Applied Thermal Engineering**, Elsevier BV, v. 112, p. 1575–1589, fev. 2017. Disponível em: <https://doi.org/10.1016/j.applthermaleng.2016.10.058>.

AGROMAYOR, R.; NORD, L. O. Preliminary design and optimization of axial turbines accounting for diffuser performance. **International Journal of Turbomachinery, Propulsion and Power**, MDPI AG, v. 4, n. 3, p. 32, set. 2019. Disponível em: <https://doi.org/10.3390/ijtpp4030032>.

AMANO, R. S.; XU, C. Aerodynamic blade optimal design of turbomachinery. In: **International Gas Turbine Congress**. [S.l.]: Gas Turbine Society of Japan, 2003.

AMINYAVARI, M.; MAMAGHANI, A. H.; SHIRAZI, A.; NAJAFI, B.; RINALDI, F. Exergetic, economic, and environmental evaluations and multi-objective optimization of an internal-reforming SOFC-gas turbine cycle coupled with a rankine cycle. **Applied Thermal Engineering**, Elsevier BV, v. 108, p. 833–846, set. 2016. Disponível em: <https://doi.org/10.1016/j.applthermaleng.2016.07.180>.

ARABNIA, M. **Aerodynamic Shape Optimization of Axial Turbines in Three Dimensional Flow**. Tese (Doutorado) — Concordia University, Montréal, Québec, Canada, mar. 2012.

ARABNIA, M.; GHALY, W. A strategy for multi-point shape optimization of turbine stages in three-dimensional flow. In: **Volume 7: Turbomachinery, Parts A and B**. ASMEDC, 2009. Disponível em: <https://doi.org/10.1115/gt2009-59708>.

ASGARSHAMSI, A.; BENISI, A. H.; ASSEMPOUR, A.; POURFARZANEH, H. Multi-objective optimization of lean and sweep angles for stator and rotor blades of an axial turbine. **Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering**, SAGE Publications, v. 229, n. 5, p. 906–916, jun. 2014. Disponível em: <https://doi.org/10.1177/0954410014541080>.

BA, W.; WANG, Z.; LI, X.; GU, C. Three-dimensional optimal design of a cooled turbine considering the coolant-requirement change. **Open Physics**, Walter de Gruyter GmbH, v. 17, n. 1, p. 768–778, dez. 2019. Disponível em: <https://doi.org/10.1515/phys-2019-0080>.

BELYAEV E. N., T. V. K.; TCHERVAKOV, V. V. **Working Processes of Liquid Propellant Rocket Engines**. [S.l.]: MAI, Moscow, 1999.

CERVANTES, N. **An Evolutionary Strategies Method to Optimize Turbine and Compressor Blades**. Dissertação (Mestrado) — Auburn University, Auburn, Alabama, maio 2018.

CERVANTES, N.; HARTFIELD, R. J. An evolutionary strategies method for optimizing compressor and turbine blades. In: **2018 Joint Propulsion Conference**. American Institute of Aeronautics and Astronautics, 2018. Disponível em: <https://doi.org/10.2514/6.2018-4736>.

CHO, S.-Y.; CHO, C.-H.; KIM, C. Effect of solidities and nozzle flow angles on a partially admitted small axial-type turbine. **International Journal of Turbo and Jet Engines**, Walter de Gruyter GmbH, v. 25, n. 2, jan. 2008. Disponível em: <https://doi.org/10.1515/tjj.2008.25.2.111>.

CRAIG, H. R. M.; COX, H. J. A. Performance estimation of axial flow turbines. **Proceedings of the Institution of Mechanical Engineers**, SAGE Publications, v. 185, n. 1, p. 407–424, jun. 1970. Disponível em: <https://doi.org/10.1243/pime_proc_1970_185_048_02>.

DEB, K. **Optimization for Engineering Design - Algorithms and Examples**. [S.l.]: PHI Learning Private Limited, 2012. ISBN 978-81-203-4678-9.

DEB, K.; PRATAP, A.; AGARWAL, S.; MEYARIVAN, T. A fast and elitist multiobjective genetic algorithm: NSGA-II. **IEEE Transactions on Evolutionary Computation**, Institute of Electrical and Electronics Engineers (IEEE), v. 6, n. 2, p. 182–197, abr. 2002. Disponível em: <https://doi.org/10.1109/4235.996017>.

DENTON, J. D. Loss mechanisms in turbomachines. In: . American Society of Mechanical Engineers, 1993. Disponível em: <https://doi.org/10.1115/93-gt-435>.

DENTON, J. D.; XU, L. The exploitation of three-dimensional flow in turbomachinery design. **Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science**, SAGE Publications, v. 213, n. 2, p. 125–137, fev. 1998. Disponível em: <https://doi.org/10.1243/0954406991522220>.

JENKINS, J. M. A comprehensive method for preliminary design optimization of axial gas turbine stages. In: **18th AIAA/SAE/ASME Joint Propulsion Conference**. [S.l.]: American Institute of Aeronautics and Astronautics, 1982.

JENKINS, J. M. A comprehensive method for preliminary design optimization of axial gas turbine stages, part ii: Code verification. In: **19th AIAA/SAE/ASME Joint Propulsion Conference**. [S.l.]: American Institute of Aeronautics and Astronautics, 1983.

JUANGPHANICH, P.; MAESSCHALCK, C. D.; PANIAGUA, G. Turbine passage design methodology to minimize entropy production—a two-step optimization strategy. **Entropy**, MDPI AG, v. 21, n. 6, p. 604, jun. 2019. Disponível em: <https://doi.org/10.3390/e21060604>.

JUANGPHANICH, P.; MAESSCHALCK, C. G. D.; PANIAGUA, G. From conceptual 1d design towards full 3d optimization of a highly loaded turbine stage. In: **55th AIAA Aerospace Sciences Meeting**. American Institute of Aeronautics and Astronautics, 2017. Disponível em: <https://doi.org/10.2514/6.2017-0110>.

KADHIM, H. T. K. **Effect of non-axisymmetric casing on flow and performance of an axial turbine**. Tese (Doutorado) — University of Leicester, 2018.

LEACH, K. P. **Energy Efficient Engine High-Pressure Turbine Component Rig Performance Test Report**. maio 1983.

LEE, H. G.; SHIN, J. H.; CHOI, C.-H.; JEONG, E.; KWON, S. Partial admission effect on the performance and vibration of a supersonic impulse turbine. **Acta Astronautica**, Elsevier BV, v. 145, p. 105–115, abr. 2018. Disponível em: <https://doi.org/10.1016/j.actaastro.2018.01.025>.

LÉONARD, O.; ADAM, O. A quasi-one-dimensional CFD model for multistage turbomachines. **Journal of Thermal Science**, Springer Science and Business Media LLC, v. 17, n. 1, p. 7–20, mar. 2008. Disponível em: <https://doi.org/10.1007/s11630-008-0007-z>.

MAIA, A. A. G.; SILVA, J. F. da; TOMITA, J. T.; BRIGHENTI, C. Preliminary design of axial flow turbine for a small jet engine. In: **Proceedings of the 5th World Congress on Mechanical, Chemical, and Material Engineering**. Avestia Publishing, 2019. Disponível em: <https://doi.org/10.11159/htff19.189>.

MENGISTU, T.; GHALY, W. Single and multipoint shape optimization of gas turbine blade cascades. In: **10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference**. American Institute of Aeronautics and Astronautics, 2004. Disponível em: <https://doi.org/10.2514/6.2004-4446>.

MENGISTU, T. T. **Aerodynamic Design and Optimization of Turbomachinery Blading**. Tese (Doutorado) — Concordia University, Montréal, Québec, Canada, abr. 2005.

NOH, J.-G.; JEONG, E.-H.; LEE, E.-S.; KIM, J.-H. Numerical flow analysis of a partial admission turbine using a frozen rotor method. **Journal of Fluid Machinery**, Korean Fluid Machinery Association, v. 7, n. 6, p. 15–20, dez. 2004. Disponível em: <https://doi.org/10.5293/kfma.2004.7.6.015>.

OHLSSON, G. O. Partial-admission turbines. **Journal of the Aerospace Sciences**, American Institute of Aeronautics and Astronautics (AIAA), v. 29, n. 9, p. 1017–1023, set. 1962. Disponível em: <https://doi.org/10.2514/8.9686>.

ÖKSüZ Özhan; AKMANDOR, İ. S. Multi-objective aerodynamic optimization of axial turbine blades using a novel multilevel genetic algorithm. **Journal of Turbomachinery**, ASME International, v. 132, n. 4, maio 2010. Disponível em: <https://doi.org/10.1115/1.3213558>.

OLIVEIRA, I.; SILVA, G. P.; TONON, D.; BRINGHENTI, C.; TOMITA, J. T. Interactive learning platform for turbine design using reduced order methods. In: . American Society of Mechanical Engineers, 2020. Disponível em: <https://doi.org/10.1115/gt2020-16028>.

OVSYANNIKOV, B. V.; BOROVSKIY, B. I. Theory and calculation of feed units of liquid-propellant rocket engines. In: ______. Moscow: Moscow State Aviation Institute (Technical University), 1971. cap. 3, 4 and 5.

RAO, R. V.; SAVSANI, V. J. **Mechanical Design Optimization Using Advanced Optimization Techniques**. Springer London, 2012. Disponível em: <https://doi.org/10.1007/978-1-4471-2748-2>.

SARAVANAMUTTOO, H. I. H.; ROGERS, G. F. C.; COHEN, H.; STRAZNICKY, P. V.; NIX, A. C. **Gas Turbine Theory (7th Edition)**. Pearson, 2017. ISBN 1292093099. Disponível em: <https://www.xarg.org/ref/a/1292093099/>.

SCHOBEIRI, M. T. **Gas Turbine Design, Components and System Design Integration**. Springer International Publishing, 2018. Disponível em: <https://doi.org/10.1007/978-3-319-58378-5>.

SIVASHANMUGAM, V. K. **Three Dimensional Aero-Structural Shape Optimization of Turbomachinery Blades**. Dissertação (Mestrado) — Concordia University, Montréal, Québec, Canada, jan. 2011.

THORN, C. R.; HARTFIELD, R. J. Three dimensional turbine blade optimization using evolutionary algorithm with viscous flow analysis. In: **54th AIAA Aerospace Sciences Meeting**. American Institute of Aeronautics and Astronautics, 2016. Disponível em: <https://doi.org/10.2514/6.2016-0115>.

VARMA, A.; SOUNDRANAYAGAM, S. Experimental study of a small partial admission axial turbine with low aspect ratio blade. **Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering**, SAGE Publications, v. 228, n. 1, p. 20–34, nov. 2012. Disponível em: <https://doi.org/10.1177/0954410012466779>.

WALLIS, A. M.; DENTON, J. D. Comparison of design intent and experimental measurements in a low aspect ratio axial flow turbine with three-dimenisonal blading. In: **International Gas Turbine and Aeroengine Congress and Exibition**. [S.l.]: American Society of Mechanical Engineers, 1998.

Appendix A -

First appendix

A.1 First section of Appendix

if needed

Annex A -

First annex

A.1 First section of Annex

If needed.