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Ronaldo Chaves Reis

**OPTIMIZATION OF PRELIMINARY DESIGN OF
AXIAL FLOW TURBINES FOR TURBOPUMP
APPLICATIONS BASED ON REDUCED ORDER
METHODOLOGY**

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OPTIMIZATION OF PRELIMINARY DESIGN OF AXIAL FLOW TURBINES FOR TURBOPUMP APPLICATIONS BASED ON REDUCED ORDER METHODOLOGY

Ronaldo Chaves Reis

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ITA

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*"If I have seen farther than others,
it is because I stood on the shoulders of giants."*

— SIR ISAAC NEWTON

Resumo

Aqui começa o resumo do referido trabalho. Não tenho a menor idéia do que colocar aqui. Sendo assim, vou inventar. Lá vai: Este trabalho apresenta uma metodologia de controle de posição das juntas passivas de um manipulador subatuado de uma maneira subótima. O termo subatuado se refere ao fato de que nem todas as juntas ou graus de liberdade do sistema são equipados com atuadores, o que ocorre na prática devido a falhas ou como resultado de projeto. As juntas passivas de manipuladores desse tipo são indiretamente controladas pelo movimento das juntas ativas usando as características de acoplamento da dinâmica de manipuladores. A utilização de redundância de atuação das juntas ativas permite a minimização de alguns critérios, como consumo de energia, por exemplo. Apesar da estrutura cinemática de manipuladores subatuados ser idêntica a do totalmente atuado, em geral suas características dinâmicas diferem devido a presença de juntas passivas. Assim, apresentamos a modelagem dinâmica de um manipulador subatuado e o conceito de índice de acoplamento. Este índice é utilizado na sequência de controle ótimo do manipulador. A hipótese de que o número de juntas ativas seja maior que o número de passivas ($n_a > n_p$) permite o controle ótimo das juntas passivas, uma vez que na etapa de controle destas há mais entradas (torques nos atuadores das juntas ativas), que elementos a controlar (posição das juntas passivas).

Abstract

Well, the book is on the table. This work presents a control methodology for the position of the passive joints of an underactuated manipulator in a suboptimal way. The term underactuated refers to the fact that not all the joints or degrees of freedom of the system are equipped with actuators, which occurs in practice due to failures or as design result. The passive joints of manipulators like this are indirectly controlled by the motion of the active joints using the dynamic coupling characteristics. The utilization of actuation redundancy of the active joints allows the minimization of some criteria, like energy consumption, for example. Although the kinematic structure of an underactuated manipulator is identical to that of a similar fully actuated one, in general their dynamic characteristics are different due to the presence of passive joints. Thus, we present the dynamic modelling of an underactuated manipulator and the concept of coupling index. This index is used in the sequence of the optimal control of the manipulator.

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List of Abbreviations and Acronyms

CTq	computed torque
DC	direct current
EAR	Equação Algébrica de Riccati
GDL	graus de liberdade
ISR	interrupção de serviço e rotina
LMI	linear matrices inequalities
MIMO	multiple input multiple output
PD	proporcional derivativo
PID	proporcional integrativo derivativo
PTP	point to point
UARMII	Underactuated Robot Manipulator II
VSC	variable structure control

List of Symbols

a	Distância
\mathbf{a}	Vetor de distâncias
\mathbf{e}_j	Vetor unitário de dimensão n e com o j -ésimo componente igual a 1
\mathbf{K}	Matriz de rigidez
m_1	Massa do cumpim
δ_{k-k_f}	Delta de Kronecker no instante k_f

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1

Introduction

1.1 Motivation

Turbines play an important role when it comes to propulsion systems for both aeronautical and space and also power generation systems. Therefore, several countries including their thousands of companies, scientists and engineers have already dedicated lots of financial, temporal and computational resources to the development of turbines, seeking above all to improve their efficiency.

It is known that any gain in the efficiency of a gas turbine has direct effects on the reduction of fuel consumption and consequent financial savings, mainly when comes to axial flow turbines as they are present in several applications and have an exceptional high scalability. Such benefit can also be pursued and achieved if turbine preliminary design tools were more efficient and speeded up the conceptual phase of its development. Then, there is a demand and need for tools of this type.

1.2 Thesis Structure

The chapter 1 aims to introduce the project, where are exposed a brief identification of the problem, overall scene and the motivation to solve it, in order to support and justify the need for this work.

In the following, chapter 2 puts in the general and specific objectives, theme identification and contributions of the present work.

Chapter 3 presents the literature review on two fronts, along the presentation of the nomenclature used. The first is the physics that describes the axial flow turbines and its design, which basically includes concepts of fluid and thermodynamics, based on a reduced order methodology. Then, the second presents a description of the genetic algorithms and their characteristics that try to mimic the concepts of natural and genetic evolution, as a tool to find optimal solutions of the problem.

Chapter 4 brings the methodology used for the development of the computational tool, with the definition of the optimization process.

Chapter 5 provides the results obtained from simulations, considering a turbine XX as a validation case. Then, an axial flow turbine is designed as being the optimal solution of the problem described in the previous chapter. Discussion and literature comparison also compound this chapter, along the critical evaluation of the tool performance capability.

Chapter 6 and last delivers final consideration and proposes future work of the studied theme.

2

Objective

2.1 General Objective

The main objective of the study is to design a computational tool for optimization of preliminary design of axial flow turbines with focus on turbopump usage. Project aims to use a reduced order methodology on turbine physics and genetic algorithm to handle the optimization process.

The process to find the optimal solution will be guided by maximizing the pressure ratio through turbine stage and also maximizing the overall efficiency in the operational envelope of the axial flow turbine.

2.2 Theme Identification

2.3 Contribution of the Present Work

This computer-aided engineering software aimed to be developed in this work has already been developed by other companies and countries. However, as this theme has strategical and state sovereignty characteristics due to the multidisciplinary technological dependencies, either this tool is restrictly available to be used in foreign countries were it was developed, either they are very expensive.

That stated, this tool has great potential to bring benefits to governamental organizations as Brazilian Air Force (FAB) and to companies and groups of the Defense Industrial

Base (BID) of Brazil.

3

Literature Review

3.1 Turbines

A meanline and simplified approach for turbomachine design can produce qualitative information of great value to the designers and engineers in the first phase of development with small computational cost as drawback. For instance, specific trends about turbine performance and operation ease the definition of machine basic geometry (dimensions and shapes), with low usage of temporal and computational resources.

The first phase consists of a detailed analysis of main requirements of the project. The engine cycle analysis provides information as inlet conditions, pressure ratio, mass flow and rotation. These variables are good candidates to be part of the inlet vector of a **black box** scheme.

The following relation of previous work are intended to be studied in this part of the dissertation:

1. (SARAVANAMUTTOO *et al.*, 2017)
2. ME-211 class notes
3. (OLIVEIRA *et al.*, 2020)
4. (SCHOBELI, 2018)
5. (DENTON, 1993)
6. (MAIA *et al.*, 2019)

7. (DENTON; XU, 1998)
8. (CRAIG; COX, 1970)
9. (LEACH, 1983)
10. (KADHIM, 2018)
11. (WALLIS; DENTON, 1998)
12. (OVSYANNIKOV; BOROVSKIY, 1971)
13. (BELYAEV E. N.; TCHERVAKOV, 1999)

3.1.1 Loss Models

3.2 Optimization

The procedure of problem formulation starts on creating a mathematical model of the optimal design problem, in order to solve it by an optimization algorithm.

According to (DEB, 2012) about problem formulation, it is necessary to realize firstly the need of using an optimization in this specific problem. The author also illustrate that optimization algorithms are routinely used in aerospace design activities to minimize the overall weight, since every element or component adds to the overall weight of the aircraft or spacecraft. Thus, the first step of this problem formulation is already satisfied.

Then, the next step is to choose the important design variables associated with the turbine problem. As said before, these variables can be fluid inlet thermodynamics properties conditions, pressure ratio through turbine stage, mass flow and rotation, since they came from requirements of high order components after engine cycle analysis.

In addition, the formulation of optimal design problems involves other considerations, such as constraints, objective function, and variable bounds, as suggested by (DEB, 2012).

The figure 3.1 shows a summarizing flowchart about the procedure illustrated previously. Once declared and defined the optimal design problem, it is chosen the suitable optimization algorithm.

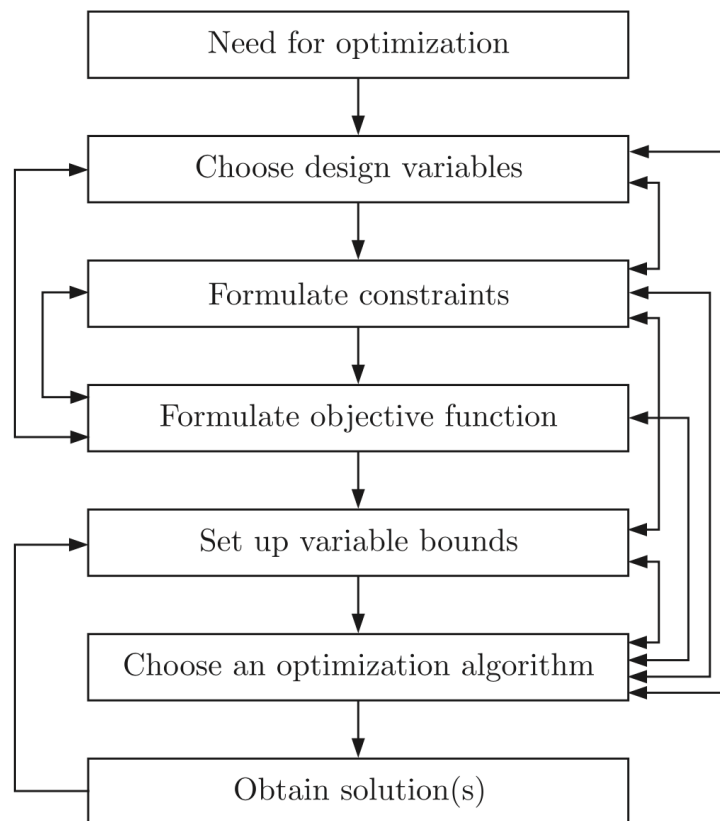


Figure 1.1 A flowchart of the optimal design procedure.

FIGURE 3.1 – Flowchart of the optimal design procedure. Source (DEB, 2012).

3.2.1 Design Variables

3.2.1.1 Sensibility of design variables

How impactant can be a variable in relation to an objective function. It is possible that a variable is more sensitive to an aspect than others.

3.2.2 Objective Function

It is suggested by (DEB, 2012) the avoidance of several objective functions. Usually, it is selected the main of them and set it as unique objective function. Modification of the other functions comes along in order to force restrictions to the problem, bounding them into specific range, that was *a priori* analised.

3.2.3 Constraints

3.2.4 Design Variables Bounds

It is set the bounds for design variables based on the previous analysis, and considering that the optimal solution is inside this range. Then, it is analysed the firsts results and reset the interval for more precise results.

3.2.5 Modeling

Normalmente, a modelagem matemática do problema de otimização não é facilmente atingível. Uma alternativa é fazer uma modificação nas equações governantes, principalmente quando se possui valores experimentais e observáveis do sistema. Assim, a formulação vai levar em consideração valores simulados *versus* valores observados, onde cria-se variáveis de projeto de maneira auxiliar (espaço de busca β , nesse caso), sendo a função objetivo $f(\beta)$ da equação 3.1, a qual deve ser minimizada. Tal processo tem muita similaridade com a metodologia de regressão linear, onde f representa o erro.

$$f(\beta) = (E_{observed} - E_{simulated})^2 \quad (3.1)$$

De acordo com [ref] Optimization for Engineering Design Algorithms and Examples, a inclusão de variáveis de projeto artificiais frequentemente com que o entendimento do problema seja simplificado, incluindo sua modelagem matemática.

Lista de referências pretendida para elaboração desta seção:

1. (MENGISTU; GHALY, 2004)
2. (ASGARSHAMSI *et al.*, 2014)
3. (THORN; HARTFIELD, 2016)
4. (ARABNIA; GHALY, 2009)
5. (CERVANTES; HARTFIELD, 2018)
6. (JUANGPHANICH *et al.*, 2019)

7. (JUANGPHANICH *et al.*, 2017)
8. (AGROMAYOR; NORD, 2019)
9. (ABADI *et al.*, 2017)
10. (AMINYAVARI *et al.*, 2016)
11. (BA *et al.*, 2019)
12. (LÉONARD; ADAM, 2008)
13. (ÖKSÜZ; AKMANDOR, 2010)
14. (ARABNIA, 2012)
15. (JENKINS, 1982)
16. (JENKINS, 1983)
17. (CERVANTES, 2018)
18. (SIVASHANMUGAM, 2011)
19. (MENGISTU, 2005)
20. (AMANO; XU, 2003)
21. (RAO; SAVSANI, 2012)
22. (DEB, 2012)

3.3 Problem Declaration

A tarefa de otimização será cumprida com o uso de algoritmos genéticos, enquanto que o projeto preliminar da turbina será simplificado e uni-dimensional.

Pretende-se utilizar um estudo de caso mais simples com o objetivo de entender o funcionamento da ferramenta de otimização e familiarização do método. Tal exemplo base pode ser feito considerando apenas uma variável de projeto. Após essa fase, o processo formulado como um todo deverá ser resolvido. Pretende-se limitar a quantidade

de variáveis de projeto para X , sendo estas facilmente adimensionalizadas para fim de comparação.

3.4 Genetic Algorithms

De acordo com a [ref] Optimization for Engineering Design Algorithms and Examples, Algoritmos Genéticos (AG, daqui em diante) permitem uma maneira fácil de encontrar múltiplas soluções ótimas simultaneamente em uma única rodada de simulação. Em se tratando de problemas de otimização multiobjetivo, surge um conjunto de soluções ótimas conhecida como Soluções Ótimas de Pareto ou Fronteira de Pareto. Também, AG demonstra um maneira de lidar com múltiplos objetivos e ajuda a encontrar multiplas soluções ótimas de Pareto simultaneamente.

3.4.1 NSGA-II

Lista de referências pretendida para elaboração desta seção:

1. (DEB *et al.*, 2002)

4

Methodology

Methodology

5

Discussion

Discussion

6

Conclusion

Conclusion

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Appendix A - First appendix

A.1 First Section of Appendix

if needed

Annex A -

First annex

A.1 First section of Annex

If needed.