$H(X) \leq \log_2(|suppX|)$

$$\begin{aligned}
H(X,Y) &= -\sum_{i,j} p(a_i,b_j)\log_2 p(a_i,b_j) \\
&= -\sum_{i,j} p(a_i,b_j)\log_2 p(a_i)p(b_j|a_i) \\
&= -\sum_{i,j} p(a_i,b_j)[\log_2 p(a_i) + \log_2 p(b_j|a_i)] \\
&= -\sum_{i,j} p(a_i,b_j)\log_2 p(a_i) - \sum_{i,j} p(a_i,b_j)\log_2 p(b_j|a_i) \\
&= -\sum_{i,j} p(a_i,b_j)\log_2 p(a_i) - \sum_{i,j} p(a_i).p(b_j|a_i)\log_2 p(b_j|a_i) \\
&= -\sum_{i=1}^{m} p(a_i)\log_2 p(a_i)\sum_{j=1}^{n} p(b_j|a_i) + H(Y|X) \\
&= -\sum_{i=1}^{m} p(a_i)\log_2 p(a_i) + H(Y|X) = H(X) + H(Y|X)
\end{aligned}$$

(Do $\sum_{j=1}^{n} p(b_j|a_i) = \sum_{j=1}^{n} \frac{P(b_j.a_i)}{P(a_i)} = \frac{1}{P(a_i)}\sum_{j=1}^{n} P(b_j.a_i) = \frac{1}{P(a_i)}P(a_i) = 1$)

$H(Y|X) \leq -\sum_{j=1}^{d} Prob(X \in E_j)\log_2 j.$

$$\begin{aligned}
H(X,Y) &= \sum_{j=1}^{d}\sum_{a\in E_j} p(a)H(Y|a) \\
&\leq \sum_{j=1}^{d}\sum_{a\in E_j} p(a)\log_2 j \qquad (1) \\
&= \sum_{j=1}^{d} Prob(X \in E_j)\log_2 j.
\end{aligned}$$

$\sigma(\tau_1), \sigma(\tau_2), ..., \sigma(\tau_n)$

$H(\sigma(\tau_1), .., \sigma(\tau_n)) = \sum_{i=1}^{n} H(\sigma(\tau_i)|\sigma(\tau_1), ..., \sigma(\tau_{k_i-1}))$

$H(\sigma(\tau_1), .., \sigma(\tau_n)) = \frac{1}{n!}\sum_{\tau}\left(\sum_{i=1}^{n} H(\sigma(\tau_i)|\sigma(\tau_1), ..., \sigma(\tau_{k_i-1}))\right)$

$\Theta(n^{\log_2 2})$