Computation Tree Logic

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Previously on Temporal Logic Week...

Temporal Logic

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Motivation

- Needing of uncertainty;

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- Different paths of the future;

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

Syntax
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$$\mathtt{AF}\phi \mid \mathtt{EF}\phi \mid \mathtt{AG}\phi \mid \mathtt{EG}\phi \mid \mathtt{A}[\phi \mathtt{U} \phi] \mid \mathtt{E}[\phi \mathtt{U} \phi]$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- $\varphi\mathtt{U}\psi$: φ is true at least until ψ becomes true;

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- We assume that, similarly to the $\neg$ operator, the "new" unary operators ($\mathtt{AX}, \mathtt{EX}, \mathtt{AF}, \mathtt{EF}, \mathtt{AG}$, and $\mathtt{EG}$) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, $\mathtt{AU}$ and $\mathtt{EU}$

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Definition (1)

A **transition system** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to say that the model $\mathcal{M}, s$ satisfies the formula φ

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$

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Satisfaction

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- $M, s, \vDash \mathtt{AG}\varphi$ iff exists some path $s_1 \to s_2 \to s_3 \to \ldots$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$ Thus, EG says: "Exists a path from now on in all next states..."

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- $M, s, \vDash \mathtt{A}[\varphi_1 \mathtt{U} \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, this path satisfies $\varphi_1 \mathtt{U} \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $M, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- "A certain process is enabled infinitely often on every computation path": $\mathtt{AG}(\mathtt{AF}\mathit{enabled})$
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $\mathtt{AG}(\mathit{floor}2 \land \mathit{directionUp} \land \mathit{button}5 \rightarrow \mathtt{A}[\mathit{directionUp}\mathtt{U}\mathit{floor}5])$