Computation Tree Logic

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- Some practical applications

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- Different paths of the future;

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- *G* "Globally (in all future states)";
- $\varphi U \psi$: φ is true at least until ψ becomes true;

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- Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as $AU$and EU, respectively;
- We assume that, similarly to the $\neg$ operator, the "new" unary operators (AX, EX, AF, EF, AG, and EG) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, AU and EU;

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Definition (1)

A **transition system** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $M, s, \vDash AG\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."

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- $M, s, \vDash AG\varphi$ iff exists some path $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$ Thus, EG says: "Exists a path from now on in all next states..."

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- $M, s, \vDash AF\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, exists s_i, $M, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

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- $M, s, \vDash A[\varphi_1 U \varphi_2]$ iff for all paths $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $M, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- $M, s, \vDash E[\varphi_1 U \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$

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- "A certain process is enabled infinitely often on every computation path": $AG(AF enabled)$
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $AG(floor2 \wedge directionUp \wedge button5 \rightarrow A[directionUp U floor5])$

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Notation: we denote the equivalence of φ and ψ by $\varphi \equiv \psi$

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Let φ be an arbitrary CTL formula.

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Minimum set of CTL connectives

Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Backus-Naur formalism below:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \rightarrow \phi \mid AX\phi \mid A[\phi U \phi] \mid E[\phi U \phi]$$

Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed. For example, we cannot say, as in LTL: "All paths in which have p on them, also have q on them".
This expression can be translated in LTL as follows:

$$Fp \to Fq$$

That's all we need?

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We can try to translate it as $AG(p \rightarrow AFq)$ which is closer, but not exactly the same. This one means "for all paths, in all states on the future, if they hold p then, all paths will eventualy hold q"

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This allows us to generate some statements:

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$

Statements only possible with CTL*

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \lor pUr]$
- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$

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- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$
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The syntax of CTL* can be defined with the BNF bellow:

$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid A[\alpha] \mid E[\alpha] \mid$$

$$\alpha ::= \phi \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \rightarrow \alpha \mid \alpha U \alpha \mid G\alpha \mid F\alpha \mid X\alpha \mid$$

With the same meanings of each operator.

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For CTL, it is trivial;