Computation Tree Logic

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Motivation

- Needing of uncertainty;

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- Different paths of the future;

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- $\mathtt{G}\varphi$: Globally (in all future states) φ is true;
- $\varphi\mathtt{U}\psi$: φ is true at least until ψ becomes true;

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- We assume that, similarly to the $\neg$ operator, the “new” unary operators ($\mathtt{AX}, \mathtt{EX}, \mathtt{AF}, \mathtt{EF}, \mathtt{AG}$, and $\mathtt{EG}$) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, AU and EU

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A **transition system** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $M, s, \vDash \texttt{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$. Thus, `AG` says: "In all possible paths from now on in all next states..."

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- $M, s, \vDash \texttt{AG}\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$ Thus, EG says: "Exists a path from now on in all next states..."

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- $M, s, \vDash \mathtt{AF}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, exists s_i, $M, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

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- $M, s, \vDash \mathtt{A}[\varphi_1 \mathtt{U} \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, this path satisfies $\varphi_1 \mathtt{U} \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $M, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- $M, s, \vDash \mathtt{E}[\varphi_1 \mathtt{U} \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$

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- "A certain process is enabled infinitely often on every computation path": $\mathtt{AG}(\mathtt{AF}\mathit{enabled})$
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $\mathtt{AG}(\mathit{floor}2 \wedge \mathit{directionUp} \wedge \mathit{button}5 \rightarrow \mathtt{A}[\mathit{directionUp}\mathtt{U}\mathit{floor}5])$

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Notation: we denote the equivalence of φ and ψ by $\varphi \equiv \psi$

Let φ be an arbitrary CTL formula.

- $\neg \mathtt{AF}\varphi \equiv \mathtt{EG}\neg\varphi$
- $\neg \mathtt{EF}\varphi \equiv \mathtt{AG}\neg\varphi$
- $\neg \mathtt{AX}\varphi \equiv \mathtt{EX}\neg\varphi$
- $\mathtt{AF}\varphi \equiv \mathtt{A}[\top \mathtt{U}\varphi]$
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