Computation
Tree Logic

Luis Tertulino &
Ronaldo Silveira

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How to
communicate
Syntax of CTL
Semantics of
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Some examples of
what we can say
More about
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- A brief introduction to Propositional Logic, its syntax and
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- Some practical applications

- Needing of uncertainty;

Motivation

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- Needing of uncertainty;
- Different paths of the future;

In Computation Tree Logic (CTL) the model of time is a
tree-like structure. This way, we cannot use Linear Temporal
Logic (LTL) to express the existence of a certain path of time
in which some event occurs.

content...

The syntax of CTL consists on the syntax of temporal logic
plus some path operators. The class of formulas can be defined
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$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid AX\phi \mid EX\phi \mid$$

$$AF\phi \mid EF\phi \mid AG\phi \mid EG\phi \mid A[\phi U\phi] \mid E[\phi U\phi]$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e
EG unary operators.

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- F "There is some state in the future (eventually)";
- G "Globally (in all future states)";
- $\varphi U \psi$: φ is true at least until ψ becomes true;

- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., AF is an atomic operator that can be read as "In all paths in the future there is some state where...";

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- Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU and EU, respectively;

- We assume that, similarly to the $\neg$ operator, the "new" unary operators (AX, EX, AF, EF, AG, and EG) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, AU and EU;

- Examples of well-formed formulas:
    - $AG(p \lor EFq)$

Examples

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- Examples of well-formed formulas:
  - $AG(p \vee EFq)$
  - $AX(q \rightarrow E[(p \vee q)Ur])$
  - $EFEGp \rightarrow AFr$ Note that this is binded as $(EFEGp) \rightarrow AFr$, not as $EFEG(p \rightarrow AFr)$

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  - $A[pUs \wedge qUs]$

Definition

Let *Atoms* be a set of atomic formulas. A **transition system**
or **model** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of
states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) such that for
every state $s \in S$, exists a s' that $s \rightarrow s'$ and
$L : S \rightarrow \mathcal{P}(Atoms)$ (or $L : S \rightarrow (Atoms \rightarrow \{0, 1\})$) is a
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CTL formulas are satisfied by a transition system and a specific state.

Notation: we will use $\mathcal{M}, s \vDash \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Definition

The **satisfaction** of a formula in CTL is recursive over the structure of the formula. It can be done as follows:

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$

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- $\mathcal{M}, s \vDash \varphi_1 \vee \varphi_2$ iff $\mathcal{M}, s \vDash \varphi_1$ OR $\mathcal{M}, s \vDash \varphi_2$

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- $\mathcal{M}, s \vDash EX\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $M, s_1 \vDash \varphi$. Thus, EX says: "in some next state..."

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- $\mathcal{M}, s, \vDash AG\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \models AX\varphi$ iff for all s_1 that $s \to s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus, AX says: "in every next state..."

- $\mathcal{M}, s \models EX\varphi$ iff exists s_1 that $s \to s_1$ and $M, s_1 \models \varphi$. Thus, EX says: "in some next state..."

- $\mathcal{M}, s, \models AG\varphi$ iff for all paths $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \models \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."

- $\mathcal{M}, s, \models AG\varphi$ iff exists some path $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \models \varphi$ Thus, EG says: "Exists a path from now on in all next states..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s, \vDash AF\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

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- $\mathcal{M}, s, \vDash EF\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, that exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, EF says: "In some path from now on, in some next state..."

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- $\mathcal{M}, s, \vDash A[\varphi_1 U \varphi_2]$ iff for all paths $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $\mathcal{M}, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s, \vDash E[\varphi_1 U \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$. Thus, EU says: "In some path from now on, until some state..."

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- "It's possible to get to a state where something has started but it's not ready": $EF(started \wedge \neg ready)$

- "It's possible to get to a state where something has started but it's not ready": *EF*(*started* ∧ ¬*ready*)
- "A certain process is enabled infinitely often on every computation path": *AG*(*AFenabled*)

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- "It's possible to get to a state where something has started but it's not ready": $EF(started \land \neg ready)$
- "A certain process is enabled infinitely often on every computation path": $AG(AFenabled)$
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor":
  $AG(floor2 \land directionup \land button5 \rightarrow A[directionup U floor5])$

Definition

Two CTL formulas φ and ψ are said to be **semantically equivalent** if any state in any model which satisfies one of them also satisfies the other;

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Notation: we denote the semantic equivalence of φ and ψ by $\varphi \equiv \psi$

Let φ be an arbitrary CTL formula.

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Example of equivalences

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- $AF\varphi \equiv A[\top U\varphi]$

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- $AF\varphi \equiv A[\top U\varphi]$
- $EF\varphi \equiv E[\top U\varphi]$

Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Backus-Naur formalism below:

$$\phi ::= \perp \mid p \mid \neg\phi \mid \phi \wedge \phi \mid EX\phi \mid AF\phi \mid E[\phi U\phi]$$

Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed. For example, we cannot say, as in LTL: "All paths in which have p on them, also have q on them".
This expression can be translated in LTL as follows:

$$Fp \rightarrow Fq$$

That's all we need?

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We can try expressing it as $AFp \rightarrow AFq$ but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then"

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the same meaning. This one statement means "If all paths
have a p along them, then all paths have a q along then"
We can try to translate it as $AG(p \rightarrow AFq)$ which is closer, but
not exactly the same. This one means "for all paths, in all
states on the future, if they hold p then, all paths will
eventualy hold q"

Presenting CTL*

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For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).
This allows us to generate some statements:

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \lor pUr]$

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \lor pUr]$

- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \lor pUr]$
- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$
- "In all paths, p will occur in the next state or in the next of the next": $A[Xp \lor XXp]$

The syntax of CTL* can be defined with the BNF bellow:

$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid A[\alpha] \mid E[\alpha] \mid$$

$$\alpha ::= \phi \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \rightarrow \alpha \mid \alpha U\alpha \mid G\alpha \mid F\alpha \mid X\alpha \mid$$

With the same meanings of each operator.

Although we don't define path operators to LTL we can assume
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For CTL, it is trivial;