

Computation Tree Logic

Luis Tertulino & Ronaldo Silveira

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■ Needing of uncertainty;

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In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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The syntax of CTL consists on the syntax of temporal logic plus some path operators. The class of formulas can be defined in Backus-Naur form. If ϕ is a formula:

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$$\phi ::= \perp \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid \text{AX}\phi \mid \text{EX}\phi \mid$$

$$\text{AF}\phi \mid \text{EF}\phi \mid \text{AG}\phi \mid \text{EG}\phi \mid \text{A}[\phi \text{U} \phi] \mid \text{E}[\phi \text{U} \phi]$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.

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The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.

The temporal operators can be read (if φ is a formula) as follows:

- $A\varphi$: φ is true in all possible paths;

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The path-specific operators can be read, considering φ and ψ formulas, as:

- $X\varphi$: φ is true until next state;
- $F\varphi$: There is some state in the future where φ is true;

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- $X\varphi$: φ is true until next state;
- $F\varphi$: There is some state in the future where φ is true;
- $G\varphi$: Globally (in all future states) φ is true;

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- $X\varphi$: φ is true until next state;
- $F\varphi$: There is some state in the future where φ is true;
- $G\varphi$: Globally (in all future states) φ is true;
- $\varphi U\psi$: φ is true at least until ψ becomes true;

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- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e., AF is a operator that can be read as “In all paths in the future there is some state where...”

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- Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU

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- We assume that, similarly to the \neg operator, the “new” unary operators (AX , EX , AF , EF , AG , and EG) have the first precedence. Next comes the \wedge and \vee operators. And at last the \rightarrow , AU and EU

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- Examples of well-formed formulas:
 - $AG(p \vee EFq)$

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- $AX(q \rightarrow E[(p \vee q)Ur])$

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- Examples of well-formed formulas:
 - $AG(p \vee EFq)$
 - $AX(q \rightarrow E[(p \vee q)Ur])$
 - $EFEGp \rightarrow AFr$ Note that this is binded as $(EFEGp) \rightarrow AFr$,
not as $EFEG(p \rightarrow AFr)$

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- Example of formulas that are not well-formed:
 - $A \neg G \neg p$

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 - $A[pUs \wedge qUs]$

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Different from usual logics, CTL formulas are interpreted by a transition system. Given an set of atoms:

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Different from usual logics, CTL formulas are interpreted by a transition system. Given an set of atoms:

Definition (1)

A **transition system** \mathcal{M} is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, \rightarrow is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Definition (2)

A **model** is a duple \mathcal{M}, s in which \mathcal{M} is a transition system and s is a state.

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Notation: we will use $\mathcal{M}, s \models \varphi$ to denote that the model \mathcal{M}, s satisfies the formula φ

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Take an arbitrary model \mathcal{M} . Let s, s_1, s_2, s_3 be states in S . Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \perp$ for all $s \in S$

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- $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \perp$ for all $s \in S$
- $\mathcal{M}, s \models p$ iff $p \in L(s)$

Take an arbitrary model \mathcal{M} . Let s, s_1, s_2, s_3 be states in S . Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $\mathcal{M}, s \models \neg\varphi$ iff $\mathcal{M}, s \not\models \varphi$

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- $\mathcal{M}, s \models \varphi_1 \wedge \varphi_2$ iff $\mathcal{M}, s \models \varphi_1$ AND $\mathcal{M}, s \models \varphi_2$

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- $\mathcal{M}, s \models \varphi_1 \vee \varphi_2$ iff $\mathcal{M}, s \models \varphi_1$ OR $\mathcal{M}, s \models \varphi_2$

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- $\mathcal{M}, s \models \text{AX}\varphi$ iff for all s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$.
Thus, AX says: “in every next state...”

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Thus, AX says: “in every next state...”
- $\mathcal{M}, s \models \text{EX}\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus,
EX says: “in some next state...”

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Thus, AX says: “in every next state...”
- $\mathcal{M}, s \models \text{EX}\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus,
EX says: “in some next state...”
- $\mathcal{M}, s \models \text{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which
 $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, AG says: “In all possible
paths from now on in all next states...”

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Thus, AX says: “in every next state...”
- $\mathcal{M}, s \models \text{EX}\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus, EX says: “in some next state...”
- $\mathcal{M}, s \models \text{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, AG says: “In all possible paths from now on in all next states...”
- $\mathcal{M}, s \models \text{EG}\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, EG says: “Exists a path from now on in all next states...”

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Take an arbitrary model \mathcal{M} . Let s, s_1, s_2, s_3 be states in S . Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \models \text{AX}\varphi$ iff for all s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$.
Thus, AX says: “in every next state...”
- $\mathcal{M}, s \models \text{EX}\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus,
EX says: “in some next state...”
- $\mathcal{M}, s \models \text{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which
 $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, AG says: “In all possible
paths from now on in all next states...”
- $\mathcal{M}, s \models \text{EG}\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in
which $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, EG says: “Exists a
path from now on in all next states...”

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- $M, s, \models \text{A}[\varphi_1 \text{U} \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, this path satisfies $\varphi_1 \text{U} \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \models \varphi_2$ and, for all $j < i$, $M, s_j \models \varphi_1$. Thus, AU says: “For all paths from now on, until some state...”

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- $M, s, \models \text{E}[\varphi_1 \text{U} \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$

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- “It’s possible to get to a state where something has started but it’s not ready”: $EF(started \wedge \neg ready)$

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- “It’s possible to get to a state where something has started but it’s not ready”: $EF(started \wedge \neg ready)$
- “A certain process is enabled infinitely often on every computation path”: $AG(AF enabled)$

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- “It’s possible to get to a state where something has started but it’s not ready”: $EF(started \wedge \neg ready)$
- “A certain process is enabled infinitely often on every computation path”: $AG(AF enabled)$
- “An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor”:
 $AG(floor2 \wedge directionUp \wedge button5 \rightarrow A[directionUp U floor5])$

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Definition

Two CTL formulas φ and ψ are said to be **semantically equivalent** if any state in any model which satisfies one of them also satisfies the other;

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Definition

Two CTL formulas φ and ψ are said to be **semantically equivalent** if any state in any model which satisfies one of them also satisfies the other;

Notation: we denote the equivalence of φ and ψ by $\varphi \equiv \psi$

Example of equivalences

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Let φ be an arbitrary CTL formula.

$$\blacksquare \neg \text{AF} \varphi \equiv \text{EG} \neg \varphi$$

Example of equivalences

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Let φ be an arbitrary CTL formula.

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Example of equivalences

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$$\blacksquare \neg \text{AF}\varphi \equiv \text{EG}\neg\varphi$$

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$$\blacksquare \neg \text{AX}\varphi \equiv \text{EX}\neg\varphi$$

$$\blacksquare \text{AF}\varphi \equiv \text{A}[\top \text{U}\varphi]$$

Example of equivalences

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$$\blacksquare \text{AF}\varphi \equiv \text{A}[\top \text{U}\varphi]$$

$$\blacksquare \text{EF}\varphi \equiv \text{E}[\top \text{U}\varphi]$$

Minimum set of CTL connectives

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Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of connectives for the CTL syntax. One of them is defined in Backus-Naur formalism below:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \rightarrow \phi \mid \mathbf{AX}\phi \mid \mathbf{A}[\phi\mathbf{U}\phi] \mid \mathbf{E}[\phi\mathbf{U}\phi]$$

That's all we need?

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Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed. For example, we cannot say, as in LTL: "All paths in which have p on them, also have q on them".

This expression can be translated in LTL as follows:

$$Fp \rightarrow Fq$$

That's all we need?

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We can try expressing it as $AFp \rightarrow AFq$ but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then"

That's all we need?

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We can try expressing it as $AFp \rightarrow AFq$ but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then"

We can try to translate it as $AG(p \rightarrow AFq)$ which is closer, but not exactly the same. This one means "for all paths, in all states on the future, if they hold p then, all paths will eventually hold q "

Presenting CTL*

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For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).

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For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).

This allows us to generate some statements:

Statements only possible with CTL*

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- “In all possible paths, q is true until r is true or p is true until r is true”: $A[qUr \vee pUr]$

Statements only possible with CTL*

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- “In all possible paths, q is true until r is true or p is true until r is true”: $A[qUr \vee pUr]$
- “There is a path in which p eventually occurring will occur in all states”: $E[GFp]$

Statements only possible with CTL*

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- “There is a path in which p eventually occurring will occur in all states”: $E[GFp]$
- “In all paths, p will occur in the next state or in the next of the next”: $A[Xp \vee XXp]$

Statements only possible with CTL*

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CTL* syntax

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The syntax of CTL* can be defined with the BNF bellow:

$$\phi ::= \perp \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid A[\alpha] \mid E[\alpha] \mid$$

$$\alpha ::= \phi \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \rightarrow \alpha \mid \alpha U \alpha \mid G\alpha \mid F\alpha \mid X\alpha \mid$$

With the same meanings of each operator.

LTL \subset CTL* and CTL \subset CTL*

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Although we don't define path operators to LTL we can assume that it consider in all paths. Therefore, we can say that a formula ϕ in LTL is a formula $A[\phi]$ in CTL*;

LTL \subset CTL* and CTL \subset CTL*

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Although we don't define path operators to LTL we can assume that it consider in all paths. Therefore, we can say that a formula ϕ in LTL is a formula $A[\phi]$ in CTL*;
For CTL, it is trivial;