

Computation Tree Logic

Luis Tertulino & Ronaldo Silveira

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In previous chapters...

Temporal Logic

Motivation and Intuition

Motivation

Intuition

How to communicate

Syntax of CTL

Semantics of CTL

In previous chapters...

Temporal Logic

Motivation and Intuition

Motivation

Intuition

How to communicate

Syntax of CTL

Semantics of CTL

Previously on Temporal Logic Week...

Temporal Logic

In previous chapters...

Temporal Logic

Motivation and Intuition

Motivation

Intuition

How to communicate

Syntax of CTL

Semantics of CTL

Motivation

Needing of uncertainty;

Motivation

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Different paths of the future;

In previous chapters...

Temporal Logic

Motivation and Intuition

Motivation

Intuition

How to communicate

Syntax of CTL

Semantics of CTL

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

In previous chapters...

Temporal Logic

Motivation and Intuition

Motivation

Intuition

How to communicate

Syntax of CTL

Semantics of CTL

Syntax

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$$\begin{aligned} \phi ::= & \perp \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid \mathbf{AX}\phi \mid \mathbf{EX}\phi \mid \\ & \mathbf{AF}\phi \mid \mathbf{EF}\phi \mid \mathbf{AG}\phi \mid \mathbf{EG}\phi \mid \mathbf{A}[\phi\mathbf{U}\phi] \mid \mathbf{E}[\phi\mathbf{U}\phi] \end{aligned}$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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$X\phi$: φ is true until next state;

$F\varphi$: There is some state in the future where φ is true;

$G\varphi$: Globally (in all future states) φ is true;

$\varphi U \psi$: φ is true at least until ψ becomes true;

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Notes

Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e., AF is a operator that can be read as "In all paths in the future there is some state where..."

Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU

We assume that, similarly to the \neg operator, the "new" unary operators (AX , EX , AF , EF , AG , and EG) have the first precedence. Next comes the \wedge and \vee operators. And at last the \rightarrow , AU and EU

In previous chapters...

Temporal Logic

Motivation and Intuition

Motivation

Intuition

How to communicate

Syntax of CTL

Semantics of CTL

Semantics