Computation Tree Logic

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October 21, 2015

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- Needing of uncertainty;

Motivation

- Needing of uncertainty;
- Different paths of the future;

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- $\varphi\mathtt{U}\psi$: φ is true at least until ψ becomes true;

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- We assume that, similarly to the $\neg$ operator, the "new" unary operators ($\mathtt{AX}, \mathtt{EX}, \mathtt{AF}, \mathtt{EF}, \mathtt{AG}$, and $\mathtt{EG}$) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, $\mathtt{AU}$ and $\mathtt{EU}$

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A **transition system** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $M, s, \vDash \texttt{AF}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, exists s_i, $M, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

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- $M, s, \vDash \mathtt{A}[\varphi_1 \mathtt{U} \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, this path satisfies $\varphi_1 \mathtt{U} \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $M, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- $M, s, \vDash \texttt{E}[\varphi_1 \texttt{U} \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$

- "It's possible to get to a state where something has started but it's not ready": $\mathtt{EF}(\mathit{started} \wedge \neg \mathit{ready})$

Examples

- "It's possible to get to a state where something has started but it's not ready": $\mathtt{EF}(\mathit{started} \wedge \neg \mathit{ready})$
- "A certain process is enabled infinitely often on every computation path": $\mathtt{AG}(\mathtt{AF}\mathit{enabled})$

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- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $\mathtt{AG}(\mathit{floor}2 \wedge \mathit{directionUp} \wedge \mathit{button}5 \rightarrow \mathtt{A}[\mathit{directionUp}\mathtt{U}\mathit{floor}5])$

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Equivalences

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Notation: we denote the equivalence of φ and ψ by $\varphi \equiv \psi$

Example of equivalences

Let φ be an arbitrary CTL formula.

- $\neg \mathtt{AF}\varphi \equiv \mathtt{EG}\neg\varphi$
- $\neg \mathtt{EF}\varphi \equiv \mathtt{AG}\neg\varphi$
- $\neg \mathtt{AX}\varphi \equiv \mathtt{EX}\neg\varphi$
- $\mathtt{AF}\varphi \equiv \mathtt{A}[\top \mathtt{U}\varphi]$
- $\mathtt{EF}\varphi \equiv \mathtt{E}[\top \mathtt{U}\varphi]$

Minimum set of CTL connectives

Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Extended Backus-Naur formalism below:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \rightarrow \phi \mid \mathtt{AX}\phi \mid \mathtt{A}[\phi\mathtt{U}\phi] \mid \mathtt{E}[\phi\mathtt{U}\phi]$$

Needing some more?

Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed.