Computation Tree Logic

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Motivation

Needing of uncertainty;

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Different paths of the future;

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Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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Syntax

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$$\mathtt{AF}\phi \mid \mathtt{EF}\phi \mid \mathtt{AG}\phi \mid \mathtt{EG}\phi \mid \mathtt{A}[\phi\mathcal{U}\phi] \mid \mathtt{E}[\phi\mathcal{U}\phi]$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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The path-specific operators can be read, considering φ and ψ formulas, as:

- $\mathtt{X}\phi$: φ is true until next state;
- $\mathtt{F}\varphi$: There is some state in the future where φ is true;
- $\mathtt{G}\varphi$: Globally (in all future states) φ is true;
- $\varphi\mathcal{U}\psi$: φ is true at least until ψ becomes true;

Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e., `AF`is one operator that can be read as "In all paths in the future there is some state where..."

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