Computation Tree Logic

Luis Tertulino & Ronaldo Silveira

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Motivation

- Needing of uncertainty;

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- Different paths of the future;

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- $\mathtt{G}\varphi$: Globally (in all future states) φ is true;
- $\varphi\mathtt{U}\psi$: φ is true at least until ψ becomes true;

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- We assume that, similarly to the $\neg$ operator, the "new" unary operators ($\mathtt{AX}, \mathtt{EX}, \mathtt{AF}, \mathtt{EF}, \mathtt{AG}$, and $\mathtt{EG}$) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, $\mathtt{AU}$ and $\mathtt{EU}$

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  - $\mathtt{AG}(p \vee \mathtt{EF}q)$

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A **transition system** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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In previous chapters...
Motivation and Intuition
How to communicate
Syntax of CTL
Semantics of CTL
Some examples of what we can say
More about semantics
Equivalences

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $M, s, \vDash \mathtt{A}[\varphi_1 \mathtt{U} \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, this path satisfies $\varphi_1 \mathtt{U} \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $M, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- $M, s, \vDash \texttt{E}[\varphi_1 \texttt{U} \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$

Examples

- "It's possible to get to a state where something has started but it's not ready": $\mathtt{EF}(\mathit{started} \land \neg \mathit{ready})$

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- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $\mathtt{AG}(\mathit{floor}2 \wedge \mathit{directionUp} \wedge \mathit{button}5 \rightarrow \mathtt{A}[\mathit{directionUp}\,\mathtt{U}\,\mathit{floor}5])$

Equivalences

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Notation: we denote the equivalence of φ and ψ by $\varphi \equiv \psi$

Example of equivalences

Let φ be an arbitrary CTL formula.

- $\neg \mathtt{AF}\varphi \equiv \mathtt{EG}\neg\varphi$
- $\neg \mathtt{EF}\varphi \equiv \mathtt{AG}\neg\varphi$
- $\neg \mathtt{AX}\varphi \equiv \mathtt{EX}\neg\varphi$
- $\mathtt{AF}\varphi \equiv \mathtt{A}[\top \mathtt{U} \varphi]$
- $\mathtt{EF}\varphi \equiv \mathtt{E}[\top \mathtt{U} \varphi]$

Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Extended Backus-Naur formalism below:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \rightarrow \phi \mid \mathtt{AX}\phi \mid \mathtt{A}[\phi \mathtt{U} \phi] \mid \mathtt{E}[\phi \mathtt{U} \phi]$$