

Computation Tree Logic

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Motivation

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How to communicate

Syntax of CTL

Semantics of CTL

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Motivation

Needing of uncertainty;

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Different paths of the future;

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In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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$$\begin{aligned} \phi ::= & \perp \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid \mathbf{AX}\phi \mid \mathbf{EX}\phi \mid \\ & \mathbf{AF}\phi \mid \mathbf{EF}\phi \mid \mathbf{AG}\phi \mid \mathbf{EG}\phi \mid \mathbf{A}[\phi\mathbf{U}\phi] \mid \mathbf{E}[\phi\mathbf{U}\phi] \end{aligned}$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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$F\varphi$: There is some state in the future where φ is true;

$G\varphi$: Globally (in all future states) φ is true;

$\varphi U \psi$: φ is true at least until ψ becomes true;

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Notes

Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e., AF is a operator that can be read as "In all paths in the future there is some state where..."

Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU

We assume that, similarly to the \neg operator, the "new" unary operators (AX , EX , AF , EF , AG , and EG) have the first precedence. Next comes the \wedge and \vee operators. And at last the \rightarrow , AU and EU

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Definition (1)

A transition system \mathcal{M} is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, \rightarrow is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \models \varphi$ to say that the model \mathcal{M}, s satisfies the formula φ

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Satisfaction

Take an arbitrary model \mathcal{M} . Let s, s_1, s_2, s_3 be states in S . Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

$$\mathcal{M}, s \models \top \text{ and } \mathcal{M}, s \not\models \perp \text{ for all } s \in S$$

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$\mathcal{M}, s \models \text{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, AG says: “In all possible paths from now on in all next states...”

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$\mathcal{M}, s \models \text{EG}\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, EG says: “Exists a path from now on in all next states...”

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