Computation Tree Logic

Luis Tertulino & Ronaldo Silveira

October 19, 2015

In previous chapters...

- Temporal Logic

Motivation and Intuition

- Motivation
- Intuition

How to communicate

- Syntax of CTL
- Semanthics of CTL

Previously on Temporal Logic Week...

Temporal Logic

Motivation

Needing of uncertainty;

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Different paths of the future;

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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$$\mathtt{AF}\phi \mid \mathtt{EF}\phi \mid \mathtt{AG}\phi \mid \mathtt{EG}\phi \mid \mathtt{A}[\phi \mathtt{U} \phi] \mid \mathtt{E}[\phi \mathtt{U} \phi]$$

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With p as a literal (atomic formula), $\mathtt{AX}$, $\mathtt{EX}$, $\mathtt{AF}$, $\mathtt{EF}$, $\mathtt{AG}$ e $\mathtt{EG}$ unary operators.

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>
> $\varphi\mathtt{U}\psi$: φ is true at least until ψ becomes true;

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Notes

Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e., $\mathtt{AF}$ is a operator that can be read as "In all paths in the future there is some state where..."

Notice as well that the binary operators $\mathtt{A}[\varphi \mathtt{U} \psi]$ and $\mathtt{E}[\varphi \mathtt{U} \psi]$ can be represented as $\mathtt{AU}$

We assume that, similarly to the $\neg$ operator, the "new" unary operators ($\mathtt{AX}, \mathtt{EX}, \mathtt{AF}, \mathtt{EF}, \mathtt{AG}$, and $\mathtt{EG}$) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, $\mathtt{AU}$ and $\mathtt{EU}$

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$A[p\mathtt{U}s \wedge q\mathtt{U}s]$

Different from usual logics, CTL formulas are interpreted by a transition system.

Definition
A model