

# Computation Tree Logic

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# Motivation

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Different paths of the future;

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# Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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With  $p$  as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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$\varphi U \psi$ :  $\varphi$  is true at least until  $\psi$  becomes true;

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## Notes

Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e.,  $AF$  is a operator that can be read as "In all paths in the future there is some state where..."

Notice as well that the binary operators  $A[\varphi U \psi]$  and  $E[\varphi U \psi]$  can be represented as  $AU$

We assume that, similarly to the  $\neg$  operator, the "new" unary operators ( $AX$ ,  $EX$ ,  $AF$ ,  $EF$ ,  $AG$ , and  $EG$ ) have the first precedence. Next comes the  $\wedge$  and  $\vee$  operators. And at last the  $\rightarrow$ ,  $AU$  and  $EU$

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A transition system  $\mathcal{M}$  is a triple  $\mathcal{M} = (S, \rightarrow, L)$  in which  $S$  is a set of states,  $\rightarrow$  is a binary relation over  $S$  ( $\rightarrow \subseteq S \times S$ ) and  $L : S \rightarrow \mathcal{P}(\text{Atoms})$  is a labelling function.



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A model is a duple  $\mathcal{M}, s$  in which  $\mathcal{M}$  is a transition system and  $s$  is a state.

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**Notation:** we will use  $\mathcal{M}, s \models \varphi$  to say that the model  $\mathcal{M}, s$  satisfies the formula  $\varphi$

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