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Luis Tertulino & Ronaldo Silveira

October 21, 2015

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■ Needing of uncertainty;

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- Needing of uncertainty;
- Different paths of the future;

# Intuition

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In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

# Syntax Definition

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$$\begin{split} \phi ::= \bot \mid \top \mid \rho \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \to \phi \mid \mathsf{AX}\phi \mid \mathsf{EX}\phi \mid \\ \mathsf{AF}\phi \mid \mathsf{EF}\phi \mid \mathsf{AG}\phi \mid \mathsf{EG}\phi \mid \mathsf{A}[\phi \mathsf{U}\phi] \mid \mathsf{E}[\phi \mathsf{U}\phi] \end{split}$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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The propositional operators:  $\neg, \lor, \land, \rightarrow$  have the same meaning of in the propositional logic.

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The path-specific operators can be read, considering  $\varphi$  and  $\psi$ formulas, as:

 $\blacksquare$  X $\varphi$  :  $\varphi$  is true until next state;

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- $G\varphi$  : Globally (in all future states)  $\varphi$  is true;
- $\blacksquare \varphi U \psi$ :  $\varphi$  is true at least until  $\psi$  becomes true;

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■ Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e., AF is a operator that can be read as "In all paths in the future there is some state where..."

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- Notice as well that the binary operators  $A[\varphi U\psi]$  and  $E[\varphi U\psi]$  can be represented as AU

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- We assume that, similarly to the ¬ operator, the "new" unary operators (AX, EX, AF, EF, AG, and EG) have the first precedence. Next comes the ∧ and ∨ operators. And at last the →, AU and EU

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■ Examples of well-formed formulas:

 $\blacksquare \ \operatorname{AG}(p \vee \operatorname{EF}q)$ 

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- $\blacksquare \ \operatorname{AG}(p \vee \operatorname{EF}q)$
- $\blacksquare \ \mathtt{AX}(q \to \mathtt{E}[(p \lor q)\mathtt{U}r])$

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  - EFEGp o AFr Note that this is binded as (EFEGp) o AFr, not as EFEG(p o AFr)

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- Example of formulas that are not well-formed:
  - A $\neg$ G $\neg$ p

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  - $\blacksquare$  AG( $p \lor EFq$ )
  - $\blacksquare$  AX $(q \to E[(p \lor q)Ur])$
  - $\mathsf{EFEG}p \to \mathsf{AF}r$  Note that this is binded as  $(\mathsf{EFEG}p) \to \mathsf{AF}r$ , not as EFEG( $p \rightarrow AFr$ )
- Example of formulas that are not well-formed:
  - $\blacksquare$  A $\neg$ G $\neg$ p
  - $\blacksquare F[pUs]$

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  - $A[pUs \land qUs]$

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Different from usual logics, CTL formulas are interpreted by a transition system. Given an set of atoms:

# Definition (1)

A transition system  $\mathcal{M}$  is a triple  $\mathcal{M} = (S, \rightarrow, L)$  in which S is a set of states,  $\rightarrow$  is a binary relation over S ( $\rightarrow \subseteq S \times S$ ) and  $L: S \rightarrow \mathcal{P}(Atoms)$  is a labelling function.

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# Definition (1)

A transition system  $\mathcal{M}$  is a triple  $\mathcal{M}=(S,\to,L)$  in which S is a set of states,  $\to$  is a binary relation over S ( $\to\subseteq S\times S$ ) and  $L:S\to\mathcal{P}(Atoms)$  is a labelling function.

# Definition (2)

A **model** is a duple  $\mathcal{M}, s$  in which  $\mathcal{M}$  is a transition system and s is a state.

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# Definition (2)

A **model** is a duple  $\mathcal{M}, s$  in which  $\mathcal{M}$  is a transition system and s is a state.

**Notation:** we will use  $\mathcal{M}, s \vDash \varphi$  to say that the model  $\mathcal{M}, s$  satisfies the formula  $\varphi$ 

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Take an arbitrary model  $\mathcal{M}$ . Let  $s, s_1, s_2, s_3$  be states in S. Let  $\varphi, \varphi_1, \varphi_2$  be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

■  $\mathcal{M}$ ,  $s \vDash \top$  and  $\mathcal{M}$ ,  $s \not\vDash \bot$  for all  $s \in S$ 

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- $\mathcal{M}$ ,  $s \models p$  iff  $p \in L(S)$

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- $\mathcal{M}, s \vDash \varphi_1 \land \varphi_2$  iff  $\mathcal{M}, s \vDash \varphi_1$  AND  $\mathcal{M}, s \vDash \varphi_2$

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- $\mathcal{M}$ ,  $s \vDash \varphi_1 \lor \varphi_2$  iff  $\mathcal{M}$ ,  $s \vDash \varphi_1$  OR  $\mathcal{M}$ ,  $s \vDash \varphi_2$

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- $\mathcal{M}$ ,  $s \vDash \varphi_1 \lor \varphi_2$  iff  $\mathcal{M}$ ,  $s \vDash \varphi_1$  OR  $\mathcal{M}$ ,  $s \vDash \varphi_2$
- $\blacksquare \mathcal{M}, s \vDash \varphi_1 \rightarrow \varphi_2 \text{ iff } \mathcal{M}, s \not\vDash \varphi_1 \text{ OR } \mathcal{M}, s \vDash \varphi_2$

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- $\mathcal{M}, s \models \mathsf{EX}\varphi$  iff exists  $s_1$  that  $s \to s_1$  and  $M, s_1 \models \varphi$ . Thus, EX says: "in some next state…"

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- $M, s, \models AG\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , for all  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AG says: "In all possible paths from now on in all next states..."

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- $\mathcal{M}, s \vDash \mathtt{EX}\varphi$  iff exists  $s_1$  that  $s \to s_1$  and  $M, s_1 \vDash \varphi$ . Thus, EX says: "in some next state..."
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- $M, s, \models AG\varphi$  iff exists some path  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , for all  $s_i$ ,  $M, s_i \models \varphi$  Thus, EG says: "Exists a path from now on in all next states..."

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- $\mathcal{M}, s \vDash \mathsf{AX}\varphi$  iff for all  $s_1$  that  $s \to s_1$  and  $\mathcal{M}, s_1 \vDash \varphi$ . Thus, AX says: "in every next state..."
- $\mathcal{M}, s \vDash \mathtt{EX}\varphi$  iff exists  $s_1$  that  $s \to s_1$  and  $M, s_1 \vDash \varphi$ . Thus, EX says: "in some next state..."
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Take an arbitrary model  $\mathcal{M}$ . Let  $s, s_1, s_2, s_3$  be states in S. Let  $\varphi, \varphi_1, \varphi_2$  be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

■  $M, s, \models AF\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AF says: "In all possible paths from now on, in some next state..."

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- $M, s, \models AF\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AF says: "In all possible paths from now on, in some next state..."
- $M, s, \models \text{EF}\varphi$  iff exists some path  $s_1 \to s_2 \to s_3 \to ...$  in which  $s = s_1$ , that exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, EF says: "In some path from now on, in some next state..."

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- $M, s, \models A[\varphi_1 U \varphi_2]$  iff for all paths  $s_1 \to s_2 \to s_3 \to ...$  in which  $s = s_1$ , this path satisfies  $\varphi_1 U \varphi_2$ , i.e., exists  $s_i$  in the path such that  $\mathcal{M}, s_i \models \varphi_2$  and, for all j < i,  $M, s_j \models \varphi_1$ . Thus, AU says: "For all paths from now on, until some state..."

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- $M, s, \models E[\varphi_1 U \varphi_2]$  iff exists some path  $s_1 \mapsto s_2 \mapsto s_3 \mapsto \ldots \circ s_2 \mapsto s_3 \mapsto$

# Examples

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- "It's possible to get to a state where something has started but it's not ready": EF(started ∧ ¬ready)
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- "It's possible to get to a state where something has started but it's not ready": EF(started ∧ ¬ready)
- "A certain process is enabled infinitely often on every computation path": AG(AFenabled)
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor":

 $\texttt{AG}(\textit{floor2} \land \textit{directionUp} \land \textit{button5} \rightarrow \texttt{A}[\textit{directionUp} \texttt{Ufloor5}])$ 

# Equivalences

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