Computation Tree Logic

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Motivation

- Needing of uncertainty;

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- Different paths of the future;

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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$$\mathtt{AF}\phi \mid \mathtt{EF}\phi \mid \mathtt{AG}\phi \mid \mathtt{EG}\phi \mid \mathtt{A}[\phi \mathtt{U} \phi] \mid \mathtt{E}[\phi \mathtt{U} \phi]$$

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With p as a literal (atomic formula), $\mathtt{AX}$, $\mathtt{EX}$, $\mathtt{AF}$, $\mathtt{EF}$, $\mathtt{AG}$ e $\mathtt{EG}$ unary operators.

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- $\mathtt{F}\varphi$: There is some state in the future where φ is true;
- $\mathtt{G}\varphi$: Globally (in all future states) φ is true;
- $\varphi\mathtt{U}\psi$: φ is true at least until ψ becomes true;

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A **transition system** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to say that the model $\mathcal{M}, s$ satisfies the formula φ

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $\mathcal{M}, s \vDash \mathtt{AX}\varphi$ iff for all s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \vDash \varphi$. Thus, $\mathtt{AX}$ says: "in every next state..."

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- $M, s, \vDash \mathtt{A}[\varphi_1 \mathtt{U} \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, this path satisfies $\varphi_1 \mathtt{U} \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $M, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- $M, s, \models \mathtt{E}[\varphi_1 \mathtt{U} \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$

Examples

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- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $\mathtt{AG}(\mathit{floor}2 \land \mathit{directionUp} \land \mathit{button}5 \rightarrow \mathtt{A}[\mathit{directionUp}\,\mathtt{U}\,\mathit{floor}5])$

Definition