Computation Tree Logic

Luis Tertulino & Ronaldo Silveira

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- A brief introduction to Propositional Logic, its syntax and its semantics

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- Some practical applications

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- Different paths of the future;

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \to \phi \mid AX\phi \mid EX\phi \mid$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- $\varphi U \psi$: φ is true at least until ψ becomes true;

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- Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU and EU, respectively;
- We assume that, similarly to the $\neg$ operator, the "new" unary operators (AX, EX, AF, EF, AG, and EG) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, AU and EU;

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A **transition system** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $\mathcal{M}, s \vDash EX\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $M, s_1 \vDash \varphi$. Thus, EX says: "in some next state..."

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- $\mathcal{M}, s, \vDash AG\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."

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- $\mathcal{M}, s, \vDash AG\varphi$ iff for all paths $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."
- $\mathcal{M}, s, \vDash AG\varphi$ iff exists some path $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$ Thus, EG says: "Exists a path from now on in all next states..."

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- $\mathcal{M}, s, \vDash AF\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

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- $\mathcal{M}, s, \vDash EF\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, that exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, EF says: “In some path from now on, in some next state...”

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- $\mathcal{M}, s, \vDash EF\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, that exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, EF says: "In some path from now on, in some next state..."
- $\mathcal{M}, s, \vDash A[\varphi_1 U \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $\mathcal{M}, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- $\mathcal{M}, s, \vDash E[\varphi_1 U \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$. Thus, EU says: "In some path from now on, until some state..."

- "It's possible to get to a state where something has started but it's not ready": $EF(started \land \neg ready)$

Examples

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- "A certain process is enabled infinitely often on every computation path": $AG(AFenabled)$

- "It's possible to get to a state where something has started but it's not ready": $EF(started \wedge \neg ready)$
- "A certain process is enabled infinitely often on every computation path": $AG(AF enabled)$
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $AG(floor2 \wedge directionup \wedge button5 \rightarrow A[directionup\, U\, floor5])$

Equivalences

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Notation: we denote the semantic equivalence of φ and ψ by $\varphi \equiv \psi$

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- $AF\varphi \equiv A[\top U\varphi]$
- $EF\varphi \equiv E[\top U\varphi]$

Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Backus-Naur formalism below:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \rightarrow \phi \mid AX\phi \mid A[\phi U \phi] \mid E[\phi U \phi]$$

Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed. For example, we cannot say, as in LTL: "All paths in which have p on them, also have q on them".
This expression can be translated in LTL as follows:

$$Fp \to Fq$$

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That's all we need?

We can try expressing it as $AFp \rightarrow AFq$ but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then"
We can try to translate it as $AG(p \rightarrow AFq)$ which is closer, but not exactly the same. This one means "for all paths, in all states on the future, if they hold p then, all paths will eventualy hold q"

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This allows us to generate some statements:

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$

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- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$

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- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$
- "In all paths, p will occur in the next state or in the next of the next": $A[Xp \vee XXp]$

The syntax of CTL* can be defined with the BNF bellow:

$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid A[\alpha] \mid E[\alpha] \mid$$

$$\alpha ::= \phi \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \rightarrow \alpha \mid \alpha U \alpha \mid G\alpha \mid F\alpha \mid X\alpha \mid$$

With the same meanings of each operator.

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For CTL, it is trivial;

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- There are two different ways of working with model checking:
  Given a model, a state and a formula, tells if the model and the state satisfies the formula;
  Given a model and a formula, returns the set of states that satisfies the formula.

We present an algorithm that, given a model and a CTL formula, outputs the set of states of the model that satisfy the formula.

The algorithm deals explicitly only with some of the CTL connectives; for the others, it transforms them to their equivalent form in terms of the minimal set of connectives previously defined: $\{\perp, \neg, \wedge, AF, EU, EX\}$

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- First, rewrite ϕ in terms of $\perp, \neg, \wedge, AF, EU$ and EX.
- Next, label the states of $\mathcal{M}$ with the subformulas of ϕ that are satisfied there, starting with the smallest subformulas and working outwards towards ϕ.

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- p: then label s with p if $p \in L(s)$.

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- $\perp$: then no states are labeled with $\perp$.
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- $\neg\psi_1$: label s with $\neg\psi_1$ if s is not already labeled with ψ_1.
- $AF\psi_1$:
  - If any state s is labeled with ψ_1, label it with $AF\psi_1$.
  - Repeat: label any state with $AF\psi_1$ if all successor states are labeled with $AF\psi_1$, until there is no change.

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- $E[\psi_1 U \psi_2]$:
  - If any state s is labeled with ψ_2, label it with $E[\psi_1 U \psi_2]$.
  - Repeat: label any state with $E[\psi_1 U \psi_2]$ if it is labeled with ϕ_1 and at least one of its successors is labeled with $E[\psi_1 U \psi_2]$, until there is no change.

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- $EX\psi_1$: label any state with $EX\psi_1$ if one of its successors is labeled with ψ_1.

- Having performed the labeling for all the subformulas of ϕ (including ϕ itself), we output the states which are labeled ϕ.

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- The complexity of this algorithm is $O(fV(V+E))$, where f is the number of connectives in the formula, V is the number of states and E is the number of transitions.

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- Here, we present a simple, pretty pseudocode for the labeling algorithm.
- The program *SAT* expects a tree-structured CTL formula constructed by means of the BNF showed earlier.

function SAT(ϕ) **begin**

case

ϕ is $\top$: **return** S

ϕ is $\perp$: **return** $\emptyset$

ϕ is atomic: **return** $\{s \in S | \phi \in L(s)\}$

ϕ is $\neg\phi_1$: **return** $S - SAT(\phi_1)$

ϕ is $\phi_1 \wedge \phi_2$: **return** $SAT(\phi_1) \cap SAT(\phi_2)$

ϕ is $\phi_1 \vee \phi_2$: **return** $SAT(\phi_1) \cup SAT(\phi_2)$

ϕ is $\phi_1 \rightarrow \phi_2$: **return** $SAT(\neg\phi_1 \vee \phi_2)$

ϕ is $AX\phi_1$: **return** $SAT(\neg EX\neg\phi_1)$

ϕ is $EX\phi_1$: **return** $SAT_{EX}(\phi_1)$

ϕ is $A[\phi_1 U \phi_2]$: **return**
$SAT(\neg(E[\neg\phi_2 U(\neg\phi_1 \wedge \phi_2)] \vee EG\neg\phi_2))$

ϕ is $E[\phi_1 U \phi_2]$: **return** $SAT_{EU}(\phi_1, \phi_2)$

ϕ is $EF\phi_1$: **return** $SAT(E(\top U\phi_1))$
ϕ is $EG\phi_1$: **return** $SAT(\neg AF\neg\phi_1)$
ϕ is $AF\phi_1$: **return** $SAT_{AF}(\phi_1)$
ϕ is $AG\phi_1$: **return** $SAT(\neg EF\neg\phi_1)$
end case
end function

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- 'pre' denotes travelling backwards along the transition relation.
- $pre_{\exists}$ returns set of states of S which *can* make a transition into S.
- $pre_{\forall}$ returns the set of states of S which make transitions *only* into Y.

The pseudocode for the special procedures of SAT are the following.

function $SAT_{EX}(\phi)$
local var X, Y
begin
    $X := SAT(\phi)$
    $Y := pre_{\exists}(X)$
    return Y
end

function $SAT_{AF}(\phi)$
local var X, Y
begin
    $X := S$
    $Y := SAT(X)$
    repeat until $X = Y$
    begin
        $X := Y$
        $Y := Y \cup pre_{\forall}(Y)$
    end
    return Y
end

function $SAT_{EU}(\phi, \psi)$
local var W, X, Y
begin
    $W := SAT(\phi)$
    $X := S$
    $Y := SAT(\psi)$
    repeat until $X = Y$
    begin
        $X := Y$
        $Y := Y \cup (W \cap pre_{\exists}(Y))$
    end
    return Y
end