

Computation
Tree Logic

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& Ronaldo
Silveira

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Some
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Luis Tertulino & Ronaldo Silveira

October 21, 2015

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- Needing of uncertainty;

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- Different paths of the future;

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In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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The syntax of CTL consists on the syntax of temporal logic plus some path operators. The class of formulas can be defined in Backus-Naur form. If ϕ is a formula:

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$$\phi ::= \perp \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid \mathbf{AX}\phi \mid \mathbf{EX}\phi \mid$$

$$\mathbf{AF}\phi \mid \mathbf{EF}\phi \mid \mathbf{AG}\phi \mid \mathbf{EG}\phi \mid \mathbf{A}[\phi\mathbf{U}\phi] \mid \mathbf{E}[\phi\mathbf{U}\phi]$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.

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- $G\varphi$: Globally (in all future states) φ is true;

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- $X\phi$: ϕ is true until next state;
- $F\varphi$: There is some state in the future where φ is true;
- $G\varphi$: Globally (in all future states) φ is true;
- $\varphi U\psi$: φ is true at least until ψ becomes true;

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- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, i.e., AF is a operator that can be read as “In all paths in the future there is some state where...”

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- Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU

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- We assume that, similarly to the \neg operator, the “new” unary operators (AX , EX , AF , EF , AG , and EG) have the first precedence. Next comes the \wedge and \vee operators. And at last the \rightarrow , AU and EU

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- Examples of well-formed formulas:
 - $AG(p \vee EFq)$

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Different from usual logics, CTL formulas are interpreted by a transition system. Given an set of atoms:

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Definition of model

Different from usual logics, CTL formulas are interpreted by a transition system. Given an set of atoms:

Definition (1)

A **transition system** \mathcal{M} is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, \rightarrow is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Definition (2)

A **model** is a duple \mathcal{M}, s in which \mathcal{M} is a transition system and s is a state.

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Notation: we will use $\mathcal{M}, s \models \varphi$ to say that the model \mathcal{M}, s satisfies the formula φ

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Take an arbitrary model \mathcal{M} . Let s, s_1, s_2, s_3 be states in S . Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \perp$ for all $s \in S$

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- $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \perp$ for all $s \in S$
- $\mathcal{M}, s \models p$ iff $p \in L(s)$

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- $\mathcal{M}, s \models \neg\varphi$ iff $\mathcal{M}, s \not\models \varphi$

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- $\mathcal{M}, s \models p$ iff $p \in L(s)$
- $\mathcal{M}, s \models \neg\varphi$ iff $\mathcal{M}, s \not\models \varphi$
- $\mathcal{M}, s \models \varphi_1 \wedge \varphi_2$ iff $\mathcal{M}, s \models \varphi_1$ AND $\mathcal{M}, s \models \varphi_2$

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- $\mathcal{M}, s \models \varphi_1 \wedge \varphi_2$ iff $\mathcal{M}, s \models \varphi_1$ AND $\mathcal{M}, s \models \varphi_2$
- $\mathcal{M}, s \models \varphi_1 \vee \varphi_2$ iff $\mathcal{M}, s \models \varphi_1$ OR $\mathcal{M}, s \models \varphi_2$

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- $\mathcal{M}, s \models \text{AX}\varphi$ iff for all s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$.
Thus, AX says: “in every next state...”

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- $\mathcal{M}, s \models \text{EX}\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus,
EX says: “in some next state...”

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Thus, AX says: “in every next state...”
- $\mathcal{M}, s \models \text{EX}\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus,
EX says: “in some next state...”
- $\mathcal{M}, s \models \text{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which
 $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, AG says: “In all possible
paths from now on in all next states...”

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- $\mathcal{M}, s \models \text{EX}\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \models \varphi$. Thus, EX says: “in some next state...”
- $\mathcal{M}, s \models \text{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, AG says: “In all possible paths from now on in all next states...”
- $\mathcal{M}, s \models \text{EG}\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, for all s_i , $\mathcal{M}, s_i \models \varphi$. Thus, EG says: “Exists a path from now on in all next states...”

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Take an arbitrary model \mathcal{M} . Let s, s_1, s_2, s_3 be states in S . Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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- $M, s, \models \text{AF}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \dots$ in which $s = s_1$, exists s_i , $M, s_i \models \varphi$. Thus, AF says: “In all possible paths from now on, in some next state...”

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- “A certain process is enabled infinitely often on every computation path”: $AG(AF enabled)$
- “An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor”:
 $AG(floor2 \wedge directionUp \wedge button5 \rightarrow A[directionUp U floor5])$