Computation Tree Logic

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In previous chapters...

- Temporal Logic

Motivation and Intuition

- Motivation
- Intuition

How to communicate

- Syntax of CTL
- Semantics of CTL

Previously on Temporal Logic Week...

Temporal Logic

Motivation

Needing of uncertainty;

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Different paths of the future;

Intuition

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

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Semantics of CTL

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$$\mathtt{AF}\phi \mid \mathtt{EF}\phi \mid \mathtt{AG}\phi \mid \mathtt{EG}\phi \mid \mathtt{A}[\phi\mathtt{U}\phi] \mid \mathtt{E}[\phi\mathtt{U}\phi]$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- $\varphi\mathtt{U}\psi$: φ is true at least until ψ becomes true;

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We assume that, similarly to the $\neg$ operator, the "new" unary operators ($\mathtt{AX}, \mathtt{EX}, \mathtt{AF}, \mathtt{EF}, \mathtt{AG}$, and $\mathtt{EG}$) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, $\mathtt{AU}$ and $\mathtt{EU}$

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Definition (1)

A transition system $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) and $L : S \rightarrow \mathcal{P}(Atoms)$ is a labelling function.

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Notation: we will use $\mathcal{M}, s \vDash \varphi$ to say that the model $\mathcal{M}, s$ satisfies the formula φ

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Satisfaction

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

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$\mathcal{M}, s \models \varphi_1 \wedge \varphi_2$ iff $\mathcal{M}, s \models \varphi_1$ AND $\mathcal{M}, s \models \varphi_2$

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- $\mathcal{M}, s \vDash \mathtt{EX}\varphi$ iff exists s_1 that $s \to s_1$ and $M, s_1 \vDash \varphi$. Thus, EX says: "in some next state..."

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- $\mathcal{M}, s \vDash \texttt{EX}\varphi$ iff exists s_1 that $s \to s_1$ and $M, s_1 \vDash \varphi$. Thus, $\texttt{EX}$ says: "in some next state..."
- $M, s, \vDash \texttt{AG}\varphi$ iff for all paths $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$. Thus, $\texttt{AG}$ says: "In all possible paths from now on in all next states..."

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$M, s, \vDash \texttt{AG}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$. Thus, AG says: “In all possible paths from now on in all next states...”

$M, s, \vDash \texttt{AG}\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $M, s_i \vDash \varphi$ Thus, EG says: “Exists a path from now on in all next states...”

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$M, s, \vDash \mathtt{AF}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, exists s_i, $M, s_i \vDash \varphi$. Thus, $\mathtt{AF}$ says: "In all possible paths from now on, in some next state..."

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$M, s, \models \mathtt{AF}\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, exists s_i, $M, s_i \models \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

$M, s, \models \mathtt{EF}\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, that exists s_i, $M, s_i \models \varphi$. Thus, EF says: "In some path from now on, in some next state..."

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$M, s, \vDash \mathtt{A}[\varphi_1 \mathtt{U} \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, this path satisfies $\varphi_1 \mathtt{U} \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $M, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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