Computation Tree Logic

Luis Tertulino & Ronaldo Silveira

October 23, 2015

- A brief introduction to Propositional Logic, its syntax and its semantics

- A brief introduction to Propositional Logic, its syntax and its semantics
- Formal models of time

- A brief introduction to Propositional Logic, its syntax and its semantics
- Formal models of time
  - Frames and Flows of time

- A brief introduction to Propositional Logic, its syntax and its semantics
- Formal models of time
  - Frames and Flows of time
- Temporal Logic extends the Propositional Logic
  - The connectives H and G

- A brief introduction to Propositional Logic, its syntax and its semantics
- Formal models of time
  - Frames and Flows of time
- Temporal Logic extends the Propositional Logic
  - The connectives H and G
- Some practical applications

- "What good is Temporal Logic?"

- “What good is Temporal Logic?”
  - Answer: “Temporal Logic is a good method for specifying and reasoning about a concurrent program”.

- "What good is Temporal Logic?"
  - Answer: "Temporal Logic is a good method for specifying and reasoning about a concurrent program".
- Some mathematical definitions about states of a program and (concurrent) programs

- "What good is Temporal Logic?"
  - Answer: "Temporal Logic is a good method for specifying and reasoning about a concurrent program".
- Some mathematical definitions about states of a program and (concurrent) programs
- The Peterson's algorithm

- "What good is Temporal Logic?"
  - Answer: "Temporal Logic is a good method for specifying and reasoning about a concurrent program".
- Some mathematical definitions about states of a program and (concurrent) programs
- The Peterson's algorithm
- How the model checking works?

- "What good is Temporal Logic?"
  - Answer: "Temporal Logic is a good method for specifying and reasoning about a concurrent program".
- Some mathematical definitions about states of a program and (concurrent) programs
- The Peterson's algorithm
- How the model checking works?
- Some strengths and weaknesses of model checking

- Syntax and Semantics of LTL

- Syntax and Semantics of LTL
- ω-languages, Kripke structures, paths and traces
- Buchi automata and LTL model checking

- Needing of expressing uncertainty;

- Needing of expressing uncertainty;
- Different paths of the future;

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

CTL was defined by:

Figure 1: Mordechai Ben-Ari

Figure 2: Amir Pnueli

Figure 3: Zohar Manna

And, at the same time by:

Figure 4: Ernest Allen Emerson

Figure 5: Edmund Clarke

The syntax of CTL consists on the syntax of temporal logic plus some path operators. The class of formulas can be defined in Backus-Naur form. If ϕ is a formula:

The syntax of CTL consists on the syntax of temporal logic plus some path operators. The class of formulas can be defined in Backus-Naur form. If ϕ is a formula:

$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid AX\phi \mid EX\phi \mid$$

$$AF\phi \mid EF\phi \mid AG\phi \mid EG\phi \mid A[\phi U \phi] \mid E[\phi U \phi]$$

The syntax of CTL consists on the syntax of temporal logic plus some path operators. The class of formulas can be defined in Backus-Naur form. If ϕ is a formula:

$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid AX\phi \mid EX\phi \mid$$

$$AF\phi \mid EF\phi \mid AG\phi \mid EG\phi \mid A[\phi U \phi] \mid E[\phi U \phi]$$

With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.
The path-specific operators can be read as:

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.
The path-specific operators can be read as:

- A: is the universal quantifier over paths. Read as: "in all possible paths";

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.
The path-specific operators can be read as:

- A: is the universal quantifier over paths. Read as: "in all possible paths";
- E:is the existential quantifier over paths. Read as: "exists a path in which";

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.
The path-specific operators can be read as:

- A: is the universal quantifier over paths. Read as: "in all possible paths";
- E:is the existential quantifier over paths. Read as: "exists a path in which";

The temporal operators, as in LTL, can be read as:

- X: "in the next state";

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.
The path-specific operators can be read as:

- *A*: is the universal quantifier over paths. Read as: “in all possible paths”;
- *E*:is the existential quantifier over paths. Read as: “exists a path in which”;

The temporal operators, as in LTL, can be read as:

- *X*: “in the next state”;
- *F* “There is some state in the future (eventually)”;

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.
The path-specific operators can be read as:

- *A*: is the universal quantifier over paths. Read as: "in all possible paths";
- *E*:is the existential quantifier over paths. Read as: "exists a path in which";

The temporal operators, as in LTL, can be read as:

- *X*: "in the next state";
- *F* "There is some state in the future (eventually)";
- *G* "Globally (in all future states)";

The propositional operators: $\neg, \vee, \wedge, \rightarrow$ have the same meaning of in the propositional logic.
The path-specific operators can be read as:

- *A*: is the universal quantifier over paths. Read as: "in all possible paths";
- *E*:is the existential quantifier over paths. Read as: "exists a path in which";

The temporal operators, as in LTL, can be read as:

- *X*: "in the next state";
- *F* "There is some state in the future (eventually)";
- *G* "Globally (in all future states)";
- $\varphi U \psi$: φ is true at least until ψ becomes true;

- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., AF is an atomic operator that can be read as "In all paths in the future there is some state where..";

- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., AF is an atomic operator that can be read as "In all paths in the future there is some state where..";
- Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU and EU, respectively;

- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., AF is an atomic operator that can be read as "In all paths in the future there is some state where..";
- Notice as well that the binary operators $A[\varphi U \psi]$ and $E[\varphi U \psi]$ can be represented as AU and EU, respectively;
- We assume that, similarly to the $\neg$ operator, the "new" unary operators (AX, EX, AF, EF, AG, and EG) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, AU and EU;

- Examples of well-formed formulas:
  - $AG(p \vee EFq)$

- Examples of well-formed formulas:
  - $AG(p \vee EFq)$
  - $AX(q \rightarrow E[(p \vee q)Ur])$

Examples

- Examples of well-formed formulas:
  - $AG(p \vee EFq)$
  - $AX(q \rightarrow E[(p \vee q)Ur])$
  - $EFEGp \rightarrow AFr$ Note that this is binded as $(EFEGp) \rightarrow AFr$, not as $EFEG(p \rightarrow AFr)$

Examples

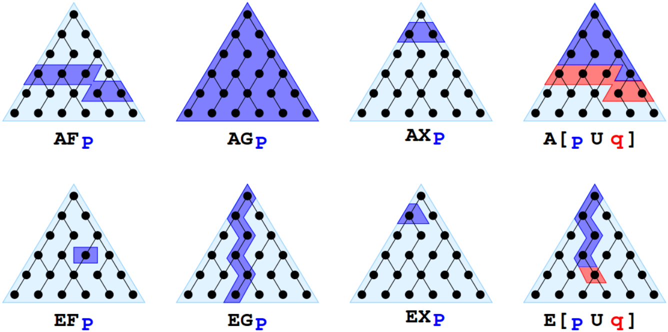
- Examples of well-formed formulas:
  - $AG(p \vee EFq)$
  - $AX(q \rightarrow E[(p \vee q)Ur])$
  - $EFEGp \rightarrow AFr$ Note that this is binded as $(EFEGp) \rightarrow AFr$, not as $EFEG(p \rightarrow AFr)$
- Example of formulas that are not well-formed:
  - $A\neg G\neg p$

Examples

- Examples of well-formed formulas:
  - $AG(p \vee EFq)$
  - $AX(q \to E[(p \vee q)Ur])$
  - $EFEGp \to AFr$ Note that this is binded as $(EFEGp) \to AFr$, not as $EFEG(p \to AFr)$
- Example of formulas that are not well-formed:
  - $A\neg G\neg p$
  - $F[pUs]$

Examples

- Examples of well-formed formulas:
  - $AG(p \vee EFq)$
  - $AX(q \to E[(p \vee q)Ur])$
  - $EFEGp \to AFr$ Note that this is binded as $(EFEGp) \to AFr$, not as $EFEG(p \to AFr)$
- Example of formulas that are not well-formed:
  - $A\neg G\neg p$
  - $F[pUs]$
  - $A[pUs \wedge qUs]$


AF p
AG p
AX p
A [p U q]
EF p
EG p
EX p
E [p U q]

Definition

Let *Atoms* be a set of atomic formulas. A **transition system** or **model** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) such that for every state $s \in S$, exists a s' that $s \rightarrow s'$ and $L : S \rightarrow \mathcal{P}(Atoms)$ (or $L : S \rightarrow (Atoms \rightarrow \{0, 1\})$) is a labelling function.

Definition

Let *Atoms* be a set of atomic formulas. A **transition system** or **model** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) such that for every state $s \in S$, exists a s' that $s \rightarrow s'$ and $L : S \rightarrow \mathcal{P}(Atoms)$ (or $L : S \rightarrow (Atoms \rightarrow \{0, 1\})$) is a labelling function.

CTL formulas are satisfied by a transition system and a specific state.
Notation: we will use $\mathcal{M}, s \models \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Definition

The **satisfaction** of a formula in CTL is recursive over the structure of the formula. It can be done as follows:

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$
- $\mathcal{M}, s \vDash p$ iff $p \in L(S)$

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$
- $\mathcal{M}, s \vDash p$ iff $p \in L(S)$
- $\mathcal{M}, s \vDash \neg\varphi$ iff $\mathcal{M}, s \nvDash \varphi$

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \bot$ for all $s \in S$
- $\mathcal{M}, s \models p$ iff $p \in L(S)$
- $\mathcal{M}, s \models \neg\varphi$ iff $\mathcal{M}, s \not\models \varphi$
- $\mathcal{M}, s \models \varphi_1 \wedge \varphi_2$ iff $\mathcal{M}, s \models \varphi_1$ AND $\mathcal{M}, s \models \varphi_2$

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$
- $\mathcal{M}, s \vDash p$ iff $p \in L(S)$
- $\mathcal{M}, s \vDash \neg\varphi$ iff $\mathcal{M}, s \nvDash \varphi$
- $\mathcal{M}, s \vDash \varphi_1 \wedge \varphi_2$ iff $\mathcal{M}, s \vDash \varphi_1$ AND $\mathcal{M}, s \vDash \varphi_2$
- $\mathcal{M}, s \vDash \varphi_1 \vee \varphi_2$ iff $\mathcal{M}, s \vDash \varphi_1$ OR $\mathcal{M}, s \vDash \varphi_2$

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$
- $\mathcal{M}, s \vDash p$ iff $p \in L(S)$
- $\mathcal{M}, s \vDash \neg\varphi$ iff $\mathcal{M}, s \nvDash \varphi$
- $\mathcal{M}, s \vDash \varphi_1 \wedge \varphi_2$ iff $\mathcal{M}, s \vDash \varphi_1$ AND $\mathcal{M}, s \vDash \varphi_2$
- $\mathcal{M}, s \vDash \varphi_1 \vee \varphi_2$ iff $\mathcal{M}, s \vDash \varphi_1$ OR $\mathcal{M}, s \vDash \varphi_2$
- $\mathcal{M}, s \vDash \varphi_1 \rightarrow \varphi_2$ iff $\mathcal{M}, s \nvDash \varphi_1$ OR $\mathcal{M}, s \vDash \varphi_2$

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash AX\varphi$ iff for all s_1 that $s \to s_1$ and $\mathcal{M}, s_1 \vDash \varphi$. Thus, AX says: "in every next state..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash AX\varphi$ iff for all s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \vDash \varphi$. Thus, AX says: "in every next state..."
- $\mathcal{M}, s \vDash EX\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $M, s_1 \vDash \varphi$. Thus, EX says: "in some next state..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash AX\varphi$ iff for all s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \vDash \varphi$. Thus, AX says: "in every next state..."
- $\mathcal{M}, s \vDash EX\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $M, s_1 \vDash \varphi$. Thus, EX says: "in some next state..."
- $\mathcal{M}, s, \vDash AG\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash AX\varphi$ iff for all s_1 that $s \rightarrow s_1$ and $\mathcal{M}, s_1 \vDash \varphi$. Thus, AX says: "in every next state..."
- $\mathcal{M}, s \vDash EX\varphi$ iff exists s_1 that $s \rightarrow s_1$ and $M, s_1 \vDash \varphi$. Thus, EX says: "in some next state..."
- $\mathcal{M}, s, \vDash AG\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."
- $\mathcal{M}, s, \vDash AG\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$ Thus, EG says: "Exists a path from now on in all next states..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s, \vDash AF\varphi$ iff for all paths $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s, \vDash AF\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."
- $\mathcal{M}, s, \vDash EF\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, that exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, EF says: "In some path from now on, in some next state..."

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s, \vDash AF\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."
- $\mathcal{M}, s, \vDash EF\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, that exists s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, EF says: "In some path from now on, in some next state..."
- $\mathcal{M}, s, \vDash A[\varphi_1 U \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $\mathcal{M}, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

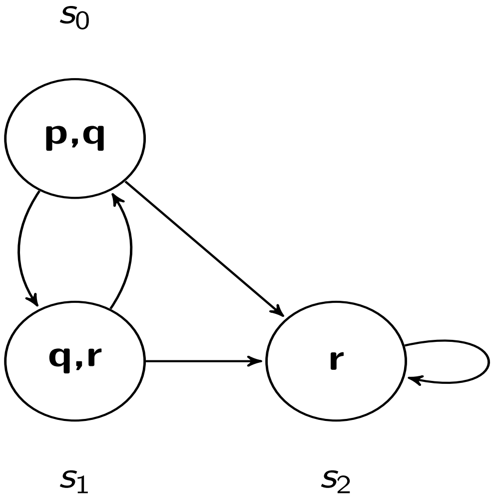
Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

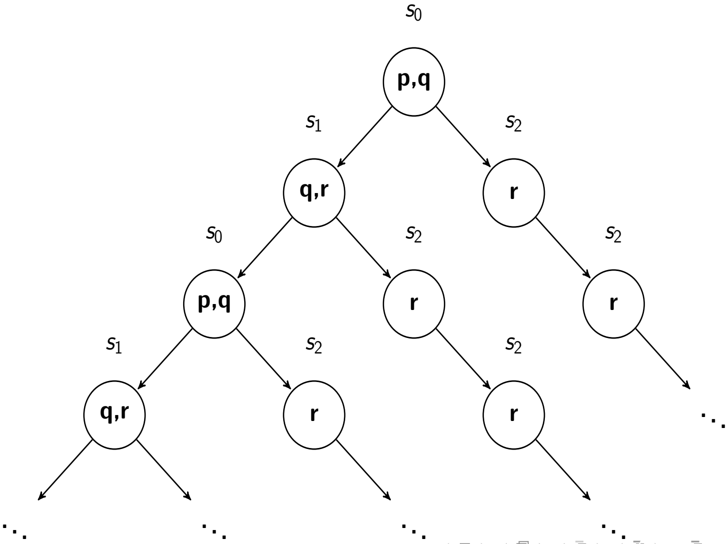
- $\mathcal{M}, s, \vDash E[\varphi_1 U \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$. Thus, EU says: "In some path from now on, until some state..."

- "It's possible to get to a state where something has started but it's not ready": $EF(started \wedge \neg ready)$

- "It's possible to get to a state where something has started but it's not ready": $EF(started \wedge \neg ready)$
- "A certain process is enabled infinitely often on every computation path": $AG(AFenabled)$

- "It's possible to get to a state where something has started but it's not ready": $EF(started \wedge \neg ready)$
- "A certain process is enabled infinitely often on every computation path": $AG(AFenabled)$
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $AG(floor2 \wedge directionup \wedge button5 \rightarrow A[directionup\, U floor5])$


s_0
p,q
q,r
r
s_1
s_2


s_0
p,q
s_1
s_2
q,r
r
s_0
s_2
s_2
p,q
r
r
s_1
s_2
s_2
q,r
r
r
…

Example of formulas that are satisfied by that model:

- $\mathcal{M}, s_0 \vDash p \wedge q$
- $\mathcal{M}, s_0 \vDash \neg r$
- $\mathcal{M}, s_0 \vDash EX(q \wedge r)$
- $\mathcal{M}, s_0 \vDash \neg AX(q \wedge r)$
- $\mathcal{M}, s_0 \vDash \neg EF(p \wedge q)$
- $\mathcal{M}, s_2 \vDash EGr$
- $\mathcal{M}, s_0 \vDash AFr$
- $\mathcal{M}, s_0 \vDash E[(p \wedge q)Ur]$
- $\mathcal{M}, s_0 \vDash A[pUr]$
- $\mathcal{M}, s_0 \vDash AG(p \vee q \vee r \rightarrow EFEGr)$

Definition

Two CTL formulas φ and ψ are said to be **semantically equivalent** if any state in any model which satisfies one of them also satisfies the other.

Equivalences

Definition

Two CTL formulas φ and ψ are said to be **semantically equivalent** if any state in any model which satisfies one of them also satisfies the other.

Notation: we denote the semantic equivalence of φ and ψ by $\varphi \equiv \psi$

Let φ be an arbitrary CTL formula.

- $\neg AF\varphi \equiv EG\neg\varphi$

Let φ be an arbitrary CTL formula.

- $\neg AF\varphi \equiv EG\neg\varphi$
- $\neg EF\varphi \equiv AG\neg\varphi$

Example of equivalences

Let φ be an arbitrary CTL formula.

- $\neg AF\varphi \equiv EG\neg\varphi$
- $\neg EF\varphi \equiv AG\neg\varphi$
- $\neg AX\varphi \equiv EX\neg\varphi$

Example of equivalences

Let φ be an arbitrary CTL formula.

- $\neg AF\varphi \equiv EG\neg\varphi$
- $\neg EF\varphi \equiv AG\neg\varphi$
- $\neg AX\varphi \equiv EX\neg\varphi$
- $AF\varphi \equiv A[\top U\varphi]$

Let φ be an arbitrary CTL formula.

- $\neg AF\varphi \equiv EG\neg\varphi$
- $\neg EF\varphi \equiv AG\neg\varphi$
- $\neg AX\varphi \equiv EX\neg\varphi$
- $AF\varphi \equiv A[\top U\varphi]$
- $EF\varphi \equiv E[\top U\varphi]$

Minimum set of CTL connectives

Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Backus-Naur formalism below:

$$\phi ::= \bot \mid p \mid \neg\phi \mid \phi \wedge \phi \mid EX\phi \mid AF\phi \mid E[\phi U \phi]$$

That's all we need?

Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed. For example, we cannot say, as in LTL: "All paths in which have p on them, also have q on them".
This expression can be translated in LTL as follows:

$$Fp \rightarrow Fq$$

That's all we need?

We can try expressing it as $AFp \rightarrow AFq$ but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then"

We can try expressing it as $AFp \rightarrow AFq$ but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then"
We can try to translate it as $AG(p \rightarrow AFq)$ which is closer, but not exactly the same. This one means "for all paths, in all states on the future, if they hold p then, all paths will eventualy hold q"

For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).

For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).
This allows us to generate some statements:

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$
- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$
- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$
- "In all paths, p will occur in the next state or in the next of the next": $A[Xp \vee XXp]$

The syntax of CTL* can be defined with the BNF bellow:

$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid A[\alpha] \mid E[\alpha] \mid$$

$$\alpha ::= \phi \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \rightarrow \alpha \mid \alpha U \alpha \mid G\alpha \mid F\alpha \mid X\alpha \mid$$

With the same meanings of each operator.

Although we don't define path operators to LTL we can assume that it consider in all paths. Therefore, we can say that a formula ϕ in LTL is a formula $A[\phi]$ in CTL*;

Although we don't define path operators to LTL we can assume that it consider in all paths. Therefore, we can say that a formula ϕ in LTL is a formula $A[\phi]$ in CTL*;
For CTL, it is trivial;

The CTL model-checking algorithm

- Given a property of a program expressed in a temporal logic, the model checker checks if the states of the program satisfy the formula.

The CTL model-checking algorithm

- Given a property of a program expressed in a temporal logic, the model checker checks if the states of the program satisfy the formula.
- Obviously, we use CTL formulas.

- Given a property of a program expressed in a temporal logic, the model checker checks if the states of the program satisfy the formula.
- Obviously, we use CTL formulas.
- There are two different ways of working with model checking:

- Given a property of a program expressed in a temporal logic, the model checker checks if the states of the program satisfy the formula.
- Obviously, we use CTL formulas.
- There are two different ways of working with model checking:
  Given a model, a state and a formula, tells if the model and the state satisfies the formula;

The CTL model-checking algorithm

- Given a property of a program expressed in a temporal logic, the model checker checks if the states of the program satisfy the formula.
- Obviously, we use CTL formulas.
- There are two different ways of working with model checking:
  Given a model, a state and a formula, tells if the model and the state satisfies the formula;
  Given a model and a formula, returns the set of states that satisfies the formula.

We present an algorithm that, given a model and a CTL formula, outputs the set of states of the model that satisfy the formula.

The algorithm deals explicitly only with some of the CTL connectives; for the others, it transforms them to their equivalent form in terms of the minimal set of connectives previously defined: $\{\perp, \neg, \wedge, AF, EU, EX\}$

Here is the algorithm:

Here is the algorithm:
INPUT: a CTL model $\mathcal{M} = (S, \rightarrow, L)$ and a CTL formula ϕ.
OUTPUT: the set of states of $\mathcal{M}$ which satisfies ϕ.

Here is the algorithm:
INPUT: a CTL model $\mathcal{M} = (S, \rightarrow, L)$ and a CTL formula ϕ.
OUTPUT: the set of states of $\mathcal{M}$ which satisfies ϕ.

- First, rewrite ϕ in terms of $\perp, \neg, \wedge, AF, EU$ and EX.

Here is the algorithm:
INPUT: a CTL model $\mathcal{M} = (S, \rightarrow, L)$ and a CTL formula ϕ.
OUTPUT: the set of states of $\mathcal{M}$ which satisfies ϕ.

- First, rewrite ϕ in terms of $\bot, \neg, \wedge, AF, EU$ and EX.
- Next, label the states of $\mathcal{M}$ with the subformulas of ϕ that are satisfied there, starting with the smallest subformulas and working outwards towards ϕ.

Suppose ψ is a subformula of ϕ and states satisfying all the immediate subformulas of ψ have already been labeled. We determine by a case analysis which states to label with ψ. If ψ is

- $\perp$: then no states are labeled with $\perp$.

Suppose ψ is a subformula of ϕ and states satisfying all the immediate subformulas of ψ have already been labeled. We determine by a case analysis which states to label with ψ. If ψ is

- $\perp$: then no states are labeled with $\perp$.
- p: then label s with p if $p \in L(s)$.

Suppose ψ is a subformula of ϕ and states satisfying all the immediate subformulas of ψ have already been labeled. We determine by a case analysis which states to label with ψ. If ψ is

- $\perp$: then no states are labeled with $\perp$.
- p: then label s with p if $p \in L(s)$.
- $\psi_1 \wedge \psi_2$: label s with $\psi_1 \wedge \psi_2$ if s is already labeled both with ψ_1 and with ψ_2.

Suppose ψ is a subformula of ϕ and states satisfying all the immediate subformulas of ψ have already been labeled. We determine by a case analysis which states to label with ψ. If ψ is

- $\perp$: then no states are labeled with $\perp$.
- p: then label s with p if $p \in L(s)$.
- $\psi_1 \wedge \psi_2$: label s with $\psi_1 \wedge \psi_2$ if s is already labeled both with ψ_1 and with ψ_2.
- $\neg\psi_1$: label s with $\neg\psi_1$ if s is not already labeled with ψ_1.

Suppose ψ is a subformula of ϕ and states satisfying all the immediate subformulas of ψ have already been labeled. We determine by a case analysis which states to label with ψ. If ψ is

- $\perp$: then no states are labeled with $\perp$.
- p: then label s with p if $p \in L(s)$.
- $\psi_1 \wedge \psi_2$: label s with $\psi_1 \wedge \psi_2$ if s is already labeled both with ψ_1 and with ψ_2.
- $\neg\psi_1$: label s with $\neg\psi_1$ if s is not already labeled with ψ_1.
- $AF\psi_1$:
  - If any state s is labeled with ψ_1, label it with $AF\psi_1$.
  - Repeat: label any state with $AF\psi_1$ if all successor states are labeled with $AF\psi_1$, until there is no change.

Suppose ψ is a subformula of ϕ and states satisfying all the immediate subformulas of ψ have already been labeled. We determine by a case analysis which states to label with ψ. If ψ is

- $E[\psi_1 U \psi_2]$:
  - If any state s is labeled with ψ_2, label it with $E[\psi_1 U \psi_2]$.
  - Repeat: label any state with $E[\psi_1 U \psi_2]$ if it is labeled with ϕ_1 and at least one of its successors is labeled with $E[\psi_1 U \psi_2]$, until there is no change.

Suppose ψ is a subformula of ϕ and states satisfying all the immediate subformulas of ψ have already been labeled. We determine by a case analysis which states to label with ψ. If ψ is

- $E[\psi_1 U \psi_2]$:
  - If any state s is labeled with ψ_2, label it with $E[\psi_1 U \psi_2]$.
  - Repeat: label any state with $E[\psi_1 U \psi_2]$ if it is labeled with ϕ_1 and at least one of its successors is labeled with $E[\psi_1 U \psi_2]$, until there is no change.
- $EX\psi_1$: label any state with $EX\psi_1$ if one of its successors is labeled with ψ_1.

- Having performed the labeling for all the subformulas of ϕ (including ϕ itself), we output the states which are labeled ϕ.

- Having performed the labeling for all the subformulas of ϕ (including ϕ itself), we output the states which are labeled ϕ.
- The complexity of this algorithm is $O(fV(V+E))$, where f is the number of connectives in the formula, V is the number of states and E is the number of transitions.

- Here, we present a simple, pretty pseudocode for the labeling algorithm.

- Here, we present a simple, pretty pseudocode for the labeling algorithm.
- The program *SAT* expects a tree-structured CTL formula constructed by means of the BNF showed earlier.

function SAT(ϕ) **begin**

case

ϕ is $\top$: **return** S

ϕ is $\perp$: **return** $\emptyset$

ϕ is atomic: **return** $\{s \in S | \phi \in L(s)\}$

ϕ is $\neg\phi_1$: **return** $S - SAT(\phi_1)$

ϕ is $\phi_1 \wedge \phi_2$: **return** $SAT(\phi_1) \cap SAT(\phi_2)$

ϕ is $\phi_1 \vee \phi_2$: **return** $SAT(\phi_1) \cup SAT(\phi_2)$

ϕ is $\phi_1 \rightarrow \phi_2$: **return** $SAT(\neg\phi_1 \vee \phi_2)$

ϕ is $AX\phi_1$: **return** $SAT(\neg EX \neg\phi_1)$

ϕ is $EX\phi_1$: **return** $SAT_{EX}(\phi_1)$

ϕ is $A[\phi_1 U \phi_2]$: **return** $SAT(\neg(E[\neg\phi_2 U(\neg\phi_1 \wedge \phi_2)] \vee EG\neg\phi_2))$

ϕ is $E[\phi_1 U \phi_2]$: **return** $SAT_{EU}(\phi_1, \phi_2)$

ϕ is $EF\phi_1$: **return** $SAT(E(\top U\phi_1))$
ϕ is $EG\phi_1$: **return** $SAT(\neg AF\neg\phi_1)$
ϕ is $AF\phi_1$: **return** $SAT_{AF}(\phi_1)$
ϕ is $AG\phi_1$: **return** $SAT(\neg EF\neg\phi_1)$
end case
end function

- SAT handles with the easy cases (the propositional) directly and passes more complicated cases (the temporal) on to special procedures.

- SAT handles with the easy cases (the propositional) directly and passes more complicated cases (the temporal) on to special procedures.
- These special procedures uses the following functions:

- SAT handles with the easy cases (the propositional) directly and passes more complicated cases (the temporal) on to special procedures.
- These special procedures uses the following functions:
$pre_{\exists}(Y) = \{s \in S | \exists s'(s \rightarrow s' \wedge s' \in Y)\}$
$pre_{\forall}(Y) = \{s \in S | \forall s'(s \rightarrow s' \longrightarrow s' \in Y)\}$

- SAT handles with the easy cases (the propositional) directly and passes more complicated cases (the temporal) on to special procedures.
- These special procedures uses the following functions:
  $pre_{\exists}(Y) = \{s \in S | \exists s'(s \rightarrow s' \wedge s' \in Y)\}$
  $pre_{\forall}(Y) = \{s \in S | \forall s'(s \rightarrow s' \longrightarrow s' \in Y)\}$
- 'pre' denotes travelling backwards along the transition relation.

- SAT handles with the easy cases (the propositional) directly and passes more complicated cases (the temporal) on to special procedures.
- These special procedures uses the following functions:
  $pre_{\exists}(Y) = \{s \in S | \exists s'(s \to s' \wedge s' \in Y)\}$
  $pre_{\forall}(Y) = \{s \in S | \forall s'(s \to s' \longrightarrow s' \in Y)\}$
- 'pre' denotes travelling backwards along the transition relation.
- $pre_{\exists}$ returns set of states of S which *can* make a transition into S.

- SAT handles with the easy cases (the propositional) directly and passes more complicated cases (the temporal) on to special procedures.
- These special procedures uses the following functions:
  $pre_{\exists}(Y) = \{s \in S | \exists s'(s \to s' \land s' \in Y)\}$
  $pre_{\forall}(Y) = \{s \in S | \forall s'(s \to s' \longrightarrow s' \in Y)\}$
- 'pre' denotes travelling backwards along the transition relation.
- $pre_{\exists}$ returns set of states of S which *can* make a transition into S.
- $pre_{\forall}$ returns the set of states of S which make transitions *only* into Y.

The pseudocode for the special procedures of SAT are the following.

function $SAT_{EX}(\phi)$
local var X, Y
begin
  $X := SAT(\phi)$
  $Y := pre_{\exists}(X)$
  return Y
end

function $SAT_{AF}(\phi)$
local var X, Y
begin
    $X := S$
    $Y := SAT(X)$
    repeat until $X = Y$
    begin
        $X := Y$
        $Y := Y \cup pre_{\forall}(Y)$
    end
    return Y
end

function $SAT_{EU}(\phi, \psi)$
local var W, X, Y
begin
    $W := SAT(\phi)$
    $X := S$
    $Y := SAT(\psi)$
    repeat until $X = Y$
    begin
        $X := Y$
        $Y := Y \cup (W \cap pre_{\exists}(Y))$
    end
    return Y
end

In this week we leaned a lot.

- Important way of speaking about time and it's properties with the Introduction to Temporal Logic.

In this week we leaned a lot.

- Important way of speaking about time and it's properties with the Introduction to Temporal Logic.
- How to know if your program is really working with Model Checking

In this week we leaned a lot.

- Important way of speaking about time and it's properties with the Introduction to Temporal Logic.
- How to know if your program is really working with Model Checking
- How to specify time as a linear structure of states and how to reason about it in a logic way with Linear Temporal Logic

In this week we leaned a lot.

- Important way of speaking about time and it's properties with the Introduction to Temporal Logic.
- How to know if your program is really working with Model Checking
- How to specify time as a linear structure of states and how to reason about it in a logic way with Linear Temporal Logic
- How to think about time not as a linear structure but as a tree with choices that may modify the state of the future with Computation Tree Logic

In this week we leaned a lot.

- Important way of speaking about time and it's properties with the Introduction to Temporal Logic.
- How to know if your program is really working with Model Checking
- How to specify time as a linear structure of states and how to reason about it in a logic way with Linear Temporal Logic
- How to think about time not as a linear structure but as a tree with choices that may modify the state of the future with Computation Tree Logic

This was new to us all. We (in the name of all the seven) hope you enjoyed and learned as much as us.

[1] Mordechai Ben-Ari. *Mathematical logic for computer science*. Springer Science & Business Media, 2012.

[2] Michael Huth and Mark Ryan. *Logic in Computer Science*. Cambridge University Press, 2004.

[3] Mordechai Ben-Ari, Zohar Manna, and Amir Pnueli. The temporal logic of branching time. 1981.

[4] Alessandro Artale. Formal methods lecture iv: Computation tree logic (ctl). URL `http://www.inf.unibz.it/~artale/FM/slide4.pdf`.

SEE YOU SPACE COWBOY...