Computation Tree Logic

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October 22, 2015

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- Some practical applications

- Needing of uncertainty;

Motivation

- Needing of uncertainty;
- Different paths of the future;

In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

CTL was defined by:

Figure 1: Mordechai Ben-Ari

Figure 2: Amir Pnueli

Figure 3: Zohar Manna

And, at the same time by:

Figure 4: Ernest Allen Emerson

Figure 5: Edmund Clarke

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$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \to \phi \mid AX\phi \mid EX\phi \mid$$

$$AF\phi \mid EF\phi \mid AG\phi \mid EG\phi \mid A[\phi U \phi] \mid E[\phi U \phi]$$

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With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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- G "Globally (in all future states)";
- $\varphi U \psi$: φ is true at least until ψ becomes true;

- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., AF is an atomic operator that can be read as "In all paths in the future there is some state where..." ;

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- We assume that, similarly to the $\neg$ operator, the "new" unary operators (AX, EX, AF, EF, AG, and EG) have the first precedence. Next comes the $\wedge$ and $\vee$ operators. And at last the $\rightarrow$, AU and EU;

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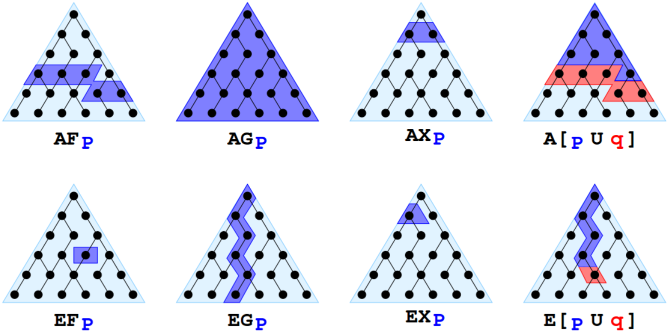
  - $A[pUs \wedge qUs]$


AF p
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AX p
A [p U q]
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EX p
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Definition

Let *Atoms* be a set of atomic formulas. A **transition system** or **model** $\mathcal{M}$ is a triple $\mathcal{M} = (S, \rightarrow, L)$ in which S is a set of states, $\rightarrow$ is a binary relation over S ($\rightarrow \subseteq S \times S$) such that for every state $s \in S$, exists a s' that $s \rightarrow s'$ and $L : S \rightarrow \mathcal{P}(Atoms)$ (or $L : S \rightarrow (Atoms \rightarrow \{0, 1\})$) is a labelling function.

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CTL formulas are satisfied by a transition system and a specific state.

Notation: we will use $\mathcal{M}, s \vDash \varphi$ to denote that the model $\mathcal{M}, s$ satisfies the formula φ

Definition

The **satisfaction** of a formula in CTL is recursive over the structure of the formula. It can be done as follows:

Take an arbitrary model $\mathcal{M}$. Let s, s_1, s_2, s_3 be states in S. Let $\varphi, \varphi_1, \varphi_2$ be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

- $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$ for all $s \in S$

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- $\mathcal{M}, s, \vDash AG\varphi$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$. Thus, AG says: "In all possible paths from now on in all next states..."

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- $\mathcal{M}, s, \vDash AG\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, for all s_i, $\mathcal{M}, s_i \vDash \varphi$ Thus, EG says: "Exists a path from now on in all next states..."

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- $\mathcal{M}, s, \models AF\varphi$ iff for all paths $s_1 \to s_2 \to s_3 \to ...$ in which $s = s_1$, exists s_i, $\mathcal{M}, s_i \models \varphi$. Thus, AF says: "In all possible paths from now on, in some next state..."

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- $\mathcal{M}, s, \models EF\varphi$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, that exists s_i, $\mathcal{M}, s_i \models \varphi$. Thus, EF says: "In some path from now on, in some next state..."

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- $\mathcal{M}, s, \vDash A[\varphi_1 U \varphi_2]$ iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$, i.e., exists s_i in the path such that $\mathcal{M}, s_i \vDash \varphi_2$ and, for all $j < i$, $\mathcal{M}, s_j \vDash \varphi_1$. Thus, AU says: "For all paths from now on, until some state..."

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- $\mathcal{M}, s, \vDash E[\varphi_1 U \varphi_2]$ iff exists some path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$ in which $s = s_1$, this path satisfies $\varphi_1 U \varphi_2$. Thus, EU says: "In some path from now on, until some state..."

- "It's possible to get to a state where something has started but it's not ready": $EF(started \wedge \neg ready)$

Examples

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- "A certain process is enabled infinitely often on every computation path": $AG(AFenabled)$

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- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor": $AG(floor2 \wedge directionup \wedge button5 \rightarrow A[directionup\,U\,floor5])$

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Notation: we denote the semantic equivalence of φ and ψ by $\varphi \equiv \psi$

Example of equivalences

Let φ be an arbitrary CTL formula.

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Minimum set of CTL connectives

Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Backus-Naur formalism below:

$$\phi ::= \bot \mid p \mid \neg\phi \mid \phi \wedge \phi \mid EX\phi \mid AF\phi \mid E[\phi U \phi]$$

That's all we need?

Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed. For example, we cannot say, as in LTL: "All paths in which have p on them, also have q on them".
This expression can be translated in LTL as follows:

$$Fp \to Fq$$

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We can try to translate it as $AG(p \rightarrow AFq)$ which is closer, but not exactly the same. This one means "for all paths, in all states on the future, if they hold p then, all paths will eventualy hold q"

For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).

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This allows us to generate some statements:

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$

- "In all possible paths, q is true until r is true or p is true until r is true": $A[qUr \vee pUr]$
- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$

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- "There is a path in which p eventually occurring will occur in all states": $E[GFp]$
- "In all paths, p will occur in the next state or in the next of the next": $A[Xp \vee XXp]$

The syntax of CTL* can be defined with the BNF bellow:

$$\phi ::= \bot \mid \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid A[\alpha] \mid E[\alpha] \mid$$

$$\alpha ::= \phi \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \rightarrow \alpha \mid \alpha U \alpha \mid G\alpha \mid F\alpha \mid X\alpha \mid$$

With the same meanings of each operator.

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