## Computation Tree Logic

#### Luis Tertulino & Ronaldo Silveira

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# Computation Tree Logic

Luis Tertulino & Ronaldo Silveira

October 22, 2015

## Computation Tree Logic

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- Temporal Logic extends the Propositional Logic
  - $\blacksquare$  The connectives H and G

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- Some practical applications

# Motivation

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■ Needing of uncertainty;

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- Needing of uncertainty;
- Different paths of the future;

## Intuition

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In Computation Tree Logic (CTL) the model of time is a tree-like structure. This way, we cannot use Linear Temporal Logic (LTL) to express the existence of a certain path of time in which some event occurs.

# Syntax Definition

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Improving ou language The syntax of CTL consists on the syntax of temporal logic plus some path operators. The class of formulas can be defined in Backus-Naur form. If  $\phi$  is a formula:

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$$\phi ::= \bot \mid \top \mid p \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \to \phi \mid AX\phi \mid EX\phi \mid$$
$$AF\phi \mid EF\phi \mid AG\phi \mid EG\phi \mid A[\phi U\phi] \mid E[\phi U\phi]$$

# Syntax Definition

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$$AF\phi \mid EF\phi \mid AG\phi \mid EG\phi \mid A[\phi U\phi] \mid E[\phi U\phi]$$

With p as a literal (atomic formula), AX, EX, AF, EF, AG e EG unary operators.

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The propositional operators:  $\neg$ ,  $\lor$ ,  $\land$ ,  $\rightarrow$  have the same meaning of in the propositional logic.

The path-specific operators can be read, considering  $\varphi$  and  $\psi$  formulas, as:

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■ X: "in the next state";

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- *G* "Globally (in all future states)";
- $\blacksquare \varphi U\psi$ :  $\varphi$  is true at least until  $\psi$  becomes true;

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■ Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., AF is an atomic operator that can be read as "In all paths in the future there is some state where...";

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- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., AF is an atomic operator that can be read as "In all paths in the future there is some state where...";
- Notice as well that the binary operators  $A[\varphi U\psi]$  and  $E[\varphi U\psi]$  can be represented as AU and EU, respectively;

# Syntax Notes

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- Notice that, in CTL, the combination of path specific operators and temporal operators are atomic, e.g., *AF* is an atomic operator that can be read as "In all paths in the future there is some state where...":
- Notice as well that the binary operators  $A[\varphi U\psi]$  and  $E[\varphi U\psi]$  can be represented as AU and EU, respectively;
- We assume that, similarly to the  $\neg$  operator, the "new" unary operators (AX, EX, AF, EF, AG, and EG) have the first precedence. Next comes the  $\land$  and  $\lor$  operators. And at last the  $\rightarrow$ , AU and EU;

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- Examples of well-formed formulas:
  - $\blacksquare \ \textit{AG}(\textit{p} \lor \textit{EFq})$

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- Examples of well-formed formulas:
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  - **■** *F*[*pUs*]
  - $A[pUs \land qUs]$

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Different from usual logics, CTL formulas are interpreted by a transition system. Given an set of atoms:

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Different from usual logics, CTL formulas are interpreted by a transition system. Given an set of atoms:

# Definition (1)

A transition system  $\mathcal{M}$  is a triple  $\mathcal{M} = (S, \rightarrow, L)$  in which S is a set of states,  $\rightarrow$  is a binary relation over S ( $\rightarrow \subseteq S \times S$ ) and  $L: S \rightarrow \mathcal{P}(Atoms)$  is a labelling function.

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A transition system  $\mathcal{M}$  is a triple  $\mathcal{M}=(S,\to,L)$  in which S is a set of states,  $\to$  is a binary relation over S ( $\to\subseteq S\times S$ ) and  $L:S\to\mathcal{P}(Atoms)$  is a labelling function.

# Definition (2)

A **model** is a duple  $\mathcal{M}, s$  in which  $\mathcal{M}$  is a transition system and s is a state.

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**Notation:** we will use  $\mathcal{M}, s \vDash \varphi$  to denote that the model  $\mathcal{M}, s$  satisfies the formula  $\varphi$ 

# Semantics Satisfaction

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Take an arbitrary model  $\mathcal{M}$ . Let  $s, s_1, s_2, s_3$  be states in S. Let  $\varphi, \varphi_1, \varphi_2$  be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

■  $\mathcal{M}$ ,  $s \vDash \top$  and  $\mathcal{M}$ ,  $s \not\vDash \bot$  for all  $s \in S$ 

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- $\mathcal{M}$ ,  $s \models p$  iff  $p \in L(S)$

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- $\blacksquare \mathcal{M}, s \vDash p \text{ iff } p \in L(S)$
- $\blacksquare \mathcal{M}, s \vDash \neg \varphi \text{ iff } \mathcal{M}, s \not\vDash \varphi$

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- $\mathcal{M}, s \vDash \varphi_1 \land \varphi_2$  iff  $\mathcal{M}, s \vDash \varphi_1$  AND  $\mathcal{M}, s \vDash \varphi_2$

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- $\mathcal{M}, s \vDash \varphi_1 \lor \varphi_2$  iff  $\mathcal{M}, s \vDash \varphi_1$  OR  $\mathcal{M}, s \vDash \varphi_2$

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- $\mathcal{M}, s \vDash EX\varphi$  iff exists  $s_1$  that  $s \to s_1$  and  $\mathcal{M}, s_1 \vDash \varphi$ . Thus, EX says: "in some next state…"

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- $M, s, \models AG\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , for all  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AG says: "In all possible paths from now on in all next states..."

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- $M, s, \models AG\varphi$  iff exists some path  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , for all  $s_i$ ,  $M, s_i \models \varphi$  Thus, EG says: "Exists a path from now on in all next states..."

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- $\mathcal{M}, s \vDash AX\varphi$  iff for all  $s_1$  that  $s \to s_1$  and  $\mathcal{M}, s_1 \vDash \varphi$ . Thus, AX says: "in every next state..."
- $\mathcal{M}, s \vDash EX\varphi$  iff exists  $s_1$  that  $s \to s_1$  and  $\mathcal{M}, s_1 \vDash \varphi$ . Thus, EX says: "in some next state…"
- $M, s, \models AG\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , for all  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AG says: "In all possible paths from now on in all next states..."
- $M, s, \models AG\varphi$  iff exists some path  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , for all  $s_i$ ,  $M, s_i \models \varphi$  Thus, EG says: "Exists a path from now on in all next states..."

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More about semantics Equivalences Take an arbitrary model  $\mathcal{M}$ . Let  $s, s_1, s_2, s_3$  be states in S. Let  $\varphi, \varphi_1, \varphi_2$  be well-formed formulas of CTL. And let p be an atom. The satisfaction of CTL formulas can be defined as follows:

■  $M, s, \models AF\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AF says: "In all possible paths from now on, in some next state..."

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- $M, s, \models AF\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AF says: "In all possible paths from now on, in some next state..."
- $M, s, \models EF\varphi$  iff exists some path  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , that exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, EF says: "In some path from now on, in some next state..."

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- $M, s, \models AF\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , exists  $s_i$ , M,  $s_i \models \varphi$ . Thus, AF says: "In all possible paths from now on, in some next state..."
- $M, s, \models EF\varphi$  iff exists some path  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , that exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, EF says: "In some path from now on, in some next state..."
- $\blacksquare$   $M, s, \models A[\varphi_1 U\varphi_2]$  iff for all paths  $s_1 \to s_2 \to s_3 \to ...$  in which  $s = s_1$ , this path satisfies  $\varphi_1 U \varphi_2$ , i.e., exists  $s_i$  in the path such that  $\mathcal{M}, s_i \models \varphi_2$  and, for all i < i,  $M, s_i \models \varphi_1$ . Thus, AU says: "For all paths from now on, until some state..."

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- $M, s, \models AF\varphi$  iff for all paths  $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow ...$  in which  $s = s_1$ , exists  $s_i$ ,  $M, s_i \models \varphi$ . Thus, AF says: "In all possible paths from now on, in some next state..."
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- $M, s, \models E[\varphi_1 U \varphi_2]$  iff exists some path  $s_{\mathbb{P}} \to s_2 \to s_3 \to ...$

# Examples

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■ "It's possible to get to a state where something has started but it's not ready":  $EF(started \land \neg ready)$ 

# Examples

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- "It's possible to get to a state where something has started but it's not ready":  $EF(started \land \neg ready)$
- "A certain process is enabled infinitely often on every computation path": AG(AFenabled)

# Examples

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- "It's possible to get to a state where something has started but it's not ready":  $EF(started \land \neg ready)$
- "A certain process is enabled infinitely often on every computation path": AG(AFenabled)
- "An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor":  $AG(floor2 \land directionUp \land button5 \rightarrow A[directionUpUfloor5])$

# Equivalences

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## Definition

Two CTL formulas  $\varphi$  and  $\psi$  are said to be **semantically equivalent** if any state in any model which satisfies one of them also satisfies the other;

# Equivalences

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## Definition

Two CTL formulas  $\varphi$  and  $\psi$  are said to be **semantically equivalent** if any state in any model which satisfies one of them also satisfies the other;

**Notation:** we denote the equivalence of  $\varphi$  and  $\psi$  by  $\varphi \equiv \psi$ 

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$$\neg AF\varphi \equiv EG\neg \varphi$$

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$$\blacksquare \ \neg \mathit{AF}\varphi \equiv \mathit{EG}\neg \varphi$$

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$$\blacksquare \neg AF\varphi \equiv EG\neg \varphi$$

$$\blacksquare \neg \mathit{EF}\varphi \equiv \mathit{AG}\neg\varphi$$

$$\blacksquare \neg AX\varphi \equiv EX\neg \varphi$$

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$$\blacksquare \neg \mathit{EF}\varphi \equiv \mathit{AG}\neg\varphi$$

$$\blacksquare \neg AX\varphi \equiv EX\neg \varphi$$

$$AF\varphi \equiv A[\top U\varphi]$$

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$$AF\varphi \equiv A[\top U\varphi]$$

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## Minimum set of CTL connectives

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Improving ou language Because of the equivalences shown and the ones in propositional logic, we can have some minimum sets of conectives for the CTL syntax. One of them is defined in Backus-Naur formalism below:

$$\phi ::= \top \mid p \mid \neg \phi \mid \phi \rightarrow \phi \mid AX\phi \mid A[\phi U\phi] \mid E[\phi U\phi]$$

## That's all we need?

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More about semantics Equivalences

Improving our language Even if CTL allow explicit quantification over paths, it cannot allow some expressions to be formed. For example, we cannot say, as in LTL: "All paths in which have p on them, also have q on them".

This expression can be translated in LTL as follows:

$$Fp \rightarrow Fq$$

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More about semantics Equivalences Improving our language We can try expressing it as  $AFp \rightarrow AFq$  but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then"

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semantics Equivalences Improving our language We can try expressing it as  $AFp \to AFq$  but it does not have the same meaning. This one statement means "If all paths have a p along them, then all paths have a q along then" We can try to translate it as  $AG(p \to AFq)$  which is closer, but not exactly the same. This one means "for all paths, in all states on the future, if they hold p then, all paths will eventually hold q"

# Presenting CTL\*

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For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).

# Presenting CTL\*

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For this, we can extend the CTL by dropping the constraint that every temporal operator (X, U, F, G) has to be associated with an unique path quantifier (A, E).

This allows us to generate some statements:

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■ "In all possible paths, q is true until r is true or p is true until r is true":  $A[qUr \lor pUr]$ 

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- "In all possible paths, q is true until r is true or p is true until r is true":  $A[qUr \lor pUr]$
- "There is a path in which p eventually occurring will occur in all states": E[GFp]

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- "In all possible paths, q is true until r is true or p is true until r is true":  $A[qUr \lor pUr]$
- "There is a path in which p eventually occurring will occur in all states": E[GFp]
- "In all paths, p will occur in the next state or in the next of the next":  $A[Xp \lor XXp]$

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- "In all possible paths, q is true until r is true or p is true until r is true":  $A[qUr \lor pUr]$
- "There is a path in which p eventually occurring will occur in all states": E[GFp]
- "In all paths, p will occur in the next state or in the next of the next":  $A[Xp \lor XXp]$

# CTL\* syntax

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The syntax of CTL\* can be defined with the BNF bellow:

$$\phi ::= \bot \mid \top \mid p \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \rightarrow \phi \mid A[\alpha] \mid E[\alpha] \mid$$

$$\alpha ::= \phi | \ \neg \alpha \ | \ \alpha \wedge \alpha \ | \ \alpha \vee \alpha \ | \ \alpha \rightarrow \alpha \ | \ \alpha \textit{U}\alpha \ | \ \textit{G}\alpha \ | \ \textit{F}\alpha \ | \ \textit{X}\alpha |$$

With the same meanings of each operator.

## LTL ⊂ CTL\* and CTL ⊂ CTL\*

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Although we don't define path operators to LTL we can assume that it consider in all paths. Therefore, we can say that a formula  $\phi$  in LTL is a formula  $A[\phi]$  in CTL\*;

## $\mathsf{LTL} \subset \mathsf{CTL}^*$ and $\mathsf{CTL} \subset \mathsf{CTL}^*$

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Equivalences Improving our language Although we don't define path operators to LTL we can assume that it consider in all paths. Therefore, we can say that a formula  $\phi$  in LTL is a formula  $A[\phi]$  in CTL\*; For CTL, it is trivial:

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