$$Q\ 1\text{-}2a)\quad \sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

Base: $n=1$

$$\sum_{k=1}^{n=1} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

$$(1)(2) = \frac{(1)(2)(3)}{3}$$

$$2 = 2 \checkmark$$

$$\text{so: } \sum_{k=1}^{k} k(k+1) = \frac{k(k+1)(k+2)}{3}$$

$n=k+1$

$$\sum_{k=1}^{k+1} k(k+1) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$= \underbrace{\sum_{k=1}^{k} k(k+1)} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= (k+1)(k+2)\left(\frac{k}{3}+1\right)$$

$$= (k+1)(k+2)\left(\frac{k+3}{3}\right)$$

$$= \frac{(k+1)(k+2)(k+3)}{3} = \frac{(k+1)(k+2)(k+3)}{3} \checkmark$$

(Q1-2b) $n! > 3n$, $n \geq 7$

Base: $n=7$

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 > 3 \cdot 7$

$5040 > 21$ ✓

$n=k$

$k! > 3k$, $k \geq 7$

$n=k+1$

$(k+1)! > 3(k+1)$

$= (k+1)(k!) > 3(k+1)$ $\cancel{(k+1)(3k)}$

$3k+3 < k!+3 < k!(k+1) = (k+1)!$ ✓

$k! = 5040$

min values $\Big\}$ $k+1 = 8$

Q1-3) 3^n in n weighing?

base: $n=1$

$3^1 = 3$ coins in 1 weighing

put two coins on scale. If one is fake, scale will show. ✓
If scale doesn't show, 3^{rd} coin is fake.

Must know if fake coin
will be lighter or heavier.
Otherwise can't be done in n weighings.

$n=k$

3^k in k

$n=k+1$

3^{k+1} in $k+1$

Split 3^{k+1} into 3 groups of 3^k and weigh two groups. Same as in base case. 1 weigh to find the group containing the fake.

Now must do for the group of 3^k coins, which takes k steps.

So, will take total of $k+1$ steps

Thus, 3^{k+1} takes $k+1$ steps ✓