

# Overview of My Research Activities in Applied Mathematics

Presented by Dr. Ronan Dupont  
July 1, 2025

## Interview for the Position:

### Postdoctoral Position in Numerical Linear Algebra *Nagoya University, Japan*

Moonshot R&D Program – “Backcasting digital system by super-dimensional state engineering”

Supervised by Dr. Shao-Liang Zhang and Dr. Tomohiro Sogabe

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- ① Presentation of Myself
- ② Pre-thesis Research Projects
- ③ Thesis Research Projects
- ④ Post-Thesis Research Projects

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## 1 Presentation of Myself

- ▷ It starts with 3 years of living in Japan...
- ▷ Academic Origins
- ▷ PhD
- ▷ Post-PhD Phase
- ▷ Beyond Mathematics
- ▷ Any questions?

## 2 Pre-thesis Research Projects

## 3 Thesis Research Projects

## 4 Post-Thesis Research Projects

It starts with 3 years of living in Japan...

- Born to a nuclear engineer (father) and a Mathematics teacher (mother).

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- Born to a nuclear engineer (father) and a Mathematics teacher (mother).
- First expatriation: Rokkasho nuclear power plant project, Japan.



Figure: Me at Japanese School.

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- A passion for water sports.



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*After these graduations, the desire to continue abroad was there, but it was the COVID year.*

# PhD in Montpellier – Coupling Waves and Morphodynamics

*A natural continuation of my desire to stay at the interface between mathematics and marine sciences.*



## PhD in Applied Mathematics – Montpellier (2021–2024)

- Wave-morphodynamic coupling of the coastline by minimization principle.
- Published results in peer-reviewed journals; presented at international conferences.
- Supervised by Pr. Bijan Mohammadi and Pr. Frédéric Bouchette (Interface of Mathematics and Geosciences).
- Teaching experience at the University of Montpellier in : Algebra, Calculus, Cardinality, Geometry, Python, Coastal Modeling.

*I felt the need to work on more mathematical subjects.*

# Post-PhD Path: Teaching and Mobility

## Teaching in Bordeaux (2025)

- High-school mathematics teacher in France.
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## Move to Asia – Cambodia (2025–)

- Currently teaching at the French International School of Phnom Penh.
- Classes for both **French** (Brevet, Baccalauréat) and **British** (A-level Further Maths) curricula.



*My first expatriation was a rewarding success, but I now seek renewed intellectual challenges in Asia.*

# Passions and Personal Interests

## Personal Interests

- **Surfing, swimming, snorkeling** — strong personal connection to the sea; spent 8 summers as a beach lifeguard.
- **Travel** — lived in Japan as a child; currently in Cambodia; completed a solo tour across Europe after my PhD.
- **Languages** — fluent in French and English and almost Italian; currently studying Khmer since my arrival in Phnom Penh.
- **Scientific curiosity** — I enjoy learning new algorithms, coding daily, and sharing math–code challenges with my students.

*This path naturally led me to apply for a position in Japan — where my scientific story began.*

# Any questions?



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## More details:

- 🌐 My Website and my CV:  
[ronan-dupont.github.io](http://ronan-dupont.github.io)  
[ronan-dupont.github.io/files/Curriculum\\_Vitae\\_Ronan\\_Dupont.pdf](http://ronan-dupont.github.io/files/Curriculum_Vitae_Ronan_Dupont.pdf)

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## ① Presentation of Myself

## ② Pre-thesis Research Projects

- ▷ Numerical Linear Algebra
- ▷ Modeling and Numerical Simulation of Epidemics and Diffusion
- ▷ Stochastic Computation
- ▷ Mesh Generation and Visualization
- ▷ Any questions?

## ③ Thesis Research Projects

## ④ Post-Thesis Research Projects

# Linear Solvers, Iterative Methods and Equation Solving

## Direct and Iterative Methods for Linear Systems

- LU factorization implemented from scratch.
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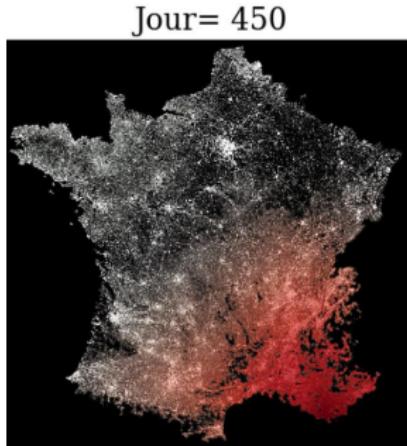
## Numerical Schemes for AD Equations (1D/2D)

- Explicit/implicit schemes for AD equations and FreeFem++.
- Resolution using a deterministic approach (finite difference resolution, finite element, finite volume) and a stochastic approach (Monte Carlo, quasi-Monte Carlo, Brownian motion).

# Modeling Epidemics

## SIR and SZR Models

- Resolution of PDEs with diffusion using finite differences.
- Classical SIR model, playful extension to SZR (Zombie outbreaks).
- Visual simulations with spatial propagation and mapping.



*Figure:* Day 450 of a Zombie epidemic starting from Toulon (south of France).

# Optimization, Stochastic Methods and Quantization Algorithms

CFD optimization of the performance of windsurf sails intended for high speeds

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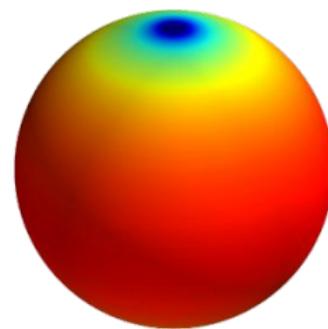
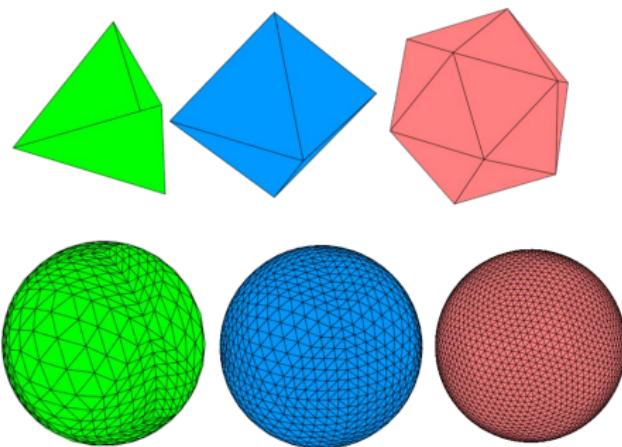
## Stochastic Problem Solving and Optimization

- Solving Sudoku puzzles using genetic algorithms.
- Solving function minima by genetic algorithm.
- Integral computation via Monte Carlo / Quasi Monte Carlo integration.
- Stochastic resolution of PDEs.

# Development of a Meshing Tool for Spheres

## Mesh Generation in Fortran

- Surface meshing of a sphere (2D) in Fortran 90.
- Custom types and coordinate storage.
- Visualization in Python.
- Application to thermal diffusion equations.



# Any questions?



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## More details:



Master projects reports:

[ronan-dupont.github.io/publication/2021-master-reports](https://ronan-dupont.github.io/publication/2021-master-reports)

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## 1 Presentation of Myself

## 2 Pre-thesis Research Projects

## 3 Thesis Research Projects

- ▷ Modelling beaches morphodynamic by Hadamard sensitivity analysis
- ▷ How does the model (OptiMorph) work?
- ▷ How to compute  $\nabla_{\psi} \mathcal{J}$ ?
- ▷ Some extensions of the model
- ▷ Any questions?

## 4 Post-Thesis Research Projects

# Modelling beaches morphodynamic by Hadamard sensitivity analysis

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## Publications:

-  Ronan Dupont, Frédéric Bouchette, and Bijan Mohammadi. (2024). "Beaches morphodynamic modeling based on Hadamard sensitivity analysis." In: *Ocean Modelling*, doi.org/10.1016/j.ocemod.2024.102370.
-  Ronan Dupont, Megan Cook, Frédéric Bouchette, Bijan Mohammadi, and Samuel Meulé. (2023). "Sandy beach dynamics by constrained wave energy minimization." In: *Ocean Modelling*, doi.org/10.1016/j.ocemod.2023.102197.

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## Main model assumption

Nature adapts the seabed  $\psi$  so that wave energy is minimized  $E_H$ , ie:

$$\min_{\psi} \mathcal{J}(\psi) \quad \text{with} \quad \mathcal{J}(\psi) = \int_{\Omega} E_H(\psi),$$

# Workflow based on the Hadamard shape derivative

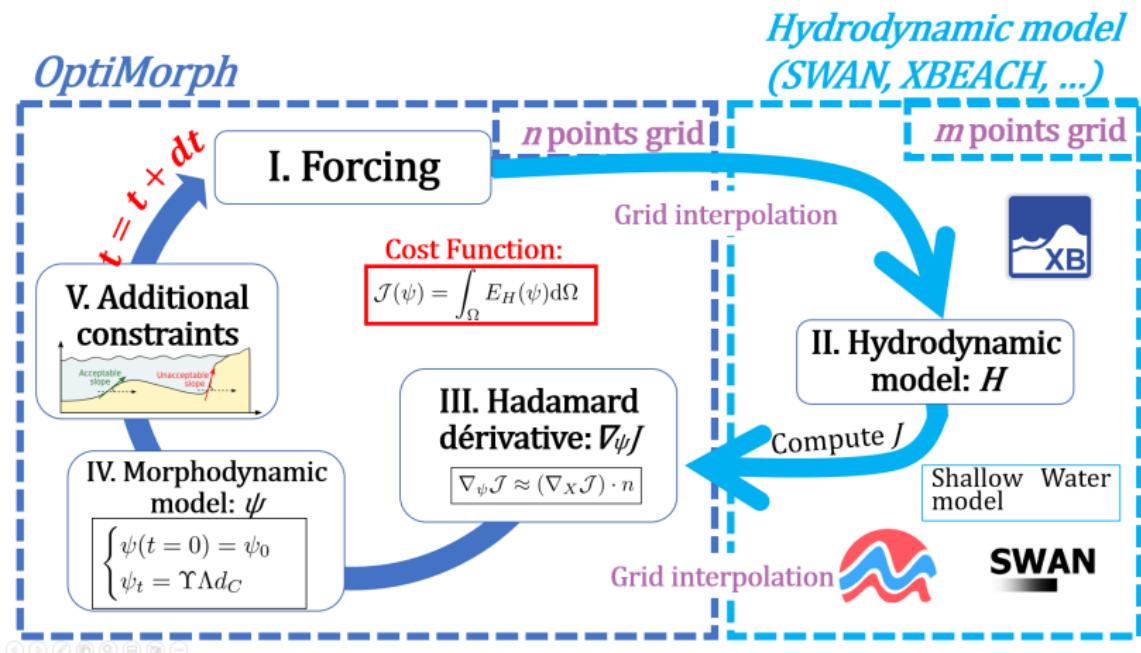


Figure: OptiMorph workflow based on Hadamard shape derivative.

# Morphodynamic model

Problem reminder:

We aim to solve  $\min_{\psi} \mathcal{J}(\psi)$  under physical constraints.

# Morphodynamic model

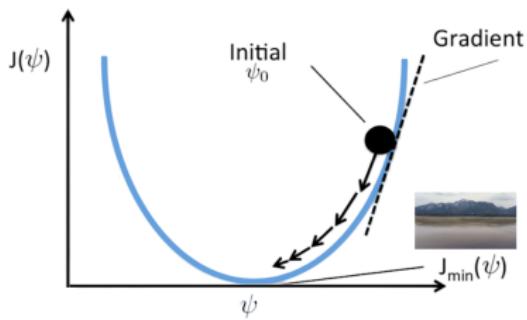
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Without physical constraints:

$$\begin{cases} \psi_t = \Upsilon d \\ \psi(t=0) = \psi_0 \end{cases}$$

where:



- $\psi(t)$ : seabed profile at time  $t$ ,
- $\Upsilon$ : sediment mobility,
- $d$ : descent direction. Here,  $d = -\nabla_{\psi} \mathcal{J}$ .

**Figure:** Gradient descent at  $t = 0$  s.

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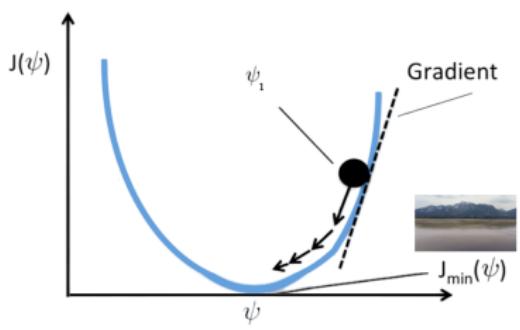


Figure: Gradient descent at  $t = \Delta t$  s.

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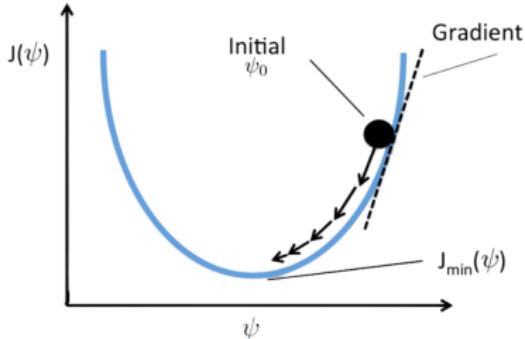


Figure: Gradient descent with constraints.

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where:

- $\psi(t)$ : seabed profile at time  $t$ ,
- $\Upsilon$ : sediment mobility,
- $d_C$ : descent direction including constraints,
- $\Delta(x)$ : wave-induced excitation of the seabed.

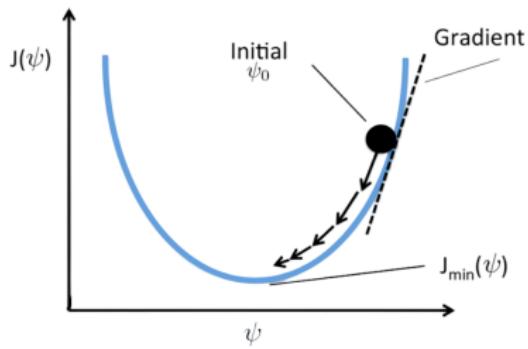


Figure: Gradient descent.

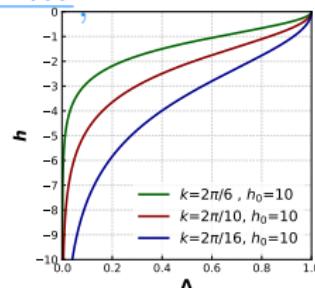
# Adding physical constraints

- Sand excitation. From (Soulsby, 1987):

$$\begin{aligned}\varphi : \quad \Omega \times [0, h_0] &\longrightarrow \mathbb{R}^+ \\ (x, z) &\longmapsto \frac{\cosh(k(x)(h(x) - (h_0 - z)))}{\cosh(k(x)h(x))}\end{aligned}$$

and at  $z = \psi$ :

$$\Lambda(x) = \varphi(x, \psi(x)) = \frac{1}{\cosh(k(x)h(x))}$$



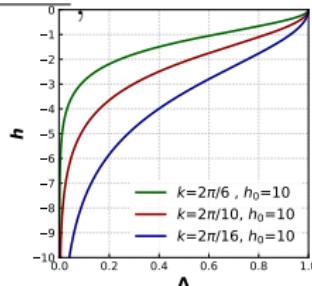
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- Maximum slope:

$$\left| \frac{\partial \psi}{\partial x} \right| \leq M_{\text{slope}}$$

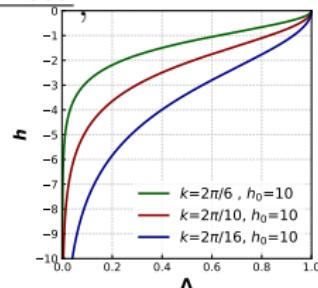
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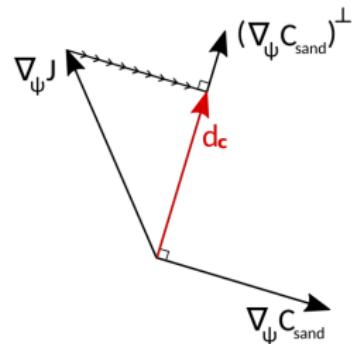
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- Sandstock conservation (in closed domain only):

$$\int_{\Omega} \psi(t, x) dx = \int_{\Omega} \psi_0(x) dx \quad \forall t \in [0, T_f]$$

# Projection of the sediment conservation constraint

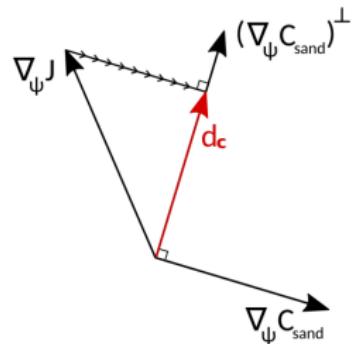
We aim to compute the descent direction  $d_C$  without violating the sediment conservation constraint. We define a residue  $C_{\text{sand}}(t)$  which we want to be zero  $\forall t \in [0, T_f]$ .



*Figure: Projection. Cook (2021).*

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So we have:

$$d_C = \nabla_\psi \mathcal{J} - \left\langle \nabla_\psi \mathcal{J}, \frac{\nabla_\psi C_{\text{sand}}}{\|\nabla_\psi C_{\text{sand}}\|} \right\rangle \frac{\nabla_\psi C_{\text{sand}}}{\|\nabla_\psi C_{\text{sand}}\|}.$$

# How to compute $\nabla_\psi \mathcal{J}$ ? Different strategies

Reminder of the governing equation:

$$\left\{ \begin{array}{l} \psi_t = \gamma \wedge d_C \\ \psi(t=0) = \psi_0 \end{array} \right. \quad \left\{ \begin{array}{l} d_C = -\nabla_\psi \mathcal{J} + \text{constraints}, \\ \mathcal{J}(\psi) = \int_{\Omega} E_H(\psi) \end{array} \right.$$

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- ✓ Exact solution.
- ✓ Fast.
- ✗ Feasible only on simple models.

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## Automatic Differentiation



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- ✓ Robust.
  - ✗ Heavy pre-processing.
  - ✗ Depends on C / Fortran 90.

# Hadamard derivative of $\nabla_{\psi} \mathcal{J}$

We consider:

$$\nabla_{\psi} \mathcal{J} = \lim_{\varepsilon \rightarrow 0} \frac{\mathcal{J}(\psi + \varepsilon n) - \mathcal{J}(\psi)}{\varepsilon}$$

with  $n$ : normal vector to the shape.

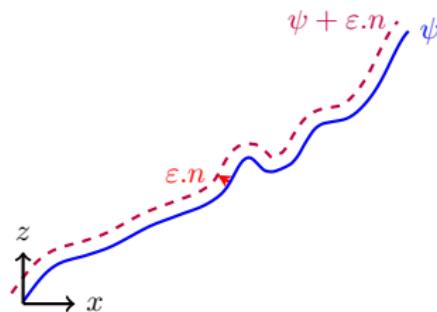


Figure: Illustration of Hadamard derivative.

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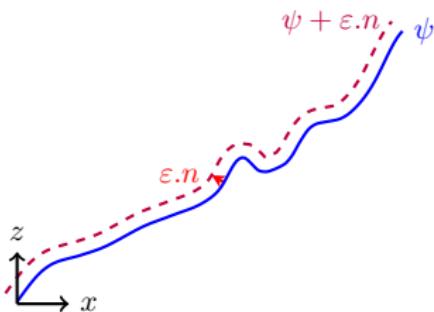


Figure: Illustration of Hadamard derivative.

At first order:

$$\nabla_{\psi}\mathcal{J} \approx \lim_{\varepsilon \rightarrow 0} \frac{\cancel{\mathcal{J}(\psi)} + \varepsilon \nabla_X \mathcal{J} \cdot n - \cancel{\mathcal{J}(\psi)}}{\varepsilon}, \quad \text{with } X = (x, z)^T.$$

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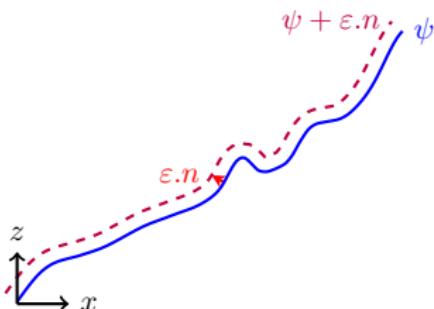


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This gives:

$$\boxed{\nabla_{\psi} \mathcal{J} \approx (\nabla_X \mathcal{J}) \cdot n}$$

# Numerical validation on an analytical case

We consider:

$$\psi = \{(x, y) \in \mathbb{R}^2 \mid y = ax + b\}$$

and  $\mathcal{J} = \cos(\psi)$ , with  $\boxed{\nabla_\psi \mathcal{J} = -\sin(\psi)\sqrt{a^2 + 1}}.$

$\nabla_\psi \mathcal{J}$  using Hadamard approximation with the following problem:

$$\psi = ax + b, \mathcal{J} = \cos(\psi), \text{ with } a = 0.02, b = -2$$

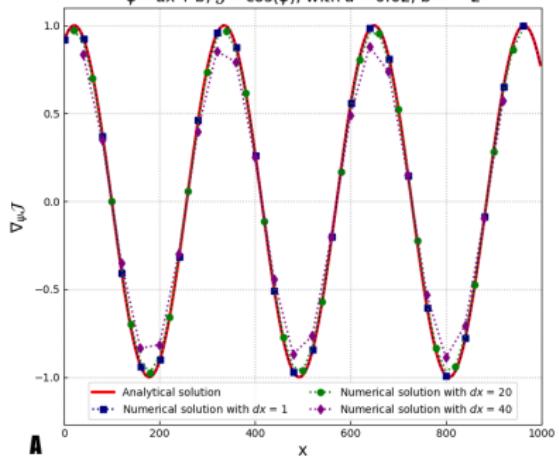
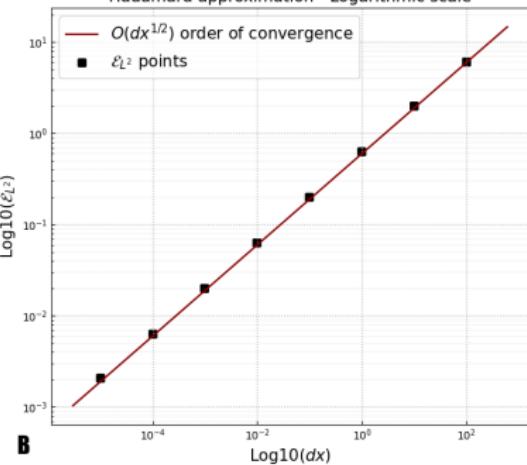


Figure: A) Analytical and approximate solution using the derivative...

Error  $\mathcal{E}_{L^2}$  between the analytical and numerical solution of the Hadamard approximation - Logarithmic scale

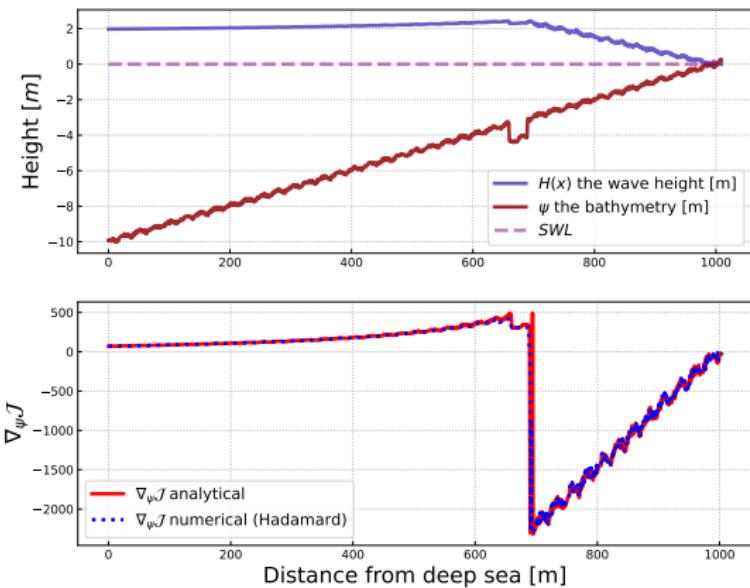


# Numerical verification on an application case with perturbations

With  $\nabla_{\psi} \mathcal{J} \approx \nabla_X \mathcal{J} \cdot n$

## Parameters:

- Shoaling model with:
  - $H_0 = 2$  m,
  - $T_0 = 10$  s,
  - $h_0 = 10$  m.
- Linear bottom with perturbations.



*Figure: Computation of  $\nabla_{\psi} \mathcal{J}$  using the Hadamard approach (blue) and analytical calculation (red). Linear bottom with perturbations.*

# Adding lateral transport

- New governing equation:

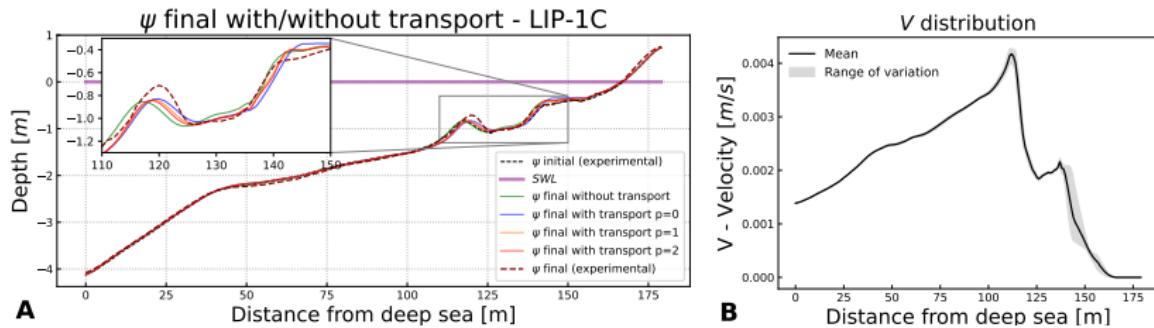
$$\psi_t + V \nabla_s \psi = \Gamma \Lambda dc$$

# Adding lateral transport

- New governing equation:  $\psi_t + V \nabla_s \psi = \Gamma \Lambda d_c$
- A possible velocity:  $V \sim U_b \left( \frac{H}{H_{\max}} \right)^p$  with  $U_b = \frac{\pi H}{T_0 \sinh(kh)}$

# Adding lateral transport

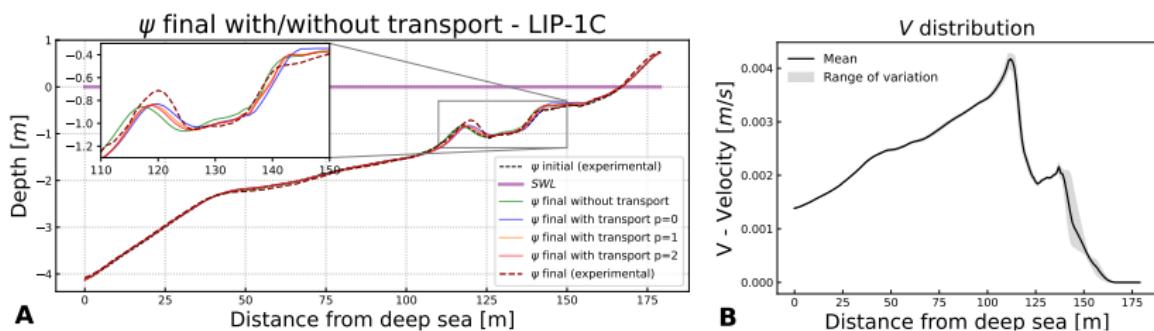
- New governing equation:  $\psi_t + V \nabla_s \psi = \Gamma \Lambda d\zeta$
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*Figure: A) OptiMorph morphodynamic results with lateral transport for  $p = 0, 1, 2$ , and XBeach wave model, for the LIP 1C channel experiment. B) Velocity distribution for  $p = 1$ .*

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*Figure: A) OptiMorph morphodynamic results with lateral transport for  $p = 0, 1, 2$ , and XBeach wave model, for the LIP 1C channel experiment. B) Velocity distribution for  $p = 1$ .*

- But  $V \geq 0$  for this choice  $\Rightarrow$  no offshore sediment bar displacement (as in LIP-1B).

# Towards a better definition of velocity?

- A new system of governing equations?

$$\begin{cases} \psi_t + V \nabla_s \psi = \Upsilon \Lambda d_C & (a) \\ V_t = -\rho \nabla_v \mathcal{J} & (b) \end{cases}$$

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- How to compute  $\nabla_V \mathcal{J}$ ?

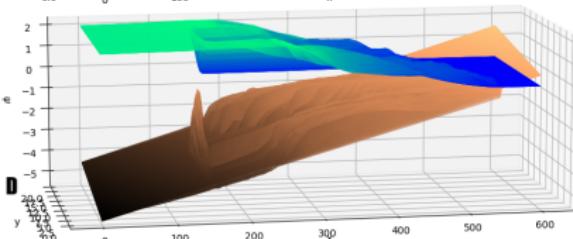
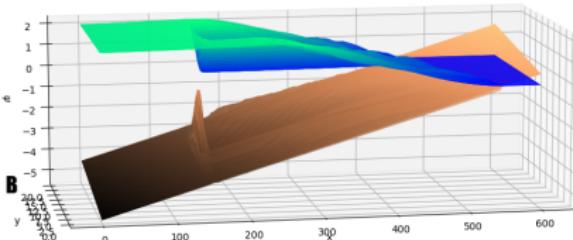
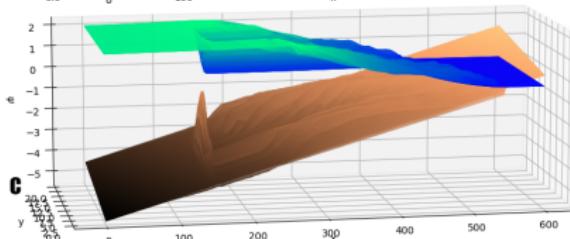
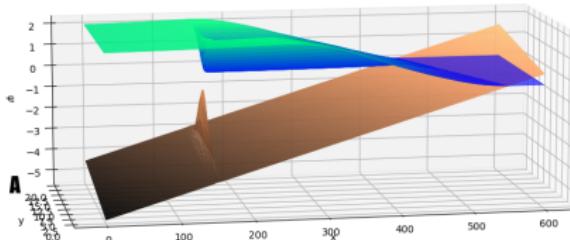
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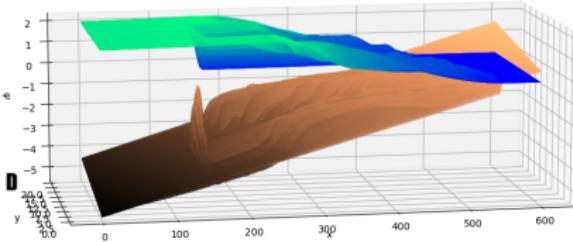
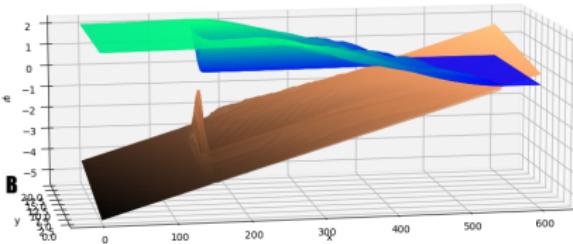
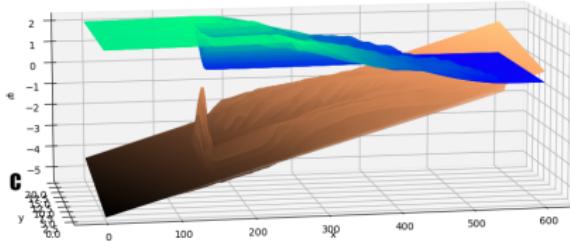
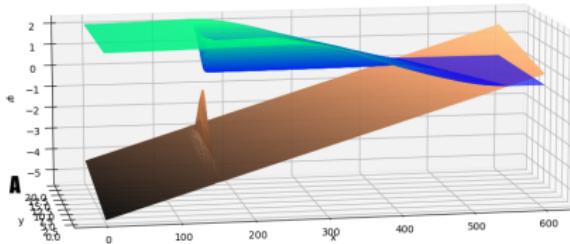
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- Equation (b) represents a velocity that minimizes  $\mathcal{J}$ .
- How to compute  $\nabla_V \mathcal{J}$ ?
- Can lateral morphodynamic transport minimize  $\mathcal{J}$ ?

# 2D Morphodynamic results

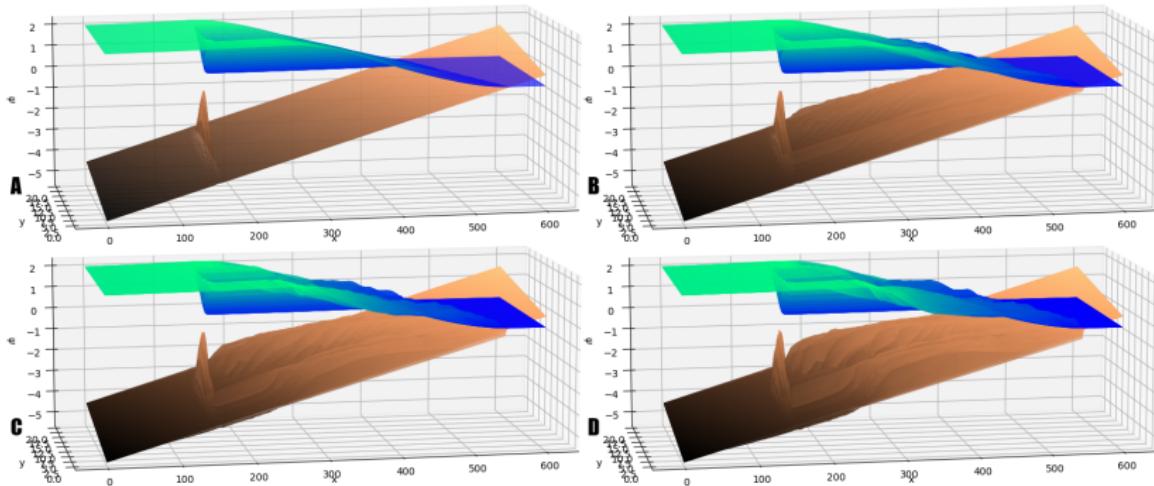


# 2D Morphodynamic results



- Observation of a sewage pit behind the geotextile tube.

# 2D Morphodynamic results



- Observation of a sewage pit behind the geotextile tube.
- Seabed modification  $\Rightarrow$  appropriate morphodynamic response.

# Any questions?



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## More details:

-  PhD manuscript:  
[ronan-dupont.github.io/publication/2024\\_manuscript](https://ronan-dupont.github.io/publication/2024_manuscript)
  
-  PhD defense (oral support):  
[ronan-dupont.github.io/talks/phd\\_defense](https://ronan-dupont.github.io/talks/phd_defense)

# Table of Contents

## ① Presentation of Myself

## ② Pre-thesis Research Projects

## ③ Thesis Research Projects

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- ▷ Problem configuration
- ▷ Analytical solution
- ▷ Validation and Convergence
- ▷ Application: Slope sensitivity, Helmholtz vs Mild-Slope
- ▷ Any questions?

# An arbitrary-order Virtual Element Method for the Helmholtz equation applied to wave field calculation in port

A subject to get me closer to numerical mathematics while retaining coastal skills.

- A Resolution of the Helmholtz Equation Using the Virtual Element Formalism.

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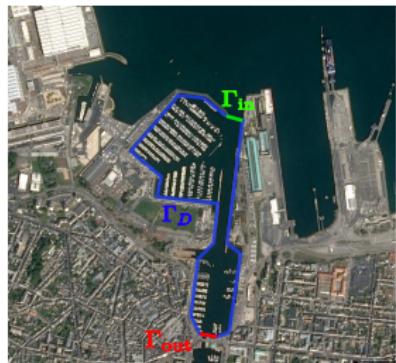
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## Publication:

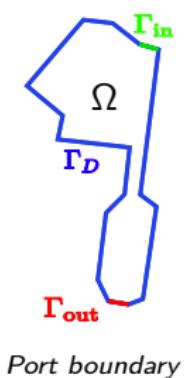
- 
- Ronan Dupont. (2025). "An Arbitrary-Order Virtual Element Method for the Helmholtz Equation Applied to Wave Field Calculation in Port."
- Results in Applied Mathematics*
- , doi.org/10.1016/j.rinam.2025.100598.

# Problem configuration

A reference to my home town:  
Cherbourg in France.



Port location



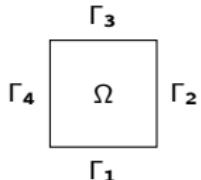
Port boundary

The Helmholtz or Mild-Slope equation:

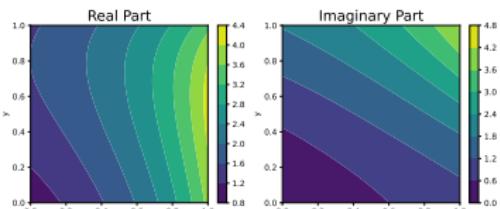
$$\left\{ \begin{array}{ll} \nabla(C_p C_g \nabla a) + k^2 C_p C_g a = 0, & \text{in } \Omega, \\ \frac{\partial a}{\partial n} = 0, & \text{in } \Gamma_{in}, \\ \frac{\partial a}{\partial n} + ik a = 0, & \text{in } \Gamma_{out}, \\ a = \gamma a_i & \text{in } \Gamma_D \end{array} \right.$$

# Analytical solution

We consider,



$$\begin{cases} \Delta u + k^2 u = f(x, y) & , \text{ in } \Omega, \\ u = u_{\text{exact}} & , \text{ on } \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \\ \frac{\partial u}{\partial n} + i k u = g(x, y) & , \text{ on } \Gamma_1, \end{cases}$$



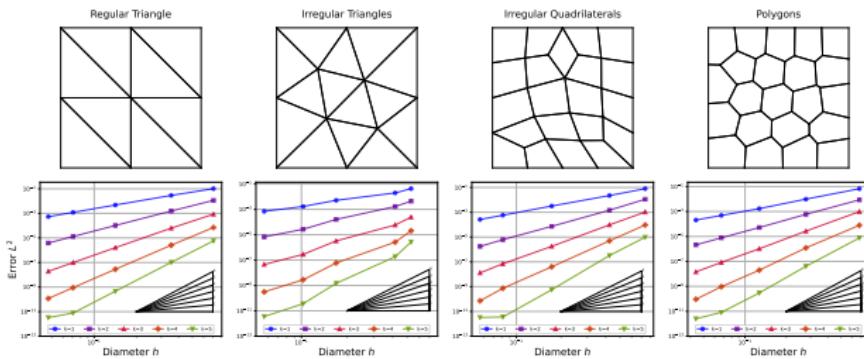
*Figure:* Real and Imaginary part of  $u_{\text{exact}}$ .

with:

$$\begin{cases} u_{\text{exact}}(x, y) = (x + y) \cdot (1 + i) + \exp(x^2 + i y^2), \\ f(x, y) = -((2x)^2 + (2iy)^2 + 2(1+i)) \cdot \exp(x^2 + iy^2) + k^2 \cdot u_{\text{exact}}(x, y), \\ g(x, y) = (1+i) + (2iy) \cdot \exp(x^2 + iy^2) + ik \cdot u_{\text{exact}}(x, y). \end{cases}$$

## Validation and Convergence

- Manufactured solution method: exact solution + right-hand side  $f$ .
  - Implementation tested with  $k = 1, 2, 3, 4, 5$  using quadrilateral and polygonal meshes.
  - Observed convergence rate:
    - $\|u - u_h\|_{L^2} \sim \mathcal{O}(h^{k+1})$



**Figure: Convergence curves with different orders  $k$  and different types of elements.**

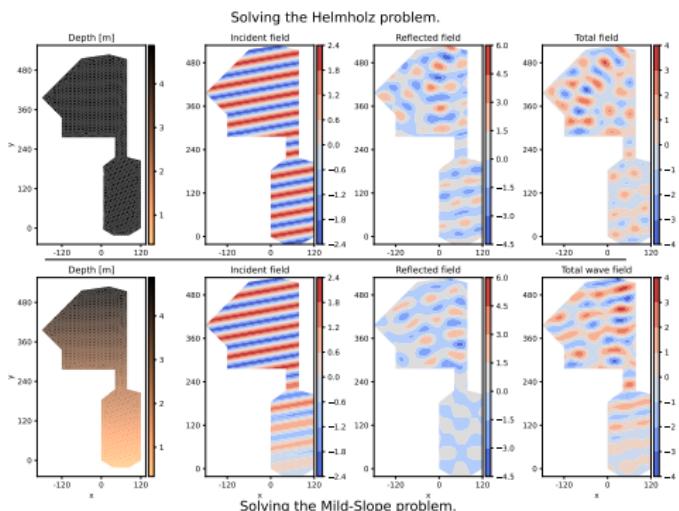
# Slope sensitivity, Helmholtz vs Mild-Slope

Problem conditions:

- $a_{\max} = 2 \text{ m}$ ,
- $T_0 = 8 \text{ s}$ ,
- $\theta = 280^\circ$ .

Points of interest:

- Eigenmode position.



# Any questions?



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## More details:

- Ronan Dupont. (2025). "An Arbitrary-Order Virtual Element Method for the Helmholtz Equation Applied to Wave Field Calculation in Port." *Results in Applied Mathematics*, doi.org/10.1016/j.rinam.2025.100598.
- Presentation at the NuMerics2024 conference where I was invited (oral support):  
[ronan-dupont.github.io/talks/NuMerics2024](https://ronan-dupont.github.io/talks/NuMerics2024)