Contents: Chapter 8

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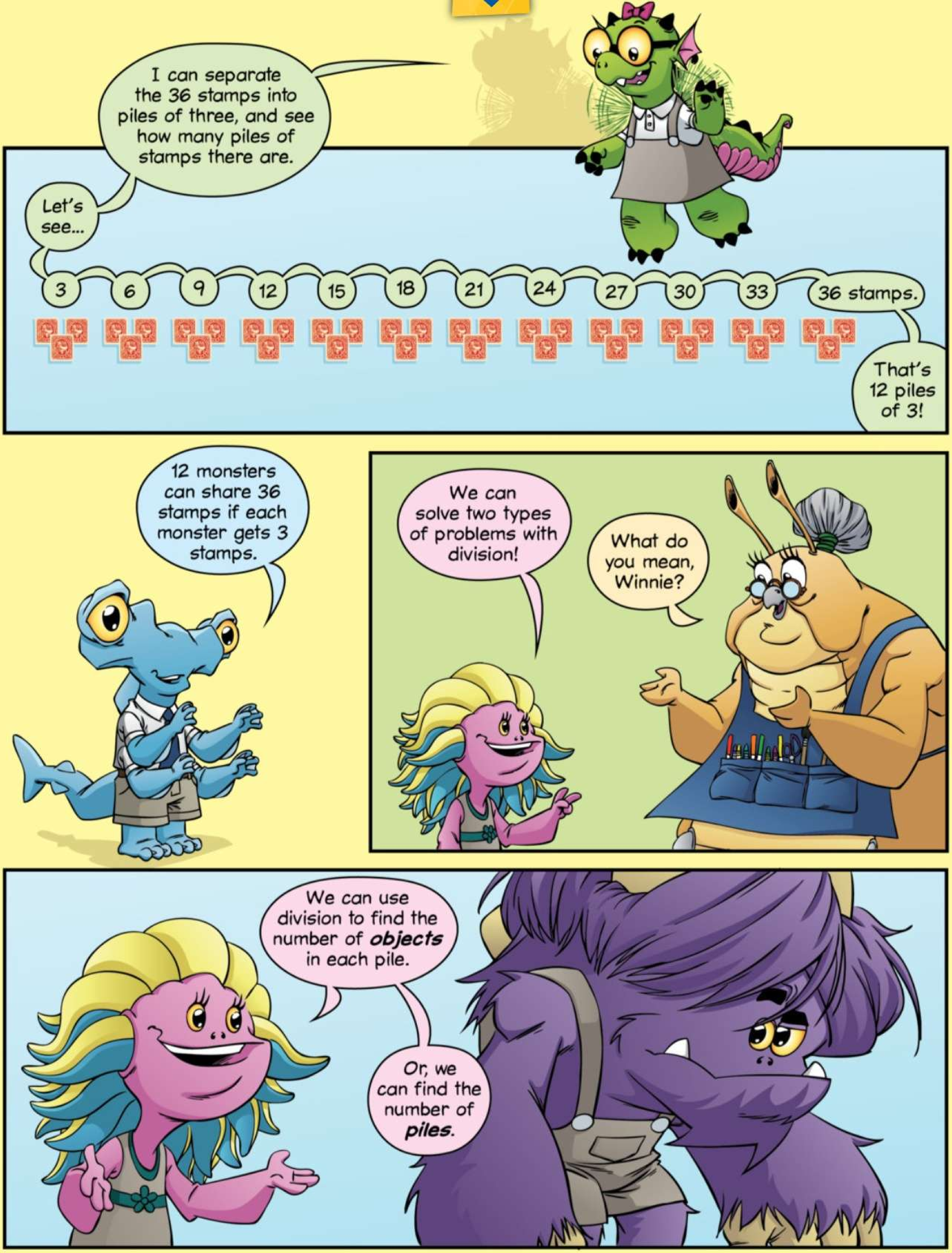
Chapter 8:
Division

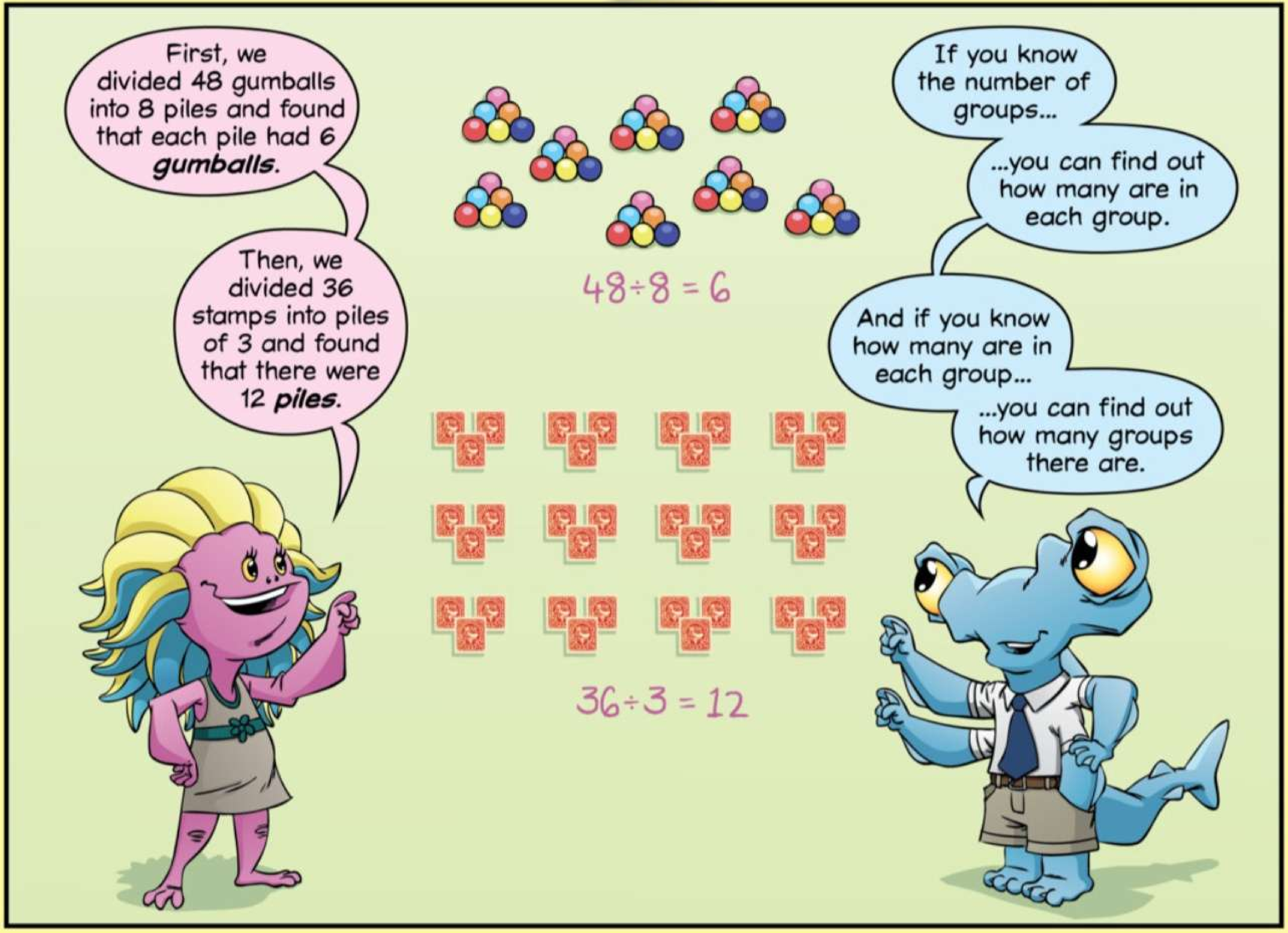

Ms. Q.
Sharing
Grogg, are you chewing gum?
Hmmb?
Do you remember the class rules?
You thed I could chew gub if I brobb enuff fuh ebrybobby.
I brought enough for everybody!
YAY!
Alright, Grogg. How many gumballs did you bring to share?
48!
There are 8 of you.
How can we make sure everyone gets the same number of gumballs?
We could split the gum into 8 equal piles.
Each of us gets one pile!

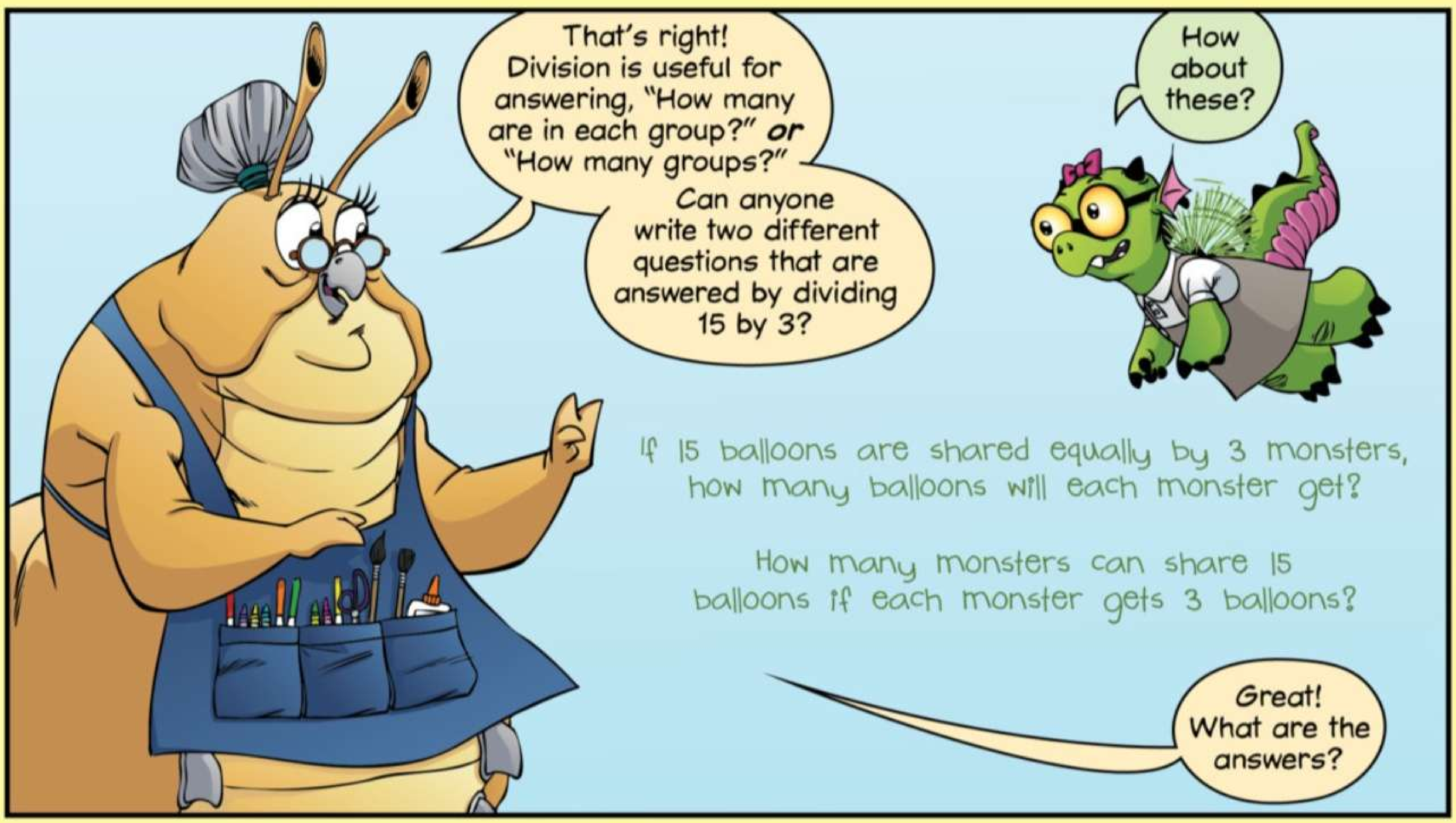

How could you make sure all of the piles are the same?
How would you split 48 gumballs into 8 equal piles?
We could just deal out the gumballs one at a time...
...the same way you deal a deck of cards.
After all the gum is dealt, we'll all have the same amount.
Deal 48 cards into 8 equal piles. How many are in each pile?
We each got six pieces of gum!
It's called *division*!
What is division?

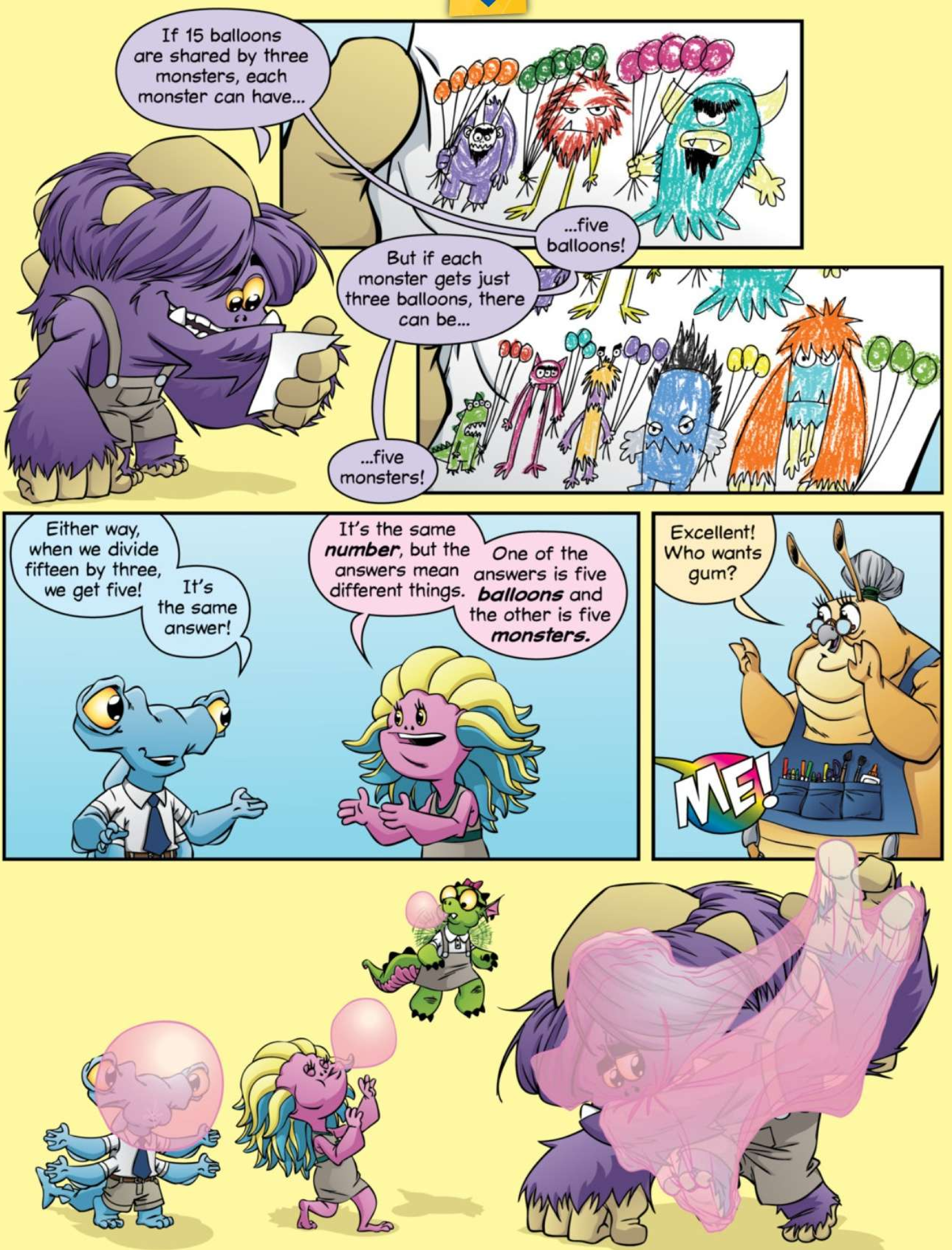

Division is the separation of a quantity into a number of equal parts.
Like when you split a pile of gum into smaller piles that are the same size.
Division means equal sharing.
We divided 48 gumballs into 8 equal piles, and there were 6 gumballs in each pile.
Very good! We say that 48 divided by 8 is 6.
Does anyone have anything else they want to share?
I have stamps!
IN MATH, WE WRITE "48 DIVIDED BY 8 IS 6" LIKE THIS: 48÷8=6. THE SYMBOL FOR DIVISION IS A DASH AND TWO DOTS.
Stamps?
Before bot-mail, you needed them to send a letter.
You put three of these on an envelope, and it will get delivered anywhere.
They still work!
I have 36 stamps, and everyone needs three.
How many little monsters can share 36 stamps if each gets three?
Try it.



I can separate the 36 stamps into piles of three, and see how many piles of stamps there are.
Let's see...
3
6
9
12
15
18
21
24
27
30
33
36 stamps.
That's 12 piles of 3!
12 monsters can share 36 stamps if each monster gets 3 stamps.
We can solve two types of problems with division!
What do you mean, Winnie?
We can use division to find the number of ***objects*** in each pile.
Or, we can find the number of ***piles***.


First, we divided 48 gumballs into 8 piles and found that each pile had 6 ***gumballs***.
Then, we divided 36 stamps into piles of 3 and found that there were 12 ***piles***.
48÷8 = 6
36÷3 = 12
If you know the number of groups...
...you can find out how many are in each group.
And if you know how many are in each group...
...you can find out how many groups there are.


That's right! Division is useful for answering, "How many are in each group?" ***or*** "How many groups?"
Can anyone write two different questions that are answered by dividing 15 by 3?
How about these?
If 15 balloons are shared equally by 3 monsters, how many balloons will each monster get?
How many monsters can share 15 balloons if each monster gets 3 balloons?
Great! What are the answers?


If 15 balloons are shared by three monsters, each monster can have...
...five balloons!
But if each monster gets just three balloons, there can be...
...five monsters!
Either way, when we divide fifteen by three, we get five!
It's the same answer!
It's the same **number**, but the answers mean different things.
One of the answers is five **balloons** and the other is five **monsters.**
Excellent! Who wants gum?
ME!


R & G
× and ÷
What are you doing?
I'm making treat bags to hand out to the little monsters!
I have 72 treats, and I want to put 8 treats in each bag.
I need to know how many bags of treats I can make.
What you are trying to do is called ***division.***
Tell me more.
Division answers the question, "How many eights are added to make 72?"
This is called "72 divided by 8." It usually looks like this:
72÷8
So, how do I figure out 72÷8?
What is 72÷8?

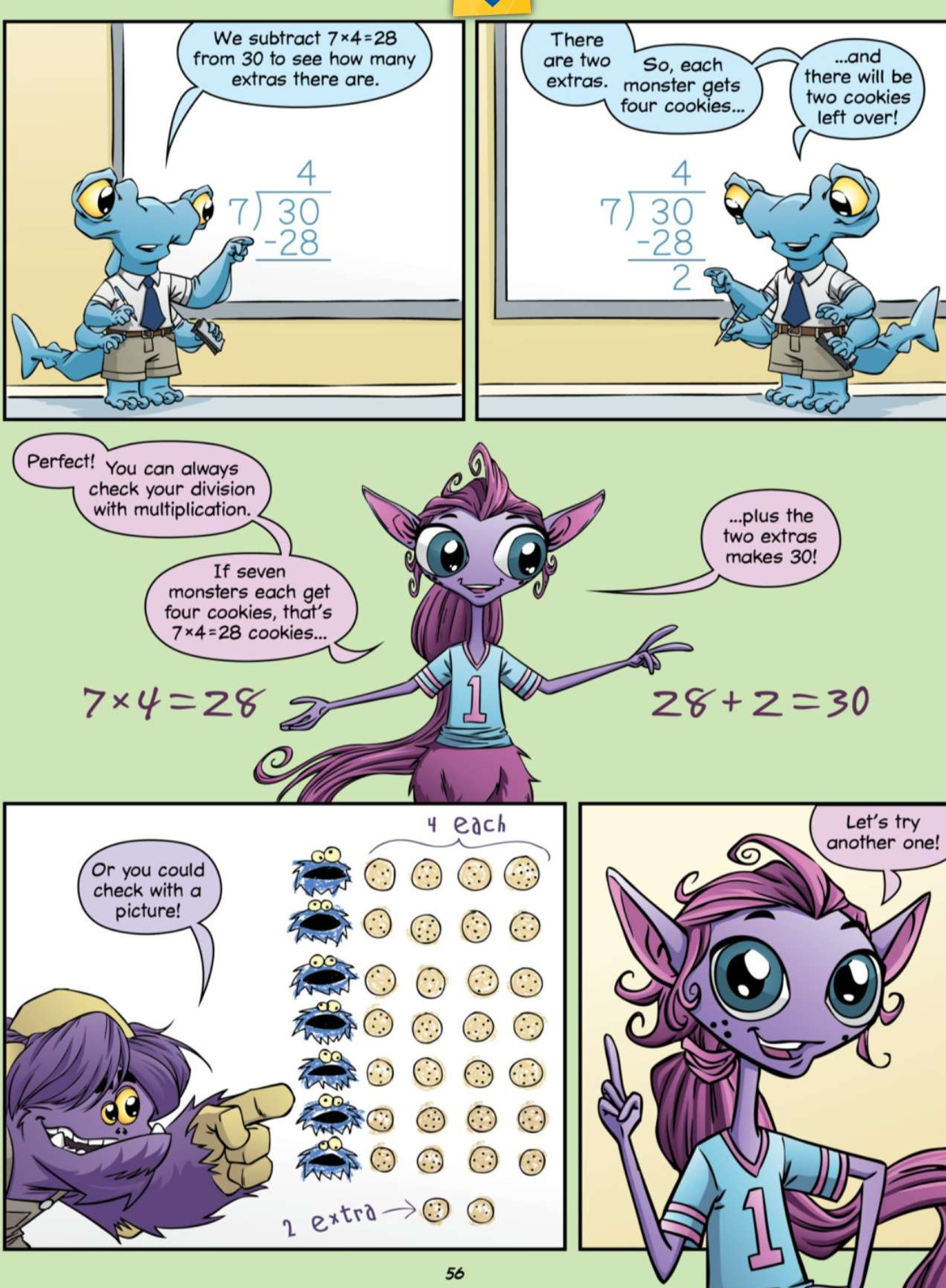

One way is by skip-counting. You can count the number of eights it takes to get to 72!
8
16
24
32
40
48
56
64
72!
It took nine fingers, so that's *nine* 8's.
Right, but the easiest way to solve a division problem...
...is with multiplication.
!?!
The best way to solve 72 ÷ 8 is to ask yourself, "What number can I multiply by 8 to get 72?"
n × 8 = 72
Hmmm... *9* times 8 is 72.
Right, and since 9×8=72...
9 × 8 = 72
...*nine* 8's add up to 72...
8+8+8+8+8+8+8+8+8=72
...so 72 ÷ 8 is 9!
72 ÷ 8 = 9

To answer a division problem, it helps to be able to multiply.
Right! To answer 42 ÷ 7, you can answer "What number times 7 is 42?"
6 times 7 is 42, so 42÷7 is 6!
Good. Try 54÷9.
What is 54÷9?
6×9=54, so 54÷9=6.
Good. How about 35÷7?
5×7=35, so 35÷7=5.
32÷8?
4×8=32, so 32÷8=4.
49÷7?
7×7=49, so 49÷7=7.
I think you've got it!
Great! Now, hand me that bag of batteries so I can finish filling these treat bags.
Wait, what's in these treat bags?
The usual stuff.
Popcorn balls, band-aids, sporks, coupons, some spaghetti,
Spaghetti!?
soy sauce packets, turnips, ice cream sandwiches,
But, you know those are gonna...
tube socks, twist ties, hex wrenches, cassettes, keys, chopsticks, electrical tape, paper clips, a few shiny pebbles,
COUPONS
SOY


MATH TEAM
Leftovers
Who wants *pizza?!*
How many slices do we get?
I brought two pizzas, and each pizza has eight slices.
That's 2×8=16 slices!
Perfect! There are four of us, so we can each have 16÷4=4 slices!
Ahem.
Let's eat!
There are *five* of us. You forgot about Fiona!
Oh, right.
We need to divide the 16 slices between the *five* of us.
16÷5=???

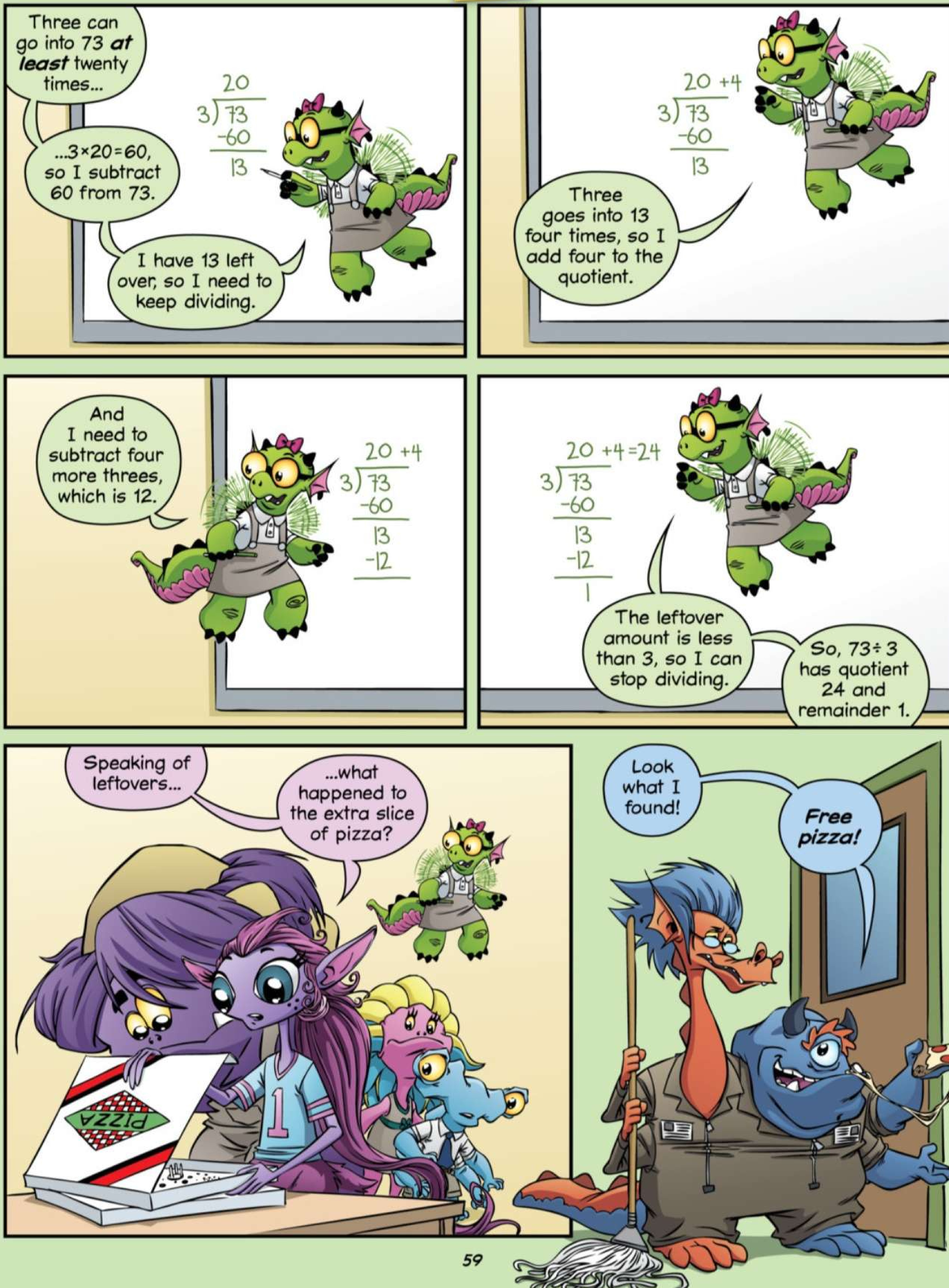
I don't think we can divide 16 by 5.
Five monsters can each have three slices, and there will be one extra.
That's right!
Sometimes when you divide, there are extras.
The extra amount is called a ***remainder.***
Division problems can be written like this:
16 ÷ 5
Sixteen is the number you are dividing. It is called the ***dividend.***
Five is the number you are dividing by. It is called the ***divisor.***
16 ÷ 5
We can write the same division problem like this:
5) 16
It means the same thing as 16÷5, but we usually think of it as "How many times does five ***go into*** sixteen?"
WHEN WE ASK HOW MANY TIMES FIVE "GOES INTO" SIXTEEN, WE ARE ASKING HOW MANY FIVES CAN BE ADDED WITHOUT GOING OVER SIXTEEN.


Five goes into sixteen three times.
Three is called the ***quotient***.
3
5) 16
But 3×5 is only 15, so there is something left over.
3
5) 16
−15
If we subtract three fives from sixteen...
...there is one extra.
3
5) 16
−15
1
THIS PROCESS IS CALLED ***LONG DIVISION***.
When we divide 16 by 5, we get a ***quotient*** of 3 and a ***remainder*** of 1.
When we use long division, we count how many times we can subtract the ***divisor*** from the ***dividend***...
...and keep track of the extras.
Try it!
If 30 cookies are divided equally among 7 monsters, how many extra cookies will there be?
Solve it!
We need to divide 30 by 7.
7) 30
Since 7×4=28, seven goes into 30 ***four*** times.
4
7) 30


We subtract 7×4=28 from 30 to see how many extras there are.
4
7) 30
-28
There are two extras.
So, each monster gets four cookies...
...and there will be two cookies left over!
4
7) 30
-28
2
Perfect! You can always check your division with multiplication.
If seven monsters each get four cookies, that's 7×4=28 cookies...
...plus the two extras makes 30!
7×4=28
28+2=30
1
Or you could check with a picture!
4 each
2 extra
Let's try another one!
1


This rope is 45 feet long.
We need to cut it into pieces that are 4 feet long.
If I cut the rope into as many 4-foot pieces as possible...
...how many extra feet of rope will be left over?
Solve it!
Let's see...
I need to divide 45 by 4.
Since 4×10=40, four can go into 45 ten times.
10
4)45
We subtract 40 from 45 to see how many extra feet of rope there are.
There are 5 extra feet of rope!
10
4)45
-40
5
If there are ***five*** extra feet of rope, we can cut one more piece that is ***four*** feet long.


You can't divide by four and get a remainder of five!
The remainder has to be ***less*** than the number you are dividing by!
THE REMAINDER WILL ALWAYS BE *LESS* THAN THE DIVISOR (THE NUMBER YOU ARE DIVIDING BY).
We can cut another piece of rope!
So I just add one piece of rope to my answer up here...
10 +1
4) 45
-40
5
...and I need to subtract another four feet of rope.
That makes the new remainder one foot!
10 +1
4) 45
-40
5
-4
1
We can cut eleven pieces of rope that are four feet long...
...and still have one foot of rope left.
10 +1 = 11
4) 45
-40
5
-4
1
Nice work!
If you get a remainder that is larger than the number you are dividing by...
...you can just keep dividing!
Try using long division to divide 73 by 3.
3) 73


Three can go into 73 **at least** twenty times...
...3×20=60, so I subtract 60 from 73.
I have 13 left over, so I need to keep dividing.
Three goes into 13 four times, so I add four to the quotient.
And I need to subtract four more threes, which is 12.
The leftover amount is less than 3, so I can stop dividing.
So, 73÷3 has quotient 24 and remainder 1.
Speaking of leftovers...
...what happened to the extra slice of pizza?
Look what I found!
Free pizza!
PIZZA

Long Division

Lizzie

$89 \div 7$ means the same thing as $7\overline{)89}$

dividend divisor divisor dividend

To divide 89 by 7 using long division:

$7\overline{)89}$

First, guess how many times 7 can go into 89. If you guess too high, start over.
$7 \times 10 = 70$, so 7 goes into 89 at least 10 times.

$$\begin{array}{r} 10 \\ 7\overline{)89} \\ -70 \\ \hline 19 \end{array}$$

The 10 goes here. This is where we keep track of how many 7's we've subtracted.
Subtract ten 7's ($10 \times 7 = 70$) from 89. This leaves 19.
Since 19 is bigger than 7, we can subtract more 7's.

$$\begin{array}{r} 10+2 \\ 7\overline{)89} \\ -70 \\ \hline 19 \\ -14 \\ \hline 5 \end{array}$$

7 goes into 19 two times, so we add two up here.

We subtract $2 \times 7 = 14$ to get 5.
Since 5 is less than 7, we can't subtract any more 7's.

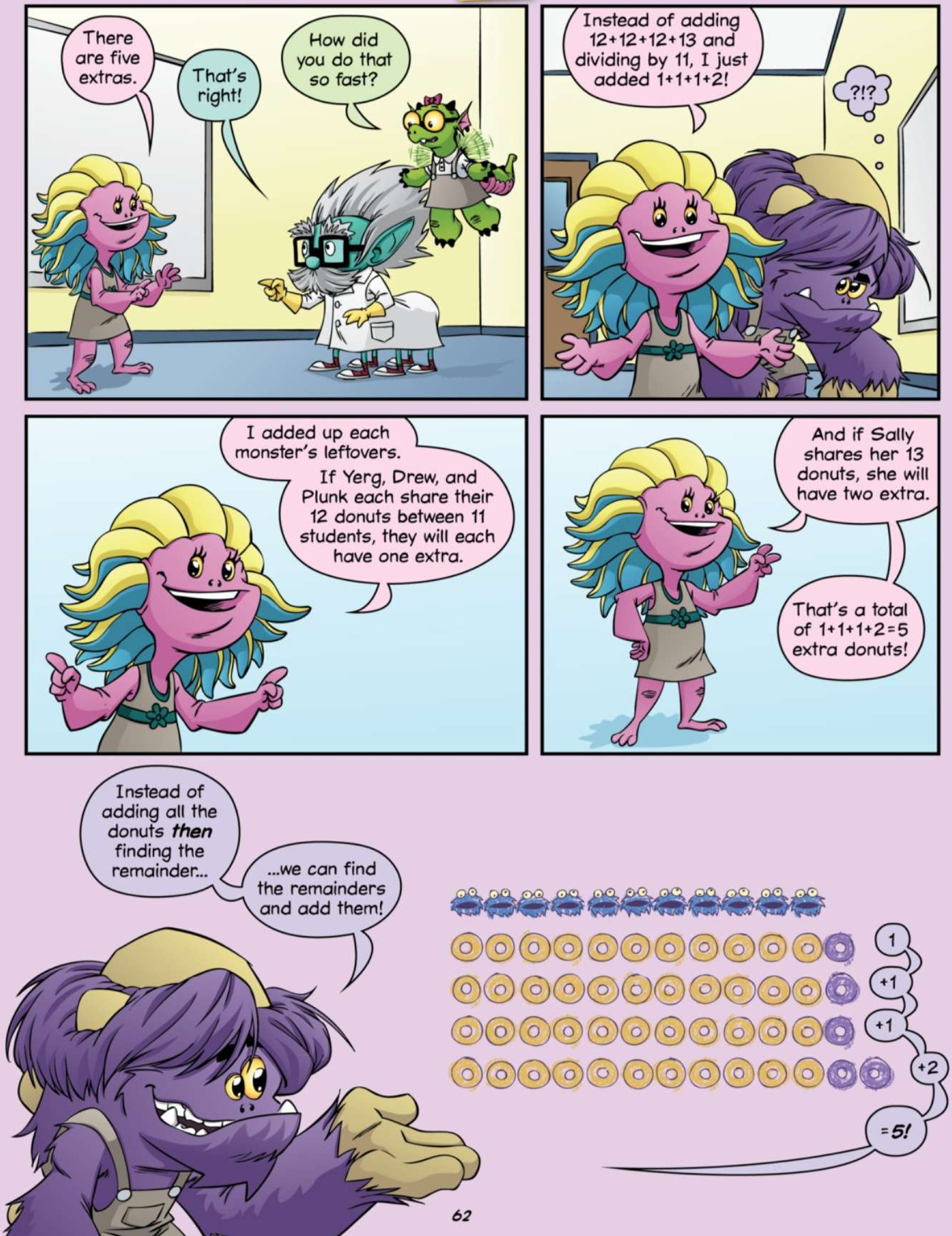
$$\begin{array}{r} 10+2=12 \\ 7\overline{)89} \\ -70 \\ \hline 19 \\ -14 \\ \hline 5 \end{array}$$

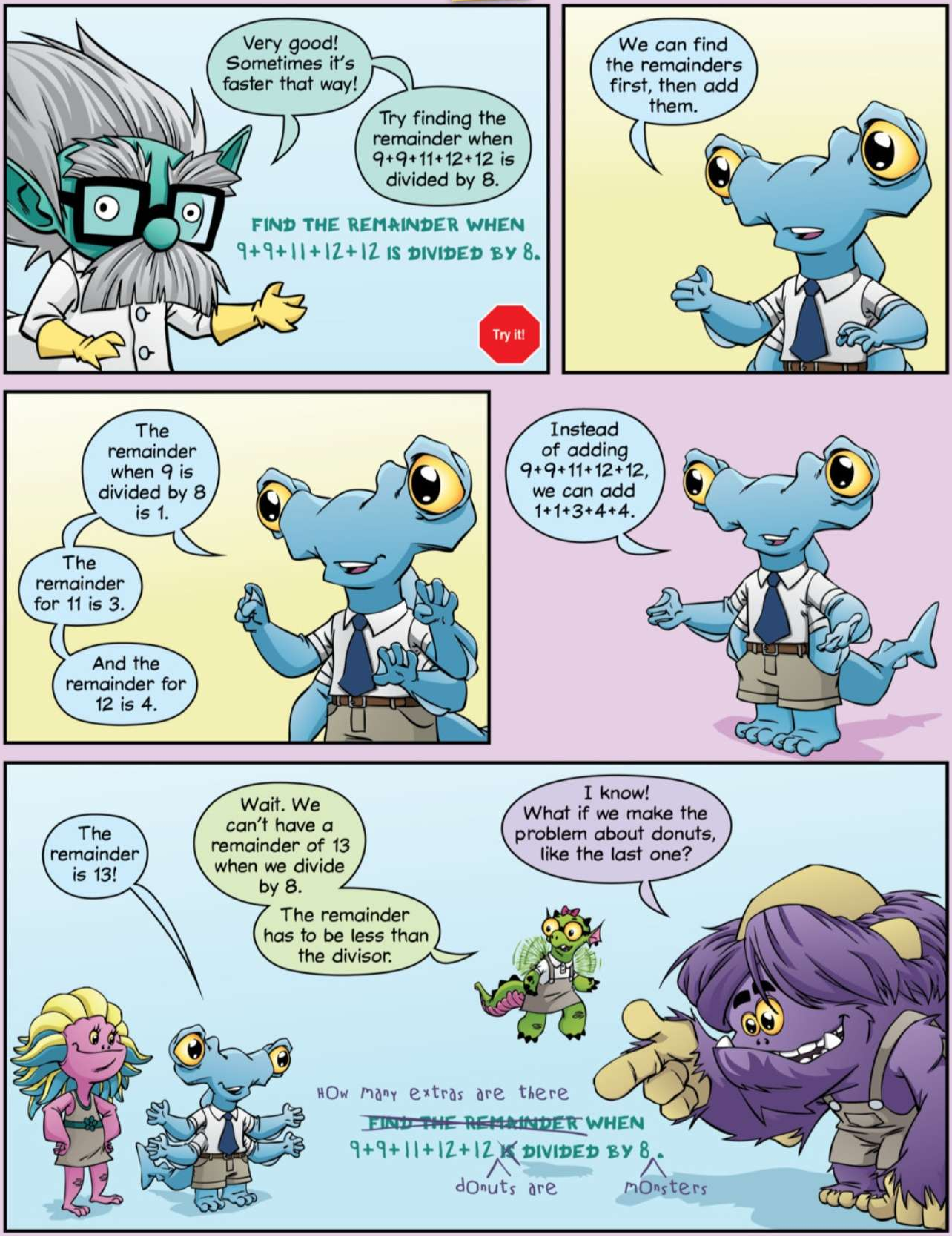
All together, we subtracted $10+2=12$ sevens. The number of 7's we subtracted is called the quotient.

The leftover is called the remainder.
So, $89 \div 7$ has quotient 12 and remainder 5.

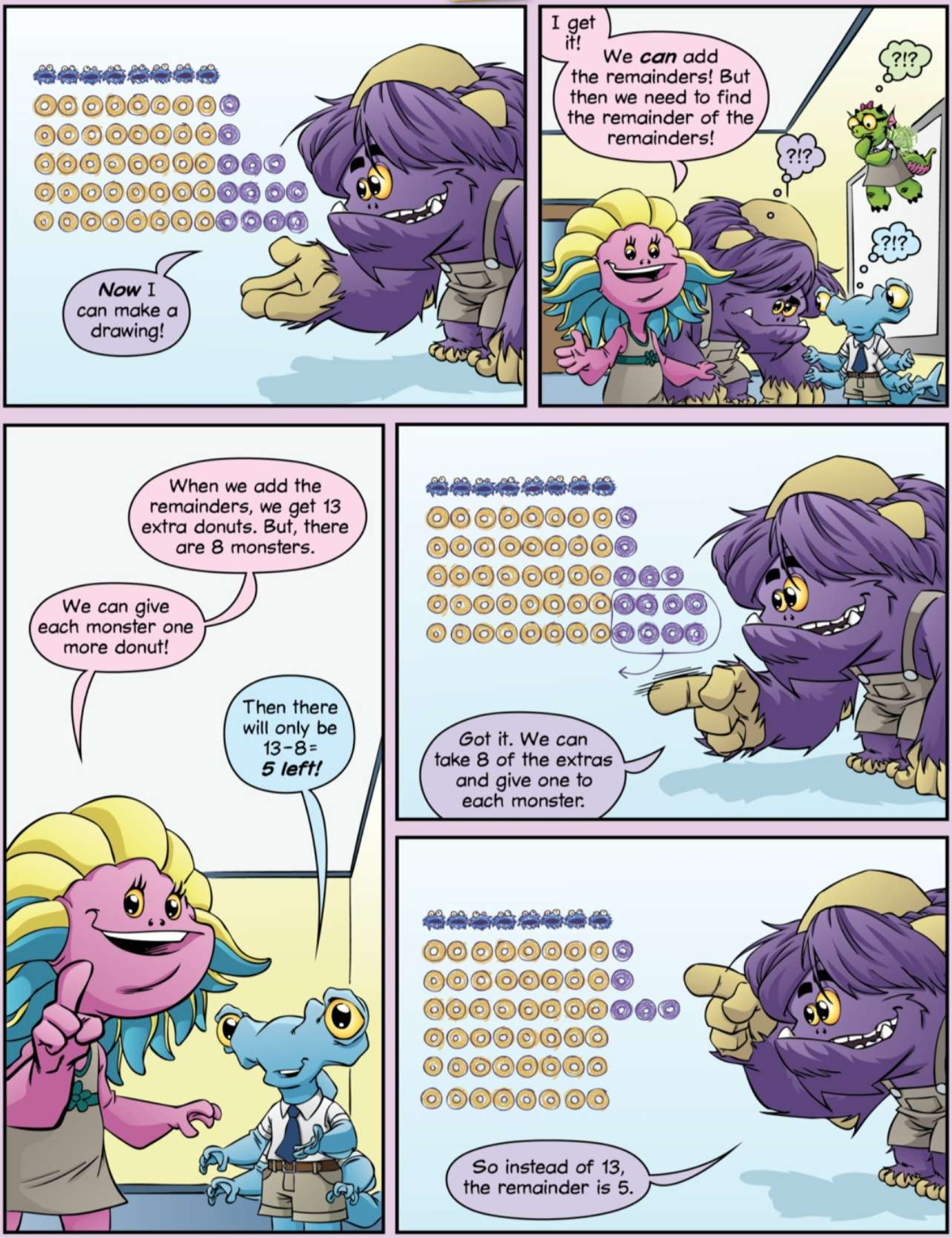
Check: $12 \times 7 + 5 = 84 + 5 = 89$ ✓


THE LAB
REMAINDERS
Today, we will talk about remainders.
For many division problems, we can find the remainder without finding the quotient!
Sometimes, you don't even need to know what number you are dividing!
Try this remainder problem.
Yerg, Drew, and Plunk each bring a dozen donuts to class.
Sally brings a baker's dozen.*
RANDY'S DONUTS
*ONE DOZEN MEANS 12. A BAKER'S DOZEN HAS ONE EXTRA (13).
If the eleven students in the class divide the donuts equally...
...how many extra donuts will there be?
We have to add 12+12+12+13...
...then divide by 11 and find the--
I know!
Can you find the remainder without finding the total number of donuts?

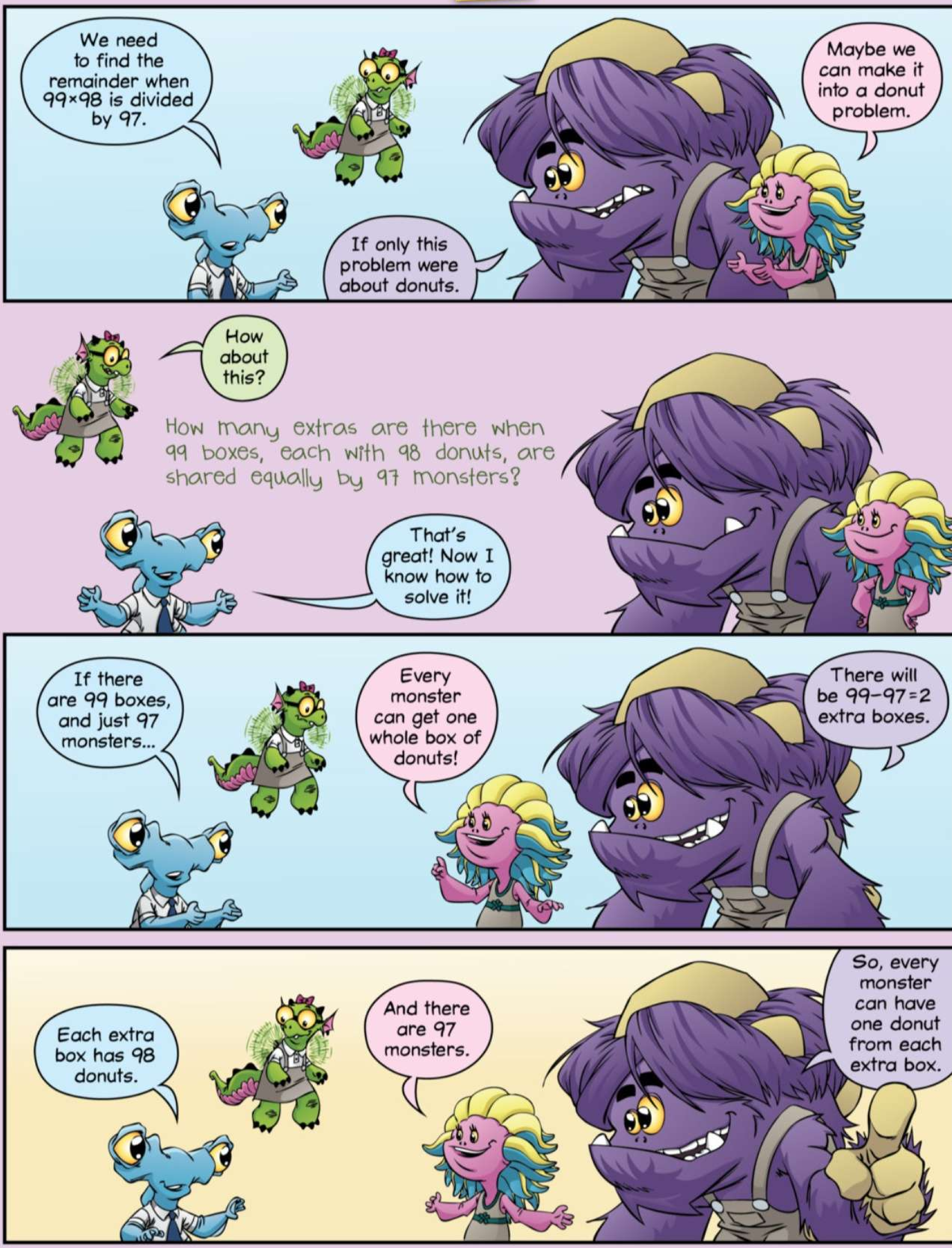

There are five extras.
That's right!
How did you do that so fast?
Instead of adding 12+12+12+13 and dividing by 11, I just added 1+1+1+2!
?!?
I added up each monster's leftovers.
If Yerg, Drew, and Plunk each share their 12 donuts between 11 students, they will each have one extra.
And if Sally shares her 13 donuts, she will have two extra.
That's a total of 1+1+1+2=5 extra donuts!
Instead of adding all the donuts *then* finding the remainder...
...we can find the remainders and add them!
1
+1
+1
+2
=5!



Very good! Sometimes it's faster that way!
Try finding the remainder when 9+9+11+12+12 is divided by 8.
FIND THE REMAINDER WHEN 9+9+11+12+12 IS DIVIDED BY 8.
Try it!
We can find the remainders first, then add them.
The remainder when 9 is divided by 8 is 1.
The remainder for 11 is 3.
And the remainder for 12 is 4.
Instead of adding 9+9+11+12+12, we can add 1+1+3+4+4.
The remainder is 13!
Wait. We can't have a remainder of 13 when we divide by 8.
The remainder has to be less than the divisor.
I know! What if we make the problem about donuts, like the last one?
HOw many extras are there
~~FIND THE REMAINDER~~ WHEN
9+9+11+12+12 ~~IS~~ DIVIDED BY 8.
dOnuts are
mOnsters


Now I can make a drawing!
I get it!
We **can** add the remainders! But then we need to find the remainder of the remainders!
?!?
?!?
?!?
When we add the remainders, we get 13 extra donuts. But, there are 8 monsters.
We can give each monster one more donut!
Then there will only be 13−8= **5 left!**
Got it. We can take 8 of the extras and give one to each monster.
So instead of 13, the remainder is 5.


Brilliant!
Remember that the remainder is always smaller than the divisor.
We can use a similar strategy when solving--
PFLUNK!
Professor Grok?
Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something diabolically difficult!
A Remarkably Ridiculous Remainder Riddle!
To procure the position of your Professor's imprisonment, you must reveal the remainder when 99 times 98 is divided by 97!
Pressing the proper pushbutton in the east elevator will take you to your teacher.
But, while the right one will release him...
...the wrong one will fill his floor with
fetid fumes,
venomous vapors,
treacherous toxins,
contagious contaminants, and
Pestiferous Poisons!
Can you find the remainder when 99×98 is divided by 97?


We need to find the remainder when 99×98 is divided by 97.
If only this problem were about donuts.
Maybe we can make it into a donut problem.
How about this?
How many extras are there when 99 boxes, each with 98 donuts, are shared equally by 97 monsters?
That's great! Now I know how to solve it!
If there are 99 boxes, and just 97 monsters...
Every monster can get one whole box of donuts!
There will be 99−97=2 extra boxes.
Each extra box has 98 donuts.
And there are 97 monsters.
So, every monster can have one donut from each extra box.


That leaves just 98 − 97 = 1 donut in each extra box.
And since there are two extra boxes, there must be ***two*** extra donuts!
Professor Grok is on the 2nd floor.
Let's go!
3
2
1
B
Ding!
Professor Grok!
2
RAIL ROAD CROSSING
Ding! Ding! Ding!
All aboard!

Remainders

Team Name: ~~Grinnie~~

~~Power Princess and the Purple Furball~~

~~team Mega-awesome~~

Unicorn Explosion

PRACTICE | Work with a partner to solve the problem below. Explain your work.

1. What is the remainder when 19×24 is divided by 7?

step 1: make the problem about donuts.

How many extra donuts are there when 19 boxes of 24 donuts are shared by 7 monsters?

step 2: Divide the boxes between the monsters. How many extra boxes are there?

Find the remainder of 19÷7

If each monster gets 2 boxes, that's 2×7=14 boxes.

So, there will be 19-14=5 extra boxes.

step 3: Divide the donuts. How many extra donuts are in each extra box?

Find the remainder of 24÷7.

If each monster eats 3 donuts, that's 3×7=21 donuts.

So, there are 24-21=3 extra donuts in each extra box.

step 4: figure how many extra donuts are in the extra boxes.

Multiply the remainders.

5 extra boxes times 3 extra donuts in each extra box = 15 extra donuts.

step 5: Since 15 is bigger than 7, each monster can have more donuts. Find the remainder of the remainder.

Each monster can have 2 more donuts. 7×2=14.

That leaves 15-14=1 extra donut. So, (19×24)÷7 has remainder 1.

RECESS

NIM

Nim is an ancient game with many variations. In each variation, players take turns removing stones from one or more piles. The variations below, in which all of the stones begin in one pile, are sometimes called "subtraction games."

5|2: The game begins with 5 stones or other objects. Players take turns removing 1 or 2 stones. The player who picks up the last stone wins. The first player can always win if he or she plays correctly. Should the first player remove 1 or 2 stones to guarantee a win?

8|3: The game begins with 8 stones, and players take turns removing 1, 2, or 3 stones. The player who picks up the last stone wins. This time, the second player can always win. Can you find a strategy that will allow the second player to win every time?

21|3: The game begins with 21 stones, and players take turns removing 1, 2, or 3 stones. The player who picks up the last stone wins. Can you discover whether it is better to play first or second? What is the winning strategy?

In each of the numbered versions above, the first number gives you the number of stones to begin with. The second number gives you the maximum number of stones a player may remove each turn.
Try your own variations.

Find a partner and play!

Find more Nim games and winning strategies at BeastAcademy.com.

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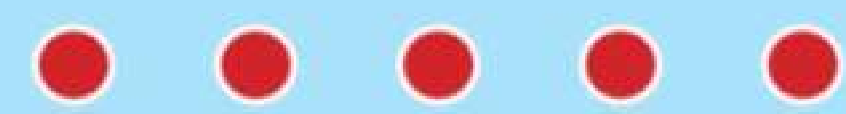

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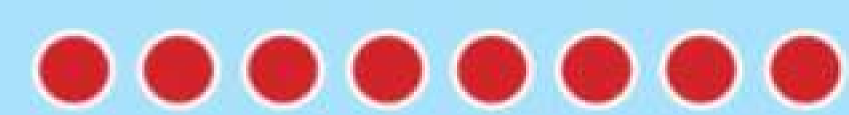

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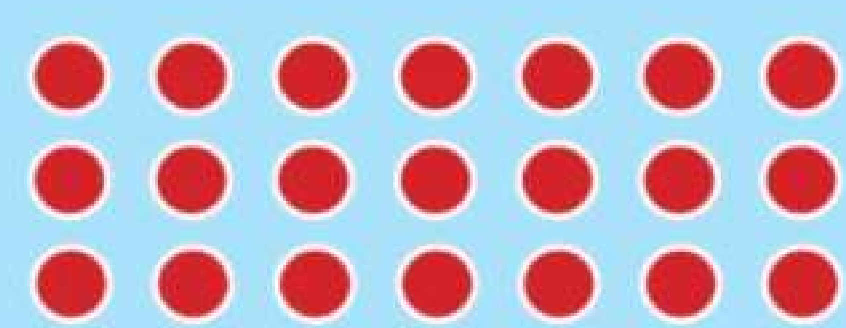

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