Contents: Chapter 4

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Chapter 4:
Statistics

R&G
The Middle

How much does a rhinoceraptor weigh?

FEED

Which rhinoceraptor?

Just a typical, common, normal, ordinary, everyday, run-of-the-mill, garden-variety, adult rhinoceraptor.

Rhinoceraptors come in all different shapes and sizes.

Small adult rhinoceraptors weigh about 1,000 pounds...

...but the biggest ones weigh almost 3,000 pounds.

That's a big difference!

Let's take a look at some data.

POP

Data?

Data. You know, facts.

Oh, right.

We have nine rhinoceraptors.

Here are their weights.

Name:	Weight:
Hildi	1,986 lbs
Mort	2,098 lbs
Herb	1,030 lbs
Mitzi	2,449 lbs
Bob	1,301 lbs
Curt	1,864 lbs
Lola	2,865 lbs
Clive	2,718 lbs
Jarvis	1,187 lbs

If we list the rhinoceraptors from smallest to largest, Hildi is right in the middle.

She's the mediumest!

Name:	Weight:
1. Herb	1,030 lbs
2. Jarvis	1,187 lbs
3. Bob	1,301 lbs
4. Curt	1,864 lbs
5. Hildi	1,986 lbs
6. Mort	2,098 lbs
7. Mitzi	2,449 lbs
8. Clive	2,718 lbs
9. Lola	2,865 lbs

Mediumest isn't a real word.

But, there is a name for the middle number in an ordered list.

The middle number is called the ***median***.

The ***median*** weight of these nine rhinoceraptors is 1,986 pounds.

Name: Weight:
1. Herb 1,030 lbs
2. Jarvis 1,187 lbs
3. Bob 1,301 lbs
4. Curt 1,864 lbs
5. Hildi 1,986 lbs
6. Mort 2,098 lbs
2,449 lbs

Cool.

Hildi has always seemed like a pretty ordinary rhinoceraptor.

Let's fill their feed bags.

The gals have already been fed. We need to fill the feed bags for the fellows.

MITZI

munch munch munch

FEED

munch munch munch

FEED

Hila

Munch Munch munch

Here's how much feed each of the male rhinoceraptors gets.

Can you find the median amount of feed for these six male rhinoceraptors?

Name:	Feed:
Herb	76 lbs
Jarvis	90 lbs
Bob	93 lbs
Curt	119 lbs
Mort	123 lbs
Clive	164 lbs

Try it.

Hmmm... There isn't one middle number.

There are *two* middle numbers.

In that case, the ***median*** is the number halfway between the two middle numbers.

Name:	Feed:
Herb	76 lbs
Jarvis	90 lbs
Bob	93 lbs
Curt	119 lbs
Mort	123 lbs
Clive	164 lbs

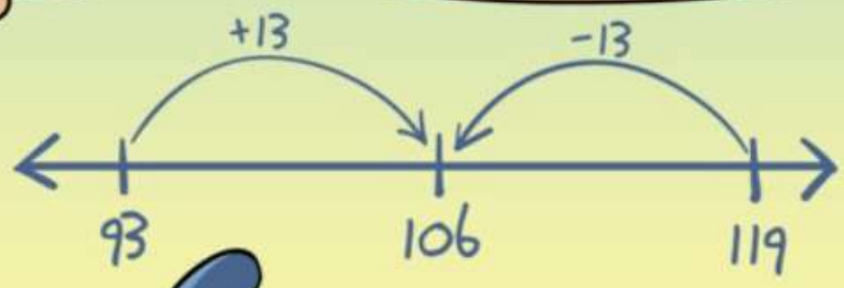
Let's see. The difference between 93 and 119 is 119−93=26. We can split this difference in half: 26÷2=13.

If we add this to 93, or subtract it from 119, we get 106. So, 106 is halfway between 93 and 119.

Is there another way?

There is.

To find the number halfway between two numbers, we add the two numbers and then divide the result by 2.



So, to find the number halfway between 93 and 119, we divide (93+119) by 2?

Yep. 93+119 =212.

And 212÷2 =106.

$$\frac{93+119}{2} = \frac{212}{2} = 106$$

Cool. 106 is 13 away from 93 and 13 away from 119.

Why did that work?

Think of it this way:

I have a 93-pound bag of feed, and a 119-pound bag of feed. I want to find the weight of a bag of feed that is right in the middle.

93 lbs

119 lbs

I can scoop feed from the big bag into the little bag until the two bags are the same weight.

You removed the same amount from the big bag as you added to the little one...

So, the weight of each of the equal bags is halfway between the weights of the two original bags.

13 lbs

Right. And, since the total weight didn't change...

...the total weight of the two equal bags is still 93+119=212 pounds.

So, each equal bag weighs 212÷2=106 pounds.

106 lbs

106 lbs

106 pounds is the median weight of the male rhinoceraptor feed bags!

Yep. These guys sure can eat!

Name:	Feed:
Herb	76 lbs
Jarvis	90 lbs
Bob	93 lbs
	106 lbs
Curt	119 lbs
Mort	123 lbs
Clive	164 lbs

Ever try to ride one?

Nope, never.

Doesn't seem very safe.

MITZI

munch munch munch

FEED

munch munch munch

FEED

Hile

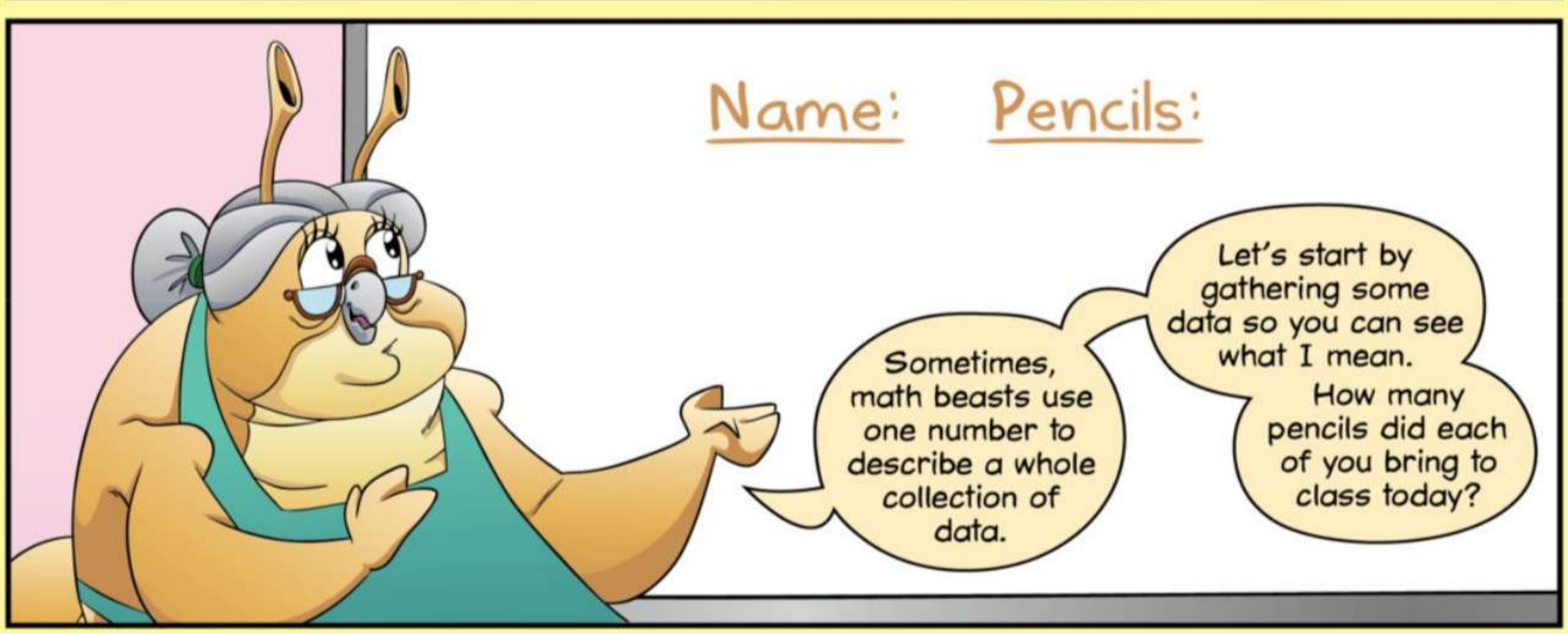
Munch Munch munch

C'mon. They're harmless.

Where's your sense of adventure?

Hand me that saddle.


Ms. Q. Average
Grogg, if you had to describe yourself in one word, what word would you use?
Awesomagnific-incredibrilliant!
That is *not* a word.


Name: Pencils:
Let's start by gathering some data so you can see what I mean.
How many pencils did each of you bring to class today?
Sometimes, math beasts use one number to describe a whole collection of data.


Here's our class data.
I listed the numbers of pencils we each brought to class from least to greatest.
Name: Pencils:
Ralph 1
Winnie 1
Cammie 2
Lizzie 3
Alex 10
Plunk 11
Grogg 14
Perfect. Now, if we share all of these pencils equally, how many pencils will each of you have?
How many?

We have 42 pencils all together.

Name:	Pencils:
Ralph	1
Winnie	1
Cammie	2
Lizzie	3
Alex	10
Plunk	11
Grogg	14
Total =	42

42 ÷ 7 = 6 each

If we divide them equally, we will each have 6 pencils.

Adding all of the numbers in a list and then dividing by the number of numbers gives us their ***mean***.

The mean is usually called the ***average***.

So, the ***average*** number of pencils each monster brought to class today is 6.

IN THIS BOOK, "AVERAGE" ALWAYS REFERS TO THE MEAN FOR A SET OF DATA.

Very good. To find the ***average*** of a list of numbers, you add them all, then divide by how many numbers there are.

What does the average tell us about a list of numbers?

The average is the number each monster gets when the total is shared equally.

Alex, Plunk, and I can give Ralph, Winnie, Cammie, and Lizzie enough pencils so that everyone has 6.

Ralph Winnie Cammie Lizzie Alex Plunk GrOgg

That's because the total number of pencils that Alex, Plunk, and Grogg have above the average...

4+5+8=17

Ralph Winnie Cammie Lizzie Alex Plunk GrOgg

...equals the total number of pencils that Winnie, Ralph, Cammie and I need to reach the average.

5+5+4+3 =17

Ralph Winnie Cammie Lizzie Alex Plunk GrOgg

The average is the number that could replace every number in a group...

Ralph	1	6
Winnie	1	6
Cammie	2	6
Lizzie	3	6
Alex	10	6
Plunk	11	6
Grogg	+14	+6
	42	42

...without changing their sum.

And if you look at the difference between each number and the average...

...the differences balance.

What do you mean, Winnie?

Ralph	1	6	
Winnie	1	6	
Cammie	2	6	
Lizzie	3	6	
Alex	10	6	
Plunk	11	6	
Grogg	+14	+6	
	42	42	

We can subtract the average from each number to find its difference from the average.

For example, I have 5 ***less*** than the average...

...Lizzie has 3 ***less*** than the average...

...but Grogg has 8 ***more*** than the average.

Difference from the average:

Ralph	1	6	-5
Winnie	1	6	-5
Cammie	2	6	-4
Lizzie	3	6	-3
Alex	10	6	+4
Plunk	11	6	+5
Grogg	+14	+6	+8
	42	42	

The numbers ***above*** the average balance the numbers ***below*** the average.

I see.

My +8 balances Winnie's -5 and Lizzie's -3.

Difference from the average:

Ralph	1	6	-5
Winnie	1	6	-5
Cammie	2	6	-4
Lizzie	3	6	-3
Alex	10	6	+4
Plunk	11	6	+5
Grogg	+14	+6	+8
	42	42	

Ralph and Plunk balance each other.

So do Cammie and Alex.

The sum of *all* the differences is 0!

			Difference from the average:
Ralph	1	6	-5
Winnie	1	6	-5
Cammie	2	6	-4
Lizzie	3	6	-3
Alex	10	6	+4
Plunk	11	6	+5
Grogg	+14	+6	+8
	42	42	0

Excellent! Let's solve a problem involving average.

What number can fill the blank so that the average of the five numbers is 90?

84 85 88 91 ___

Five numbers with an average of 90 have a sum of $5 \cdot 90 = 450$.

$$\begin{array}{r} 90 \\ 90 \\ 90 \\ 90 \\ +\ 90 \\ \hline 450 \end{array}$$

The sum of the four numbers we know is $84+85+88+91=348$.

$$\begin{array}{r} 84 \\ 85 \\ 88 \\ +91 \\ \hline 348 \end{array}$$

So, the missing number is $450-348=102$.

$$\begin{array}{r} 450 \\ -348 \\ \hline 102 \end{array}$$

I tried something different.

What did you do, Winnie?

I looked at the differences between each of the four numbers we know and the average, 90.

The differences between 84, 85, 88, and 91 and the average are -6, -5, -2, and +1.

-6 -5 -2 +1
84 85 88 91 ___

The sum of these four differences is (-6)+(-5)+(-2)+1 =-12.

-12

-6 -5 -2 +1

84 85 88 91 __

The numbers we know are a total of 12 below the average.

So, we need a number that is above the average by 12!

-6 -5 -2 +1 +12

84 85 88 91 __

The average is 90, so the missing number is 90+12=102!

Hey! Those first four numbers match my four test scores for this term.

-6 -5 -2 +1 +12

84 85 88 91 102

That means you'll need a 102 on the next test to bring your test average up to 90.

That's gonna be tough.

Better hope there's extra credit!


AVERAGING
Ahoy, little buccaneers!
Ahoy, Cap'n!
What's all this?
These be the spoils o' me most recent voyage.
Whoa. These are pretty!
Aye. Two strands o' giant pearls.
Each o' the 10 pearls on this strand be 3 grams...
...'n' each o' the 30 pearls on this strand be 11 grams.
What if you put them all on one strand?
Excellent inquiry.
If I put all the pearls on a single strand, what be the average weight o' the 40 pearls on the strand?
Try it.

The small pearls on the first strand weigh 3 grams each, and the big pearls on the second strand weigh 11 grams each.

The average of 3 grams and 11 grams is 7 grams.

But there are way more 11-gram pearls than 3-gram pearls.

The average must be closer to 11 grams than to 3 grams.

Huh?

We can't just take the average of the weights.

What if there were ninety-nine 11-gram pearls, but just one 3-gram pearl?

Ninety-nine 11-gram pearls

One 3-gram pearl

I see.

The average weight of all 100 pearls would be really close to 11 grams.*

*THE TOTAL WEIGHT OF ALL 100 PEARLS ABOVE IS $(99 \cdot 11)+(1 \cdot 3)=1{,}089+3=1{,}092$ GRAMS. SO, THE AVERAGE WEIGHT OF THE 100 PEARLS IS $\frac{1092}{100}=10\frac{92}{100}=10.92$ GRAMS.

Right. The weight of the pearl that could replace every one of the 100 pearls without changing the total weight is really close to 11 grams.

So, how do we find the average weight of thirty 11-gram pearls and ten 3-gram pearls?

We can find the total weight of all the pearls, and divide by 40.

Each of the 30 pearls on this strand weighs 11 grams, so their total weight is 30 • 11 = 330 grams.

And each of the 10 pearls on this strand weighs 3 grams, for a total weight of 10 • 3 = 30 grams.

So, the total weight of all 40 pearls on both strands is 330 + 30 = 360 grams.

That makes the average weight of all 40 pearls 360 ÷ 40 = 9 grams.

330 + 30 = 360 grams

$\frac{360}{40}$ = 9 grams

Aye. Excellent figurin'.

What's in here?

In this box be five gems...

...two identical rubies, 'n' three identical sapphires.

The average weight o' the five gemstones be 10 carats.

If the weight o' each ruby be 13 carats, what be the weight o' each sapphire?

A CARAT IS A MEASURE OF WEIGHT USUALLY USED TO MEASURE JEWELS. 1 CARAT = 200 MILLIGRAMS (0.2 GRAMS).

Try it.

Well, if all five stones have an average weight of 10 carats, then they have a total weight of $5 \cdot 10 = 50$ carats.

And if the two rubies weigh 13 carats each, then they have a total weight of $2 \cdot 13 = 26$ carats.

So, the weight of the three sapphires is $50 - 26 = 24$ carats.

And since there are three of them, that makes the weight of each sapphire $24 \div 3 = 8$ carats!

I did it differently.

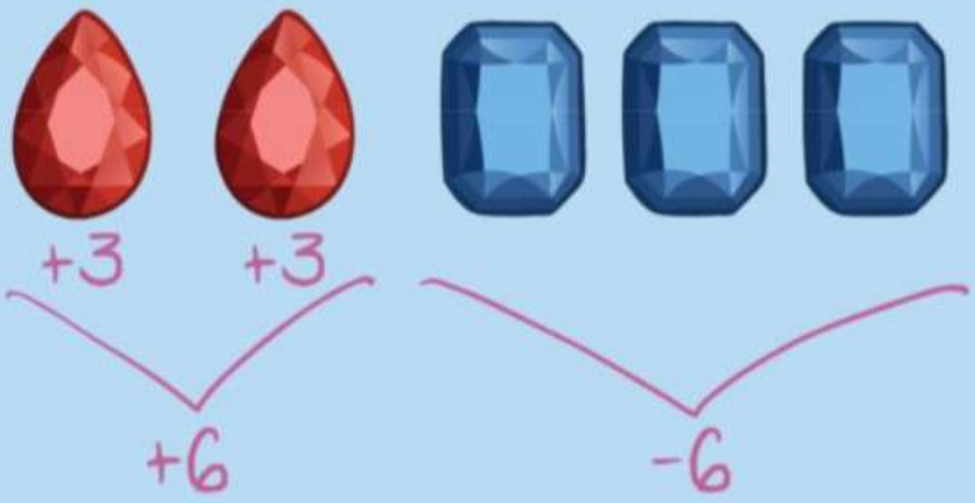
How did ye do it, lass?

I looked at the ***differences*** between the gemstones and the average.

The rubies are each 3 carats larger than the average.

There are 2 of them, so they are a total of $2 \cdot 3 = 6$ carats ***above*** the average.

So, the sapphires need to be a total of 6 carats ***below*** the average.



So, each of the 3 sapphires has to be 6÷3=2 carats below the average.

That's 10−2=8 carats each.

13 13 8 8 8

+3 +3 −2 −2 −2

+6 −6

Aye. 'Tis a fine method.

Each o' the sapphires be 8 carats in weight.

Let's look at one more treasure.

In this satchel be a number o' coins.

One o' the coins be worth $50. The rest be worth $5 each.

The average value o' the coins be $8. How many coins be there in this satchel?

One coin be worth $50.
All others be $5 each.
The average value o' the coins be $8.

Scratch Scratch Scratch

How many coins are in Captain Kraken's satchel?

I know! We can solve this coin problem the same way Winnie solved the sapphire problem.

The $50 coin is 50−8=42 dollars **above** the average.

Each $5 coin is $3 **below** the average.

We need to know how many -3's are needed to balance the +42.

All together, the $5 coins are $42 below the average.

+42

-3 -3 -3 . . . -3

+42

-42

It will take 42÷3=14 five-dollar coins.

That makes sense. Let's check our work.

14 five-dollar coins are worth 14•5=70 dollars.

Plus one fifty-dollar coin makes 70+50=120 dollars worth of coins.

And since there are 15 coins, their average value is 120÷15=8 dollars.

It works out!

There are 15 coins.

I did something different.

I wrote an equation.

We can write the total value of the coins two ways.

One coin be worth $50.
All others be $5 each.

$50+5n$

If n is the number of $5 coins...

...then the value of all the coins is $50+5n$ dollars.

And since n is the number of $5 coins...

...there are $n+1$ coins all together.

If $n+1$ coins have an average value of $8, their total value is $8(n+1)$ dollars.

The average value o' the coins be $8.

$50+5n=8(n+1)$

So, $50+5n=8(n+1)$

$50+5n=8(n+1)$

$50+5n=8n+8$

To solve, we start by distributing the 8 to get $50+5n=8n+8$.

$50+5n=8(n+1)$

$50+5n=8n+8$

$-5n \quad -5n$

$50 \quad = 3n+8$

We subtract $5n$ from both sides so that we only have n on one side of the equation.

Then, we subtract 8 from both sides.

$50+5n=8(n+1)$

$50+5n=8n+8$

$-5n \quad -5n$

$50 \quad = 3n+8$

$-8 \quad -8$

$42 \quad = 3n$

And we divide both sides by 3 to isolate n.

$n=14$.

$50+5n=8(n+1)$

$50+5n=8n+8$

$-5n \quad -5n$

$50 \quad = 3n+8$

$-8 \quad -8$

$\frac{42}{3} = \frac{3n}{3}$

$14 = n$

Since n represents the number of $5 coins, there are 14 $5 coins.

So, there are a total of 15 coins in the satchel.

Aye.

There be 15 coins in the satchel.

Oooh... What's in here?

Wait--!

Practice: Pages 19-23

Arrr. Ye weren't s'posed to be seein' those.

Pirate Booty

MATH TEAM
Range and Mode

What statistics have you learned about so far?

Statistics?

STATISTICS

A ***statistic*** is a number used to describe a list of data.

Like the median, or the average?

Exactly.

We learned that the ***median*** is the middle number in a list of data.

And the ***average*** for a list of data is the number that could replace all of the numbers in a group without changing the group's sum.

STATISTICS IS THE FIELD OF MATHEMATICS INVOLVING THE STUDY OF DATA.

Great.
What I'd like each of you to do is to create a list of seven numbers whose median is 8 and whose average is 11.

Try it.

So, we need 7 numbers...

...with a middle number of 8.

And for the average to be 11, the sum of the 7 numbers must be 7•11=77.

__+__+__+8+__+__+__=77

I have my list!

8, 8, 8, 8, 15, 15, 15.

I wrote 2, 2, 2, 8, 10, 10, 43.

My numbers are 2, 4, 6, 8, 11, 18, and 28.

And I have 3, 3, 3, 8, 20, 20, 20.

8	8	8	8	15	15	15
2	2	2	8	10	10	43
2	4	6	8	11	18	28
3	3	3	8	20	20	20

Whoa. All of our lists have the same median...

...and the same average.

But our lists are soooo different!

That's right.

The median and the average for a list of data only give a small amount of information about the list.

What other statistics could help describe how these four lists of numbers are different?

8	8	8	8	15	15	15
2	2	2	8	10	10	43
2	4	6	8	11	18	28
3	3	3	8	20	20	20

How could you describe the differences between these lists?

Some of the lists are much more spread out than others.

My numbers only go from 8 to 15...

...but Grogg's start at 2 and go all the way up to 43!

8	8	8	8	15	15	15
2	2	2	8	10	10	43
2	4	6	8	11	18	28
3	3	3	8	20	20	20

We can label the difference between the largest and smallest value in each list.

Very good. The difference between the smallest and largest numbers in a list is called its ***range***.

							Range:
8	8	8	8	15	15	15	15 - 8 = 7
2	2	2	8	10	10	43	43 - 2 = 41
2	4	6	8	11	18	28	28 - 2 = 26
3	3	3	8	20	20	20	20 - 3 = 17

We can list the most common number in each list.

The number that occurs most often in a list is called its ***mode***.

							Range:	Mode:
8	8	8	8	15	15	15	15 - 8 = 7	8
2	2	2	8	10	10	43	43 - 2 = 41	2
2	4	6	8	11	18	28	28 - 2 = 26	
3	3	3	8	20	20	20	20 - 3 = 17	

But, what do we write if no number occurs more than once...

...or if there is a tie?

If no number in a list occurs more than once, we say that the list has no mode.

But, a list can have more than one mode if there is a tie. In Lizzie's list, there are two modes: 3 and 20.

Got it.

							Range:	Mode:
8	8	8	8	15	15	15	15 - 8 = 7	8
1	1	1	8	10	10	43	43 - 1 = 42	1
2	4	6	8	11	18	28	28 - 2 = 26	no mode
3	3	3	8	20	20	20	20 - 3 = 17	3 & 20

Good. You've named the four most common statistics used to describe a list of data: the median, average, range, and mode.

Median: Middle number

Average: $\frac{\text{Sum of numbers}}{\text{Number of numbers}}$

Range: Biggest minus smallest

Mode: Occurs most

Let's put these statistics to use.

The ages of the five monsters in a group are 11, 9, 10, 4, and 11 years.

Four new monsters join the group. This increases the average, median, and mode ages of the group by 1.

What is the new ***range*** of ages for the nine monsters in the group?

Ages: 11 9 10 4 11 ? ? ? ?

Try it.

We can start by putting the ages of the five monsters in order.

The median age is 10.

The mode is 11.

And the average age is (4+9+10+11+11)÷5 =45÷5=9.

median

4 9 10 11 11

mode

average: $\frac{4+9+10+11+11}{5} = \frac{45}{5} = 9$

So, for the new group, the average is 9+1=10, the median is 10+1=11, and the mode is 11+1=12.

For the mode of the new group to be 12, there have to be ***more*** monsters who are 12 than monsters who are 11!

So, there must be ***at least*** three 12-year-old monsters who joined the group.

4 9 10 11 11 12 12 12 ?

Grogg is right!

Now, all we need is the age of the ninth monster.

How old is the missing monster?

4 + 9 + 10 + 11 + 11 + 12 + 12 + 12 + ? = 90

The average age of the nine monsters is 10, so the sum of their ages is 10 • 9 = 90.

4 + 9 + 10 + 11 + 11 + 12 + 12 + 12 + ? = 90

81 + ? = 90

81 + (9) = 90

The ages we know add up to 81.

So, the missing age must be 90 − 81 = **9**.

The ages in order are 4, 9, 9, 10, 11, 11, 12, 12, and 12.

The average is 10, the median is 11, and the mode is 12.

So, the age ***range*** for the nine monsters is 12 − 4 = **8**.

4 9 9 10 11 11 12 12 12

Great job, guys. Look who's here.

Max!

Hi, team!
I came to talk about the World Math Olympiad.
The qualifying round is in 7 weeks, and I think Beast Academy should enter a team.
If you join, we'll have the best team on Beast Island!
The five of us will be really tough to beat.
That's part of what I wanted to talk about.
Only **four** monsters can compete.
But, every team gets to bring an alternate.
I'm your guy!
No way, Grogg! The team won't be the same without you.
Alex is right.
None of us see problems quite the way you do.
But Max gives Beast Academy our best chance at qualifying for World's.
Plus, I'll still be a part of the team.
If one of you can't compete, I'll be ready to step in.
If not, I'll be cheering louder than anyone from the stands.
not Max's actual tail

RECESS

Stat Stumped

Stat Stumped is a game for 2 or more players played with a standard shuffled deck of cards with the face cards removed. Aces are 1's.

Setup:

Three cards are dealt onto the board to begin piles marked Mean, Median, and Range as shown. *If a 1 or a 10 is dealt onto the board, it is immediately moved below the board.*

Cards are then dealt so that there are a total of 7 cards below the board.

Stat!

Players simultaneously look for a group of three or more cards below the board that has at least two of the three statistics on the board.

A player who spots such a group calls out "Stat!" If the player can select a group of three or more cards from below the board that has at least two of the three statistics, the player gets to keep those cards in their score pile. Cards are then dealt from the stock so that there are 7 cards below the board, and play continues.

If the selected cards do *not* satisfy two of the three statistics, they are returned, and the other players are given the chance to call out "Stat!"

Stumped.

Any player who thinks it is impossible to create a group of cards with at least two of the stats says "Stumped."

Once all players are stumped, the first stumped player chooses one of the three stat piles and places the top card from the shuffled deck onto that pile. If the dealt card is an Ace, a 10, or matches the previous card, it is immediately moved below the board and play continues until someone calls "Stat!" or all players are stumped again.

Winning:

The game ends when all cards have been dealt from the stock and all players are stumped. The player who collects the most cards wins.

Samples:

Find a group of 3 or more cards below the board that has at least two of the three statistics on the board.

Answers are given at the bottom of this page.

1.

2.

3.

4.

Find a partner and

RECESS

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Once all players are stumped, the first stumped player chooses one of the three stat piles and places the top card from the shuffled deck onto that pile. If the dealt card is an Ace, a 10, or matches the previous card, it is immediately moved below the board and play continues until someone calls "Stat!" or all players are stumped again.

Winning:

The game ends when all cards have been dealt from the stock and all players are stumped. The player who collects the most cards wins.

Samples:

Find a group of 3 or more cards below the board that has at least two of the three statistics on the board.

Answers are given at the bottom of this page.

1.

2.

3.

4.

Find a partner and