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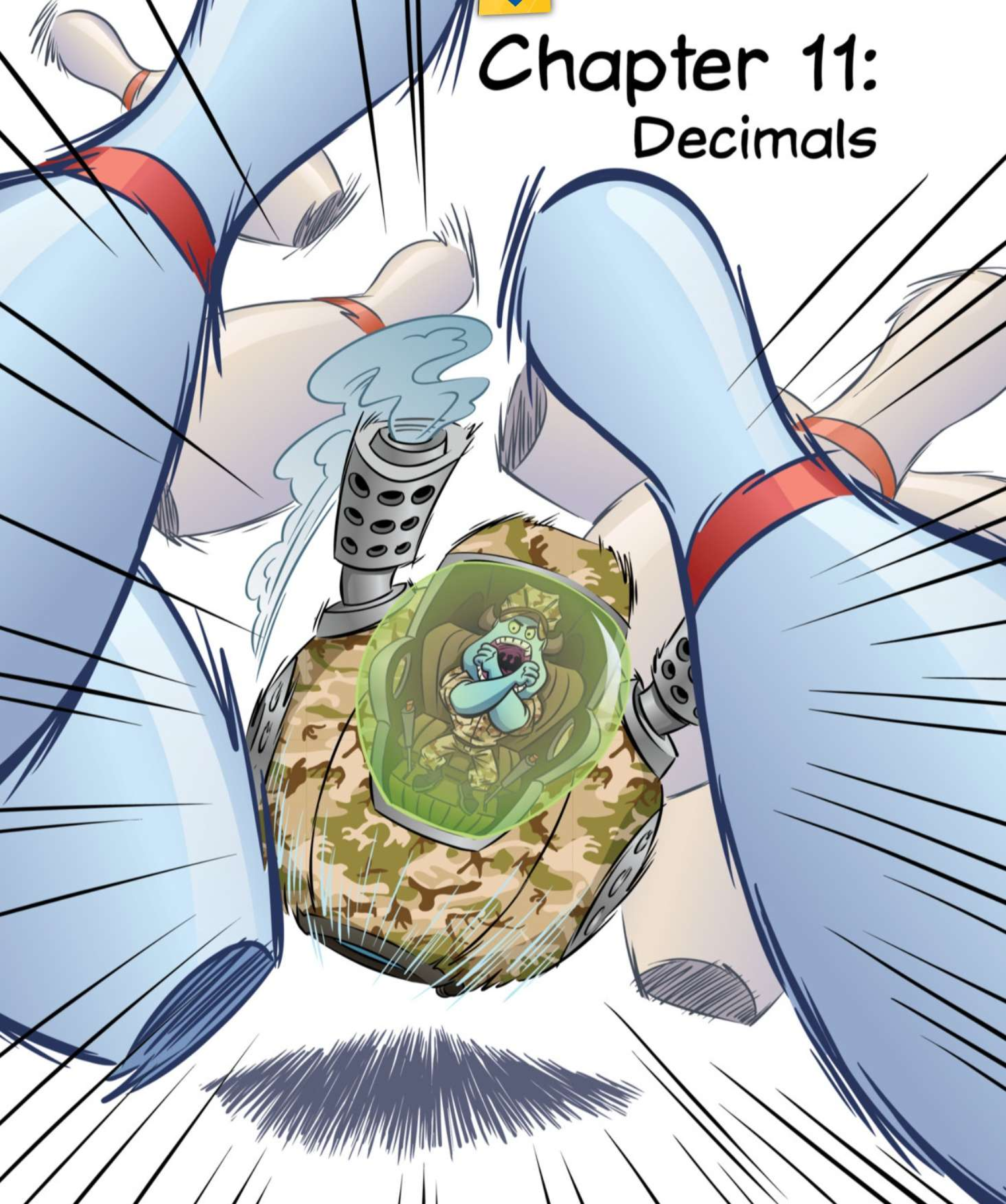
Chapter 11: Decimals

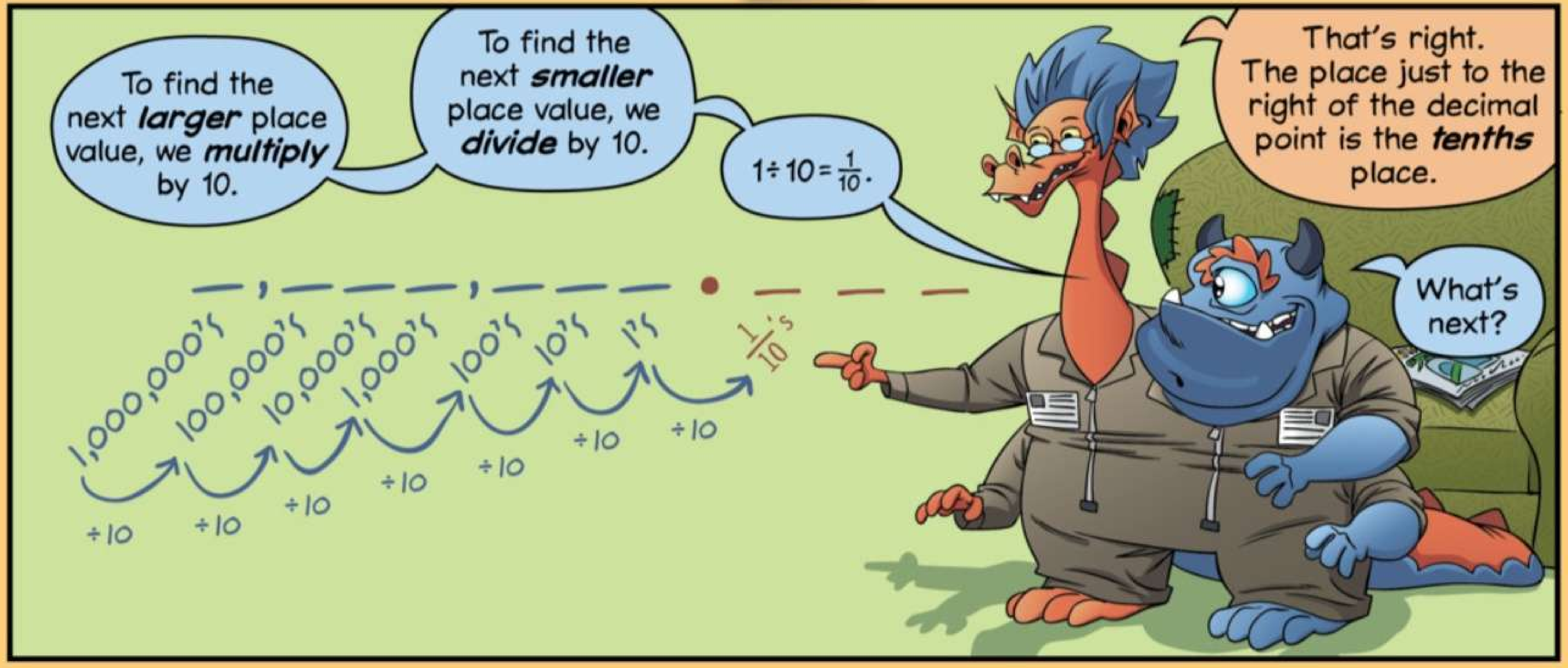


7.3 IS READ "SEVEN POINT THREE." SIMILARLY, 12.2 IS READ "TWELVE POINT TWO," AND 2.05 IS READ "TWO POINT ZERO FIVE."

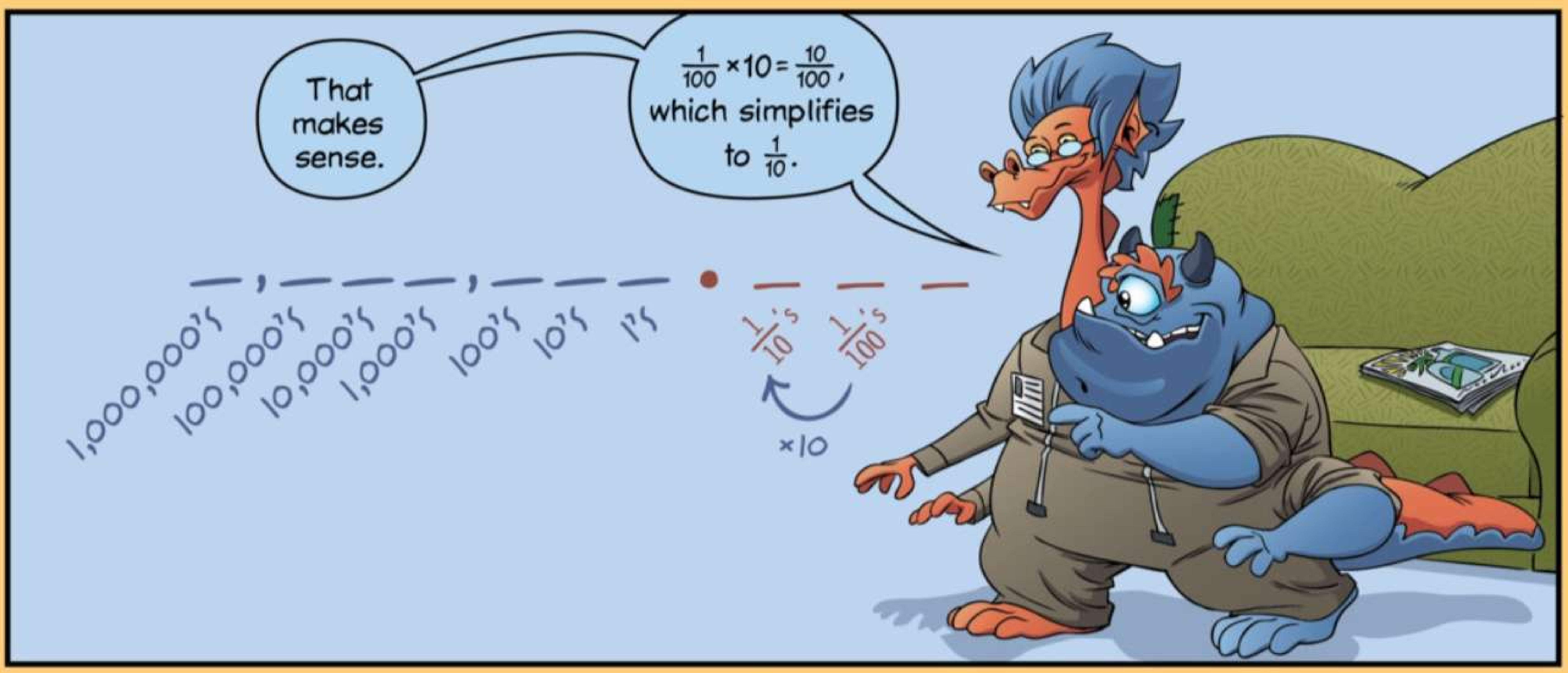
The dot is called a ***decimal point***.
The decimal point separates the whole-number part from the fractional part.
Fractional part?
7.3
decimal point

The number 7.3 is a little more than 7.
To understand how much more, you need to understand ***place value***.
I remember learning about place value!
HANG IN THERE
FRIDAY'S COMIN'
CAUTION
WET FLOOR
Bessie

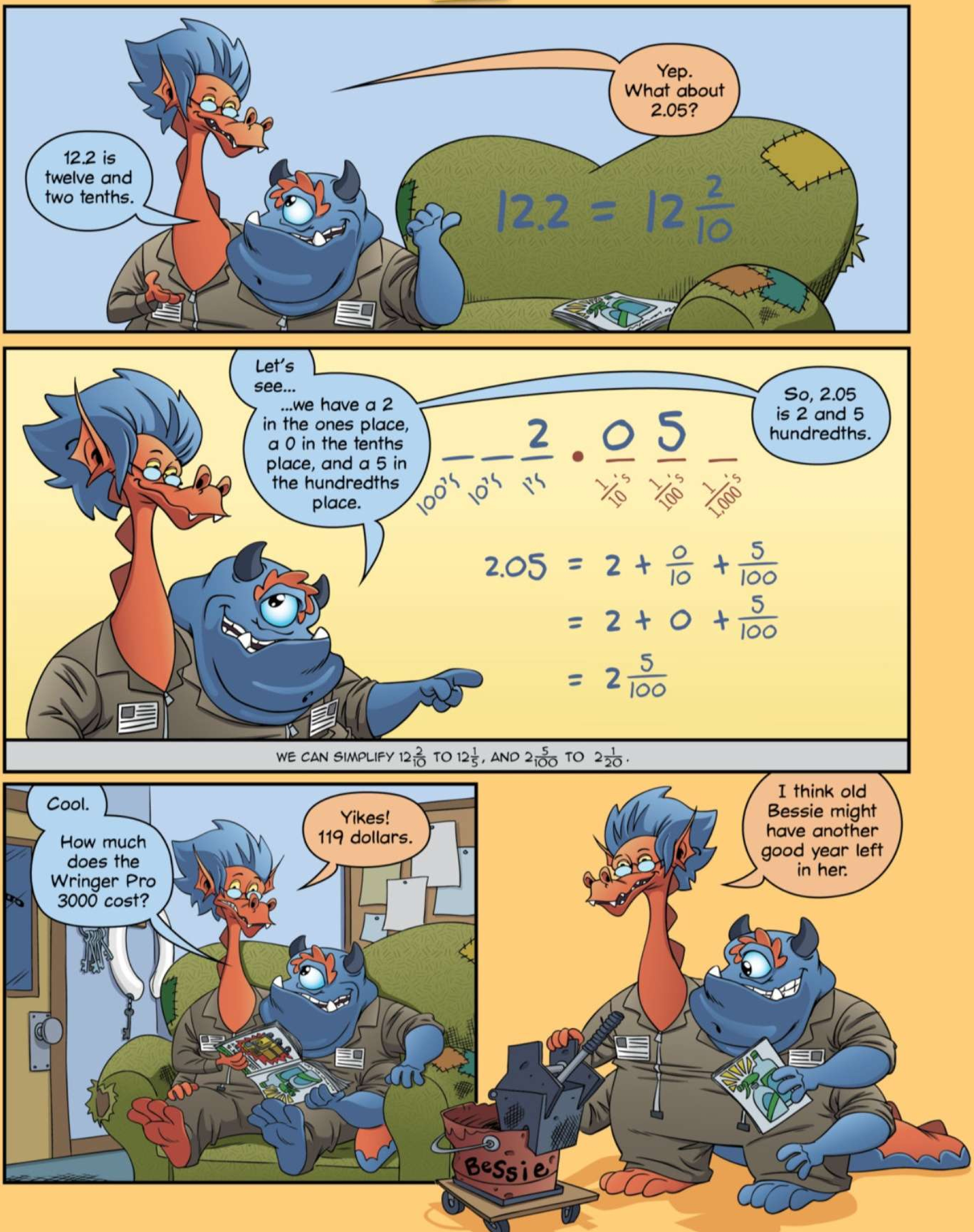
Each place value is ten times the place value to its right.
That's right. But, how would you find the place values to the ***right*** of the ones place?
1,000,000's
100,000's
10,000's
1,000's
100's
10's
1's
×10
×10
×10
×10
×10
×10
Try to fill in the place values to the right of the ones place.

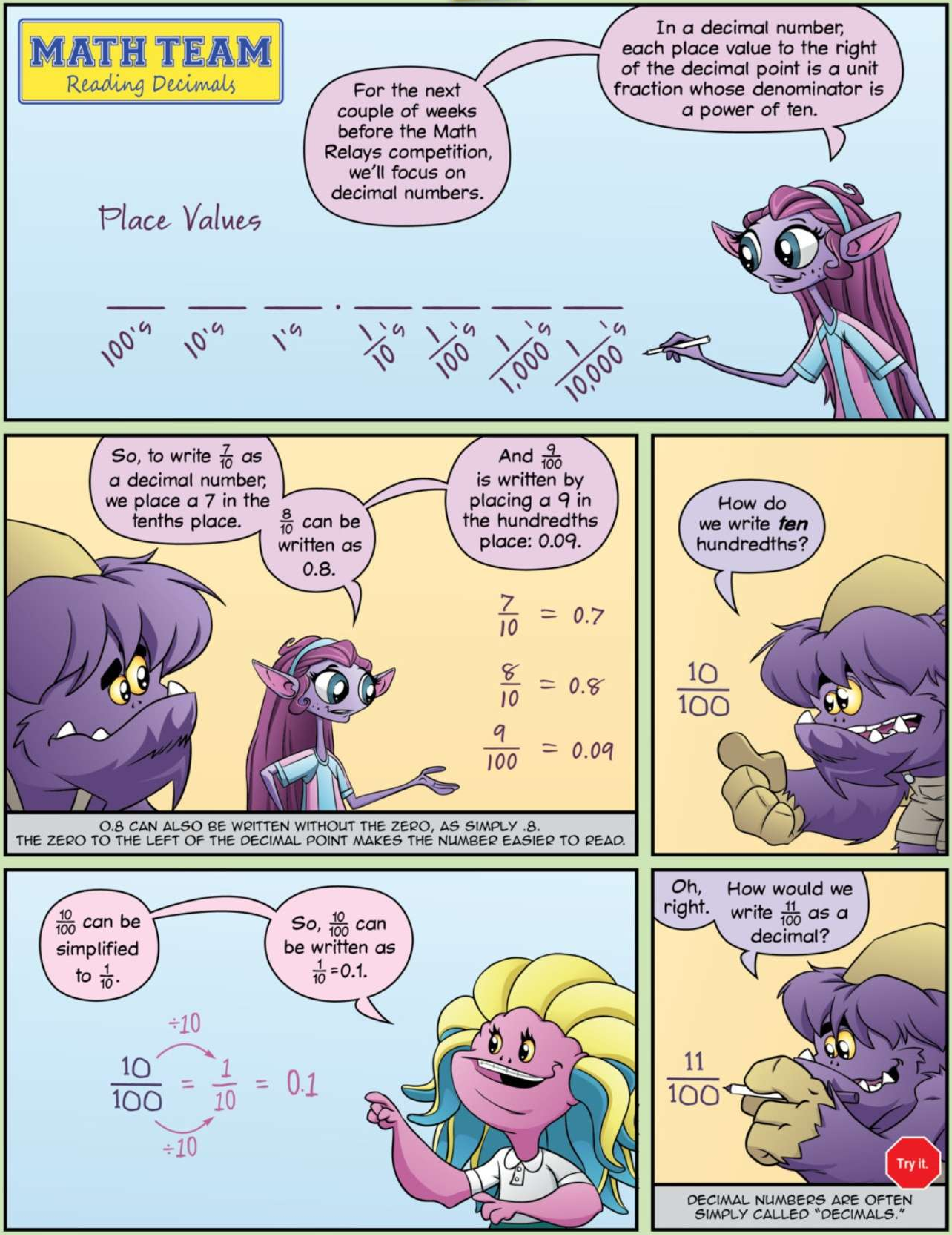

To find the next *larger* place value, we *multiply* by 10.
To find the next *smaller* place value, we *divide* by 10.
$1 \div 10 = \frac{1}{10}$.
That's right. The place just to the right of the decimal point is the *tenths* place.
What's next?
1,000,000's 100,000's 10,000's 1,000's 100's 10's 1's $\frac{1}{10}$'s
÷10 ÷10 ÷10 ÷10 ÷10 ÷10 ÷10

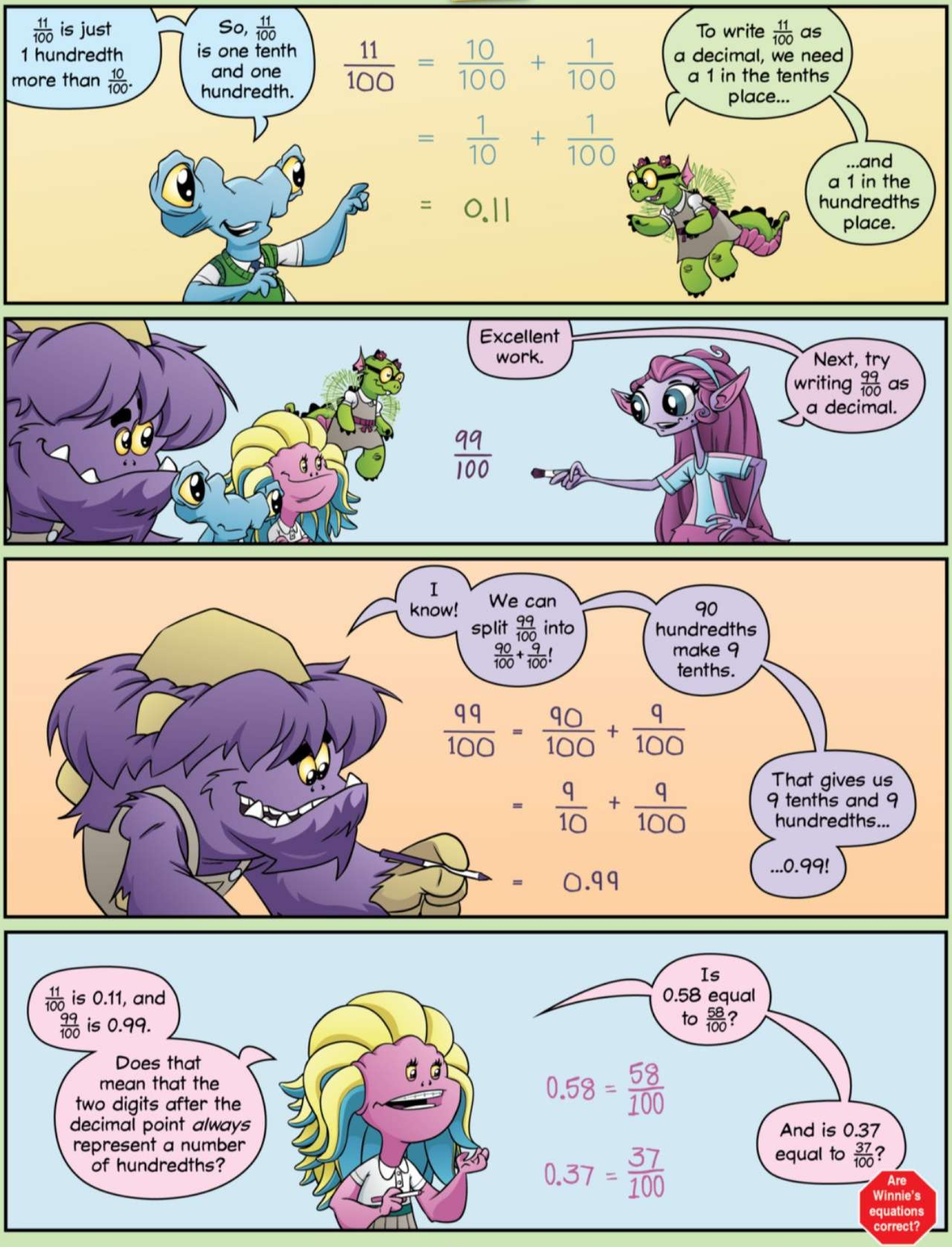

Next is the *hundredths* place.
1,000,000's 100,000's 10,000's 1,000's 100's 10's 1's $\frac{1}{10}$'s $\frac{1}{100}$'s


$\frac{1}{100} \times 10 = \frac{10}{100}$, which simplifies to $\frac{1}{10}$.
That makes sense.
1,000,000's 100,000's 10,000's 1,000's 100's 10's 1's $\frac{1}{10}$'s $\frac{1}{100}$'s
×10

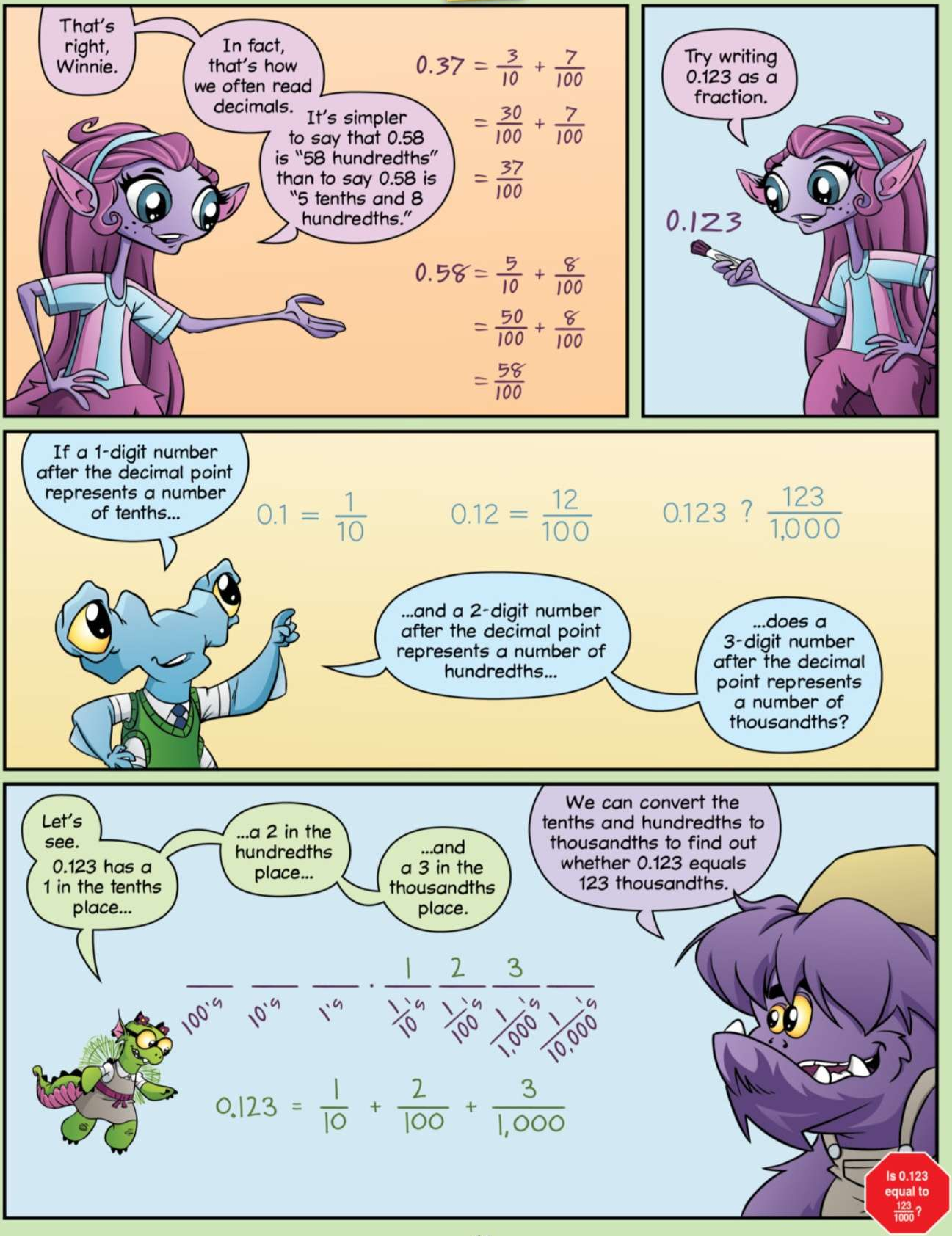

After the hundredths place comes the thousandths place...
...then ten-thousandths, hundred-thousandths, millionths, and so on.
1,000,000's 100,000's 10,000's 1,000's 100's 10's 1's
$\frac{1}{10}$'s $\frac{1}{100}$'s $\frac{1}{1,000}$'s
Cool. So, 7.3 means 7 ones and 3 tenths!
7 . 3
100's 10's 1's
$\frac{1}{10}$'s $\frac{1}{100}$'s $\frac{1}{1,000}$'s
$7.3 = 7\frac{3}{10}$
That's right.
7.3 is 7 and 3 tenths.
Try writing 12.2 as a mixed number.
WET FLOOR
Bessie
Try it.

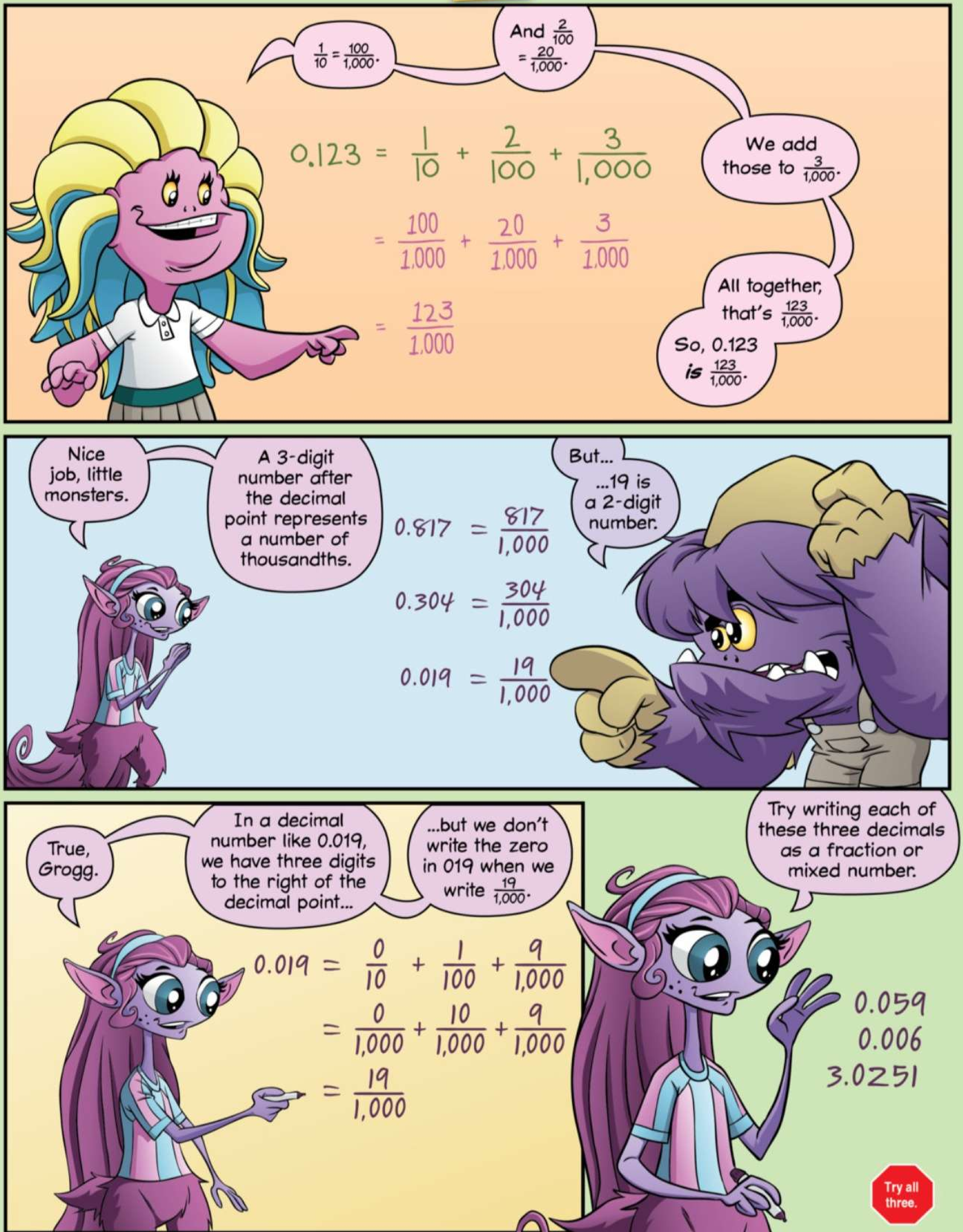

12.2 is twelve and two tenths.
Yep. What about 2.05?
12.2 = 12 2/10
Let's see...
...we have a 2 in the ones place, a 0 in the tenths place, and a 5 in the hundredths place.
So, 2.05 is 2 and 5 hundredths.
2 . 0 5
100's 10's 1's 1/10's 1/100's 1/1,000's
2.05 = 2 + 0/10 + 5/100
= 2 + 0 + 5/100
= 2 5/100
WE CAN SIMPLIFY 12 2/10 TO 12 1/5, AND 2 5/100 TO 2 1/20.
Cool.
How much does the Wringer Pro 3000 cost?
Yikes! 119 dollars.
I think old Bessie might have another good year left in her.
Bessie


MATH TEAM
Reading Decimals
For the next couple of weeks before the Math Relays competition, we'll focus on decimal numbers.
In a decimal number, each place value to the right of the decimal point is a unit fraction whose denominator is a power of ten.
Place Values
100's 10's 1's . $\frac{1}{10}$'s $\frac{1}{100}$'s $\frac{1}{1,000}$'s $\frac{1}{10,000}$'s
So, to write $\frac{7}{10}$ as a decimal number, we place a 7 in the tenths place.
$\frac{8}{10}$ can be written as 0.8.
And $\frac{9}{100}$ is written by placing a 9 in the hundredths place: 0.09.
$\frac{7}{10} = 0.7$
$\frac{8}{10} = 0.8$
$\frac{9}{100} = 0.09$
0.8 CAN ALSO BE WRITTEN WITHOUT THE ZERO, AS SIMPLY .8. THE ZERO TO THE LEFT OF THE DECIMAL POINT MAKES THE NUMBER EASIER TO READ.
How do we write ***ten*** hundredths?
$\frac{10}{100}$
$\frac{10}{100}$ can be simplified to $\frac{1}{10}$.
So, $\frac{10}{100}$ can be written as $\frac{1}{10}$=0.1.
$\frac{10}{100} \xrightarrow{\div 10} \frac{1}{10} = 0.1$
Oh, right.
How would we write $\frac{11}{100}$ as a decimal?
$\frac{11}{100}$
Try it.
DECIMAL NUMBERS ARE OFTEN SIMPLY CALLED "DECIMALS."

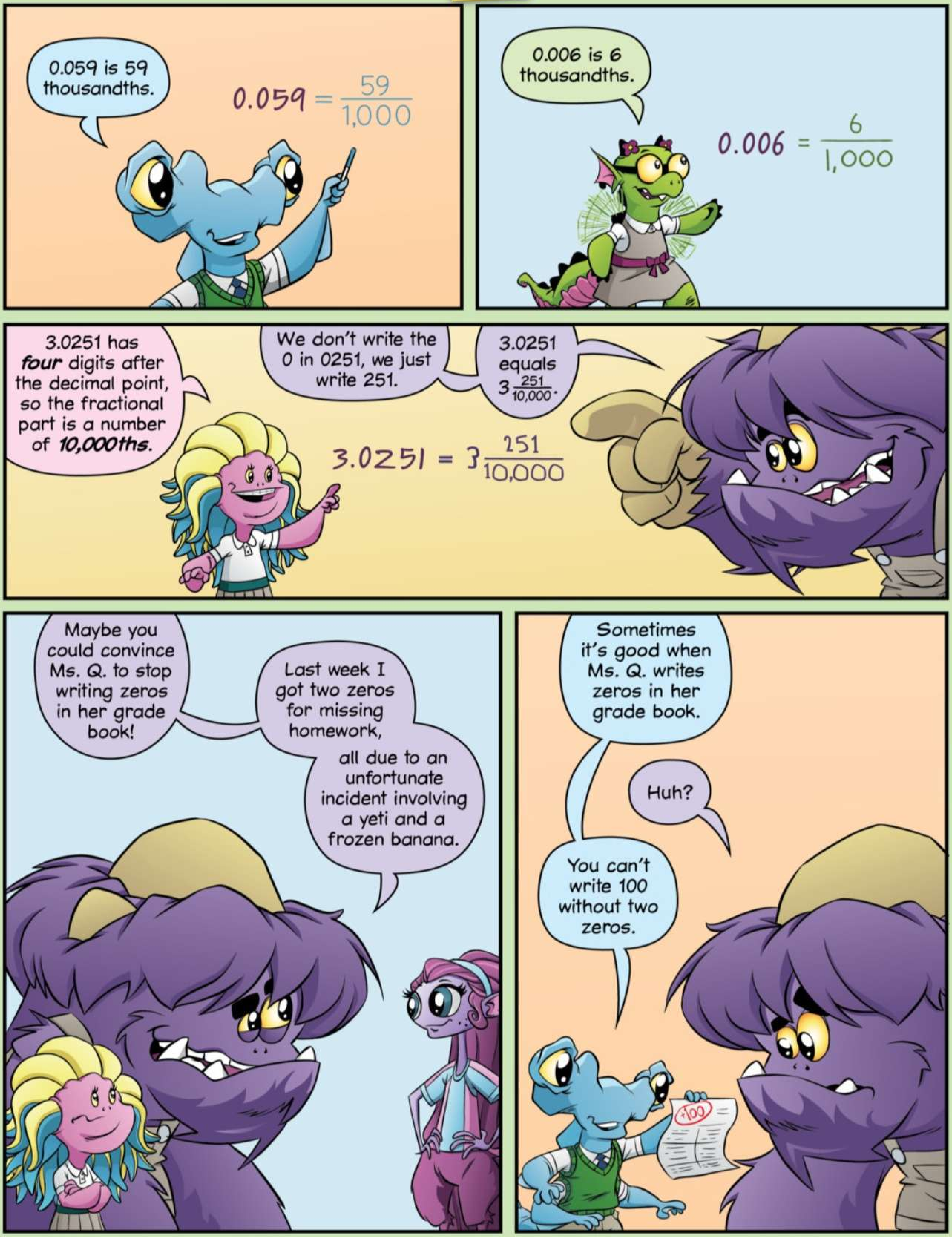


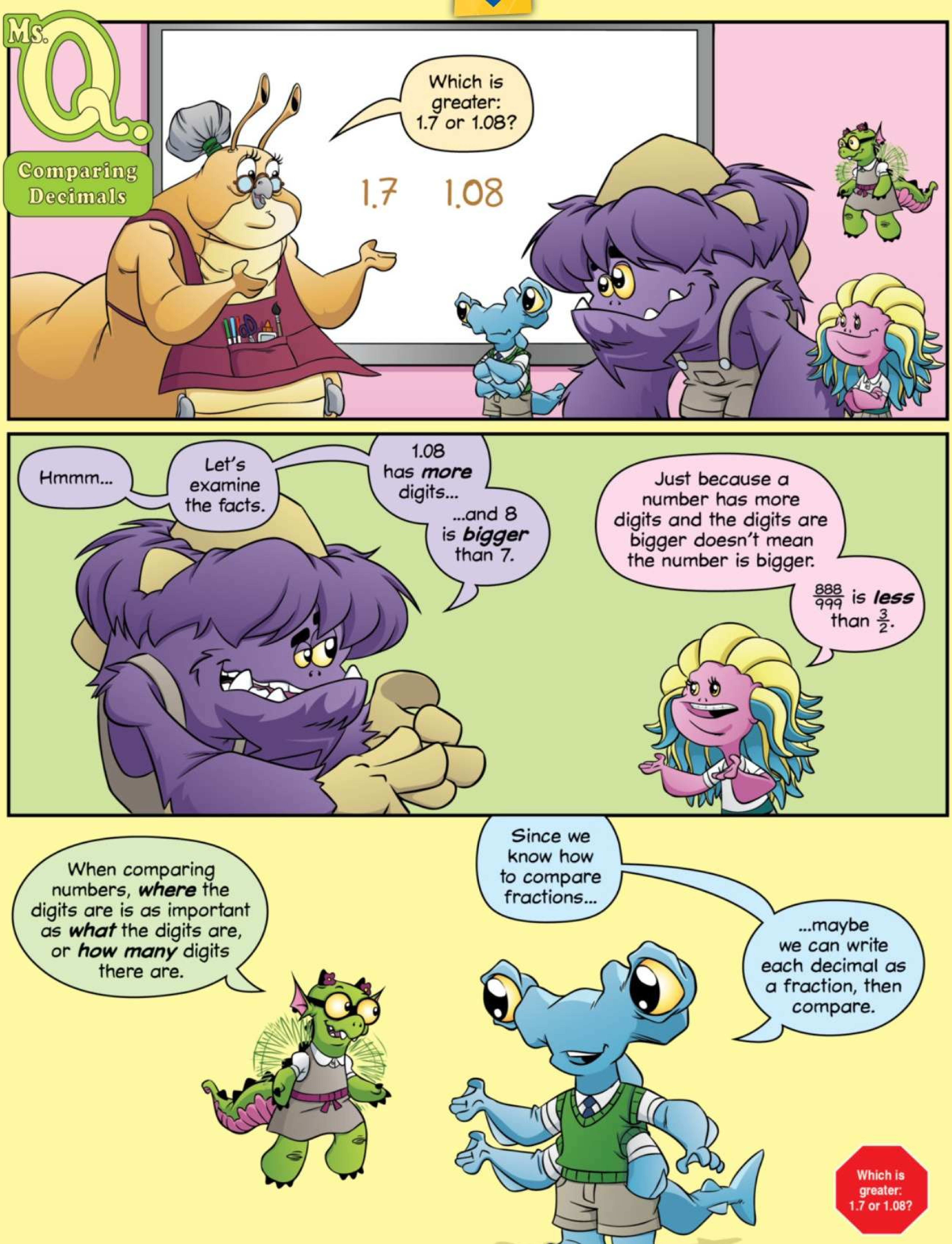
11/100 is just 1 hundredth more than 10/100.
So, 11/100 is one tenth and one hundredth.
11/100 = 10/100 + 1/100
= 1/10 + 1/100
= 0.11
To write 11/100 as a decimal, we need a 1 in the tenths place...
...and a 1 in the hundredths place.
Excellent work.
Next, try writing 99/100 as a decimal.
99/100
I know!
We can split 99/100 into 90/100 + 9/100!
90 hundredths make 9 tenths.
99/100 = 90/100 + 9/100
= 9/10 + 9/100
= 0.99
That gives us 9 tenths and 9 hundredths...
...0.99!
11/100 is 0.11, and 99/100 is 0.99.
Does that mean that the two digits after the decimal point *always* represent a number of hundredths?
Is 0.58 equal to 58/100?
0.58 = 58/100
0.37 = 37/100
And is 0.37 equal to 37/100?
Are Winnie's equations correct?

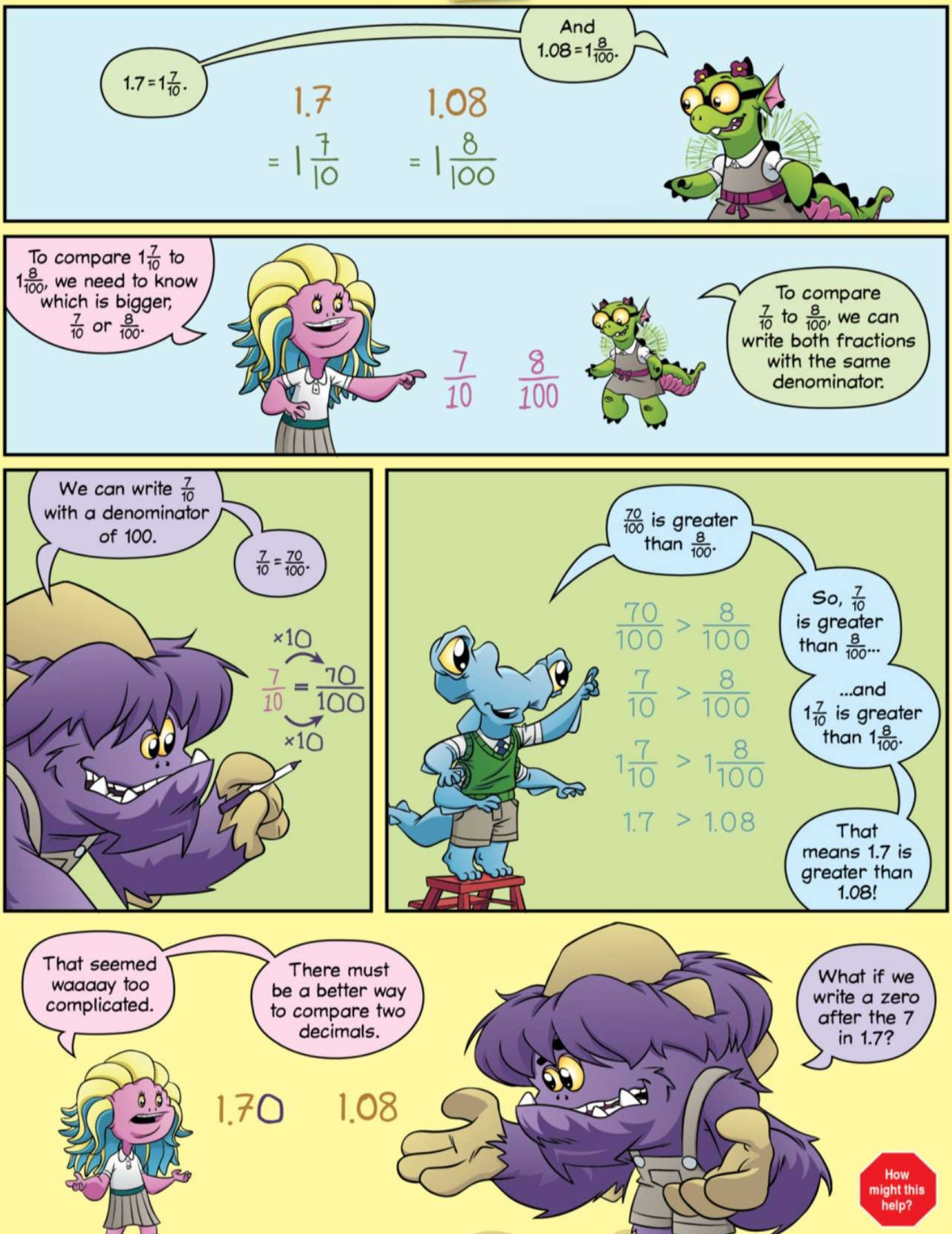

That's right, Winnie.
In fact, that's how we often read decimals.
It's simpler to say that 0.58 is "58 hundredths" than to say 0.58 is "5 tenths and 8 hundredths."
$0.37 = \frac{3}{10} + \frac{7}{100}$
$= \frac{30}{100} + \frac{7}{100}$
$= \frac{37}{100}$
$0.58 = \frac{5}{10} + \frac{8}{100}$
$= \frac{50}{100} + \frac{8}{100}$
$= \frac{58}{100}$
Try writing 0.123 as a fraction.
0.123
If a 1-digit number after the decimal point represents a number of tenths...
$0.1 = \frac{1}{10}$
$0.12 = \frac{12}{100}$
$0.123 \ ? \ \frac{123}{1{,}000}$
...and a 2-digit number after the decimal point represents a number of hundredths...
...does a 3-digit number after the decimal point represents a number of thousandths?
Let's see. 0.123 has a 1 in the tenths place...
...a 2 in the hundredths place...
...and a 3 in the thousandths place.
We can convert the tenths and hundredths to thousandths to find out whether 0.123 equals 123 thousandths.
100's 10's 1's . 1/10's 1/100's 1/1,000's 1/10,000's
1 2 3
$0.123 = \frac{1}{10} + \frac{2}{100} + \frac{3}{1{,}000}$
Is 0.123 equal to $\frac{123}{1000}$?



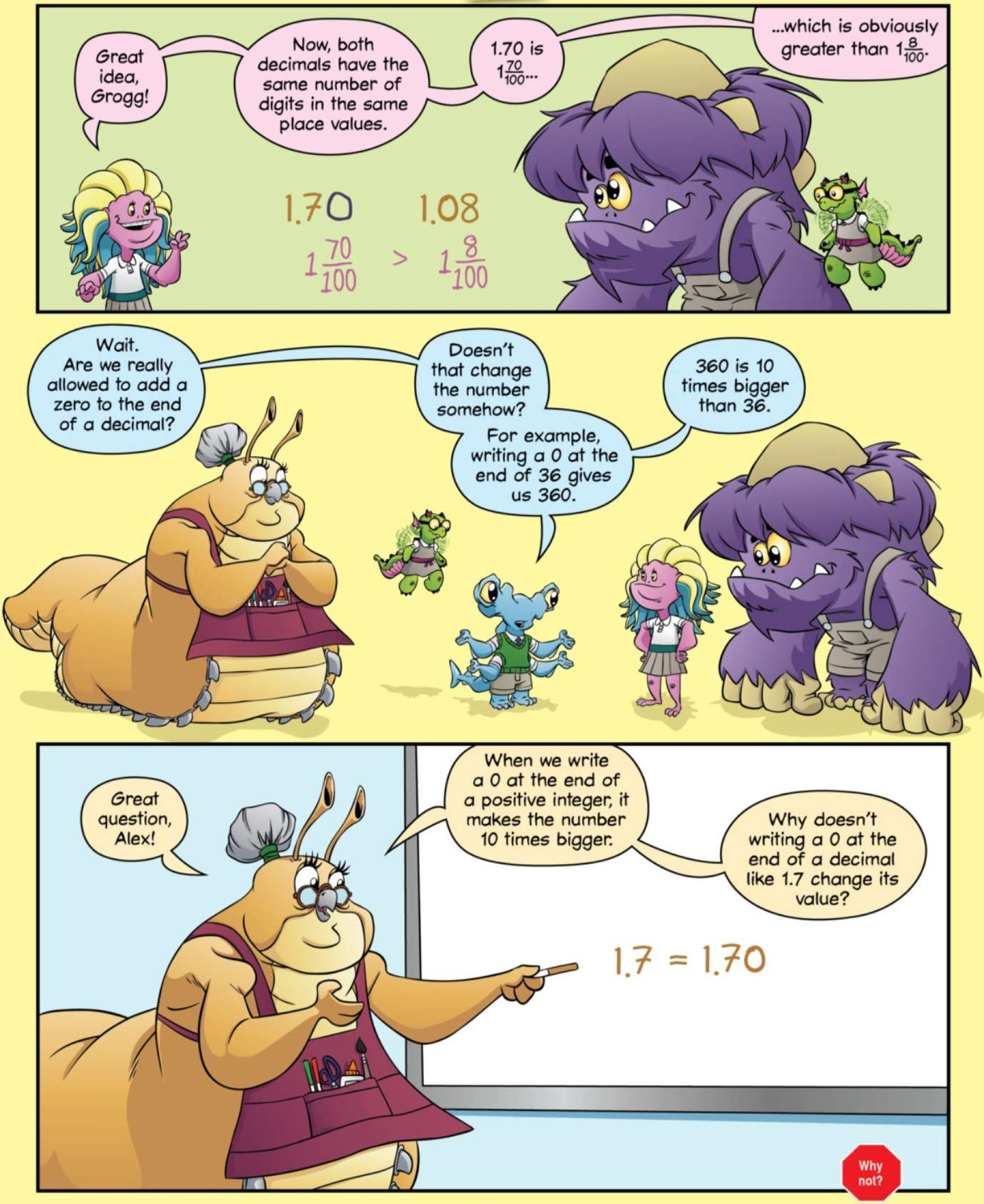
$\frac{1}{10} = \frac{100}{1,000}$.
And $\frac{2}{100} = \frac{20}{1,000}$.
$0.123 = \frac{1}{10} + \frac{2}{100} + \frac{3}{1,000}$
$= \frac{100}{1,000} + \frac{20}{1,000} + \frac{3}{1,000}$
$= \frac{123}{1,000}$
We add those to $\frac{3}{1,000}$.
All together, that's $\frac{123}{1,000}$.
So, 0.123 *is* $\frac{123}{1,000}$.
Nice job, little monsters.
A 3-digit number after the decimal point represents a number of thousandths.
$0.817 = \frac{817}{1,000}$
$0.304 = \frac{304}{1,000}$
$0.019 = \frac{19}{1,000}$
But...
...19 is a 2-digit number.
True, Grogg.
In a decimal number like 0.019, we have three digits to the right of the decimal point...
...but we don't write the zero in 019 when we write $\frac{19}{1,000}$.
$0.019 = \frac{0}{10} + \frac{1}{100} + \frac{9}{1,000}$
$= \frac{0}{1,000} + \frac{10}{1,000} + \frac{9}{1,000}$
$= \frac{19}{1,000}$
Try writing each of these three decimals as a fraction or mixed number.
0.059
0.006
3.0251
Try all three.

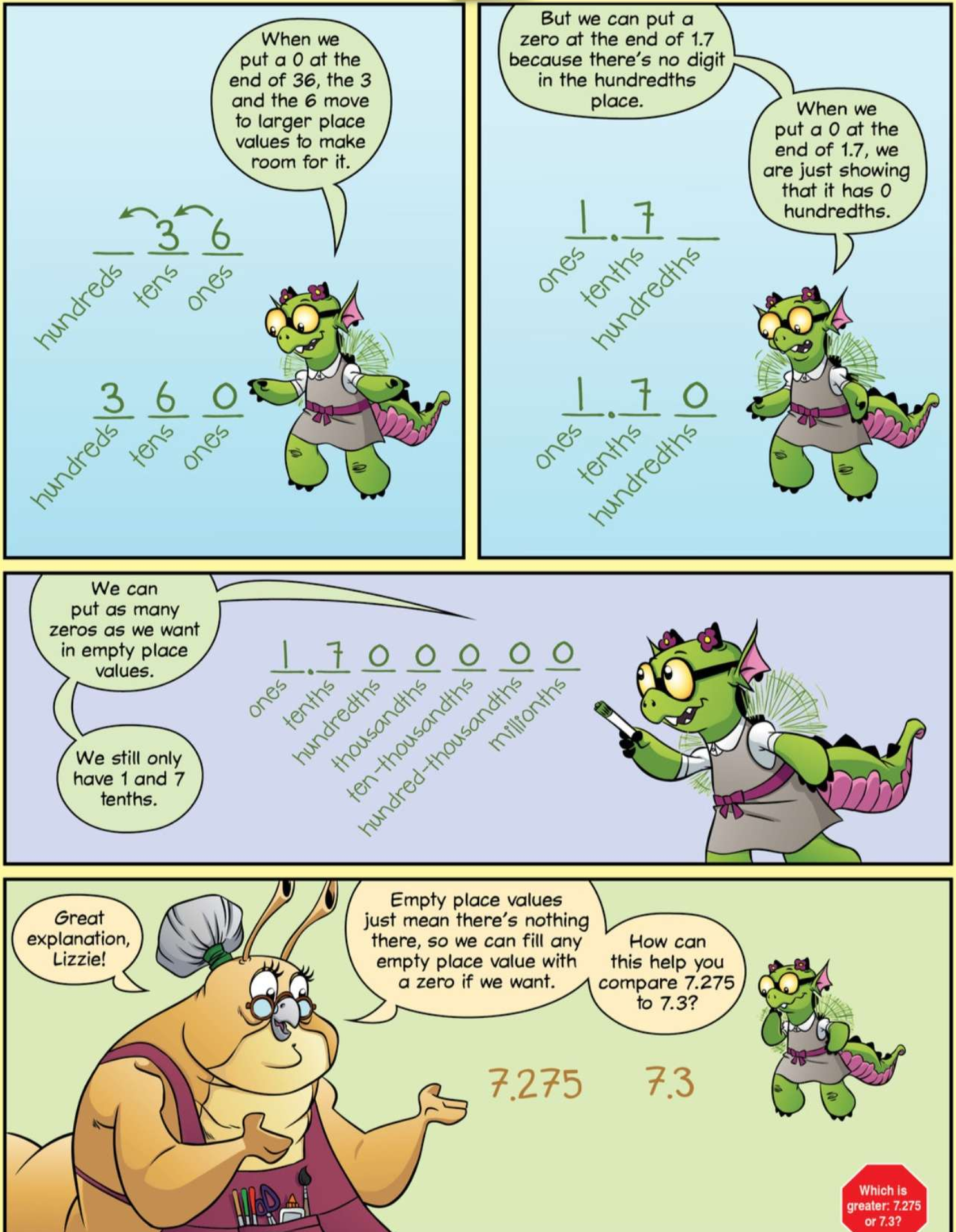

0.059 is 59 thousandths.
$0.059 = \frac{59}{1{,}000}$
0.006 is 6 thousandths.
$0.006 = \frac{6}{1{,}000}$
3.0251 has **four** digits after the decimal point, so the fractional part is a number of ***10,000ths***.
We don't write the 0 in 0251, we just write 251.
3.0251 equals $3\frac{251}{10{,}000}$.
$3.0251 = 3\frac{251}{10{,}000}$
Maybe you could convince Ms. Q. to stop writing zeros in her grade book!
Last week I got two zeros for missing homework,
all due to an unfortunate incident involving a yeti and a frozen banana.
Sometimes it's good when Ms. Q. writes zeros in her grade book.
Huh?
You can't write 100 without two zeros.
+100

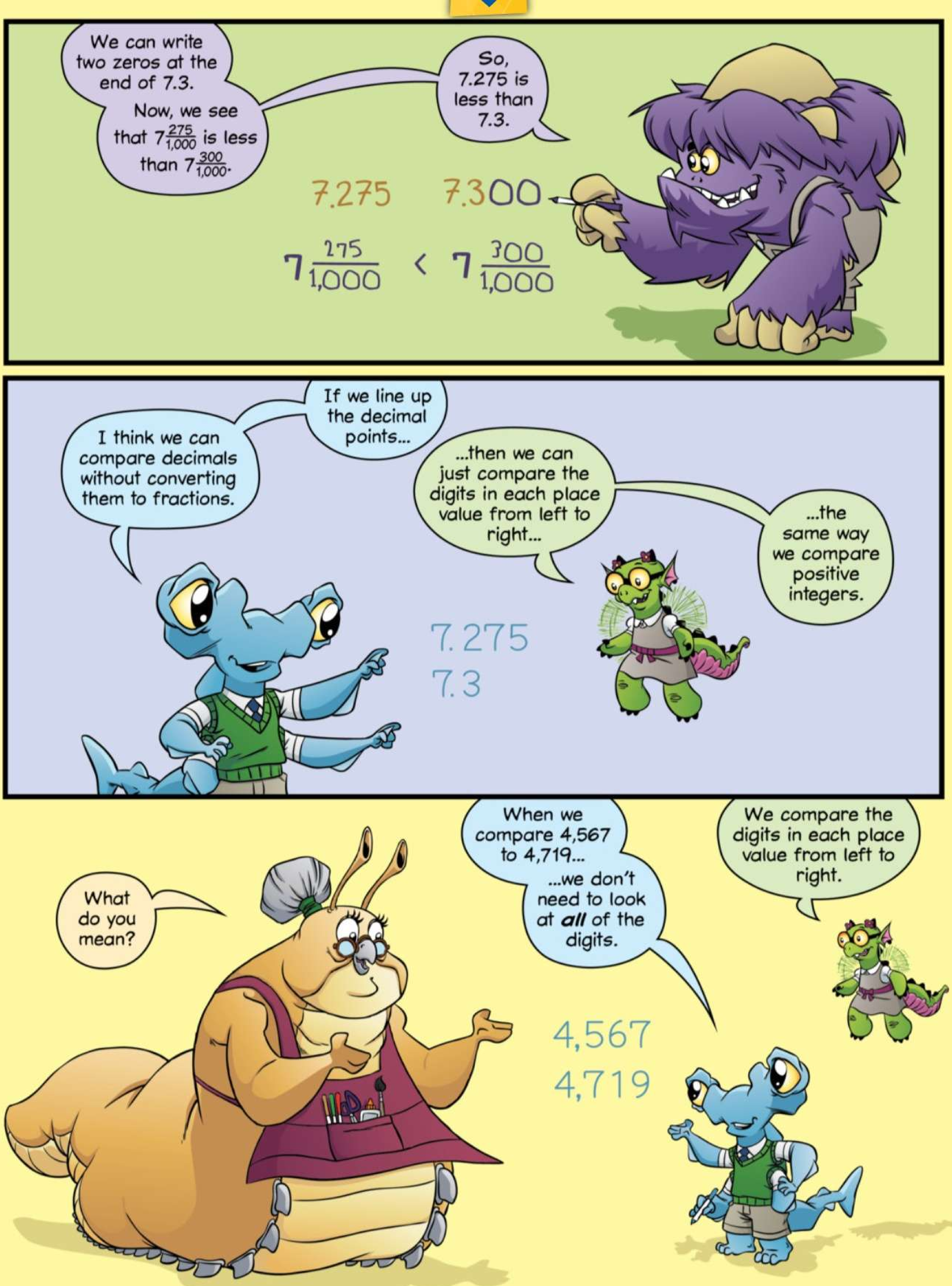

Ms. Q.
Comparing Decimals
Which is greater: 1.7 or 1.08?
1.7 1.08
Hmmm...
Let's examine the facts.
1.08 has ***more*** digits...
...and 8 is ***bigger*** than 7.
Just because a number has more digits and the digits are bigger doesn't mean the number is bigger.
$\frac{888}{999}$ is ***less*** than $\frac{3}{2}$.
When comparing numbers, ***where*** the digits are is as important as ***what*** the digits are, or ***how many*** digits there are.
Since we know how to compare fractions...
...maybe we can write each decimal as a fraction, then compare.
Which is greater: 1.7 or 1.08?



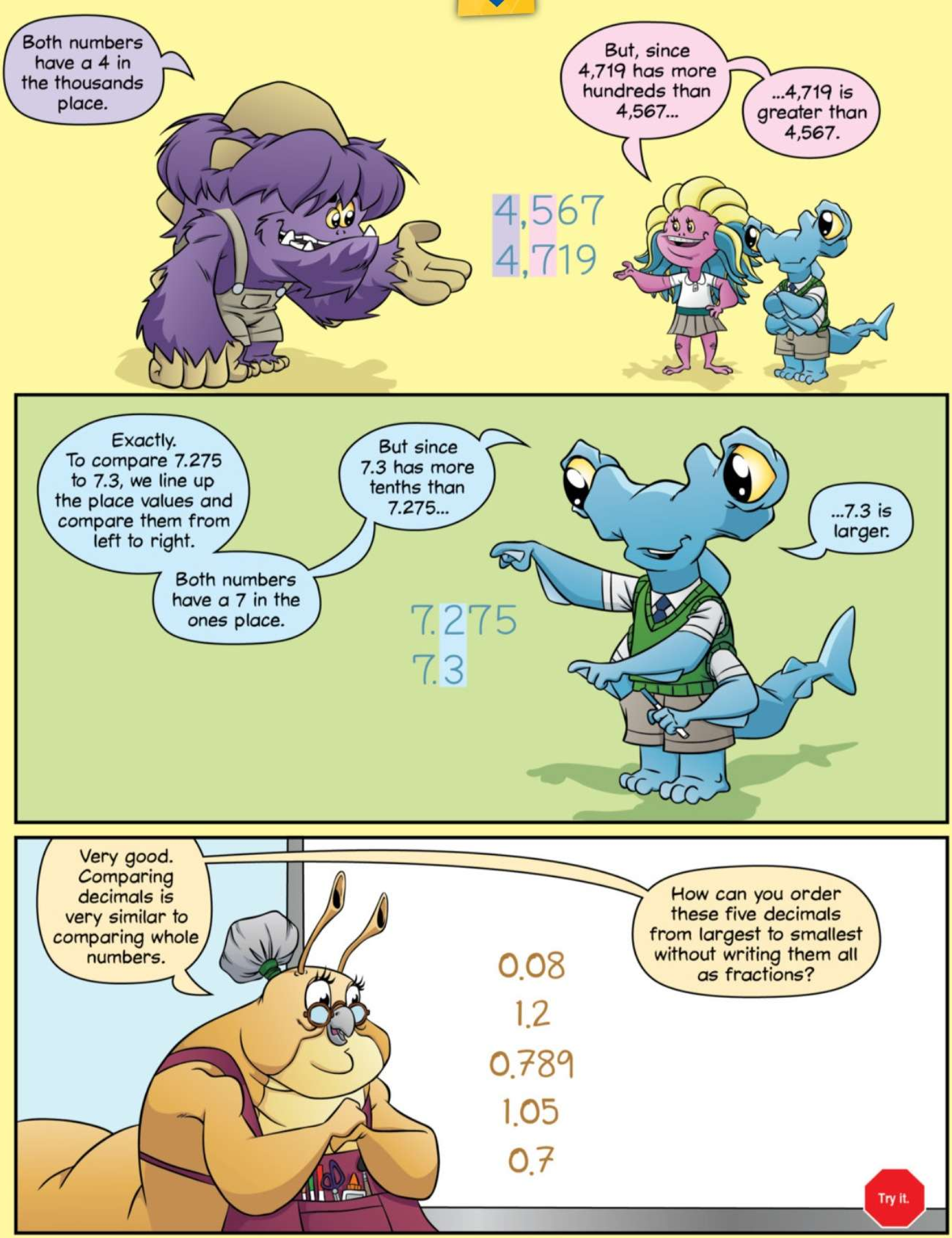
1.7 = 1 7/10.
And 1.08 = 1 8/100.
1.7 = 1 7/10
1.08 = 1 8/100
To compare 1 7/10 to 1 8/100, we need to know which is bigger, 7/10 or 8/100.
7/10 8/100
To compare 7/10 to 8/100, we can write both fractions with the same denominator.
We can write 7/10 with a denominator of 100.
7/10 = 70/100.
×10
7/10 = 70/100
×10
70/100 is greater than 8/100.
70/100 > 8/100
7/10 > 8/100
1 7/10 > 1 8/100
1.7 > 1.08
So, 7/10 is greater than 8/100...
...and 1 7/10 is greater than 1 8/100.
That means 1.7 is greater than 1.08!
That seemed waaaay too complicated.
There must be a better way to compare two decimals.
1.70 1.08
What if we write a zero after the 7 in 1.7?
How might this help?


Great idea, Grogg!
Now, both decimals have the same number of digits in the same place values.
1.70 is $1\frac{70}{100}$...
...which is obviously greater than $1\frac{8}{100}$.
1.70 1.08
$1\frac{70}{100} > 1\frac{8}{100}$
Wait. Are we really allowed to add a zero to the end of a decimal?
Doesn't that change the number somehow?
For example, writing a 0 at the end of 36 gives us 360.
360 is 10 times bigger than 36.
Great question, Alex!
When we write a 0 at the end of a positive integer, it makes the number 10 times bigger.
Why doesn't writing a 0 at the end of a decimal like 1.7 change its value?
1.7 = 1.70
Why not?

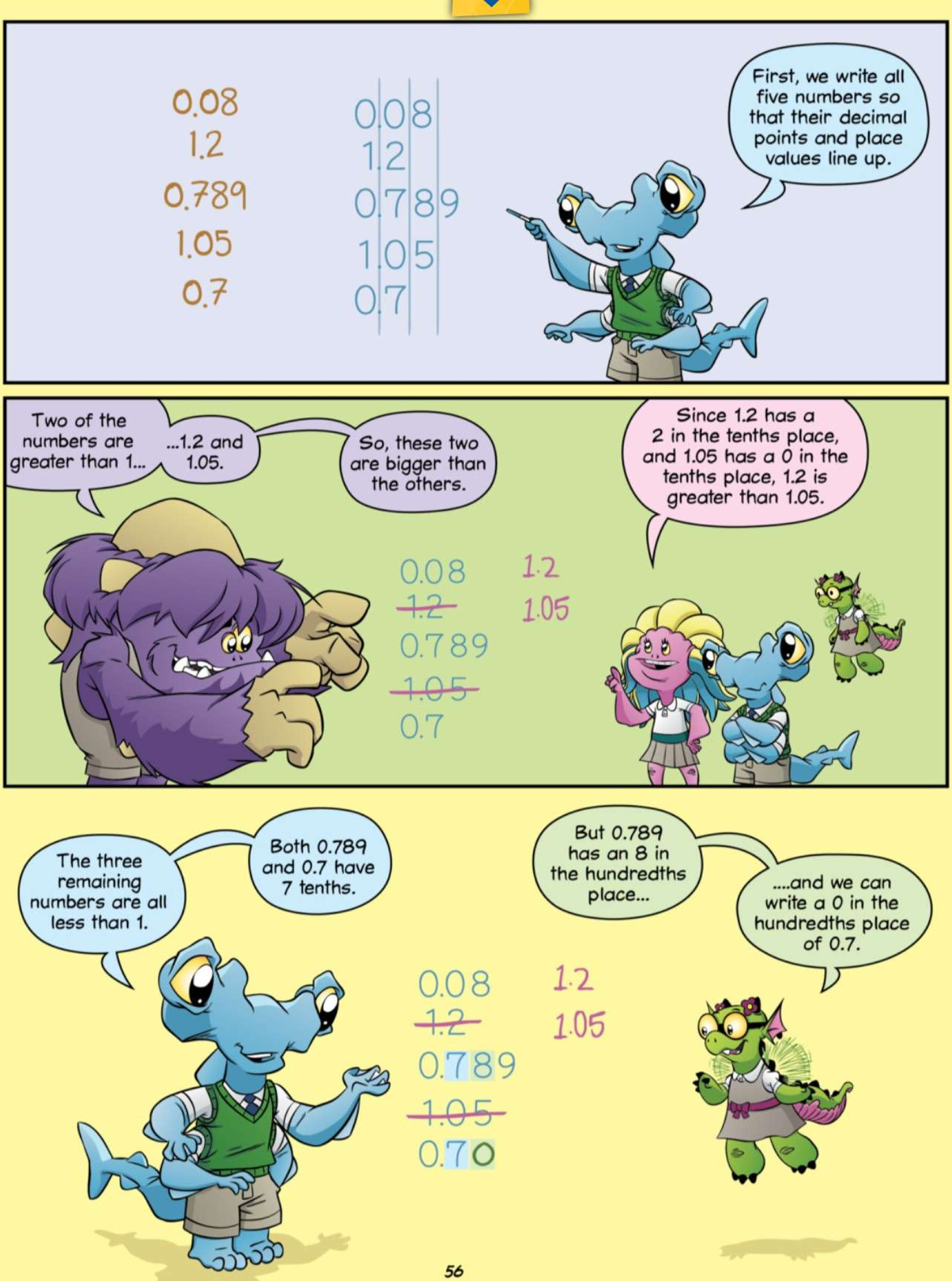

When we put a 0 at the end of 36, the 3 and the 6 move to larger place values to make room for it.
_ 3 6
hundreds tens ones
3 6 0
hundreds tens ones
But we can put a zero at the end of 1.7 because there's no digit in the hundredths place.
When we put a 0 at the end of 1.7, we are just showing that it has 0 hundredths.
1.7 _
ones tenths hundredths
1.7 0
ones tenths hundredths
We can put as many zeros as we want in empty place values.
We still only have 1 and 7 tenths.
1.7 0 0 0 0 0
ones tenths hundredths thousandths ten-thousandths hundred-thousandths millionths
Great explanation, Lizzie!
Empty place values just mean there's nothing there, so we can fill any empty place value with a zero if we want.
How can this help you compare 7.275 to 7.3?
7.275
7.3
Which is greater: 7.275 or 7.3?

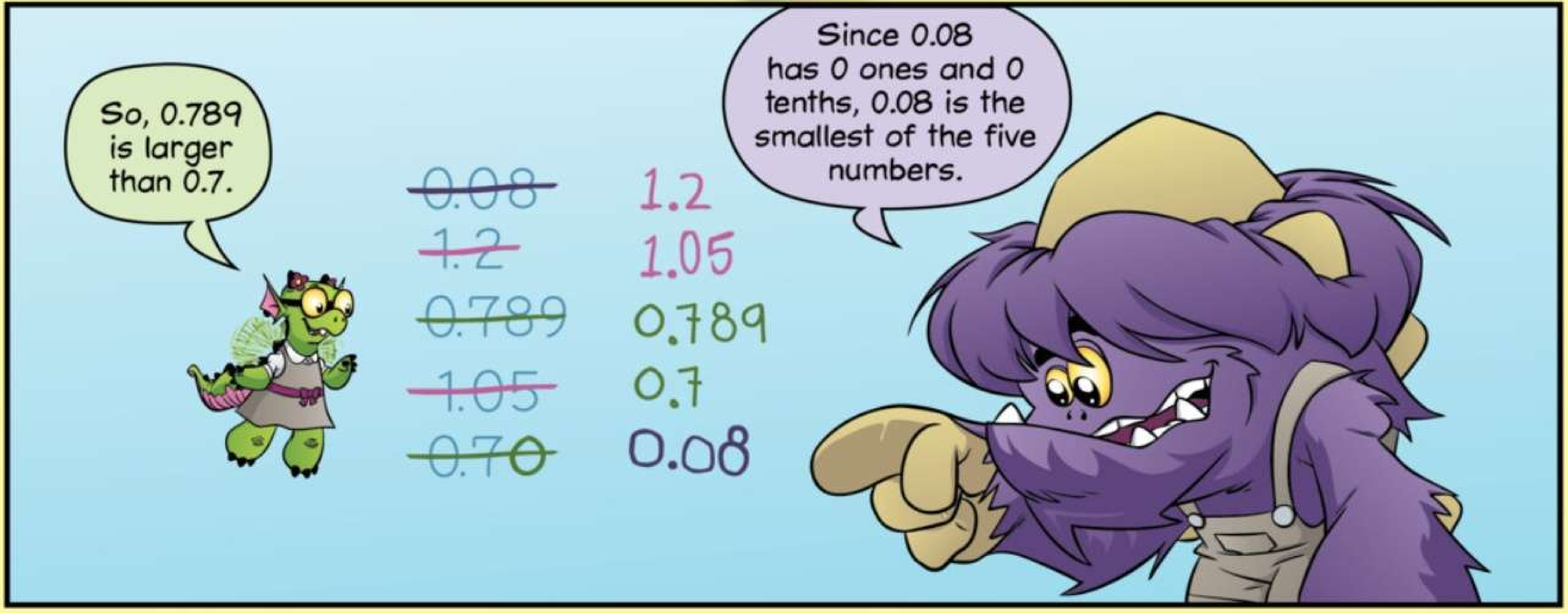


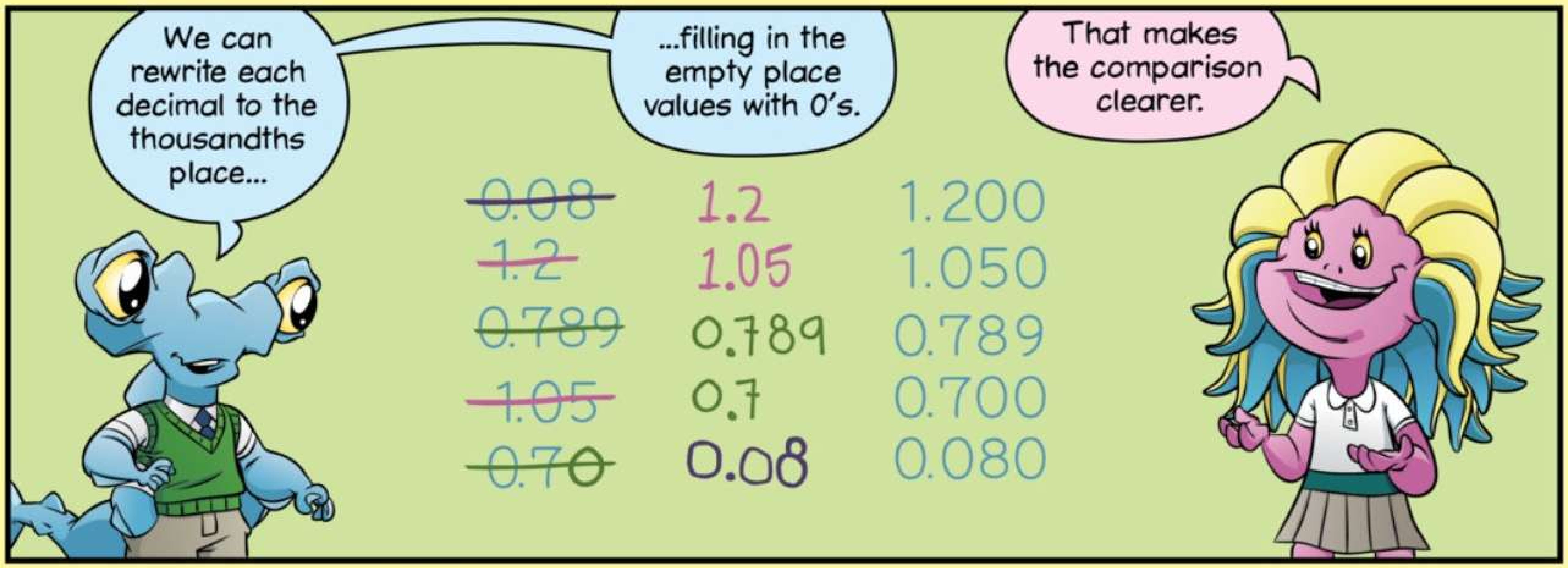
We can write two zeros at the end of 7.3.
Now, we see that $7\frac{275}{1,000}$ is less than $7\frac{300}{1,000}$.
So, 7.275 is less than 7.3.
7.275 7.300
$7\frac{275}{1,000} < 7\frac{300}{1,000}$
I think we can compare decimals without converting them to fractions.
If we line up the decimal points...
...then we can just compare the digits in each place value from left to right...
...the same way we compare positive integers.
7.275
7.3
What do you mean?
When we compare 4,567 to 4,719...
...we don't need to look at ***all*** of the digits.
We compare the digits in each place value from left to right.
4,567
4,719

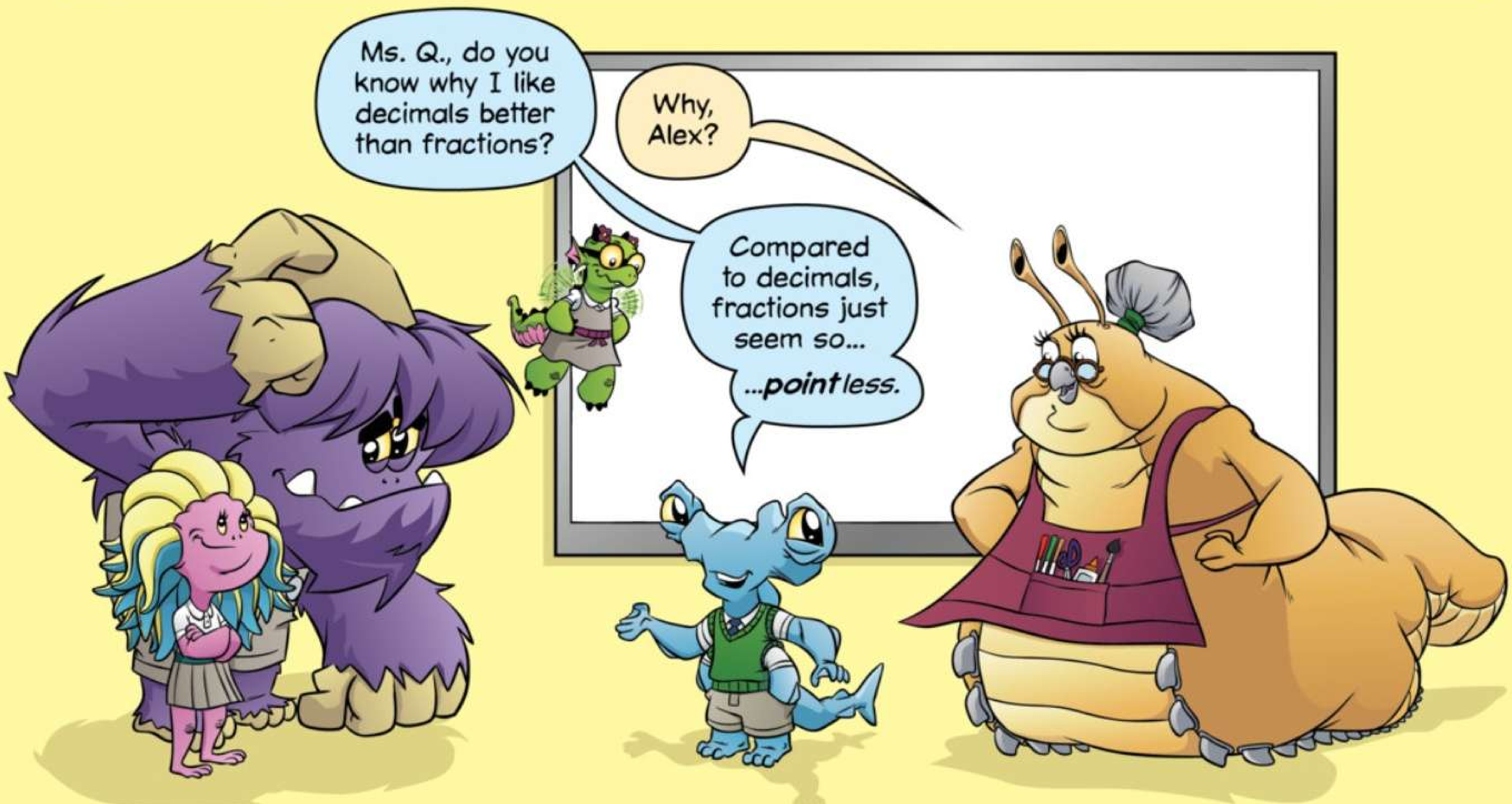


Both numbers have a 4 in the thousands place.
But, since 4,719 has more hundreds than 4,567...
...4,719 is greater than 4,567.
4,567
4,719
Exactly. To compare 7.275 to 7.3, we line up the place values and compare them from left to right.
Both numbers have a 7 in the ones place.
But since 7.3 has more tenths than 7.275...
...7.3 is larger.
7.275
7.3
Very good. Comparing decimals is very similar to comparing whole numbers.
How can you order these five decimals from largest to smallest without writing them all as fractions?
0.08
1.2
0.789
1.05
0.7
Try it.


0.08
1.2
0.789
1.05
0.7
0.08
1.2
0.789
1.05
0.7
First, we write all five numbers so that their decimal points and place values line up.
Two of the numbers are greater than 1...
...1.2 and 1.05.
So, these two are bigger than the others.
Since 1.2 has a 2 in the tenths place, and 1.05 has a 0 in the tenths place, 1.2 is greater than 1.05.
0.08
~~1.2~~
0.789
~~1.05~~
0.7
1.2
1.05
The three remaining numbers are all less than 1.
Both 0.789 and 0.7 have 7 tenths.
But 0.789 has an 8 in the hundredths place...
....and we can write a 0 in the hundredths place of 0.7.
0.08
~~1.2~~
0.789
~~1.05~~
0.70
1.2
1.05

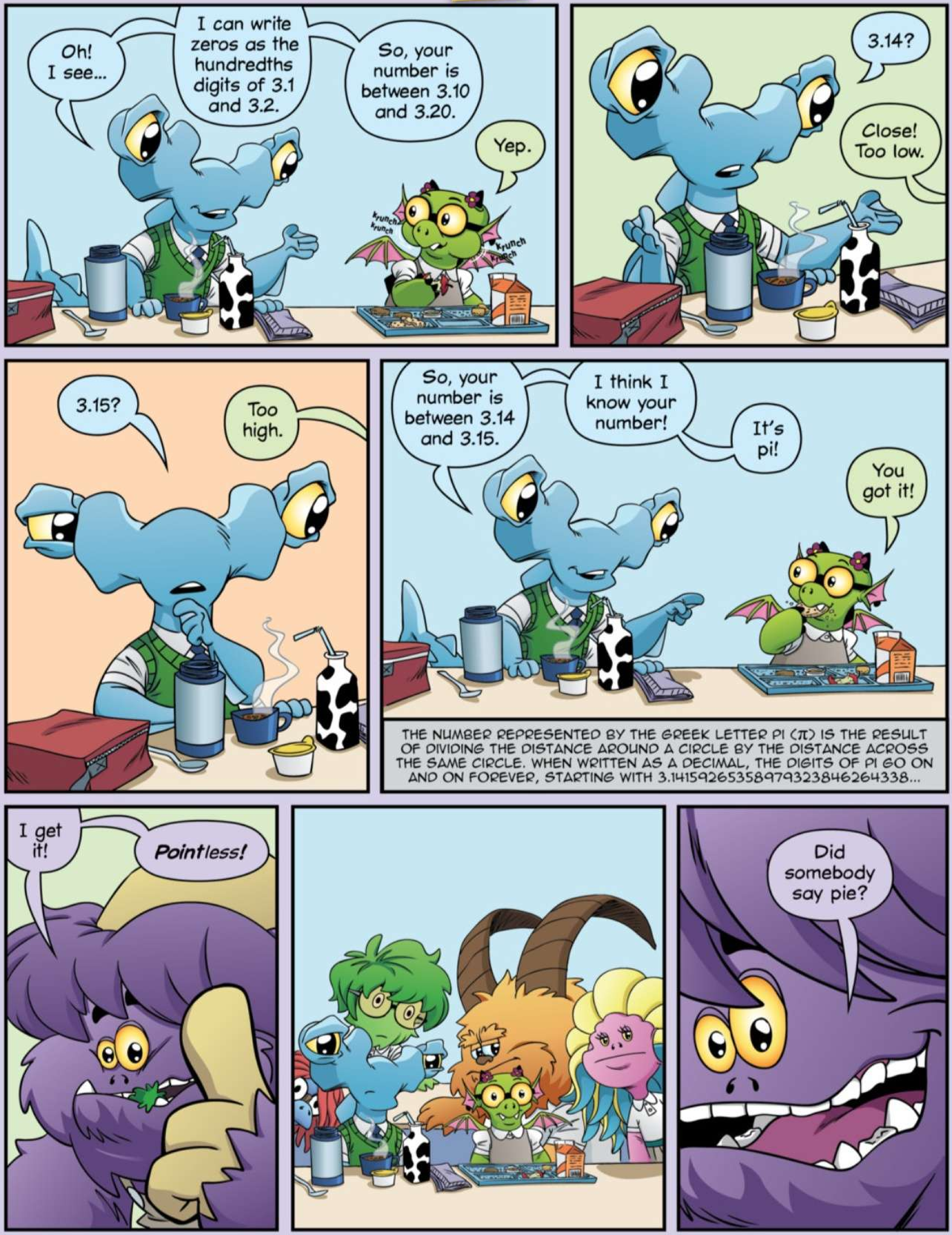

So, 0.789 is larger than 0.7.
~~0.08~~
~~1.2~~
~~0.789~~
~~1.05~~
~~0.70~~
1.2
1.05
0.789
0.7
0.08
Since 0.08 has 0 ones and 0 tenths, 0.08 is the smallest of the five numbers.


We can rewrite each decimal to the thousandths place...
...filling in the empty place values with 0's.
That makes the comparison clearer.
~~0.08~~
~~1.2~~
~~0.789~~
~~1.05~~
~~0.70~~
1.2
1.05
0.789
0.7
0.08
1.200
1.050
0.789
0.700
0.080

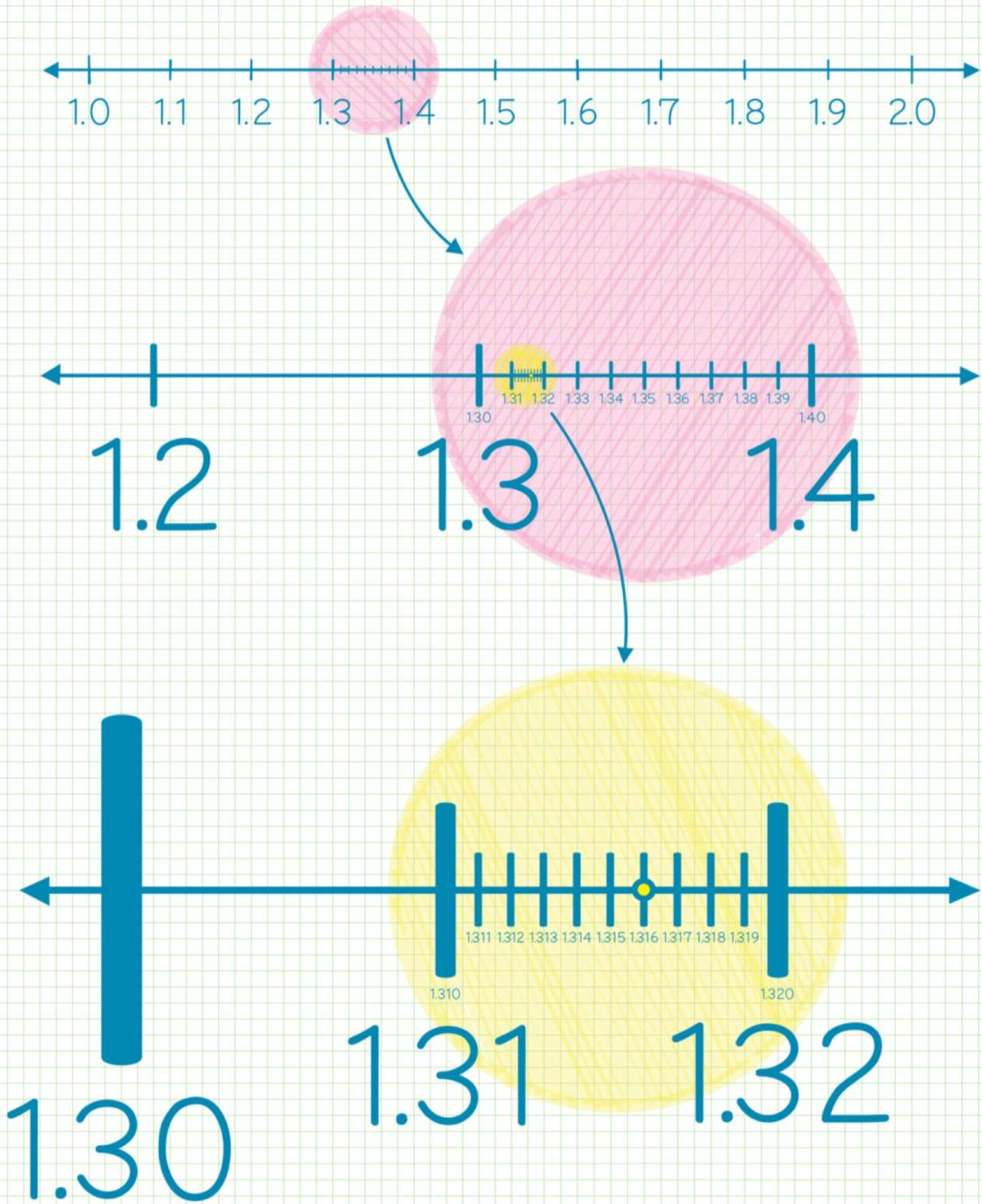

Ms. Q., do you know why I like decimals better than fractions?
Why, Alex?
Compared to decimals, fractions just seem so...
...*point*less.

Lunch
Want to play a guessing game?
Sure.
I'll think of a number, and you try to guess it in as few tries as--
10.
Whoa.
How did you...?
Why don't I pick a number, and you try to guess it.
Okay.
I'll give you a hint.
It's between 3 and 4.
Huh?
munch munch
munch munch munch
It's a decimal!
Oh, right. Let's see...
Name a decimal that is between 3 and 4.


3.3?
Nope, lower.
3.2?
Lower.
3.1?
Higher.
Higher!?
It's *higher* than 3.1, but *lower* than 3.2?
So, your number is ***between*** 3.1 and 3.2.
That's right.
Yep.
How?
My decimal has a hundredths digit.
Can you name a number that is between 3.1 and 3.2?



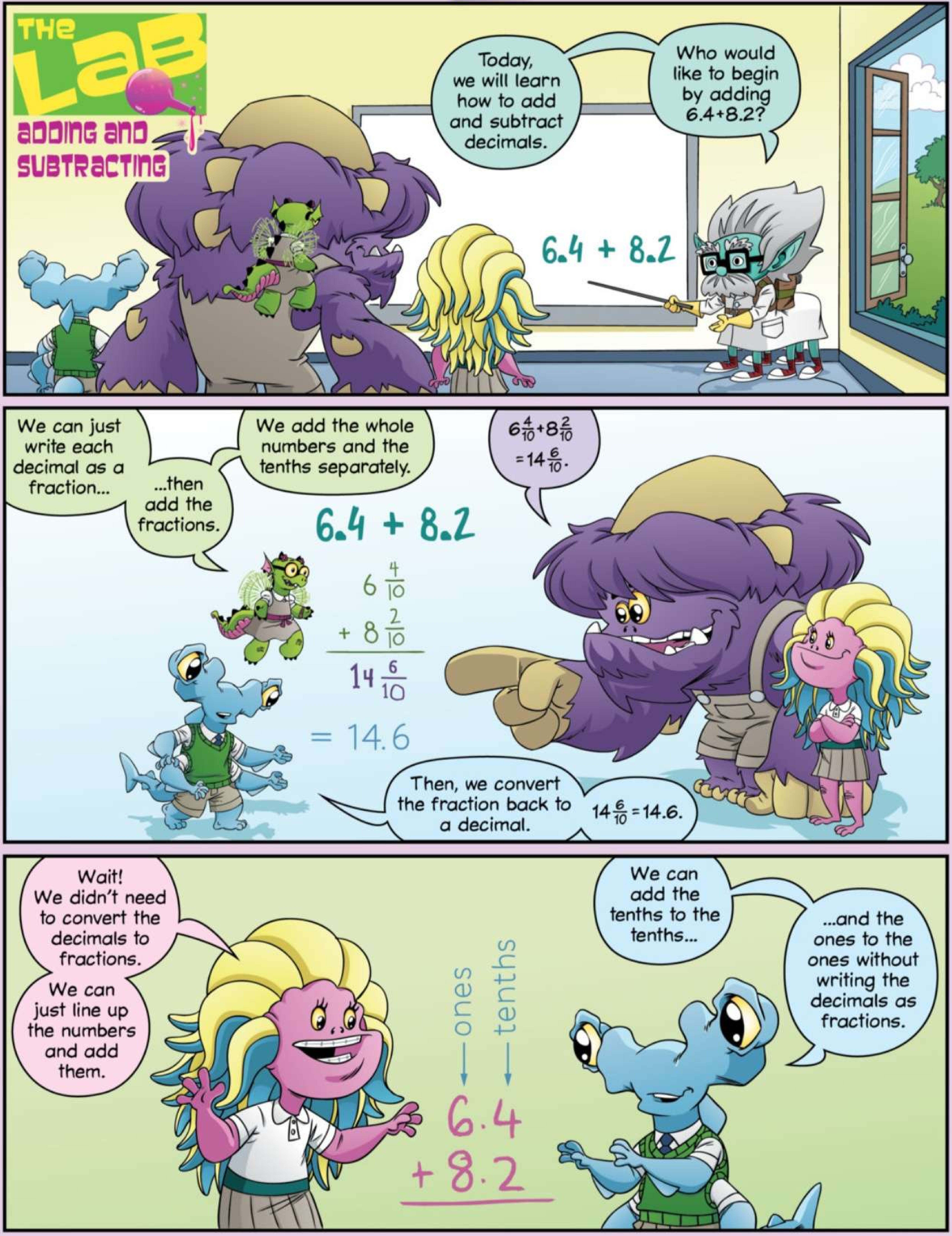
Oh! I see...
I can write zeros as the hundredths digits of 3.1 and 3.2.
So, your number is between 3.10 and 3.20.
Yep.
Krunch krunch
Krunch krunch
3.14?
Close! Too low.
3.15?
Too high.
So, your number is between 3.14 and 3.15.
I think I know your number!
It's pi!
You got it!
THE NUMBER REPRESENTED BY THE GREEK LETTER PI (π) IS THE RESULT OF DIVIDING THE DISTANCE AROUND A CIRCLE BY THE DISTANCE ACROSS THE SAME CIRCLE. WHEN WRITTEN AS A DECIMAL, THE DIGITS OF PI GO ON AND ON FOREVER, STARTING WITH 3.14159265358979323846264338...
I get it!
*Point*less!
Did somebody say pie?

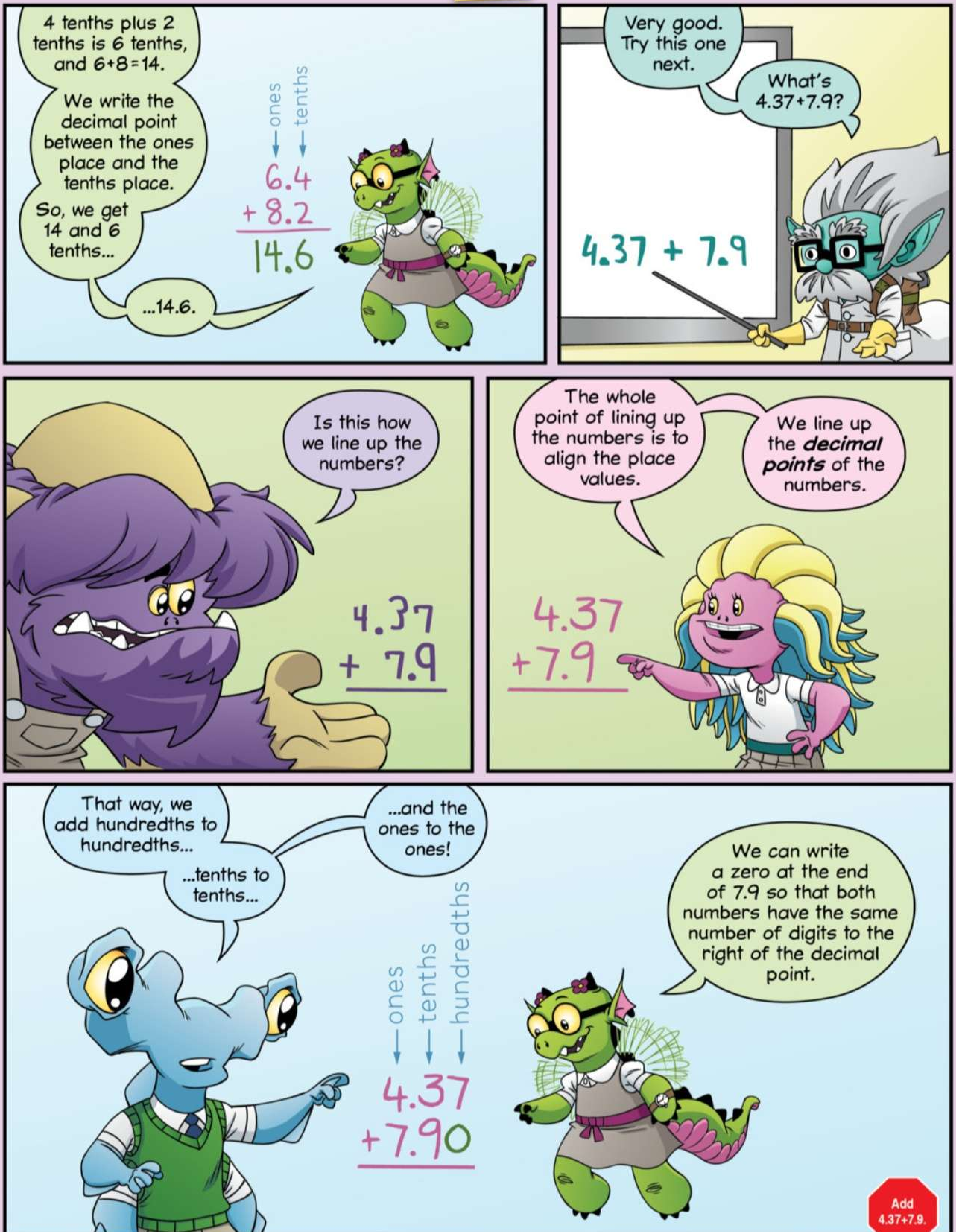


TITLE Location of 1.316

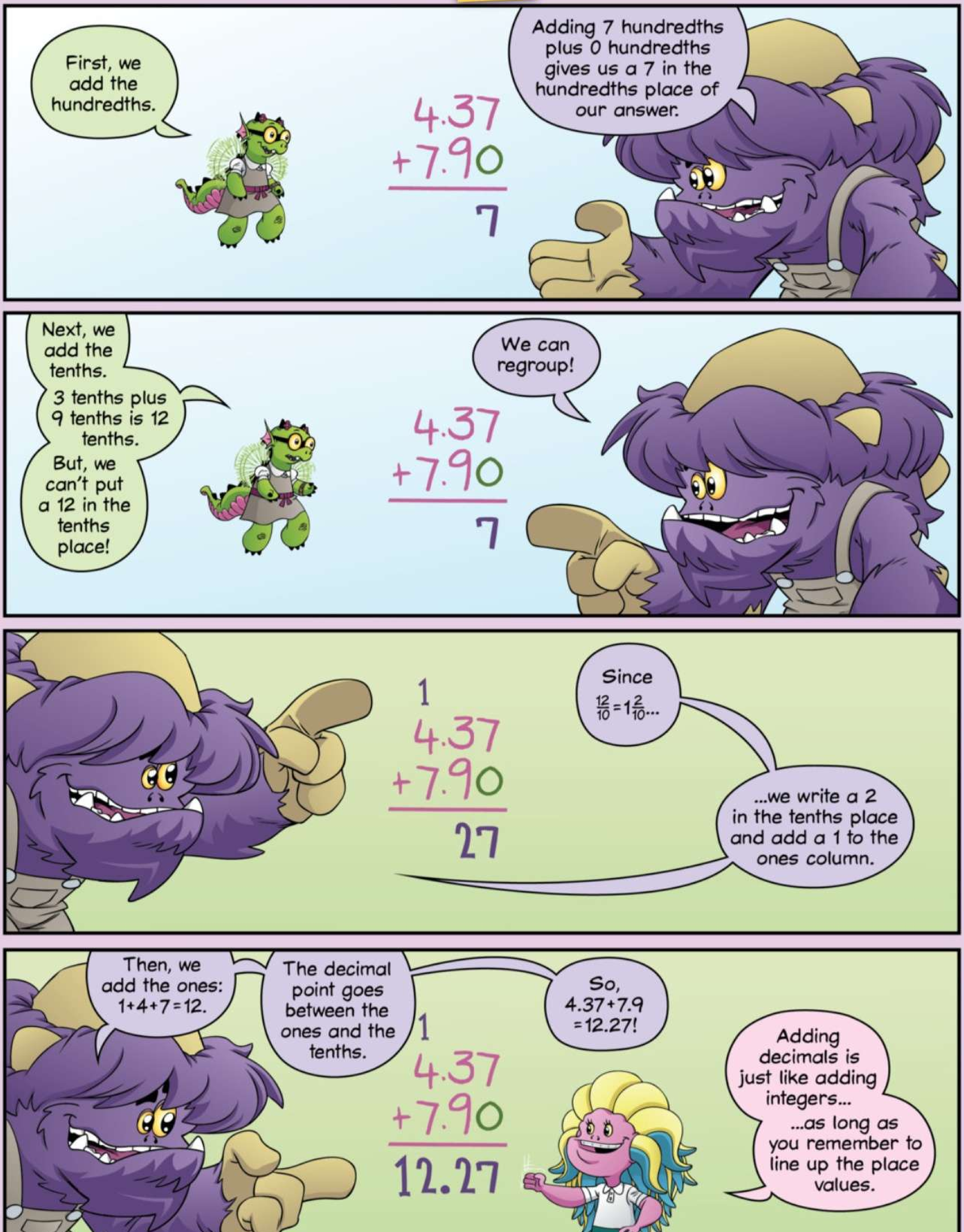
NAME Alex

DATE 3/14

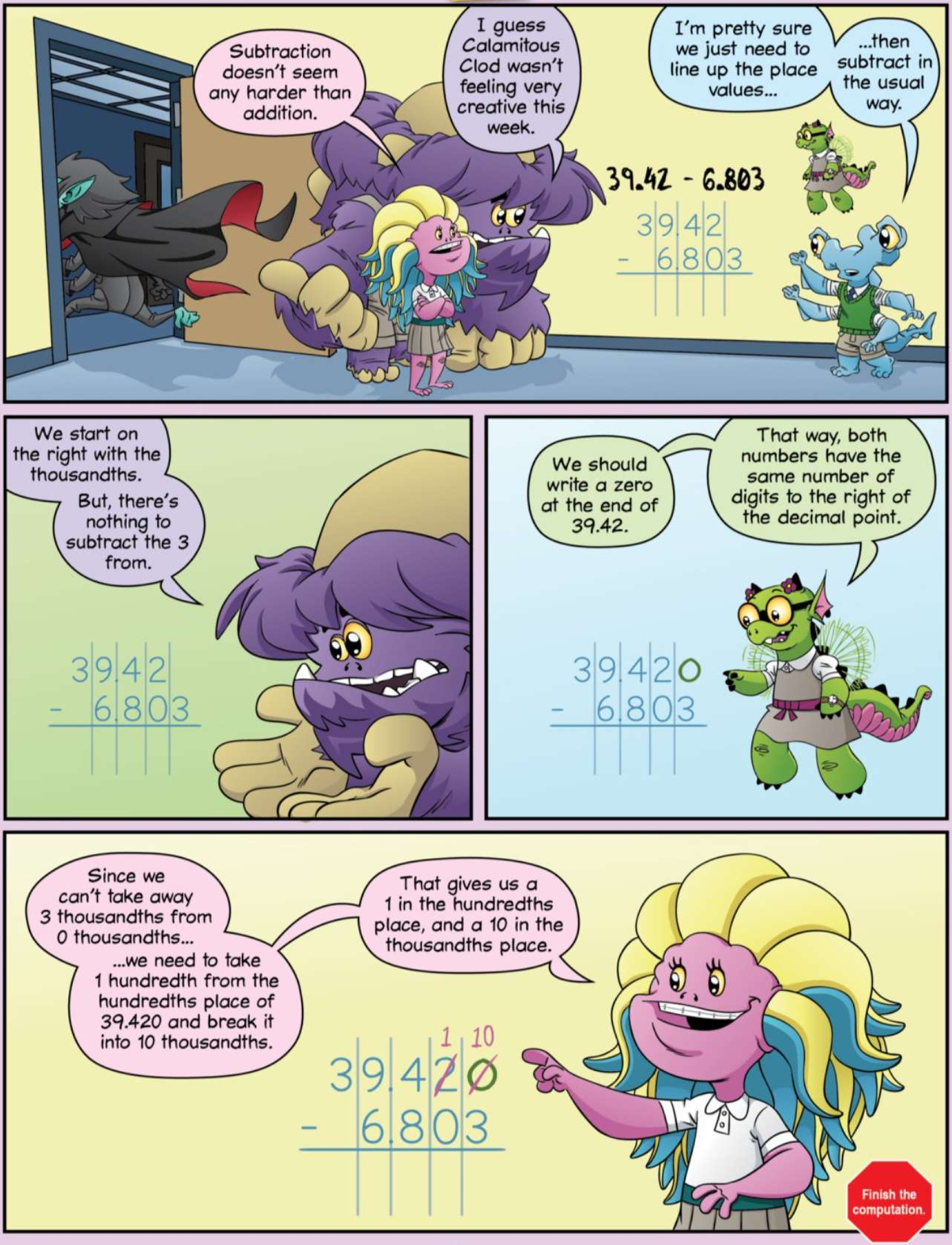

THE LAB
ADDING AND SUBTRACTING
Today, we will learn how to add and subtract decimals.
Who would like to begin by adding 6.4+8.2?
6.4 + 8.2
We can just write each decimal as a fraction...
...then add the fractions.
We add the whole numbers and the tenths separately.
$6\frac{4}{10}+8\frac{2}{10}=14\frac{6}{10}$.
6.4 + 8.2
$6\frac{4}{10}$
$+\ 8\frac{2}{10}$
$14\frac{6}{10}$
= 14.6
Then, we convert the fraction back to a decimal.
$14\frac{6}{10}=14.6$.
Wait! We didn't need to convert the decimals to fractions.
We can just line up the numbers and add them.
We can add the tenths to the tenths...
...and the ones to the ones without writing the decimals as fractions.
ones
tenths
6.4
+ 8.2

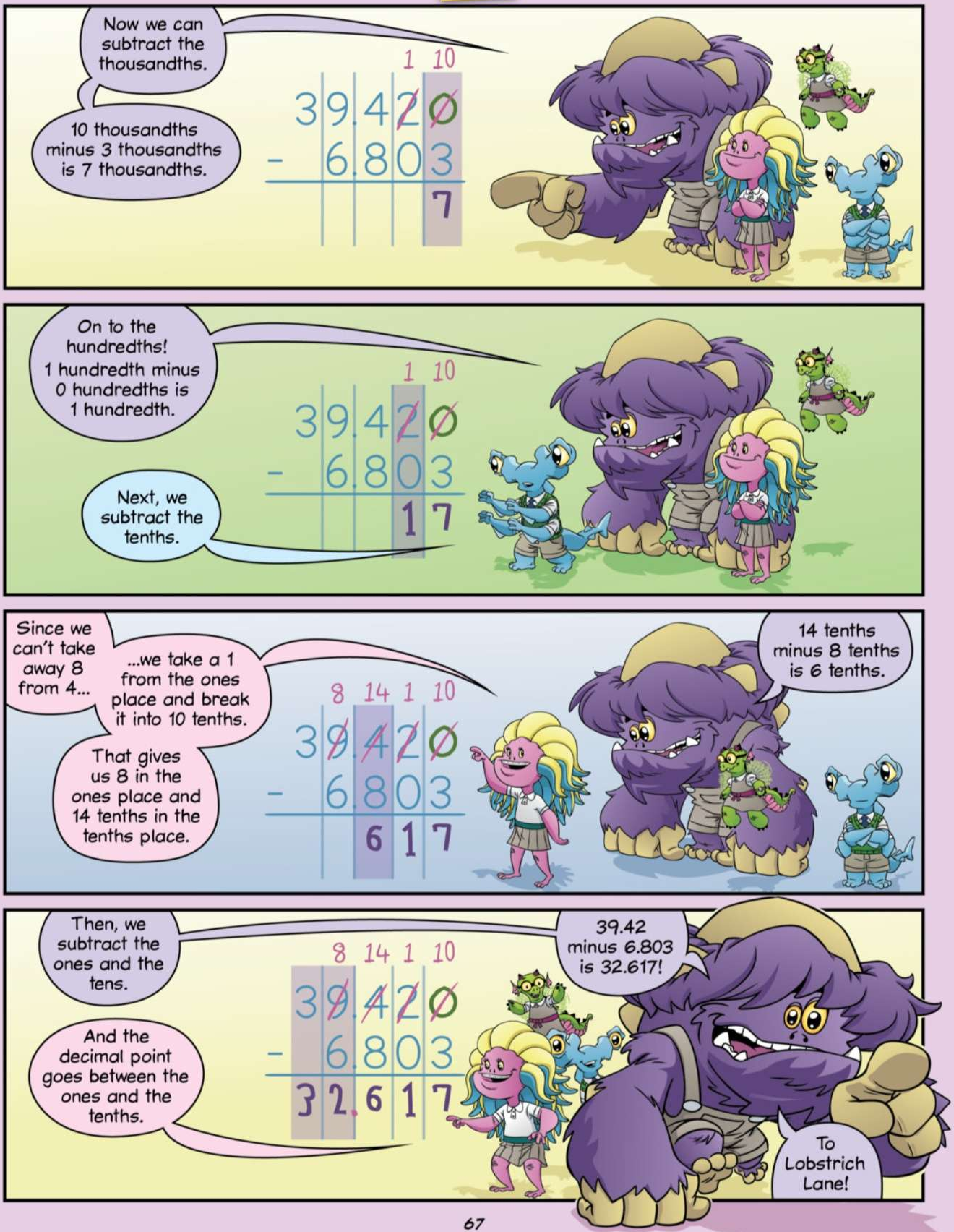


4 tenths plus 2 tenths is 6 tenths, and 6+8=14.
We write the decimal point between the ones place and the tenths place.
So, we get 14 and 6 tenths...
...14.6.
ones
tenths
6.4
+ 8.2
14.6
Very good. Try this one next.
What's 4.37+7.9?
4.37 + 7.9
Is this how we line up the numbers?
4.37
+ 7.9
The whole point of lining up the numbers is to align the place values.
We line up the ***decimal points*** of the numbers.
4.37
+7.9
That way, we add hundredths to hundredths...
...tenths to tenths...
...and the ones to the ones!
We can write a zero at the end of 7.9 so that both numbers have the same number of digits to the right of the decimal point.
ones
tenths
hundredths
4.37
+7.90
Add 4.37+7.9.


First, we add the hundredths.
Adding 7 hundredths plus 0 hundredths gives us a 7 in the hundredths place of our answer.
4.37
+7.90
7
Next, we add the tenths.
3 tenths plus 9 tenths is 12 tenths.
But, we can't put a 12 in the tenths place!
We can regroup!
4.37
+7.90
7
1
4.37
+7.90
27
Since $\frac{12}{10} = 1\frac{2}{10}$...
...we write a 2 in the tenths place and add a 1 to the ones column.
Then, we add the ones: 1+4+7=12.
The decimal point goes between the ones and the tenths.
So, 4.37+7.9 =12.27!
1
4.37
+7.90
12.27
Adding decimals is just like adding integers...
...as long as you remember to line up the place values.

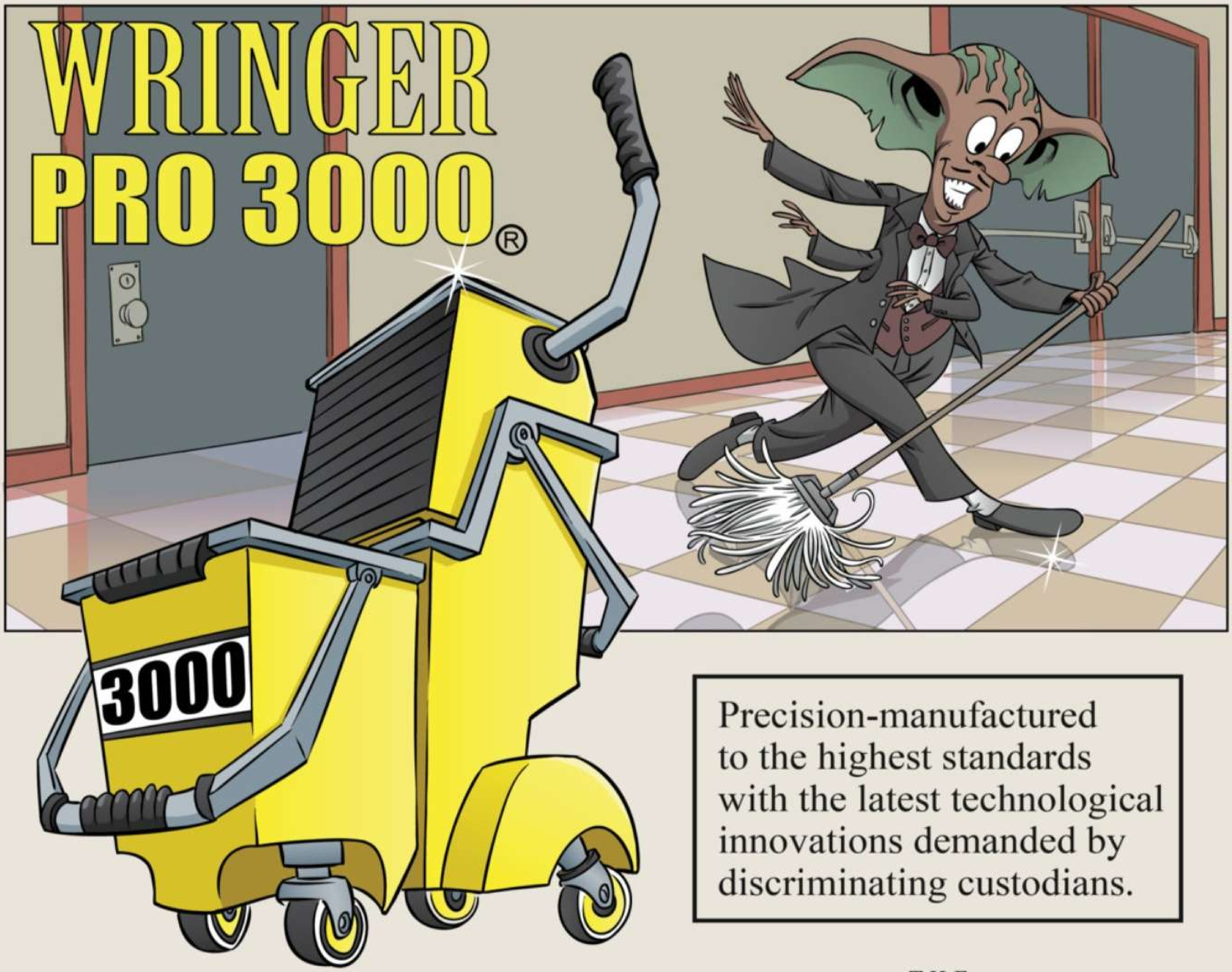
That's right.
The process of adding decimals is really no different than the process of adding inte--
ThWAAANNG
FWUP FWOP FWUP FWOP FWUP FWOP FWUP FWOP
FWOP FWUP FWUP FWOP FWUP FWUP FWOP FWOP
Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something diabolically difficult!
huff *huff*
Decimal addition demands no deft deductions.
But subtracting decimals requires ***resolute reasoning.***
huff *huff*
Determine the difference between these two decimals to discover the latitude of your lecturer's location along Lobstrich Lane.
Subtract swiftly to spare your schoolteacher ***significant suffering.***
39.42 - 6.803
huff
Try it.


Subtraction doesn't seem any harder than addition.
I guess Calamitous Clod wasn't feeling very creative this week.
I'm pretty sure we just need to line up the place values...
...then subtract in the usual way.
39.42 - 6.803
39.42
- 6.803
We start on the right with the thousandths.
But, there's nothing to subtract the 3 from.
39.42
- 6.803
We should write a zero at the end of 39.42.
That way, both numbers have the same number of digits to the right of the decimal point.
39.420
- 6.803
Since we can't take away 3 thousandths from 0 thousandths...
...we need to take 1 hundredth from the hundredths place of 39.420 and break it into 10 thousandths.
That gives us a 1 in the hundredths place, and a 10 in the thousandths place.
1 10
39.420
- 6.803
Finish the computation.


Now we can subtract the thousandths.
1 10
39.420
- 6.803
7
10 thousandths minus 3 thousandths is 7 thousandths.
On to the hundredths! 1 hundredth minus 0 hundredths is 1 hundredth.
1 10
39.420
- 6.803
17
Next, we subtract the tenths.
Since we can't take away 8 from 4...
...we take a 1 from the ones place and break it into 10 tenths.
That gives us 8 in the ones place and 14 tenths in the tenths place.
8 14 1 10
39.420
- 6.803
617
14 tenths minus 8 tenths is 6 tenths.
Then, we subtract the ones and the tens.
And the decimal point goes between the ones and the tenths.
8 14 1 10
39.420
- 6.803
32.617
39.42 minus 6.803 is 32.617!
To Lobstrich Lane!

LOBSTRICH LN
STOP
What does your GPS bot say, Alex?
Here's Lobstrich Lane.
It's this way!
LOBSTRICH LANE LANES
Great work, little monsters!
Now, go get some shoes and we'll bowl a round.

Imagine your floors. . . only cleaner!



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