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Chapter 7:
Variables

R&G n

Do you have a favorite number?

I do.

Mine is 42. What's yours?

n.

?!? Your favorite *NUM-ber.*

I told you, my favorite number is *n*.

Today's program was brought to you by the letter *n*.

Letter n

MONSTERPÆDIA

Nn

Nn Nn

N is the fourteenth letter in the alphabet.

ABCDEFGHIJKLMN
OPQRSTUVWXYZ

n is a *letter.*

Sure. Math beasts use letters all the time.

What for?

Letters can stand for *numbers*... *n* can be *any* number!

?!?

You know how on some math worksheets, you fill in little boxes with the answers?

Like this?

Right. What does each box equal?

Let's see.

The first box is 8.

2 + 6 = 8

5 + □ = 9

12 - □ = 5

The next box is 4.

6 = 8

5 + 4 = 9

12 - □ = 5

And this last box is 7.

Each box is different.

2 + 6 = 8

4 = 9

12 - 7 = 5

Right. You don't always have to use the same number in every box.

Instead of using an empty box in place of a number you don't know, you can use a letter...

...like *n!*

These problems are like the ones from before, but instead of a ***box***, they have an ***n***.

2 + n = 7

n - 4 = 9

12 - n = 3

Can you figure out what each *n* equals?

LETTERS CAN BE USED TO STAND FOR UNKNOWN NUMBERS. THESE LETTERS ARE CALLED ***VARIABLES***.

Instead of figuring out what each **box** equals, I figure out what each ***n*** equals.

Right.

$2 + n = 7$

$n - 4 = 9$

$12 - n = 3$

In the first problem, since 2 plus 5 is 7...

$2 + n = 7$

...*n* must be 5.

$n = 5$

In the second problem, *n* has to be 13, because 13 minus 4 is 9.

$n - 4 = 9$

$n = 13$

And in the last problem, *n* must be 9.

$12 - n = 3$

$n = 9$

That's right!

You can see why *n* is my favorite number.

It's quite versatile.

It can be anything?

It is.

Anything!

Can it be 42?

Yep.

But, it's not ***always*** 42?

Nope.

Then I still like 42 best.

Snap!

Ms. Q

Variables

What is a variable?

My mom says I'm very able.

Sometimes I just don't apply myself.

Not very able, ***variable!***

Vary-Uh-Bull.

"A variable is a symbol, usually a lower-case letter, that is used to represent an unknown quantity."

It's a letter that stands for a number.

How would a ***letter*** be useful in ***math***?

If you don't know a ***number***, you can use a ***letter*** in its place.

Right! A letter can be used to represent a number.

Can anyone guess why I usually like to use *n*?

Variables represent ***numbers***, and the word ***number*** starts with the letter ***n!***

Right! Now, here is a math ***expression*** that uses a variable. What does this expression equal?

4 + n

A MATH ***EXPRESSION*** IS A PHRASE THAT USES NUMBERS, VARIABLES, AND OPERATIONS (LIKE ADDITION, SUBTRACTION, AND MULTIPLICATION) INSTEAD OF WORDS.

Can't it equal anything?

We don't know what number n is.

Since n can be anything, we don't know what $4+n$ is.

$4+n$ is just four more than whatever n is.

So, we need to know n to find $4+n$.

That's right! What is the value of $4+n$ when $n=7$?

4 + n

4 + 7 = 11

If n is 7, then 4+n is 11!

How about when $n=9$?

4 + n

4+9=13

4+n is thirteen!

When $n=16$?

4 + n

4 + 16 = 20

Twenty!

And what if $n=96$?

4 + n

4 + 96=100

100!

THE LITTLE MONSTERS ARE ***EVALUATING*** THE EXPRESSION $4 + n$. WHEN WE EVALUATE AN EXPRESSION, WE REPLACE THE VARIABLE WITH A NUMBER. THEN, WE FIND THE VALUE OF THE EXPRESSION.

In an expression, we can replace n with a number, then figure out what the expression equals.

The value of $4+n$ depends on what number we pick for n.

$4 + n$

if $n = 10$, $n = 16$, $n = 0$, $n = 99$

then $4+10 = 14$, $4+16 = 20$, $4+0 = 4$, $4+99 = 103$

(by Winnie)

Great! A variable can appear more than once in the same expression.

When a variable is used more than once in the same expression, it represents the same number each time it is used.

All of the n's in this expression represent the same number.

How could we ***simplify*** this expression?

$n + n + n + n + n + n + n + n + n + n$

When you ***simplify*** an expression, you write it in a different way that means the same thing, but is easier to use.

$n + n + n + n + n + n + n + n + n + n$

$= 10 \times n$

Adding ten n's is the same as multiplying 10 times n.

It's simpler to write $10 \times n$.

Great! Now, here is an ***equation***.

4 + n=23

How is an ***equation*** different from an ***expression***?

"An equation is a mathematical statement in which two expressions are equal."

Equations have ***equals*** signs.

Expressions don't.

Equations:
4+n=23
10−n=3
n−6=11
11+n=14

Expressions:
4+n
10−n
n−6
11+n

In an ***equation***, we can figure out what number the variable is!

FIGURING OUT WHAT NUMBER A VARIABLE REPRESENTS IN AN EQUATION IS CALLED ***SOLVING*** THE EQUATION. THE NUMBER THAT THE VARIABLE REPRESENTS IS THE ***SOLUTION*** TO THE EQUATION.

Equations are like puzzles.

To solve a math ***equation***, you have to figure out what number the ***variable*** is.

4 + n = 23

For example, if $4+n=23$, then n is 19, because $4+\mathbf{19}=23$.

4 + n = 23

4 + 19 = 23 ✓

so ☺ n = 19

Wonderful!

Who else would like to try solving an equation?

$10 - n = 3$

$n - 6 = 11$

$11 + n = 14$

Solve all three.

If $10-n=3$, then n must be...

$10 - n = 3$

$10 - 7 = 3$ ✓

$n = 7$

7!

If $n-6=11$, then n must be...

$n - 6 = 11$

$17 - 6 = 11$ ✓

$n = 17$

17!

If $11+n=14$, then n must be...

$11 + n = 14$

~~$11+1=12$~~

~~$11+2=13$~~

$11+3=14$ ✓

$n=3$

3!

Great! Try one more.

What's the solution to ***this*** equation?

$5 + n + n - n = 16$

Try it.

Is there anything you can simplify first?

$5+n+n-n=16$

I know!

$5+n+n-n=16$

Adding n, then taking n away is the same as not doing anything.

Over here, we add one n and then we take n away.

So, we can just erase these.

$5+n$ $=16$

That leaves $5+n=16$. And since $5+11=16$...

$5+n$ $=16$

$n=11$!

Marvelous. How will I ever stump you little monsters?

Simplifying Expressions

Winnie

Original Expression	Simplified Expression
6-3	3
16+4	20
5+6-2+1-9	1
4×9+2×4	44

Simplify just means to write the expression in an easier way.

6-6	0
7-7	0
28-28	0
n-n	0

When you subtract a number from itself, you always get zero. So, n-n is 0

6+1-1	6
17+5-5	17
12+n-n	12
n+7-7	n

Adding a number and subtracting the same number is the same as doing nothing.

4+4+4+4+4	5×4 (or 20)
7+7+7+7	4×7 (or 28)
n+n+n+n+n+n	6×n
(n+5)+(n+5)+(n+5)	3×(n+5)

It's usually simpler to write repeated addition as multiplication.

more examples.

6+197-6	197
33+w-33	w
16+43-43-16	0
h+j-j-h	0
3+3+3+3-3-3-3	3
m+m+m+m-m-m-m	m

Winnie

MATH TEAM

Writing Equations

Do any of you speak another language?

I know a little Monchhichi.

I learned some Snork last summer.

I speak Smurf.

Draconic! All dragons speak it.

Great! When you know two languages, you can translate from one language into the other.

What we are learning today is a lot like translating. We are going to turn ***words*** into ***math***.

Let's start by writing some equations for these sentences that include variables.

Nine minus x is 4.

The product of eight and j is thirty-two.

If you subtract 17 from c, you get 15.

Try to write an equation for each.

Nine minus x is 4.

$9 - x = 4$

Nailed it!

"Product" means you are ***multiplying*** eight and *j*...

...and the word "is" means equals.

The product of eight and j is thirty-two.

$8 \times j = 32$

At first, I wrote $17-c=15$, but that means you are subtracting c from 17.

If you subtract 17 from c, you get 15.

17 - c = 15

C - 17 = 15

We want to subtract 17 ***from*** c, so it should be $c-17=15$.

That's right! Always check to make sure your equation means the same thing as the sentence.

Sometimes, you need to change the order of the numbers and variables. For example, what equation would you write for this sentence?

Fifty-nine is eighteen less than w.

Try it.

Since fifty-nine is eighteen less than w...

...that means that if we take eighteen away from w, we get fifty-nine!

So, w minus eighteen equals fifty-nine!

w - 18 = 59

Great!

THE EQUATION $w-18=59$ MEANS THE SAME THING AS THE EQUATION $59=w-18$. SINCE THE TWO SIDES OF AN EQUATION ARE EQUAL, YOU CAN SWITCH THEM.

Solve for the variables in these equations. Answers are on page 28.

These next problems are a little harder. You need to choose a variable to represent the unknown number.

Seven more than a number is 31.

Five times Dugg's age is 35.

Mick needs three more points to reach 100.

Try to write an equation for each.

Seven more than a number is 31.

7 plus n is 31

7 + n = 31

For the first sentence, we can use n to represent the number.

So, 7 plus n is 31.

For the second sentence, we can use d for Dugg's age.

Five times Dugg's age is 35.

5 times d is 35

5 × d = 35

For the last sentence, m can represent the number of points that Mick has.

If he gets 3 more, he will have 100.

So, $m+3=100$.

Mick needs three more points to reach 100.

m plus 3 is 100

m + 3 = 100

Great!

It's usually a good idea to choose variables that help you remember what each represents...

...like n for a number, d for Dugg's age, and m for Mick's points.

There are three more apples than there are pears in the bowl.

Sometimes, you need **two** variables.

What equation could you write to show how the number of apples relates to the number of pears?

Try to write the sentence above as an equation.

We can use a to represent the number of apples...

...and p to represent the number of pears.

a = the number of apples
p = the number of pears

Since the number of apples is 3 more than the number of pears...

...we need to add 3 to the number of pears to equal the number of apples.

a is 3 more than p
$a = 3 + p$

WE CAN'T SOLVE THE EQUATION $a=3+p$ UNLESS WE KNOW THE VALUE OF ONE OF THE VARIABLES. SOLUTIONS TO THE EARLIER EQUATIONS ARE IN LIZZIE'S NOTES ON THE NEXT PAGE.

Now, the number of apples is **two** more than the number of pears!

a is ~~3~~ 2 more than p
$a = \cancel{3}\ 2 + p$

Grogg! That was the wax fruit from art class!

BL-aaakkh!

Writing and Solving Equations

Lizzie

Nine minus x is 4.
equation: $9 - x = 4$
Since $9 - \boxed{5} = 4$
$x = 5$

The product of 8 and j is 32. product means multiply
equation: $8 \times j = 32$
$8 \times \boxed{4} = 32$
$j = 4$

If you subtract 17 from c, you get 15.
equation: ~~17 - c~~ $c - 17 = 15$
$\boxed{32} - 17 = 15$
$c = 32$

$30 \xrightarrow{+2} \boxed{32}$
$-\ 17 \qquad -\ 17$
$13 \xrightarrow{+2} \boxed{15}$

59 is 18 less than w.
equation: $w - 18 = 59$
$\boxed{77} - 18 = 59$
$w = 77$

$80 \xrightarrow{-3} \boxed{77}$
$-\ 18 \qquad -\ 18$
$62 \xrightarrow{-3} \boxed{59}$

7 more than a number is 31. the variable n represents a number
equation: $7 + n = 31$
$7 + \boxed{24} = 31$
$n = 24$

5 times Dugg's age is 35. d represents Dugg's age (in years)
equation: $5 \times d = 35$
$5 \times \boxed{7} = 35$
$d = 7$

Mick needs 3 more points to reach 100. m is the number of points Mick has now
equation: $m + 3 = 100$
$\boxed{97} + 3 = 100$
$m = 97$

SOLVING EQUATIONS

Arrrr! X always marks the spot o' the pirate booty.

Each o' me buried treasures be marked with a different x on this map.

Here be the treasure o' me most recent exploration...

...a satchel o' silver dollars.

Who can guess the number o' dollars within'?

Coney Island

Cinnabar

Myst

Kokomo

Genosha

Lilliput

Membata

Skull Island

Koholint

Tortuga

N

W

E

S

132.

100.

196.

One dollar!

One?!

GrOgg

132

100

196

1

You all overbid!

Here be a hint:

If I be addin' fifteen more...

...the sack be holdin' two hundred 'n four.

Hmmm...

What number gives us 204 when we add 15?

Can you find the number of silver dollars in the sack?

It's close to 200.

200+15 is 215.

Too big.

200
+15
215

190+15 is 205.

Missed by 1...

...still just a little too big.

190
+15
205

189+15 is 204!

Bingo!

Aye, there be 189 silver dollars.

Is that the only way?

We just guess the answer and check our guess until it's right?

189
+15
204

Guessin' and checkin' can be useful...

...but there always be more than one way to solve a problem.

I got the answer with a drawing!

If we add fifteen coins to the sack, that makes 204 coins.

So, if we take fifteen coins away from 204, we get the number of coins that are in the sack.

You just subtract fifteen from 204!

It's the same answer we got by guessing and checking!

$$\begin{array}{r} 204 \\ -15 \\ \hline 189 \end{array}$$

Excellent figurin'!

How can we make Grogg's doodle into a proper math equation?

Write an equation that matches Grogg's drawing.

We are trying to find out how many coins are in the sack.

We can use n to represent the number of coins: $n+15=204$!

$n + 15 = 204$

Aye. A picture be worth a thousand words...

...and sometimes a math equation be just as good.

But we shouldn't use the letter "n" to represent me pirate loot.

What be a more proper symbol to use for pirate treasure?

x!!!

Aye! We'll be usin' x as a variable to represent the number o' coins in me satchel.

How do we solve this equation?

We do the same thing Grogg did in his drawing...

...we subtract 15 on both sides of the equals sign.

Aye! If we take fifteen coins away on the left, we also take fifteen away on the right.

$x + 15 = 204$

$-15 \quad -15$

Every equation be havin' two sides...

...the left side o' the equals sign...

..and the right side o' the equals sign.

$$x + 15 = 204$$
$$-15 \quad -15$$

For the two sides to stay equal, whatever be done on one side...

...must also be done on the other.

After we subtract 15 from both sides...

$$x + 15 = 204$$
$$-15 \quad -15$$
$$x = 189$$

...we're left with x equals 189!

Aye, 189 coins.

For a pirate, searchin' for x means lookin' for the location o' buried treasure.

For a math beast, searchin' for x means lookin' for an unknown number.

Here be a chest full o' rubies.

How many rubies be there in the chest?

If ye' be removin' nine rubies, left inside be ninety-three.

Use the clue written on the chest to figure out how many rubies it holds.

We can write an equation!

If x is the number of rubies...

...and we take nine away...

...there are 93 rubies left.

$x - 9 = 93$

If 93 is 9 ***less*** than the number of rubies...

...then the number of rubies must be 9 ***more*** than 93!

This time we ***add*** 9 to both sides of the equation!

So, x equals 102. There are 102 rubies!

Excellent work!

$$\begin{array}{r} x - 9 = 93 \\ +9 \quad +9 \\ \hline x = 102 \end{array}$$

But x was the number of ***coins*** in the sack. We should have used a different variable for the number of ***rubies!***

Like what?

Like Arrrr!

Yo ho ho! You'll make a fine pirate one day!

RECESS

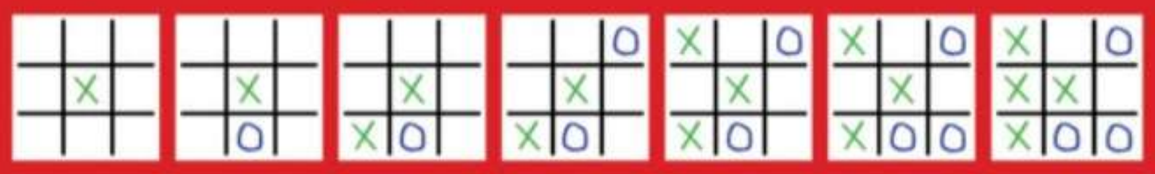
Wild Tic-tac-toe

Standard Tic-tac-toe:

Tic-tac-toe is a popular game for two players played with pencil and paper. The game begins with a standard 3×3 board as shown below.

Players take turns placing X's and O's on the board. In standard tic-tac-toe, one player places the X's and the other places the O's. The player who gets three of his or her marks in a line (horizontally, vertically, or diagonally) is the winner. If all the spaces on the board are filled and neither player has three in a row, the result is a draw (tie).

Below is an example of a standard game of tic-tac-toe in which the first player (Lizzie, marking X's) wins.



If both players play perfectly, a game of standard tic-tac-toe will always end in a draw.

Wild Tic-tac-toe:

Wild tic-tac-toe is a variation of standard tic-tac-toe with one important difference. On each turn, a player may choose to place an X or an O. The player to complete a line of three X's or three O's is the winner.

Below is an example of a game of wild tic-tac-toe won by the first player.

The first player can always win a game of wild tic-tac-toe if he or she plays correctly. Can you find a strategy that will allow you to win every time if you play first?

Find a partner and play!

THE LAB

CAPSULES

Good afternoon, little monsters.

Let's begin today's lesson with a problem.

I have placed a total of 51 capsules in these two mugs.

The second mug contains 7 more capsules than the first.

How many capsules are there in each mug?

Can you figure it out?

1st Mug	2nd Mug	Total
1	8	9
2	9	11
3	10	13
4	11	15
5	12	17
6	13	19
7	14	21
8	15	23
9	16	25
10	17	27
11	18	

Isn't there a way we can use a variable to solve this problem?

Indeed! Suppose we use c to represent the number of capsules in the first mug.

The second mug has 7 more capsules than the first.

So, if there are c capsules in the first mug...

...then there are $c+7$ capsules in the second mug!

C

C+7

gone FISSION

I GOT MY ION U!

Brilliant!

Now, how could you write an equation that will help you find the value of c?

So, c...

...plus $c+7$...

...equals 51!

There are a total of 51 capsules in both mugs.

$$c+(c+7)=51$$

Extraordinary!

We can erase the parentheses, since it's all addition.

$$c+\ c+7\ =51$$

Squeek Squeek Squeek

THE ASSOCIATIVE PROPERTY OF ADDITION LETS US MOVE OR REMOVE PARENTHESES IN A SUM.

If we subtract 7 from both sides of the equation, we get $c+c=44$.

$$\begin{array}{rrr} c+\ c+7 & = & 51 \\ -7 & & -7 \\ \hline c+c & = & 44 \end{array}$$

If two c's add up to 44, then c must be half of 44.

c=22!

Wait, what does that mean?

Skratch Skratch

We used c to represent the number of capsules in the first mug...

...and $c+7$ to represent the number of capsules in the second mug.

C

C+7

Since $c=22$, there are 22 capsules in the first mug...

...and 22+7=29 capsules in the second mug.

C =22

C+7 =29

Got it.

And since 22+29=51, our answer checks out.

There are 51 capsules all together.

IT'S ALWAYS A GOOD IDEA TO CHECK YOUR ANSWER!

Wait! Grogg, don't mix those!

Yikes!

Klonk!

Koff

Koff

Koff

Koff

Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something diabolically difficult!

A Combination Conundrum!

I've locked Professor Grok in this stainless steel steam sterilization cylinder!

I offer these three clues to help you crack the combination code.

1. The second number is five more than the first.

2. The last number is six more than the second.

3. The three numbers sum to seventy-six.

The countdown clock is set to start the steamy sterilization shortly. Only a swift solution can save your schoolteacher from a...

...Scorching Steambath!

POOF!

We need to use these three clues to find the combination code.

The second number is 5 more than the first.
The last number is 6 more than the second.
The three numbers sum to 76.

And we only have 3 minutes!

Can you find the three-number code?

Maybe we can use a variable again.

We can use n to represent the first number.

n

n+5

Since the second number is 5 more than the first...

...we can label the second number n+5!

The last number is six more than the second.

So, we need to add 6 to the second number to get the third number.

We can label the third number n+5+6!

Or, n+11!

n

n+5

n + 11

The last clue says that the three numbers sum to 76.

So, if we add n, n+5 and n+11, we get 76!

$$n+(n+5)+(n+11)=76$$

n+ n+5 + n+11 =76

Since it's all addition, we don't need the parentheses.

Squeek Squeek Squeek

n+ n+5 + n+11 =76
n + n + n + 16 = 76

We simplify the left side of the equation by adding 5 and 11.

n+ n+5 + n+11 = 76
n + n + n + 16 = 76
-16 -16
n + n + n =60

Then, we can subtract 16 from both sides.

Now, $n+n+n$ is the same as $3\times n$.

n + n + n =60
3×n =60
3×20 =60
n=20

Since $3\times 20=60$, $n=20$!

n

n+5

n + 11

So, the first number is 20...

...the second number is 20+5=25...

...and the third number is 20+11=31!

20+25+31=76! It's right!

Quick! Open the cabinet!

Great work, little monsters!

You've saved me from this steamy sauna!