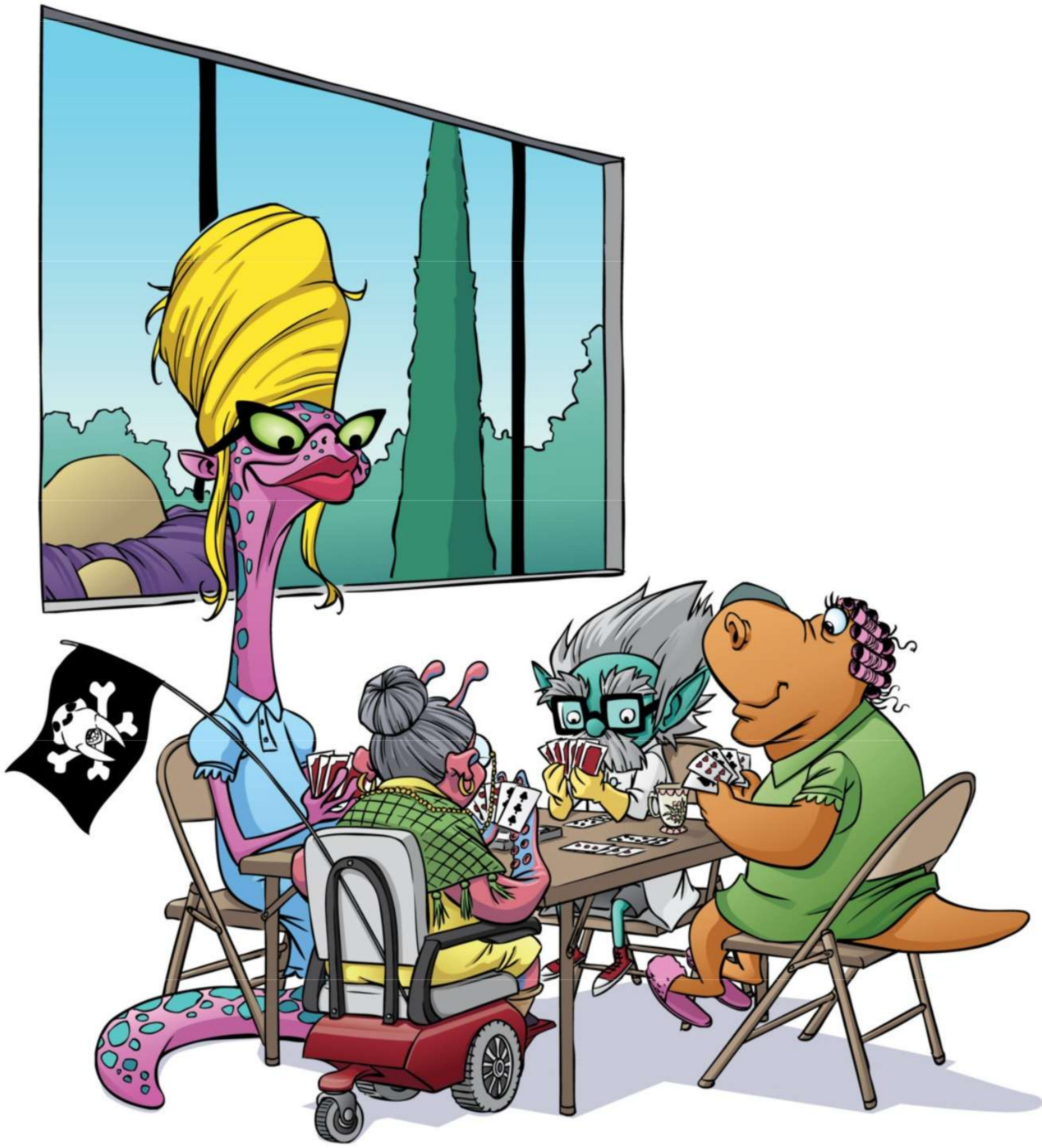
Contents: Chapter 3

Click the Play List tab in the top-left to view a recommended reading/practice sequence.

Chapter 3:
Perimeter and Area



Woodshop

PERIMETER AND AREA

Behold, The Lazy Susan!

'Twas once a great, grand ship.

What happened?

She was dashed to bits on a small rocky isle in the north seas.

I spent two years on that shore with only a stitched leather sphere to keep me company.

How did you escape?

'Tis a tale for another day.

Today, you'll each begin buildin' your own pirate ship, poop deck 'n' all. Begin by selectin' a plank in one of three sizes...

...but choose wisely, little buccaneers. 'Tis all the wood you'll be gettin'!

How do we know which piece is the biggest?

The tall one on the left ***looks*** the biggest.

It's the ***tallest***, but I'm not sure that makes it the ***biggest***.

We could weigh them.

That'd be brilliant... ***if we had a scale!*** We don't even have a ruler!

So, how could we measure each plank to find out which is the largest?

There must be *something* we can use.

If we light all three planks on fire...

Huh?

Huh?

Huh?

Huh?!

...we could see which one takes the longest to burn.

But all we'd have left is a pile of ashes!

If we grind all three planks into sawdust, we could see which one fills the most trash cans!

How are we going to ***build a boat*** out of ***sawdust?!***

Termite

Termites!

I've got an idea!

We could put the planks on the floor, wrap a string around each one, and see which needs the most string.

I've got some string!

The tallest plank needs more string than the other two.

I *thought* it was the biggest!

What are you *doing,* Alex?

These planks are all crooked. I'm lining them up with the floor tiles.

That gives me an idea! I know another way to compare the size of the planks.

If we line up all of the planks with the floor tiles--

The more tiles a plank covers, the bigger it is!

Right! We just need to count how many tiles are under each plank.

But how can we count the tiles if they are under the planks?

We just use a marker to trace each plank.

Then, we can move the planks and count the number of tiles in the marked rectangles.

Don't marker me floors!

What if we use tape?

I'll have you swabbin' the decks! It leaves an awful, sticky mess!

Gum?

No!

No!

No!

No!

We need a way to count the tiles under the planks without moving them.

I know!

This plank covers two rows of tiles, and each row has:

1 2 3 4 5 6 7 tiles!

That makes 7+7=14 tiles all together!

This one has three rows of five tiles...

...or is it five rows of three?

5+5+5=15, and 3+3+3+3+3=15. Five 3's and three 5's both add up to ***15!***

This one has four rows of four tiles: 4+4+4+4=***16!***

So, the one that needed the most string covers the smallest number of tiles.

And the one covering the most tiles was the shortest when they were leaning against the wall.

I guess how big something is doesn't just depend on how tall it is.

A tall, skinny piece of wood can be smaller than a short, fat one.

I get it! Glob over there is way ***heavier*** than Mantis, but Mantis is a lot ***taller*** than Glob.

Aye kids, there be lots o' ways to measure the size of a thing.

For the planks, ye be usin' the measures of ***perimeter*** 'n' ***area***.

When ye be wrappin' the string 'round the planks, ye be comparin' ***perimeter***.

The tall plank had the largest perimeter.

And when ye be countin' the tiles, ye be comparin' ***area***.

The short plank had the largest area... 16 tiles!

Ms. Q.

Perimeter

What is *perimeter?*

It's the distance around the outside of a shape.

Good. How do you measure perimeter?

With a string, or a measuring tape...

...or Cammie's tongue!

You wrap a string around the outside of the shape, then hold the string up to a ruler to see how long it is.

Great! What is the perimeter of this stop sign?

STOP

I know!

How do you know? We're not done measuring!

How did Winnie figure out the perimeter so quickly?

The sides are ***all*** the same. I only needed to measure one of them.

It was 11 inches long, so I added eight 11's by skip-counting: 11, 22, 33, 44, 55, 66, 77, ***88!***

STOP

A STOP SIGN IS AN OCTAGON WITH 8 EQUAL SIDES.

Winnie is right. The perimeter of the sign ***is*** 88 inches.

So, instead of using a string or a measuring tape to wrap ***around*** a shape, we can just measure the sides and ***add*** them.

Terrific! How would we measure the perimeter of one of these rectangular tables?

We can measure all four sides and add them.

This short side is...

...30 inches.

This long side is 60 inches.

The perimeter is 180 inches!

We haven't measured the other two sides!

I know, but it's a ***rectangle***. The other sides are the ***same*** as the ones we already measured.

She's right!

The perimeter is 30+60+30+60 =180 inches!

60

30

30

60

I guess sometimes you need to measure ***all*** of the sides, but other times there are shortcuts.

Like what?

Like when the sides are ***all*** the same length, we only need to measure ***one*** of them.

In an equilateral triangle you can measure one side, then add three of that length: 7+7+7=21.

7

EQUILATERAL MEANS EQUAL-SIDED.

For a square, you measure one side and add it four times. 6+6+6+6=24.

6

And for a ***regular*** chiliagon... ...you measure one side and add it 1,000 times!

A SHAPE IS "REGULAR" IF ALL ITS SIDES AND ANGLES ARE THE SAME. A REGULAR CHILIAGON (PRONOUNCED KILLY-UH-GONE) HAS 1,000 EQUAL SIDES.

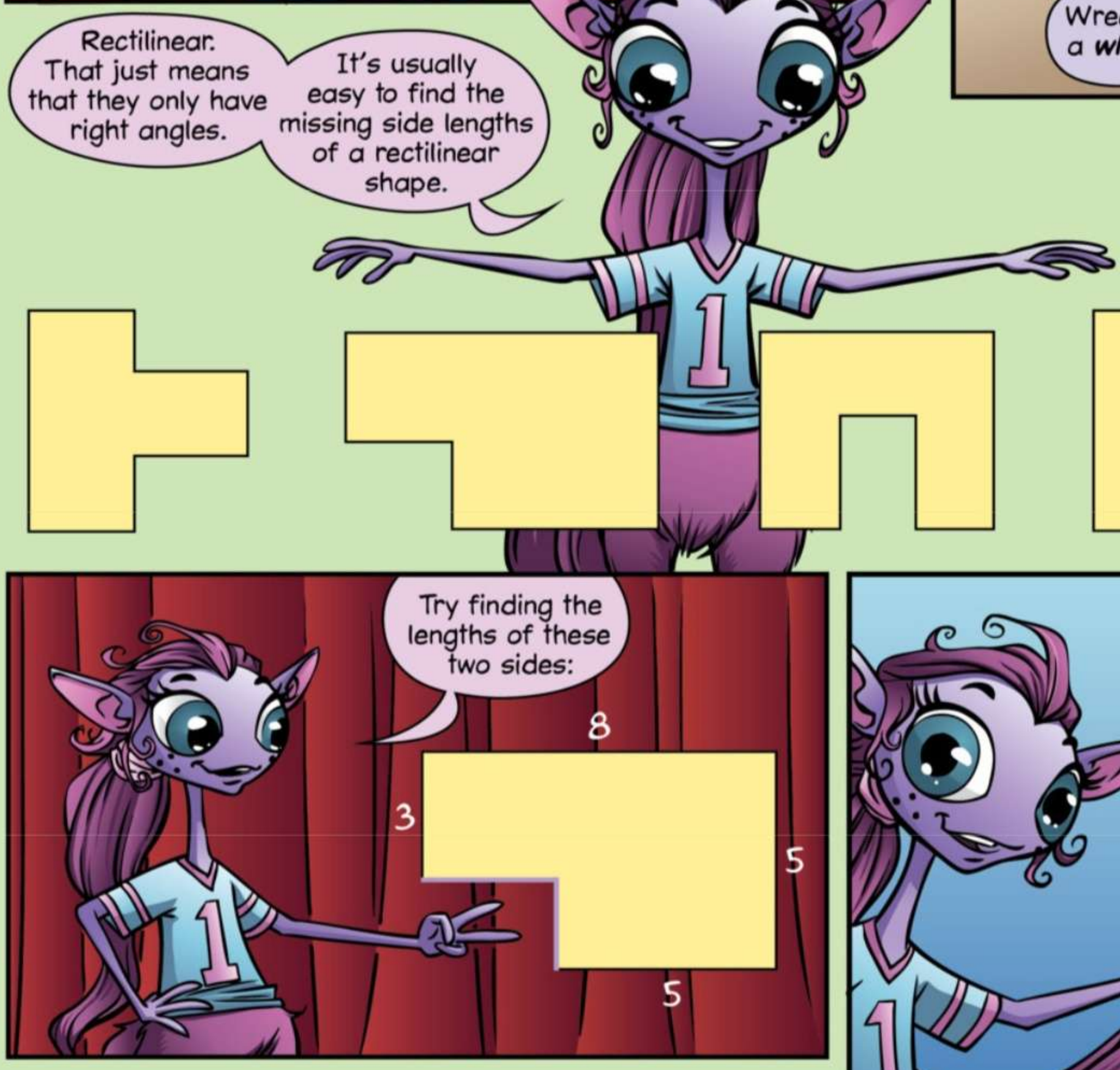
But if a polygon had a thousand sides and they were all nine inches long, you wouldn't want to add a thousand 9's.

You could add nine 1000's. It's nine thousand!

What?! Nine thousand! There's no way that can be right.

9,000 ***is*** right! Great work. Ready for this year's first math meet?

MATH TEAM ...with Fiona!



The two short sides on the bottom look like the long side on top, only broken into two pieces.

8

5

The bottom sides have to add up to 8, so the shorter one must be 3.

8

3

5

And these two have to add up to 5, so the shorter one is 2.

3

2

5

Now, we can find the perimeter: 8+5+5+2+3+3=26.

8

3

3

2

5

5

Right! There's another way to find the perimeter.

What is it?

And now...

Uh oh...

...no time...

...get out there!

But? Wait!

Please welcome the Little Monsters of Beast Academy and their opponents... the Bots!

Good luck!

For today's math meet, we will ask six perimeter questions. The first five are each worth one point, and the final question is worth two. The team with the most points wins the meet. Is everyone ready for the first question?

Question 1: Find the perimeter of a regular pentagon whose sides are 17 inches long.

17

We just add 17, five times.

Or 5, seventeen times!

Skip-count by 5's!

5, 10, 15, 20--

BZZZZZT!

Wait! How did...

EIGHTY-FIVE INCHES

Correct! Bots earn the first point.

Question 2: What is the perimeter of the right triangle shown?

600

1000

800

1,000 is ten hundreds, so we have 10 hundreds +8 hundreds +6 hundreds--

BZZZZZT!

TWO THOUSAND FOUR HUNDRED

Correct! Bots lead, two to zero.

Question 3: Each team has been given a box that is 7 inches tall, 5 inches long, and 3 inches wide. What is the length of ribbon needed to wrap the box as shown, not including the bow?

7

3

5

The ribbon goes around the box three ways.

We just need to solve three rectangle perimeter problems.

The rectangle that is 7 inches tall and 5 inches long has a perimeter of (7+5)+(7+5)=12+12 =24 inches.

5

7

The rectangle that is 7 inches tall and 3 inches wide has a perimeter of (7+3)+(7+3) =10+10=20 inches.

3

7

The rectangle that is 5 inches long and 3 inches wide has a perimeter of (5+3)+(5+3)=8+8 =16 inches.

3

5

Now add those three together! 24+20+16--

BZZZZZT!

Urrrgh!

60 INCHES

60 inches is *Correct!*

The score is now Bots: 3, Little Monsters: 0.

Question 4: Four equilateral triangles are arranged as shown to form a larger equilateral triangle. If each of the smaller triangles has a perimeter of 10 feet, then what is the perimeter of the large triangle they make?

Can you find the perimeter of the large equilateral triangle?

If we can find the side length of a small triangle, we can add it 6 times.

The perimeter of the big triangle is made from 6 sides of the small triangles.

The small triangles have three equal sides that add up to 10 feet...

...but what number can you add three times to get 10? 3+3+3=9 and 4+4+4=12.

There isn't one!*

*THERE IS A NUMBER THAT WORKS, BUT IT IS A ***FRACTION***, WHICH YOU WILL LEARN ABOUT IN BOOK 3D.

INVALID.

BONK BONK

ERROR 400
BAD REQUEST

DOES NOT COMPUTE.

Maybe we don't ***need*** to know the side lengths to find the perimeter.

That's right, we don't!

Lizzie, you said it takes six sides of the small triangle to make the big triangle.

But we don't know how ***long*** they are.

No, but we know it takes ***3*** sides to make the small triangle and ***6*** of those sides to make the big triangle!

If we double the number of sides, then we double the perimeter!

20 feet!

DING!

CORRECT!
Bots: 3
Little Monsters: 1

Question 5: Each team has been given four metal poles, measuring 1, 3, 5, and 7 feet long. Choose three of these poles to make a triangle. What is its perimeter?

Quick, the bots are already welding!

How do we know which poles to use?

I got it!

15 feet!

DING!

Correct!
Bots: 3
Little Monsters: 2

Can you figure out why 15 feet is the only possible perimeter?

That was incredible, Grogg. How did you do that?

See what the bots are doing over there?

Skratch
Skratch

They'll never get the 5-foot pole to touch the 1-foot pole.

The two short poles have to add up to ***more*** than the long pole.

Or the two short poles won't be able to reach each other!

THIS IS CALLED THE ***TRIANGLE INEQUALITY***: THE SUM OF THE LENGTHS OF THE TWO SHORTEST SIDES OF A TRIANGLE MUST BE GREATER THAN THE LENGTH OF THE LONGEST SIDE.

You can't make a triangle with the 1, the 3, and the 5 either.

And it's impossible to make a triangle with the 1, the 3, and the 7.

The only way to make a triangle is with the 3, 5, and 7-foot poles!

ANY THREE NUMBERS THAT SATISFY THE TRIANGLE INEQUALITY CAN BE USED AS THE SIDE LENGTHS OF A TRIANGLE.

I still don't know how you got it so fast, Grogg.

Last week, I made the same mistake as the bots while I was building a new roll cage for my shopping cart.

This last question is worth 2 points and will decide the winner of the math meet.

What is the perimeter of the rectilinear figure shown?

7

8

We were just talking about rectilinear shapes!

But we didn't get to finish.

I don't think we have enough information!

Can you find the perimeter of the figure shown?

Hurry! The bots are stacking up to measure it.

There *must* be a better way.

When we were working earlier, we made the opposite sides the same color.

Maybe that will help us on this problem.

7

8

Got it!

All of the short blue lines add up to 7, and all of the short red lines add up to 8.

If we slide all of the short red lines up...

...and we slide all of the short blue lines to the right...

It's the same as solving the perimeter of a simple rectangle!

8+7+8+7=...

Ding!

30 feet.

Correct! Little Monsters win!

SMAK!

R&G Area

Which is *more*: a ladder, or a yeti?

More *what?*

What?

Yes.

Huh?

I mean, when you say *more*, do you mean more furry? More bouncy? More adorable as a stuffed toy? More likely to be strapped to the roof of a van? More--

Okay, I get it.

You need to know *what* you're comparing about the yeti and the ladder.

Exactly!

If I ask, "Which is taller, a yeti or a ladder?" you would probably pick the ladder...

...but if I ask, "Which weighs more?" you would probably pick the yeti.

That depends.

On what?

Mostly on the yeti.

What do you mean?

Well, is it a bigfoot or an abominable snowman?

I'd guess those are both heavier than a ladder, wouldn't you?

Male or female?

Okay, you've made your point.

Is it full grown or just a cub?

Can we move on?

Where is it standing?

Oh, come on! Now you're just being silly. What does ***that*** have to do with anything?

Well, a yeti wouldn't weigh as much standing on the ***moon,*** would it?

That's one small step for a yeti...

...one giant leap for yetikind.

How about this... which of these ***rectangles*** is bigger?

The one on top is wider, but the one on the bottom is taller.

But to figure out which was ***bigger***, you probably want to know which one takes up more space. This is called ***area***.

I'd guess the bottom one is bigger. I mean, it has more area.

How could you be sure?

Which one looks bigger to you?

Well, it takes two candy bars to cover the top rectangle, and four to cover the bottom rectangle.

BEAST BEAST

BEAST BEAST BEAST BEAST

That's how math beasts compare area!

With candy bars?

Actually, they use same-sized squares called "square units," but it's the same idea.

WE WILL EXPLAIN UNITS AND SQUARE UNITS IN BOOK 3C. FOR NOW, WE'LL JUST CALL THEM "SQUARES."

What about these two rectangles, which do you think has more area?

Hmmm... The wide one is wider, but the tall one is taller.

Take a guess.

But, if we split each of the rectangles into squares that are the same size--

Then, we can count which has more squares!

This one has:

8

16

24

32 squares.

And this one has:

5

10

15

20

25

30 squares.

Yours has more area.

So, to find the area of a rectangle, all I need are the lengths of its sides!

Right!

If a rectangle is 3 squares tall and 8 squares wide...

...I can split it into three rows of eight squares.

Or eight columns of three squares.

Then, I just add three 8's: 8+8+8=24.

Or eight 3's: 3+3+3+3+3+3+3+3=24.

Except that you wouldn't say that it has an area of 24, you would say that it has an area of 24 ***squares.***

Cool! My candy bar has 24 squares.

Really?

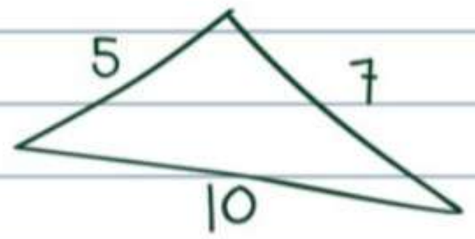
I only count 12!

KRUNCH!

Hey!

Lizzie

Perimeter is the distance around a shape.
To find a shape's perimeter, add its side lengths.

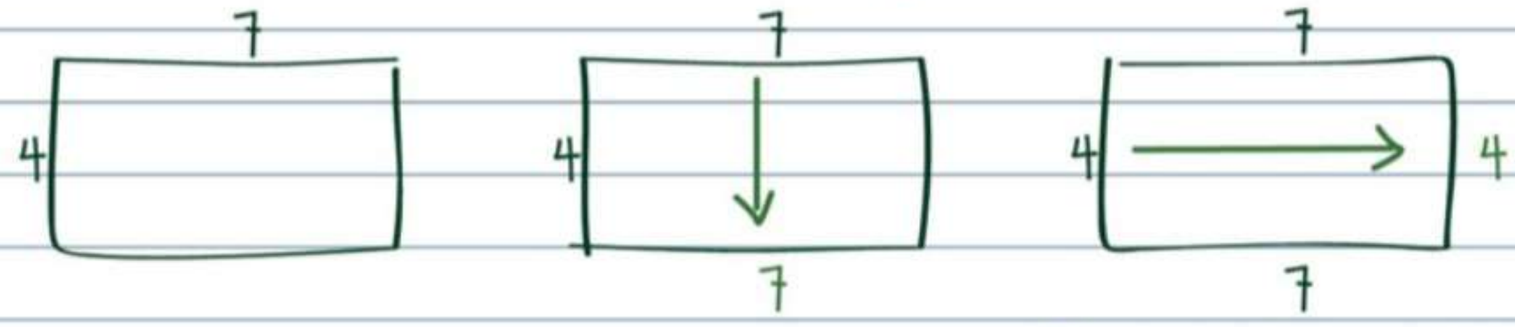


Perimeter=5+7+10=22

Equilateral Polygons (all sides the same length):
Find the length of a side.
Add this number once for each side.

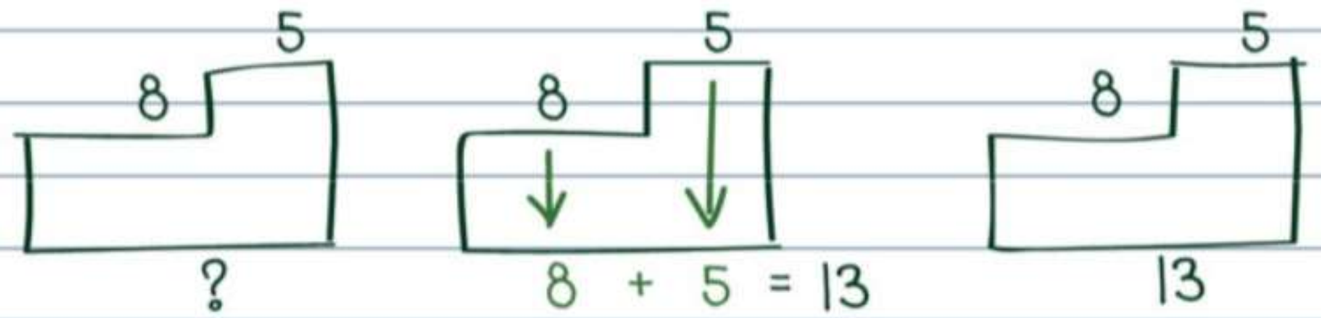
Perimeter=3+3+3+3+3=15

Rectangles:
Opposite sides of a rectangle are the same length.



Perimeter=4+7+4+7=22

Rectilinear Shapes only have right angles.
You can find the missing sides of a rectilinear shape using the opposite sides.

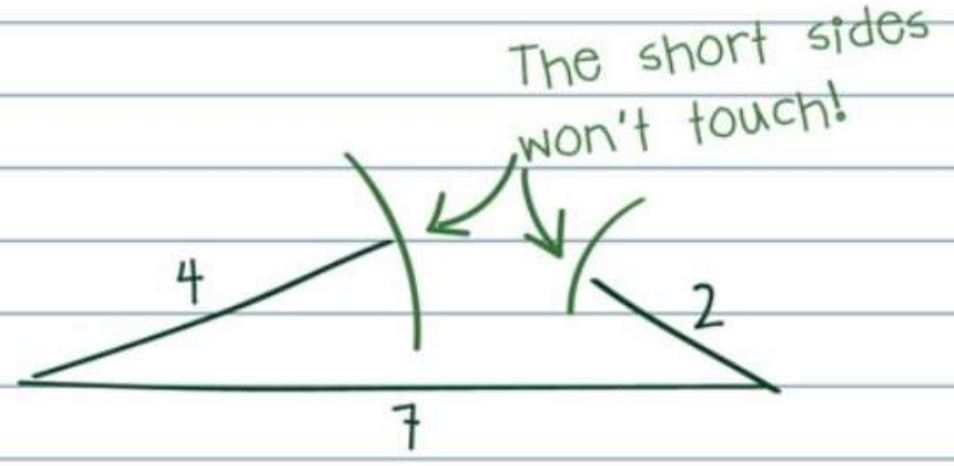


Turn Over
(more on back)

The Triangle Inequality:

The lengths of the two short sides of a triangle add up to more than the length of the long side.

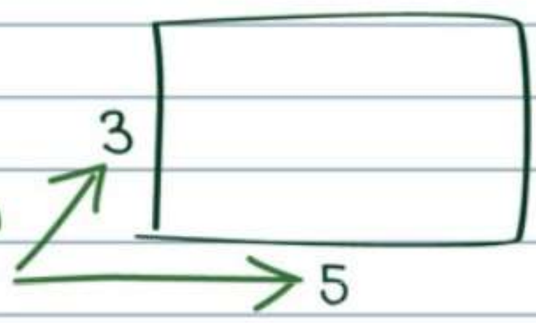
Ex. We can't make a triangle with sides that are 2, 4, and 7, because 2+4 is less than 7.



Area is how much space a shape takes up.

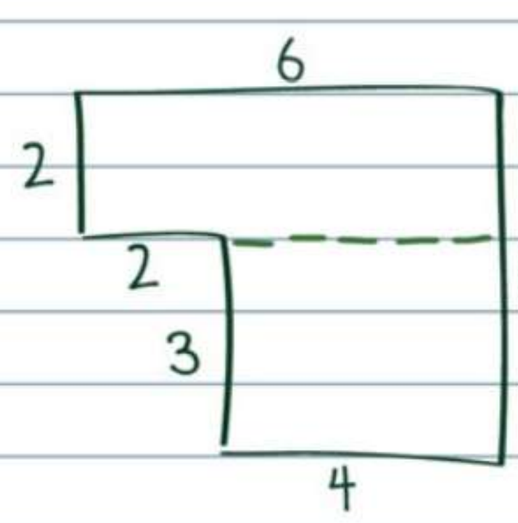
To find the area of a rectangle, split it into squares and count the number of squares.

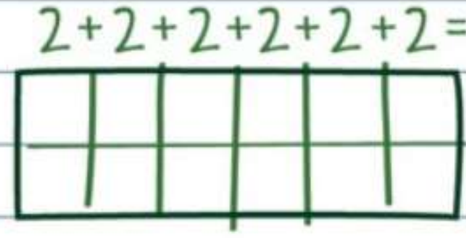
The side lengths tell you how many squares to make



This rectangle's area is 3+3+3+3+3=15 squares.

To find the area of a rectilinear shape, split it into rectangles!



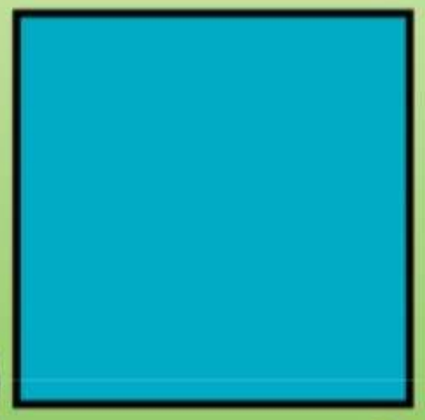


2+2+2+2+2+2=12 squares on top.

3+3+3+3=12 squares on the bottom.

Area=12+12=24 squares.

THE LAB REP-TILES



This is a square. The square has four equal sides and four right angles. Captain Kraken might argue that the triangle is superior, but I have always been most fond of the square.

One great thing about the square is that there are many ways to divide it into pieces that are the same size ***and*** the same shape.

The possibilities are ***limitless!***

POLYGONS THAT ARE THE SAME SIZE AND SHAPE ARE CALLED ***CONGRUENT***.

Each of the squares has been split into four congruent parts. The one we are ***most*** interested in today is the one farthest to the left.

What is special about that one?

We split the square into more squares!

We could keep going!

The little squares could be split into smaller squares and ***those*** squares could be split into even ***smaller*** squares, and we could go on and on forever, dividing the ***teeniest tiniest*** squares into ***more and more*** squares, until--

Grogg!

Grogg is right. We can continue dividing the square forever... but the square is not the ***only*** shape that can be divided into smaller versions of itself.

Who can discover a way to split an equilateral triangle into four smaller copies of itself?

Does it count if one of the triangles is upside-down?

Indeed! A triangle can be flipped over or turned around without changing its size or shape.

That's not too hard then. We can split it into four triangles like this.

Or, four hundred triangles, like this!

Very good! The equilateral triangle can be divided into four congruent equilateral triangles.

What about an isosceles right triangle? Can you find a way to divide it into four congruent isosceles right triangles?

Do you see how it can be done?

Got it!

Can you split an isosceles right triangle into just two smaller copies of itself?

Well done, little monsters! Shapes like these are called rep-tiles.

Lizzie is a reptile!

Actually, a rep-tile is any shape that can be split into smaller copies of itself. The "rep" in rep-tile is short for "replicate," which is just another word for copy.

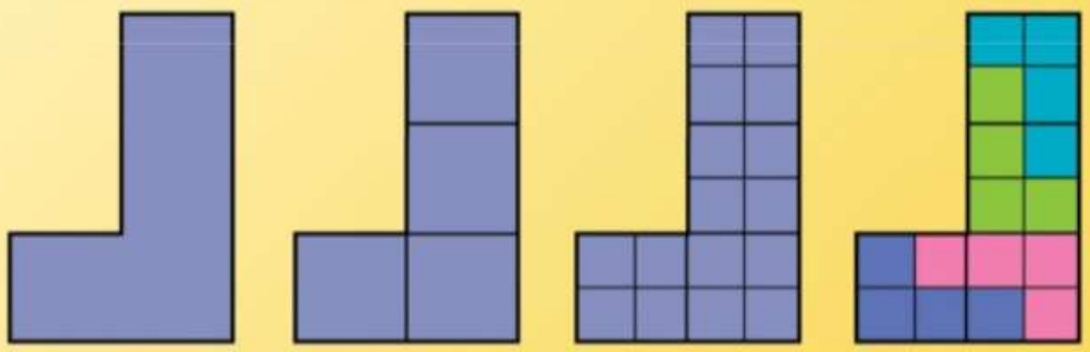
Lots of shapes are rep-tiles.* The most common rep-tiles split into ***four*** smaller copies like the ones below. These are called 4-rep-tiles.

Sometimes it can be very difficult to figure out how to split a rep-tile into congruent parts.

*ALL TRIANGLES AND RECTANGLES ARE REP-TILES, BUT NOT *ALL* SHAPES ARE REP-TILES. FOR EXAMPLE, IT IS NOT POSSIBLE TO SPLIT A CIRCLE INTO SMALLER CIRCLES.

There are some tricks which can make it easier.

Splitting a rep-tile into squares or triangles usually makes it easier to see how to divide it into smaller copies of itself.



Eeegad! Did someone turn off the--

EAAAAAAH!

Bwah Ha Ha! Professor Grok is gone. I've abducted your educator. It's time for something much more diabolically difficult! ***Brain-Rattling Rep-tiles!***

Recognizing repeated triangle tilings is terribly trivial!

Squares are certainly simple!

But pinpointing pentagonal partitions can baffle the most brilliant beast.

And hexagonal rep-tile recognition can be ***completely confounding!***

To pinpoint the position of your professor, first find these five humble homes in a nearby neighborhood.

Each has a simple shape. Four are rep-tiles while one is not. Captured in the closet of the home whose profile proves impossible to rep-tile is your imperiled professor.

You must determine the right residence, because within every other edifice resides...

...a Rabble of Ravenous Reptiles!

Those houses are on my block!

We need to find which of these five shapes is ***not*** a rep-tile.

What did Professor Grok say right before the lights went out?

Eeegad!

No, ***before*** that.

He said that splitting rep-tiles into squares or triangles makes them easier to divide.

Can you discover which of the five shapes is not a rep-tile?

I see! The first two shapes can be split into squares.

The next two can be split into right triangles.

And this last one can be split into equilateral triangles!

But how does ***that*** help?

Huh?

Grogg! What are you doing?!

You're always doodling! Why don't you ever--

Wait! Grogg, you're a genius!

The first shape ***is*** a rep-tile!

Grogg divided it into 12 squares.

We can split it into four shapes if each shape has three squares.

Right, 3+3+3+3=12. But how can we make all four small shapes the same as the big one?

Try to figure out which four shapes are rep-tiles.

Like this!

We can tile the second one like this.

Or, like this!

They **both** work!

Shape 3 is a rep-tile too!

So is shape 5!

That leaves just the fourth one.

That must be where Professor Grok is!

To my neighbor's house!

Hi, Ms. Levans, may we look in your closets?

Sure, try the one in the kitchen.

Thank goodness you found me, little monsters!

Notes: reptile/rep-tile

① reptile:

has scales

lays eggs

GrOgg

② Rep-Tile: a shape that can be split into copies of itself.

Example:

grrrrrr

not a rep-tile