Contents: Chapter 6

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Chapter 6:
The Distributive Property

R&G
Order of Operations

Hey!

Huh?

Did you *forget* something?

I think I forgot how to add and multiply.

Take a look at this.

✓2+7×3 = 23

✓4×4-3×2 = 10

-6×5-1 = 33

✓17-5×2 = 7

What's wrong with it?

When I solve 2+7×3, I get 27. This paper says it's 23, and it's marked ***correct!***

Let me try again.

I add 2+7 to get 9, then multiply by 3 to get 27.

I see your problem. You ***added*** before you ***multiplied.***

Of course I added before I multiplied. The addition was ***first!***

2+7×3
=9×3
=27

Yes, but there are rules for solving math problems.* You don't ***always*** go from left to right.

In problems without parentheses, we multiply before we add or subtract.

~~2+7×3~~
= ~~9×3~~
= ~~27~~

2+7×3
= 2+21
= 23

*THESE RULES ARE CALLED THE *ORDER OF OPERATIONS.*

Why can't we just work from left to right?

Try this... Remember our cousin Horatio?

We call him "Spike."

Right. He's covered with them.

He ***scares*** me.

Horatio has three tails, each with 7 spikes.

He also has four shoulder plates, each with 2 spikes.

How many spikes does Horatio have all together?

Let's see. He has three tails, each with 7 spikes...

...plus four shoulder plates, each with 2 spikes.

3×7 + 4×2
= 21 + 8
= 29

That's 29 spikes!

That's right, but look what you did. You didn't work from left to right, you multiplied first!

You're right! I found the number of tail spikes, then I found the number of shoulder spikes, and I added those two numbers together.

That makes sense. If you just work from left to right, you get the ***wrong*** answer.

3×7 + 4×2
= 21 + 8
= 29

3×7+4×2

Hmmm...

...50?! That's not right.

$3\times7+4\times2$
$= 21 + 8$
$= 29$

$3\times7+4\times2$
$= 21 + 4\times2$
$= 25 \times 2$
$= 50$

You get two different answers. They can't ***both*** be correct.

That's why we need a rule!

Right! In ancient times, math beasts decided that we multiply before we add or subtract.

Does addition come before subtraction?

Nope. After multiplying, you add and subtract from left to right.

Ahhh! Let me try these problems again.

$2+7\times3$
$= 2+21$
$= 23$

$4\times4-3\times2$
$= 16-6$
$= 10$

$4+6\times5-1$
$= 4+30-1$
$= 34-1$
$= 33$

$17-5\times2$
$= 17-10$
$= 7$

Nice work! How about this one...

...remember the tiger lilies we planted last month?

They started a turf war with the snapdragons!

We planted ten lilies, and each one had four petals.

Sure. Each lily grew three more petals...

...but two lilies uprooted and moved to the park.

How many total petals are on the tiger lilies we have left?

We multiply the number of lilies we have left: 10-2...

...times the number of petals on each lily: 4+3.

10-2 is 8 lilies, and 4+3 is 7 petals each, so we get 8×7=56 petals.

How would you write that in math?

Easy.

Wait! *That* doesn't work!

10−2×4+3

Why not?

Because you would multiply first.

How could you fix it?

Parentheses!

Without parentheses, we get 5 petals.

We need to group the 10-2 and the 4+3!

10−2×4+3
= 10−8+3
= 2+3
= 5

(10−2)×(4+3)
= 8×7
= 56

You got it! Now, let's clean up this hallway!

No problem. We'll just go back the way we came.

Great. Your side is filthy!

Hey!

Ms. Q.

Big Rectangles

Have you all finished drawing your rectangles?

Here's mine.

Almost done... ...I want to make sure my lines are straight.

Rectangle?

I drew a tangled wreck.

Can you find the area of your rectangles?

What could we do to make finding the area of Lizzie's rectangle easier?

Mine is 7 squares tall...

...and 13 squares wide...

. . .

...so I need to multiply 7 times 13.

Can you think of a way to make finding the area easier?

What if we split Lizzie's rectangle into two smaller rectangles?

We can find the areas of both small rectangles.

Then, we can just add those areas together!

Great idea! What is the best way to split Lizzie's rectangle in two?

How would you split it?

This line separates her 7 by 13 rectangle into two rectangles.

One is 7 by 10, and the other is 7 by 3.

Both of ***those*** areas are easy to figure out!

7

10

3

7×10 is 70.

7×3 is 21.

So 7×13 is 70+21=91!

Do mine! Do mine!

My rectangle is 3 by ummm...

...52!

Can you find the area of Grogg's rectangle?

We can split it into smaller rectangles like this.

Five of them are the same... 3×10=30.

This little one on the end is 3×2=6.

3

10 10 10 10 10 2

So the five big rectangles have a total of 5×30=150 squares...

...plus these six on the end makes 156 total!

5×30=150

3×2=6

Instead of breaking it into lots of rectangles, we can just use two!

This big one is 3×50=150 squares...

...and then we add the six squares on the end to get 156.

3×50 = 150

3×2=6

It doesn't matter how you split up the rectangle, as long as you remember to add all the pieces to get the total area.

And when you split it into pieces, it is usually best to use ***multiples*** of 10.

Very good! Who's next?

Me! My rectangle is 9 by 18.

WHEN YOU MULTIPLY A NUMBER BY 10, THE NUMBER YOU GET IS CALLED A ***MULTIPLE*** OF 10. FOR EXAMPLE, THE NUMBERS 10, 60, AND 140 ARE ALL ***MULTIPLES*** OF 10.

How would you split Winnie's rectangle to find its area?

You can split it into one rectangle that is 9 by 10, and one that is 9 by 8.

9×10=90

9×8=72

If we add these two areas together, we get 90+72=162!

90+72=162

I did it differently. You can split the rectangle in half.

That makes two 9 by 9 squares...

9×9=81

9×9=81

...and doubling 81 gives us 162.

81×2 = 162

I guess you don't ***always*** need to use multiples of ten.

Not always, but it's ***usually*** a good idea. Can we find the area of yours next, Alex?

Sure. Mine is really easy!

Why is that?

It's a perfect 10 by 10 square.

PIRATE BOOTY

Shiver me timbers! You little monsters have all finished your sailin' ships!

'Tis time to christen your ships 'n set sail for a maiden voyage!

Squawk. Bon voyage!

Each of your ships will be loaded with treasure. If there be one thing every pirate ship needs, 'tis a hull full o' pirate booty!

Snicker Snicker

In this chest be gold doubloons of varyin' worth. Your goal be to transport coins 'cross the school fountain.

I don't think these are ***real*** gold.

Arrrr! They ***taste*** like chocolate!

Real gold doubloons don't have big numbers and smiley faces on them.

Each coin be stamped with a number tellin' you its worth. The bigger coins be worth more than the smaller ones. Choose a coin 'n start loadin' your ships.

I'll use these little 3's.

I'll use 7's.

I'll take the 9's!

Arrrrr! I'll load these 18's onto the SS Denominator.

Set sail, little buccaneers!

Grogg! Is your ship ***biting*** mine?!

Aye, you lily-livered landlubber! Prepare to be plundered!

Quit it! Give me that!

Time to tally your treasure 'n figure whose haul be worth the most.

My ship carried fifty-one of these 3's.

I have twenty-three 7's...

...I'm sure I loaded 25.

My ship carried nineteen 9's!

Arrrr! The SS Denominator hauled seven 18's...

...and plundered two 7's from the Coral Princess.

Can you figure out whose treasure is worth the most?

I'm not used to multiplying such big numbers.

Let's see, what is 3×51?

Arrrr! I'll make it easy! Hand over a coin, scurvy reptile!

Why?

Then, you'll have only 50 coins... and 3×50 is easy!

That's right! If I take one coin away, it's easy to multiply 3×50=150. Then, I can add the coin back to get 150+3=153!

My coins are worth 153!

Aye. Multiplyin' 3×51 be the same as multiplyin' 3×50 and addin' one more 3.

Now, what be the value of your twenty-three 7's, Winnie?

7×23...

...well, If I only had twenty 7's, I would have 7×20=140...

...but I have three more coins worth 7×3=21.

That makes the total value of all 23 coins 140+21=161!

'Tis the *distributive property!*

7×23 be equal to 7×20 plus 7×3.

Twenty-three 7's is the same as twenty 7's plus three 7's.

Aye. What be the value of your treasure, Alex?

My ship carried nineteen 9's.

If I had ten 9's, that would make 90. Nine more 9's is 81. Added together, I get...

...90+81 =171.

Arrrr! There be an easier way!

What if you had twenty 9's and I plundered one?

I get it! 9×20=180, so to solve 9×19, I can just take away a 9... 180-9=171!

That *is* easier!

Aye.

Excellent figurin'! Now, we'll examine Grogg's booty.

Oh, my *treasure!*

'Tis a mighty haul! Seven 18's and two 7's.

How much is Grogg's treasure worth?

'Tis a troublesome task. 7×18 is 7×10 plus 7×8, then we add these two 7's...

Arrrr! I've lost track. Be there an easier way?

I know. We can trade Grogg's seven 18's for eighteen of my 7's.

Hands off, *scallywag!*

They're worth the same, because 18×7=7×18!

All right, but I'll be keepin' me good eye on you.

If we add the two 7's he stole from ***my*** ship to the eighteen 7's here, that makes twenty 7's, or 20×7=140!

Arrrr! 'Tis the smallest haul of them all...

...I've failed as a pirate.

Don't beat yourself up about it, lad. Proper plunderin' takes plenty o' practice.

MATH TEAM ...with Fiona!

The semester's final math meet is this afternoon. We need to cover one important thing...

...the ***distributive property.***

It's good for multiplying big numbers.

For example, how would you multiply 6 times 27?

You could start with 6 times 20, then add 6 times 7.

6×20 is 120, and 6×7 is 42...

...so 6 times 27 is 120+42=162!

6×27
6×20 120
6×7 42
162

Great! You just used the distributive property!

Since 27 is the same as 20+7...

...6×27 equals 6×(20+7).

6×27
= 6×(20+7)

*REMEMBER TO USE PARENTHESES! 6×20+7 IS NOT THE SAME AS 6×27.

We can rewrite 6×(20+7) as 6×20+6×7.

This is called ***distributing*** the 6.

We multiply the 6 by the numbers we are adding inside the parentheses.

6×27
= 6×(20+7)
= 6×20+6×7

THE DISTRIBUTIVE PROPERTY TELLS US THAT 6×(20+7) IS EQUAL TO 6×20+6×7, SO WE CAN REWRITE 6×(20+7) AS 6×20+6×7.

Then, we add.

6×27

= 6×(20+7)

= 6×20+6×7

= 120+42

= 162

It's just like when we found the areas of big rectangles!

And when we counted our pirate treasure!

How would you solve this one?

41×8

Try it!

41×8

= (40+1)×8

= 40×8+1×8

= 320+8

= 328

We can distribute the 8 to the 40 and the 1.

It's 328!

Good! Try this one.

7×29

7×29

= 7×(20+9)

= 7×20+7×9

= 140+63

= 203

203!

Both ways work. You can replace the 29 with (20+9), or with (30-1), because they both equal 29.

How did you know 140+63 so fast?

Because 140+63 is the same as 140+60+3.

140+63

= 140+60+3

= 200+3

= 203

What about solving 7×29 this way?

7×29

= 7×(30-1)

= 7×30-7×1

= 210-7

= 203

Let's try something different.

How would you solve 17×5+3×5?

17×5 is the same as seventeen 5's...

...and 3×5 is three 5's.

We can add seventeen 5's plus three 5's to get twenty 5's!

20×5 is 100!

What does this have to do with the distributive property?

It *is* the distributive property!

Since we can rewrite (17+3)×5 as 17×5+3×5...

...we can also change 17×5+3×5 to (17+3)×5...

...which is just 20×5=100!

$= 17\times5+3\times5$

$= (17+3)\times5$

$= 20\times5$

$= 100$

Try this one...

...what do you get when you add nine 1's plus nine 2's plus nine 3's plus nine 4's?

WHEN YOU USE THE DISTRIBUTIVE PROPERTY TO WRITE 17×5+3×5 AS (17+3)×5, IT IS CALLED *FACTORING*.

$9\times1+9\times2+9\times3+9\times4$

90!

$9\times1 + 9\times2 + 9\times3 + 9\times4$

$= 9 + 18 + 27 + 36$

Can you find the shortcut that Winnie used?

9×1 + 9×2 + 9×3 + 9×4
= 9 + 18 + 27 + 36

$$\begin{array}{r} {}^{3}9 \\ 18 \\ 27 \\ +36 \\ \hline 90 \end{array}$$

She's right! How did you get that so fast, Winnie?

9×1 + 9×2 + 9×3 + 9×4
= 9×(1+2+3+4)
= 9×10
= 90

Instead of multiplying first and then adding, I rewrote the problem using the distributive property.

So, for this problem...

8×9+8×10+8×11

...instead of solving it this way...

8×9+8×10+8×11
= 72 + 80 + 88
= 152 + 88
= 240

$$\begin{array}{r} {}^{1\,1}152 \\ +88 \\ \hline 240 \end{array}$$

...I can rewrite it like this!

8×9+8×10+8×11
= 8×(9+10+11)
= 8×(30)
= 240

And now...

Time for the math meet.

Good luck!

...please welcome the Little Monsters of Beast Academy and their opponents, the Bots!

Today's math meet will consist of 6 questions...

...covering all the topics you have learned so far this year.*

*THIS INCLUDES TOPICS COVERED IN GUIDE AND PRACTICE 3A.

The first five questions are each worth one point. The final question is worth two. The team with the most points wins the meet. Is everyone ready?

Question 1: A scalene triangle has whole number side lengths. One side has length 2, and another has length 3. What is the perimeter of the triangle?

Try it!

We need to know the third side of the triangle.

The triangle is scalene, so all three sides have to be different lengths.

So the third side can't be 2 or 3...

...can it be 1?

1 is too short... 1+2=3.

See page 83 of Guide 3A to review the Triangle Inequality.

That means the third side has to be ***more*** than 3. The unknown side is the ***longest*** side of the triangle.

The short sides are 2 and 3...

...so the long side has to be ***less*** than 2+3=5.

It has to be 4!

Is 4 the correct answer to question 1?

The perimeter is 2+3+4=9.

DING!

Nine is correct! The Monsters take the early lead: one to zero.

Question 2: Compute 6×5×4×3×2×1.

We can ignore the 1. Multiplying a number by 1 gives us the number.

6×5×4×3×2

We can pair the 2 and the 5 to make 10, since multiplying by 10 is easy.

6×4×3×(2×5)
= 6×4×3×10

We should start with 6×3 to get 18.

(6×3)×4×10
= 18×4×10

Then, we can double twice to multiply by 4!

18×4×10
= 18×2×2×10

18×2 is 36, 36×2 is 72, and--

BZZZZZT!

720.

Correct! Bots tie the score at 1.

4
7
8
2
3
10

Question 3: Each team has been given a rectilinear shape with the dimensions shown. The shape can be cut into two pieces that can then be rearranged to form a square. What is the side length of the square?

Try it!

The bots are already cutting their shape!

I don't think we actually need to know how to cut it into two pieces that make a square.

Maybe there is an easier way to solve the problem.

SKRATCH SKRATCH

Do you see an easier way?

If we find the area of the original shape...

It's the same as the area of the square!

Because rearranging the pieces doesn't change the area of a shape!

We can split the shape into rectangles.

Its area is the sum of the areas of these three rectangles...

...50+12+2 is 64!

4
3 3×4 =12
1
5 5×10 =50
2 2
10

(5×10)+(3×4)+(2×1)
= 50 + 12 + 2
= 64

The area of the square is 64!

Since 8×8=64, the side length of the square is...

...8!

DING!

Correct! The Little Monsters retake the lead, 2 to 1!

Can you find the two pieces that can be rearranged to make an 8×8 square?

Question 4:
Rick skip-counts by 7's, starting at 7. Nick skip-counts by 9's, starting at 9. What is the only 2-digit number that Rick and Nick will both say?

BZZZZT!

Huh!?

Try it!

63.

Correct!

Of course! 63 is 7×9... ...or 9×7.

It's seven 9's...

...or nine 7's.

The score is tied again, with just two questions left.

Question 5:
Compute (1×44)+(3×44)+(5×44)+(7×44)+(9×44).

Try it!

We can use the distributive property!

One 44 plus three 44's plus five 44's plus seven 44's plus nine 44's...

...is the same as (1+3+5+7+9)×44!

$1\times44 + 3\times44 + 5\times44 + 7\times44 + 9\times44 = (1+3+5+7+9)\times44$

1+3+5+7+9! That's the sum of the first five odd numbers!

The sum of the first five odd numbers is 5 squared... 25!

$(1+3+5+7+9)\times44 = 25\times44$

SEE PAGES 64-65 FOR REVIEW.

Now we just need to multiply 25×44.

I got it!

What is it!?!

I cut two pieces that can be rearranged to make a square!

Grogg! That was ten minutes ago! We need to multiply 25×44!

Did you try 25×40+25×4?

25×44
= 25×(40+4)
= 25×40 + 25×4

That's a great idea! 25×4=100...

...and 25×40 is--

BZZZZZT!

= 25×40 + 25×4
= + 100

1,100.

Correct!

The bots have taken the lead! The last question is worth 2 points and will determine the math meet winner. *Good* luck, teams!

19

Question 6:
Four congruent rectangles are arranged to form a square that has side length 19 as shown. What is the perimeter of one of the rectangles?

Try it!

SHAPES THAT ARE THE SAME SIZE AND SHAPE ARE CONGRUENT.

How can we find the side lengths of the rectangles?

There are too many possibilities!

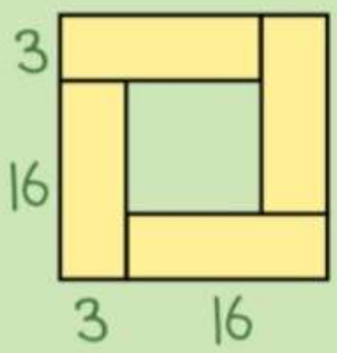
The short side of the rectangle could be 3 if the long side is 16...

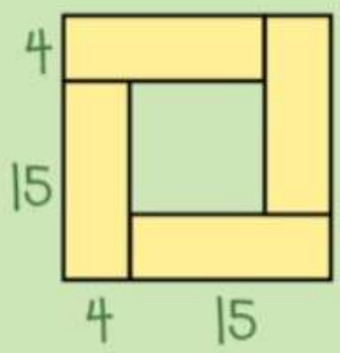
...or the short side could be 4 if the long side is 15...

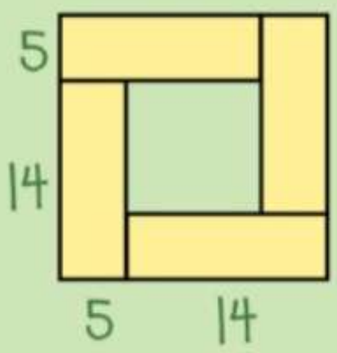
...or the sides could be 5 and 14...

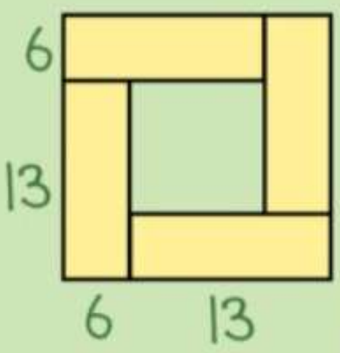
...or 6 and 13...

...how do we know which one to use?









IF YOU'RE STUCK ON A PROBLEM, DO SOMETHING!
TRYING SOME POSSIBLE ANSWERS CAN HELP YOU SEE PATTERNS THAT HELP YOU GET UNSTUCK.

It doesn't matter!

The rectangles you labeled all have the ***same perimeter!***

Find the perimeter of each rectangle Lizzie labeled.

Are you sure? For the first one, the rectangle has a perimeter of 16+3+16+3=38.

16

3

And this one has a perimeter of 15+4+15+4=38.

15

4

This one has a perimeter of 14+5+14+5 =38 too!

How did you know they all have the same perimeter, Winnie?

14

5

One short side and one long side of each rectangle always add up to the side length of the square...

...19!

19

short side →

long side →

long side

short side

Each rectangle has two short sides and two long sides...

...so the perimeter of each rectangle is 19+19!

DING!

38!

Correct!

Great Job, Little Monsters!

Today's math meet champions are the Little Monsters of Beast Academy!