Chapter 8:
Strategies (+&−)

WORKSHOP

ADDING AND SUBTRACTING

Welcome aboard, little buccaneers.

Cool boat, Captain Kraken!

Do you ever use your ship for anything besides treasure hunting, Captain Kraken?

Aye. Sometimes I ferry passengers around Beast Island.

Just last weekend, I set sail from Beast Bay with 36 passengers.

Our first stop be Hydra Harbor, where we dropped off 17 monsters 'n' picked up 17.

After that, you had 36-17+17 passengers?

Aye. How many passengers be left on me ship?

36-17+17=

How many?

If you started with 36 passengers...

...and the same number got off as got on...

...then you still had 36 passengers.

Oh, yeah. Subtracting 17, then adding 17 is the same as doing nothing.

36-17+17=36

Well done. We left Hydra Harbor with 36 passengers aboard.

Alas, a huge wave washed 18 passengers into the sea.

Splash!

We quickly threw rescue nets 'n' ropes...

...'n' 19 monsters climbed aboard.

So, you had 36-18+19 passengers after that?

Aye. How many passengers be on me ship after the rescue?

36-18+19=

How many?

You rescued more passengers than you lost!

Since 1 more monster climbed on than washed off, the number of passengers aboard increased by 1.

So, you went from 36 to 37 passengers.

36-18+19 is 37.

36-18+19=37

Aye. We rescued all 18 of our passengers, plus 1 monster whose boat be flipped by the wave.

We arrived at Pixie Port with 37 passengers.

25 new passengers climbed aboard...

...'n' 37 passengers left the ship.

After that, you had 37+25-37 passengers?

Aye. How many passengers be aboard, then?

37+25-37

How many?

37+25 is 62...

...and 62−37 is 25.

So, you had 25 passengers.

I think there's an easier way to figure out 37+25−37.

$$37+25-37$$

$$= 62-37$$

$$= 25$$

Instead of having 25 passengers climb aboard, ***then*** dropping 37 passengers off...

...you could have dropped the 37 passengers off...

...***before*** the 25 new passengers climbed aboard.

That wouldn't change the final number of passengers aboard.

$$37+25-37 \qquad 37-37+25$$

So, we can switch the plus 25 and the minus 37.

That gives us 37−37+25.

Are these two expressions equal?

That does make the subtraction easier.

Since 37−37 is 0, 37−37+25 is 25.

$$37+25-37 = 62-37 = 25$$

$$37-37+25 = 0+25 = 25$$

Aye.

Addin' 25 then takin' away 37 be the same as takin' away 37 then addin' 25.

We left for the Yeti Cove with 25 passengers aboard.

At Yeti Cove, 9 passengers got off, 'n' 25 climbed aboard.

After that, you had 25−9+25 passengers?

Aye. How many passengers be on me ship then?

$$25-9+25$$

How many?

25-9 is 16...

...and 16+25 is 41.

$$25-9+25 = 16+25 = 41$$

I tried switching the numbers around, but I got a different answer.

$$25-25+9 = 0+9 = 9$$

What was Alex's mistake?

Taking away 9 then adding 25 is ***not*** the same as taking away 25 then adding 9.

Is there a different way you can switch the order?

Taking away 9 then adding 25 ***is*** the same as adding 25 then taking away 9.

I see.

You can only switch the order of the 25 and the 9 if you keep the + and the − signs with them.

$$25-9+25 = 16+25 = 41$$

$$25+25-9 = 50-9 = 41$$

So, we can change the order of addition and subtraction...

...as long as we keep the + and − signs with the numbers that are being added or subtracted.

Aye, 'n' if you're unsure, 'tis always safe to work from left to right.

WELCOME to BEAST BAY

BROCHURES

We returned to Beast Bay safely with 41 passengers aboard.

Sounds like the trip was a success!

Aye. 'Tis good to get a break from piratin' work now 'n' then.

This weekend, I'll be takin' tourists on a manatee-rex watching tour!

CAPTAIN KRAKEN'S MANATEE-REX WATCHING

View the majestic Manatee-Rex natural habitat!

VISIT THE GIFT SHOP!

Novelty Tees! 3 for $15 or $6/ea.

Dino-Cow of the Sea

I ♥

RECESS

Zero-Sum Game

Zero-Sum is a 2-player card game in which players score points by creating expressions that equal zero.

Setup

The game uses a standard deck of playing cards with the face cards (K, Q, J) removed. Aces are treated as 1's.

Black cards stand for addition. For example, a black 9 stands for +9.
Red cards stand for subtraction. For example, a red 7 stands for −7.
Or, find printable cards with + and − symbols at BeastAcademy.com.

Shuffle the deck and deal 4 cards face down to each player. Place the remaining cards face down in a pile called the stock. Turn four cards from the stock face up. These are the draw cards.

Play

Players take turns. On a player's turn, the goal is to create an expression that equals zero using one of the draw cards and one or more cards in their hand. The number of cards used in the expression is the number of points scored on their turn.

For example, a player can combine the red 2♦ (−2) from the draw cards above with cards from their hand to create an expression that equals zero. Several examples are given below.

If a player can create an expression that equals zero, the cards used in the expression are placed in their score pile. Cards from the stock are used to replace the draw card and to fill the player's hand to 4 cards. This ends their turn.

If a player cannot create an expression that equals zero, they draw one card from the stock and end their turn.

Winning

The game ends after all of the cards in the stock have been drawn and neither player can play an expression.

The winner is the player with the most cards in their score pile.

MATH TEAM

Evaluating Expressions

What rules have you all learned for adding and subtracting?

You can ***add*** numbers in any order.

$$44+39+56$$
$$= 100+39$$
$$= 139$$

If an expression has addition and subtraction...

$$94+18-34$$
$$= 112-34$$
$$= 78$$

...you can always get the right answer by working from left to right.

You can even rearrange addition and subtraction...

...as long as you keep the + and − signs with the numbers you are adding and subtracting.

$$94 + 18 - 34$$
$$= 94 - 34 + 18$$
$$= 60 + 18$$
$$= 78$$

It helps me to think of + and − signs attached to the numbers that come after them.

94 +18 −34

Then, I can rearrange them to make the addition and subtraction easier.

94 −34 +18

Good.

How could you make this expression easier to evaluate?

$$48 + 43 + 12 - 43$$

Try it.

We could switch the minus 43 and the plus 12.

48 +43 +12 -43

That way, the −43 is right after the +43.

48 +43 -43 +12

Adding 43 then subtracting 43 is the same as doing nothing.

48 +43 -43 +12

So, we can just get rid of these.

All that's left is 48+12=60.

48 +12

= 60

Excellent. Many expressions can be simplified by ***canceling***.

Adding 43 and subtracting 43 are opposites, so they cancel each other.

We can cross them out, then evaluate.

$$48 \cancel{+43} + 12 \cancel{-43}$$
$$= 48 + 12$$
$$= 60$$

The +43 and the −43 don't need to be next to each other to cancel?

That's right.

48 $+43$ -43 $+12$

How could we evaluate this expression?

$$87 + 88 - 78 - 88 + 77$$

Try it.

Adding 88 and subtracting 88 is the same as doing nothing.

So, we can cancel the +88 and the −88.

$87 \cancel{+88} - 78 \cancel{-88} + 77$

$= 87 - 78 + 77$

That leaves 87−78+77.

Subtracting 78 is *almost* the opposite of adding 77.

We're subtracting one more than we're adding.

So, subtracting 78 then adding 77 is the same as subtracting 1.

That gives us 87−1=86.

$87 \cancel{+88} - 78 \cancel{-88} + 77$

$= 87 - 78 + 77$

$= 87 - 1$

$= 86$

Great! Sometimes it's useful to pair addition with subtraction that *almost* cancels.

How could we evaluate this expression?

$54 - 43 + 33 + 44 - 34$

Try it.

We can pair the −43 and the +44.

Since we're adding 1 more than we are subtracting...

...subtracting 43 and adding 44 is the same as adding 1.

We can also pair the +33 with the −34.

Since we're subtracting 1 more than we are adding...

...adding 33 and subtracting 34 is the same as subtracting 1.

$$54 \underbrace{-43}_{} \underbrace{+33}_{} \underbrace{+44}_{} \underbrace{-34}_{}$$

$$= 54 \quad +1 \quad -1$$

And since adding 1 then subtracting 1 is the same as doing nothing...

...54+1−1 is just 54.

$$54 \underbrace{-43}_{} \underbrace{+33}_{} \underbrace{+44}_{} \underbrace{-34}_{}$$

$$= 54 \quad +1 \quad -1$$

$$= 54$$

I paired different numbers, but I still got the same answer.

Subtracting 43 then adding 33 is the same as subtracting 10.

Adding 44 then subtracting 34 is the same as adding 10.

Then, 54−10+10 is 54.

$$54 \underbrace{-43}_{} \underbrace{+33}_{} \underbrace{+44}_{} \underbrace{-34}_{}$$

$$= 54 \quad -10 \quad +10$$

$$= 54$$

Excellent!

Try one more. How would you evaluate this expression?

$10-1+20-2+30-3+40$

$10-1+20-2+30-3+40$

$= 9 \quad +18 \quad +27 \quad +40$

This doesn't help much.

$10-1+20-2+30-3+40$

$= 10 \quad +19 \quad +28 \quad +37$

Neither does this.

$10-1+20-2+30-3+40$

I guess we can just work from left to right.

10−1 is 9...

...plus 20 is 29...

...minus 2 is 27...

I know a way to make it easier!

What would you do?

We can do all of the addition first.

10+20+30+40 is 100.

Then, subtracting 1, then 2, then 3, we subtract a total of 6.

And 100−6 is 94!

$$10-1+20-2+30-3+40$$

$$= 10+20+30+40-1-2-3$$

$$= 100 - 6$$

$$= 94$$

Great work, Winnie.

Did you find a different way to rearrange the expression, Grogg?

Nope, but I did rearrange some of my drawings!

Ms. Q. Parentheses

What are parentheses used for in math?

(23+34+45)-(45+34+23)

Parentheses tell you what to evaluate first.

That's right.

How would you evaluate this expression?

(23+34+45)-(45+34+23)

We can start by adding the numbers in the parentheses on the left.

23+34+45 is...

(23+34+45)-(45+34+23)

Wait. You don't need to add!

How would you evaluate it?

We're adding the same numbers in the left parentheses as in the right parentheses.

So, we just end up subtracting a number from itself.

That *always* gives us zero!

(23+34+45)-(45+34+23)

= a number - the same number

= 0

It's good to look carefully at what's in parentheses before you start evaluating.

That made things a lot easier!

Nice job, Grogg!

How would you evaluate this expression?

85-(71-28)+(71-28)

Try it.

Subtracting (71−28) then adding (71−28) is the same as doing nothing.

So, we can cancel these. We're left with 85!

$$85 - (71 - 28) + (71 - 28) = 85$$

Good. Since the expressions in parentheses are the same...

...the addition and subtraction cancel.

Even when the expressions in parentheses are different...

...you can sometimes find a clever way to evaluate.

How about this expression?

How would you evaluate it?

$$(27+28+29) - (26+27+28)$$

Try it.

Both parentheses have 27+28 in them.

On the left is (27+28+**29**)...

...and on the right is (**26**+27+28).

$$(27+28+29)-(26+27+28)$$

Since 29 is 3 more than 26...

...(27+28+**29**) is 3 more than (**26**+27+28).

So, (27+28+29) minus (26+27+28) is 3!

$$(27+28+29)-(26+27+28) = 3$$

I got the same answer a different way.

Each of the numbers on the left is 1 more than a number on the right.

So, the difference between (27+28+29) and (26+27+28) is 1+1+1=3.

$$(27+28+29)-(26+27+28) = 3$$

Parentheses are a lot like eyebrows.

How so, Grogg?

They're shaped a lot like eyebrows...

...and moving them can ***really*** change an expression.

(27+28+29) - 26+27+28

R & G

Skip-Counting

Mopping isn't going to help this floor.

Nope.

It's time to replace these tiles. Help me count them.

I'll start with the green ones. There are...

...1, 2...

...3, 4, 5...

...6...

...7, 8...

...10, 12, 14, 16, 18, 20, 22, 24!

Since all of the green tiles are in pairs...

...I counted them 2 at a time.

Whoa! That was quick.

How'd you do that?

To count by 2's, you just skip every other number.

Oooh. That's smart.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32...

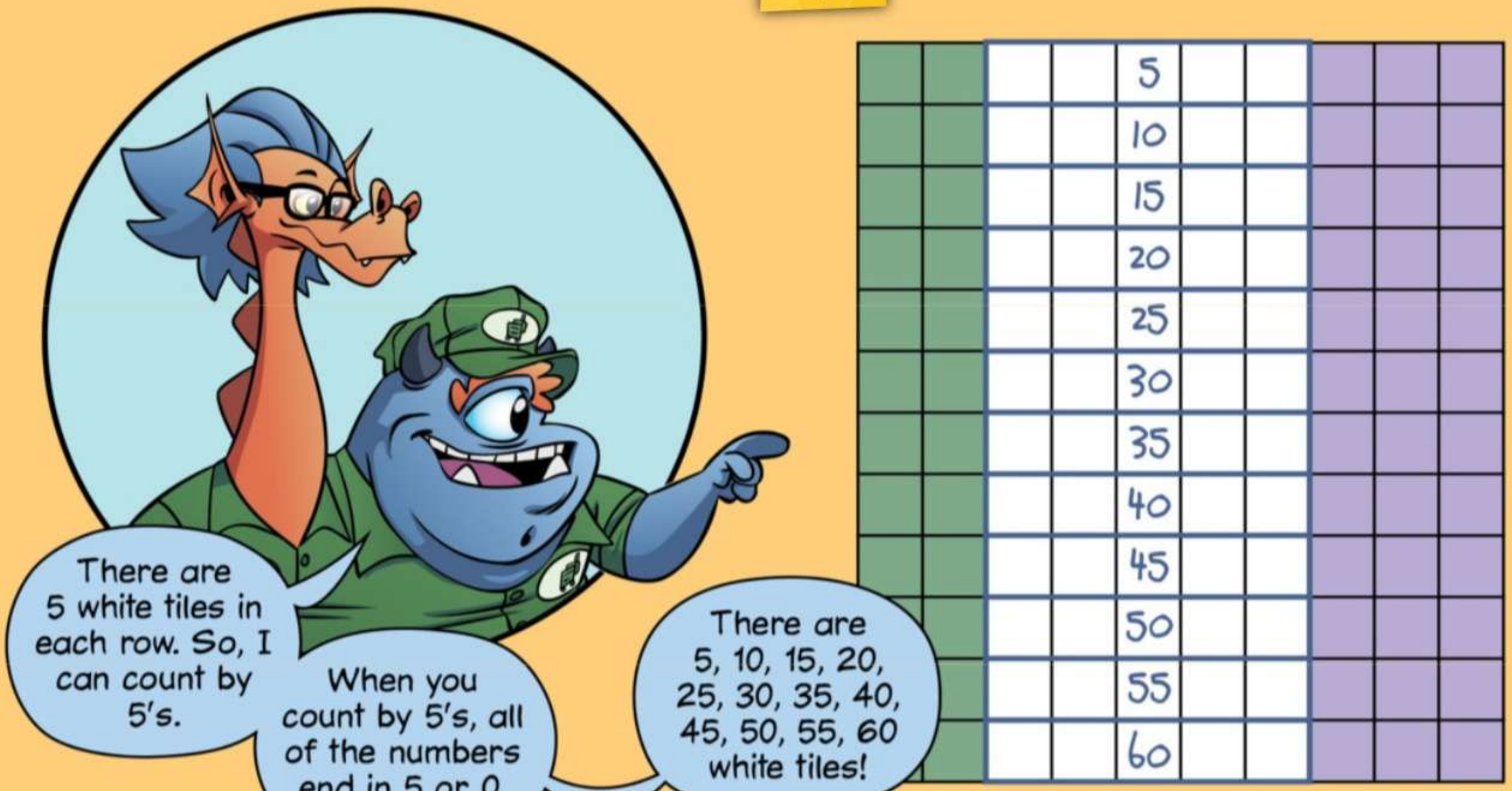
...I could go on forever.

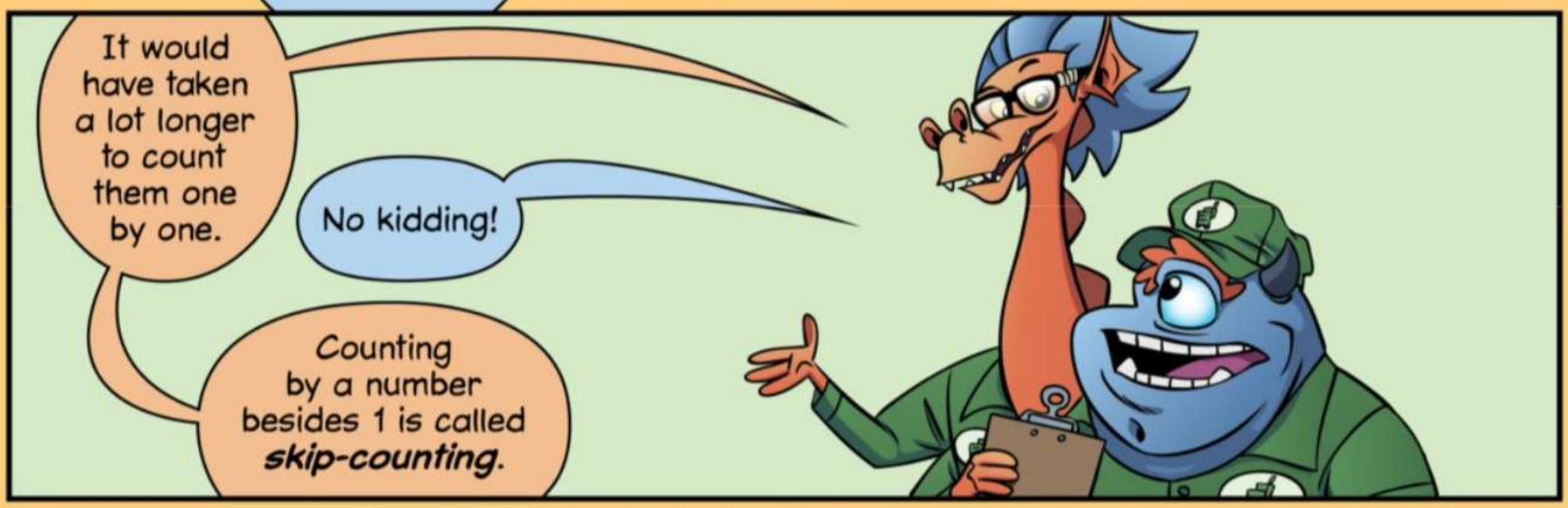
Yep. When we count by 2's, we skip every number that ends in 1, 3, 5, 7, or 9.

You can count by other numbers, too.

Try counting the white tiles.

Try it.


5
10
15
20
25
30
35
40
45
50
55
60
There are 5 white tiles in each row. So, I can count by 5's.
When you count by 5's, all of the numbers end in 5 or 0.
There are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60 white tiles!


It would have taken a lot longer to count them one by one.
No kidding!
Counting by a number besides 1 is called *skip-counting*.


I'll count the purple tiles next.
Try it.

Counting by 3's is not as easy as counting by 2's or 5's.

3, 6, 9...

...12, 15, 18...

...21, 24, 27, 30...

...33, 36.

I count 36 purple tiles.

3
6
9
12
15
18
21
24
27
30
33
36

That's 24 green tiles, 60 white tiles, and 36 purple tiles.

Counting by 2's, 5's, and 10's is easy.

3's are a little harder, but with a little practice, you can learn to skip-count by just about any number.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ...
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ...
5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, ...
10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, ...

9+9+9+9+9+9+9=

You can skip-count to solve addition problems, too.

For example, you can skip-count to add seven 9's.

Try it.

+10
-1
9
18 19
+9
Hmmm... Adding 9 is the same as adding 10, then taking 1 away.
9+9+9+9+9+9+9=63
9
18
27
36
45
54
63
So, to skip-count by 9's, I increase the tens digit by 1 and decrease the ones digit by 1.*
Adding seven 9's gives me... 9, 18, 27, 36, 45, 54, 63!
*THIS ONLY WORKS UP TO 90. WHAT HAPPENS AFTER 90?
You got it. Now, let's get rid of these old tiles.