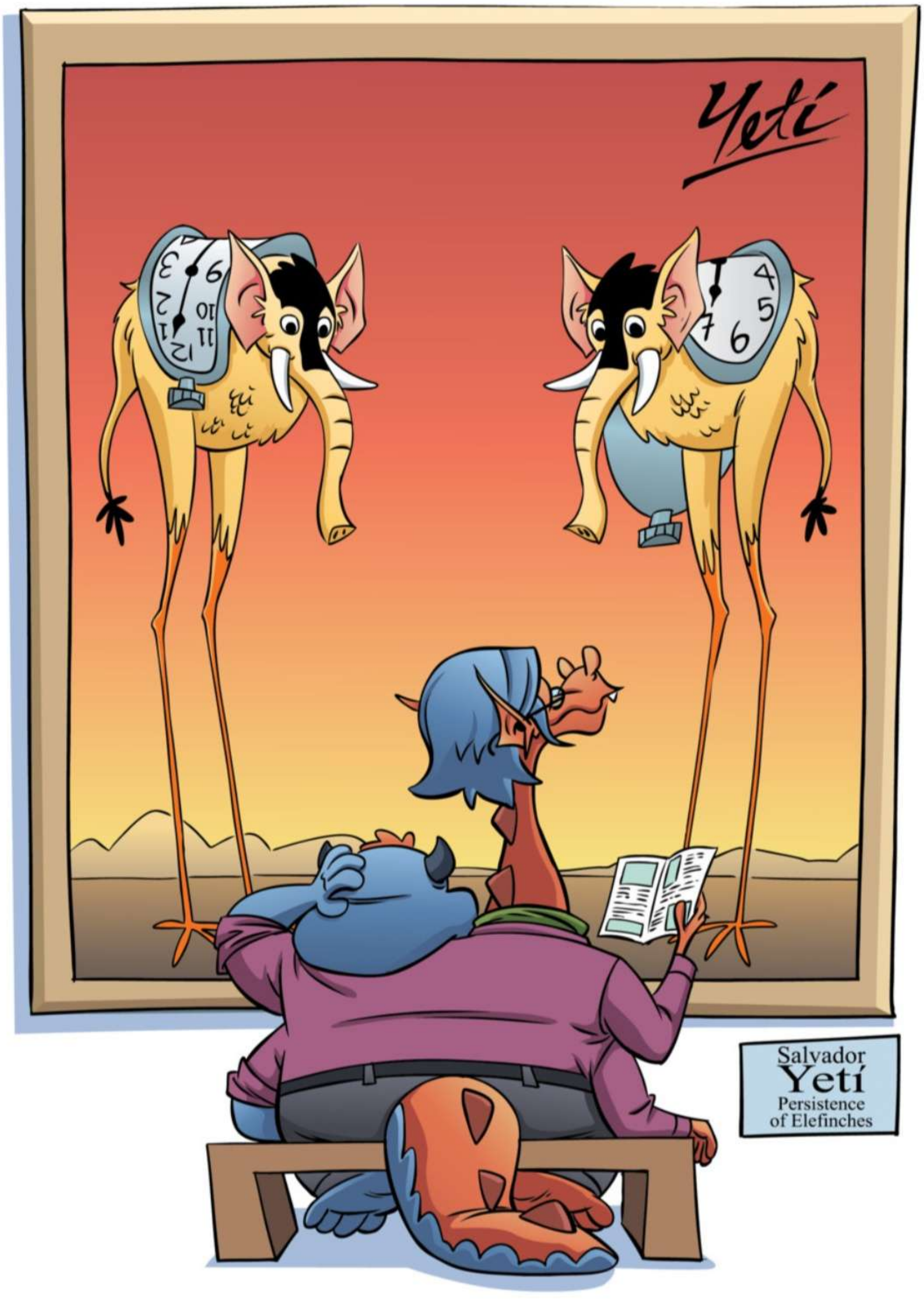
Contents: Chapter 1

Click the Play List tab in the top-left to view a recommended reading/practice sequence.

Chapter 1:
3D Solids




R&G
3D
You know, we really don't get enough culture in our lives.
Yeah, this is nice.
BLOB ROSS
BEAST ISL
ART MUS
SALVADOR YETÍ
n heads
= n tickets
One ticket, please.
Read the sign.
SALVADOR
YETÍ
THE EXHIBITION
Aw, man!
I hear they've opened a fantastic new 3D exhibit.
3D?
3D stands for three-dimensional.
Huh?
BLOB ROSS
HAPPY TREES
EXHIBIT

A *dimension* is a measurement in one direction.

Example?

1D EXHIBIT

This line segment has one dimension. We can only measure its length.

Other objects have *two* dimensions.

2D EXHIBIT

Take this rectangle, for example...

...it has two dimensions: height *and* width.

I get it. So all of the shapes in this display are two-dimensional.

You got it!

2D OR NOT 2D?

So, flat shapes are all two-dimensional?

Right.

A flat, 2D shape is sometimes called a ***plane shape***.

Plane shapes have height and width, but no thickness.

PLANE IS ANOTHER WORD FOR AN ENDLESS FLAT SURFACE.

I see. So, a ***3D*** object has ***three*** dimensions.

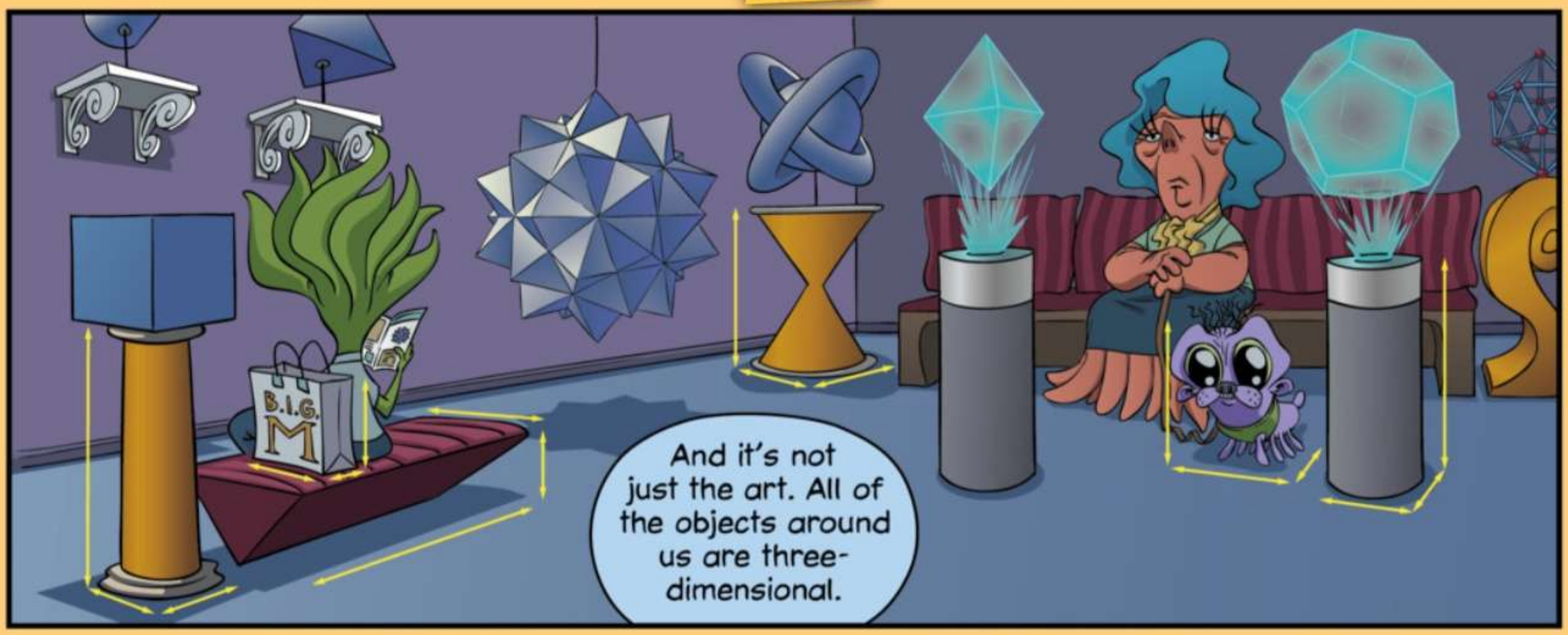
3D EXHIBIT

Yep. Let's check out the 3D exhibit!

Cooool.

They take up space in three dimensions!

All of these pieces are three-dimensional ***solids***.


B.I.G. M
And it's not just the art. All of the objects around us are three-dimensional.


A N D
WARTHOG
EXHIBITION
15 MINUTES ONLY
I wonder what's in here.
ØD
EXHIBIT


It's just a tiny speck.
I think that's the *point*.
I don't get art.

G*Y*M
SOLIDS

Line up, polliwogs!

Today's physical training will help you improve your balance, stamina, and coordination.

In front of you is an obstacle course made of prisms, pyramids, and various other ***geometric solids***.

???

Some of you have not learned to identify the major groups of three-dimensional solids.

Who can tell me which one of these obstacles is a rectangular prism?

Sir, the big blue one, sir.

Very good, Tinkerbell.

What makes that solid a rectangular prism?

Sir, a ***prism*** is a solid with two congruent ***faces*** that are parallel.

These congruent faces are called the ***bases*** of the prism.

We name a prism by the shape of its bases.

The bases of this prism are rectangles, so it is a rectangular prism, sir!

THE FLAT SIDES OF A GEOMETRIC SOLID ARE ITS ***FACES***. TWO FACES ARE ***PARALLEL*** IF THEY ARE IN PLANES THAT NEVER CROSS.

Excellent. You will begin my obstacle course by climbing up one face of the rectangular prism.

After climbing down the opposite face, you will continue to this set of six smaller prisms.

Hop from the triangular prism...

...to the rectangular prism...

...to the pentagonal prism...

...to the hexagonal prism...

...to the heptagonal prism...

...and finally, onto the octagonal prism.

I see. This octagonal prism has an octagon on top...

...and the same octagon on the bottom.

The octagons are the bases...

...and the bases are connected by rectangles.

The next obstacle will test your balance.

What do we call this ***polyhedron***, Hammerhead?

What do we call it?

A GEOMETRIC SOLID WHOSE FACES ARE ALL FLAT IS CALLED A ***POLYHEDRON***.

Sir, it's a triangular prism, sir!

Excellent. Regardless of which face is on the bottom...

...a triangular prism is still a triangular prism, and its bases are still triangles.

Maintain your balance while walking across the ***edge*** of this triangular prism.

EDGE

THE LINES WHERE THE FACES OF A SOLID MEET ARE CALLED ITS ***EDGES***.

Next, you must cross this moat using these five pyramids.

The ***base*** of a ***pyramid*** can be any polygon.

The remaining faces of a pyramid are triangles that meet at a single ***vertex***.

BASE

VERTEX

A ***VERTEX*** IS A POINT WHERE EDGES MEET.

The vertex where all of the triangular faces meet is called the ***apex*** of the pyramid.

APEX

Hop from the square pyramid...

...to the pentagonal pyramid...

...to the hexagonal pyramid...

...to the heptagonal pyramid...

...to the octagonal pyramid...

...to the opposite side of the moat.

A PYRAMID IS NAMED BY THE SHAPE OF ITS BASE.

You must complete the remaining obstacles as a team.

After you have all crossed the moat, you must place this ***sphere***...

...atop this ***cylinder***.

And finally, you must work as a team to place a flag on the apex of this ***cone***.

So... ***sphere*** is the math word for ball.

And a ***cylinder*** is like a prism... only its bases are circles.

And a ***cone*** is like a pyramid... only its base is a circle.

You got it, Grape Ape.

Now, line up on the starting line.

You will be timed.

Are there any questions?

Go!

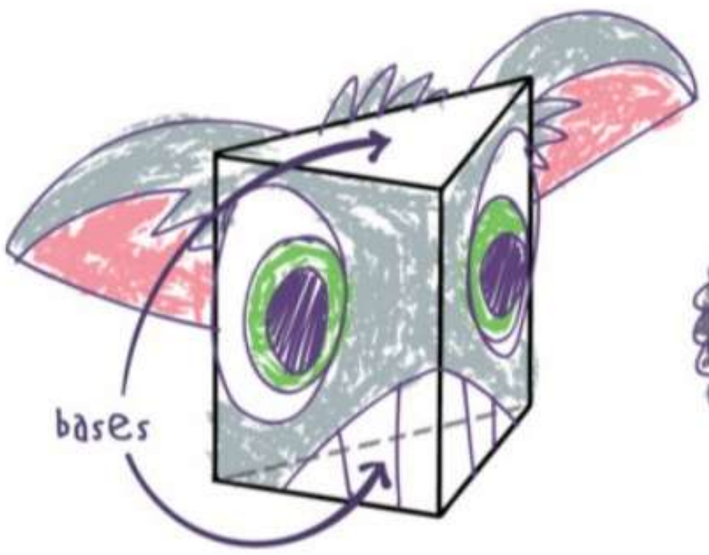
3D Solids

Prism:

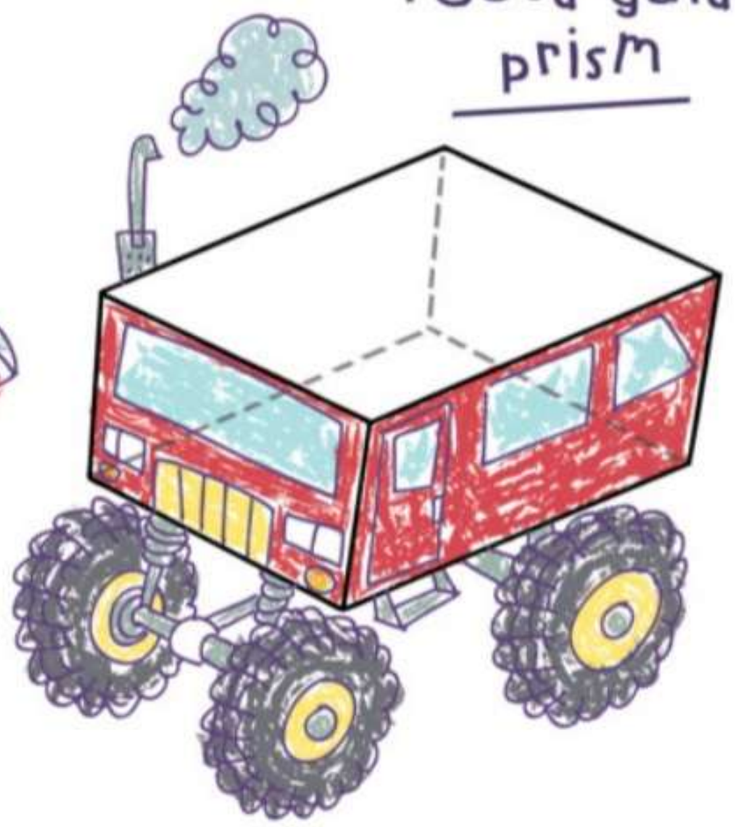
Has two congruent parallel faces.
Other faces (lateral faces) are all rectangles.
Prisms are named by the shape of their bases.

- All of the prisms in this book are called right prisms and have lateral faces that are rectangles. But, prisms can have lateral faces that are parallelograms.

triangular prism

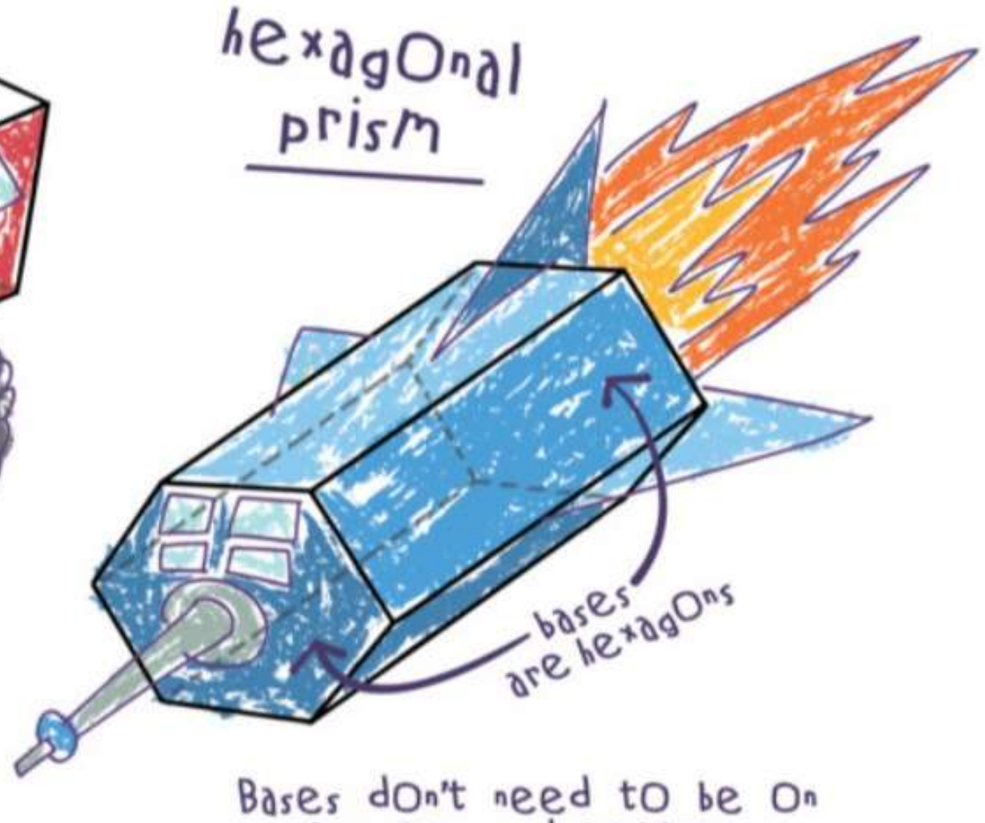


rectangular prism



Any pair of opposite faces can be used as the bases of a rectangular prism.

hexagonal prism

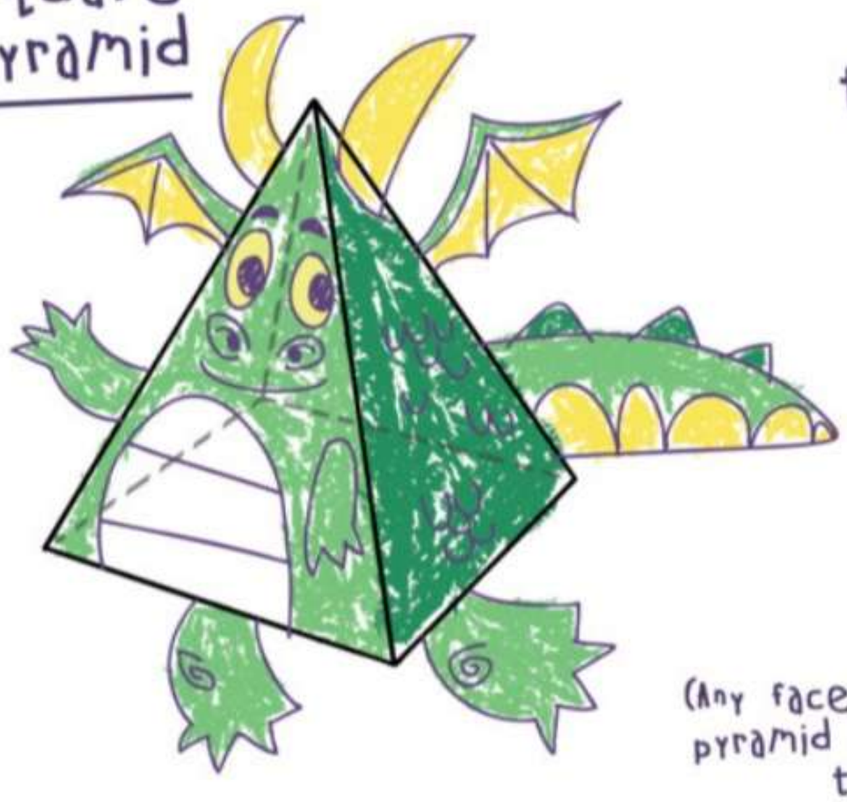


Bases don't need to be on the top and bottom.

Pyramid:

Any polygon can be the base of a pyramid.
All the other faces are triangles that meet at a vertex.

square pyramid



triangular pyramid

(Any face of a triangular pyramid can be used as the base.)

octagonal pyramid

cylinder:
Like a prism, but with a circular base.

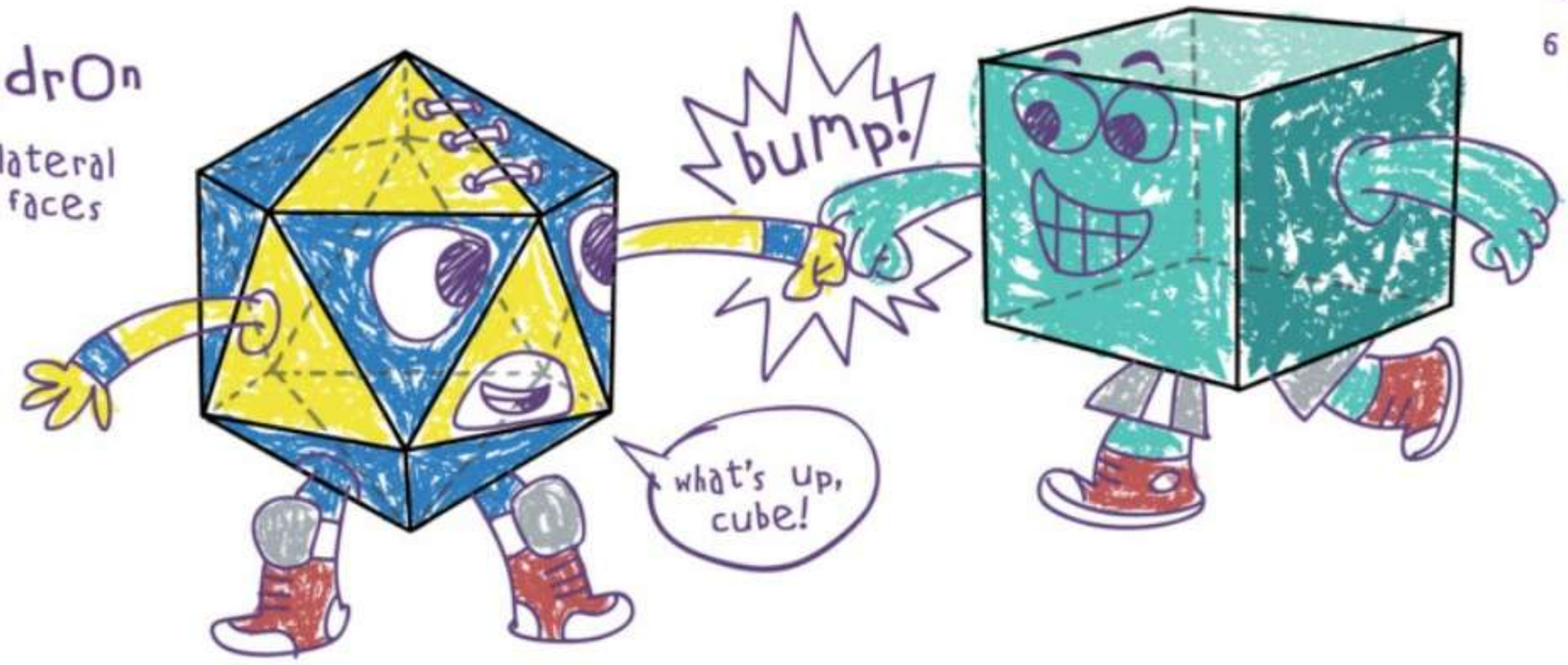
sphere:
A ball. Every point on its surface is the same distance from its center.

cone:
Like a pyramid, but with a circular base.

Five Platonic Solids:
All regular faces. All faces congruent. same # of faces meet at each vertex.

icosahedron
20 equilateral triangle faces

cube
6 square faces

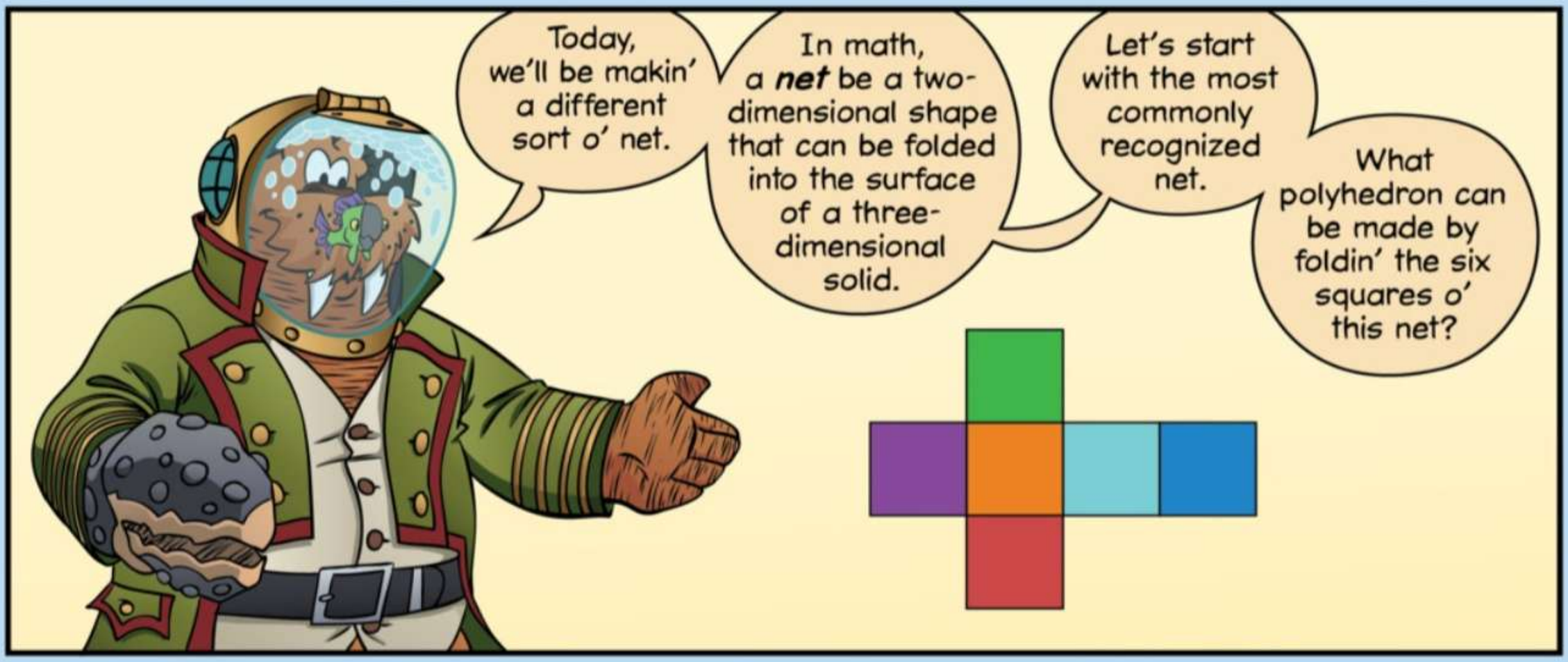


tetrahedron
4 equilateral triangle faces

octahedron
8 equilateral triangle faces

dodecahedron
12 regular pentagon faces


NETS
Ahoy, me hearties!
Out at sea, 'tis hard to find anything more useful than a good net.


Today, we'll be makin' a different sort o' net.
In math, a **net** be a two-dimensional shape that can be folded into the surface of a three-dimensional solid.
Let's start with the most commonly recognized net.
What polyhedron can be made by foldin' the six squares o' this net?

A polyhedron with six square faces is called a ***cube***.

We can fold this net into a cube.

A ***CUBE*** IS A SPECIAL TYPE OF RECTANGULAR PRISM IN WHICH ALL SIX FACES ARE SQUARES.

Aye. The net of a cube be havin' six squares.

There be many other ways to join six squares to form the net of a cube.

Which o' these four arrangements o' squares can be folded to make a cube?

Which ones?

PRINT COPIES OF THESE AND MANY OTHER NETS FOR CUTTING AND FOLDING AT BEASTACADEMY.COM.

All four can be folded into cubes.

Cool! Cubes are awesome!

Can any arrangement of six connected squares be folded into a cube?

Nope. This straight strip of six squares can't be folded into a cube.

Neither can this one.

The green square and the orange square overlap when I try to fold it into a cube.

This one doesn't work at all.

Aye. Not every arrangement o' six squares can be folded into a cube.

Let's look at some other nets.

Who can create a net for this regular ***tetrahedron***?

Try it.

A REGULAR ***TETRAHEDRON*** IS A PYRAMID WITH FOUR TRIANGULAR FACES THAT ARE ALL EQUILATERAL.

The four faces of a regular tetrahedron are equilateral triangles.

If we arrange them like this, we can fold the three outer triangles up...

...to meet at the apex.

Be there another way to create a net for a regular tetrahedron?

I found one arrangement that doesn't work.

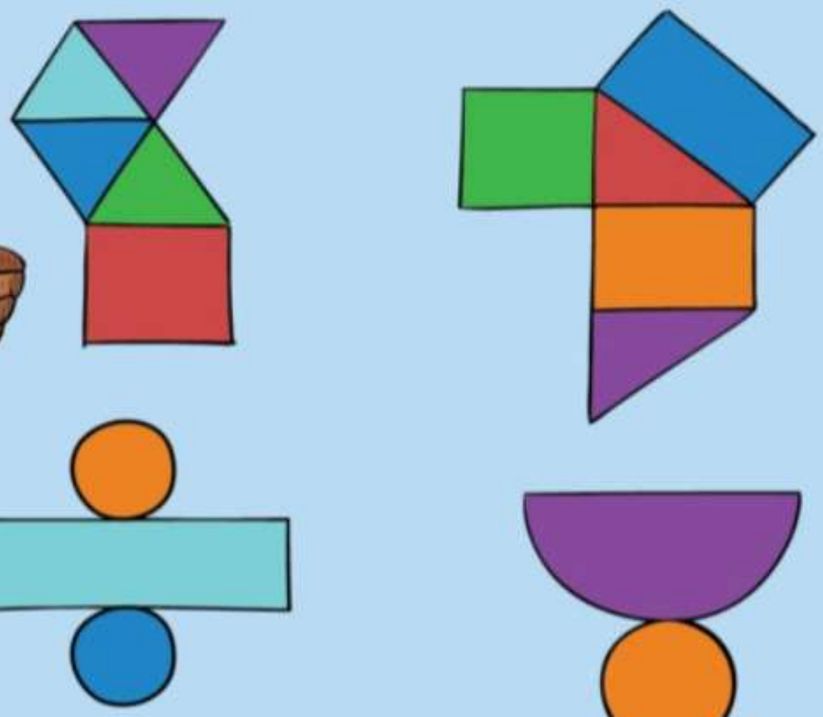
The red and green faces overlap.

But, if we arrange four triangles like this...

...they fold into a tetrahedron.

Excellent foldin'.

Here be four more nets. What solids be formed by foldin' the followin' nets?



What solids are formed by folding these nets?

We can fold the four triangles around the square to make a ***square pyramid*** with this net.

This net folds into a wedge, which is a ***triangular prism***.

We can wrap the rectangle into a tube and put circles on top and bottom to make a ***cylinder***.

And for this net, we can wrap up the half-circle and place it on top of the full circle to make a ***cone***!

Ye be a clever bunch.

Now let's build some bigger solids.

These three nets be made o' wood, hinged at the edges.

Help me fold 'em all.

The 8 faces o' this ***octahedron*** be equilateral triangles.

The 12 faces o' this ***dodecahedron*** be regular pentagons.

And this ***icosahedron*** be made from 20 equilateral triangles.

What are these all for?

Sergeant Rote be needin' more challengin' polyhedrons for his obstacle course.

PRINT THESE AND MORE NETS AT BEASTACADEMY.COM.

MATH TEAM
Surface Area

How much paper will it take to cover this box?

Yep.

The whole thing?

How many square inches of paper will it take?

It's a cube with 8-inch edges.

It has six square faces.

And each face is 8 inches by 8 inches.

So, we could cover the box with six 8-by-8 paper squares.

Each square has an area of 8×8=64 square inches.

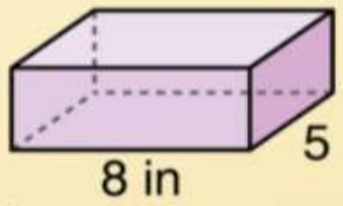
So, six squares have a total area of 6×64=384 square inches.

Very good. You've just solved your very first surface area problem.

The ***surface area*** of a solid is the total area of all its faces.

Since all six faces of a cube are the same, a cube's surface area is six times the area of one of its faces.

Let's try a slightly more difficult solid.

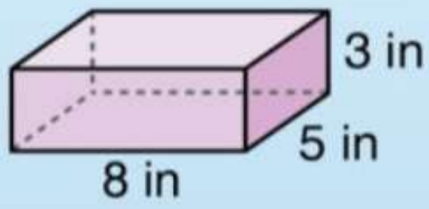


What's the surface area of this prism?

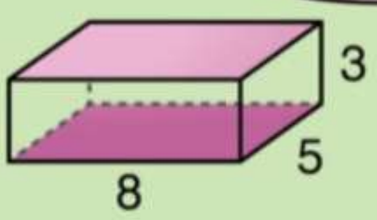
Like the cube, it has six faces...

...but the faces are not all the same.

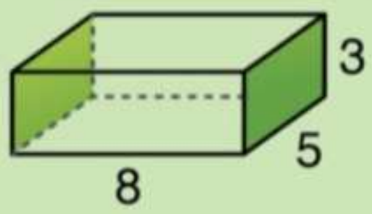
But the opposite faces of a rectangular prism are congruent.



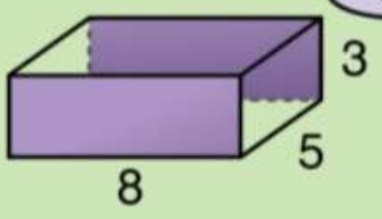
The top and bottom are both 5-by-8.



The left and right are both 3-by-5.



And the front and back are both 3-by-8.



Surfaces	Area	
Top & Bottom	5×8	×2
Left & Right	3×5	×2
Front & Back	3×8	×2

That's all six faces.

Compute the total surface area of the prism.

Now, we just compute the total area.

The top and bottom faces each have an area of 40 square inches, for a total of 40×2=80 square inches.

The total area of the left and right faces is 15×2=30 square inches.

And the front and back faces have a total area of 24×2=48 square inches.

Surfaces	Area			
Top & Bottom	5×8	×2	=	80 sq in
Left & Right	3×5	×2	=	30 sq in
Front & Back	3×8	×2	=	48 sq in

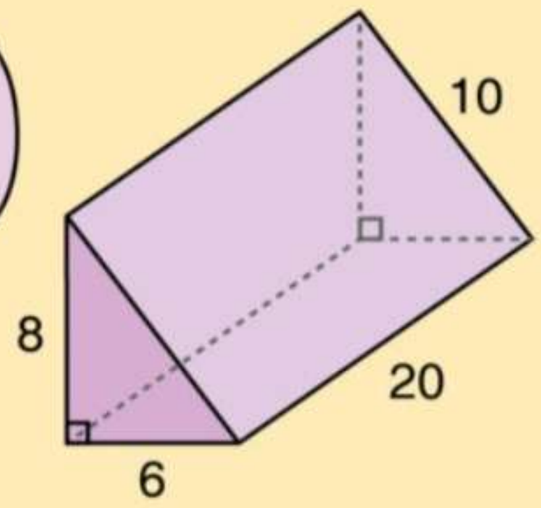
All together, that's 158 square inches.

Surfaces	Area			
Top & Bottom	5×8	×2	=	80 sq in
Left & Right	3×5	×2	=	30 sq in
Front & Back	3×8	×2	=	48 sq in
				158 sq in

Great work! Finding the surface area of solids is really all about staying organized.

This becomes even more important when finding the surface area of a triangular prism.

How would you compute the surface area of this prism?



Try it.

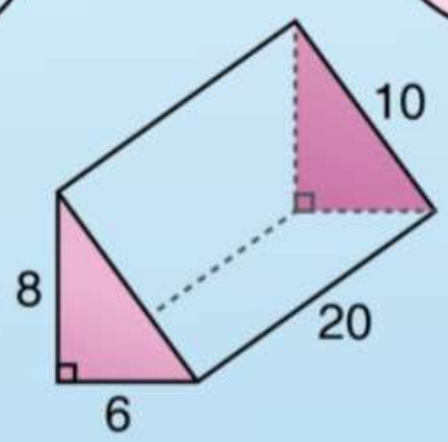
The triangular prism has five faces.

Two are right triangles...

...and three are rectangles.

We need to find the areas of all five faces and add them together.

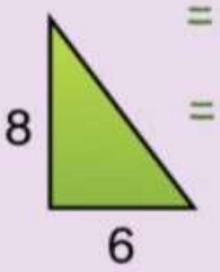
Let's start by finding the areas of the triangles.



To find the area of a triangle, we multiply the length of its base by its height and divide by two.

Each triangle has an area of 24 square units.

Area
$= 6 \times 8 \div 2$
$= 24$



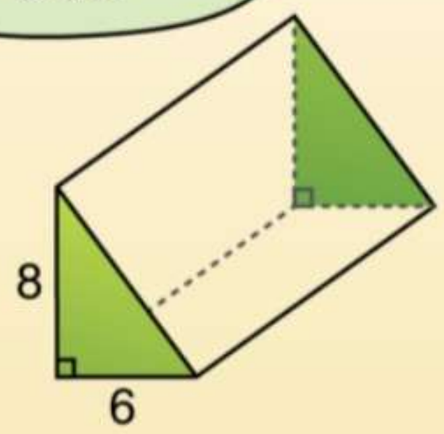
REVIEW TRIANGLE AREA IN GUIDE 3D.

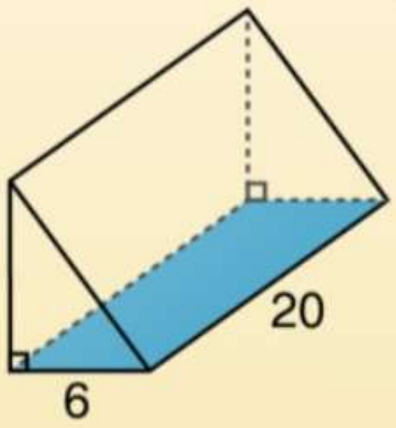
So, the two triangular bases have a total area of $24 \times 2 = 48$ square units.

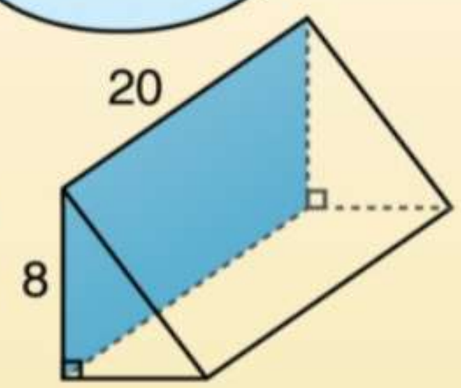
The rectangles have areas of $6 \times 20 = 120$...

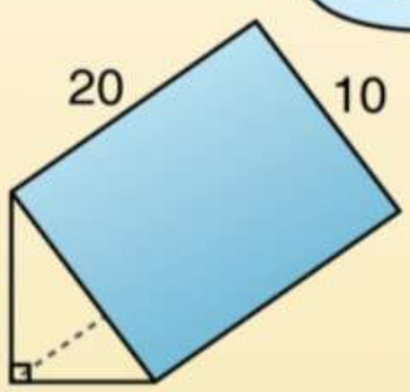
...$8 \times 20 = 160$...

...and $10 \times 20 = 200$ square units.









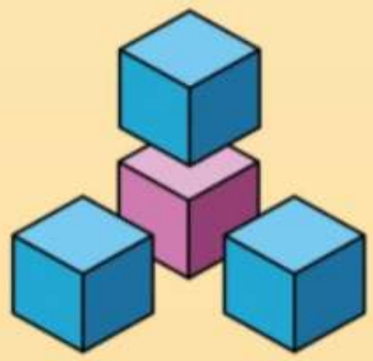
So, the combined area of all five faces is 48+120+160+200=528 square units.

Well done!

Let's try one last surface area problem.

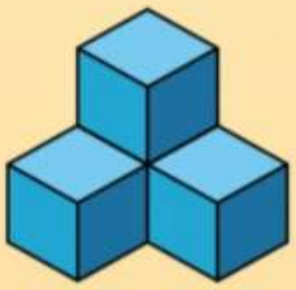
Each of these four cubes has a surface area of exactly 5 square inches.

By attaching each of the blue cubes to a face of the pink cube...



...I can make this solid.

What is its surface area?



If we find the area of one cube face...

...we can multiply that by the number of faces that are showing.

The total surface area of 6 cube faces is 5 square inches.

So, each face has an area of $5 \div 6 = \frac{5}{6}$ square inches.

Great! How many of the cube faces are showing?

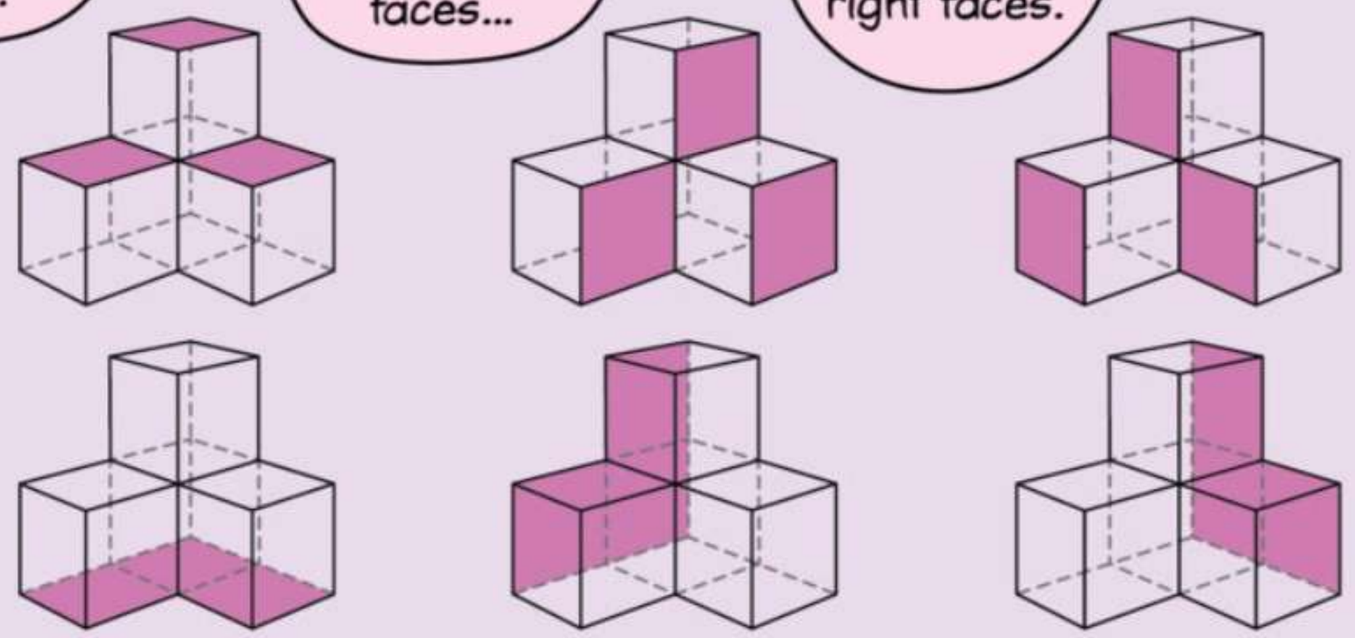
How many do you count?

There are three top faces, three bottom faces...

...three front-right faces, three back-left faces...

...three front-left faces, and three back-right faces.

That makes 18 total.

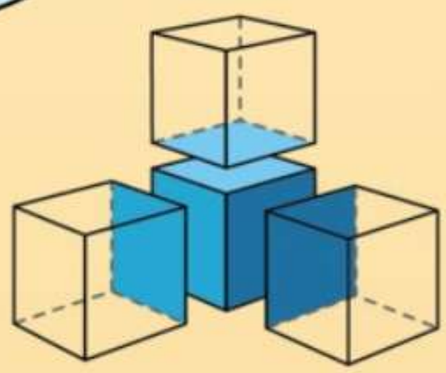


I got 18 a different way.

Before we attached them, each of the 4 cubes had 6 visible faces for a total of $4 \times 6 = 24$ faces.

Attaching the cubes hid some of their faces.

6 faces are hidden, so $24 - 6 = 18$ faces are still visible.



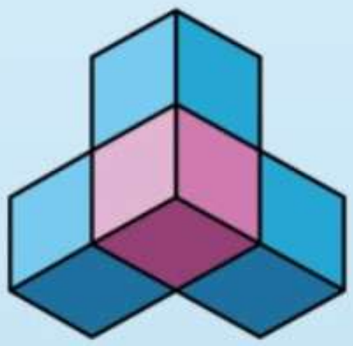
I did something else.

I counted the number of faces showing for each cube, and added to get the total.

Each blue cube has five faces showing.

But only three of the pink faces are showing.

That makes $5+5+5+3 = 18$ faces.



What is the surface area of the solid?

If each face has an area of $\frac{5}{6}$...

...and there are 18 faces...

...then the total area of the solid is $18 \times \frac{5}{6} = 15$ square inches.

$$18 \times \frac{5}{6} = \frac{18 \times 5}{6} = \frac{18}{6} \times 5 = 3 \times 5 = 15$$

We can also solve the problem without ever multiplying fractions.

If a cube with **6** square faces has a surface area of 5 square inches...

...then a solid with **18** of the same square faces will have three times the surface area.

Three times 5 square inches is 15 square inches.

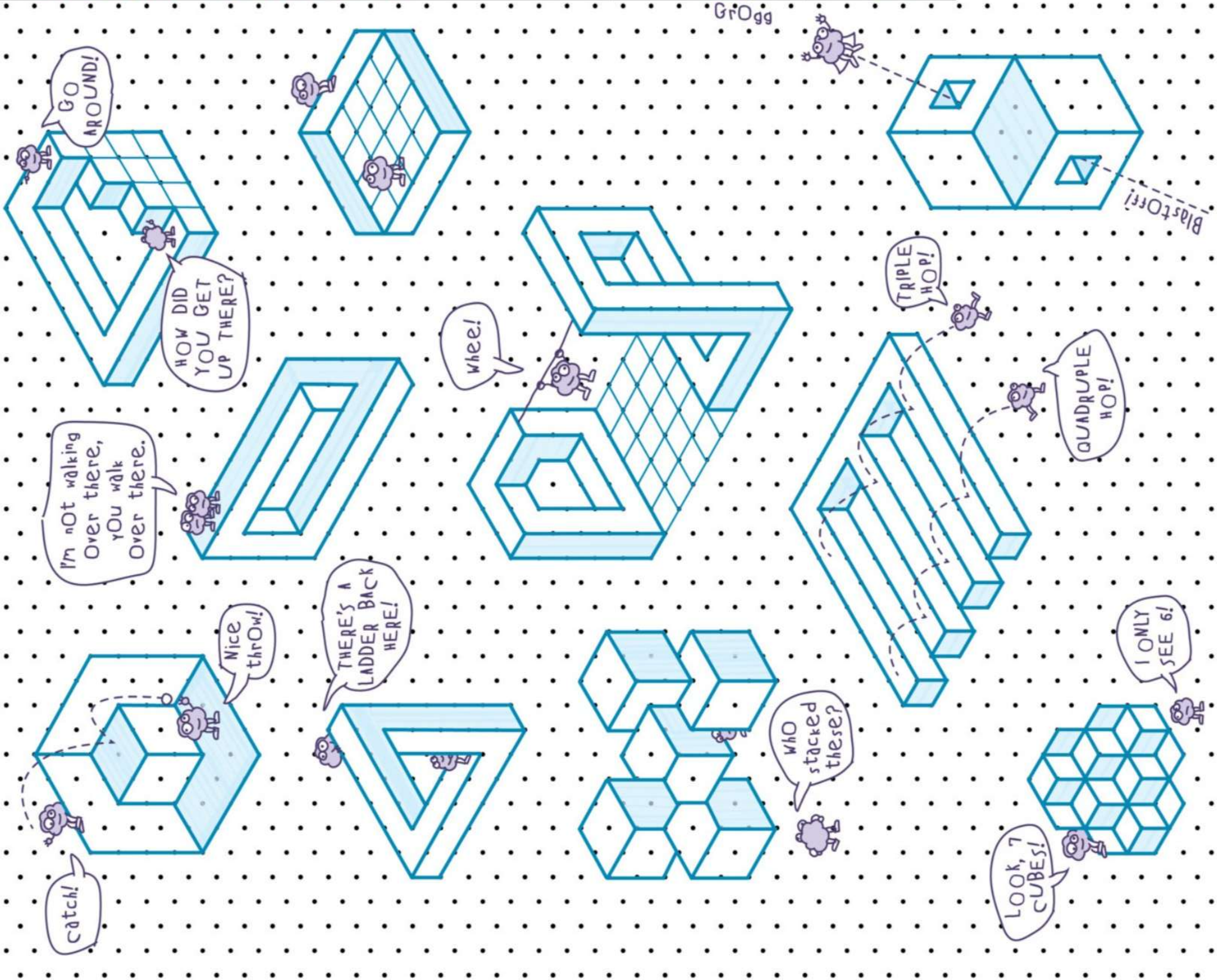
Nice work! Now, let's see what's in this box.

Our new Math Team shirts!

Uh oh.

I think that someone misjudged my surface area.

TITLE	NAME	DATE
Isometric Dot Paper Cube Drawings	Alex & GrOgg	8/19



Ms. Q. Volume

What is volume?

Volume is the amount of space a solid fills.

How do we measure volume?

With cubes.

What do you mean, Alex?

Well...

To measure length, we use one-dimensional units like inches, feet, and miles.

To measure area, we use two-dimensional units like square inches, square feet, and square miles.

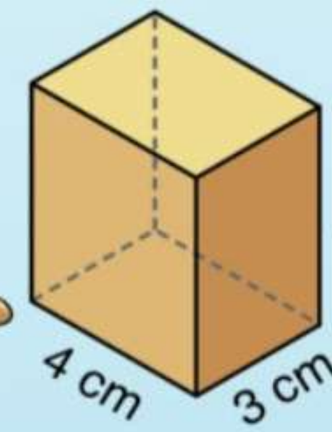
But to measure volume, we need three-dimensional units. Like cubes!

Very good, Alex.

Volume is measured in cubic units...

...like cubic inches, cubic feet, and cubic miles.

How could we find the volume of this rectangular prism in cubic centimeters?



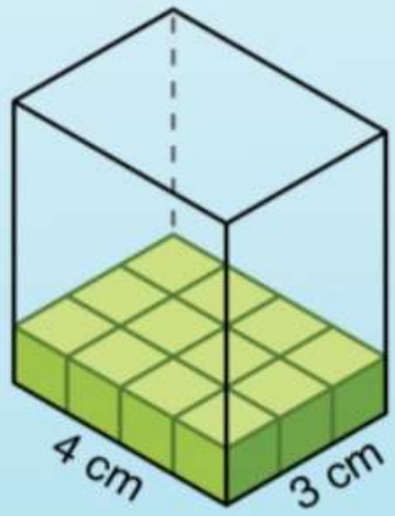
What is the volume of this prism?

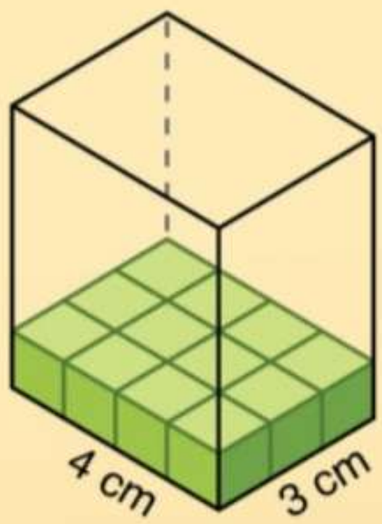
WE ALSO LEARNED TO MEASURE VOLUME USING OUNCES, GALLONS, AND LITERS IN GUIDE 3C.

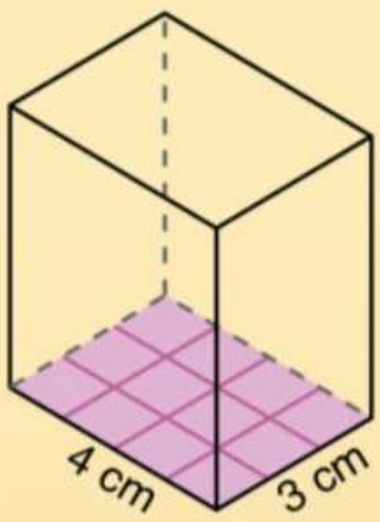
A cube with an edge length of 1 centimeter has a volume of 1 cubic centimeter.

We need to figure out how many of these cubes it will take to fill the prism.

The base of the prism is 4 cm by 3 cm. We can cover the base using 4×3=12 cubes.



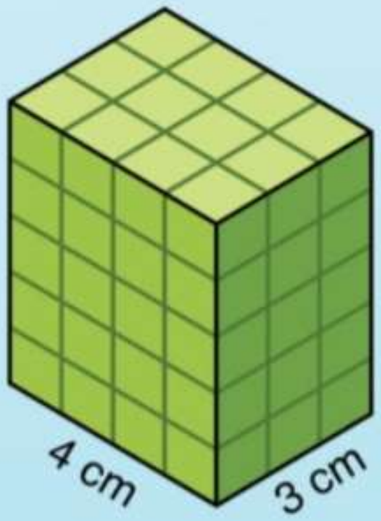




Area: 4×3 = 12 Sq Cm

The number of cubes it takes to cover the base is the same as the area of the base.

The prism is 5 centimeters high. So, we can stack 5 layers of 12 cubes.

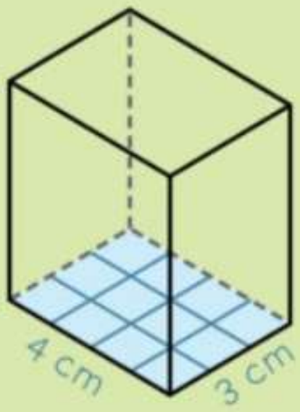


5 layers of 12 cubes makes a total of 5×12=60 cubic centimeters.

5×12=60 cubic cm

So to find the volume of a rectangular prism, we count how many cubes it takes to cover the base...

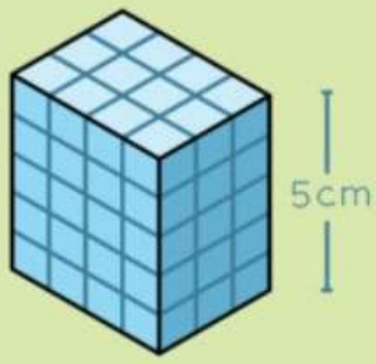
...which is the area of the base.



Area:
4×3=12 sq cm

And we multiply by the number of layers we stack to make the prism...

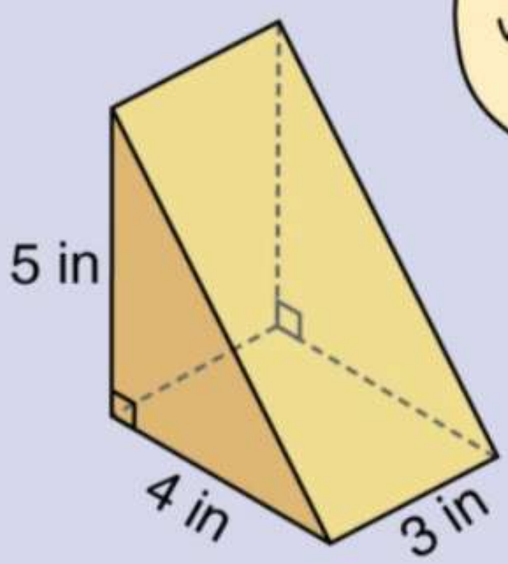
...which is the prism's height.



Volume:
12×5=60 cu cm

Very good, Alex. We use the same method to find the volume of ***any*** prism, not just rectangular prisms.

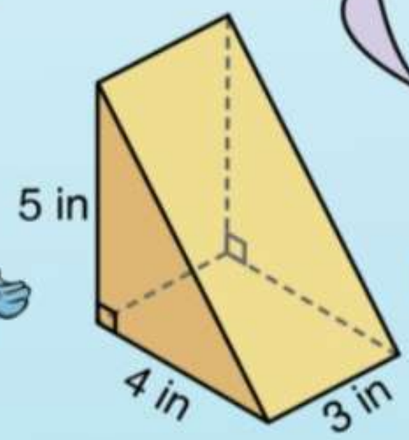
How would you find the volume of this triangular prism?



First we need to know how many cubes it takes to cover the base of the prism.

Wait, which face is the base?

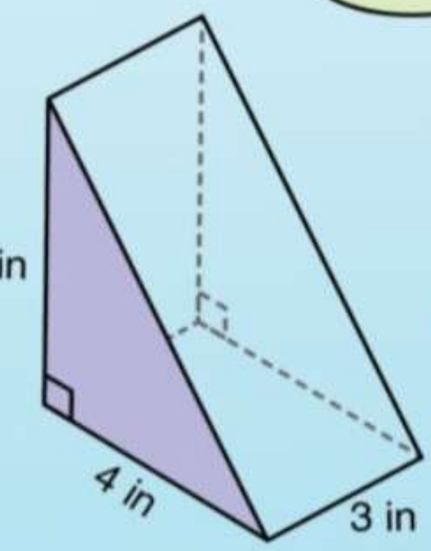
It's a ***triangular*** prism. Its bases are ***triangles***.



So, we call this triangle a base?

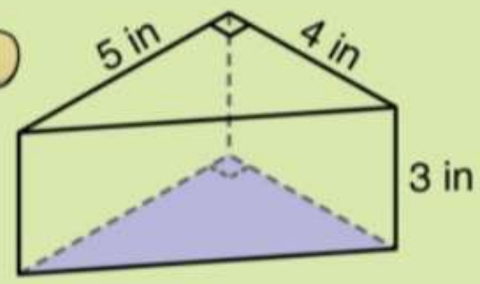
Right, Grogg.

The bases of a prism are its congruent ***parallel*** faces.

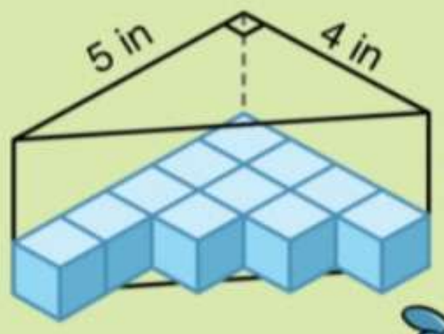


Let's flip it so that the base is on the bottom.

How many cubes does it take to cover the base?



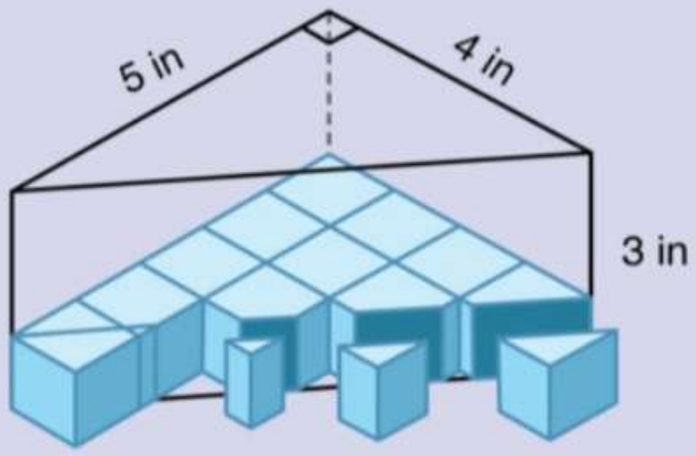
When we try to cover a triangular base with cubes, some stick out.



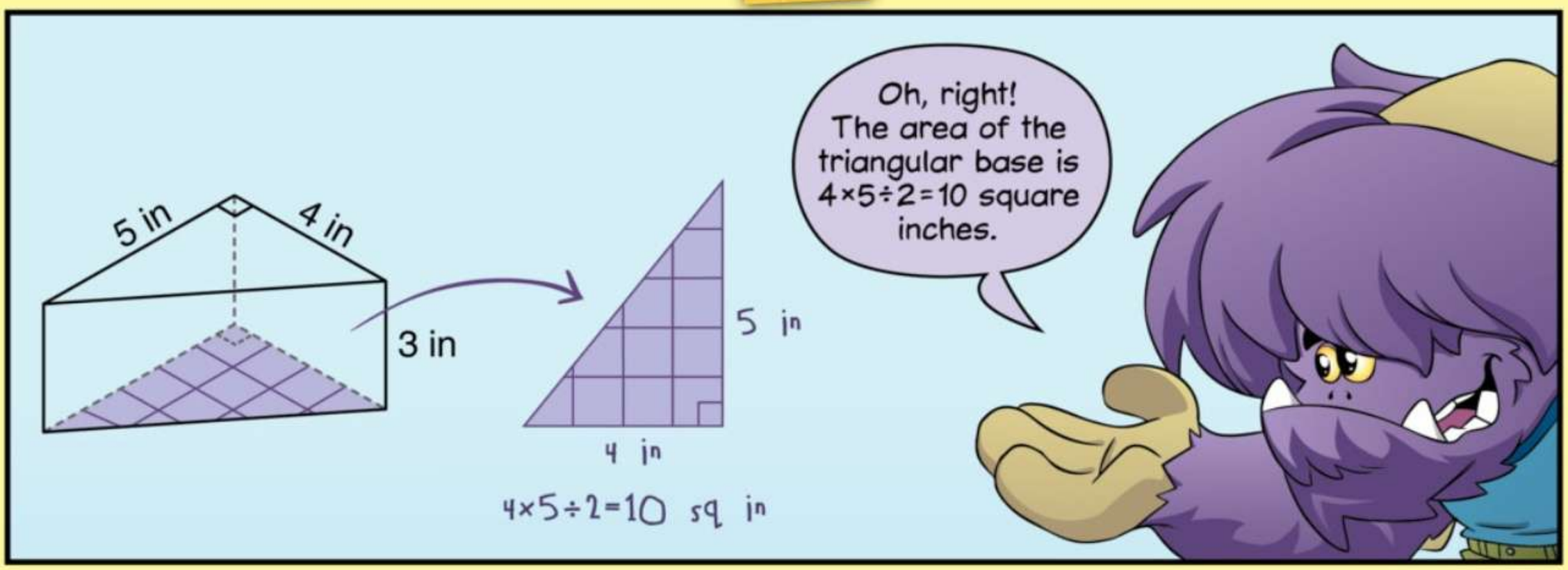
We can chop off some of the cubes that stick out and use the pieces to fill the spaces that aren't covered.

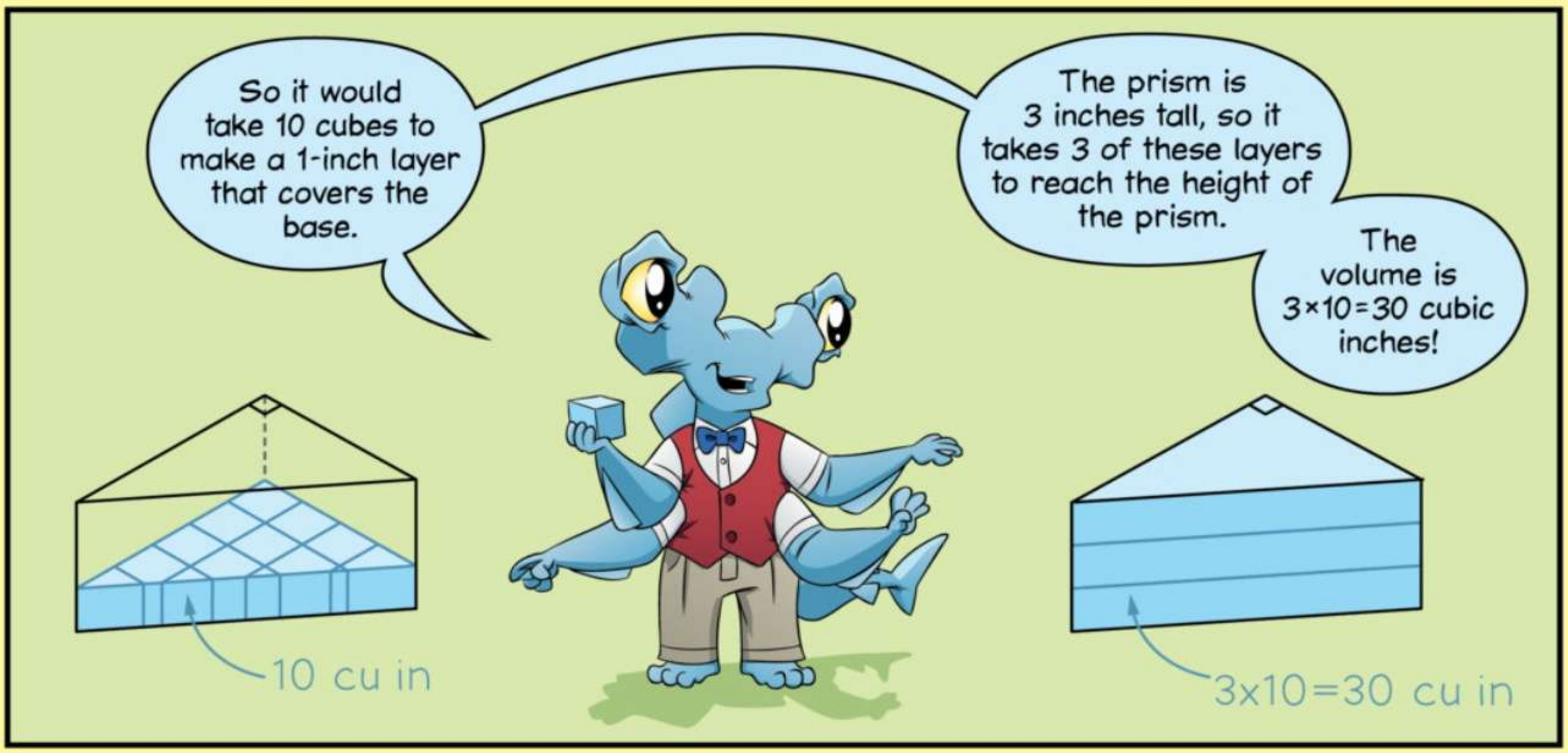
Wait a minute!

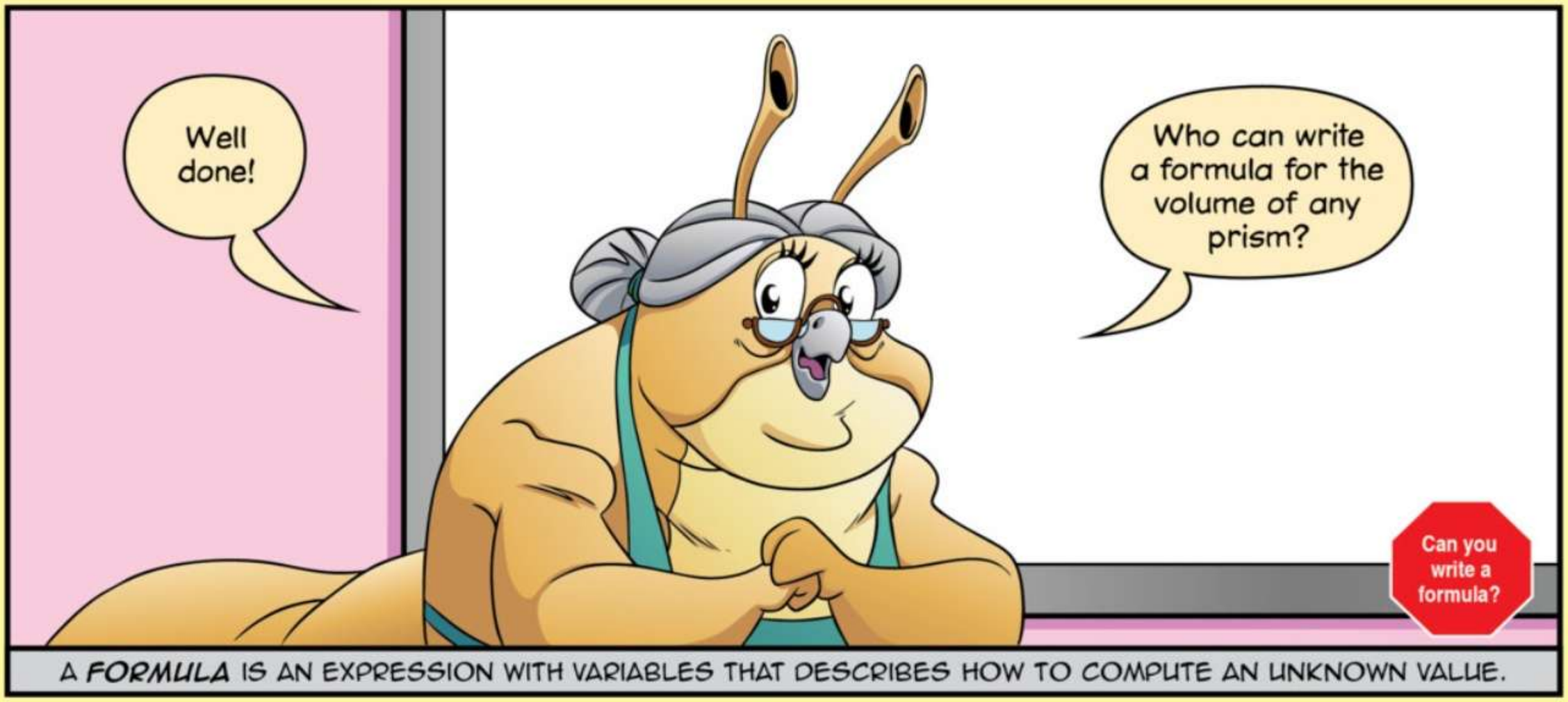
The number of cubes it takes to cover the base is the same as the ***area*** of the base.



What is the volume of this prism?







A ***FORMULA*** IS AN EXPRESSION WITH VARIABLES THAT DESCRIBES HOW TO COMPUTE AN UNKNOWN VALUE.

To find the volume of a prism, we start by finding the area of its base.

That tells us how many cubes it will take to cover the base.

Then, we multiply the area of the base by the height of the prism.

The height tells us how many layers it takes to fill the prism.

We can use a big B for the area of the prism's base.

And an h for the prism's height.

The volume of a prism is the area of its base times its height. $V = B \times h$.

B

h

Volume = B × h

Nicely done, little monsters.

What in the world are you doing, Grogg?

I'm trying to measure my volume...

...in cheese cubes!