Contents: Chapter 12

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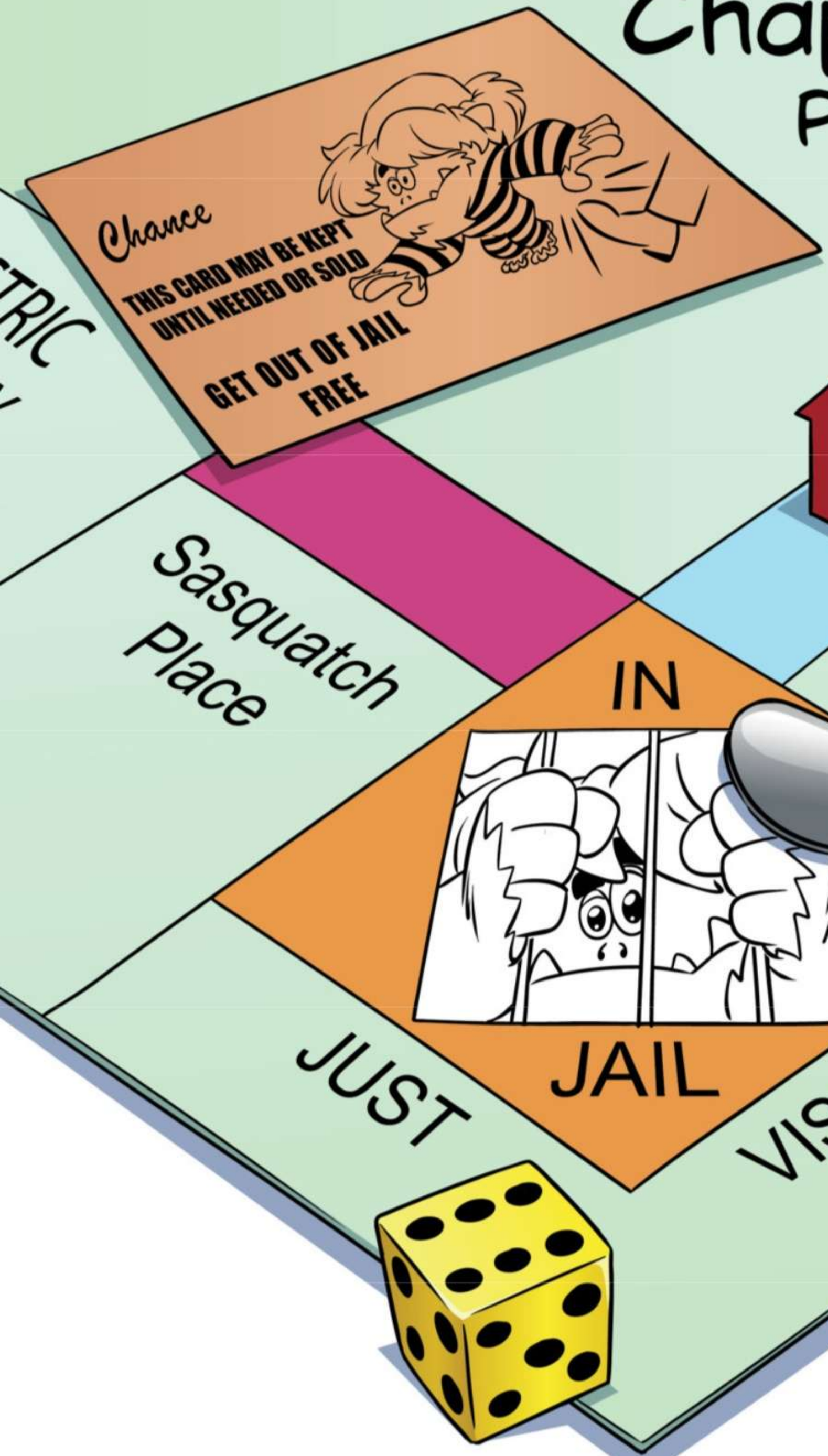
Chapter 12:
Probability



R&G Chance

Tomorrow, we get to take the Math Team to another competition.

Yep. We'd better finish getting the activity blimp tied down before that storm rolls in.

Think we'll get some rain?

I'd say the probability is high.

zzzzzz

The what?

Probability. It's another word for chance, or likelihood.

When I say that the probability of rain is high, I mean there's a good chance we'll get some rain.

So, probability describes how likely something is to happen?

Exactly. If an event is likely, we say that it has a ***high*** probability of occurring.

That's right.

But having a ***low*** probability means an event is unlikely?

munch munch munch munch

It's already starting to drizzle. I think there are a few umbrellas on board.

Here they are. Which one should we use?

Let's leave it to chance.

Three of the umbrellas are yellow, and the fourth one is...

Ridiculous.

I'll turn off the lights and mix up all the umbrellas.

You reach in and grab the first umbrella you touch.

What kind of umbrella do you think you'll get?

What do you expect to happen?

I guess I'll probably grab a yellow one.

Since there are three yellow umbrellas and only one ridiculous one...

...you are more likely to grab a yellow umbrella than the silly one. Let's see.

KLIK!

Shuffle rustle Shuffle Shuffle

Okay, pick an umbrella and I'll turn the lights back on.

KLIK!

Got it.

Hey! I got the silly one!

That's a surprise.

Understanding probability helps us make predictions based on what we ***expect*** to happen.

But, just because you were more likely to pick a yellow umbrella doesn't mean it was guaranteed.

Whoa. It's really raining now. Let's get back to the school.

I don't want to get struck by lightning!

The probability of being struck by lightning is extremely low.

Sure, but I bet that carrying a big umbrella across a large field during a thunderstorm greatly increases that probability.

KRAKOOM!!

Better not take our chances!

Ms. Q.

Computing Probability

Did everyone get a bag?

Yes!

Alex, what's in your bag?

Three red blocks and five yellow blocks...

...all the same size and shape.

If you pull a block from your bag without looking, what's the probability that the block will be yellow?

Hmmm... Since there are more yellow blocks than red blocks, I'd guess it's pretty likely.

How can we represent the probability of Alex drawing a yellow block from his bag?

An event's probability is usually expressed as a fraction.

We write the number of ways the desired event can happen over the total number of possible outcomes.

$$\frac{\text{Number of Desired Outcomes}}{\text{Number of Possible Outcomes}}$$

What is the probability that Alex will draw a yellow block?

THE WAYS A DESIRED EVENT CAN HAPPEN ARE CALLED ***SUCCESSFUL OUTCOMES***, OR ***FAVORABLE OUTCOMES***. THE NUMBER OF SUCCESSFUL OUTCOMES IS NEVER MORE THAN THE TOTAL NUMBER OF POSSIBLE OUTCOMES.

To find the probability of Alex drawing a yellow block, we need to count the ways Alex can pull a yellow block...

...and count the ways he can pull any block from his bag.

Since Alex has 5 yellow blocks, there are 5 ways he can pull a yellow block from his bag.

And there are 3+5=8 blocks all together.

So, the probability of Alex drawing a yellow block is $\frac{5}{8}$!

$$\frac{\text{Yellow Blocks}}{\text{Total blocks}} = \frac{5}{8}$$

That's right. What's the probability of Alex drawing a *red* block from his bag?

Since 3 of the 8 blocks are red, I expect to pull a red block $\frac{3}{8}$ of the time.

That makes sense!

Pulling a red block is less likely than pulling a yellow block...

...and $\frac{3}{8}$ is less than $\frac{5}{8}$.

$$\frac{\text{Red blocks}}{\text{Total blocks}} = \frac{3}{8}$$

Very good!

Winnie, what's in your bag?

I have a die.

If Winnie rolls her die, what is the probability that the number rolled will be a perfect square?

What is the probability that Winnie will roll a perfect square?

1×1=1 and 2×2=4 are the only perfect squares on the die.

So, there are 2 ways Winnie could roll a perfect square.

1 2 3 4 5 6

$$\frac{\text{Ways to roll a perfect square}}{\text{Total possible rolls}} = \frac{2}{6} = \frac{1}{3}$$

Since there are six numbers on the die, and two are perfect squares...

...the probability that Winnie will roll a perfect square is $\frac{2}{6}$.

We can simplify $\frac{2}{6}$ to $\frac{1}{3}$.

Right, because $\frac{1}{3}$ of the numbers on the die are perfect squares.

So, we expect to roll a perfect square $\frac{1}{3}$ of the time.

I see. So, if I roll this die 300 times...

...I would expect a perfect square to come up on about $\frac{1}{3} \times 300 = 100$ of the rolls.

REMEMBER, PROBABILITY IS USEFUL FOR MAKING PREDICTIONS.
WE EXPECT TO SEE ***ABOUT*** 100 PERFECT SQUARES ON 300 ROLLS, NOT EXACTLY 100.

And if Winnie rolls her die a bazillion times...

...about $\frac{1}{3}$ of her rolls would be perfect squares.

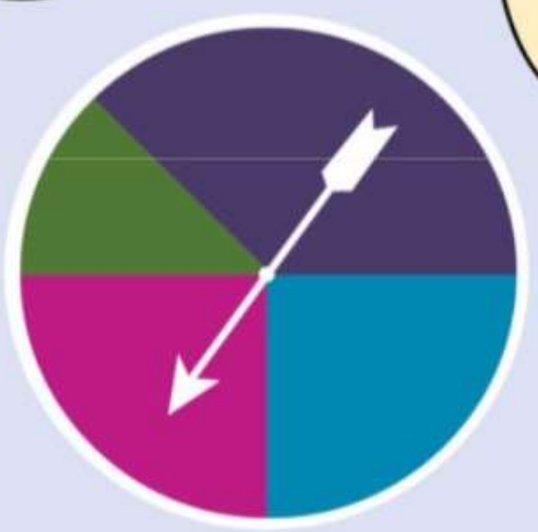
Wonderful work, little monsters.

Probability is the fraction of times we expect an event to happen.

Lizzie, what's in your bag?

A spinner.

It's divided into four regions.

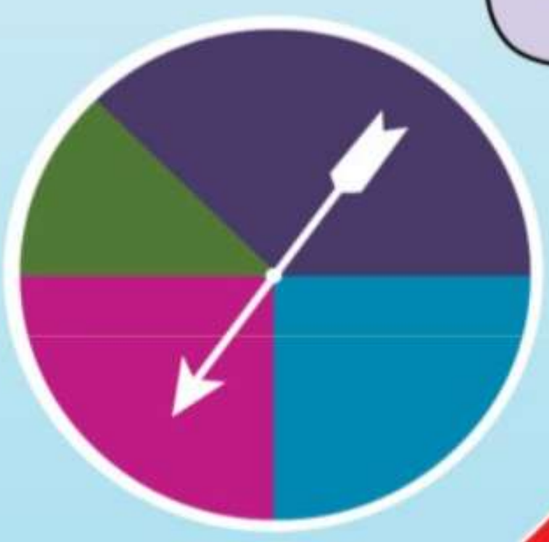


If Lizzie spins her spinner, what's the probability that the arrow will land on purple?

There are 4 regions: green, pink, blue, and purple.

Only one of the 4 regions is purple. Does that make the probability...

...$\frac{1}{4}$?

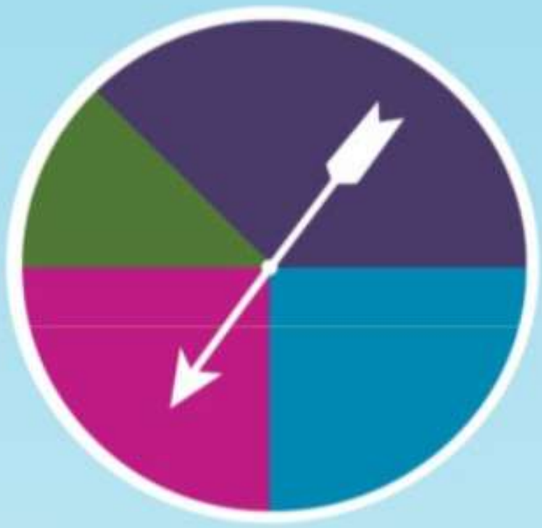


Is the probability of spinning purple $\frac{1}{4}$?

I don't think that's right. The purple region is bigger than the others.

The purple region is waaay bigger than the green section...

...they can't ***both*** have a probability of $\frac{1}{4}$.



When we compute a probability by counting outcomes, all of the outcomes must be equally likely.

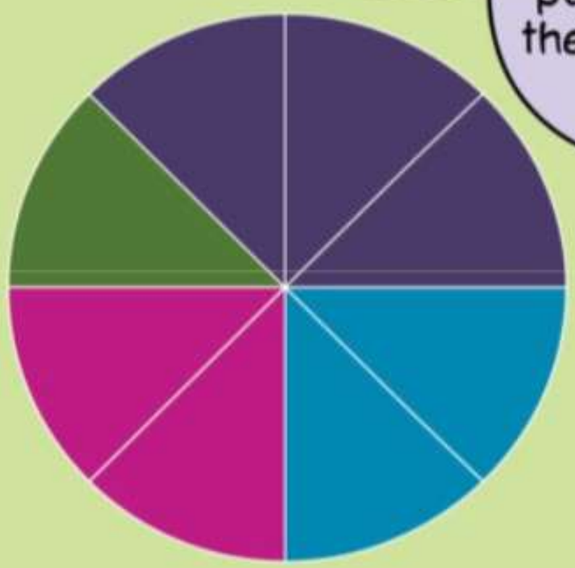
Huh?

Like when we roll a die...

...each number has the same chance of being rolled.

How can we make sure the outcomes we are counting on the spinner are equally likely?

We can divide the spinner into eight equal sections, like this.



I see.

Since 3 of the 8 ***equal*** sections are purple, the probability the spinner will land on purple is $\frac{3}{8}$.

$\frac{2}{8}$ of the spinner is pink...

...so the probability of spinning pink is $\frac{2}{8}$, which equals $\frac{1}{4}$.

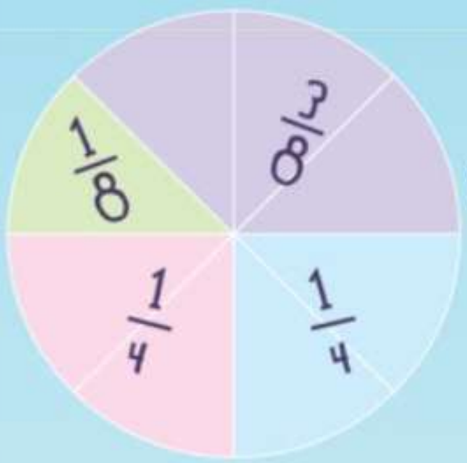
And the probability of spinning blue is also $\frac{1}{4}$.

Only $\frac{1}{8}$ of the spinner is green...

...so the probability of spinning green is $\frac{1}{8}$.

So, if you spun the spinner over and over again, you would get green about $\frac{1}{8}$ of the time, pink $\frac{1}{4}$ of the time, blue $\frac{1}{4}$ of the time...

...and purple about $\frac{3}{8}$ of the time.



Cool.

The spinner is 3 times more likely to land on purple than on green.

Probability of green: $\frac{1}{8}$

×3=

Probability of purple: $\frac{3}{8}$

An event three times more likely than spinning purple would have a probability of $\frac{9}{8}$!

Grogg! Probability can never be greater than 1!

Huh?

Why can't the probability of an event ever be greater than 1?

There can't be ***more*** successful outcomes than total possible outcomes.

But, if all of the possible outcomes are successful outcomes, then the probability is 1.

$$\frac{\text{Successful Outcomes}}{\text{Total Possible Outcomes}}$$

For example, if all 5 of the blocks in my bag are yellow...

...the probability of pulling a yellow block is $\frac{5}{5}=1$.

But, the probability of pulling a red block is $\frac{0}{5}=0$.

That's right! The probability of an event can never be more than 1, or less than 0.

Grogg, what's in your bag?

A carton of milk, a pack of jelly beans, a banana, some snack crackers, and a sandwich.

Grogg!! That's your lunch!

impOssible

very unlikely

sOrta unlikely

cOin flip

kinda likely

very likely

certain

0

1/2

1

Ms. Q. will make a statement withOut fOllOwing it up with a questiOn.

we will actually build sOmething in wOOdshOp this week.

we will win the math relays cOmpetitiOn On Tuesday.

The blue custOdian will have his sandwich stOlen this mOnth.

Lizzie will tell me sOmething cOOl abOut dragOns On the bus ride hOme.

PrOfessOr GrOk will get captured by clOd in class tOday.

mwaahahahaha

GrOgg!

winnie will Yell "GrOgg!" befOre lunch.

Alex will wear a tie tOmOrrOw.

MATH TEAM
Coins and Dice

Alright, little monsters...

...climb aboard.

We're off to the math relays.

zzzzzz

Do you think we have a chance of winning?

Some probabilities are almost impossible to compute.

But others aren't so tough.

For example, what is the probability that a tossed coin will land on heads?

It's a lot more likely to land on my head than on Lizzie's.

But, I guess it depends on where you aim it.

That is *not* what she means.

Most coins have a heads side, with the head of a famous monster...

...and a tails side, with the tail of the famous monster.

Landing heads means landing with the heads side face-up.

What is the probability of flipping heads on a coin flip?

Since 1 of the 2 possible outcomes of flipping a coin is heads, the probability of a flipped coin landing heads is $\frac{1}{2}$.

That just means that if you flip a coin a bunch of times and tally the results...

...it will land heads about half the time.

Good.

Things get a little trickier when we flip two coins.

Heads

Tails

MONSTEROPOLY

If I flip *both* of these coins, what is the probability that both will land heads?

We need to count all of the possible outcomes.

You can flip two heads...

...or two tails...

...or one of each.

Are all three outcomes equally likely?

Try flipping both of these coins 60 times and record how many times you flip two heads, two tails, and one of each.

Try it.

Here are our results.

Two Heads	Two Tails	One Each
𝍸 𝍸 𝍸 |	𝍸 𝍸 |||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |

We flipped two heads 16 times...

...two tails 13 times...

...and one of each ***31 times!***

Weird. Why did "one each" come up so often?

I know! There are two ways to flip one heads and one tails!

We can flip heads on the penny and tails on the nickel... ...or heads on the nickel and tails on the penny.

Winnie's right. We can make a tree diagram to show all of the possible outcomes.

The penny can land heads or tails...

...and the nickel can land heads or tails.

That makes ***four*** possibilities: heads-heads, heads-tails, tails-heads, and tails-tails.

All 4 possible outcomes are equally likely...

...and only 1 of the 4 outcomes has two heads.

So, the probability of flipping heads twice is $\frac{1}{4}$.

That matches our results. We flipped two heads on 16 of the 60 flips, which is about $\frac{1}{4}$ of the time.

Penny Nickel

H
- H → 1. HH
- T → 2. HT

T
- H → 3. TH
- T → 4. TT

Excellent.

Suppose I flip ***three*** coins.

What is the probability of flipping heads ***exactly*** twice with three coins?

We can add the third coin flip to our tree diagram.

That gives us ***8*** possible outcomes.

And ***3*** of the outcomes have exactly two heads.

So, on three flips of a coin, the probability you will get exactly two heads is $\frac{3}{8}$.

Penny Nickel dime

H
- H
  - H → 1. HHH
  - T → 2. HHT
- T
  - H → 3. HTH
  - T → 4. HTT

T
- H
  - H → 5. THH
  - T → 6. THT
- T
  - H → 7. TTH
  - T → 8. TTT

Awesome job!

Let's switch from coins to dice.

Have any of you played Monsteropoly?

Penny Nickel dime

H H H T

T H H T

MONSTEROPOLY

How do you get out of jail?

I always get sent to jail in that game. *Directly* to jail.

You have to roll doubles with a pair of dice.

What's the probability of rolling doubles?

GO DIRECTLY TO JAIL

PASS GO, DO NOT COLLECT $200

We need to know how many different rolls are possible.

A coin only has 2 sides, but a die has 6!

We can make another tree diagram to show every possible roll.

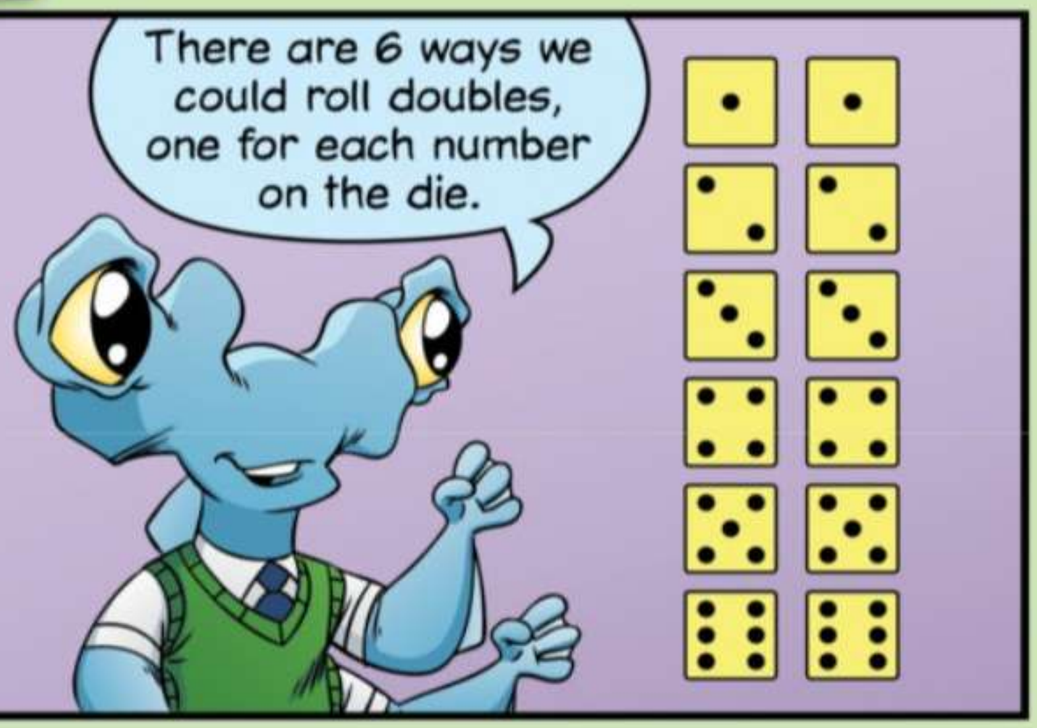
What is the probability of rolling two of the same number with a pair of dice?

There are 6 possible rolls for the first die, and 6 possible rolls for the second die.

That makes 6×6=36 possible rolls.

1st Die Roll	2nd Die Roll
1	1,1
	1,2
	1,3
	1,4
	1,5
	1,6
2	2,1
	2,2
	2,3
	2,4
	2,5
	2,6
3	3,1
	3,2
	3,3
	3,4
	3,5
	3,6
4	4,1
	4,2
	4,3
	4,4
	4,5
	4,6
5	5,1
	5,2
	5,3
	5,4
	5,5
	5,6
6	6,1
	6,2
	6,3
	6,4
	6,5
	6,6

There are 6 ways we could roll doubles, one for each number on the die.



6 of the 36 possible rolls are doubles, so the probability of rolling doubles is $\frac{6}{36}$...

...which simplifies to $\frac{1}{6}$.

$\frac{1}{6}$! That makes perfect sense!

What do you mean, Winnie?

No matter what number we roll on the first die...

...exactly 1 of the 6 numbers on the second die will match it.

So, the probability of rolling doubles is $\frac{1}{6}$.

Great thinking, Winnie!

Now, who wants to be the race car?

R&G Flip!

FLIP!

Heads.

Heads.

Heads.

Heads.

Heads.

FLIP!

Oops.

Heads.

7.

7 heads in a row!?

Let me see that.

Heads. 8.

Amazing!

Huh?

I can't believe it landed heads again!

Why not?

The probability of flipping heads 8 times in a row is tiny.*

Yes, but each flip of the coin is as likely to land heads as tails.

*EACH FLIP HAS 2 POSSIBLE OUTCOMES. SO, FOR 8 FLIPS, THERE ARE 2×2×2×2×2×2×2×2=256 POSSIBLE OUTCOMES. ONLY ONE OF THESE OUTCOMES IS 8 HEADS IN A ROW, SO THE PROBABILITY OF FLIPPING 8 HEADS IN A ROW IS $\frac{1}{256}$.

So, it shouldn't be surprising each time it lands heads.

FLIP!

Catch

Heads.

How is that possible!?

You had already flipped heads 8 times in a row!

The coin didn't know that.

The previous tosses don't change the probability of the next toss landing heads.

But, the probability of flipping heads 9 times in a row is only $\frac{1}{512}$!

Sure, but the probability of flipping 8 heads followed by 1 tails is also $\frac{1}{512}$.

FOR 9 FLIPS, THERE ARE 2×2×2×2×2×2×2×2×2=512 POSSIBLE OUTCOMES, ONLY ONE OF WHICH IS HHHHHHHHH, AND ONLY ONE OF WHICH IS HHHHHHHHT.

Ooooh. You're right.

Every sequence of 9 coin tosses is equally likely.

You are just as likely to toss 9 heads in a row as you are to toss H-H-T-H-T-T-T-H-H.

Right, but you'd probably never remember flipping H-H-T-H-T-T-T-H-H.

So, as long as you're not cheating, it's just as likely that your next flip will land heads as tails!

That's what I've been trying to tell you.

FLIP!

Catch

Heads.

10 in a row. That ties my record!

Your record? How many times have you tossed that coin?

A couple thousand, I'd guess.

Toss it again!

FLIP!

Whoops!

Plink!

Plink!

Plink!

Plink!

MATH TEAM

Math Relays: Round 1

We're almost there!

Before we land, let's review how the Math Relays competition works.

There are four problem stations in the competition hall.

I start at Station 1, Alex starts at Station 2, Winnie is at Station 3, and Grogg is at Station 4.

Other teams will have a player at each station, too.

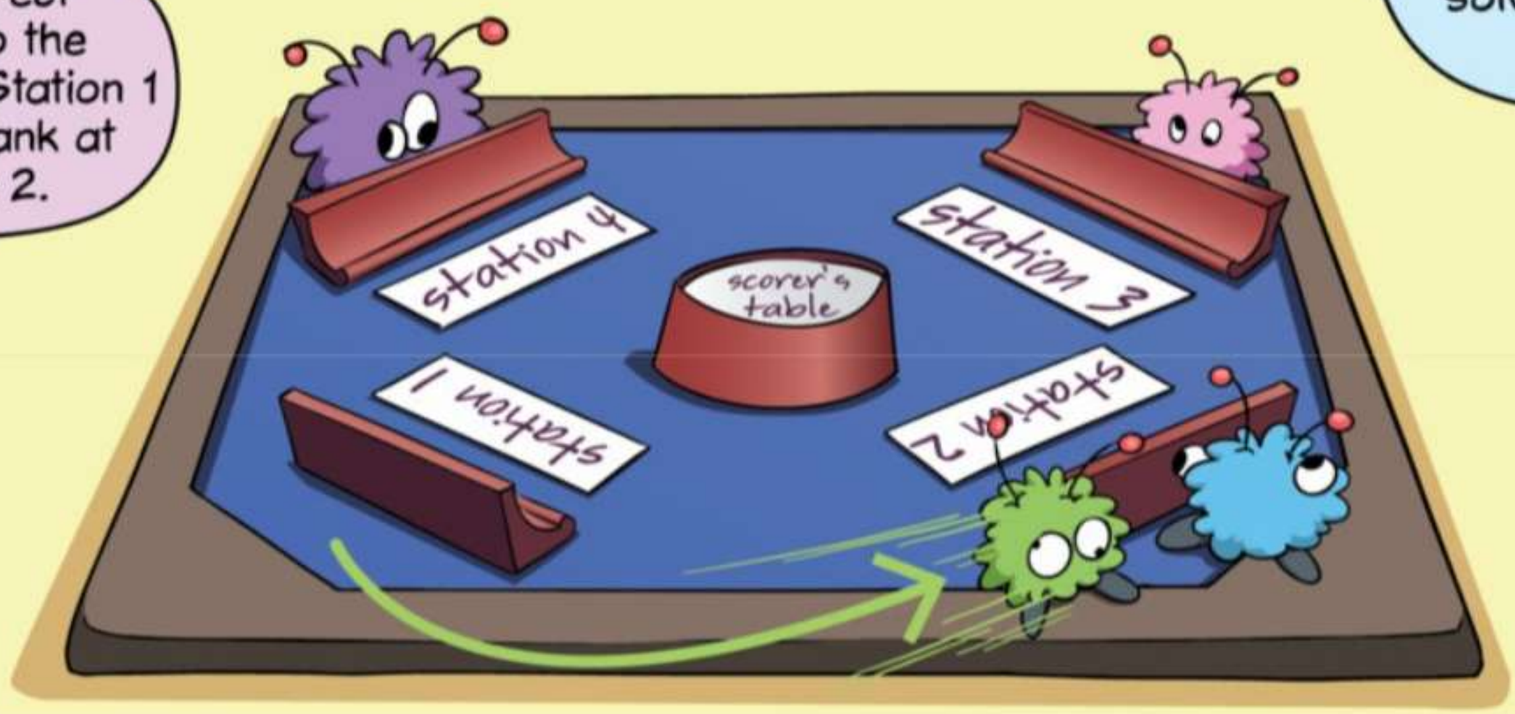
When the relay begins, a problem appears on the screen at each station.

But the problems at stations 2, 3, and 4 all have blanks in them.

The correct answer to the problem at Station 1 fills the blank at Station 2.

I answer Problem 1 and bring it to Alex at Station 2.

We use the answer to fill in the blank in Problem 2, then we solve Problem 2 together.



I bring the Problem 2 answer to fill the blank at Station 3.

Station 4
Scorer's table
Station 3
Station 1
Station 2

Alex and I work together to solve Problem 3.

Then I bring the Problem 3 answer to Station 4.

Winnie and I solve Problem 4...

...and I take the final answer to the Scorer's Table.

Four teams compete in each relay. The first two teams who deliver the correct answer from Station 4 to the scorer's table move on to the next round.

Look! We're about to land!

Here we are!

This is one of the biggest math events every year, so all of the top teams from the region are here.

There's Max and his team.

MATH RELAYS

He clobbered us all by himself at the Math Bowl.

ROUND 1

CENTRAL ELEMENTARY

HIGHLAND SCHOOL OF THE ACADEMIC YETI

TO THE MAX

MIDVALE SCHOOL FOR THE GIFTED

L.E.R.N.S. INSTITUTE

ORB ELEMENTARY

PS 254

POKEY OAKS

MATH RELAYS

ROUND 2

FINALS

ROUND 2

ROUND 1

WOOLIE PREP

COBRA KAI

DRAGON VIEW

BEAST ACADEMY

MONKU ELEMENTARY

THE AVIAN SCHOOL

ARACHNID ONLINE PREP SCHOOL

STUART ELEMENTARY

But in *this* competition, he'll need the help of his teammates.

Looks like we won't even get to compete against Max unless we make it to the finals.

The first round starts soon.

Let's watch.

Max is at Station 3.

The questions are up!

Problem 1:
Find the sum of all the integers from -1,867 to 1,869 inclusive.

Problem 2:
The number __ has exactly two prime factors. Find their difference.

Problem 3:
What is the units digit of __ raised to the $1{,}000^{th}$ power?

Problem 4:
What is the probability of rolling a total of __ with a pair of standard six-faced dice?

Can you solve all four?

I think I know the first answer!

If we pair every negative integer with its opposite, we get a bunch of sums that equal zero.

The only integers that can't be paired with their opposites are 0, 1,868, and 1,869.

$$\begin{aligned}
&(-1{,}867) + (-1{,}866) + \cdots + 1{,}866 + 1{,}867 + 1{,}868 + 1{,}869 \\
=\ &(-1{,}867+1{,}867)+(-1{,}866+1{,}866)+\cdots+(-1+1)+0+1{,}868+1{,}869 \\
=\ &0+0+\cdots+0+0+1{,}868+1{,}869 \\
=\ &3{,}737
\end{aligned}$$

That leaves 1,868+1,869 =3,737!

Whoa! What is Max doing?

He's already headed to Station 4!

How could he have an answer to Problem 3 without even knowing what number goes in the blank!?

That's impressive.

I think I see what he did.

Without knowing the answer to Problem 2, Max narrowed the answer to Problem 3 to just a few possibilities.

How?

Problem 2
3,737=3,700+37
=37×(100+1)
=37×101
3,737
101 37
101-37=64

A number ending in 0 raised to any power ends in 0.

A number ending in 5 raised to any power ends in 5.

But, since these answers are so obvious, Max probably ruled them out.

if x ends in:	then x^{1000} ends in:
0	0
1	
2	
3	
4	
5	5
6	
7	
8	
9	

Any number ending in 1, 3, 7, or 9 raised to the 1,000th power ends in 1...

...and any number ending in 2, 4, 6, or 8 raised to the 1,000th power ends in 6.

if x ends in:	then x^{1000} ends in:
0	0
1	1
2	6
3	1
4	6
5	5
6	6
7	1
8	6
9	1

REVIEW UNITS-DIGIT COMPUTATIONS IN CHAPTER 3 OF GUIDE 4A.

So, Max knows that the answer to Problem 3 is 0, 1, 5, or 6.

And he can guess that it probably isn't 0 or 5.

At Station 4, he'll know the answer to Problem 3 isn't 0 or 1...

...since you can't roll a 0 or 1 with a pair of dice.

So, the answer to Problem 3 is probably 6.

And all he has to do now is find the probability of rolling a 6 with a pair of dice.

What is the probability of rolling a 6 with a pair of dice?

I made a table to show the results of all 6×6=36 possible rolls.

There are 5 ways to roll a 6: 1+5, 2+4, 3+3, 4+2, and 5+1.

Since 5 of the 36 possible rolls give a total of 6...

...the probability of rolling a 6 is $\frac{5}{36}$.

First Die \ Second Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

THIS TABLE GIVES ANOTHER WAY TO DISPLAY THE INFORMATION IN THE TREE DIAGRAM ON PAGE 89.

His teammate is already taking the final answer to the finish table!

Unbelievable. They finished before any of the other teams reached Station 3.

Let's head down to the arena floor. You guys compete soon.

As soon as your problem shows up on the screen, start working!

CHANNEL 5

This is Mike and Phil and we're coming to you live from the 67th Annual Math Relays.

Phil, there have already been some great moments in this year's competition.

You bet, Mike. Let's get straight to the highlights from the first two rounds.

BREAKING NEWS ysterious falling coin disrupts golf

In the first relay, Max Norris stunned the crowd by winning the round with almost no help from his teammates.

Max and his team breezed into the finals.

But, there was one scary moment in Round 2.

That's right, Phil. Orb Elementary team member Erg Shortz collided with Jake "Jelly" Roger from Central Elementary on their way to Station 2.

BOOM!

Fortunately there were no injuries. Erg was extricated quickly and Orb Elementary went on to earn a spot in the finals.

BEAST ACADEMY

But the big story of the day has got to be the performance of Beast Academy.

They have been an exciting team to watch, winning both their relays in impressive fashion.

Beast Academy hasn't sent a team to the Math Relays since their runner-up performance 6 years ago.

Back then, their current coach, Fiona Fawn, led the team to a memorable second-place finish.

COURTESY OF MATH RELAYS FILMS

MATH RELAYS

ROUND 1	ROUND 2	FINALS	ROUND 2	ROUND 1
CENTRAL ELEMENTARY	TO THE MAX	BEAST ACADEMY	BEAST ACADEMY	WOOLIE PREP
HIGHLAND SCHOOL OF THE ACADEMIC YETI	CENTRAL ELEMENTARY	WOOLIE PREP	WOOLIE PREP	COBRA KAI
TO THE MAX	ORB ELEMENTARY	TO THE MAX	THE AVIAN SCHOOL	DRAGON VIEW
MIDVALE SCHOOL FOR THE GIFTED	L.E.R.N.S. INSTITUTE	ORB ELEMENTARY	ARACHNID ONLINE PREP SCHOOL	BEAST ACADEMY
L.E.R.N.S. INSTITUTE				MONKU ELEMENTARY
ORB ELEMENTARY				THE AVIAN SCHOOL
PS 254				ARACHNID ONLINE PREP SCHOOL
POKEY OAKS				STUART ELEMENTARY

Do you think they've got what it takes to top Max Norris and his team?

Max is on a great winning streak...

...but this is an event that usually takes four strong team members to win.

We'll find out soon. They're just about ready to begin the final relay.

MATH TEAM
Math Relays: The Finals

This is what you've been working for. It's time to get out there and do your best.

Get to your stations. Good luck!

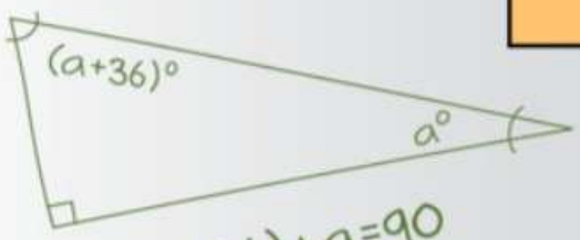
ding ding

There's the start bell. The first four competitors have to solve Problem 1 without any help from teammates.

Problem 1:
One of the acute angles in a right triangle is 36 degrees larger than the other. How many degrees are in the measure of the smallest angle?

Beast Academy team member Lizzie is off to a quick start. Let's go to her glasses-cam for a better view.

It looks like she already has an answer!



$(a+36)+a=90$
$a+a+36=90$
$-36\ \ -36$
$a+a=54$
$a=27$

Lizzie is headed to Station 2 just ahead of the other teams, Mike.

She'll join teammate Alex at Station 2.

Problem 2:
A fish tank that is $\frac{3}{4}$ full contains ___ gallons of water. How many gallons must be added to fill the tank?

Meanwhile, the monsters at Station 3 are already hard at work.

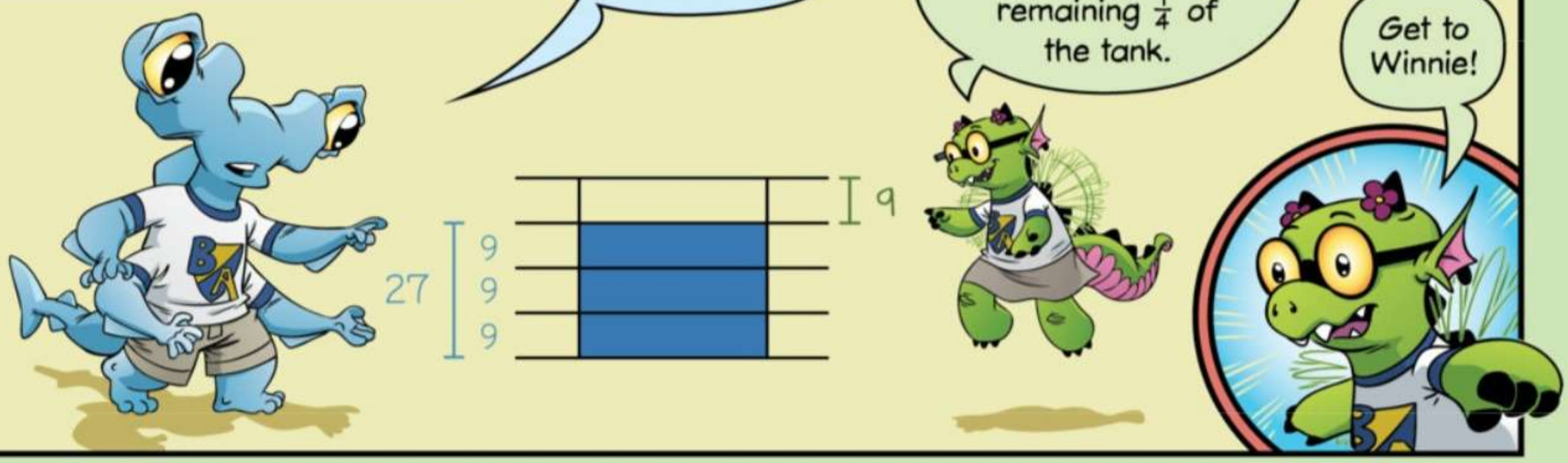
Problem 3:
Find the sum of the five smallest multiples of ___ that have a remainder of 24 when divided by 25.

And at Station 4, it looks like a few competitors are trying out some sample numbers, while Max's team member Dash Farnsworth appears to be stretching out his hamstrings.

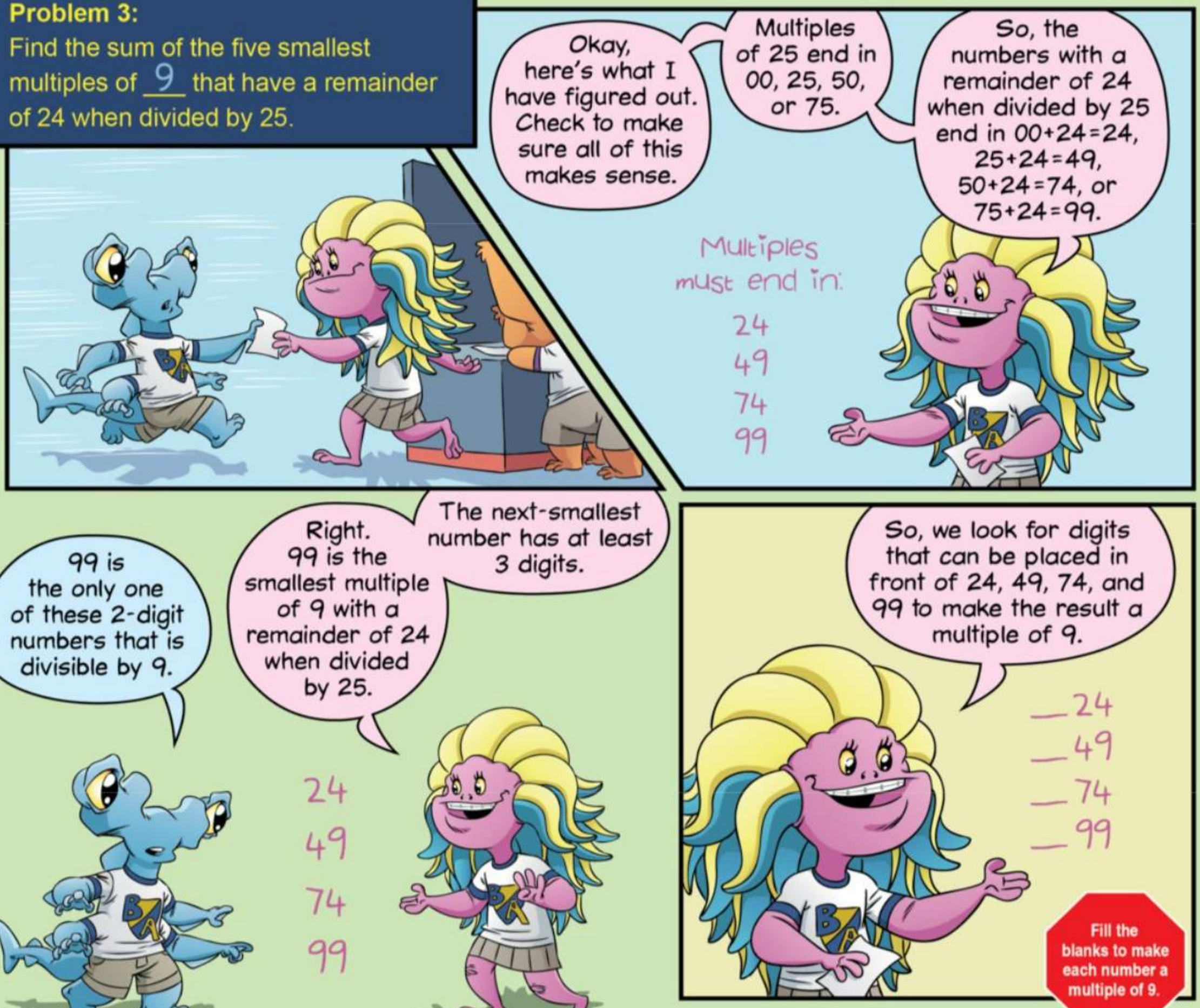
Problem 4:
Two of the digits of ____ are selected at random and multiplied. What is the probability that their product is even?

Can you solve them all?

Problem 2:
A fish tank that is $\frac{3}{4}$ full contains 27 gallons of water. How many gallons must be added to fill the tank?



Problem 3:
Find the sum of the five smallest multiples of 9 that have a remainder of 24 when divided by 25.



Every multiple of 9 has a digit sum that is a multiple of 9, so we look for numbers that have a digit sum that is divisible by 9.

I get 324, 549, 774, and 999.

These are the only hundreds digits that work.

3 24 3+2+4 = 9
5 49 5+4+9 = 18
7 74 7+7+4 = 18
9 99 9+9+9 = 27

I got the same answer!

99
324
549
774
999

The five smallest numbers are 99, 324, 549, 774, and 999.

Let's add them.

$$\begin{array}{r} {\scriptstyle 3\,3} \\ 99 \\ 324 \\ 549 \\ 774 \\ +\ 999 \\ \hline 2,745 \end{array}$$

Problem 4:
Two of the digits of 2,745 are selected at random and multiplied. What is the probability that their product is even?

What's the number?

2,745.

I know how to solve the problem!

What is the probability that two randomly selected digits from 2,745 have an even product?

Hey! Where did Max go?

He just left! I gotta run!

five smallest
have a remainder

We count all of the possible pairs of digits.

Then we figure out how many of those pairs of digits have an even product.

We can choose $4\times3\div2=6$ possible pairs of digits from the digits 2, 7, 4, and 5.*

2745

*THESE PAIRS ARE (2,7) (2,4) (2,5) (7,4) (7,5) AND (4,5). REVIEW COUNTING PAIRS IN BEAST ACADEMY 4B.

How many of those six pairs have a product that is even?

We can count the pairs that have an ***odd*** product.

The only way for a pair of digits to have an odd product is if ***both*** digits are odd.

And the only pair of odd digits is 7 and 5.

2745

$7\times5=35$

SOMETIMES IT'S EASIER TO COUNT THE POSSIBILITIES YOU ***DON'T*** WANT.

Exactly! All of the other pairs have a product that is even.

So, 5 of the 6 possible pairs of digits have a product that is even.

Which means the probability that two randomly selected digits of 2,745 have an even product is $\frac{5}{6}$.

Grogg, ***go!***

Grogg has left Station 4 and is racing to turn in a final answer.

Max sent his teammate, Dash, immediately after he saw Grogg leave.

Max is not looking too confident, though.

Remember, Max never received the answer from Station 2, so his team is relying on some guesswork.

L

That's right, Phil...

...but this kid Dash can really move!

Dash has nearly caught up with Grogg as they round the corner towards the scoring table!

4

Grogg and Dash are both ***diving*** towards the judges!

We've never seen a finish like this one!

Grogg is first to reach the scorer's table!

SLAM

Let's see what the judges have to say as they review the answer sheets.

Beast Academy wins!

It does not get any better than that, Mike. What a performance!

Great job, Beast Academy.

You're an amazing team!

Amazing!? You solved Problems 3 and 4 without even knowing the answer to Problem 2!

I made some good guesses, but it still wasn't enough to win.

I'm sure we'll see you again at the next competition.

Hopefully sooner!