Chapter 5:
Expressions

Ms. Q. Expressions

Who can show me a math expression?

These are the expressions I make when I do math.

These are *not* the math expressions Ms. Q. is looking for.

A math ***expression*** uses numbers and operations like + and − to stand for a value.

For example, 2+2, 15−6, and 9−5+3 are all expressions.

2+2

15−6

9−5+3

Excellent.

When we ***evaluate*** an expression, we find its value.

What do we get when we evaluate Lizzie's expressions?

2+2

15−6

9−5+3

The first two are easy. 2+2 is 4, and 15−6 is 9.

For the last one, do we subtract first, then add...

...or add first, then subtract?

Does it matter? Maybe we get the same answer both ways.

2+2 =4

15−6 =9

9−5+3

Does it matter?

If we start with 9−5=4, then add 3, we get 4+3=7.

9−5+3
= 4+3
= 7

But if we add 5+3 first, we get 8.

Then, 9−8=1.

9−5+3
= 9−8
= 1

Both answers can't be right.

"In an expression that has addition and subtraction, working from left to right always gives the correct answer."

So, 9−5+3 is 7.

9−5+3
= 4+3
= 7

9−5+3
= 9−8
= 1

So, the expression 9−5+3 means you start with 9, take away 5, then add 3.

What if we want to subtract 5+3 from 9?

How could we write an expression for that?

We can use ***parentheses!***

In an expression, we do what's in parentheses first.

That's right, Lizzie.

How could you write parentheses in 9−5+3 so the addition comes before the subtraction?

$9-5+3$

You could put parentheses around 5+3.

So, the parentheses mean we add 5+3 first?

Yep.

So, 5+3=8.

Then, 9−8=1.

$$9-(5+3)$$
$$= 9-8$$
$$= 1$$

Excellent. What do we get when we evaluate each of these expressions?

$16-8-4+2$

$16-(8-4)+2$

$16-8-(4+2)$

$16-(8-4+2)$

Try all four.

Without any parentheses, we just work from left to right.

16−8 is 8, 8−4 is 4, and 4+2 is 6.

$$16-8-4+2$$
$$= 8-4+2$$
$$= 4+2$$
$$= 6$$

We start in the parentheses for this one. 8−4=4.

Then, we work from left to right.

$$16-(8-4)+2$$
$$= 16-4+2$$
$$= 12+2$$
$$= 14$$

16−4 is 12, and 12+2 is 14.

Here, we start with 4+2=6. That gives us 16−8−6.

Then, 16−8 is 8, and 8−6 is 2.

$$16-8-(4+2)$$
$$= 16-8-6$$
$$= 8-6$$
$$= 2$$

For this one, we start in parentheses with 8−4+2.

8−4 is 4, and 4+2 is 6.

$$16-(8-4+2)$$
$$= 16-(4+2)$$
$$= 16-6$$
$$= 10$$

Then, 16−6 is 10.

Nice job. An expression can even contain more than one pair of parentheses.

How would you evaluate this one?

$$7-(5-(3-1))$$

Parentheses *inside* parentheses?

The outer parentheses have 5−(3−1) inside.

$7-(5-(3-1))$

$= 7-(5-2)$

$= 7-3$

$= 4$

So, we start with the inner parentheses. 3−1=2.

Then, 5−2 is 3...

...and 7−3 is 4.

Want to see a few more of my expressions?

Sure, Grogg. Why not?

What expression do you make when you realize that you left your homework folder at home on the day it's due?

How about the expression you make when you find out it's pizzarito day in the cafeteria?

Do the one where it looks like you're about to sneeze!

That's my favorite, too!

Lunch

Have you guys tried the last question on tonight's homework?

It's lunchtime, Grogg.

Why are you doing your homework *now?*

I have karate at 3:15, macrame club at 4:30...

...and my grandpa promised to show me how to charge a smartbot with potatoes after dinner!

What's the last question?

How many *different* values can you create by writing one pair of parentheses in 30−10+5−1?

Sounds like a busy day!

12. How many *different* values can you create by writing one pair of parentheses in the expression below?

30 − 10 + 5 − 1

12. _____

What have you tried so far?

I just started by writing a few different pairs of parentheses.

$$(30 - 10 + 5 - 1)$$

$$(30 - 10) + 5 - 1$$

$$(30 - 10 + 5) - 1$$

Putting parentheses around the whole expression doesn't change how we evaluate it.

We just work from left to right.

$$(30 - 10 + 5 - 1)$$

$$= 20 + 5 - 1$$

$$= 25 - 1$$

$$= 24$$

Putting parentheses around 30−10 doesn't change what we do first.

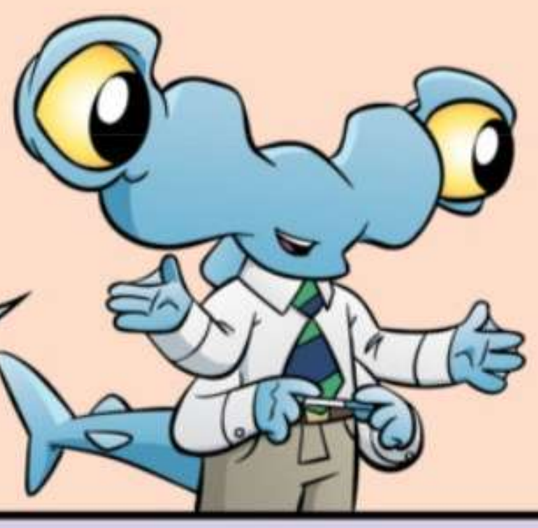
We subtract 30−10 first, with or without the parentheses.

$$(30 - 10) + 5 - 1$$

$$= 20 + 5 - 1$$

$$= 25 - 1$$

$$= 24$$



These parentheses don't change the order, either.

We get 24 again.

$$(30 - 10 + 5) - 1$$

$$= 20 + 5 - 1$$

$$= 25 - 1$$

$$= 24$$

Putting parentheses around the stuff at the ***beginning*** doesn't change what we do first.

Oh, right!

That stuff was going to come first, anyway!

$(30 - 10 + 5 - 1) = 24$

$(30 - 10) + 5 - 1 = 24$

$(30 - 10 + 5) - 1 = 24$

Maybe I'll get something different if I put the parentheses at the end, around 5−1.

$30 - 10 + (5 - 1)$

Oh, no! I got 24 again!

The order was different, but you still got 24.

Isn't there ***anywhere*** I can write parentheses that ***doesn't*** give me 24?

$30 - 10 + (5 - 1)$

$= 30 - 10 + 4$

$= 20 + 4$

$= 24$

Can you find two more ways to place parentheses?

$(30-10)+5-1$

$30-(10+5)-1$

$30-10+(5-1)$

Let's organize all the ways to write parentheses.

There are 3 ways to group two numbers...

$(30-10+5)-1$

$30-(10+5-1)$

...2 ways to group three numbers...

$(30-10+5-1)$

...and only 1 way to group all four numbers.

$(30-10)+5-1=24$

$30-(10+5)-1$

$30-10+(5-1)=24$

$(30-10+5)-1=24$

$30-(10+5-1)$

$(30-10+5-1)=24$

We already evaluated four of these.

But, we haven't tried these two.

$30-(10+5)-1$

$30-(10+5-1)$

Maybe we'll get something other than 24.

Try both.

Putting parentheses around 10+5 gives us a new value...

...14!

Putting 10+5−1 in parentheses gives us...

...16!

That's new, too!

$30-(10+5)-1$

$= 30 - 15 - 1$

$= 15 - 1$

$= 14$

$30-(10+5-1)$

$= 30-(15-1)$

$= 30-14$

$= 16$

$(30-10)+5-1 = 24$

$(30-10+5)-1 = 24$

$(30-10+5-1) = 24$

$30-(10+5)-1 = 14$

$30-(10+5-1) = 16$

$30-10+(5-1) = 24$

So, with one pair of parentheses in 30−10+5−1, we can get 14, 16, or 24.

That's 3 different values.

12. How many *different* values can you create by writing one pair of parentheses in the expression below?

$30-10+5-1$

12. 3

Thanks, guys!

Where are you going, Grogg?

To see if they have any extra potatoes in the kitchen!

SNACKS!

PIZZARITO

R&G Symbols

It's National Fitness Month again.

Time to get back in shape!

What have you got planned for us this year?

Every day this month, we'll do three exercises...

...pushups, jogging, and burpees.

What's this poster for?

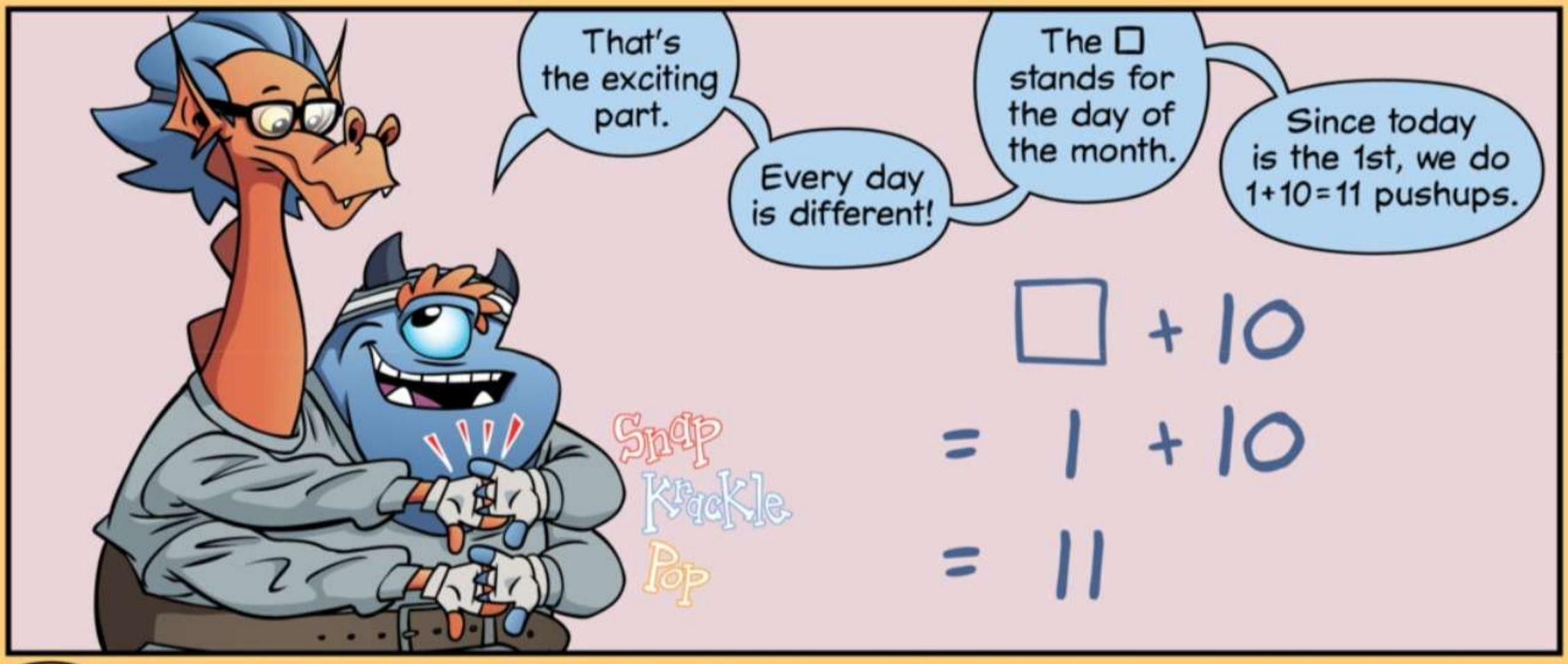
It lets us know how much of each exercise to do each day this month.

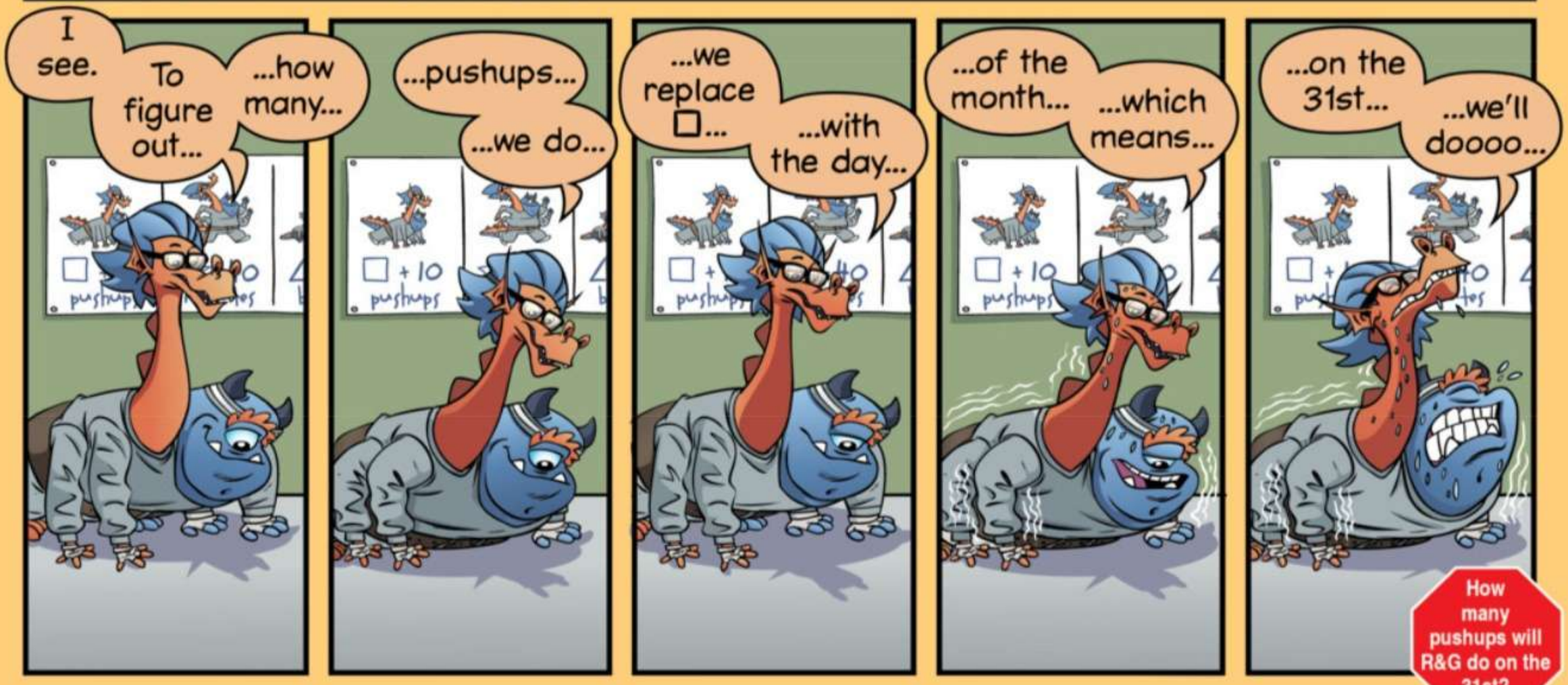
□ + 10 pushups

☆ - 40 minutes

△ + △ burpees


□ + 10 pushups
But, there's no calendar. How are we supposed to know what to do on each day?
I wrote math expressions that tell us what to do.
Hmmm... This poster says that we do □+10 pushups. What does that mean?


That's the exciting part.
Every day is different!
The □ stands for the day of the month.
Since today is the 1st, we do 1+10=11 pushups.
Snap
Krackle
Pop
□ + 10
= 1 + 10
= 11


I see.
To figure out...
...how many...
...pushups...
...we do...
...we replace □...
...with the day...
...of the month...
...which means...
...on the 31st...
...we'll doooo...
How many pushups will R&G do on the 31st?

...41 pushups!

□ + 10
= 31 + 10
= 41

dab dab dab

Yep. We're gonna look awesome!

Next is jogging.

The poster says that we jog for ☆-40 minutes.

What does ☆ stand for?

☆-40 minutes

☆ stands for the day's high temperature.

It's supposed to get up to 65 today.

Yep. So, how many minutes do we have to jog?

☆-40

How many?


We can replace the ☆ in the expression with today's high temperature, 65.
That gives us 65−40=25 minutes of jogging.
☆−40
=65−40
=25


If the high temperature is only 40 degrees, we won't have to run at all!
Yep, and it's supposed to get cooler over the weekend, so we won't have to run for as long.
Good. I don't like jogging when it's cold.
Thok
☆−40
=40−40
=0


Last, we do △+△ burpees.
What is △?
△ stands for the red Superball number from the most recent Big Bucks Lottery drawing.
Last night, the Superball was 11.
How many burpees is that?
△+△
burpees
How many?

If Δ is 11, then Δ+Δ is 11+11=22.

22 is not too bad. How high does the Superball number get?

△ + △

= 11 + 11

= 22

The Superball can be as high as 50.

50!

You mean, we might need to do 50+50=100 burpees?

Yep.

But, if the Superball is 1, we only need to do 1+1=2 burpees.

This poster will definitely keep our exercise interesting this month.

Ready to run?

You go on ahead, I'll catch up.

Just kidding.

Let's do this!

MATH TEAM
Simplifying

Expressions that are different can stand for the same value.

For example, 7+3 and 13−3 both equal 10.

$7 + 3$

$13 - 3$

To begin today, I want each of you to write a different expression that is equal to 10.

OK, let's see what you wrote.

I just wrote 10.

Does that count as an expression?

10

It does. An expression can be just a number.

What did you write, Lizzie?

That works. 1+2+3+4 is 10.

1+2+3+4.

What about you, Winnie? What did you write?

$$1+2+3+4 = 3+7 = 10$$

I started to write 10−0...

...but, that seemed boring.

Since 999−999 is 0...

...I wrote 10−(999−999) instead.

$$10-(999-999) = 10-0 = 10$$

Well done.

Adding or subtracting 0 doesn't change a number.

What's your expression, Grogg?

I wrote 10+5−5+5−5+5−5.

$$10+5-5+5-5+5-5$$

Are you sure that equals 10?

Does Grogg's expression equal 10?

Adding 5 and then taking 5 away is the same as doing nothing.

As long as we take away as many 5's as we add...

...we'll end up with 10.

$$10+5-5+5-5+5-5$$

$$= 10$$

Great!

All of your expressions stand for the same value.

Which expression is simplest?

$$10$$

$$1+2+3+4$$

$$10-(999-999)$$

$$10+5-5+5-5+5-5$$

Definitely Alex's.

It's just a number!

Right.

When we ***simplify*** an expression, we make it easier to understand.

All of your expressions simplify to 10.

Simplifying is also useful when expressions include symbols.

For example, how could we simplify this expression?

$$\square - \square + 7$$

Try it.

What does the □ stand for?

It doesn't matter!

Huh?

The first thing we do is subtract □-□.

1-1 is 0.
50-50 is 0.
999-999 is 0.
Any number minus itself is 0.

So, no matter what number □ is, □-□ is ***always*** 0.

So, □-□+7 is ***always*** 7.

$$\square - \square + 7 = 0 + 7 = 7$$

That means □-□+7 ***simplifies*** to 7.

That's right. What about this one?

How could you simplify △+6-6?

$$\triangle + 6 - 6$$

Try it.

Starting with Δ, we add 6, then take away 6.

Adding 6, then taking 6 away is the same as doing nothing.

So, Δ+6−6 simplifies to just Δ!

$$\Delta + 6 - 6$$

$$= \Delta$$

You got it.

Simplifying an expression can really save time.

Especially if you have to evaluate it.

For example, if ☆ is 75...

...what is the value of this expression?

$$☆ = 75$$

What is

$$☆ + (☆ - 19) - (☆ - 19) - 40 \ ?$$

Try it.

We can replace every ☆ with 75.

☆ + (☆ − 19) − (☆ − 19) − 40 ?

= 75 + (75 − 19) − (75 − 19) − 40

= 75 + 56 − 56 − 40

Then, we do what's in parentheses.

75 − 19 is 56.

Adding 56 then subtracting 56 is the same as doing nothing.

So, we can just cross these out.

☆ + (☆ − 19) − (☆ − 19) − 40 ?

= 75 + (75 − 19) − (75 − 19) − 40

= 75 ~~+ 56~~ ~~− 56~~ − 40

That leaves us with 75 − 40...

...which is *35!*

☆ + (☆ − 19) − (☆ − 19) − 40 ?

= 75 + (75 − 19) − (75 − 19) − 40

= 75 ~~+ 56~~ ~~− 56~~ − 40

= 75 − 40

= 35

Well done.

What does the same expression give us when ☆ is 54?

☆ = 54

☆ + (☆ − 19) − (☆ − 19) − 40

Maybe we can simplify ***before*** we replace all of the ☆'s.

Can you simplify the expression?

☆+(☆−19)−(☆−19)−40

Starting with ☆...

...we add (☆−19)...

...then subtract (☆−19).

☆+(☆−19)=(☆−19)−40

Adding (☆−19) then subtracting (☆−19) is the same as doing nothing!

Yep, so we can cross all of this out!

All we're left with is ☆−40.

☆+(☆−19)=(☆−19)−40

= ☆ − 40

= 54 − 40

= 14

And when ☆ is 54, ☆−40 is 54−40 =14!

☆+(☆−19)−(☆−19)−40

Great! Simplifying can be really helpful.

= ☆ − 40

This expression simplifies to ☆−40.

When is it ever useful to have a symbol in an expression?

The custodians are using this expression for their daily workout.

The star stands for the day's high temperature...

...and the expression tells them how many minutes to run.

☆ − 40

So, the value of the expression isn't the same every day.

It's a little chilly today, so they won't have to run for very long.

Nope. But, they'll be doing a lot of burpees.

Yesterday's Superball was 45!


WORKSHOP
EQUATIONS
Ahoy, kids.
Oooh, what's in the fancy boxes?
Each small chest be holdin' a different treasure.
Moonstones, heartcoins, 'n' skull rings.
Your task be to find how many of each treasure there be, usin' the *equation* on each chest.
Equation?

An ***equation*** shows that two expressions are equal.

Like, 1+1=2?

Or, 3+3+3=4+5?

How about 99+99+99 =300-3?

1+1 = 2
3+3+3 = 4+5
99+99+99=300-3

Aye, all of these be equations.

In every equation, there be an expression on the left...

...an equals sign in the middle...

...and an expression on the right.

1+1 = 2
3+3+3 = 4+5
99+99+99=300-3

On each treasure chest, there be an equation with a symbol.

The symbol stands for the number o' treasures in the chest.

For example, the moon shape on this chest stands for the number o' moonstones inside.

How many moonstones be there in this chest?

+ 3 = 30

How many?

To figure out what number the ☾ stands for...

...we need to know what number plus 3 equals 30.

☾ + 3 = 30

_ + 3 = 30

Since 27+3=30...

...the ☾ stands for 27.

☾ + 3 = 30

27 + 3 = 30

Aye, there be 27 moonstones in this chest.

Findin' the value o' a symbol in an equation be called ***solving*** the equation.

☾ + 3 = 30

Next, try solving the equation on this chest. The heart stands for the number o' heartcoins within.

How many heartcoins be there?

10 + 40 = ♥ + 30

How many?

On the left side of the equation, we have 10+40, which equals 50.

10+40=♥+30

50 =

The right side has to equal 50, too.

Since 20+30=50, the ♥ stands for 20.

10+40=♥+30

50 =20+30

So, there are 20 heartcoins.

Well done!

In the third chest, there be skull rings.

The skull symbol stands for the number o' rings in the chest.

This time, we have symbols on both sides o' the equation.

What number does ☠ stand for?

☠+(☠-5)=☠+12

Try it.

Both sides of the equation add something to ☠.

What we add to ☠ must be the same on both sides.

☠ + (☠ - 5) = ☠ + 12

On the left side, we add (☠ - 5)...

...and on the right, we add 12.

So, (☠ - 5) must equal 12.

☠ + (☠ - 5) = ☠ + 12

For (☠ - 5) to equal 12, ☠ has to be 17!

Let's check to make sure.

☠ + (☠ - 5) = ☠ + 12

17 + (17 - 5) = 17 + 12

29 = 29

Yep, both sides equal 29 when ☠ is 17.

So, there are 17 skull rings in the box.

Aye. Excellent figurin', little buccaneers.

Where did you find all this treasure, Captain Kraken?

The moonstones be plucked from the walls in the deep caves o' the Crescent Isles.

The heartcoins be from the wreckage o' the infamous pirate, Cutlass Pete.

The skull rings be from aisle 7 at Stop-N-Save.

SALE

BIG DEALS

AHOY! PIRATE TOYS

Doubloons

Rings

Have one!

Do they come in XXXXL?