Contents: Chapter 2

Click the Play List tab in the top-left to view a recommended reading/practice sequence.

The Number Line 42

What whole number is to the left of 25 on the number line?

Distance Between 48

What number is halfway between 54 and 98?

Iago 55

You have a penny on 21. Where can you place a second penny to capture the dime on 25?

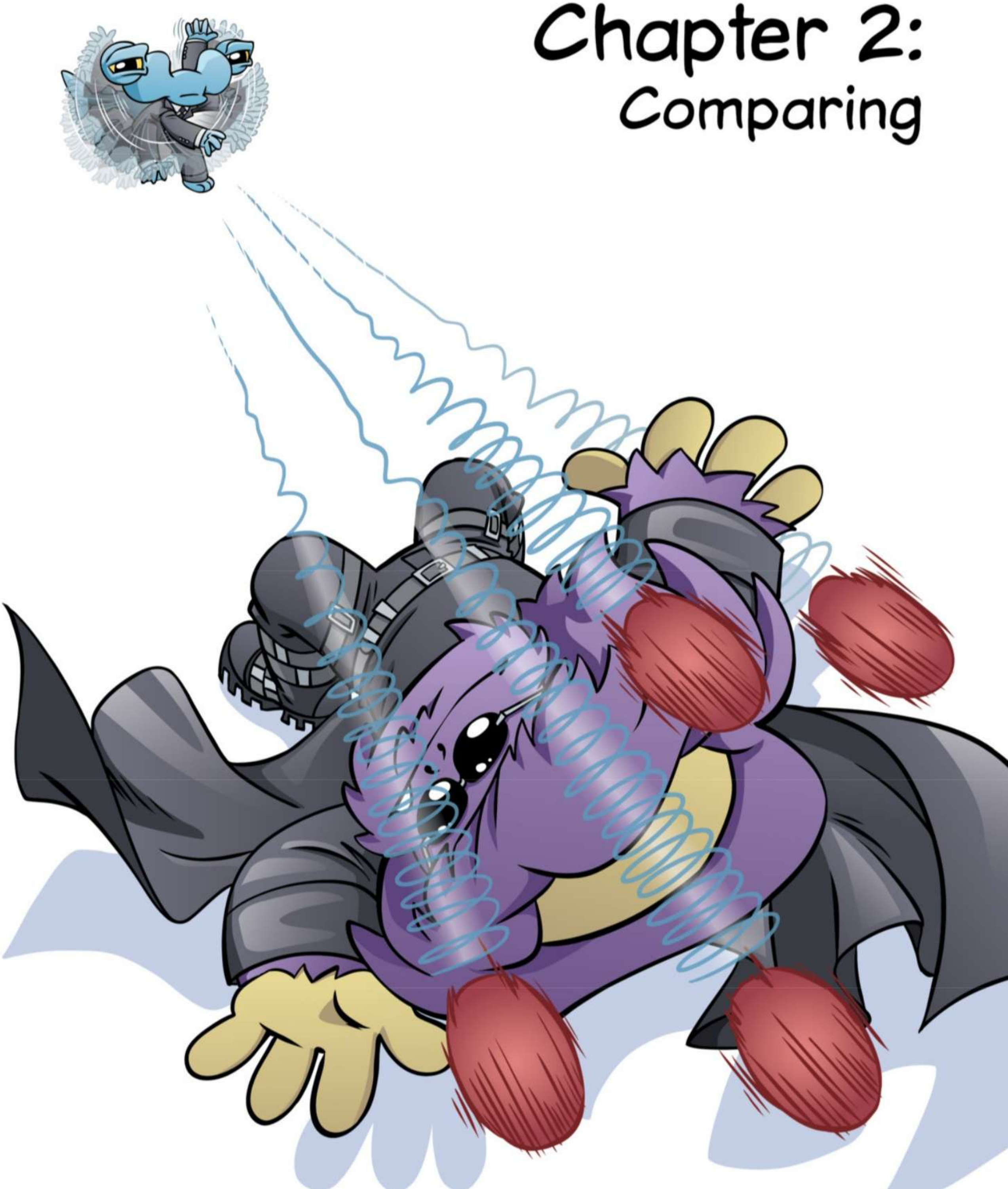
Comparing 56

What symbols are used to compare numbers that are not equal?

Ordering 60

After 888 and 889, what is the third-smallest three-digit number that uses only 8's and 9's?

Chapter 2:
Comparing



R&G
The Number Line
What's all this paint for?
The headmaster asked us to paint a giant number line on the field.
A number line?
A number line is just a straight line with numbers marked on it in order.
What numbers?
Our number line will start at 0 and include every number from 0 to 100.
Every number!?
This is going to take forever.
Every whole number from 0 to 100 will have a mark on the number line...
...but we don't need to write numbers on all of them.
WHOLE NUMBERS START AT 0 AND GO ON FOREVER: 0, 1, 2, 3, 4, 5, ...AND SO ON.

How will anyone know which mark stands for which number?

It will be easier to explain once we draw the line.

Let's start by drawing a long, straight line. Then, we can mark it.

That ought to be long enough.

Now what?

Now we can draw marks on the line.

The marks have to be equally spaced.

The distance from 0 to 1 has to be the same as the distance from 99 to 100.

We can use this rod to measure the distance between ***consecutive numbers***.

I see. We can start by drawing a mark for 0. Then, the length of this rod is the distance to the next mark.

CONSECUTIVE NUMBERS COME ONE AFTER ANOTHER. FOR EXAMPLE, 23 AND 24 ARE CONSECUTIVE.

Yep.

Uh oh. I lost track. What mark are we on?

I'm not sure.

I have an idea that will make it easier for us to keep track.

If we make every tenth mark longer than the others...

...it will be easier to know how many we've drawn.

I see. The long marks start at 0 and mark the tens...

0...

...10...

...20...

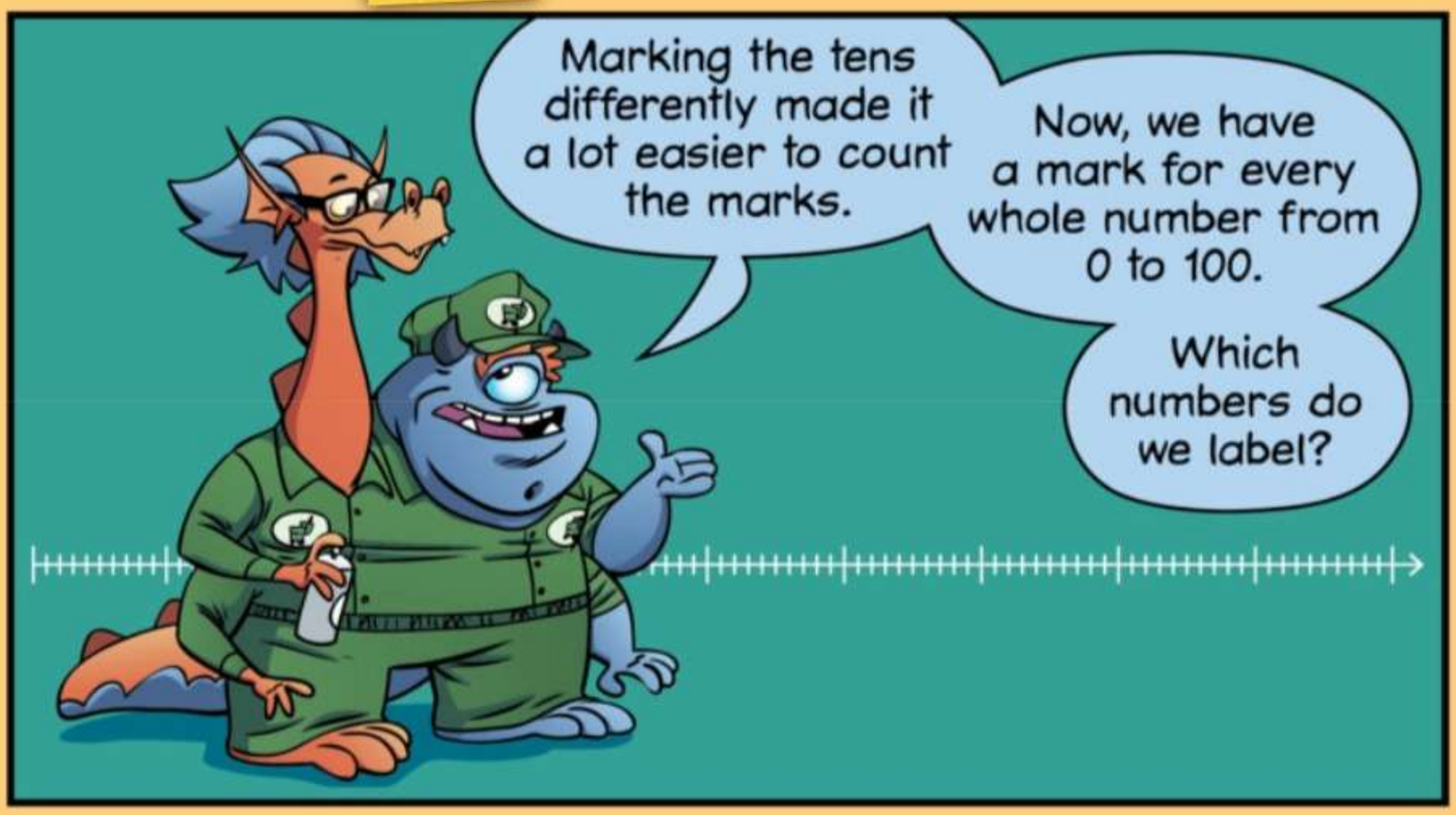
...30...

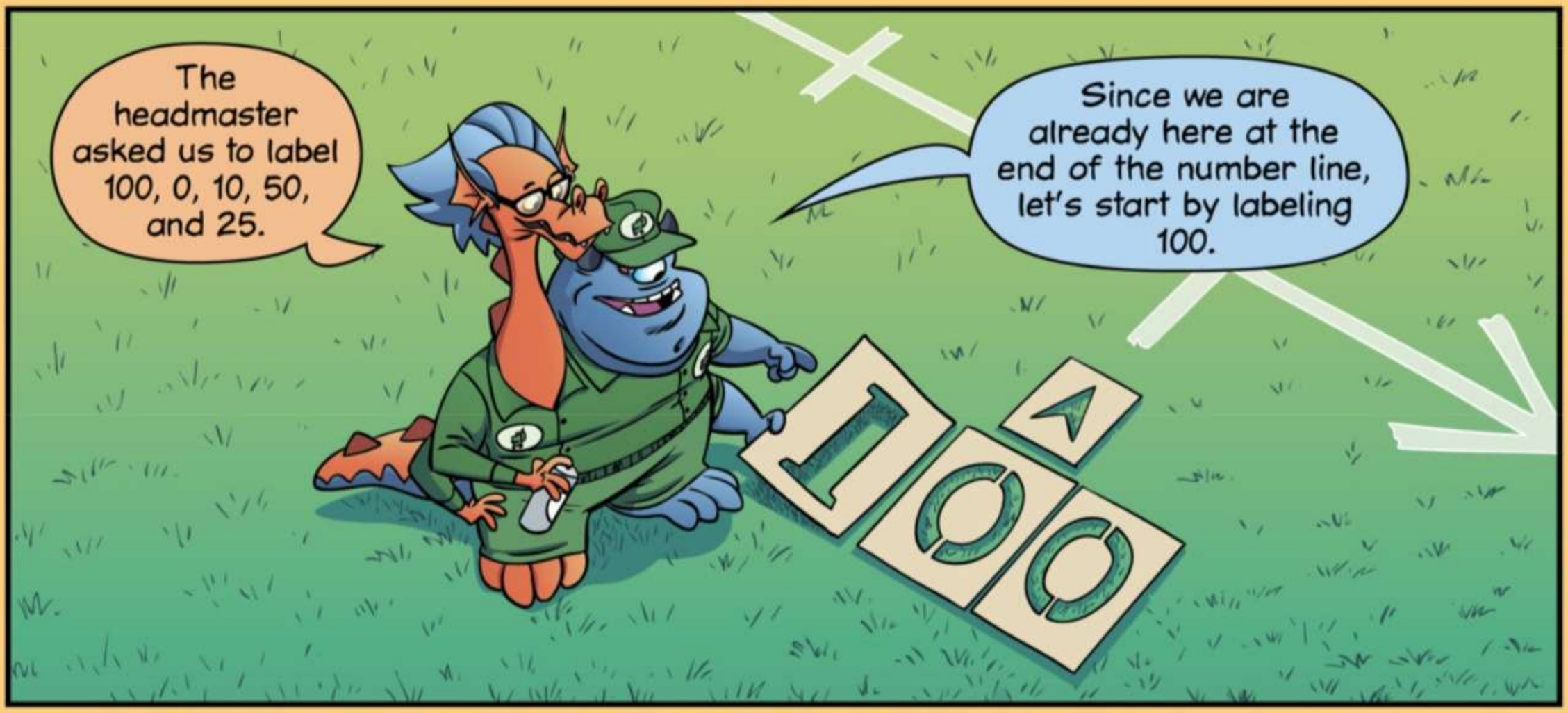
...40...

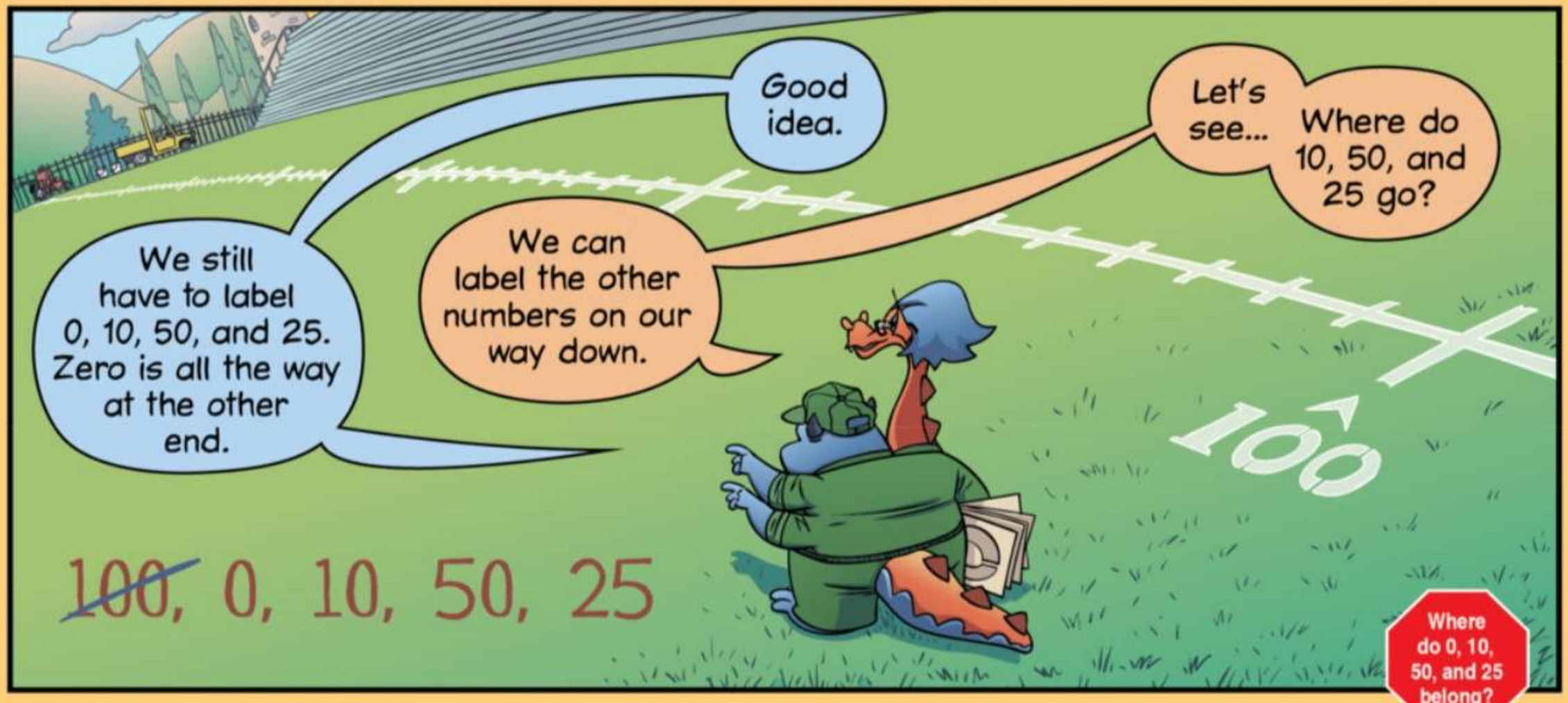
...so, we're at 44.

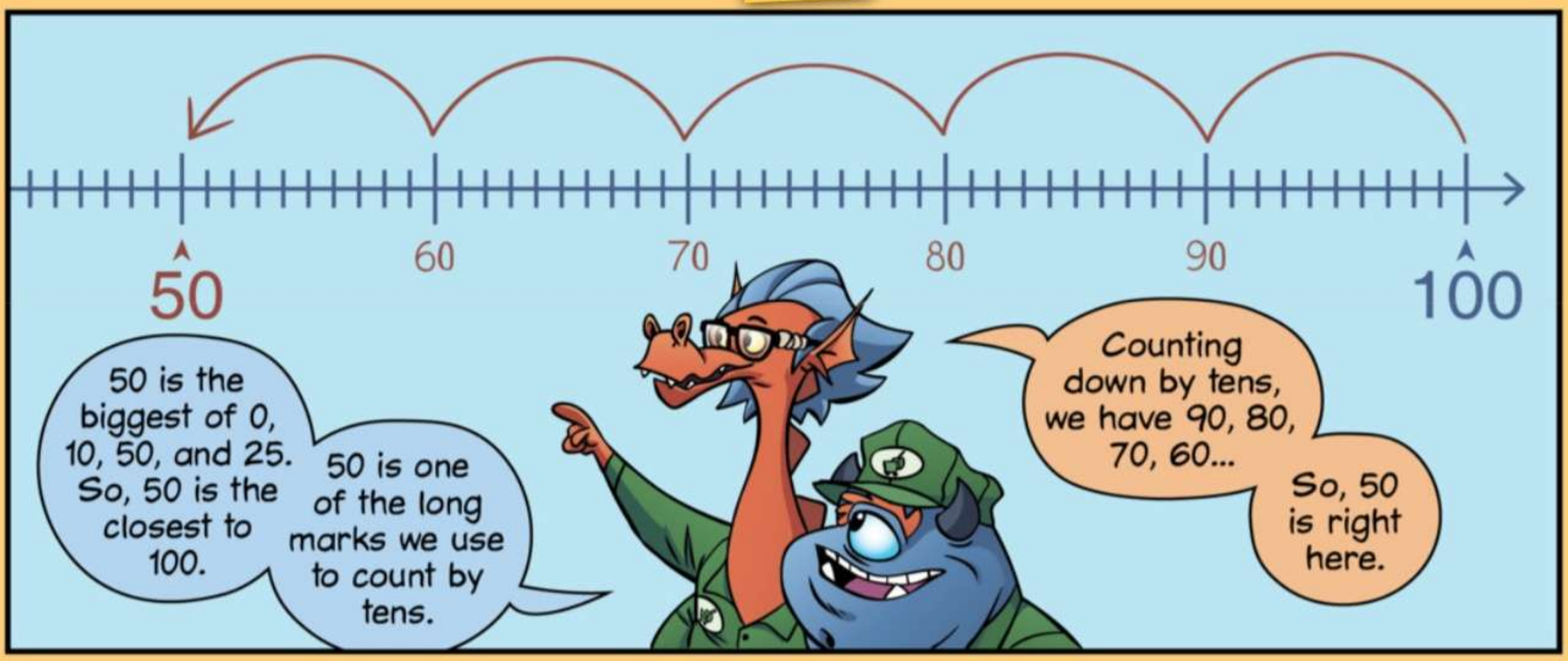
Let's finish up.

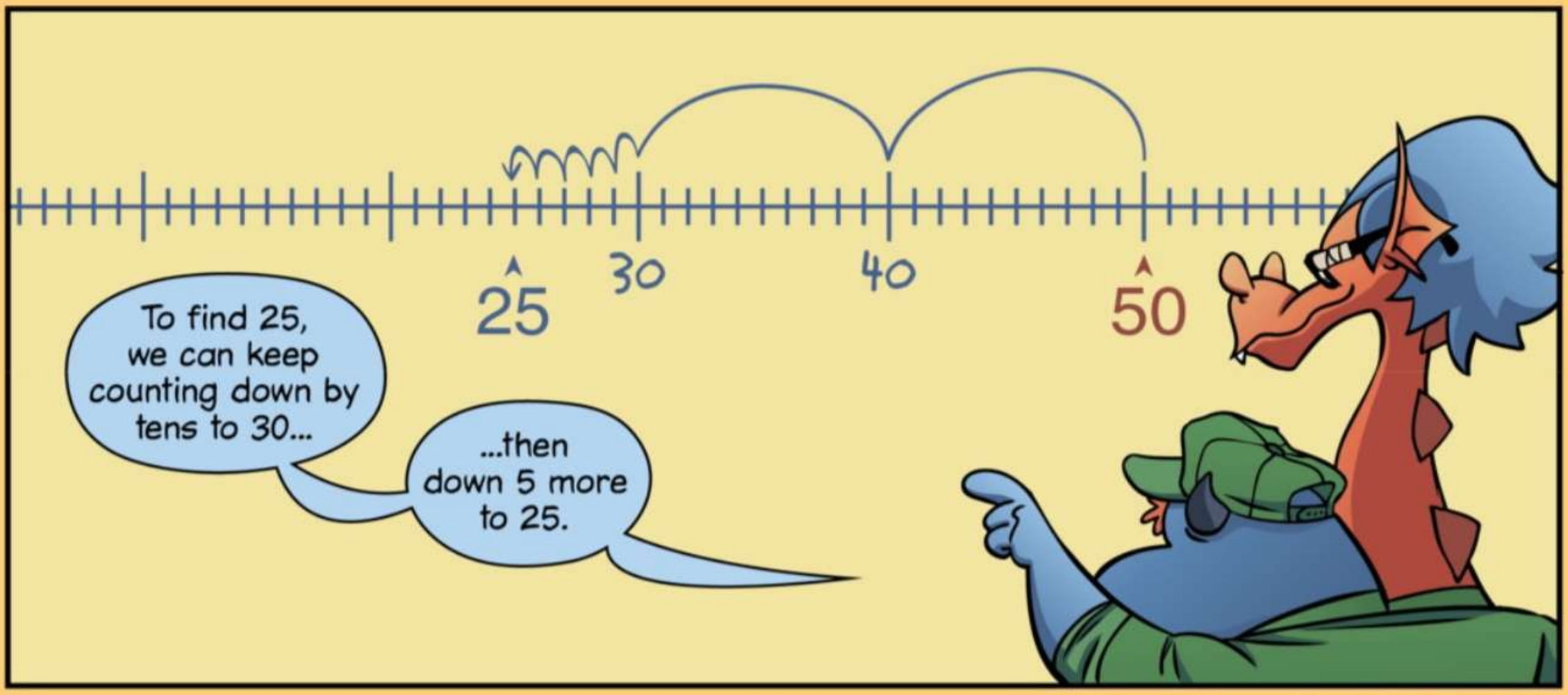
That's the last one.

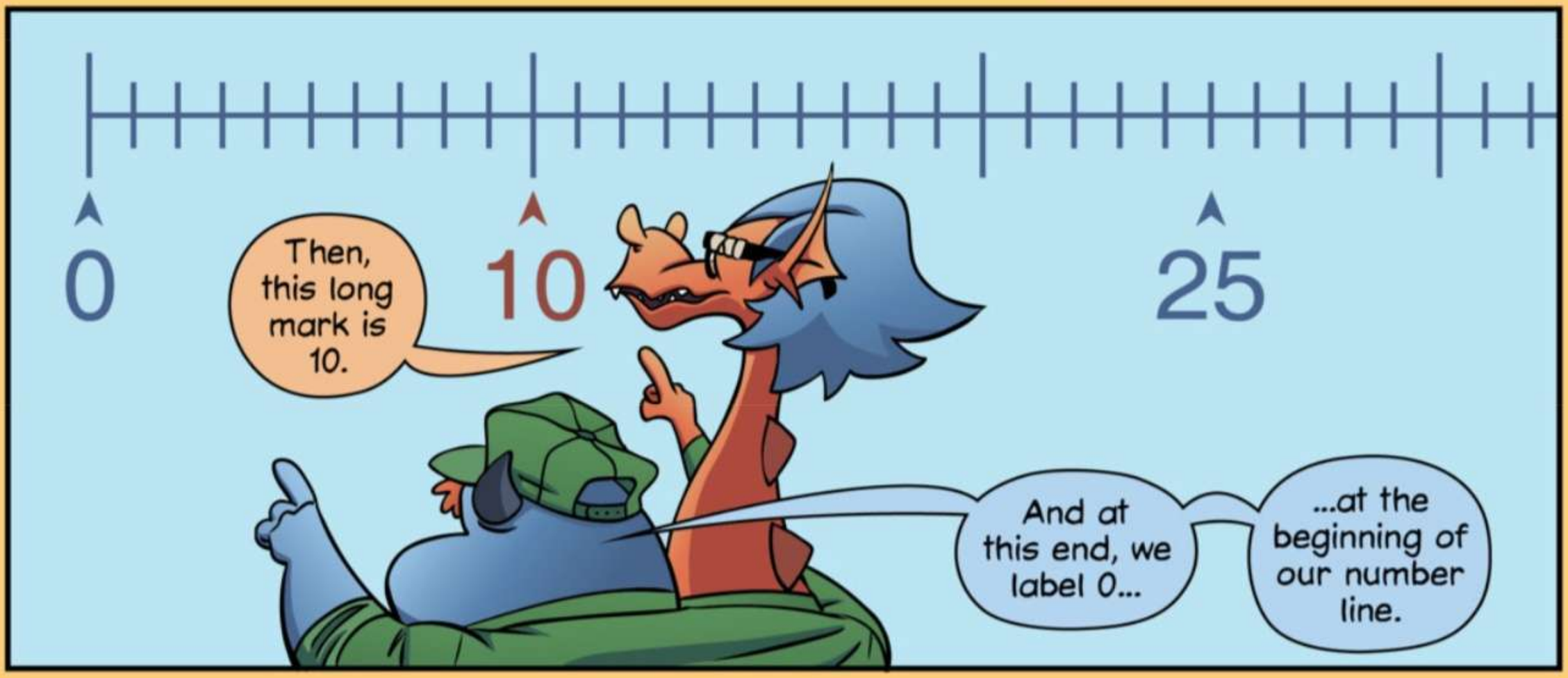

Marking the tens differently made it a lot easier to count the marks.
Now, we have a mark for every whole number from 0 to 100.
Which numbers do we label?

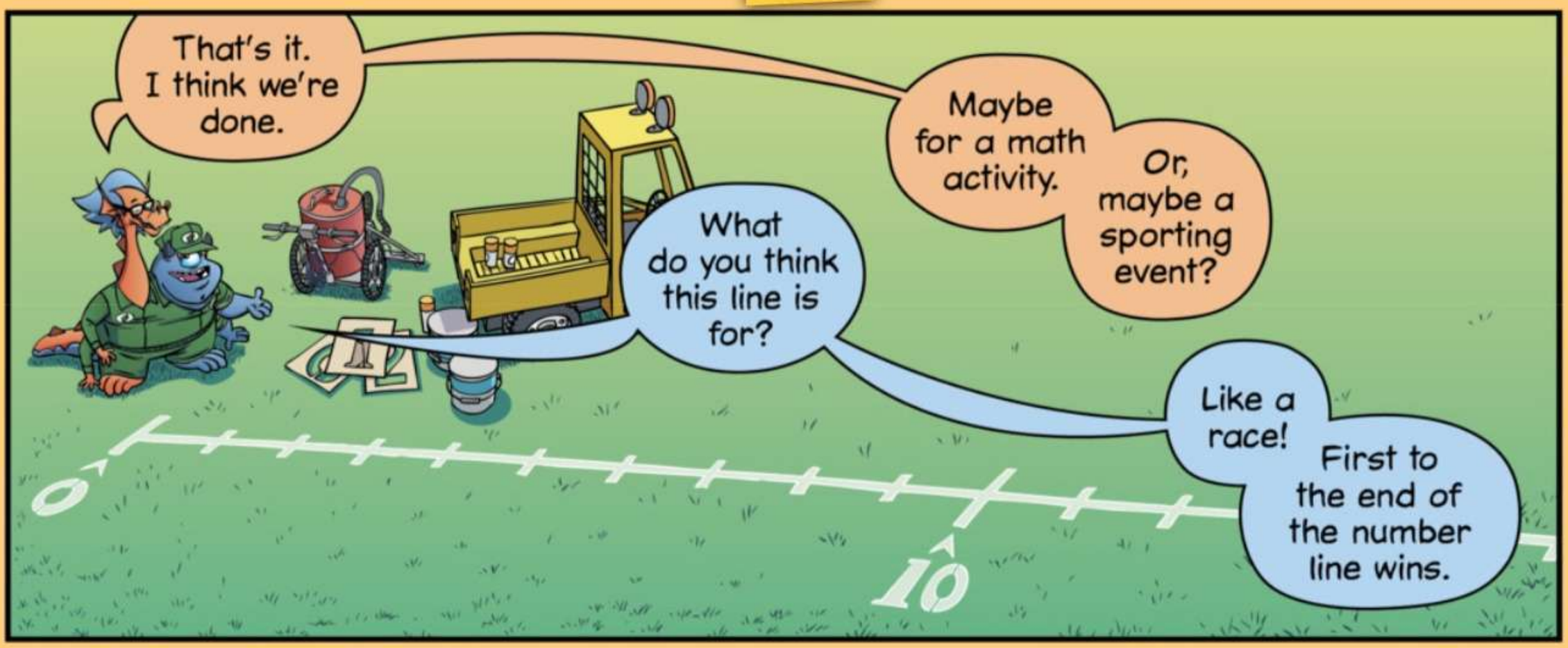

The headmaster asked us to label 100, 0, 10, 50, and 25.
Since we are already here at the end of the number line, let's start by labeling 100.
100

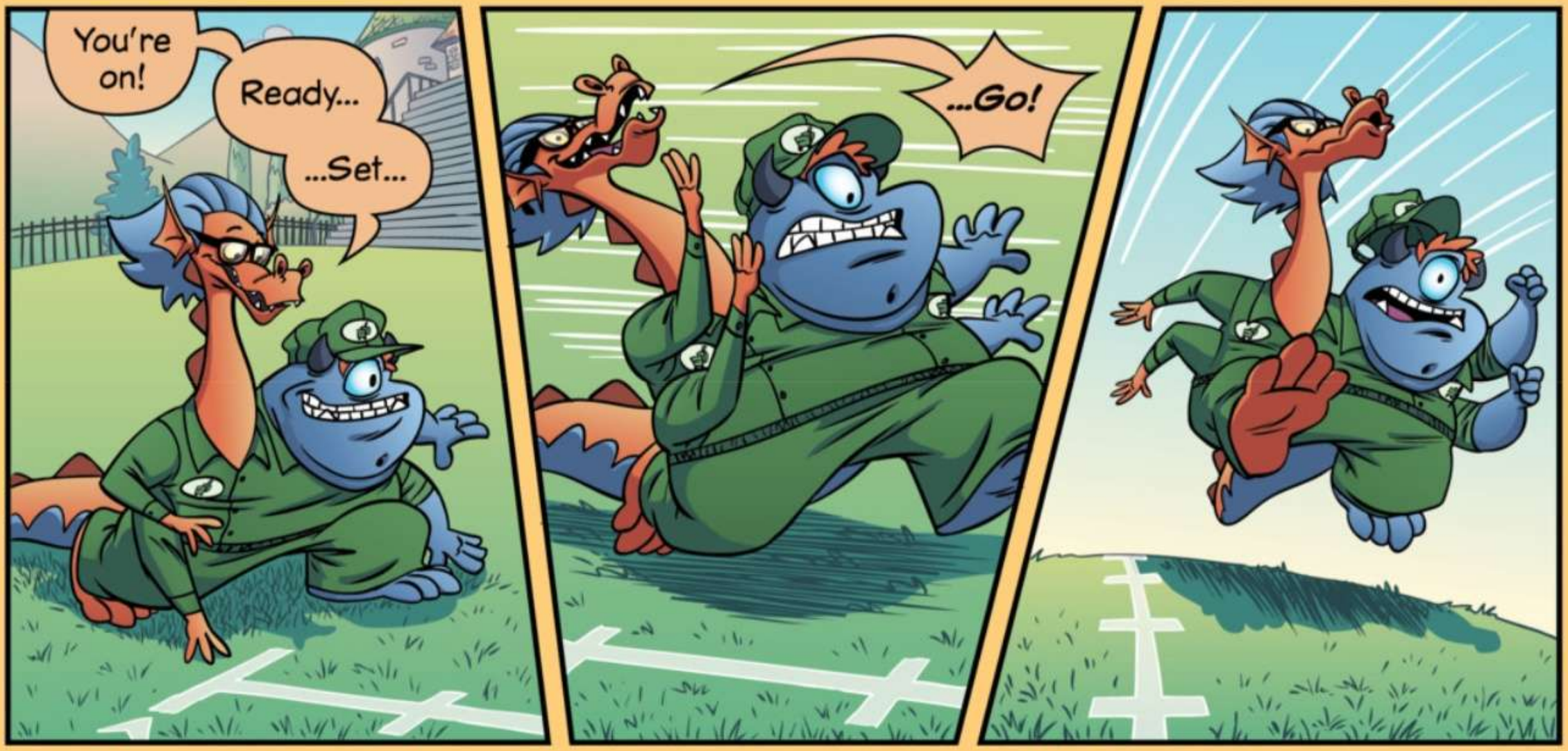

Good idea.
Let's see...
Where do 10, 50, and 25 go?
We still have to label 0, 10, 50, and 25. Zero is all the way at the other end.
We can label the other numbers on our way down.
100
~~100~~, 0, 10, 50, 25
Where do 0, 10, 50, and 25 belong?


50
60
70
80
90
100
50 is the biggest of 0, 10, 50, and 25. So, 50 is the closest to 100.
50 is one of the long marks we use to count by tens.
Counting down by tens, we have 90, 80, 70, 60...
So, 50 is right here.


25
30
40
50
To find 25, we can keep counting down by tens to 30...
...then down 5 more to 25.


0
10
25
Then, this long mark is 10.
And at this end, we label 0...
...at the beginning of our number line.


That's it. I think we're done.
What do you think this line is for?
Maybe for a math activity.
Or, maybe a sporting event?
Like a race!
First to the end of the number line wins.
0
10


You're on!
Ready...
...Set...
...Go!

100
Good race!
You always beat me by a nose.

MATH TEAM
Distance Between

Today, I'm going to introduce you to the number line.

What are we doing out here?

This number line has all of the numbers from 0 to 100.

Each whole number has its own mark, and the whole numbers are equally spaced along the line.

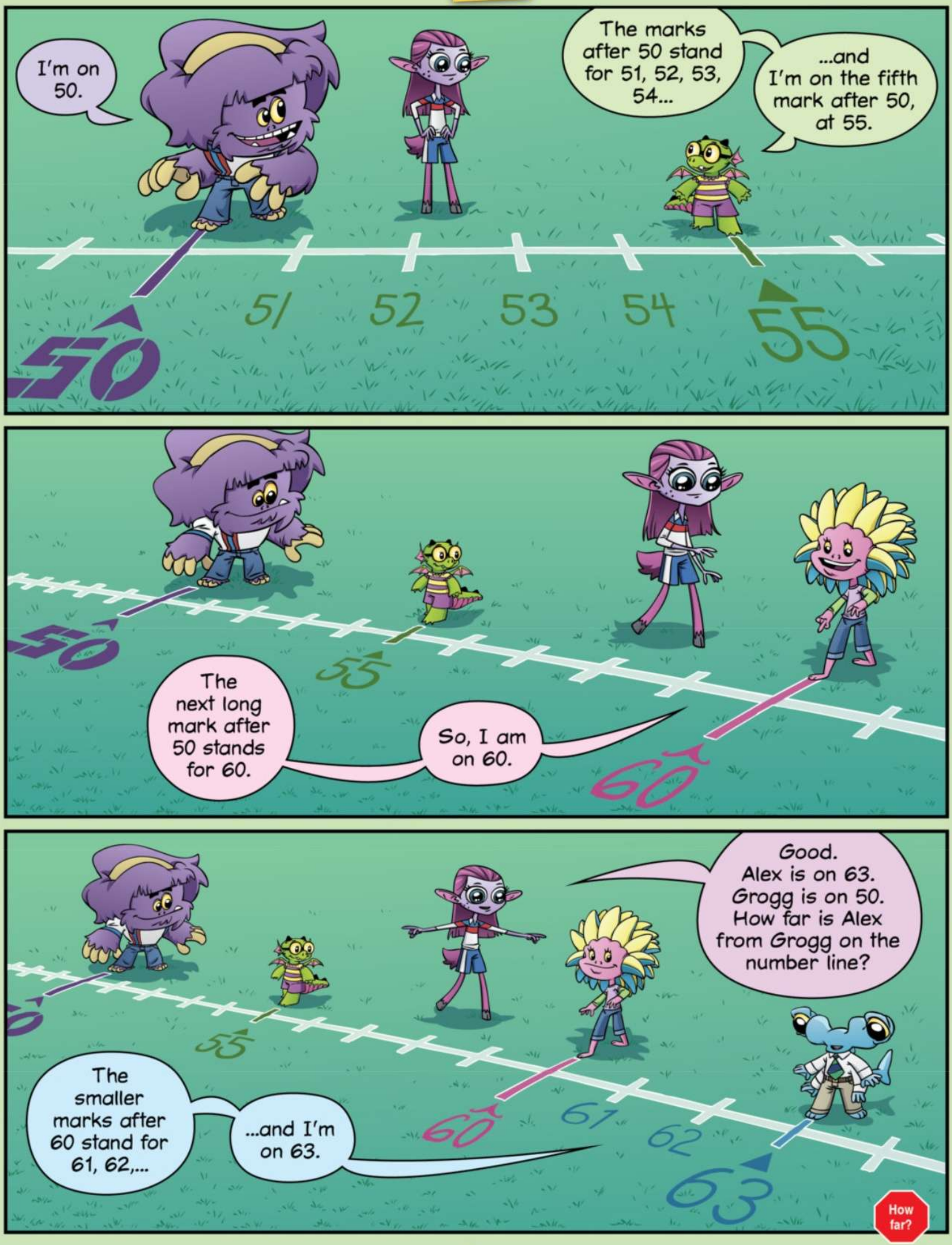
The longer marks on this number line are the numbers we get when we count by tens.

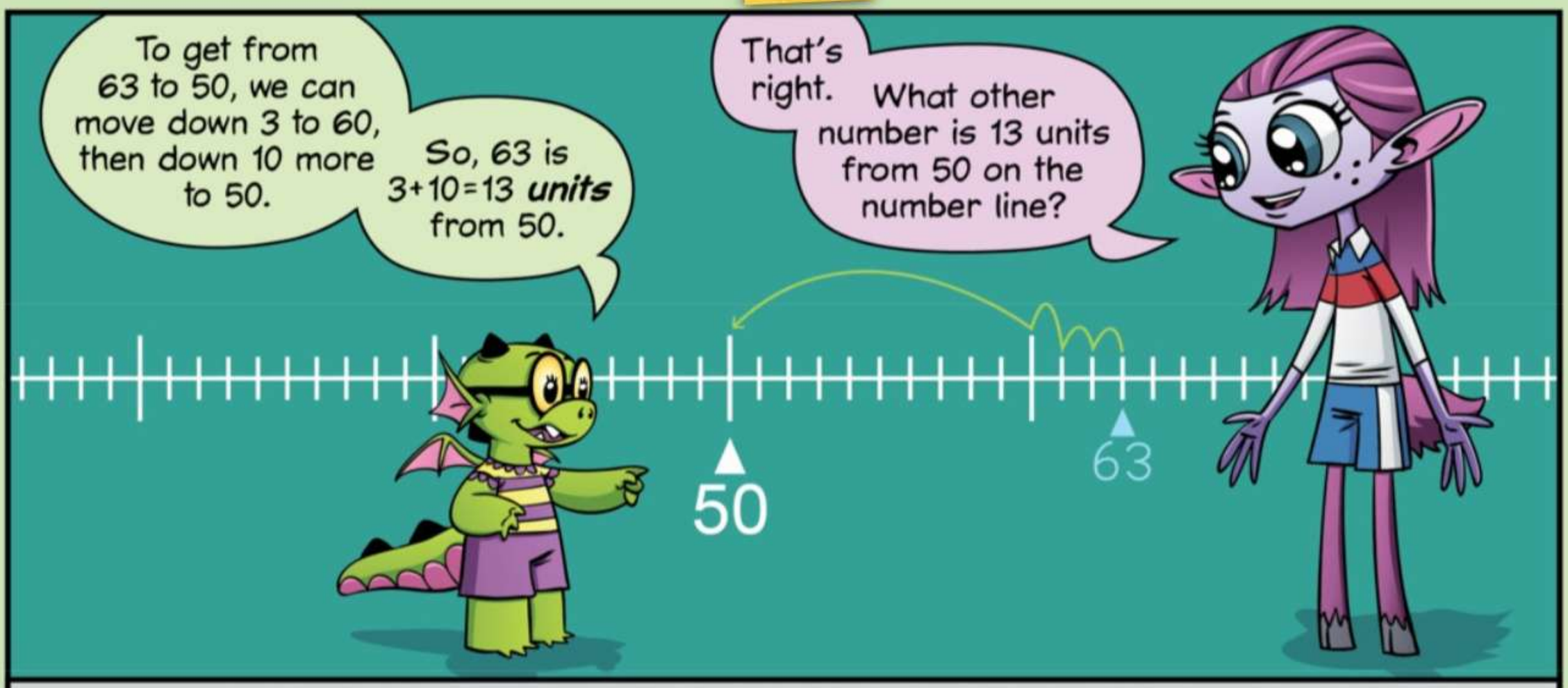
0 10 25 50 100

Everyone stand with one foot on a mark on the number line.

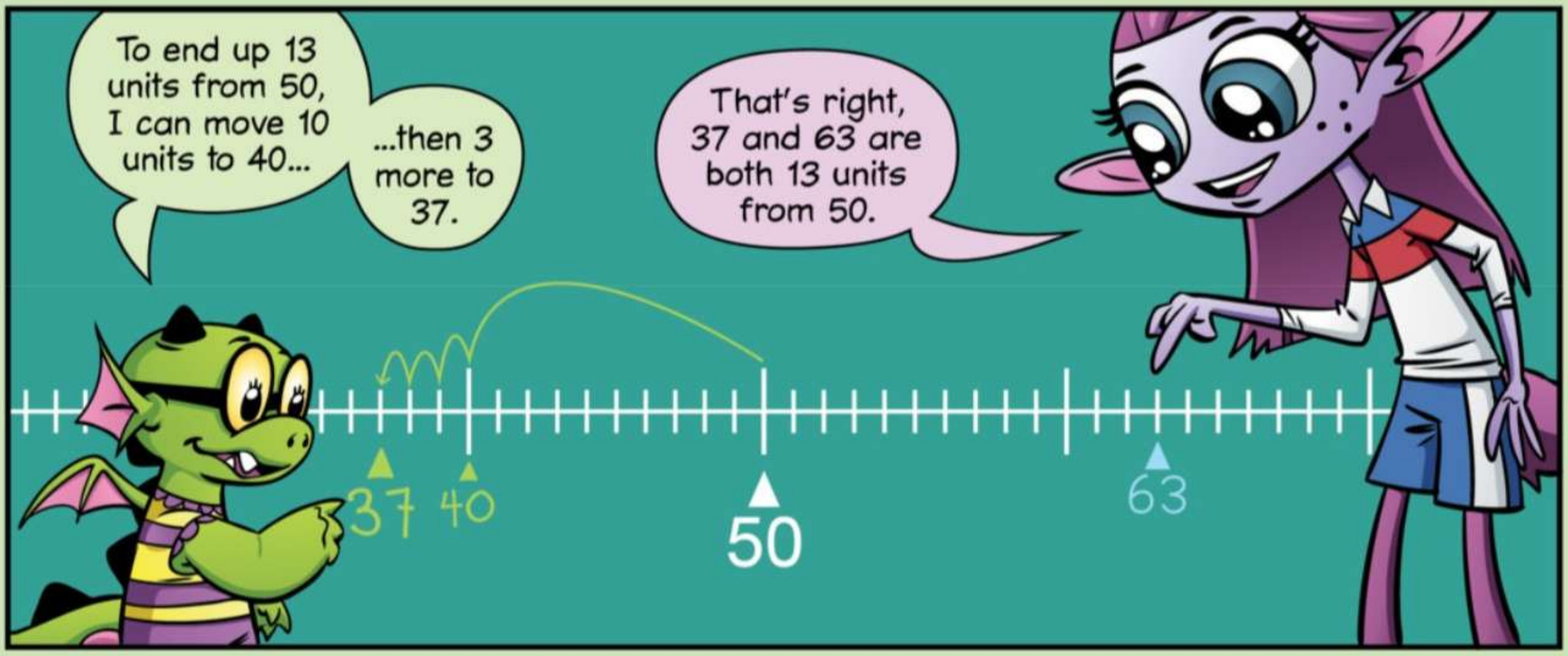
Who can tell me what number they are standing on?

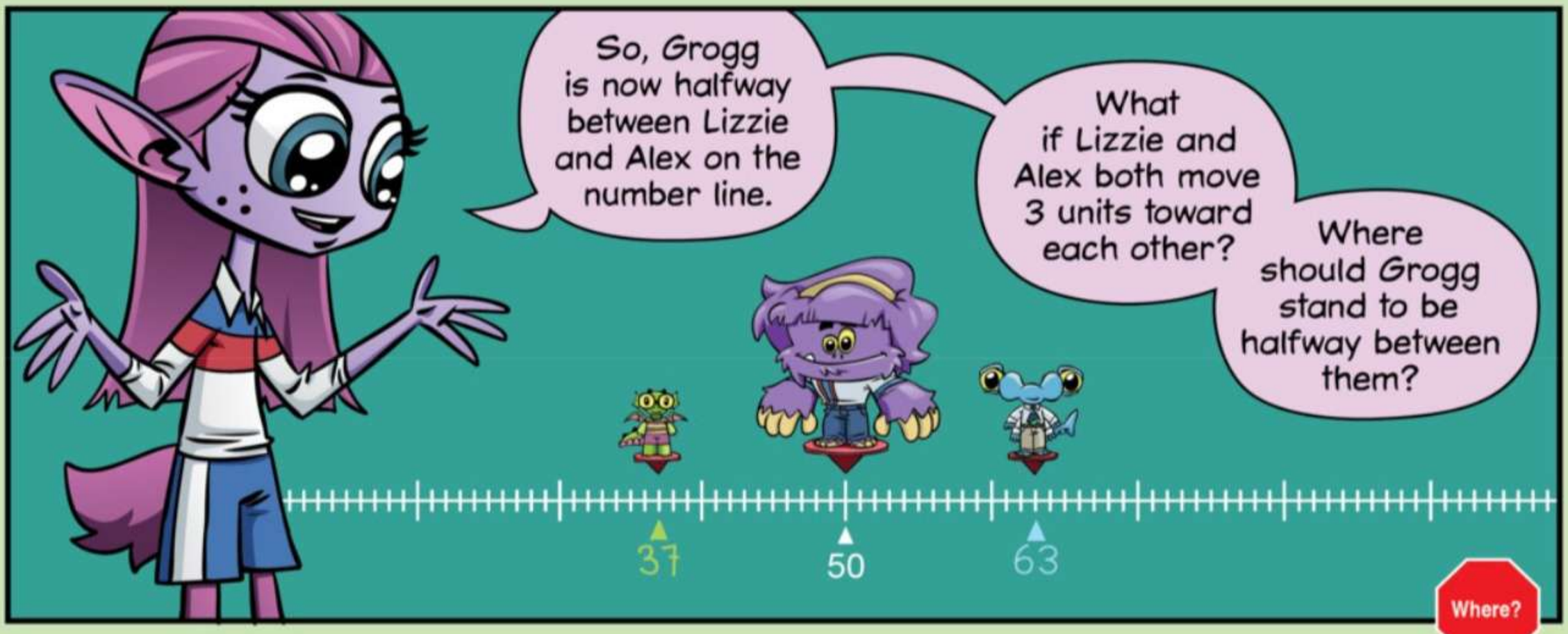
What number is each monster on?

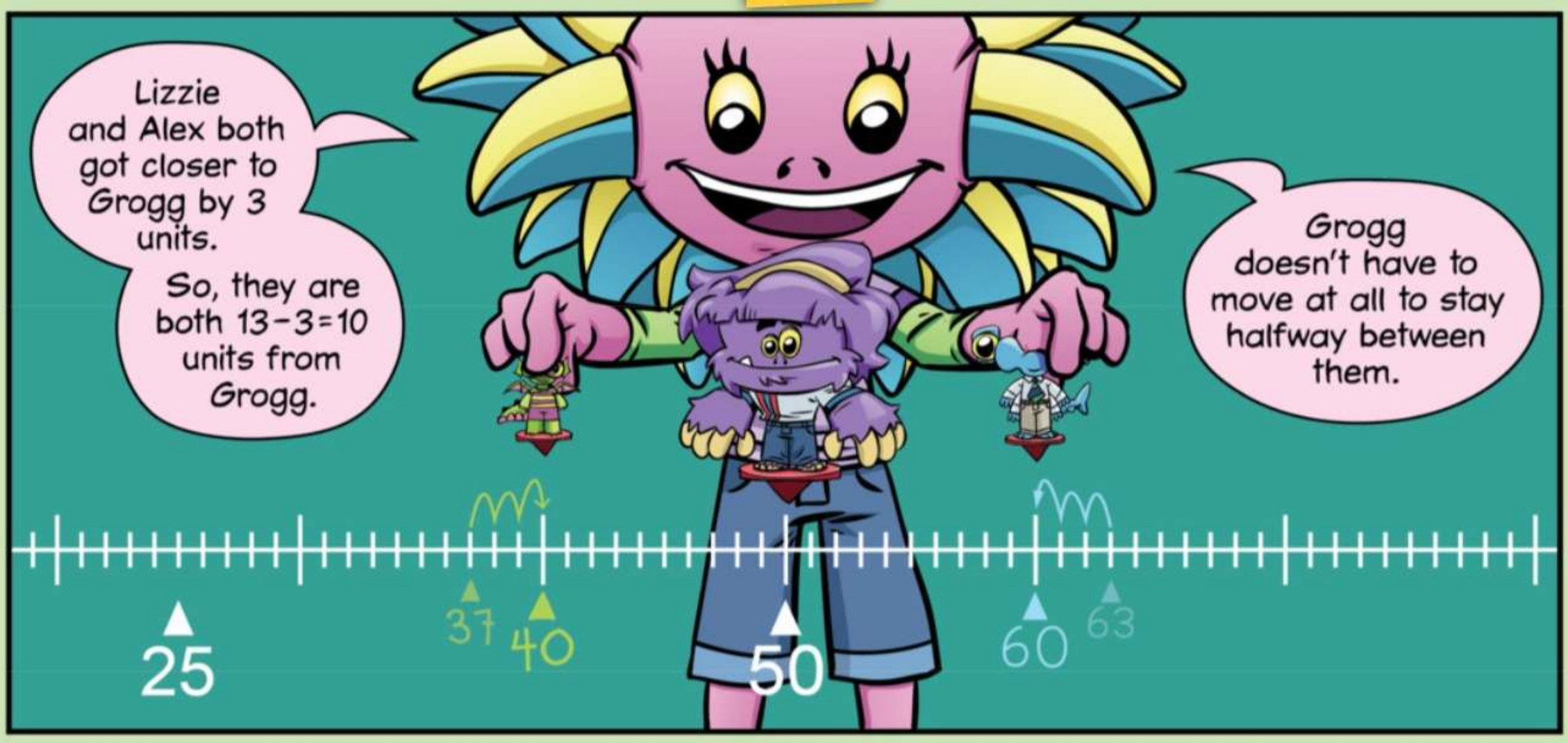

I'm on 50.
The marks after 50 stand for 51, 52, 53, 54...
...and I'm on the fifth mark after 50, at 55.
50
51
52
53
54
55
The next long mark after 50 stands for 60.
So, I am on 60.
60
Good. Alex is on 63. Grogg is on 50. How far is Alex from Grogg on the number line?
The smaller marks after 60 stand for 61, 62,...
...and I'm on 63.
61
62
63
How far?



THE DISTANCE BETWEEN CONSECUTIVE NUMBERS ON A NUMBER LINE IS CALLED *1 UNIT*.






Lizzie and Alex both got closer to Grogg by 3 units.
So, they are both 13−3=10 units from Grogg.
Grogg doesn't have to move at all to stay halfway between them.
25
37
40
50
60
63

Good. When Lizzie and Alex move toward each other by the same amount, the number that is halfway between them doesn't change.
We can use this to help us find the number that is halfway between two numbers.

For example, if Lizzie stands at 56, and Alex stands at 94, what number is halfway between them?
0
10
25
50
56
94
100

Try it.

To find the number halfway between Lizzie and Alex, they can move the same distance toward each other until they meet.

To start, they can each move 4 units. That puts them at 60 and 90.

The number halfway between 56 and 94 is the same as the number halfway between 60 and 90.

50 56 60 90 94 100

They can each move 10 more units to 70 and to 80.

And since 75 is halfway between 70 and 80...

...75 is halfway between 56 and 94.

50 56 60 70 75 80 90 94 100

We can count up from 54 and down from 98 to find the middle.

The number that is halfway between 54 and 98...

...is the same as the number that is halfway between 64 and 88...

...or 74 and 78...

...which is 76!

10 10 2 2 10 10

54 64 74 76 78 88 98

Great! We can use number line strategies without an actual number line.

For example, how could you figure out whether 44 is closer to 10 or to 80?

We can see if 44 is smaller or larger than the number that is halfway between 10 and 80.

The number that is halfway between 10 and 80 is also halfway between 20 and 70...

...and 30 and 60...

...and 40 and 50...

...which is 45.

10 10 10 5 5 10 10 10

10 20 30 40 45 50 60 70 80

44

44 is less than 45, so 44 is closer to 10 than to 80.

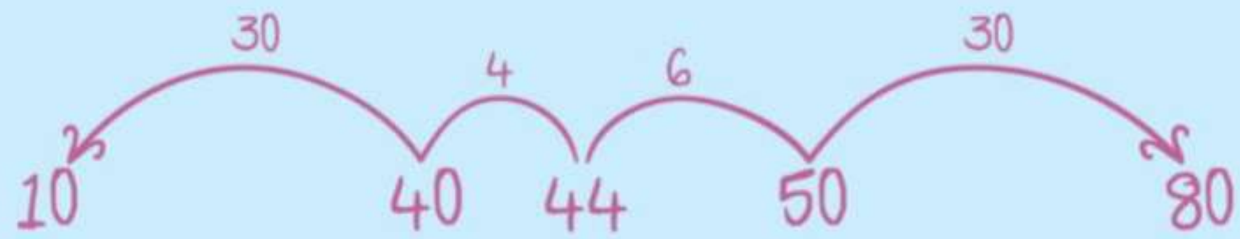
I did something different.

I started at 44 and counted ***out*** towards 10 and 80.

44 is 6 units from 50, and only 4 units from 40.

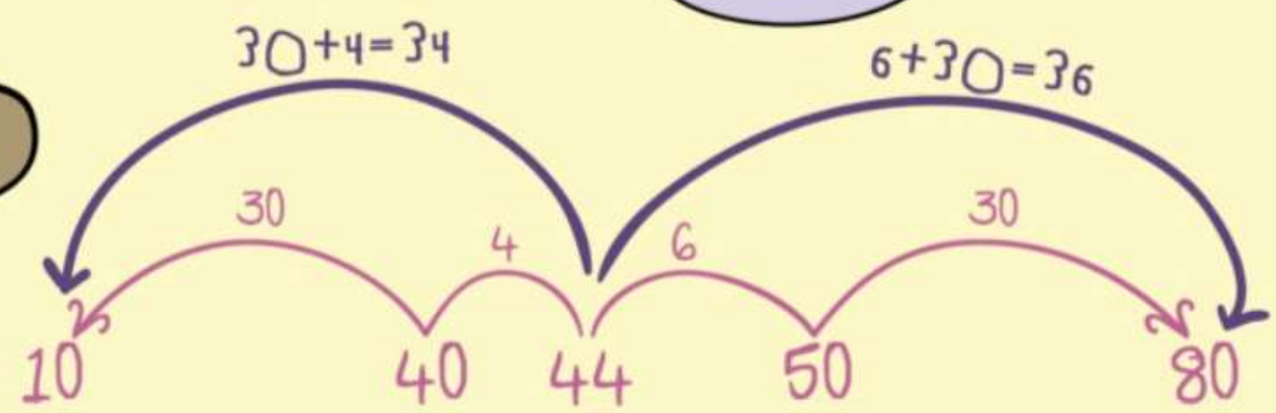
Then, from 40 to 10 is the same distance as from 50 to 80.

So, 44 is closer to 10 than to 80.



From 44 to 10 is 34...

...and from 44 to 80 is 36.



Perfect! The number line is great for helping us see how numbers are related.

It's also great for playground games, like Number Line Dodgeball Tag!

The monster with the most hands gets first throw!

Too late!

Grogg is it!

Bonk!

RECESS

Iago

In the game of Iago, two players take turns placing coins on a number line and capturing their opponent's coins. The player with the most coins on the number line when the line is full wins.

Setup

The game is played on a number line with ten numbered marks. Each player begins with a set of game pieces. For example, Player 1 can use pennies, and Player 2 can use dimes.

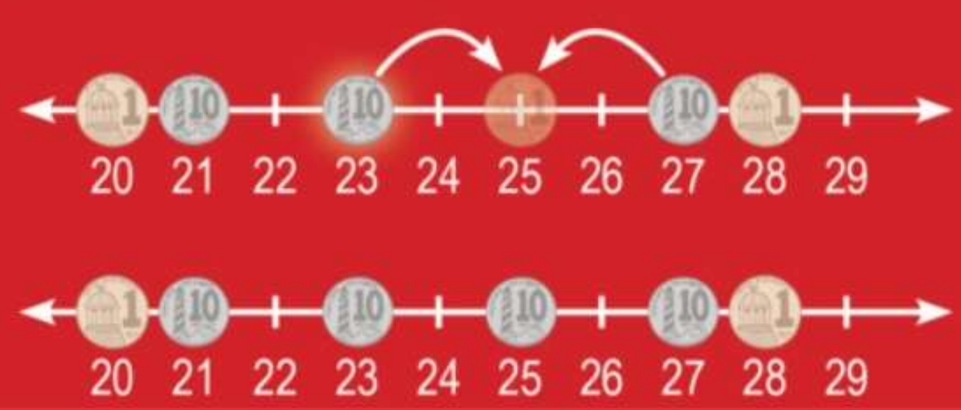
Play

Players take turns placing one of their own coins on the number line until the number line is full.

Players can capture each other's coins. To capture a dime, Player 1 must place a penny so that a dime is halfway between the new penny and a penny that was already on the number line. To capture a penny, Player 2 must place a dime so that a penny is halfway between the new dime and a dime that was already on the number line.

For example, it is Player 2's turn to place a dime on the board below:

By placing a dime on 23, Player 2 can capture the penny on 25 and replace it with a dime.



— or —

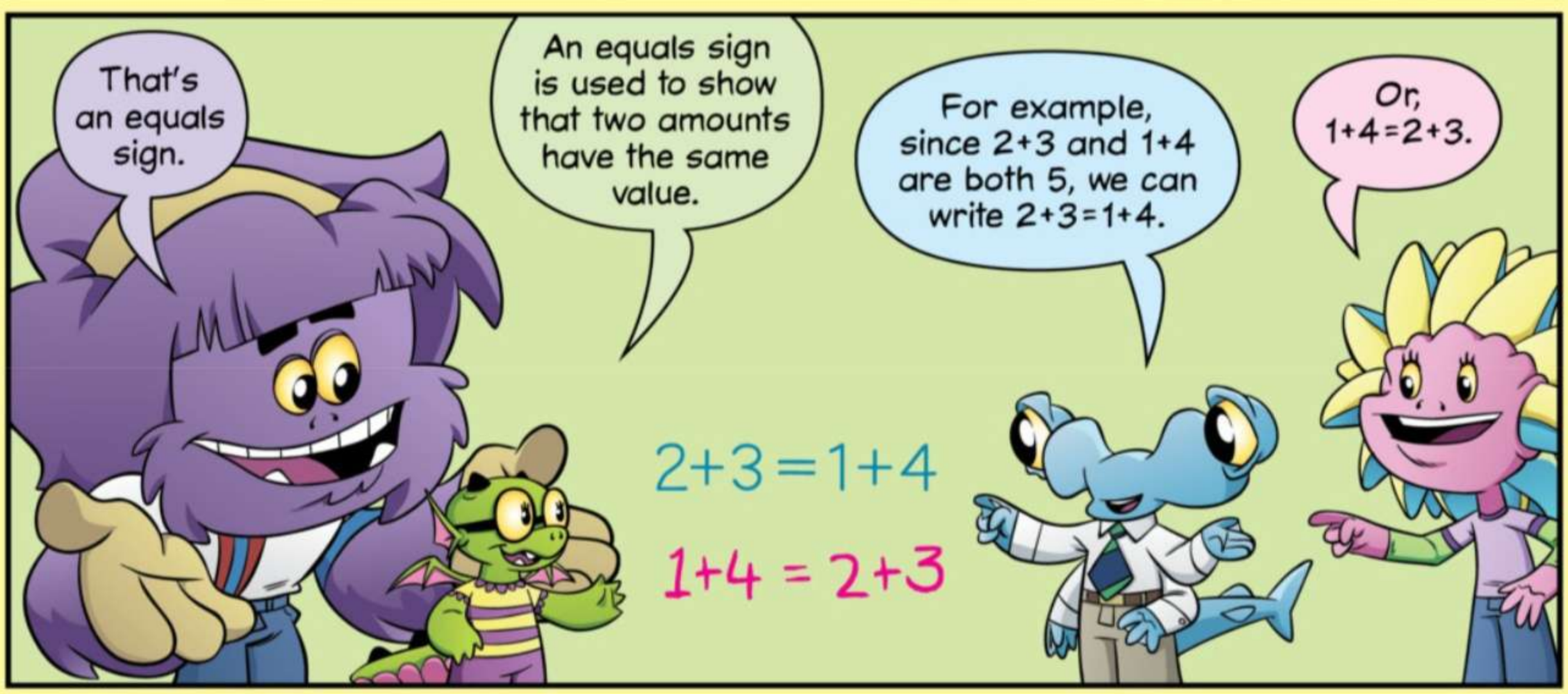
By placing a dime on 29, Player 2 can capture the pennies on 25 and 28! Both are replaced with dimes.

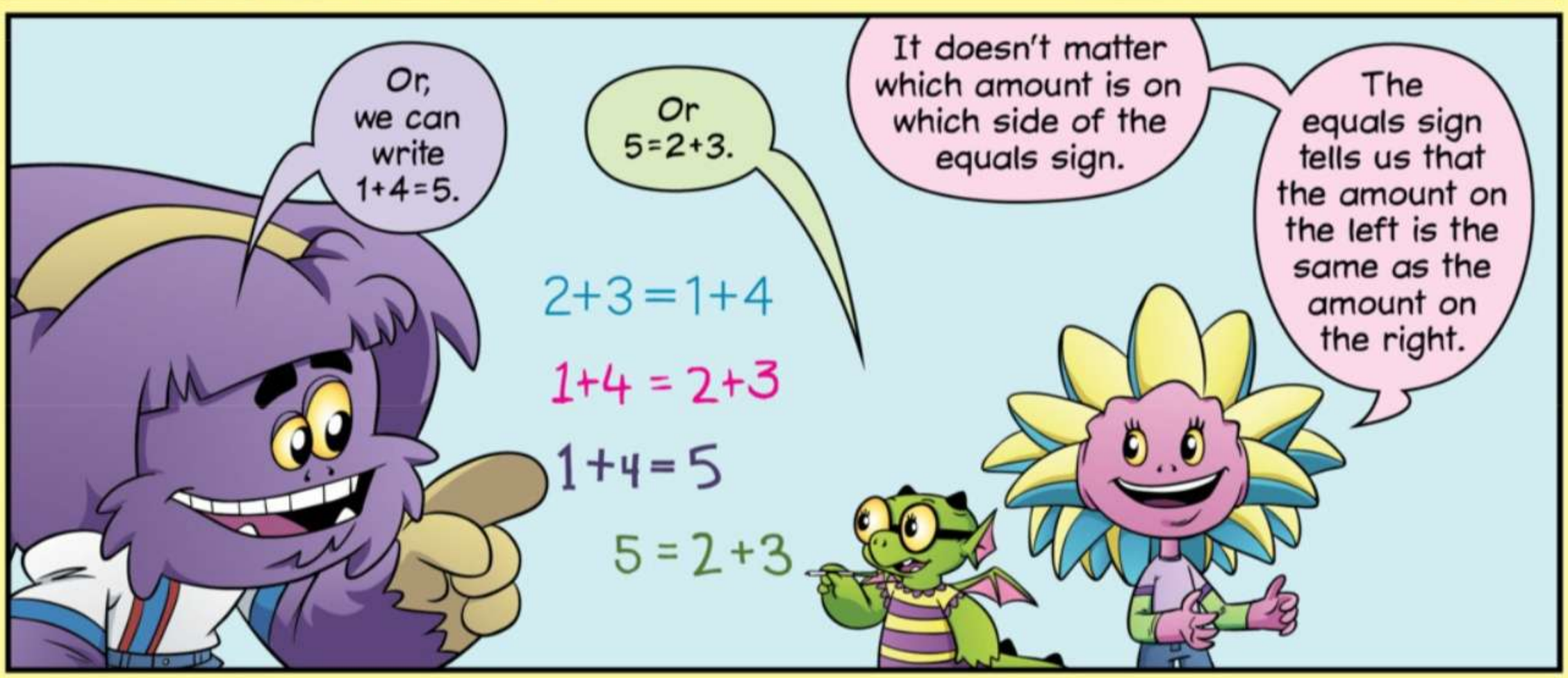
When a coin is replaced, the new coin is not used to create extra captures on the same turn.

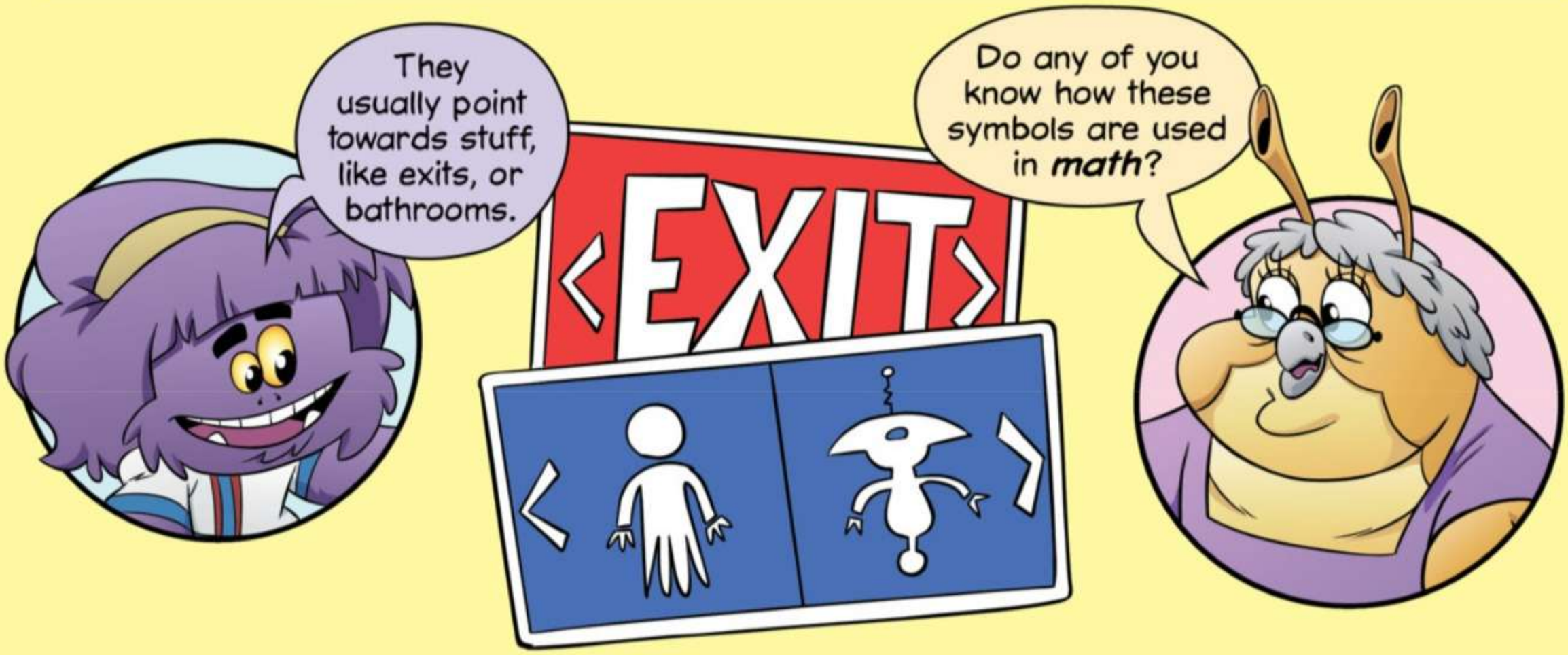
Winning

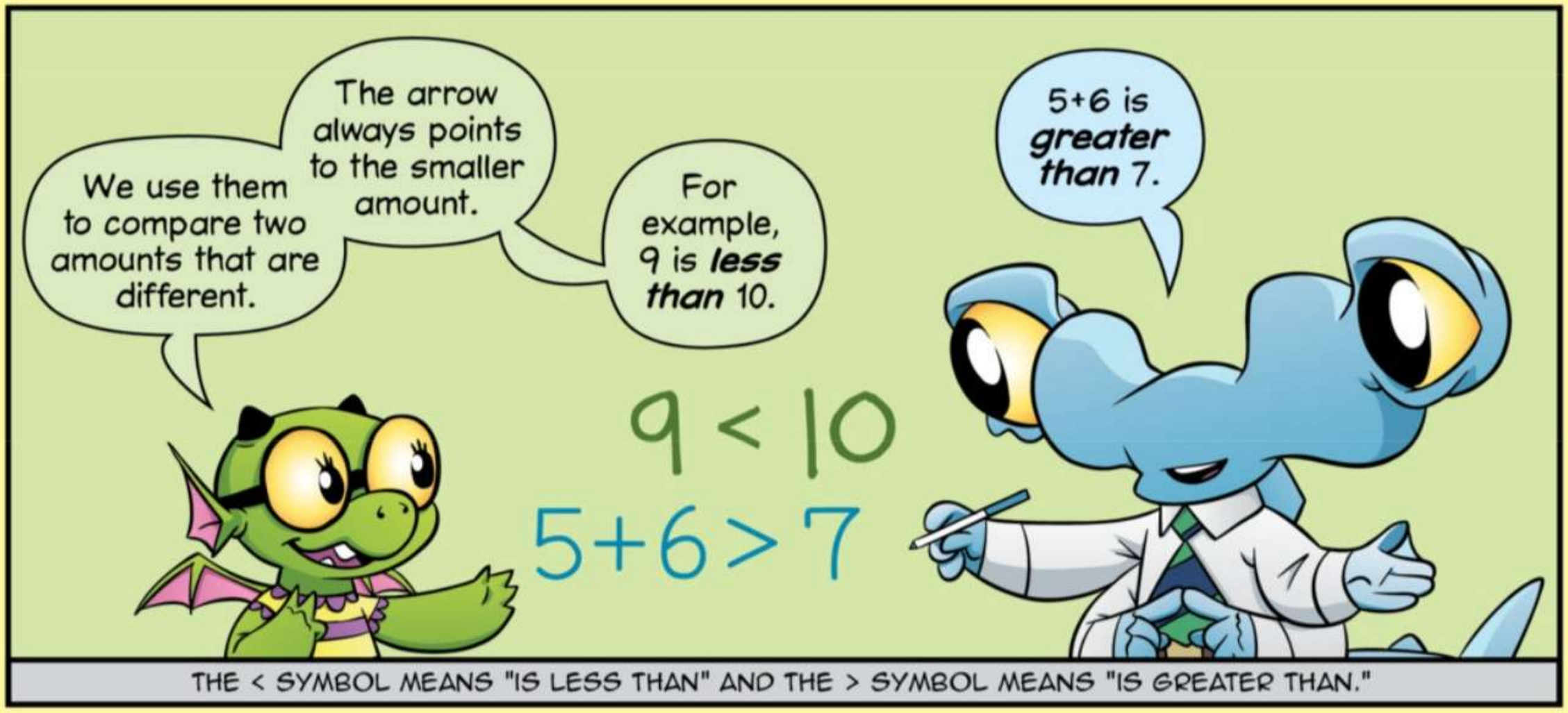
Play continues until there is one coin on each number. The player with the most coins on the number line wins. If players have the same number of coins, the game ends in a tie. *Print game boards and find variations at BeastAcademy.com.*

Ms. Q.
Comparing
What does this symbol mean?
=

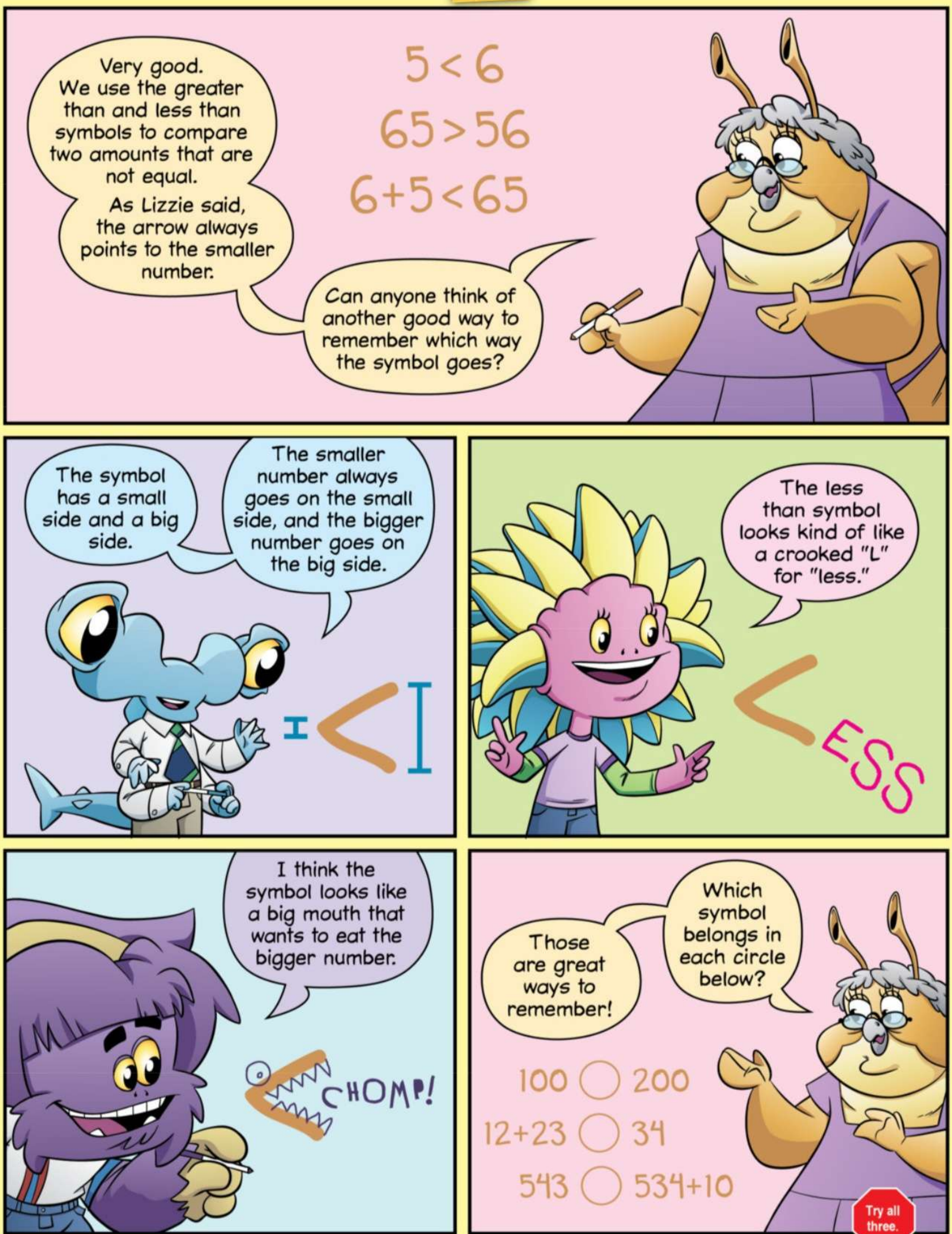

That's an equals sign.
An equals sign is used to show that two amounts have the same value.
For example, since 2+3 and 1+4 are both 5, we can write 2+3=1+4.
Or, 1+4=2+3.
2+3=1+4
1+4 = 2+3


Or, we can write 1+4=5.
Or 5=2+3.
It doesn't matter which amount is on which side of the equals sign.
The equals sign tells us that the amount on the left is the same as the amount on the right.
2+3=1+4
1+4 = 2+3
1+4=5
5 = 2+3

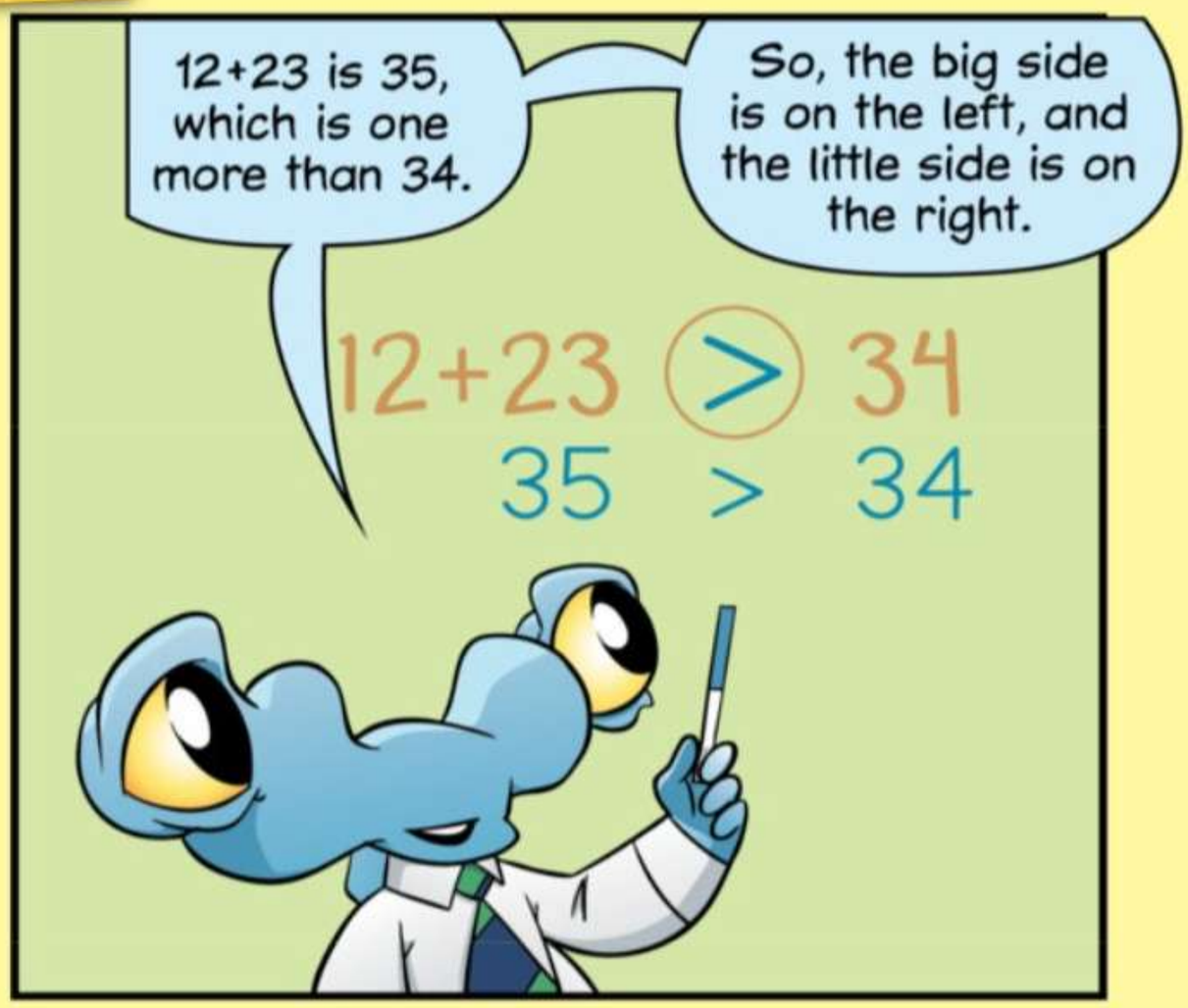




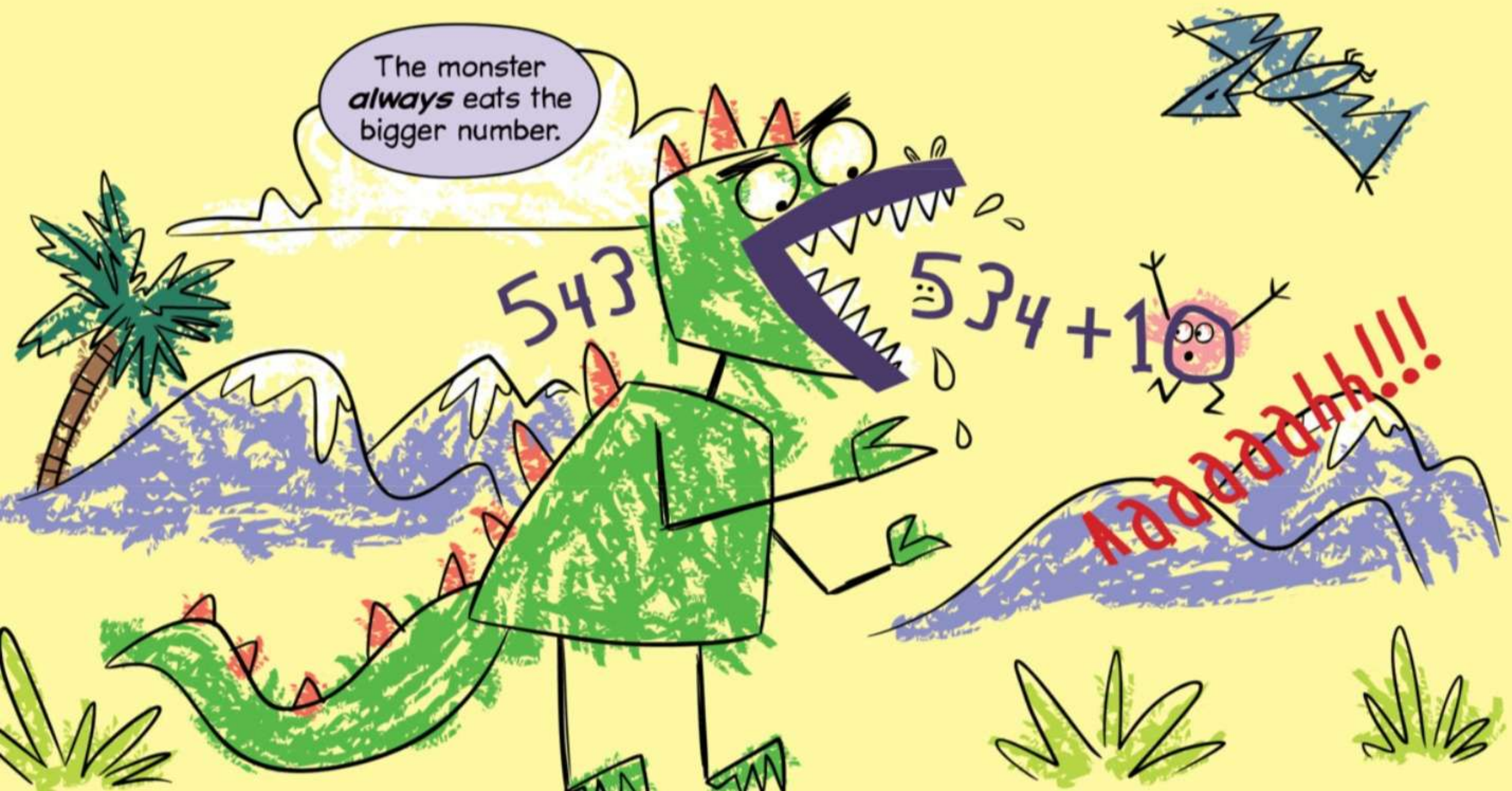
THE < SYMBOL MEANS "IS LESS THAN" AND THE > SYMBOL MEANS "IS GREATER THAN."


Very good. We use the greater than and less than symbols to compare two amounts that are not equal.
As Lizzie said, the arrow always points to the smaller number.
5 < 6
65 > 56
6+5 < 65
Can anyone think of another good way to remember which way the symbol goes?
The symbol has a small side and a big side.
The smaller number always goes on the small side, and the bigger number goes on the big side.
The less than symbol looks kind of like a crooked "L" for "less."
LESS
I think the symbol looks like a big mouth that wants to eat the bigger number.
CHOMP!
Those are great ways to remember!
Which symbol belongs in each circle below?
100 ◯ 200
12+23 ◯ 34
543 ◯ 534+10
Try all three.

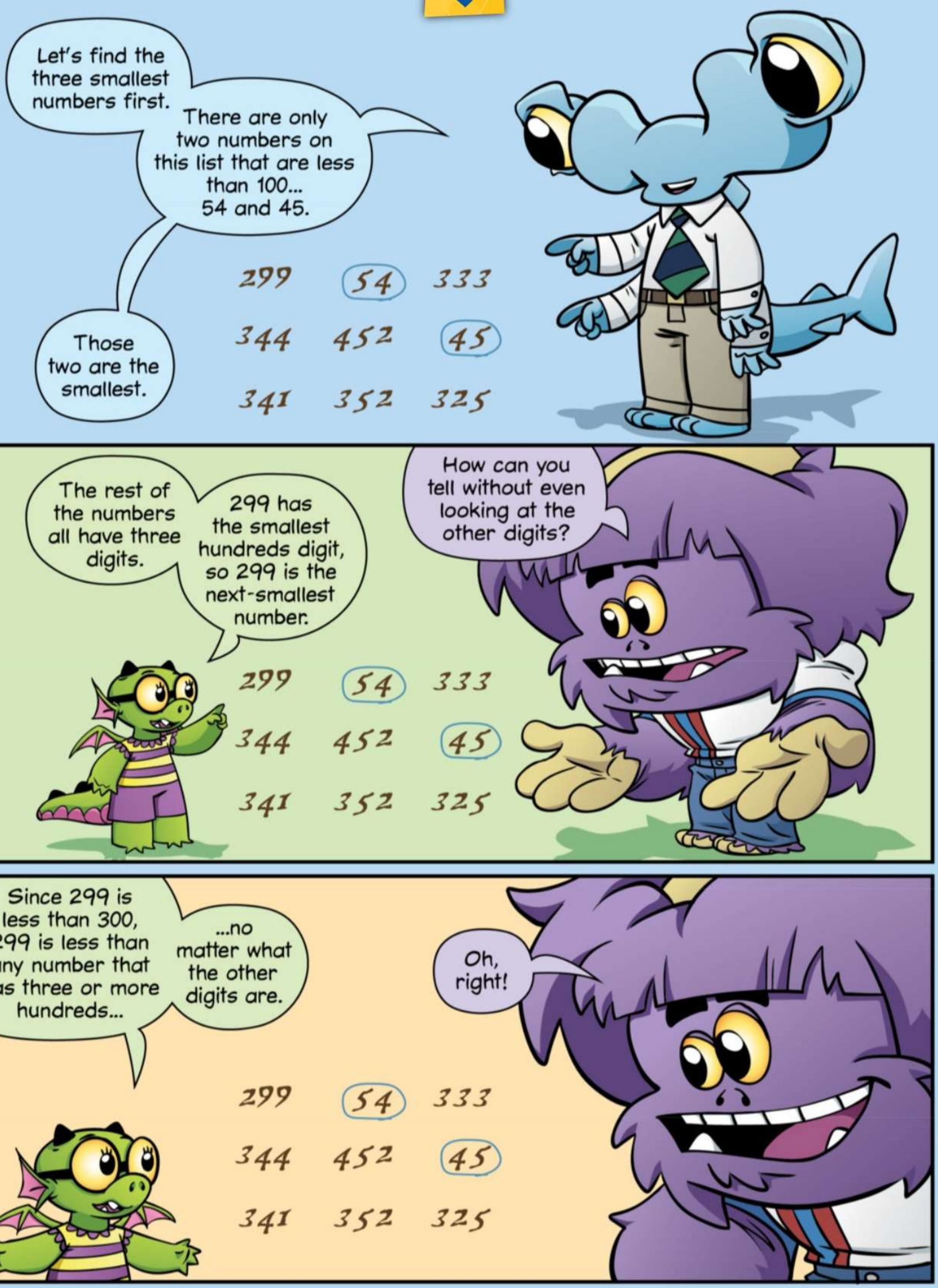
100 is less than 200.
We point the arrow at the smaller number.
100 < 200

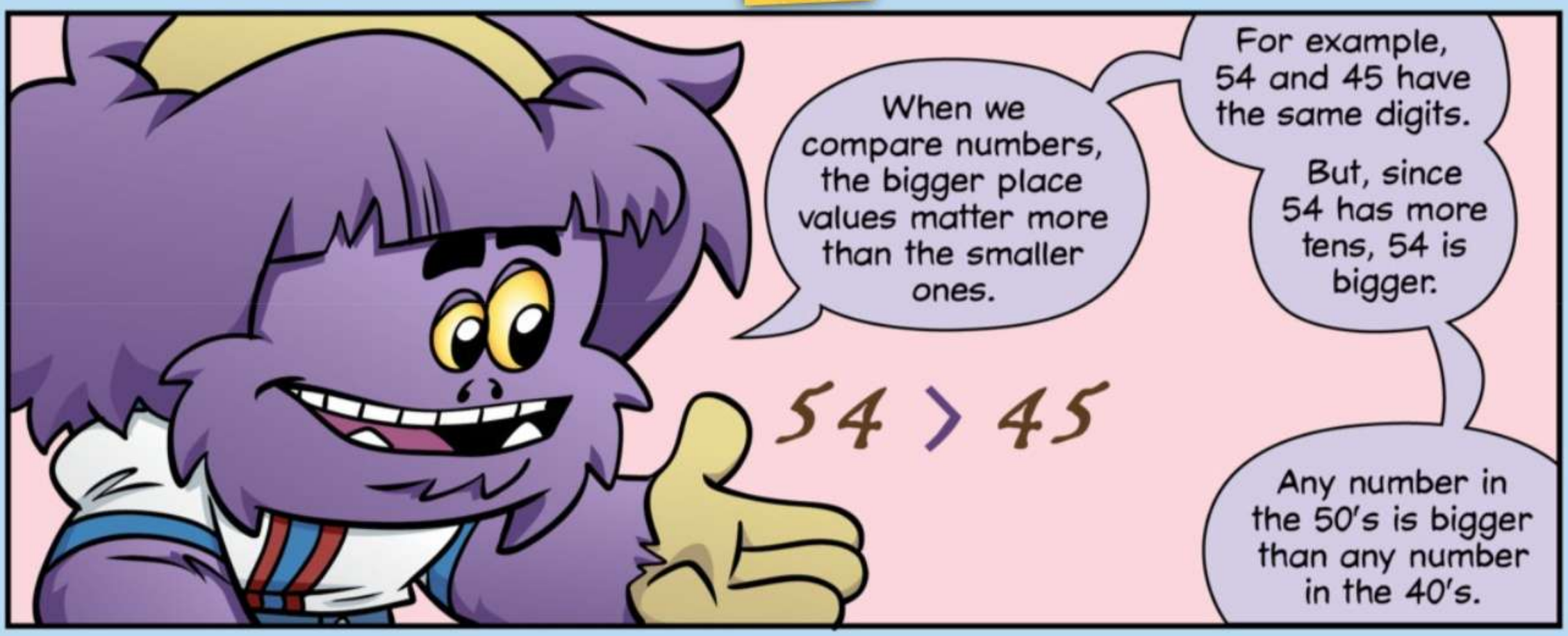

12+23 is 35, which is one more than 34.
So, the big side is on the left, and the little side is on the right.
12+23 > 34
35 > 34

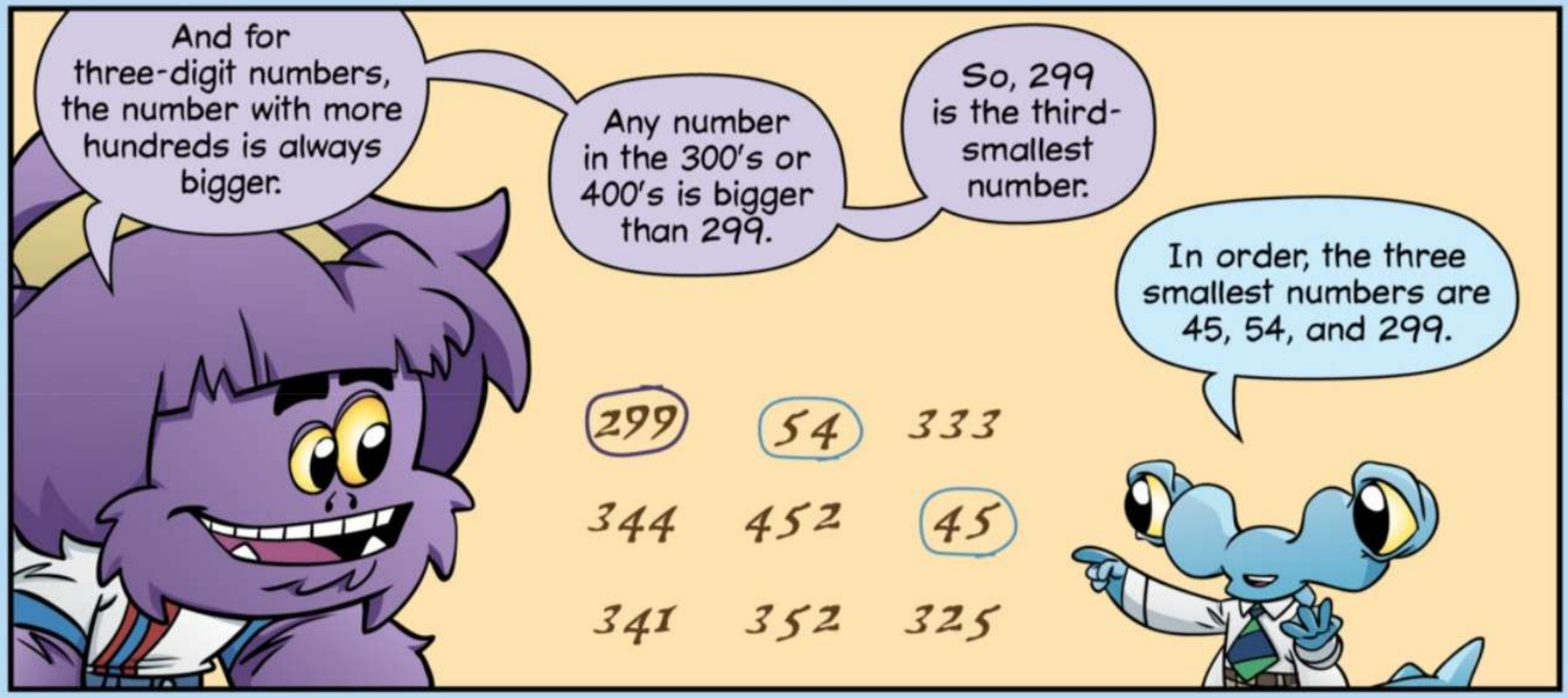
Adding 10 to 534 gives us 544.
So, 543 is less than 534+10.
543 < 534+10
543 < 544

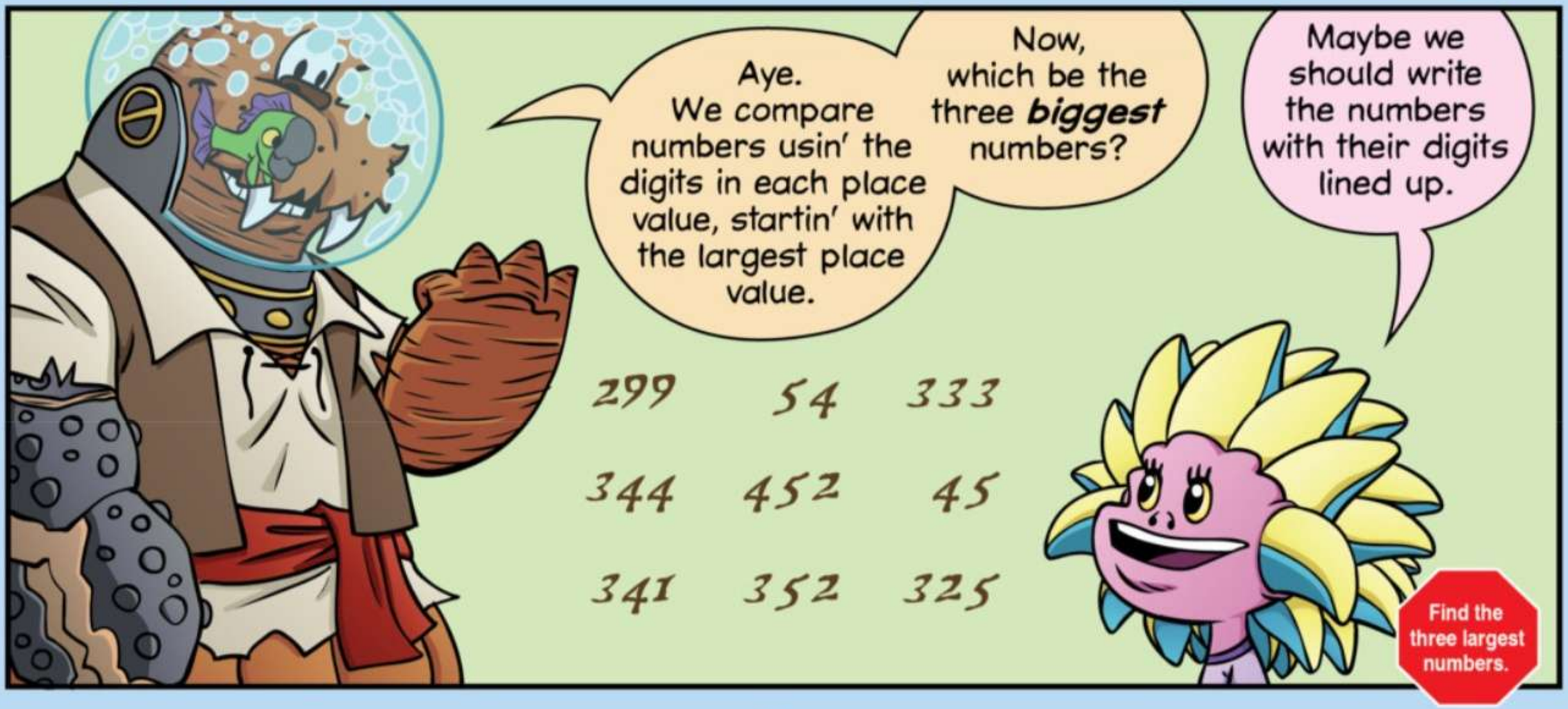

The monster *always* eats the bigger number.
543
534+10
Aaaaaahh!!!

ORDERING
What are we making today?
Today, we'll be craftin' number signs to replace the old ones on the classroom doors.
Will we get to hammer stuff?
Aye, but first we need to figure out which numbers to make.
In the arts building, there be nine numbered rooms.
Only the three smallest 'n' the three biggest room numbers need replacin'.
Which six o' these nine room numbers must be made?
299 54 333
344 452 45
341 352 325
Find the three smallest and the three largest numbers.


Let's find the three smallest numbers first.
There are only two numbers on this list that are less than 100... 54 and 45.
Those two are the smallest.
299 54 333
344 452 45
341 352 325
The rest of the numbers all have three digits.
299 has the smallest hundreds digit, so 299 is the next-smallest number.
How can you tell without even looking at the other digits?
299 54 333
344 452 45
341 352 325
Since 299 is less than 300, 299 is less than any number that has three or more hundreds...
...no matter what the other digits are.
Oh, right!
299 54 333
344 452 45
341 352 325


When we compare numbers, the bigger place values matter more than the smaller ones.
For example, 54 and 45 have the same digits.
But, since 54 has more tens, 54 is bigger.
54 > 45
Any number in the 50's is bigger than any number in the 40's.


And for three-digit numbers, the number with more hundreds is always bigger.
Any number in the 300's or 400's is bigger than 299.
So, 299 is the third-smallest number.
In order, the three smallest numbers are 45, 54, and 299.
299 54 333
344 452 45
341 352 325


Aye. We compare numbers usin' the digits in each place value, startin' with the largest place value.
Now, which be the three **biggest** numbers?
Maybe we should write the numbers with their digits lined up.
299 54 333
344 452 45
341 352 325
Find the three largest numbers.

It's easier to compare the numbers when we line them up, like this.

299
54
333
344
452
45
341
352
325

Great idea!

Then, it's easy to compare the digits in each place value.

Exactly! To order the numbers, we start by looking at their hundreds digits.

hundreds	tens	ones
2	9	9
	5	4
3	3	3
3	4	4
4	5	2
	4	5
3	4	1
3	5	2
3	2	5

Two numbers are less than 100.

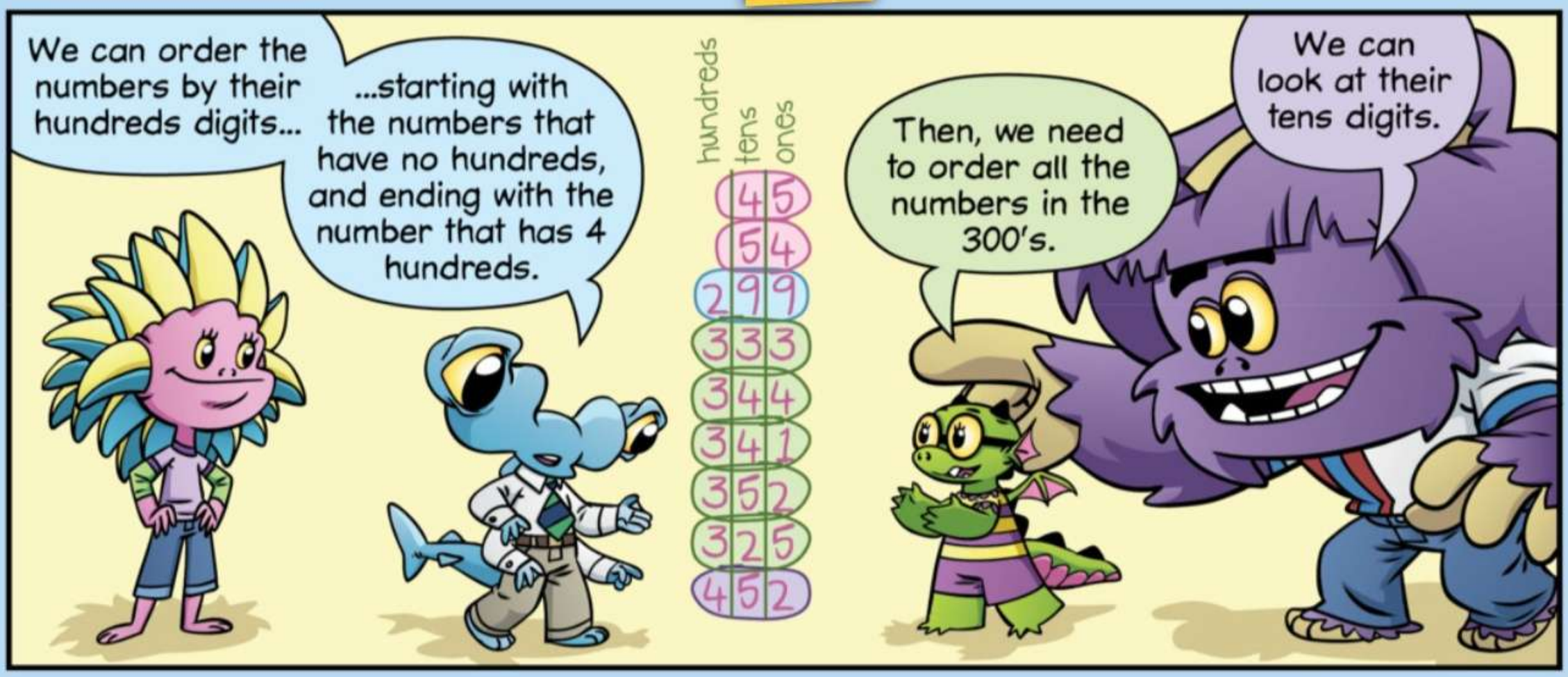
Only one number is in the 200's.

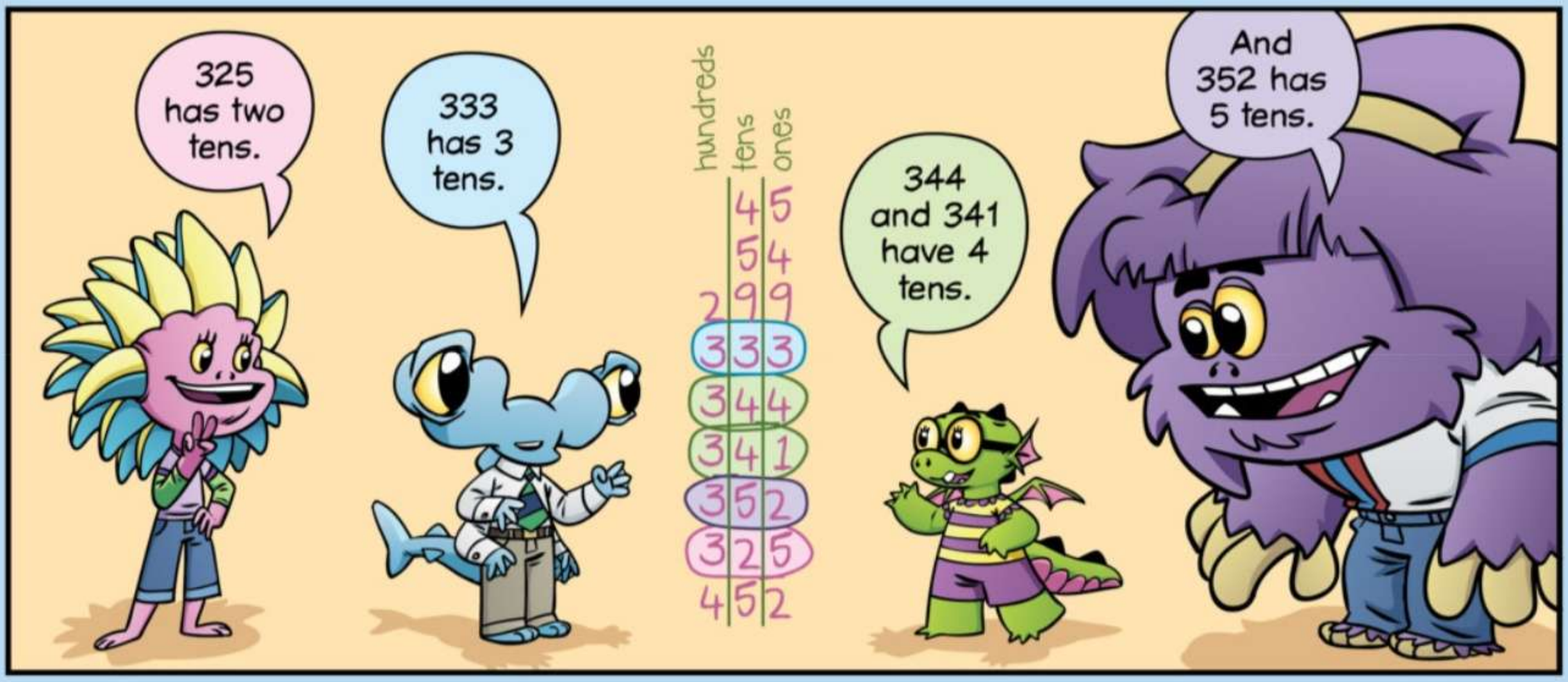
There are five numbers in the 300's.

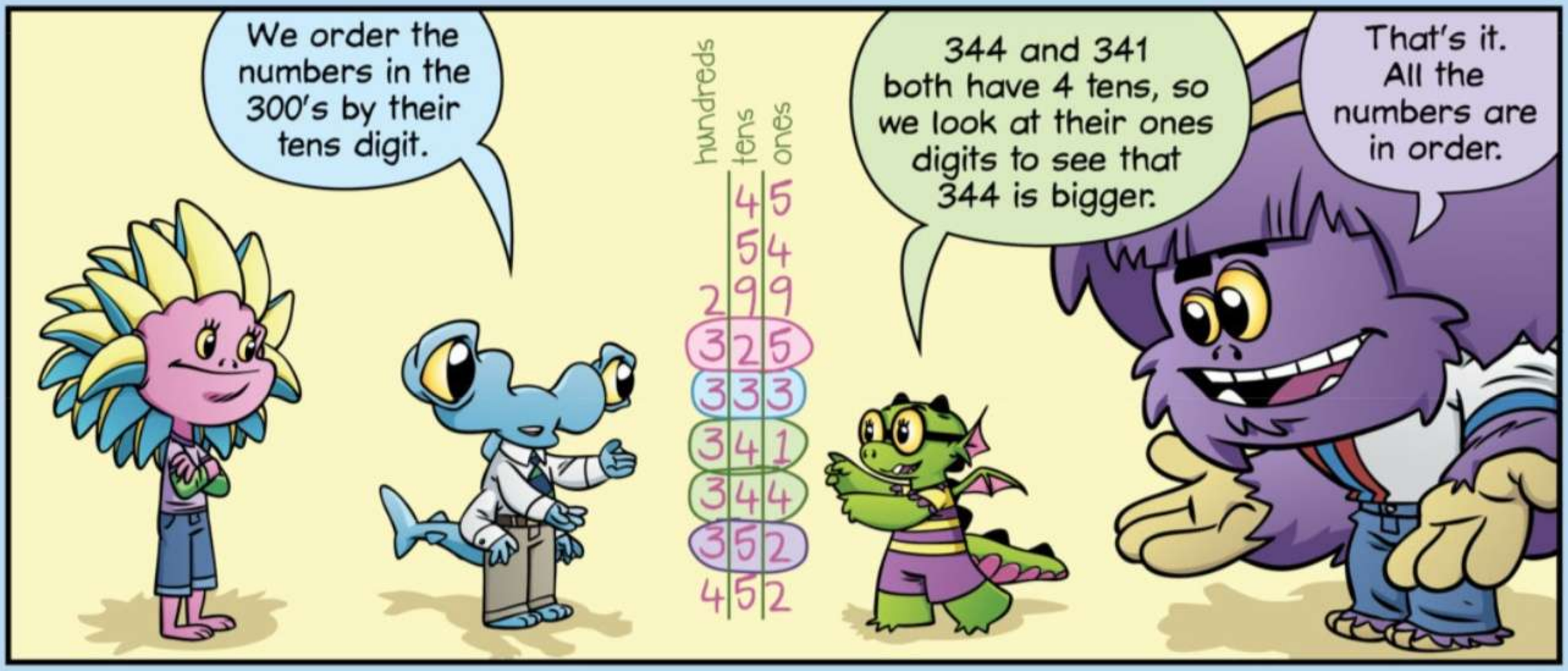
And there's only one number in the 400's.

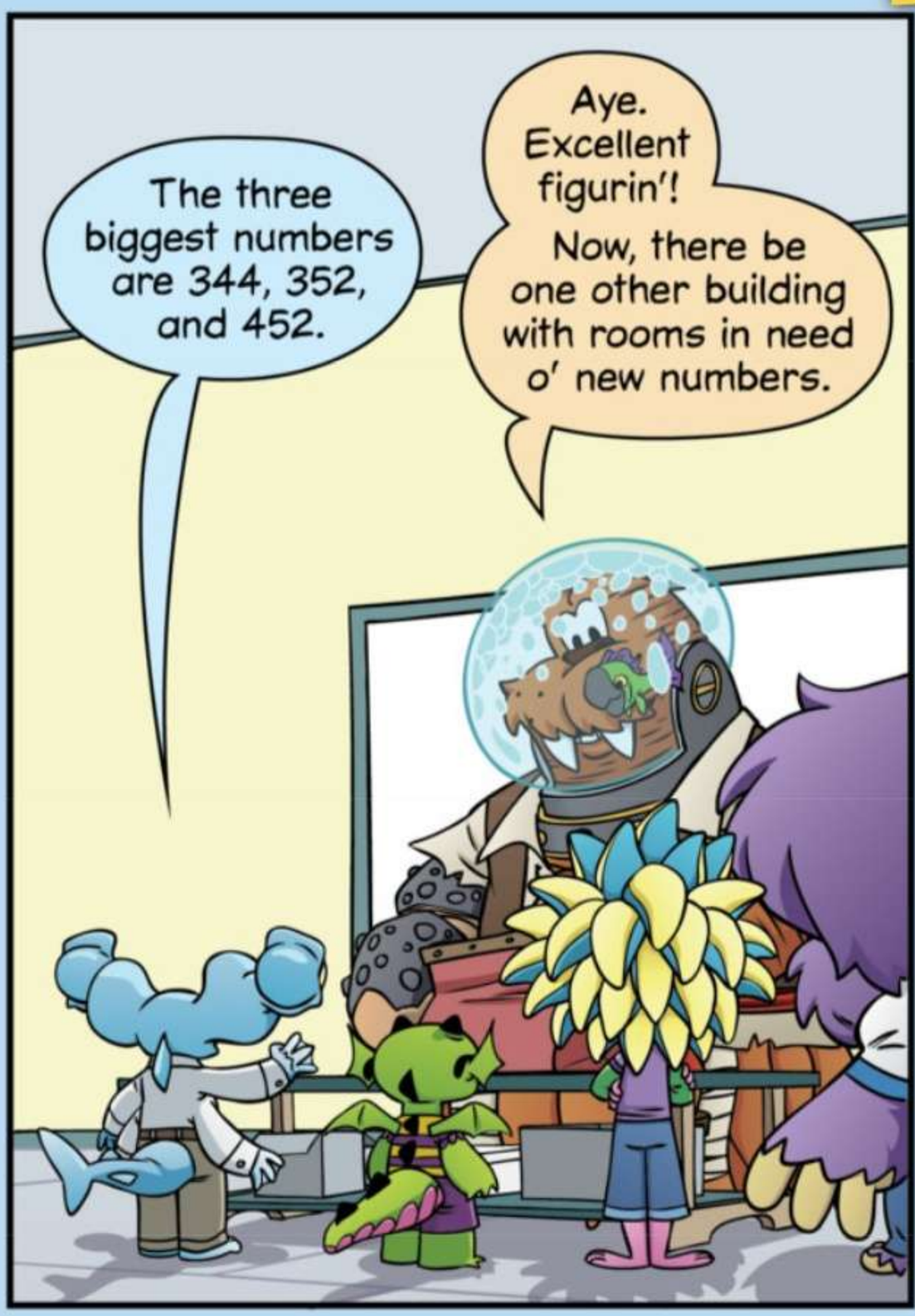
hundreds	tens	ones
2	9	9
	5	4
3	3	3
3	4	4
4	5	2
	4	5
3	4	1
3	5	2
3	2	5

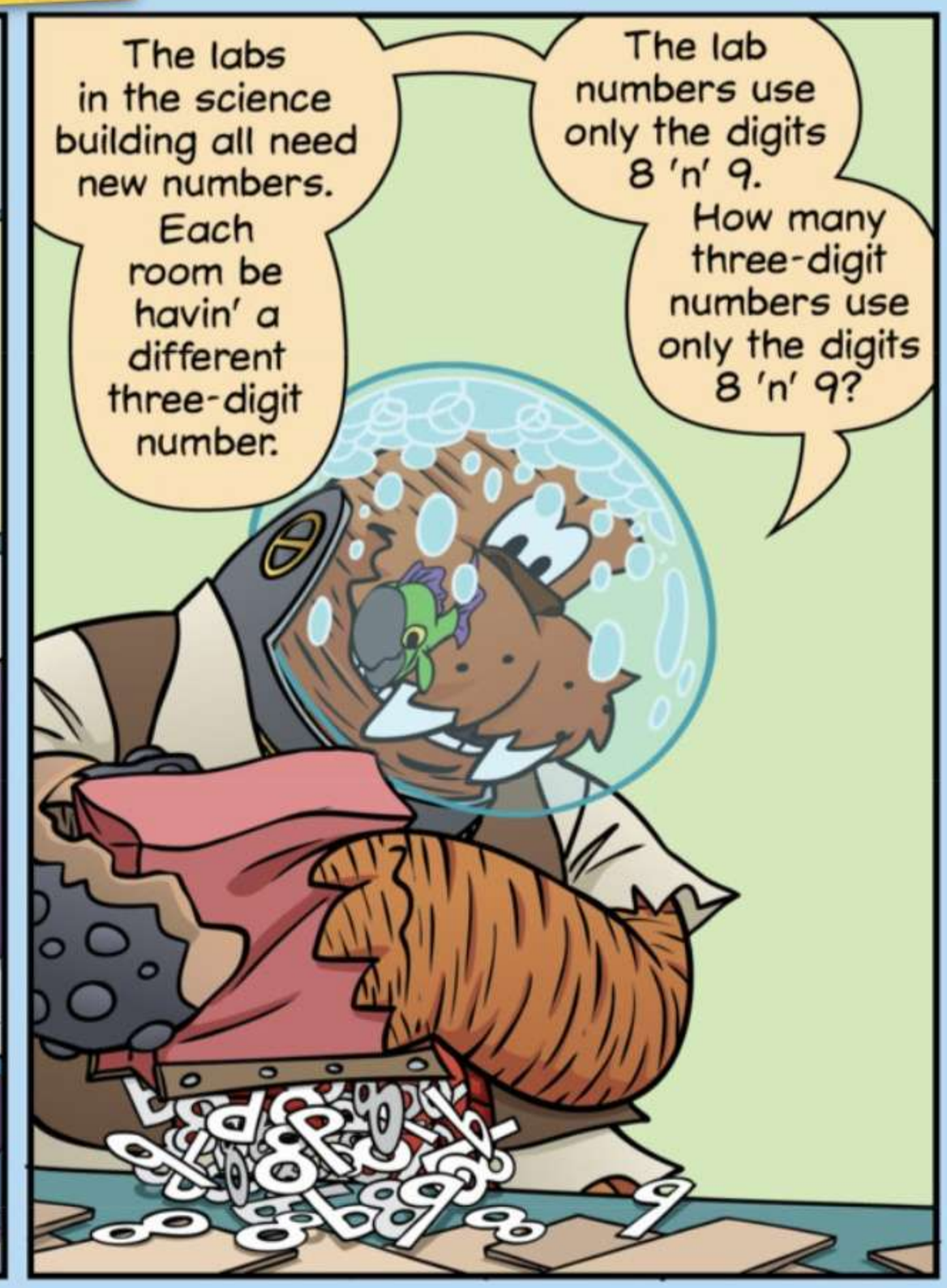
Can you put all nine numbers in order?

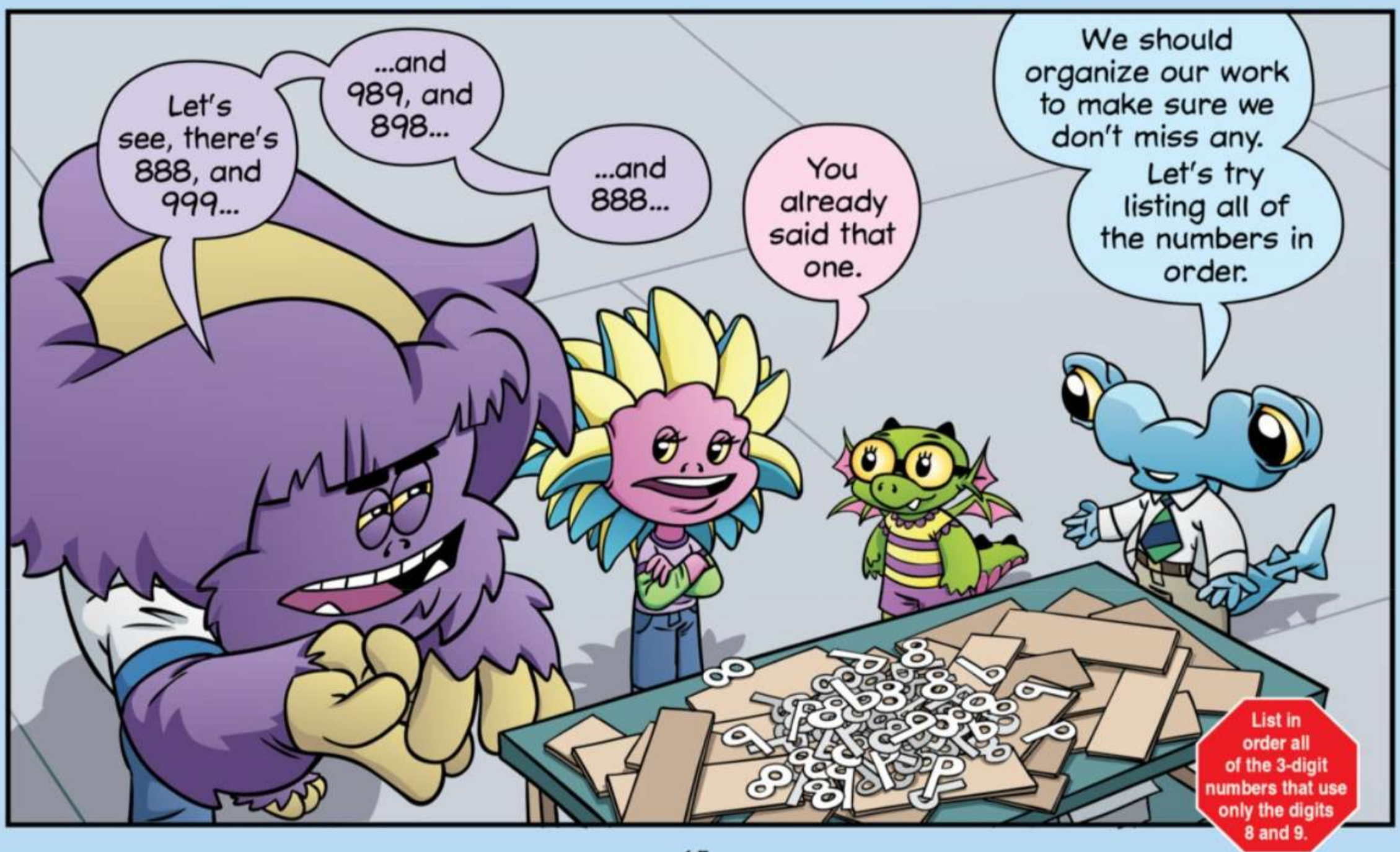

We can order the numbers by their hundreds digits...
...starting with the numbers that have no hundreds, and ending with the number that has 4 hundreds.
hundreds tens ones
45
54
299
333
344
341
352
325
452
Then, we need to order all the numbers in the 300's.
We can look at their tens digits.


325 has two tens.
333 has 3 tens.
hundreds tens ones
45
54
299
333
344
341
352
325
452
344 and 341 have 4 tens.
And 352 has 5 tens.

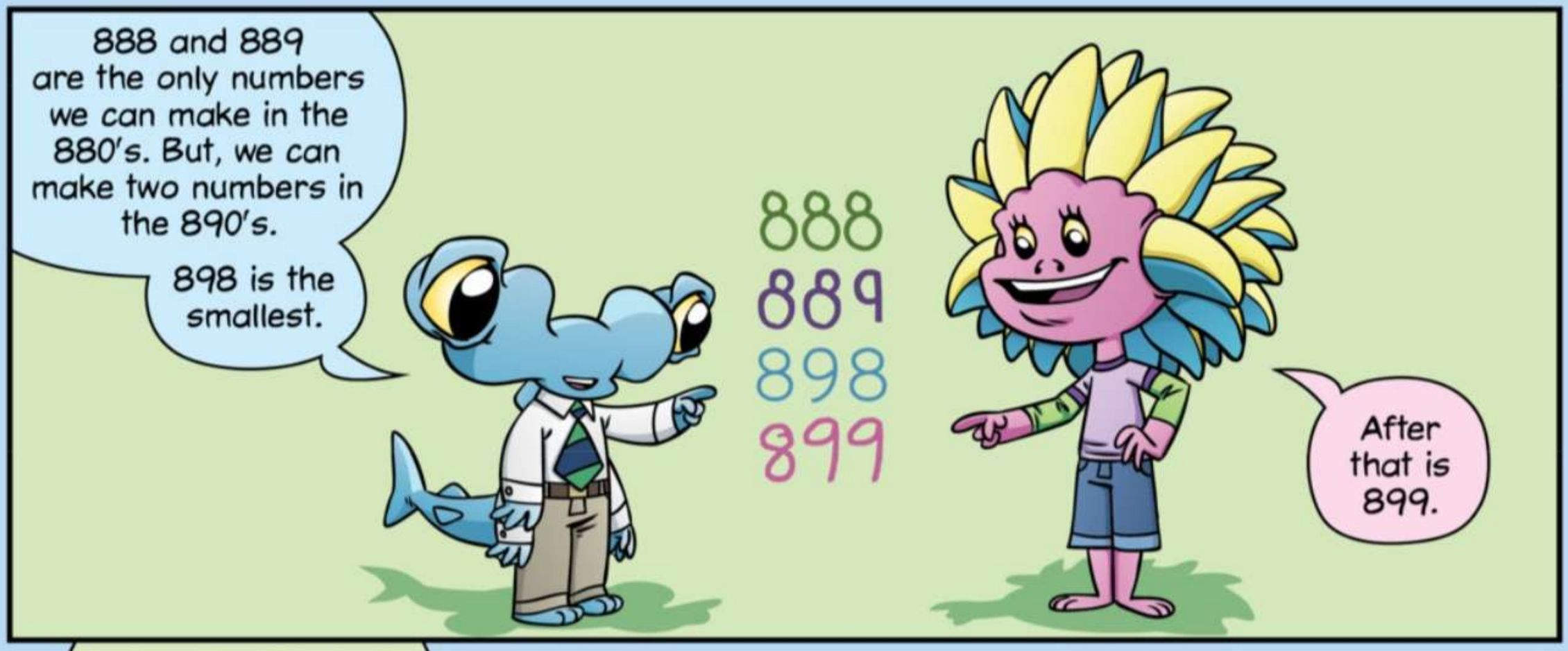

We order the numbers in the 300's by their tens digit.
hundreds tens ones
45
54
299
325
333
341
344
352
452
344 and 341 both have 4 tens, so we look at their ones digits to see that 344 is bigger.
That's it. All the numbers are in order.


The three biggest numbers are 344, 352, and 452.
Aye. Excellent figurin'!
Now, there be one other building with rooms in need o' new numbers.


The labs in the science building all need new numbers.
Each room be havin' a different three-digit number.
The lab numbers use only the digits 8 'n' 9.
How many three-digit numbers use only the digits 8 'n' 9?


Let's see, there's 888, and 999...
...and 989, and 898...
...and 888...
You already said that one.
We should organize our work to make sure we don't miss any.
Let's try listing all of the numbers in order.
List in order all of the 3-digit numbers that use only the digits 8 and 9.

The smallest 3-digit number we can make using only 8's and 9's is 888.
889 has to be next, since 889 comes right after 888.
Can we make any more numbers in the 800's?
888
889


888 and 889 are the only numbers we can make in the 880's. But, we can make two numbers in the 890's.
898 is the smallest.
888
889
898
899
After that is 899.

899 is the largest number in the 800's. The next-smallest number has to be in the 900's.
988 is the smallest number we can make in the 900's.
Then 989.
888
889
898
899
988
989
998
999
Followed by 998.
And finally 999.

Aye, well done.
There be 8 labs in the science building.
Now that we know which numbers we need, it's hammertime!
Aye, let's get to work.