Contents: Chapter 12

Click the Play List tab in the top-left to view a recommended reading/practice sequence.

Chapter 12:
Problem Solving


Workshop
COUNTING PATHS
Whoa. This is a weird set of islands.
Aye. Between Beast Beach 'n' Cyclops Shore, there be 10 islands...
...all in a grid.
Beast Beach
VORACIOUS
DANGER
VORTEX
Cyclops Shore
To safely sail from Beast Beach to Cyclops Shore, always follow two rules:
Only sail east or south between islands, never diagonally...
...'n' always avoid the Voracious Vortex.
N
W
E
S

On my last trip from Beast Beach to Cyclops Shore, we planned to visit 5 islands in search o' treasure...

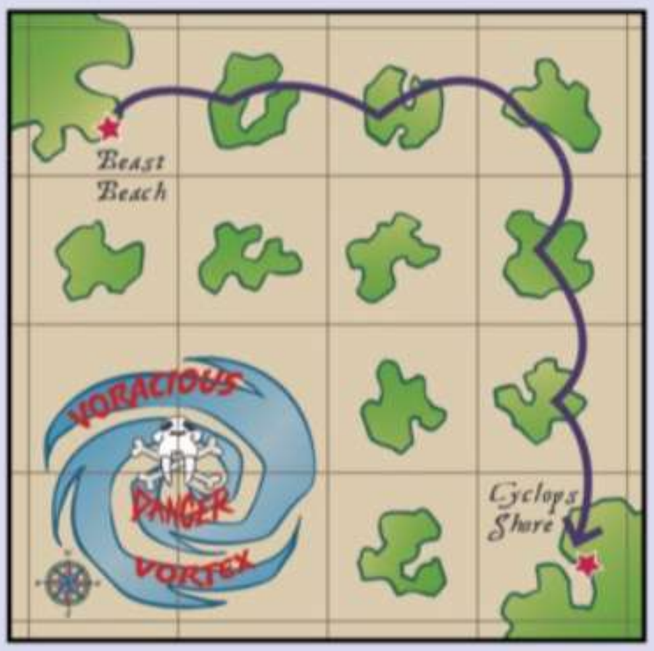
...always sailin' east or south between islands.

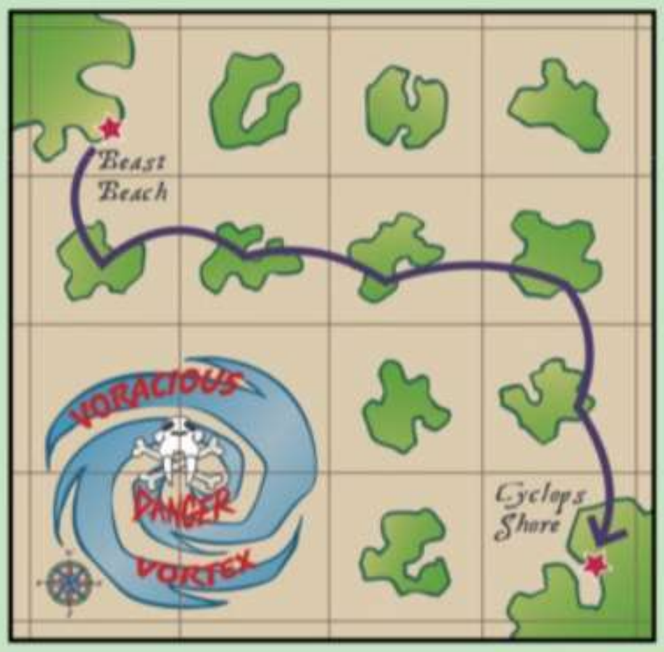
There are lots of ways to do that!

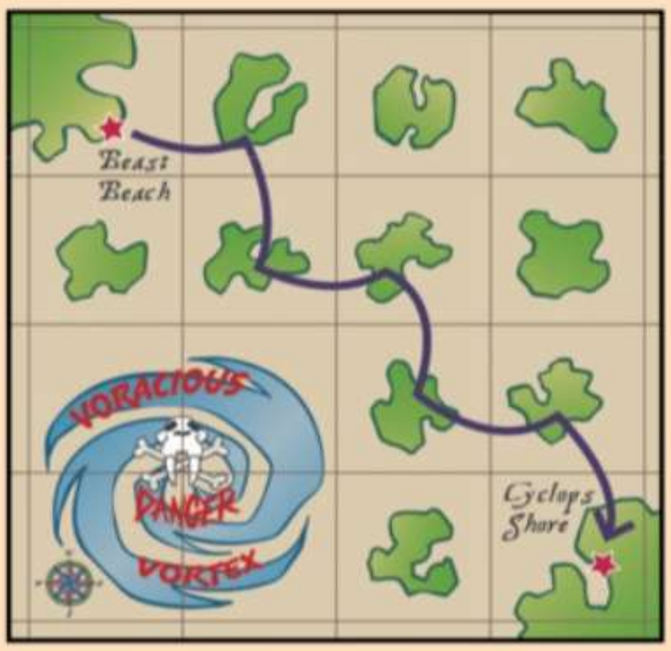
You could go all the way east first, then all the way south...

...or south, then all the way east, then south again...

...or you could go all zig-zaggy!







Aye.

How many ***different*** ways could we visit 5 islands along the way from Beast Beach to Cyclops Shore?

Try it.

I found a few more.

You could go east twice, then south three times, then east again...

Beast Beach

Cyclops Shore

...and there's this one.

How many is that?

Beast Beach

Cyclops Shore

We need to keep better track to figure out which ones we've already counted.

And which ones we've missed.

We need a way to organize all the paths.

Maybe we can organize by how far east you go at first.

You could start by traveling 3, 2, 1 or 0 islands to the east.

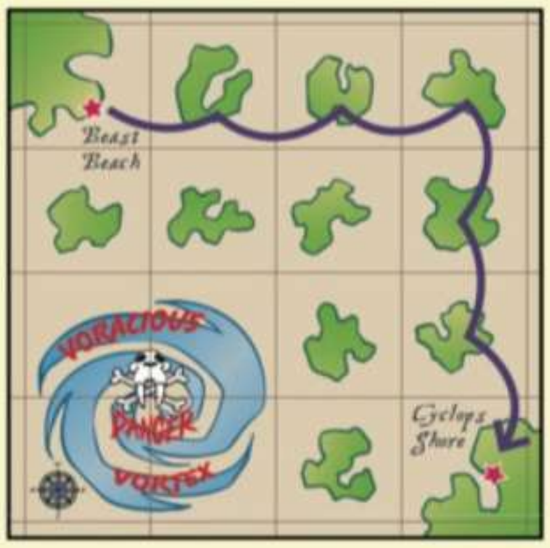
I see. That way, we turn one hard problem into four easier ones.

Beast Beach

Cyclops Shore

If you go all the way east first, there's only one way to finish...

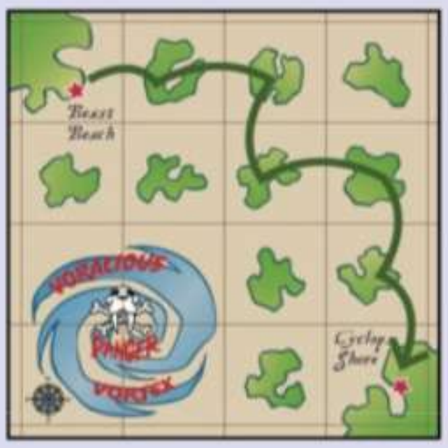
...by going south the rest of the way.



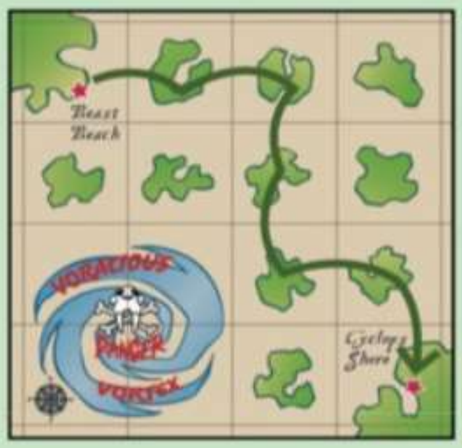
Great. That's 1 path. Now we can count the paths that start by going east twice before turning south.

Start:	East 3	East 2	East 1	East 0
Paths:	1			

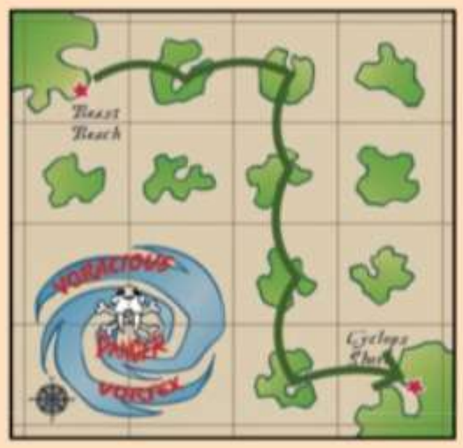
You could go east twice, south ***once***, then finish this way...



...or east twice, then south ***twice*** before turning east...



...or you could go east twice, then south ***three*** times before going east to Cyclops Shore.



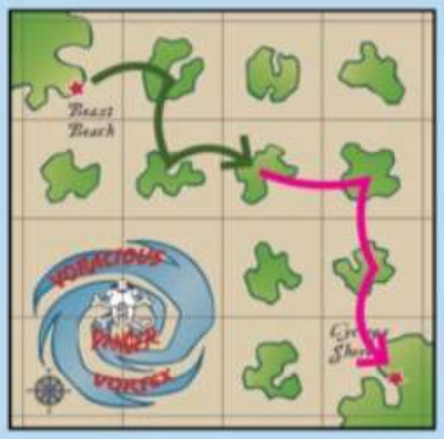
Great. That's 3 more paths. Next, let's count the paths that start by going east once, then south.

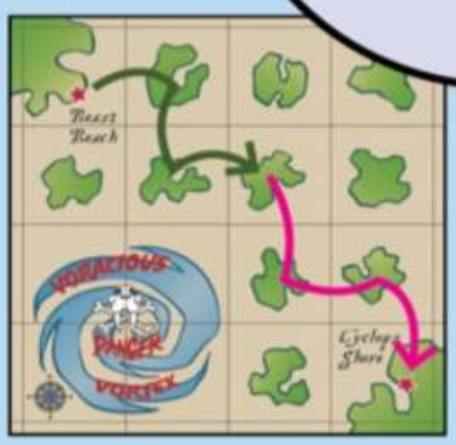
Start:	East 3	East 2	East 1	East 0
Paths:	1	3		

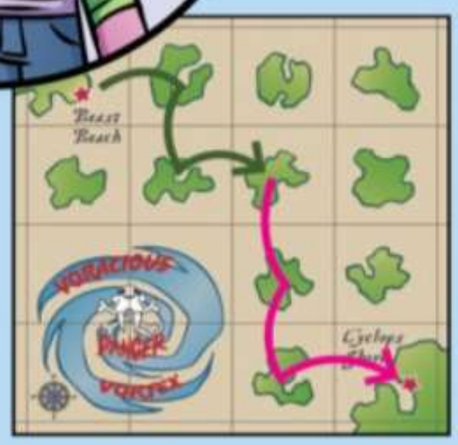
How many paths start by going east once?

If you go east once, then south, you have to go east next to this island.

There are three ways to finish the trip from there.







That's 3 more paths.

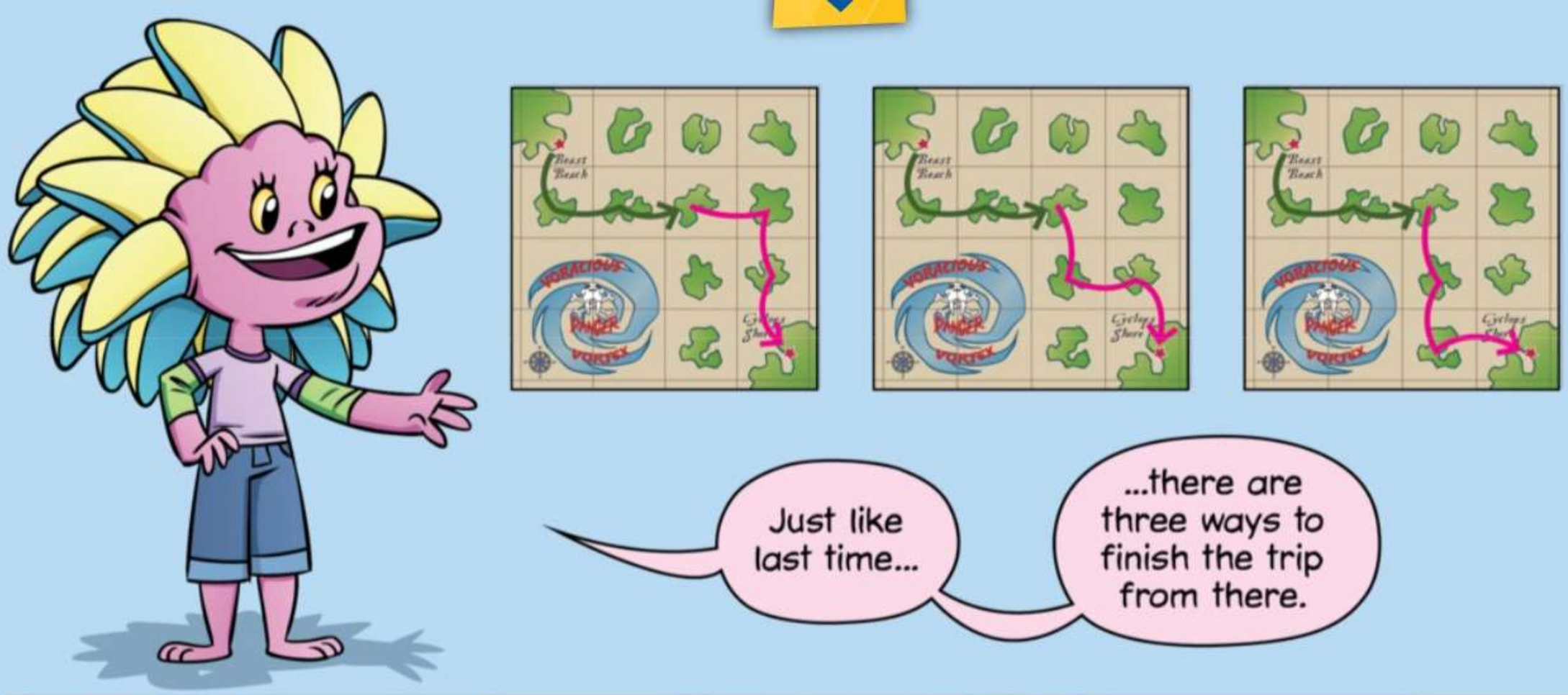
Now, we need to count the ways you can go if you start by going south instead of east.

Start:	East 3	East 2	East 1	East 0
Paths:	1	3	3	

If you start by going south, you have to go east twice from there.

Or you'll end up in this big, scary, swirly thing!

How many total paths are there?


Just like last time...
...there are three ways to finish the trip from there.
That's all of them! That makes a total of 1+3+3+3=10 paths.
Which one did you take, Captain Kraken?
East 3
East 2
East 1
East 0
1
3
3
3
Not the best one, lad...
...not the best one.

Ms. Q.

Organizing

There are four 2-digit numbers you can write using only 3's and 4's.

Can you find them all?

Let's see...

34 and 43 both work.

34

43

33

44

33 and 44 work, too!

I think that's all of them.

That's right.

The list gets longer if we try three-digit numbers.

How many three-digit numbers can you write using only 3's and 4's?

Can you list them all?

I found 9!

343 443 333 444 434 344 433 434 444

Are you sure you found them all?

Not really. There might be more.

343 443 333 444 434 344 433 434 444

You found 444 twice!

And there are two 434's on your list, Grogg.

343 443 333 444 434 344 433 434 444

If I erase the extra copies, that leaves seven numbers.

Did I miss any?

If we want to be able to tell if we found them all, we need to be organized.

How could you organize the numbers?

How?

Maybe we can write them in order.

We can start with all of the numbers in the 300's.

333
334
343
344

333 is smallest, then 334.

After that are 343 and 344.

Then, we have four numbers in the 400's.

333
334
343
344
433
434
443
444

Which number did Grogg miss?

That makes sense.

We found four 2-digit numbers.

34
43
33
44

To make a 3-digit number...

...we can either put a 3 in the hundreds place...

...or a 4 in the hundreds place.

That gives us 8 numbers.

334
343
333
344

434
443
433
444

I did something different.

I started with the only number that has three 3's and no 4's.

333

Then, I wrote the numbers that have two 3's and one 4.

334
343
433

The 4 can go in the ones, tens, or hundreds place.

Next, I found three numbers that have one 3 and two 4's.

333
334
343
433

The 3 can go in the ones, tens, or hundreds place.

443
434
344

And finally, there's just one number with three 4's.

444

I found 8 numbers, too.

Excellent! The key to solving many problems like these is being organized.

How would you organize your work for this problem?

How many different three-digit numbers have digits that add up to 3?

Try it.

I found three so far!

300
(3+0+0=3)

102
(1+0+2=3)

210
(2+1+0=3)

Grogg! You need to *organize.*

We can try listing them in order again.

The hundreds digit needs to be at least 1.

Then, if the tens digit is 0, the ones digit is 2...

102

...if the tens digit is 1, the ones digit is 1...

102
111

...and if the tens digit is 2, the ones digit is 0.

That's all the numbers in the 100's.

102
111
120

Then, with a 2 in the hundreds place...

...the tens digit can be a 0 or a 1.

102
111
120
201
210

And there's just one number with a 3 in the hundreds place.

102
111
120
201
210
300

That makes 6 numbers all together!

I got 6 too, but I organized my work a little differently.

I started by finding all of the ways three digits add up to 3.

The biggest digit you can use is a 3.

Right.

If you have a 3, then the other digits both need to be 0's.

3+0+0

If 2 is the biggest digit, then the other digits need to be 1 and 0.

2+1+0

And if 1 is the biggest digit, then *all* the digits need to be 1's.

1+1+1

So, there are just three different groups of digits you can use?

Yep.

3+0+0, 2+1+0, and 1+1+1.

Then, I found all the ways to arrange each group of digits.

3+0+0

2+1+0

1+1+1

How many ways can you arrange each group to make a three-digit number?

If the digits are 3, 0, and 0, then the 3 needs to be the hundreds digit.

If we use 2, 1, and 0, either the 2 or the 1 has to be the hundreds digit.

Then, there are two ways to arrange the other digits.

And there's only one number we can make with three 1's.

300 210 111
201
120
102

I got the same six numbers as Alex!

I love keeping things organized!

Like what, Alex?

I keep our house decorations arranged by holiday.

I order my cereals by sugar content.

I hang all my ties by length.

And I keep my full list of things to keep organized sorted alphabetically!

Item:	Sort By:
Art Supplies	Color
Books	Height
Cereals	Sugar Grams
Decorations	Holiday
Frozen Fish	Purchase Date
Hats	Size
Junk Drawer	Category
Money (bills)	Denomination
Money (coins)	Diameter
Phone Numbers	Numerical
Piano Music	Composer
Rock Collection	Hardness
Silverware	Type
Socks	Left/Right
Tax Documents	Year
Ties	Length
Toothbrushes	Firmness
Trophies	Activity

1+1+1+1+1+1+1+1+1+1=

2+2+2+2+2+2+2+2+2+2=

3+3+3+3+3+3+3+3+3+3=

Ten 1's is easy. That's 10.

Ten 2's is 20.

Ten 3's is 30.

Since I'm adding ten numbers...

...when I increase each number I'm adding by 1, the sum increases by 10.

So, ten 4's is 40.

$1+1+1+1+1+1+1+1+1+1=10$

$2+2+2+2+2+2+2+2+2+2=20$

$3+3+3+3+3+3+3+3+3+3=30$

$\overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \overset{+1}{\downarrow}(\ \ )\overset{+10}{\downarrow}$

$4+4+4+4+4+4+4+4+4+4=40$

The pattern keeps going, counting by 10's.

Counting by 10's is just like counting by 1's, but with an extra 0 at the end.

$1+1+1+1+1+1+1+1+1+1=10$

$2+2+2+2+2+2+2+2+2+2=20$

$3+3+3+3+3+3+3+3+3+3=30$

$4+4+4+4+4+4+4+4+4+4=40$

$5+5+5+5+5+5+5+5+5+5=50$

$6+6+6+6+6+6+6+6+6+6=60$

...and so on...

$25+25+25+25+25+25+25+25+25+25=250$

So, to add ten copies of a number, I can just take the number...

...and write a zero after it!

Yep!

Quick, add ten 867's!

Find the sum of ten 867's.

Ten 1's = 10

Ten 2's = 20

Ten 3's = 30

Ten 4's = 40

Ten 11's = 110

Ten 25's = 250

I just write a 0 after 867 to get 8-6-7-0.

That's eight thousand, six hundred seventy.

Ten 867's = 8,670

Yep. You figured that trick out fast!

Sometimes, if you want to solve a hard problem...

...solving easier problems can help you find a useful pattern.

Try this.

If you add forty-five 45's, what's the ones digit of the sum?

Hmmm...

I'll never be able to add forty-five 45's, but I could add a few and look for a pattern.

What's the ones digit?

45+45 is 90...

...plus another 45 is 135...

...plus 45 is 180...

...plus 45 is 225...

...and six 45's is 270.

45+45=90

45+45+45=135

45+45+45+45=180

45+45+45+45+45=225

45+45+45+45+45+45=270

The ones digit just goes from 0 to 5 and back.

Will it keep switching like that?

It will!

We only care about the ones place, so we can ignore all the other digits.

Adding 5 to a number that ends in 5 always gives us a number that ends in 0...

...and adding 5 to a number that ends in 0 always gives us a number that ends in 5.

So, the 0-5-0-5-0-5 pattern in the ones place will keep going forever.

If you add forty-five 45's, is the ones digit 0 or 5?

If we add an ***even*** number of 45's, the sum ends in 0...

...and if we add an ***odd*** number of 45's, the sum ends in 5.

So, the sum of forty-five 45's ends in 5!

*IF YOU ARE CURIOUS ABOUT THE ACTUAL SUM, FORTY-FIVE 45'S ADD UP TO 2,025.

You got it!

Oooh! Look at ***this*** pattern.

For curtains?

I have a better idea!

We look great!

HIPPOPOTAMOOSE

LODGE

Dance Night