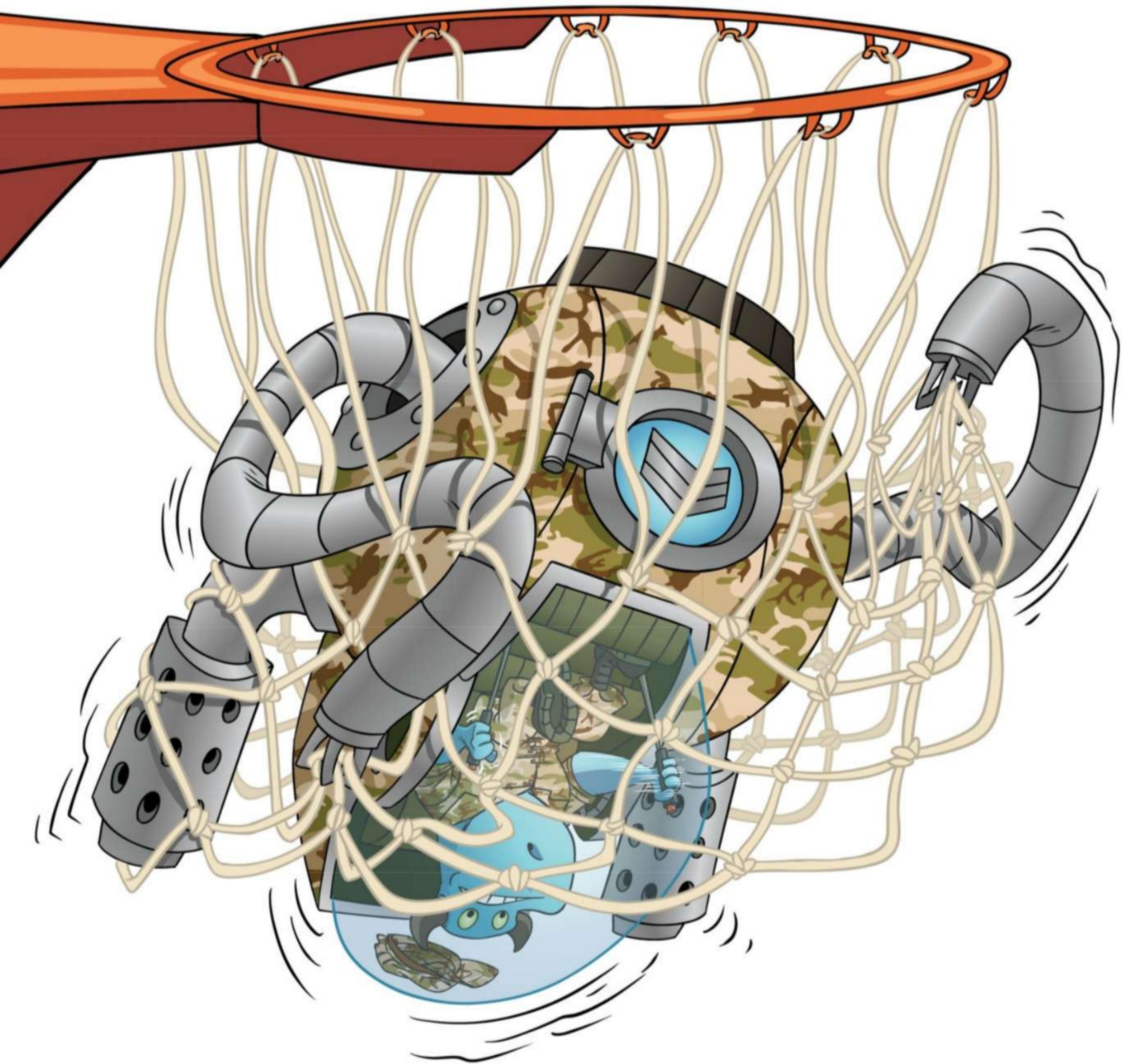




Chapter 7: Factors



Ms. Q. Factors

What
are
factors?

Factors are
the numbers
that you multiply
to get other
numbers.

Huh?

For example,
when you
multiply 2×3 ,
you get 6.

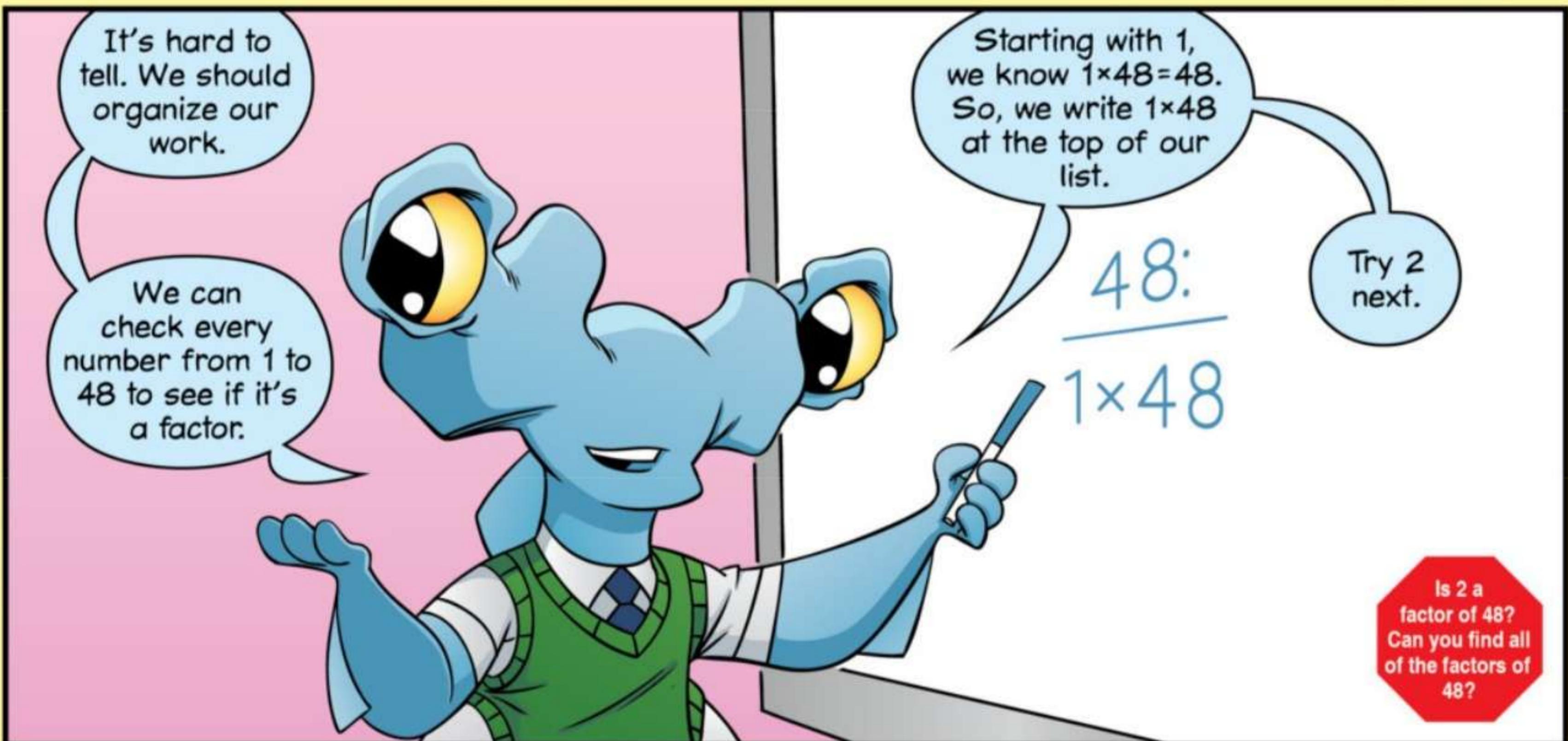
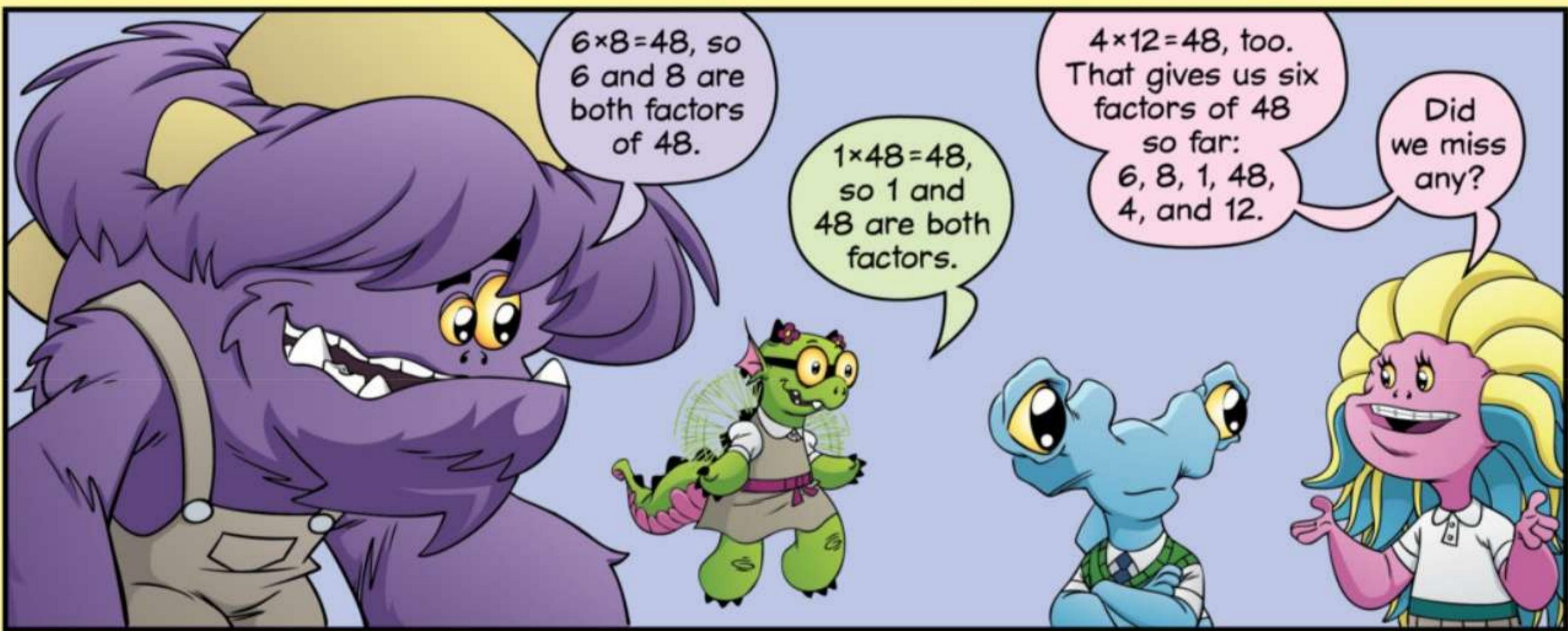
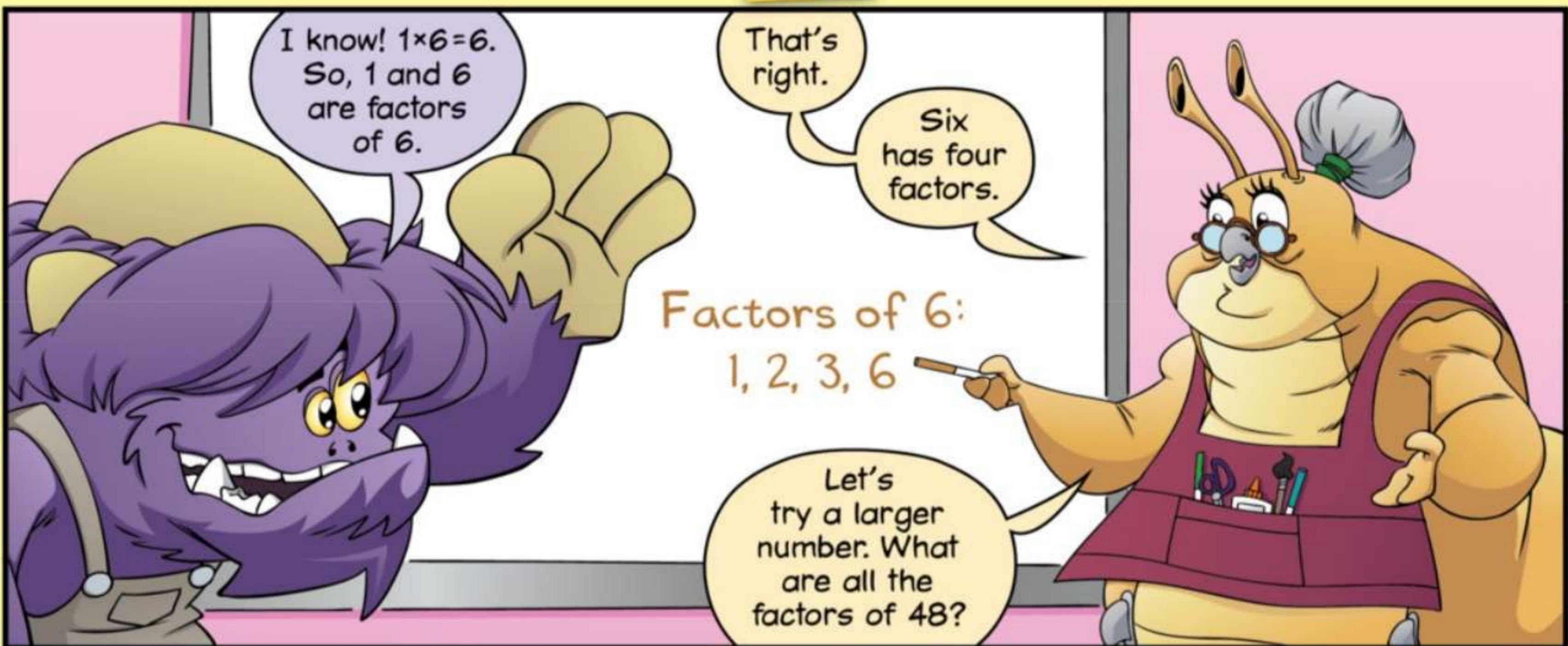
So, we
say that
2 and 3
are **factors**
of 6.

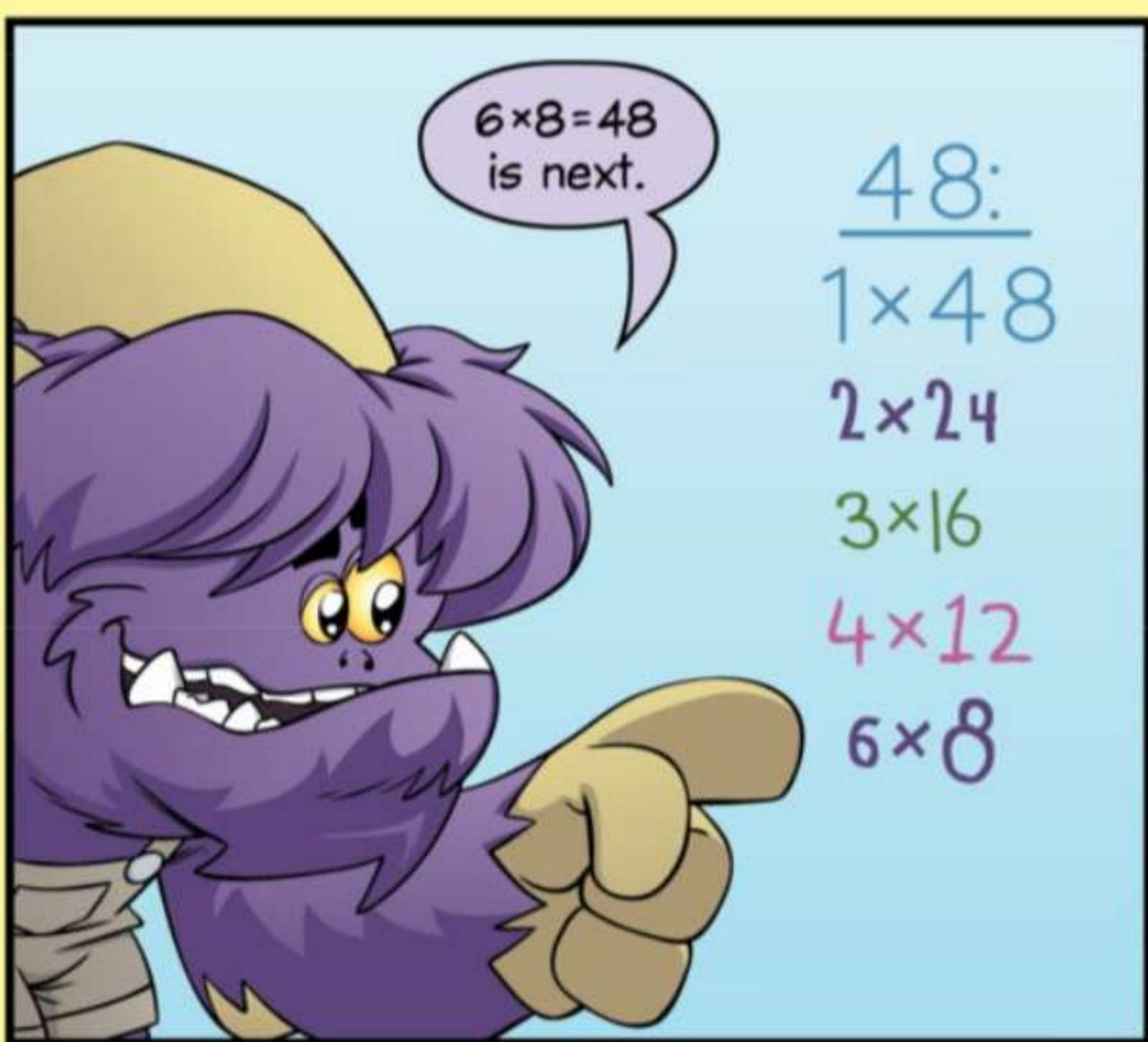
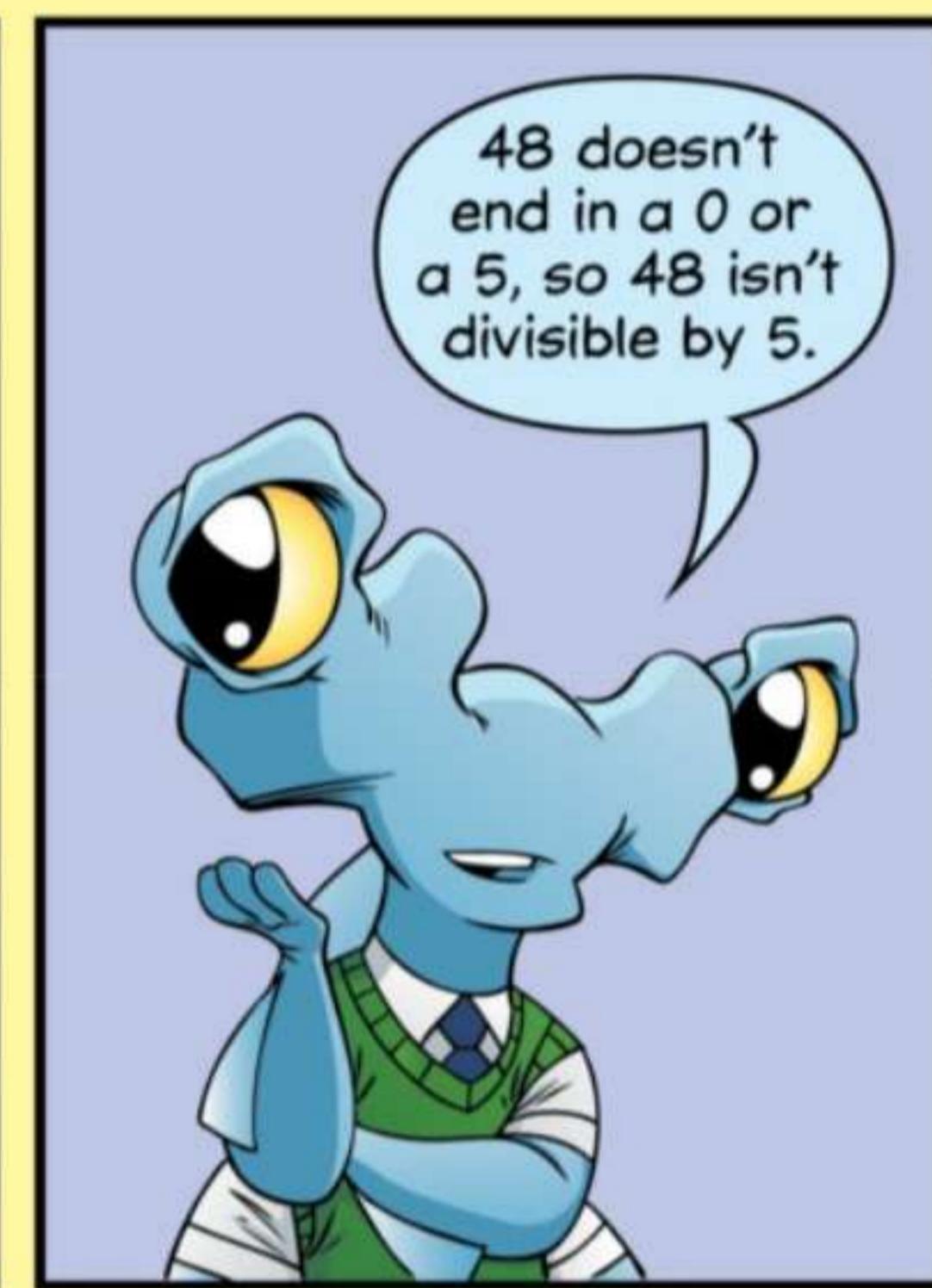
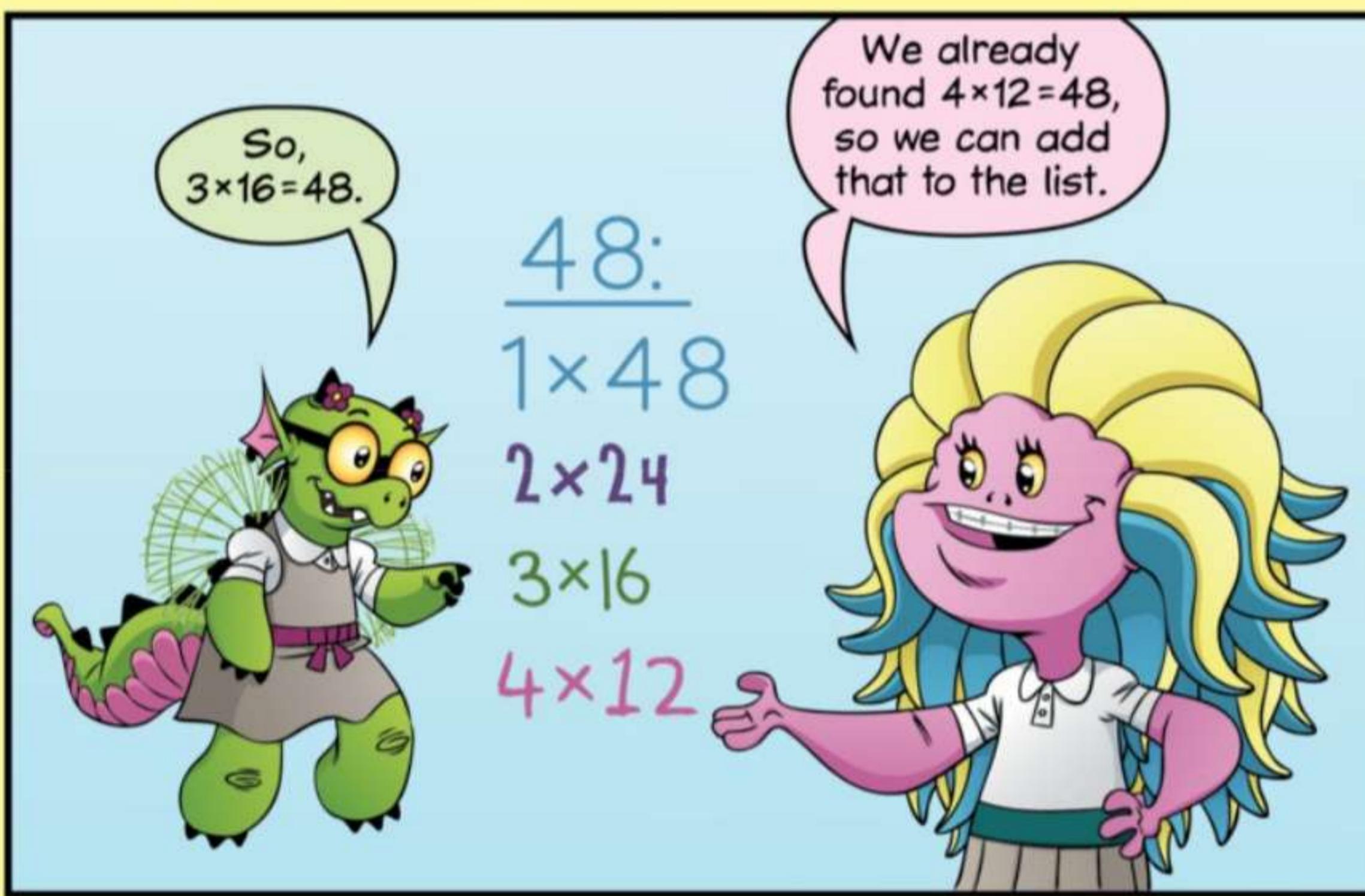
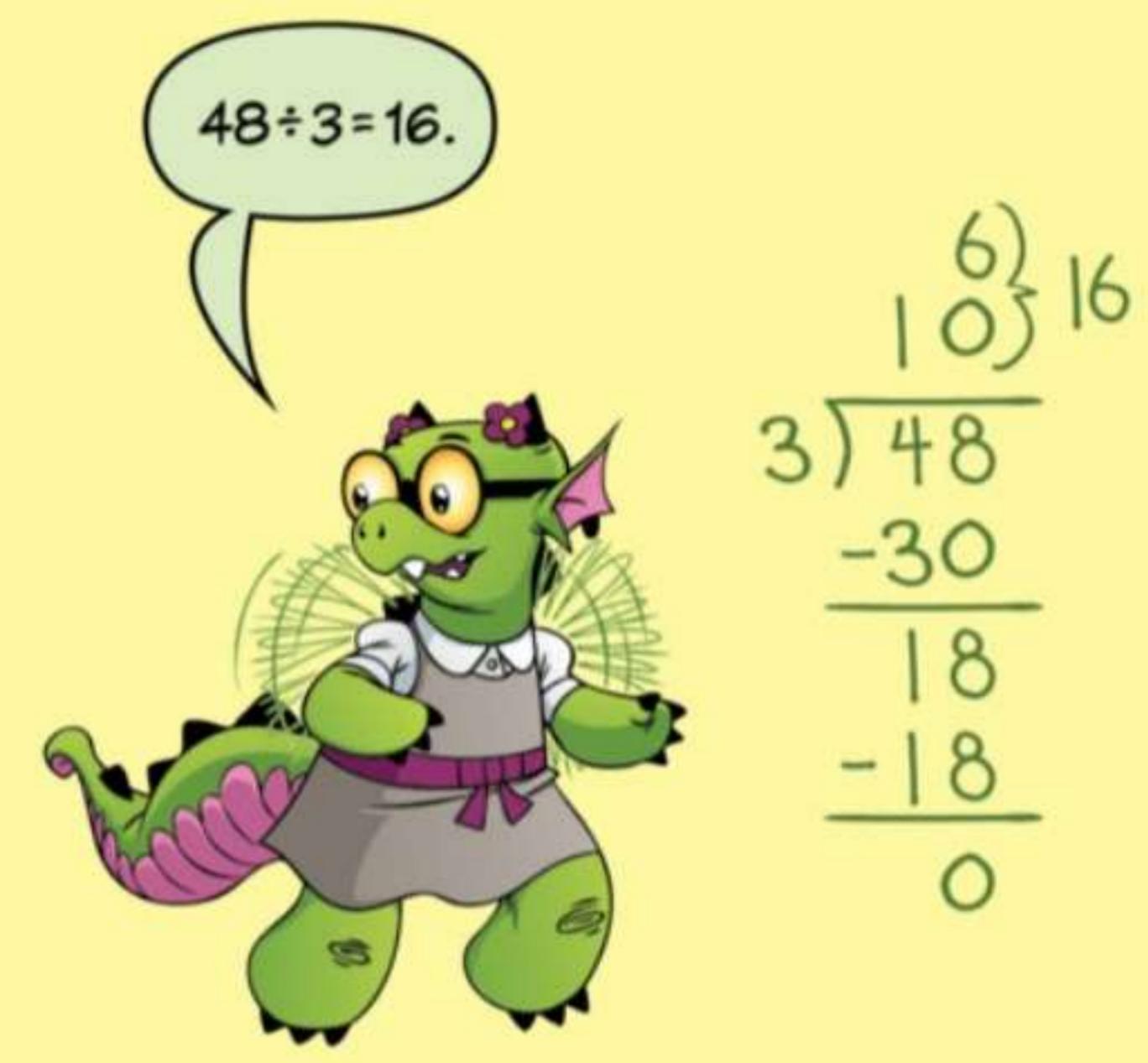
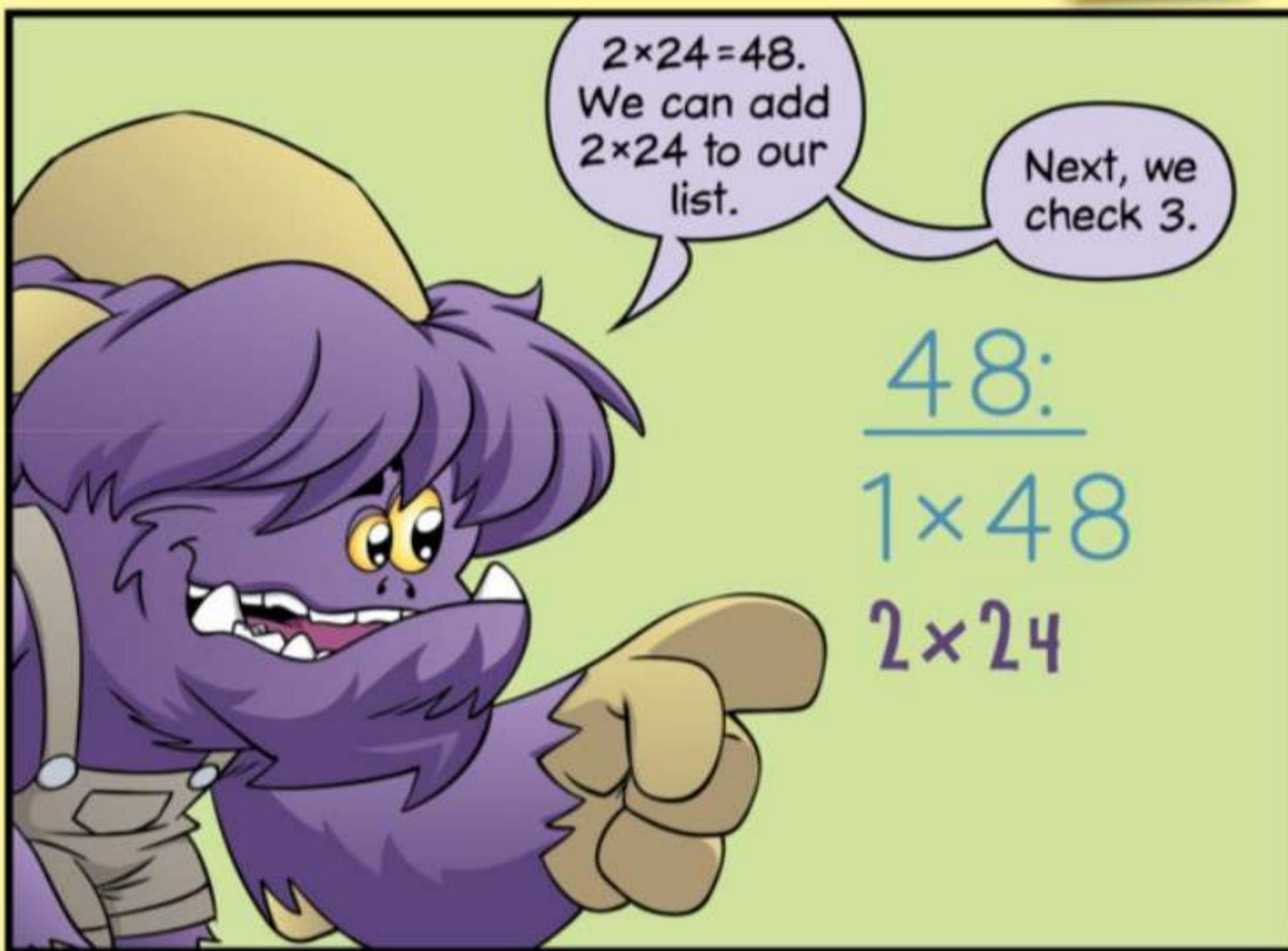
Factors are
the numbers
that a number is
divisible by.

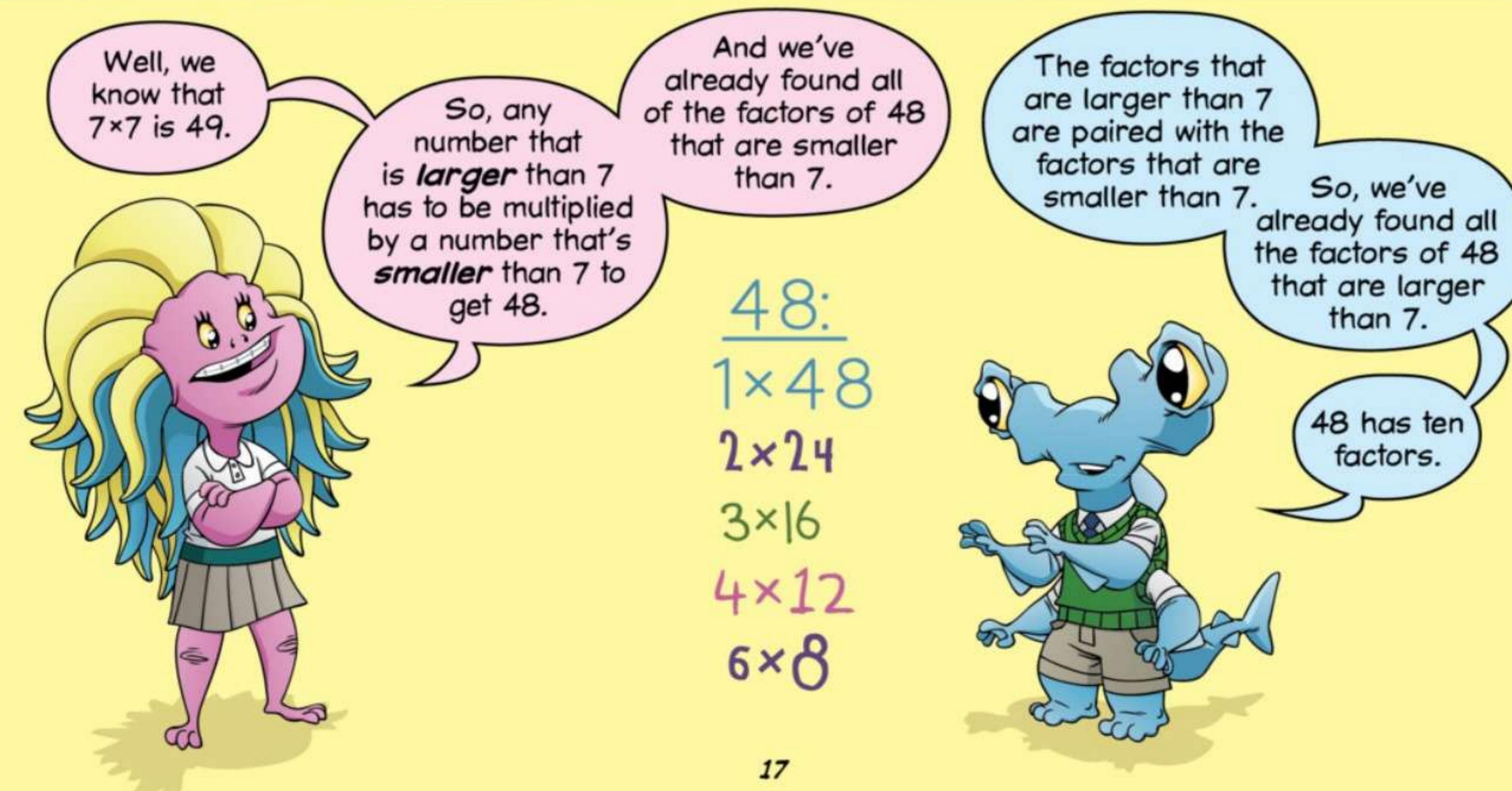
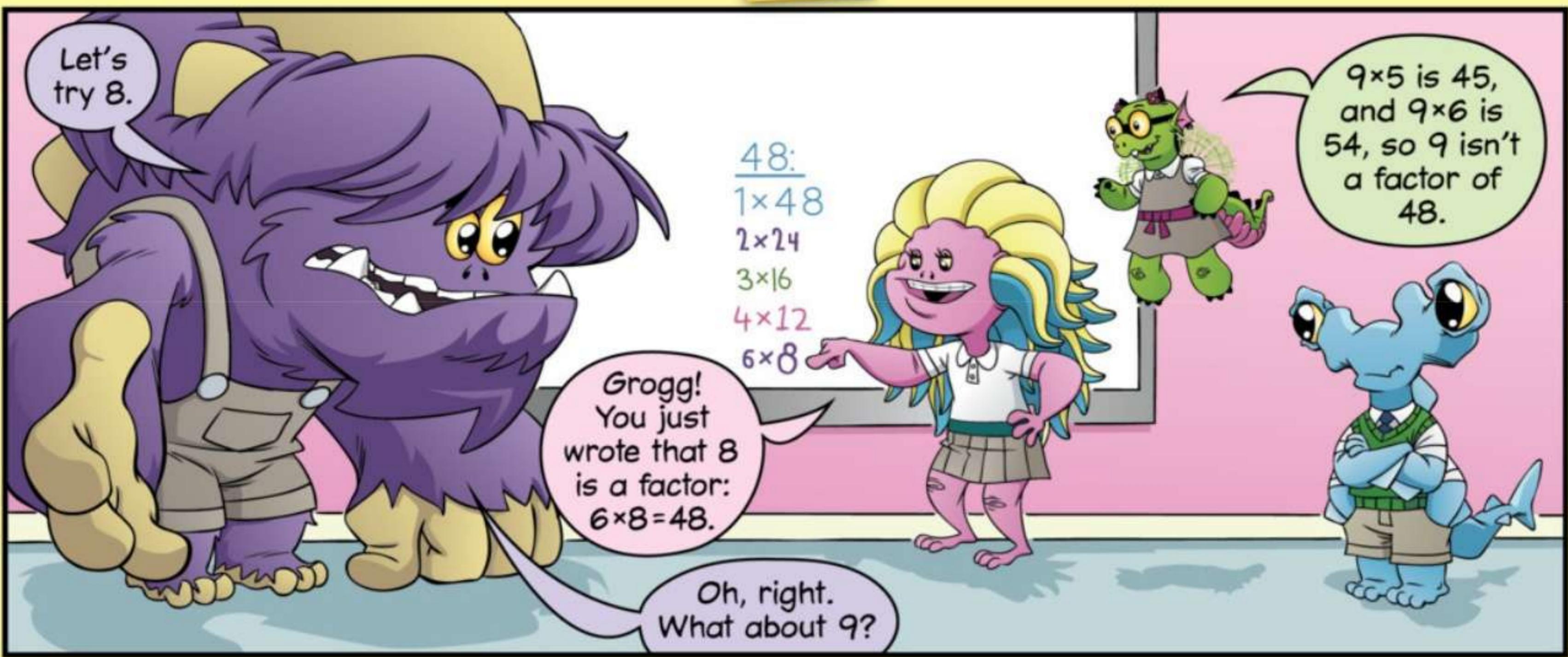
Since 6 is
divisible by 3,
we say that 3 is
a **factor** of 6.

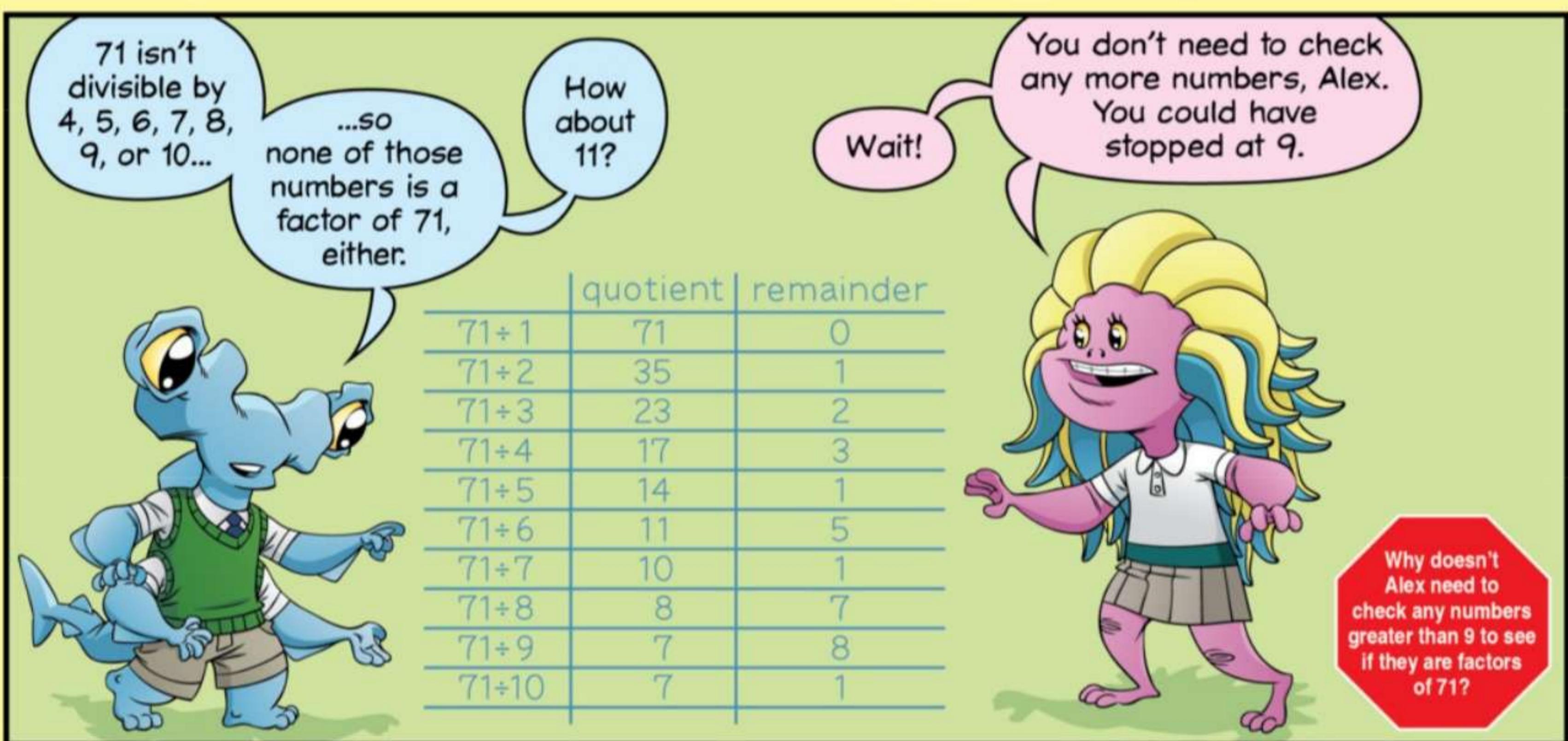
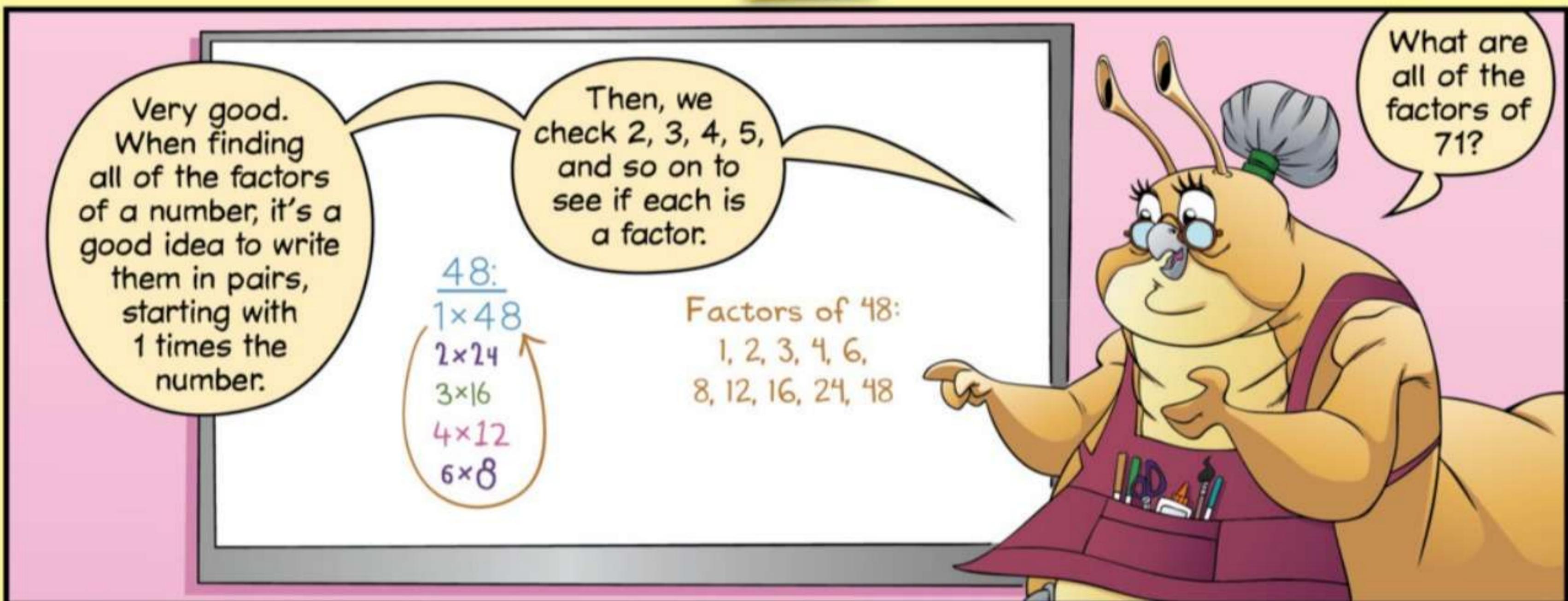
Very good.
Does 6 have
any other
factors?

Hmmm...
What other
numbers can
we multiply to
get 6?









Since $9 \times 9 = 81$, any number that is **larger** than 9 has to be multiplied by a number that's **smaller** than 9 to get a product that is less than 81.

So, to get a product of 71 using a number that is larger than 9, we have to multiply by a number that's smaller than 9.

Since 1 is the only factor of 71 that is smaller than 9...

...we know that 71 is the only factor of 71 that is larger than 9.

That means 1 and 71 are the only factors of 71.

That's right. The number 71 has only two factors, 1 and 71.

Does anyone know what we call a number whose only factors are 1 and itself?

$$\begin{array}{c} 71: \\ 1 \times 71 \end{array}$$

A number with exactly 2 factors, 1 and itself, is called a **prime number**.

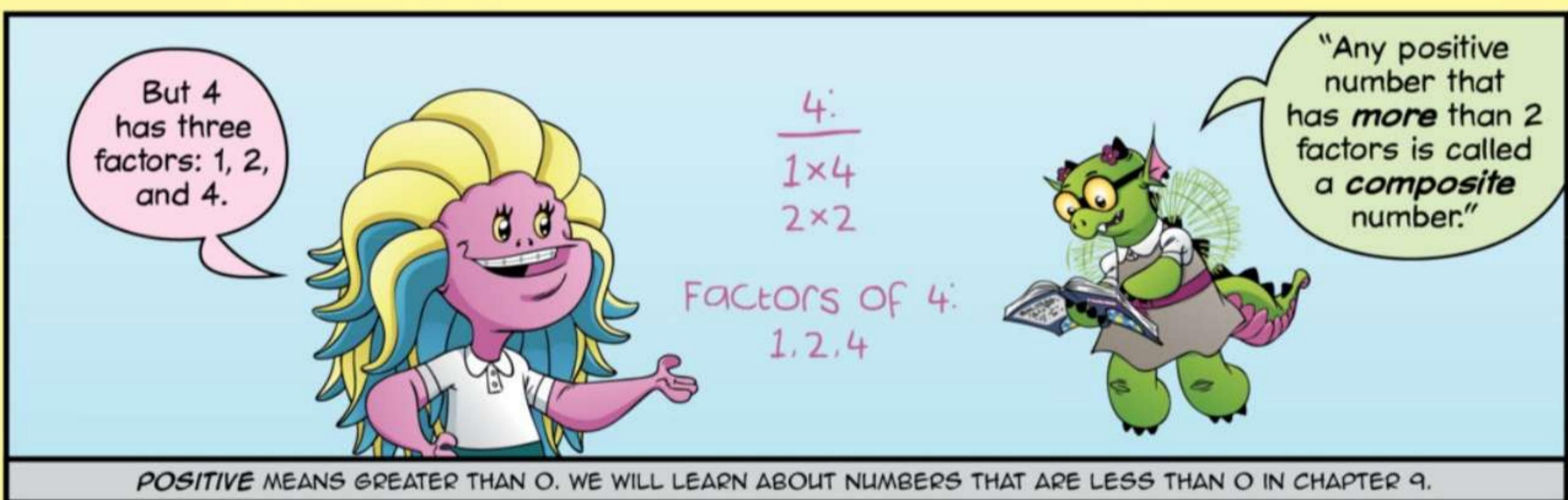
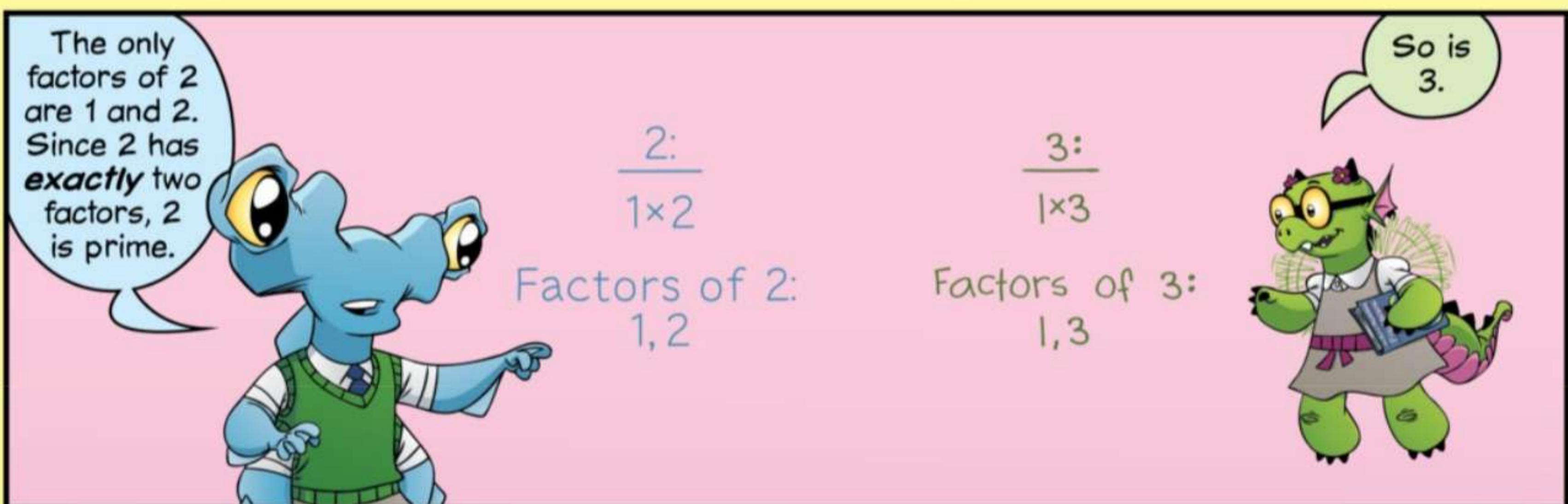
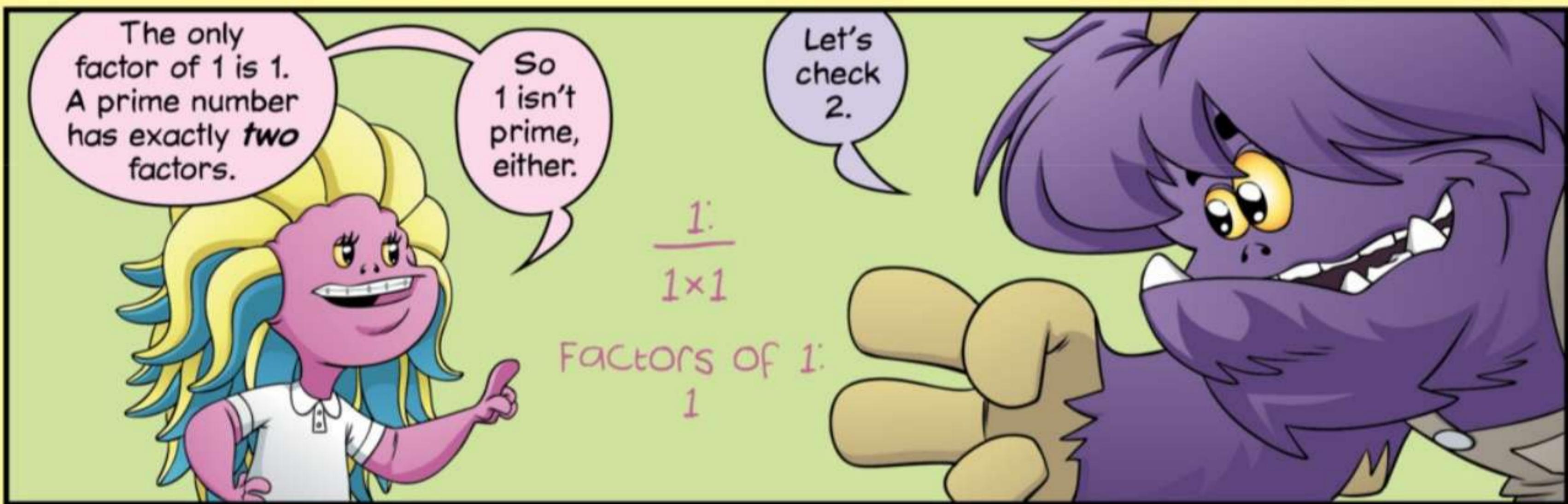
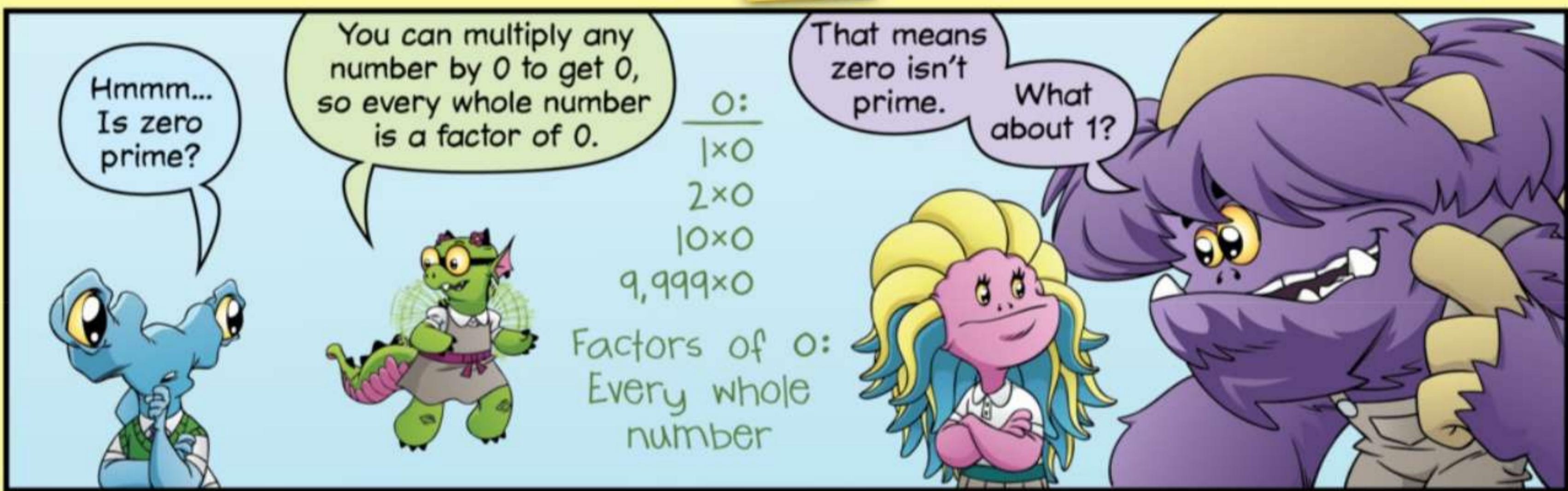
Very good, Lizzie.

Which of these 1-digit numbers are prime?

Circle the primes:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Try it.

NUMBERS THAT ARE PRIME ARE OFTEN SIMPLY CALLED "PRIMES."



POSITIVE MEANS GREATER THAN 0. WE WILL LEARN ABOUT NUMBERS THAT ARE LESS THAN 0 IN CHAPTER 9.

So, 5 is prime, 6 is composite, 7 is prime...

...8 is composite, and 9 is composite.

$$\frac{5}{1 \times 5}$$

prime

$$\frac{6}{1 \times 6}$$

2 × 3

composite

$$\frac{7}{1 \times 7}$$

prime

$$\frac{8}{1 \times 8}$$

2 × 4

composite

$$\frac{9}{1 \times 9}$$

3 × 3

composite



The 1-digit primes are 2, 3, 5, and 7.

Circle the primes:
0, 1, **2**, **3**, **4**, **5**, **6**, **7**, 8, 9



Is 41 prime, too, Ms. Q.?

It is, Grogg. Why do you ask?



Last week, my dad turned 41.

I asked him if he felt old. He said...

Not me. I'm in my prime.



G*Y*M

THE SIEVE

Ten hut!

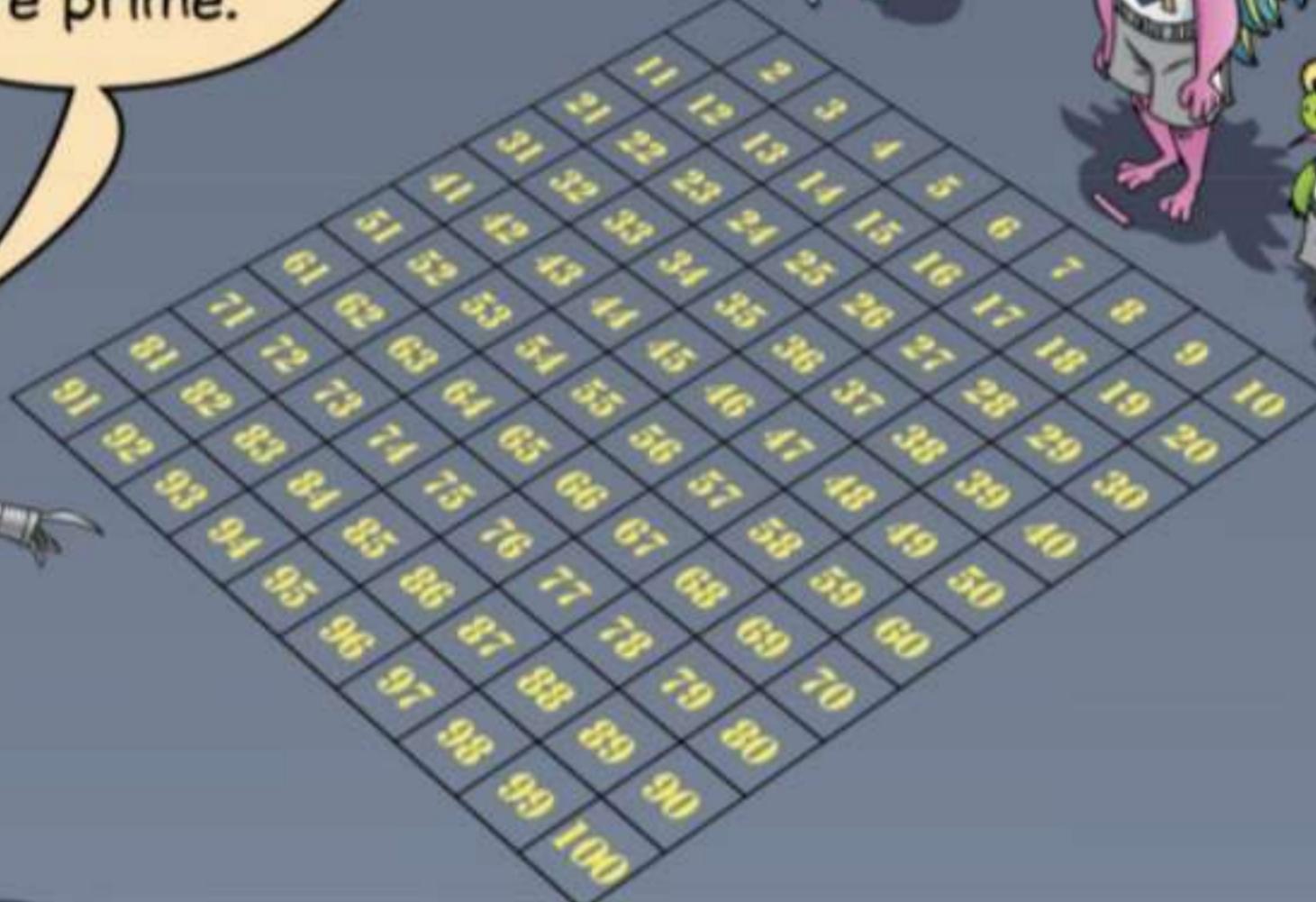
Today, we are going to find every prime number that is less than 100.

2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19

On the pavement in front of you is a 100 chart.

On this chart, you will circle all of the numbers that are prime.

Sir, how do we do that, sir?



Excellent question, furball.

You will find all of the primes by crossing out all of the numbers that are **composite**.

When you are finished with this chart, every prime number will be circled, and every composite number will be crossed out.



PRINT YOUR OWN 100 CHART AT BEASTACADEMY.COM AND FOLLOW ALONG WITH THE LITTLE MONSTERS.

As you can see, I have removed the number 1 from this 100 chart.

Which of you polliwogs can explain why I have removed the number 1?

Sir, 1 isn't prime or composite. Its only factor is 1, sir.



Good work, hammerhead. 1 is neither prime nor composite.

However, every whole number greater than 1 is either prime or composite.

Each of you has been given a piece of chalk. At precisely zero nine thirty, I expect this chart to have every prime circled, and every composite crossed out. Am I understood?

Sir, yes, sir!



There are 99 numbers on the chart! We don't have time to check them all.

What if we split the chart into 4 sections?

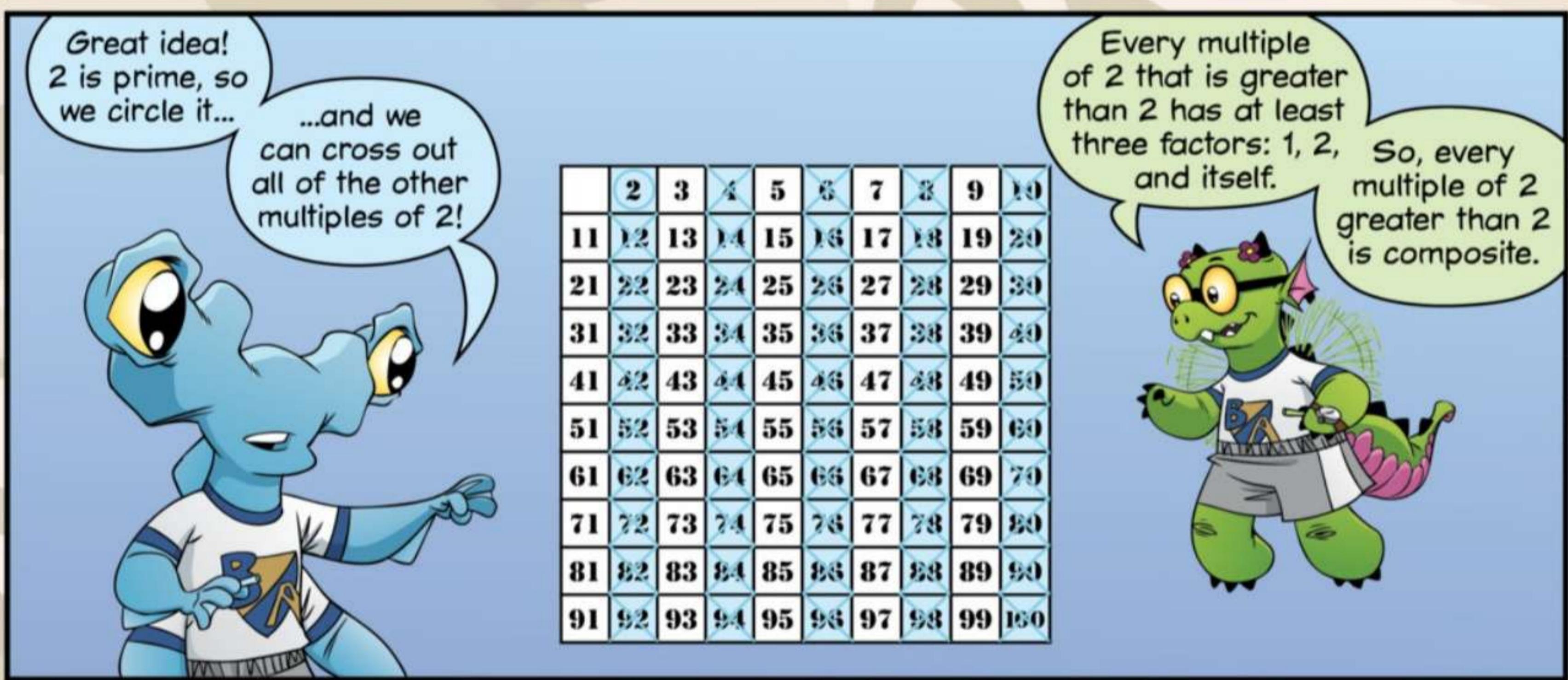
I'll take the numbers in this corner.

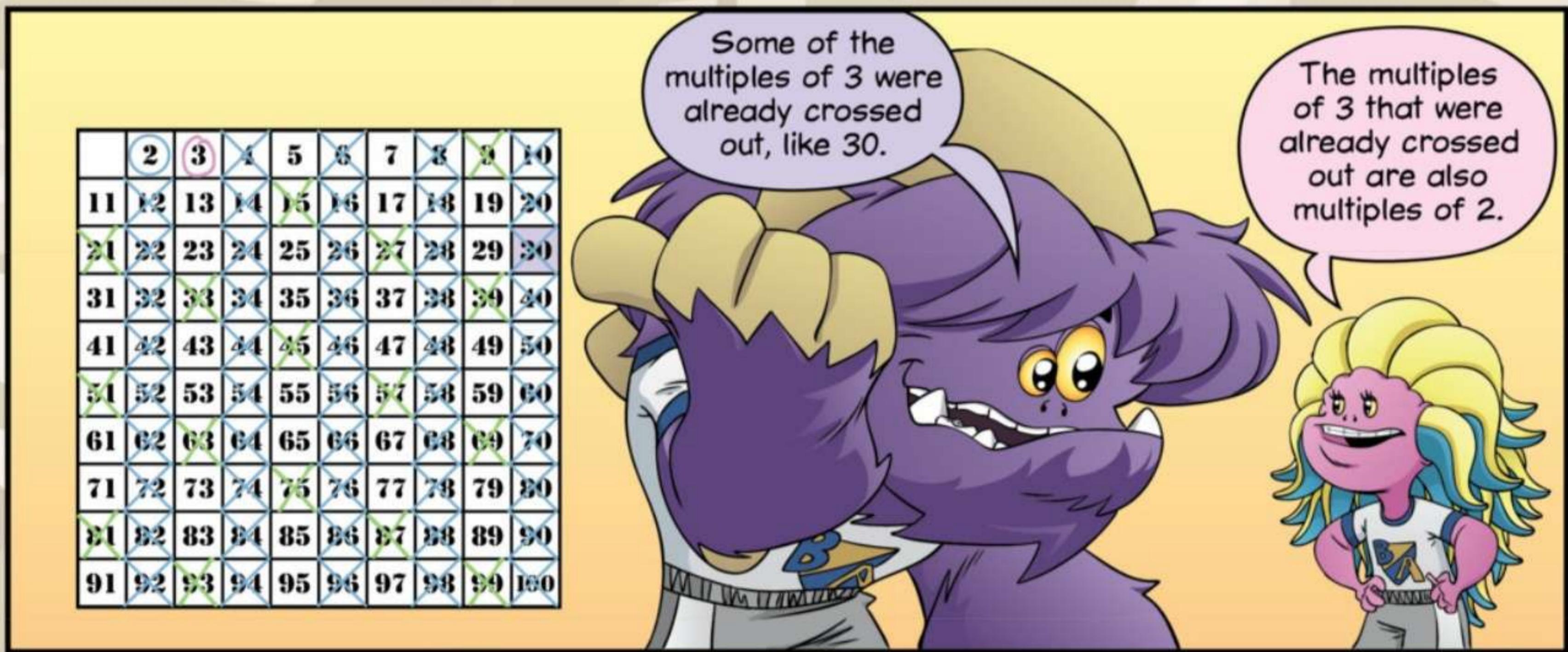
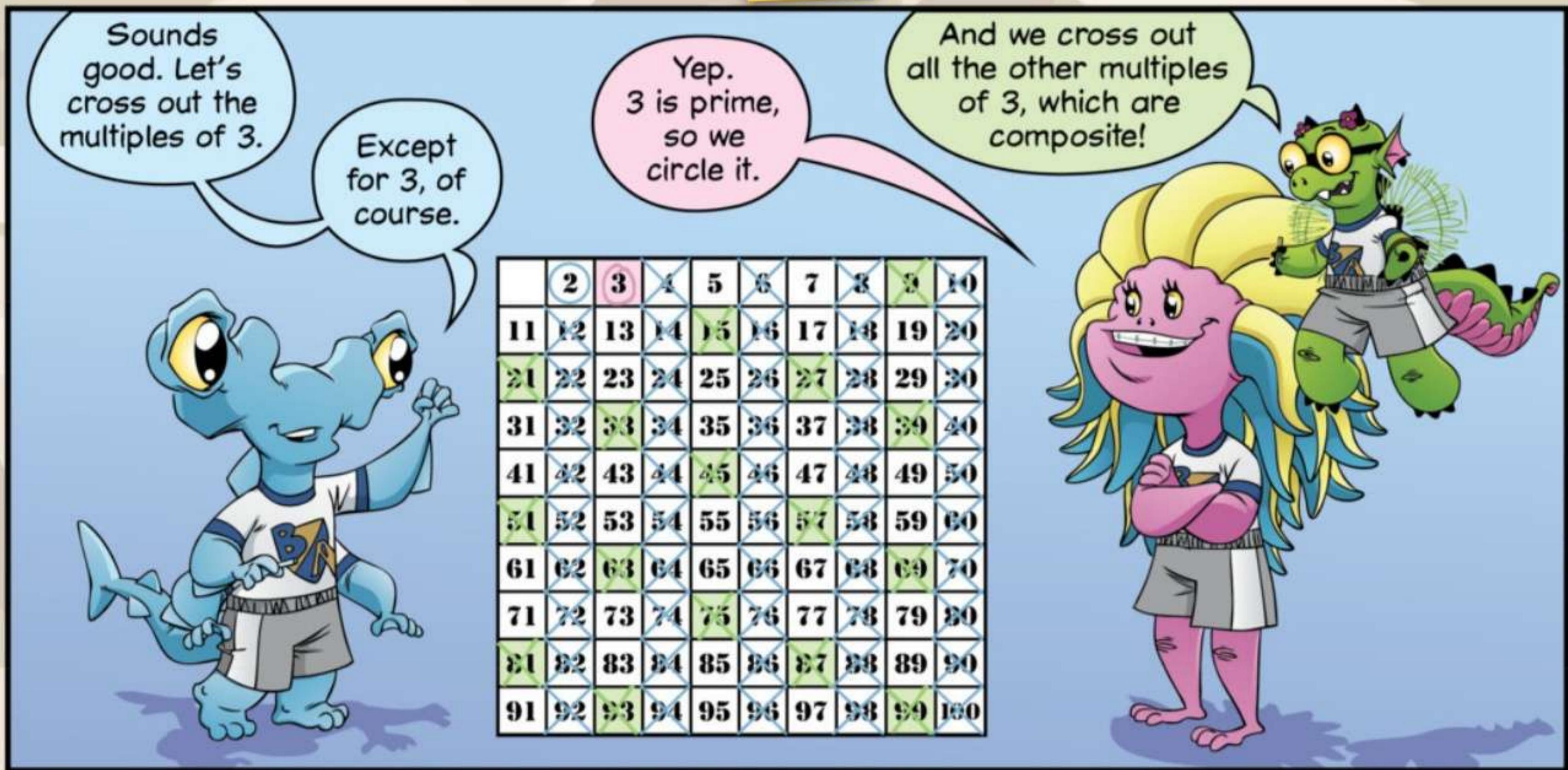
Is 34 prime or composite?



2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Is 34 prime?





We already did!
Since 4 is 2×2 ,
every multiple of 4
is a multiple of 2.



2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The multiples
of 4 got crossed
out when we
crossed out the
multiples of 2.



Five is
prime, so we
circle 5...

...and cross
out all of the
other multiples
of 5.

2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Next is
6.

2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

6 is already
crossed out, so 6
is composite...

...and since
 $6=2\times 3$, all of the
other multiples of 6 got
crossed out when we
crossed out multiples
of 2 and 3.

We
only need
to cross out
multiples of
primes!



Is Grogg
right?



Every composite number has at least one prime factor.

So, multiples of composite numbers like 4 and 6 get crossed out when we cross out multiples of their prime factors.

2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



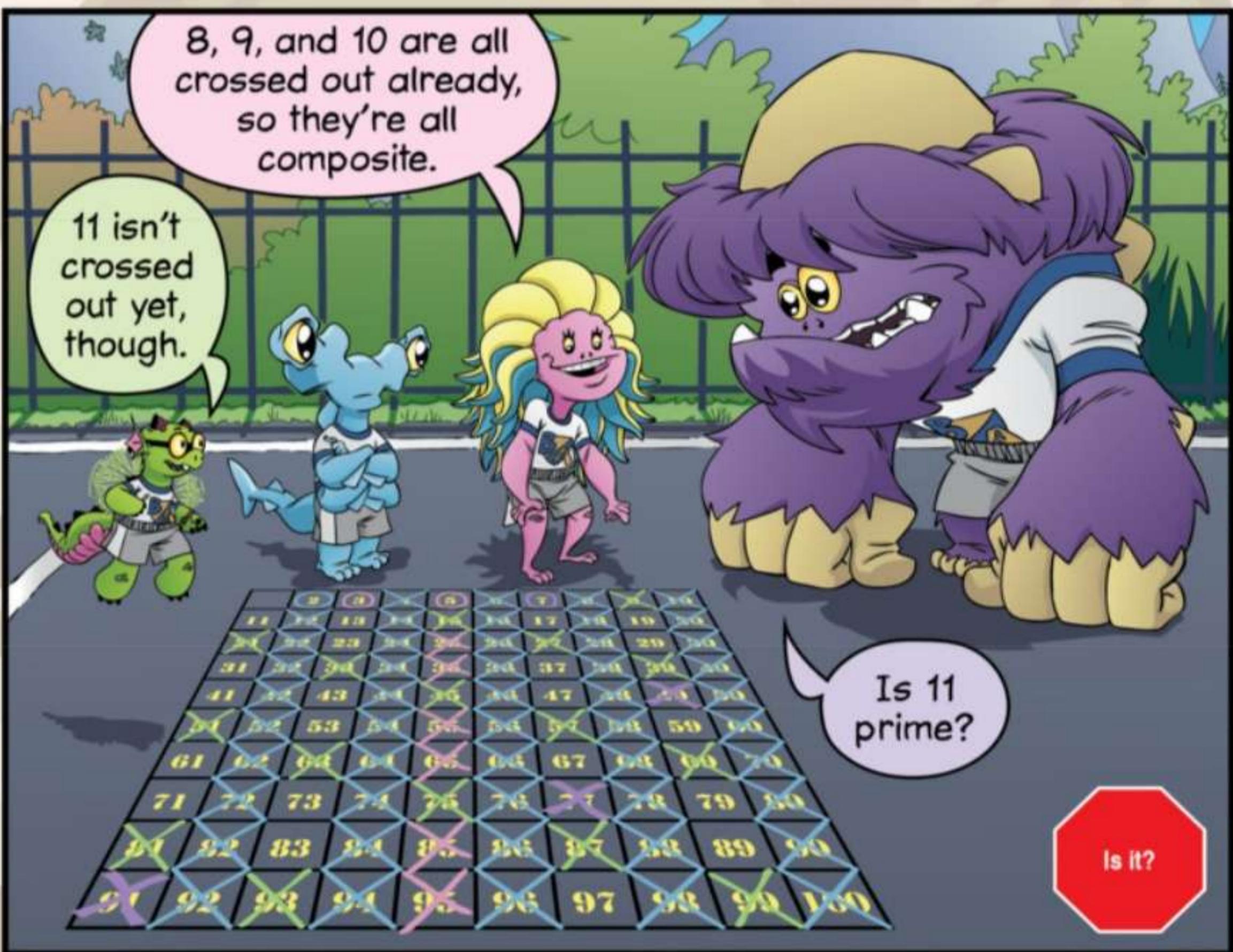
We're running out of time, and there are still a lot of numbers left.

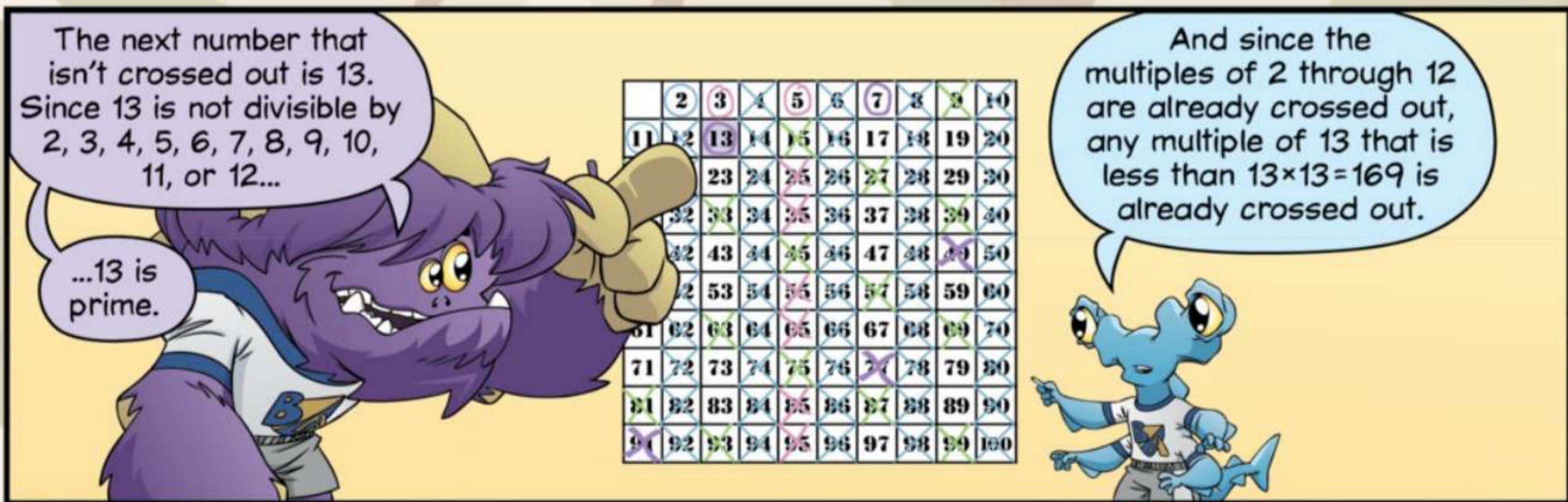
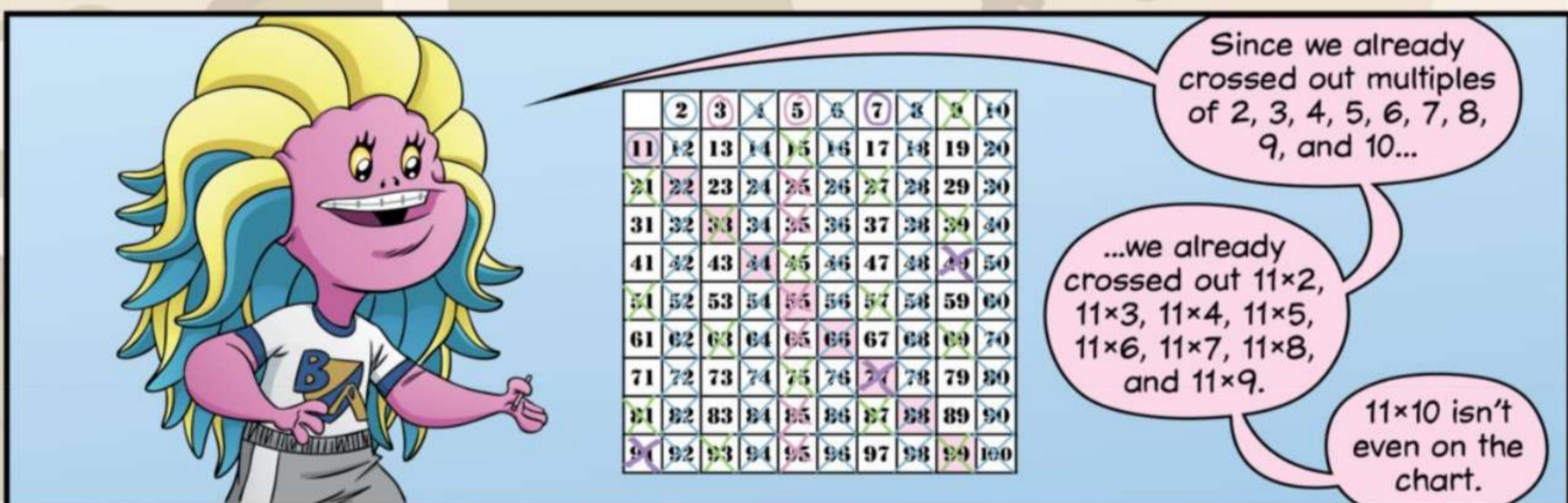
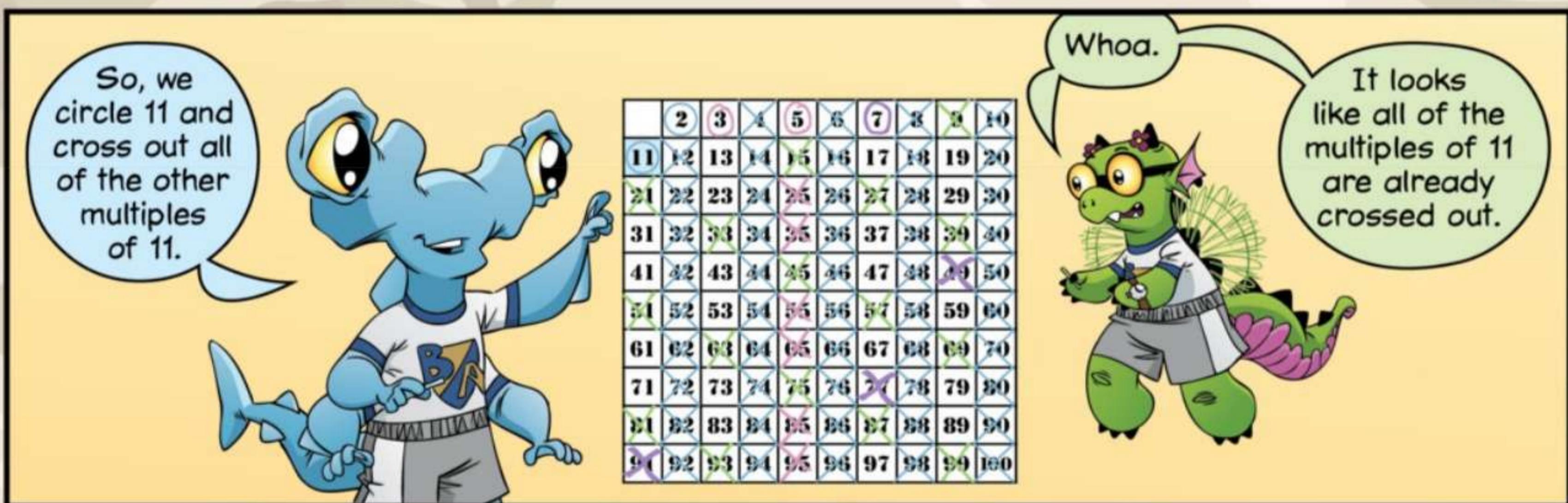
What's the next prime?

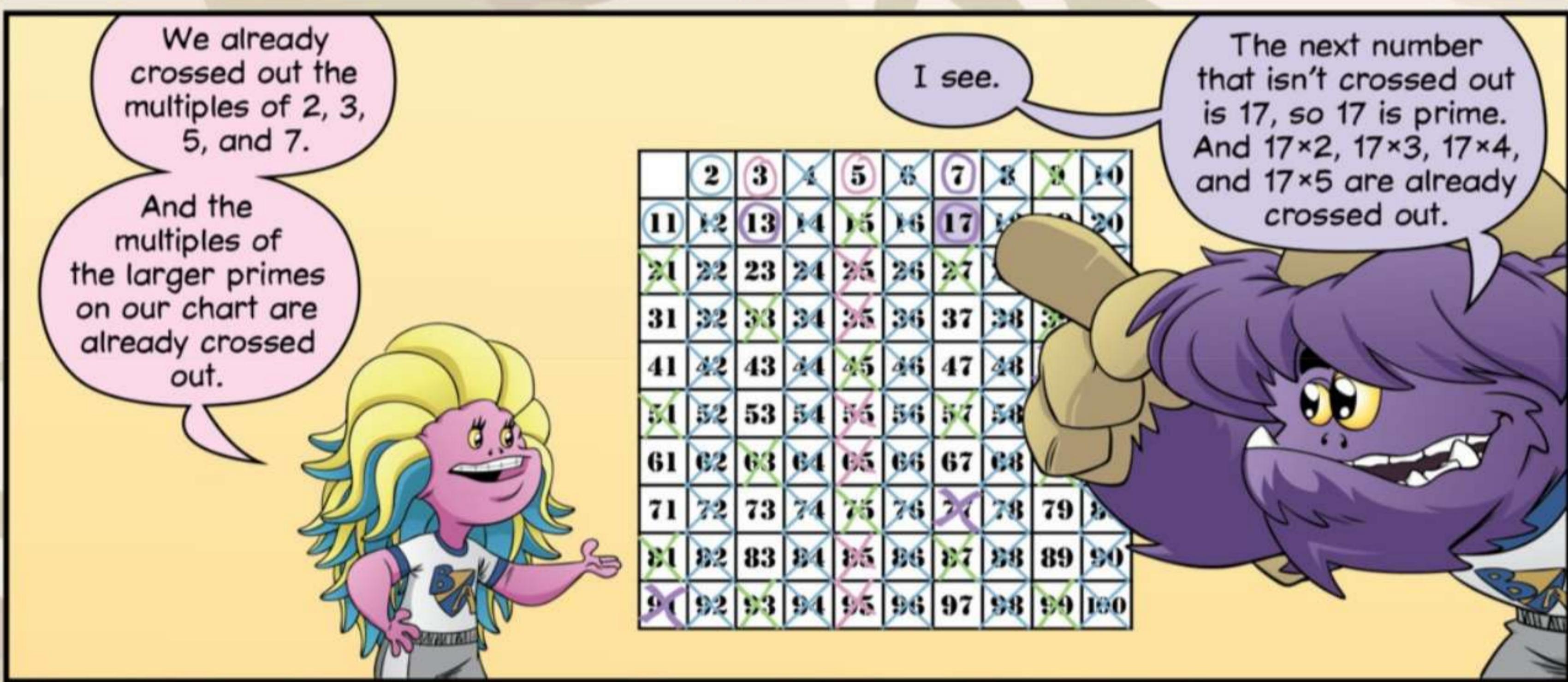


8, 9, and 10 are all crossed out already, so they're all composite.

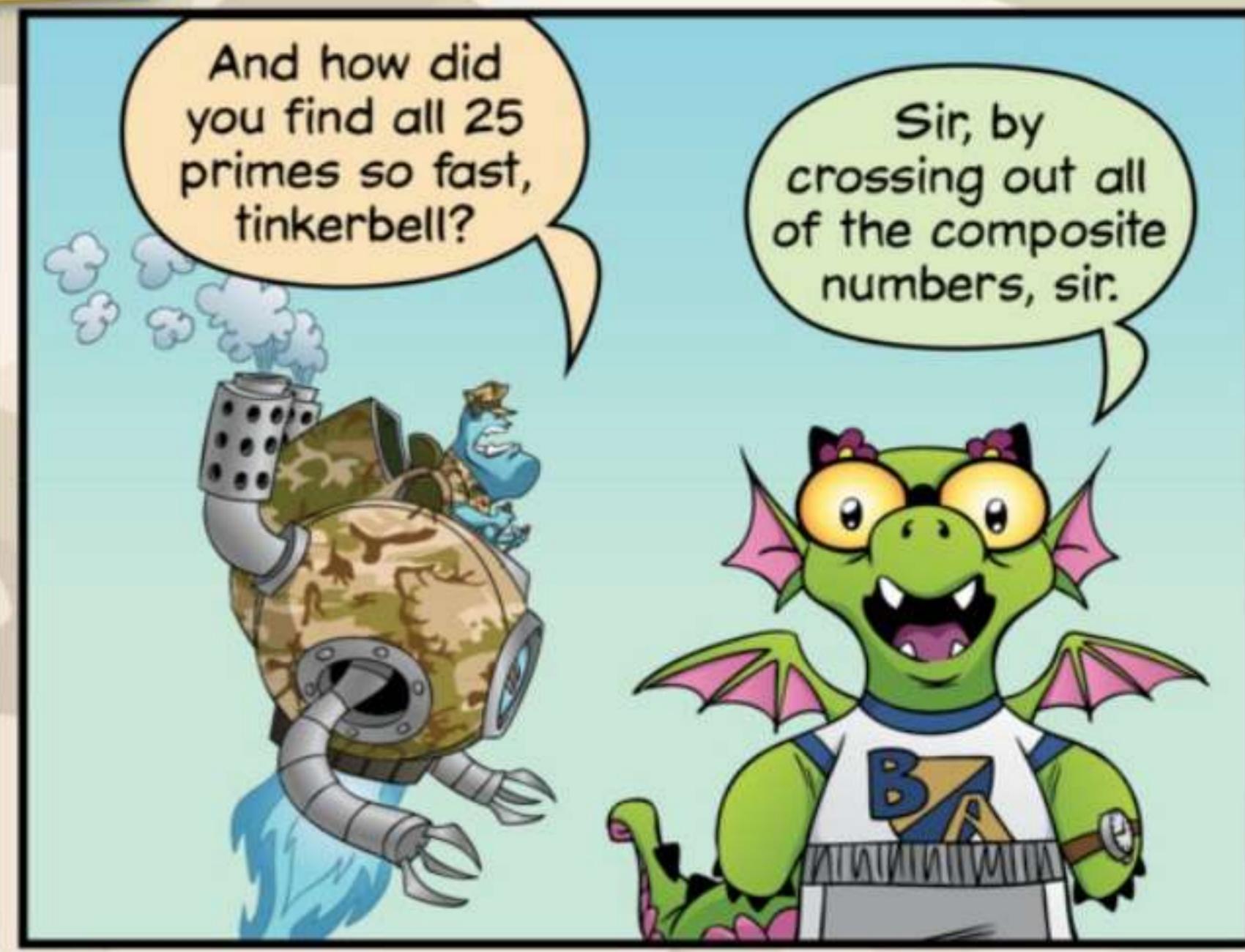
11 isn't crossed out yet, though.







	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



*THE PROCESS IS CALLED THE SIEVE OF ERATOSTHENES (PRONOUNCED SIV OF AIR-UH-TOSS-THE-KNEES).



RECESS

Fizz Buzz

Game Play:

Fizz Buzz is a counting game for two or more players, but is best when played with 5 or more. Players arrange themselves in a circle. Player 1 begins the game by saying "1." The player to the left says "2." Play continues clockwise around the circle with each player stating the next number. However, any multiple of 3 is replaced with the word "fizz," and any multiple of 5 is replaced with the word "buzz." A multiple of both 3 and 5 is replaced with the word "fizzbuzz."

For example, a standard round of Fizz Buzz begins as shown:

1, 2, fizz, 4, buzz, fizz, 7, 8, fizz, buzz, 11, fizz, 13, 14, fizzbuzz, 16, 17, fizz, 19, buzz, fizz, 22, 23, fizz, buzz, ...

Any player who hesitates or makes a mistake is eliminated, and the player to his or her left begins the next round at 1. Play continues until only one player remains. That player is declared the winner.

Variations:

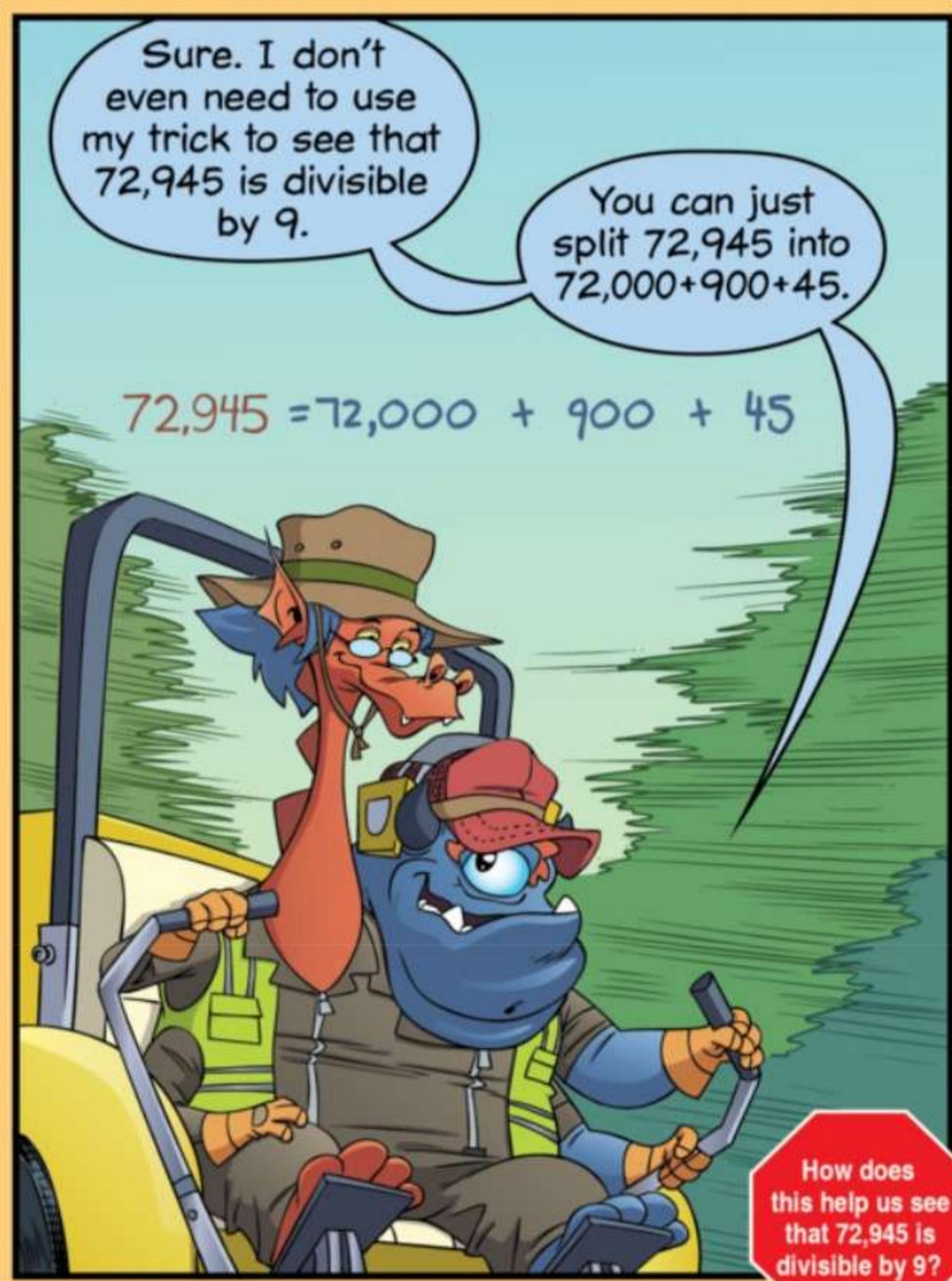
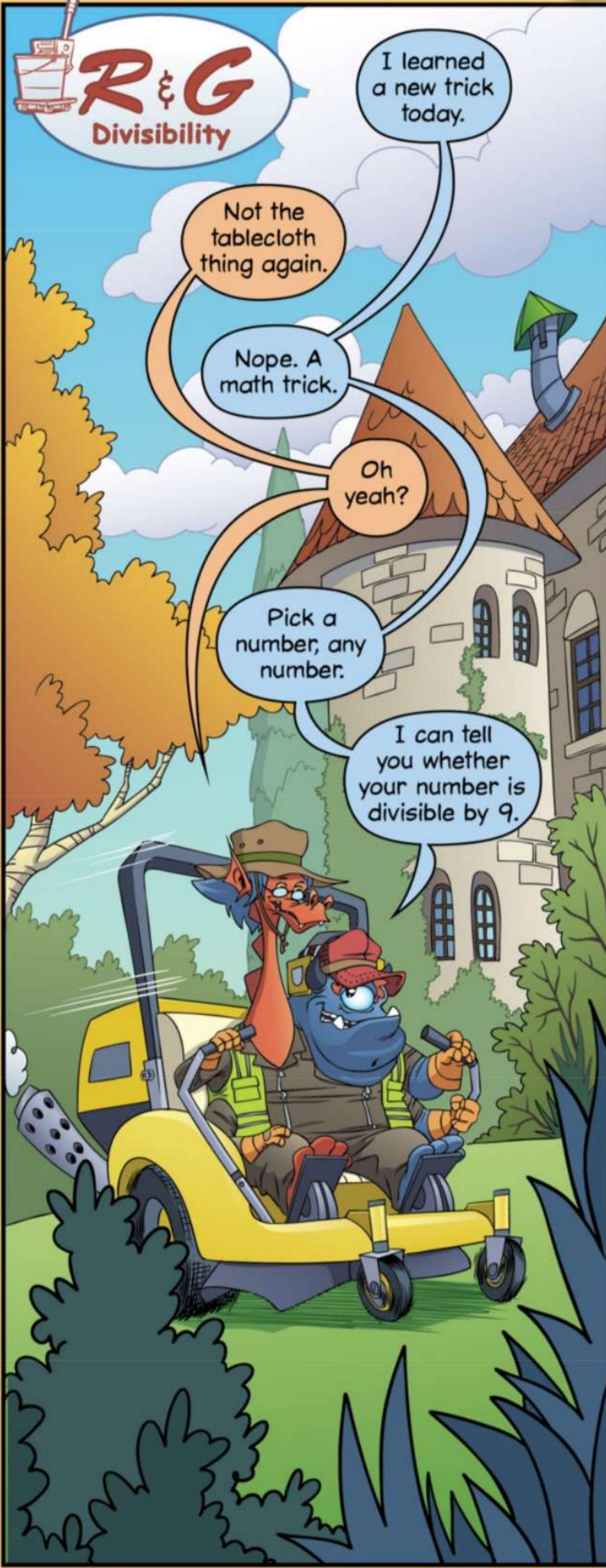
- Different factors can replace 3 and 5. Similarly, different words or gestures can replace the words "fizz" and "buzz." For example, multiples of 3 can be replaced with a clap, and multiples of 7 can be replaced with a hop, with multiples of both 3 and 7 replaced with a clap and a hop.
- Play switches direction on the word fizz or buzz, but not the word fizzbuzz.
- Additional factors with associated words and gestures can be added to make the game even more difficult.
- Instead of restarting at 1 after a mistake, play can resume at the number on which the mistake was made.

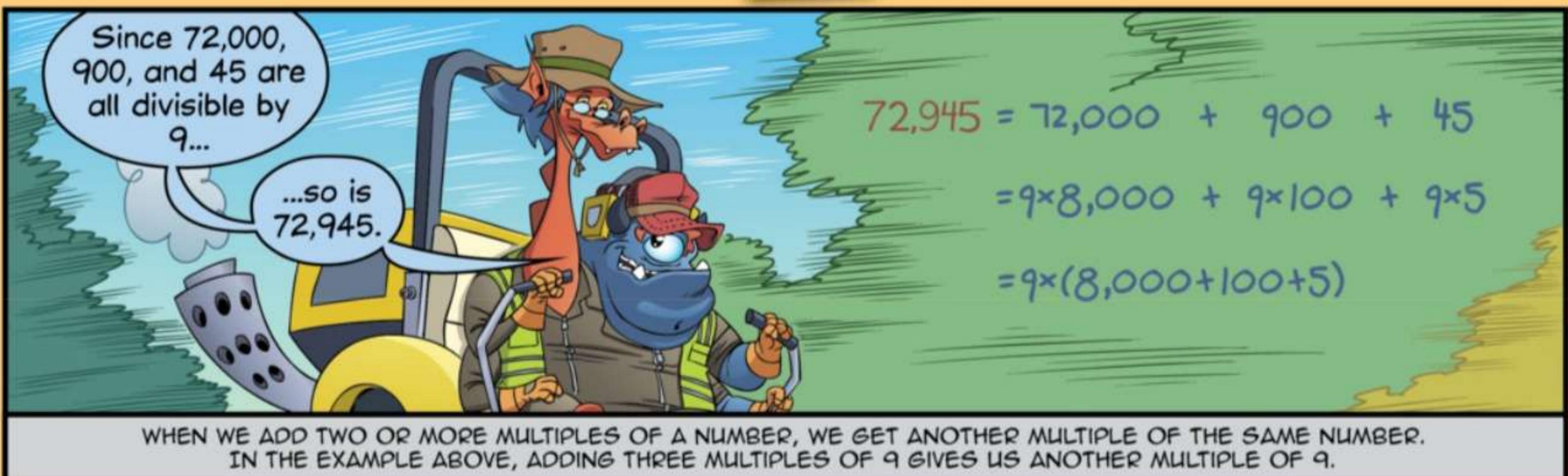
Even better
when played
in a large
group!

Cheat Sheet:

Give this list to an adult so they can referee a game of Fizz Buzz between you and your friends.

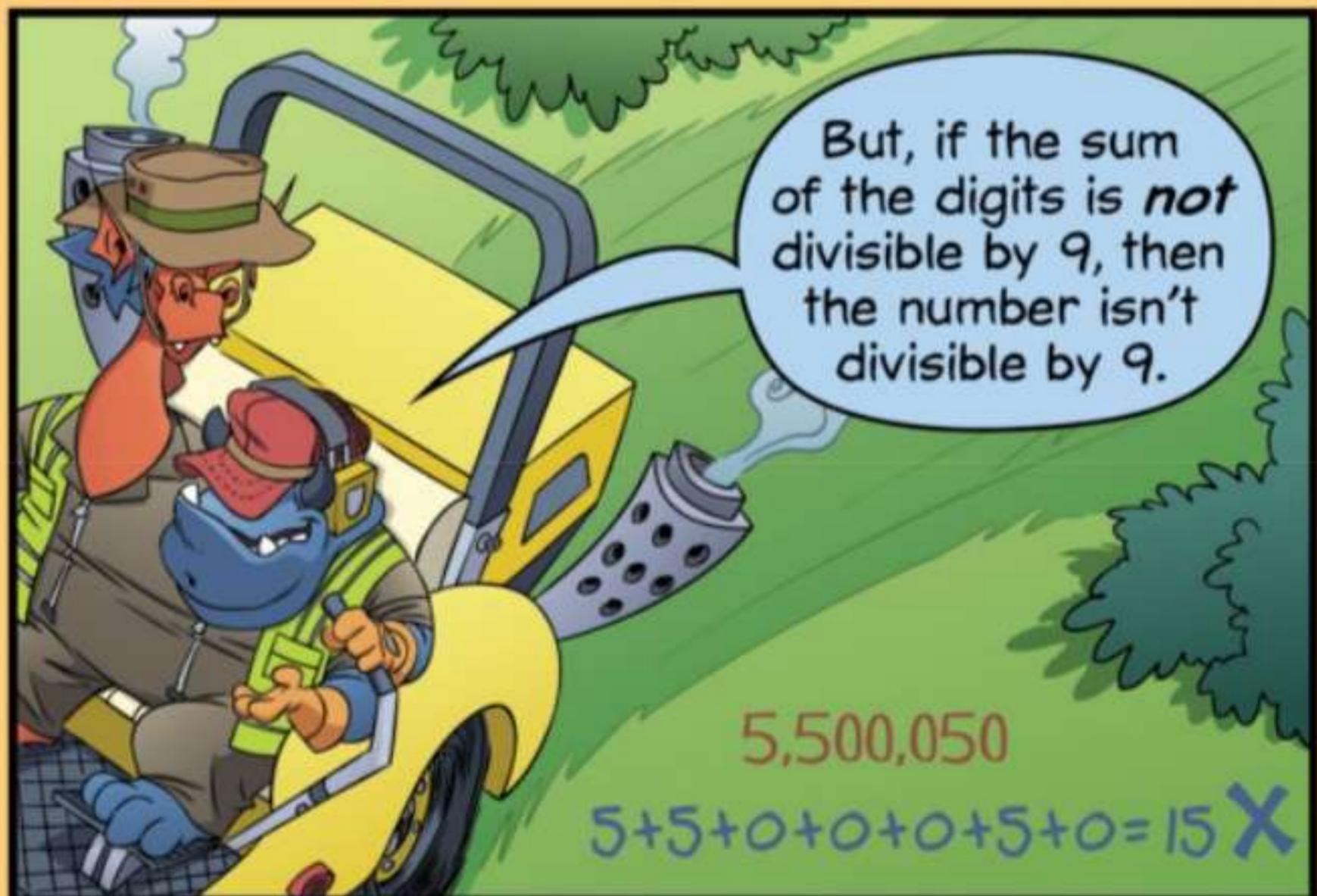
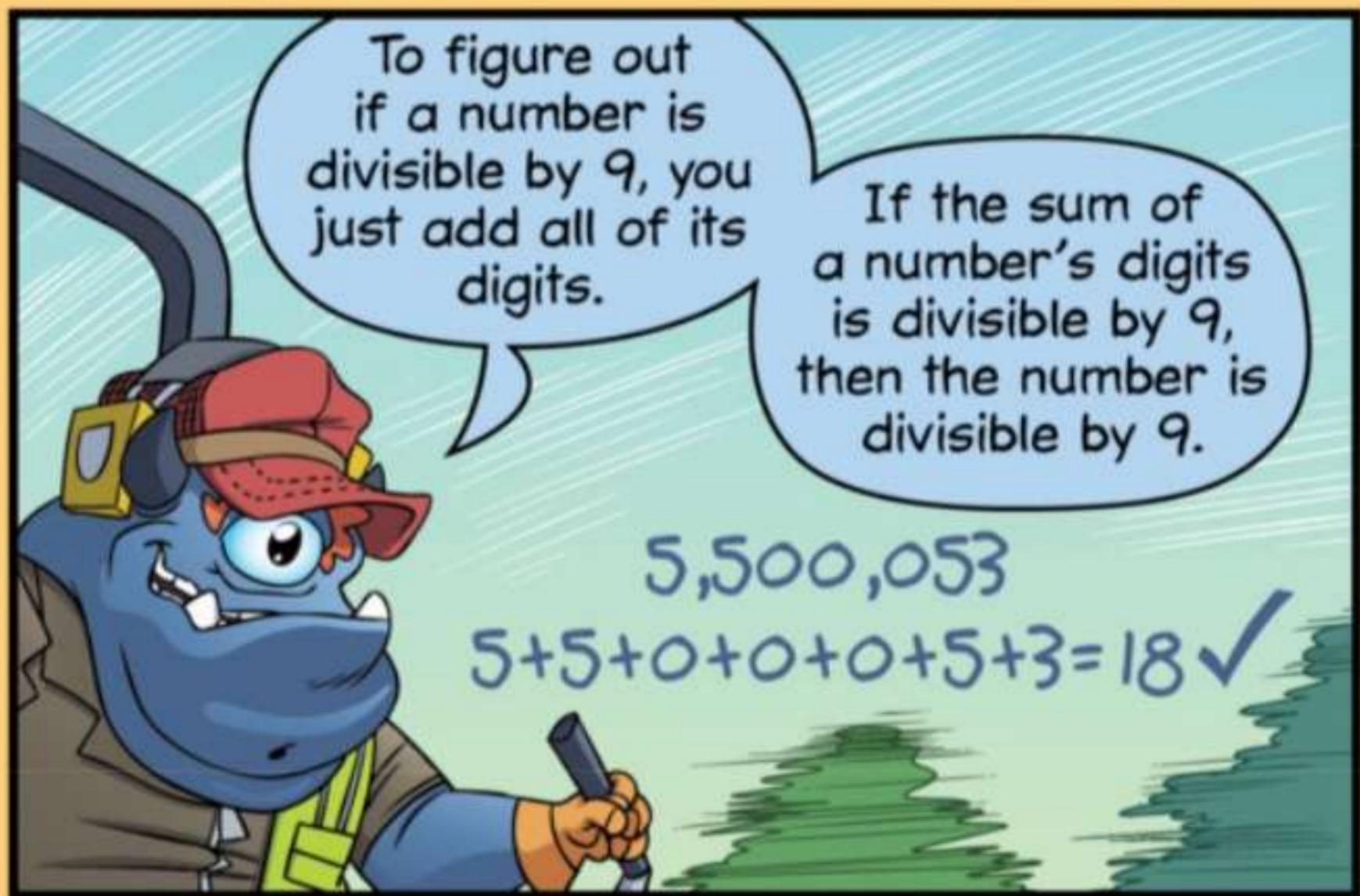
1	34	67
2	buzz	68
fizz	fizz	fizz
4	37	buzz
buzz	38	71
fizz	fizz	fizz
7	buzz	73
8	41	74
fizz	fizz	fizzbuzz
buzz	43	76
11	44	77
fizz	fizzbuzz	fizz
13	46	79
14	47	buzz
fizzbuzz	fizz	fizz
16	49	82
17	buzz	83
fizz	fizz	fizz
19	52	buzz
buzz	53	86
fizz	fizz	fizz
22	buzz	88
23	56	89
fizz	fizz	fizzbuzz
buzz	58	91
26	59	92
fizz	fizzbuzz	fizz
28	61	94
29	62	buzz
fizzbuzz	fizz	fizz
31	64	97
32	buzz	98
fizz	fizz	fizz

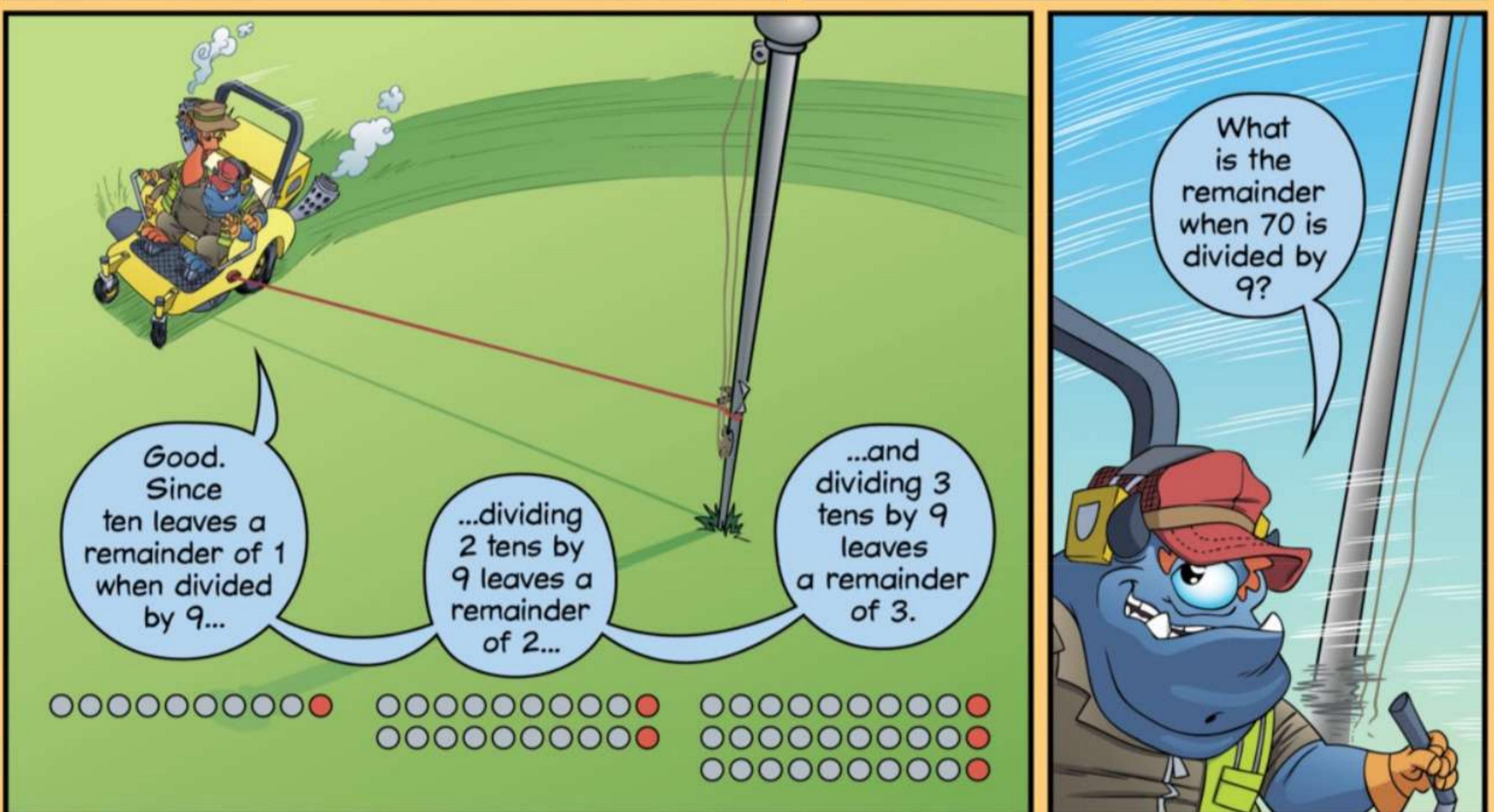
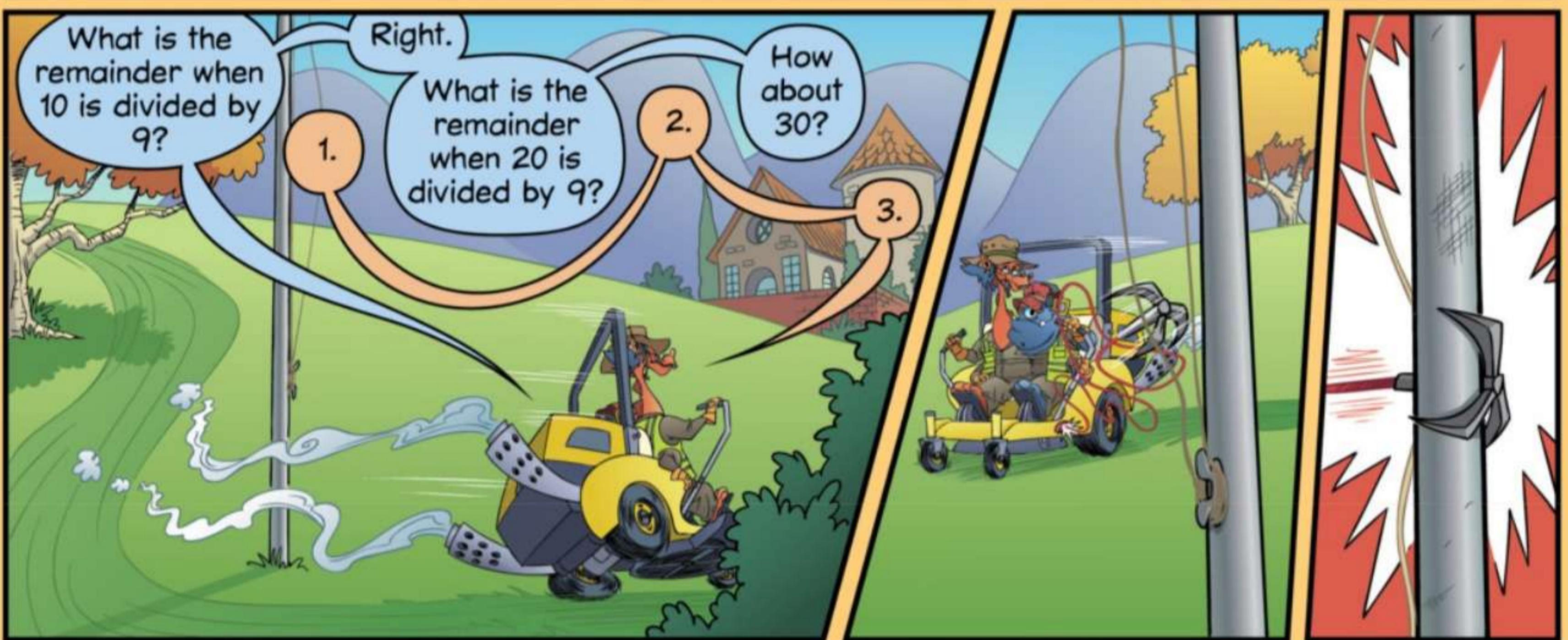




$$\begin{aligned} 72,945 &= 72,000 + 900 + 45 \\ &= 9 \times 8,000 + 9 \times 100 + 9 \times 5 \\ &= 9 \times (8,000 + 100 + 5) \end{aligned}$$

WHEN WE ADD TWO OR MORE MULTIPLES OF A NUMBER, WE GET ANOTHER MULTIPLE OF THE SAME NUMBER. IN THE EXAMPLE ABOVE, ADDING THREE MULTIPLES OF 9 GIVES US ANOTHER MULTIPLE OF 9.



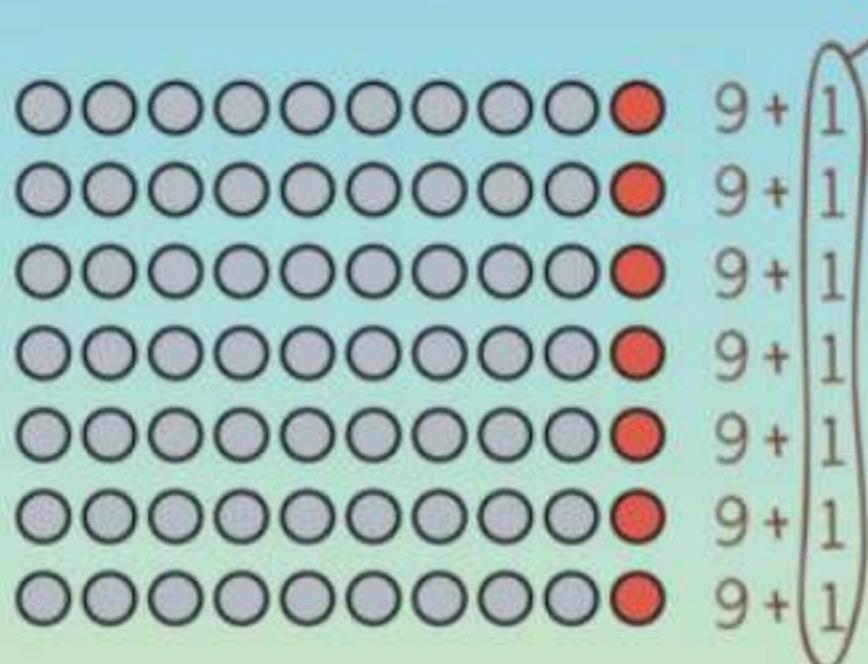


70 is seven 10's.

Each 10 gives us one 9 plus 1 extra.

That makes 7 extras.

You've got it. Next, let's try 100. What's the remainder when 100 is divided by 9?



$$\begin{aligned} 70 &= 7 \times 10 \\ &= 7 \times (9+1) \\ &= (7 \times 9) + (7 \times 1) \\ &= (7 \times 9) + 7 \end{aligned}$$

Well, since $9 \times 11 = 99$, 100 is 1 more than a multiple of 9.

So, $100 \div 9$ has a remainder of 1.

Right. Now, what is the remainder when 700 is divided by 9?

I think I see where you're going with this.



700 is seven 100's.

Each 100 gives us one 99 plus 1 extra.

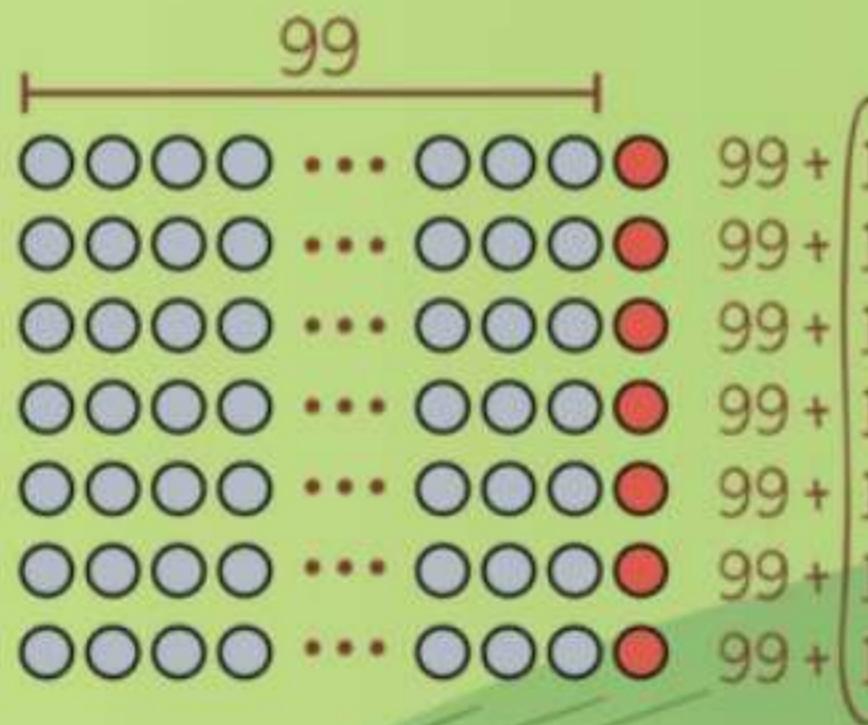
That makes 7 extras.

So, $700 \div 9$ has remainder 7.

Yep.

Try 1,000.

What's the remainder when 1,000 is divided by 9?



$$\begin{aligned} 700 &= 7 \times 100 \\ &= 7 \times (99+1) \\ &= (7 \times 99) + (7 \times 1) \\ &= (7 \times 99) + 7 \end{aligned}$$



Well, since $9 \times 111 = 999$, we know that 1,000 is 1 more than a multiple of 9.

So, $1,000 \div 9$ has a remainder of 1.

Right.

In fact, every power of 10 is 1 more than a multiple of 9!

10,000 is 1 more than 9,999.

100,000 is 1 more than 99,999.

And 1,000,000 is 1 more than 999,999.

Since 10 and 100 and 1,000,000 are each 1 more than a multiple of 9...

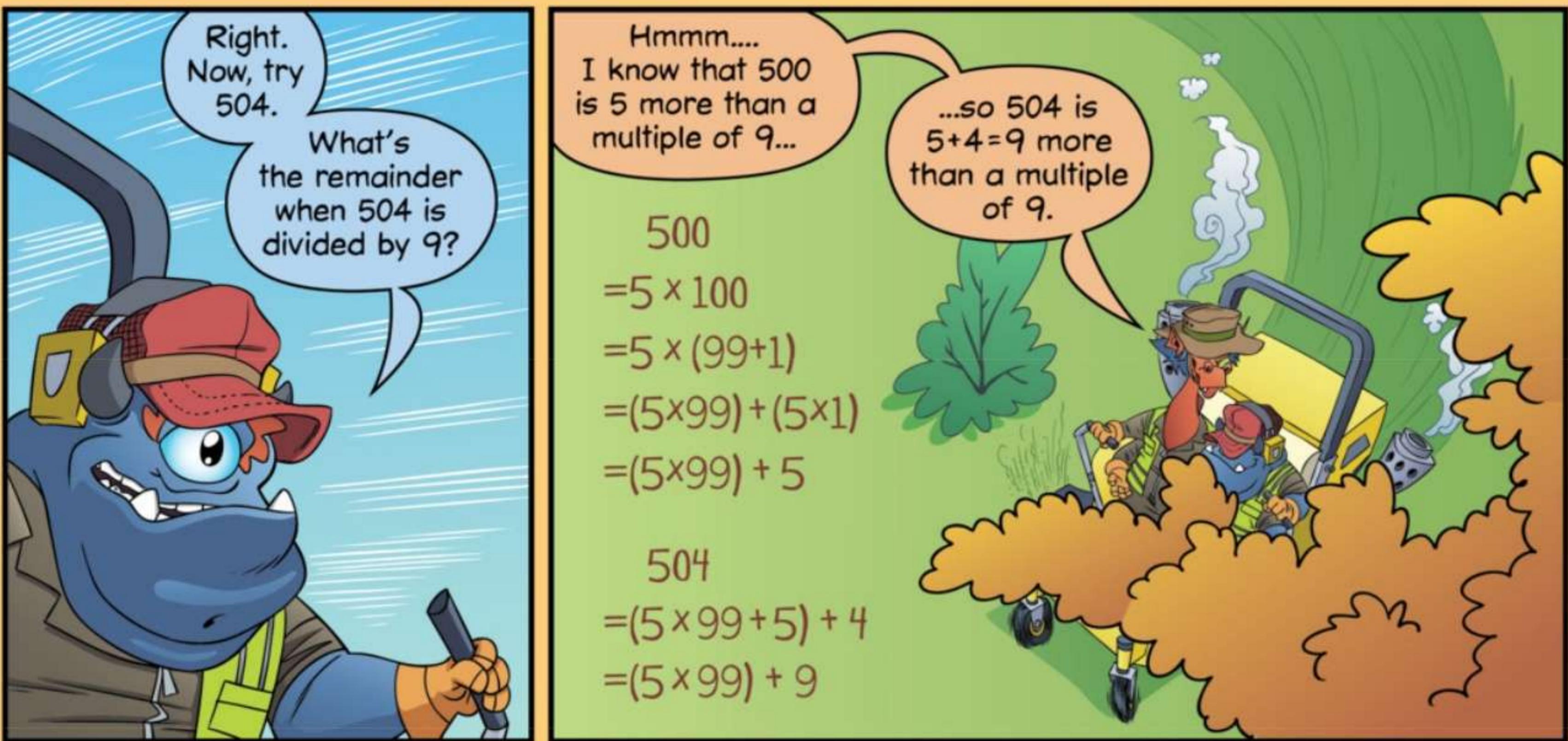
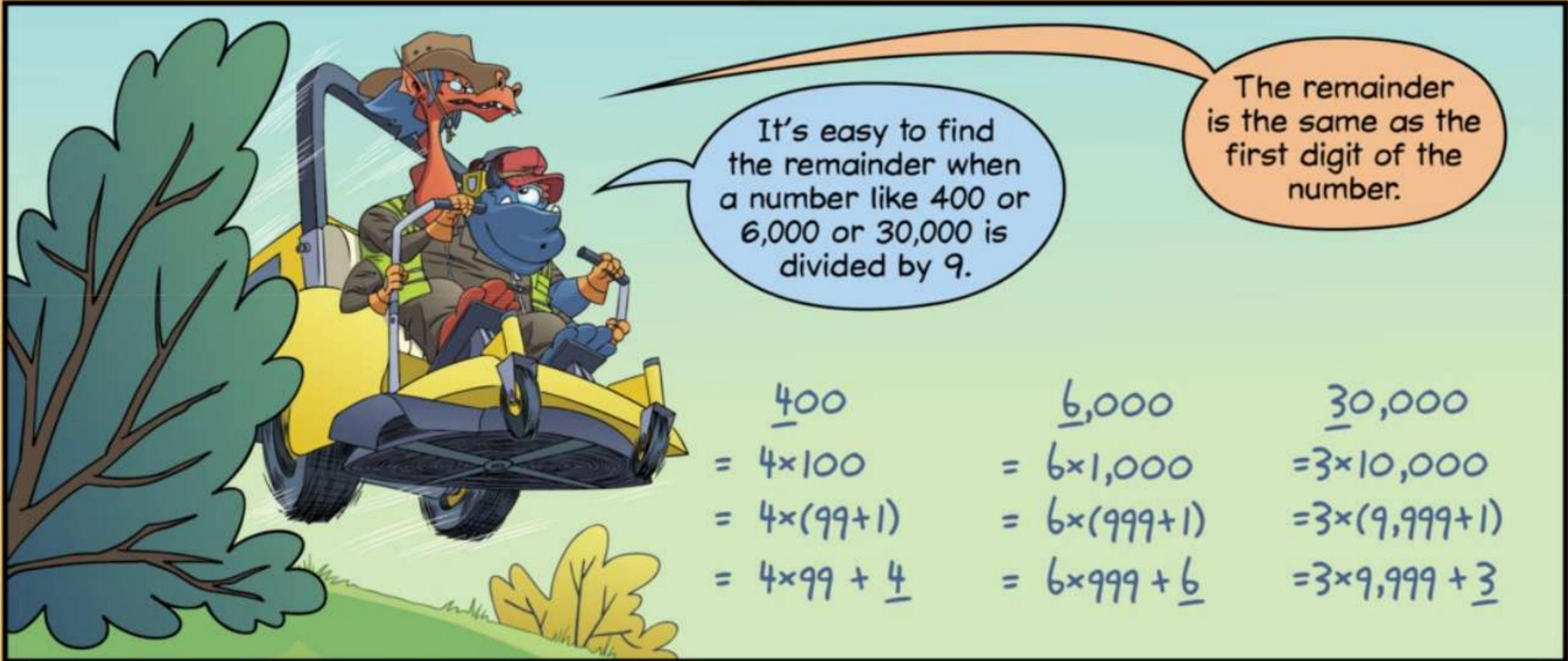
...70 and 700 and 7,000,000 are each 7 more than a multiple of 9!

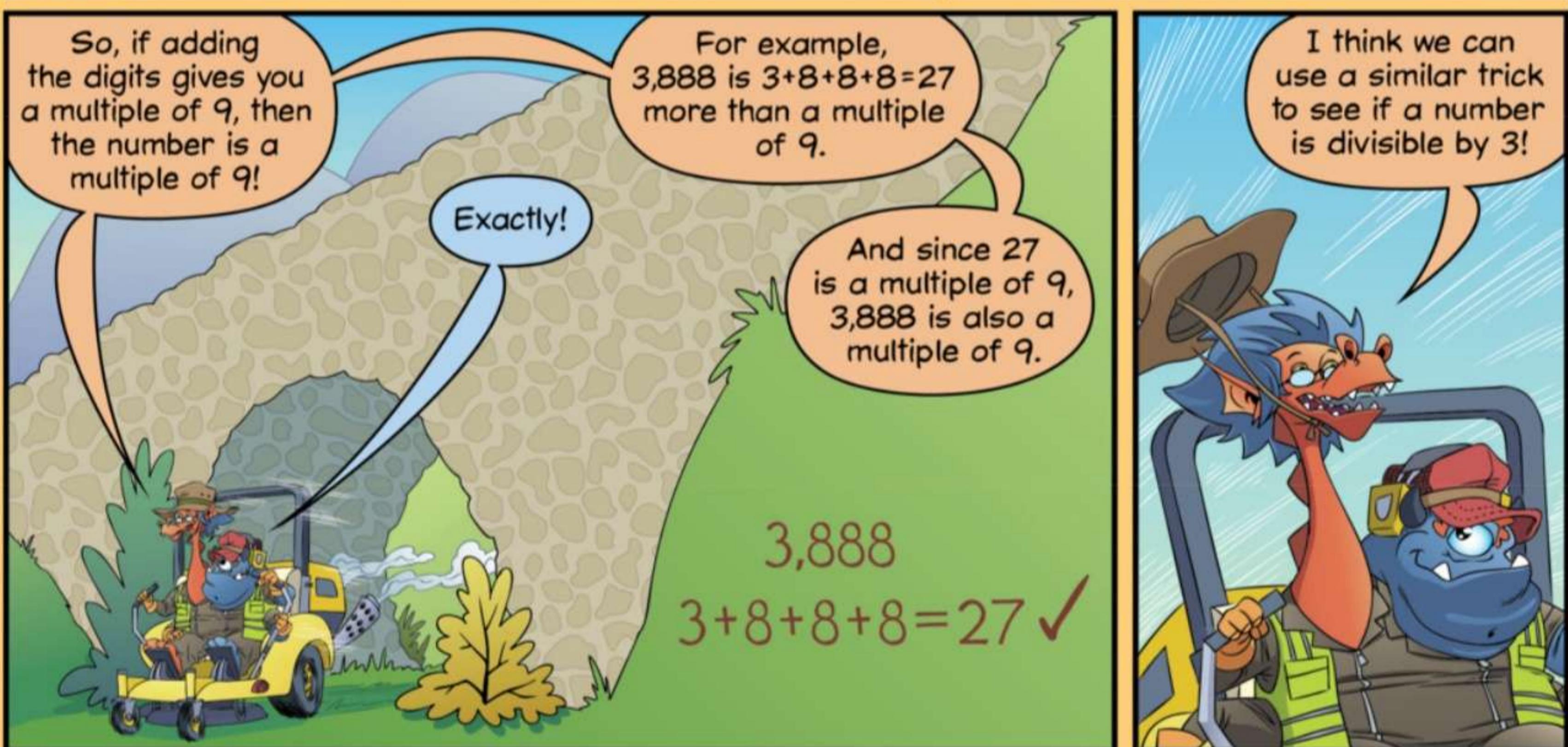
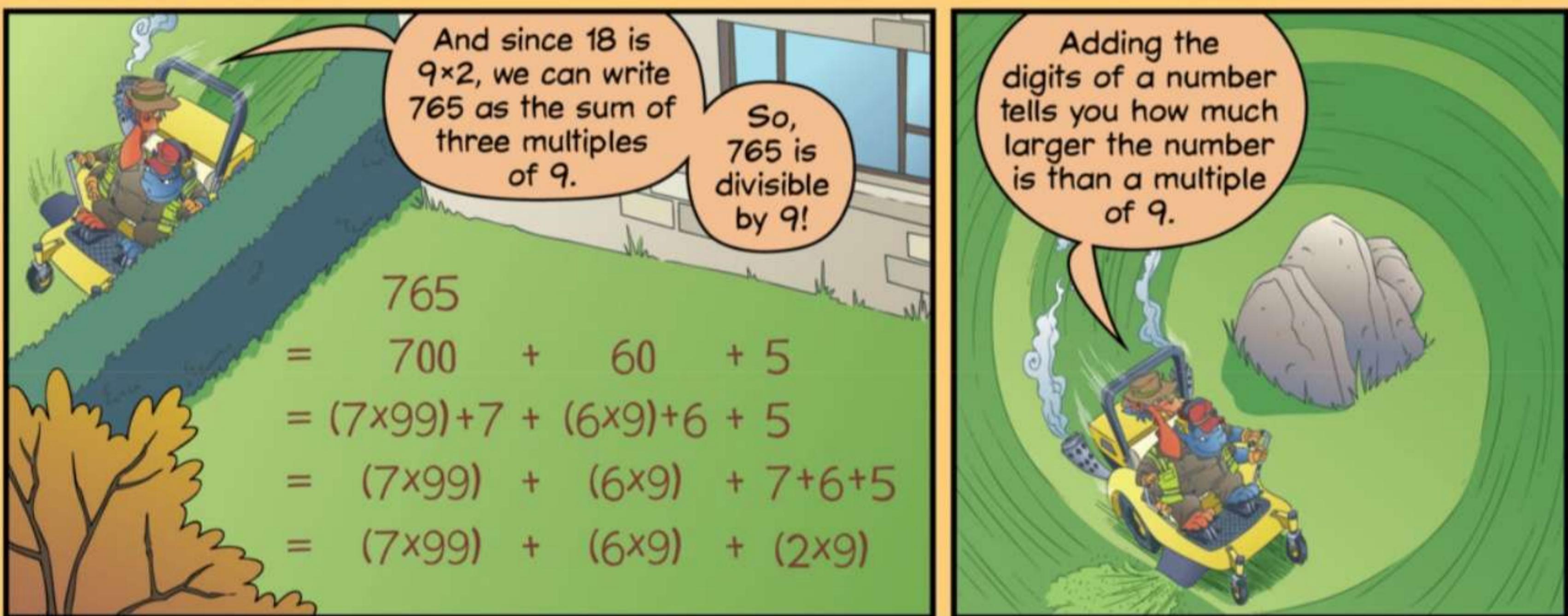
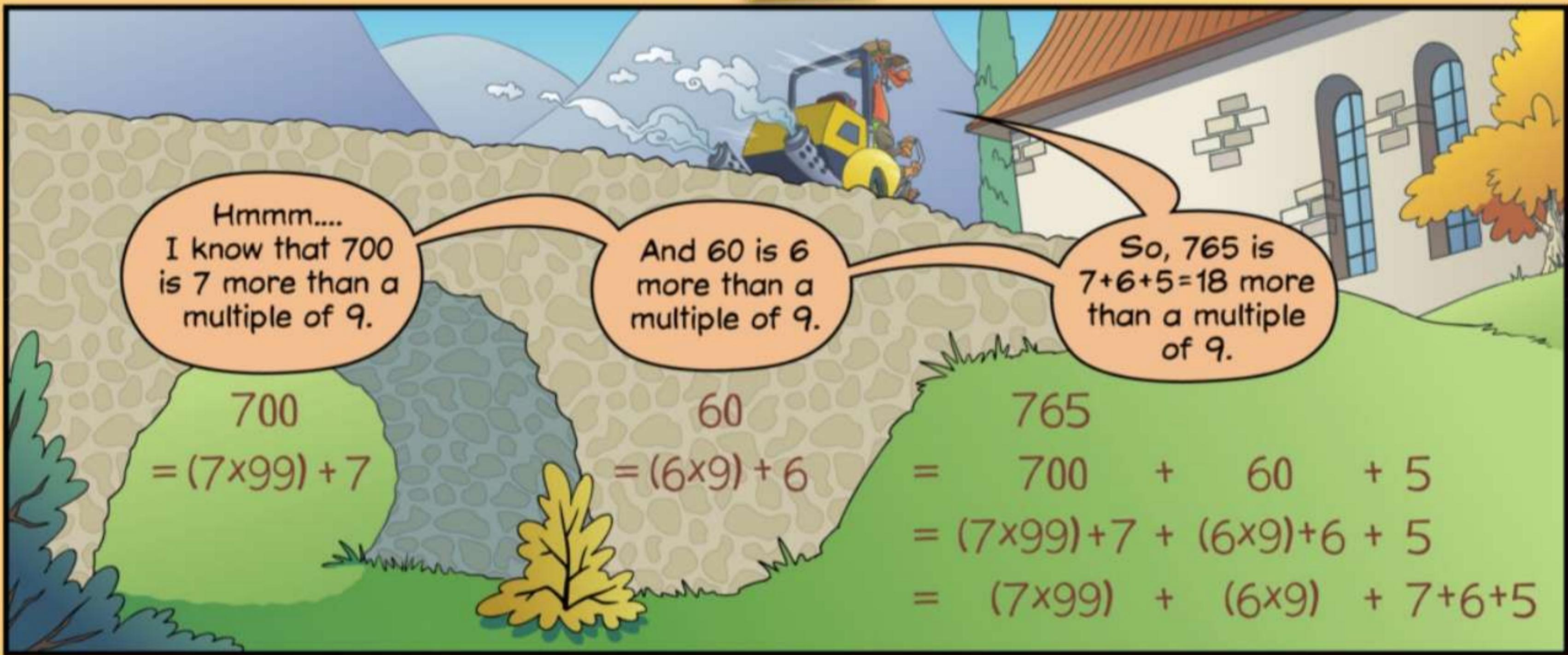
So, 70 and 700 and 7,000,000 each leave a remainder of 7 when divided by 9.

$$\begin{aligned} 70 &= 7 \times 10 \\ &= 7 \times (9+1) \\ &= (7 \times 9) + (7 \times 1) \\ &= (7 \times 9) + 7 \end{aligned}$$

$$\begin{aligned} 700 &= 7 \times 100 \\ &= 7 \times (99+1) \\ &= (7 \times 99) + (7 \times 1) \\ &= (7 \times 99) + 7 \end{aligned}$$

$$\begin{aligned} 7,000,000 &= 7 \times 1,000,000 \\ &= 7 \times (999,999+1) \\ &= (7 \times 999,999) + (7 \times 1) \\ &= (7 \times 999,999) + 7 \end{aligned}$$





Since 9 and 99 and 999 are all multiples of 9, they're also multiples of 3.

So, when we add the digits of a number, we find out how much larger the number is than a multiple of 3.

For example, 425 is $4+2+5=11$ more than a multiple of 3.

$$\begin{aligned} 425 &= 400 + 20 + 5 \\ &= (4 \times 99) + 4 + (2 \times 9) + 2 + 5 \\ &= (4 \times 99) + (2 \times 9) + 4 + 2 + 5 \end{aligned}$$





A number's prime factorization be the group o' primes you multiply to get the number.

For example, the prime factorization o' 36 be $2 \times 2 \times 3 \times 3$.

Is there another way to get 36 by multiplying primes?

$36 = 2 \times 2 \times 3 \times 3$

IN THE PRIME FACTORIZATION OF A NUMBER, THE ORDER OF THE PRIMES DOESN'T MATTER, BUT WE USUALLY ARRANGE THE PRIME FACTORS FROM LEAST TO GREATEST.

Can you find another group of primes whose product is 36?

5×7 is close.

2×17 and 2×19 are close to 36, too.

I don't think there's another group of primes that has a product of 36.

Aye!

Every whole number greater than 1 be havin' just one prime factorization.*

$2 \times 17 = 34$

$2 \times 19 = 38$

$5 \times 7 = 35$

*O AND 1 ARE SPECIAL. EVERY PRIME IS A FACTOR OF 0. NO PRIME IS A FACTOR OF 1.

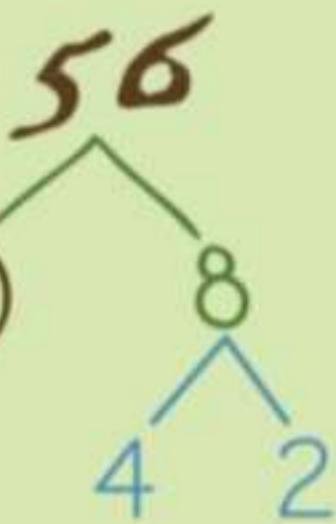


7 is prime, so we've already found one prime factor of 56.

Aye. We'll be circlin' the primes we find in our factor tree.

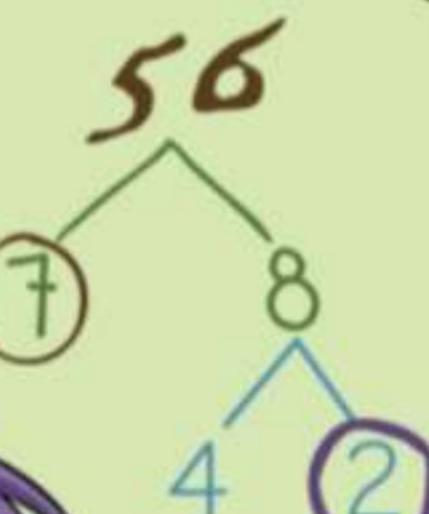


8 isn't prime. We can factor 8 into 4×2 .

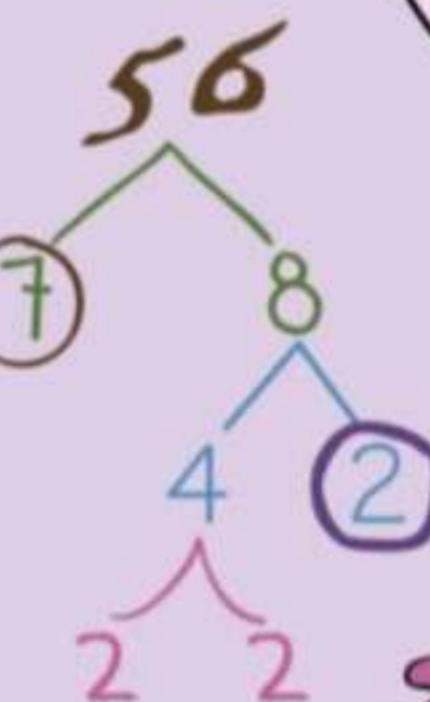


And since 2 is prime, we circle it!

Now you're gettin' the hang of it, lad!



Then, we can factor 4 into 2×2 .

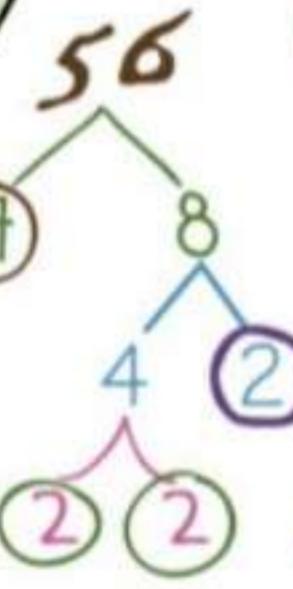


And we circle these two 2's because 2 is prime!

The circled numbers give us the prime factors of 56...

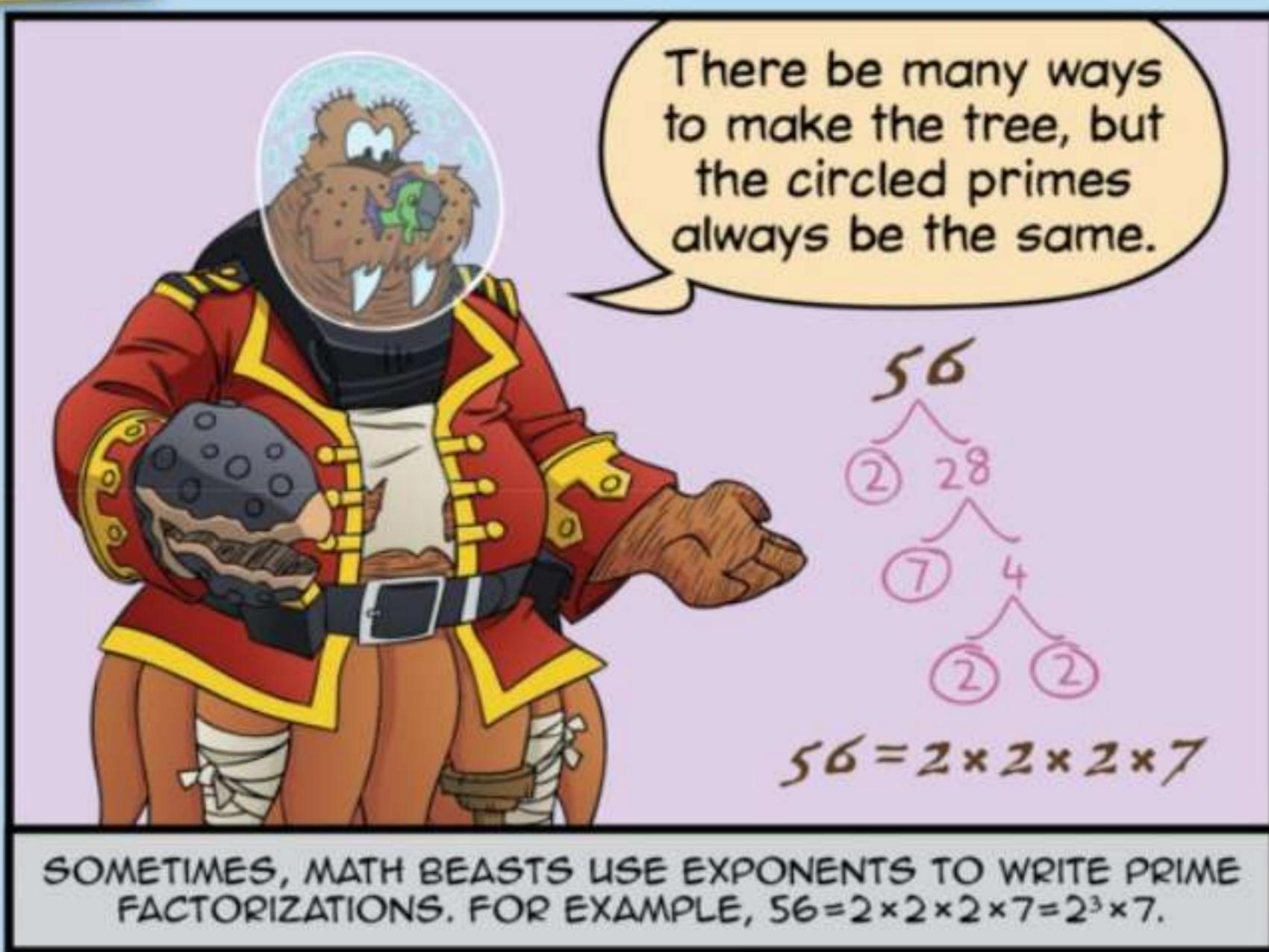
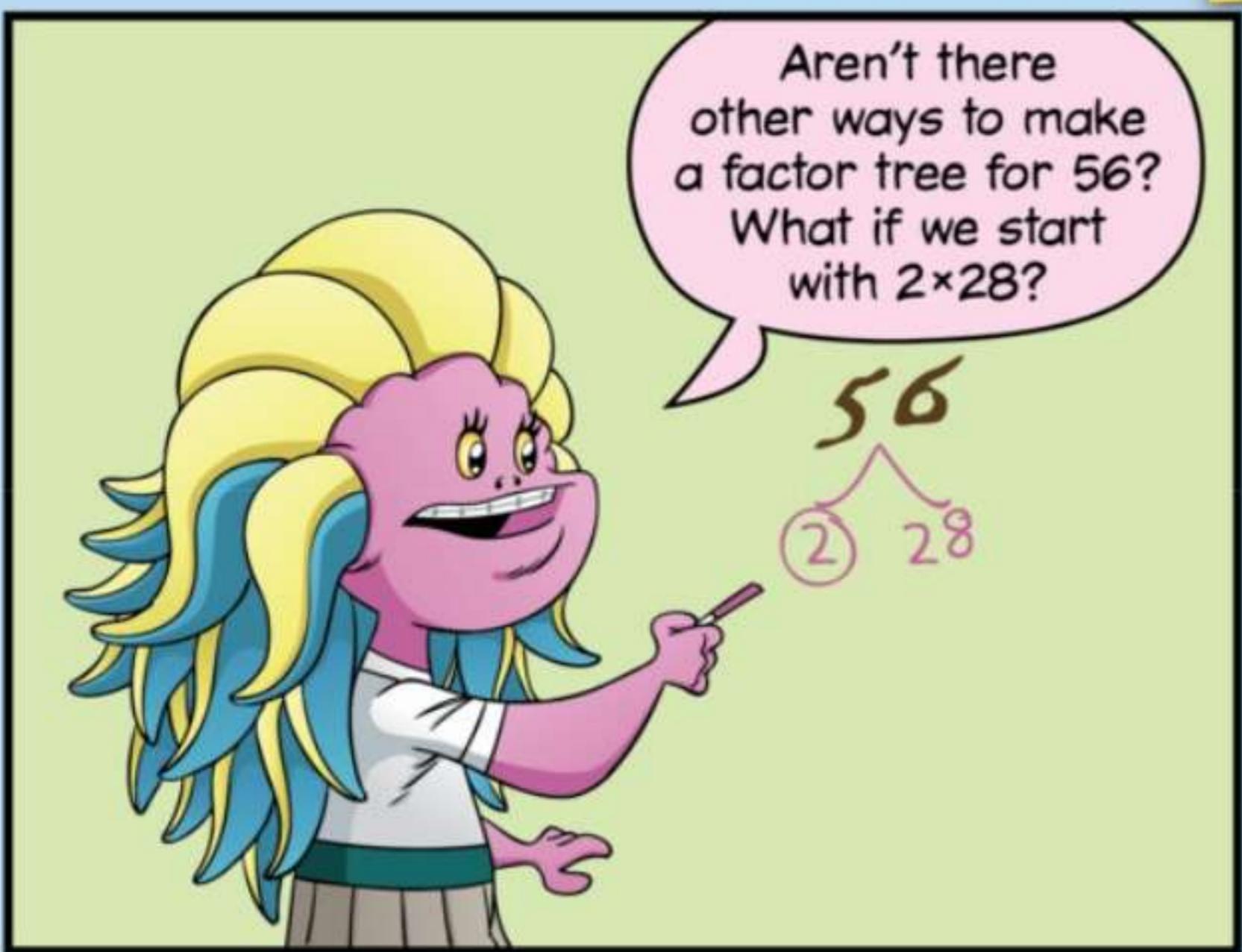
...three 2's and a 7.

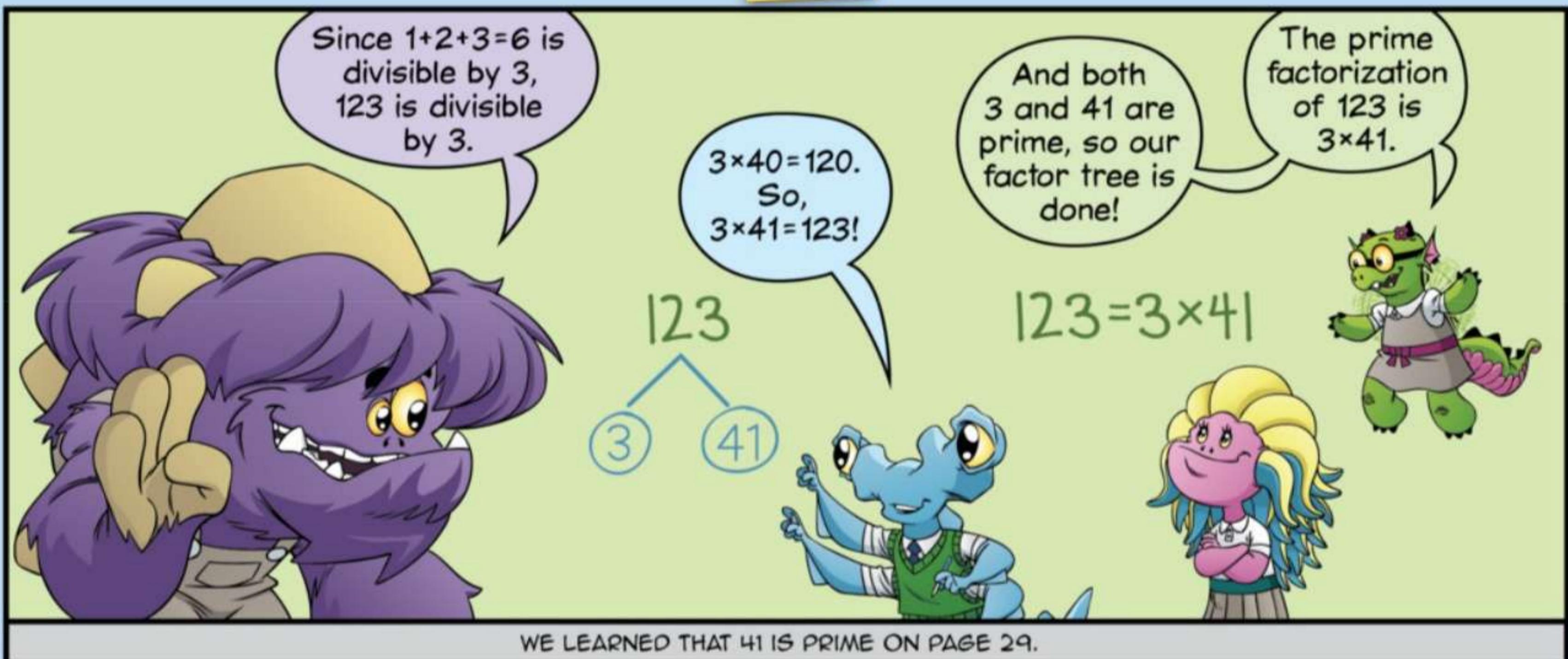
Aye. We usually write the factors in order, like so.



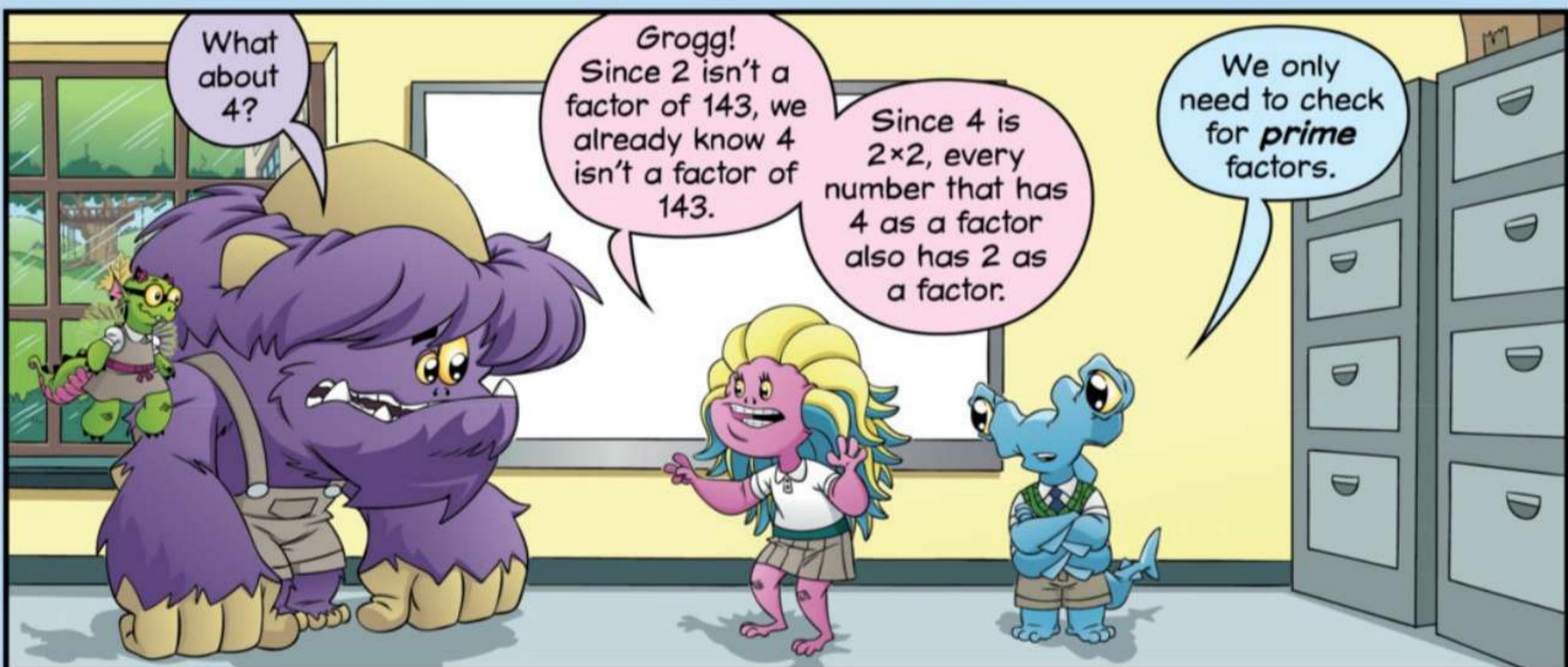
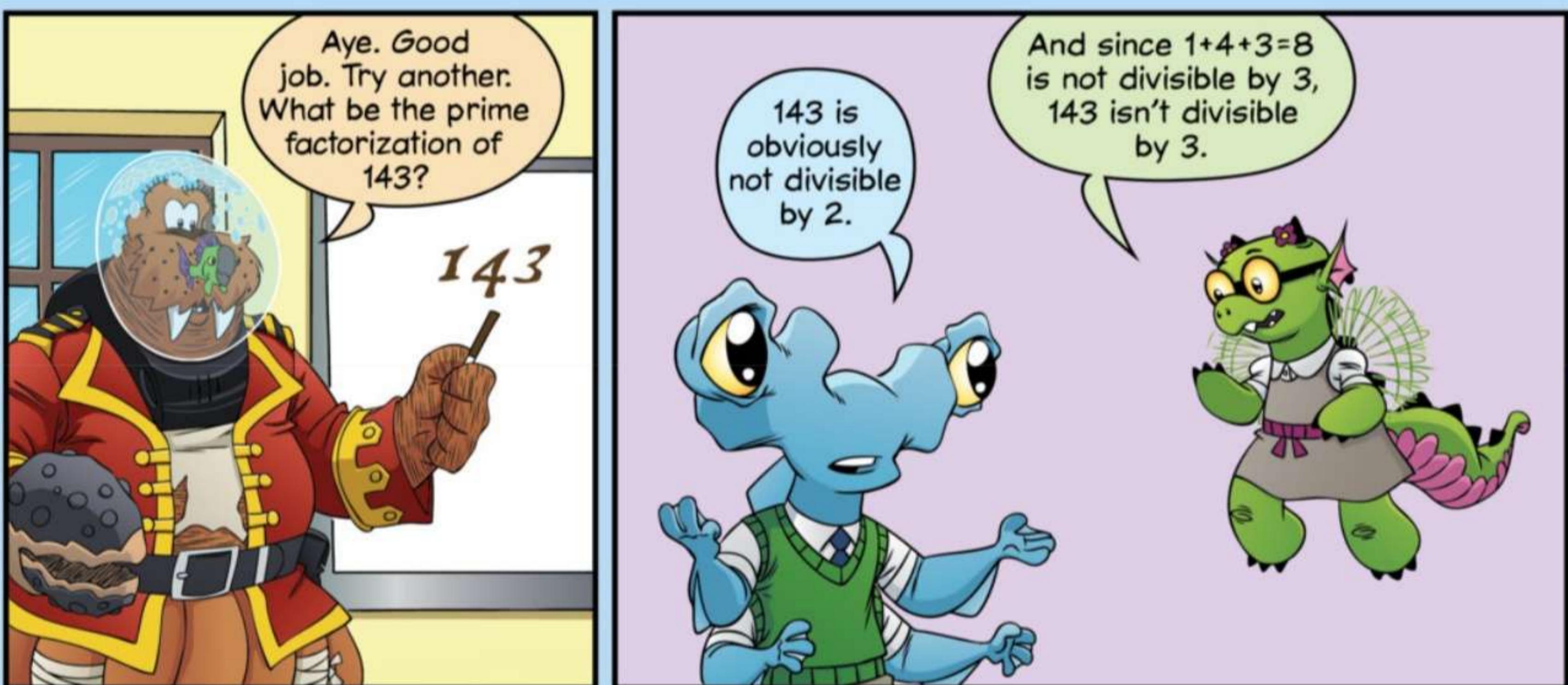
$$56 = 2 \times 2 \times 2 \times 7$$







WE LEARNED THAT 41 IS PRIME ON PAGE 29.



5 is the next prime.

Since 143 doesn't end in a 0 or a 5, we know 143 is not divisible by 5.

We don't need to check 6!

$6=2\times3$.

Since 2 and 3 are not factors of 143, we know 6 isn't a factor of 143.

Next is 7, and since $7\times20=140$, we know 143 is 3 more than a multiple of 7.

We don't need to check 8, 9, or 10, because those numbers are all composite.

11 is the next prime.

11×11 is 121.

11×12 is 132.

11×13 is 143!

$$11\times11 = 121$$

$$11\times12 = 132$$

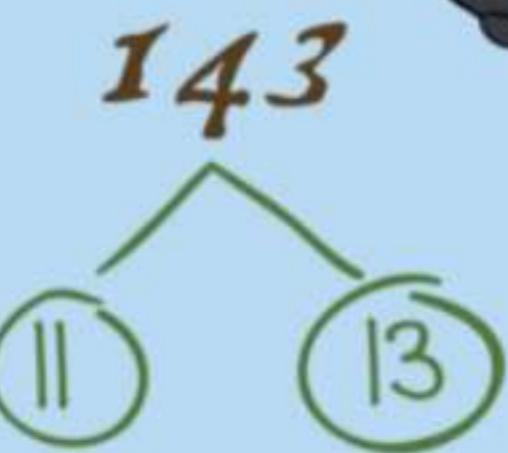
$$11\times13 = 143$$

And since 11 and 13 are both prime, the prime factorization of 143 is 11×13 !

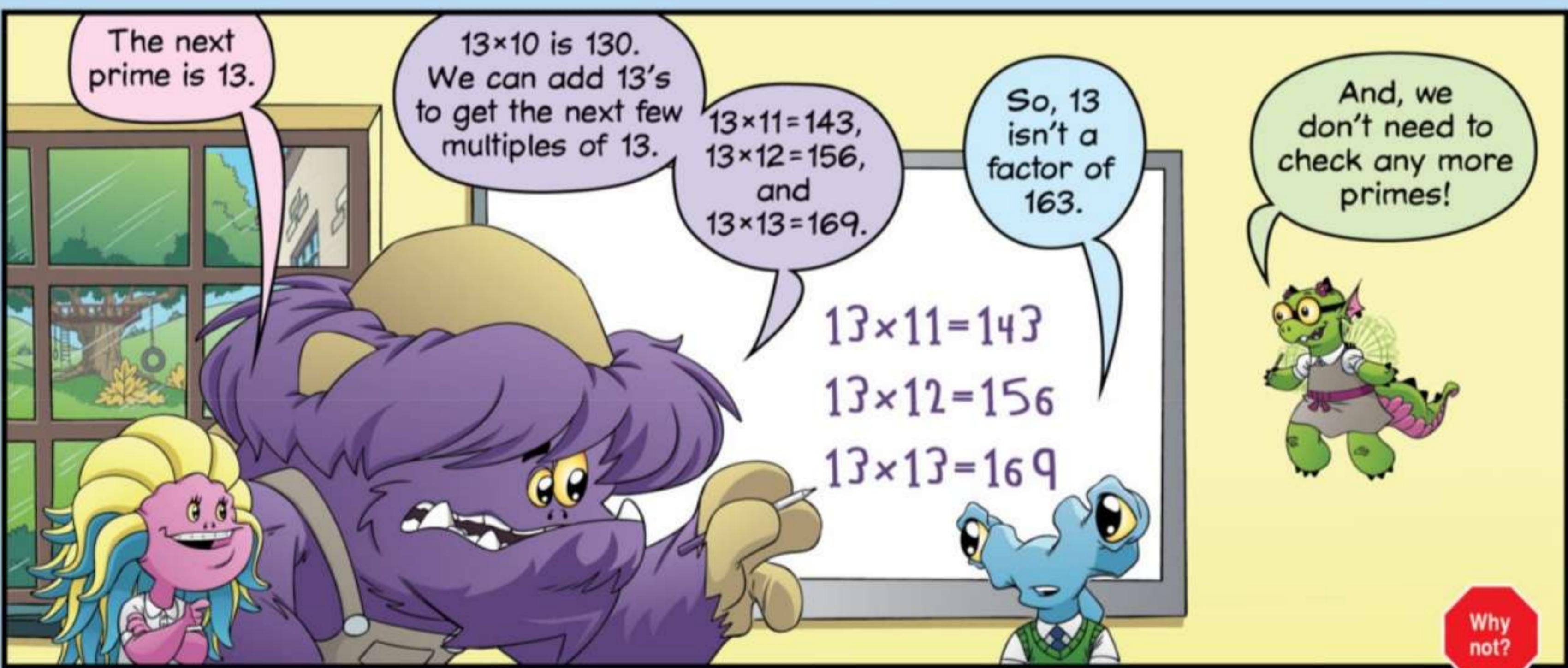
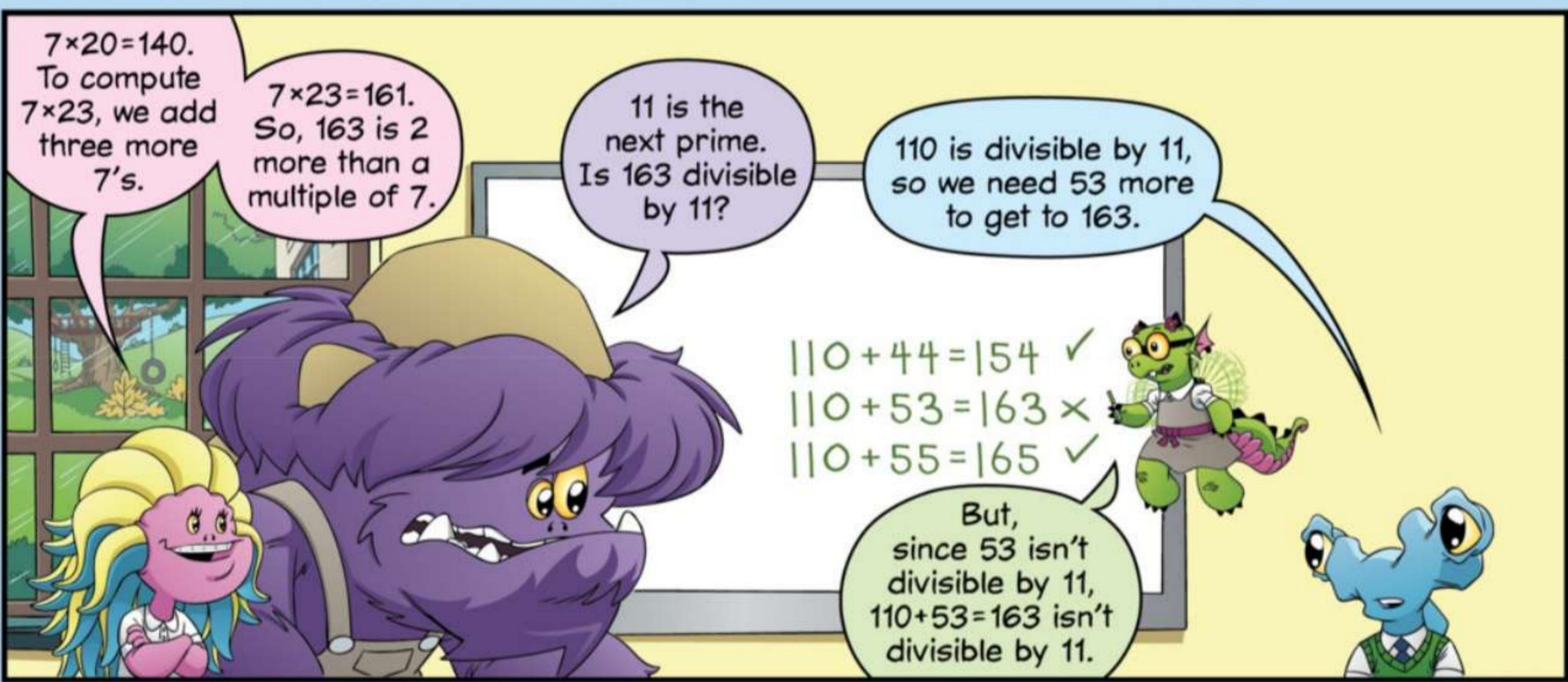
Well blow me down!

Try one more.

What be the prime factorization of 163?



What is the prime factorization of 163?



Since $13 \times 13 = 169$, any number that is **larger** than 13 has to be multiplied by a number that's **smaller** than 13 to get 163.

So, 163 doesn't have any factors besides 1 and 163.

The prime factorization of 163 is just 163.

Excellent figurin'!

163 is prime!

You've earned yourselves a break.

