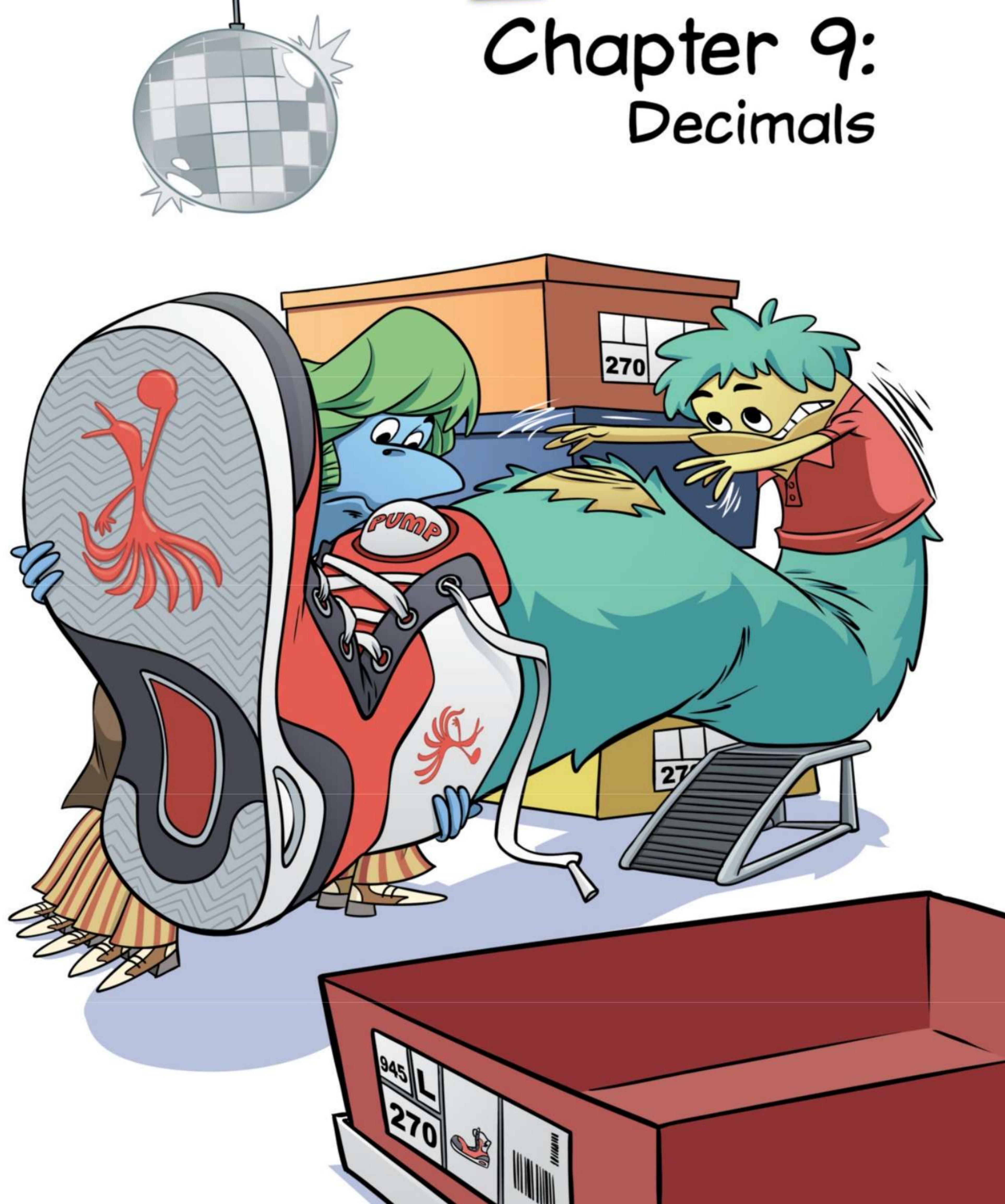


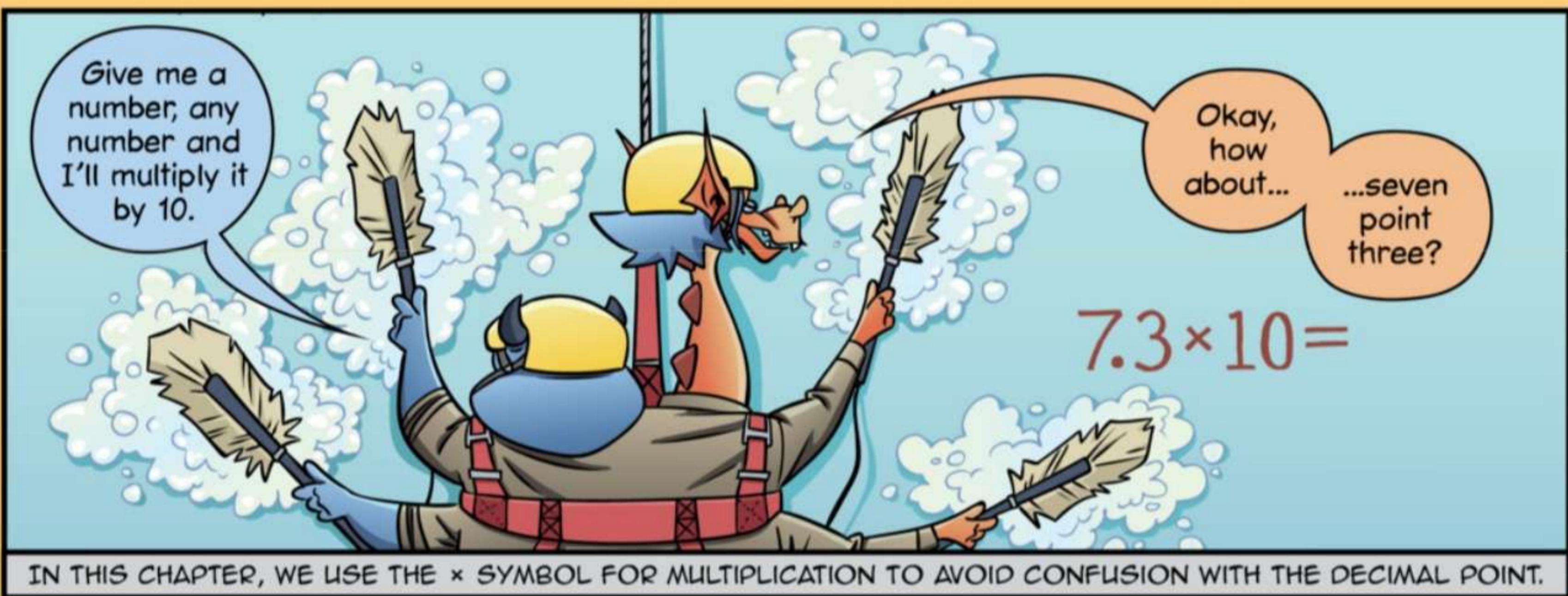
Contents: Chapter 9

Click the Play List tab in the top-left to view a recommended reading/practice sequence.

	Tens & Tenths	72
	What happens when you multiply a decimal number by 10? By 0.1?	
	Multiplying Decimals	78
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Chapter 9: Decimals





IN THIS CHAPTER, WE USE THE \times SYMBOL FOR MULTIPLICATION TO AVOID CONFUSION WITH THE DECIMAL POINT.



REVIEW DECIMAL BASICS IN CHAPTER 11 OF BEAST ACADEMY 4D.

How would you compute 7.3×10 ?



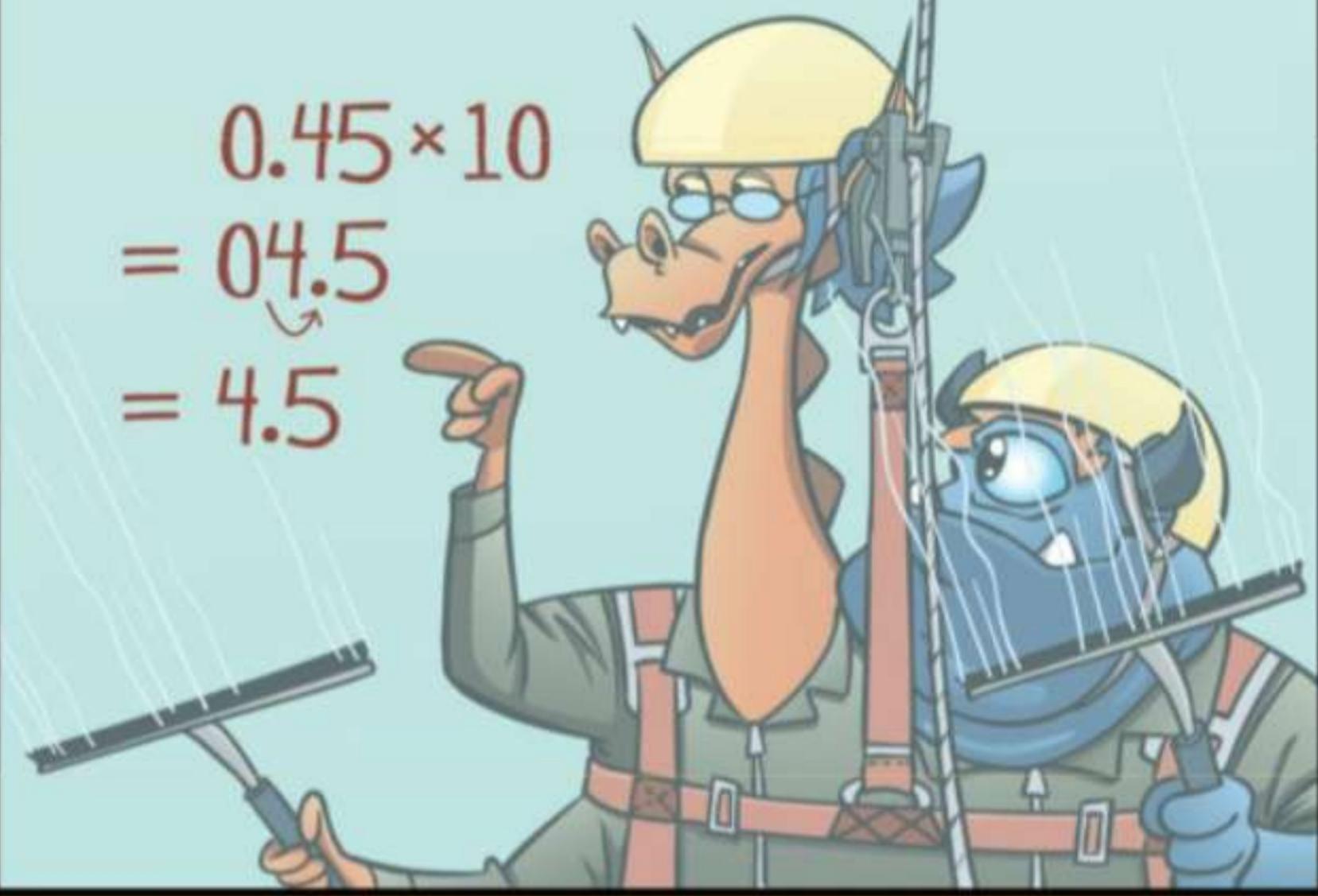
Yep. When you multiply a number by 10, each digit moves to the next larger place value.

That makes sense, since every place value is ten times the place value to its right.

$$0.45 \times 10 \\ = 4.5$$

To put it another way, multiplying a number by 10 shifts its **decimal point** one place to the right.

$$0.45 \times 10 \\ = 04.5 \\ = 4.5$$



Cool. And since multiplying by 100 is the same as multiplying by 10 twice...

...multiplying by 100 shifts the decimal point **two** places to the right!

$$0.45 \times 100 = 45.0$$

$$0.45 \times 1,000 = 450.0$$

And since $1,000 = 10 \times 10 \times 10$, multiplying by 1,000 shifts the decimal point **three** places to the right!

If multiplying by 10 moves the decimal point one place to the right...

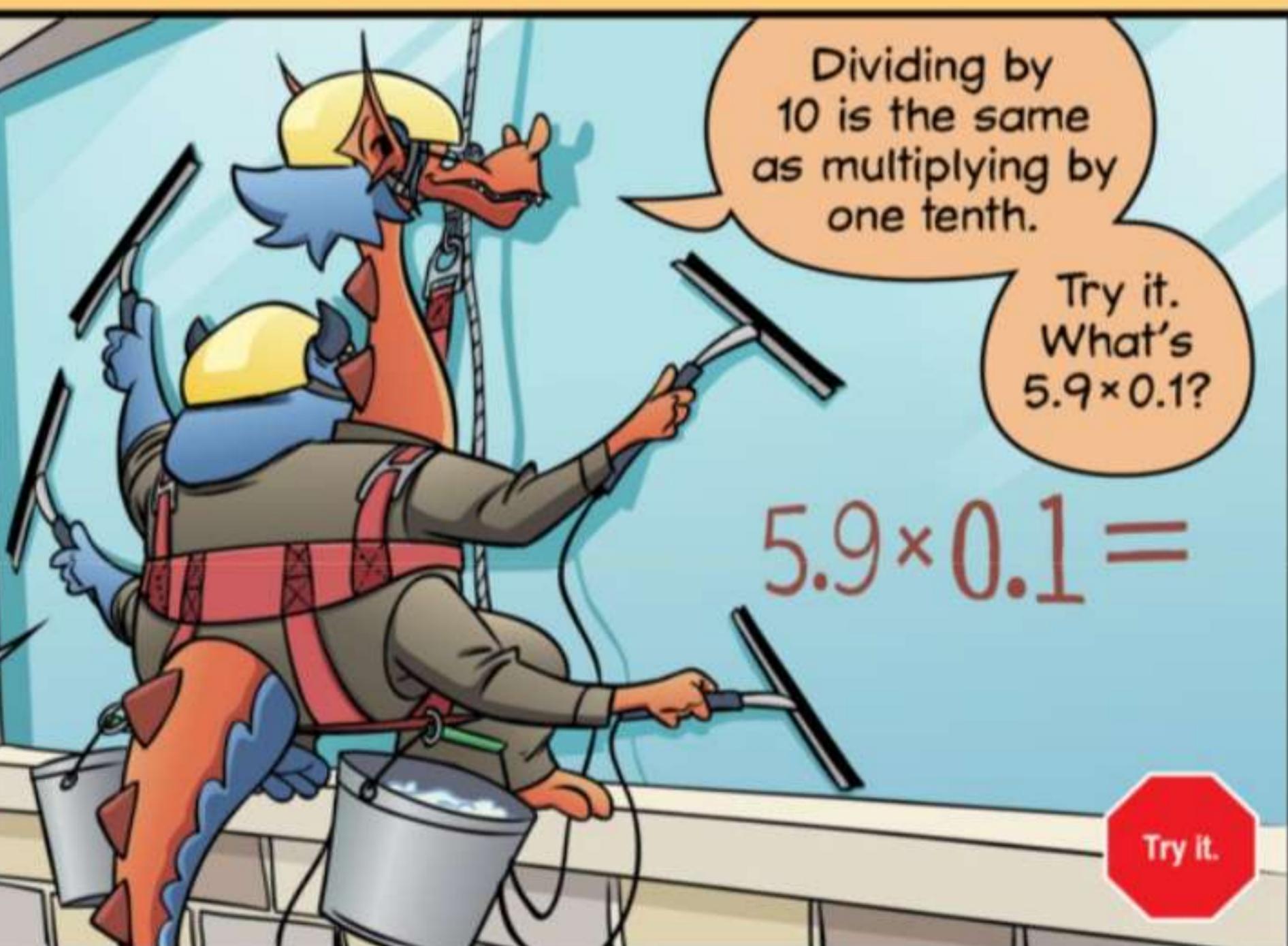
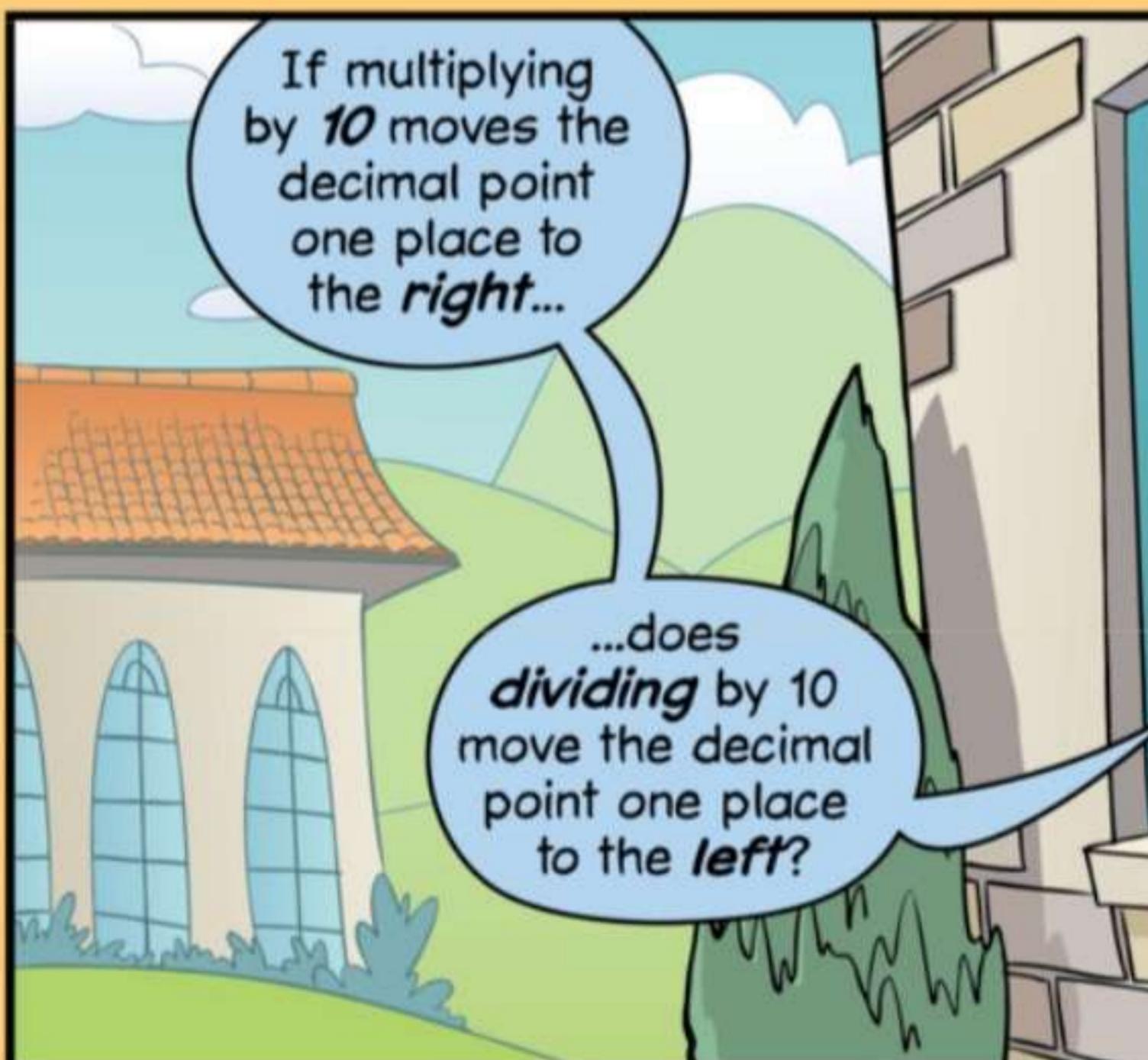
...does dividing by 10 move the decimal point one place to the left?

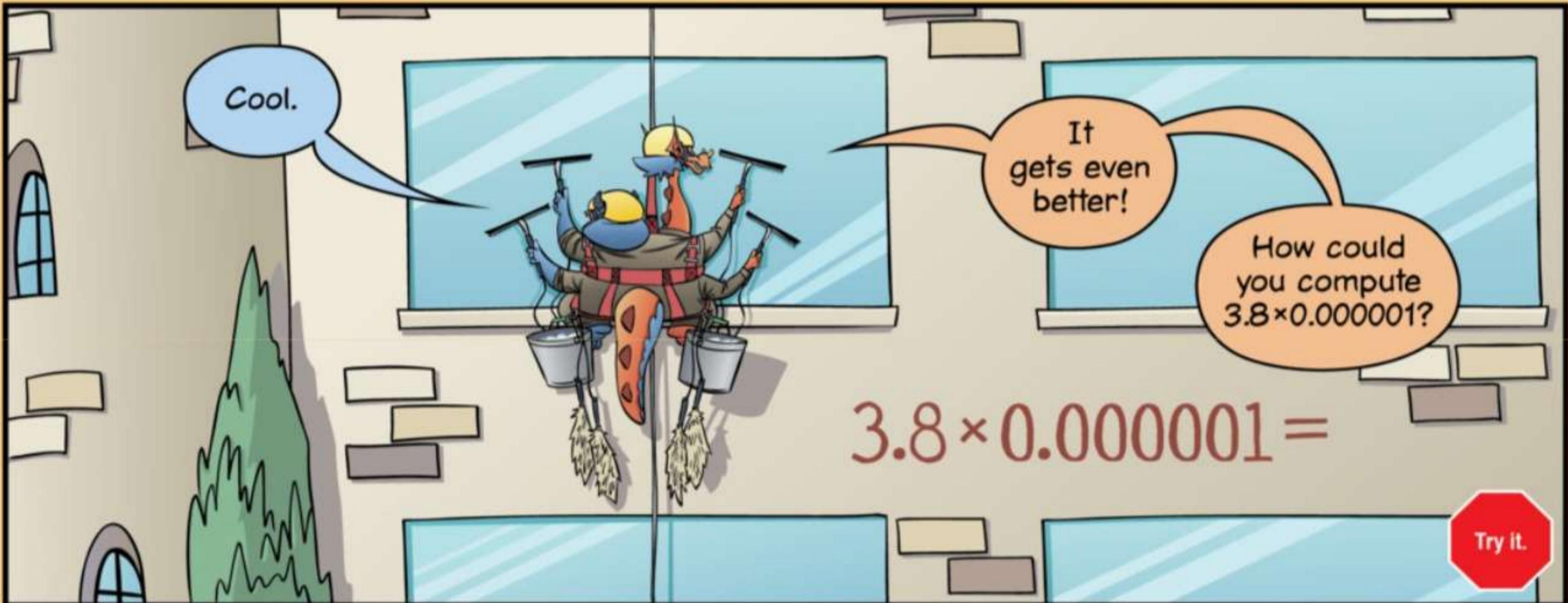
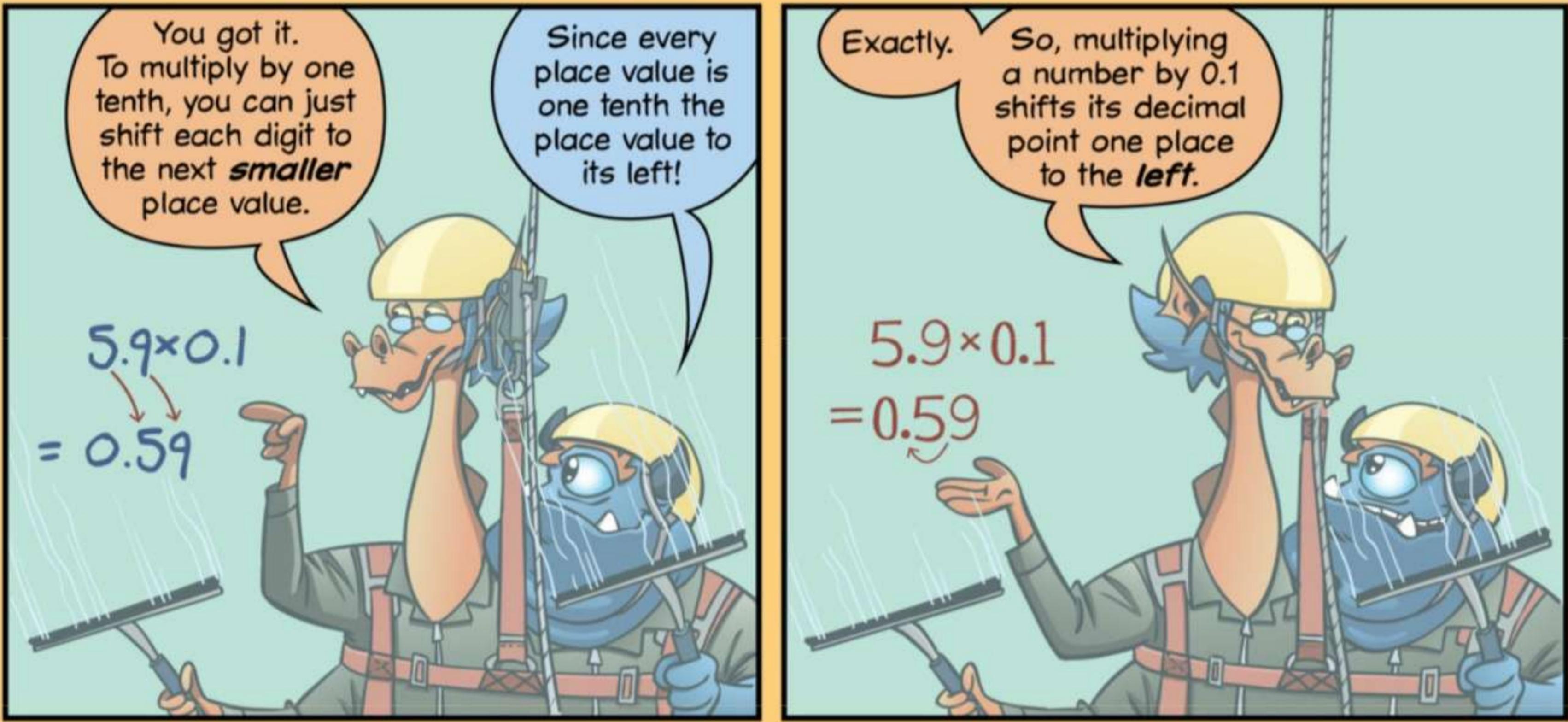
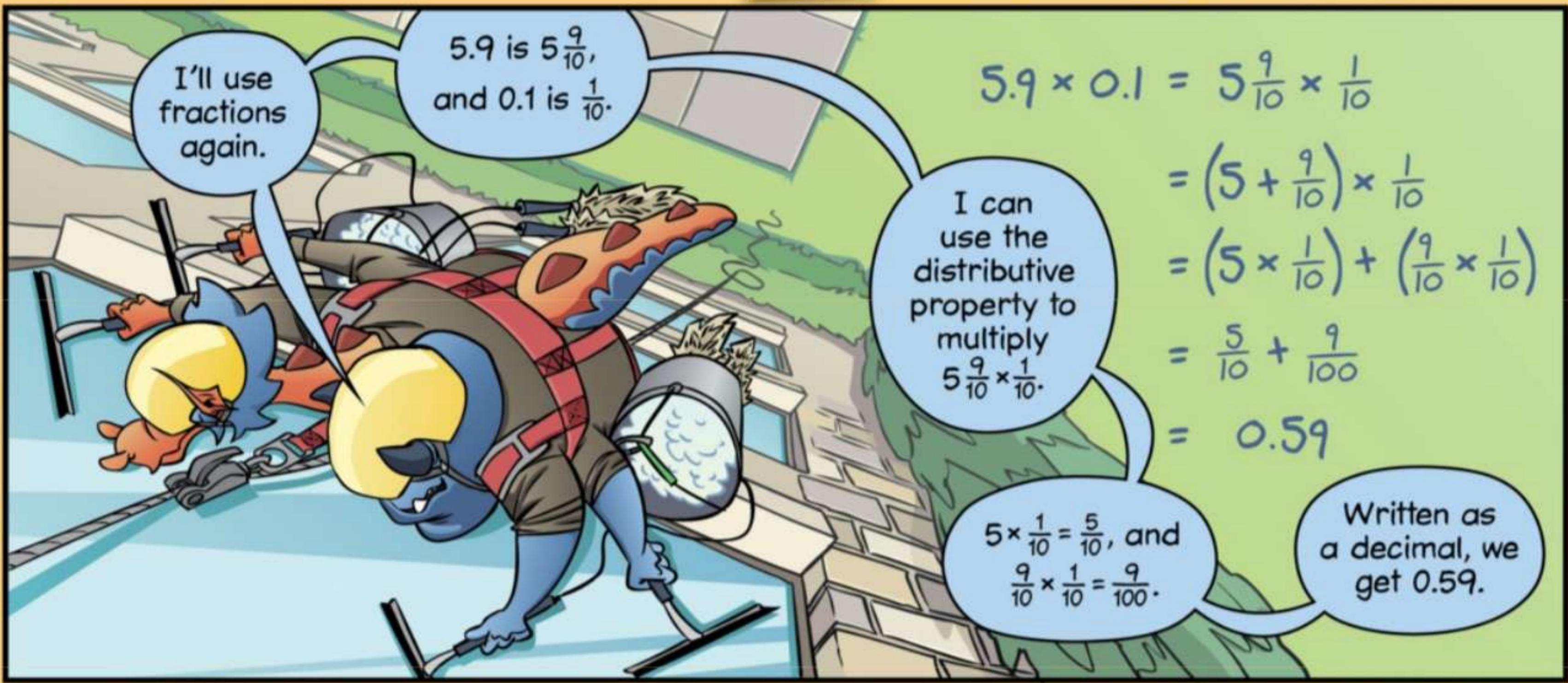
Dividing by 10 is the same as multiplying by one tenth.

Try it. What's 5.9×0.1 ?

$$5.9 \times 0.1 =$$

Try it.





I know!
I can get
0.000001 by
multiplying a
bunch of
0.1's.

So, multiplying by
0.000001 is the same
as multiplying by a
bunch of 0.1's.

How many?

Let me see...
 $(0.1)^2 = 0.01$,
 $(0.1)^3 = 0.001$,
 $(0.1)^4 = 0.0001$.

$$0.1 \times 0.1 = 0.01$$

$$0.1 \times 0.1 \times 0.1 = 0.001$$

$$0.1 \times 0.1 \times 0.1 \times 0.1 = 0.0001$$

The number
of copies of 0.1
I multiply tells
me how many
places right of the
decimal point to
place the 1.

The 1 in
0.000001 is **six**
places right of the
decimal point, so
0.000001 equals
 $(0.1)^6$.

$$0.000001 = (0.1)^6$$

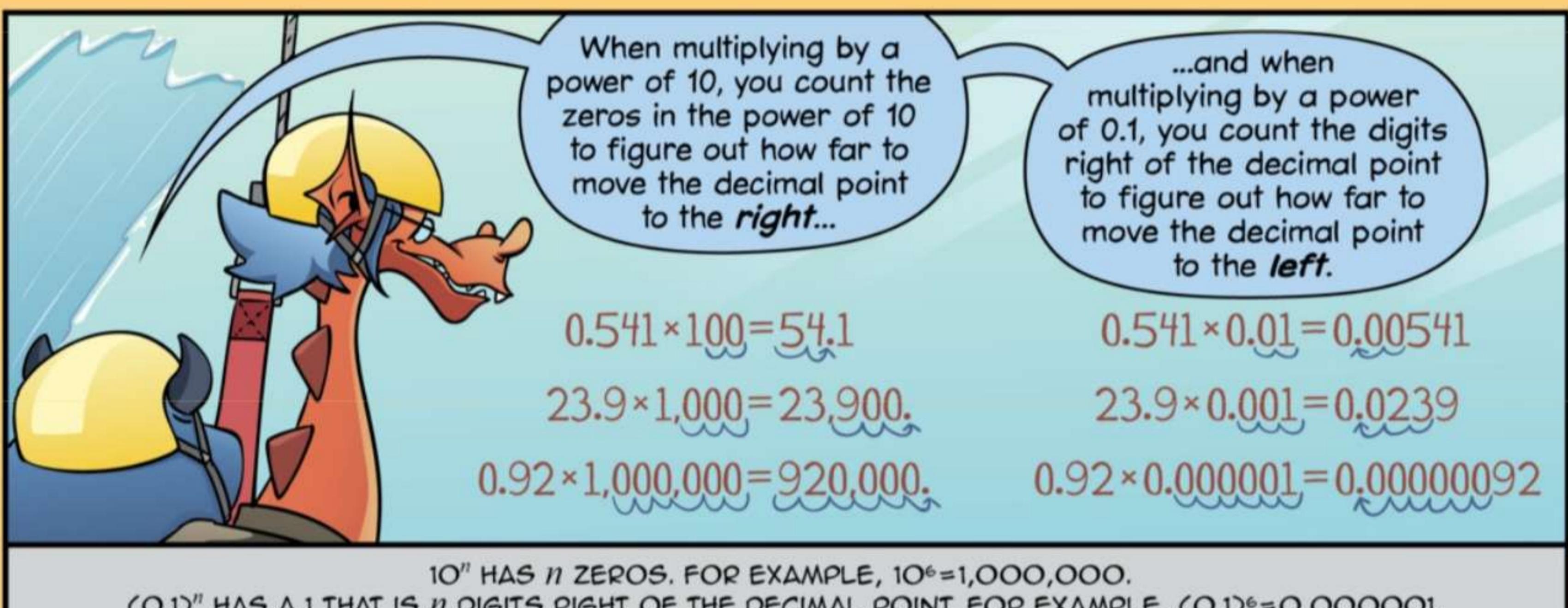
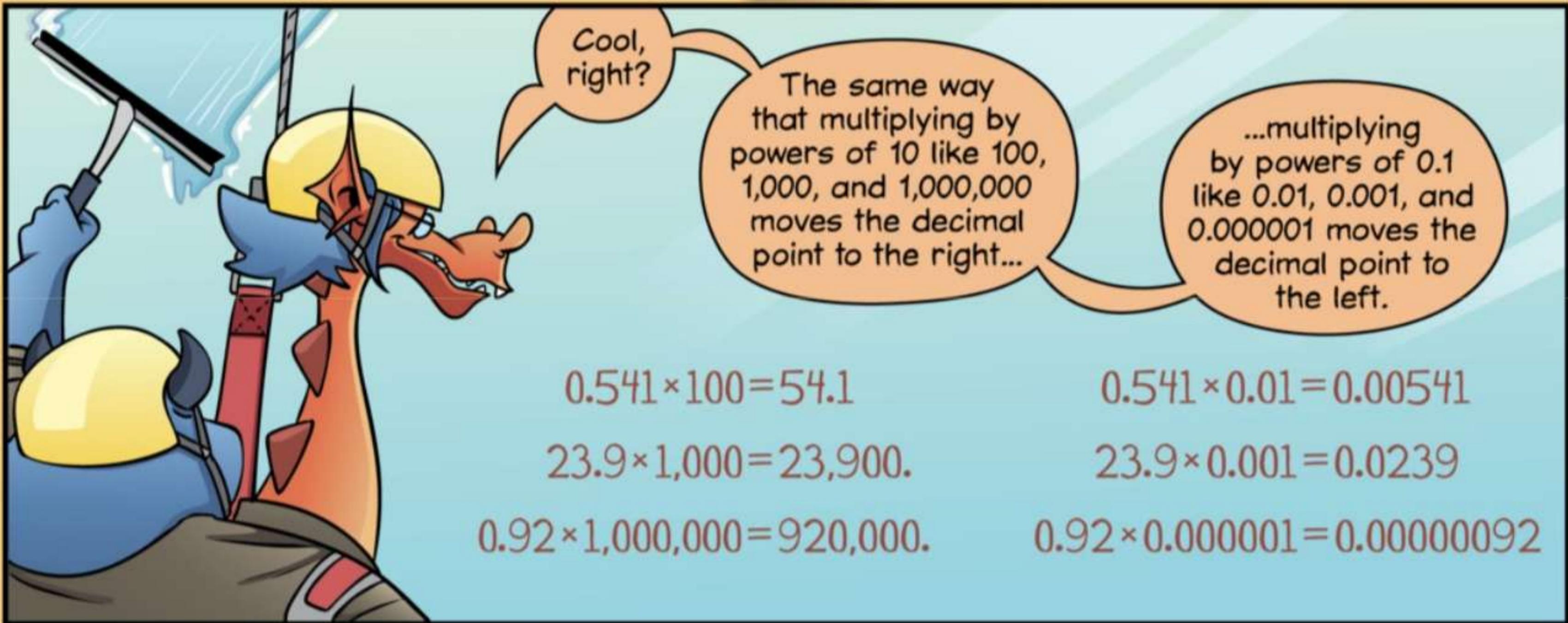
So, multiplying
by 0.000001 is
the same as
multiplying by
six copies
of 0.1!

Since
each copy of 0.1
moves the decimal
point one place to
the left...

$$3.8 \times 0.000001 = 3.8 \times (0.1)^6$$
$$= 0.0000038$$

...multiplying by
 $(0.1)^6$ moves the
decimal point **six**
places to the left.

$$3.8 \times 0.000001 = 0.0000038.$$



10^n HAS n ZEROS. FOR EXAMPLE, $10^6 = 1,000,000$.

$(0.1)^n$ HAS A 1 THAT IS n DIGITS RIGHT OF THE DECIMAL POINT. FOR EXAMPLE, $(0.1)^6 = 0.000001$.



MATH TEAM

Multiplying Decimals

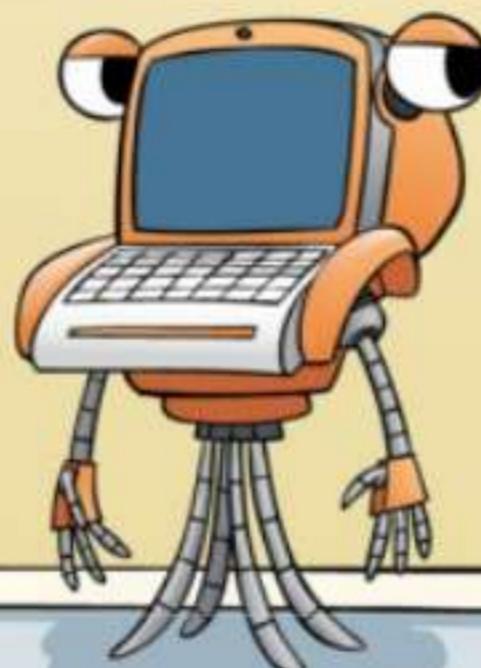
We can multiply by decimals like 0.1, or 0.01, or 0.001 just by moving the decimal point 1, 2, or 3 places to the left.

Today, we'll learn how to multiply **any** two decimal numbers.

$$39.521 \times 0.1 = 3.9521$$

$$39.521 \times 0.01 = 0.39521$$

$$39.521 \times 0.001 = 0.039521$$



This is a skill where monsters just can't compete with bots.

If you need the **exact** product of two long decimal numbers, it's best just to use a compubot.

$$39.521 \times 798.302 = 31549.693342$$

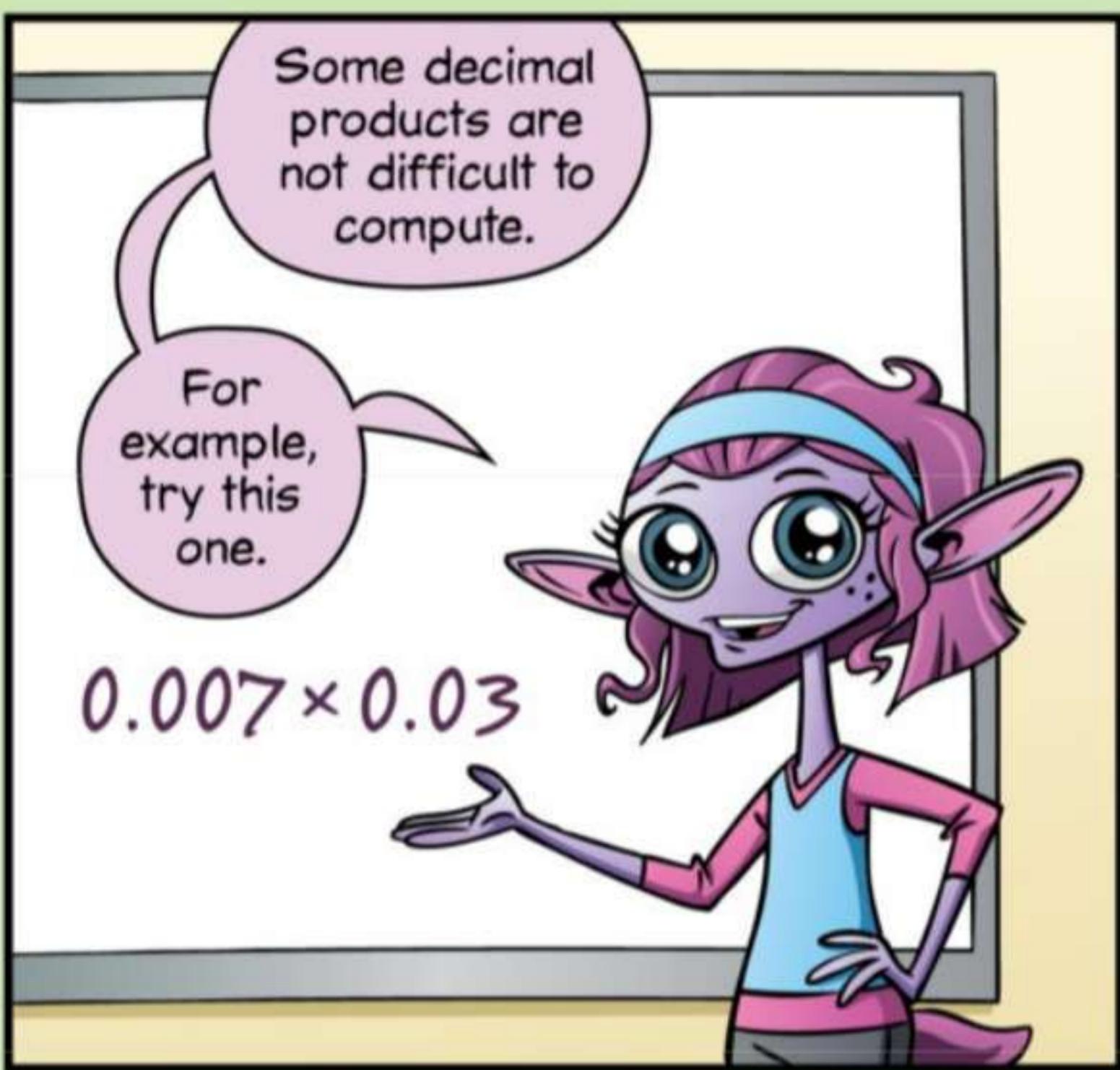
WE WILL NOT ASK YOU TO COMPUTE PRODUCTS LIKE THE ONE ABOVE.
NONE OF THE QUESTIONS WE ASK IN BEAST ACADEMY REQUIRE THE USE OF A CALCULATOR.

Fortunately, an estimate is usually good enough.

$$39.521 \times 798.302 \approx 40 \times 800 = 32,000$$



A WAVY EQUALS SIGN LIKE THE ONE ABOVE (\approx) MEANS "IS APPROXIMATELY" OR "IS ABOUT."



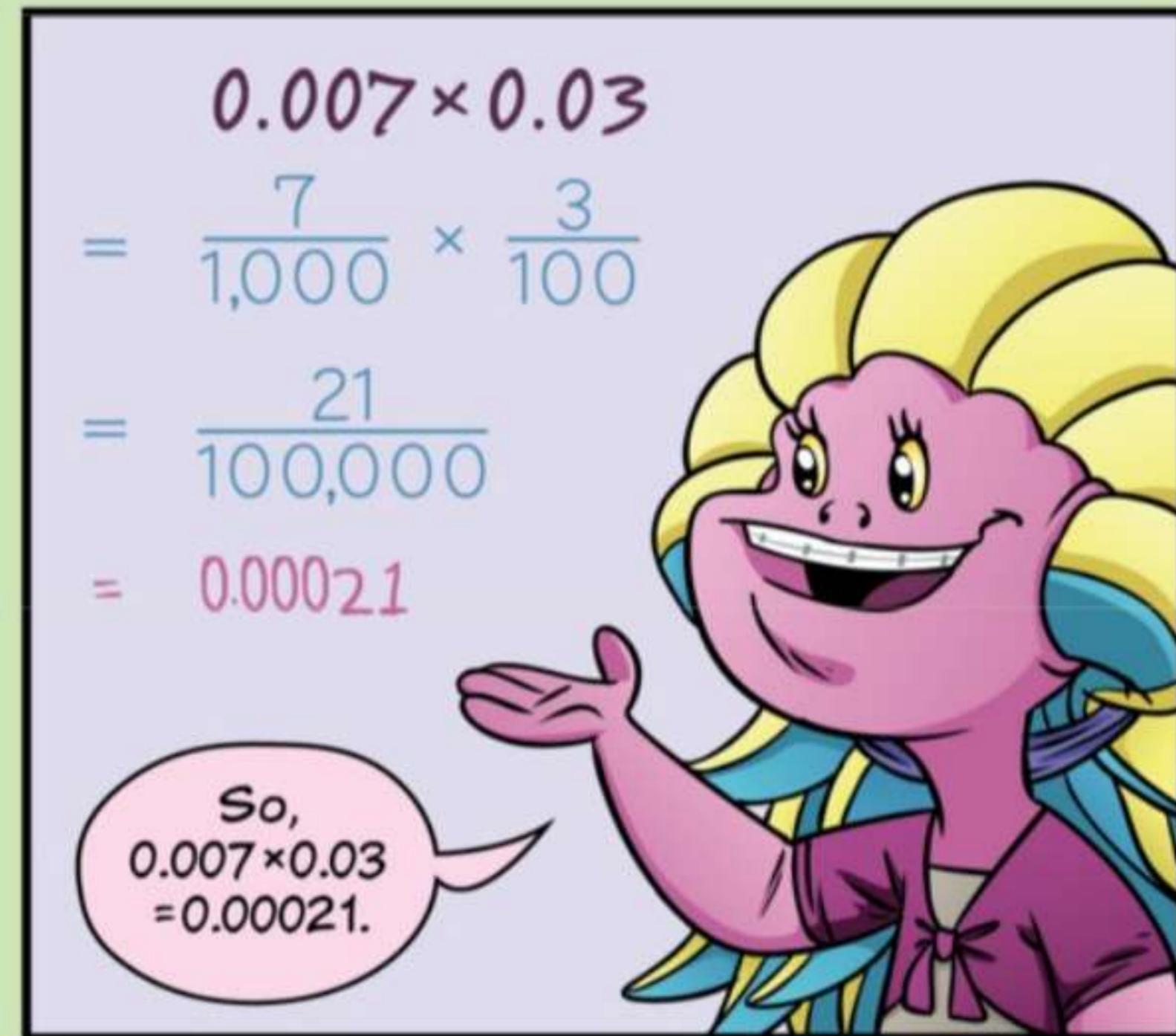
We can write both decimals as fractions.

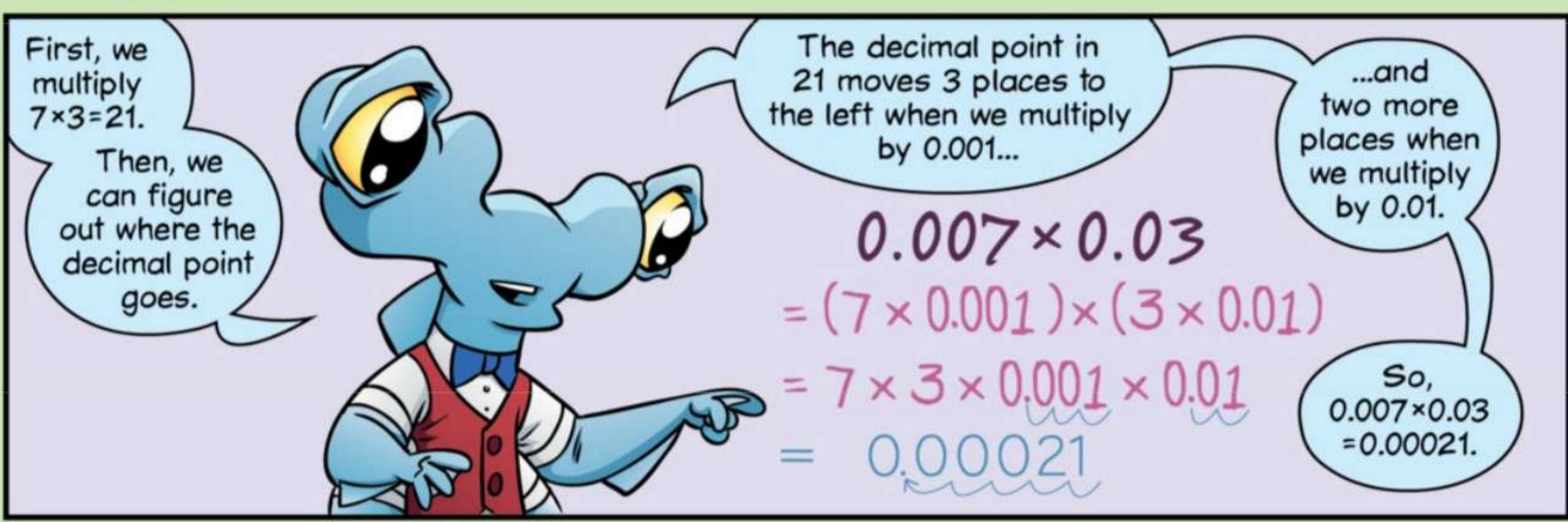
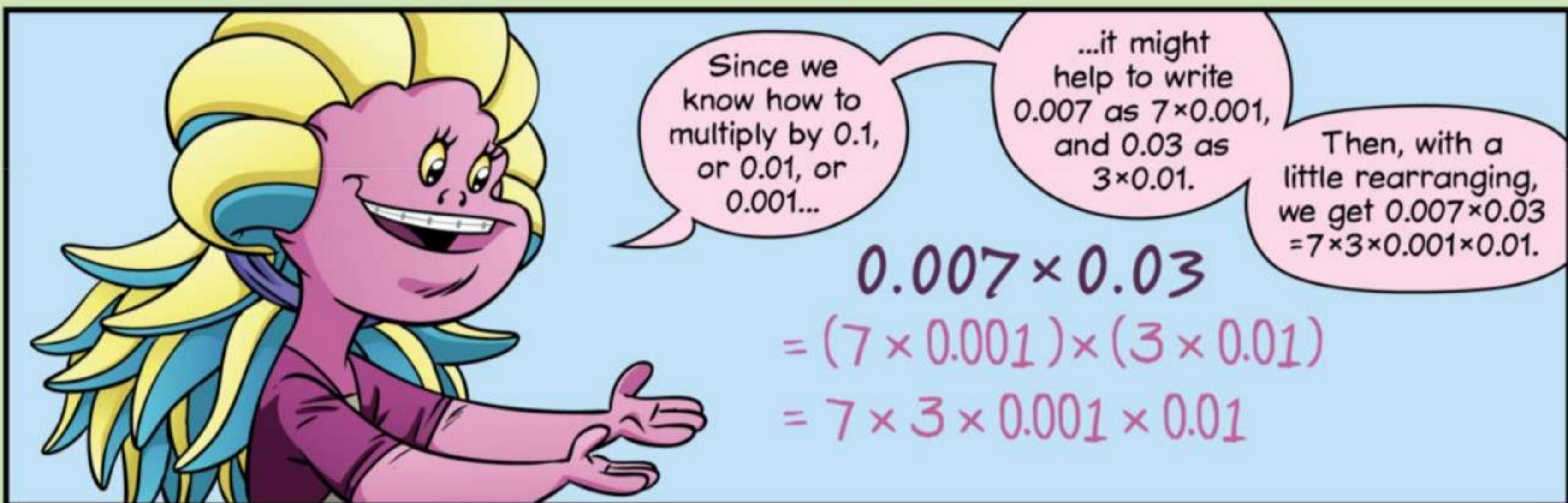
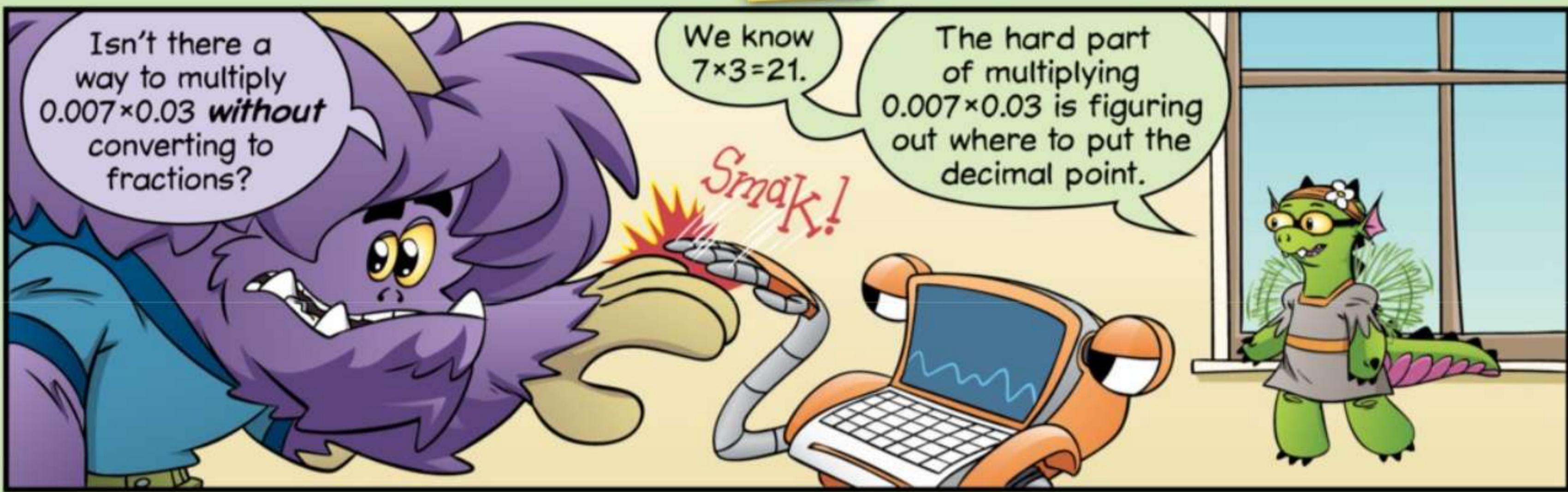
$$0.007 = \frac{7}{1,000}, \text{ and } 0.03 = \frac{3}{100}.$$

And $\frac{7}{1,000} \times \frac{3}{100}$
 $= \frac{21}{100,000}.$

$$0.007 \times 0.03$$

$$= \frac{7}{1,000} \times \frac{3}{100}$$

$$= \frac{21}{100,000}$$






$$0.024 \times 0.9$$

$$\begin{aligned} &= (24 \times 0.001) \times (9 \times 0.1) \\ &= 24 \times 9 \times 0.001 \times 0.1 \\ &= 216 \times 0.001 \times 0.1 \\ &= 0.0216 \end{aligned}$$

0.024 is
 24×0.001 , and
0.9 is 9×0.1 .

$24 \times 9 = 216$,
and we move
the decimal point
4 places to the left
to get 0.0216.

We don't
even need the
middle steps!



$$0.024 \times 0.9$$

3 + 1 = 4 digits total

$$= 0.0216$$

4 digits after decimal

We can just count
the number of digits right
of the decimal point in 0.024
and 0.9 to figure out where to
place the decimal point
in their product.

That's
right!

Careful, though.
What do you get
when you multiply
0.125 by 0.08?

$$0.125 \times 0.08$$



Try it.



To multiply 0.125×0.08 , we can start by ignoring the decimal points.
 $125 \times 8 = 1,000$.

$$125 \times 8 = 1,000$$

$$0.\underline{1}\underline{2}\underline{5} \times 0.\underline{0}\underline{8}$$

3 2

Then, we count the total number of digits right of the decimal point in 0.125 and 0.08 .

There are $3+2=5$ digits right of the decimal point.

So, we move the decimal point in $1,000$ so that there are **5** digits right of the decimal point.

$$0.\underline{1}\underline{2}\underline{5} \times 0.\underline{0}\underline{8} = 0.01000$$

3 2 5

$$= 0.01$$

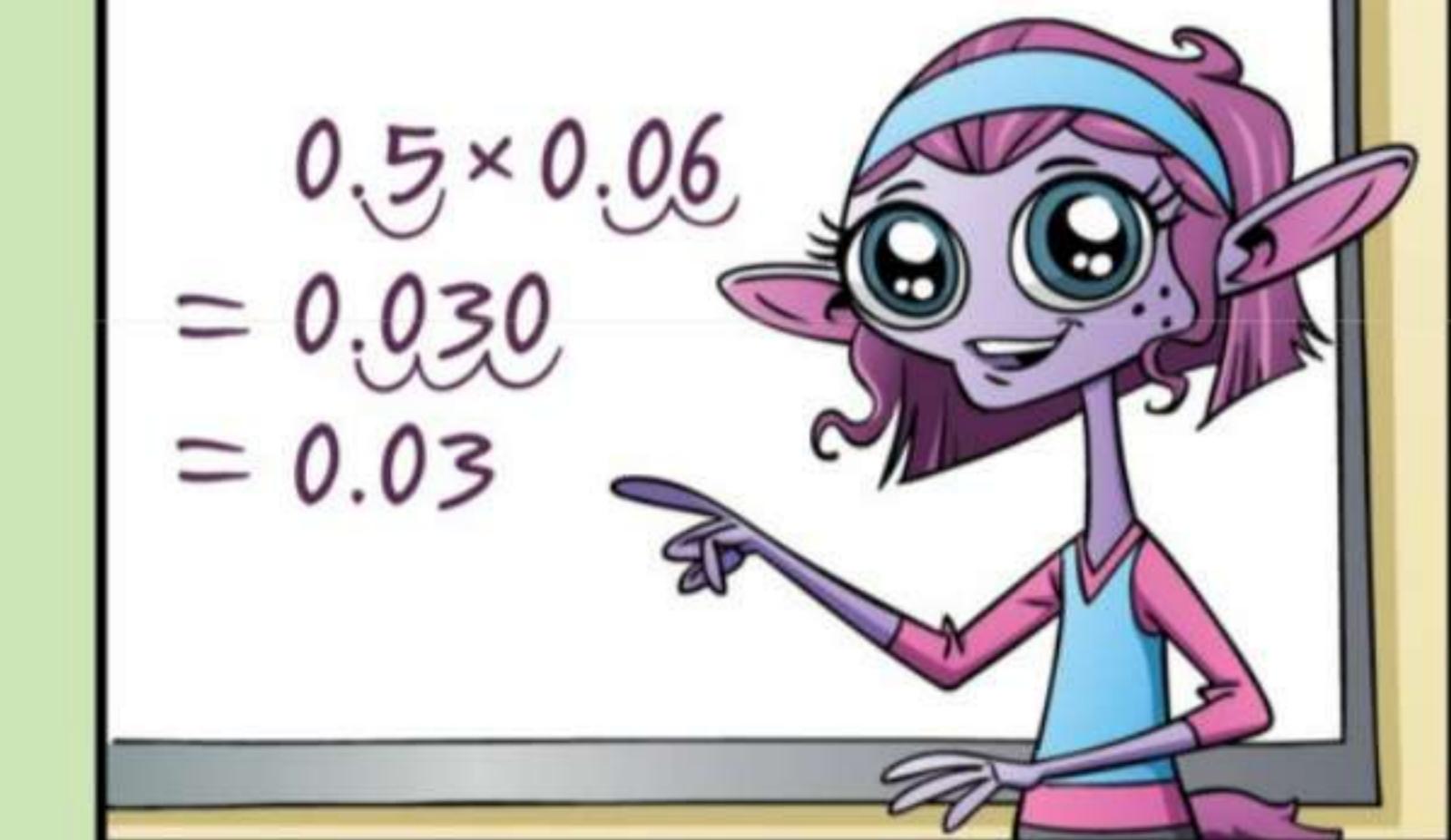


But,
 $0.01000 = 0.01$.
So, $0.125 \times 0.08 = 0.01$.



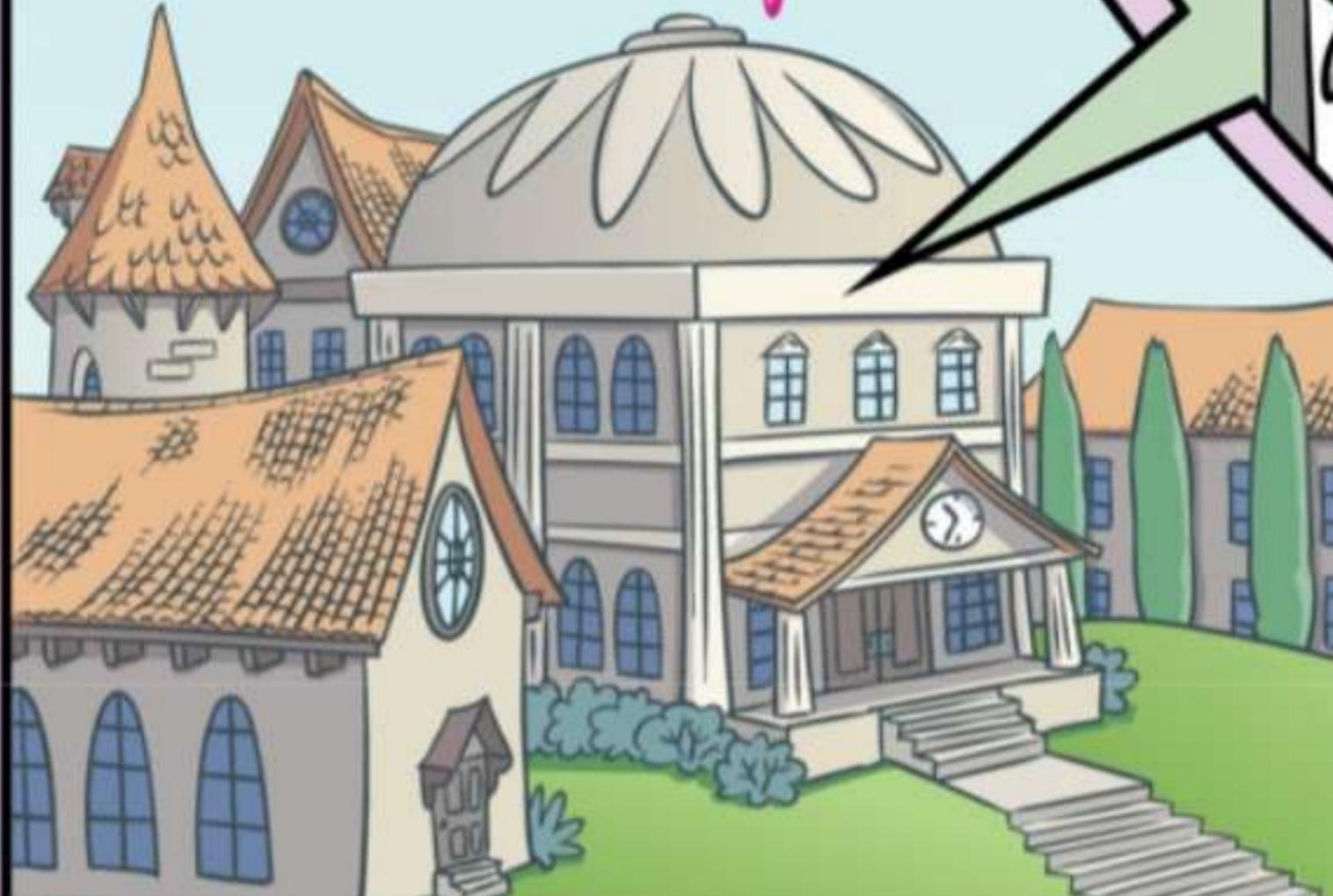
Good.
You have to include the trailing zeros when you place the decimal point.

But, it's fine to remove them afterwards. For example,
 $0.5 \times 0.06 = 0.030\dots$
...which equals 0.03.



THE LAB

CONVERSIONS



$$0.3 = \frac{3}{10}$$

$$0.12 = \frac{12}{100}$$

$$0.487 = \frac{487}{1,000}$$

$$0.0096 = \frac{96}{10,000}$$

We can write decimals as fractions using what we know about place value.

REVIEW CONVERTING FRACTIONS AND DECIMALS IN CHAPTER 11 OF BEAST ACADEMY 4D.

Similarly, we can write any fraction whose denominator is a power of 10 as a decimal.

$$\frac{9}{10} = 0.9$$

$$\frac{631}{1,000} = 0.631$$

$$\frac{23}{100} = 0.23$$

$$\frac{3}{10,000} = 0.0003$$

However, not every fraction is quite so easy to write in decimal form.

Let's start with $\frac{2}{5}$.

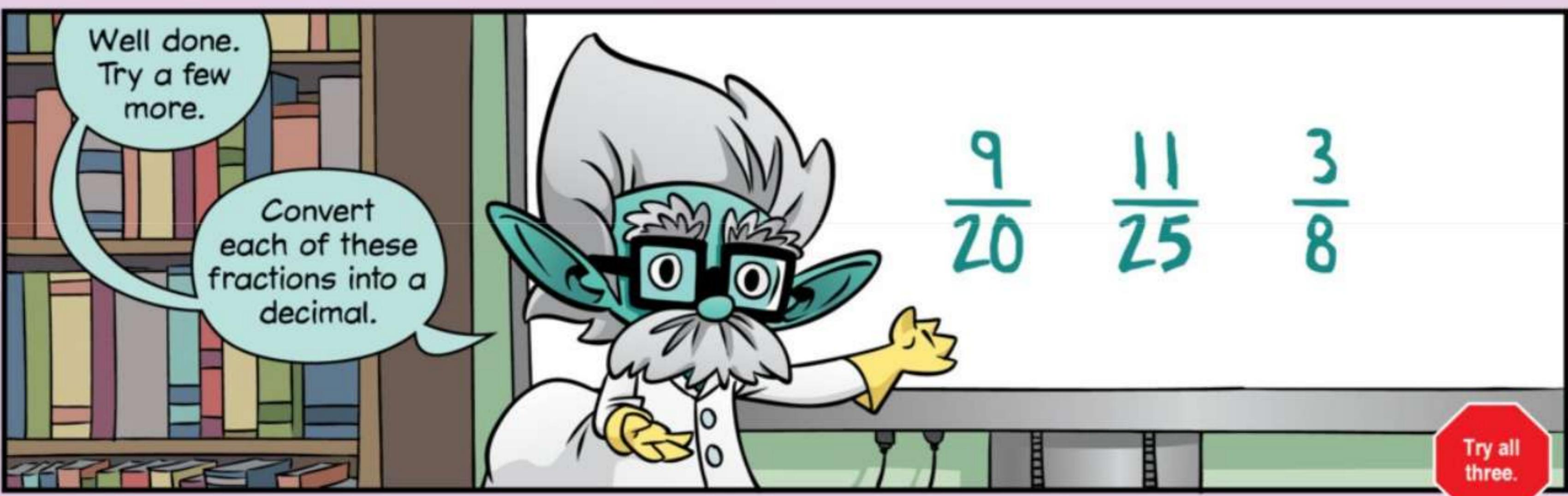
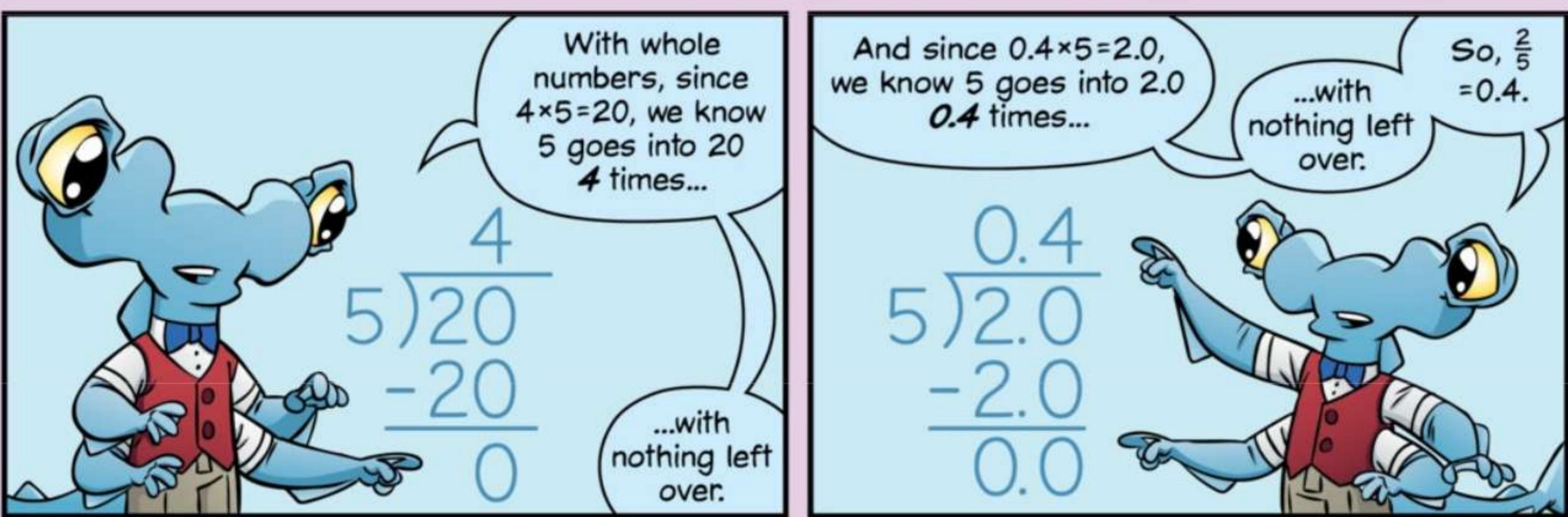
How could we write $\frac{2}{5}$ as a decimal?

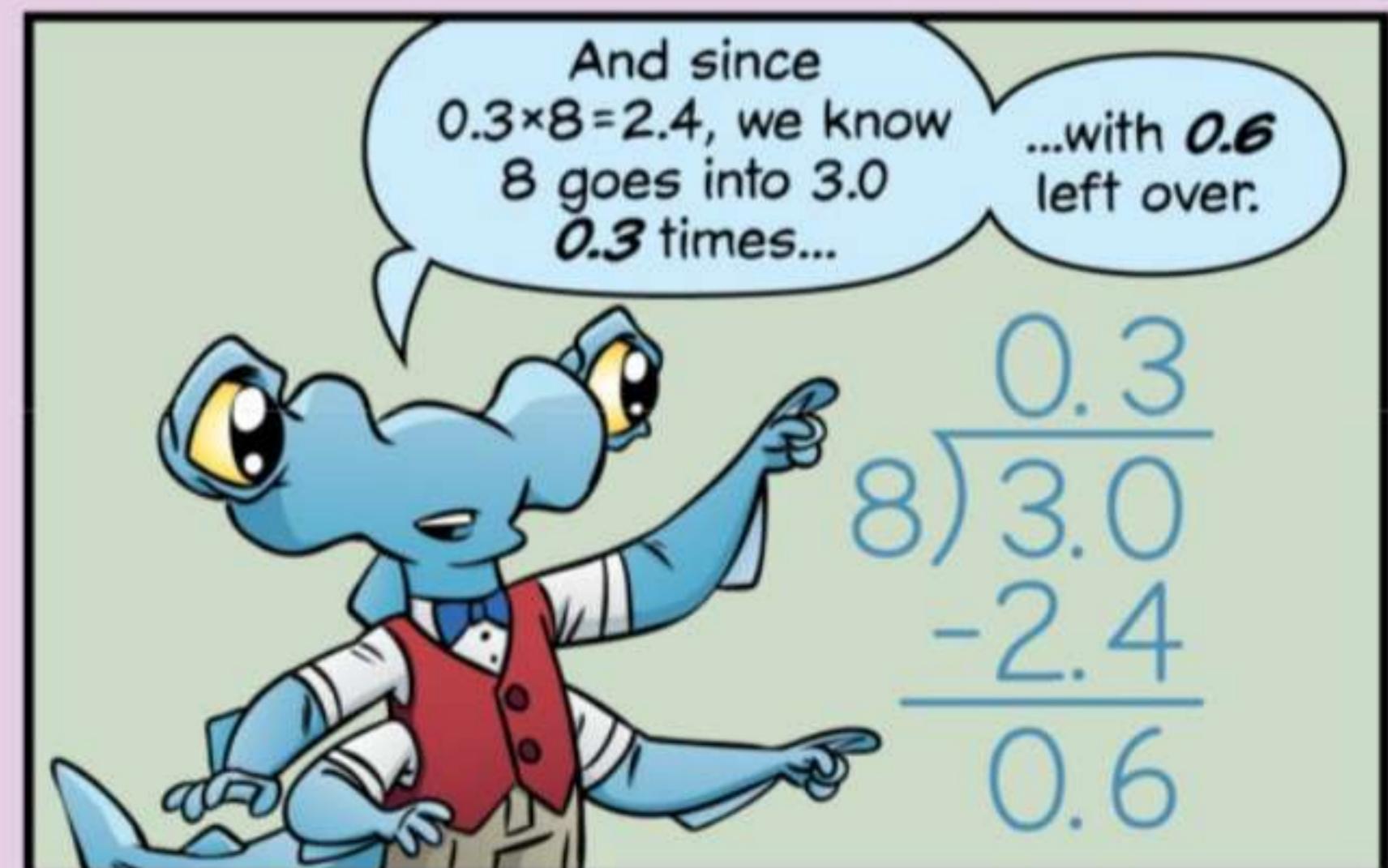
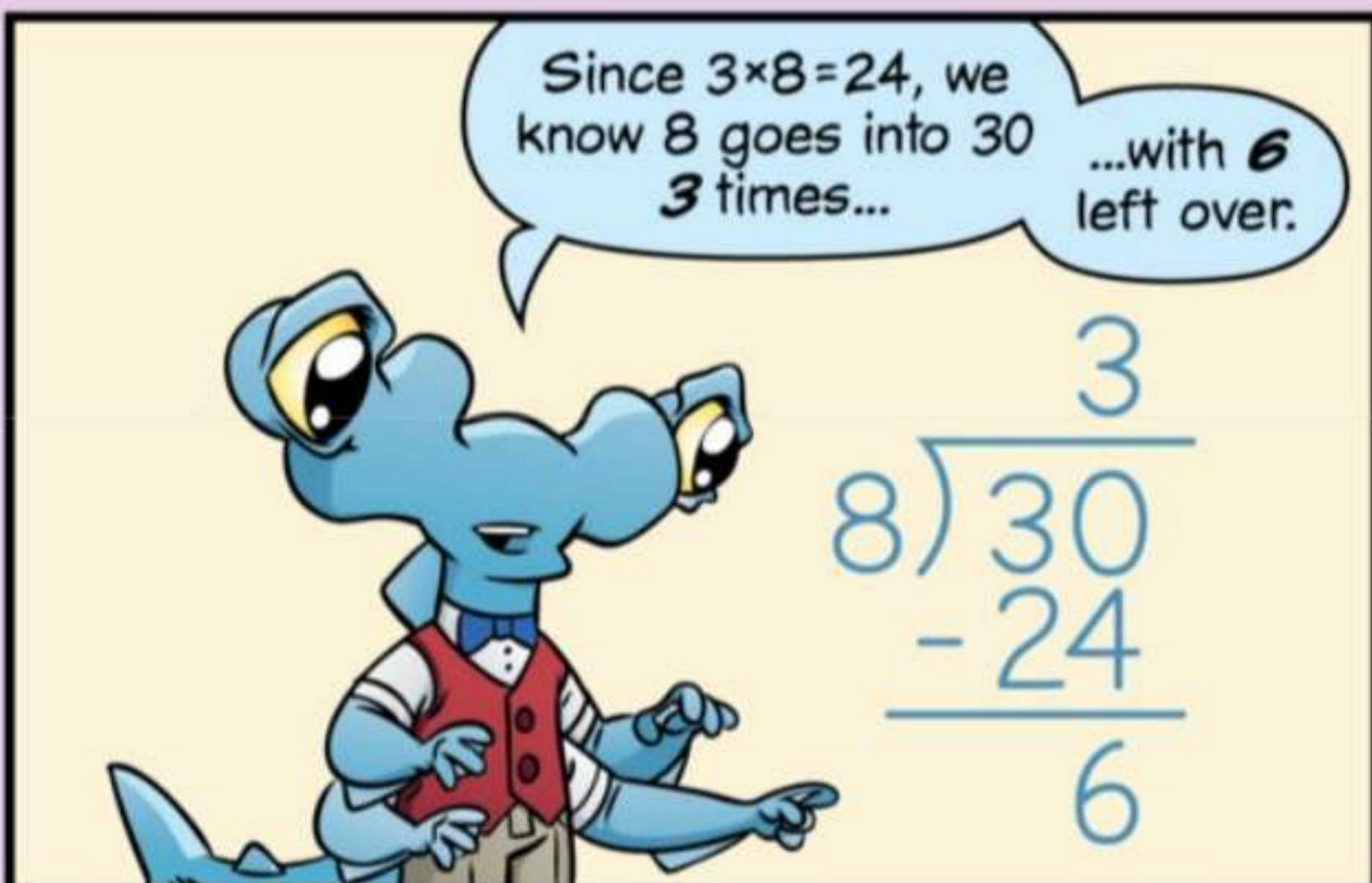
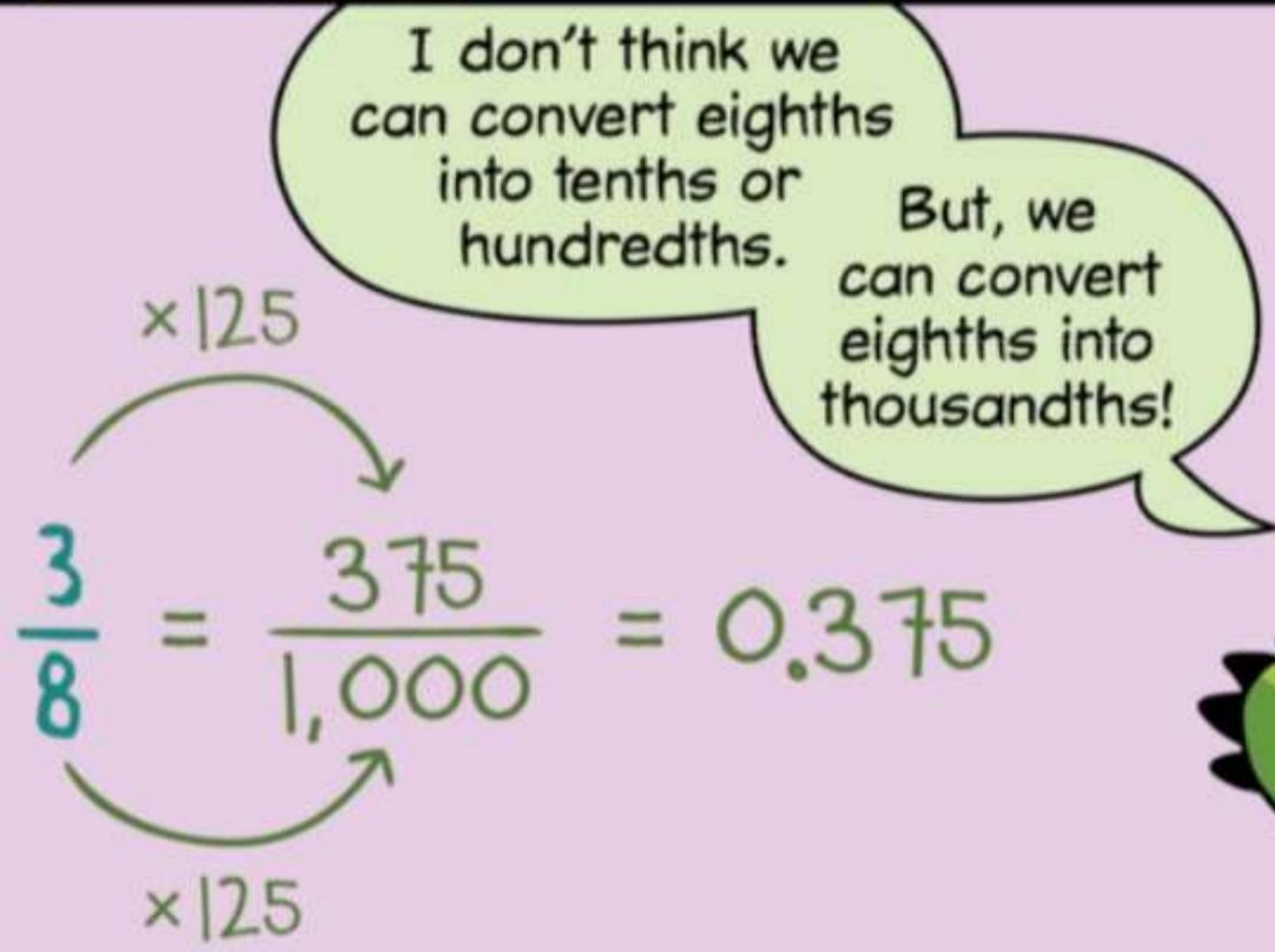
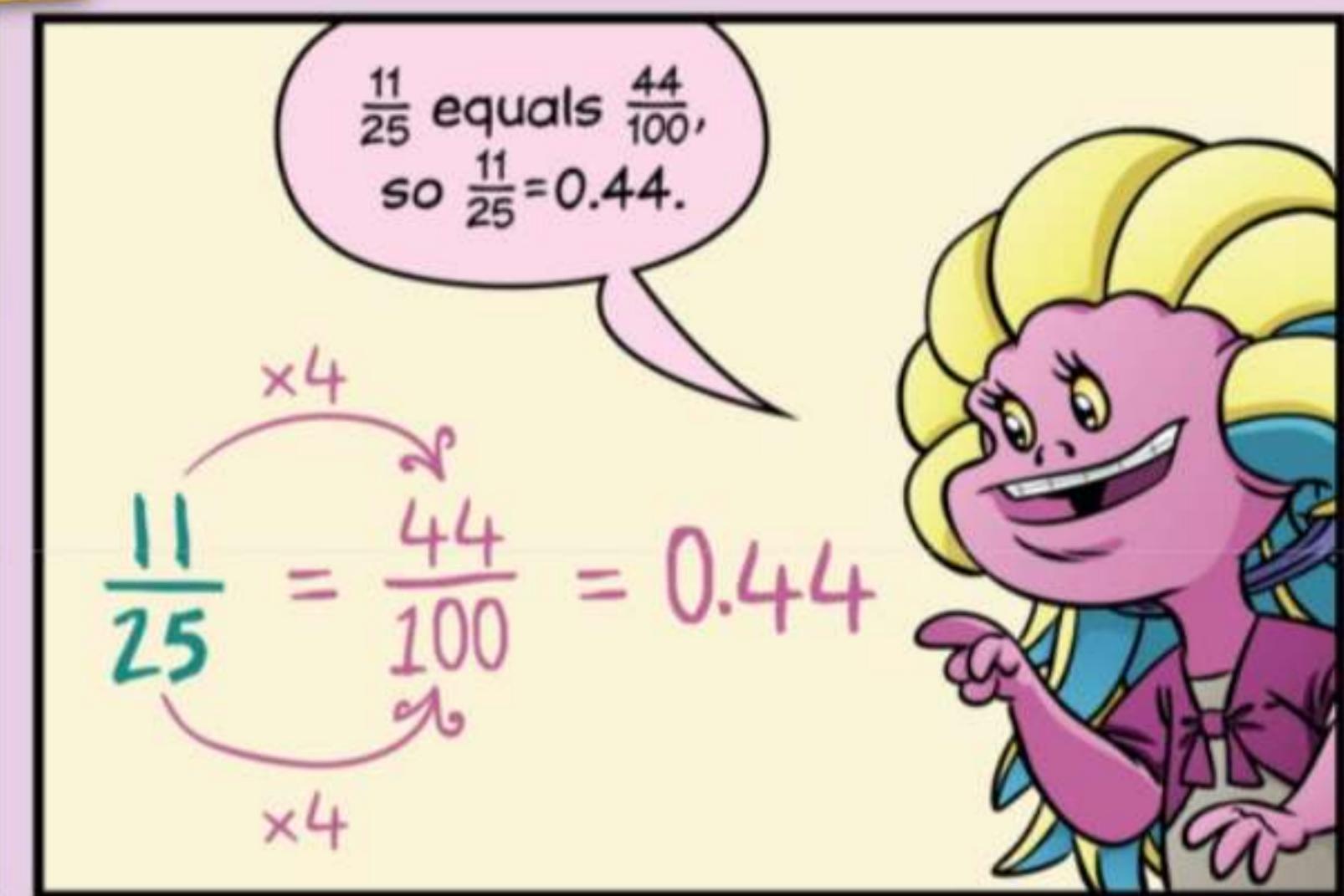
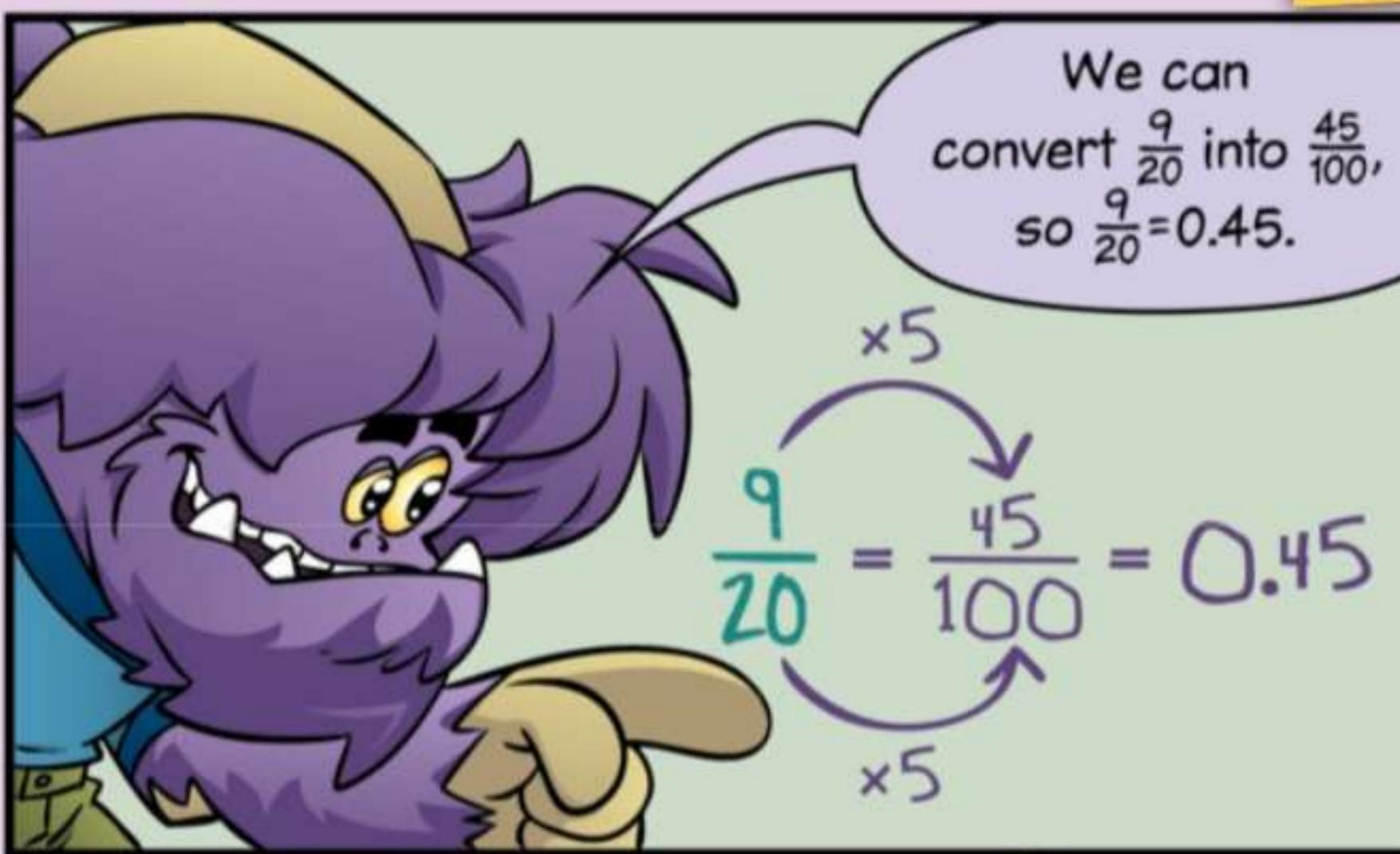
$$\frac{2}{5}$$

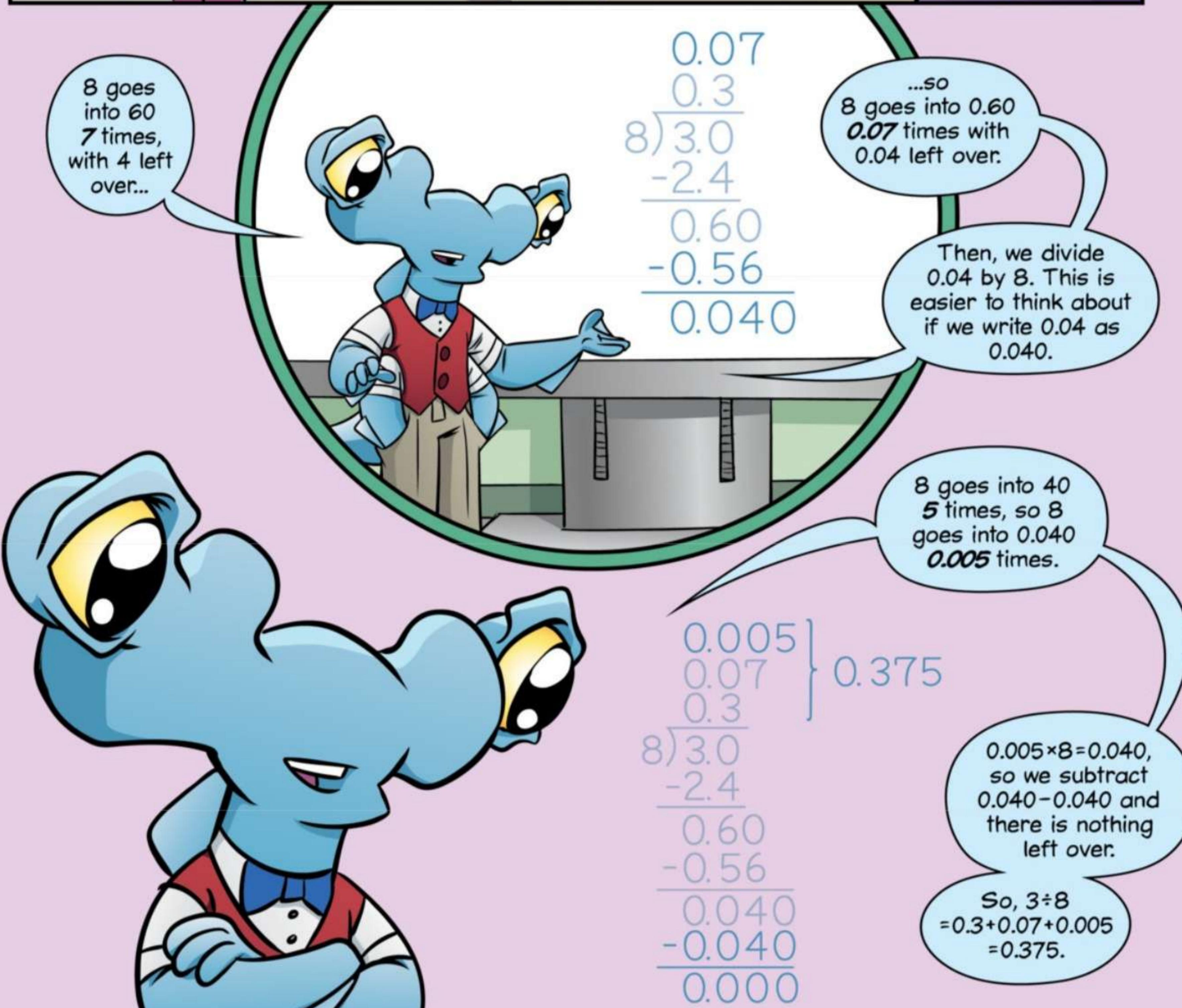
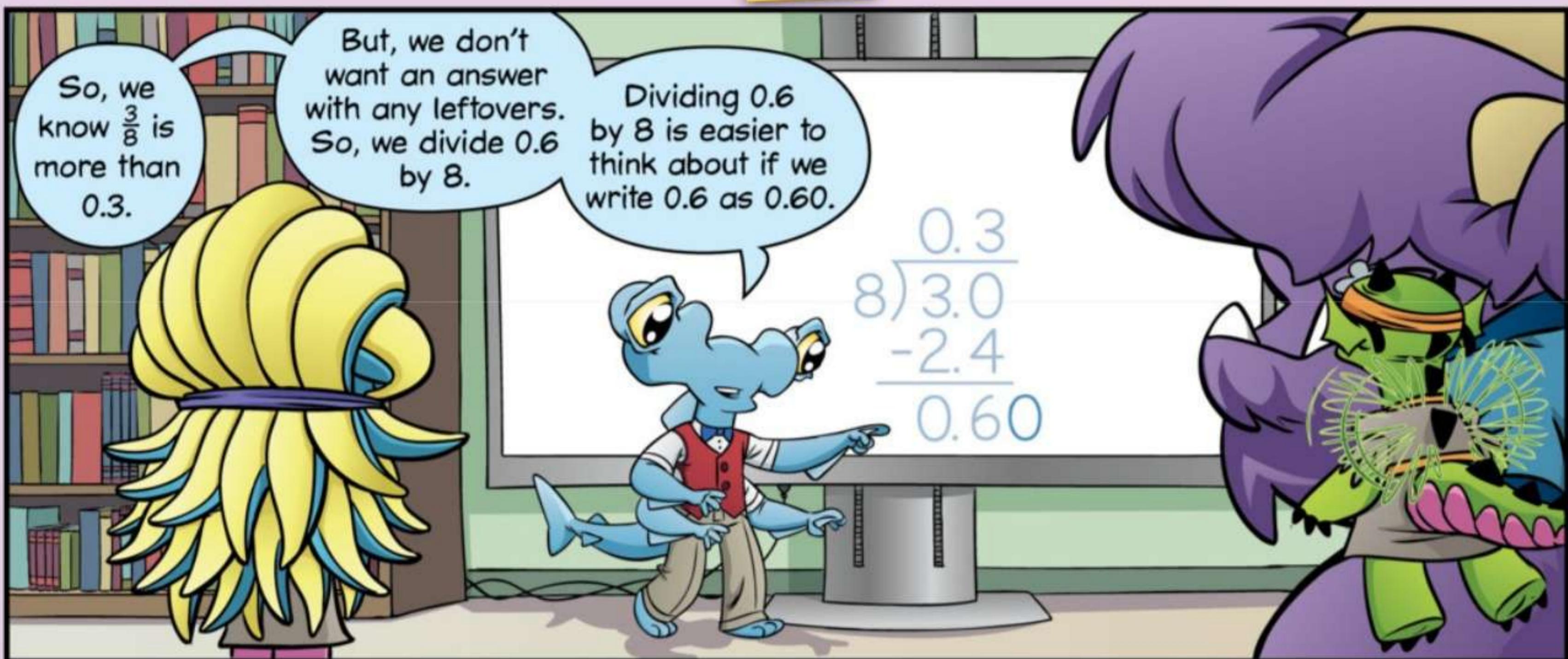
Try it.

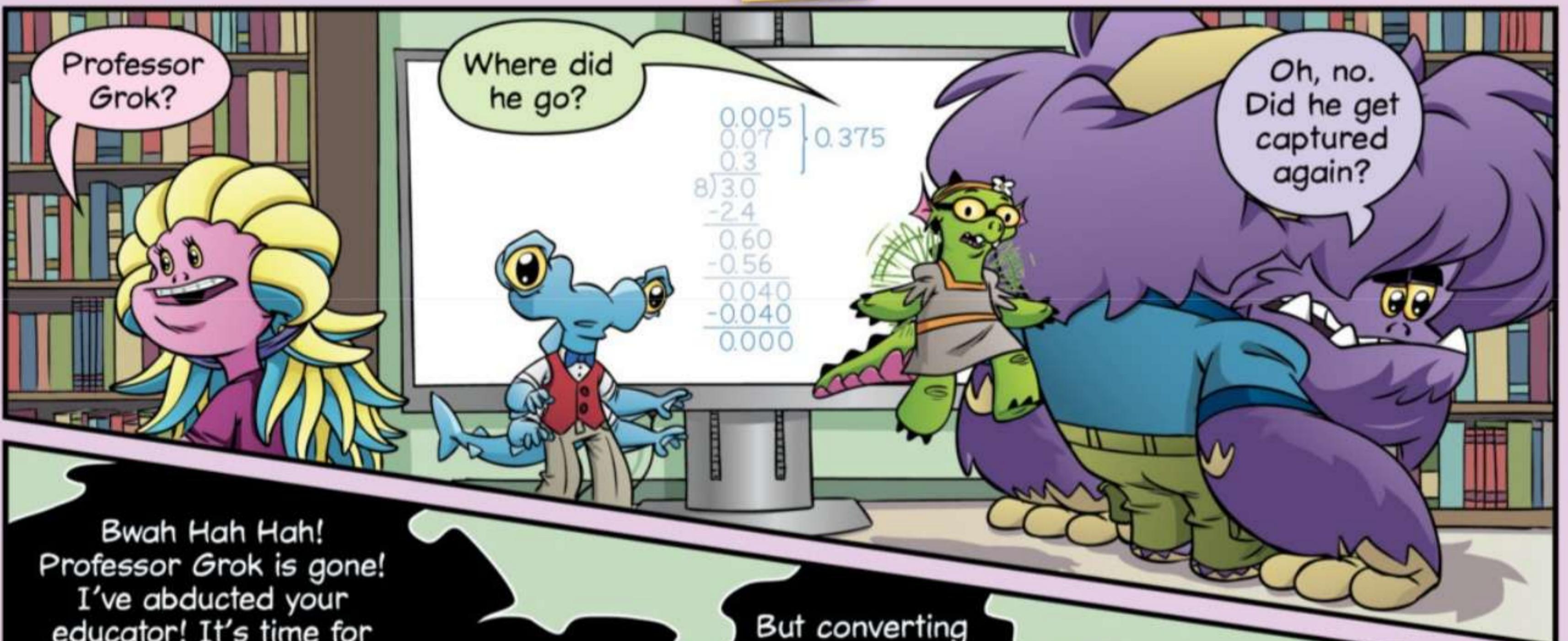


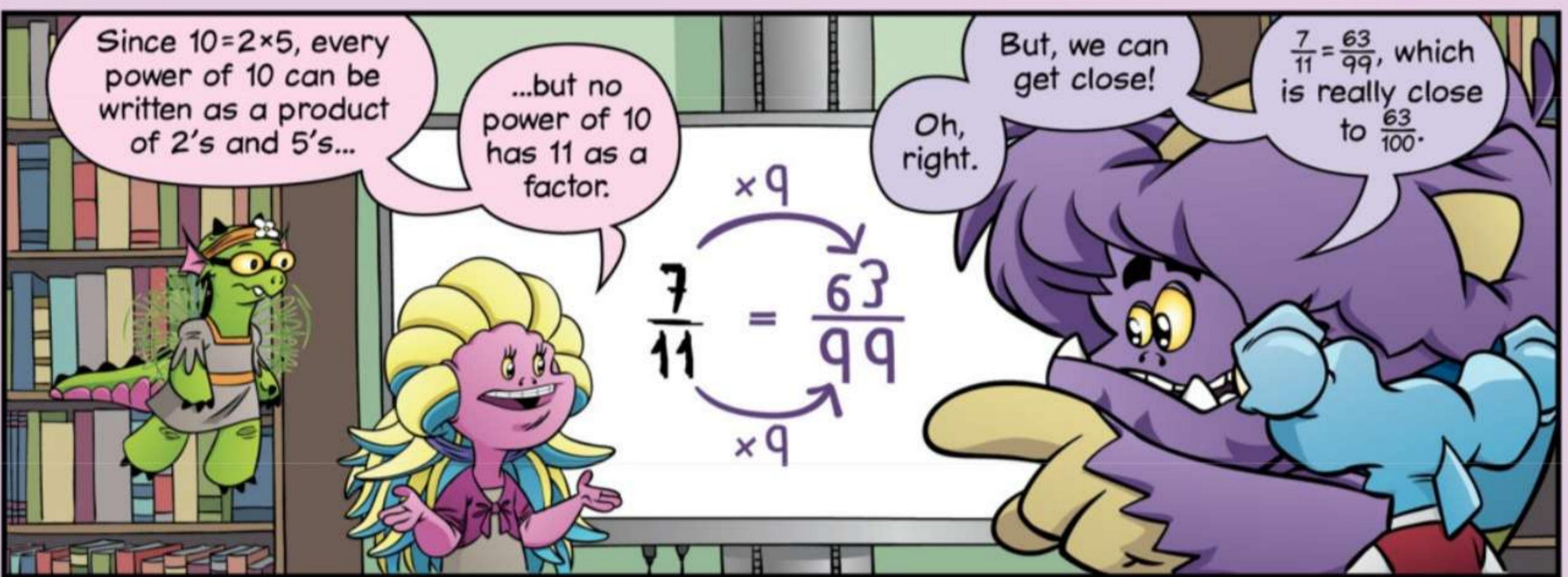
REVIEW LONG DIVISION IN CHAPTER 5 OF BEAST ACADEMY 4B.







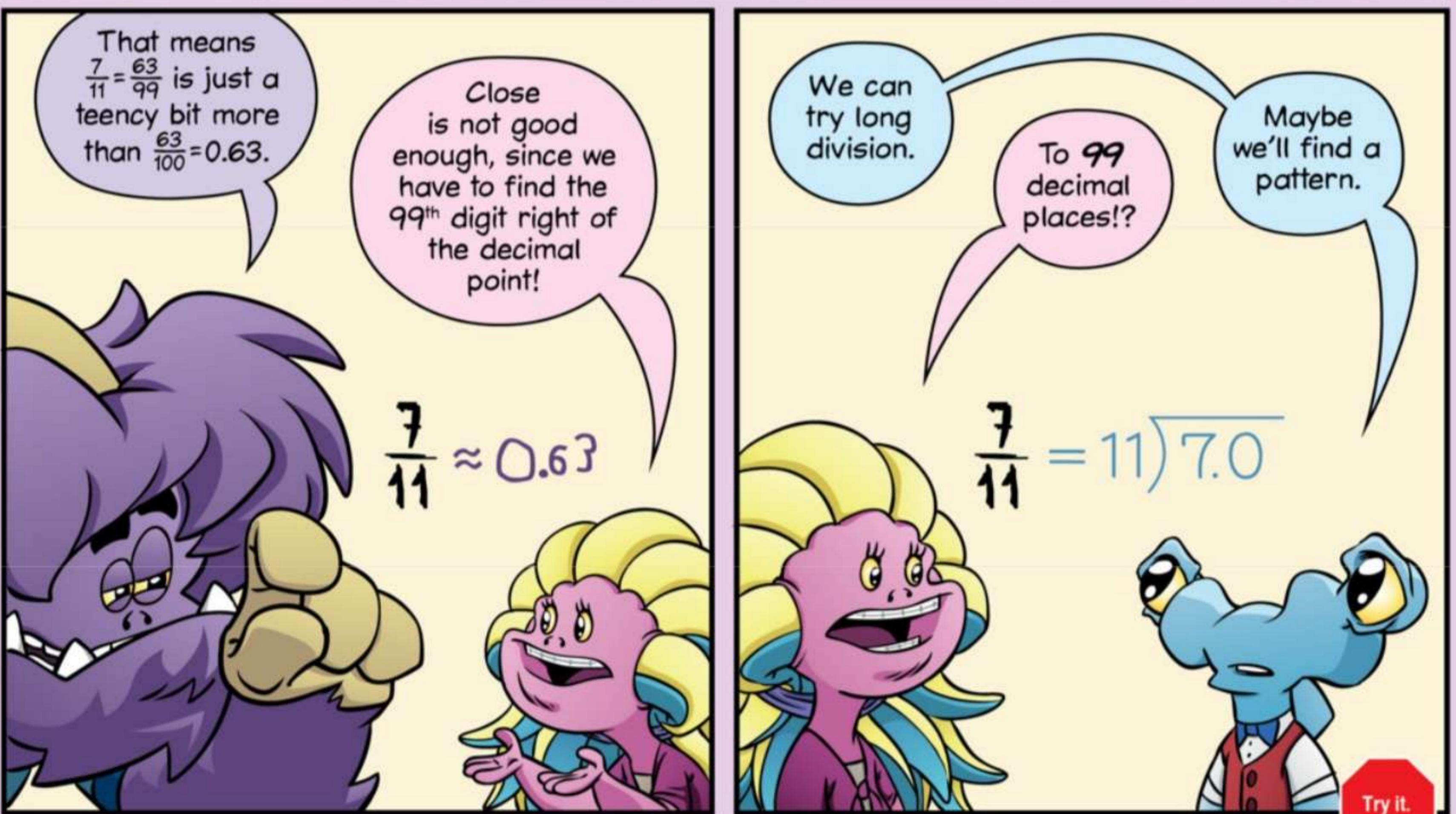


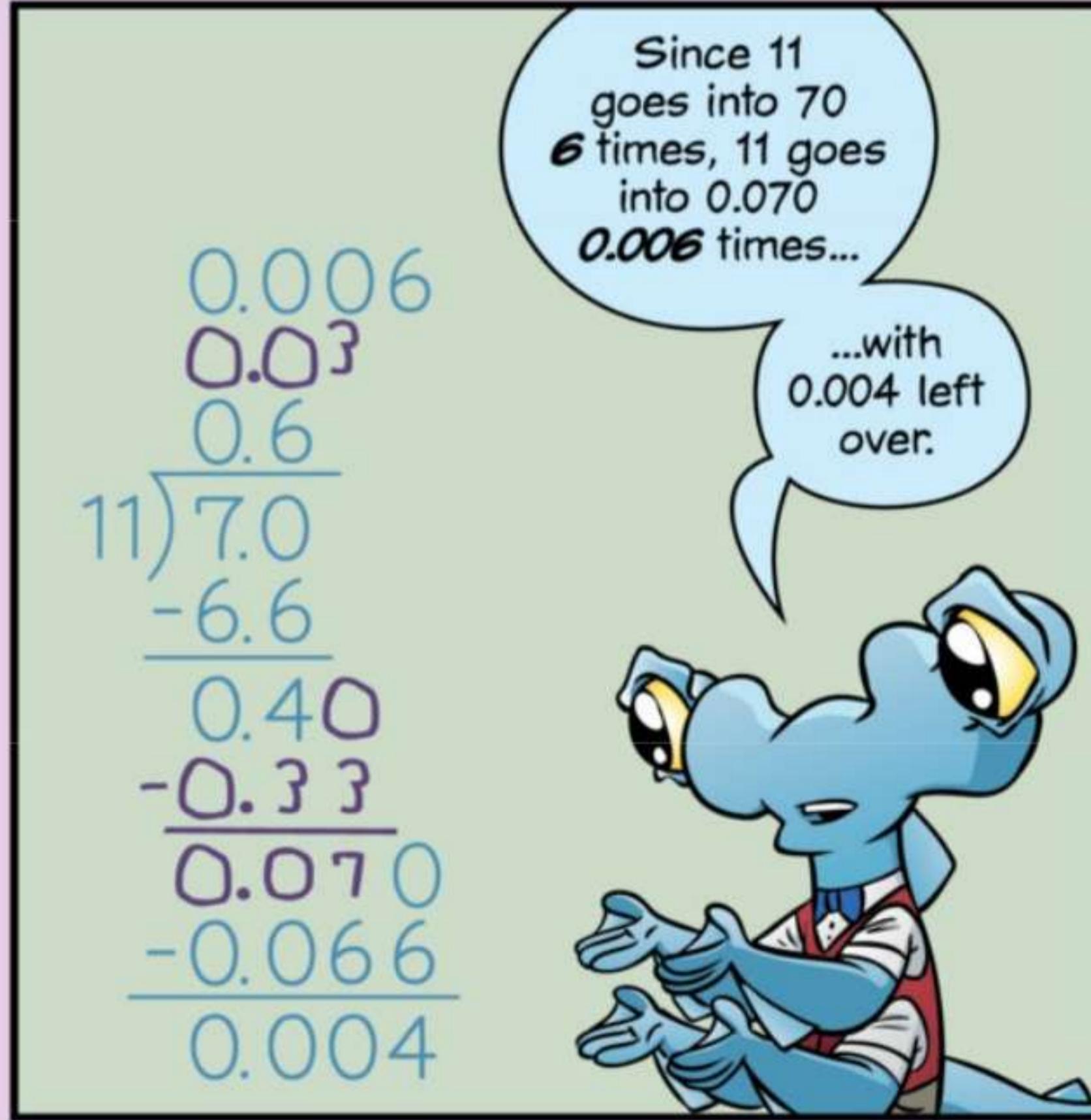
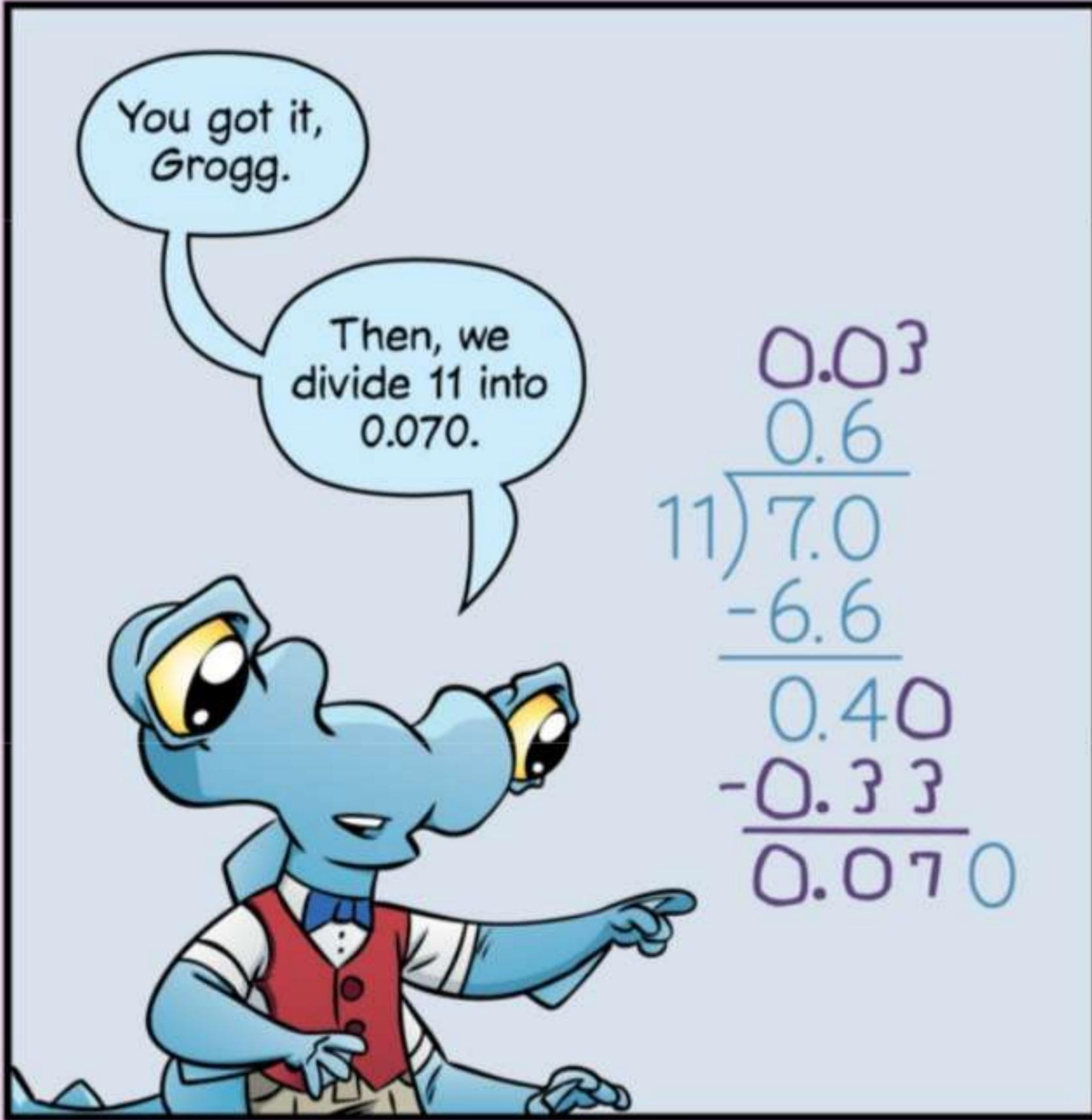
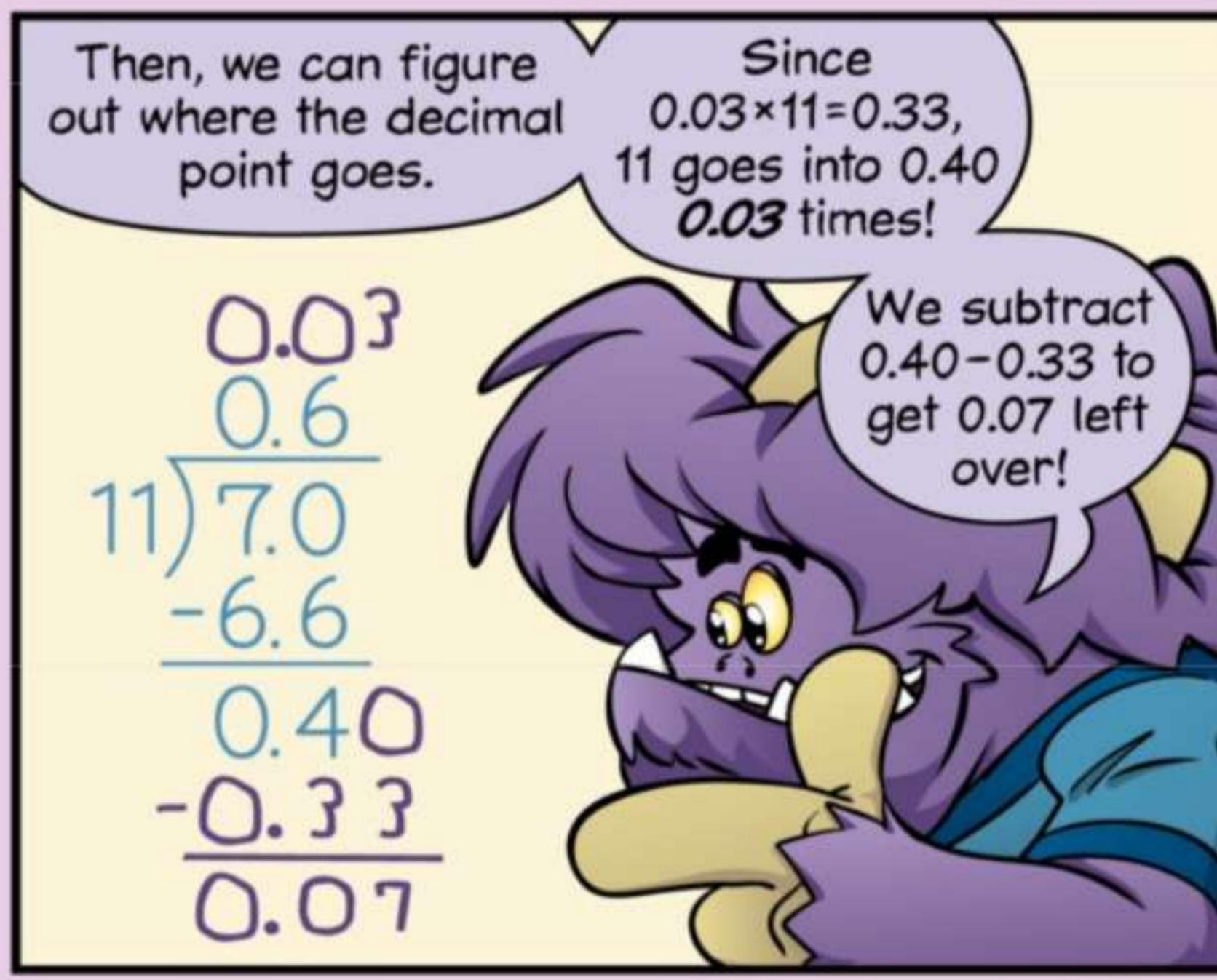
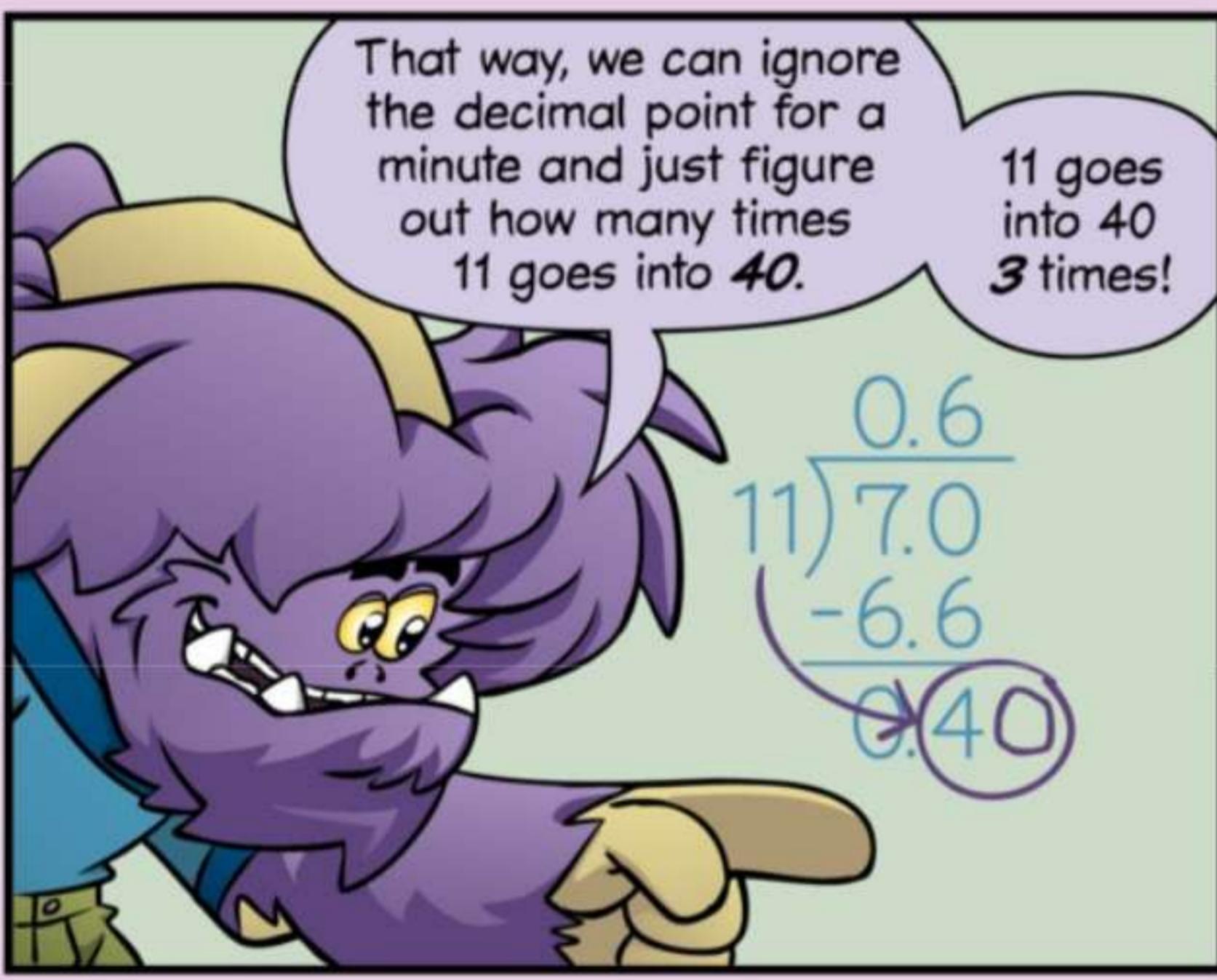
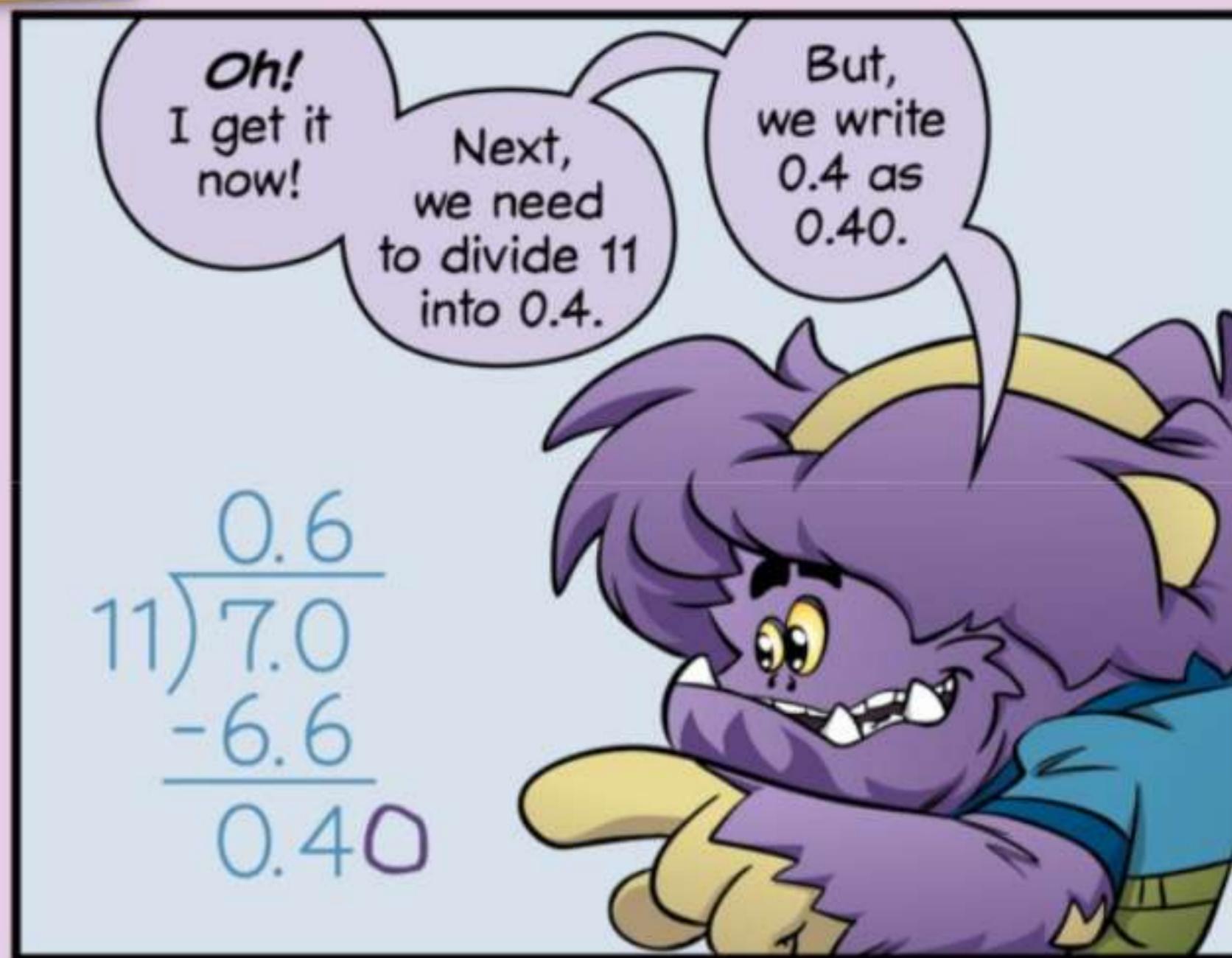
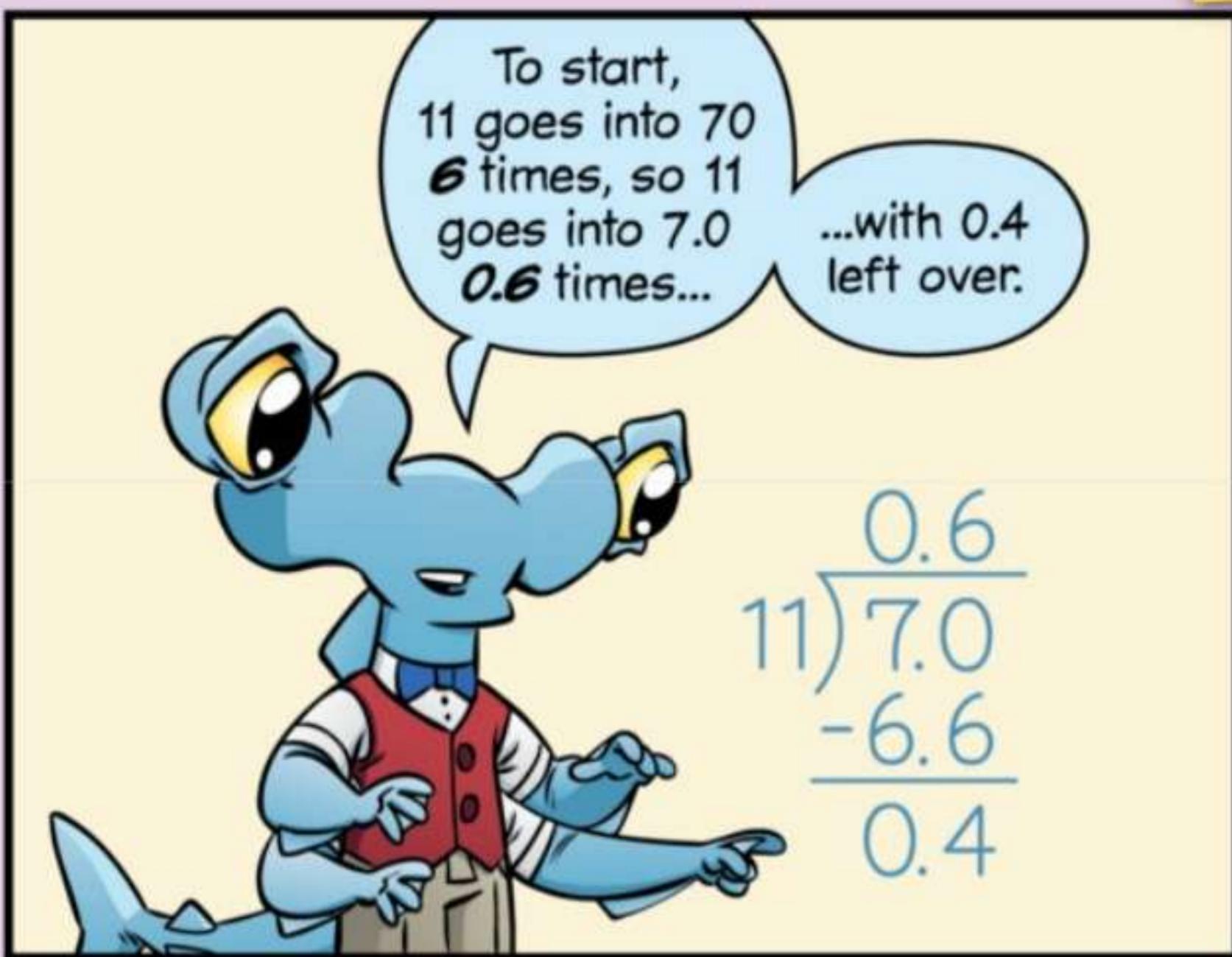


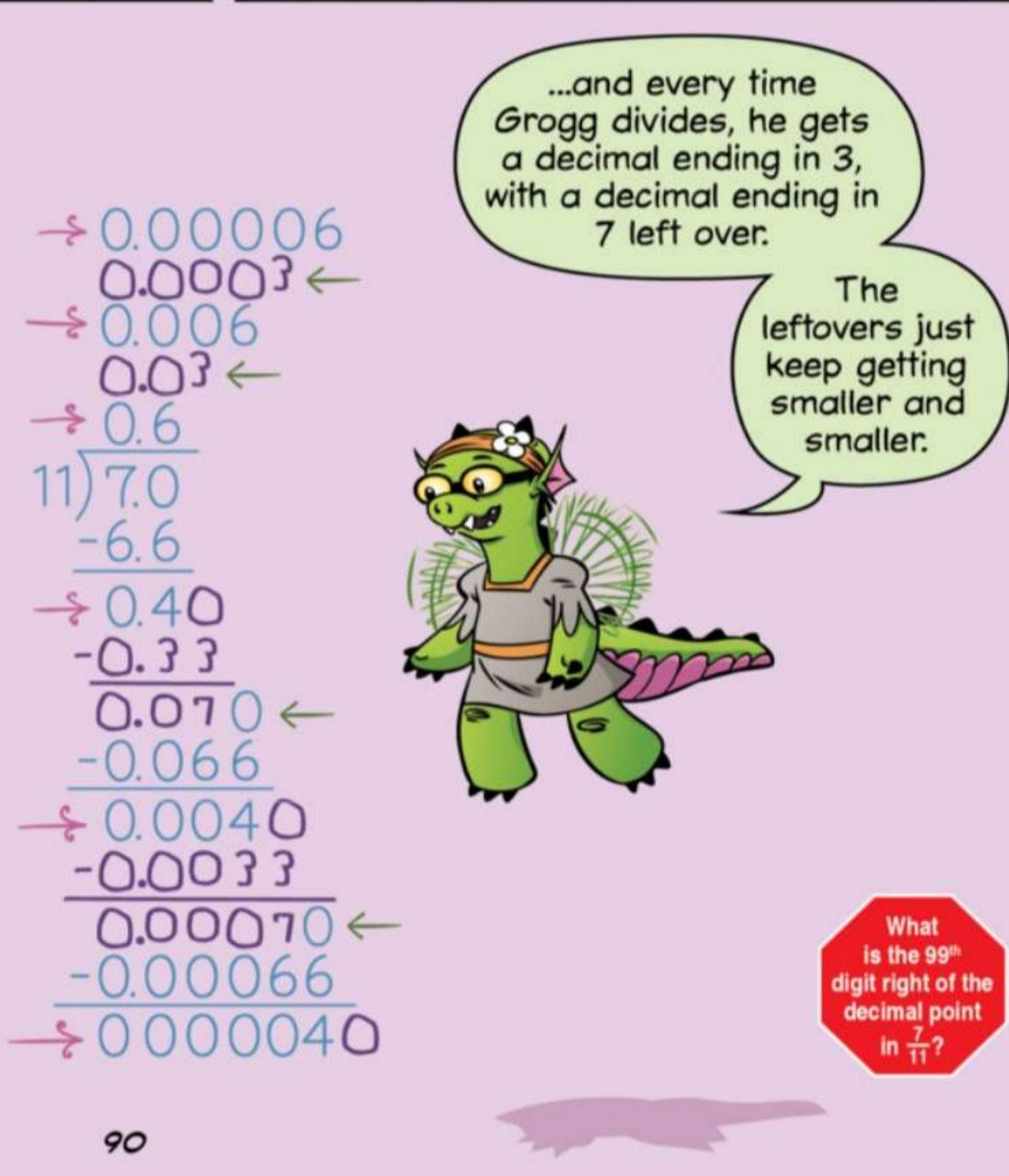
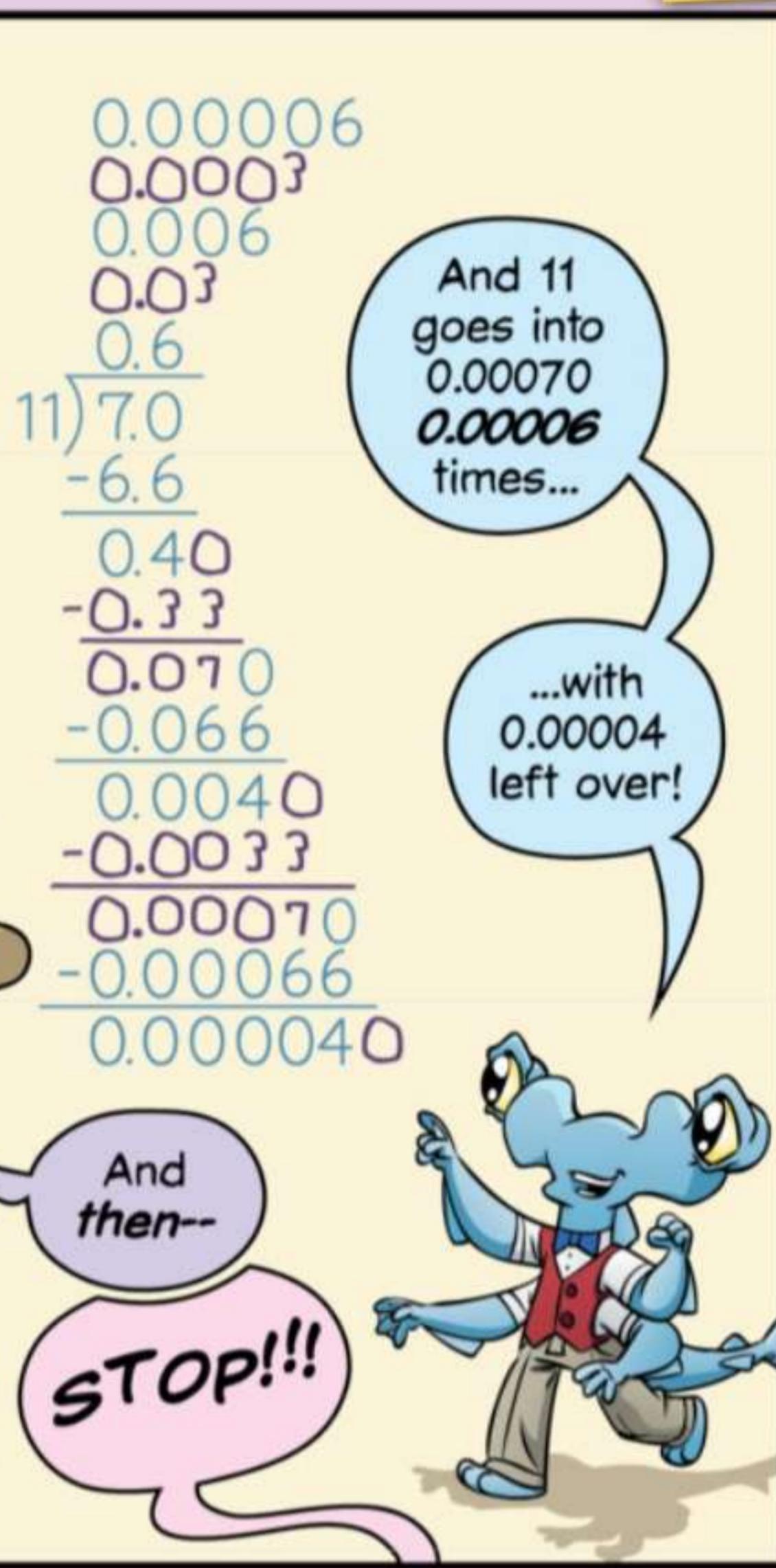
$$\frac{7}{11} = \frac{63}{99}$$

$\times 9$

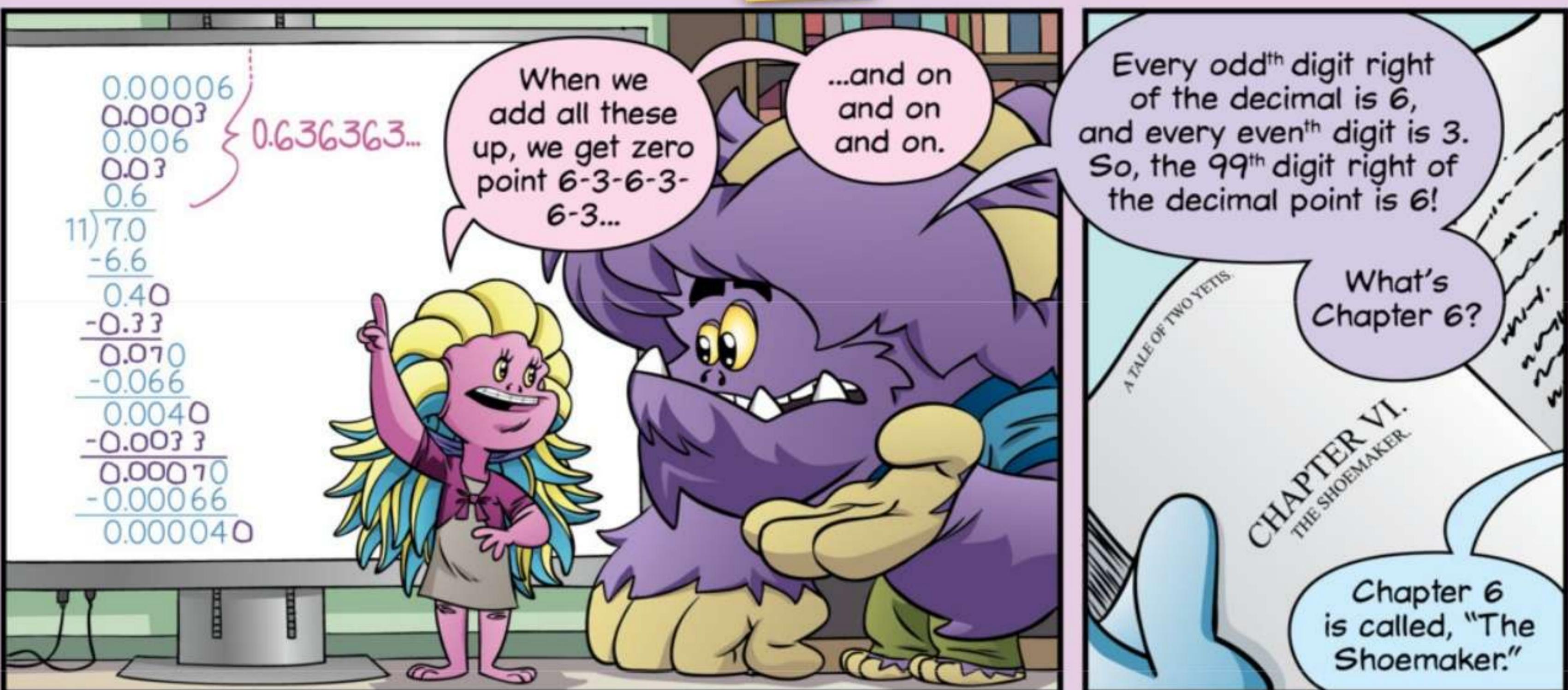
$\times 9$







What is the 99th digit right of the decimal point in $\frac{7}{11}$?



MATH TEAM

Math Meet

Alright, team.
This is our last
meet against the bots
before the World
Math Olympiad
Championships.

Where's
Max?

He just called
and said he would
be late.

The four
of you will have
to work together
again to beat
the bots.

With
the focus on
decimals, it's
not going to
be easy.

The bots are great at
computations they have
been programmed
to perform.
But, their
programming
is limited.

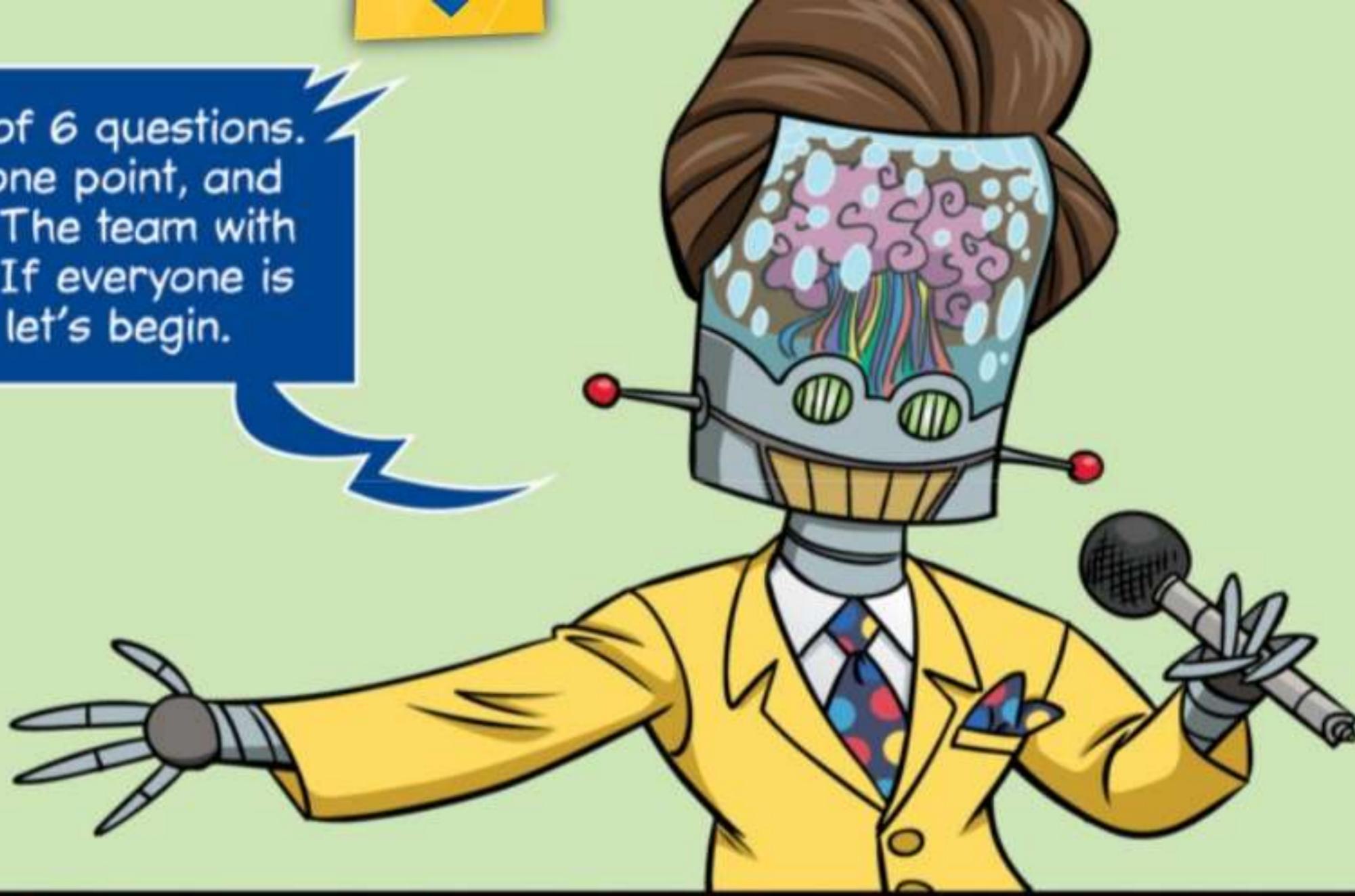
Hopefully, some
of the questions
will require some
creativity.

Good luck!

And now...

...please welcome the
Little Monsters of
Beast Academy and
their opponents...
the Bots.

Today's Math Meet will consist of 6 questions. The first five are each worth one point, and the final question is worth two. The team with the most points wins the meet. If everyone is ready for the first question, let's begin.



Divide 0.04 by 0.3.
Express your answer as a fraction in simplest form.

Using long division, I came up with the same answer as the bots.

But I don't know how to write 0.13333333... as a fraction.

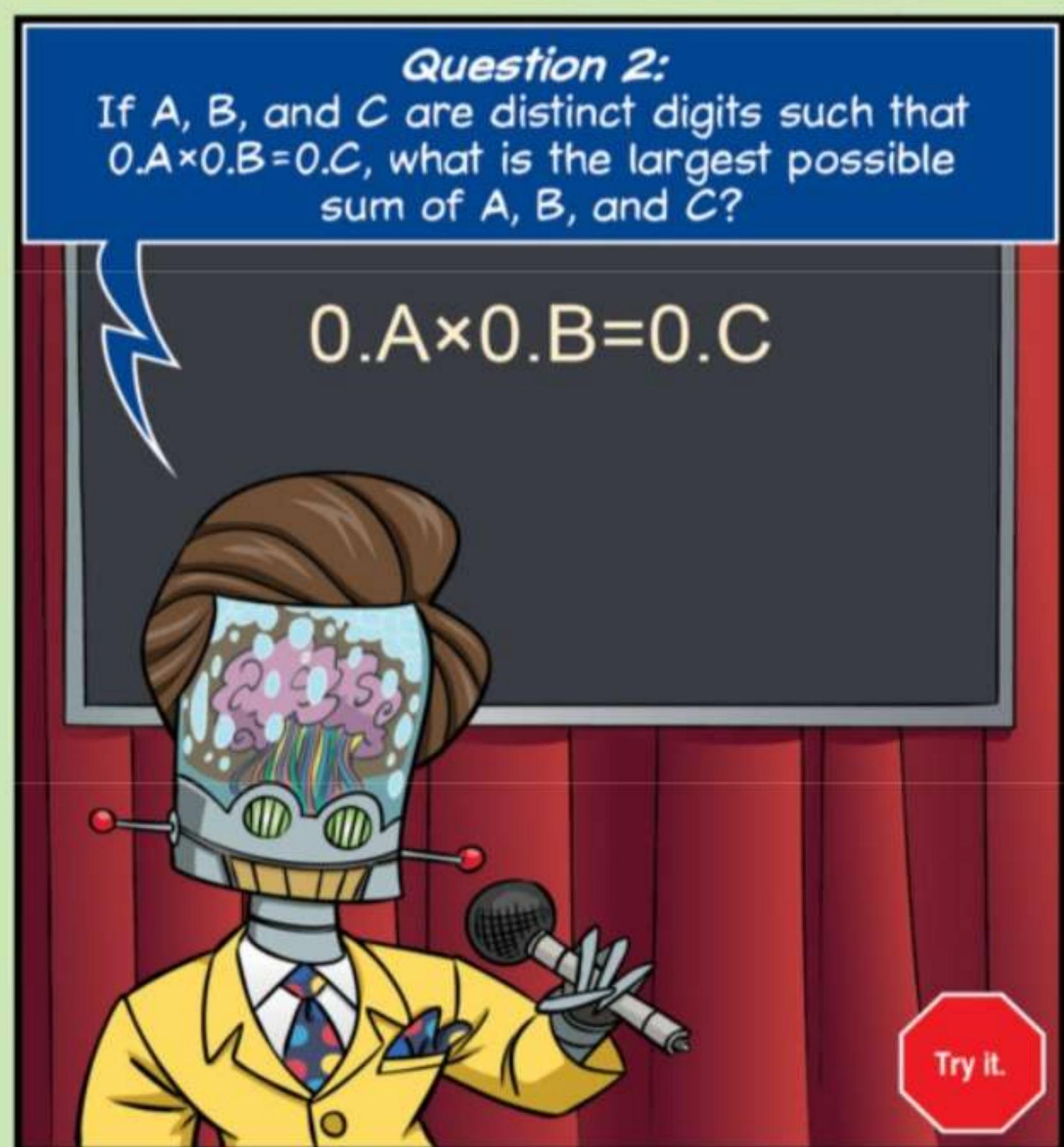
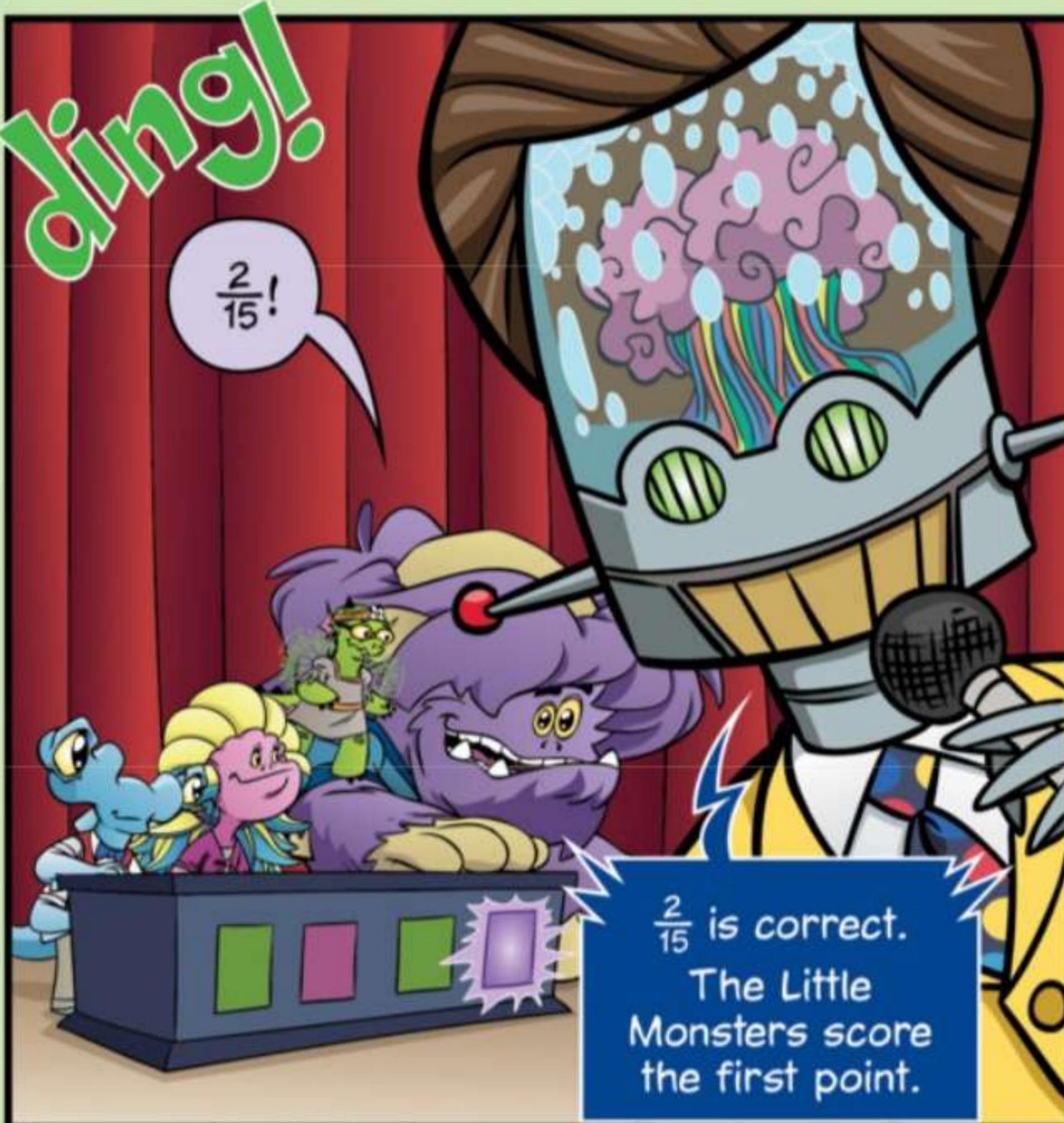
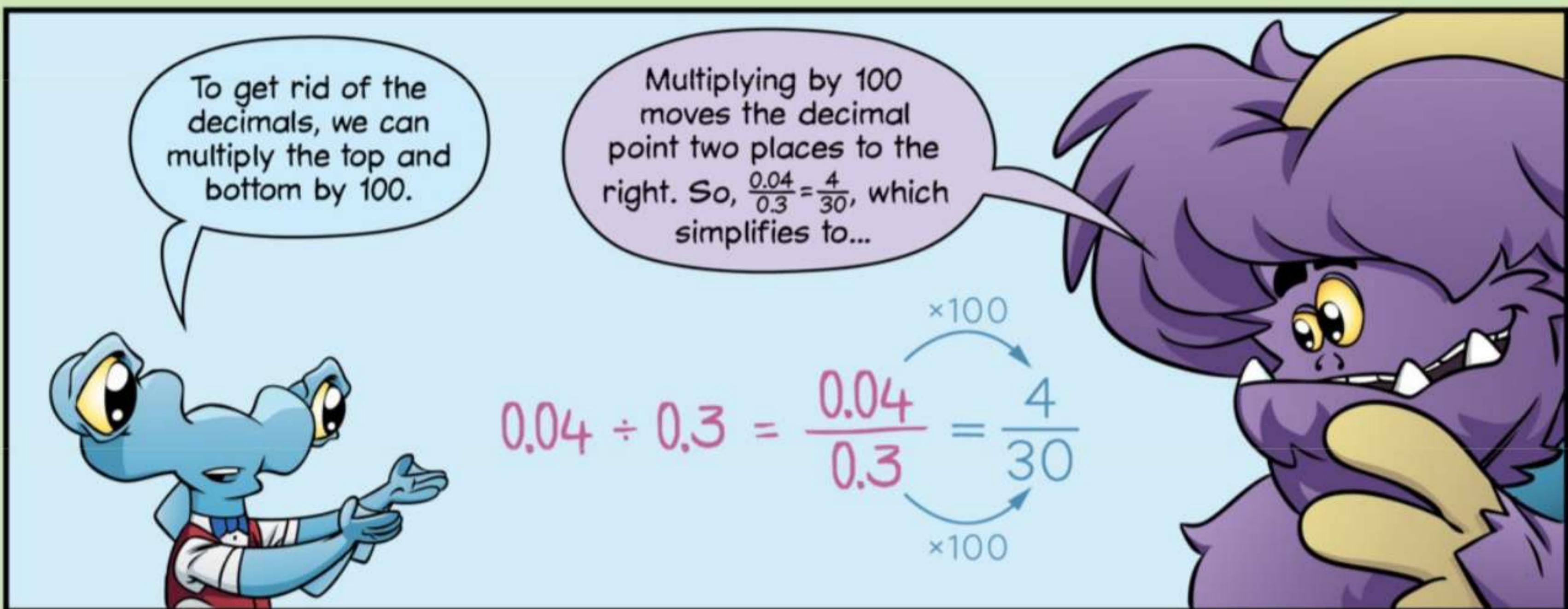
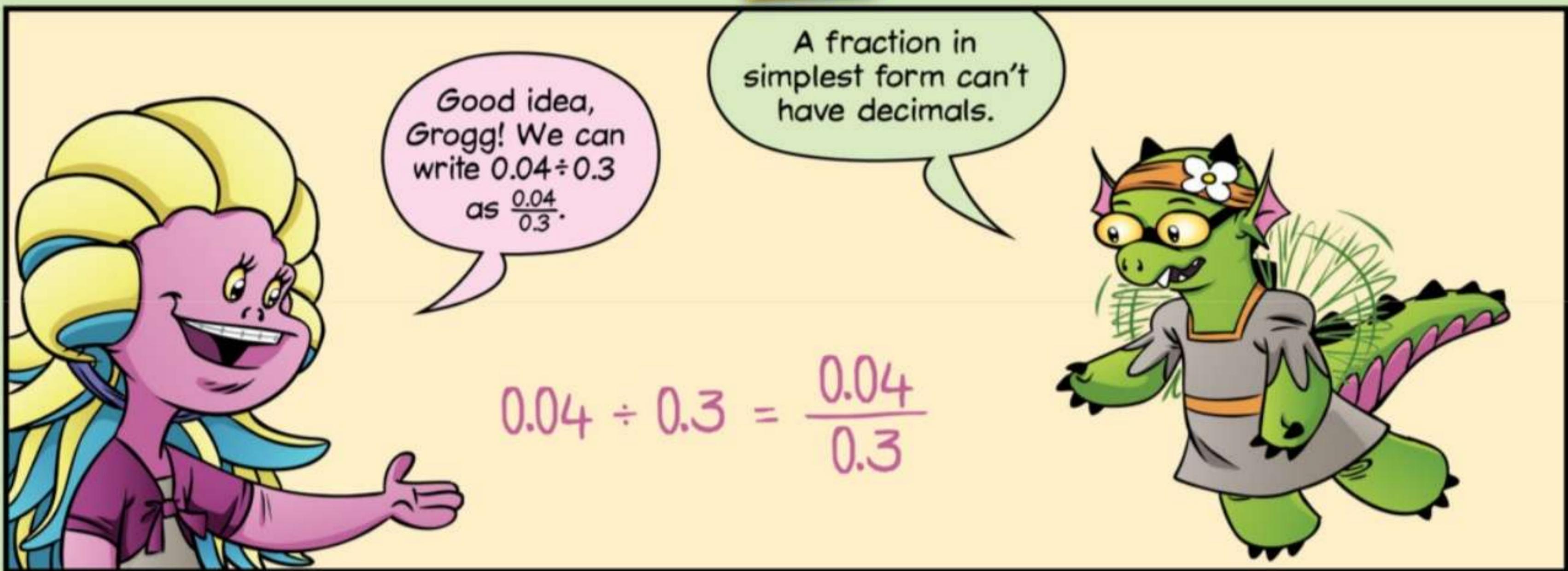
What if we **start** by writing $0.04 \div 0.3$ as a fraction?

0.0003
0.003
0.03
0.1

0.3) 0.04
- 0.03
0.010
- 0.009
0.0010
- 0.0009
0.00010
- 0.00009

A DECIMAL IN WHICH ONE OR MORE DIGITS REPEAT FOREVER IS CALLED A REPEATING DECIMAL.

Write $0.04 \div 0.3$ as a fraction in simplest form.



When we multiply $0.A \times 0.B$, we usually get *two* digits right of the decimal point.

For example,
 $0.8 \times 0.9 = 0.72$.

But, if the hundredths digit is 0, we don't write it.

For example,
 $0.2 \times 0.5 = 0.10$, which is the same as
 $0.2 \times 0.5 = 0.1$.

For $0.A \times 0.B$ to have a 0 in the hundredths place, $A \times B$ must be a multiple of 10.

For $A \times B$ to be a multiple of 10, one of the digits must be 5...

...and the other must be even.

We want to make $A+B+C$ to be as big as we can. 8 is the largest even digit. $8 \times 5 = 40$, so $0.8 \times 0.5 = 0.4$. That makes $A+B+C$...

17!

Correct!
The Little Monsters lead by a score of 2 to 0.

3, 1.5, 1, 0.75, 0.6, —

Question 3:
What decimal comes next in this pattern?

What's next?



I found something!

First, I wrote 1.5, 0.75, and 0.6 as simplified fractions.

3, 1.5, 1, 0.75, 0.6, ___

$\frac{3}{1}$, $\frac{3}{2}$, 1, $\frac{3}{4}$, $\frac{3}{5}$, ___



I didn't see a pattern, so I wrote the whole numbers as fractions, too.

3, 1.5, 1, 0.75, 0.6, ___

$\frac{3}{1}$, $\frac{3}{2}$, $\frac{1}{1}$, $\frac{3}{4}$, $\frac{3}{5}$, ___



Since all of the other fractions had 3 in the numerator, I wrote $\frac{1}{1}$ as $\frac{3}{3}$.

3, 1.5, 1, 0.75, 0.6, ___

$\frac{3}{1}$, $\frac{3}{2}$, $\frac{3}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, ___



Next in the pattern is $\frac{3}{6}$!
As a decimal, that's...

3, 1.5, 1, 0.75, 0.6, ___

$\frac{3}{1}$, $\frac{3}{2}$, $\frac{3}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{6}$

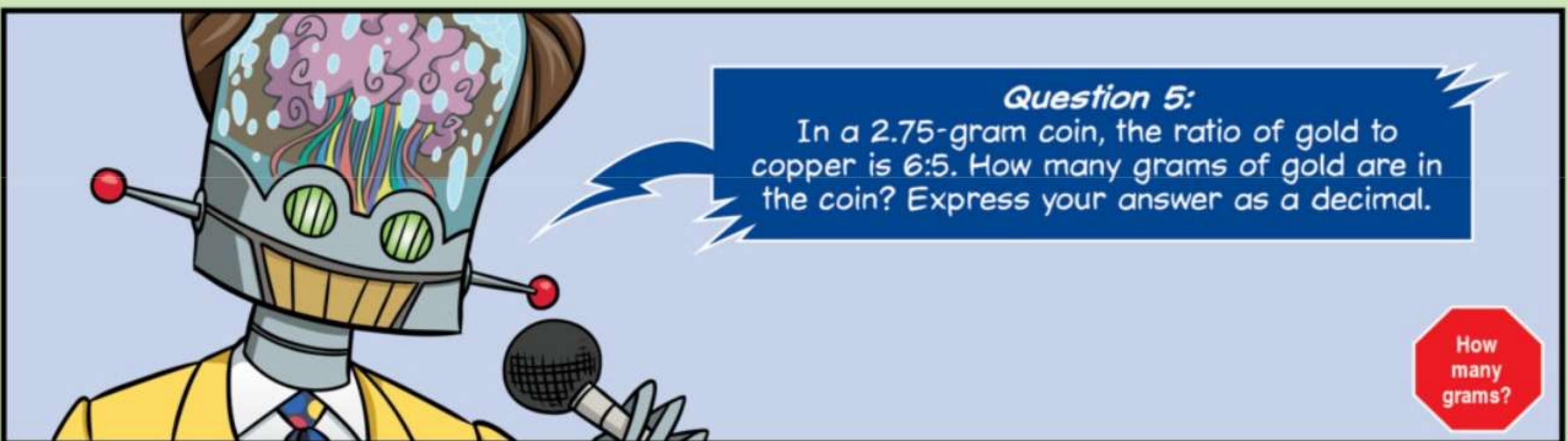
ding!



Try it.



REVIEW UNITS DIGIT COMPUTATIONS IN CHAPTER 2 OF BEAST ACADEMY 4A.



Question 5:

In a 2.75-gram coin, the ratio of gold to copper is 6:5. How many grams of gold are in the coin? Express your answer as a decimal.

How
many
grams?



The Little Monsters beat the Bots to the buzzer.

1.5 grams of gold!

Correct!
The Little Monsters lead 5 to 0 and have a chance for a perfect score!

Here is the 6th and final question:

Express $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ as a decimal.

We need to be fast. The bots will have no trouble converting all those fractions to decimals.

Maybe we can add all of the fractions first, then convert the answer to a decimal.

I think I see an even faster way!

Find the sum.

