Contents: Chapter 12

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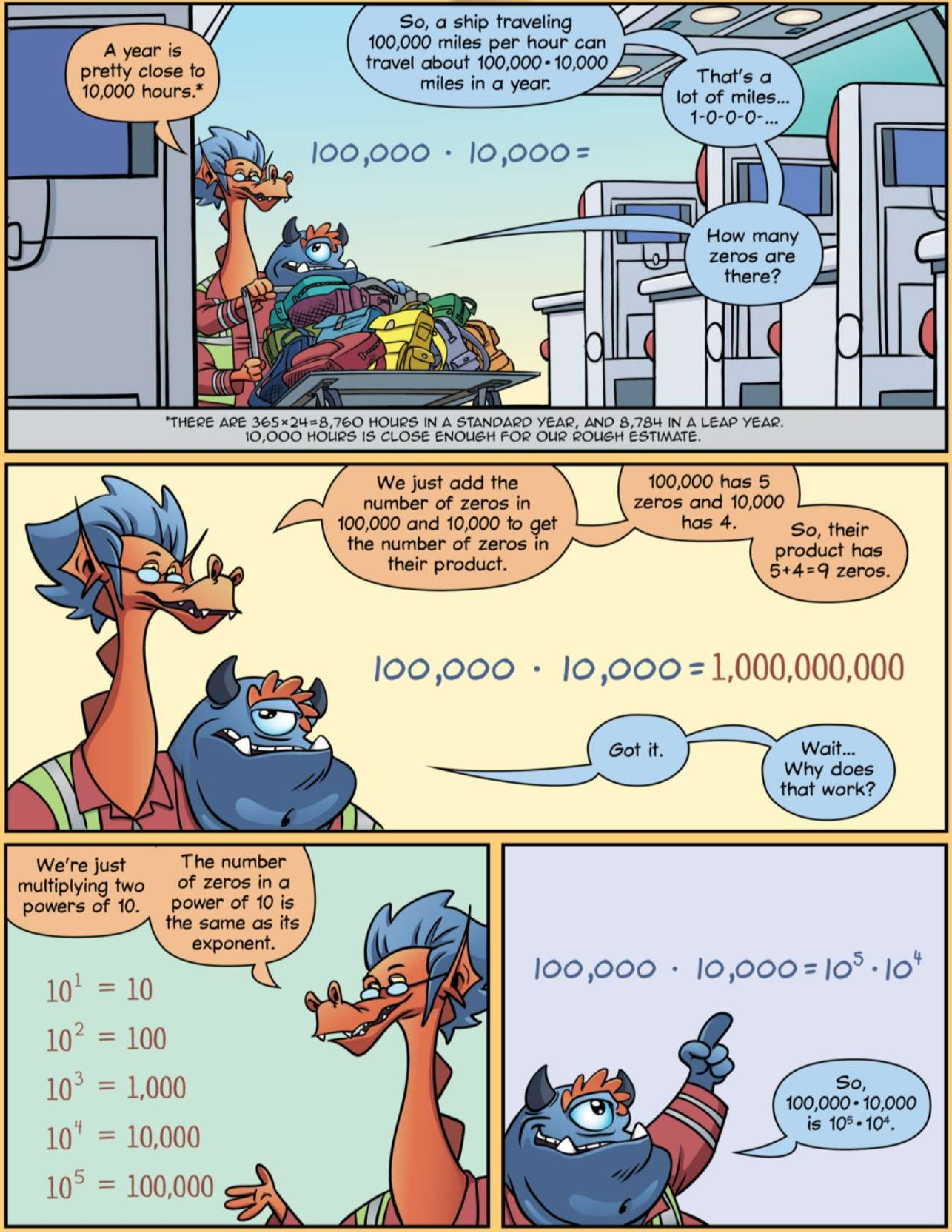
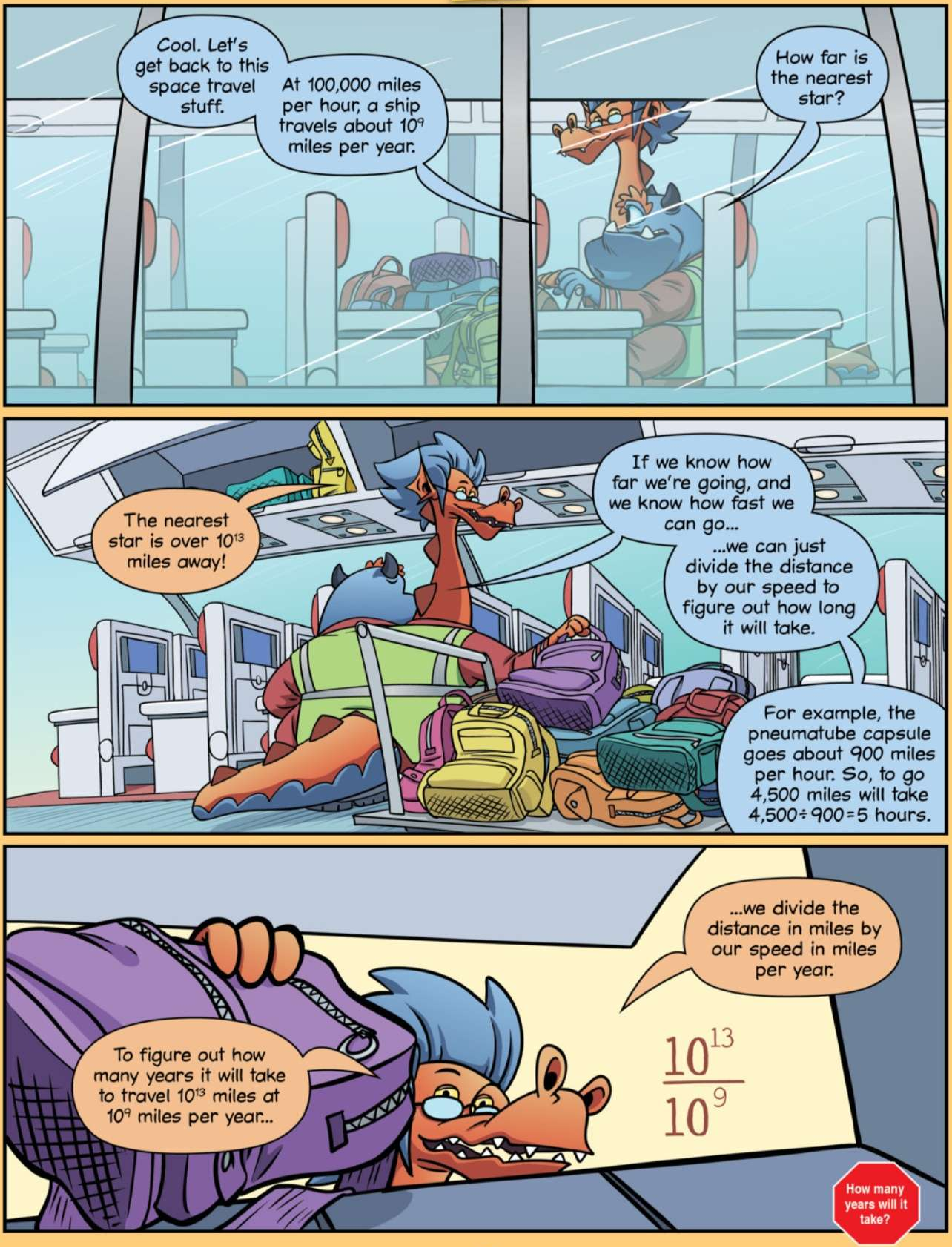
Chapter 12:
Exponents


R&G
Exponents
What are we prepping the pneumatube capsule for?
The math team is going to the World Math Olympiad!
Oh, right! Is that bushy-tailed kid still on the team?
Nope, he got invited to join the Star Search program.
The talent showcase TV show hosted by Ed McMonster where contestants compete for fame and fortune?
Nope, the space program dedicated to exploring neighboring star systems to discover extraterrestrial lifeforms.
Ohhh... *That* Star Search.
But, the nearest star outside our solar system is really, *really* far away.
Even if you could travel 100,000 miles per hour, it would take...
...a *really* long time.
Let's estimate how far you could travel in one year at 100,000 miles per hour.
Then, we can estimate how many years it would take to travel to the nearest star.

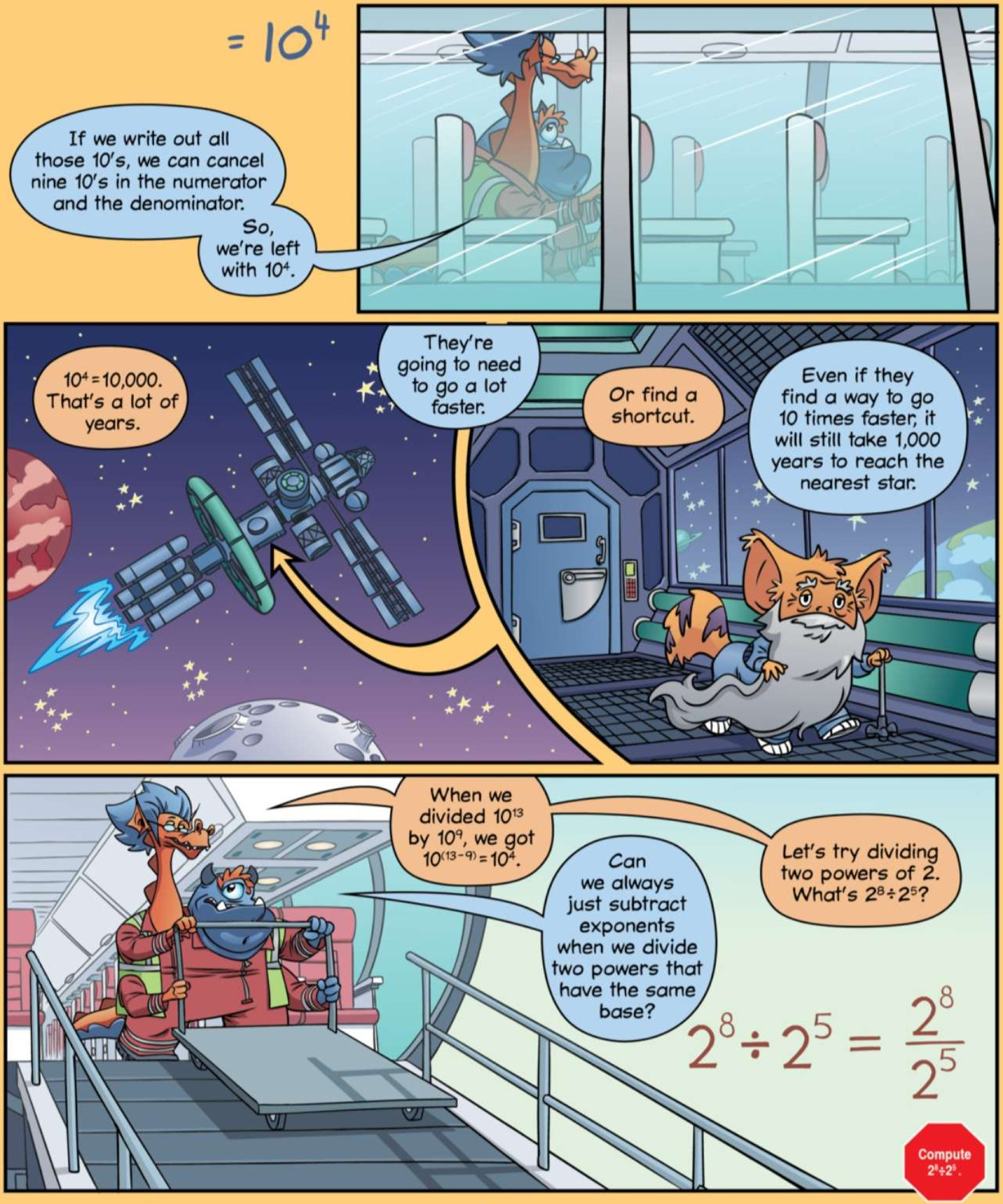

A year is pretty close to 10,000 hours.*
So, a ship traveling 100,000 miles per hour can travel about 100,000 • 10,000 miles in a year.
That's a lot of miles... 1-0-0-0-...
100,000 · 10,000 =
How many zeros are there?
*THERE ARE 365×24=8,760 HOURS IN A STANDARD YEAR, AND 8,784 IN A LEAP YEAR. 10,000 HOURS IS CLOSE ENOUGH FOR OUR ROUGH ESTIMATE.
We just add the number of zeros in 100,000 and 10,000 to get the number of zeros in their product.
100,000 has 5 zeros and 10,000 has 4.
So, their product has 5+4=9 zeros.
100,000 · 10,000 = 1,000,000,000
Got it.
Wait... Why does that work?
We're just multiplying two powers of 10.
The number of zeros in a power of 10 is the same as its exponent.
$10^1 = 10$
$10^2 = 100$
$10^3 = 1,000$
$10^4 = 10,000$
$10^5 = 100,000$
$100,000 \cdot 10,000 = 10^5 \cdot 10^4$
So, 100,000 • 10,000 is $10^5 \cdot 10^4$.

WHEN MULTIPLYING TWO POWERS OF THE SAME BASE, $a^m \cdot a^n = a^{m+n}$.


Cool. Let's get back to this space travel stuff.
At 100,000 miles per hour, a ship travels about 10^9 miles per year.
How far is the nearest star?
The nearest star is over 10^{13} miles away!
If we know how far we're going, and we know how fast we can go...
...we can just divide the distance by our speed to figure out how long it will take.
For example, the pneumatube capsule goes about 900 miles per hour. So, to go 4,500 miles will take 4,500÷900=5 hours.
To figure out how many years it will take to travel 10^{13} miles at 10^9 miles per year...
...we divide the distance in miles by our speed in miles per year.
$\frac{10^{13}}{10^9}$
How many years will it take?

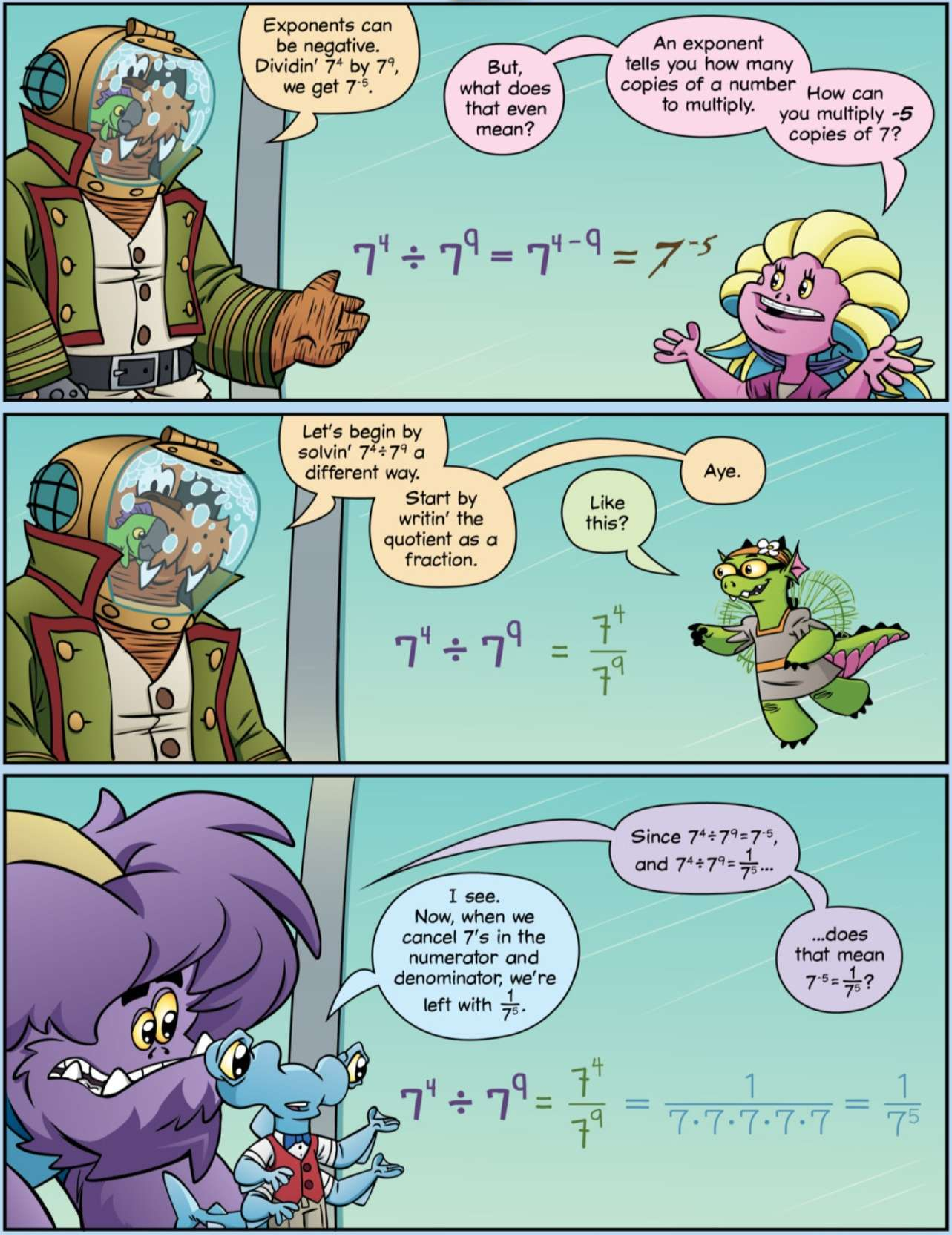
$$\frac{10^{13}}{10^{9}} = \frac{\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot 10\cdot 10\cdot 10\cdot 10}{\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}}$$

$$= 10^4$$

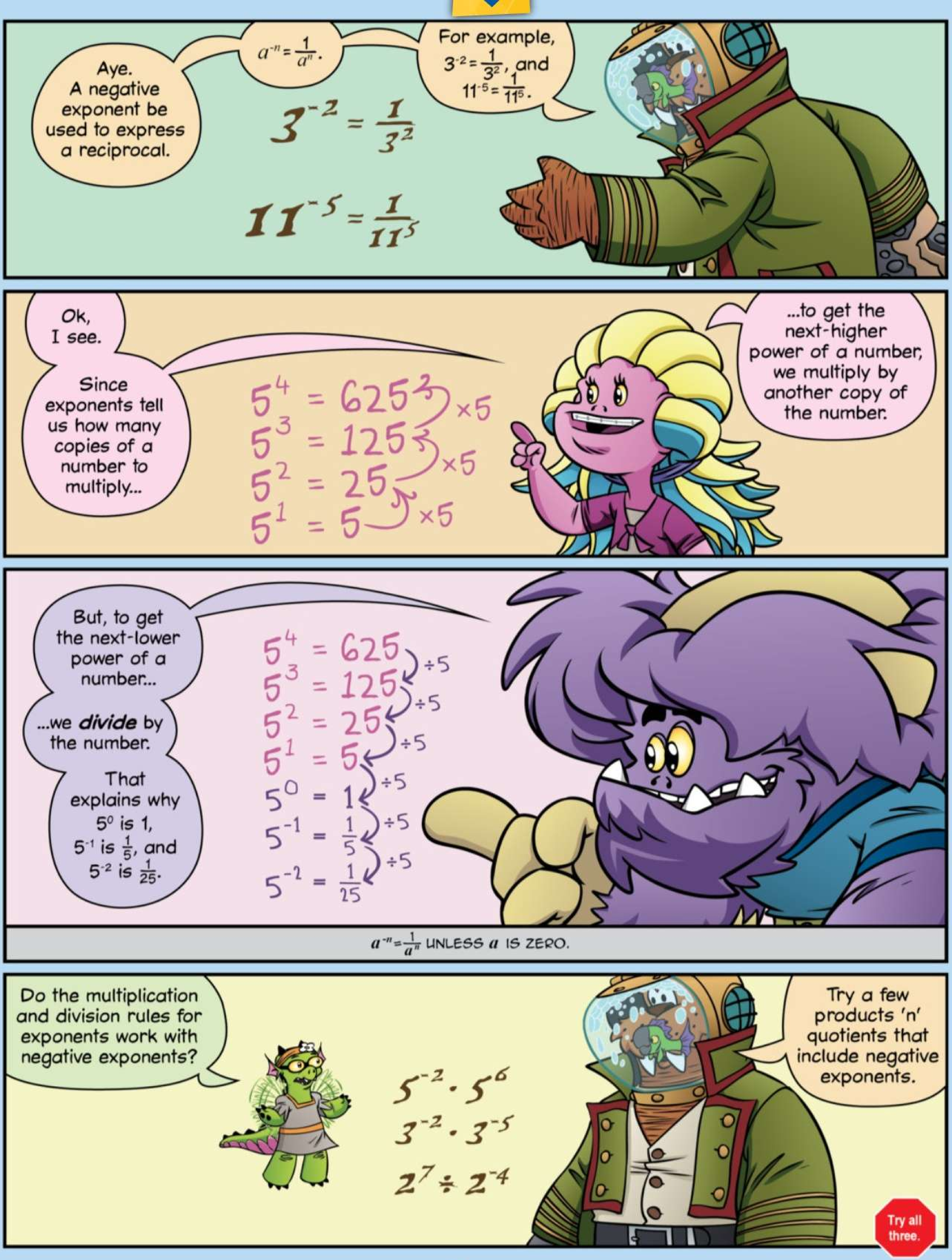



Writing out all of the 2's, we can cancel five 2's in the numerator and the denominator...
...which leaves us with $2 \cdot 2 \cdot 2 = 2^3$.
We got the same answer we would have gotten if we just subtracted exponents.
$\frac{2^8}{2^5} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}$
$= 2^3$
$2^8 \div 2^5 = 2^{8-5} = 2^3$.
When dividing two powers of the same base, we subtract their exponents.
Cool!
Do you think we'll ever find life on other planets?
$2^8 \div 2^5 = 2^{8-5} = 2^3$
WHEN DIVIDING TWO POWERS OF THE SAME BASE, $a^m \div a^n = a^{m-n}$.
I bet aliens are really freaky looking.

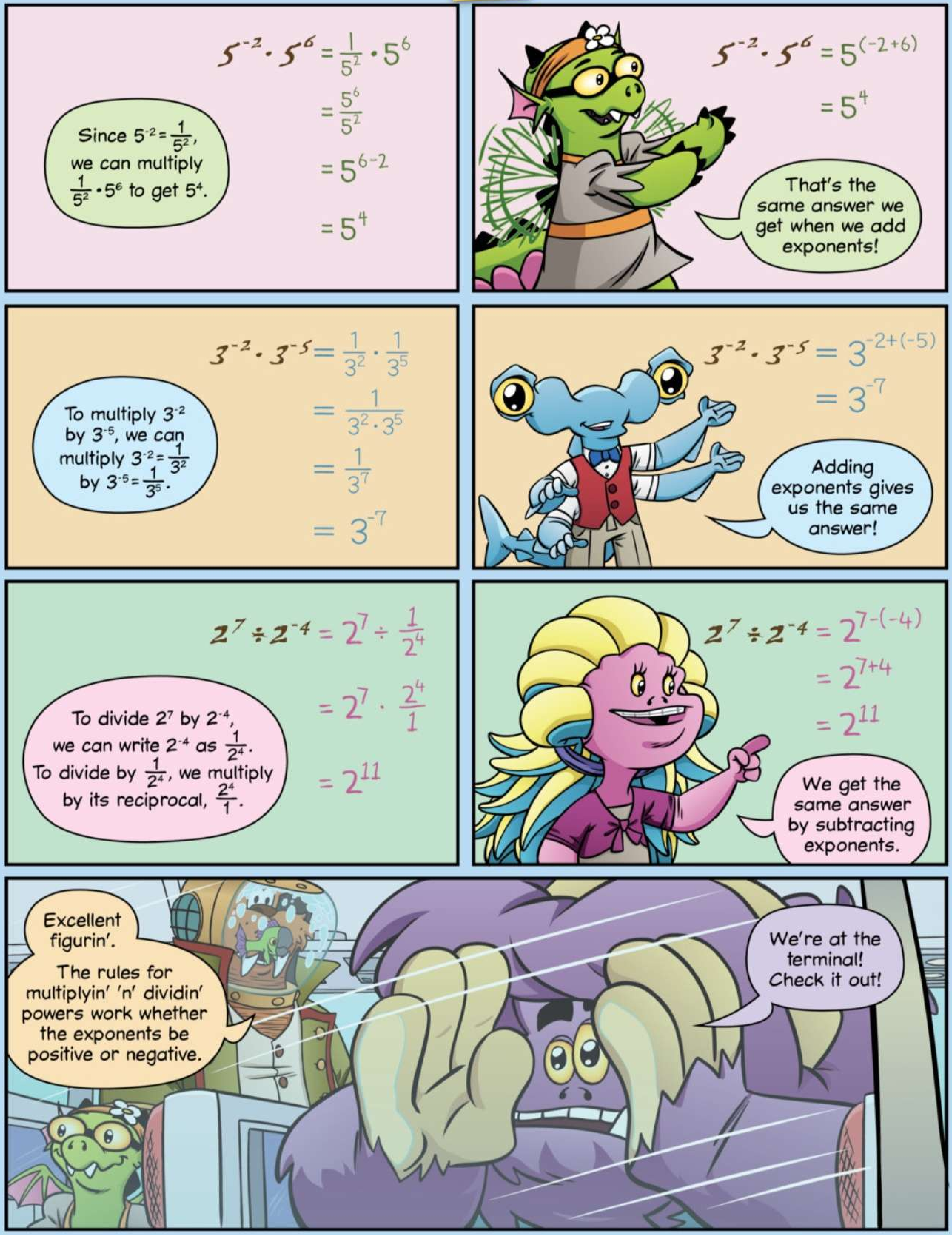

NEGATIVE EXPONENTS
Captain Kraken! I didn't know you were coming with us.
Aye. I haven't missed the World Math Olympiad Championships in 20 years.
I wasn't about to miss this one!
Are any of the other teachers coming?
Of course! But, they'll be takin' a later tube, along with your parents.
The Headmaster asked that I ride with the team to help out during the trip.
Plus, I love ridin' the pneumatube.
Perfect. Maybe you can help explain something.
I'll do my best, lad.
You can divide two powers of the same number by subtracting their exponents, right?
Aye.
But, that means that when we divide 7^4 by 7^9, we get 7^{4-9}.
Can an exponent be negative?
$7^4 \div 7^9 = 7^{4-9}$

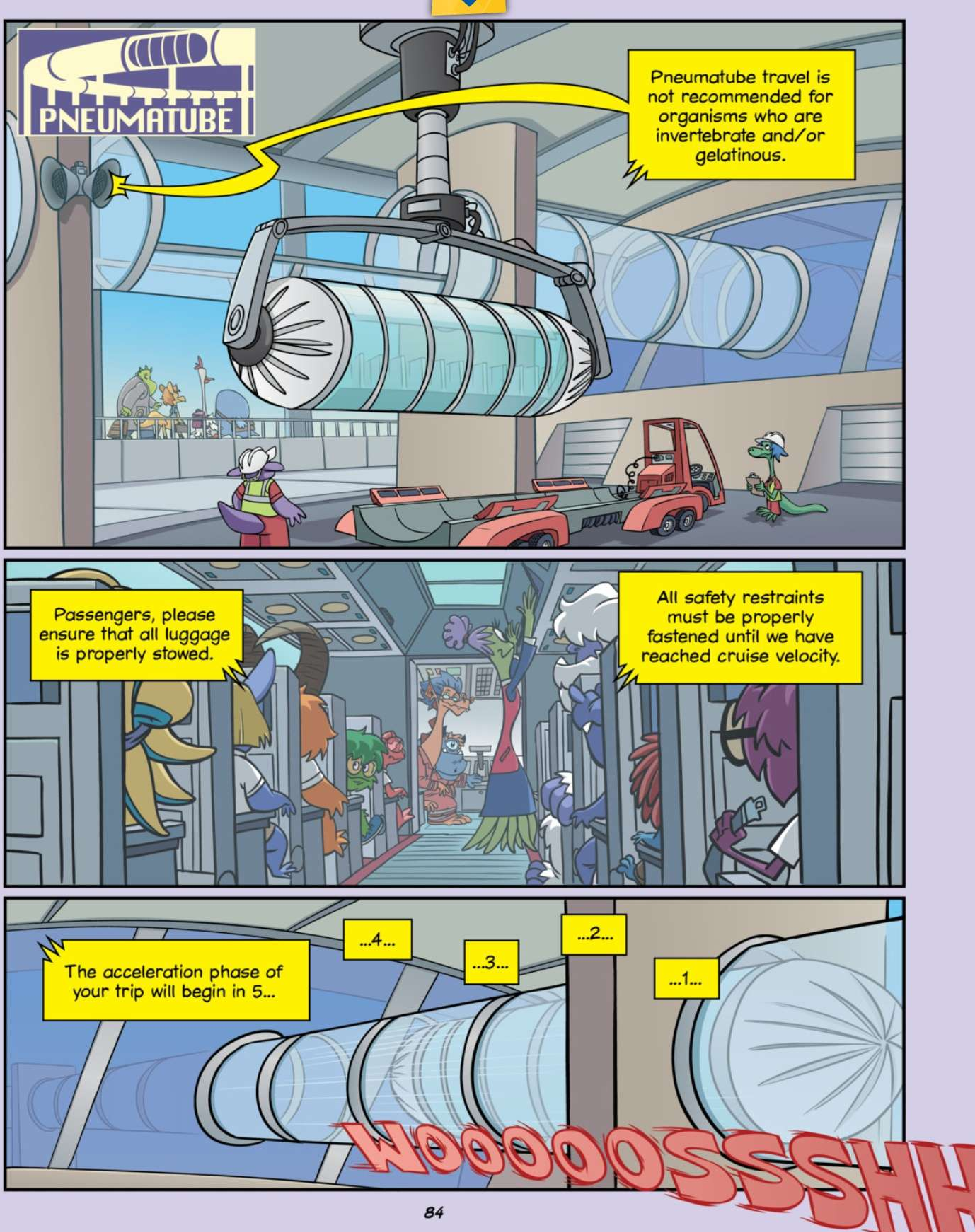


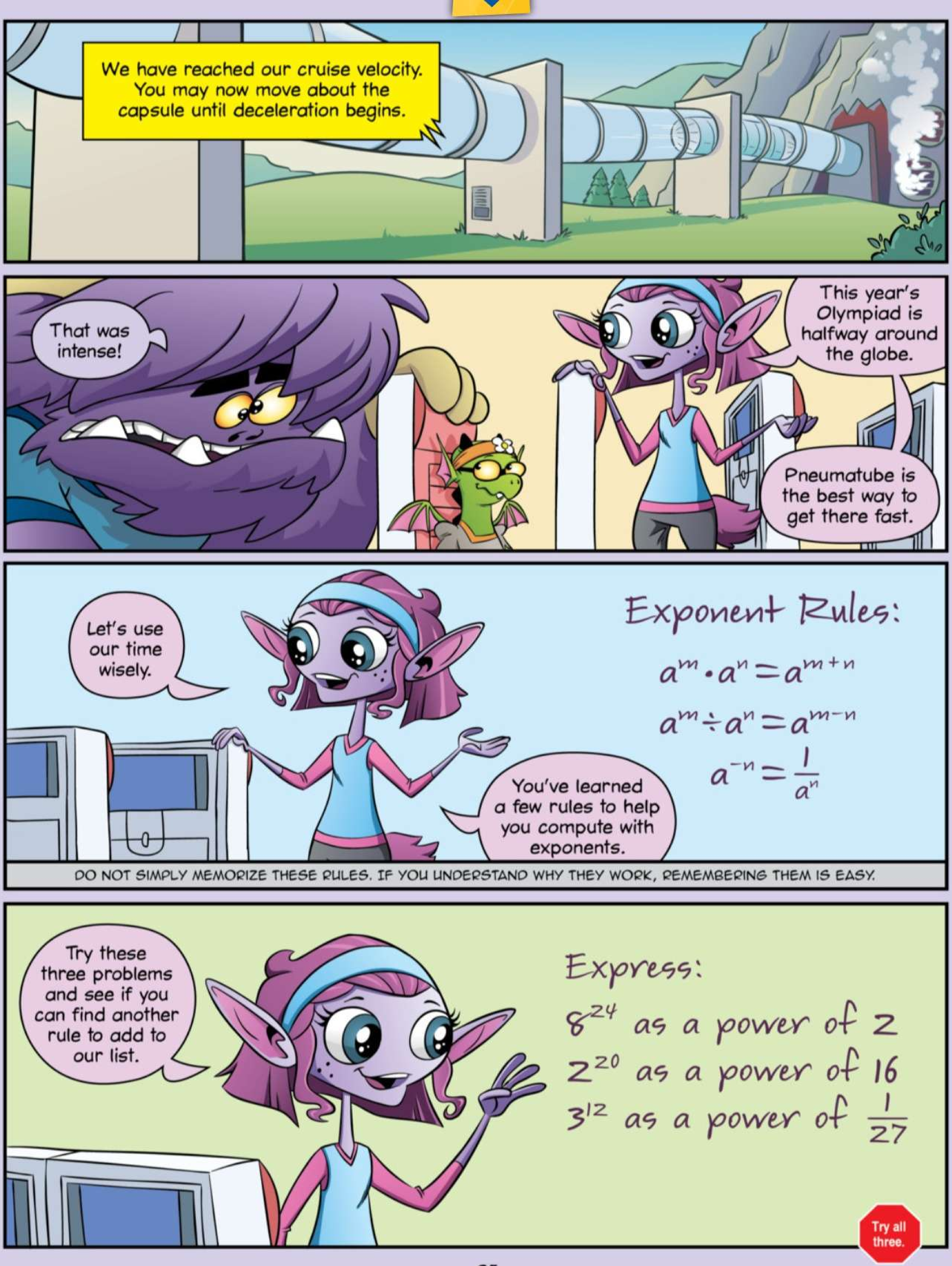
Exponents can be negative. Dividin' 7^4 by 7^9, we get 7^{-5}.
But, what does that even mean?
An exponent tells you how many copies of a number to multiply.
How can you multiply **-5** copies of 7?
$7^4 \div 7^9 = 7^{4-9} = 7^{-5}$
Let's begin by solvin' $7^4 \div 7^9$ a different way.
Start by writin' the quotient as a fraction.
Aye.
Like this?
$7^4 \div 7^9 = \frac{7^4}{7^9}$
Since $7^4 \div 7^9 = 7^{-5}$, and $7^4 \div 7^9 = \frac{1}{7^5}$...
I see. Now, when we cancel 7's in the numerator and denominator, we're left with $\frac{1}{7^5}$.
...does that mean $7^{-5} = \frac{1}{7^5}$?
$7^4 \div 7^9 = \frac{7^4}{7^9} = \frac{1}{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7} = \frac{1}{7^5}$

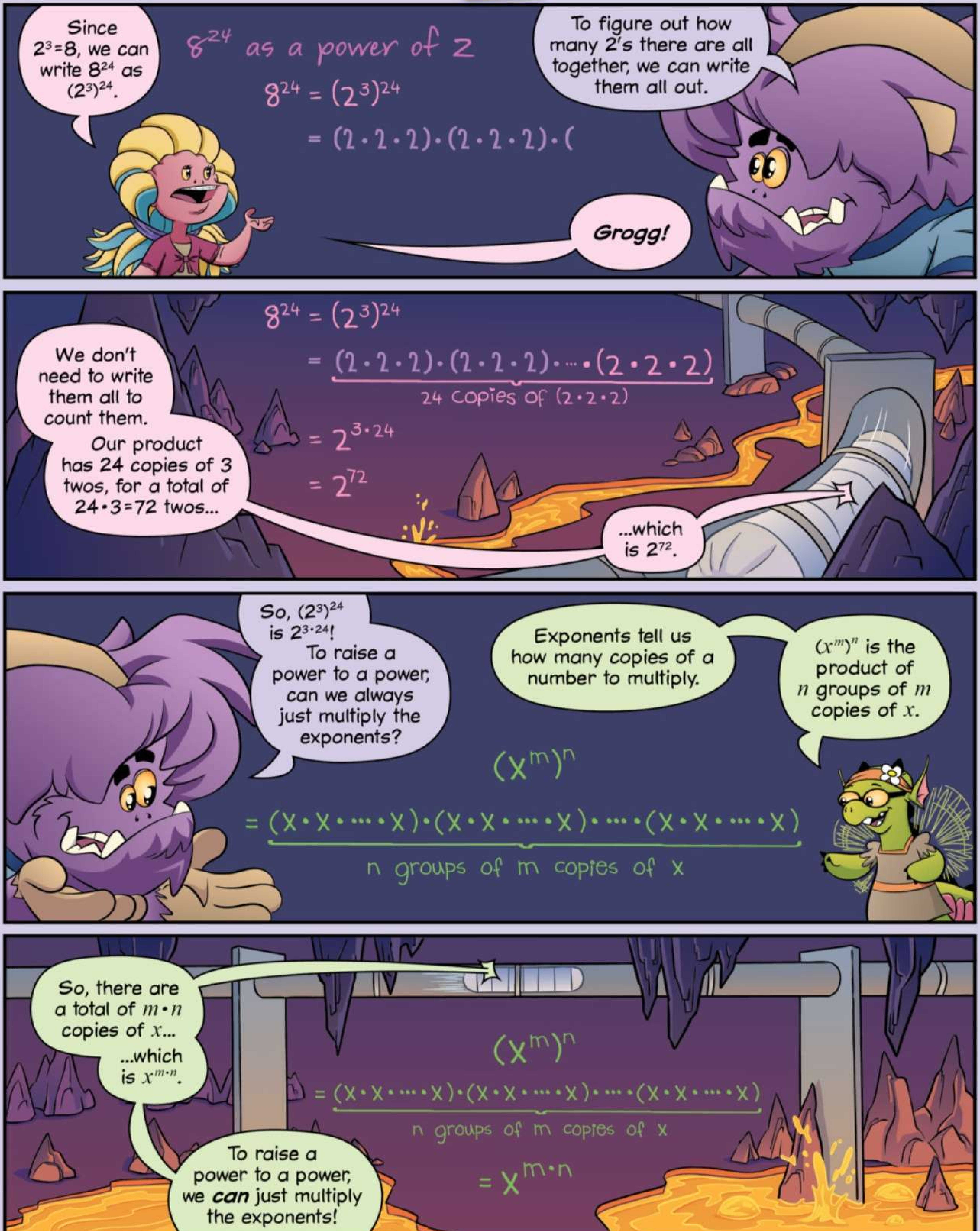


Aye. A negative exponent be used to express a reciprocal.
$a^{-n} = \frac{1}{a^n}$.
For example, $3^{-2} = \frac{1}{3^2}$, and $11^{-5} = \frac{1}{11^5}$.
$3^{-2} = \frac{1}{3^2}$
$11^{-5} = \frac{1}{11^5}$
Ok, I see.
Since exponents tell us how many copies of a number to multiply...
...to get the next-higher power of a number, we multiply by another copy of the number.
$5^4 = 625$
$5^3 = 125$
$5^2 = 25$
$5^1 = 5$
×5
×5
×5
But, to get the next-lower power of a number...
...we ***divide*** by the number.
That explains why 5^0 is 1, 5^{-1} is $\frac{1}{5}$, and 5^{-2} is $\frac{1}{25}$.
$5^4 = 625$
$5^3 = 125$
$5^2 = 25$
$5^1 = 5$
$5^0 = 1$
$5^{-1} = \frac{1}{5}$
$5^{-2} = \frac{1}{25}$
÷5
÷5
÷5
÷5
÷5
÷5
$a^{-n} = \frac{1}{a^n}$ UNLESS a IS ZERO.
Do the multiplication and division rules for exponents work with negative exponents?
Try a few products 'n' quotients that include negative exponents.
$5^{-2} \cdot 5^6$
$3^{-2} \cdot 3^{-5}$
$2^7 \div 2^{-4}$
Try all three.

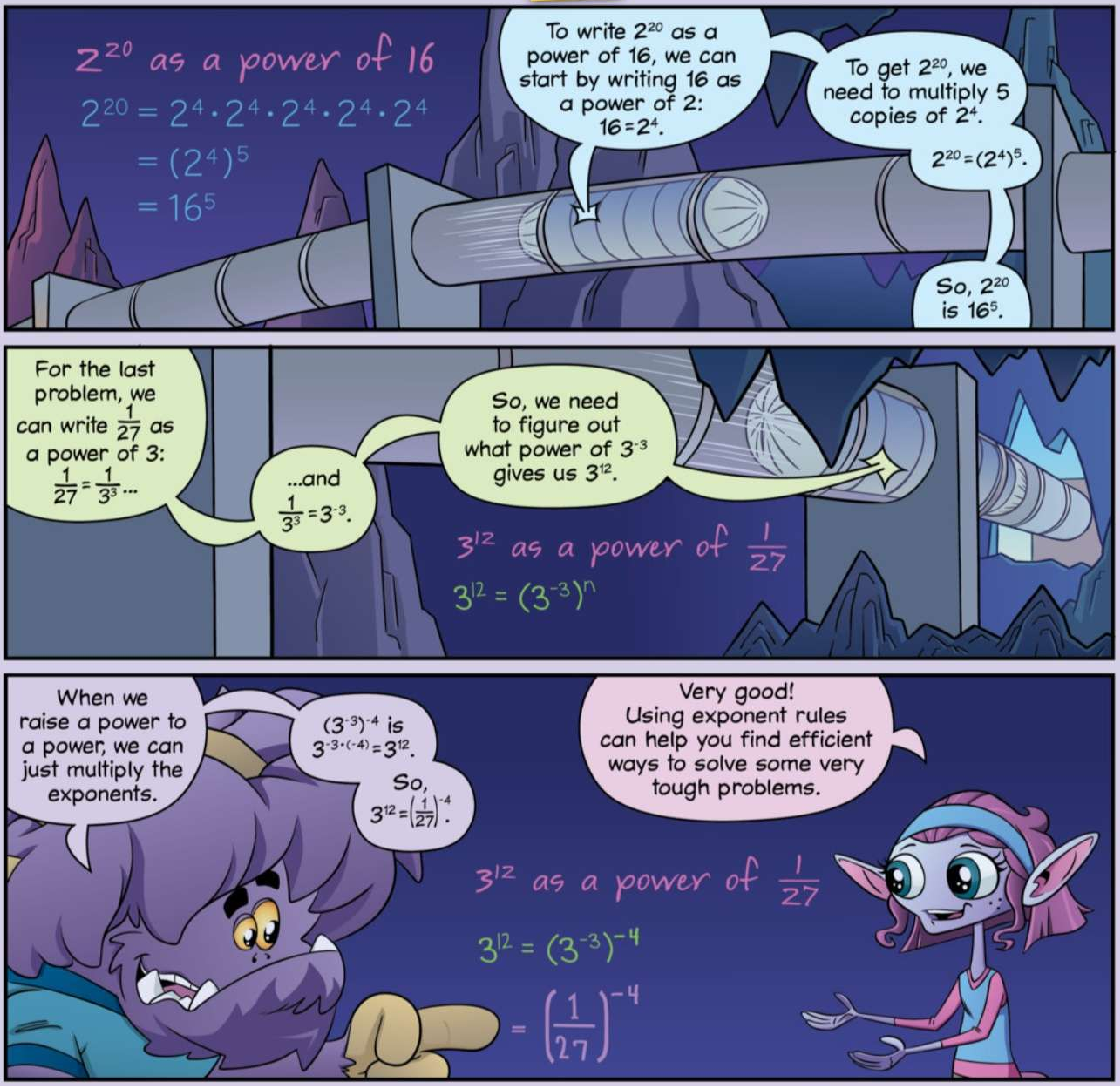

$5^{-2} \cdot 5^{6} = \frac{1}{5^2} \cdot 5^6 = \frac{5^6}{5^2} = 5^{6-2} = 5^4$
Since $5^{-2} = \frac{1}{5^2}$, we can multiply $\frac{1}{5^2} \cdot 5^6$ to get 5^4.
$5^{-2} \cdot 5^{6} = 5^{(-2+6)} = 5^4$
That's the same answer we get when we add exponents!
$3^{-2} \cdot 3^{-5} = \frac{1}{3^2} \cdot \frac{1}{3^5} = \frac{1}{3^2 \cdot 3^5} = \frac{1}{3^7} = 3^{-7}$
To multiply 3^{-2} by 3^{-5}, we can multiply $3^{-2} = \frac{1}{3^2}$ by $3^{-5} = \frac{1}{3^5}$.
$3^{-2} \cdot 3^{-5} = 3^{-2+(-5)} = 3^{-7}$
Adding exponents gives us the same answer!
$2^7 \div 2^{-4} = 2^7 \div \frac{1}{2^4} = 2^7 \cdot \frac{2^4}{1} = 2^{11}$
To divide 2^7 by 2^{-4}, we can write 2^{-4} as $\frac{1}{2^4}$. To divide by $\frac{1}{2^4}$, we multiply by its reciprocal, $\frac{2^4}{1}$.
$2^7 \div 2^{-4} = 2^{7-(-4)} = 2^{7+4} = 2^{11}$
We get the same answer by subtracting exponents.
Excellent figurin'.
The rules for multiplyin' 'n' dividin' powers work whether the exponents be positive or negative.
We're at the terminal! Check it out!


PNEUMATUBE
Pneumatube travel is not recommended for organisms who are invertebrate and/or gelatinous.
Passengers, please ensure that all luggage is properly stowed.
All safety restraints must be properly fastened until we have reached cruise velocity.
The acceleration phase of your trip will begin in 5...
...4...
...3...
...2...
...1...
WOOOOOOSSSSHH


We have reached our cruise velocity. You may now move about the capsule until deceleration begins.
That was intense!
This year's Olympiad is halfway around the globe.
Pneumatube is the best way to get there fast.
Let's use our time wisely.
Exponent Rules:
$a^m \cdot a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$a^{-n} = \frac{1}{a^n}$
You've learned a few rules to help you compute with exponents.
DO NOT SIMPLY MEMORIZE THESE RULES. IF YOU UNDERSTAND WHY THEY WORK, REMEMBERING THEM IS EASY.
Try these three problems and see if you can find another rule to add to our list.
Express:
8^{24} as a power of 2
2^{20} as a power of 16
3^{12} as a power of $\frac{1}{27}$
Try all three.



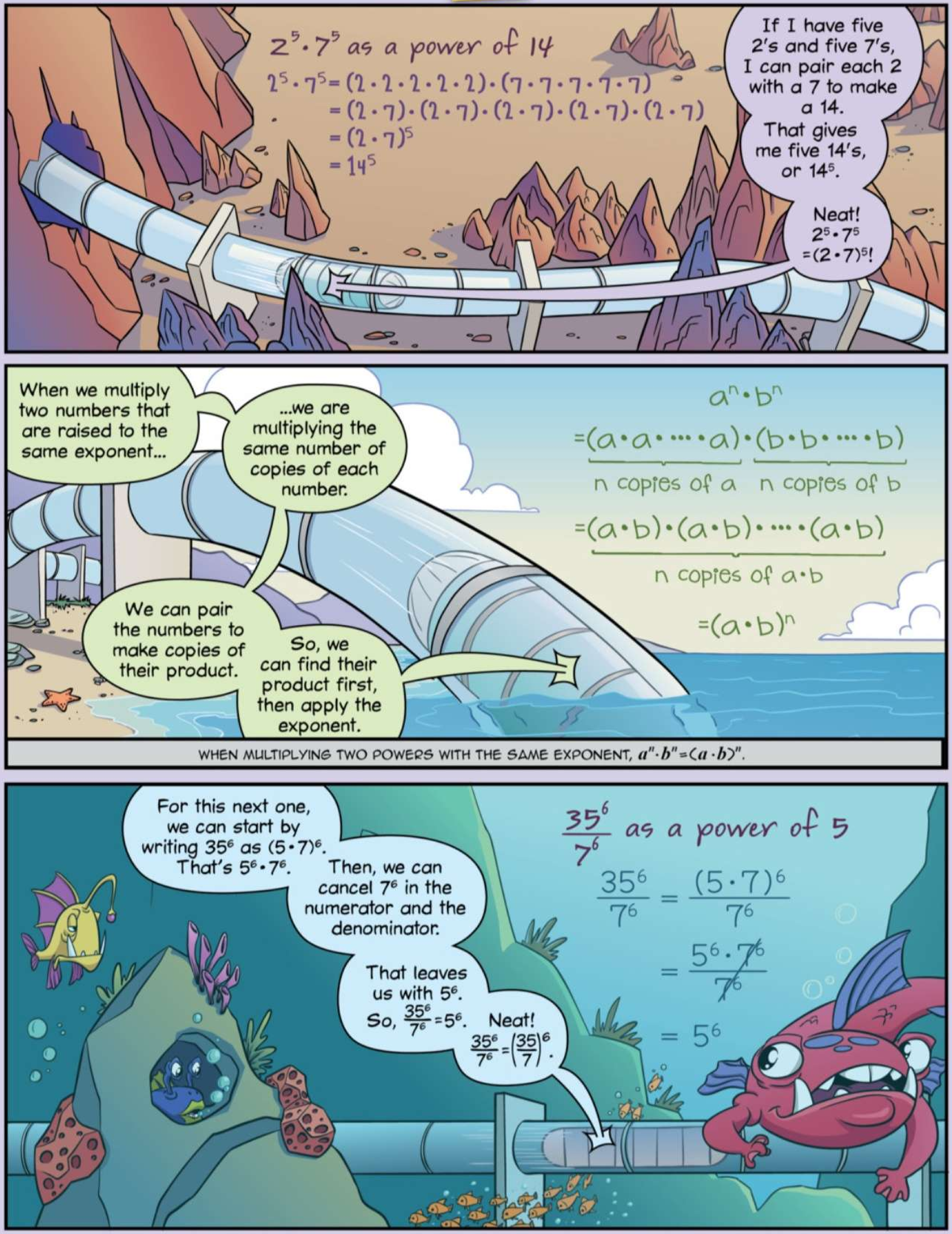
WHEN RAISING A POWER TO A POWER, $(x^m)^n = x^{m \cdot n}$.

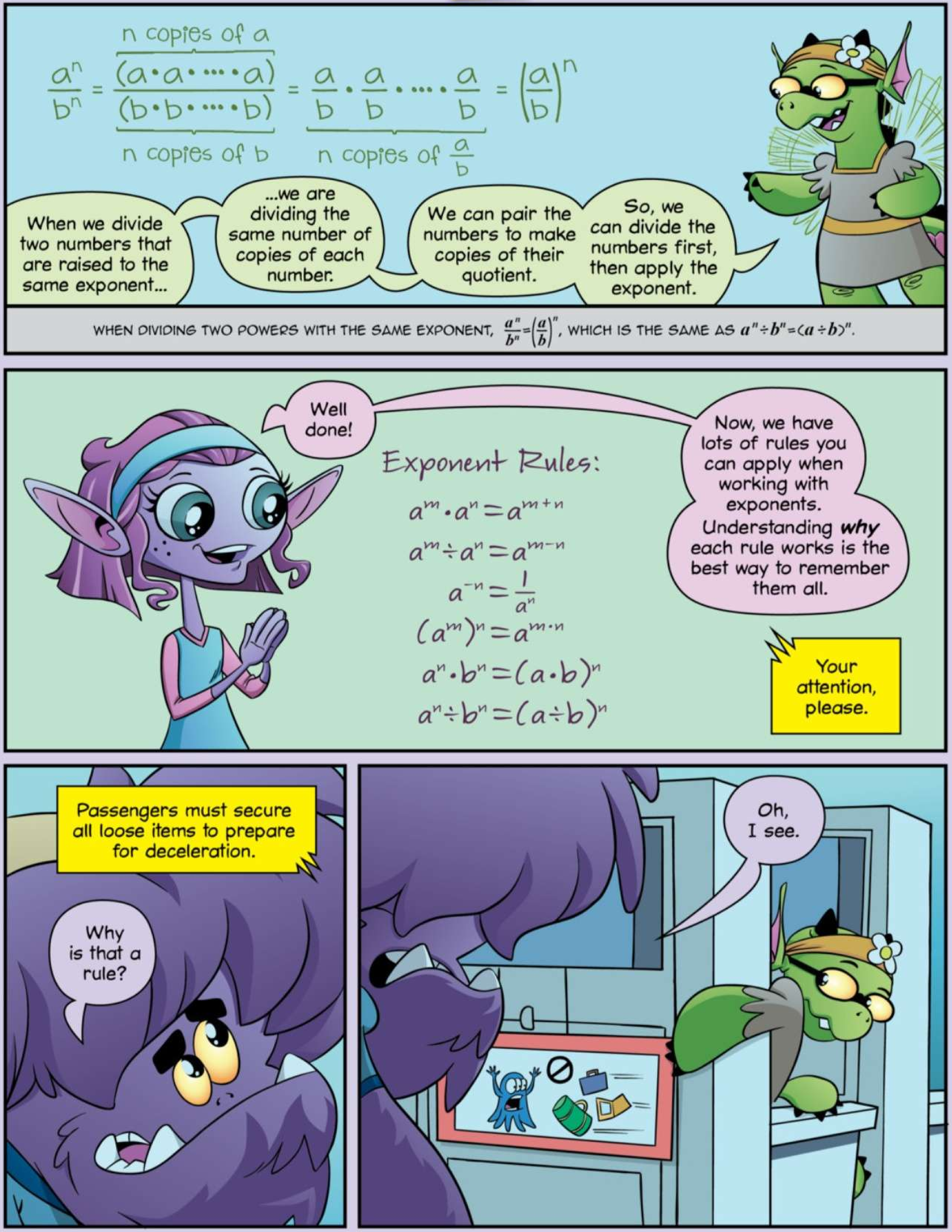


Express:

$2^5 \cdot 7^5$ as a power of 14

$\frac{35^6}{7^6}$ as a power of 5


$2^5 \cdot 7^5$ as a power of 14
$2^5 \cdot 7^5 = (2\cdot2\cdot2\cdot2\cdot2)\cdot(7\cdot7\cdot7\cdot7\cdot7)$
$= (2\cdot7)\cdot(2\cdot7)\cdot(2\cdot7)\cdot(2\cdot7)\cdot(2\cdot7)$
$= (2\cdot7)^5$
$= 14^5$
If I have five 2's and five 7's, I can pair each 2 with a 7 to make a 14. That gives me five 14's, or 14^5.
Neat! $2^5 \cdot 7^5 = (2\cdot7)^5$!
When we multiply two numbers that are raised to the same exponent...
...we are multiplying the same number of copies of each number.
We can pair the numbers to make copies of their product.
So, we can find their product first, then apply the exponent.
$a^n \cdot b^n$
$= \underbrace{(a\cdot a\cdot \cdots \cdot a)}_{n \text{ copies of } a}\cdot\underbrace{(b\cdot b\cdot \cdots \cdot b)}_{n \text{ copies of } b}$
$= \underbrace{(a\cdot b)\cdot(a\cdot b)\cdot \cdots \cdot(a\cdot b)}_{n \text{ copies of } a\cdot b}$
$= (a\cdot b)^n$
WHEN MULTIPLYING TWO POWERS WITH THE SAME EXPONENT, $a^n \cdot b^n = (a\cdot b)^n$.
For this next one, we can start by writing 35^6 as $(5\cdot7)^6$. That's $5^6\cdot7^6$.
Then, we can cancel 7^6 in the numerator and the denominator.
That leaves us with 5^6. So, $\frac{35^6}{7^6} = 5^6$.
Neat! $\frac{35^6}{7^6} = \left(\frac{35}{7}\right)^6$.
$\frac{35^6}{7^6}$ as a power of 5
$\frac{35^6}{7^6} = \frac{(5\cdot7)^6}{7^6}$
$= \frac{5^6\cdot\cancel{7^6}}{\cancel{7^6}}$
$= 5^6$



$\frac{a^n}{b^n} = \frac{\overbrace{(a \cdot a \cdot \cdots \cdot a)}^{n \text{ copies of } a}}{\underbrace{(b \cdot b \cdot \cdots \cdot b)}_{n \text{ copies of } b}} = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \cdots \cdot \frac{a}{b}}_{n \text{ copies of } \frac{a}{b}} = \left(\frac{a}{b}\right)^n$
When we divide two numbers that are raised to the same exponent...
...we are dividing the same number of copies of each number.
We can pair the numbers to make copies of their quotient.
So, we can divide the numbers first, then apply the exponent.
WHEN DIVIDING TWO POWERS WITH THE SAME EXPONENT, $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$, WHICH IS THE SAME AS $a^n \div b^n = (a \div b)^n$.
Well done!
Exponent Rules:
$a^m \cdot a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$a^{-n} = \frac{1}{a^n}$
$(a^m)^n = a^{m \cdot n}$
$a^n \cdot b^n = (a \cdot b)^n$
$a^n \div b^n = (a \div b)^n$
Now, we have lots of rules you can apply when working with exponents. Understanding ***why*** each rule works is the best way to remember them all.
Your attention, please.
Passengers must secure all loose items to prepare for deceleration.
Why is that a rule?
Oh, I see.

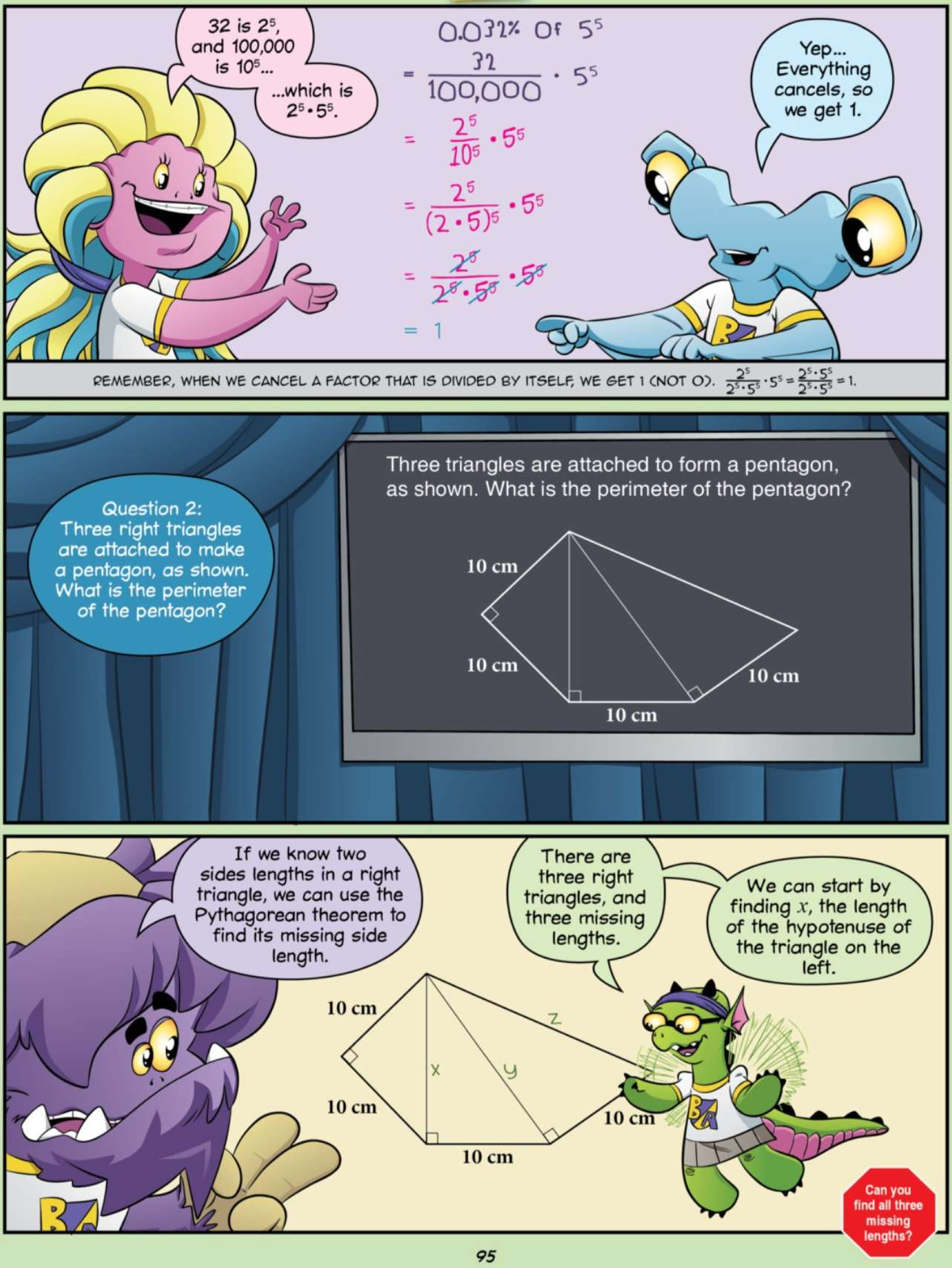

MATH TEAM
The WMOC
Wow!
Are all of these monsters here to watch *us*?
There are competitions for all levels and over a dozen different categories.
But, the competition is just one part of the World Math Olympiad Championships.
It's a huge event!


The world's greatest math beasts come every year to meet, share ideas, and celebrate math.
The top science, engineering, and computer beasts come, too...
...since math is such a big part of what they do.
STAR SEARCH
Whoa! There's Max!
Hey, team! I was hoping I'd see you here!
How's the Star Search program?
Amazing. I am learning so much!
When do you guys compete?
STAR SEARCH
The written contest starts at 4:30.
We'd better get going.
I'll find you guys after the competition. I can't wait to watch you in the finals!
STAR SEARCH

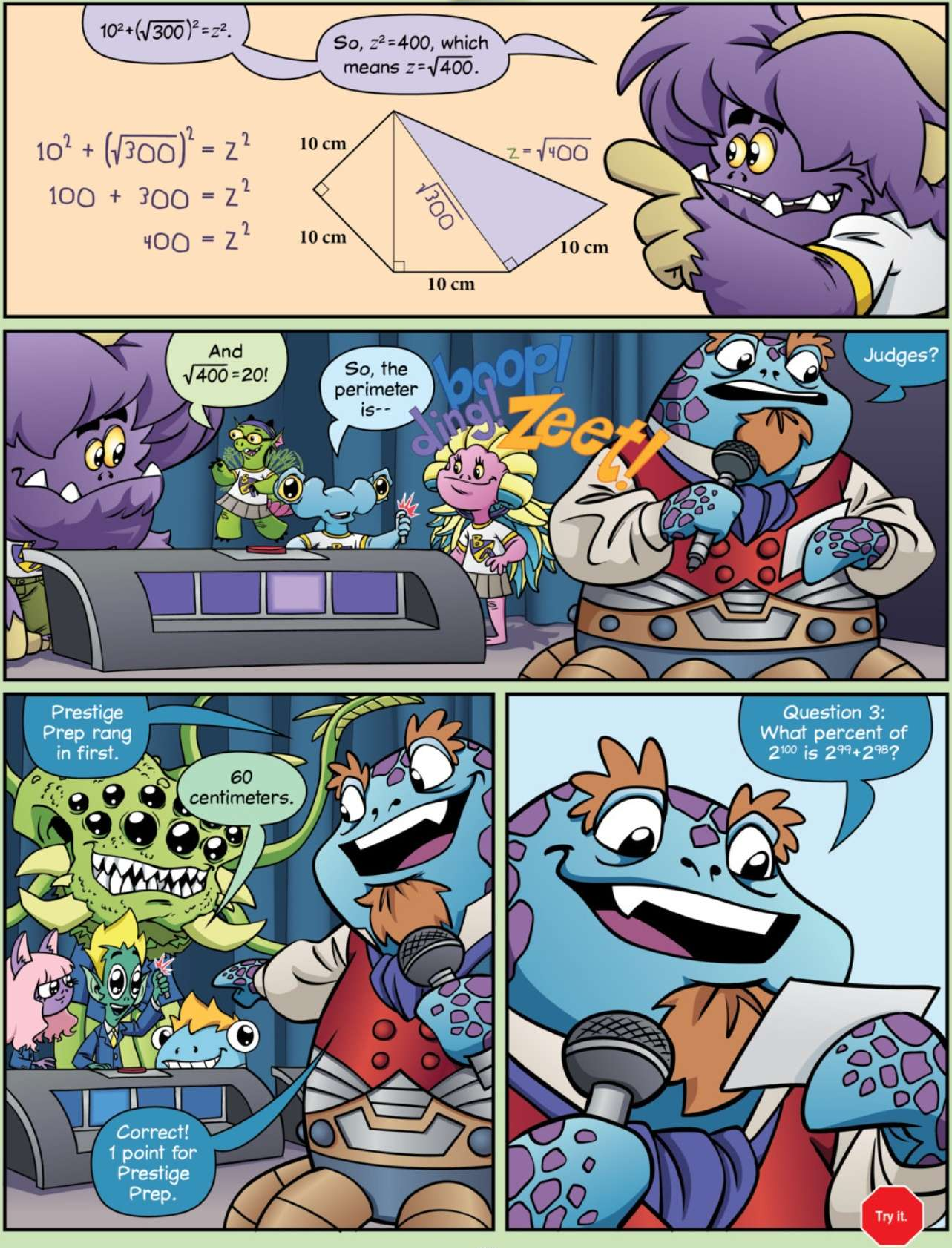

CHANNEL 5
We're back, coming to you live from the World Math Olympiad Championships.
We've just received results from the written contest for the elementary division, Mike.
No huge surprises, here.
Three of these teams are familiar to the WMOC stage.
WMO
ON
CHANNE
Excel Elementary is here to defend their championship from last year...
PRESTIGE PREP
Prestige Prep makes the finals for the ninth year in a row, along with the Talent School team that finished fourth last year.
EXCEL ES
...but few expected the team from Beast Academy to make the finals without phenom Max Norris.
TALENT SCHOOL
BEAST ACADEMY
They must be thrilled with their written round performance, Phil.
Few teams could make the finals of the WMOC after the loss of a superstar like Max.
We're in for a treat, Mike.
Let's join the action live as the top four teams make their way on to the stage.
WMO
ON
CHANNE

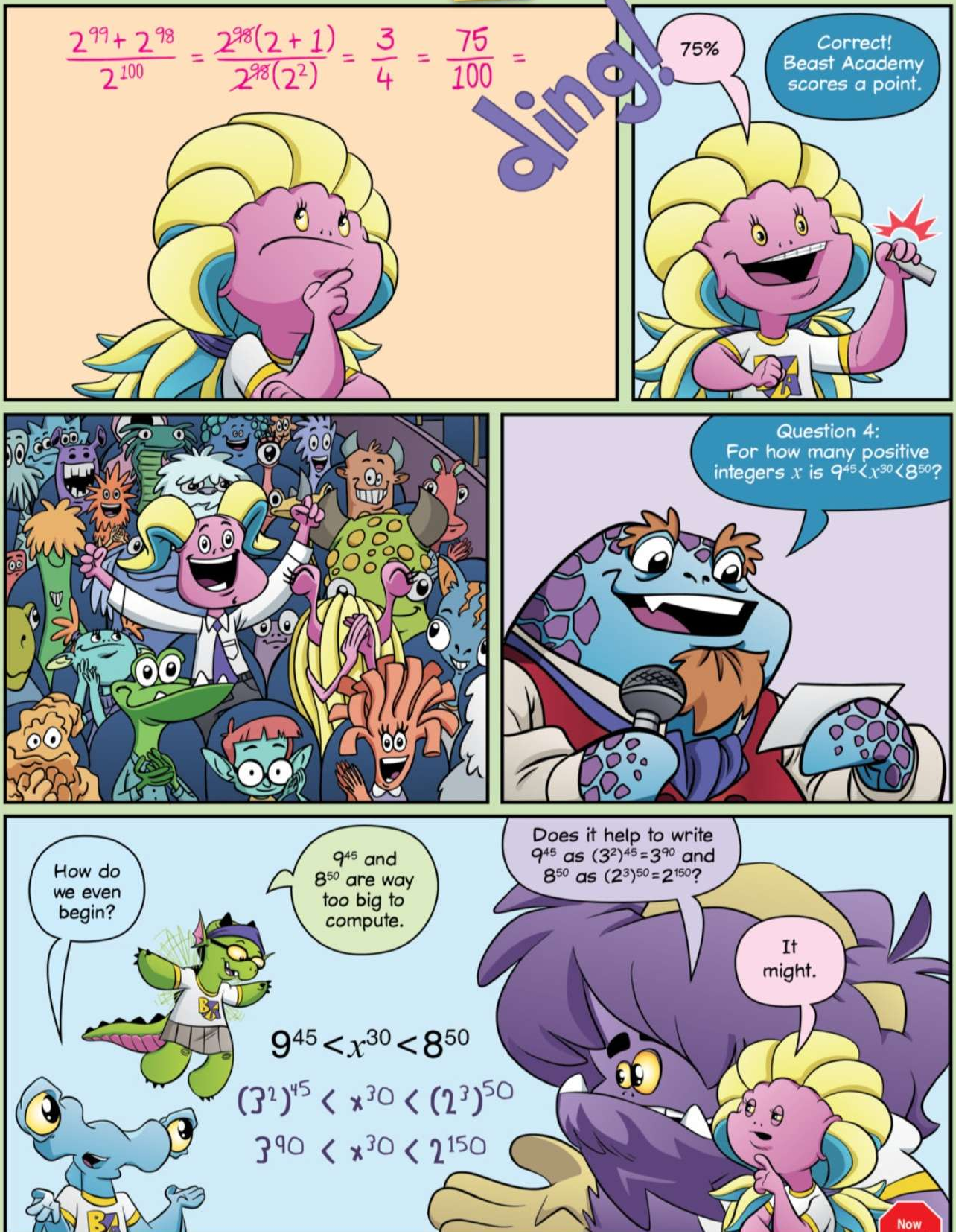

MATH TEAM
The Finals
Welcome, participants and spectators to our championship final in the elementary division.
Teams have been briefed on the rules. The first team to correctly answer 10 questions will be crowned champion. Teams, please test your buzzers before our first question.
boop!
bzzzt!
Zeet!
ding!
Excellent! Let's begin.
Question 1: What is 0.032% of 5^5?
Try it.

That's a really small percent of a big number.
If we set up the multiplication, I bet there will be lots of stuff that cancels.
0.032% is 0.032 over 100. We need to multiply that by 5^5.
$0.032\% = \frac{0.032}{100}$
First, we can eliminate the decimal in $\frac{0.032}{100}$ by multiplying its numerator and denominator by 1,000.
$0.032\% = \frac{0.032}{100} = \frac{32}{100,000}$
•1,000
•1,000
So, 0.032% of 5^5 is $\frac{32}{100,000} \cdot 5^5$.
0.032% of 5^5
$= \frac{32}{100,000} \cdot 5^5$
bzzzt!
1!
1 is correct. The first point goes to Excel Elementary.
Of course! Everything cancels!


32 is 2^5, and 100,000 is 10^5...
...which is 2^5 • 5^5.
0.032% of 5^5
= 32/100,000 · 5^5
= 2^5/10^5 • 5^5
= 2^5/(2•5)^5 • 5^5
= 2^5/(2^5•5^5) • 5^5
= 1
Yep... Everything cancels, so we get 1.
REMEMBER, WHEN WE CANCEL A FACTOR THAT IS DIVIDED BY ITSELF, WE GET 1 (NOT 0). 2^5/(2^5·5^5) · 5^5 = (2^5·5^5)/(2^5·5^5) = 1.
Question 2: Three right triangles are attached to make a pentagon, as shown. What is the perimeter of the pentagon?
Three triangles are attached to form a pentagon, as shown. What is the perimeter of the pentagon?
10 cm
10 cm
10 cm
10 cm
If we know two sides lengths in a right triangle, we can use the Pythagorean theorem to find its missing side length.
There are three right triangles, and three missing lengths.
We can start by finding x, the length of the hypotenuse of the triangle on the left.
10 cm
10 cm
10 cm
10 cm
x
y
z
Can you find all three missing lengths?


$10^2+10^2=x^2$. Since 10^2+10^2 is $100+100=200$, we have $x^2=200$.
10 cm
10 cm
$x=\sqrt{200}$
10 cm
10 cm
That means $x=\sqrt{200}$.
But, $\sqrt{200}$ isn't an integer.
It's a little more than $\sqrt{196}=14$.
We don't need to know how big $\sqrt{200}$ is to find the next missing side length.
We can just use $x=\sqrt{200}$.
$10^2+(\sqrt{200})^2=y^2$
$100+200=y^2$
$300=y^2$
10 cm
$\sqrt{200}$
$y=\sqrt{300}$
10 cm
10 cm
10 cm
With two leg lengths of the middle triangle, we can find its hypotenuse using the Pythagorean theorem. $10^2+(\sqrt{200})^2=y^2$.
So, we have $y^2=300$.
That means $y=\sqrt{300}$.
And now that we know two side lengths of the triangle on the right, we can find the missing side length z.
z
$\sqrt{300}$
10 cm
10 cm
Find the perimeter of the pentagon.


$10^2+(\sqrt{300})^2=z^2$.
So, $z^2=400$, which means $z=\sqrt{400}$.
$10^2 + (\sqrt{300})^2 = z^2$
$100 + 300 = z^2$
$400 = z^2$
10 cm
10 cm
$\sqrt{300}$
$z=\sqrt{400}$
10 cm
10 cm
And $\sqrt{400}=20$!
So, the perimeter is--
boop!
ding!
Zeet!
Judges?
Prestige Prep rang in first.
60 centimeters.
Correct! 1 point for Prestige Prep.
Question 3: What percent of 2^{100} is $2^{99}+2^{98}$?
Try it.



$\frac{2^{99}+2^{98}}{2^{100}} = \frac{\cancel{2^{98}}(2+1)}{\cancel{2^{98}}(2^2)} = \frac{3}{4} = \frac{75}{100} =$
ding!
75%
Correct! Beast Academy scores a point.
Question 4: For how many positive integers x is $9^{45}<x^{30}<8^{50}$?
How do we even begin?
9^{45} and 8^{50} are way too big to compute.
Does it help to write 9^{45} as $(3^2)^{45}=3^{90}$ and 8^{50} as $(2^3)^{50}=2^{150}$?
It might.
$9^{45}<x^{30}<8^{50}$
$(3^2)^{45} < x^{30} < (2^3)^{50}$
$3^{90} < x^{30} < 2^{150}$
Now what?


I know! Since 90 and 150 are multiples of 30, we can write 3^{90} and 2^{150} as numbers to the 30th power!
$9^{45} < x^{30} < 8^{50}$
$(3^2)^{45} < x^{30} < (2^3)^{50}$
$3^{90} < x^{30} < 2^{150}$
$(3^3)^{30} < x^{30} < (2^5)^{30}$
$3 \cdot 30 = 90$, so $(3^3)^{30} = 3^{90}$.
And since $5 \cdot 30 = 150$, $(2^5)^{30} = 2^{150}$.
That gives us $(3^3)^{30} < x^{30} < (2^5)^{30}$.
And since $3^3 = 27$ and $2^5 = 32$, we have $27^{30} < x^{30} < 32^{30}$.
$9^{45} < x^{30} < 8^{50}$
$(3^2)^{45} < x^{30} < (2^3)^{50}$
$3^{90} < x^{30} < 2^{150}$
$(3^3)^{30} < x^{30} < (2^5)^{30}$
$27^{30} < x^{30} < 32^{30}$
Since the question asks for ***positive*** integers, x has to be between 27 and 32.
bzzzt!
Zeet!
Excel Elementary was first to ring in.
8.
Sorry, 8 is not correct.
Talent School rang in next.
4.
Correct! After four questions, the score is tied at 1 all.
THE INTEGERS ARE 28, 29, 30, AND 31. RAISING A NEGATIVE NUMBER TO AN EVEN POWER GIVES A POSITIVE RESULT, SO THERE ARE ALSO FOUR NEGATIVE INTEGERS FOR WHICH $27^{30} < x^{30} < 32^{30}$: −28, −29, −30, AND −31.

Zeet!
What a competition!
55%.
Correct!
All four teams are tied again!
boop!
11 inches.
72.
bzzzt!
Excel Elementary is back in the lead!
ding!
$\sqrt{15}$.
Correct!
We've reached the third break, and after 34 correct answers, we still don't have a winner!
This is the most closely contested competition we've ever seen at the WMOC.
Let's check the scoreboard.
Remember, teams need 10 points to win.
Prestige Prep and Excel Elementary each have 9 points. Beast Academy and Talent School are tied at 8.
THE WMOC Standings
Prestige Prep 9
Excel ES 9
Beast Academy 8
Talent School 8


Teams are returning to the stage.
Let's get back to the action!
Welcome back, competitors. Please take your positions and prepare for the next question.
Question 35: The square root of an integer n is between 9 and 10. The square root of $2n$ is between 14 and 15. What is n?
n has to be less than this...
...but n is greater than that...
So, n has to be...
What is n?


ding!
99.
Correct!
Beast Academy scores their 9th point!
How did you guys...?
Numbers with a square root between 9 and 10 are between $9^2=81$ and $10^2=100$.
So, n is between 81 and 100.
Numbers with a square root between 14 and 15 are between $14^2=196$ and $15^2=225$.
So, $2n$ is between 196 and 225, which means n is between 98 and 112.5.
So, n has to be more than 98, but less than 100.
Since n is an integer, it can only be 99.
Wow, nice teamwork!

Question 36: If $3^{(x-1)}=2$, what is $9^{(x+1)}$?
If $3^{(x-1)}=2$, what is $9^{(x+1)}$?
Scratch Scratch
Scratch Scratch
All four teams are looking stumped, Phil.
This is the kind of problem most high school monsters won't be able to solve, Mike.
High school monsters? I don't even know where to begin with this one!
Try it.

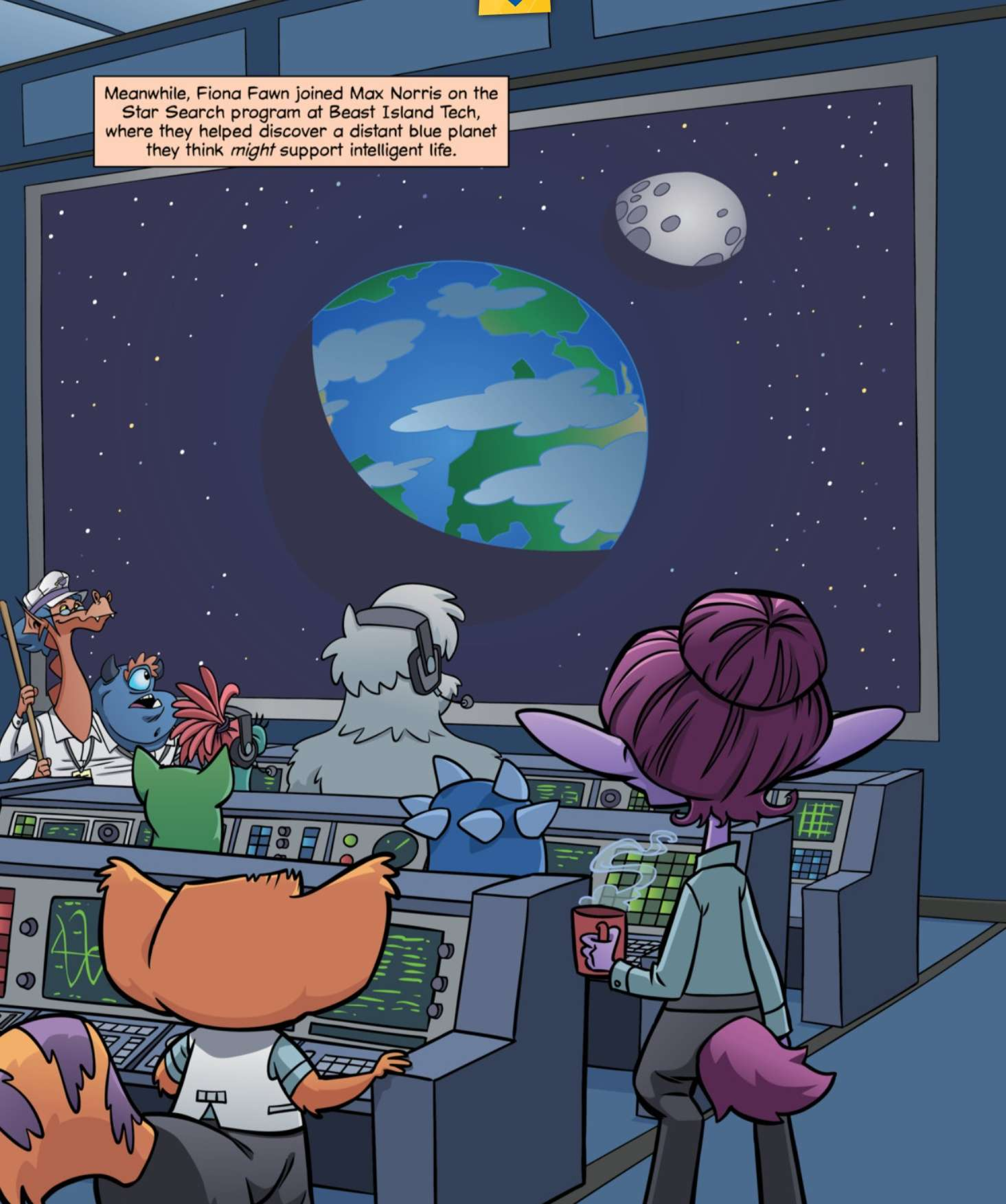
How can a power of 3 equal 2?
Since $3^0=1$ and $3^1=3$, the exponent in the power of 3 must be between 0 and 1.
$3^0 = 1$
$3^{(x-1)} = 2$
$3^1 = 3$
So, if $x-1$ is between 0 and 1, that means x is between 1 and 2.
Maybe we don't need to find x to figure out what $9^{(x+1)}$ equals.
What do you mean, Grogg?
Maybe we can use exponent rules to make the expressions simpler.
For example, in $3^{(x-1)}$, we subtract exponents. That happens when you divide two powers of the same number.
So, $3^{(x-1)}$ is the same as $3^x \div 3^1$, which is $\frac{3^x}{3}$.
And we can multiply both sides of $\frac{3^x}{3}=2$ by 3 to get $3^x=6$.
$3^{(x-1)} = 2$
$\frac{3^x}{3} = 2$
$3^x = 6$
Great idea, Grogg. That is simpler.
What we can do with $9^{(x+1)}$?
When you multiply two powers of the same number, you add their exponents. $9^{(x+1)} = 9^x \cdot 9^1$.
I know what to do next!
$9^{(x+1)}$
$= 9^x \cdot 9$


We can write 9^x as a power of 3^x!
Since $9=3^2$, $9^x=(3^2)^x$.
When raising a power to a power, we can multiply exponents.
So, $(3^2)^x = 3^{(2x)} = 3^{(x\cdot 2)}$.
Finally, we can write $3^{(x\cdot 2)}$ as a power raised to a power! $3^{(x\cdot 2)}=(3^x)^2$.
$9^{(x+1)}$
$= 9^x \cdot 9$
$= (3^2)^x \cdot 9$
$= 3^{(2x)} \cdot 9$
$= 3^{(x\cdot 2)} \cdot 9$
$= (3^x)^2 \cdot 9$
Since $3^x=6$, we can replace 3^x with 6 in $(3^x)^2 \cdot 9$ to get $(6)^2 \cdot 9$! That equals...
$9^{(x+1)}$
$= 9^x \cdot 9$
$= (3^2)^x \cdot 9$
$= 3^{(2x)} \cdot 9$
$= 3^{(x\cdot 2)} \cdot 9$
$= (3^x)^2 \cdot 9$
$= (6)^2 \cdot 9$
ding!
...324!
This could be it, Phil.
The announcer is confirming the answer with the judges.


Correct!
Clap Clap
Beast Academy wins!
Amazing!
Woohoo!
Solid!
Please join me in congratulating the team from Beast Academy, your new World Math Olympiad Champions!


This trip has been awesome.
It's so cool being surrounded by people who love math.
I really hope we can come back next year.
That may be the next time I get to see all of you again.
Huh!?
I'll be working at an outdoors program this summer, then I'm off to Beast Island Tech.
Oh, right. You won't be our coach next year.
Nope. But you'll still have lots of great teachers and coaches in the upper school.
Who is going to coach the new Beast Academy team?


Elizabeth Wyvern coached three Beast Academy teams to Beast Island championships before becoming a decorated professor at BA.
WINNERS!
Clap Clap
Alexander Gardner's investment aptitude earned him enough money to open his own theatre company, where he now directs and performs.
Groggorius George became the world's youngest Competibot champion. He now builds a variety of both extraordinary and practical robots.
And Wendolyn DeMonstre created her own robust programming language, which is now used around the world (and in all of Grogg's bots.)


Meanwhile, Fiona Fawn joined Max Norris on the Star Search program at Beast Island Tech, where they helped discover a distant blue planet they think *might* support intelligent life.