Contents: Chapter 6

Click the Play List tab in the top-left to view a recommended reading/practice sequence.

Addition & Subtraction 78
How can we add fifths to tenths?

Multiplication 84
What's $\frac{1}{7}$ of $\frac{1}{4}$?

More Multiplication 88
How could you compute $\frac{4}{5} \cdot \frac{7}{9}$?

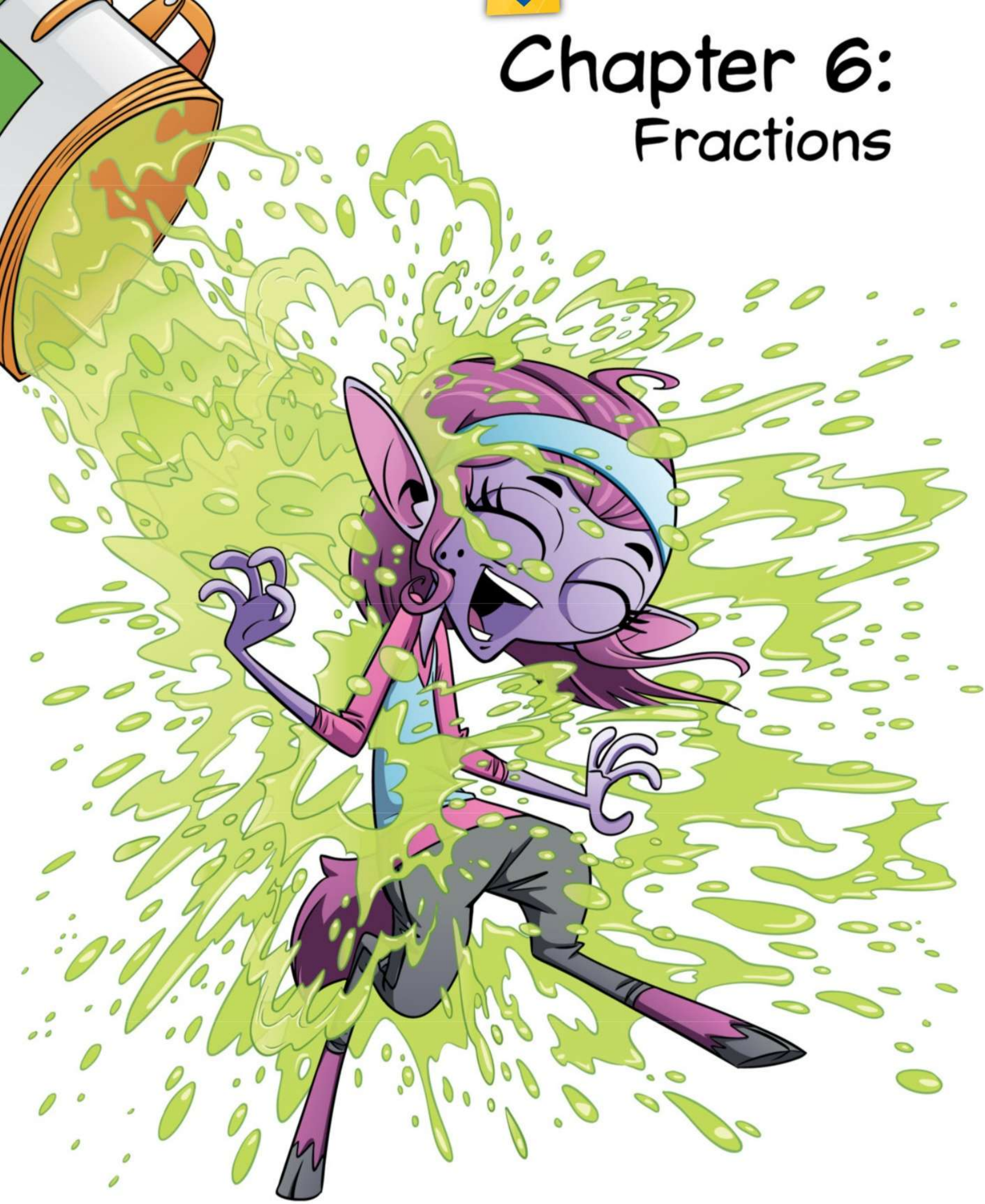
Cancelling 95
How can we simplify $\frac{6}{11} \cdot \frac{3}{10} \cdot \frac{7}{9}$ before we multiply?

Training 96
How do we divide a fraction by another fraction?

Channel 5 102
What sign would you make to cheer on the Beast Academy team?

The Final Round 103
Will the little monsters advance to the World Math Olympiad Championships?

Chapter 6:
Fractions



Ms. Q.
Addition & Subtraction
How do we add and subtract fractions?

To add two fractions with the same denominator, we add their numerators and keep the same denominator.
For example, adding 3 elevenths plus 4 elevenths equals 7 elevenths.
$\frac{3}{11} + \frac{4}{11} = \frac{7}{11}$

Subtraction is similar.
We subtract the numerators, and keep the same denominator.
$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$
If we have 5 sevenths and we take away 2 sevenths, we are left with 3 sevenths.

Very good.
What if the fractions have ***different*** denominators?
For example, how would you add these two fractions?
$\frac{2}{5} + \frac{3}{10}$
Try it.

Both numbers are a little less than $\frac{1}{2}$, so the answer is less than 1...

...but probably greater than $\frac{1}{2}$.

How could you get an ***exact*** answer?

How can we add fifths and tenths? Fifths are bigger than tenths.

What if we ***convert*** the fifths to tenths?

Then, the fractions will have the same denominator.

Great idea, Grogg!

$\frac{2}{5} = \frac{4}{10}$.

So, to compute $\frac{2}{5} + \frac{3}{10}$, we can add $\frac{4}{10} + \frac{3}{10}$.

$$\frac{2}{5} + \frac{3}{10}$$
$$= \frac{4}{10} + \frac{3}{10}$$

$\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$.

So, $\frac{2}{5} + \frac{3}{10}$ is $\frac{7}{10}$!

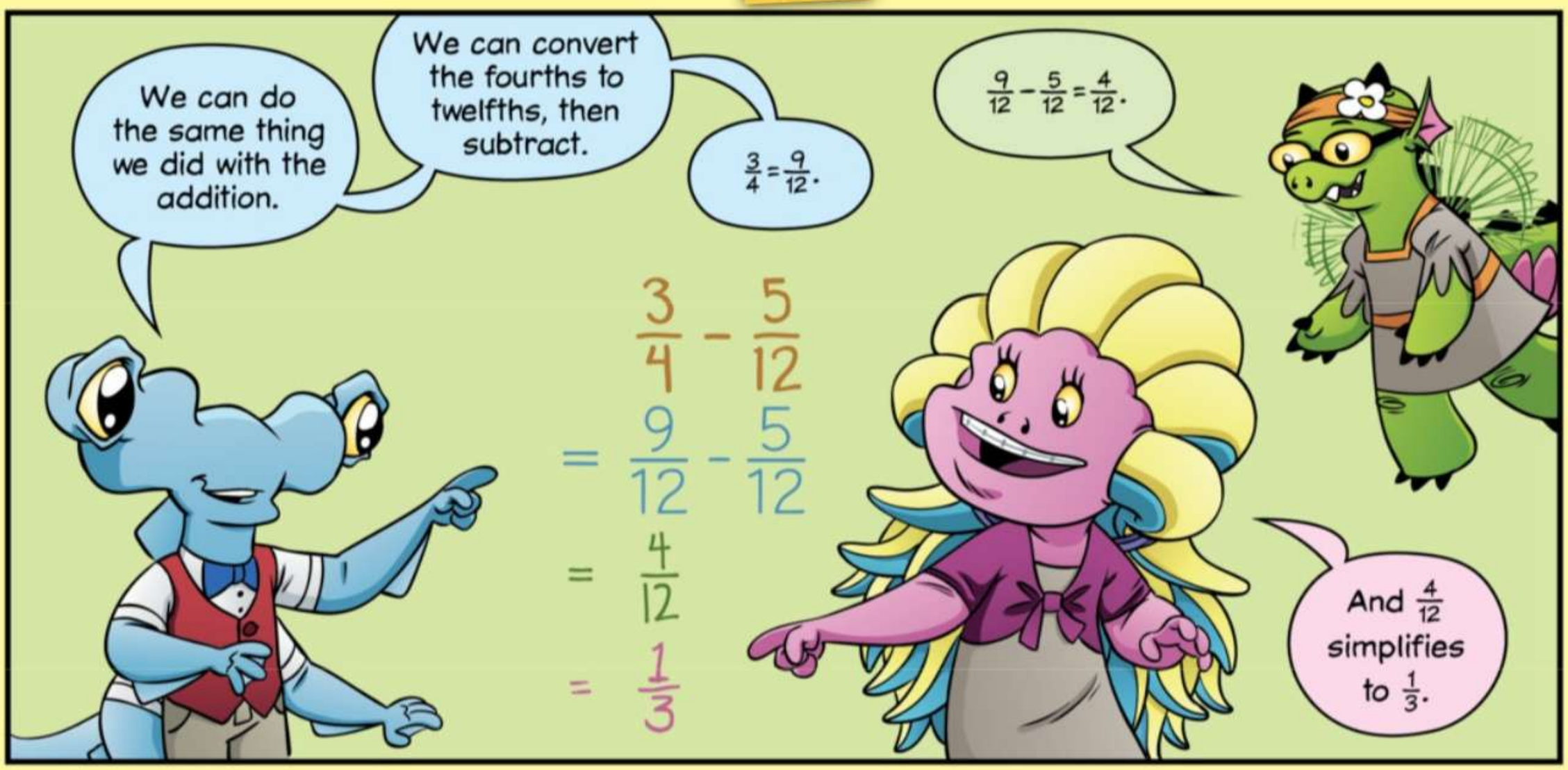
$$\frac{2}{5} + \frac{3}{10}$$
$$= \frac{4}{10} + \frac{3}{10}$$
$$= \frac{7}{10}$$

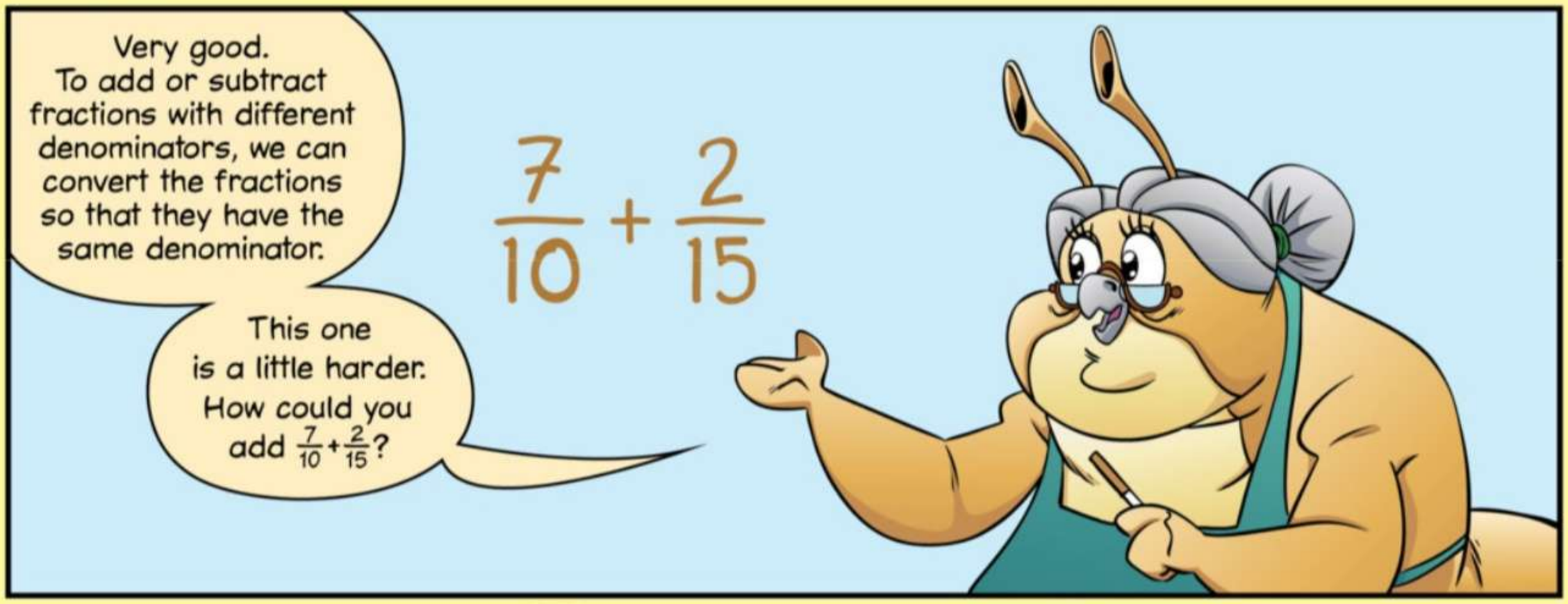
Great work!

How would you compute this difference?

$$\frac{3}{4} - \frac{5}{12}$$

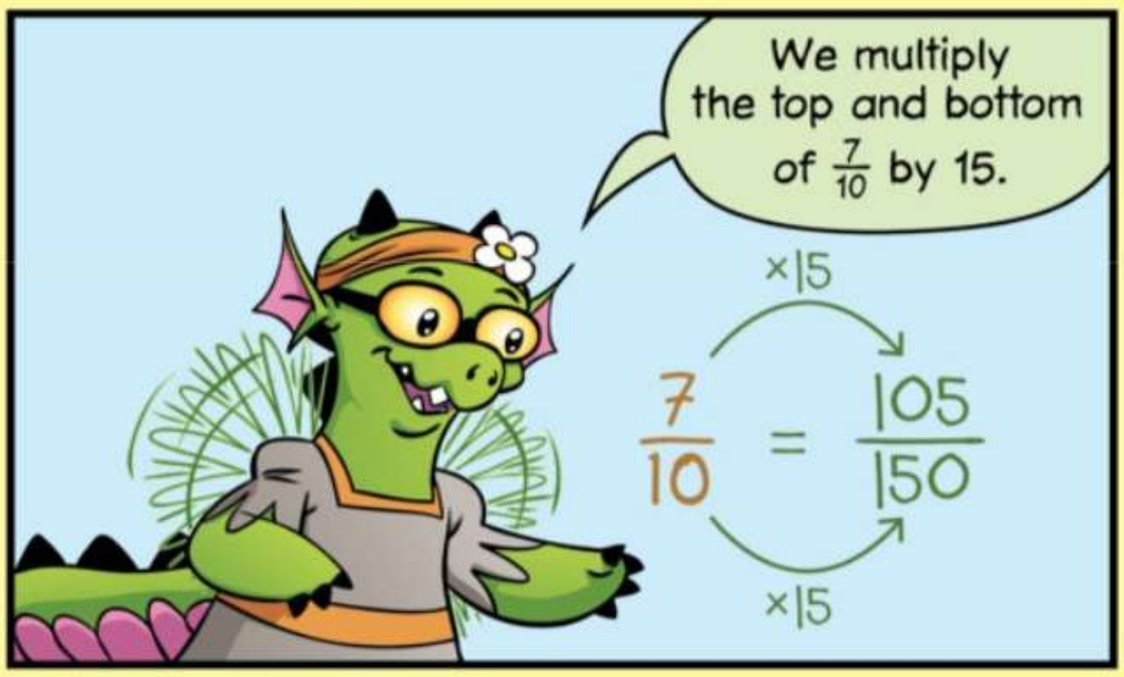
Try it.

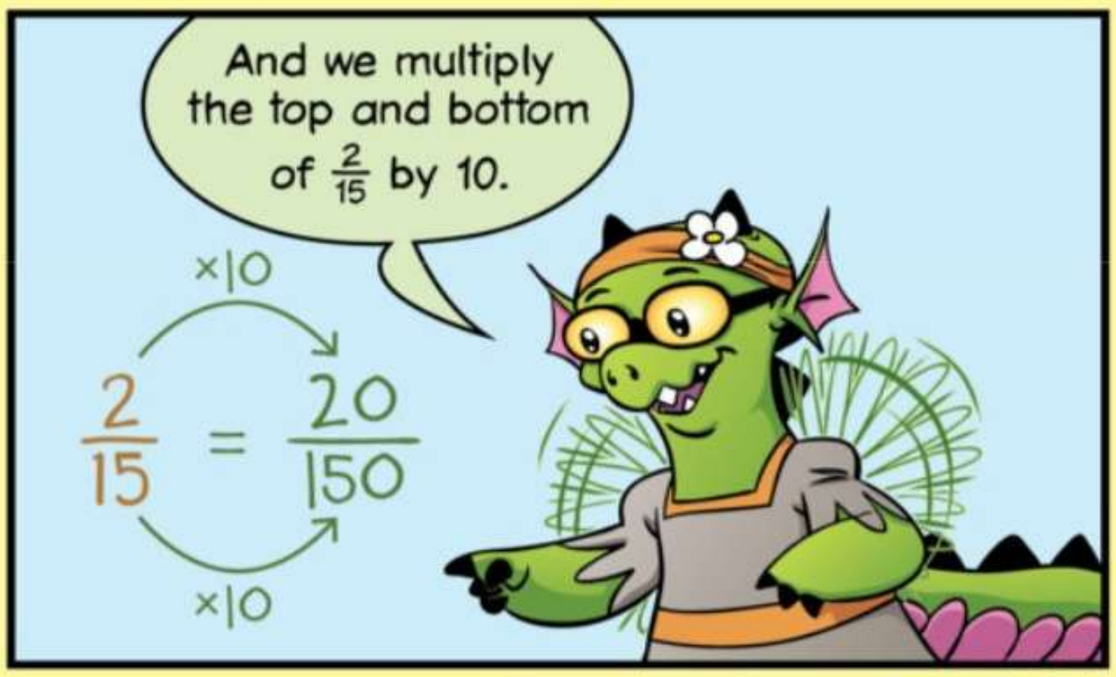

We can do the same thing we did with the addition.
We can convert the fourths to twelfths, then subtract.
$\frac{3}{4} = \frac{9}{12}$.
$\frac{9}{12} - \frac{5}{12} = \frac{4}{12}$.
$\frac{3}{4} - \frac{5}{12}$
$= \frac{9}{12} - \frac{5}{12}$
$= \frac{4}{12}$
$= \frac{1}{3}$
And $\frac{4}{12}$ simplifies to $\frac{1}{3}$.

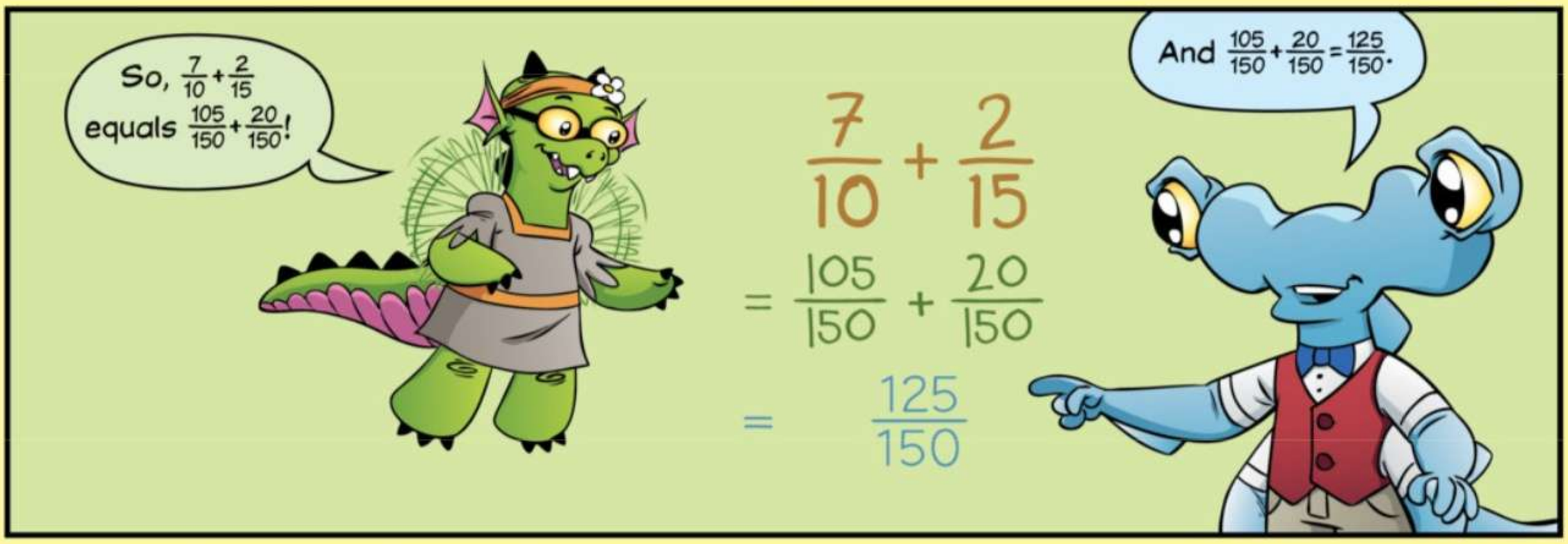

Very good. To add or subtract fractions with different denominators, we can convert the fractions so that they have the same denominator.
This one is a little harder. How could you add $\frac{7}{10} + \frac{2}{15}$?
$\frac{7}{10} + \frac{2}{15}$

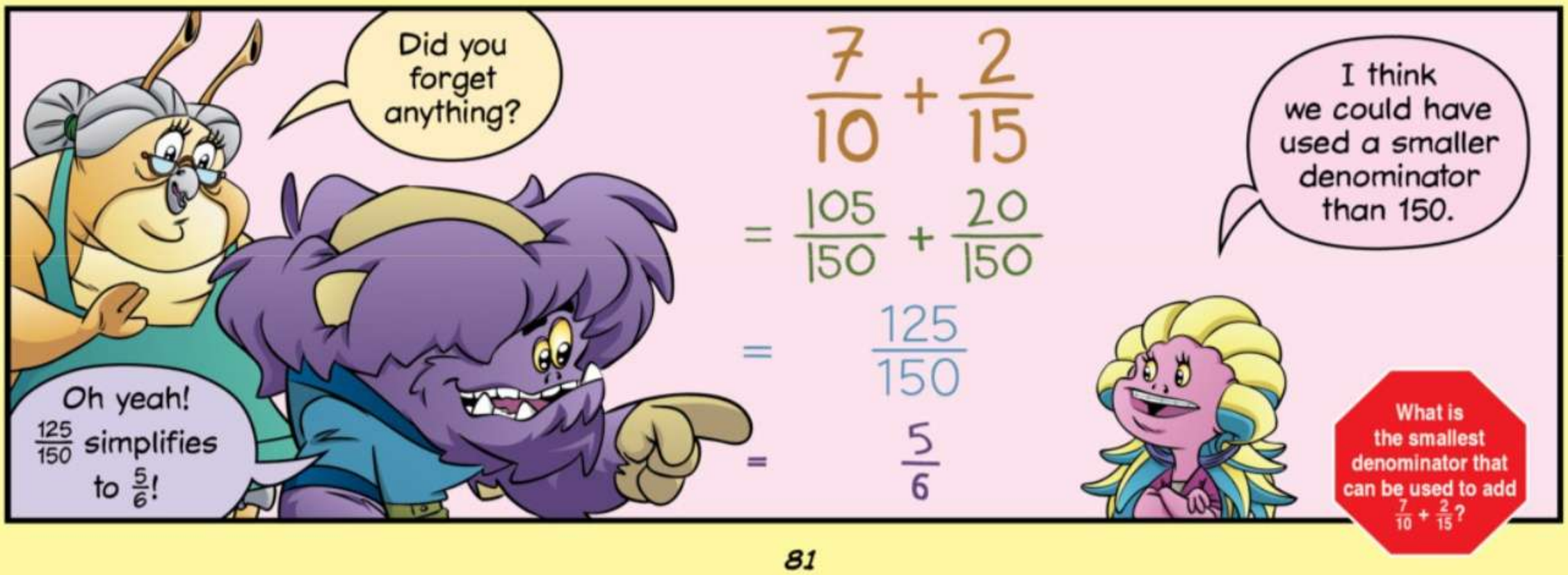
Well, we can't convert tenths to fifteenths.
Or fifteenths to tenths.
Maybe we can convert **both** fractions.
Huh?

I see. Maybe there's a different denominator that we can use for both fractions.
Like 10 • 15 = 150?
Great idea! We can convert 10ths and 15ths to 150ths.


We multiply the top and bottom of $\frac{7}{10}$ by 15.
×15
$\frac{7}{10} = \frac{105}{150}$
×15


And we multiply the top and bottom of $\frac{2}{15}$ by 10.
×10
$\frac{2}{15} = \frac{20}{150}$
×10


So, $\frac{7}{10} + \frac{2}{15}$ equals $\frac{105}{150} + \frac{20}{150}$!
And $\frac{105}{150} + \frac{20}{150} = \frac{125}{150}$.
$\frac{7}{10} + \frac{2}{15}$
$= \frac{105}{150} + \frac{20}{150}$
$= \frac{125}{150}$


Did you forget anything?
Oh yeah! $\frac{125}{150}$ simplifies to $\frac{5}{6}$!
$\frac{7}{10} + \frac{2}{15}$
$= \frac{105}{150} + \frac{20}{150}$
$= \frac{125}{150}$
$= \frac{5}{6}$
I think we could have used a smaller denominator than 150.
What is the smallest denominator that can be used to add $\frac{7}{10} + \frac{2}{15}$?

$10 = 2 \cdot 5$

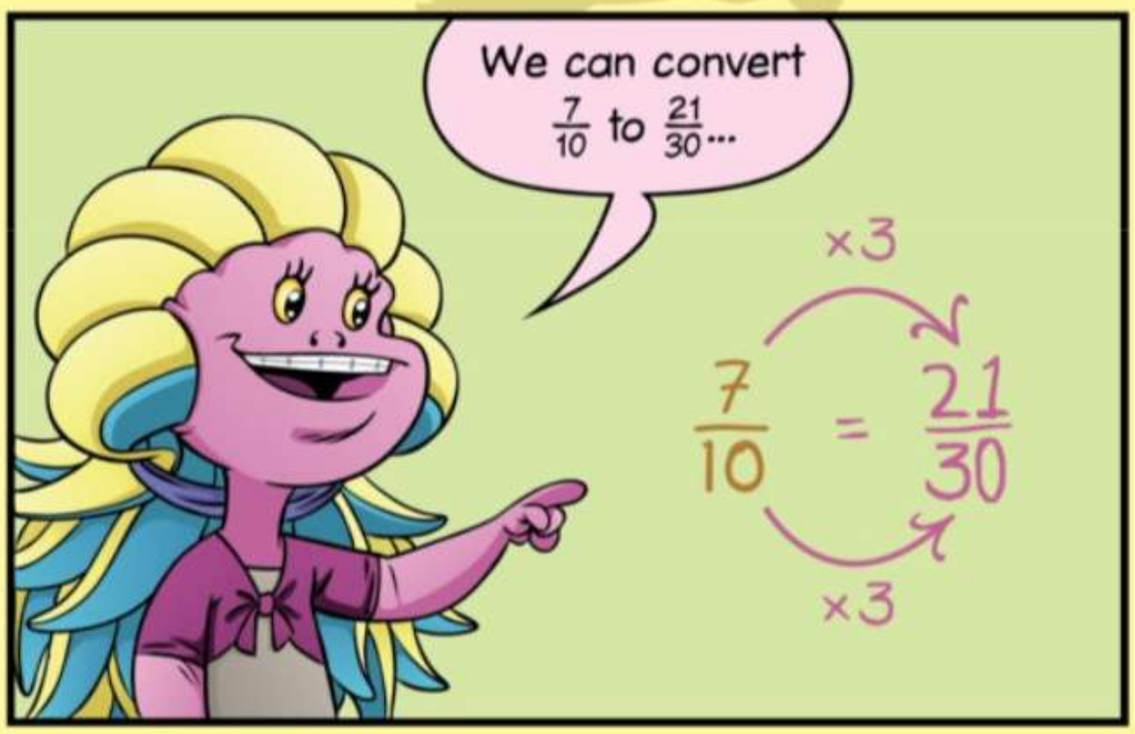
$15 = 3 \cdot 5$

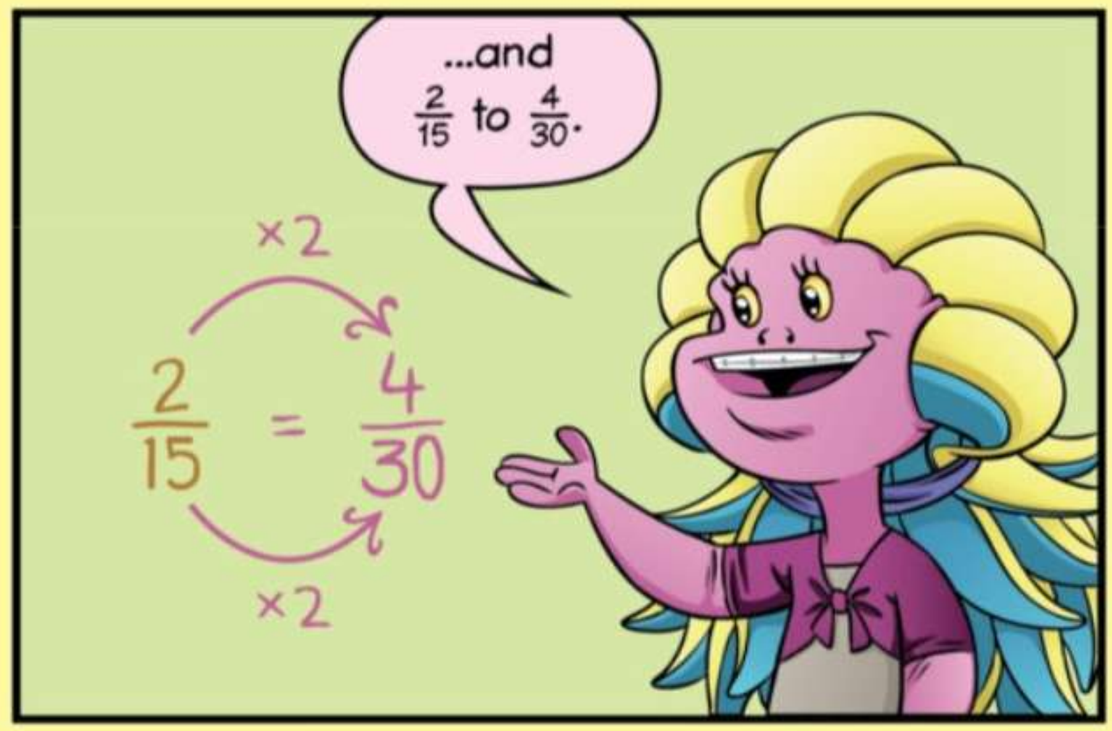
LCM:

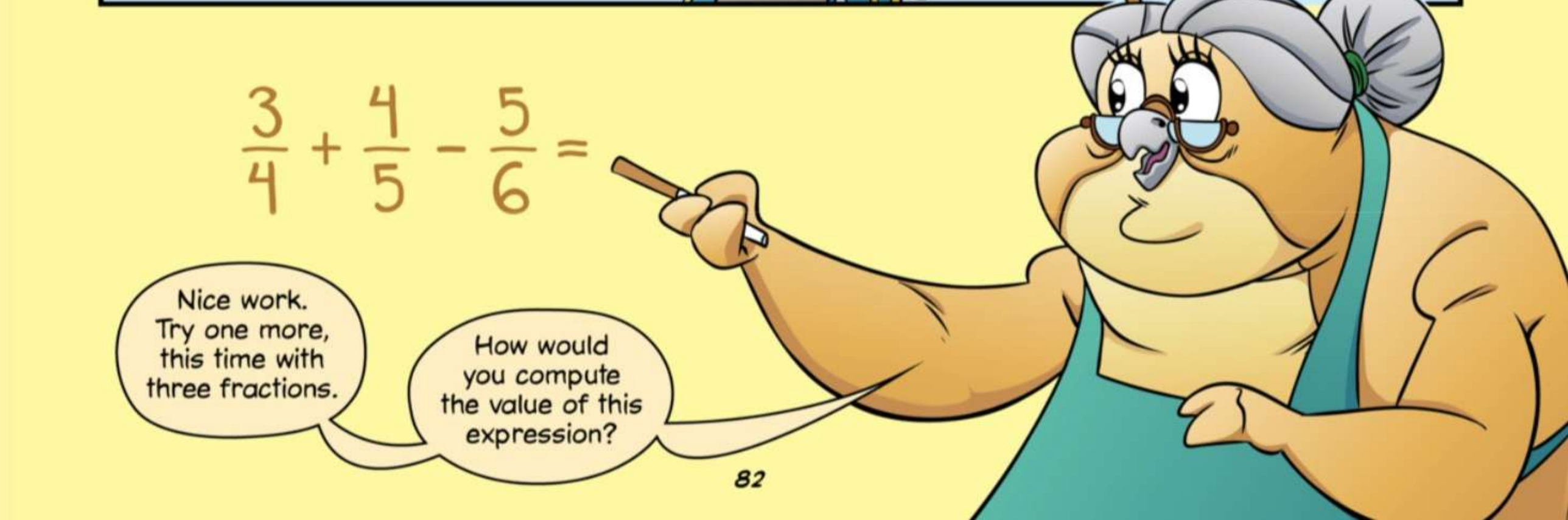
$2 \cdot 3 \cdot 5 = 30$

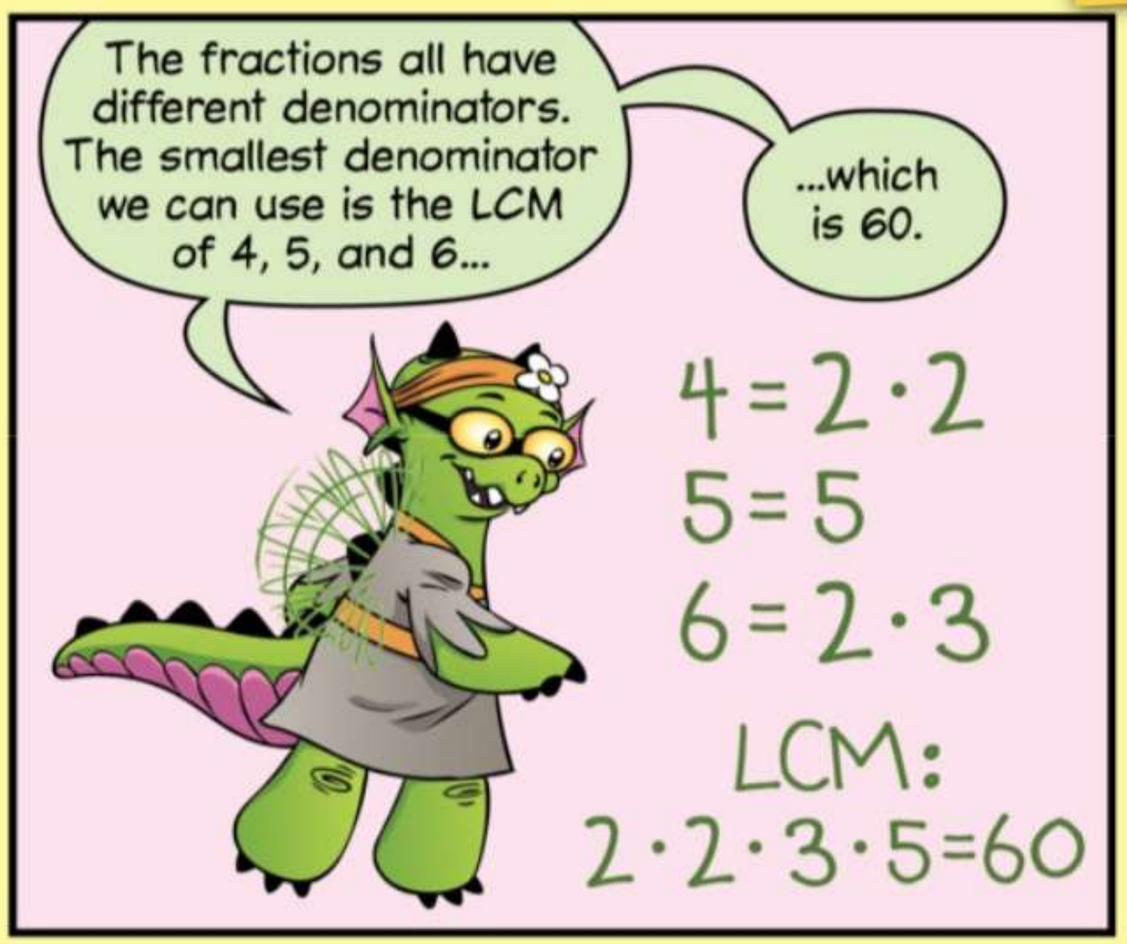
30 is the least common multiple of 10 and 15.

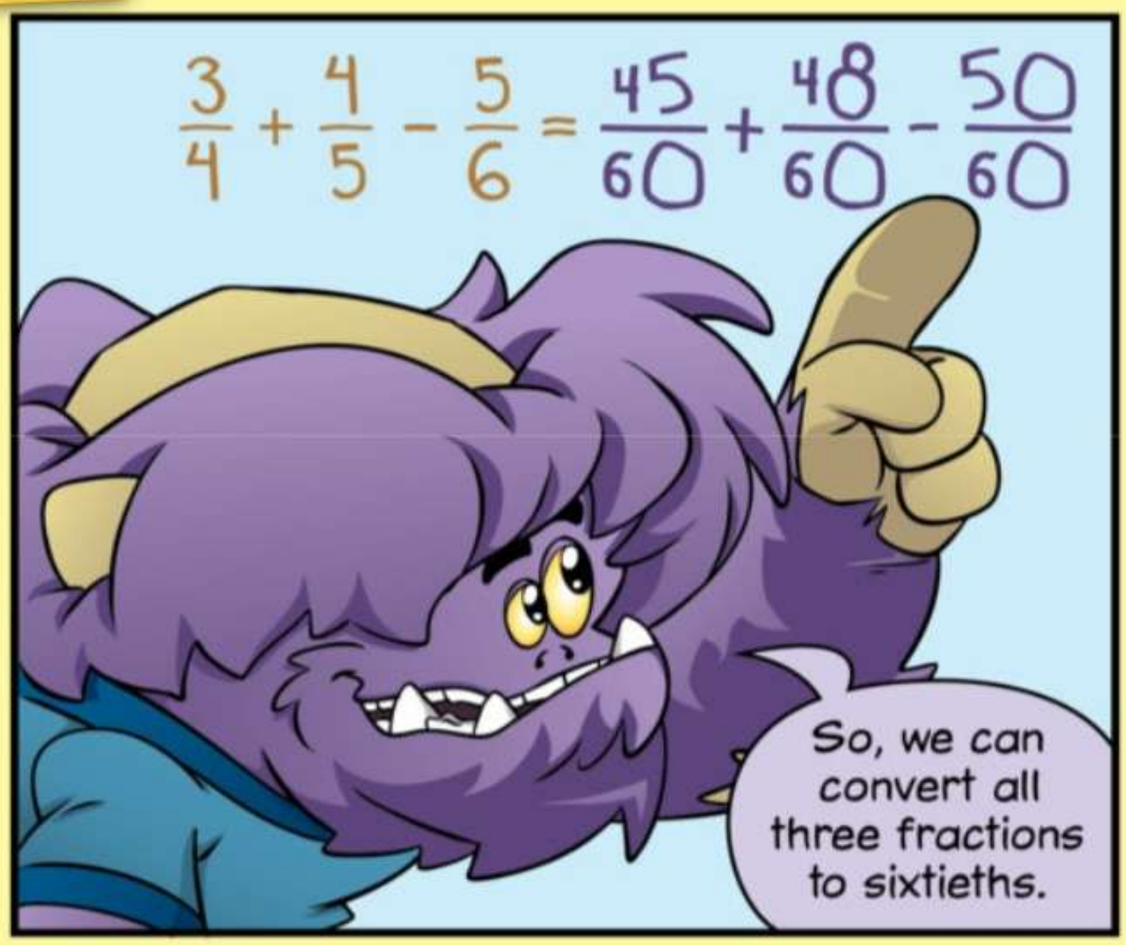
So, we can use 30 as the denominator of $\frac{7}{10}$ and $\frac{2}{15}$ to add them.

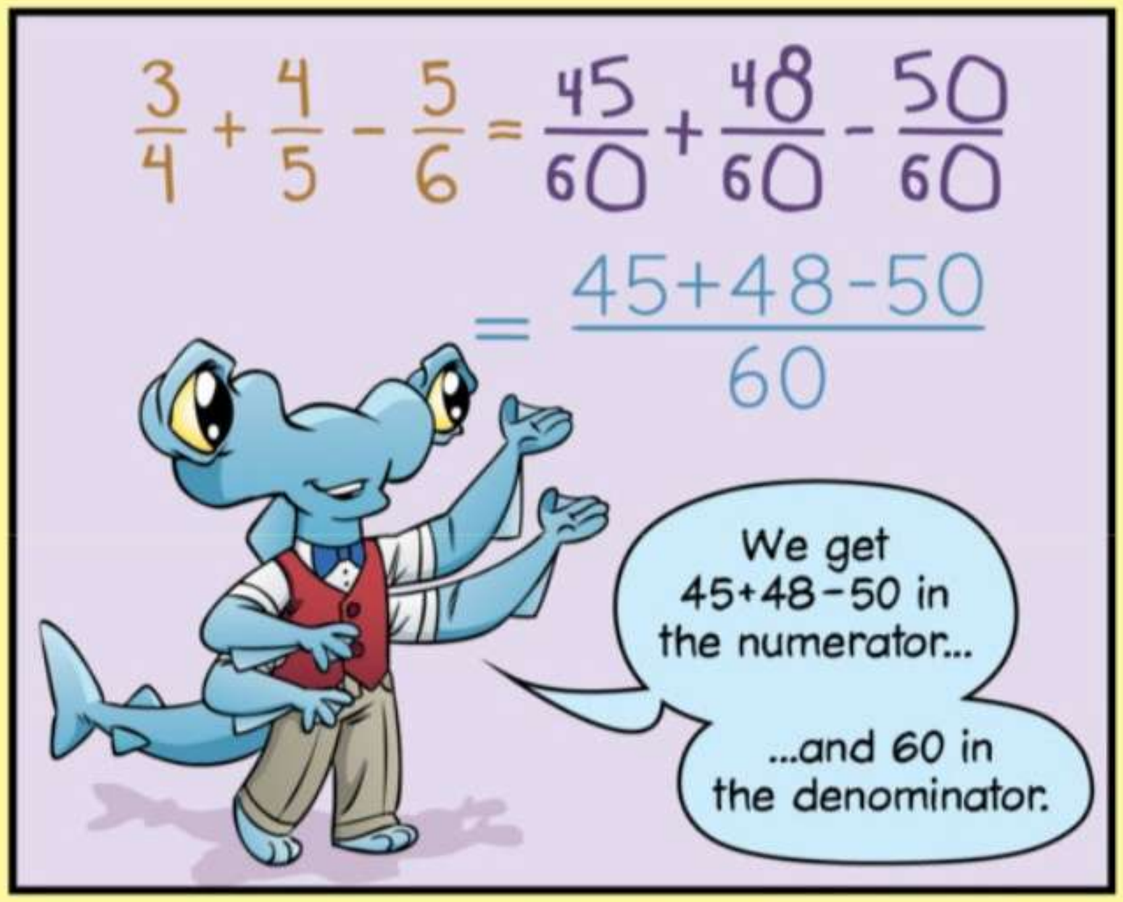


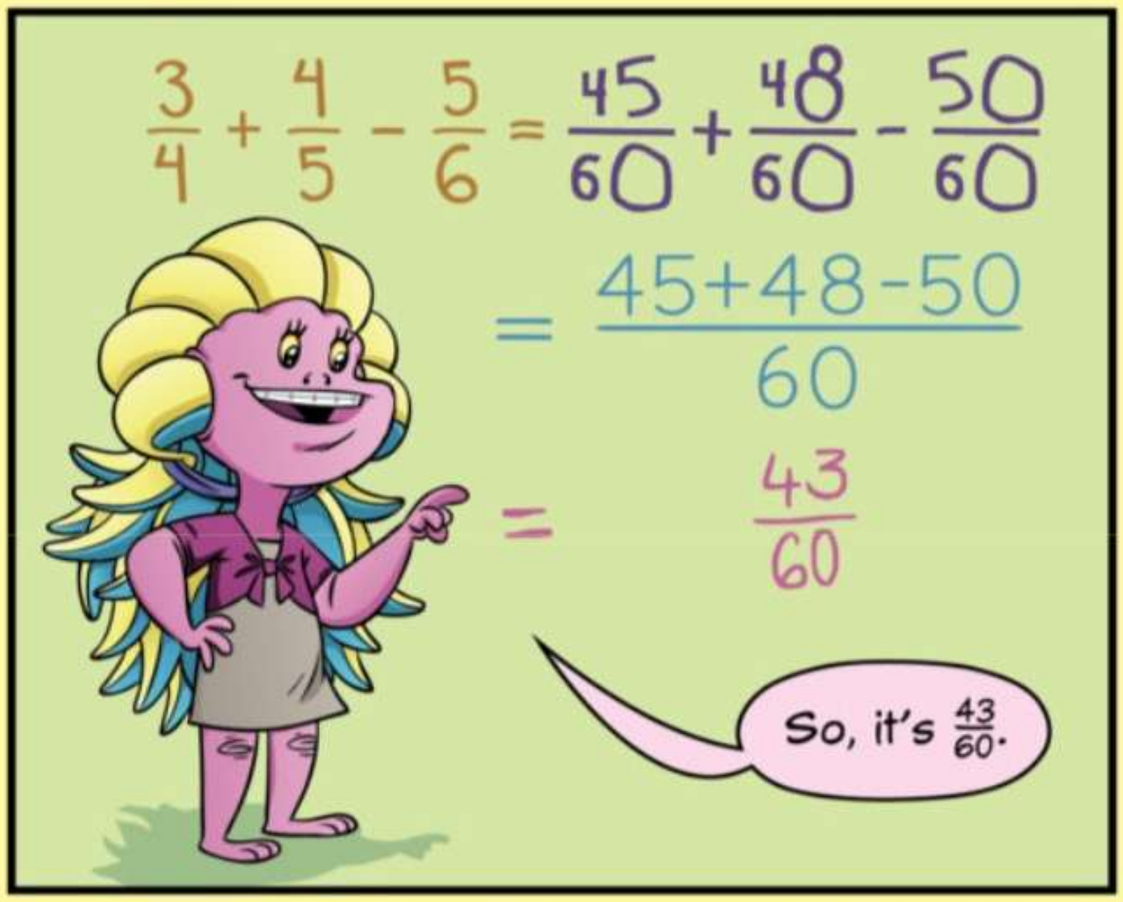





The fractions all have different denominators. The smallest denominator we can use is the LCM of 4, 5, and 6...
...which is 60.
4 = 2·2
5 = 5
6 = 2·3
LCM:
2·2·3·5=60


3/4 + 4/5 − 5/6 = 45/60 + 48/60 − 50/60
So, we can convert all three fractions to sixtieths.


3/4 + 4/5 − 5/6 = 45/60 + 48/60 − 50/60
= (45+48−50)/60
We get 45+48−50 in the numerator...
...and 60 in the denominator.


3/4 + 4/5 − 5/6 = 45/60 + 48/60 − 50/60
= (45+48−50)/60
= 43/60
So, it's 43/60.

Excellent.
When do you little monsters leave for the World Math Olympiad Qualifiers?
Tomorrow morning.
We get to ride the train!

R & G Multiplication

Are you sure you want to go through with this?

If you're jumping, I'm jumping.

OK.

Let's review our instructions.

It says that for the first half of our skydive, we'll be in free fall.

Then, we deploy our parachute.

For the second half of the jump, we float gently to the ground.

Free Fall

Under Parachute

Pull Cord

Too Late!

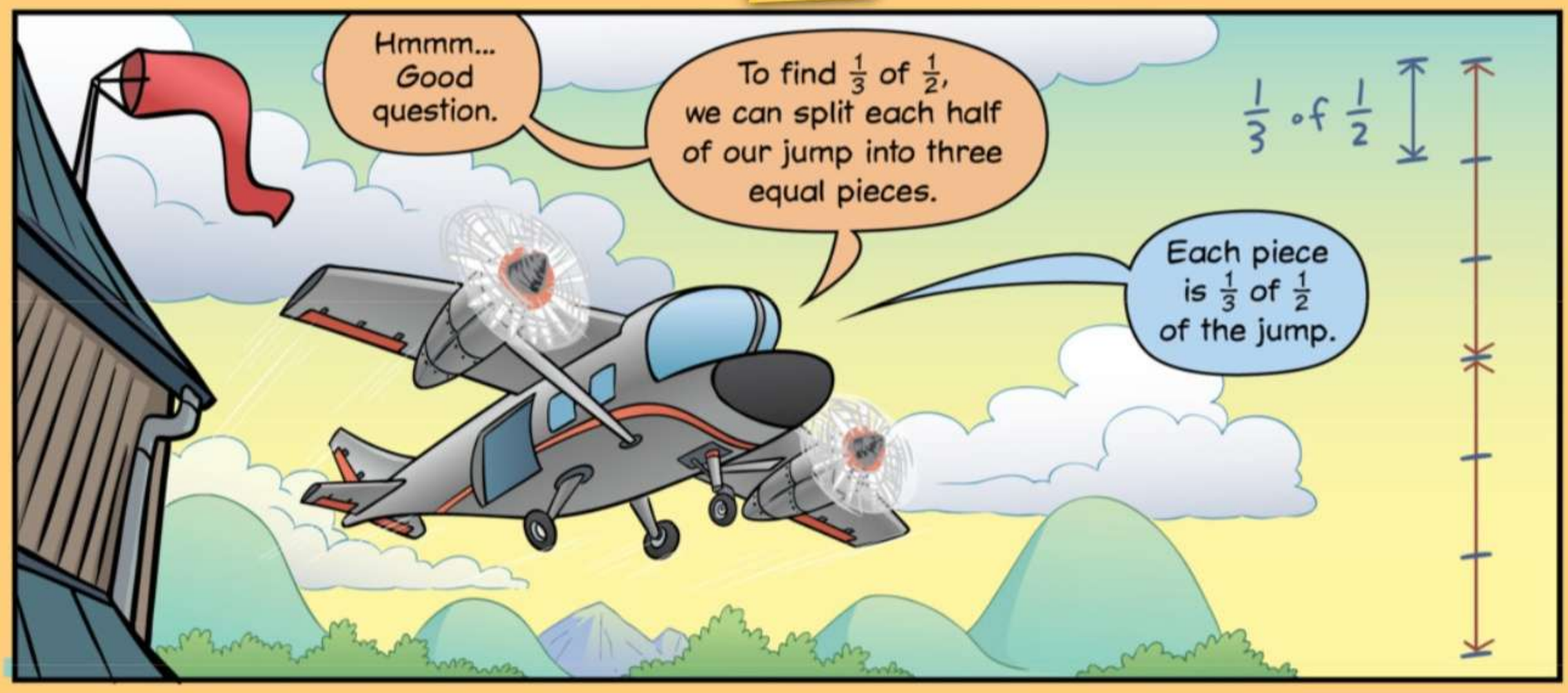
For the first third of the free fall, we'll be accelerating.

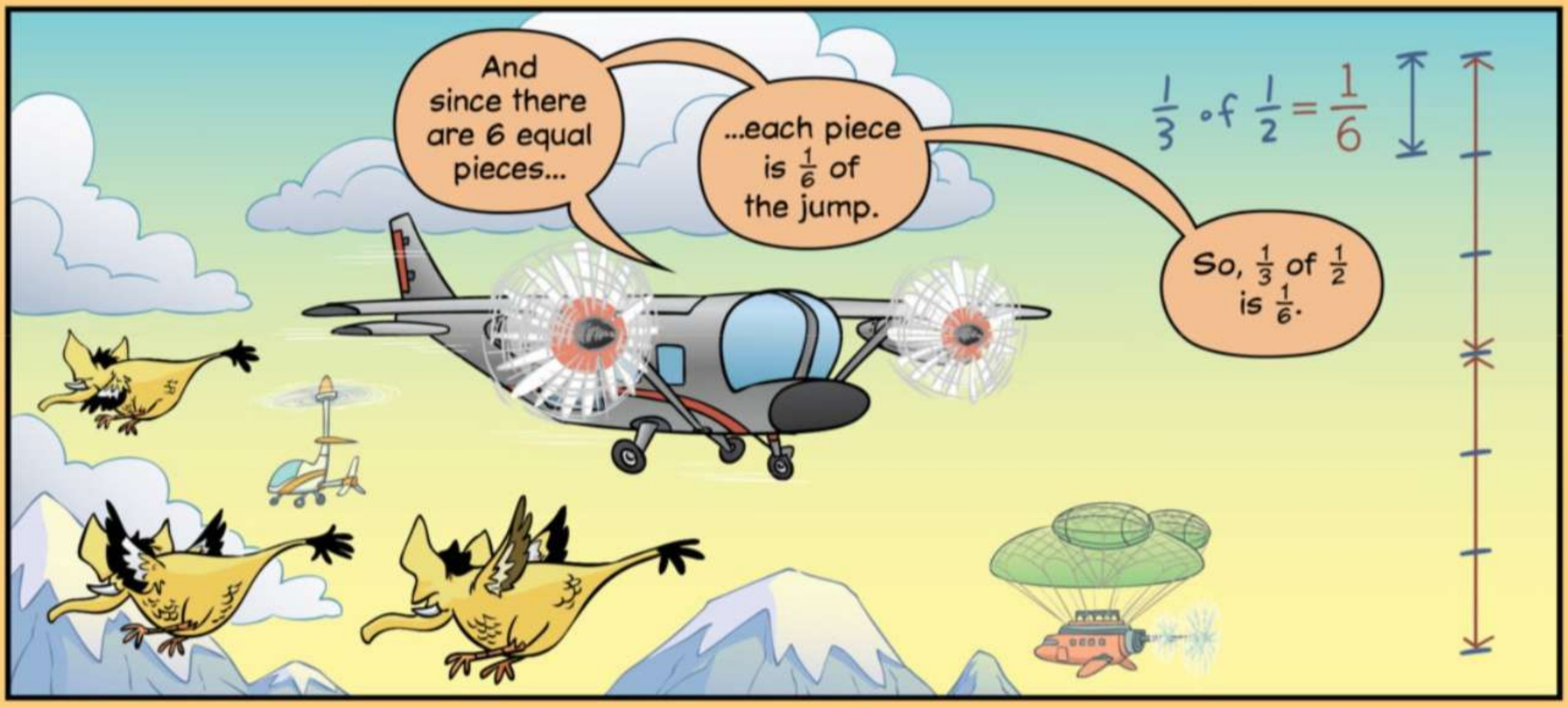
So... We accelerate for the first third of the first half of our jump?

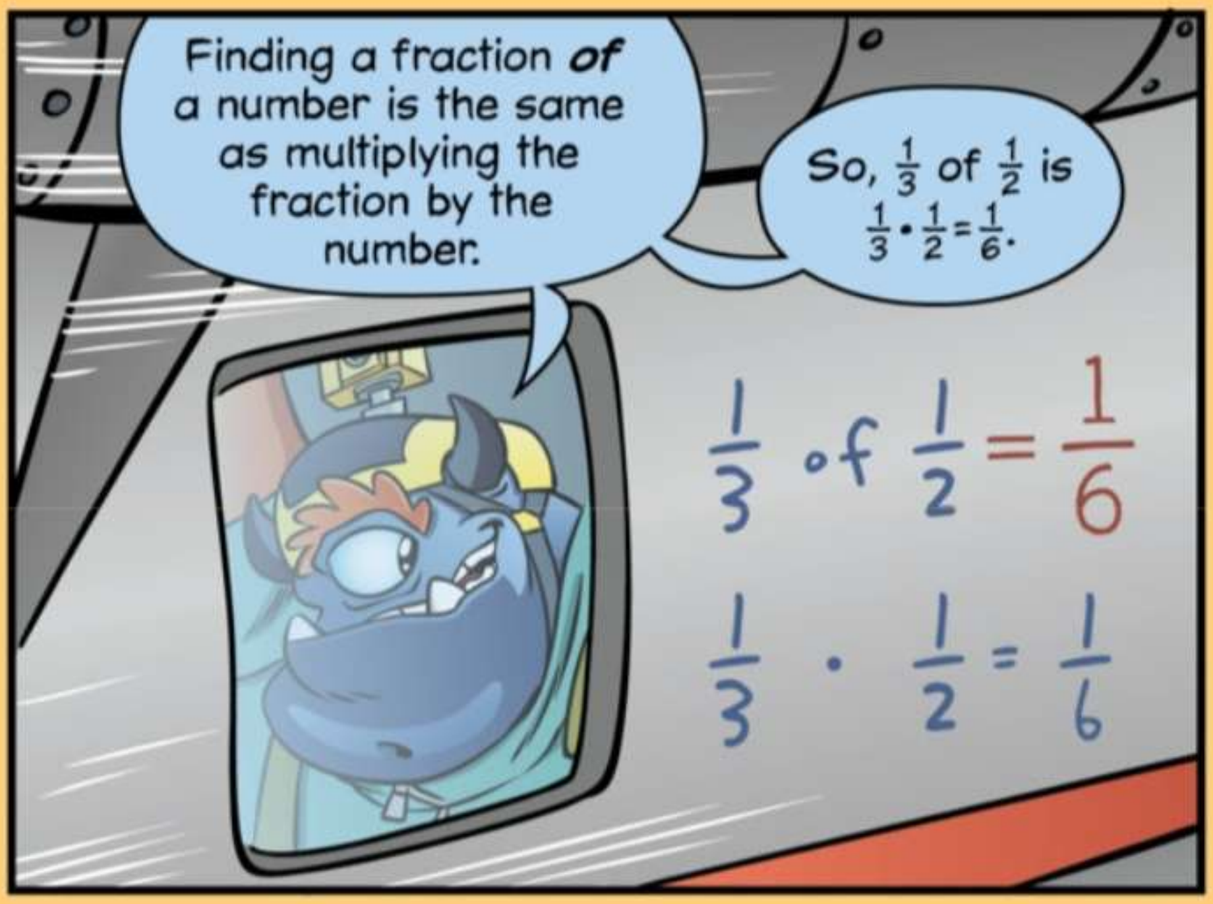
Exactly.

What fraction of the *whole* jump is that?

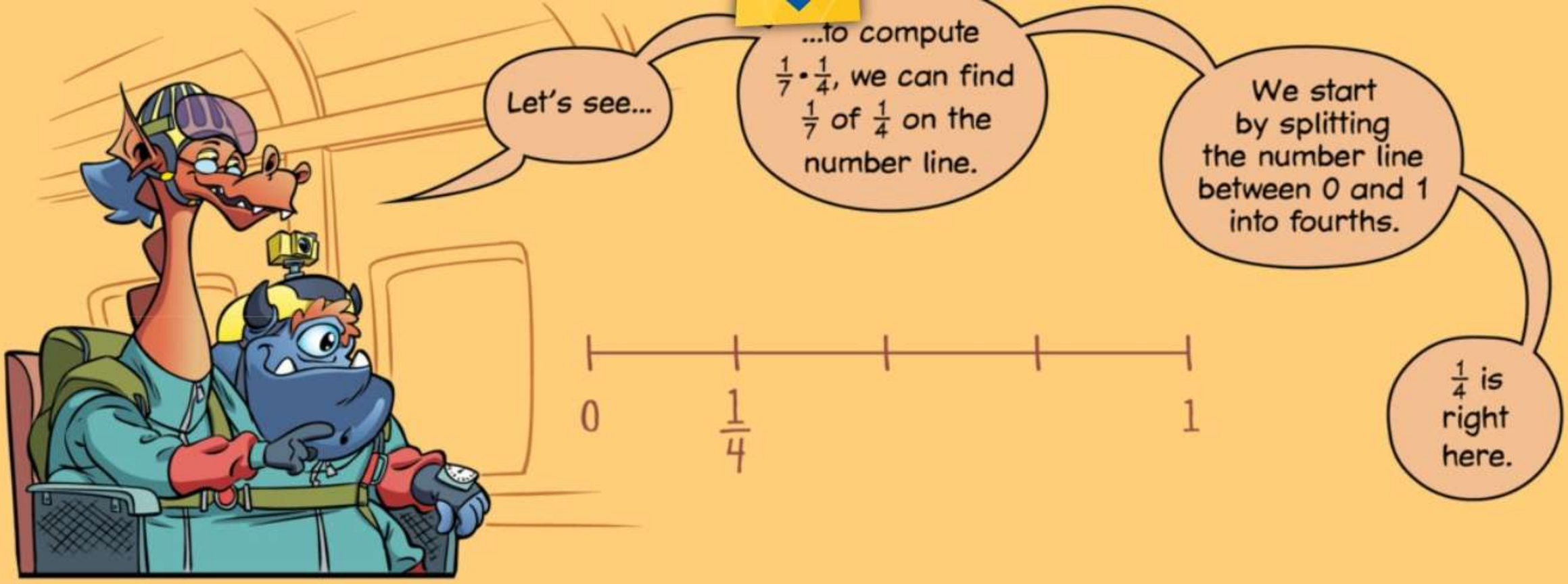
What fraction equals $\frac{1}{3}$ of $\frac{1}{2}$?

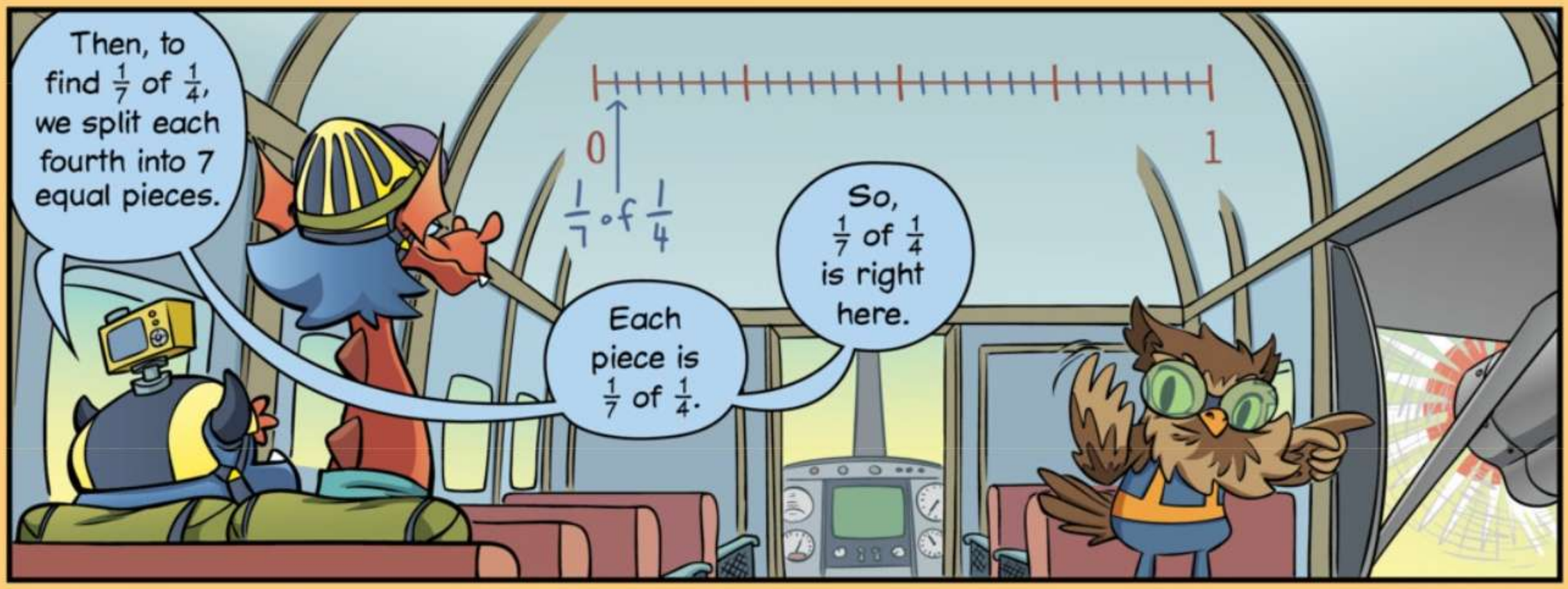

Hmmm... Good question.
To find $\frac{1}{3}$ of $\frac{1}{2}$, we can split each half of our jump into three equal pieces.
Each piece is $\frac{1}{3}$ of $\frac{1}{2}$ of the jump.
$\frac{1}{3}$ of $\frac{1}{2}$

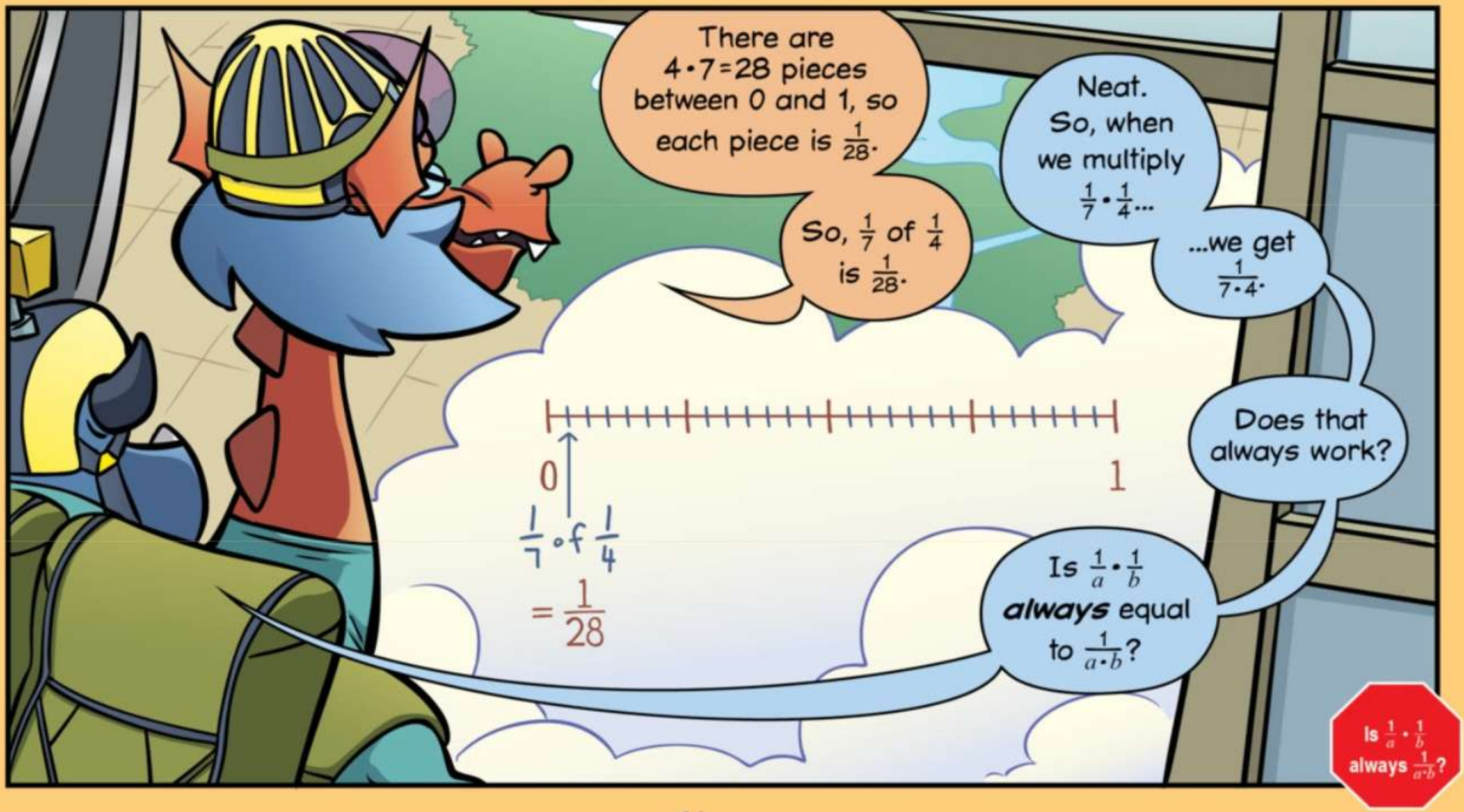

And since there are 6 equal pieces...
...each piece is $\frac{1}{6}$ of the jump.
So, $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$.
$\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$


Finding a fraction *of* a number is the same as multiplying the fraction by the number.
So, $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$.
$\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$
$\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

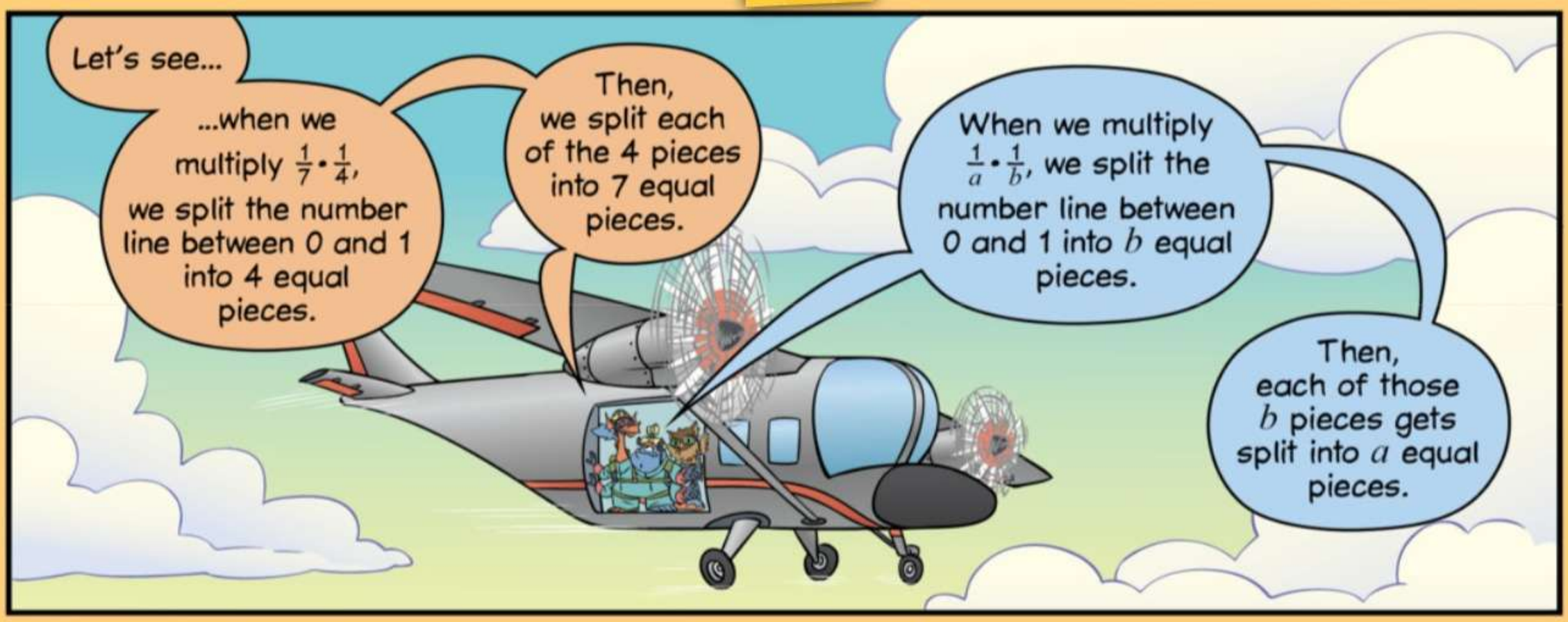
We've never multiplied two fractions before! Let's try again. What's $\frac{1}{7} \cdot \frac{1}{4}$?
$\frac{1}{7} \cdot \frac{1}{4}$







Is $\frac{1}{a} \cdot \frac{1}{b}$ always $\frac{1}{a \cdot b}$?

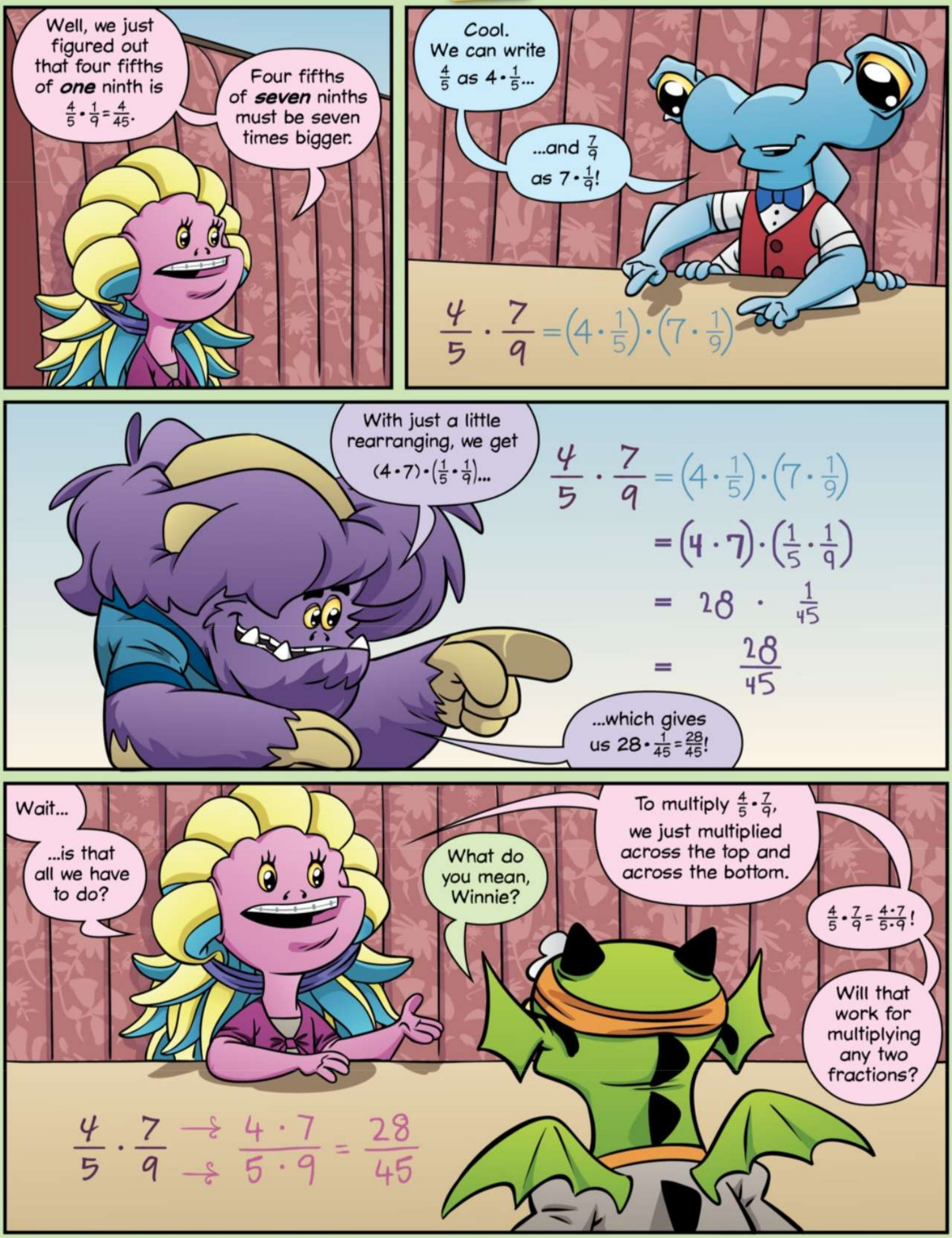

Let's see...
...when we multiply $\frac{1}{7} \cdot \frac{1}{4}$, we split the number line between 0 and 1 into 4 equal pieces.
Then, we split each of the 4 pieces into 7 equal pieces.
When we multiply $\frac{1}{a} \cdot \frac{1}{b}$, we split the number line between 0 and 1 into b equal pieces.
Then, each of those b pieces gets split into a equal pieces.

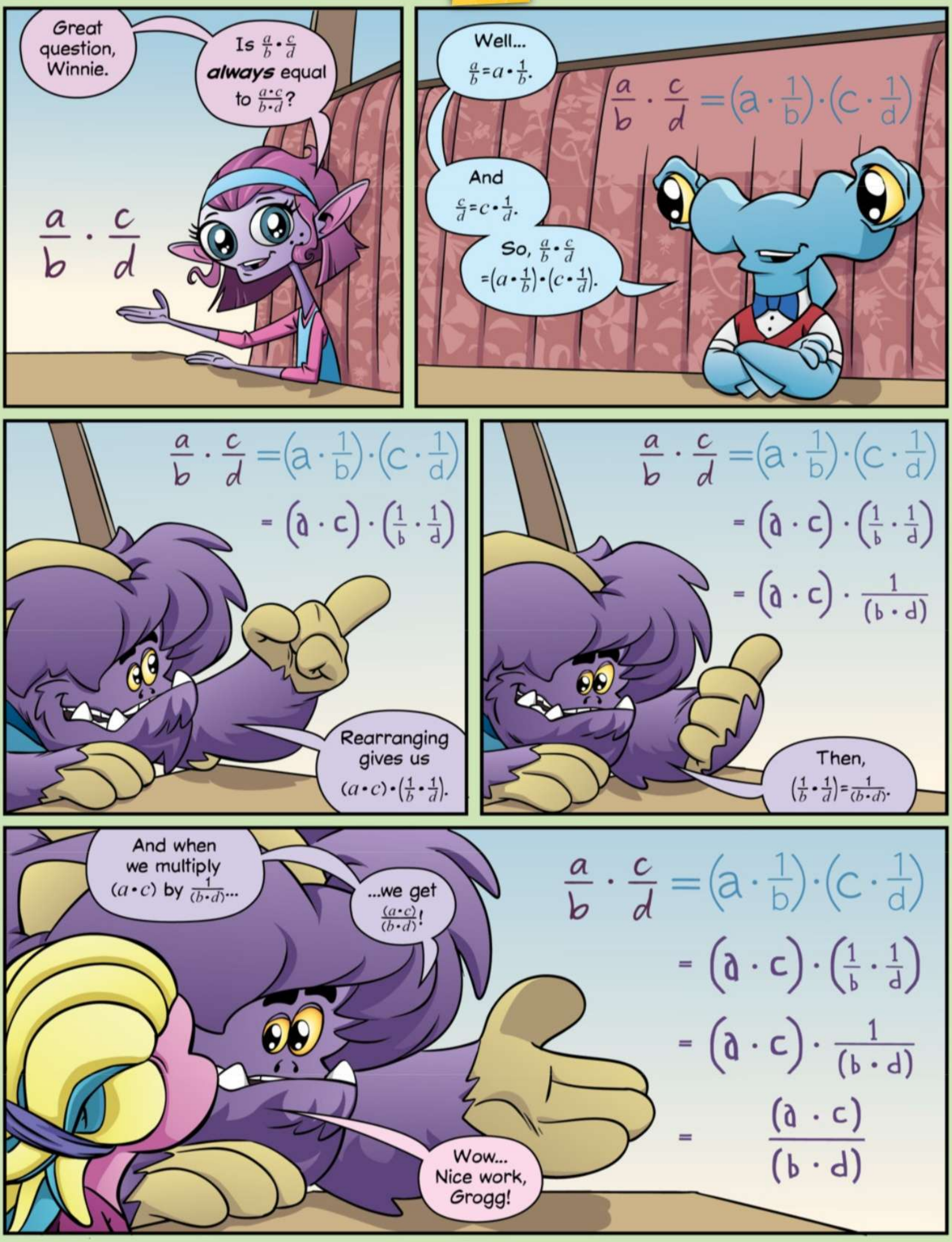
Right, so we end up with $a \cdot b$ equal pieces between 0 and 1.
So, each piece is $\frac{1}{a \cdot b}$.
$\frac{1}{a} \cdot \frac{1}{b}$ is $\frac{1}{a \cdot b}$!
Good job! Now, JUMP!

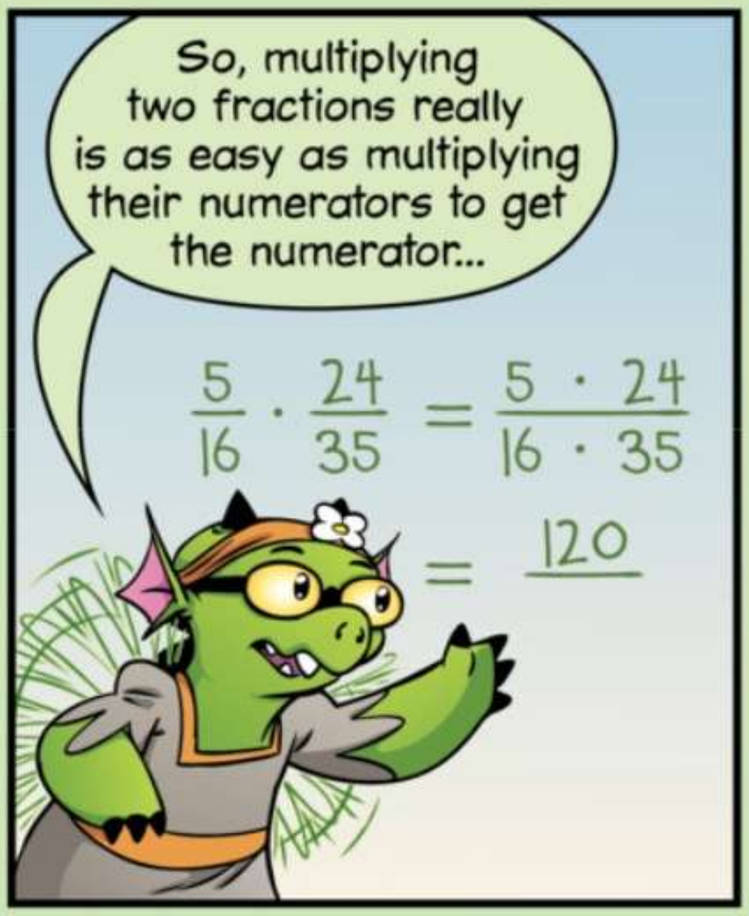
Geronimo!

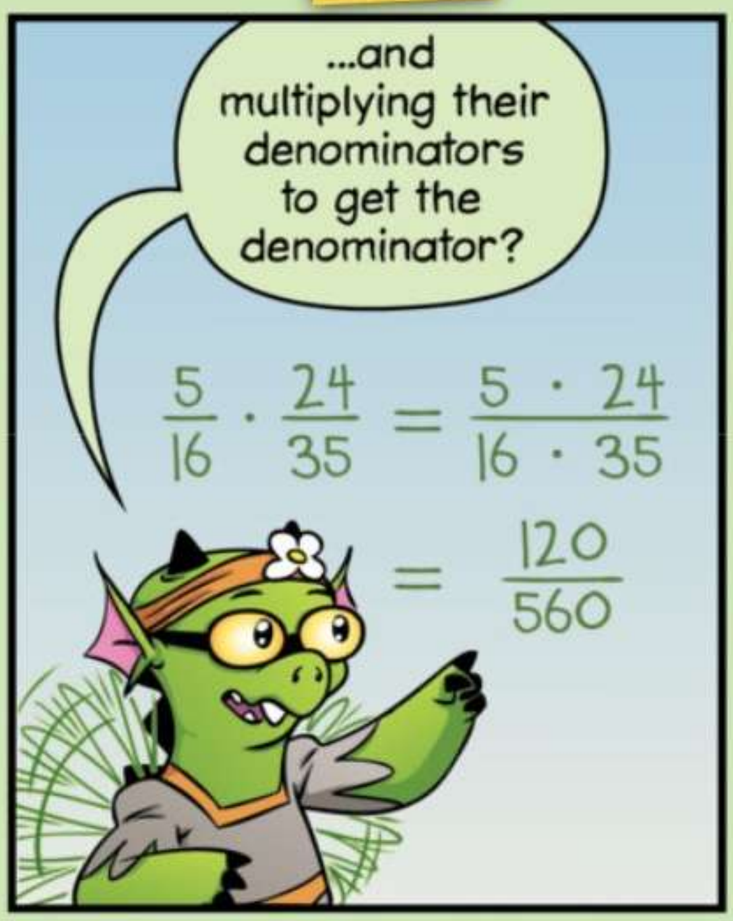
MATH TEAM
More Multiplication
All aboard, kids. We're off to the World Math Olympiad Qualifiers.
Wait! Where's Max? Isn't he coming?
He's getting on at the next stop.
Between now and then, we're going to learn how to multiply any pair of fractions.
Any pair?
Yep.
Who can start us off by multiplying $\frac{1}{5} \cdot \frac{1}{9}$?

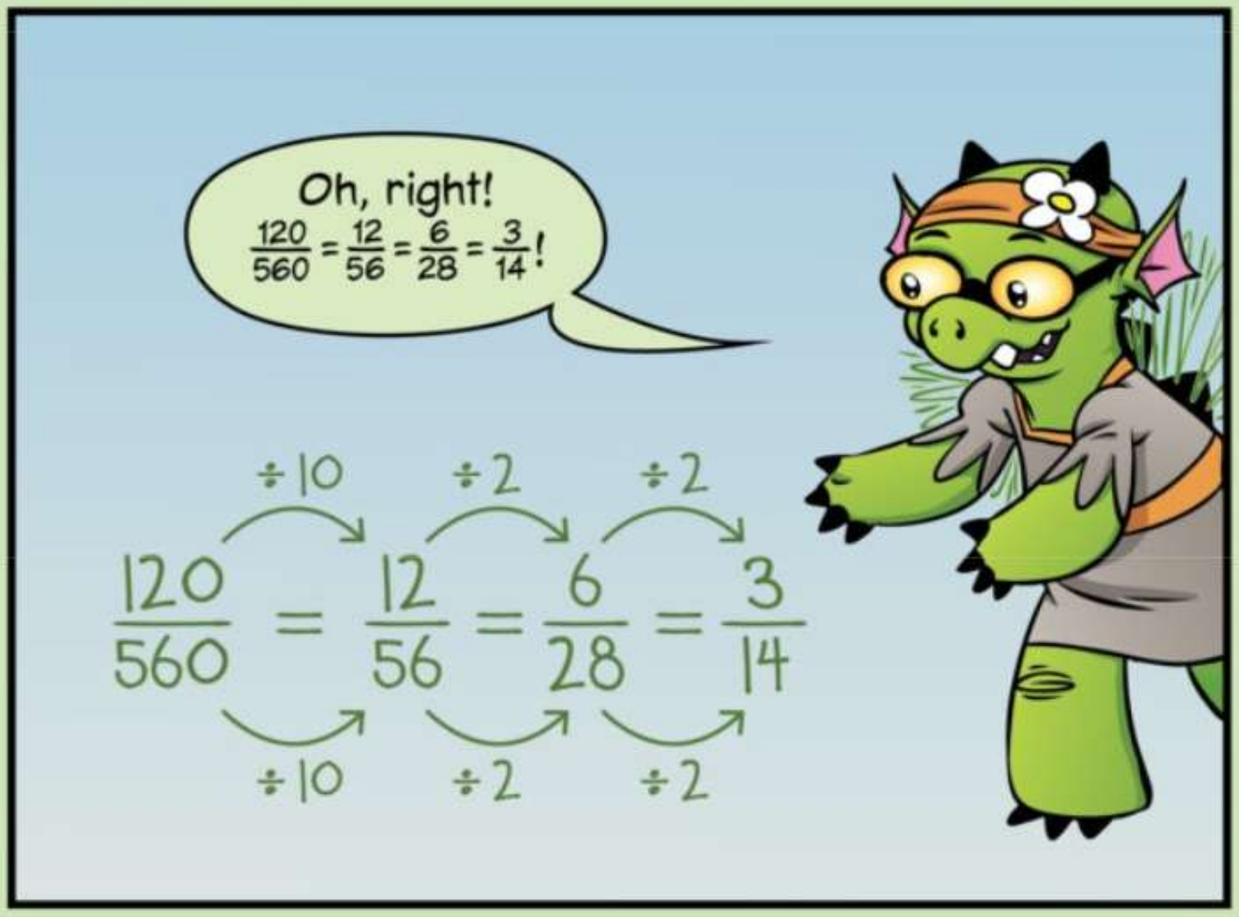
$\frac{1}{5} \cdot \frac{1}{9} = \frac{1}{5 \cdot 9}$, which is $\frac{1}{45}$.
Multiplying two unit fractions is as easy as multiplying their denominators.
$\frac{1}{5} \cdot \frac{1}{9} = \frac{1}{45}$
A FRACTION WITH 1 IN THE NUMERATOR IS CALLED A *UNIT FRACTION*.
Good. $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$.
How could we use this fact to help us multiply two fractions that are *not* unit fractions?
For example, how could we multiply $\frac{4}{5} \cdot \frac{1}{9}$?
$\frac{4}{5} \cdot \frac{1}{9}$
Well, we already know that *one* fifth of $\frac{1}{9}$ is $\frac{1}{5} \cdot \frac{1}{9} = \frac{1}{45}$.
Four fifths of $\frac{1}{9}$ is four times as much.
That makes sense, since $\frac{4}{5} = 4 \cdot \frac{1}{5}$.
So, $\frac{4}{5} \cdot \frac{1}{9} = 4 \cdot \frac{1}{5} \cdot \frac{1}{9}$!
$\frac{4}{5} \cdot \frac{1}{9} = 4 \cdot \frac{1}{5} \cdot \frac{1}{9}$
REVIEW MULTIPLYING WHOLE NUMBERS AND FRACTIONS IN BEAST ACADEMY 4D: $a \cdot \frac{1}{b} = \frac{a}{b}$.
$\frac{4}{5} \cdot \frac{1}{9} = 4 \cdot \frac{1}{5} \cdot \frac{1}{9}$
$= 4 \cdot \frac{1}{45}$
$= \frac{4}{45}$
So, $\frac{4}{5} \cdot \frac{1}{9} = \frac{4}{45}$.
Well done! Next, try $\frac{4}{5} \cdot \frac{7}{9}$.
$\frac{4}{5} \cdot \frac{7}{9}$
Try it.


Well, we just figured out that four fifths of ***one*** ninth is $\frac{4}{5} \cdot \frac{1}{9} = \frac{4}{45}$.
Four fifths of ***seven*** ninths must be seven times bigger.
Cool. We can write $\frac{4}{5}$ as $4 \cdot \frac{1}{5}$...
...and $\frac{7}{9}$ as $7 \cdot \frac{1}{9}$!
$\frac{4}{5} \cdot \frac{7}{9} = \left(4 \cdot \frac{1}{5}\right) \cdot \left(7 \cdot \frac{1}{9}\right)$
With just a little rearranging, we get $(4 \cdot 7) \cdot \left(\frac{1}{5} \cdot \frac{1}{9}\right)$...
$\frac{4}{5} \cdot \frac{7}{9} = \left(4 \cdot \frac{1}{5}\right) \cdot \left(7 \cdot \frac{1}{9}\right)$
$= (4 \cdot 7) \cdot \left(\frac{1}{5} \cdot \frac{1}{9}\right)$
$= 28 \cdot \frac{1}{45}$
$= \frac{28}{45}$
...which gives us $28 \cdot \frac{1}{45} = \frac{28}{45}$!
Wait...
...is that all we have to do?
What do you mean, Winnie?
To multiply $\frac{4}{5} \cdot \frac{7}{9}$, we just multiplied across the top and across the bottom.
$\frac{4}{5} \cdot \frac{7}{9} = \frac{4 \cdot 7}{5 \cdot 9}$!
Will that work for multiplying any two fractions?
$\frac{4}{5} \cdot \frac{7}{9} \rightarrow \frac{4 \cdot 7}{5 \cdot 9} = \frac{28}{45}$

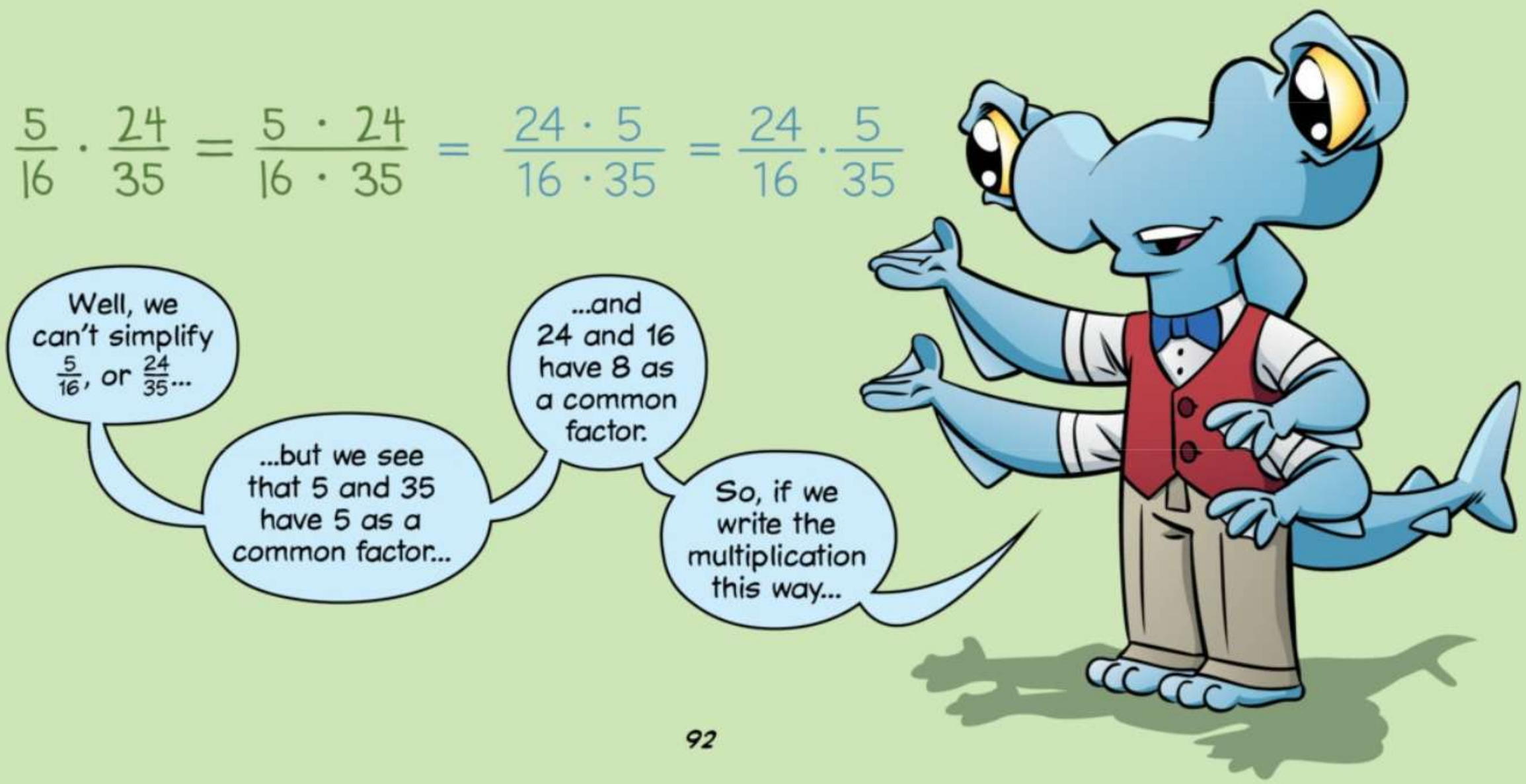

Great question, Winnie.
Is $\frac{a}{b} \cdot \frac{c}{d}$ always equal to $\frac{a \cdot c}{b \cdot d}$?
$\frac{a}{b} \cdot \frac{c}{d}$
Well... $\frac{a}{b} = a \cdot \frac{1}{b}$.
And $\frac{c}{d} = c \cdot \frac{1}{d}$.
So, $\frac{a}{b} \cdot \frac{c}{d} = \left(a \cdot \frac{1}{b}\right) \cdot \left(c \cdot \frac{1}{d}\right)$.
$\frac{a}{b} \cdot \frac{c}{d} = \left(a \cdot \frac{1}{b}\right) \cdot \left(c \cdot \frac{1}{d}\right)$
$= (a \cdot c) \cdot \left(\frac{1}{b} \cdot \frac{1}{d}\right)$
Rearranging gives us $(a \cdot c) \cdot \left(\frac{1}{b} \cdot \frac{1}{d}\right)$.
$\frac{a}{b} \cdot \frac{c}{d} = \left(a \cdot \frac{1}{b}\right) \cdot \left(c \cdot \frac{1}{d}\right)$
$= (a \cdot c) \cdot \left(\frac{1}{b} \cdot \frac{1}{d}\right)$
$= (a \cdot c) \cdot \frac{1}{(b \cdot d)}$
Then, $\left(\frac{1}{b} \cdot \frac{1}{d}\right) = \frac{1}{(b \cdot d)}$.
And when we multiply $(a \cdot c)$ by $\frac{1}{(b \cdot d)}$...
...we get $\frac{(a \cdot c)}{(b \cdot d)}$!
Wow... Nice work, Grogg!
$\frac{a}{b} \cdot \frac{c}{d} = \left(a \cdot \frac{1}{b}\right) \cdot \left(c \cdot \frac{1}{d}\right)$
$= (a \cdot c) \cdot \left(\frac{1}{b} \cdot \frac{1}{d}\right)$
$= (a \cdot c) \cdot \frac{1}{(b \cdot d)}$
$= \frac{(a \cdot c)}{(b \cdot d)}$



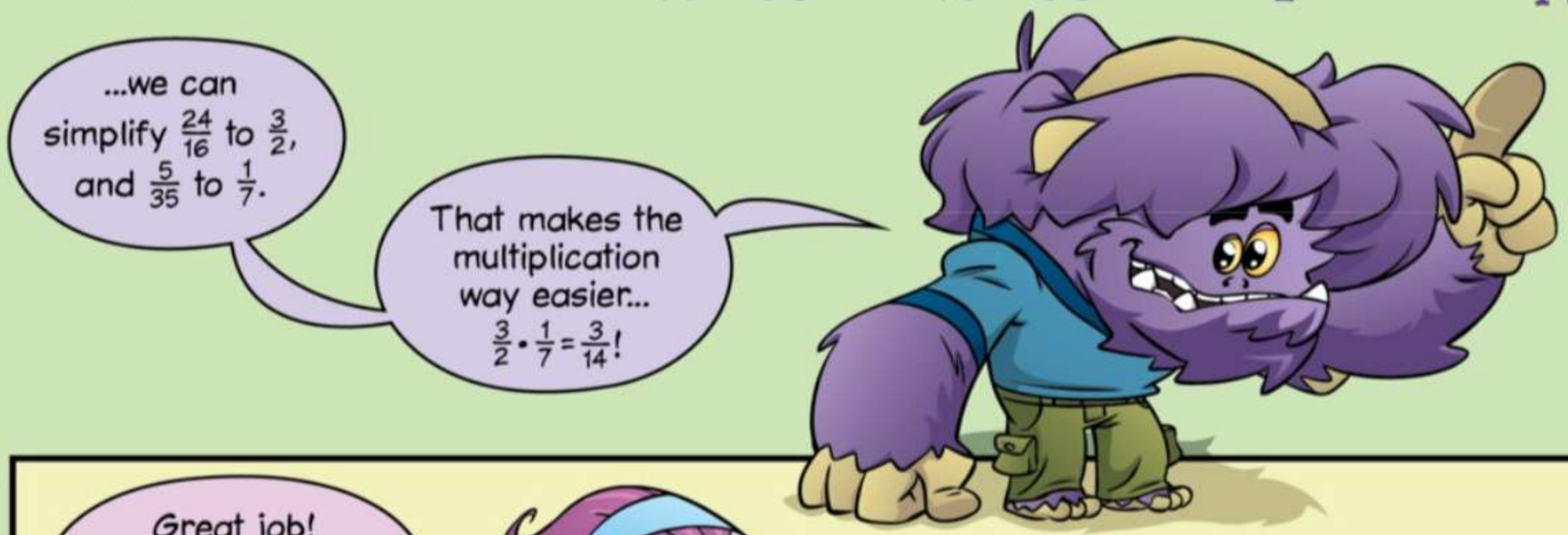




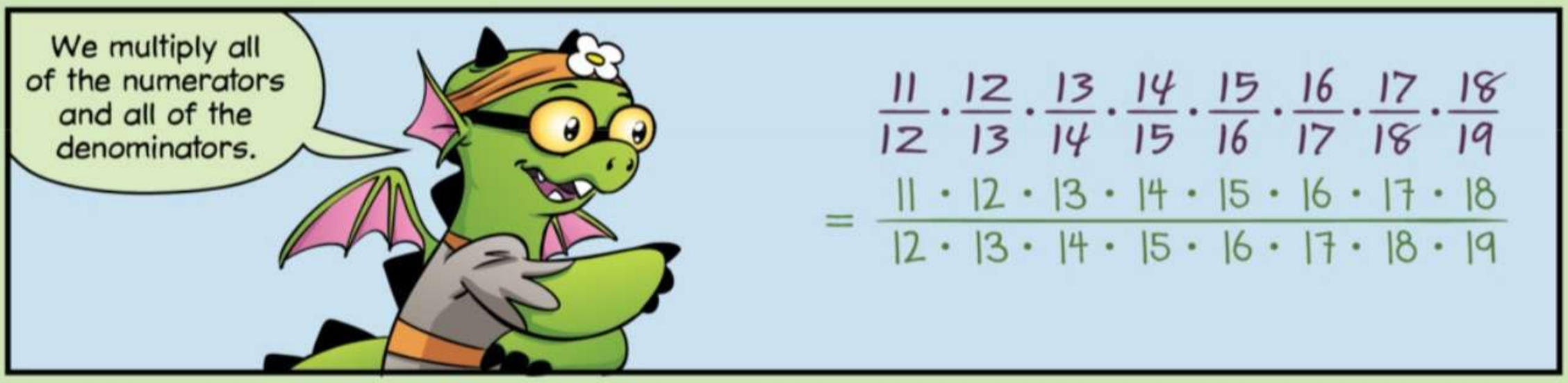
$$\frac{5}{16} \cdot \frac{24}{35} = \frac{5 \cdot 24}{16 \cdot 35} = \frac{24 \cdot 5}{16 \cdot 35} = \frac{24}{16} \cdot \frac{5}{35}$$



$$\frac{5}{16}\cdot\frac{24}{35}=\frac{5\cdot 24}{16\cdot 35}=\frac{24\cdot 5}{16\cdot 35}=\frac{24}{16}\cdot\frac{5}{35}=\frac{3}{2}\cdot\frac{1}{7}=\frac{3}{14}$$

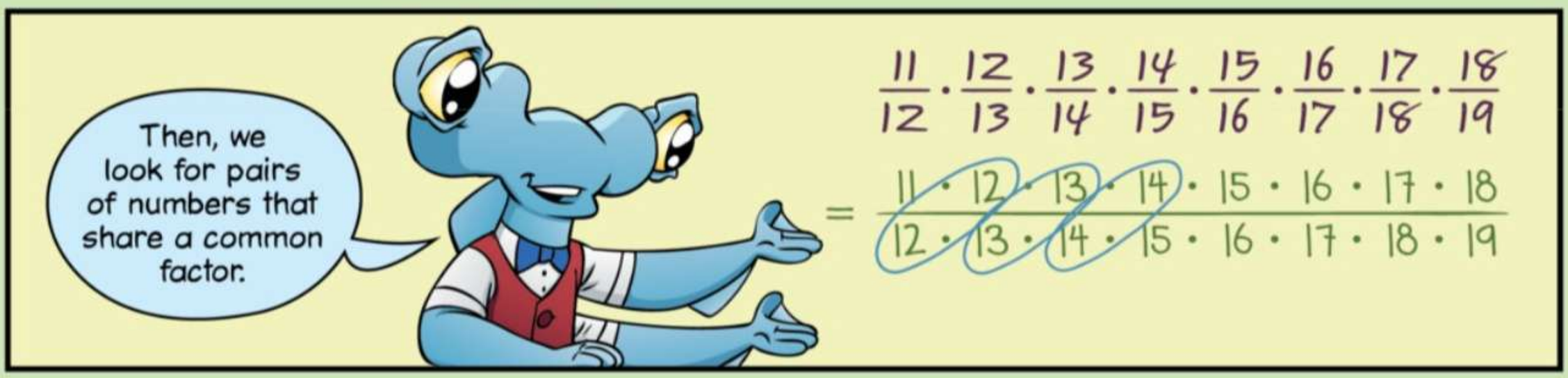


$$\frac{11}{12}\cdot\frac{12}{13}\cdot\frac{13}{14}\cdot\frac{14}{15}\cdot\frac{15}{16}\cdot\frac{16}{17}\cdot\frac{17}{18}\cdot\frac{18}{19}$$



$$\frac{11}{12}\cdot\frac{12}{13}\cdot\frac{13}{14}\cdot\frac{14}{15}\cdot\frac{15}{16}\cdot\frac{16}{17}\cdot\frac{17}{18}\cdot\frac{18}{19}$$

$$=\frac{11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18}{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19}$$



$$\frac{11}{12}\cdot\frac{12}{13}\cdot\frac{13}{14}\cdot\frac{14}{15}\cdot\frac{15}{16}\cdot\frac{16}{17}\cdot\frac{17}{18}\cdot\frac{18}{19}$$

$$=\frac{11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18}{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19}$$

$$=\frac{11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18}{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19}$$

$$=\frac{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 11}{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19}$$

$$\frac{11}{12}\cdot\frac{12}{13}\cdot\frac{13}{14}\cdot\frac{14}{15}\cdot\frac{15}{16}\cdot\frac{16}{17}\cdot\frac{17}{18}\cdot\frac{18}{19} = \frac{11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18}{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19}$$

$$= \frac{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 11}{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19}$$

$$= \frac{12}{12}\cdot\frac{13}{13}\cdot\frac{14}{14}\cdot\frac{15}{15}\cdot\frac{16}{16}\cdot\frac{17}{17}\cdot\frac{18}{18}\cdot\frac{11}{19}$$

$$= 1\cdot 1\cdot 1\cdot 1\cdot 1\cdot 1\cdot 1\cdot \frac{11}{19}$$

$$= \frac{11}{19}$$

WE DON'T ***NEED*** TO WRITE THE 1'S.

Simplifying Fractions: Cancelling

$$\frac{180}{225} \overset{\div 5}{\underset{\div 5}{=}} \frac{36}{45} \overset{\div 9}{\underset{\div 9}{=}} \frac{4}{5}$$

When we show the work like this, it is called "cancelling"

$$\frac{\overset{4}{\cancel{180}}}{\underset{5}{\cancel{225}}} = \frac{4}{5}$$

Cancelling factors in products:

$$\frac{\overset{7}{\cancel{21}} \cdot \overset{11}{\cancel{22}}}{23 \cdot \underset{4}{\cancel{24}}} = \frac{77}{92}$$

We cancel a factor of 2 from 22 and 24.
We write 11 by the 22 and 12 by the 24.
Then, we cancel a factor of 3 from 21 and 12.
We write 7 by the 21 and 4 by the 12.

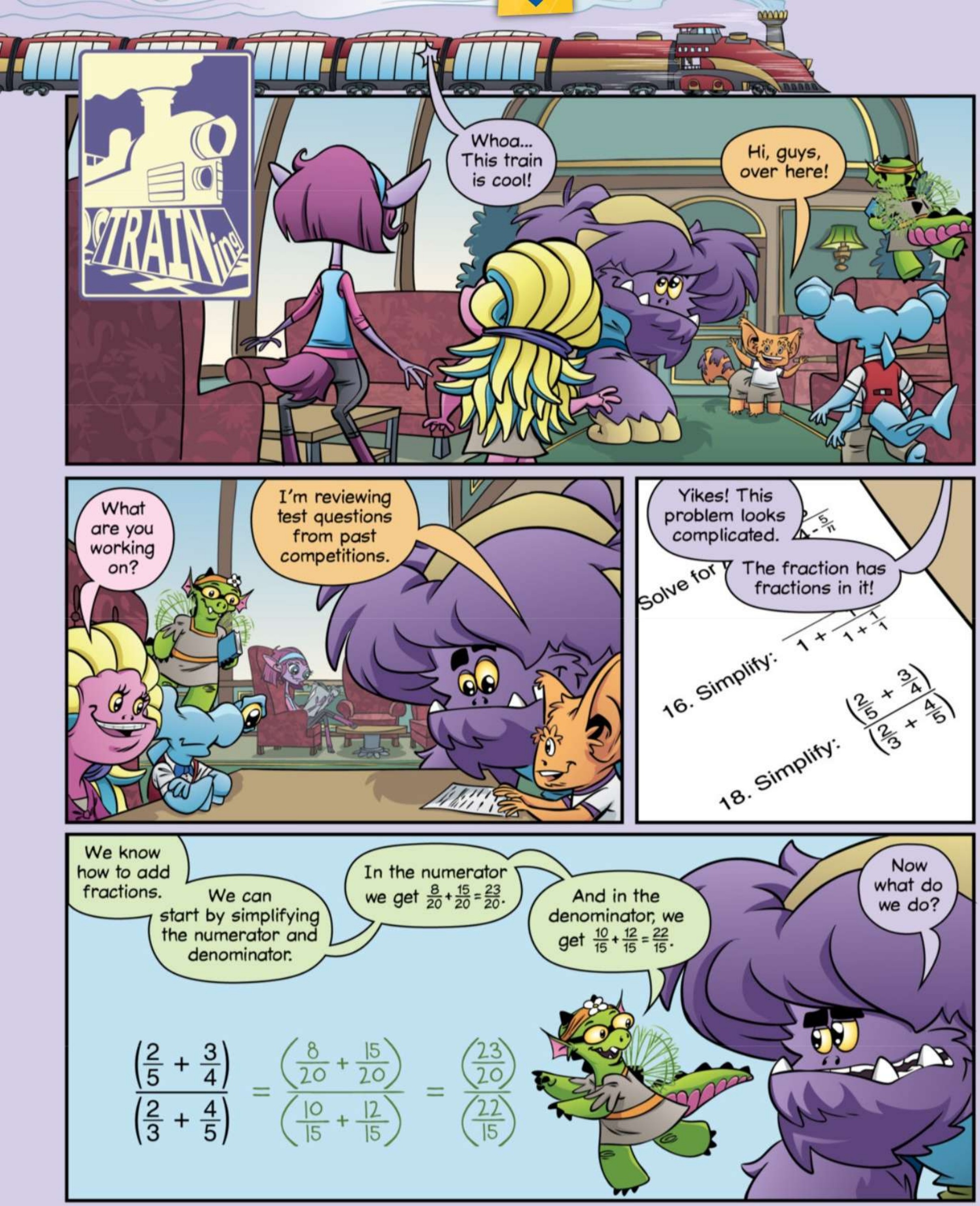
Cancelling when multiplying fractions:

$$\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{14}}} \cdot \frac{\overset{1}{\cancel{7}}}{\underset{4}{\cancel{12}}} = \frac{3}{8}$$

We cancel a factor of 3 from 9 and 12.
We write 3 by the 9 and 4 by the 12.
Then, we cancel a factor of 7 from 7 and 14.
We write 1 by the 7 and 2 by the 14.

$$\frac{\overset{3}{\cancel{6}}}{11} \cdot \frac{\cancel{3}}{\underset{5}{\cancel{10}}} \cdot \frac{7}{\underset{3}{\cancel{9}}} = \frac{7}{55}$$

We cancel a factor of 2 from 6 and 10.
We cancel a factor of 3 from 3 and 9.
Then, we cancel the 3 on top and bottom.
We don't need to write 1's by the 3's.


TRAINing
Whoa... This train is cool!
Hi, guys, over here!
What are you working on?
I'm reviewing test questions from past competitions.
Yikes! This problem looks complicated.
The fraction has fractions in it!
Solve for
16. Simplify: $1 + \frac{1}{1 + \frac{1}{1}}$
18. Simplify: $\frac{\left(\frac{2}{5} + \frac{3}{4}\right)}{\left(\frac{2}{3} + \frac{4}{5}\right)}$
We know how to add fractions.
We can start by simplifying the numerator and denominator.
In the numerator we get $\frac{8}{20} + \frac{15}{20} = \frac{23}{20}$.
And in the denominator, we get $\frac{10}{15} + \frac{12}{15} = \frac{22}{15}$.
Now what do we do?
$\frac{\left(\frac{2}{5} + \frac{3}{4}\right)}{\left(\frac{2}{3} + \frac{4}{5}\right)} = \frac{\left(\frac{8}{20} + \frac{15}{20}\right)}{\left(\frac{10}{15} + \frac{12}{15}\right)} = \frac{\left(\frac{23}{20}\right)}{\left(\frac{22}{15}\right)}$

We divide the fraction in the numerator by the fraction in the denominator.

How do we divide $\frac{23}{20}$ by $\frac{22}{15}$?

To divide by a number, we multiply by its reciprocal.

Reciprocal?

$$\frac{\left(\frac{23}{20}\right)}{\left(\frac{22}{15}\right)} = \frac{23}{20} \div \frac{22}{15}$$

THIS IS VERY IMPORTANT, SO WE WILL REPEAT IT: TO DIVIDE BY A NUMBER, WE MULTIPLY BY ITS RECIPROCAL.

A number's ***reciprocal*** is the number you multiply it by to get 1.

Oh, right...

...the reciprocal of 2 is $\frac{1}{2}$, and the reciprocal of $\frac{1}{7}$ is 7.

$$2 \cdot \frac{1}{2} = 1$$

$$\frac{1}{7} \cdot 7 = 1$$

Exactly. To divide $\frac{23}{20}$ by $\frac{22}{15}$, we multiply $\frac{23}{20}$ by the reciprocal of $\frac{22}{15}$.

RECIPROCALS AND FRACTION DIVISION ARE INTRODUCED IN CHAPTER 10 OF BEAST ACADEMY 4D.

So, we need to find the reciprocal of $\frac{22}{15}$.

What number can we multiply $\frac{22}{15}$ by to get 1?

I know!

What is the reciprocal of $\frac{22}{15}$?

The reciprocal of $\frac{22}{15}$ is $\frac{15}{22}$.

That's right.

You can find the reciprocal of any fraction $\frac{a}{b}$ by flipping the numerator and denominator to get $\frac{b}{a}$.

$$\frac{22}{15} \cdot \frac{15}{22} = \frac{22 \cdot 15}{15 \cdot 22} = 1$$

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ba} = 1$$

That makes sense. $\frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ba} = 1$.

So, the reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$.

$$\frac{\left(\frac{2}{5} + \frac{3}{4}\right)}{\left(\frac{2}{3} + \frac{4}{5}\right)} = \frac{\left(\frac{8}{20} + \frac{15}{20}\right)}{\left(\frac{10}{15} + \frac{12}{15}\right)} = \frac{\left(\frac{23}{20}\right)}{\left(\frac{22}{15}\right)} = \frac{23}{20} \div \frac{22}{15} = \frac{23}{20} \cdot \frac{15}{22}$$

So, to divide $\frac{23}{20}$ by $\frac{22}{15}$, we multiply $\frac{23}{20}$ by $\frac{15}{22}$.

20 and 15 have a common factor of 5, so we can simplify before we multiply.

Then, $23 \cdot 3 = 69$, and $4 \cdot 22 = 88$.

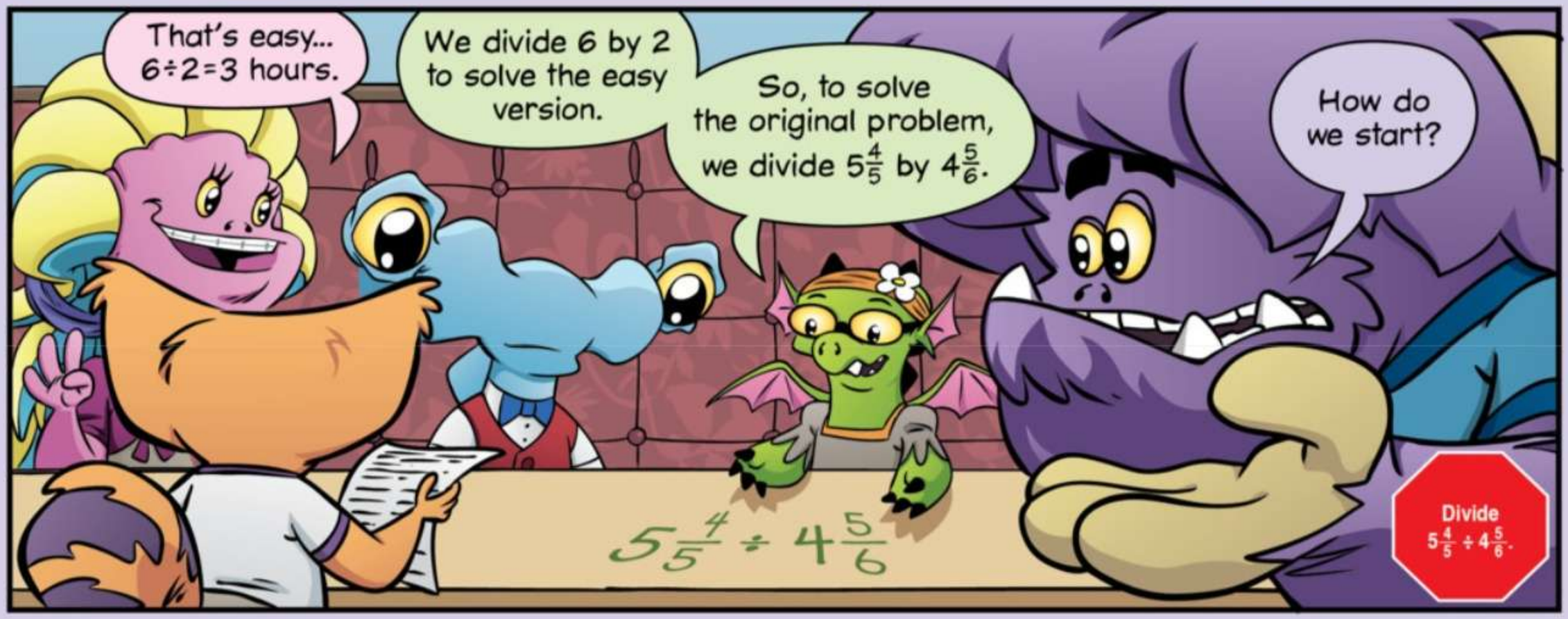
So, $\frac{23}{20} \div \frac{22}{15} = \frac{69}{88}$.

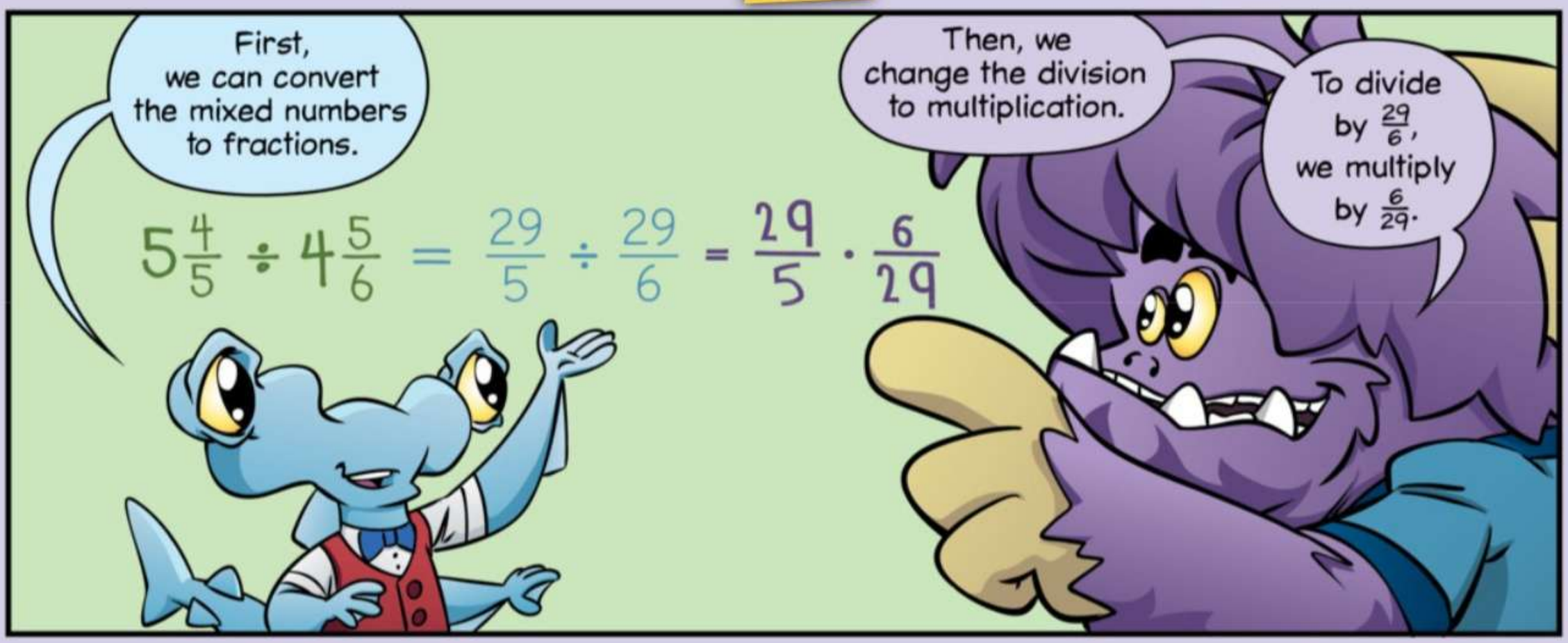
You got it!

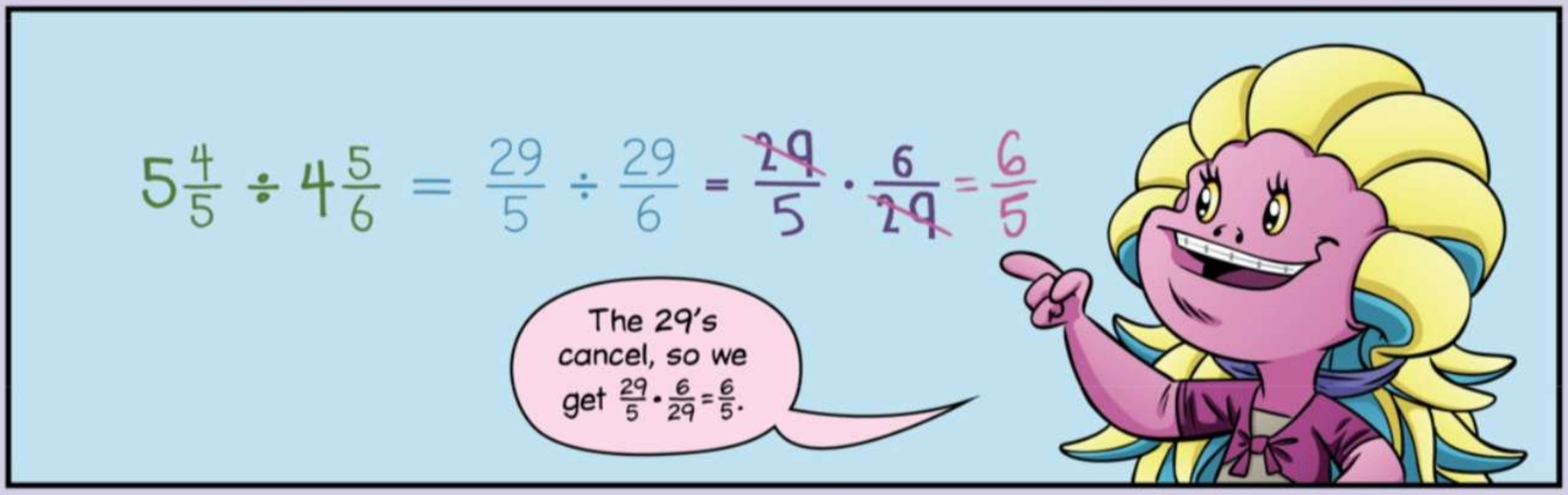
$$\frac{\left(\frac{2}{5} + \frac{3}{4}\right)}{\left(\frac{2}{3} + \frac{4}{5}\right)} = \frac{\left(\frac{8}{20} + \frac{15}{20}\right)}{\left(\frac{10}{15} + \frac{12}{15}\right)} = \frac{\left(\frac{23}{20}\right)}{\left(\frac{22}{15}\right)} = \frac{23}{20} \div \frac{22}{15} = \frac{23}{\cancel{20}_{4}} \cdot \frac{\cancel{15}^{3}}{22} = \frac{69}{88}$$

Wow!
On this train ride, we've already learned how to multiply or divide any two fractions.
Not yet.
Have you done any work with mixed numbers?
See if you can solve this one.
21. If Sally Slug can slime $4\frac{5}{6}$ meters in an hour, how many hours will it take for her to slime $5\frac{4}{5}$ meters?

I think we need to divide.
Do we divide $4\frac{5}{6}$ by $5\frac{4}{5}$, or is it the other way around?
Let's try an easier problem.
What if we change these mixed numbers to ones we know how to deal with... like 2 and 6.
21. If Sally Slug can slime $\cancel{4\frac{5}{6}}$ 2 meters in an hour, how many hours will it take for her to slime $\cancel{5\frac{4}{5}}$ 6 meters?


That's easy... 6÷2=3 hours.
We divide 6 by 2 to solve the easy version.
So, to solve the original problem, we divide $5\frac{4}{5}$ by $4\frac{5}{6}$.
How do we start?
$5\frac{4}{5} \div 4\frac{5}{6}$
Divide $5\frac{4}{5} \div 4\frac{5}{6}$.


First, we can convert the mixed numbers to fractions.
$5\frac{4}{5} \div 4\frac{5}{6} = \frac{29}{5} \div \frac{29}{6} = \frac{29}{5} \cdot \frac{6}{29}$
Then, we change the division to multiplication.
To divide by $\frac{29}{6}$, we multiply by $\frac{6}{29}$.


$5\frac{4}{5} \div 4\frac{5}{6} = \frac{29}{5} \div \frac{29}{6} = \frac{\cancel{29}}{5} \cdot \frac{6}{\cancel{29}} = \frac{6}{5}$
The 29's cancel, so we get $\frac{29}{5} \cdot \frac{6}{29} = \frac{6}{5}$.

So, it takes Sally Slug $\frac{6}{5} = 1\frac{1}{5}$ hours to slime $5\frac{4}{5}$ meters.
Nice. You guys learn fast!

Alright, team. We're almost there. Grab your stuff and let's get ready to go.

There are just a few more things to go over.

The rules are pretty simple. The first round is a written test.

The four teams with the best written test scores qualify for the final round on stage.

As alternate, Grogg, you will take the written test...

...but your score will only count as a tiebreaker.

Here we are...

...the World Math Olympiad Qualifiers.

WORLD MATH OLYMPIAD QUALIFIERS

The final round is like the Math Meets against the bots...

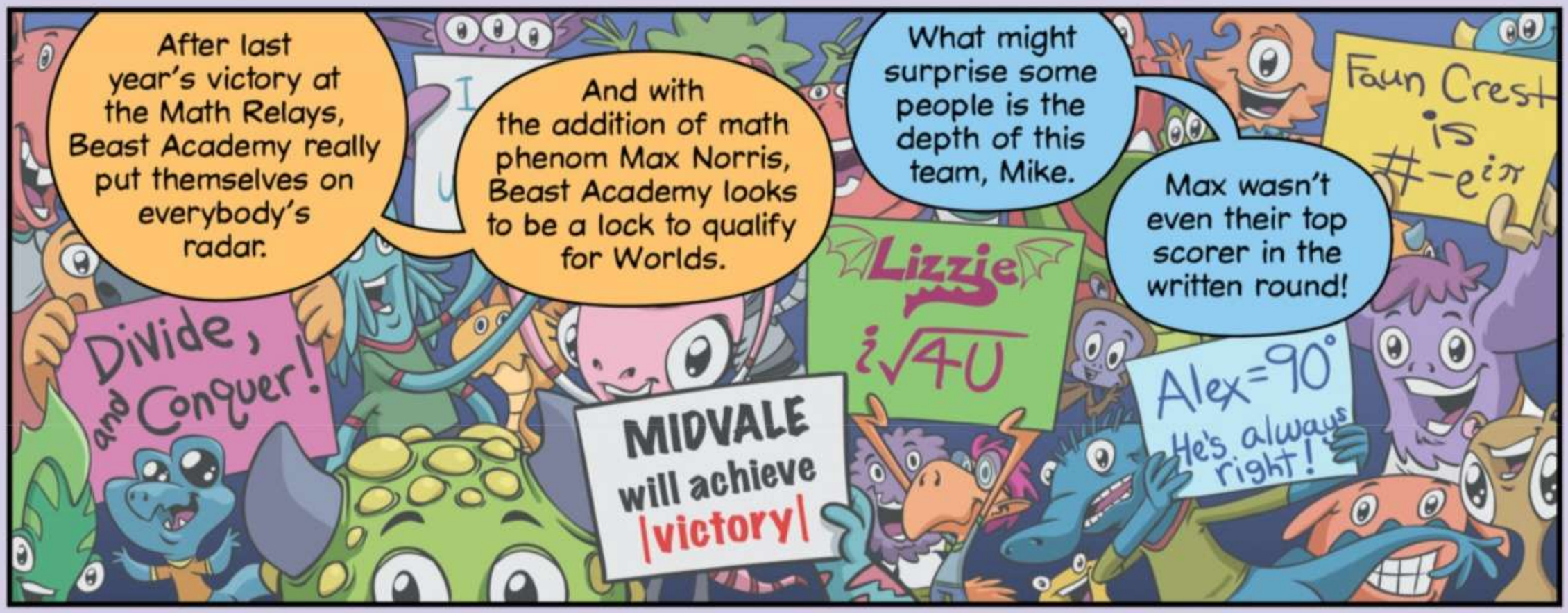
...only with four teams and even harder questions.

The top team at the end of the final round qualifies for the Championships.

Go ahead and find your seats.

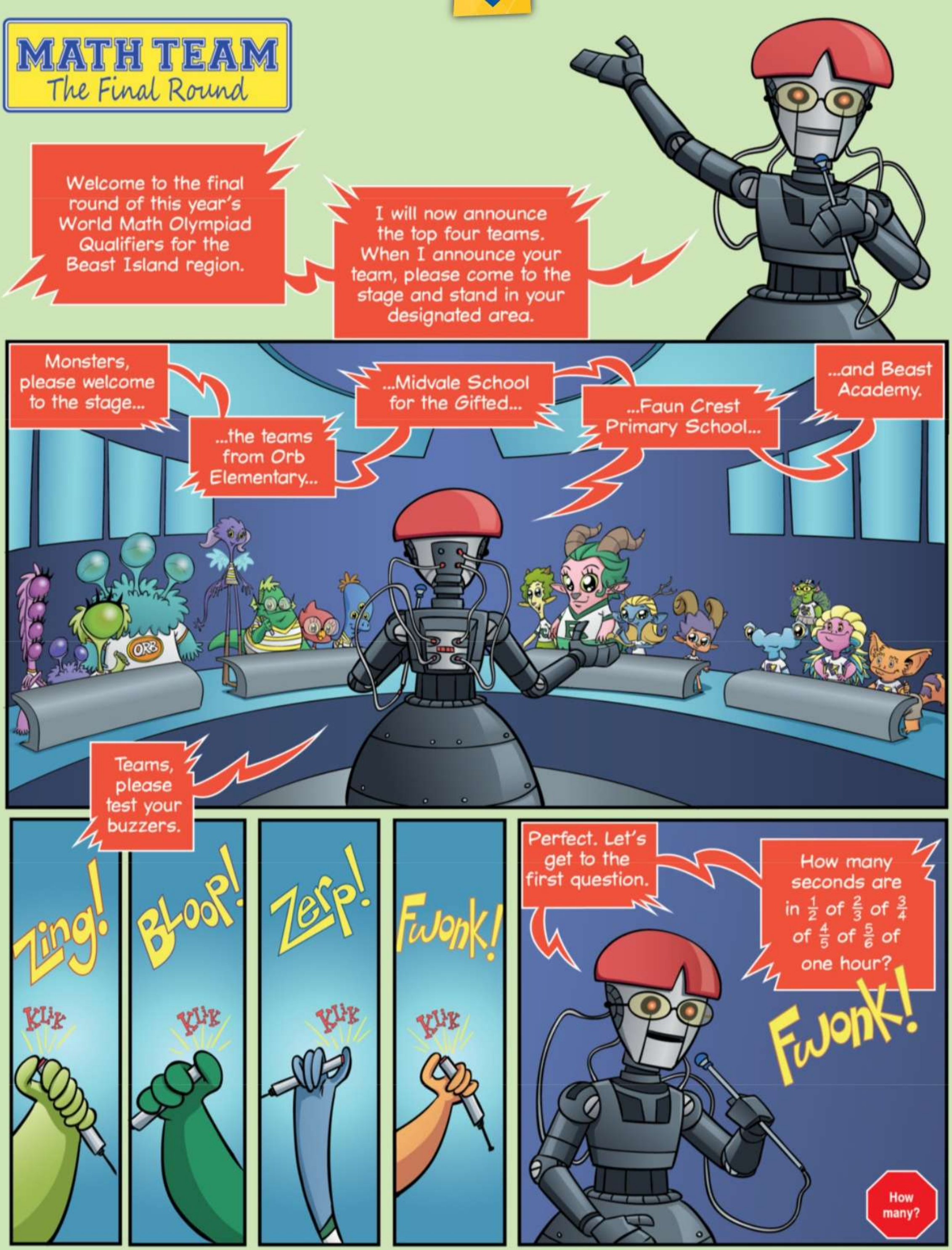
Do your best!

CHANNEL 5
Welcome back to Channel 5's live coverage of the World Math Olympiad Qualifiers.
The written test is over, and there are very few surprises, Phil.
Nobody should be surprised at the top-scoring team from the written round, Mike.

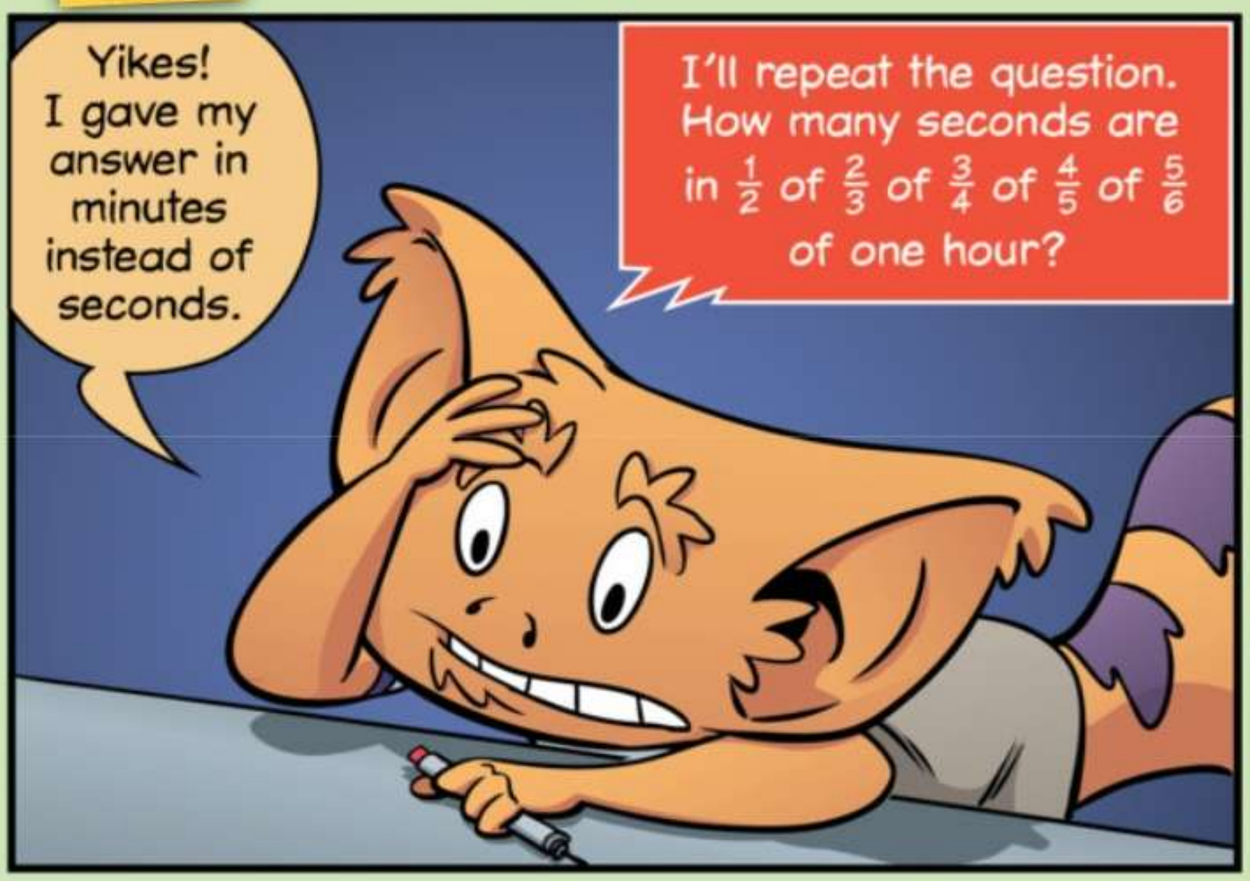

After last year's victory at the Math Relays, Beast Academy really put themselves on everybody's radar.
And with the addition of math phenom Max Norris, Beast Academy looks to be a lock to qualify for Worlds.
What might surprise some people is the depth of this team, Mike.
Max wasn't even their top scorer in the written round!
Divide, and Conquer!
MIDVALE will achieve victory
Lizzie
Alex=90°
He's always right!

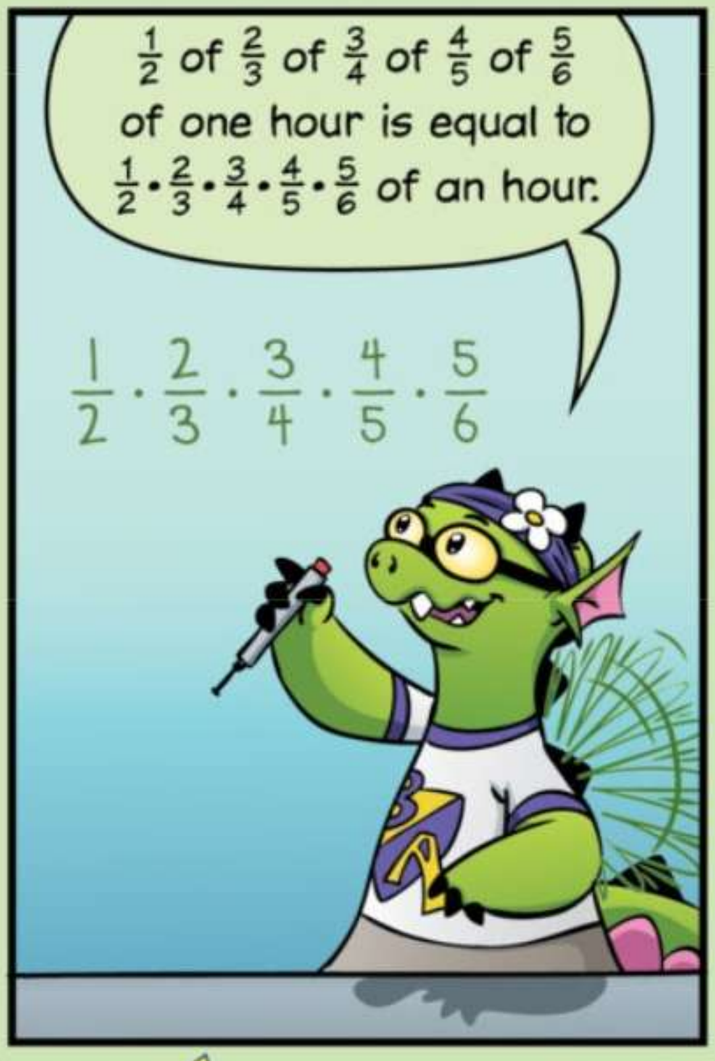
That's right, Phil. Beast Academy team member Wendolyn D'Monstre is our individual leader, with a perfect score on the written test.
Beast Academy had all of its team members place in the top 10 individually, *including* their alternate, Groggorius George.
They're about to call the top four teams to the stage. Let's get back to the action!
Here's My Sine!!
Hi Mom!
We ♥ u, WINnie!
OH NO! SignError!

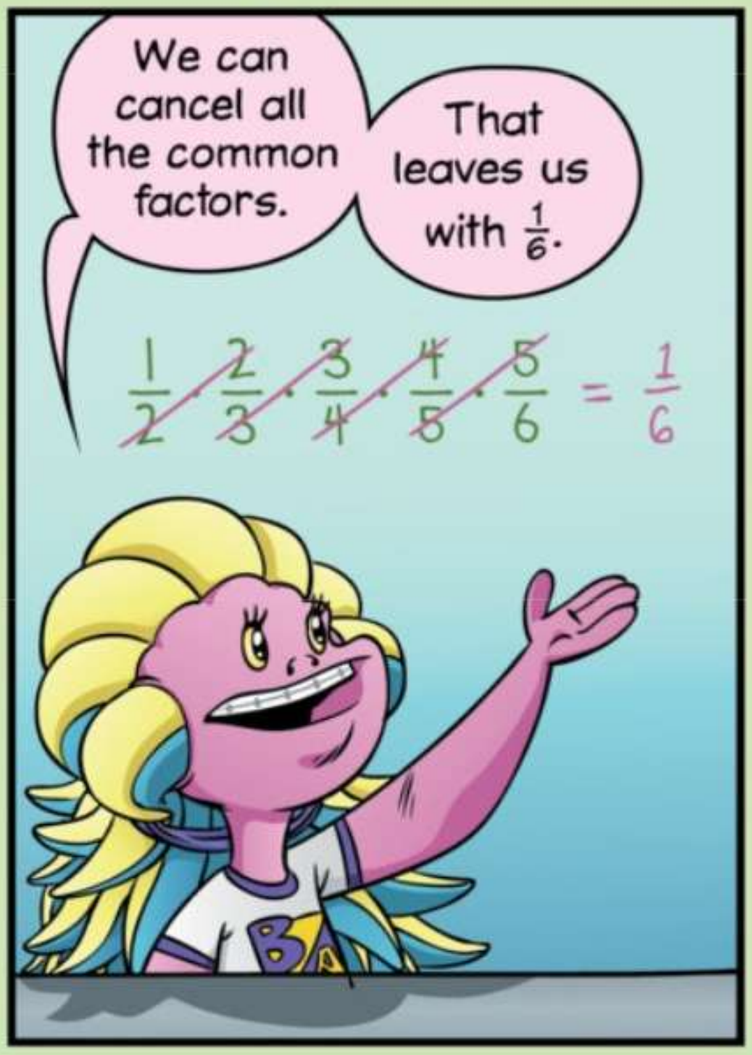
MATH TEAM
The Final Round

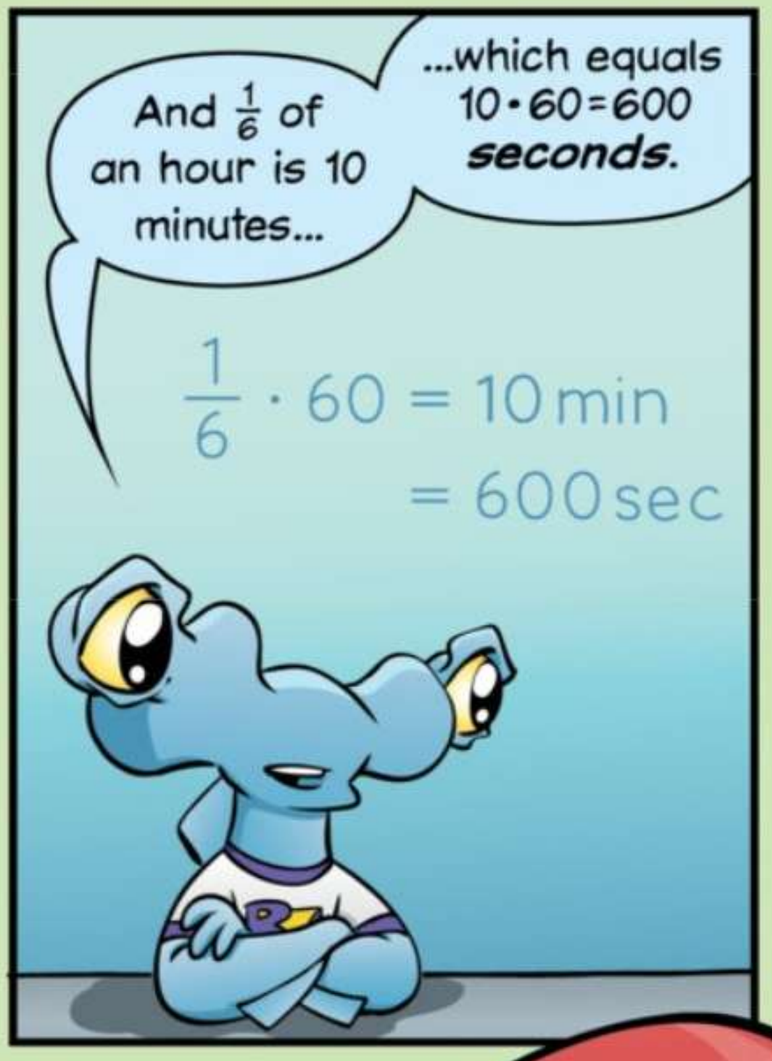


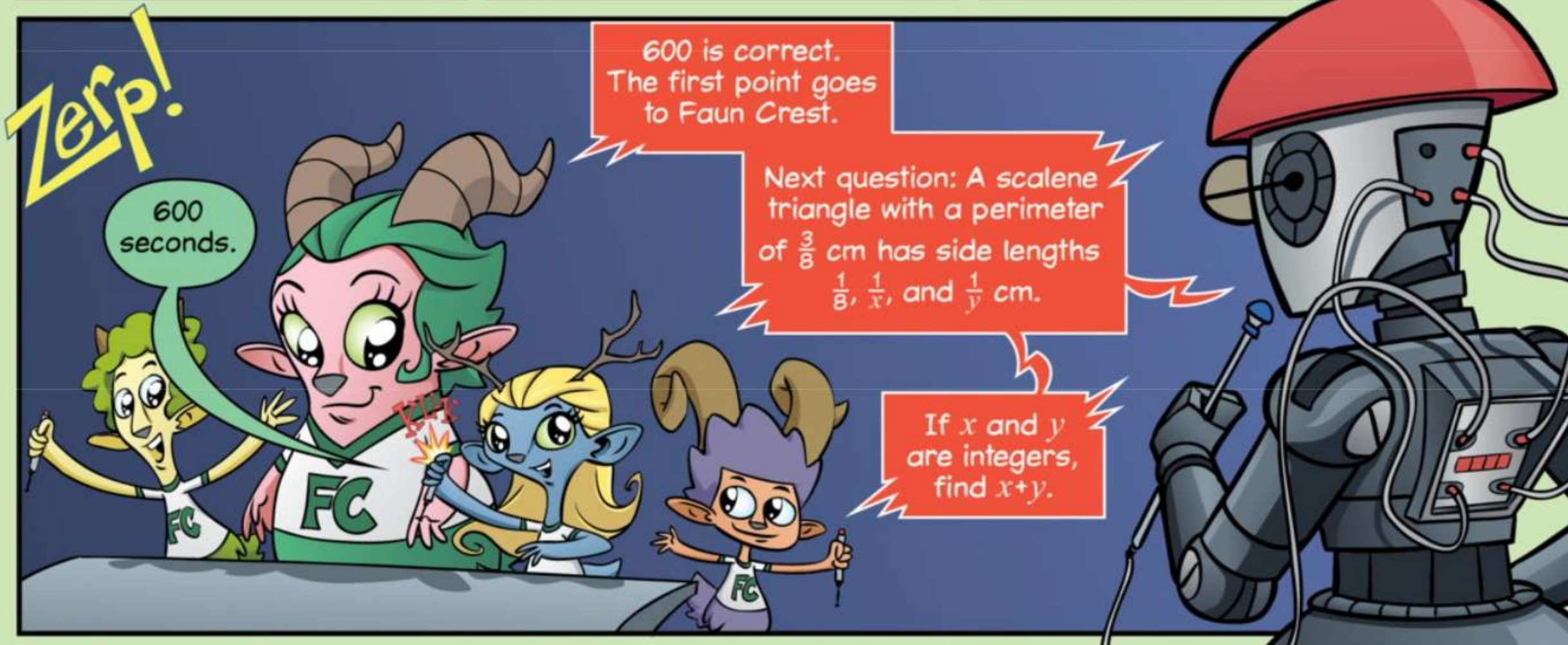
10.
Sorry, 10 is incorrect.


Yikes! I gave my answer in minutes instead of seconds.
I'll repeat the question. How many seconds are in $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of one hour?


$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of one hour is equal to $\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6}$ of an hour.
$\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6}$


We can cancel all the common factors.
That leaves us with $\frac{1}{6}$.
$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{4}}\cdot\frac{\cancel{4}}{\cancel{5}}\cdot\frac{\cancel{5}}{6} = \frac{1}{6}$


And $\frac{1}{6}$ of an hour is 10 minutes...
...which equals $10\cdot 60=600$ **seconds.**
$\frac{1}{6}\cdot 60 = 10\,\text{min}$
$= 600\,\text{sec}$


Zerp!
600 seconds.
600 is correct. The first point goes to Faun Crest.
Next question: A scalene triangle with a perimeter of $\frac{3}{8}$ cm has side lengths $\frac{1}{8}$, $\frac{1}{x}$, and $\frac{1}{y}$ cm.
If x and y are integers, find $x+y$.

A triangle with three side lengths of $\frac{1}{8}$ has a perimeter of $\frac{3}{8}$...
...but, it's not scalene.
We know one of the sides is $\frac{1}{8}$, so the remaining sides add up to $\frac{3}{8}-\frac{1}{8}=\frac{2}{8}=\frac{1}{4}$.
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
Perimeter $=\frac{3}{8}$
One of the missing side lengths must be greater than $\frac{1}{8}$, and the other must be less than $\frac{1}{8}$.
The only fractions $\frac{1}{x}$ that are greater than $\frac{1}{8}$ are $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$.
But we can't have a side length of $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$, since the two sides *together* add up to $\frac{1}{4}$.
So, the longest side must be $\frac{1}{7}$, $\frac{1}{6}$ or $\frac{1}{5}$.
Found it!
$\frac{1}{8}+\frac{1}{5}+\frac{1}{20}$
$=\frac{5}{40}+\frac{8}{40}+\frac{2}{40}$
$=\frac{15}{40}=\frac{3}{8}$.
So, $x+y=$ $5+20=$
Fwonk!
25.
Klik
Sorry, 25 is incorrect.
Why can't a triangle have sides lengths $\frac{1}{8}$, $\frac{1}{5}$, and $\frac{1}{20}$?

A triangle can't have side lengths $\frac{1}{8}$, $\frac{1}{5}$, and $\frac{1}{20}$...
...because $\frac{1}{8}+\frac{1}{20}$ is *less* than $\frac{1}{5}$.
Max missed that.
But, $\frac{1}{8}+\frac{1}{6}+\frac{1}{12}$ works!
$\frac{1}{8}=\frac{5}{40}$
$\frac{1}{20}=\frac{2}{40}$
$\frac{1}{5}=\frac{8}{40}$
$\frac{1}{8}=\frac{3}{24}$
$\frac{1}{12}=\frac{2}{24}$
$\frac{1}{6}=\frac{4}{24}$

BLOOP!
18.
KLIK
Correct!
Midvale wins the point.

Beast Academy is having some early troubles, Phil.
Max Norris looks like his confidence has been shaken.
Question 3:
Find the average of four primes whose median is 18 and whose range is 35.
Try it.

The range is the difference between the smallest and largest primes.
For two primes to have a difference of 35, one has to be odd, and the other has to be even.
Since 2 is the only even prime, the smallest and largest primes must be 2 and 37.

The median, 18, is halfway between the two middle primes.
They could be 17 and 19, or 13 and 23, or 7 and 29, or 5 and 31!
It doesn't matter which pair it is, their sum is 36.
So, the sum of all four primes is 2+36+37=75.

And the average of all four primes is $\frac{75}{4}$=...
Fwonk!
$18\frac{3}{4}$.

Correct. Beast Academy scores a point.
Yes!

Next question:
Simplify $\frac{11!+12!}{13!}$.
Try it.

We can factor 11! from both the numerator and the denominator.

Right, since 11!, 12!, and 13! all have 11! as a factor.

On top, we get (1+12)•11!.

And on the bottom, we can write 13! as 13•12•11!.

$$\frac{11! + 12!}{13!} = \frac{11! + 12 \cdot 11!}{13 \cdot 12 \cdot 11!} = \frac{(1+12) \cdot 11!}{13 \cdot 12 \cdot 11!}$$

We can simplify 1+12 to 13, then cancel the common factors.

$$\frac{11! + 12!}{13!} = \frac{11! + 12 \cdot 11!}{13 \cdot 12 \cdot 11!} = \frac{(1+12) \cdot 11!}{13 \cdot 12 \cdot 11!} = \frac{13 \cdot 11!}{13 \cdot 12 \cdot 11!}$$

The 11!'s cancel, and the 13's cancel.

$$\frac{11! + 12!}{13!} = \frac{11! + 12 \cdot 11!}{13 \cdot 12 \cdot 11!} = \frac{(1+12) \cdot 11!}{13 \cdot 12 \cdot 11!} = \frac{\cancel{13} \cdot \cancel{11!}}{\cancel{13} \cdot 12 \cdot \cancel{11!}}$$

KLIK

That leaves...

FWONK!

Correct.

Another point for Beast Academy.

$\frac{1}{12}$.

Woohoo!

Find the smallest possible sum of two integers whose GCF is 8 and whose LCM is 80.

Fwonk!

56.

Correct!

Klik

Beast Academy really seems to have found a rhythm now, Mike.

Sensational!

On a list of nine fractions whose average is $\frac{1}{8}$, the first eight fractions average $\frac{1}{9}$. What is the ninth fraction on the list?

Fwonk!

$\frac{17}{72}$.

Klik

How many perfect squares are factors of $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$?

Fwonk!

8.

Klik

After a slow start, Beast Academy is really leaving no doubt as to who the top team on Beast Island is, Phil.

Compute: $\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5}$.

Fwonk!

$\frac{1}{5}$.

Klik

FIND DETAILED SOLUTIONS TO ALL FOUR PROBLEMS ABOVE AT BEASTACADEMY.COM.

It's all over! What a performance!

Beast Academy is moving on to the World Math Olympiad Championships!