Contents: Chapter 11

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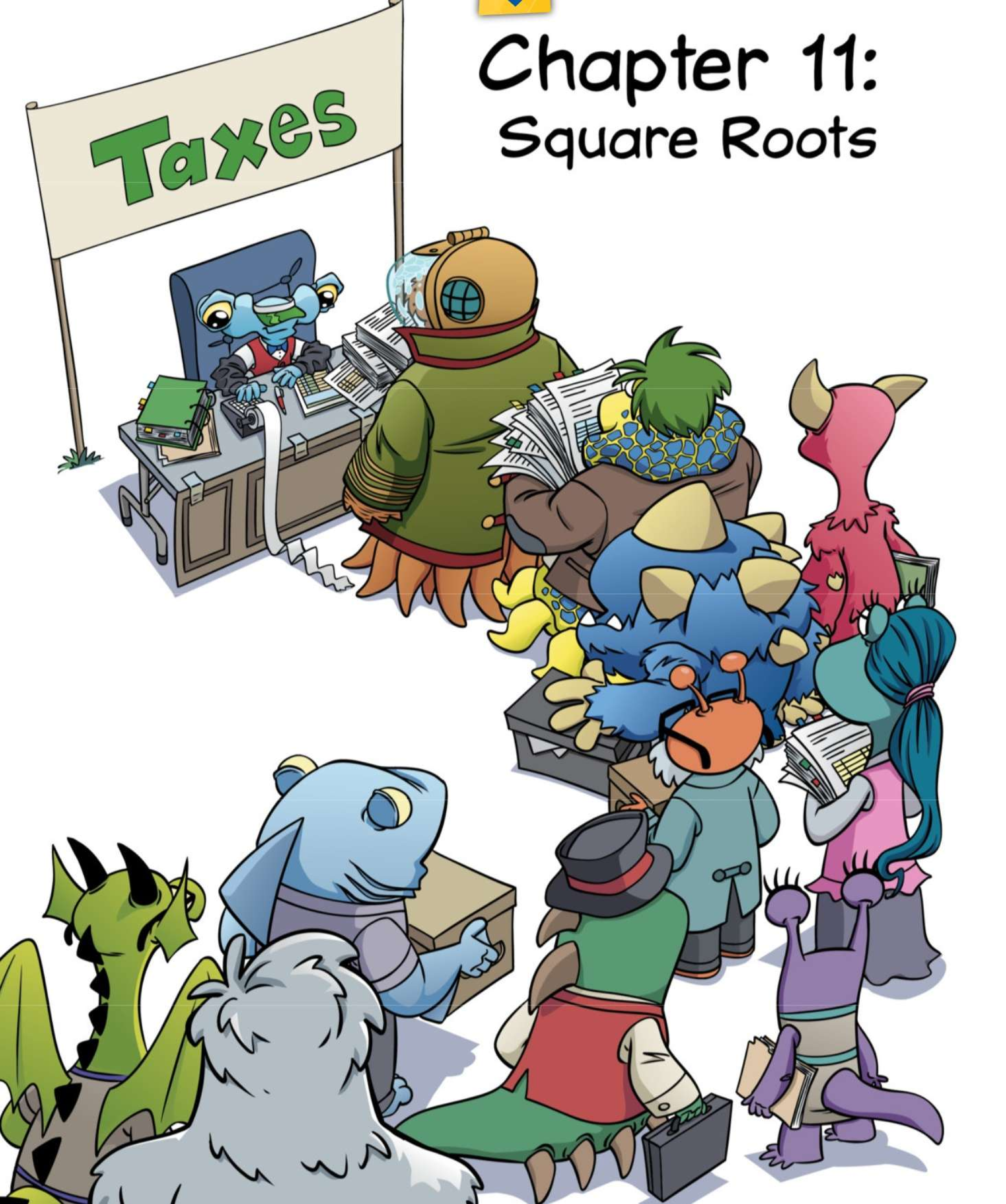
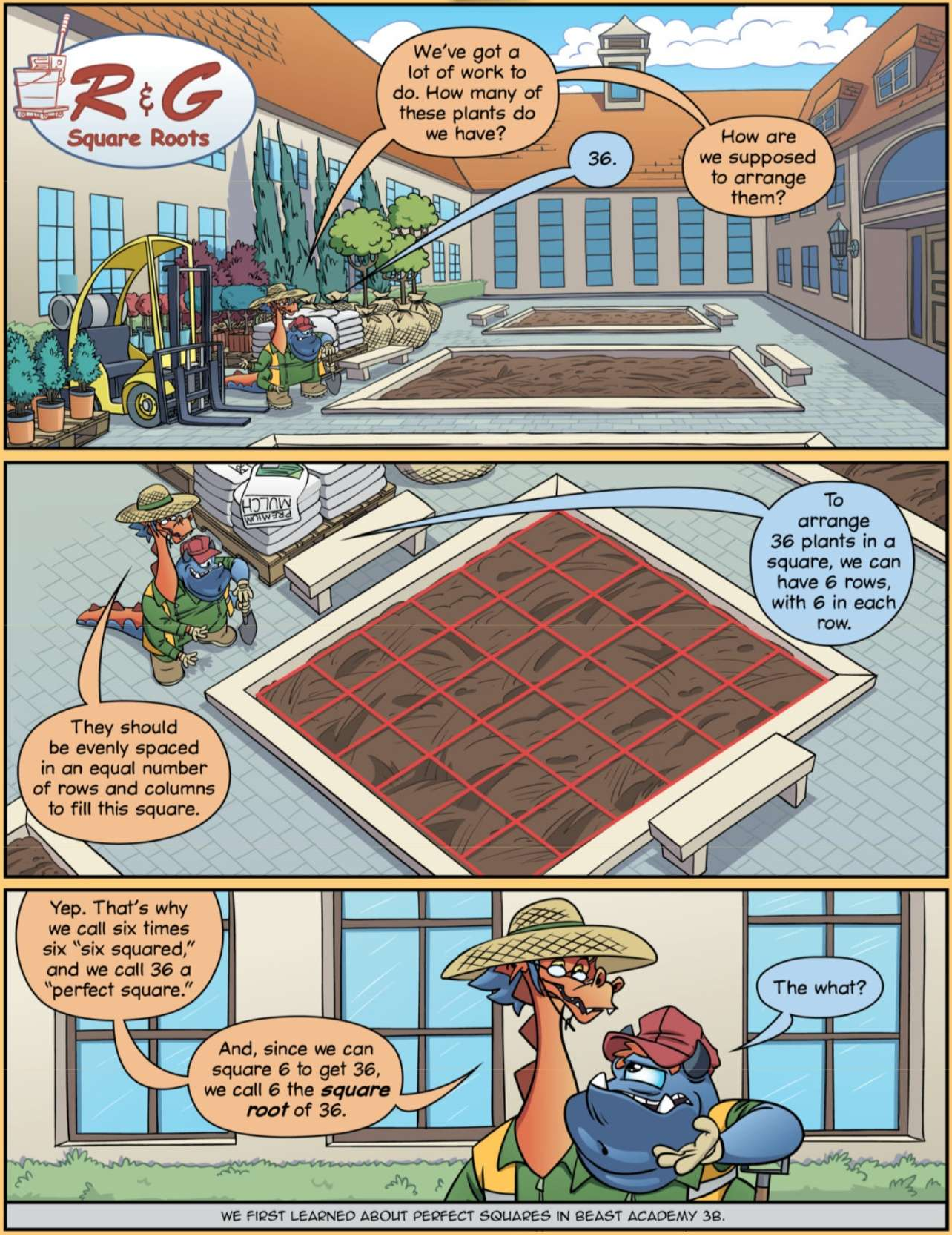
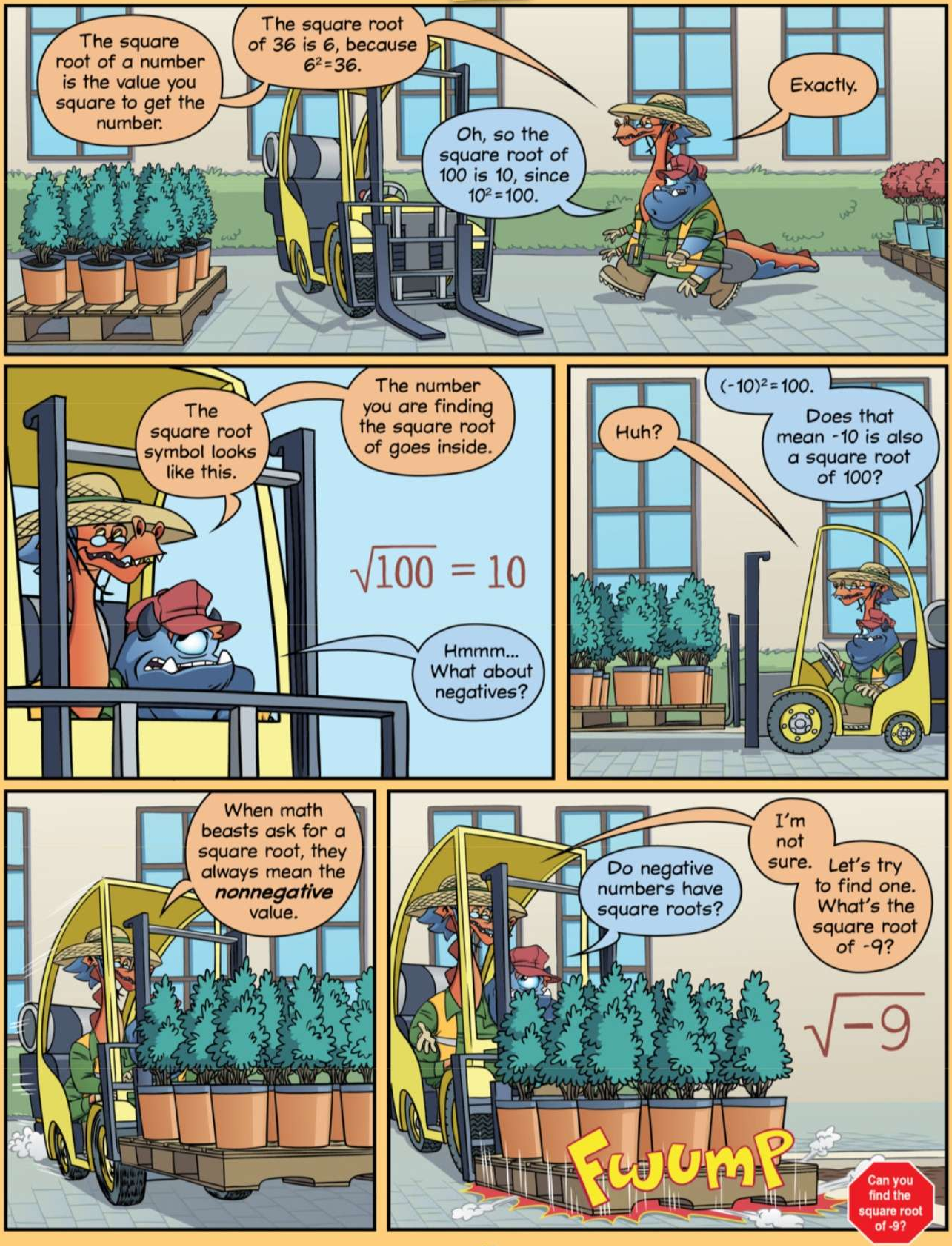
Chapter 11:
Square Roots

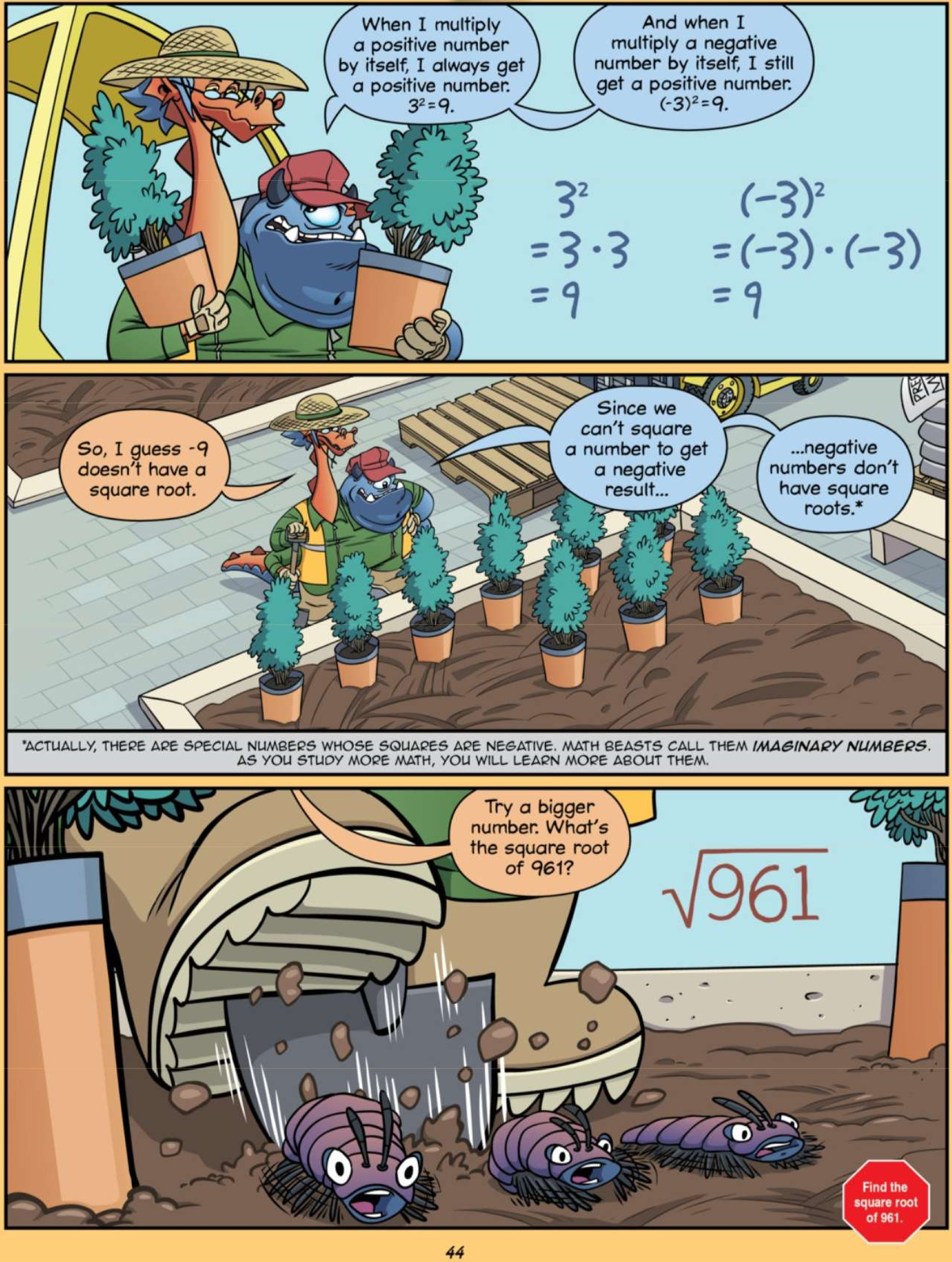


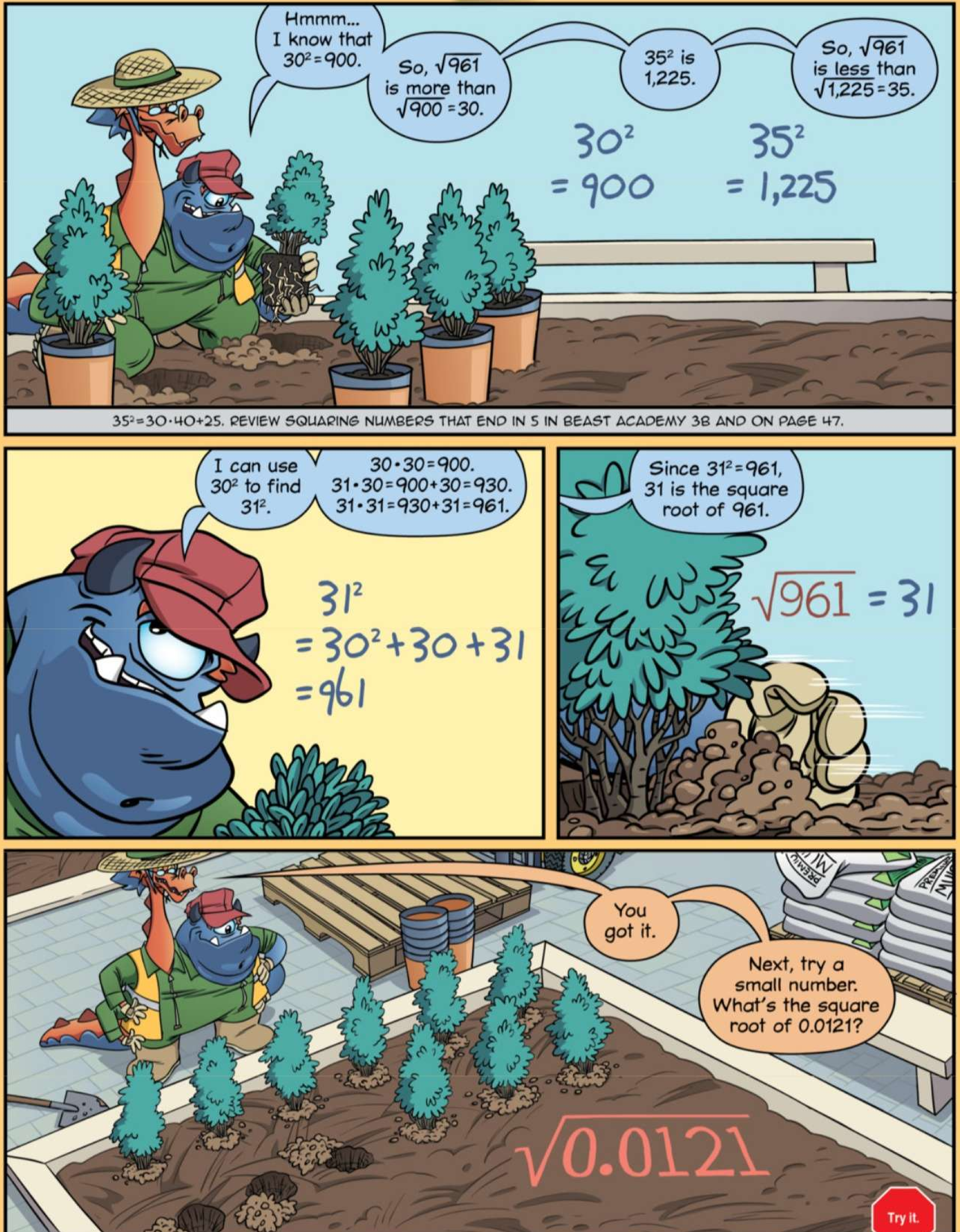


R & G
Square Roots
We've got a lot of work to do. How many of these plants do we have?
36.
How are we supposed to arrange them?
PREMIUM MULCH
To arrange 36 plants in a square, we can have 6 rows, with 6 in each row.
They should be evenly spaced in an equal number of rows and columns to fill this square.
Yep. That's why we call six times six "six squared," and we call 36 a "perfect square."
And, since we can square 6 to get 36, we call 6 the ***square root*** of 36.
The what?
WE FIRST LEARNED ABOUT PERFECT SQUARES IN BEAST ACADEMY 3B.



The square root of a number is the value you square to get the number.
The square root of 36 is 6, because $6^2 = 36$.
Oh, so the square root of 100 is 10, since $10^2 = 100$.
Exactly.
The square root symbol looks like this.
The number you are finding the square root of goes inside.
$\sqrt{100} = 10$
Hmmm... What about negatives?
$(-10)^2 = 100$.
Does that mean -10 is also a square root of 100?
Huh?
When math beasts ask for a square root, they always mean the ***nonnegative*** value.
Do negative numbers have square roots?
I'm not sure.
Let's try to find one. What's the square root of -9?
$\sqrt{-9}$
FWUMP
Can you find the square root of -9?


When I multiply a positive number by itself, I always get a positive number. $3^2=9$.
And when I multiply a negative number by itself, I still get a positive number. $(-3)^2=9$.
3^2
$=3\cdot3$
$=9$
$(-3)^2$
$=(-3)\cdot(-3)$
$=9$
So, I guess -9 doesn't have a square root.
Since we can't square a number to get a negative result...
...negative numbers don't have square roots.*
*ACTUALLY, THERE ARE SPECIAL NUMBERS WHOSE SQUARES ARE NEGATIVE. MATH BEASTS CALL THEM *IMAGINARY NUMBERS*. AS YOU STUDY MORE MATH, YOU WILL LEARN MORE ABOUT THEM.
Try a bigger number. What's the square root of 961?
$\sqrt{961}$
Find the square root of 961.


Hmmm... I know that $30^2 = 900$.
So, $\sqrt{961}$ is more than $\sqrt{900} = 30$.
35^2 is 1,225.
So, $\sqrt{961}$ is less than $\sqrt{1{,}225} = 35$.
$30^2 = 900$
$35^2 = 1{,}225$
$35^2 = 30 \cdot 40 + 25$. REVIEW SQUARING NUMBERS THAT END IN 5 IN BEAST ACADEMY 3B AND ON PAGE 47.
I can use 30^2 to find 31^2.
$30 \cdot 30 = 900$.
$31 \cdot 30 = 900 + 30 = 930$.
$31 \cdot 31 = 930 + 31 = 961$.
$31^2 = 30^2 + 30 + 31 = 961$
Since $31^2 = 961$, 31 is the square root of 961.
$\sqrt{961} = 31$
You got it.
Next, try a small number. What's the square root of 0.0121?
$\sqrt{0.0121}$
Try it.

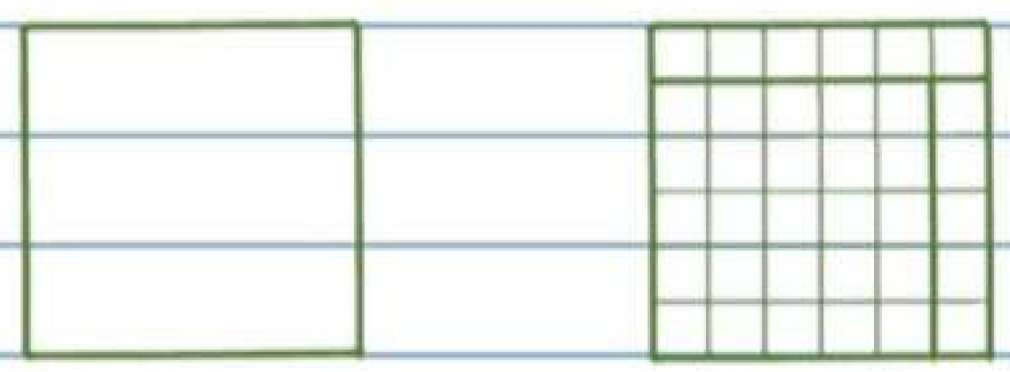
Hmmm... I know $11^2 = 121$.
So, $1.1^2 = 1.21$...
...and $0.11^2 = 0.0121$.
$11^2 = 121$
$1.1^2 = 1.1 \times 1.1 = 1.21$
$0.11^2 = 0.11 \times 0.11 = 0.0121$
$\sqrt{0.0121} = 0.11$
So, the square root of 0.0121 is 0.11.
PREMIUM MULCH
Nice job! Now, let's get the rest of these plants in the ground.
Whoa, check this out!
It's a square root.
Looks more like a cube root to me.
RAISING A NUMBER TO THE THIRD POWER (n^3) IS CALLED "CUBING" THE NUMBER. THE ***CUBE ROOT*** OF A NUMBER IS THE VALUE YOU ***CUBE*** TO GET THE NUMBER.

Squaring Numbers

Lizzie

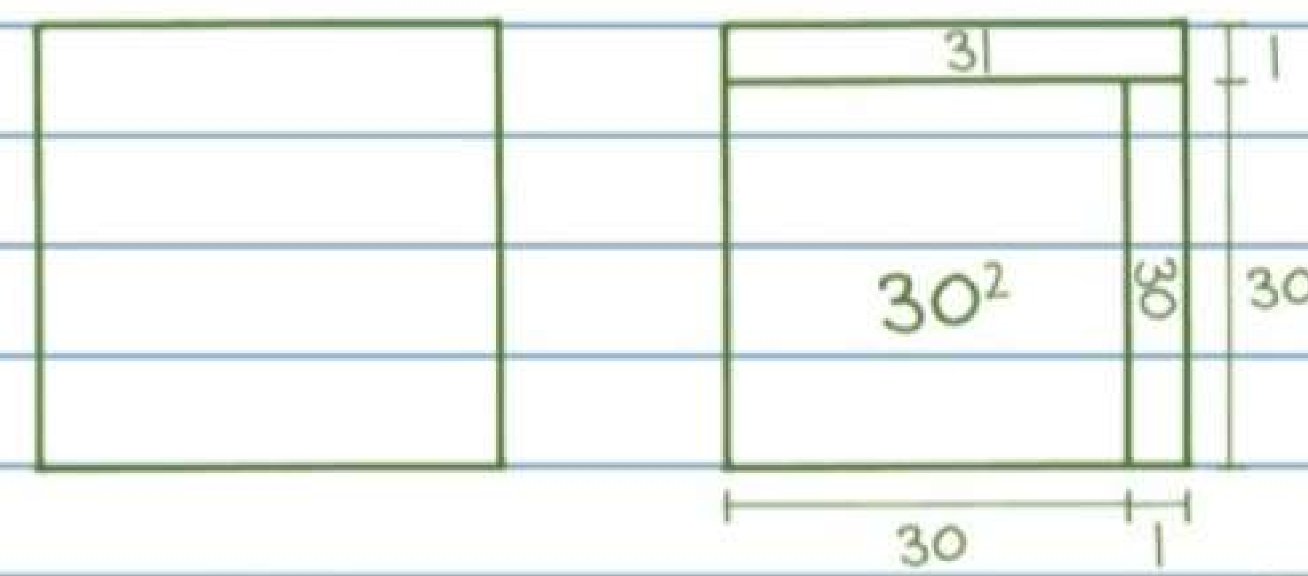
Finding the next square:
To get from 5^2 to 6^2, we add 5 and 6 to 5^2: $5^2+5+6=6^2$.
To get from 30^2 to 31^2, we add 30 and 31 to 30^2: $30^2+30+31=31^2$.

$6^2 = 5^2+5+6$

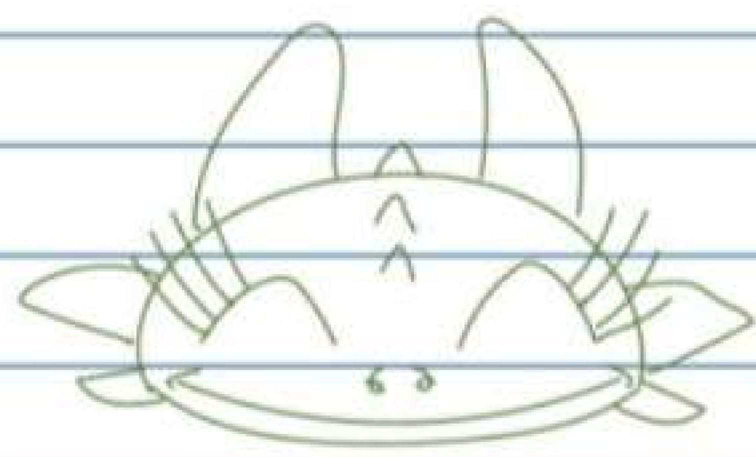


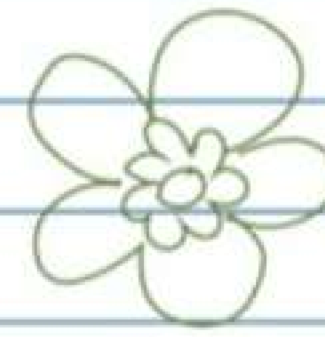
$=36$

$31^2 = 30^2+30+31$



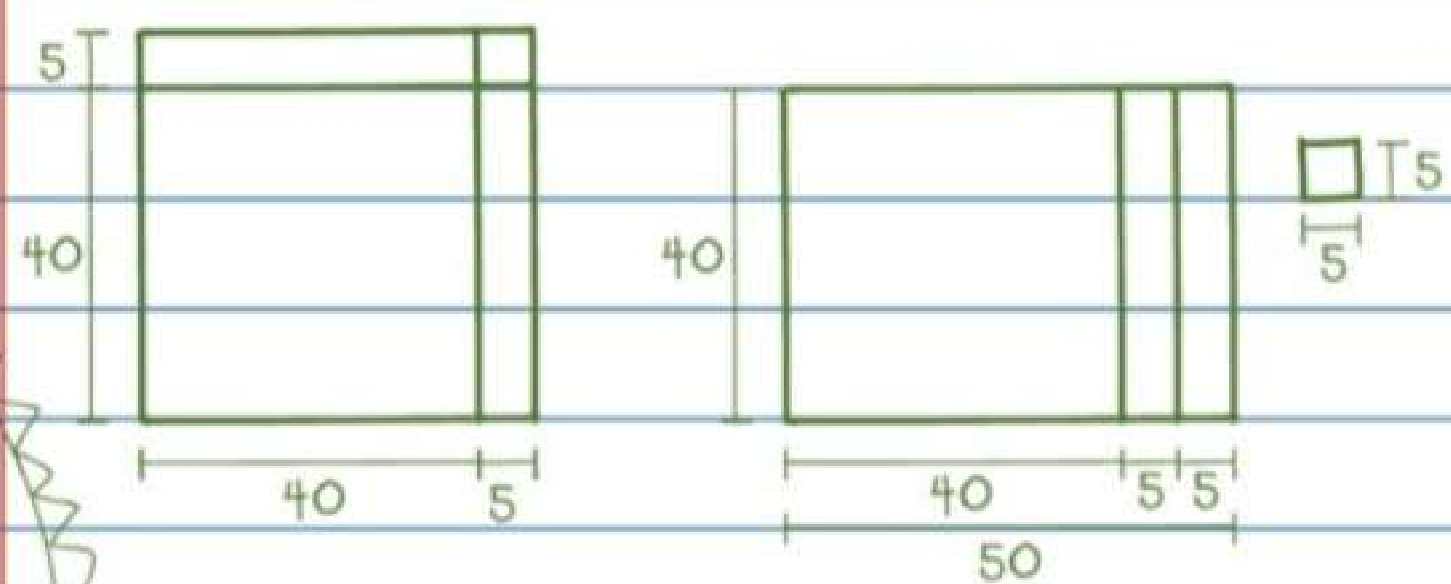
$=961$





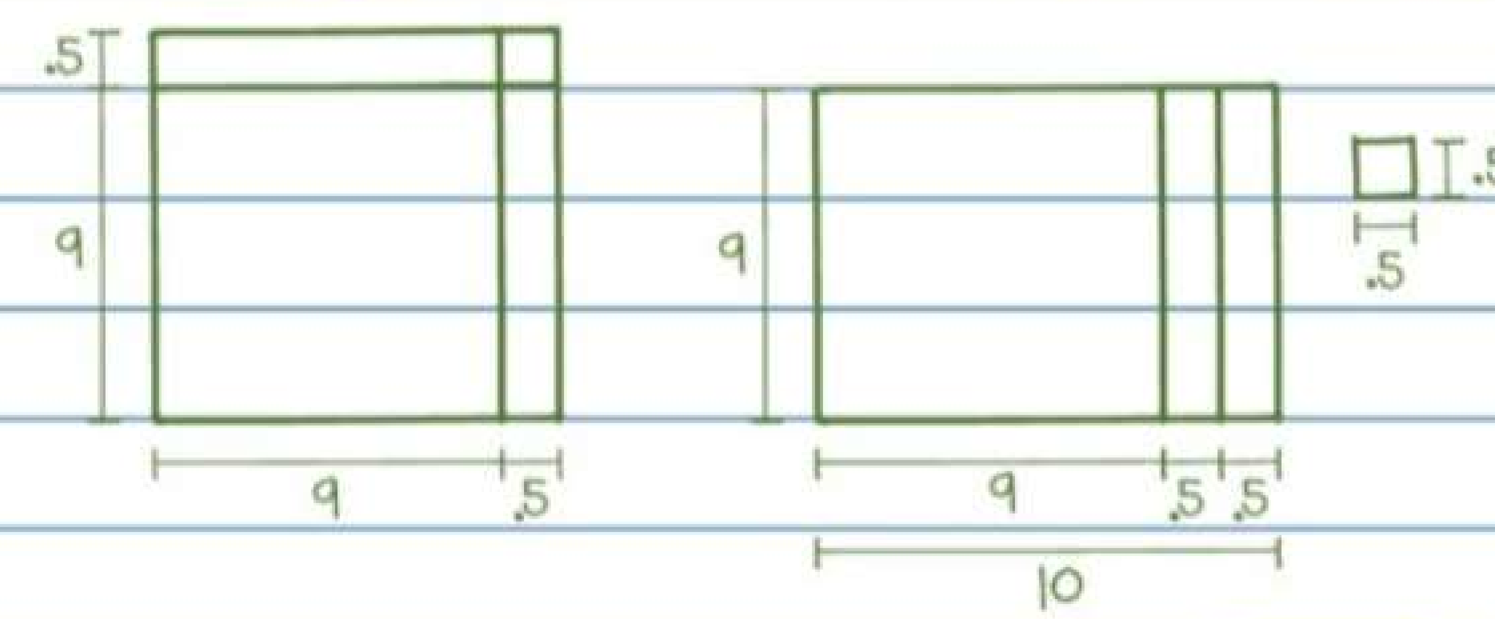
Squaring a number that ends in 5:
To square 45, we multiply 40×50 and add 25.
Works for decimals, too! 9.5^2 is 9×10 plus 0.25.

$45^2 = 40 \times 50 + 25$

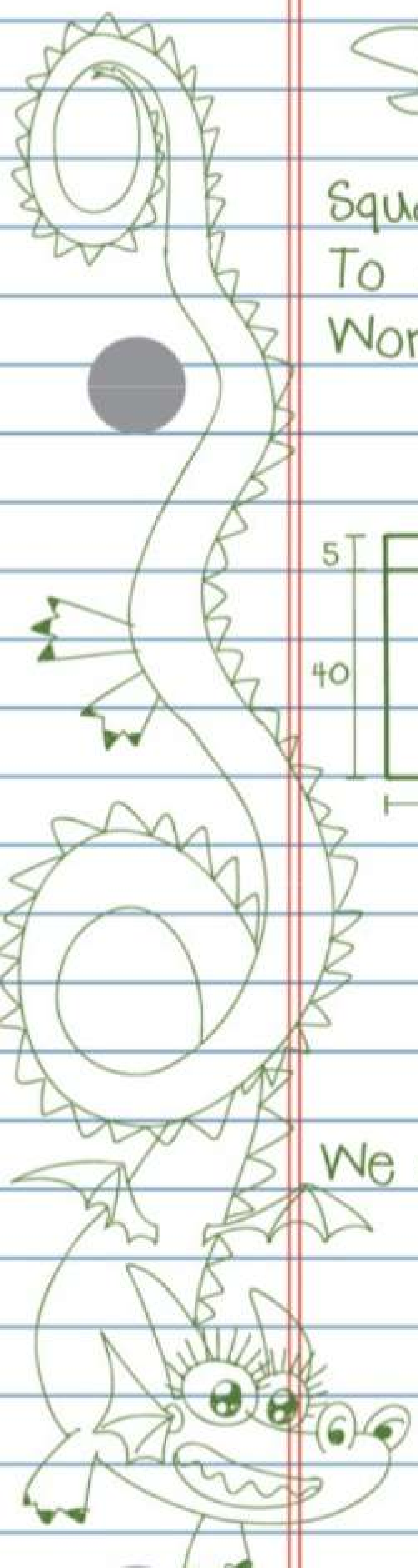


$=2{,}025$

$9.5^2 = 9 \times 10 + 0.25$

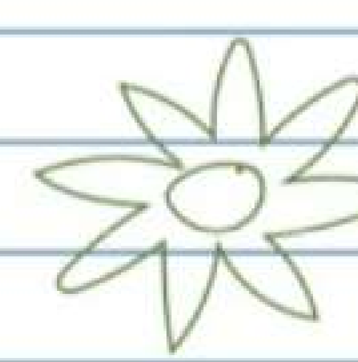


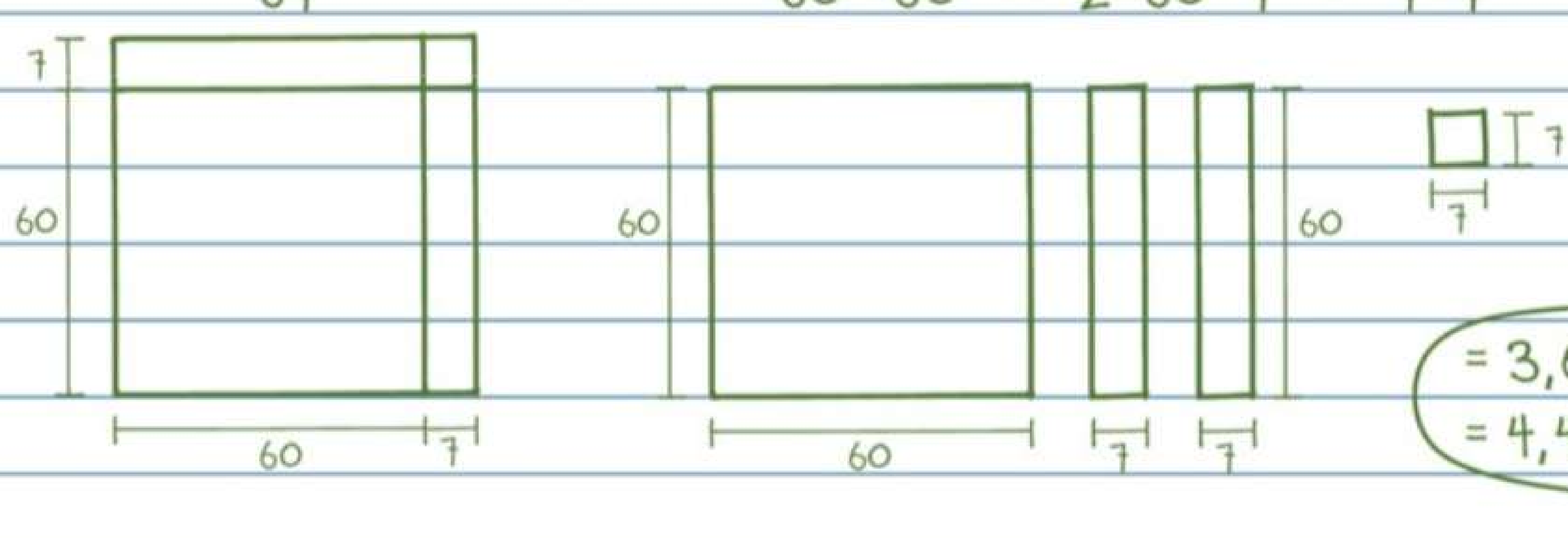
$=90.25$



We can use a similar diagram to help us square any 2-digit number:

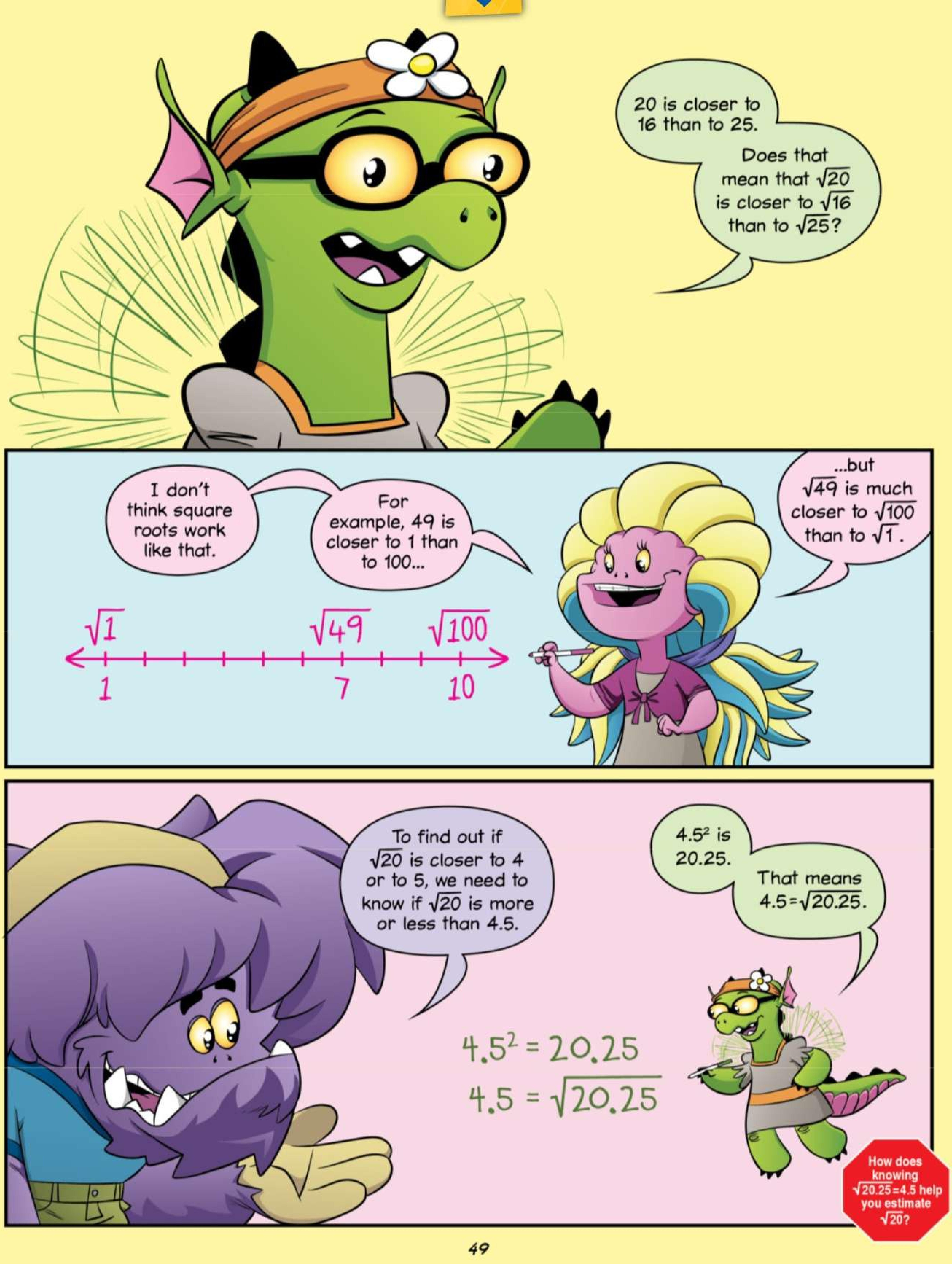
$67^2 = 60 \times 60 + 2 \times 60 \times 7 + 7 \times 7$

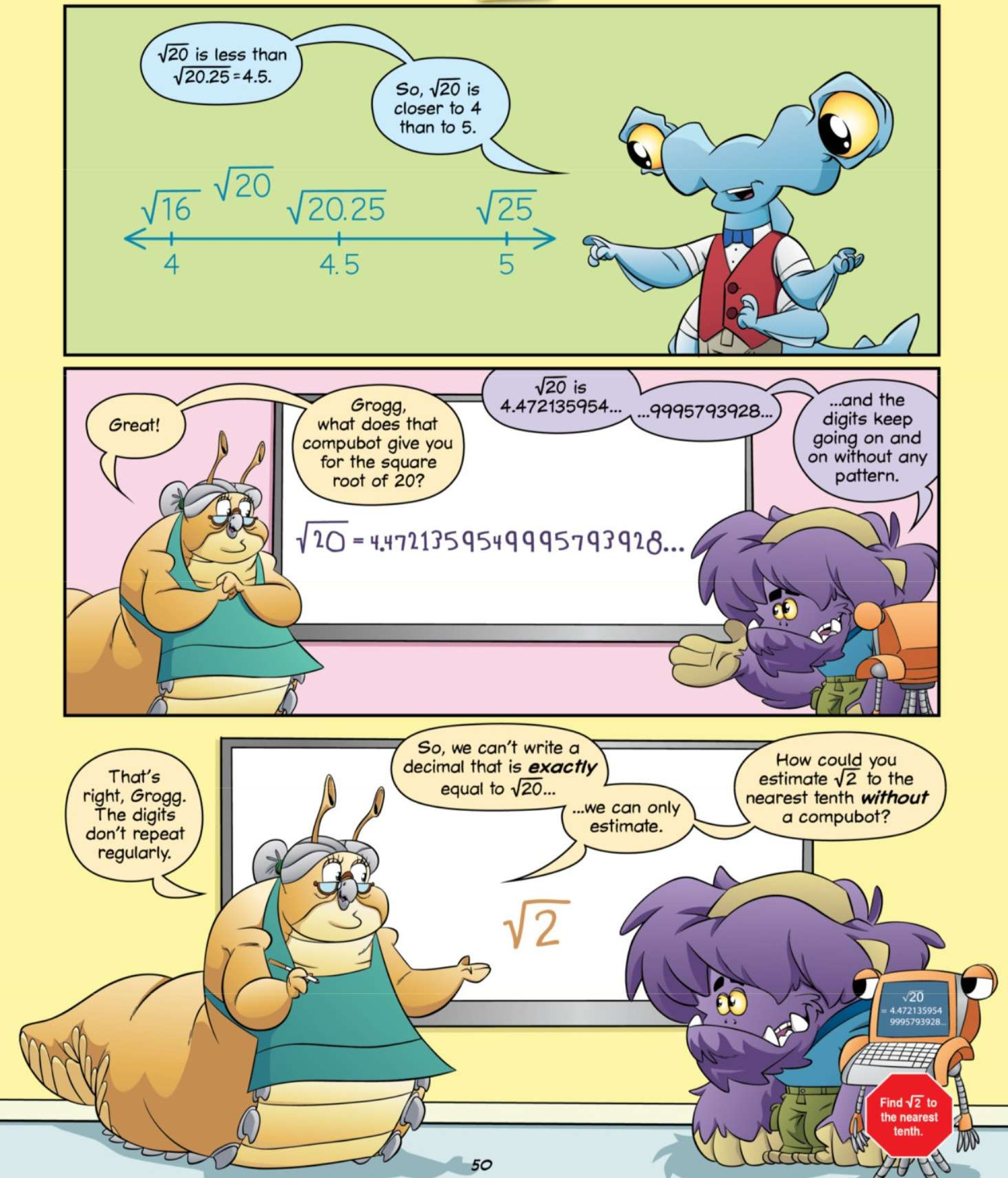




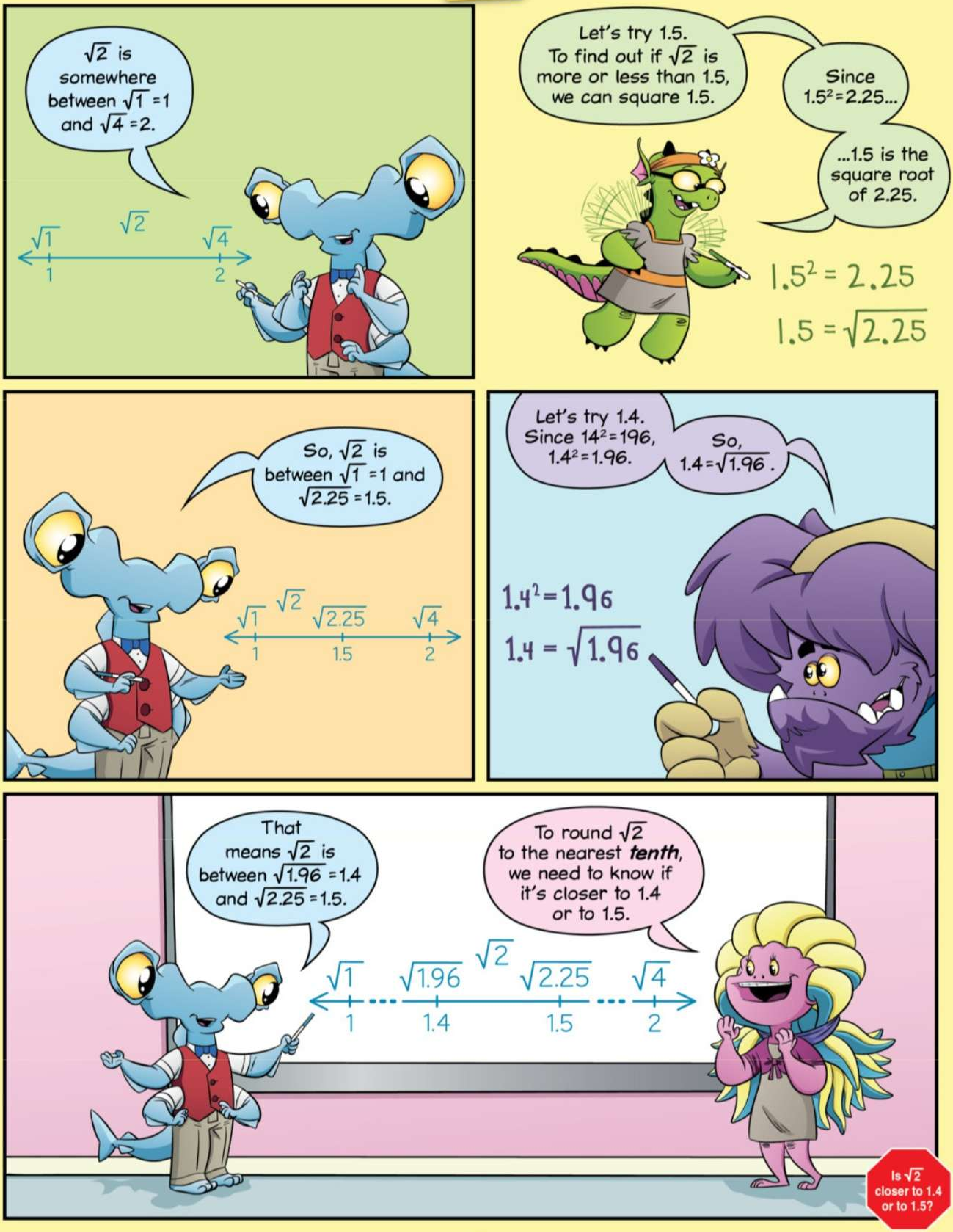
$= 3{,}600 + 840 + 49$
$= 4{,}489$

FOR ANY TWO NONNEGATIVE NUMBERS a AND b, IF $a<b$, THEN $\sqrt{a}<\sqrt{b}$. SO, $\sqrt{16}<\sqrt{20}<\sqrt{25}$.

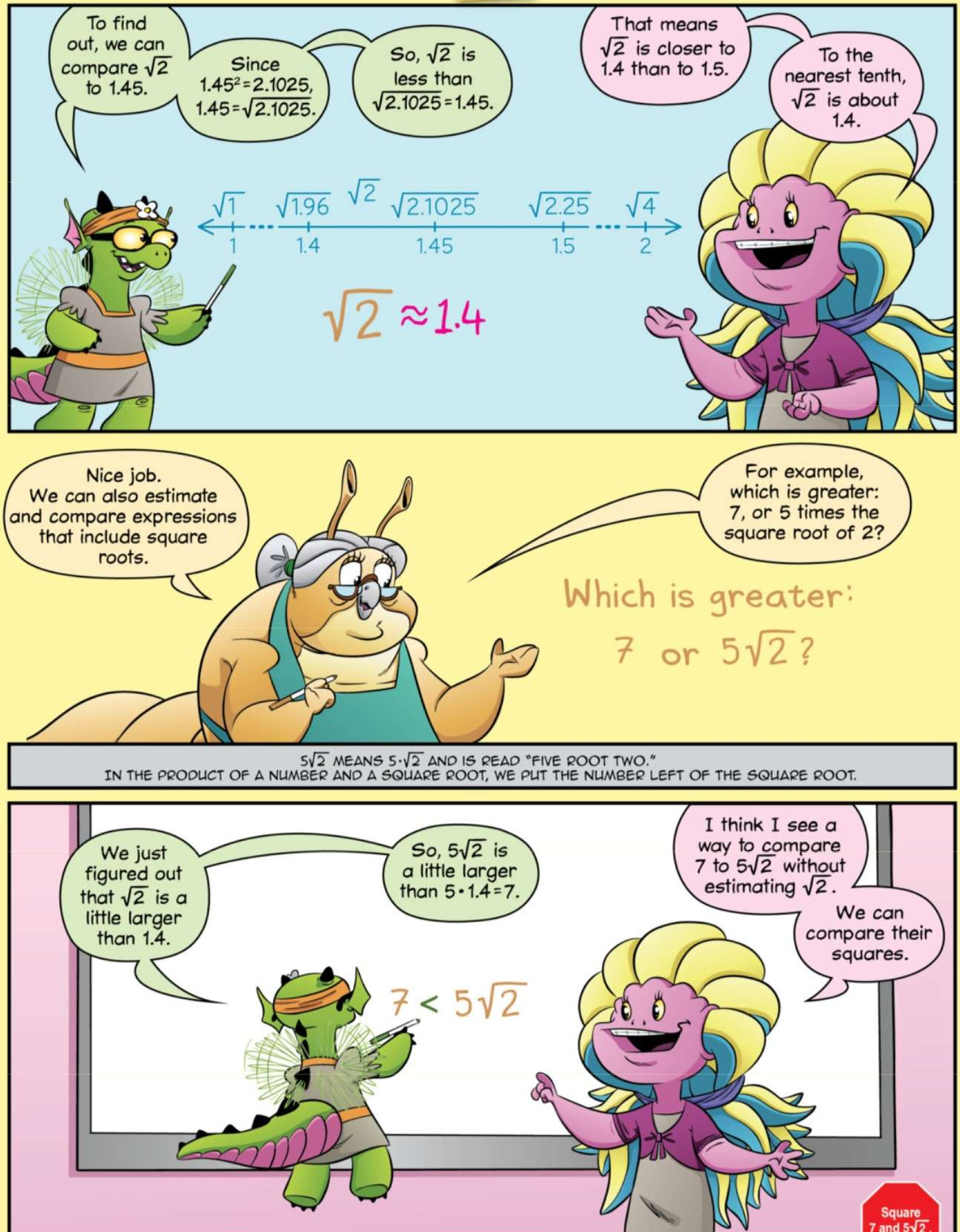

20 is closer to 16 than to 25.
Does that mean that $\sqrt{20}$ is closer to $\sqrt{16}$ than to $\sqrt{25}$?
I don't think square roots work like that.
For example, 49 is closer to 1 than to 100...
...but $\sqrt{49}$ is much closer to $\sqrt{100}$ than to $\sqrt{1}$.
$\sqrt{1}$ $\sqrt{49}$ $\sqrt{100}$
1 7 10
To find out if $\sqrt{20}$ is closer to 4 or to 5, we need to know if $\sqrt{20}$ is more or less than 4.5.
4.5^2 is 20.25.
That means $4.5 = \sqrt{20.25}$.
$4.5^2 = 20.25$
$4.5 = \sqrt{20.25}$
How does knowing $\sqrt{20.25} = 4.5$ help you estimate $\sqrt{20}$?

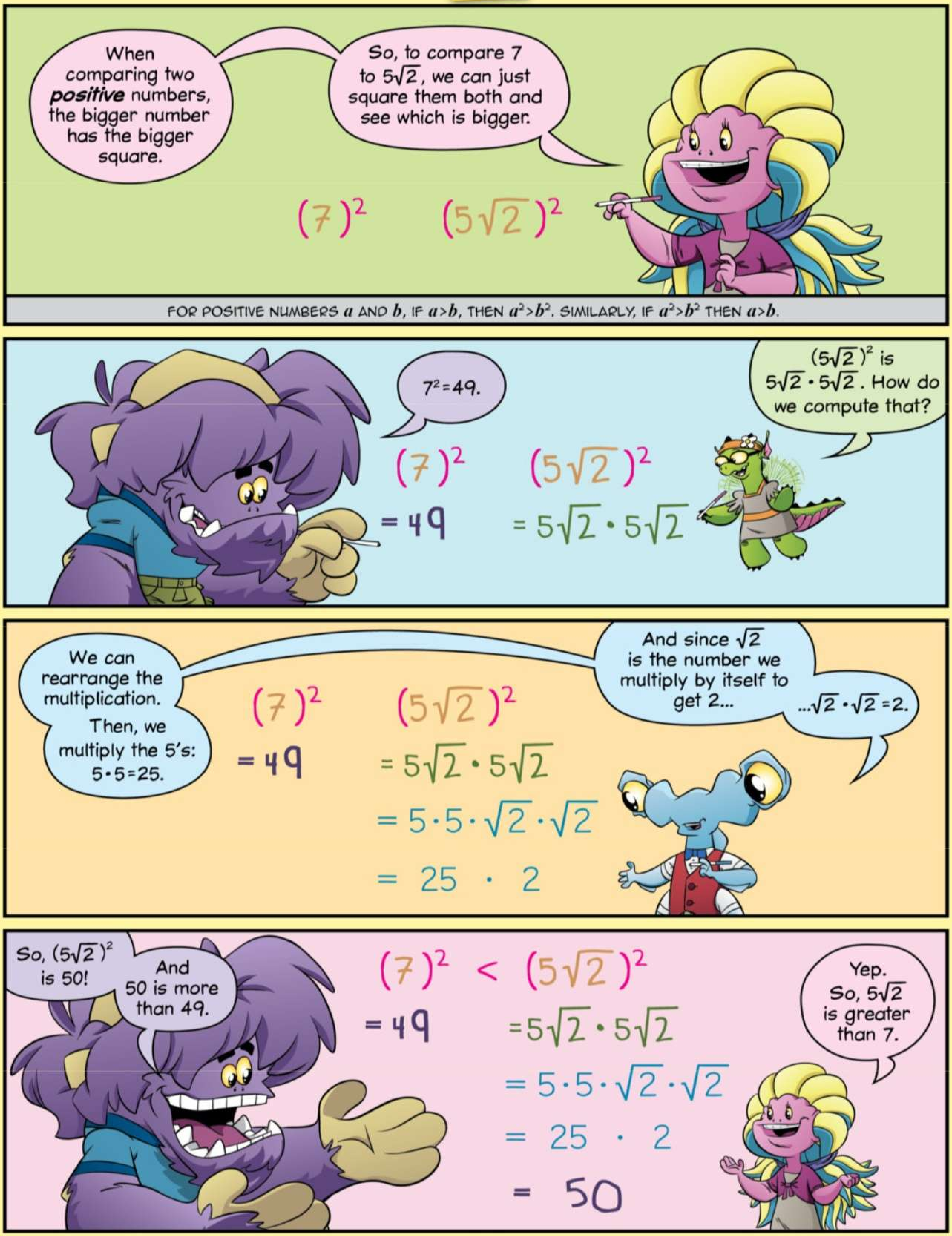


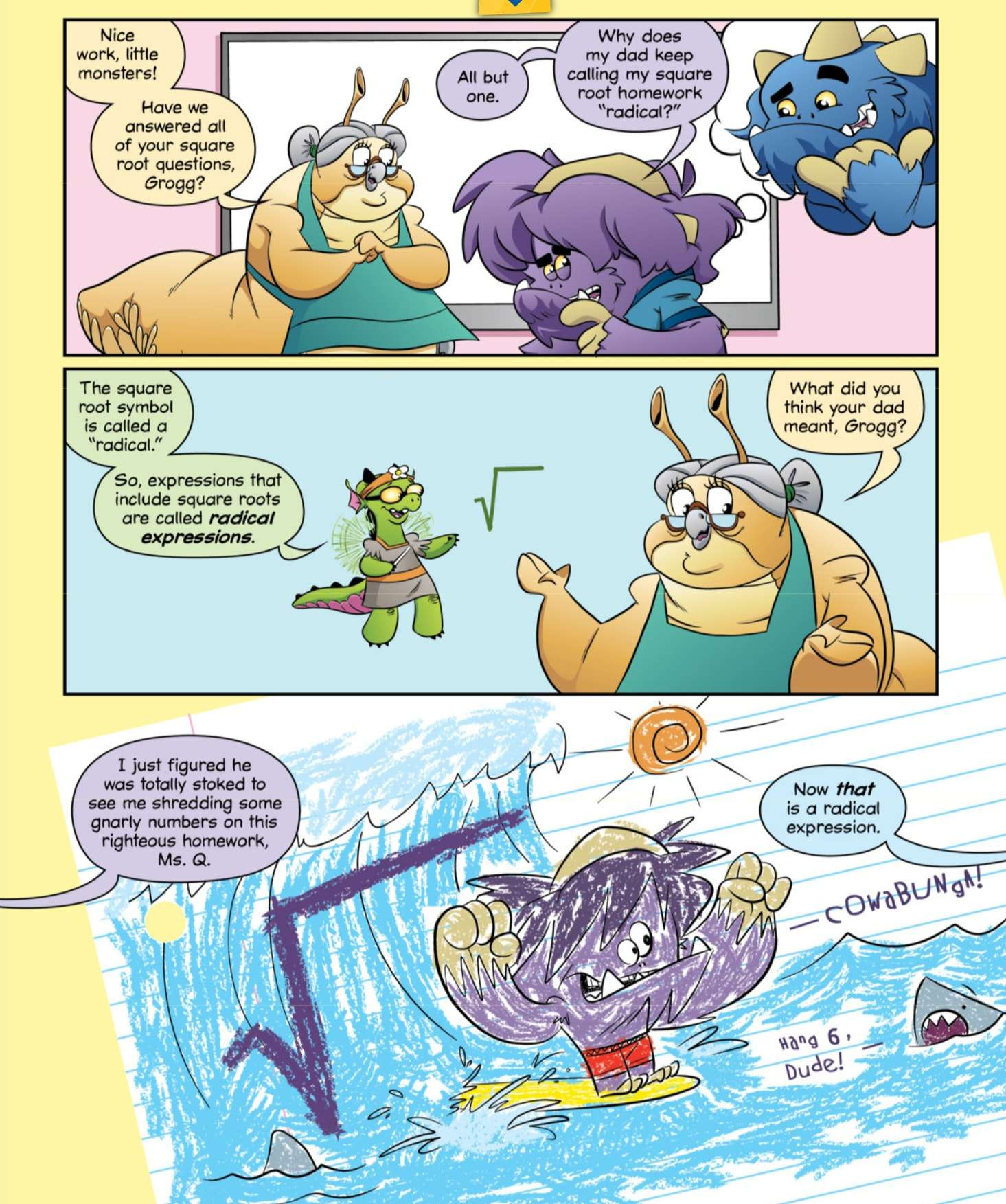
√20 is less than √20.25 = 4.5.
So, √20 is closer to 4 than to 5.
√16
√20
√20.25
√25
4
4.5
5
Great!
Grogg, what does that compubot give you for the square root of 20?
√20 is 4.472135954...
...9995793928...
...and the digits keep going on and on without any pattern.
√20 = 4.4721359549995793928...
That's right, Grogg. The digits don't repeat regularly.
So, we can't write a decimal that is ***exactly*** equal to √20...
...we can only estimate.
How could you estimate √2 to the nearest tenth ***without*** a compubot?
√2
√20 = 4.472135954 9995793928...
Find √2 to the nearest tenth.



$\sqrt{2}$ is somewhere between $\sqrt{1} = 1$ and $\sqrt{4} = 2$.
$\sqrt{1}$ $\sqrt{2}$ $\sqrt{4}$
1 2
Let's try 1.5. To find out if $\sqrt{2}$ is more or less than 1.5, we can square 1.5.
Since $1.5^2 = 2.25$...
...1.5 is the square root of 2.25.
$1.5^2 = 2.25$
$1.5 = \sqrt{2.25}$
So, $\sqrt{2}$ is between $\sqrt{1} = 1$ and $\sqrt{2.25} = 1.5$.
$\sqrt{1}$ $\sqrt{2}$ $\sqrt{2.25}$ $\sqrt{4}$
1 1.5 2
Let's try 1.4. Since $14^2 = 196$, $1.4^2 = 1.96$.
So, $1.4 = \sqrt{1.96}$.
$1.4^2 = 1.96$
$1.4 = \sqrt{1.96}$
That means $\sqrt{2}$ is between $\sqrt{1.96} = 1.4$ and $\sqrt{2.25} = 1.5$.
To round $\sqrt{2}$ to the nearest ***tenth***, we need to know if it's closer to 1.4 or to 1.5.
$\sqrt{1}$ $\sqrt{1.96}$ $\sqrt{2}$ $\sqrt{2.25}$ $\sqrt{4}$
1 1.4 1.5 2
Is $\sqrt{2}$ closer to 1.4 or to 1.5?


To find out, we can compare $\sqrt{2}$ to 1.45.
Since $1.45^2 = 2.1025$, $1.45 = \sqrt{2.1025}$.
So, $\sqrt{2}$ is less than $\sqrt{2.1025} = 1.45$.
That means $\sqrt{2}$ is closer to 1.4 than to 1.5.
To the nearest tenth, $\sqrt{2}$ is about 1.4.
$\sqrt{1}$ $\sqrt{1.96}$ $\sqrt{2}$ $\sqrt{2.1025}$ $\sqrt{2.25}$ $\sqrt{4}$
1 1.4 1.45 1.5 2
$\sqrt{2} \approx 1.4$
Nice job. We can also estimate and compare expressions that include square roots.
For example, which is greater: 7, or 5 times the square root of 2?
Which is greater: 7 or $5\sqrt{2}$?
$5\sqrt{2}$ MEANS $5 \cdot \sqrt{2}$ AND IS READ "FIVE ROOT TWO."
IN THE PRODUCT OF A NUMBER AND A SQUARE ROOT, WE PUT THE NUMBER LEFT OF THE SQUARE ROOT.
We just figured out that $\sqrt{2}$ is a little larger than 1.4.
So, $5\sqrt{2}$ is a little larger than $5 \cdot 1.4 = 7$.
$7 < 5\sqrt{2}$
I think I see a way to compare 7 to $5\sqrt{2}$ without estimating $\sqrt{2}$.
We can compare their squares.
Square 7 and $5\sqrt{2}$.


When comparing two ***positive*** numbers, the bigger number has the bigger square.
So, to compare 7 to $5\sqrt{2}$, we can just square them both and see which is bigger.
$(7)^2 \quad (5\sqrt{2})^2$
FOR POSITIVE NUMBERS a AND b, IF $a>b$, THEN $a^2>b^2$. SIMILARLY, IF $a^2>b^2$ THEN $a>b$.
$7^2=49$.
$(5\sqrt{2})^2$ is $5\sqrt{2}\cdot 5\sqrt{2}$. How do we compute that?
$(7)^2 = 49 \quad (5\sqrt{2})^2 = 5\sqrt{2}\cdot 5\sqrt{2}$
We can rearrange the multiplication.
Then, we multiply the 5's: $5\cdot 5=25$.
And since $\sqrt{2}$ is the number we multiply by itself to get 2...
...$\sqrt{2}\cdot\sqrt{2}=2$.
$(7)^2 = 49 \quad (5\sqrt{2})^2 = 5\sqrt{2}\cdot 5\sqrt{2} = 5\cdot 5\cdot\sqrt{2}\cdot\sqrt{2} = 25 \cdot 2$
So, $(5\sqrt{2})^2$ is 50!
And 50 is more than 49.
Yep. So, $5\sqrt{2}$ is greater than 7.
$(7)^2 < (5\sqrt{2})^2$
$= 49 \quad = 5\sqrt{2}\cdot 5\sqrt{2} = 5\cdot 5\cdot\sqrt{2}\cdot\sqrt{2} = 25 \cdot 2 = 50$



Nice work, little monsters!
Have we answered all of your square root questions, Grogg?
All but one.
Why does my dad keep calling my square root homework "radical?"
The square root symbol is called a "radical."
So, expressions that include square roots are called ***radical expressions***.
What did you think your dad meant, Grogg?
I just figured he was totally stoked to see me shredding some gnarly numbers on this righteous homework, Ms. Q.
Now ***that*** is a radical expression.
COWABUNGA!
Hang 6, Dude!

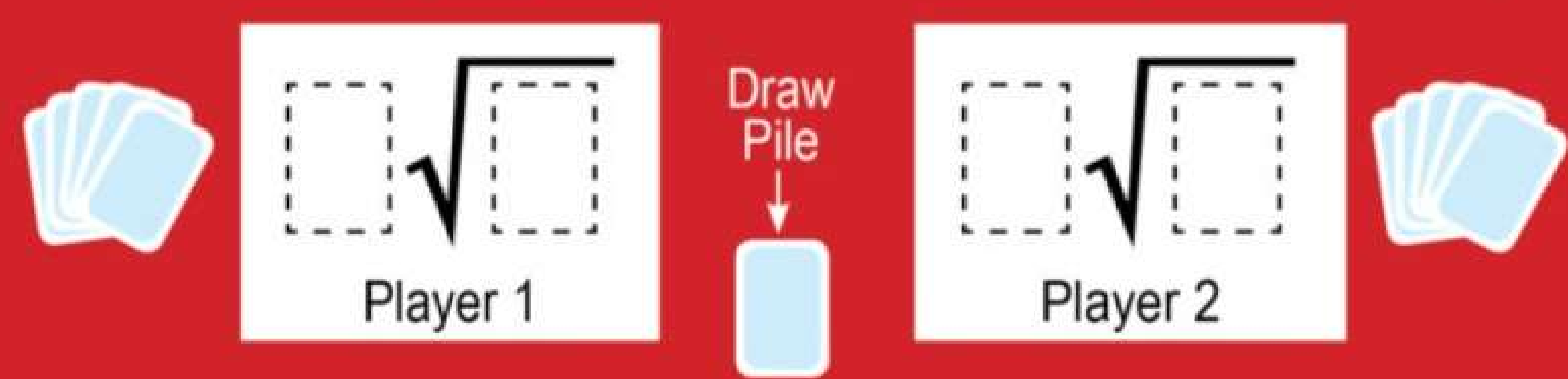
RECESS

More or Less

More or Less is a card game for 2 players which uses two game boards, a $>$ marker, and 20 cards from a standard deck (Ace through 5 in all four suits; Aces count as 1's). The goal is to be the first player to play all the cards in your hand.

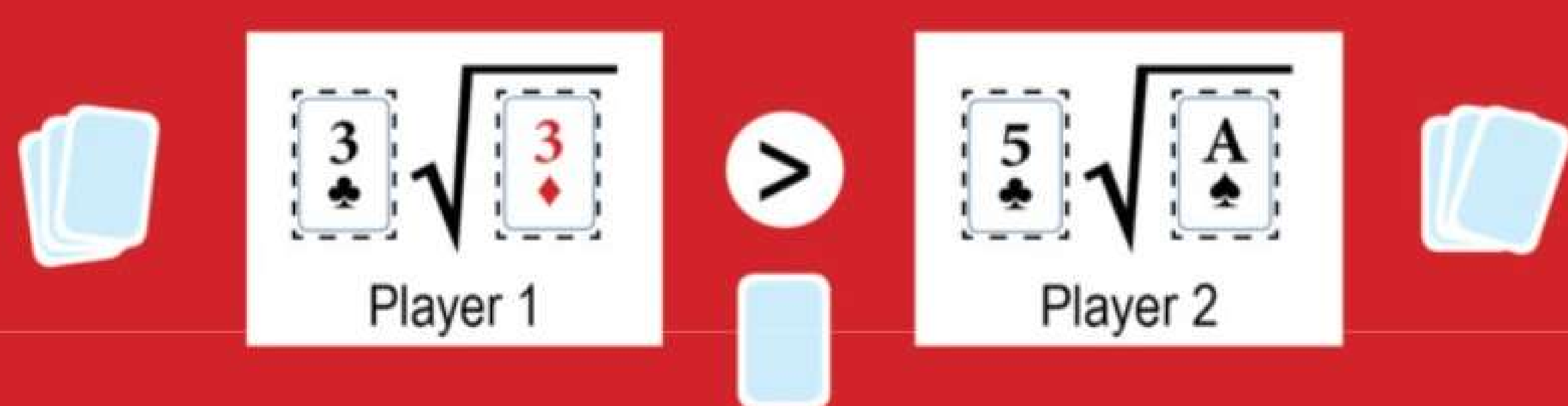
Setup

Each player begins with a square root game board and is dealt 5 cards from the shuffled deck. The remaining cards are placed in a draw pile as shown. You may print game boards at BeastAcademy.com, or make your own.



Beginning

Player 1 places two cards from their hand on the dashed rectangles of their game board to create an expression. Player 2 then places two cards from their hand on their game board to create an expression that is not equal to Player 1's. Players place a marker between the expressions to show which number is greater, pointing $>$ or $<$. For example, in the game below, we compare $3\sqrt{3}$ to $5\sqrt{1}$ by comparing their squares. Since $(3\sqrt{3})^2=27$, and $(5\sqrt{1})^2=25$, we have $3\sqrt{3} > 5\sqrt{1}$.



Play

Players take turns, starting with Player 1. On each turn, a player draws one card from the draw pile, then places 1 or 2 cards on their game board so that the relationship between the two expressions ($<$ or $>$) does not change. In the example above, Player 1 must create an expression that is greater than $5\sqrt{1}$. For example, Player 1 could use a 4 and a 2 to play $4\sqrt{2}$, since $4\sqrt{2}>5\sqrt{1}$. Or, Player 1 could simply place a 5 on top of either 3 to create $5\sqrt{3}$ or $3\sqrt{5}$, since both are greater than $5\sqrt{1}$.

If a Player creates an expression that does not work, they must pick up the cards played and forfeit their turn.

If a Player cannot create an expression that works, they must pass, and play continues with the other player.

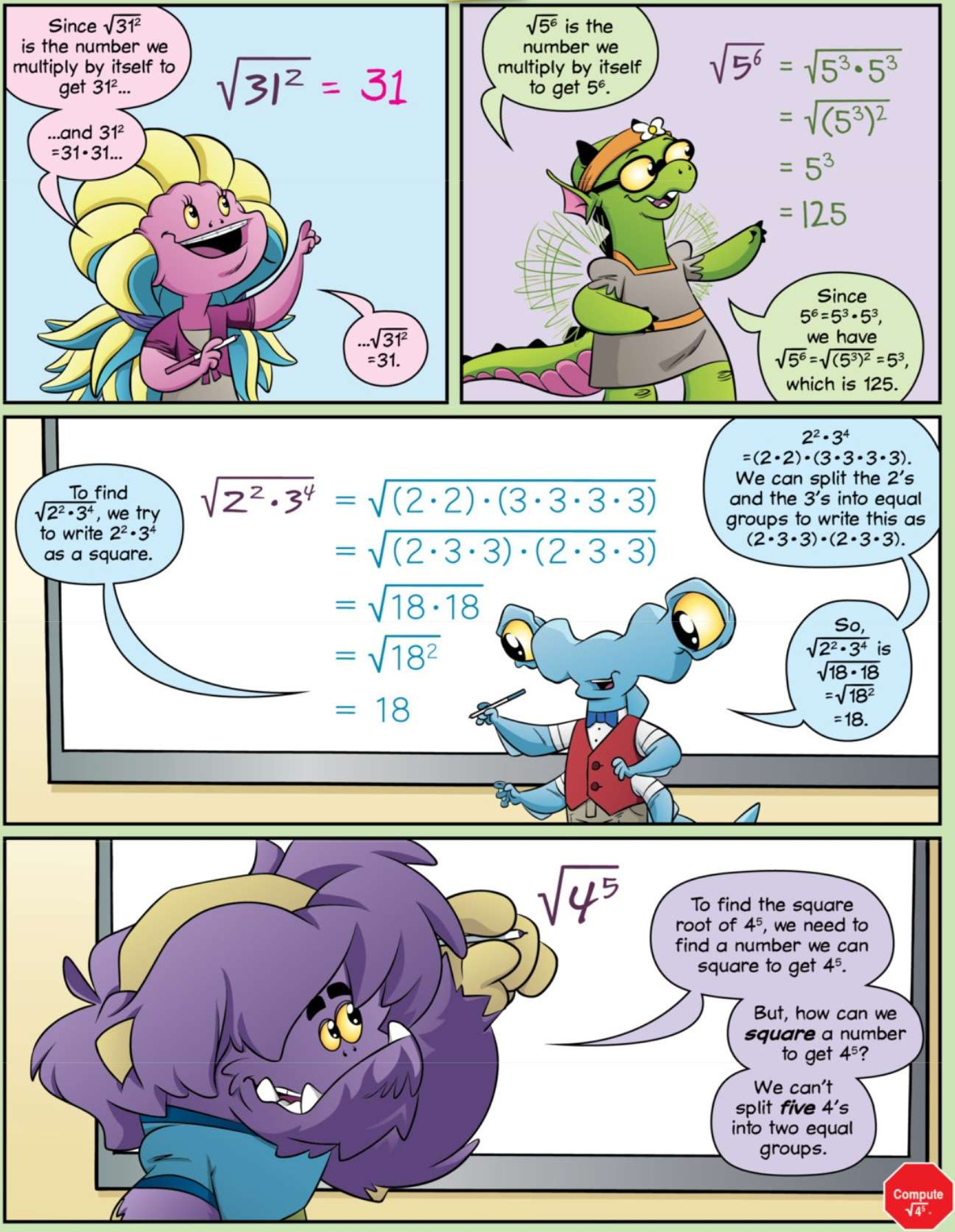
Winning

The first player to play all of their cards wins. If neither player can play and the draw pile is empty, the game ends in a draw.

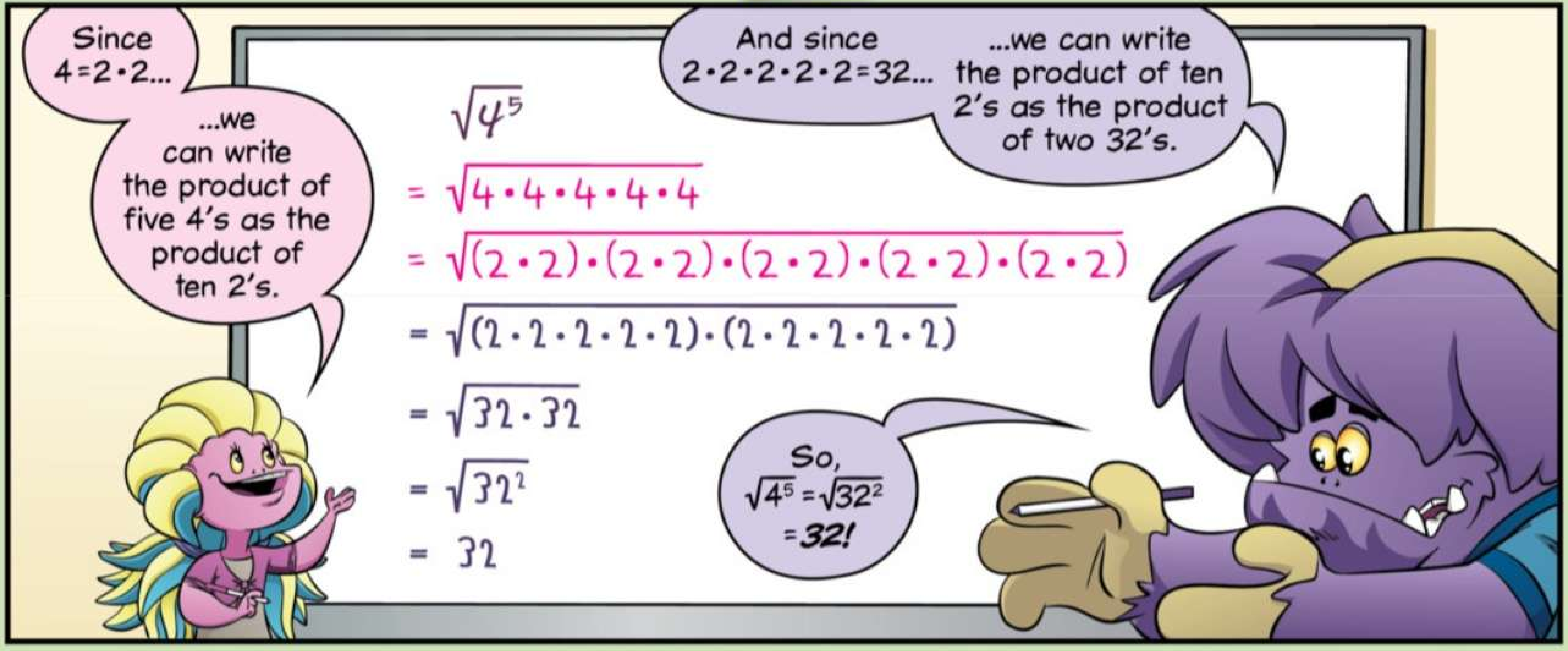
Variations

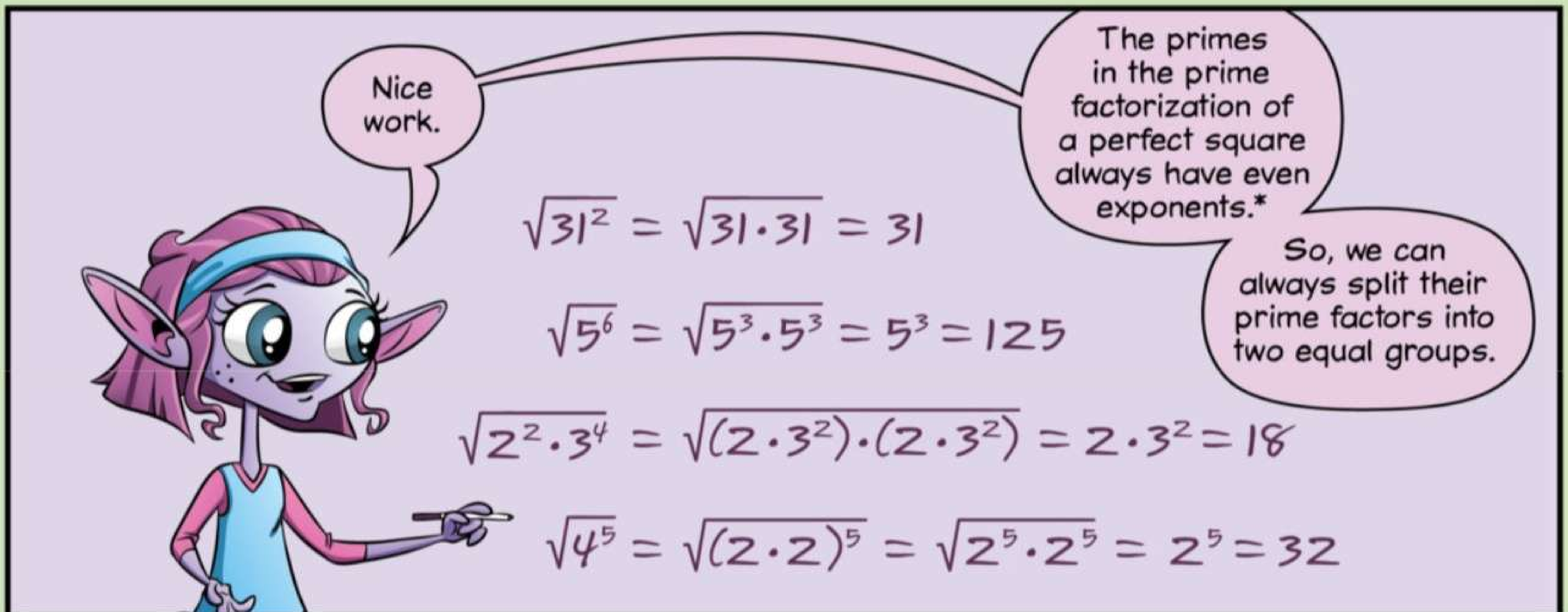
You may add additional digit cards in the deck (like 6's or 7's), or change the number of cards dealt to begin the game.

MATH TEAM
Tricky Square Roots
In just a few weeks, we'll be on our way to the World Math Olympiad.
It's going to be tough to do well without Max.
He knows so much math!
Max is great, but you guys are an amazing team, even without him.
You've been working together for a long time.
But, we do have a lot to learn before the competition.
Let's start today with some square root practice.
Find each of these square roots.
$\sqrt{31^2}$
$\sqrt{5^6}$
$\sqrt{2^2 \cdot 3^4}$
$\sqrt{4^5}$
Find each square root.

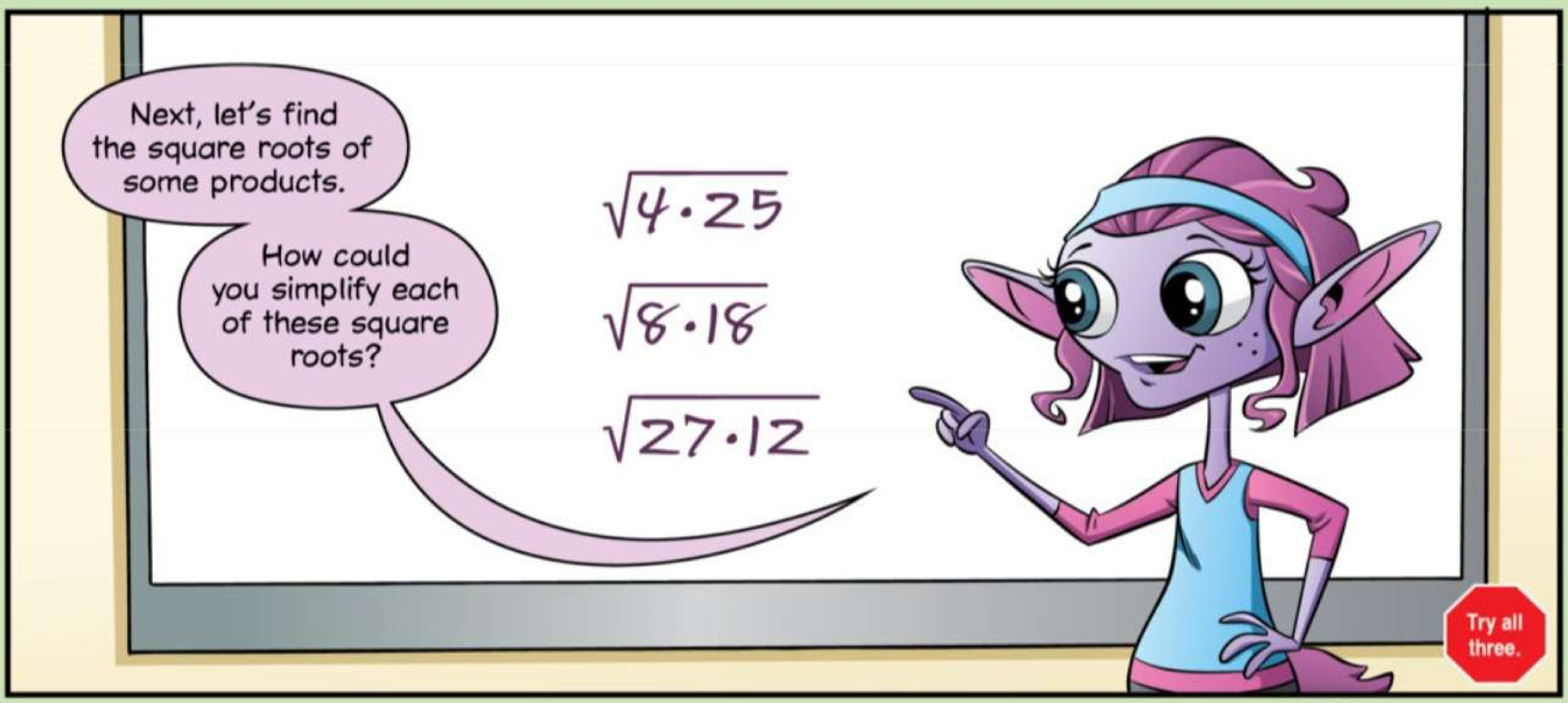


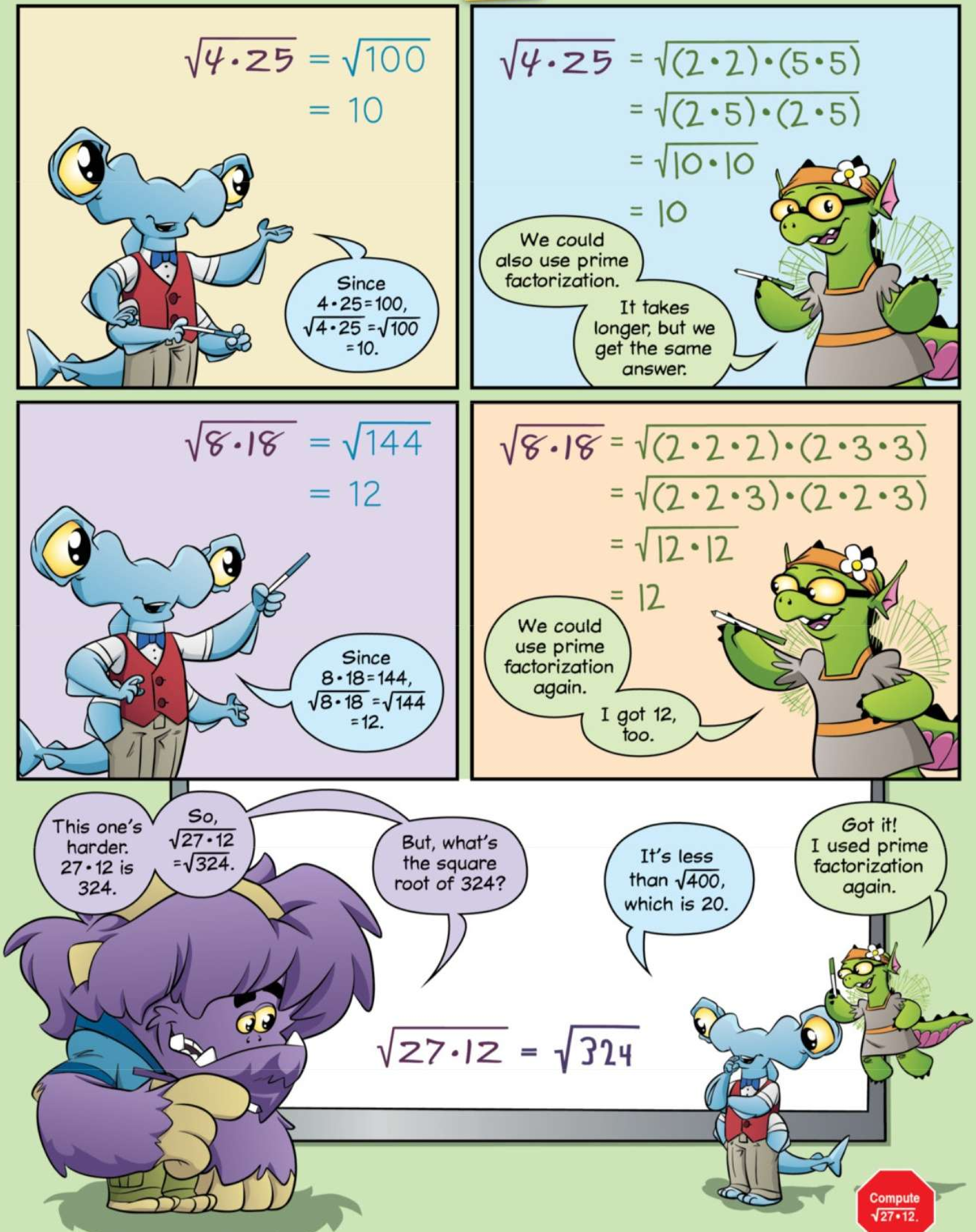
Since $\sqrt{31^2}$ is the number we multiply by itself to get 31^2...
...and $31^2 = 31 \cdot 31$...
$\sqrt{31^2} = 31$
...$\sqrt{31^2} = 31$.
$\sqrt{5^6}$ is the number we multiply by itself to get 5^6.
$\sqrt{5^6} = \sqrt{5^3 \cdot 5^3} = \sqrt{(5^3)^2} = 5^3 = 125$
Since $5^6 = 5^3 \cdot 5^3$, we have $\sqrt{5^6} = \sqrt{(5^3)^2} = 5^3$, which is 125.
To find $\sqrt{2^2 \cdot 3^4}$, we try to write $2^2 \cdot 3^4$ as a square.
$\sqrt{2^2 \cdot 3^4} = \sqrt{(2\cdot2)\cdot(3\cdot3\cdot3\cdot3)} = \sqrt{(2\cdot3\cdot3)\cdot(2\cdot3\cdot3)} = \sqrt{18\cdot18} = \sqrt{18^2} = 18$
$2^2 \cdot 3^4 = (2\cdot2)\cdot(3\cdot3\cdot3\cdot3)$. We can split the 2's and the 3's into equal groups to write this as $(2\cdot3\cdot3)\cdot(2\cdot3\cdot3)$.
So, $\sqrt{2^2 \cdot 3^4}$ is $\sqrt{18\cdot18} = \sqrt{18^2} = 18$.
$\sqrt{4^5}$
To find the square root of 4^5, we need to find a number we can square to get 4^5.
But, how can we ***square*** a number to get 4^5?
We can't split ***five*** 4's into two equal groups.
Compute $\sqrt{4^5}$.



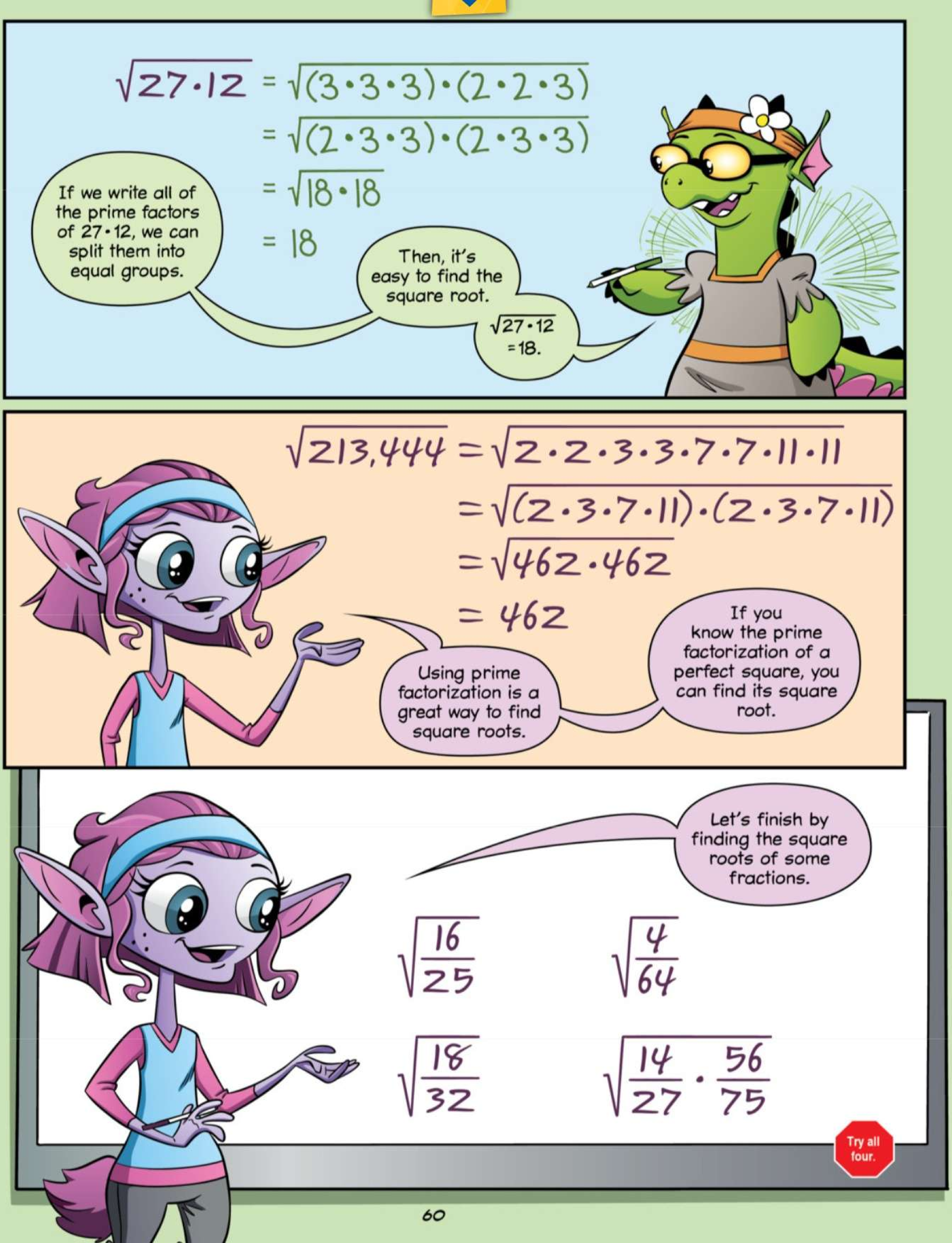


*REVIEW PRIME FACTORIZATIONS OF PERFECT SQUARES IN CHAPTER 5 OF BEAST ACADEMY 5B.

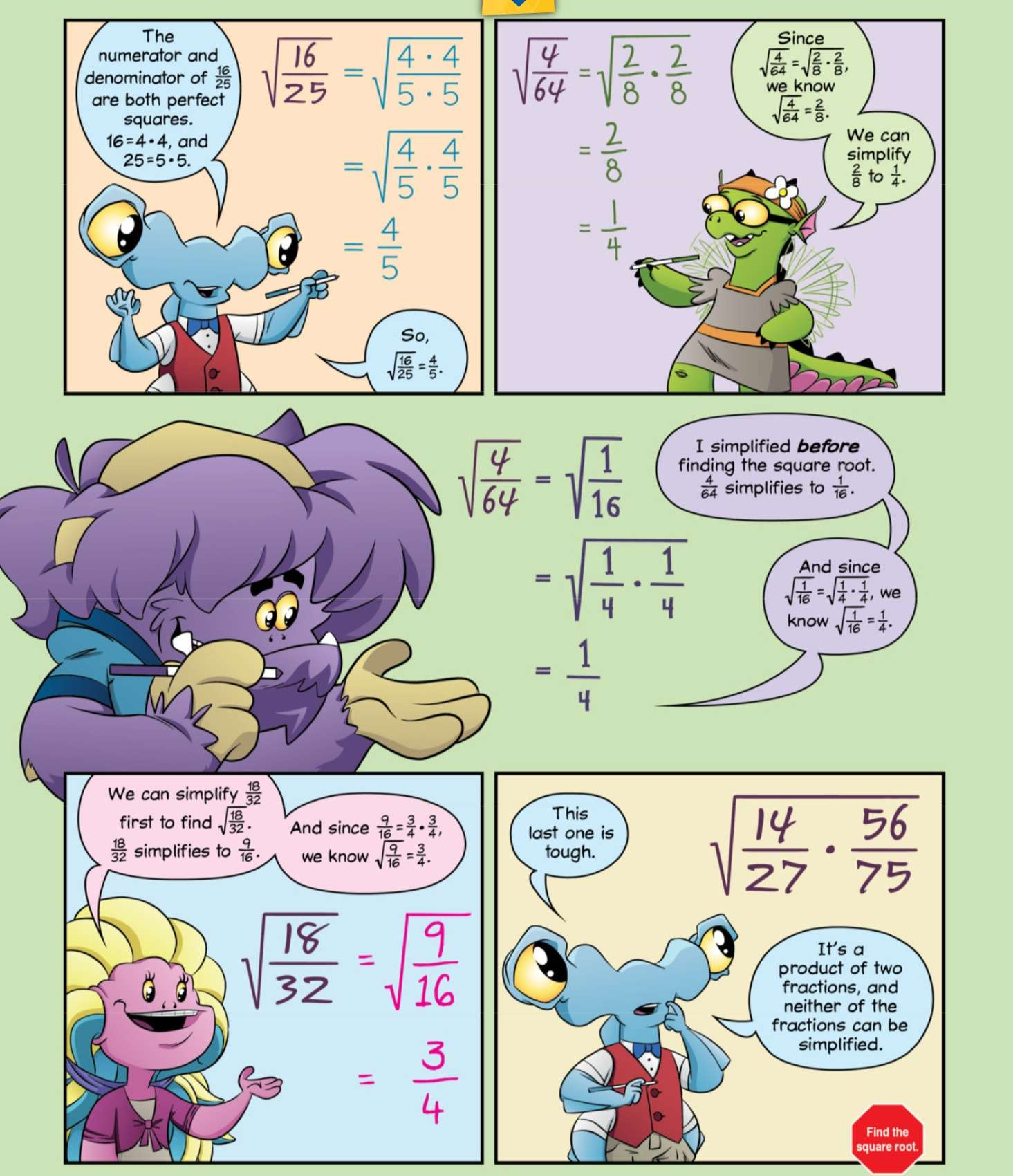




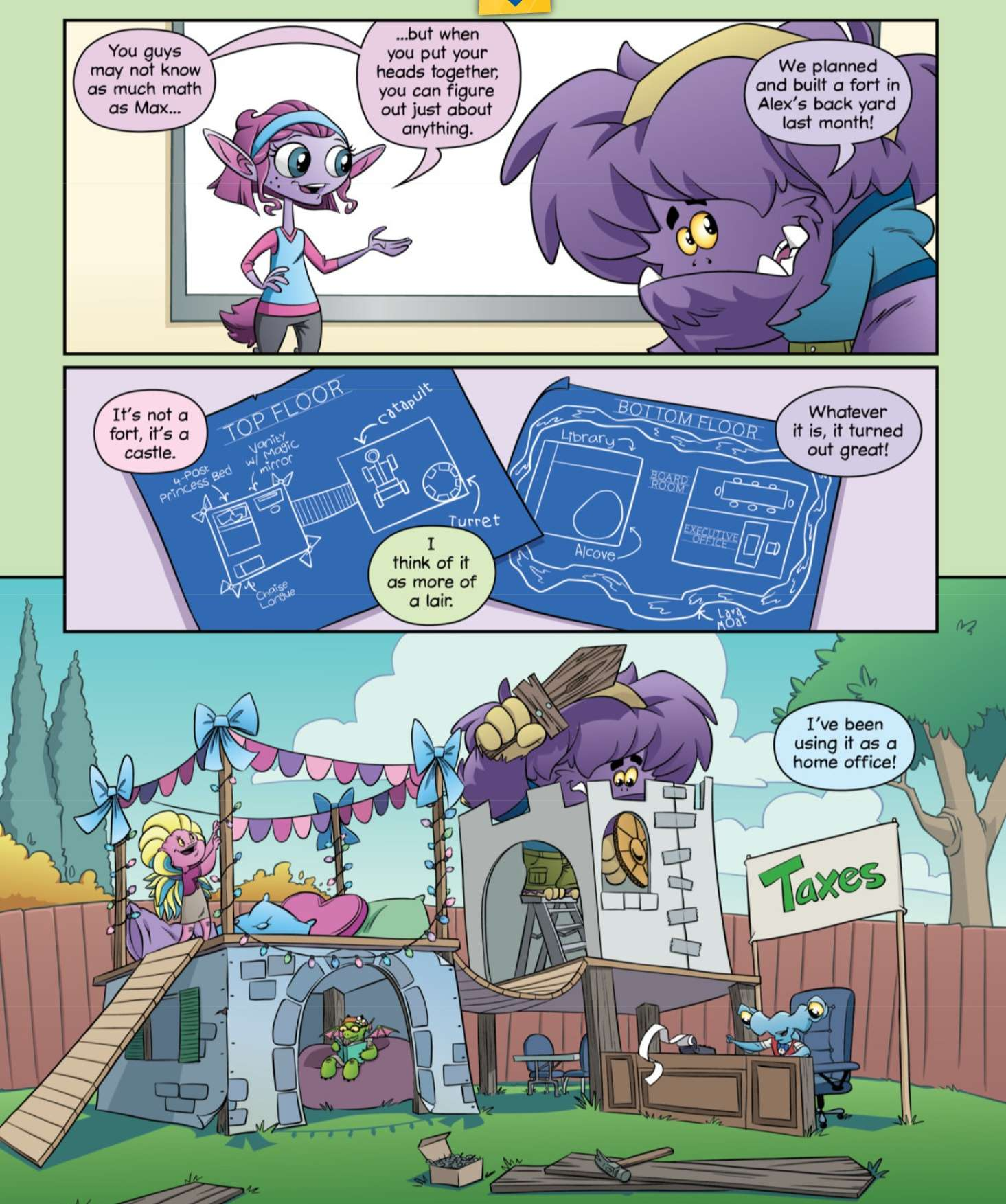
$\sqrt{4 \cdot 25} = \sqrt{100}$
$= 10$
Since $4 \cdot 25 = 100$, $\sqrt{4 \cdot 25} = \sqrt{100} = 10$.
$\sqrt{4 \cdot 25} = \sqrt{(2 \cdot 2) \cdot (5 \cdot 5)}$
$= \sqrt{(2 \cdot 5) \cdot (2 \cdot 5)}$
$= \sqrt{10 \cdot 10}$
$= 10$
We could also use prime factorization.
It takes longer, but we get the same answer.
$\sqrt{8 \cdot 18} = \sqrt{144}$
$= 12$
Since $8 \cdot 18 = 144$, $\sqrt{8 \cdot 18} = \sqrt{144} = 12$.
$\sqrt{8 \cdot 18} = \sqrt{(2 \cdot 2 \cdot 2) \cdot (2 \cdot 3 \cdot 3)}$
$= \sqrt{(2 \cdot 2 \cdot 3) \cdot (2 \cdot 2 \cdot 3)}$
$= \sqrt{12 \cdot 12}$
$= 12$
We could use prime factorization again.
I got 12, too.
This one's harder. $27 \cdot 12$ is 324.
So, $\sqrt{27 \cdot 12} = \sqrt{324}$.
But, what's the square root of 324?
It's less than $\sqrt{400}$, which is 20.
Got it! I used prime factorization again.
$\sqrt{27 \cdot 12} = \sqrt{324}$
Compute $\sqrt{27 \cdot 12}$.

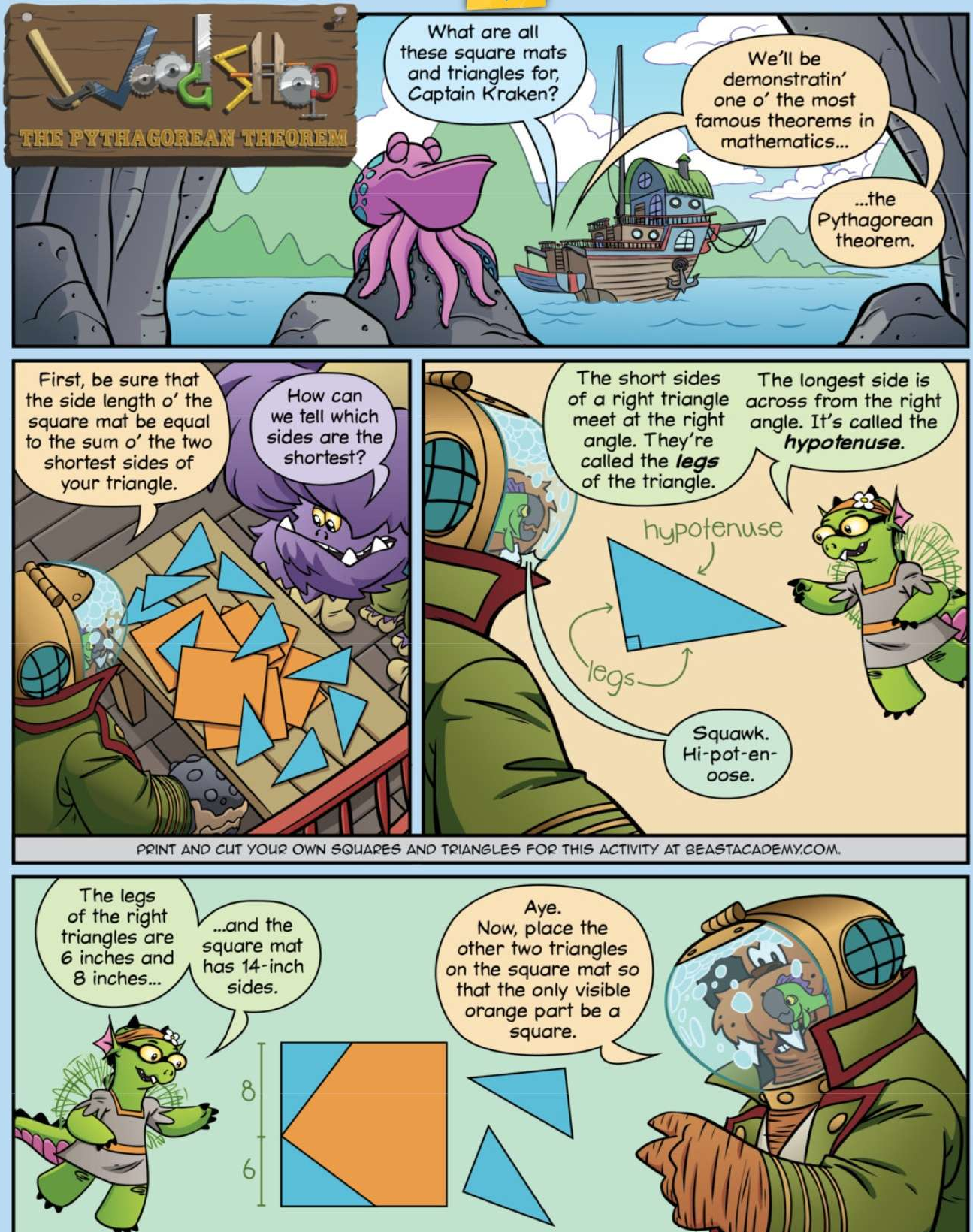


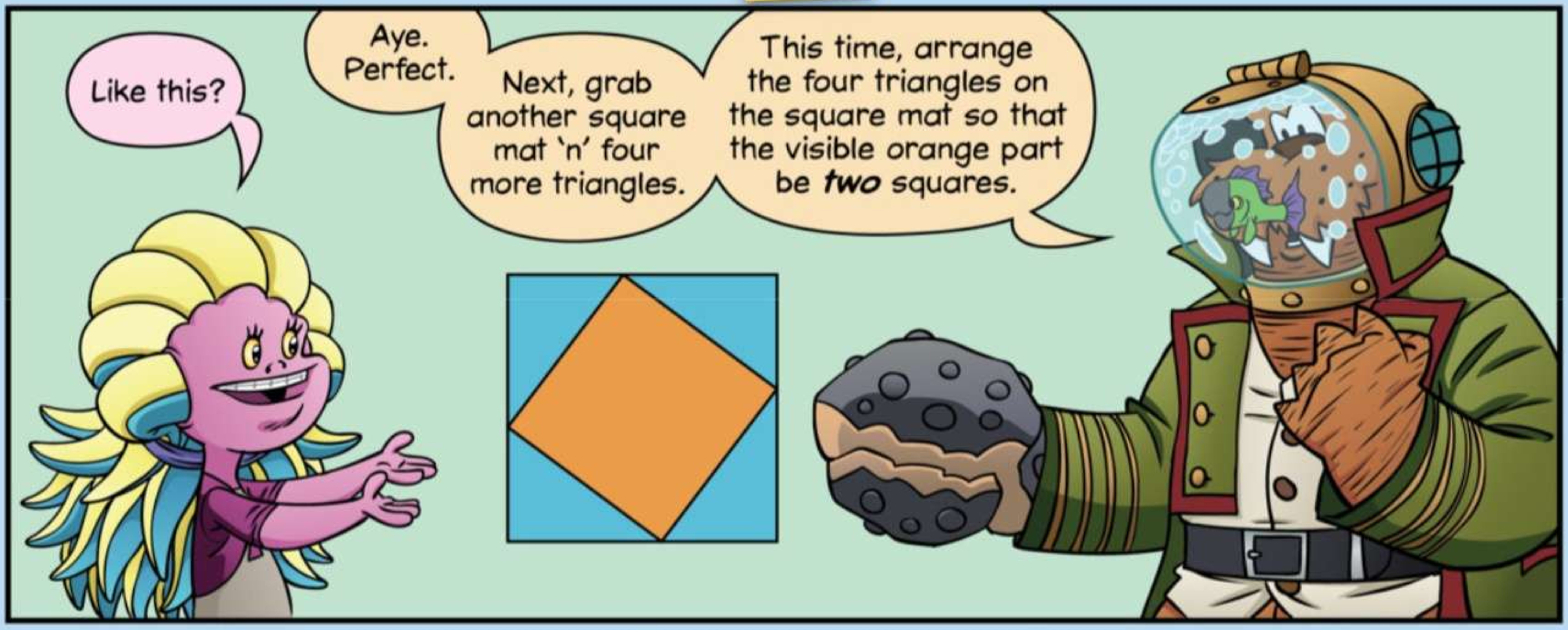
√27•12 = √(3•3•3)•(2•2•3)
= √(2•3•3)•(2•3•3)
= √18•18
= 18
If we write all of the prime factors of 27•12, we can split them into equal groups.
Then, it's easy to find the square root.
√27•12 = 18.
√213,444 = √2•2•3•3•7•7•11•11
= √(2•3•7•11)•(2•3•7•11)
= √462•462
= 462
Using prime factorization is a great way to find square roots.
If you know the prime factorization of a perfect square, you can find its square root.
Let's finish by finding the square roots of some fractions.
√(16/25)
√(4/64)
√(18/32)
√(14/27 • 56/75)
Try all four.


The numerator and denominator of $\frac{16}{25}$ are both perfect squares. $16 = 4 \cdot 4$, and $25 = 5 \cdot 5$.
$\sqrt{\frac{16}{25}} = \sqrt{\frac{4 \cdot 4}{5 \cdot 5}}$
$= \sqrt{\frac{4}{5} \cdot \frac{4}{5}}$
$= \frac{4}{5}$
So, $\sqrt{\frac{16}{25}} = \frac{4}{5}$.
$\sqrt{\frac{4}{64}} = \sqrt{\frac{2}{8} \cdot \frac{2}{8}}$
$= \frac{2}{8}$
$= \frac{1}{4}$
Since $\sqrt{\frac{4}{64}} = \sqrt{\frac{2}{8} \cdot \frac{2}{8}}$, we know $\sqrt{\frac{4}{64}} = \frac{2}{8}$.
We can simplify $\frac{2}{8}$ to $\frac{1}{4}$.
$\sqrt{\frac{4}{64}} = \sqrt{\frac{1}{16}}$
$= \sqrt{\frac{1}{4} \cdot \frac{1}{4}}$
$= \frac{1}{4}$
I simplified ***before*** finding the square root. $\frac{4}{64}$ simplifies to $\frac{1}{16}$.
And since $\sqrt{\frac{1}{16}} = \sqrt{\frac{1}{4} \cdot \frac{1}{4}}$, we know $\sqrt{\frac{1}{16}} = \frac{1}{4}$.
We can simplify $\frac{18}{32}$ first to find $\sqrt{\frac{18}{32}}$. $\frac{18}{32}$ simplifies to $\frac{9}{16}$.
And since $\frac{9}{16} = \frac{3}{4} \cdot \frac{3}{4}$, we know $\sqrt{\frac{9}{16}} = \frac{3}{4}$.
$\sqrt{\frac{18}{32}} = \sqrt{\frac{9}{16}}$
$= \frac{3}{4}$
This last one is tough.
$\sqrt{\frac{14}{27} \cdot \frac{56}{75}}$
It's a product of two fractions, and neither of the fractions can be simplified.
Find the square root.

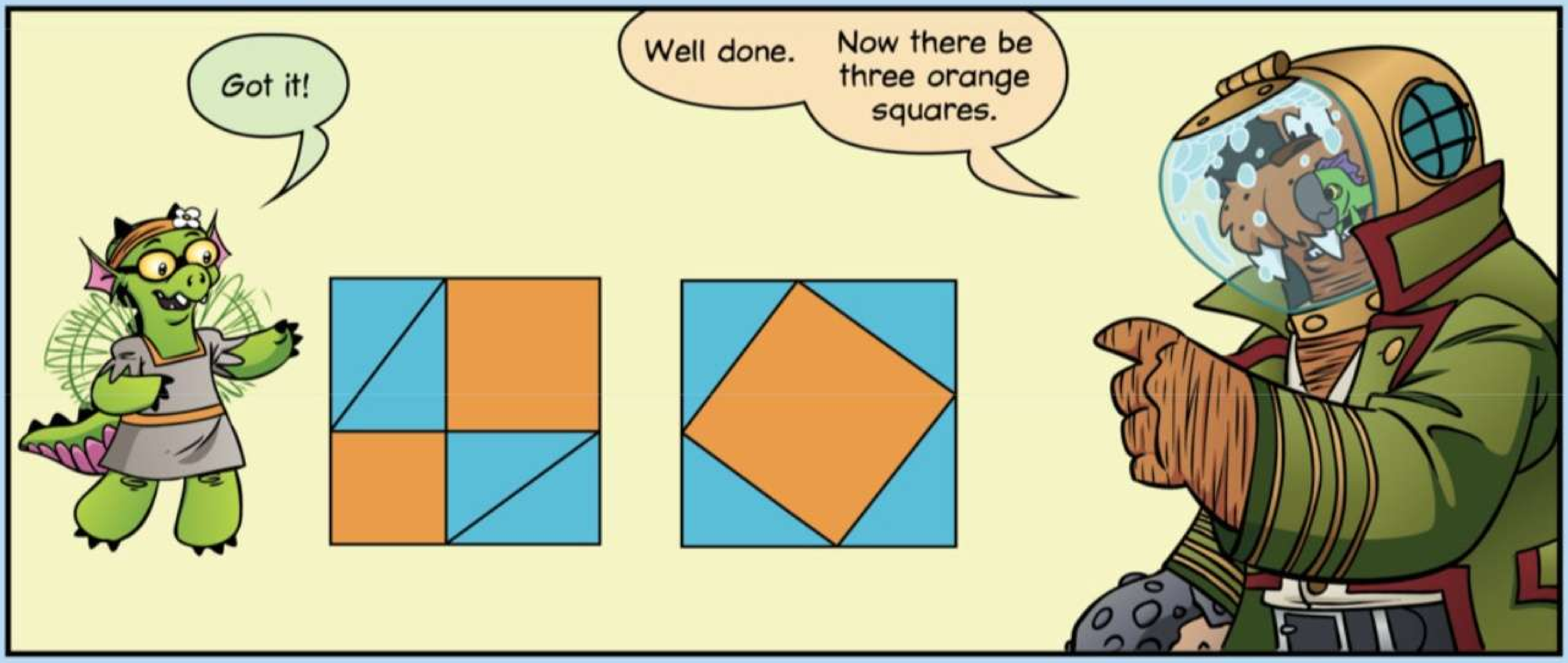

Let's see if we can use prime factorizations again.
In the numerator, we have $14 \cdot 56 = (2 \cdot 7) \cdot (2 \cdot 2 \cdot 2 \cdot 7)$.
And in the denominator, we have $27 \cdot 75 = (3 \cdot 3 \cdot 3) \cdot (3 \cdot 5 \cdot 5)$.
$\sqrt{\frac{14}{27} \cdot \frac{56}{75}} = \sqrt{\frac{(2 \cdot 7) \cdot (2 \cdot 2 \cdot 2 \cdot 7)}{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 5 \cdot 5)}}$
Then, we can split the prime factors into two equal groups.
$\sqrt{\frac{14}{27} \cdot \frac{56}{75}} = \sqrt{\frac{(2 \cdot 7) \cdot (2 \cdot 2 \cdot 2 \cdot 7)}{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 5 \cdot 5)}}$
$= \sqrt{\frac{(2 \cdot 2 \cdot 7) \cdot (2 \cdot 2 \cdot 7)}{(3 \cdot 3 \cdot 5) \cdot (3 \cdot 3 \cdot 5)}}$
In the numerator, we have $(2 \cdot 2 \cdot 7) \cdot (2 \cdot 2 \cdot 7)$.
And in the denominator, we have $(3 \cdot 3 \cdot 5) \cdot (3 \cdot 3 \cdot 5)$.
We can turn this into a product of two fractions.
$\sqrt{\frac{14}{27} \cdot \frac{56}{75}} = \sqrt{\frac{(2 \cdot 7) \cdot (2 \cdot 2 \cdot 2 \cdot 7)}{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 5 \cdot 5)}}$
$= \sqrt{\frac{(2 \cdot 2 \cdot 7) \cdot (2 \cdot 2 \cdot 7)}{(3 \cdot 3 \cdot 5) \cdot (3 \cdot 3 \cdot 5)}}$
$= \sqrt{\left(\frac{2 \cdot 2 \cdot 7}{3 \cdot 3 \cdot 5}\right) \cdot \left(\frac{2 \cdot 2 \cdot 7}{3 \cdot 3 \cdot 5}\right)}$
$= \sqrt{\frac{28}{45} \cdot \frac{28}{45}}$
$= \frac{28}{45}$
So, $\sqrt{\frac{14}{27} \cdot \frac{56}{75}} = \sqrt{\frac{28}{45} \cdot \frac{28}{45}} = \frac{28}{45}$.


You guys may not know as much math as Max...
...but when you put your heads together, you can figure out just about anything.
We planned and built a fort in Alex's back yard last month!
It's not a fort, it's a castle.
TOP FLOOR
catapult
4-Post Princess Bed
Vanity w/ Magic mirror
Turret
Chaise Longue
BOTTOM FLOOR
Library
BOARD ROOM
EXECUTIVE OFFICE
Alcove
Lava Moat
Whatever it is, it turned out great!
I think of it as more of a lair.
I've been using it as a home office!
Taxes

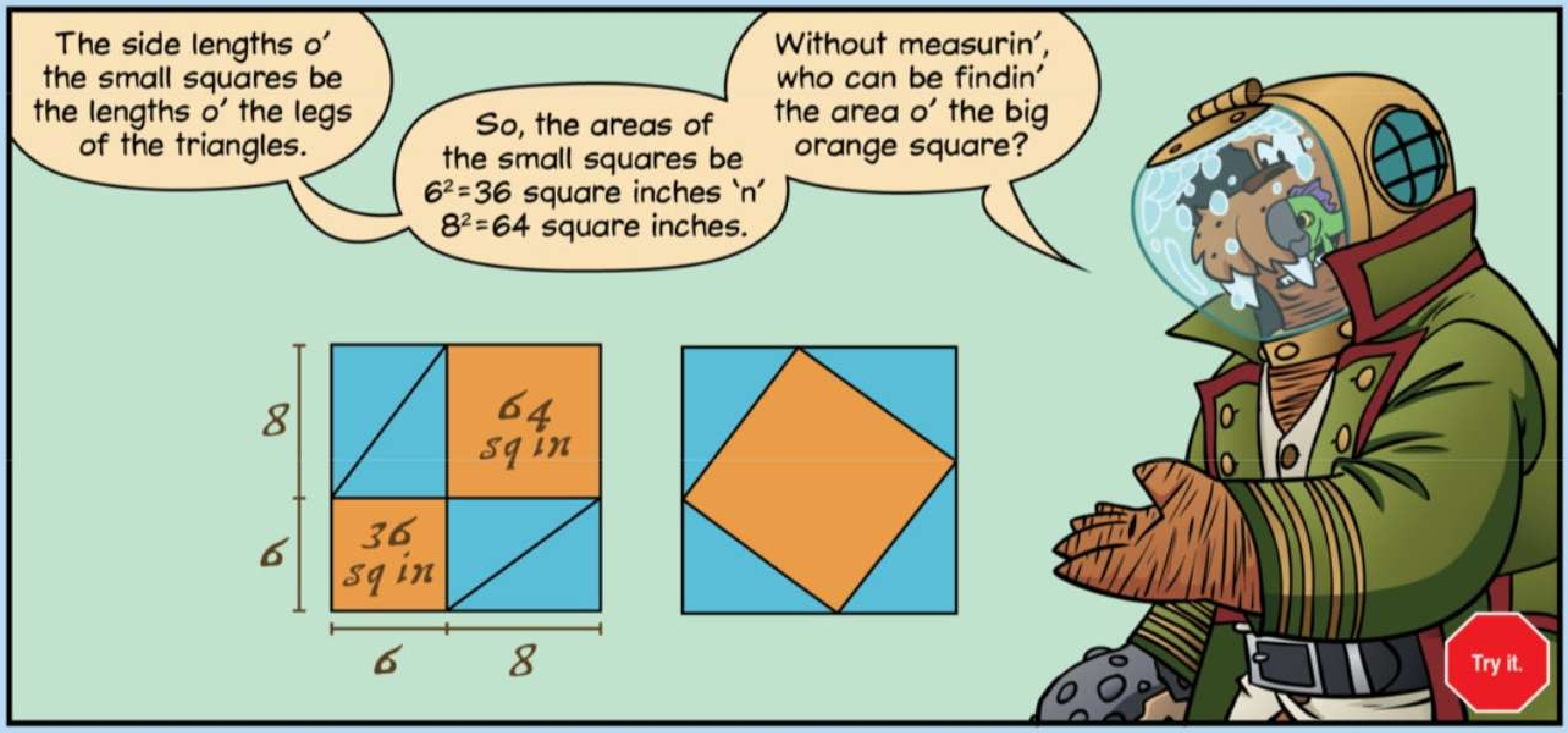

WORKSHOP
THE PYTHAGOREAN THEOREM
What are all these square mats and triangles for, Captain Kraken?
We'll be demonstratin' one o' the most famous theorems in mathematics...
...the Pythagorean theorem.
First, be sure that the side length o' the square mat be equal to the sum o' the two shortest sides of your triangle.
How can we tell which sides are the shortest?
The short sides of a right triangle meet at the right angle. They're called the ***legs*** of the triangle.
The longest side is across from the right angle. It's called the ***hypotenuse***.
hypotenuse
legs
Squawk. Hi-pot-en-oose.
PRINT AND CUT YOUR OWN SQUARES AND TRIANGLES FOR THIS ACTIVITY AT BEASTACADEMY.COM.
The legs of the right triangles are 6 inches and 8 inches...
...and the square mat has 14-inch sides.
Aye. Now, place the other two triangles on the square mat so that the only visible orange part be a square.
8
6



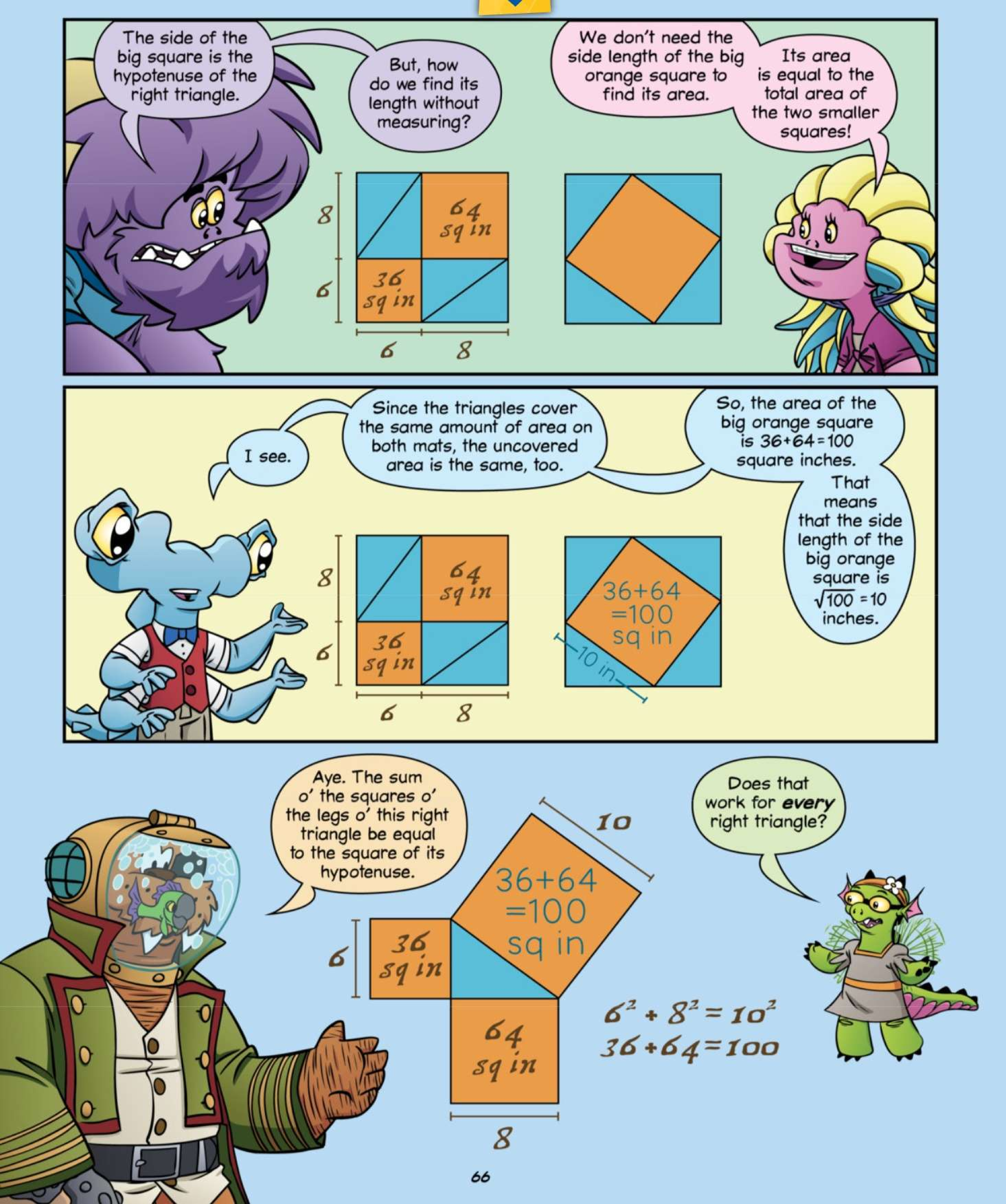
Like this?
Aye. Perfect.
Next, grab another square mat 'n' four more triangles.
This time, arrange the four triangles on the square mat so that the visible orange part be ***two*** squares.

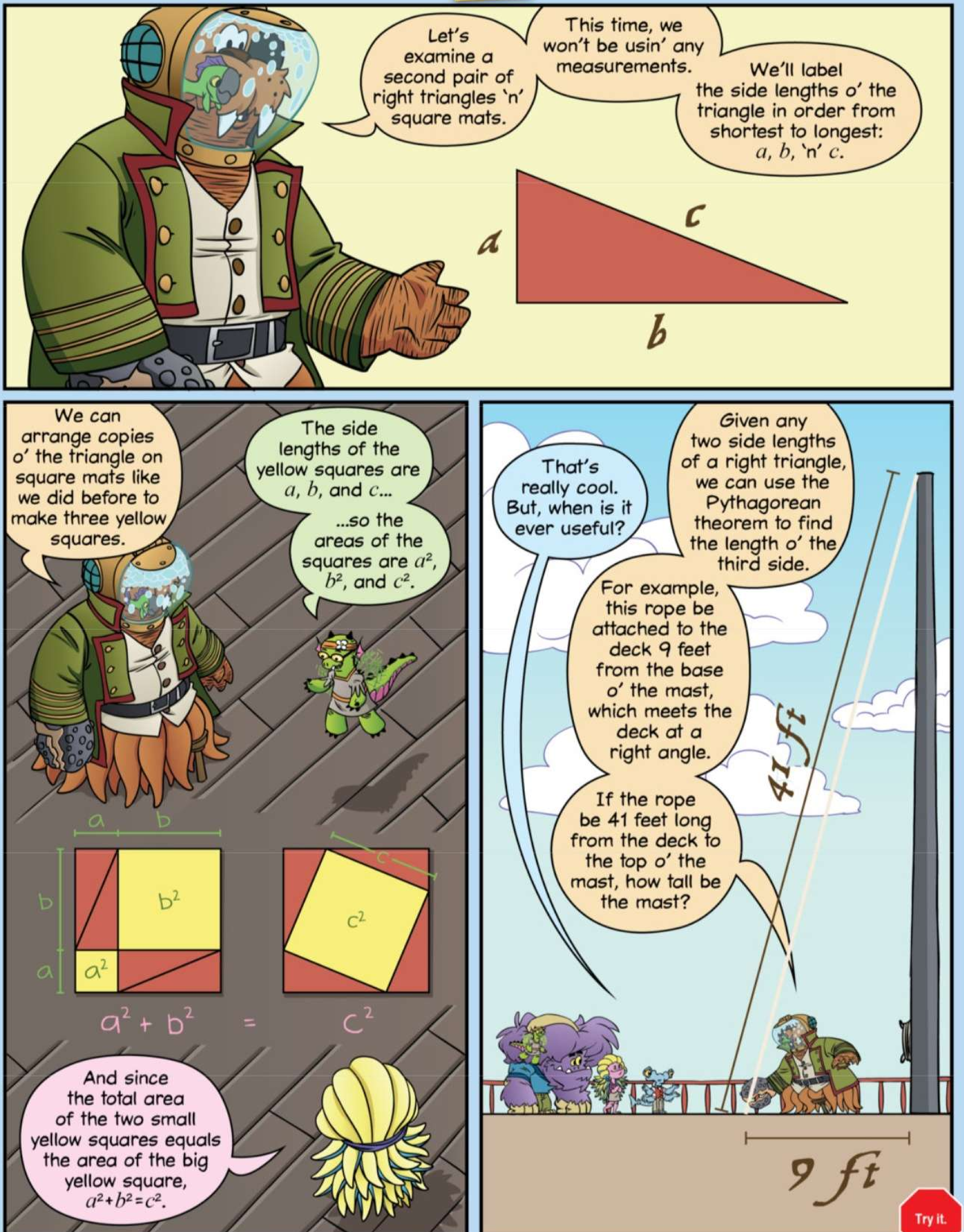


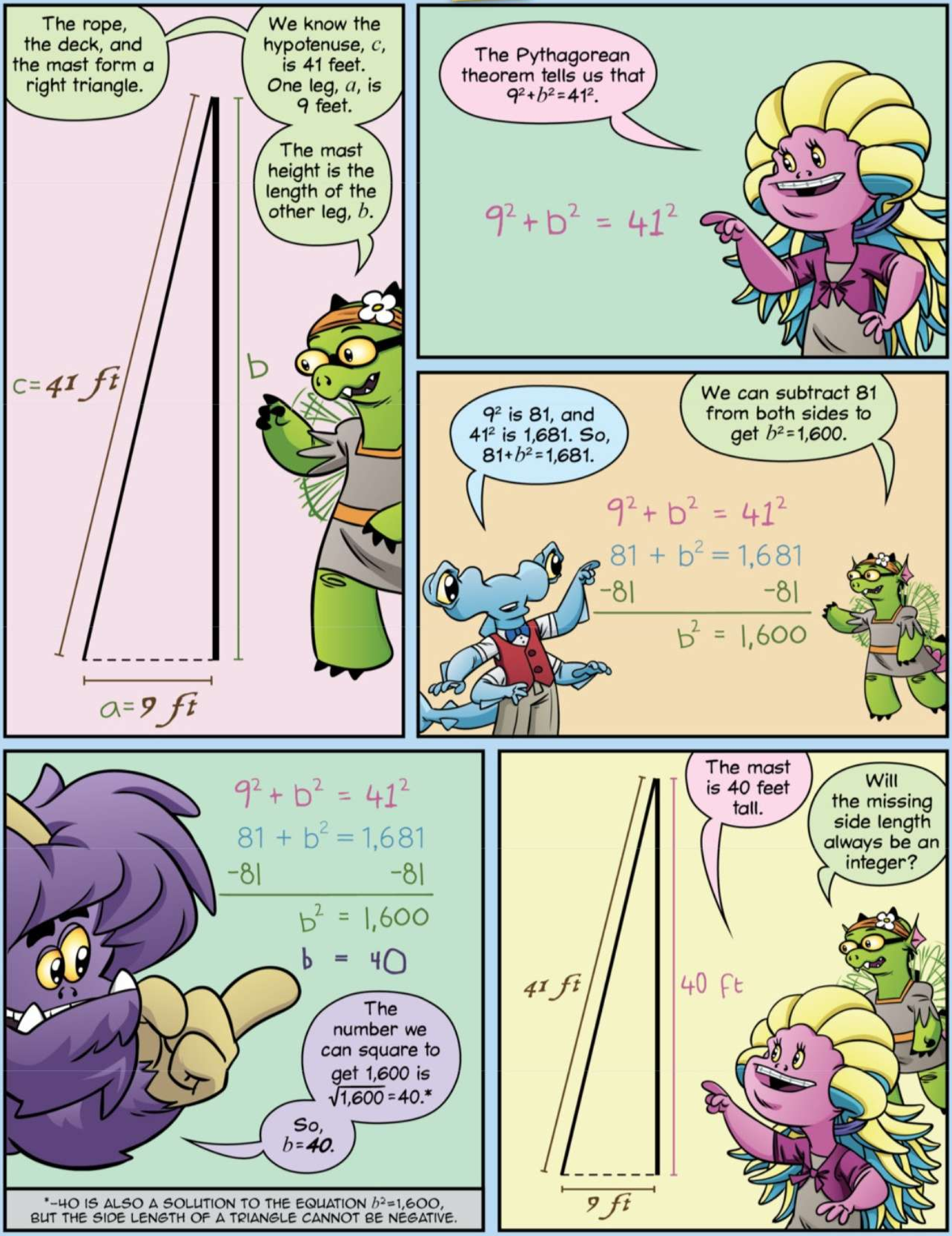
Got it!
Well done.
Now there be three orange squares.



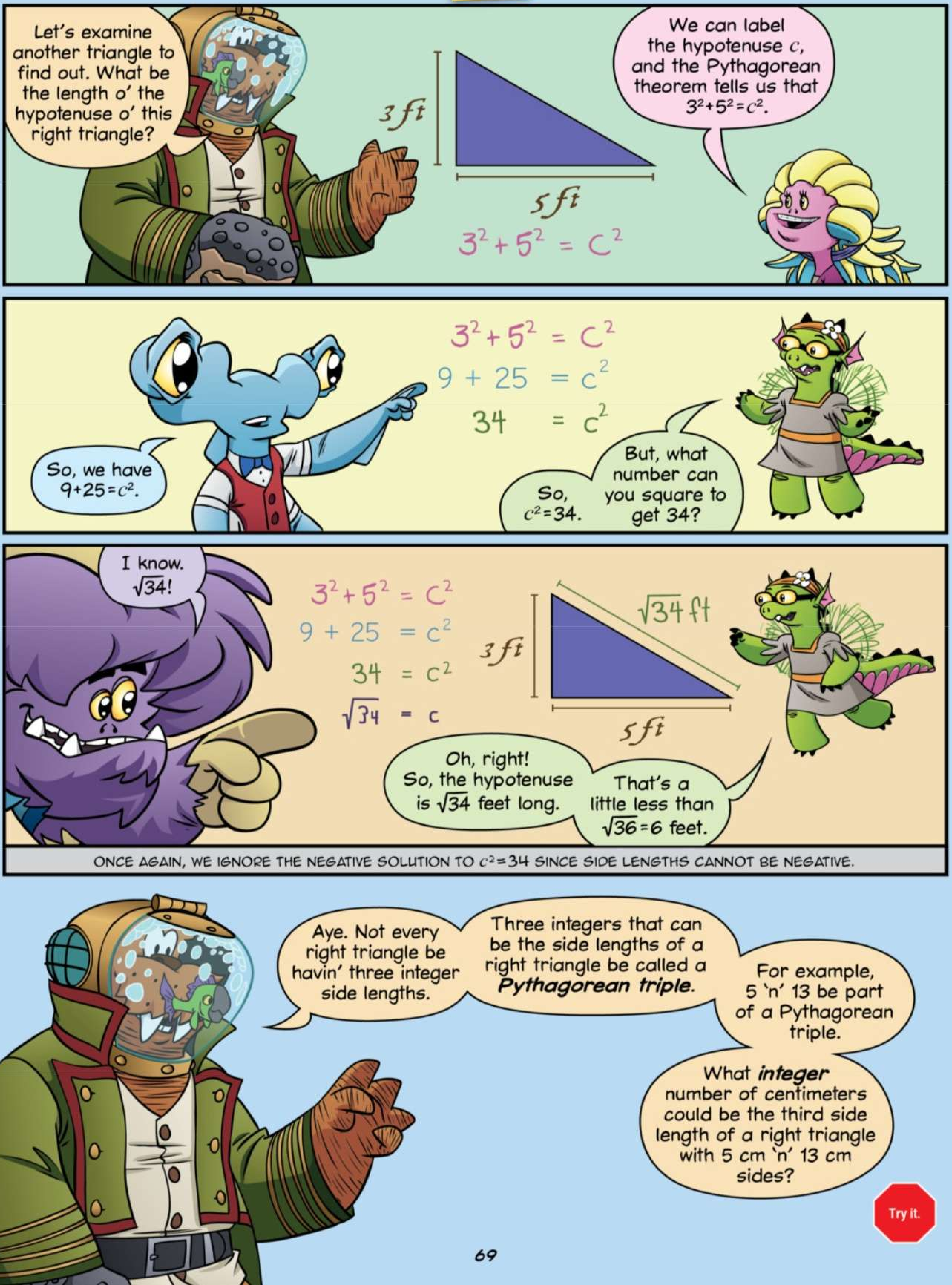
The side lengths o' the small squares be the lengths o' the legs of the triangles.
So, the areas of the small squares be $6^2=36$ square inches 'n' $8^2=64$ square inches.
Without measurin', who can be findin' the area o' the big orange square?
8
6
64 sq in
36 sq in
6
8
Try it.


The side of the big square is the hypotenuse of the right triangle.
But, how do we find its length without measuring?
We don't need the side length of the big orange square to find its area.
Its area is equal to the total area of the two smaller squares!
8
6
6
8
64 sq in
36 sq in
I see.
Since the triangles cover the same amount of area on both mats, the uncovered area is the same, too.
So, the area of the big orange square is 36+64=100 square inches.
That means that the side length of the big orange square is √100 = 10 inches.
8
6
6
8
64 sq in
36 sq in
36+64 =100 sq in
10 in
Aye. The sum o' the squares o' the legs o' this right triangle be equal to the square of its hypotenuse.
Does that work for ***every*** right triangle?
10
6
8
36 sq in
64 sq in
36+64 =100 sq in
6² + 8² = 10²
36 + 64 = 100

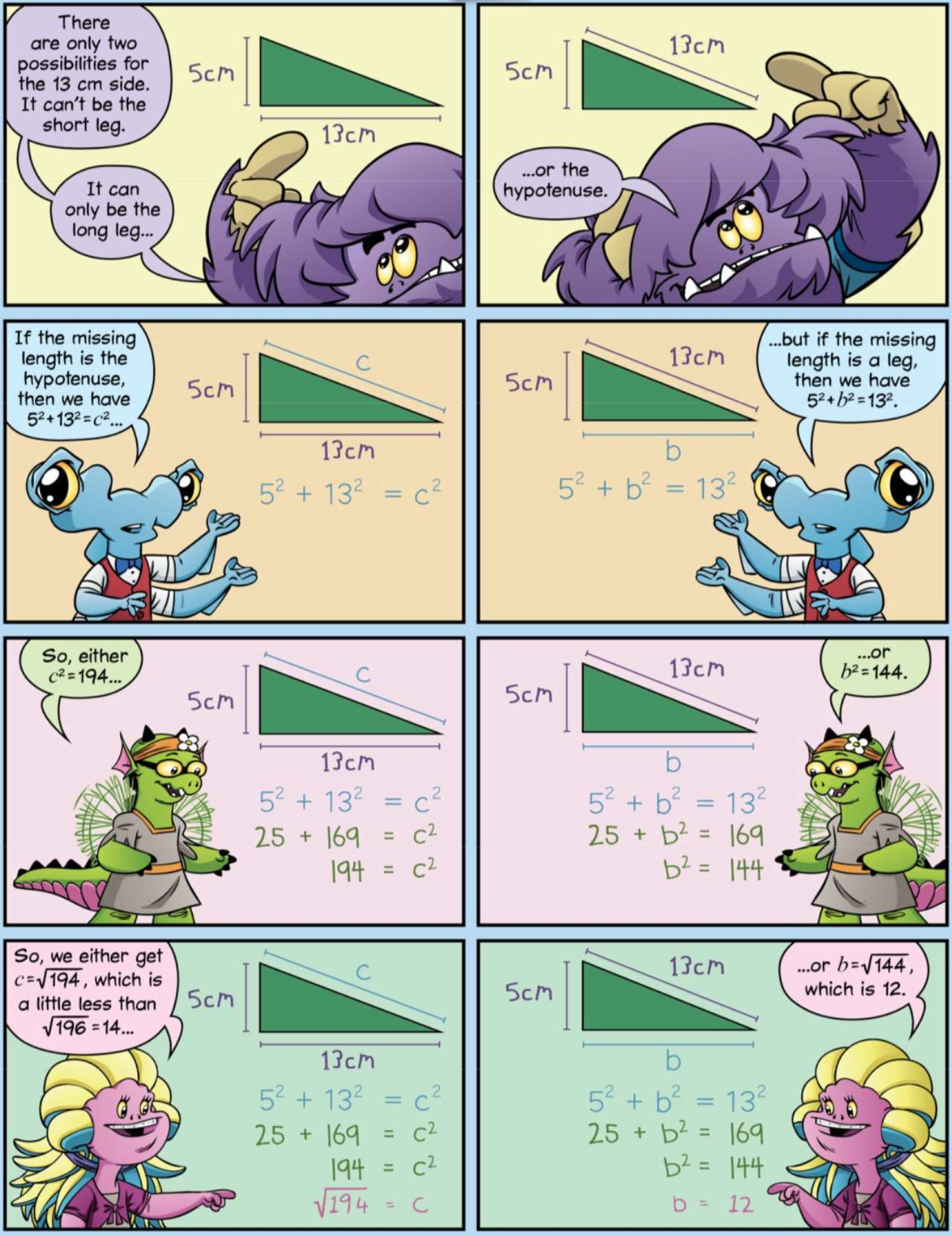

Let's examine a second pair of right triangles 'n' square mats.
This time, we won't be usin' any measurements.
We'll label the side lengths o' the triangle in order from shortest to longest: a, b, 'n' c.
a
c
b
We can arrange copies o' the triangle on square mats like we did before to make three yellow squares.
The side lengths of the yellow squares are a, b, and c...
...so the areas of the squares are a^2, b^2, and c^2.
a
b
b
a
b^2
a^2
c
c^2
$a^2 + b^2 = c^2$
And since the total area of the two small yellow squares equals the area of the big yellow square, $a^2+b^2=c^2$.
That's really cool. But, when is it ever useful?
Given any two side lengths of a right triangle, we can use the Pythagorean theorem to find the length o' the third side.
For example, this rope be attached to the deck 9 feet from the base o' the mast, which meets the deck at a right angle.
If the rope be 41 feet long from the deck to the top o' the mast, how tall be the mast?
41 ft
9 ft
Try it.



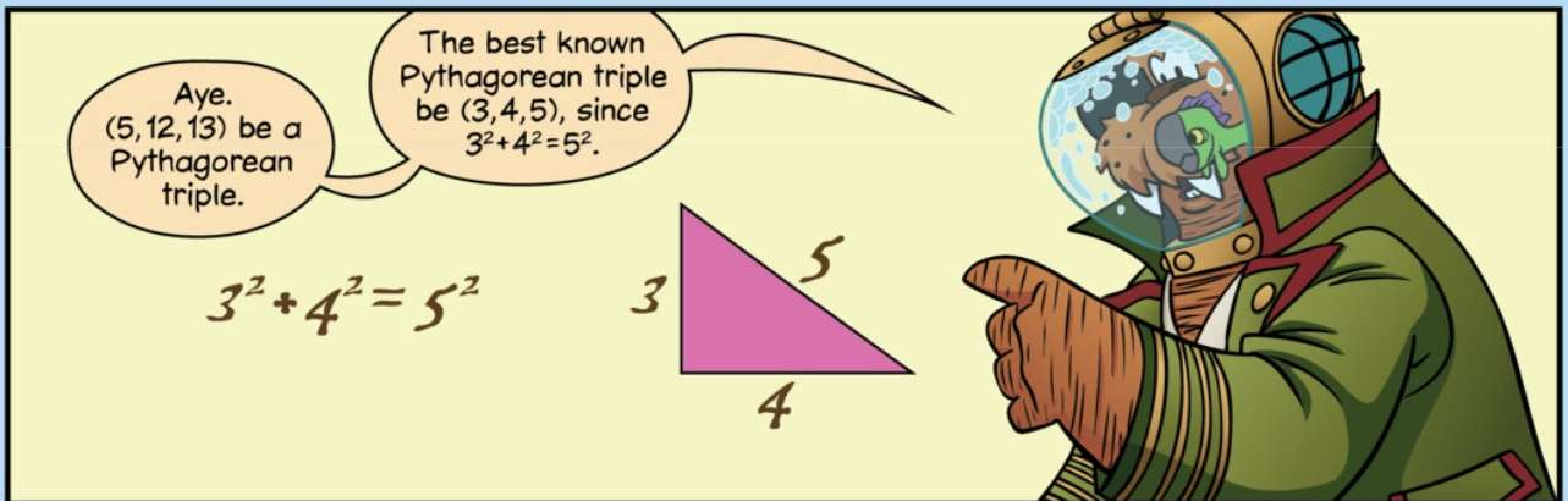
The rope, the deck, and the mast form a right triangle.
We know the hypotenuse, c, is 41 feet. One leg, a, is 9 feet.
The mast height is the length of the other leg, b.
c=41 ft
b
a=9 ft
The Pythagorean theorem tells us that $9^2+b^2=41^2$.
$9^2 + b^2 = 41^2$
9^2 is 81, and 41^2 is 1,681. So, $81+b^2=1{,}681$.
We can subtract 81 from both sides to get $b^2=1{,}600$.
$9^2 + b^2 = 41^2$
$81 + b^2 = 1{,}681$
$-81 \quad -81$
$b^2 = 1{,}600$
$9^2 + b^2 = 41^2$
$81 + b^2 = 1{,}681$
$-81 \quad -81$
$b^2 = 1{,}600$
$b = 40$
The number we can square to get 1,600 is $\sqrt{1{,}600}=40$.*
So, $b=40$.
*−40 IS ALSO A SOLUTION TO THE EQUATION $b^2=1{,}600$, BUT THE SIDE LENGTH OF A TRIANGLE CANNOT BE NEGATIVE.
The mast is 40 feet tall.
Will the missing side length always be an integer?
41 ft
40 ft
9 ft



Let's examine another triangle to find out. What be the length o' the hypotenuse o' this right triangle?
3 ft
5 ft
$3^2 + 5^2 = c^2$
We can label the hypotenuse c, and the Pythagorean theorem tells us that $3^2+5^2=c^2$.
$3^2 + 5^2 = c^2$
$9 + 25 = c^2$
$34 = c^2$
So, we have $9+25=c^2$.
So, $c^2=34$.
But, what number can you square to get 34?
I know. $\sqrt{34}$!
$3^2 + 5^2 = c^2$
$9 + 25 = c^2$
$34 = c^2$
$\sqrt{34} = c$
3 ft
$\sqrt{34}$ ft
5 ft
Oh, right! So, the hypotenuse is $\sqrt{34}$ feet long.
That's a little less than $\sqrt{36}=6$ feet.
ONCE AGAIN, WE IGNORE THE NEGATIVE SOLUTION TO $c^2=34$ SINCE SIDE LENGTHS CANNOT BE NEGATIVE.
Aye. Not every right triangle be havin' three integer side lengths.
Three integers that can be the side lengths of a right triangle be called a ***Pythagorean triple.***
For example, 5 'n' 13 be part of a Pythagorean triple.
What ***integer*** number of centimeters could be the third side length of a right triangle with 5 cm 'n' 13 cm sides?
Try it.



There are only two possibilities for the 13 cm side. It can't be the short leg.
5cm
13cm
It can only be the long leg...
...or the hypotenuse.
13cm
5cm
If the missing length is the hypotenuse, then we have $5^2+13^2=c^2$...
5cm
c
13cm
$5^2 + 13^2 = c^2$
...but if the missing length is a leg, then we have $5^2+b^2=13^2$.
13cm
5cm
b
$5^2 + b^2 = 13^2$
So, either $c^2=194$...
5cm
c
13cm
$5^2 + 13^2 = c^2$
$25 + 169 = c^2$
$194 = c^2$
...or $b^2=144$.
13cm
5cm
b
$5^2 + b^2 = 13^2$
$25 + b^2 = 169$
$b^2 = 144$
So, we either get $c=\sqrt{194}$, which is a little less than $\sqrt{196}=14$...
5cm
c
13cm
$5^2 + 13^2 = c^2$
$25 + 169 = c^2$
$194 = c^2$
$\sqrt{194} = c$
...or $b=\sqrt{144}$, which is 12.
13cm
5cm
b
$5^2 + b^2 = 13^2$
$25 + b^2 = 169$
$b^2 = 144$
$b = 12$



MULTIPLYING ALL OF THE NUMBERS IN A PYTHAGOREAN TRIPLE BY THE SAME POSITIVE INTEGER GIVES ANOTHER PYTHAGOREAN TRIPLE. FOR EXAMPLE, DOUBLING (3,4,5) GIVES (6,8,10), AND $6^2+8^2=10^2$.