Contents: Chapter 5

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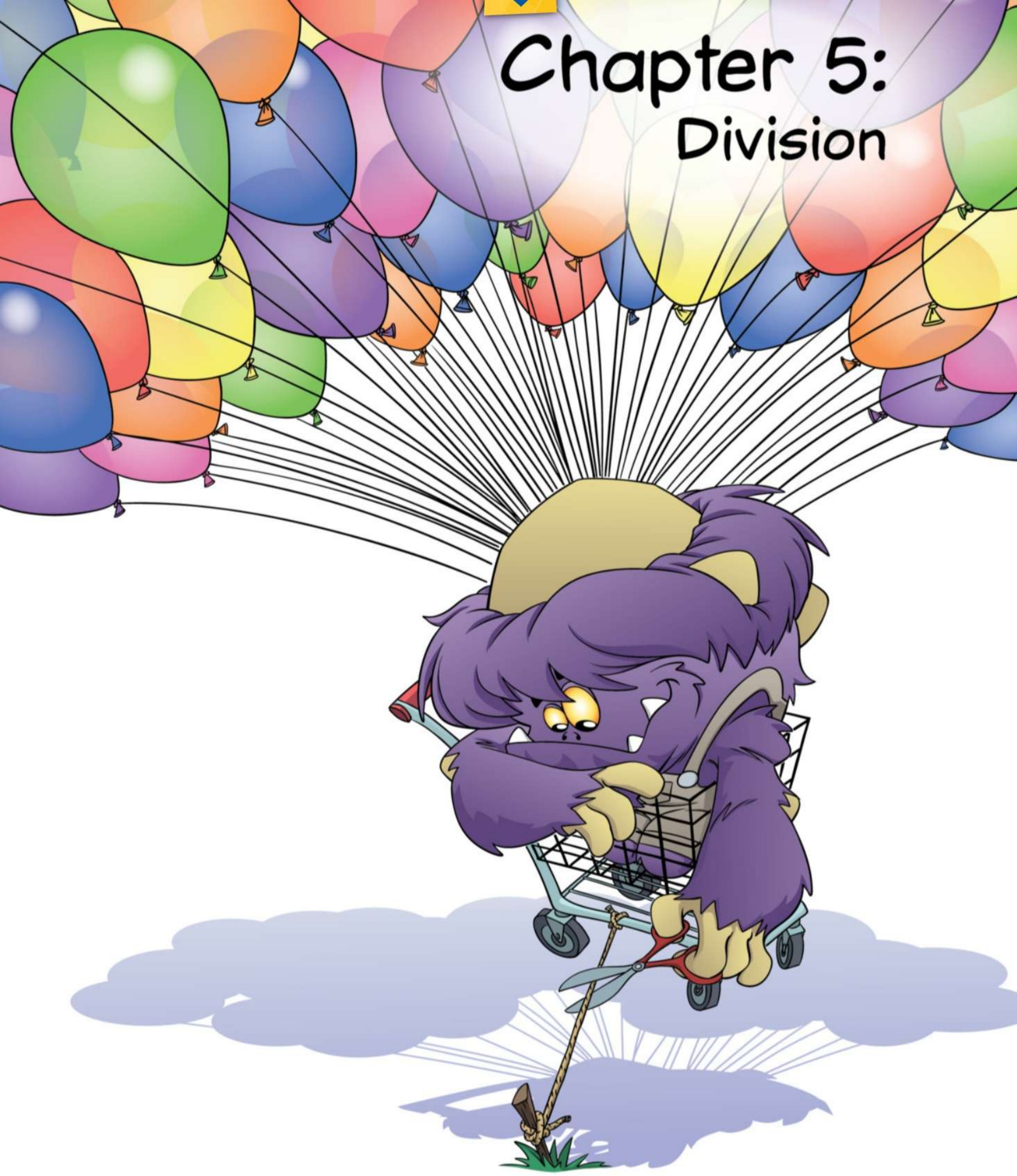
Chapter 5:
Division



R&G Special Quotients

Grogg, do you have your division homework?

I did it...

...but on one problem, I tried to divide by zero and my homework burst into flames.

ha ha ha ha ha ha ha ha

I don't get it.

Maybe we should try dividing by zero.

Alright, but we should go outside, just in case.

Agreed.

Let's try to divide six by zero.

We can use these pebbles.

All we need to do is split these six pebbles into *zero* piles.

?!?

?!?

That doesn't make any sense.

Nope.

We can't split six pebbles into *zero* piles.

There's more than one way to think about division. Maybe instead of making zero ***piles***, we can make piles with zero ***pebbles***.

Right, and we could see how many piles we can make.

?!?

?!?

We can make as many piles of zero pebbles as we want.

That doesn't make any sense, either.

And we'll ***never*** run out of pebbles!

I know! We can answer a division problem with multiplication.

Right!

To divide six by ***three***, we could just answer, "What number times three is six?"

Two!

Perfect!

So, to divide six by ***zero***, we need to answer, "What number times zero is six?"

???

Is there a number you can multiply by 0 to get 6?

Multiplying any number by zero gives you zero!

So there *isn't* a number that you can multiply by zero to get six!

You can't divide by zero. It doesn't make any sense!

DIVISION BY ZERO IS "UNDEFINED," WHICH IS ANOTHER WAY OF SAYING THAT IT DOESN'T MAKE ANY SENSE.

Let's try something else.

We can't divide six by zero.

Can we divide zero by six?

Let's see. If I have zero pebbles, and I want to place them into 6 cups...

...how many pebbles will there be in each cup?

That's easy! Each cup will have zero pebbles.

If we have zero pebbles, then no matter how many cups we try to place them into, we'll always end up with zero pebbles in each cup.

So zero divided by any number is zero, except for 0÷0, which doesn't make sense.

Got it!

0÷*n*=0 FOR ALL NONZERO VALUES OF *n*.

Dividing by 1 is easy, too.

Don't tell me! I want to figure it out.

I'll start with 6÷1.

If I have 6 pebbles and I put them into one cup, the cup will contain all 6 pebbles...

...so 6÷1=6.

And no matter how many pebbles I start with, if I put them into just one cup, the cup will contain the same number I started with.

That's right!

Dividing any number by 1 gives you the same number you started with.

$n \div 1 = n$ FOR ALL VALUES OF n.

I know another one!

What do you get when you divide a number by itself?

Try it.

I'll try 6÷6.

If I divide 6 pebbles equally among 6 cups, there will be 1 pebble in each cup.

So, 6÷6=1.

If we have the same number of pebbles and cups, we'll always end up with one pebble in each cup.

That's right! Any number divided by itself is 1, except 0÷0.

FOR ALL NONZERO VALUES OF n, $n \div n = 1$.

This is a lot to remember. I better write these down.

If we divide a number by itself, we get--

1.

And any number divided by 1 is--

The number.

Zero divided by anything is--

Zero.

Unless we try to divide by zero, which gives us--

Yikes!

Whoa!

That big purple monster wasn't kidding.

Ms. Q.

Multiples of 10

How can we divide 3,500,000 by 7?

We can use multiplication to solve division problems!

We need to find the number that we multiply by 7 to get 3,500,000.

Since 5×7=35...

...500,000×7 is 3,500,000.

$\_\_ \times 7 = 3{,}500{,}000$

$5 \times 7 = 35$

$500{,}000 \times 7 = 3{,}500{,}000$

So, 3,500,000÷7 =500,000!

Great job! Let's try another.

How can we divide 3,500 by 700?

$\_\_ \times 7 = 3{,}500{,}000$

$5 \times 7 = 35$

$500{,}000 \times 7 = 3{,}500{,}000$

$3{,}500{,}000 \div 7 = 500{,}000$

Try it.

We can use multiplication to solve this division problem, too.

We need to find the number that we multiply by 700 to get 3,500.

___ × 700 = 3,500

5 × 700 = 3,500

Since 5×700 =3,500...

___ × 700 = 3,500

5 × 700 = 3,500

3,500 ÷ 700 = 5

...3,500 ÷ 700 = 5!

Good. Did anyone solve 3,500 ÷ 700 a different way?

I just divided 35 by 7.

How did you know that would work, Grogg?

3,500 is 35 hundreds, and 700 is 7 hundreds.

Dividing hundreds is just like dividing anything else.

If we have 35 giffles, we can split them into groups of 7 giffles. There will be 35÷7=5 groups of 7 giffles.

If we have 35 hundreds, we can split them into groups of 7 hundreds.

There will be 35÷7=5 groups of 7 hundreds.

So, 3,500÷700 equals 5.

Marvelous! How could you use a similar strategy to compute 36,000÷9,000?

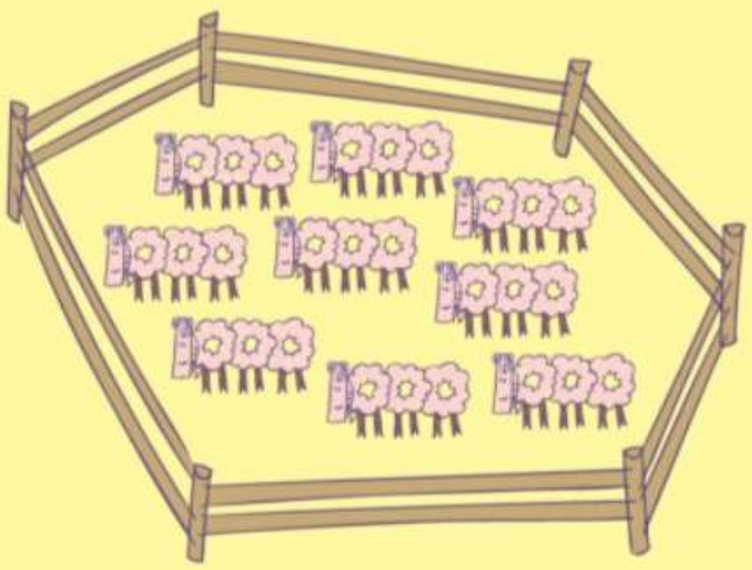
36,000 ÷ 9,000

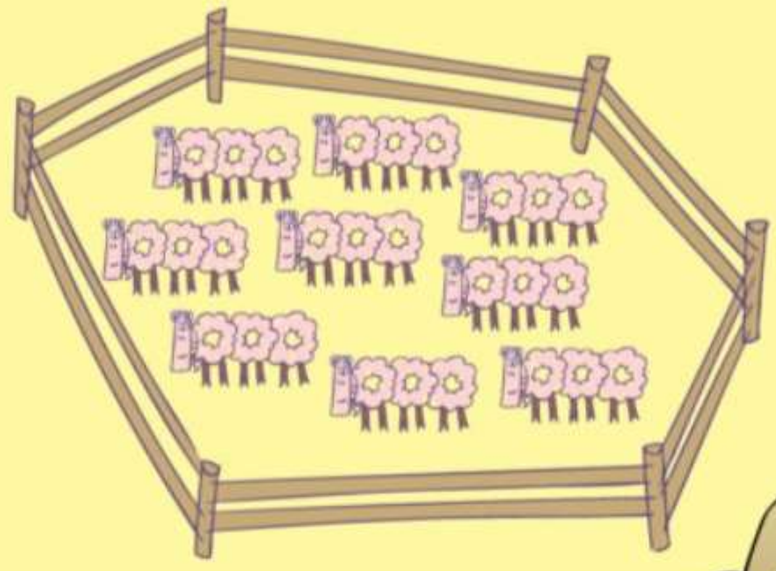
Try it.

We can just divide 36 thousands into groups of 9 thousands.

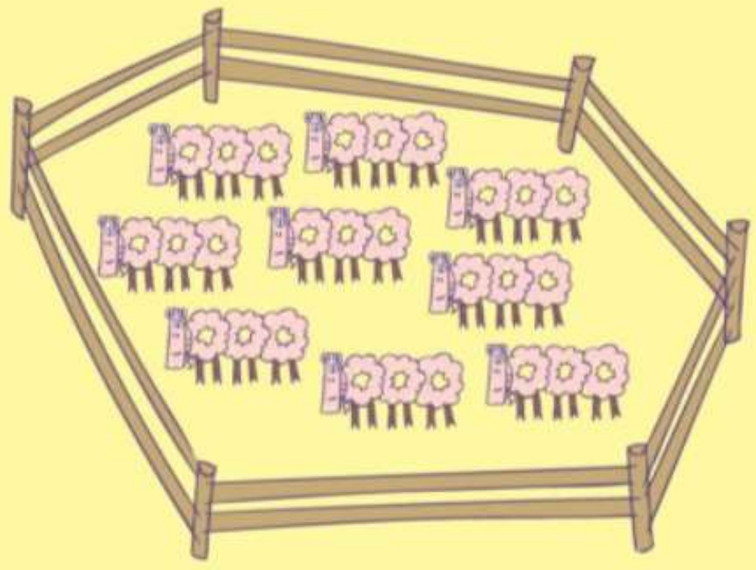
$$36{,}000 \div 9{,}000$$
$$= 36 \div 9$$
$$= 4$$

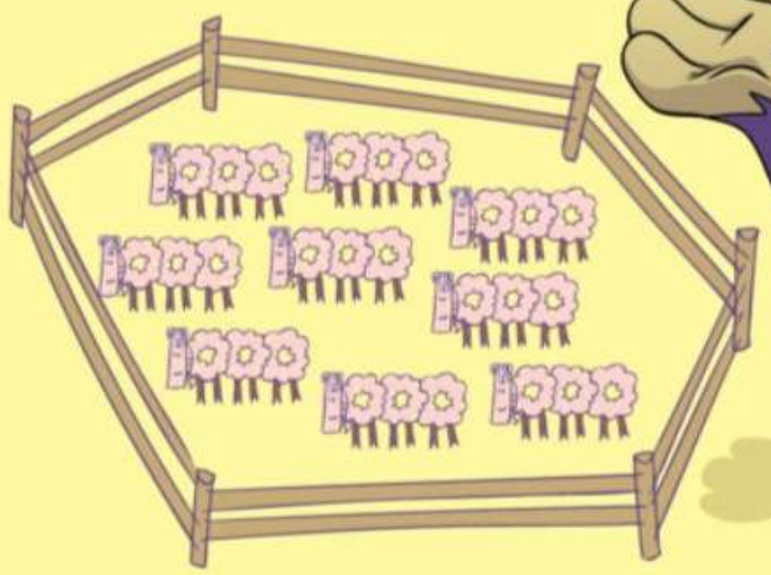
36 thousands divided into groups of 9 thousands gives us 36÷9=4 groups!

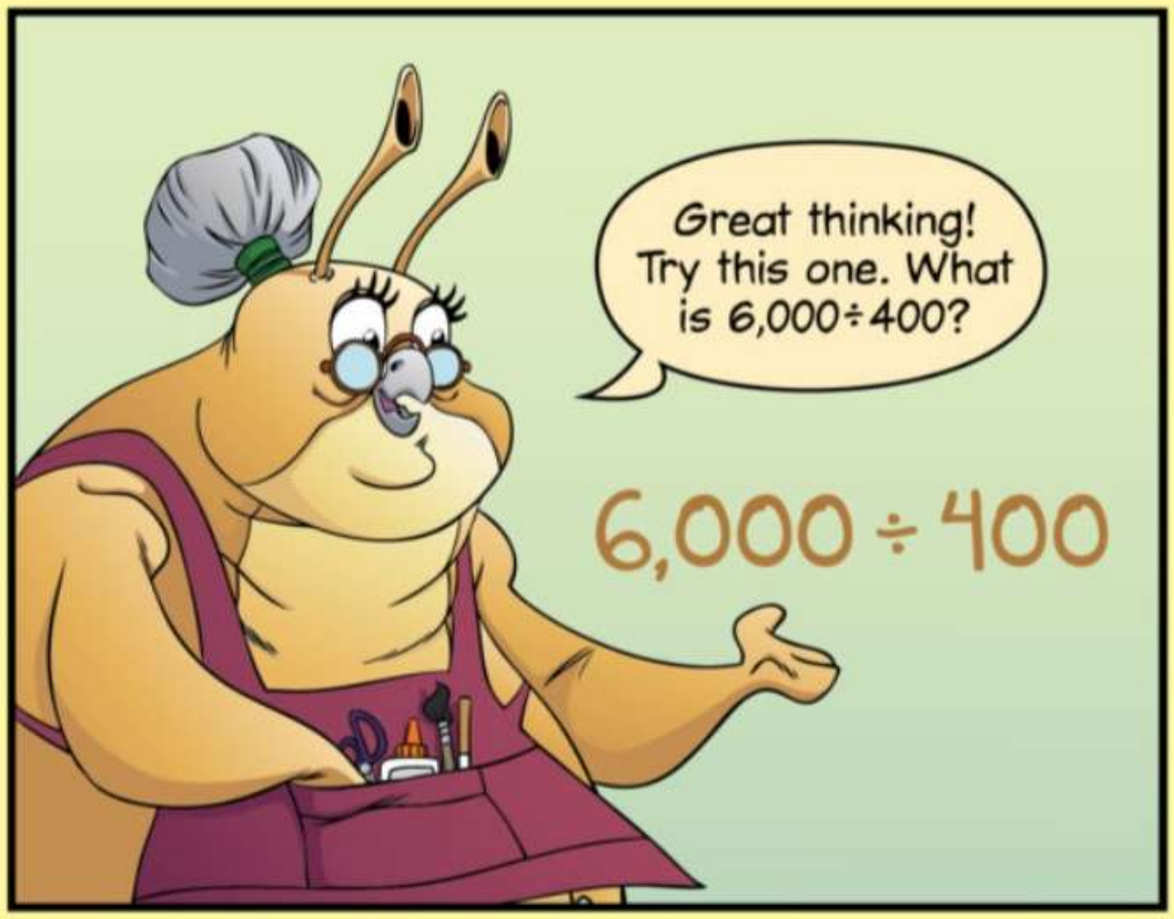




Like this!







Instead of grouping 6,000 into ***thousands***, and 400 into ***hundreds***, maybe we can group both numbers the same way.

6 thousands is the same as ***60*** hundreds!

We can compute 6,000÷400 by dividing 60 ***hundreds*** into groups of 4 ***hundreds!***

So, we can just divide 60÷4.

$$6{,}000 \div 400$$

$$= 60 \div 4$$

$$= 15$$

60 hundreds can be divided into 60÷4=15 groups of 4 hundreds.

So, 6,000÷400 equals 15!

Yep. That answer checks out.

$15 \times 400 = 6{,}000$

We can remove the same number of zeros from the end of each number in a division problem without changing the quotient!

$3{,}500 \div 700$
$= 35 \div 7$
$= 5$

$54{,}000 \div 6{,}000$
$= 54 \div 6$
$= 9$

$6{,}000 \div 400$
$= 60 \div 4$
$= 15$

You're right, Winnie. Why does that work?

If both numbers in a division problem end in zero, then we can group both numbers into tens.

For example, 5,000 is 500 tens, and 200 is 20 tens.

So, dividing 5,000÷200 is the same as dividing 500 ***tens*** into groups of 20 ***tens***.

So, 5,000÷200 =500÷20.

$5{,}000 \div 200$

$= 500 \div 20$

And dividing 500÷20 is the same as dividing 50 tens into groups of 2 tens.

So, 500÷20 =50÷2.

$5{,}000 \div 200$

$= 500 \div 20$

$= 50 \div 2$

I see. We can just keep removing a zero from each number until there are no more zeros to remove from one of the numbers.

Then, we divide.

$5{,}000 \div 200$

$= 500 \div 20$

$= 50 \div 2$

$= 25$

Very good. Removing the same number of zeros from the dividend and the divisor won't change the quotient.

Now, who can find this quotient?

$15{,}000{,}000 \div 2{,}500{,}000$

Try it.

We can start by taking five zeros off the end of each number.

15,000,000÷2,500,000

= 150÷25

Then, we just divide 150÷25.

6×25=150, so 150÷25=6.

Superb!

Who can sum up what we've learned?

15,000,000÷2,500,000

= 150÷25

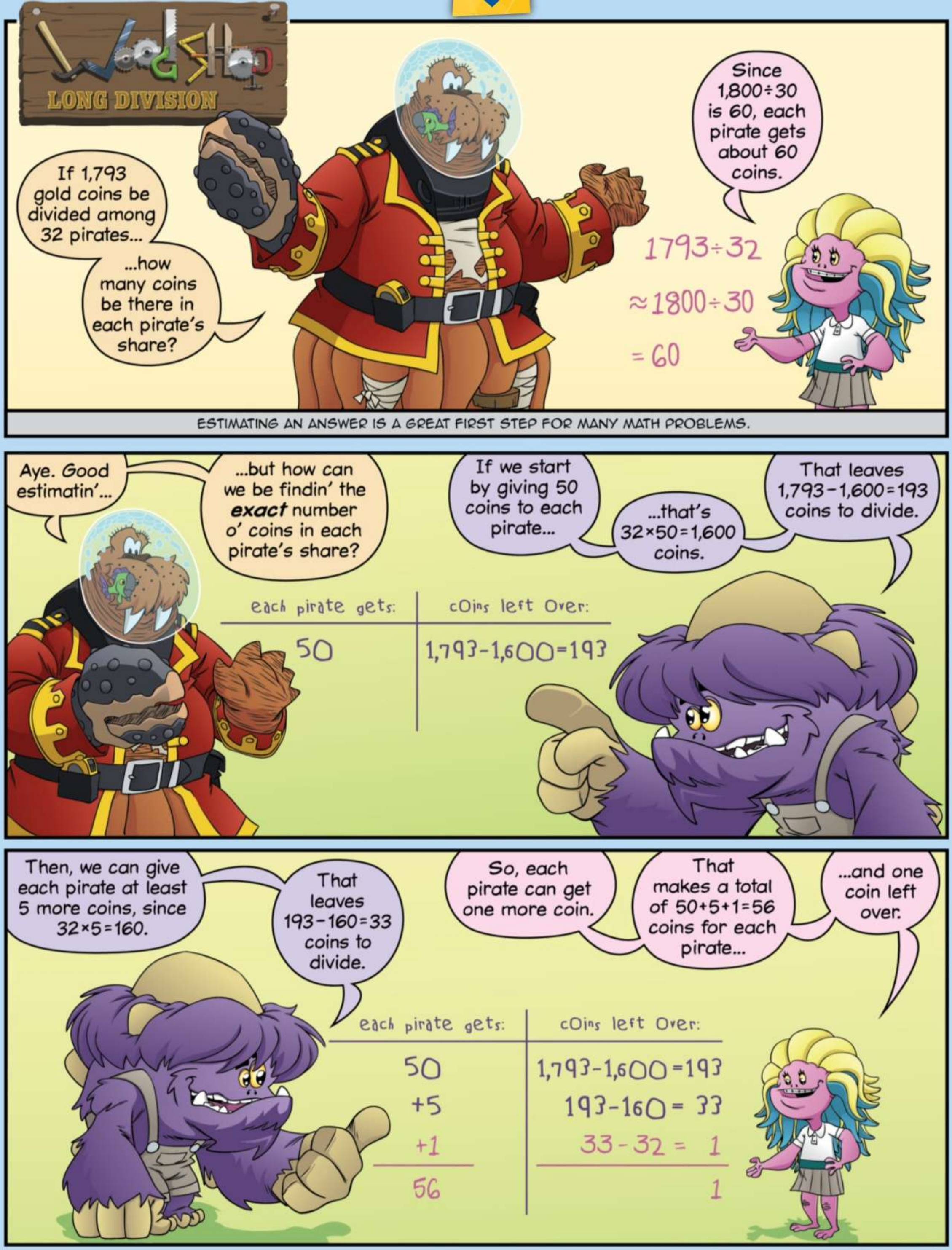
= 6

If you want to be a division hero...

...all you have to do is drop the zeros.

Grogg, you're a poet.

You know it!


WOODSHOP
LONG DIVISION
If 1,793 gold coins be divided among 32 pirates...
...how many coins be there in each pirate's share?
Since 1,800÷30 is 60, each pirate gets about 60 coins.
1793÷32
≈1800÷30
=60
ESTIMATING AN ANSWER IS A GREAT FIRST STEP FOR MANY MATH PROBLEMS.
Aye. Good estimatin'...
...but how can we be findin' the exact number o' coins in each pirate's share?
If we start by giving 50 coins to each pirate...
...that's 32×50=1,600 coins.
That leaves 1,793−1,600=193 coins to divide.
each pirate gets:
coins left over:
50
1,793−1,600=193
Then, we can give each pirate at least 5 more coins, since 32×5=160.
That leaves 193−160=33 coins to divide.
So, each pirate can get one more coin.
That makes a total of 50+5+1=56 coins for each pirate...
...and one coin left over.
each pirate gets:
coins left over:
50
+5
+1
56
1,793−1,600=193
193−160=33
33−32=1
1

We could organize our work like this.

The number of pirates goes out here.

$32\overline{)1{,}793}$

And the number of coins goes here.

On top, we keep track of how many coins each pirate gets.

$32\overline{)1{,}793}$

And down here, we keep track of how many coins are left over.

We can give each pirate at least 50 coins.

$$\begin{array}{r} 50 \\ 32\overline{)1{,}793} \\ -1{,}600 \\ \hline 193 \end{array}$$

After we subtract $32\times50=1{,}600$ from 1,793...

...there are 193 coins left to divide.

Then, we can give each pirate 5 more coins.

$$\begin{array}{r} 5 \\ 50 \\ 32\overline{)1{,}793} \\ -1{,}600 \\ \hline 193 \\ -160 \\ \hline 33 \end{array}$$

We subtract $32\times5=160$ from 193.

And we have 33 coins left to divide.

Finally, we can give each pirate one more coin.

That makes 56 coins for each pirate.

$$\begin{array}{r} 1 \\ 5 \\ 50 \end{array}\!\!\Big\}\,56$$

$$\begin{array}{r} 50 \\ 32\overline{)1{,}793} \\ -1{,}600 \\ \hline 193 \\ -160 \\ \hline 33 \\ -32 \\ \hline 1 \end{array}$$

And one coin left over.

WE SAY THAT $1{,}793\div32$ HAS QUOTIENT 56 AND REMAINDER 1.

Aye. Excellent figurin'. Try another.

If 7,972 coins be divided among 24 pirates...

...how many coins be there in each pirate's share?

Try it.

To start, we can give 200 coins to each pirate.

That's 24×200=4,800 coins. So, there are 3,172 coins left over.

```
   100
   200
24)7,972
 -4,800
  3,172
 -2,400
    772
```

Then, we give 100 more to each pirate, which leaves 3,172−2,400=772 coins to divide.

We give each pirate 20 more, then 10 more, and finally 2 more coins...

...for a total of 200+100+20+10+2 =332 coins each, with 4 left over.

```
   2
  10
  20 } 332
 100
 200
24)7,972
 -4,800
  3,172
 -2,400
    772
  -480
    292
  -240
     52
   - 48
      4
```

My work looks completely different, but I got the same answer.

Me, too.

Same here.

Arrr.... Let's have a look at your work, little monsters.

2
10
20 } 332
100
200

24)7,972
-4,800
3,172
-2,400
772
-480
292
-240
52
- 48
4

2
30
100 } 332
100
100

24) 7,972
-2,400
5,572
-2,400
3,172
-2,400
772
-720
52
-48
4

2
30 } 332
300

24)7,972
- 7,200
772
-720
52
-48
4

4
5
20 } 332
300
3

24)7,972
- 72
7,900
-7,200
700
-480
220
- 120
100
- 96
4

Lizzie gave each pirate 200, then 100, then 20, 10, and 2 coins...

...but Alex gave each pirate 100 coins three times, then 30, and 2 coins.

Winnie and I found other ways to give out the coins.

Does your work look like any of these?

We all kept track of how many coins were given to each pirate...

...and subtracted the coins we gave until there were fewer coins than pirates.

Since none of us made any errors with our arithmetic, we all got the same answer.

Aye. To avoid arithmetic errors, 'tis good to choose numbers that be easy to multiply by 24.

This usually means choosin' multiples of th' powers o' 10.

Like 10, 30, or 200.

That's why I used 100.

Because 100×24 is easy to compute.

'Tis also a good idea to limit the number o' steps ye be takin'.

That's why I started with 300. It's not quite as easy to multiply by 24 as 100 is.

But, I didn't need to take as many steps as Alex.

Aye. Fewer steps means fewer chances to make mistakes.

My mom says that every mistake is an opportunity!

She also says that I've had ***way*** more opportunities than most little monsters my age.

CAR POOL

For last night's homework, I figured out 36,063÷9...

...in my *head!*

In your head?

Yep.

Whoa. How?

I made it a problem about donuts, like I often do with division problems.

Huh?

You see, if I have 36,063 donuts to divide equally among 9 little monsters...

...I don't try to divide them all at once.

RANDY'S DONUTS 10 mi.

I start by dividing 36,000 donuts.

Then, I divide the other 63.

Because 36,000 and 63 are both easy to divide by 9!

Exactly.

Try to compute 36,063÷9 in your head.

If I divide 36,000 donuts between 9 monsters, each monster gets 36,000÷9=4,000 donuts.

Then, when you divide the remaining 63 donuts...

...each monster gets 63÷9=7 more donuts.

That makes a total of 4,000+7=4,007 donuts for each monster.

RANDY'S DONUTS

Good thinking, Grogg!

Some numbers are easiest to divide one part at a time.

There's only one problem with using donuts to solve division problems.

RANDY'S DONUTS

Now I'm starving for a donut!

Oh yeah? What's that?

THE LAB

divisibility

When one whole number can be divided by another with no remainder...

...we say that the first number is ***divisible*** by the second.

For example, since 430÷10=43, we say that 430 is divisible by 10.

430÷10=43
430
IS DIVISIBLE
BY 10.

What are some other numbers that are divisible by 10?

Twenty!

Fifty!

One hundred!

Every ***multiple*** of 10 is ***divisible*** by 10.

That's right!

Saying that one number is ***divisible*** by another is the same as saying that the first number is a ***multiple*** of the second.

Since 560 is a multiple of 10...

...560 is divisible by 10.

Precisely!

How can you quickly tell whether a number is divisible by 10?

450
923
5,789
19,200

THE STATEMENTS "X IS DIVISIBLE BY Y" AND "X IS MULTIPLE OF Y" MEAN THE SAME THING.

How can you tell?

You can look at the last digit.

Every number that ends in zero is a multiple of ten.

For example, 450 is 45×10, and 19,200 is 1,920×10.

But 923 and 5,789 are not multiples of 10.

450 = 45×10

923 ×

5,789 ×

19,200 = 1,920×10

Any number that ends in zero is divisible by 10.

Very good.

How can you quickly tell if a number is divisible by 100?

You can look at the last ***two*** digits of the number.

1,500

9,001

58,000

192,010

When you multiply a number by 100, you put two zeros at the end.

Every number that ends in two zeros is a multiple of 100.

1,500 = 15×100

9,001 ×

58,000 = 580×100

192,010 ×

Excellent!

How can you quickly tell if a number is divisible by ***five?***

How?

Easy!

Multiples of 5 end in 0 or 5.

5×0=0
5×1=5
5×2=10
5×3=15
5×4=20
5×5=25
5×6=30
5×7=35

Any number that is divisible by 5 ends in 0 or 5.

Is **every** number that ends in 0 or 5 divisible by 5?

Huh?

What if someone told you that 5,393,680 is ***not*** divisible by 5.

How could you convince them that 5,393,680 ***is*** divisible by 5?

5,393,680

Are you sure that 5,393,680 is divisible by 5?

We know that 5,393,680 is a multiple of 10, since it ends in zero.

539,368×10=5,393,680.

5,393,680 = 10×539,368

And since 5×2=10...

...we know that (5×2)×539,368 =5,393,680.

5,393,680 is 5 times (2×539,368).

So, 5,393,680 is definitely a multiple of 5!

Since 2×5=10, every multiple of 10 is a multiple of 5.

So, any number that ends in 0 is divisible by 5.

5,393,680 = 10×539,368
=(5×2)×539,368
=5×(2×539,368)

Very good.

How could you convince me that 76,345 is divisible by 5?

Since 76,340 is a multiple of 5...

...adding 5 more gives us the next multiple of 5, which is 76,345.

So, 76,345 is definitely divisible by 5.

Every number that ends in 5 is five more than a number that ends in 0.

And since every number that ends in 0 is a multiple of 5, every number that ends in 5 is a multiple of 5.

So, every number that ends in 0 or 5 is divisible by 5.

NUMBER	DIVISIBILITY RULE
10	ENDS IN 0
100	ENDS IN 00
5	ENDS IN 0 OR 5

Outstanding!

This is called a ***divisibility rule.***

Are there divisibility rules for other numbers?

Indeed!

For example, there is a quick way to tell whether or not a number is divisible by 25.

Can you tell which of these numbers is divisible by 25?

7,841,955
7,841,965
7,841,975
7,841,985

I'm pretty sure that multiples of 25 always end in 25, 50, 75, or 00.

$25 \times 1 = 25$
$25 \times 2 = 50$
$25 \times 3 = 75$
$25 \times 4 = 100$
$25 \times 5 = 125$
$25 \times 6 = 150$
$25 \times 7 = 175$

So, 7,841,975 is divisible by 25!

7,841,955
7,841,965
7,841,975
7,841,985

How can you be completely sure that 7,841,975 is divisible by 25?

We know that 7,841,900 is a multiple of 100, since it ends in 00.

And since $100 = 25 \times 4$, every multiple of 100 is also a multiple of 25!

$7,841,900 = 100 \times 78,419$
$= (25 \times 4) \times 78,419$
$= 25 \times (4 \times 78,419)$

Since 7,841,900 is a multiple of 25, we can add 25's to 7,841,900 to find the next few multiples of 25.

7,841,900 +25
7,841,925 +25
7,841,950 +25
7,841,975

7,841,975 is a multiple of 25, so it's divisible by 25.

To see if a number is divisible by 25, we only need to look at its last two digits.

If the last two digits of a number are 00, 25, 50, or 75...

...then the number is divisible by 25.

NUMBER	DIVISIBILITY RULE
10	ENDS IN 0
	ENDS IN 00
	ENDS IN 0 OR 5
	ENDS

Very good! Let's add that rule to our list.

Yikes!

Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something much more diabolically difficult!

Determining divisibility rules for 5 and 10 is indubitably obvious.

But discovering the divisibility rule for 4 demands a deft deduction.

One of these five phone numbers can be used to dial your educator directly.

Only finding the number that is divisible by 4 can foil my fiendish folly.

776-2323
111-2222
552-3278
555-2368
867-5309

Dialing any one of the other four phone numbers will precipitate your professor's perilous predicament.

To start, we can write all of the phone numbers like regular numbers.

7,762,323
1,112,222
5,523,278
5,552,368
8,675,309

We need to figure out which one of these numbers is divisible by 4.

Which number is divisible by 4?

I'm pretty sure that multiples of 4 are always even.

4
8
12
16
20
24
28
32

Since 2×2=4, every multiple of 4 is also a multiple of 2!

And every multiple of 2 is even, so every multiple of 4 is even.

We can cross out these two choices, since they're both odd.

~~7,762,323~~
1,112,222
5,523,278
5,552,368
~~8,675,309~~

But how can we tell which of the other three choices is divisible by 4?

We could try using long division.

Uuuuggggh... Do we have to?

$4\overline{)1,112,222}$

Maybe we can figure out a divisibility rule for 4.

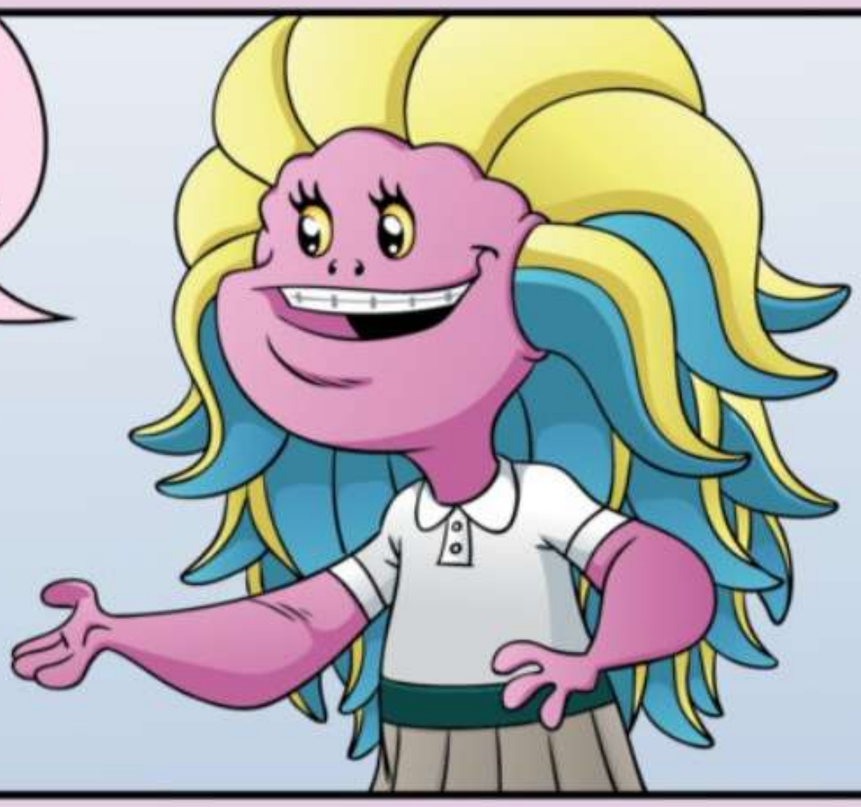
Since 100=4×25, every multiple of 100 is also a multiple of 4!

That means that any number ending in 00 is a multiple of 4.

For example, 1,112,200 is a multiple of 4, so it's divisible by 4.

$1,112,200 = 100 \times 11,122$

$= 4 \times 25 \times 11,122$



Since we know that 1,112,200 is divisible by 4...

...we can count up by 4's to get the next few multiples of 4.

1,112,200 +4
1,112,204 +4
1,112,208 +4
1,112,212 +4
1,112,216 +4
1,112,220 +4
1,112,224

We skip right over 1,112,222.

So, 1,112,222 is ***not*** a multiple of 4.

How about 5,523,278?

~~7,762,323~~
~~1,112,222~~
5,523,278
5,552,368
~~8,675,309~~

5,523,200 +4
5,523,204 +4
5,523,208 +4
5,523,212 +4
5,523,216 +4
5,523,220
5,523,

5,523,200 is divisible by 4.

We can count by 4's to see if 5,523,278 is a multiple of 4.

5,52
5,5
5,52
5,52 3,212 +4
5, 23,216 +4
5 23,220 +4
5 523,224 +4
,523,228 +4
,523,232
36
40

Wait, Alex. You don't need to write the whole numbers.

Just write the last two digits.

Since we start at 00 and add 4's--

The last 2 digits make multiples of 4.

5,523,200 +4
5,523,204 +4
5,523,208 +4
5,523,212 +4
5,523,216 +4
5,523,220 +4
5,523,224 +4
5,523,228 +4
5,523,232
36
40

You'll eventually get up to 5,523,2***80***, but you'll skip right over 5,523,2***78***.

72
76
80

Since 78 isn't a multiple of 4, neither is 5,523,2***78***.

That only leaves 5,552,368... that must be the number.

We should check to be safe.

~~7,762,323~~
~~1,112,222~~
~~5,523,278~~
5,552,368
~~8,675,309~~

We know that 5,552,300 is a multiple of 4.

If we add 15 more 4's, we get 5,552,3**60**, since 15×4=60.

Adding two more 4's gives us 5,552,3**68**.

Since 68 is a multiple of 4, so is 5,552,3**68**!

That means 5,552,368 is divisible by 4.

~~7,762,323~~
~~1,112,222~~
~~5,523,278~~
5,552,368
~~8,675,309~~

Quick, dial the number!

Professor Grok?

Great work, little monsters!

Your phone call triggered the lock to my room!