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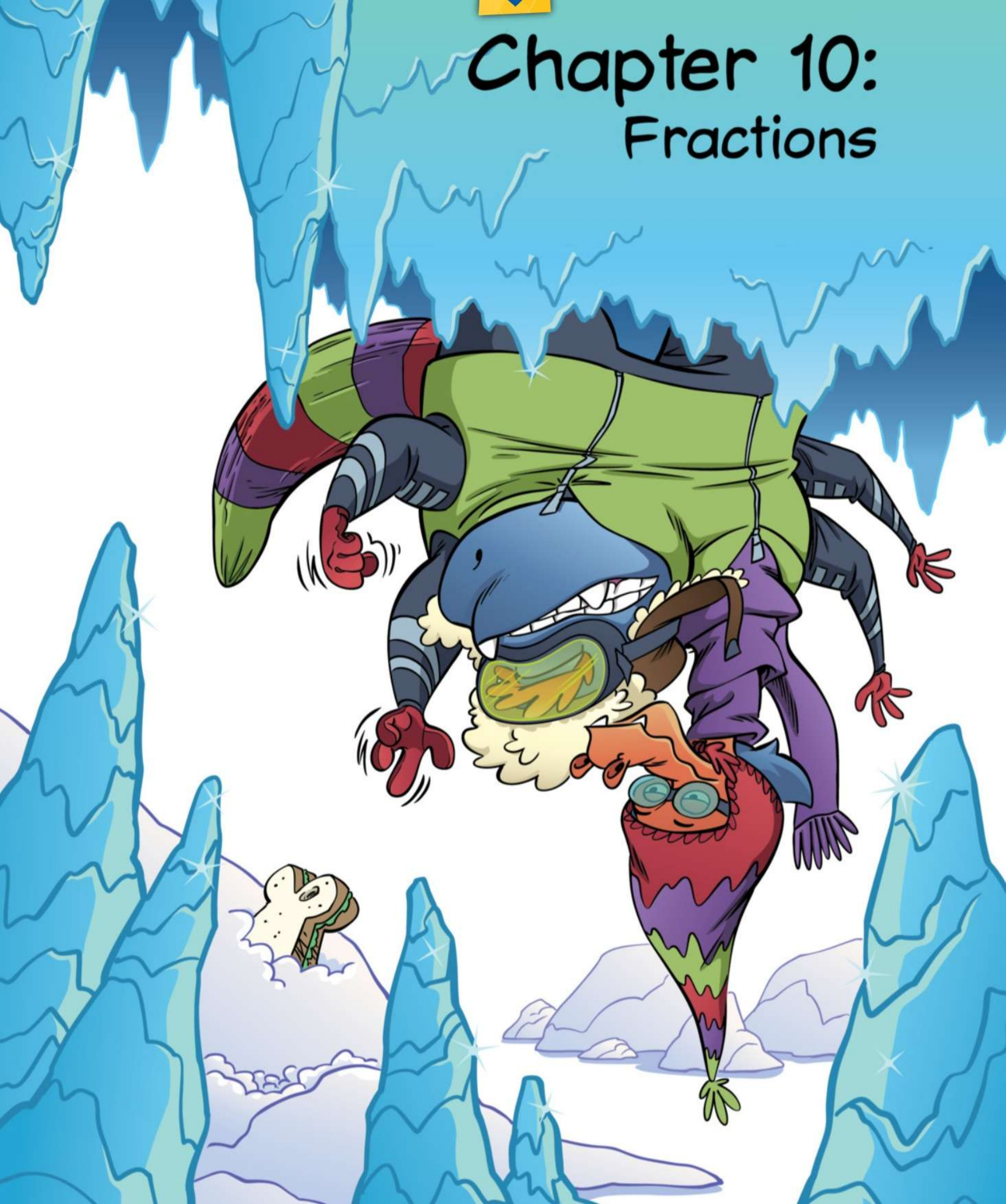
Chapter 10:
Fractions



R&G "Of"

There's no better workout than a 15-kilometer trek on cross-country skis.

15 kilometers!?

Yep. We're going all the way around Wampa Reservoir.

Hope we don't see any wampas!

We're already $\frac{1}{3}$ of the way around!

What's $\frac{1}{3}$ of 15?

If you split 15 into three equal parts, each part is $\frac{1}{3}$ of 15.

So, $\frac{1}{3}$ of 15 is 5?

Yep.

How do we write that in math?

0 5 10 15

To find a fraction of a number, we ***multiply*** the fraction by the number.

$\frac{1}{3}$ ***of*** 15 means $\frac{1}{3}$ ***times*** 15.

$\frac{1}{3}$ of 15 means $\frac{1}{3} \times 15$

Neat.
$\frac{1}{3} \times 15 = 5$.
That's right.
How would you compute $\frac{2}{3} \times 15$?
Since *one* third of 15 is 5...
...*two* thirds of 15 must be twice as much.
So, $\frac{2}{3} \times 15 = 10$.
$\frac{2}{3} \times 15 = 10$
0
5
10
15
Try this one...
...what's $\frac{3}{7}$ of 28?
That's right.
Try it.

Hmmm... First I'll find $\frac{1}{7}$ of 28.
I can split the number line between 0 and 28 into seven equal parts.
Each part is 4 units long.
So, $\frac{1}{7}$ of 28 is 4.
0 4 8 12 16 20 24 28
And if $\frac{1}{7}$ of 28 is 4...
...then $\frac{2}{7}$ of 28 is 8...
...and $\frac{3}{7}$ of 28 is 12!
0 4 8 12 16 20 24 28
You got it.
That's *two* things I've learned today.
What else have you learned?
Skis are really meant for going *down*hill.

Ms. Q.

Multiplying Fractions

What do you get when you multiply $6 \times \frac{1}{7}$?

I get...

...confused.

$6 \times \frac{1}{7}$

Multiplication can be used as a shortcut for repeated addition.

$6 \times \frac{1}{7}$

To solve $6 \times \frac{1}{7}$, we can add six copies of $\frac{1}{7}$.

$6 \times \frac{1}{7}$

$= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$

$\frac{1}{7} + \frac{1}{7} = \frac{2}{7}$. And $\frac{2}{7} + \frac{1}{7} = \frac{3}{7}$.

Then, we add another seventh to get $\frac{4}{7}$--

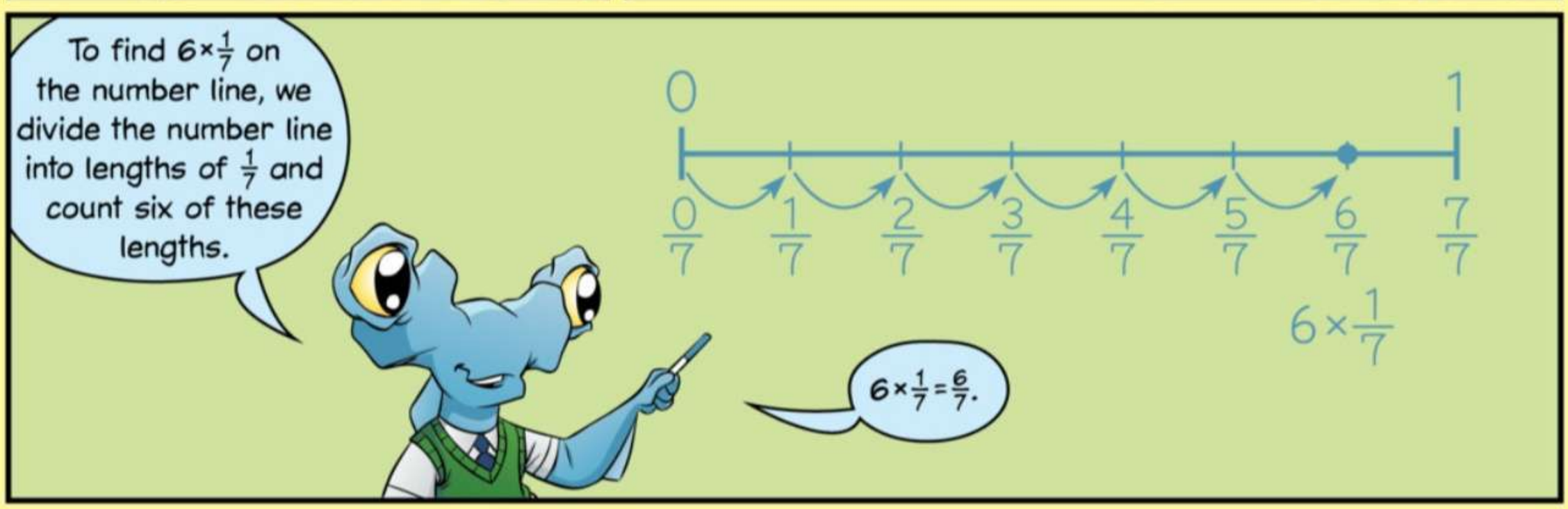
$6 \times \frac{1}{7}$

$= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$

Grogg! Adding ***6 sevenths*** gives us $\frac{6}{7}$!

Oh, right. That makes sense.

6 times $\frac{1}{7}$ is 6 sevenths.
$6 \times \frac{1}{7}$
$= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$
$= \frac{6}{7}$


To find $6 \times \frac{1}{7}$ on the number line, we divide the number line into lengths of $\frac{1}{7}$ and count six of these lengths.
0
1
$\frac{0}{7}$ $\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$ $\frac{7}{7}$
$6 \times \frac{1}{7}$
$6 \times \frac{1}{7} = \frac{6}{7}$.

Very good.
Let's try multiplying by a fraction whose numerator is not 1.
Who would like to try this one?
$5 \times \frac{7}{8}$
Try it.

To multiply $5\times\frac{7}{8}$, we can add five copies of $\frac{7}{8}$.

$$5 \times \frac{7}{8} = \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$$

The numerator is 7+7+7+7+7, and the denominator is 8.

$$= \frac{7+7+7+7+7}{8}$$

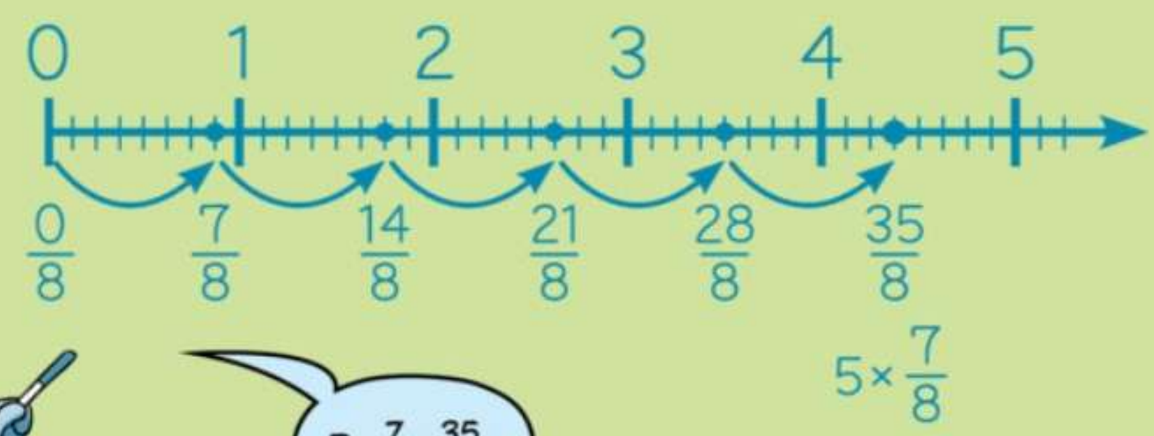
$$= \frac{5\times7}{8}$$

$$= \frac{35}{8}$$

Adding five 7's in the numerator gives us 5×7=35.

So, $5\times\frac{7}{8}=\frac{35}{8}$!

To find $5\times\frac{7}{8}$ on the number line, we divide the number line into eighths and count five lengths of $\frac{7}{8}$.



$5\times\frac{7}{8}=\frac{35}{8}$.

So, to compute $5\times\frac{7}{8}$, we multiply 5×7 and put the result over 8!

$5\times\frac{7}{8}=\frac{5\times7}{8}$.

$$5 \times \frac{7}{8}$$

$$= \frac{5\times7}{8}$$

$$= \frac{35}{8}$$

That's right.

Who can write an expression that is equal to $a\times\frac{b}{c}$?

$$a \times \frac{b}{c}$$

Try it.

When we multiply an integer by a fraction, we multiply the integer by the numerator and put the result over the denominator.

So, $a \times \frac{b}{c}$ equals $\frac{a \times b}{c}$.

$$a \times \frac{b}{c}$$

$$= \frac{a \times b}{c}$$

THIS IS TRUE EXCEPT WHEN C=0, SINCE DIVISION BY ZERO IS UNDEFINED.

Very good.

How could you rewrite $\frac{b}{c} \times a$?

$$\frac{b}{c} \times a$$

Since multiplication is commutative, $\frac{b}{c} \times a = a \times \frac{b}{c}$.

$$\frac{b}{c} \times a = a \times \frac{b}{c} = \frac{a \times b}{c} = \frac{b \times a}{c}$$

And we just learned that $a \times \frac{b}{c} = \frac{a \times b}{c}$.

And if we want, we can switch the b and the a.

All four of these expressions are equal!

Great work!

Try a few more.

How would you compute each of these products?

$$3 \times \frac{5}{4}$$

$$\frac{3}{20} \times 6$$

$$6 \times \frac{7}{8}$$

Try all three. Simplify your answers.

$3\times\frac{5}{4}=\frac{3\times5}{4}$...
...$\frac{15}{4}$!
As a mixed number, we get $3\frac{3}{4}$.
$3\times\frac{5}{4}$
$=\frac{3\times5}{4}$
$=\frac{15}{4}$
$=3\frac{3}{4}$
$\frac{3}{20}\times6=\frac{3\times6}{20}$.
We multiply to get $\frac{18}{20}$, then simplify to get $\frac{9}{10}$.
$\frac{3}{20}\times6$
$=\frac{3\times6}{20}$
$=\frac{18}{20}$
$=\frac{9}{10}$
$6\times\frac{7}{8}=\frac{6\times7}{8}$.
6×7 is 42.
So, $6\times\frac{7}{8}=\frac{42}{8}$.
$6\times\frac{7}{8}$
$=\frac{6\times7}{8}$
$=\frac{42}{8}$
$=5\frac{2}{8}$
$=5\frac{1}{4}$
We can convert $\frac{42}{8}$ into a mixed number: $5\frac{2}{8}$.
And $5\frac{2}{8}$ simplifies to $5\frac{1}{4}$.
See, Grogg. That wasn't so confusing, was it?
Nope.
Besides, I kind of like feeling a little discombobulated.
Huh?
Whu?
Dis com-what??
See! Feels good, doesn't it?

MATH TEAM

Multiplying by a Fraction

What number is $\frac{5}{8}$ of 3?

$\frac{5}{8}$ of 3

I know that $\frac{5}{8}$ is a little more than $\frac{1}{2}$.

So, $\frac{5}{8}$ of 3 is a little more than $\frac{1}{2}$ of 3.

$\frac{1}{2}$ of 3 is $1\frac{1}{2}$, so $\frac{5}{8}$ of 3 is a little more than $1\frac{1}{2}$.

Good estimating! Does anyone know how we could get an exact answer?

ESTIMATING THE ANSWER IS OFTEN A GREAT WAY TO BEGIN A PROBLEM. YOU CAN ALSO USE YOUR ESTIMATE TO CHECK YOUR ANSWER WHEN YOU ARE DONE.

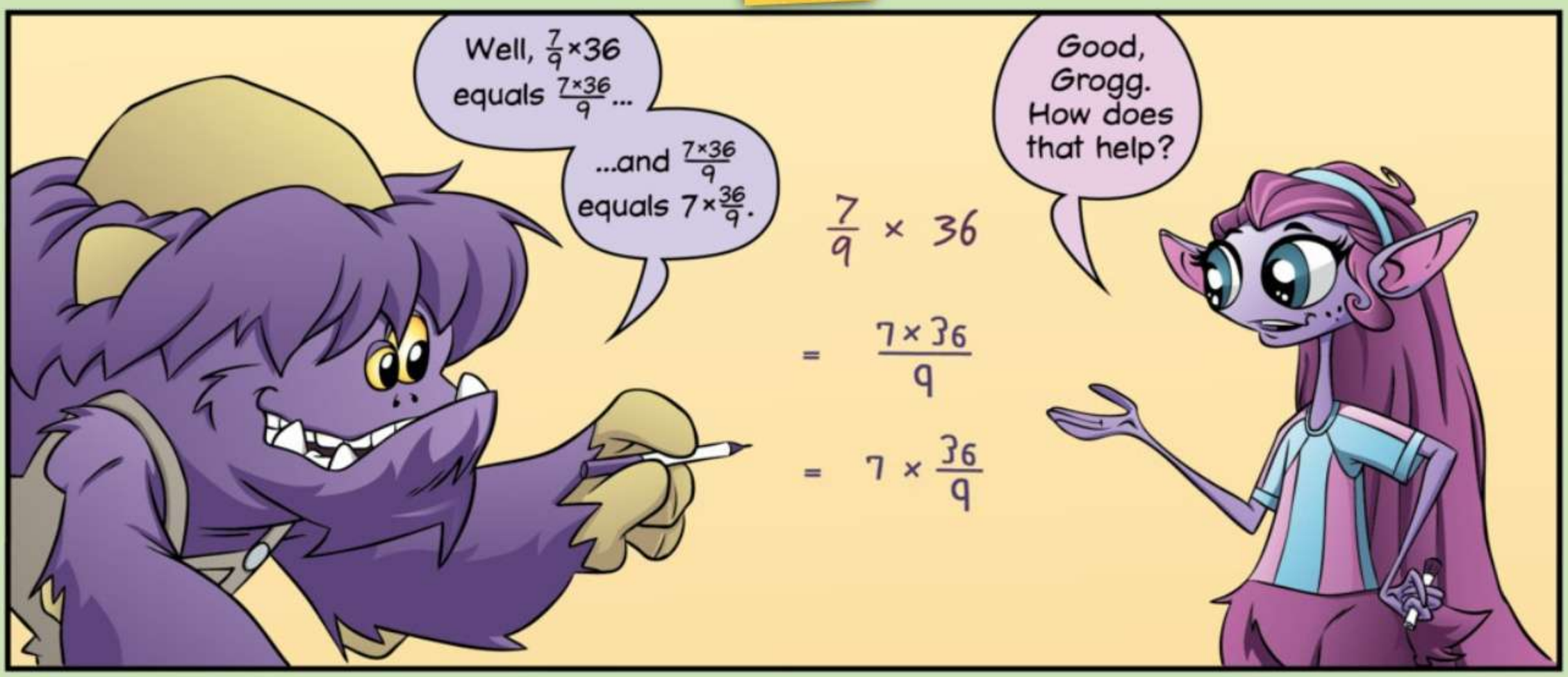
To find $\frac{5}{8}$ of 3, we multiply $\frac{5}{8} \times 3$.

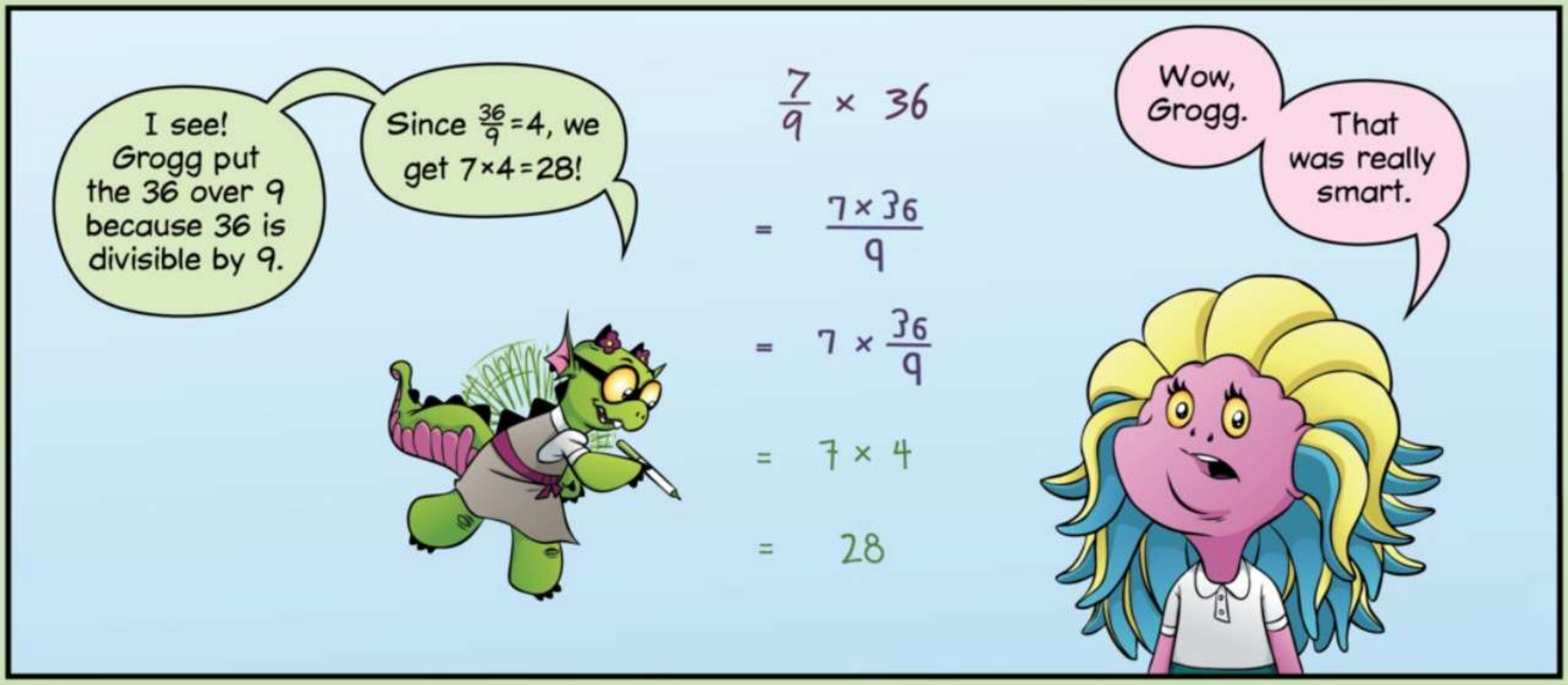
$$\frac{5}{8} \text{ of } 3$$
$$= \frac{5}{8} \times 3$$
$$= \frac{5 \times 3}{8}$$
$$= \frac{15}{8}$$
$$= 1\frac{7}{8}$$

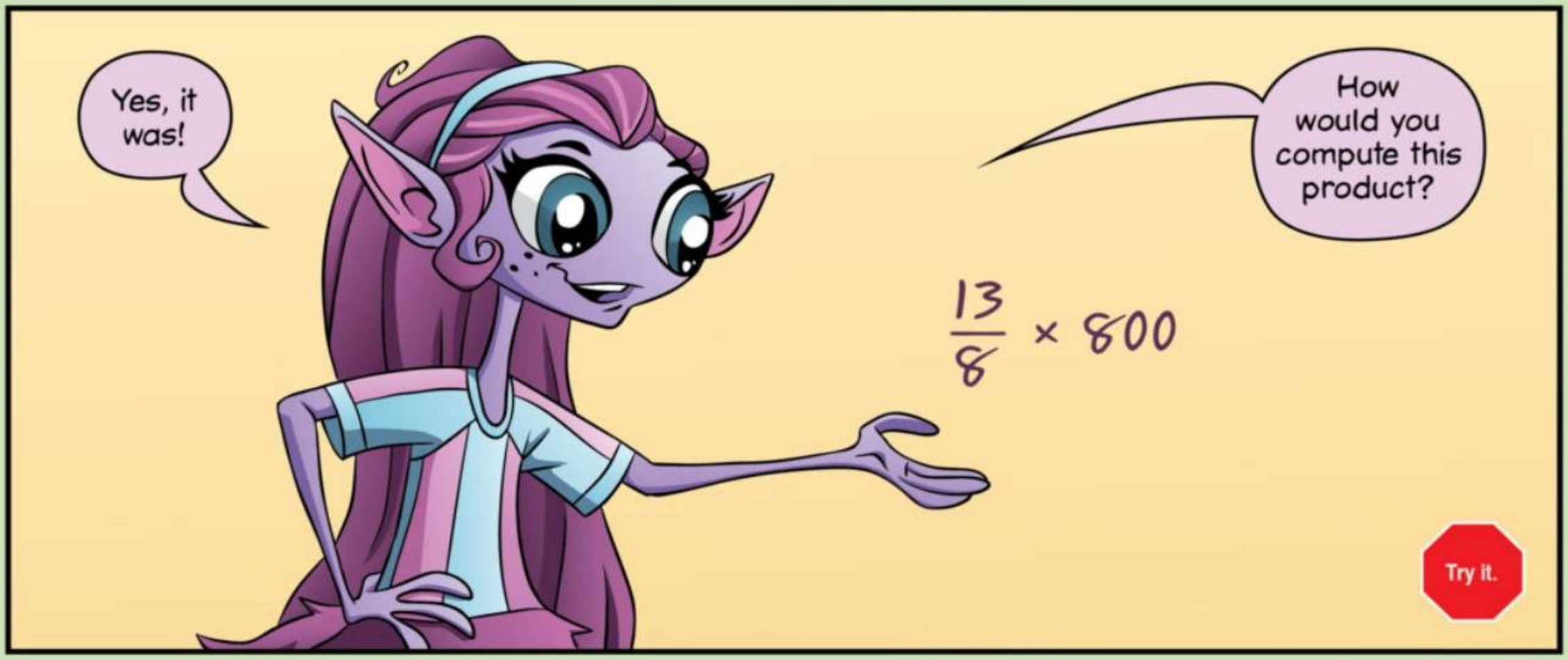
$\frac{5}{8} \times 3$ is $\frac{15}{8}$. We can write $\frac{15}{8}$ as a mixed number: $1\frac{7}{8}$.

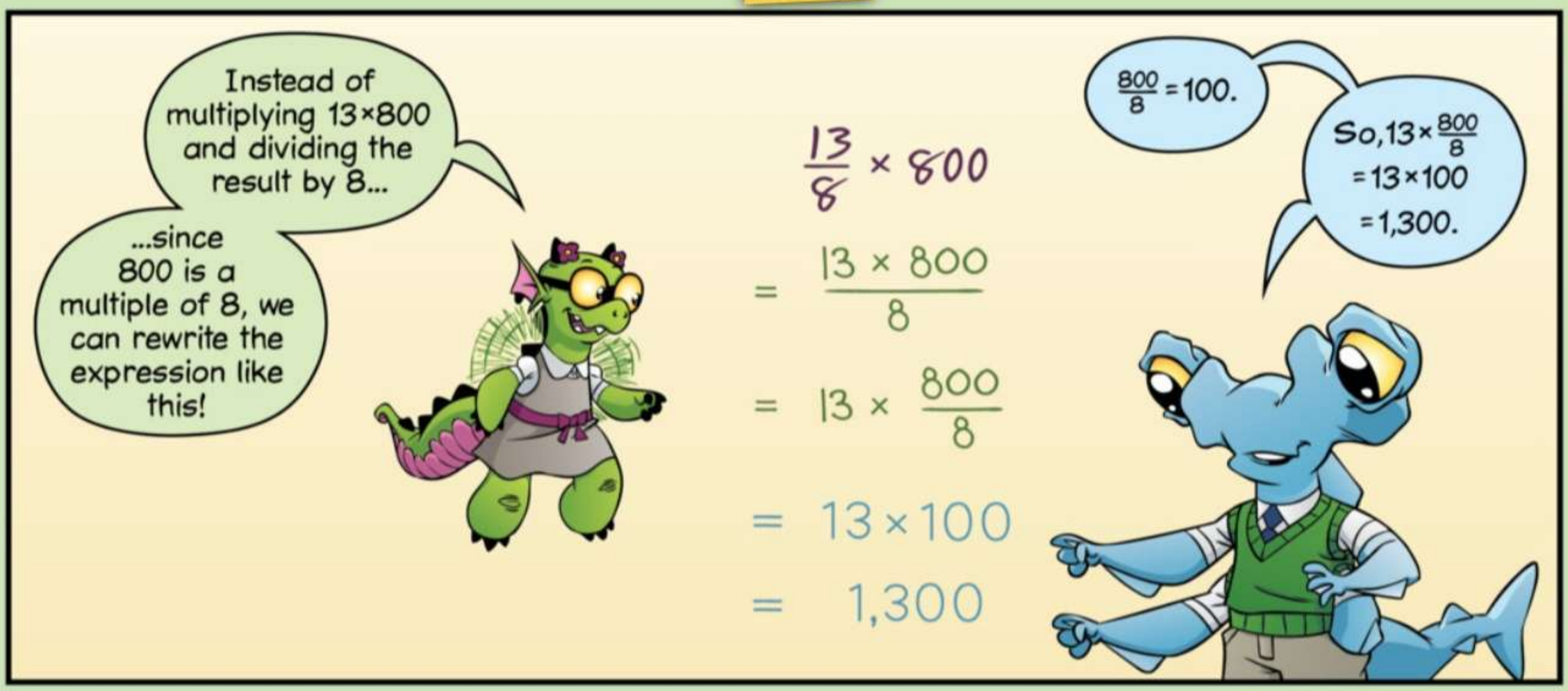
That fits our estimate.

Great job! We find a fraction of a number by multiplying.
Try this one next...
...what is $\frac{7}{9}\times 36$?
$\frac{7}{9}\times 36$
$\frac{7}{9}\times 36 = \frac{7\times 36}{9}$.
7×36 is... 252.
So, $\frac{7}{9}\times 36 = \frac{252}{9}$.
$\frac{7}{9}\times 36$
$= \frac{7\times 36}{9}$
$= \frac{252}{9}$
=
Since 2+5+2=9, we know 252 is divisible by 9.
$\frac{252}{9}$ equals 252÷9=... Ummm...
IF THE SUM OF A NUMBER'S DIGITS IS DIVISIBLE BY 9, THE NUMBER ITSELF IS DIVISIBLE BY 9. SEE BEAST ACADEMY 4C FOR AN EXPLANATION OF WHY THIS IS TRUE.
Wait!
Grogg! We were almost done!
$\frac{7}{9}\times 36$
But I think I see an easier way to compute $\frac{7}{9}\times 36$.
How, Grogg?
Can you find an easier way?


Well, $\frac{7}{9}\times 36$ equals $\frac{7\times 36}{9}$...
...and $\frac{7\times 36}{9}$ equals $7\times\frac{36}{9}$.
$\frac{7}{9}\times 36$
$=\frac{7\times 36}{9}$
$=7\times\frac{36}{9}$
Good, Grogg. How does that help?


I see! Grogg put the 36 over 9 because 36 is divisible by 9.
Since $\frac{36}{9}=4$, we get $7\times 4=28$!
$\frac{7}{9}\times 36$
$=\frac{7\times 36}{9}$
$=7\times\frac{36}{9}$
$=7\times 4$
$=28$
Wow, Grogg.
That was really smart.

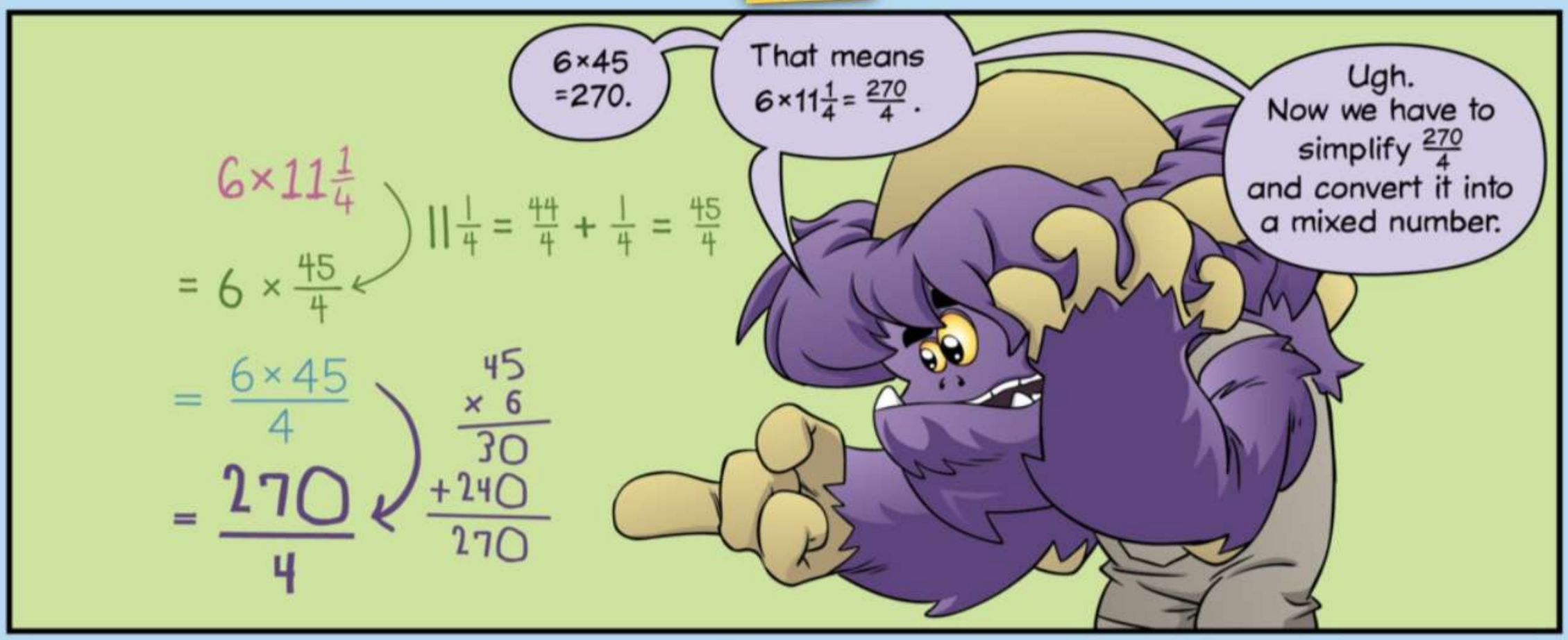

Yes, it was!
How would you compute this product?
$\frac{13}{8}\times 800$
Try it.

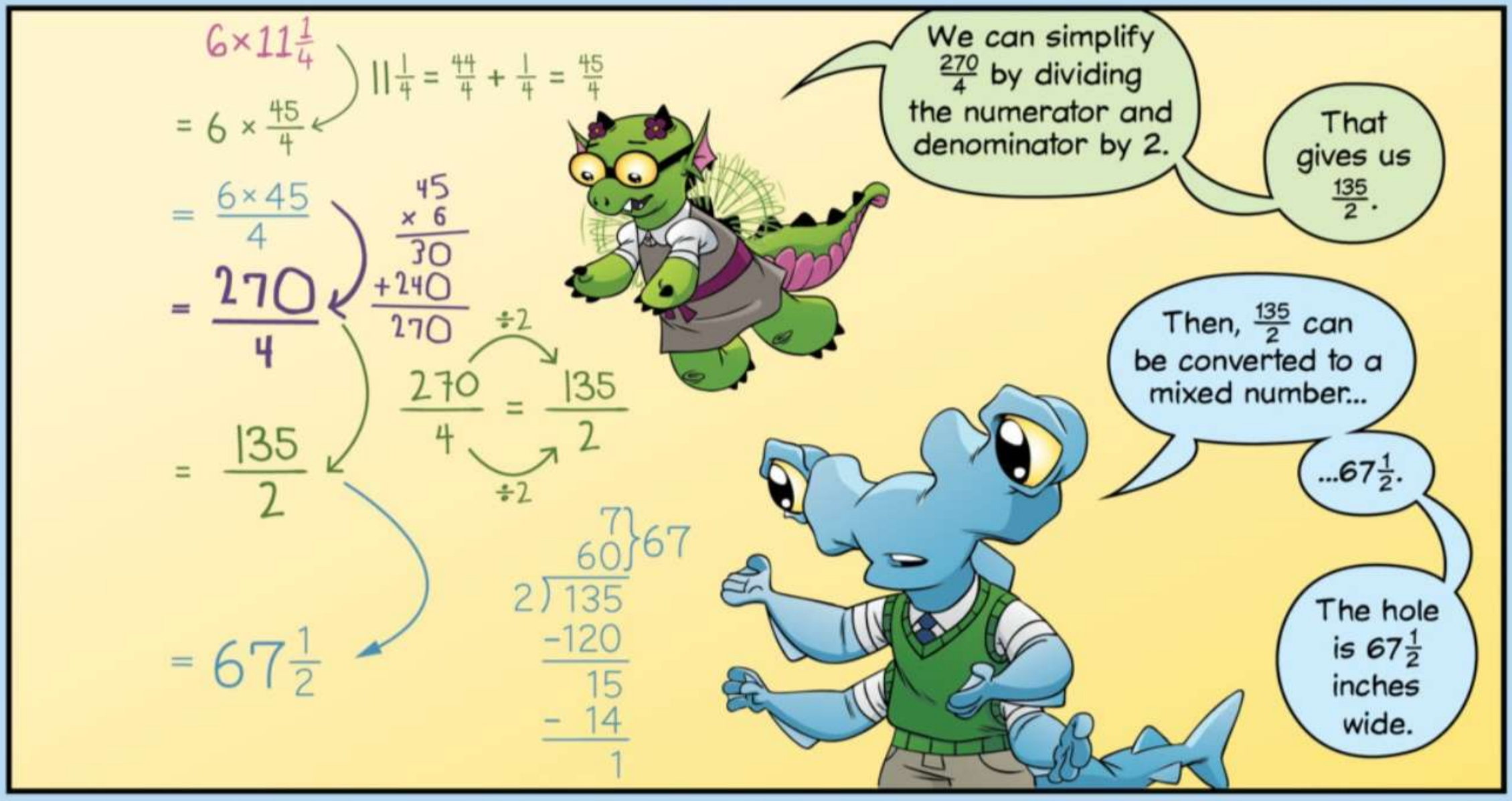

Instead of multiplying 13×800 and dividing the result by 8...
...since 800 is a multiple of 8, we can rewrite the expression like this!
$\frac{13}{8} \times 800$
$= \frac{13 \times 800}{8}$
$= 13 \times \frac{800}{8}$
$= 13 \times 100$
$= 1{,}300$
$\frac{800}{8} = 100.$
So, $13 \times \frac{800}{8}$ $= 13 \times 100$ $= 1{,}300.$

These sorts of strategies not only speed up your computations...
...they also make it easier to avoid errors.
"Avoiding Errors" is my middle name.
That's *two* middle names, Grogg.

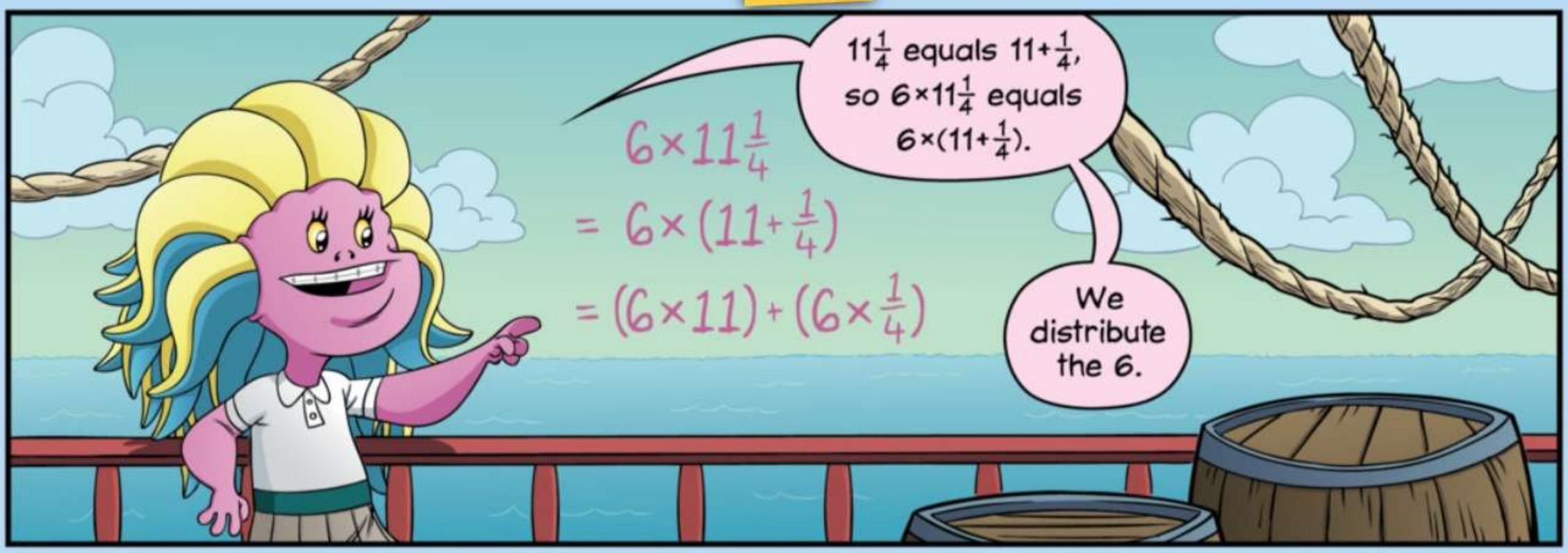
Last week, you said your middle name was "Jalapeño"...
...right before you swallowed three jalapeños.
That was an error you definitely should have avoided.
My nostrils still burn when I burp!

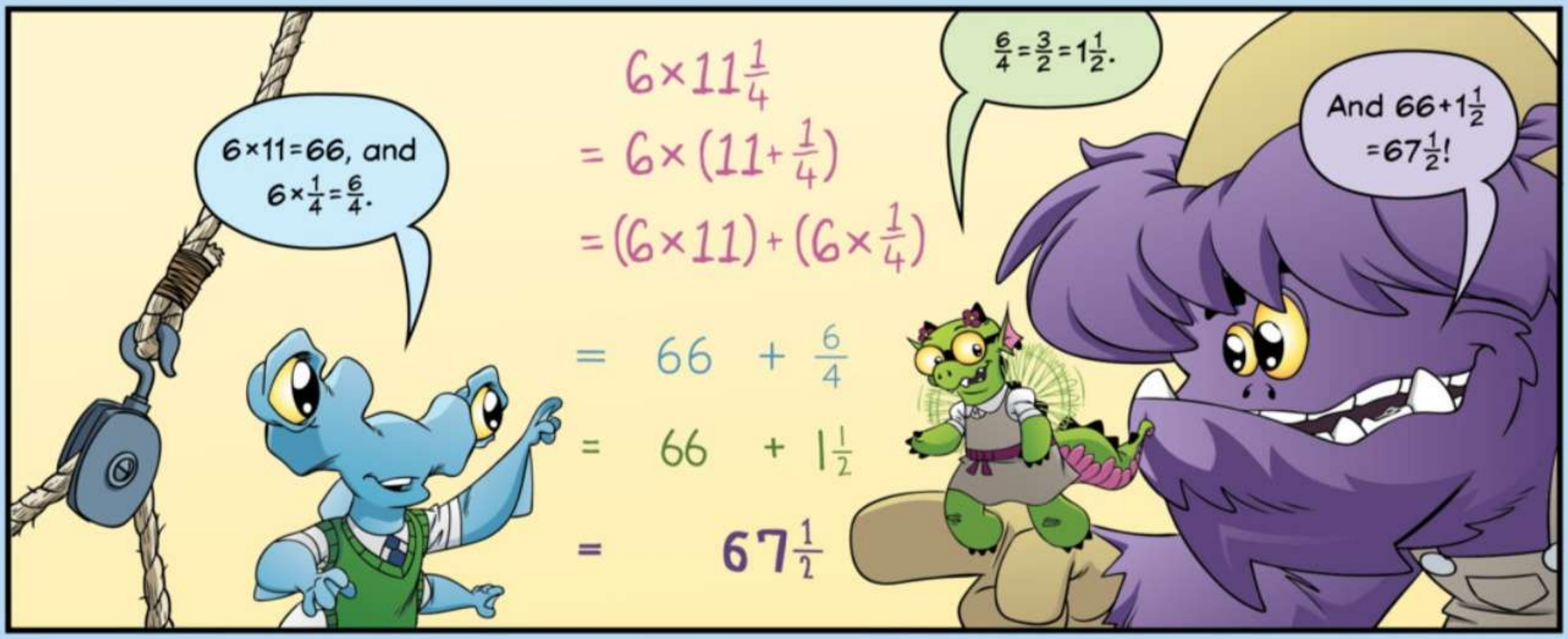
WoodShop
MIXED NUMBERS
What are we doing on your ship, Captain Kraken?
Today, we'll be replacin' some o' the deck planks.
Whoa! That's a big hole.
How wide is it?
Arrr... I've forgotten me measurin' tape.
We can compute the width o' the hole usin' the width o' the broken planks.
Each o' these 6 planks be $11\frac{1}{4}$ inches wide.
6 planks
$6 \times 11\frac{1}{4}$
$11\frac{1}{4}$ in.
To find the width of the hole in inches, we can multiply $6 \times 11\frac{1}{4}$.
To multiply $6 \times 11\frac{1}{4}$, we can convert $11\frac{1}{4}$ into a fraction.
$11\frac{1}{4} = \frac{45}{4}$.
$6 \times 11\frac{1}{4}$
$= 6 \times \frac{45}{4}$
$= \frac{6 \times 45}{4}$
$11\frac{1}{4} = \frac{44}{4} + \frac{1}{4} = \frac{45}{4}$
Then, we multiply 6×45 and put the result over 4.
Finish the computation.





Can you find an easier way to compute the product?


$11\frac{1}{4}$ equals $11+\frac{1}{4}$, so $6\times11\frac{1}{4}$ equals $6\times(11+\frac{1}{4})$.
$6\times11\frac{1}{4}$
$=6\times(11+\frac{1}{4})$
$=(6\times11)+(6\times\frac{1}{4})$
We distribute the 6.


$6\times11=66$, and $6\times\frac{1}{4}=\frac{6}{4}$.
$6\times11\frac{1}{4}$
$=6\times(11+\frac{1}{4})$
$=(6\times11)+(6\times\frac{1}{4})$
$=66+\frac{6}{4}$
$=66+1\frac{1}{2}$
$=67\frac{1}{2}$
$\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$.
And $66+1\frac{1}{2}=67\frac{1}{2}$!

Aye, the total width o' the hole be $67\frac{1}{2}$ inches.
There be a second hole belowdecks in need o' repair.
Let's go have a look.

These planks be $4\frac{3}{8}$-inches wide, and there be 14 broken planks.

What be the width o' the hole?

We can multiply the width of each plank by the number of planks...

...$4\frac{3}{8} \times 14$.

$4\frac{3}{8} \times 14$ is between $4 \times 14 = 56$ and $5 \times 14 = 70$.

We can write $4\frac{3}{8}$ as $4 + \frac{3}{8}$, then distribute the 14.

$4\frac{3}{8} \times 14$
$= (4 + \frac{3}{8}) \times 14$
$= (4 \times 14) + (\frac{3}{8} \times 14)$
$= 56 + \frac{42}{8}$

4×14 is 56, and $\frac{3}{8} \times 14$ is $\frac{42}{8}$.

And $\frac{42}{8} = 5\frac{2}{8}$, which simplifies to $5\frac{1}{4}$.

$56 + 5\frac{1}{4} = 61\frac{1}{4}$.

$4\frac{3}{8} \times 14$
$= (4 + \frac{3}{8}) \times 14$
$= (4 \times 14) + (\frac{3}{8} \times 14)$
$= 56 + \frac{42}{8}$
$= 56 + 5\frac{1}{4}$
$= 61\frac{1}{4}$

Aye, the second hole be $61\frac{1}{4}$ inches wide.

What made these giant holes, Captain Kraken?

Let's go down to the hull 'n' I'll show you.

Rocky took a tumble from the crow's nest 'n' fell through both decks on our last voyage.
Lucky for us, he didn't make it all the way through.

Sadly, we've not found a good way to get him out.
How long has he been stuck down here?

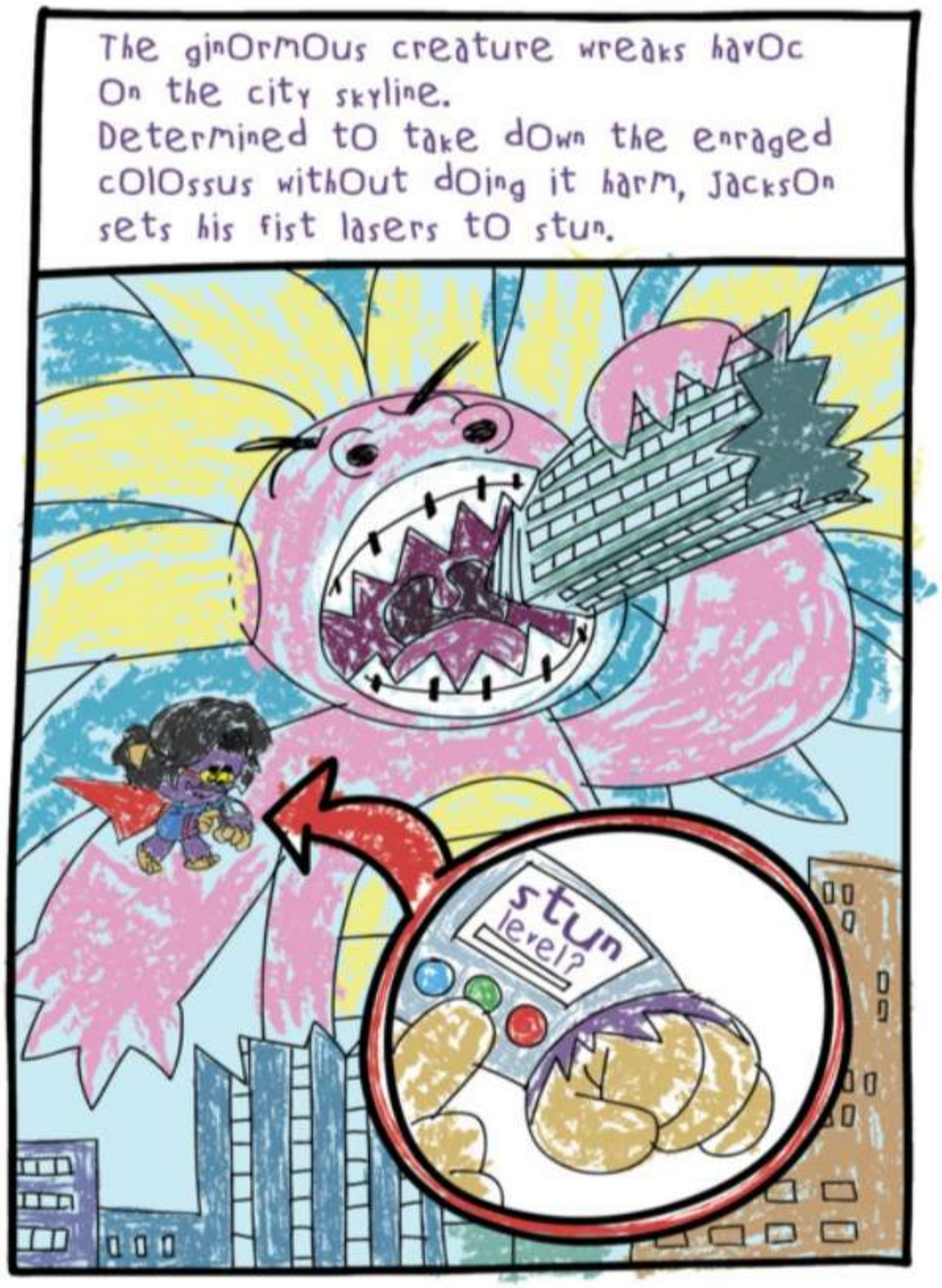
Too long, lad.
Far too long.

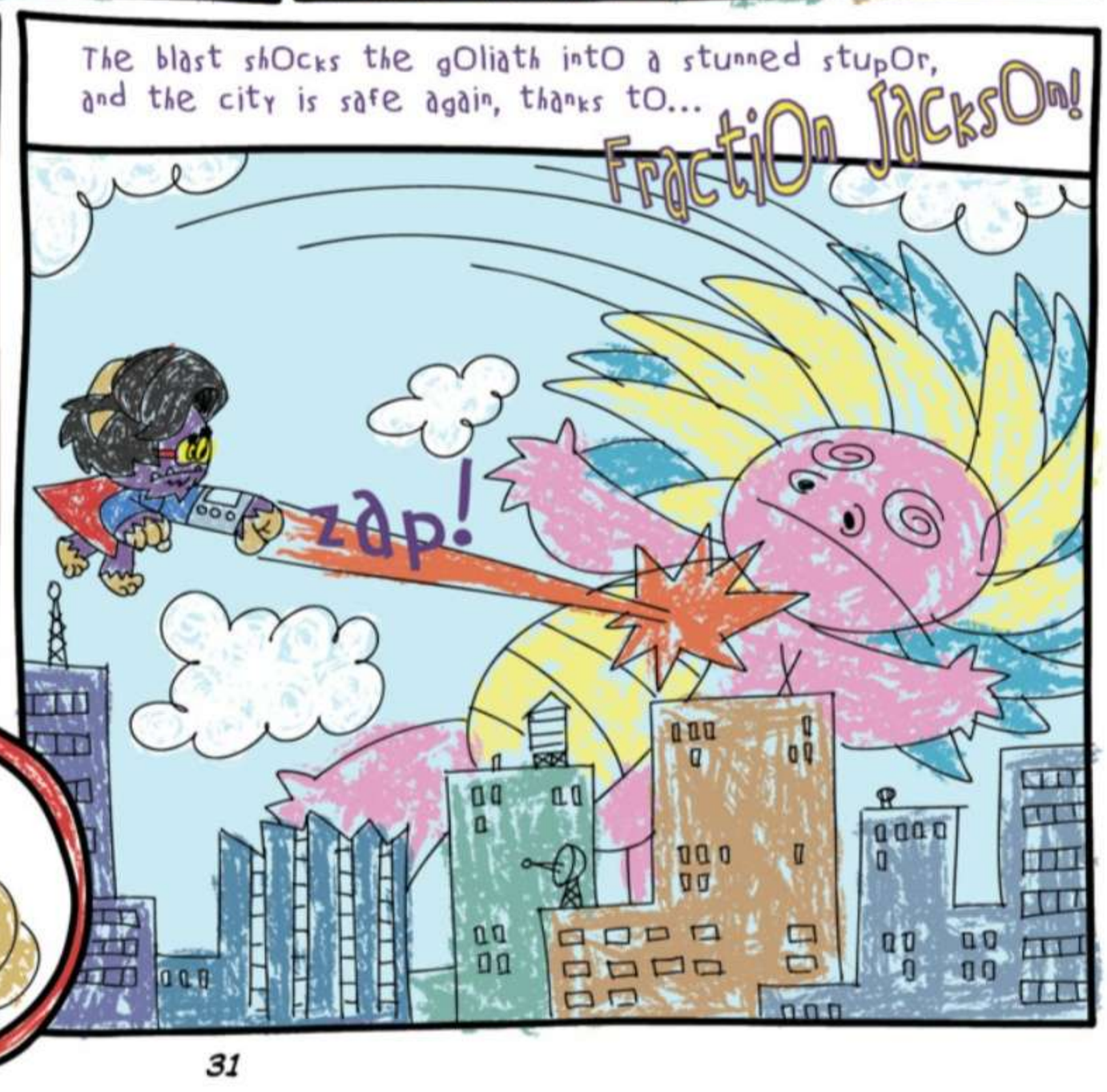
name: GrOgg

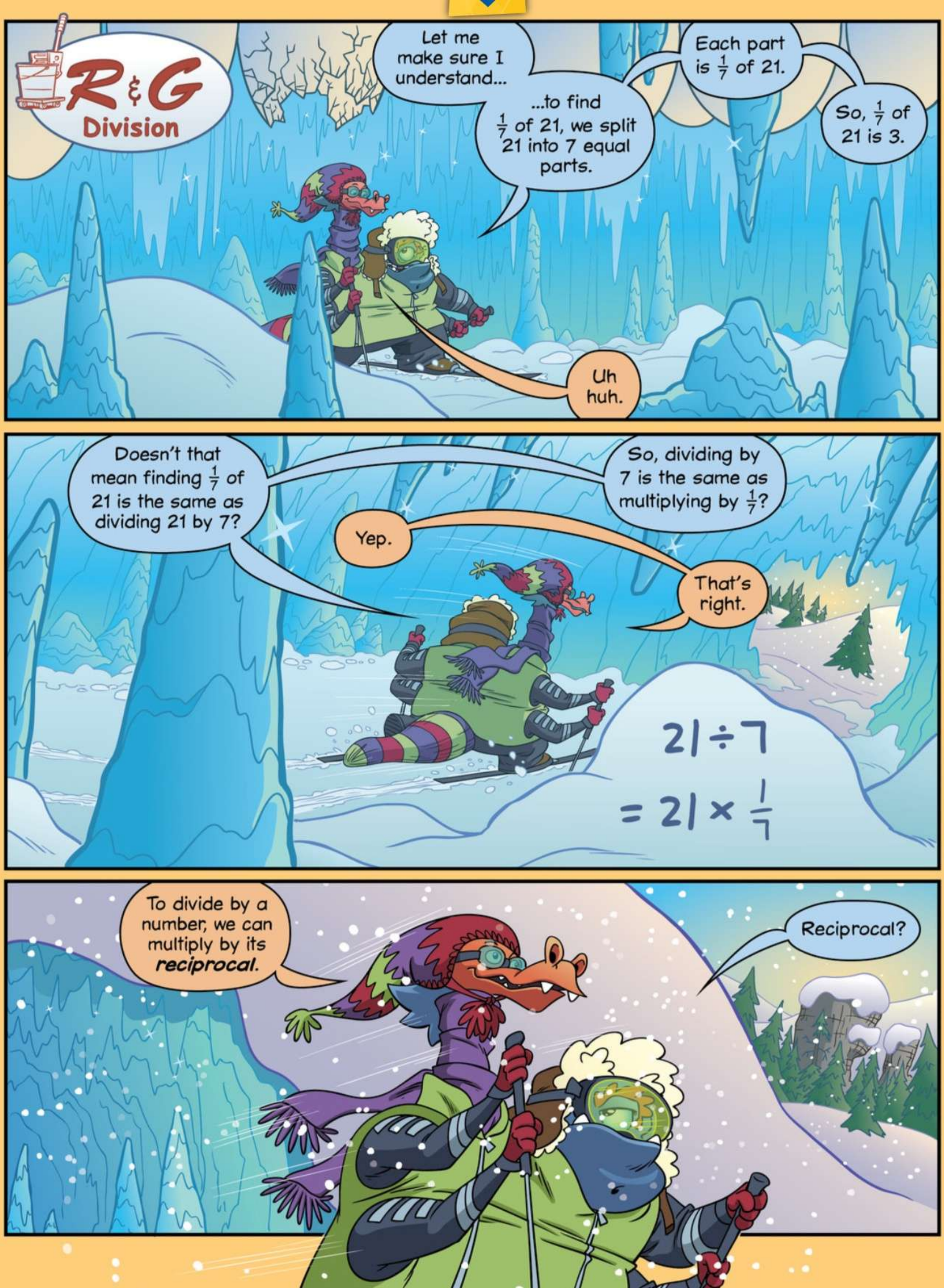
Homework: Write and solve a story problem involving fractions in the space below.

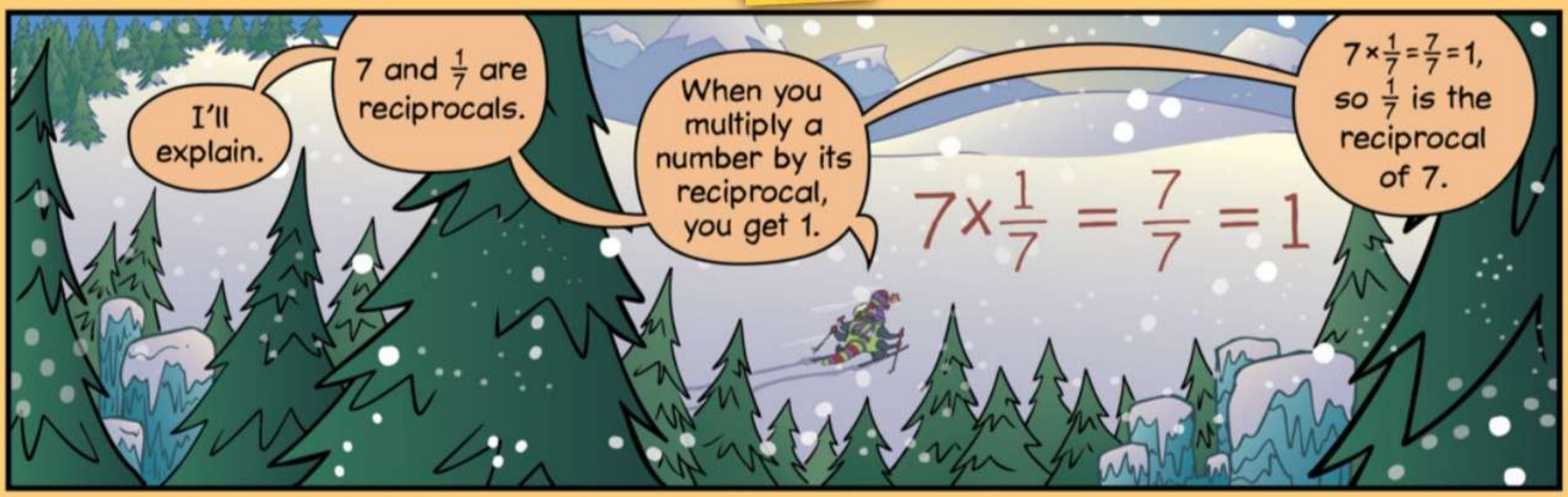
FractiOn JacksOn

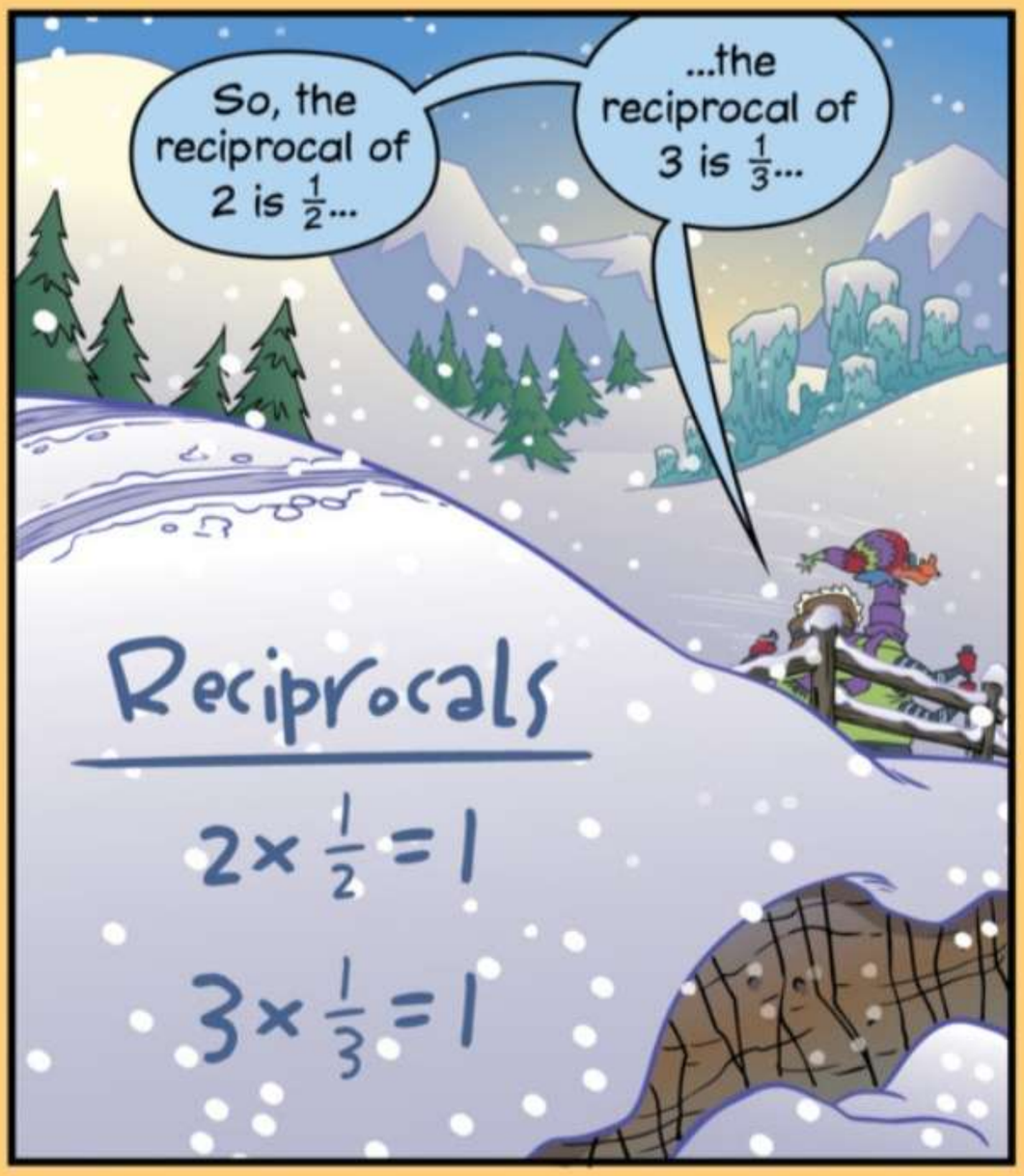
½ MOnster, ½ amazing!

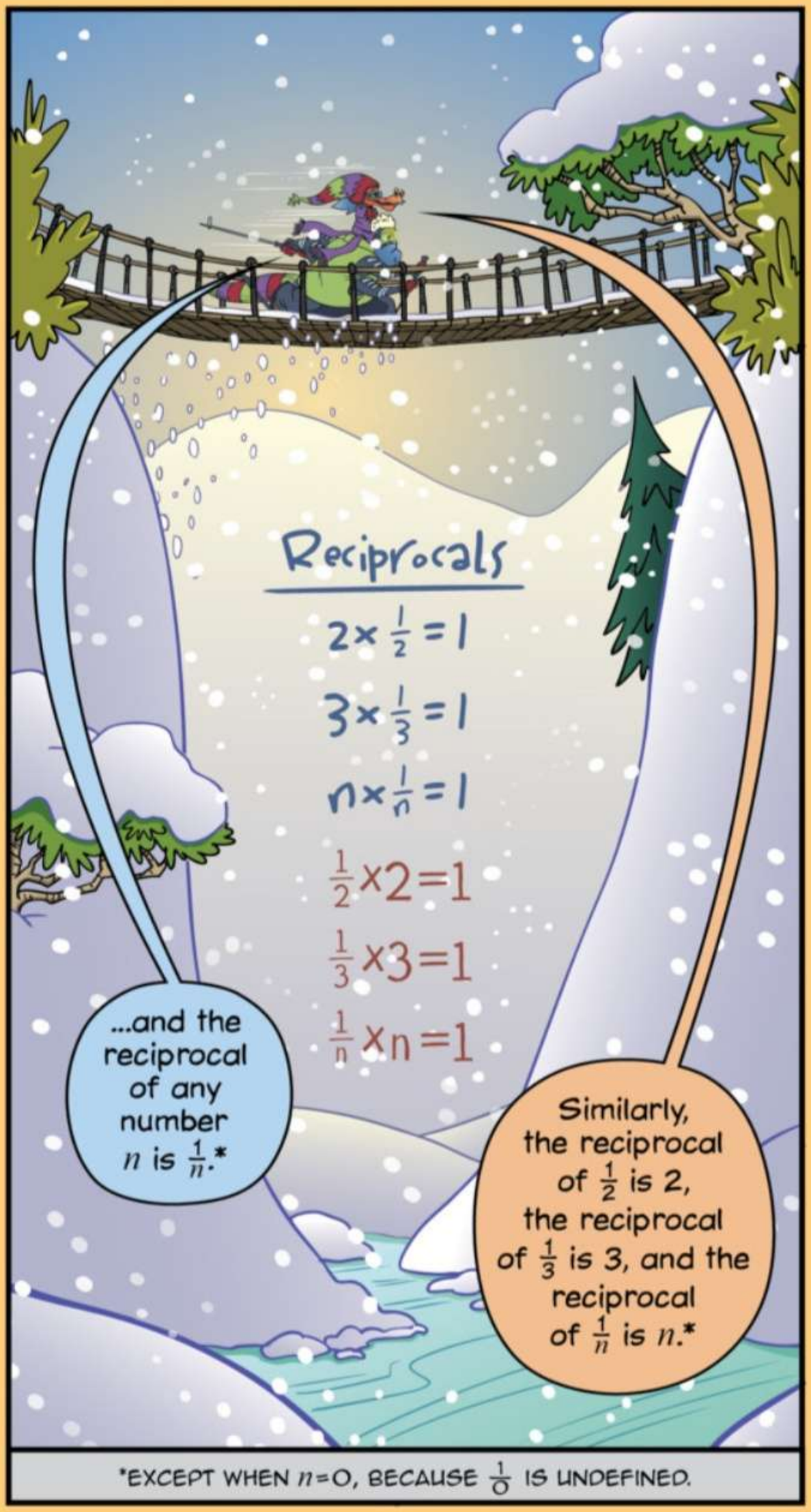




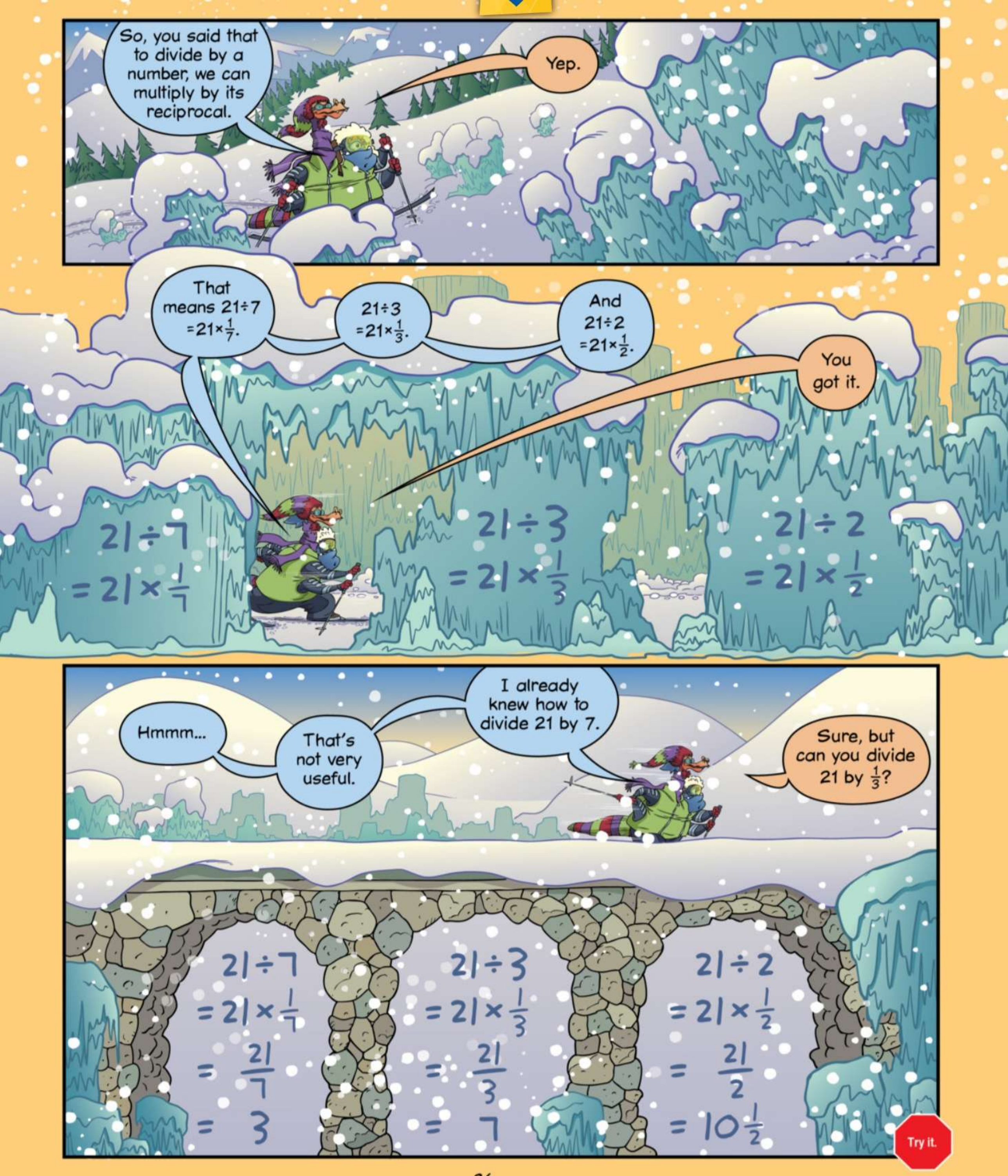

R&G
Division
Let me make sure I understand...
...to find $\frac{1}{7}$ of 21, we split 21 into 7 equal parts.
Each part is $\frac{1}{7}$ of 21.
So, $\frac{1}{7}$ of 21 is 3.
Uh huh.
Doesn't that mean finding $\frac{1}{7}$ of 21 is the same as dividing 21 by 7?
Yep.
So, dividing by 7 is the same as multiplying by $\frac{1}{7}$?
That's right.
$21 \div 7$
$= 21 \times \frac{1}{7}$
To divide by a number, we can multiply by its ***reciprocal***.
Reciprocal?

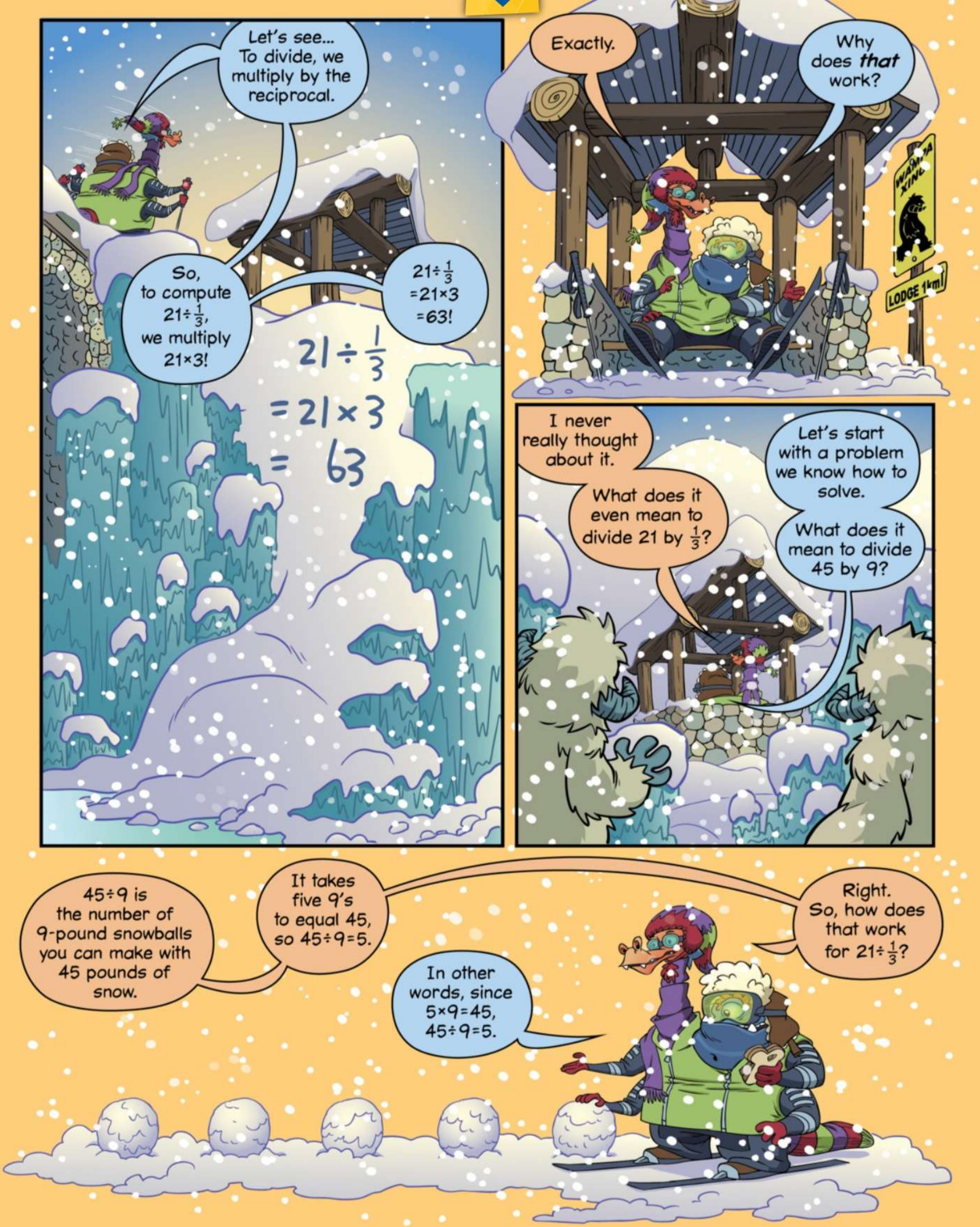






*EXCEPT WHEN $n=0$, BECAUSE $\frac{1}{0}$ IS UNDEFINED.


So, you said that to divide by a number, we can multiply by its reciprocal.
Yep.
That means 21÷7 =21×1/7.
21÷3 =21×1/3.
And 21÷2 =21×1/2.
You got it.
21÷7 =21×1/7
21÷3 =21×1/3
21÷2 =21×1/2
Hmmm...
That's not very useful.
I already knew how to divide 21 by 7.
Sure, but can you divide 21 by 1/3?
21÷7 =21×1/7 = 21/7 = 3
21÷3 =21×1/3 = 21/3 = 7
21÷2 =21×1/2 = 21/2 = 10 1/2
Try it.


Let's see... To divide, we multiply by the reciprocal.
So, to compute $21 \div \frac{1}{3}$, we multiply 21×3!
$21 \div \frac{1}{3} = 21 \times 3 = 63$!
$21 \div \frac{1}{3} = 21 \times 3 = 63$
Exactly.
Why does *that* work?
LODGE 1km
I never really thought about it.
What does it even mean to divide 21 by $\frac{1}{3}$?
Let's start with a problem we know how to solve.
What does it mean to divide 45 by 9?
45÷9 is the number of 9-pound snowballs you can make with 45 pounds of snow.
It takes five 9's to equal 45, so 45÷9=5.
In other words, since 5×9=45, 45÷9=5.
Right. So, how does that work for $21 \div \frac{1}{3}$?

To divide 21 by $\frac{1}{3}$, we can find out how many $\frac{1}{3}$-pound snowballs we can make with 21 pounds of snow.

Since we can make three $\frac{1}{3}$-pound snowballs with 1 pound of snow...

...we can make $21 \times 3 = 63$ $\frac{1}{3}$-pound snowballs with 21 pounds of snow!

So, $21 \div \frac{1}{3}$ is $21 \times 3 = 63$.

Let's get going...

...there's only one kilometer left to ski.

WAMPA XING

LODGE 1km

All that math kept my mind off of how tired I am.

Ask me a few more questions to distract me from the pain.

Okay, what's $7 \div \frac{1}{5}$?

$7 \div \frac{1}{5}$ $= 7 \times 5$ $= 35$.

WAMPA XING

GE 1km

What's $4 \div \frac{1}{11}$?

Try it.

$4 \div \frac{1}{11}$
$= 4 \times 11$
$= 44.$
What's $6 \div \frac{1}{2}$?

$6 \div \frac{1}{2}$
$= 6 \times 2$
$= 12.$
What's *that?*
Huh?

Oh, *that!*
That's a *wampa!*

AAAAAA'HH!!!!

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