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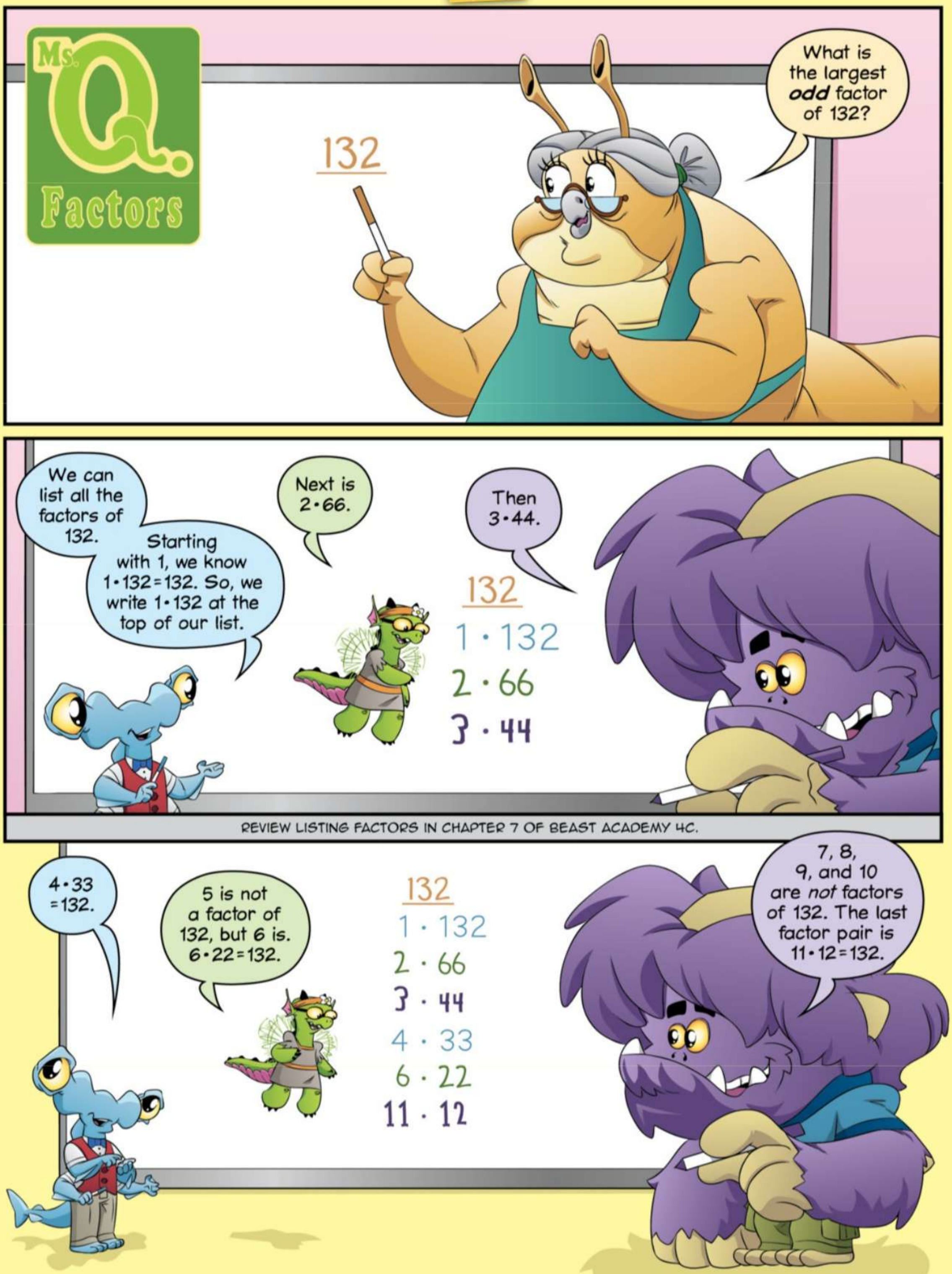
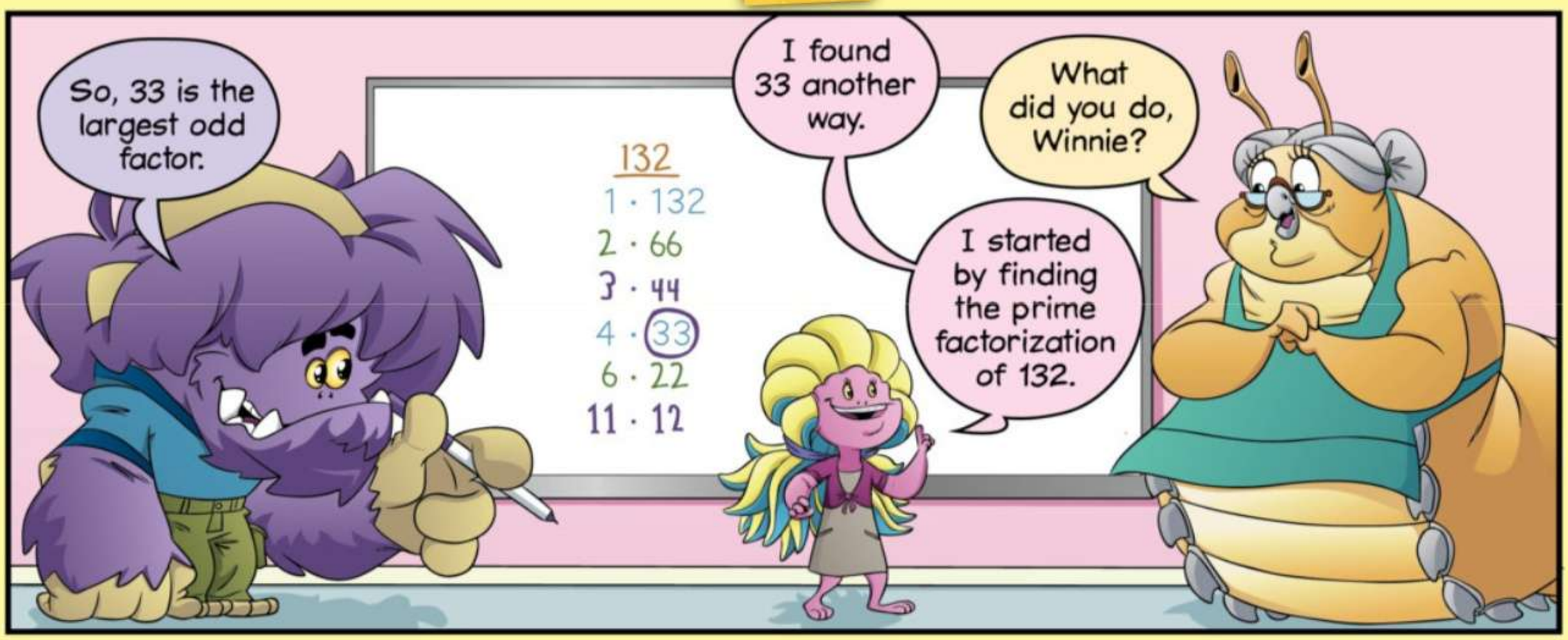
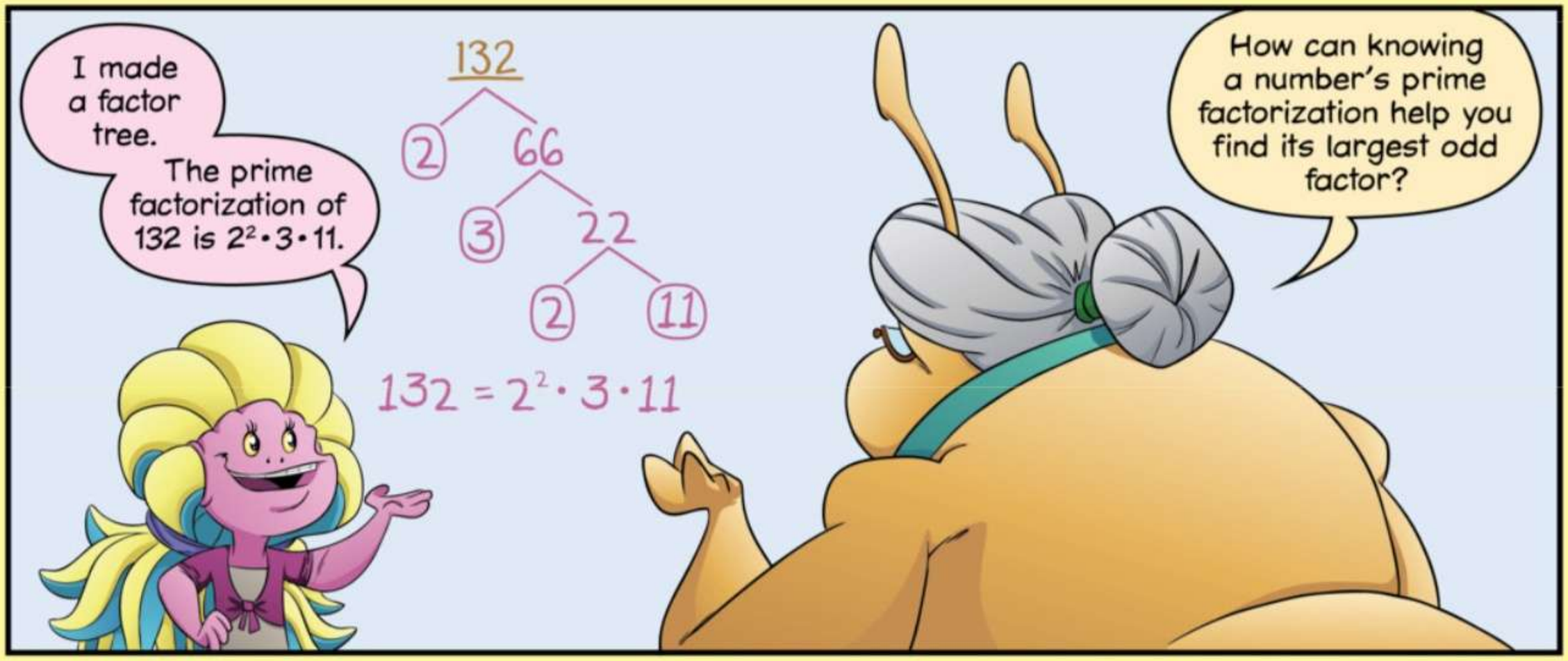
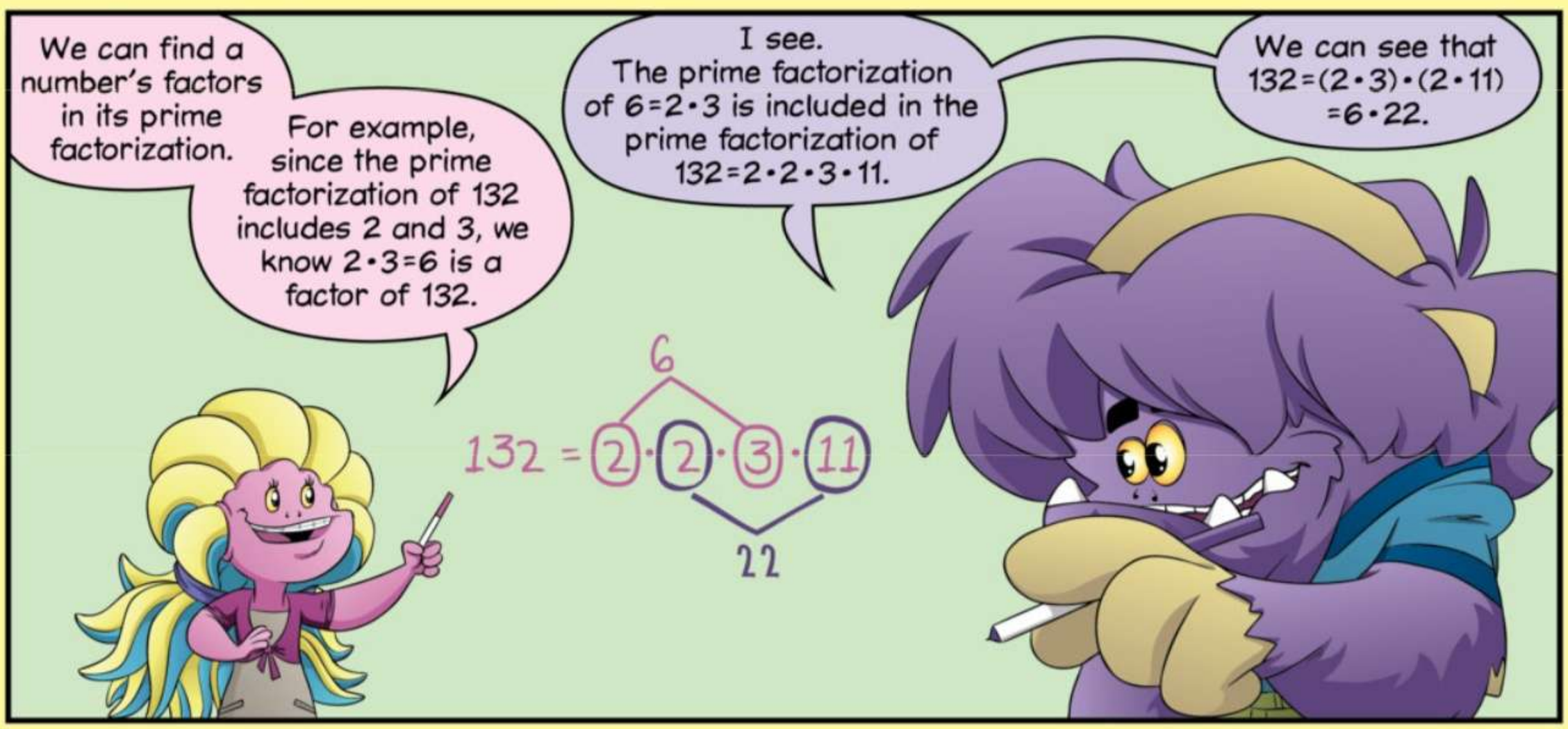
Chapter 5:
Factors & Multiples


Ms. Q. Factors
132
What is the largest odd factor of 132?
We can list all the factors of 132.
Starting with 1, we know 1•132=132. So, we write 1•132 at the top of our list.
Next is 2•66.
Then 3•44.
132
1 · 132
2 · 66
3 · 44
REVIEW LISTING FACTORS IN CHAPTER 7 OF BEAST ACADEMY 4C.
4•33 =132.
5 is not a factor of 132, but 6 is. 6•22=132.
132
1 · 132
2 · 66
3 · 44
4 · 33
6 · 22
11 · 12
7, 8, 9, and 10 are *not* factors of 132. The last factor pair is 11•12=132.


So, 33 is the largest odd factor.
132
1 · 132
2 · 66
3 · 44
4 · 33
6 · 22
11 · 12
I found 33 another way.
I started by finding the prime factorization of 132.
What did you do, Winnie?


I made a factor tree.
The prime factorization of 132 is $2^2 \cdot 3 \cdot 11$.
132
2
66
3
22
2
11
$132 = 2^2 \cdot 3 \cdot 11$
How can knowing a number's prime factorization help you find its largest odd factor?


We can find a number's factors in its prime factorization.
For example, since the prime factorization of 132 includes 2 and 3, we know $2 \cdot 3 = 6$ is a factor of 132.
I see. The prime factorization of $6 = 2 \cdot 3$ is included in the prime factorization of $132 = 2 \cdot 2 \cdot 3 \cdot 11$.
We can see that $132 = (2 \cdot 3) \cdot (2 \cdot 11) = 6 \cdot 22$.
6
$132 = 2 \cdot 2 \cdot 3 \cdot 11$
22

To find the largest ***odd*** factor, we look for the biggest factor of 132 that is ***not*** divisible by 2.

Right. To find the largest odd factor of 132, we multiply all of the odd primes in its prime factorization.

$132 = 2 \cdot 2 \cdot 3 \cdot 11$

33

3•11 =33!

Well done. A number's prime factorization tells us a lot about the number.

For example, look at the prime factorizations of these three numbers.

Which of these numbers is divisible by 12?

$784 = 2^4 \cdot 7^2$

$858 = 2 \cdot 3 \cdot 11 \cdot 13$

$1{,}044 = 2^2 \cdot 3^2 \cdot 29$

None of those numbers has 12 as a prime factor.

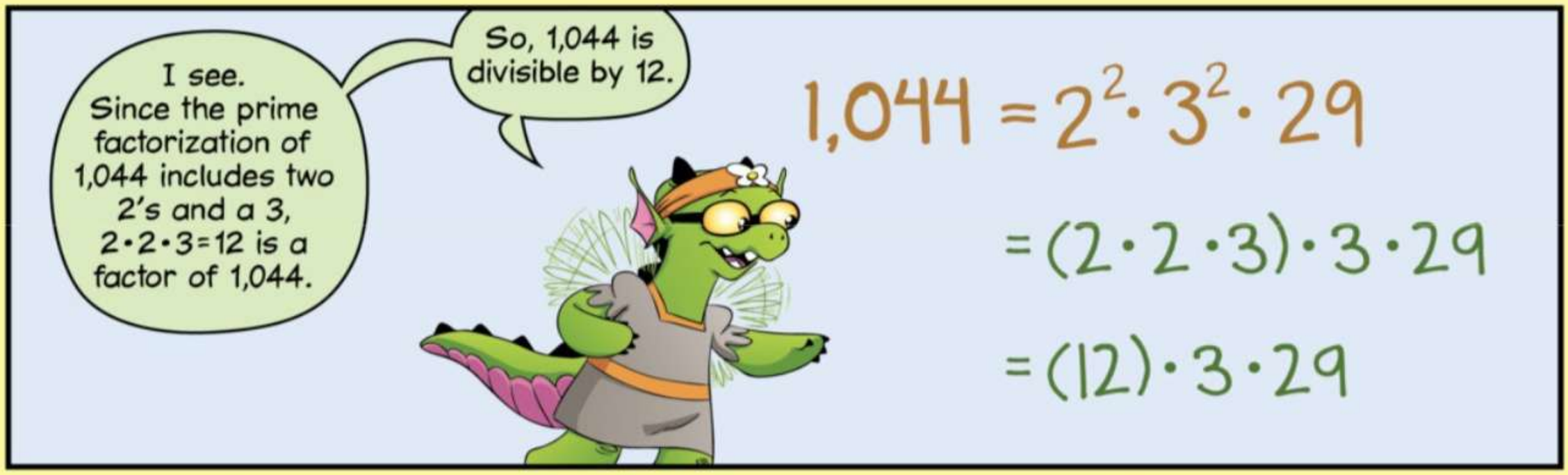
That's because 12 isn't ***prime.***

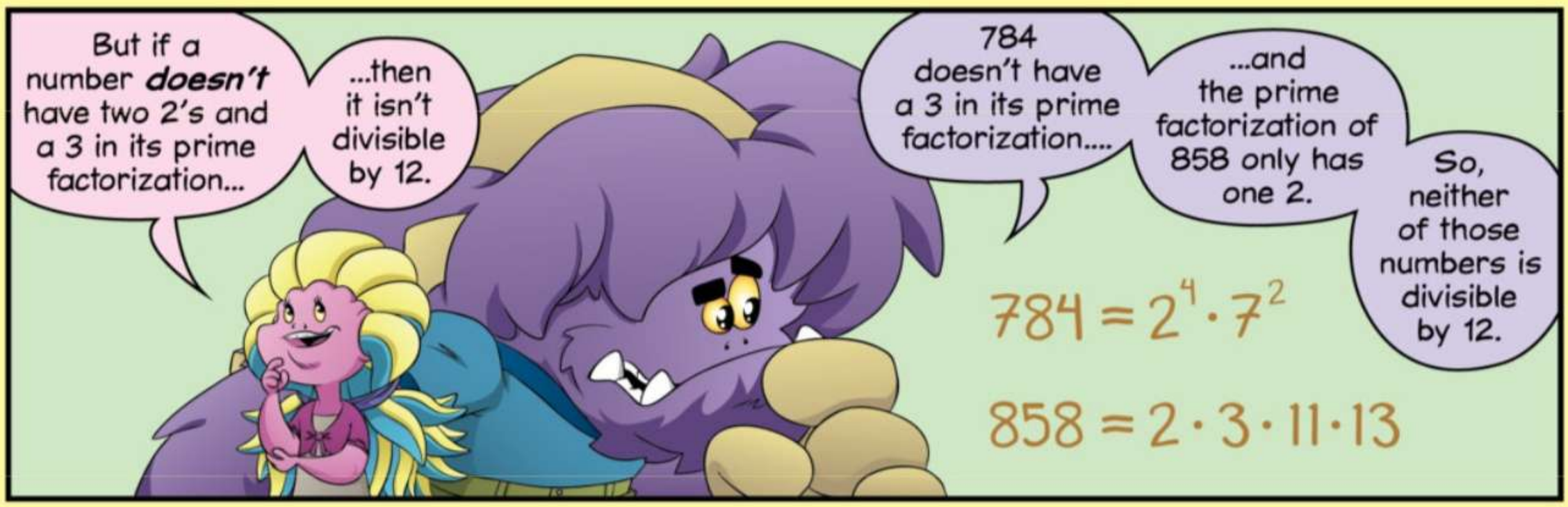
But, we know that the prime factorization of 12 is 2•2•3.

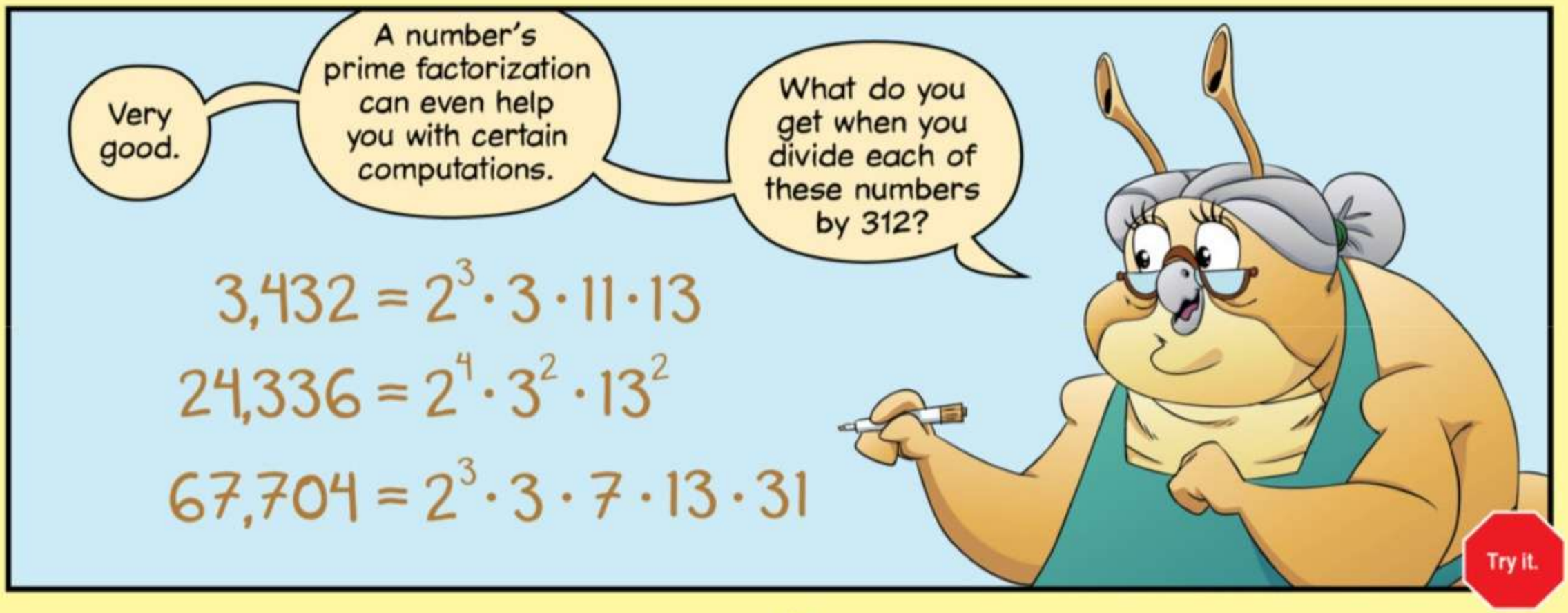
How does that help?

Which number above is divisible by 12?

Any number that has at least two 2's and one 3 in its prime factorization is divisible by 12.
$12 = 2^2 \cdot 3$


I see. Since the prime factorization of 1,044 includes two 2's and a 3, 2•2•3=12 is a factor of 1,044.
So, 1,044 is divisible by 12.
$1{,}044 = 2^2 \cdot 3^2 \cdot 29$
$= (2 \cdot 2 \cdot 3) \cdot 3 \cdot 29$
$= (12) \cdot 3 \cdot 29$


But if a number *doesn't* have two 2's and a 3 in its prime factorization...
...then it isn't divisible by 12.
784 doesn't have a 3 in its prime factorization....
...and the prime factorization of 858 only has one 2.
So, neither of those numbers is divisible by 12.
$784 = 2^4 \cdot 7^2$
$858 = 2 \cdot 3 \cdot 11 \cdot 13$


Very good.
A number's prime factorization can even help you with certain computations.
What do you get when you divide each of these numbers by 312?
$3{,}432 = 2^3 \cdot 3 \cdot 11 \cdot 13$
$24{,}336 = 2^4 \cdot 3^2 \cdot 13^2$
$67{,}704 = 2^3 \cdot 3 \cdot 7 \cdot 13 \cdot 31$

Try it.

To start, we can find the prime factorization of 312: $312 = 2^3 \cdot 3 \cdot 13$.

312
4 78
2 2 3 26
2 13

The prime factorization of 312 includes three 2's, a 3, and a 13.

Each of these three numbers includes three 2's, a 3, and a 13 in its prime factorization...

...so they are all divisible by 312.

$3{,}432 = 2^3 \cdot 3 \cdot 11 \cdot 13$

$24{,}336 = 2^4 \cdot 3^2 \cdot 13^2$

$67{,}704 = 2^3 \cdot 3 \cdot 7 \cdot 13 \cdot 31$

If we separate three 2's, a 3, and a 13 from the rest of each prime factorization...

...we can write each number as a product of 312 and another number.

$$\begin{aligned} 3{,}432 &= 2^3 \cdot 3 \cdot 11 \cdot 13 \\ &= (2 \cdot 2 \cdot 2 \cdot 3 \cdot 13) \cdot 11 \\ &= 312 \cdot 11 \end{aligned}$$

3,432 is 312 • 11.

So, 3,432 ÷ 312 = 11.

$3{,}432 \div 312 = 11$

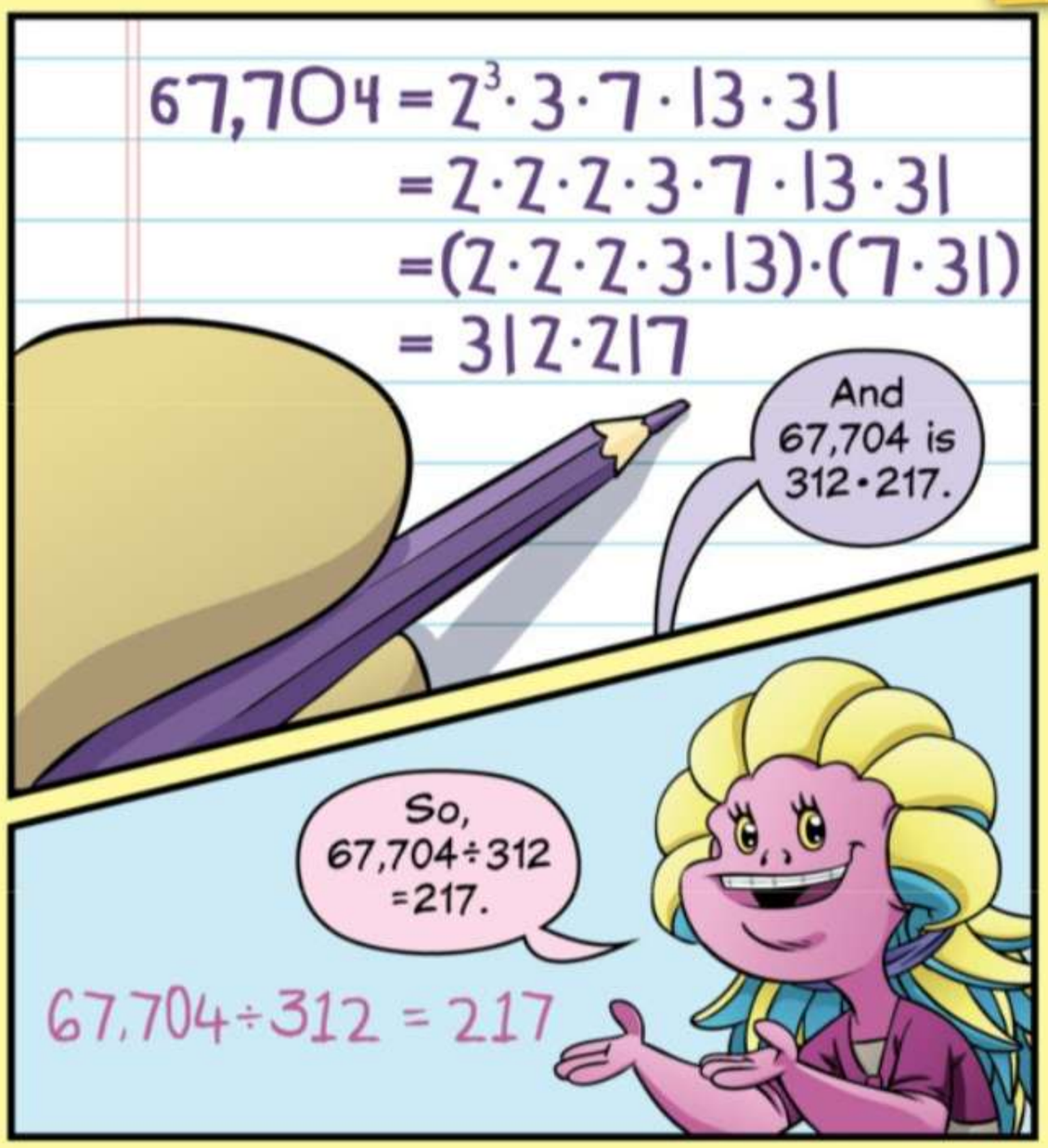
$$\begin{aligned} 24{,}336 &= 2^4 \cdot 3^2 \cdot 13^2 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 13 \cdot 13 \\ &= (2 \cdot 2 \cdot 2 \cdot 3 \cdot 13) \cdot (2 \cdot 3 \cdot 13) \\ &= 312 \cdot 78 \end{aligned}$$

24,336 is 312 • 78.

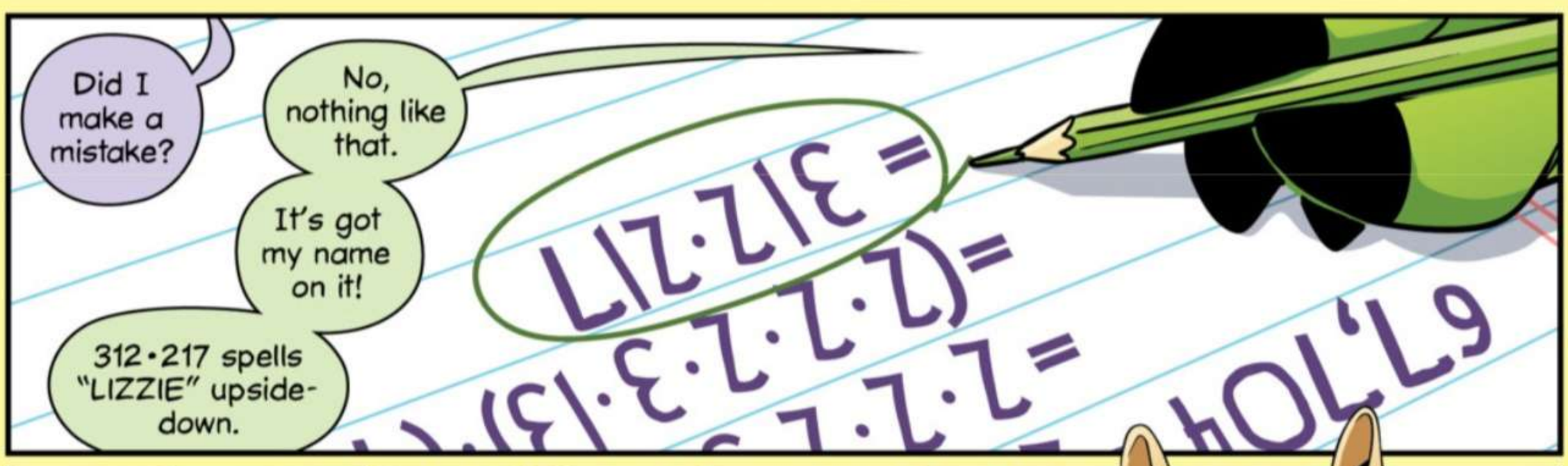
So, 24,336 ÷ 312 = 78.

$24{,}336 \div 312 = 78$

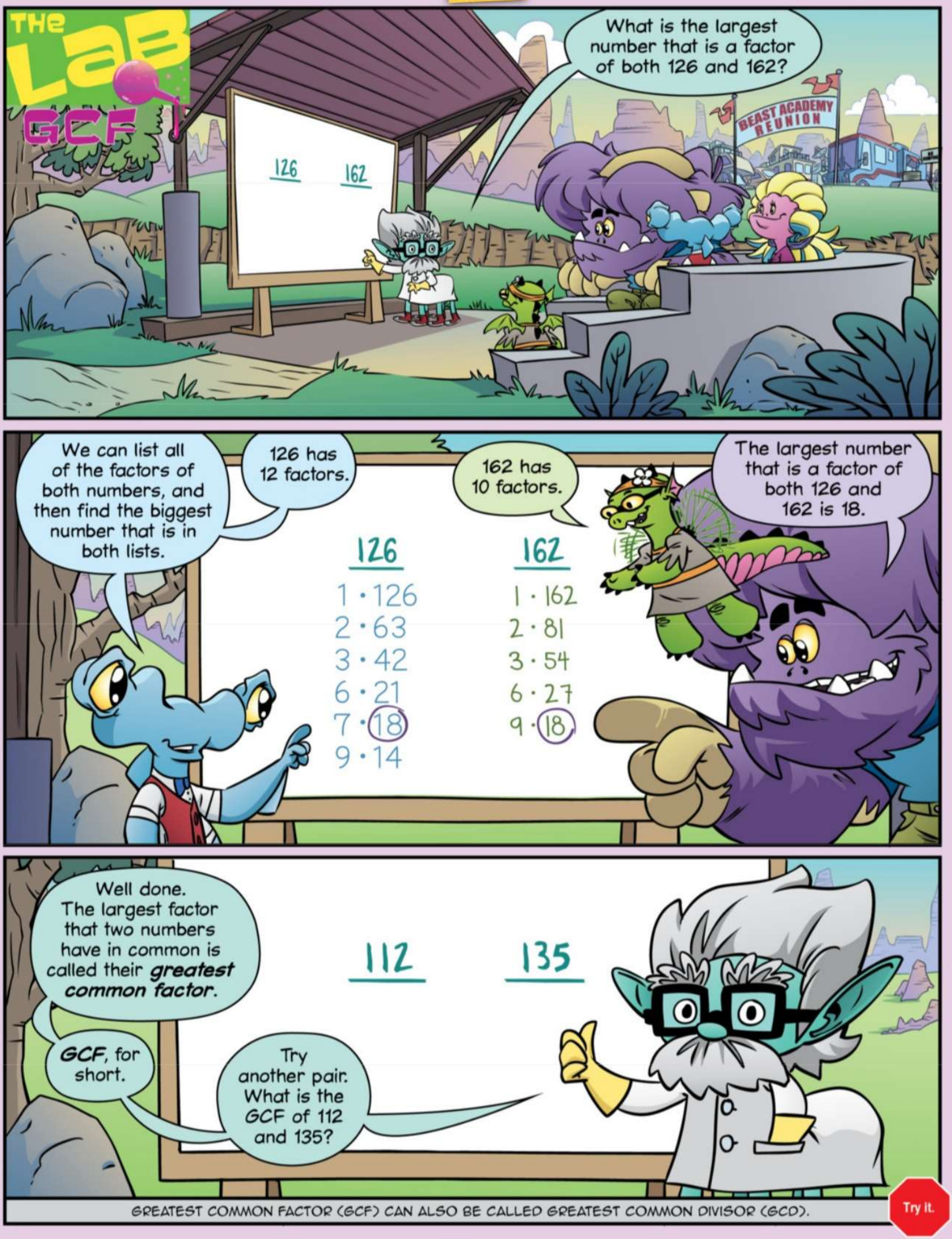
What is 67,704 ÷ 312?


67,704 = 2³·3·7·13·31
= 2·2·2·3·7·13·31
= (2·2·2·3·13)·(7·31)
= 312·217
And 67,704 is 312•217.

Whoa!
Look at your work, Grogg!

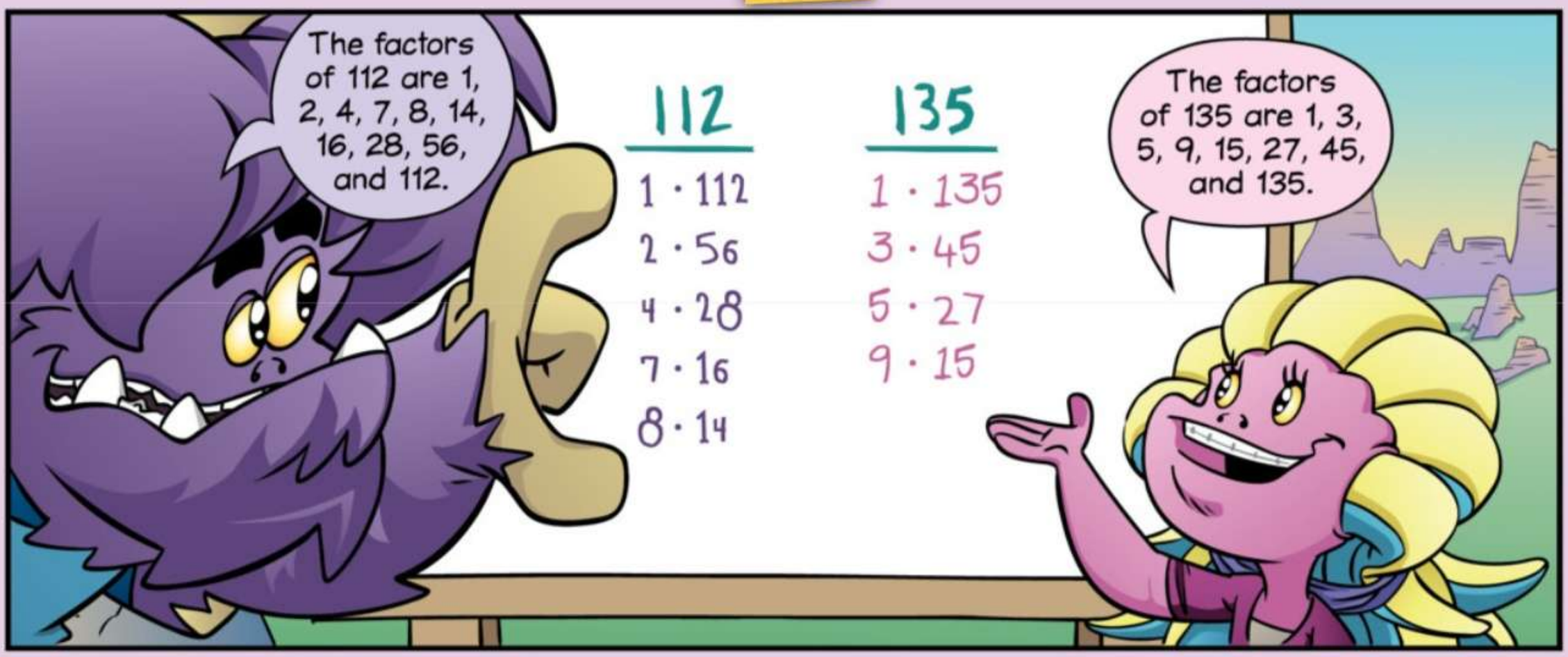

Did I make a mistake?
No, nothing like that.
It's got my name on it!
312•217 spells "LIZZIE" upside-down.

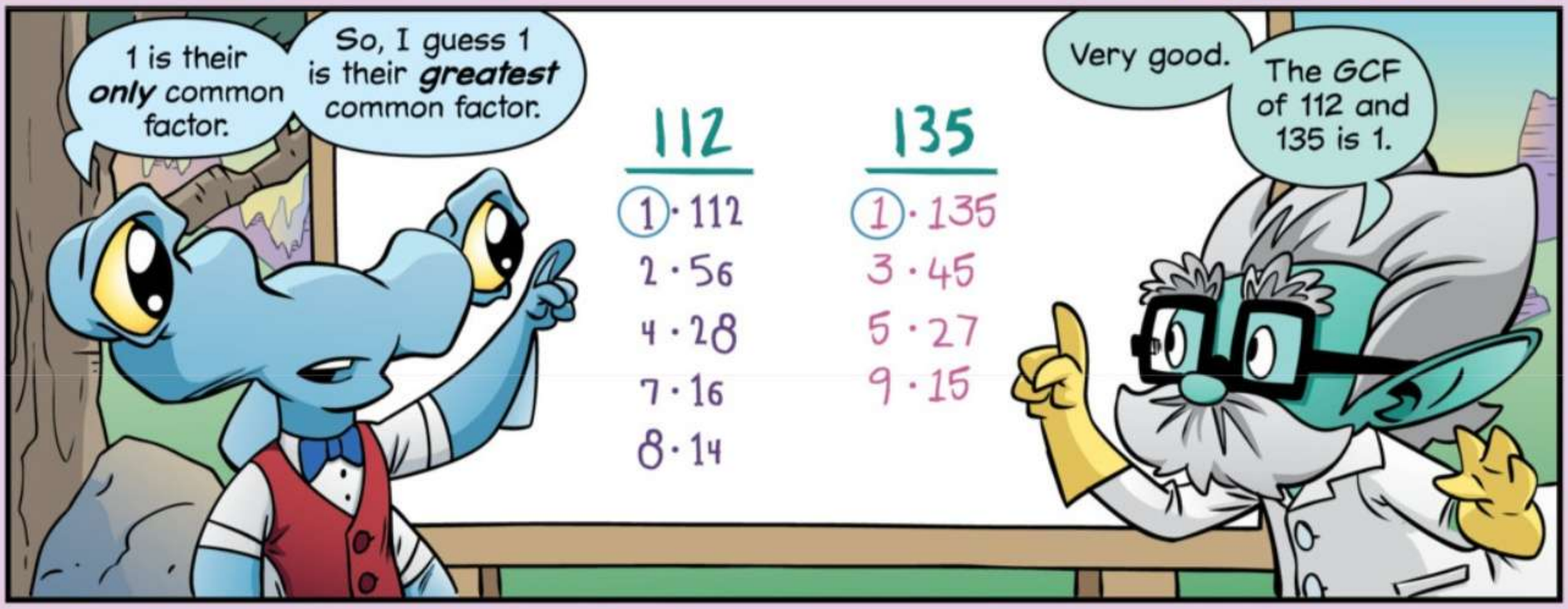
You planned that, didn't you, Ms. Q.?
You're mathemagical!



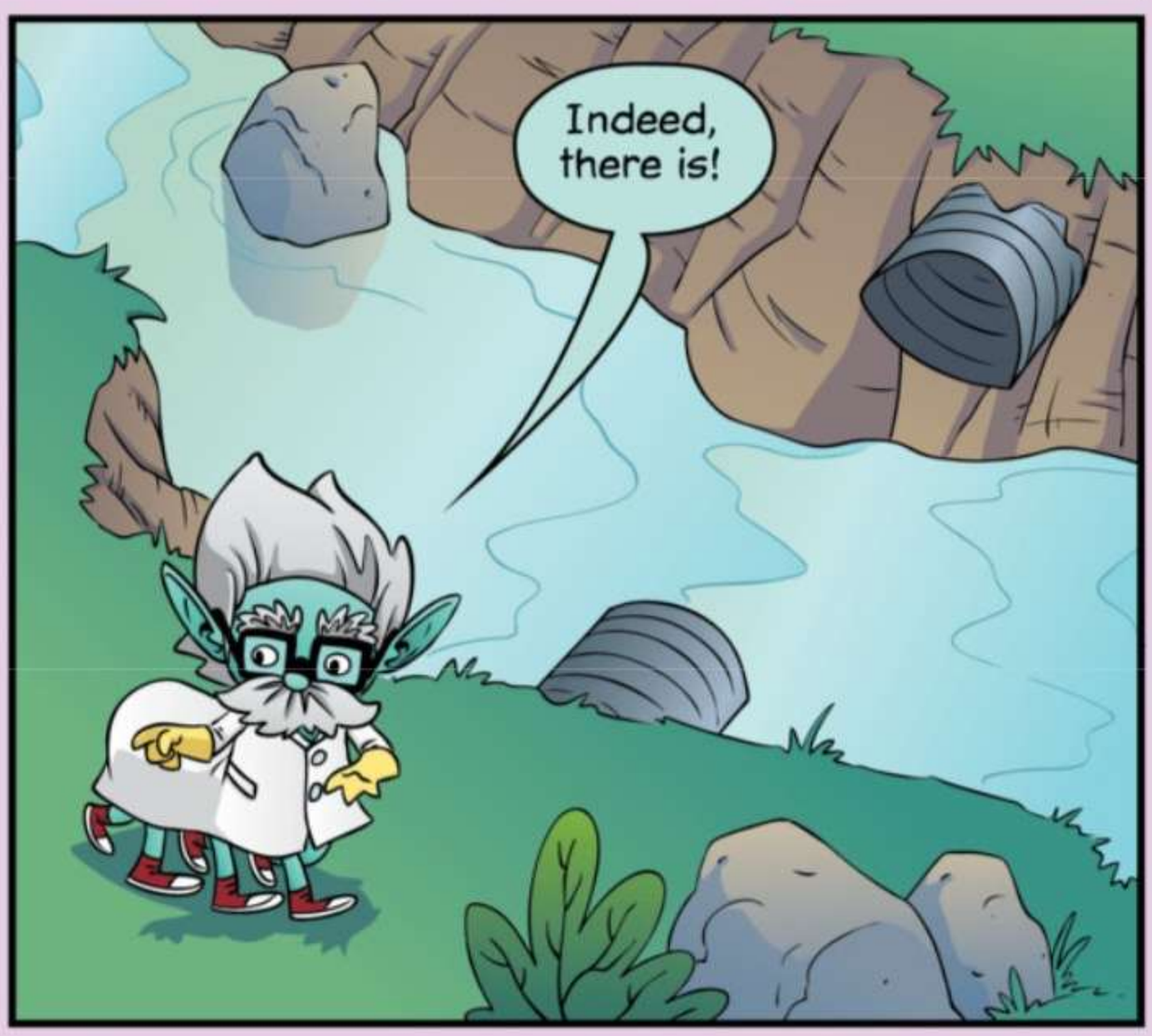
GREATEST COMMON FACTOR (GCF) CAN ALSO BE CALLED GREATEST COMMON DIVISOR (GCD).

Try it.

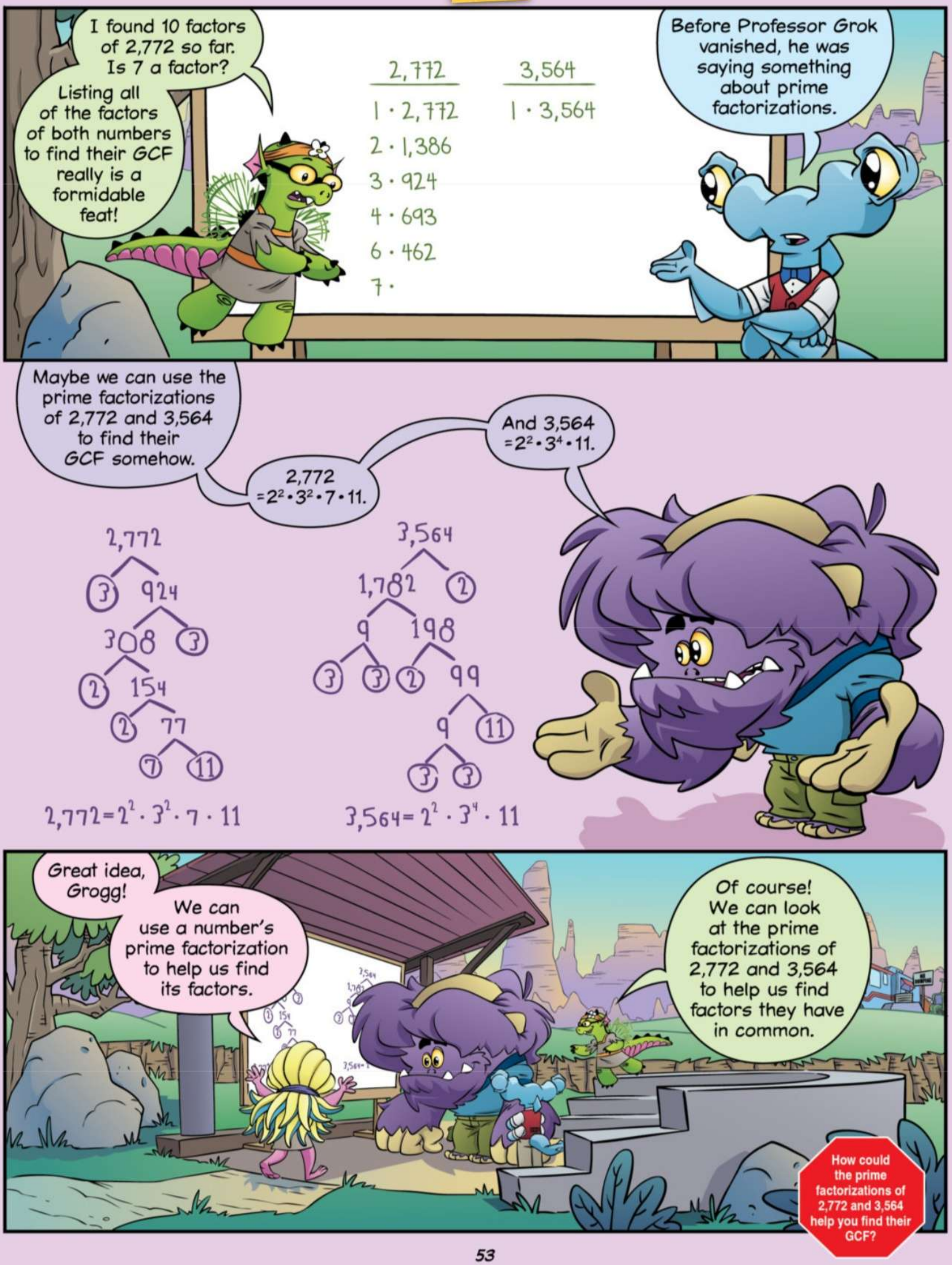

The factors of 112 are 1, 2, 4, 7, 8, 14, 16, 28, 56, and 112.
112
1 · 112
2 · 56
4 · 28
7 · 16
8 · 14
135
1 · 135
3 · 45
5 · 27
9 · 15
The factors of 135 are 1, 3, 5, 9, 15, 27, 45, and 135.


1 is their *only* common factor.
So, I guess 1 is their *greatest* common factor.
112
1 · 112
2 · 56
4 · 28
7 · 16
8 · 14
135
1 · 135
3 · 45
5 · 27
9 · 15
Very good.
The GCF of 112 and 135 is 1.

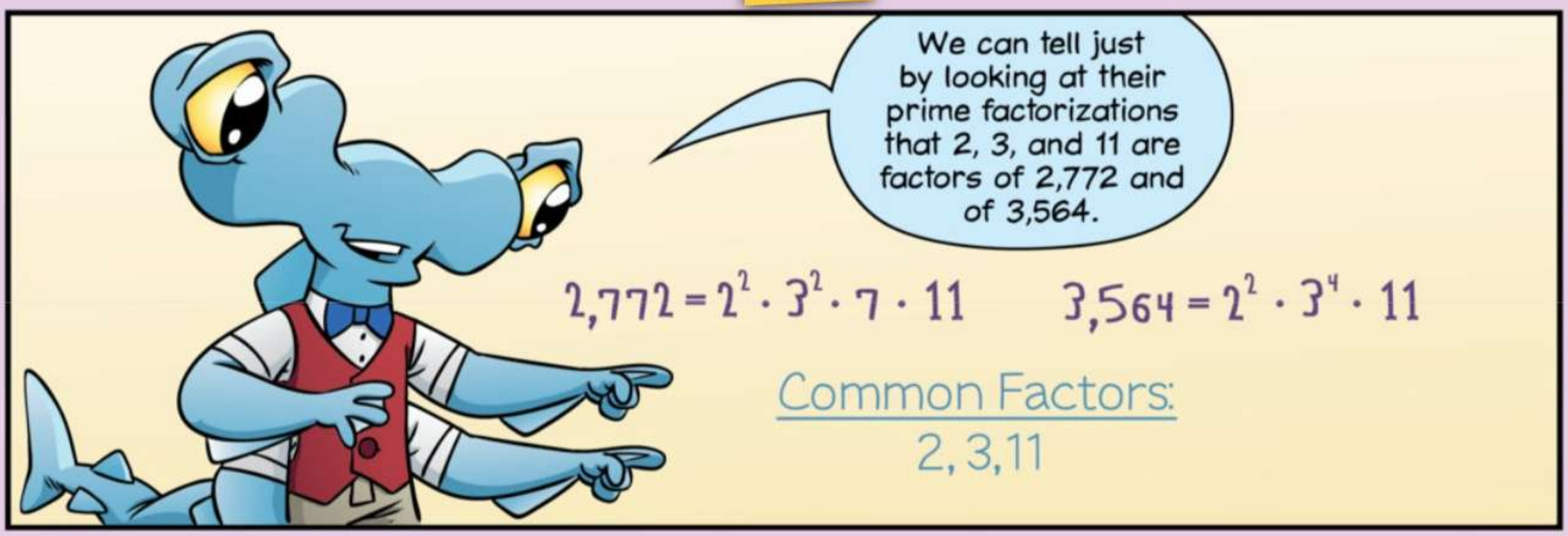
Is there a better way to find the GCF of two numbers...
...*without* listing all of their factors?

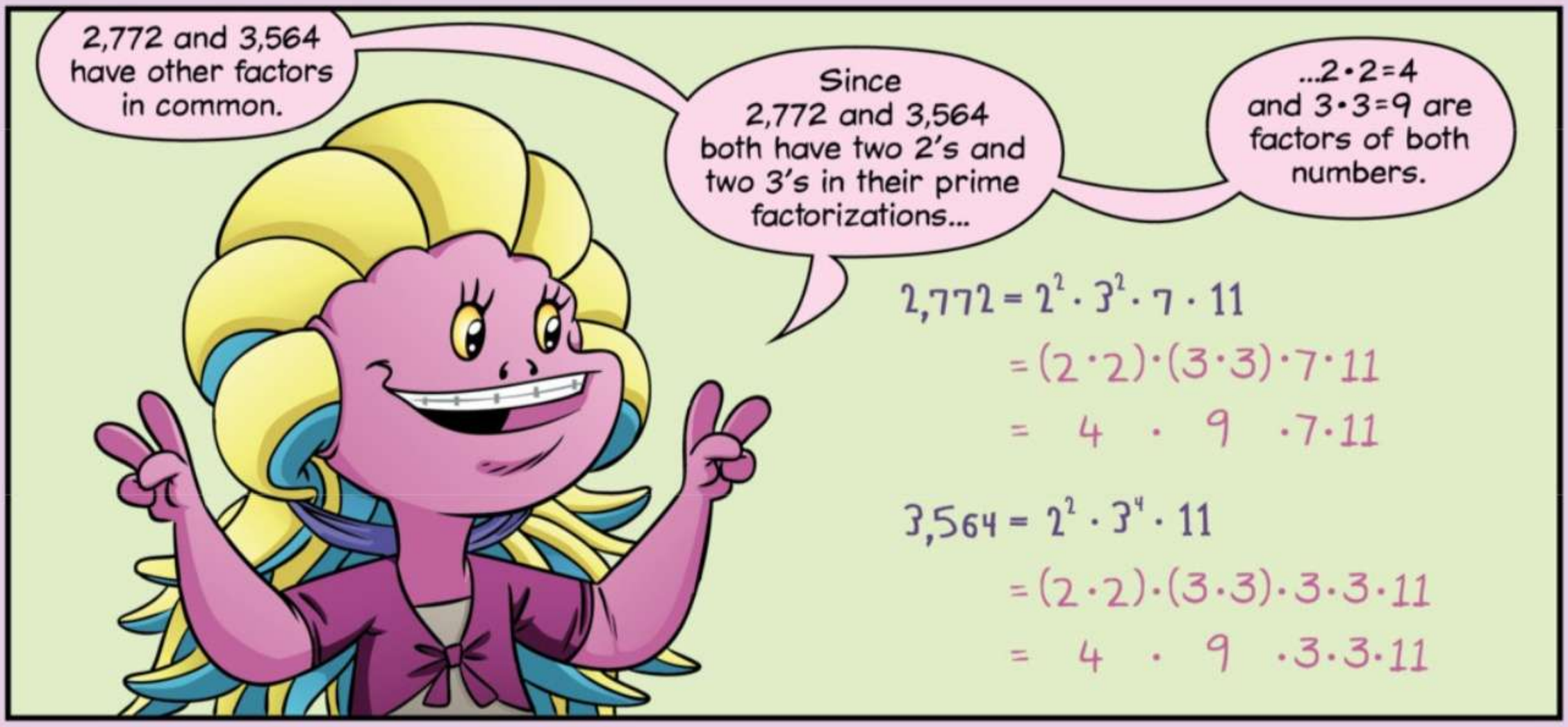

Indeed, there is!

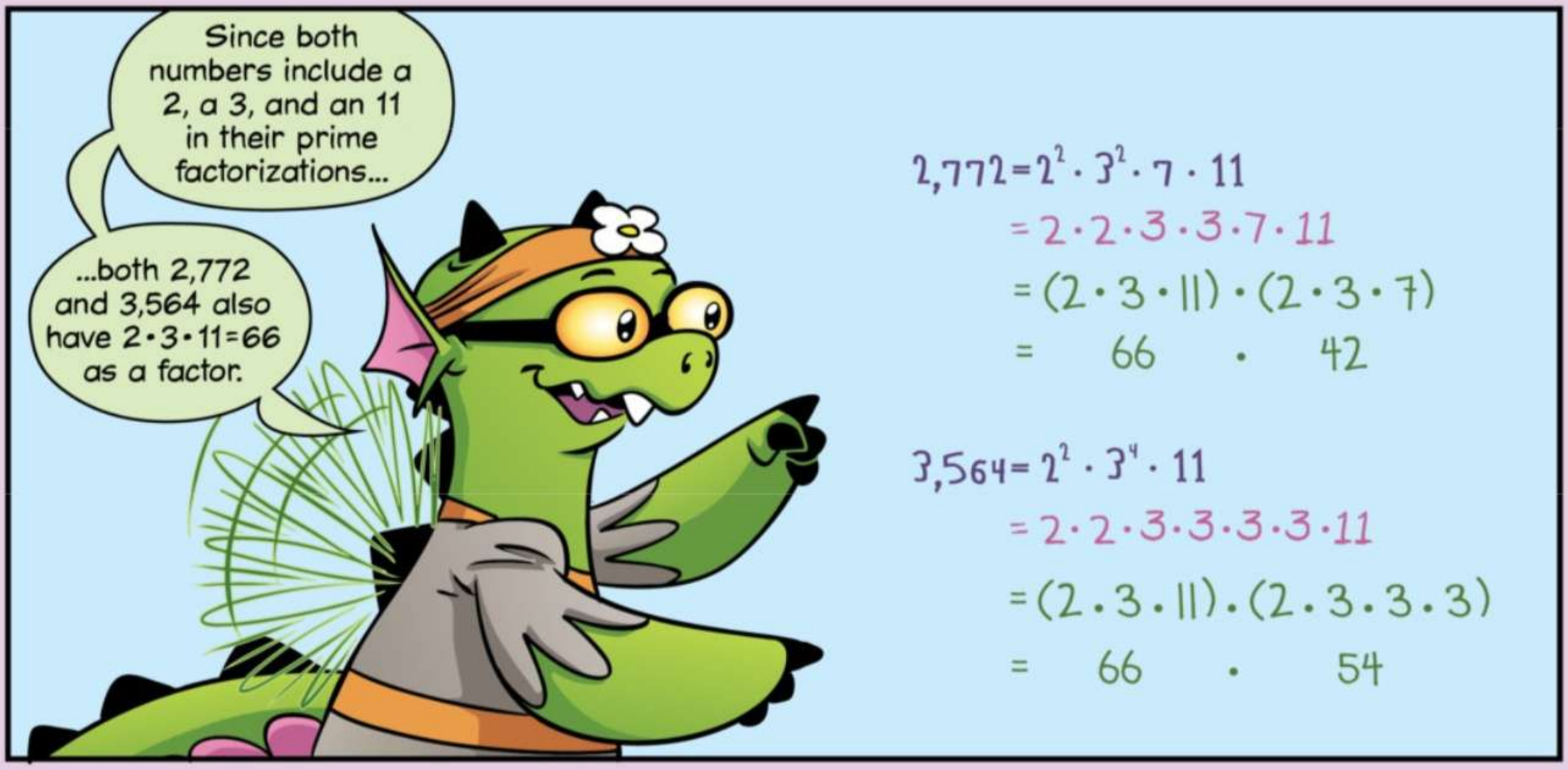
We begin by looking at each number's prime factori--
yoink!
Professor Grok?
Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something much more diabolically difficult!
The folly of finding the largest factor that divides two diminutive numerals is ***undoubtedly dull.***
But finding the GCF of two four-digit numbers is a ***formidable feat.***
BEAST ACADEMY REUNION
The greatest integer that divides both 2,772 and 3,564 is located on the license plate of the livingmobile in the lower lot where I've locked your lecturer.
2,772 & 3,564
Liberate your professor lickety-split to ***limit his laments.***
Thunk
What is the GCF of 2,772 and 3,564?



How could the prime factorizations of 2,772 and 3,564 help you find their GCF?


We can tell just by looking at their prime factorizations that 2, 3, and 11 are factors of 2,772 and of 3,564.
$2{,}772 = 2^2 \cdot 3^2 \cdot 7 \cdot 11$
$3{,}564 = 2^2 \cdot 3^4 \cdot 11$
Common Factors:
2, 3, 11


2,772 and 3,564 have other factors in common.
Since 2,772 and 3,564 both have two 2's and two 3's in their prime factorizations...
...2•2=4 and 3•3=9 are factors of both numbers.
$2{,}772 = 2^2 \cdot 3^2 \cdot 7 \cdot 11$
$= (2 \cdot 2) \cdot (3 \cdot 3) \cdot 7 \cdot 11$
$= 4 \cdot 9 \cdot 7 \cdot 11$
$3{,}564 = 2^2 \cdot 3^4 \cdot 11$
$= (2 \cdot 2) \cdot (3 \cdot 3) \cdot 3 \cdot 3 \cdot 11$
$= 4 \cdot 9 \cdot 3 \cdot 3 \cdot 11$


Since both numbers include a 2, a 3, and an 11 in their prime factorizations...
...both 2,772 and 3,564 also have 2•3•11=66 as a factor.
$2{,}772 = 2^2 \cdot 3^2 \cdot 7 \cdot 11$
$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 11$
$= (2 \cdot 3 \cdot 11) \cdot (2 \cdot 3 \cdot 7)$
$= 66 \cdot 42$
$3{,}564 = 2^2 \cdot 3^4 \cdot 11$
$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 11$
$= (2 \cdot 3 \cdot 11) \cdot (2 \cdot 3 \cdot 3 \cdot 3)$
$= 66 \cdot 54$

2,772 and 3,564 both have two 2's, two 3's, and one 11.

That's all of their shared primes.

So, both numbers are divisible by 2•2•3•3•11!

$2{,}772 = 2^2 \cdot 3^2 \cdot 7 \cdot 11$

$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 11$

$= (2 \cdot 2 \cdot 3 \cdot 3 \cdot 11) \cdot 7$

$3{,}564 = 2^2 \cdot 3^4 \cdot 11$

$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 11$

$= (2 \cdot 2 \cdot 3 \cdot 3 \cdot 11) \cdot 3 \cdot 3$

The product of all these shared primes is the GCF of 2,772 and 3,564.

$2{,}772 = 2^2 \cdot 3^2 \cdot 7 \cdot 11$

$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 11$

$= (2 \cdot 2 \cdot 3 \cdot 3 \cdot 11) \cdot 7$

$= 396 \cdot 7$

So, 2•2•3•3•11=396 is the GCF of 2,772 and 3,564!

$3{,}564 = 2^2 \cdot 3^4 \cdot 11$

$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 11$

$= (2 \cdot 2 \cdot 3 \cdot 3 \cdot 11) \cdot 3 \cdot 3$

$= 396 \cdot 9$

396•7=2,772 and 396•9=3,564!

To the parking lot!

Look at all these license plates! What's a "livingmobile?"
Maybe it's one of those big RVs over there.
We're looking for a plate with 396 on it.
BEAST ACADEMY REUNION
NO DUMPING

Found it!
GCF 396

Extraordinary! You've rescued me again!

RECESS

GCF ⅃⅃⅁

Setup

The game uses a standard deck of playing cards with the face cards and 10's (K, Q, J, 10) removed. Aces are treated as 1's. Players sit on opposite sides of a table. Shuffle and deal 2 cards to each player. Place the remaining cards face down in a pile called the stock. Flip two cards face-up on either side of the stock, as shown below. This creates two 2-digit numbers. In the example below, the player on one side of the table sees the numbers 42 and 15. The player on the opposite side of the table sees the numbers 51 and 24.

Player 1 Sees: 42 Stock 15

Player 2 sees: 51 Stock 24

Play

Players take turns placing one of the two cards in their hand on top of one of the four face-up cards. In the example below, Player 1 places the 4 of spades on the 5 of hearts. Now, Player 1 sees 42 and 14. Player 2 sees 41 and 24.
Each player finds the GCF of the pair of numbers they see, and adds the GCF to their score.

The GCF of 42 and 14 is 14, so Player 1 scores 14 points this turn. The GCF of 41 and 24 is 1, so Player 2 scores 1 point.

After playing a card and scoring, a player draws a card from the stock. This completes their turn.

Player 1 Sees: 42 Stock 14

Player 2 sees: 41 Stock 24

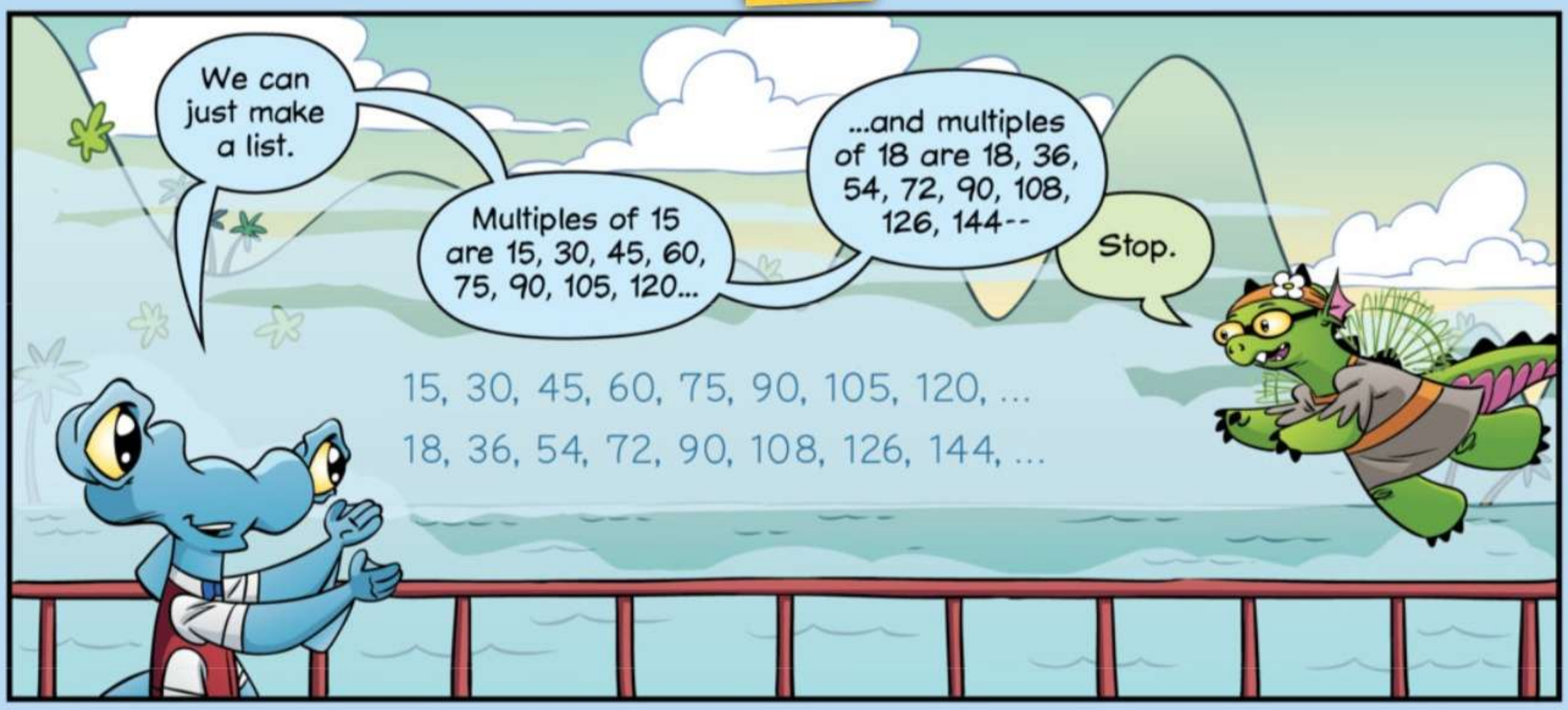
Winning

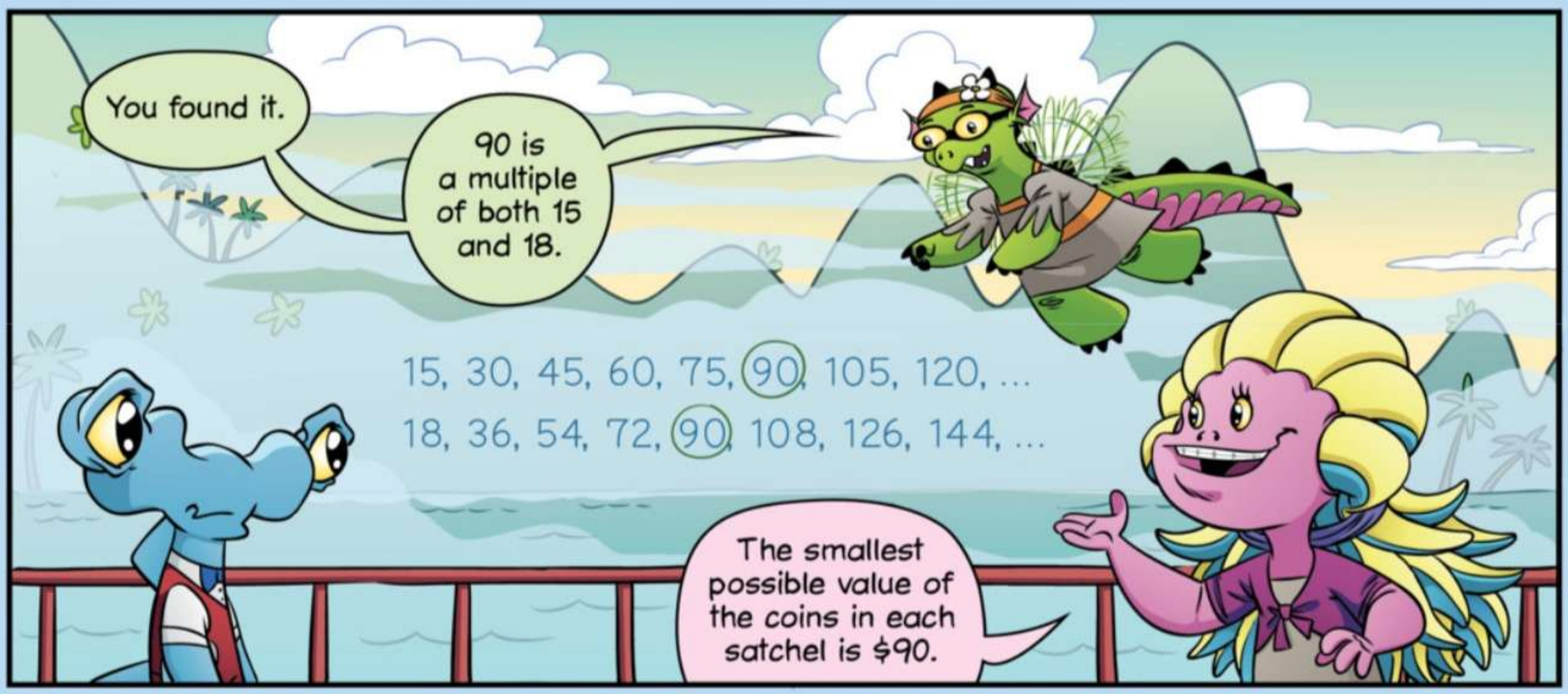
Players take turns playing a single card, scoring, and drawing until the stock is emptied and all the cards are played.
The player with the most points after all of the cards have been played wins.

Variations

For a simpler game, players are not given cards to begin, but instead take turns drawing from the stock and playing the card immediately. For a shorter game, choose a winning score. For example, the first player to go over 100 point wins.

LCM
Each o' these satchels be holdin' a different type o' coin.
The coins in the red satchel be worth $15 each...
...'n' the coins in the purple satchel be worth $18 each.
Each satchel be holdin' the same total value o' coins.
What be the smallest possible total value o' the coins in each satchel?
A satchel full of $15 coins can hold any multiple of $15.
And a satchel full of $18 coins can hold any multiple of $18.
We need to find the smallest positive value that is a multiple of both $15 and $18.
Try it.


We can just make a list.
Multiples of 15 are 15, 30, 45, 60, 75, 90, 105, 120...
...and multiples of 18 are 18, 36, 54, 72, 90, 108, 126, 144--
Stop.
15, 30, 45, 60, 75, 90, 105, 120, ...
18, 36, 54, 72, 90, 108, 126, 144, ...


You found it.
90 is a multiple of both 15 and 18.
15, 30, 45, 60, 75, 90, 105, 120, ...
18, 36, 54, 72, 90, 108, 126, 144, ...
The smallest possible value of the coins in each satchel is $90.

Aye. The red satchel be holdin' six $15 coins...
...'n' the purple satchel be holdin' five $18 coins...
...for a total o' $90 in each satchel.

Let's look at some more valuable treasures.

The coins in this blue satchel be worth $30 each...

...'n' the coins in this gold satchel be worth $54 each.

Each satchel be holdin' the same total value o' coins.

What be the smallest possible value o' the coins in each o' these satchels?

30, 60, 90, 120, 150, 180, 210, ...

We need to find the smallest value that is a multiple of both $30 and $54.

We can start by listing the multiples of 30:

30, 60, 90, 120, 150, 180, 210,...

Then, the multiples of 54 are...

...54, 108, ummm...

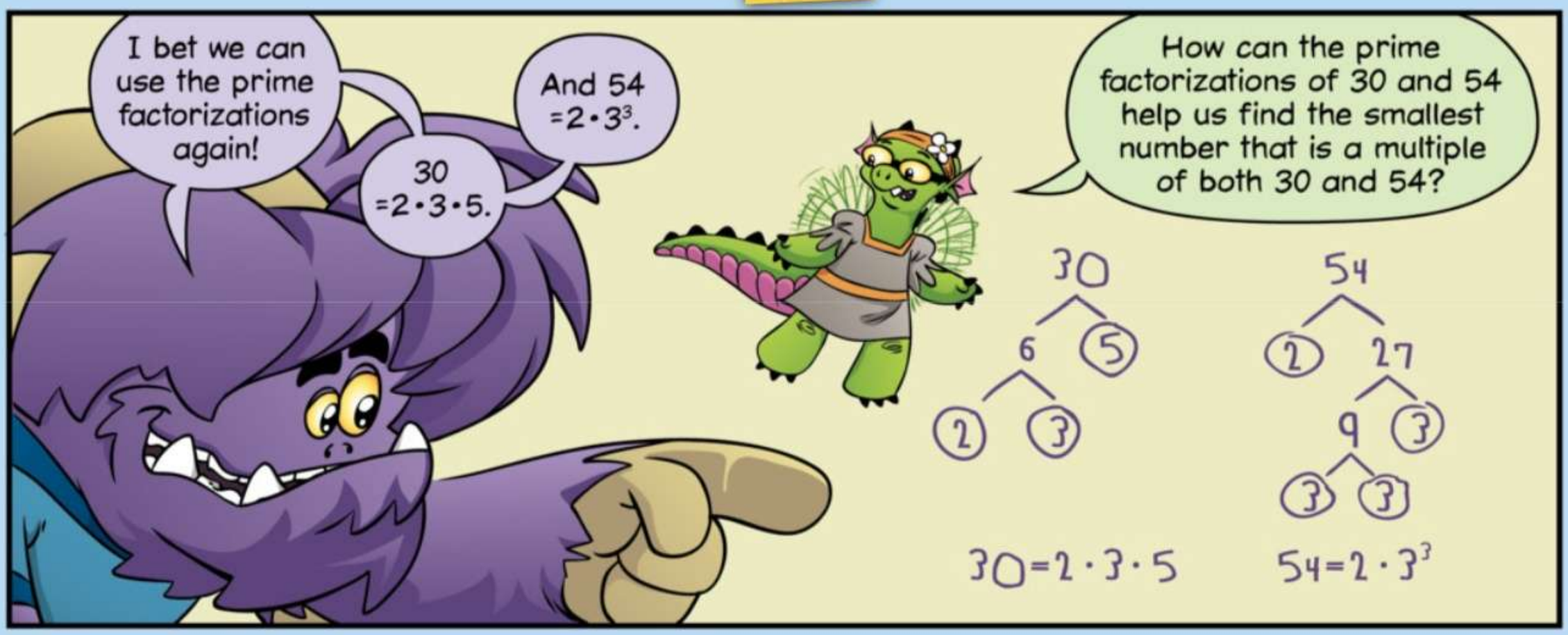
...162...

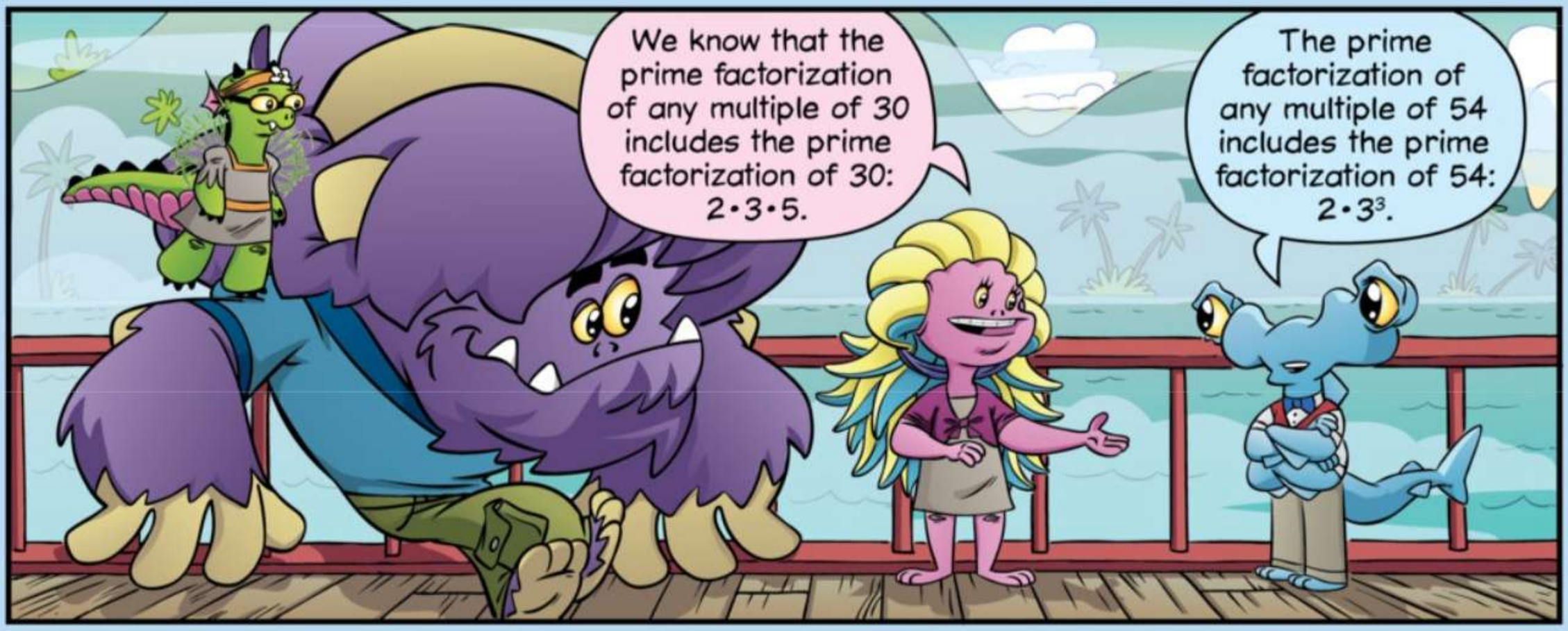
Counting by 54's is a lot harder.

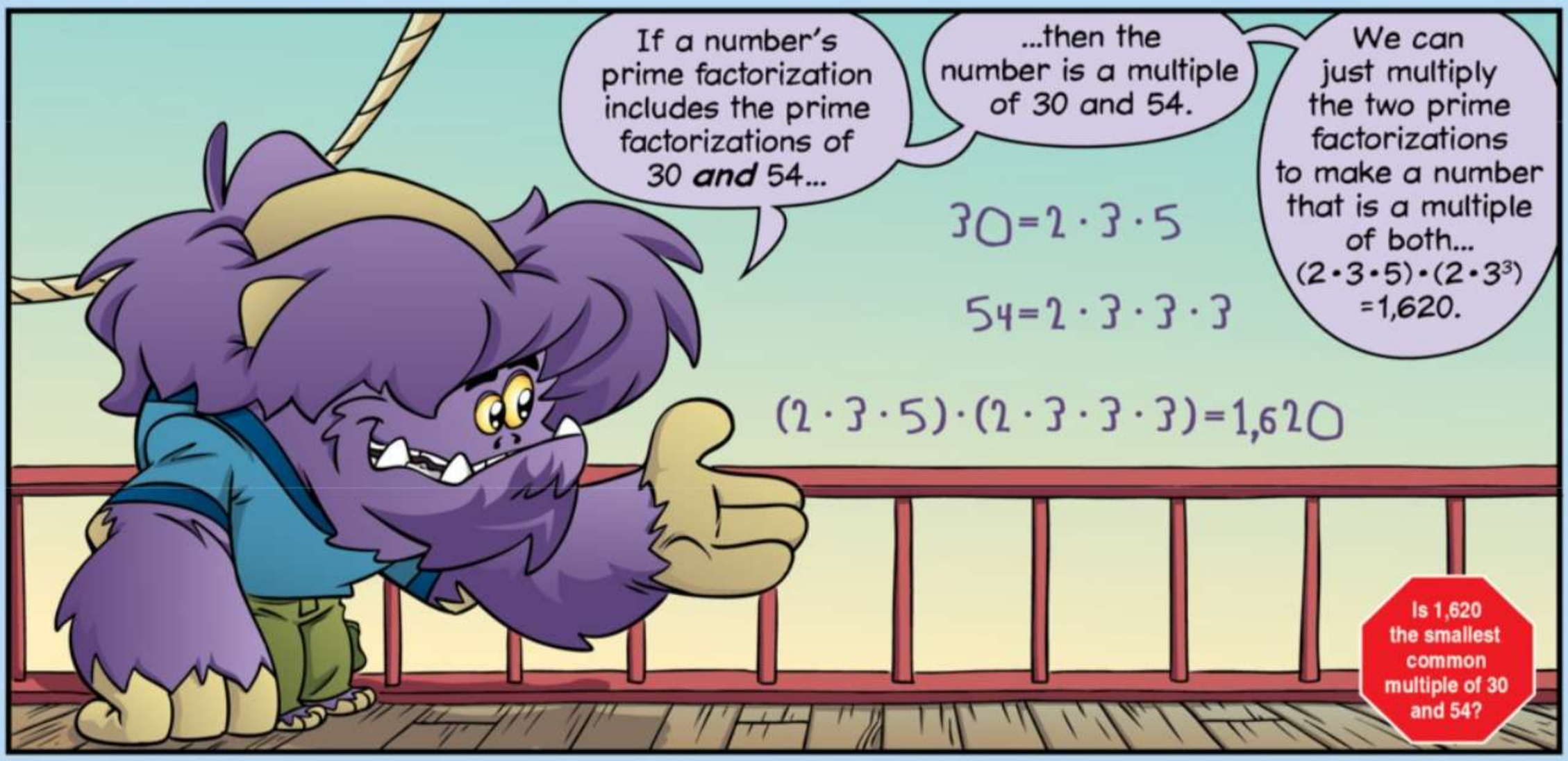
30, 60, 90, 120, 150, 180, 210,...

54, 108, 162,

There must be a way to find the smallest multiple of two numbers without listing all of their multiples.


I bet we can use the prime factorizations again!
30 =2•3•5.
And 54 =2•3³.
How can the prime factorizations of 30 and 54 help us find the smallest number that is a multiple of both 30 and 54?
30
6
5
2
3
54
2
27
9
3
3
3
30=2·3·5
54=2·3·3·3


We know that the prime factorization of any multiple of 30 includes the prime factorization of 30: 2•3•5.
The prime factorization of any multiple of 54 includes the prime factorization of 54: 2•3³.


If a number's prime factorization includes the prime factorizations of 30 and 54...
...then the number is a multiple of 30 and 54.
We can just multiply the two prime factorizations to make a number that is a multiple of both... (2•3•5)•(2•3³) =1,620.
30=2·3·5
54=2·3·3·3
(2·3·5)·(2·3·3·3)=1,620
Is 1,620 the smallest common multiple of 30 and 54?

We want to know the ***smallest*** multiple of 30 and 54.

1,620 isn't the smallest.

How can you tell?

We can remove a 2.

3•5•2•3•3•3=810 still includes the prime factorizations of 30 and 54.

$$\underbrace{(\cancel{2}\cdot 3\cdot 5)}_{30}\cdot\overbrace{(2\cdot 3\cdot 3\cdot 3)}^{54}=\cancel{1{,}620}\ 810$$

Oh, yeah! We only need ***one*** 2 because we can use the same 2 in the prime factorization of 30 and of 54.

$$\underbrace{(\cancel{2}\cdot 3\cdot 5)}_{30}\cdot\overbrace{(2\cdot 3\cdot 3\cdot 3)}^{54}$$

We don't need ***four*** 3's, either!

Any multiple of 54 needs at least ***three*** 3's in its prime factorization, since 54=2•3•3•3.

We can use any one of those 3's to help make 2•3•5=30.

$$\underbrace{(\cancel{2}\cdot\cancel{3}\cdot 5)}_{30}\cdot\overbrace{(2\cdot 3\cdot 3\cdot 3)}^{54}$$

What about the 5?

We need at least one 5...

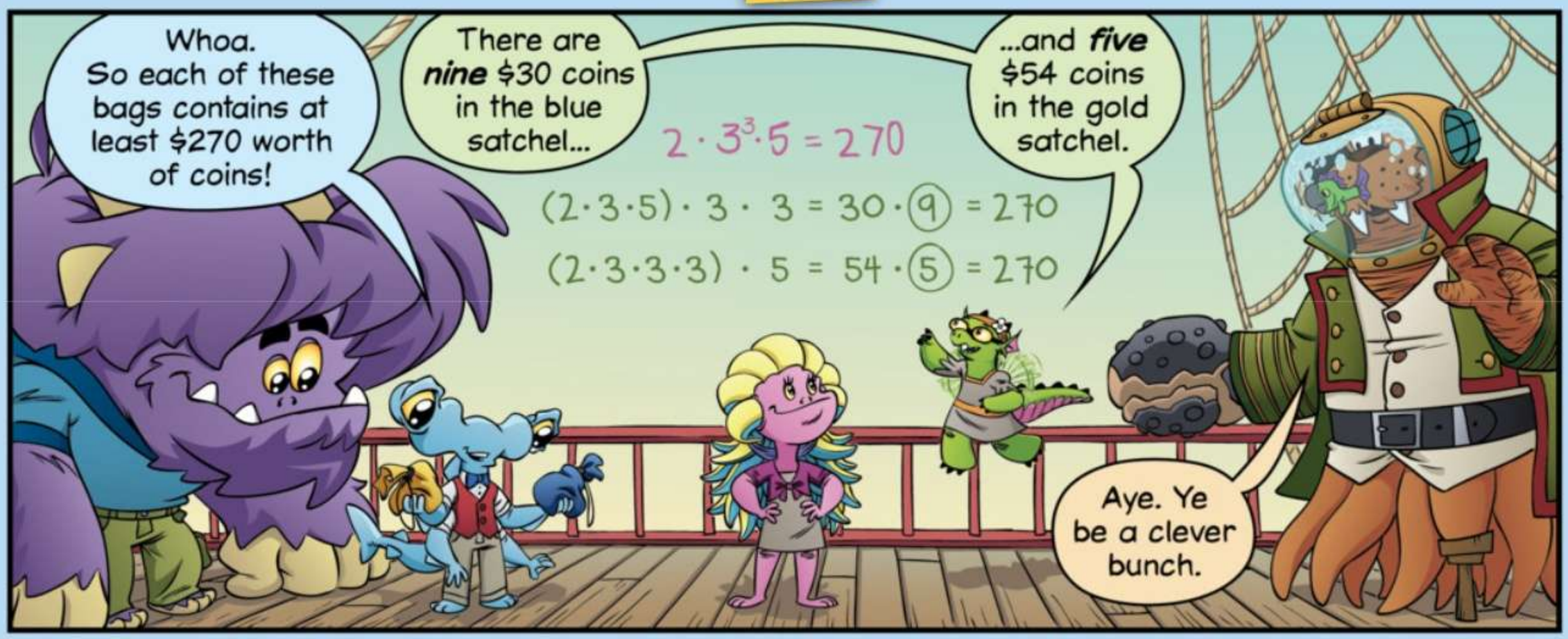
...since 30 has a 5 in its prime factorization.

So, the smallest number that is a multiple of both 30 and 54 has one 2, three 3's, and one 5 in its prime factorization.

$2\cdot 3^3\cdot 5$ =270!

$$\underbrace{(\cancel{2}\cdot\cancel{3}\cdot 5)}_{30}\cdot\overbrace{(2\cdot 3\cdot 3\cdot 3)}^{54}$$

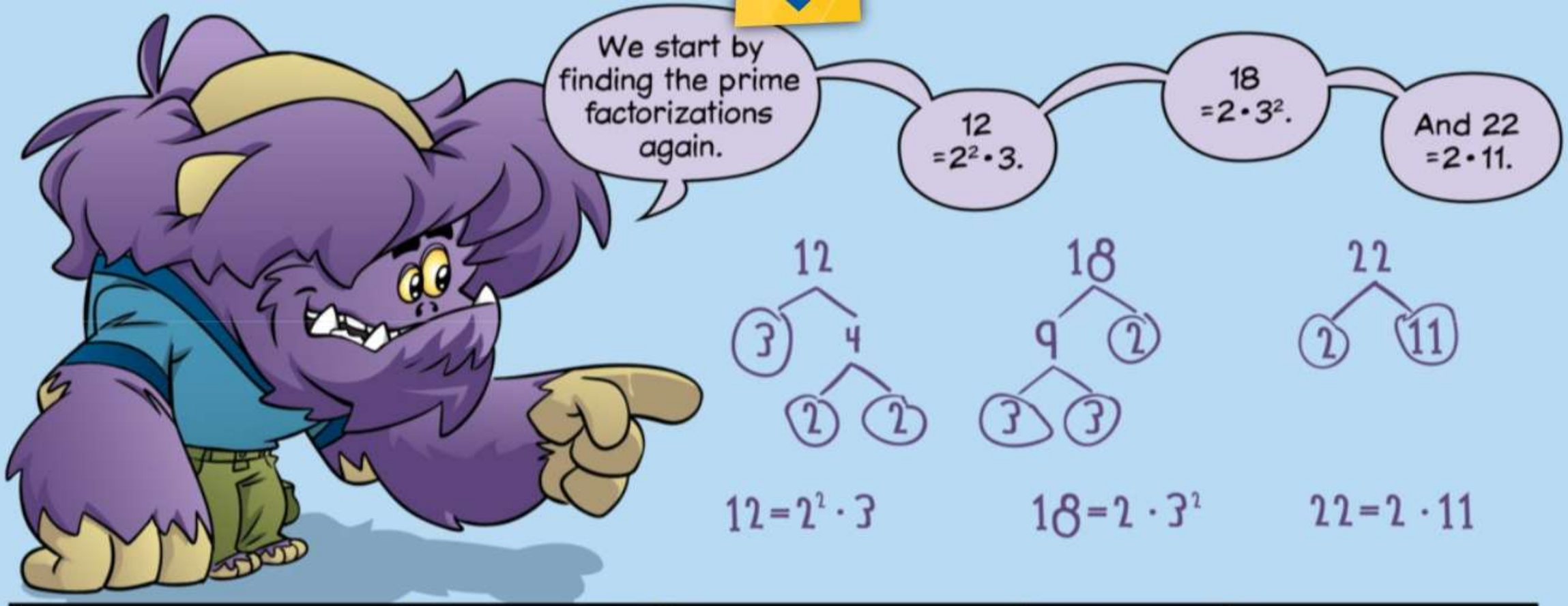
$$2\cdot 3^3\cdot 5=270$$

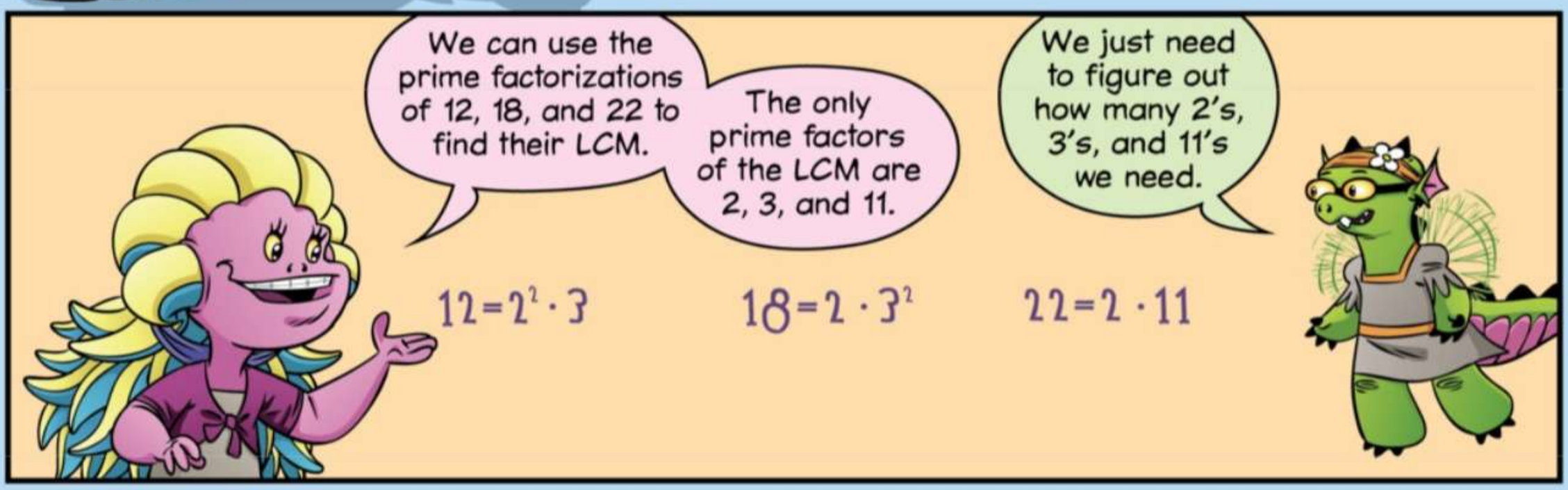


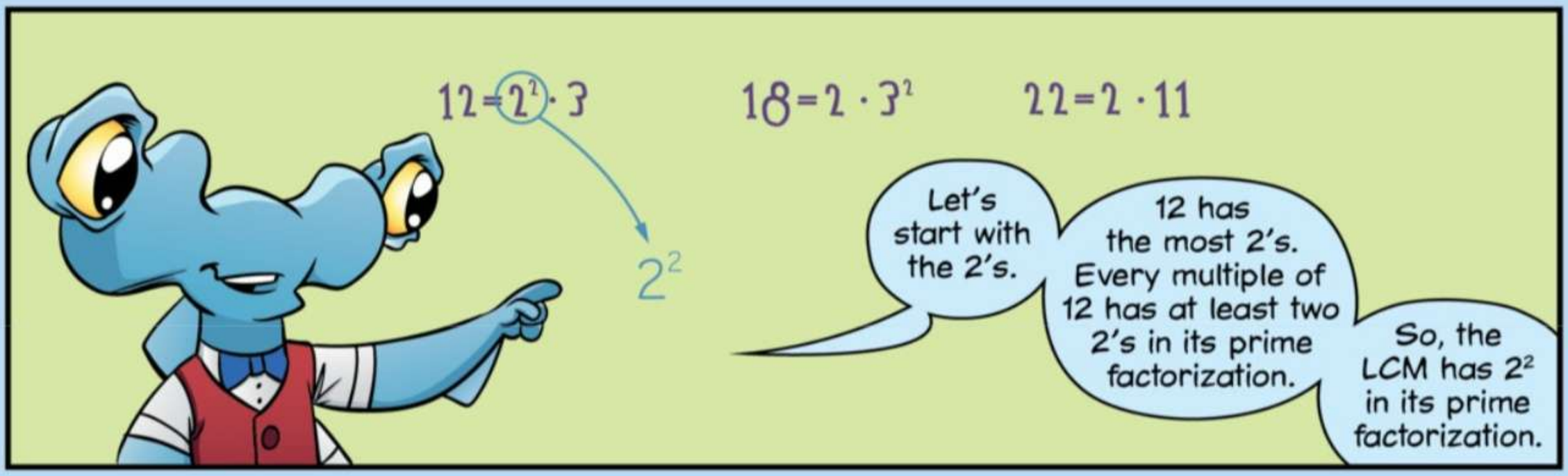
THE LEAST COMMON MULTIPLE (LCM) OF a AND b IS THE SMALLEST POSITIVE NUMBER THAT IS A MULTIPLE OF BOTH a AND b.

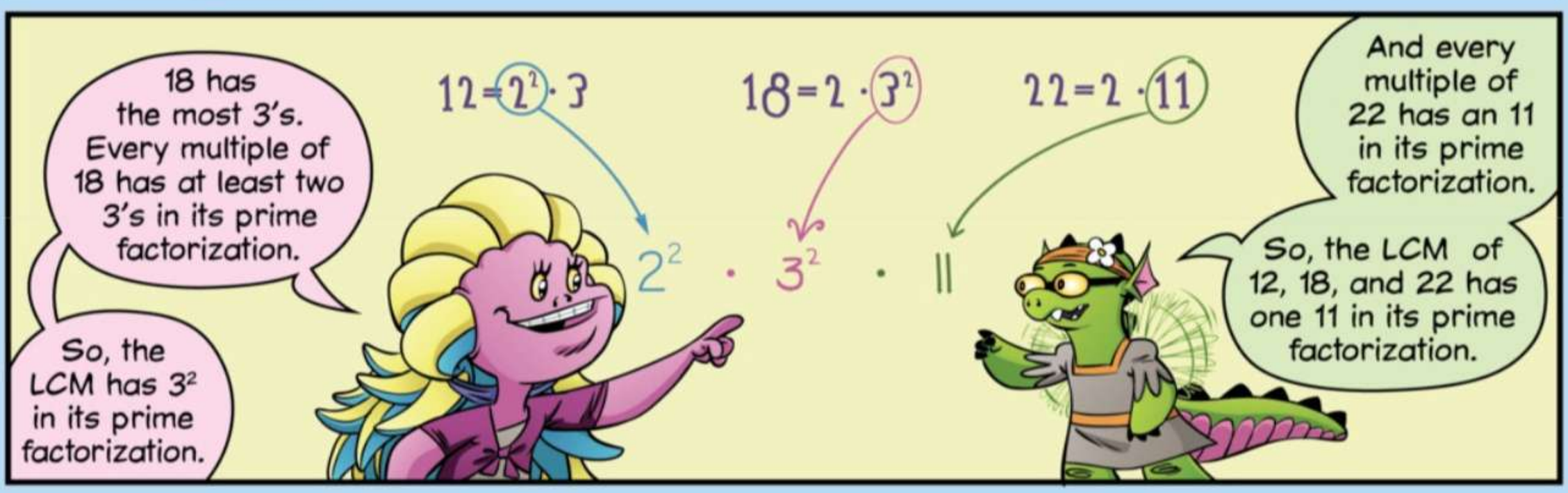
*THE LCM OF a AND b IS THE SMALLEST POSITIVE INTEGER WHOSE PRIME FACTORIZATION INCLUDES THE PRIME FACTORIZATIONS OF BOTH a AND b.

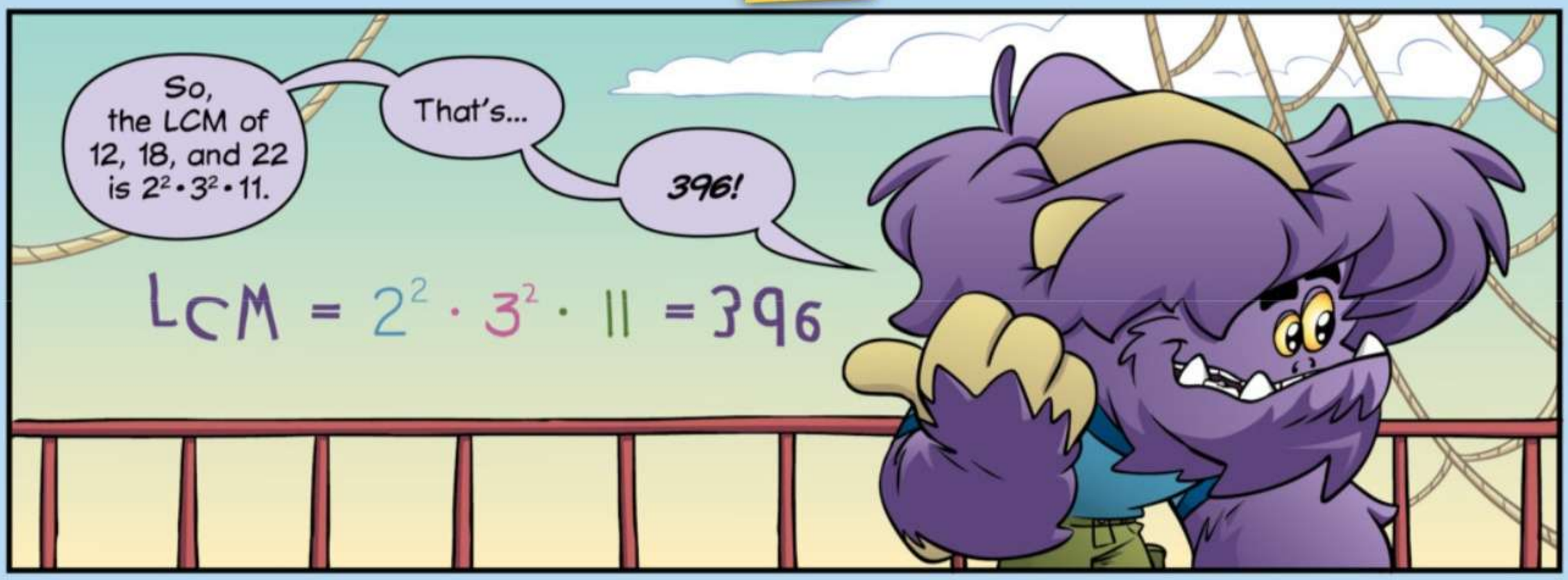
Find the LCM.


We start by finding the prime factorizations again.
12 =2²•3.
18 =2•3².
And 22 =2•11.
12
3 4
2 2
18
9 2
3 3
22
2 11
12=2²·3
18=2·3²
22=2·11


We can use the prime factorizations of 12, 18, and 22 to find their LCM.
The only prime factors of the LCM are 2, 3, and 11.
We just need to figure out how many 2's, 3's, and 11's we need.
12=2²·3
18=2·3²
22=2·11


12=2²·3
18=2·3²
22=2·11
2²
Let's start with the 2's.
12 has the most 2's. Every multiple of 12 has at least two 2's in its prime factorization.
So, the LCM has 2² in its prime factorization.


18 has the most 3's. Every multiple of 18 has at least two 3's in its prime factorization.
So, the LCM has 3² in its prime factorization.
12=2²·3
18=2·3²
22=2·11
2² · 3² · 11
And every multiple of 22 has an 11 in its prime factorization.
So, the LCM of 12, 18, and 22 has one 11 in its prime factorization.

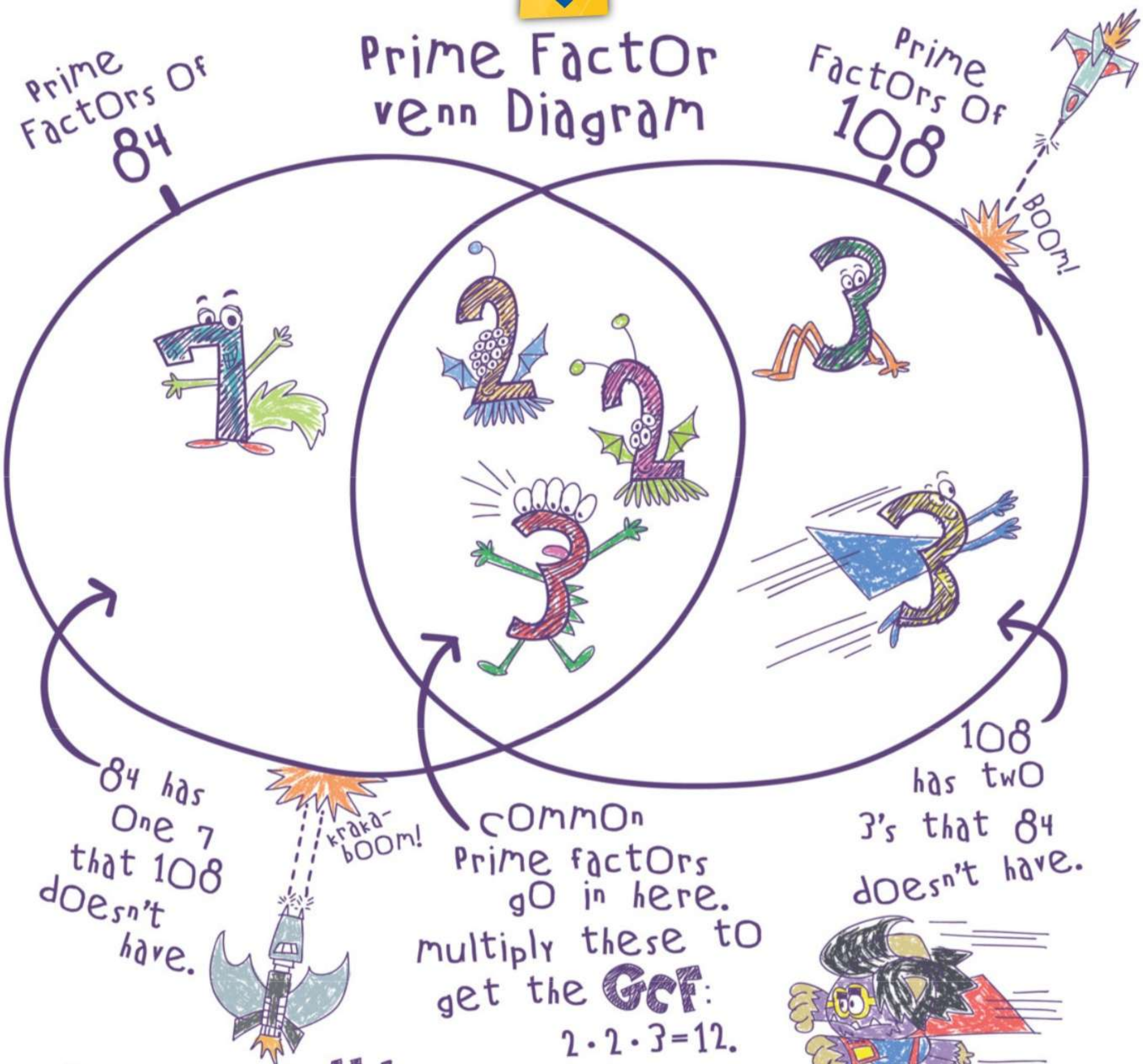

So, the LCM of 12, 18, and 22 is $2^2 \cdot 3^2 \cdot 11$.
That's...
396!
$LCM = 2^2 \cdot 3^2 \cdot 11 = 396$

Aye. The smallest number that be a multiple o' 12, 18, 'n' 22 be 396.
Not bad for a few junior buccaneers!

So, what are you planning to do with all of this loot, Captain Kraken?

Arrrr... I've been savin' up for a new basketball court on me deck.

Prime Factor venn Diagram



84 has one 7 that 108 doesn't have.

common prime factors go in here. multiply these to get the **GCF**: $2 \cdot 2 \cdot 3 = 12$.

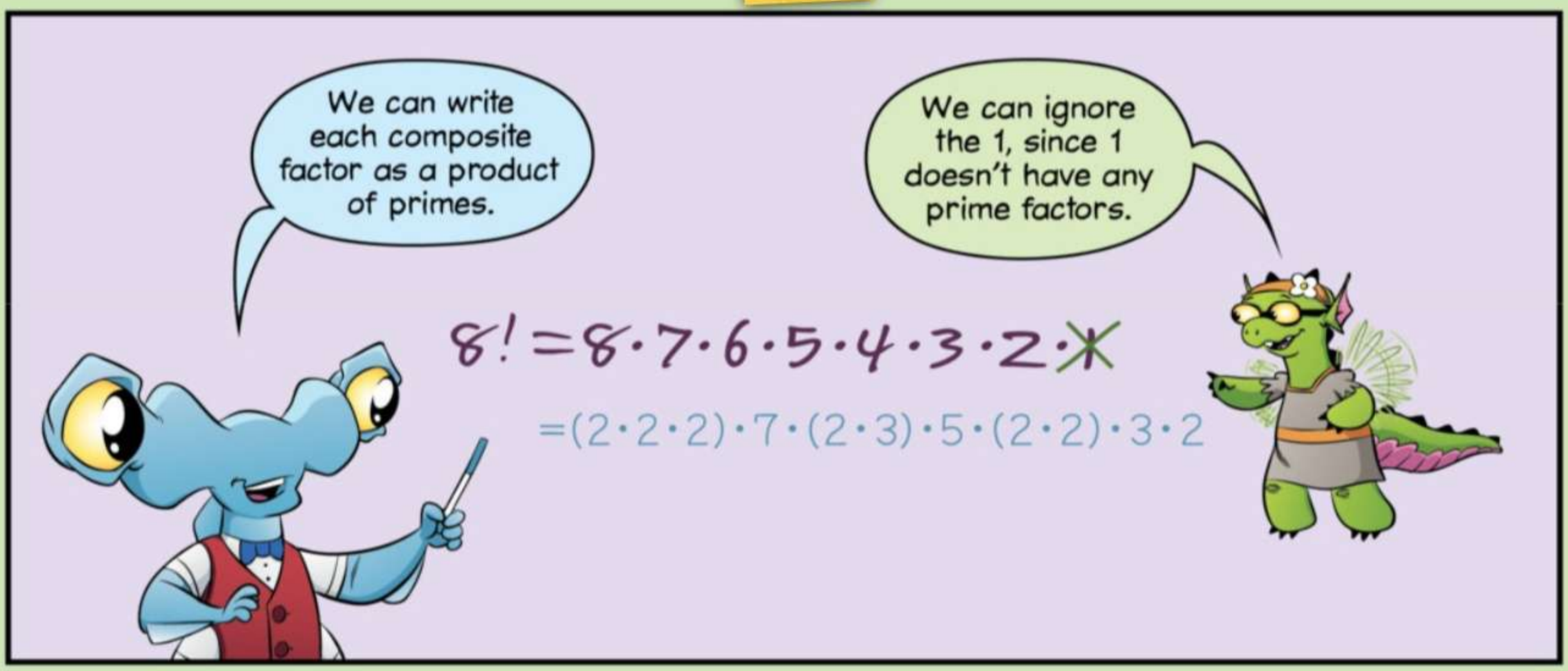
108 has two 3's that 84 doesn't have.

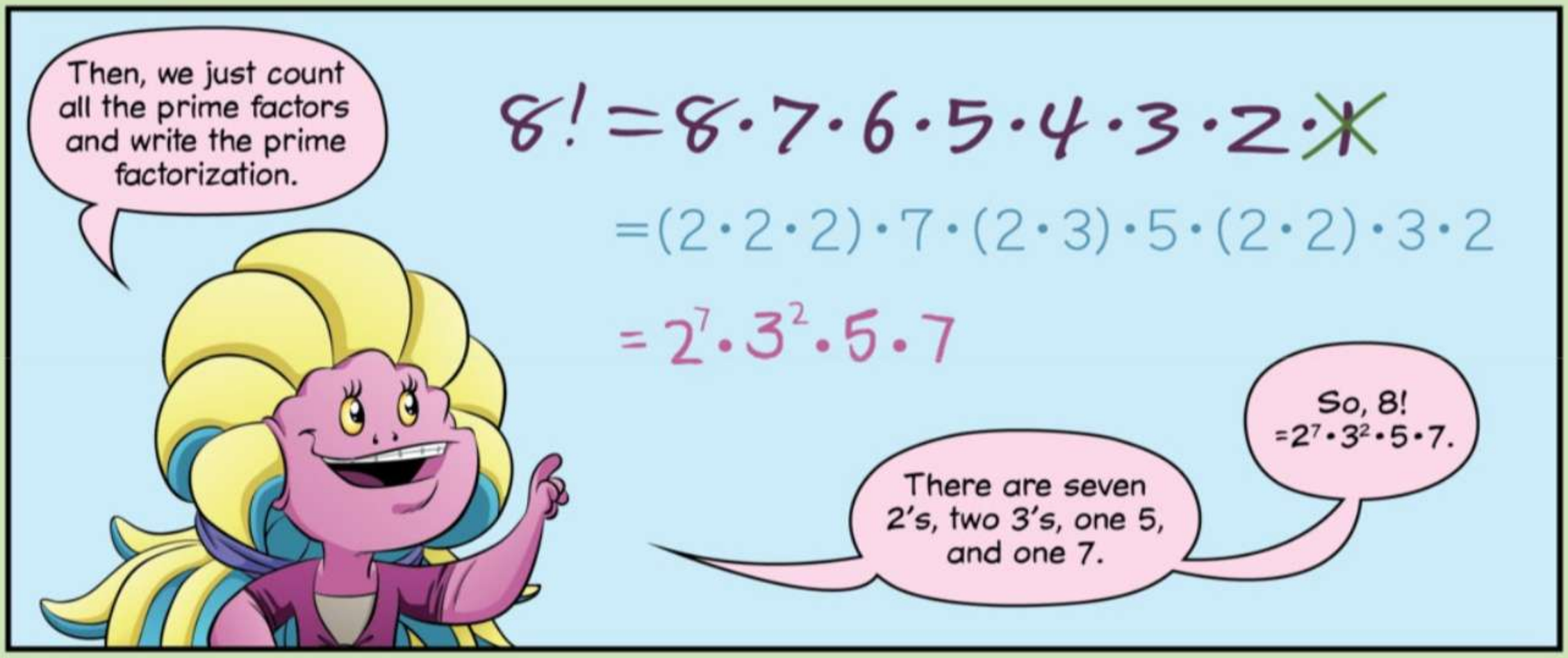
If we multiply **ALL** of the prime factors, we get 756, which is a multiple of 84 and of 108:

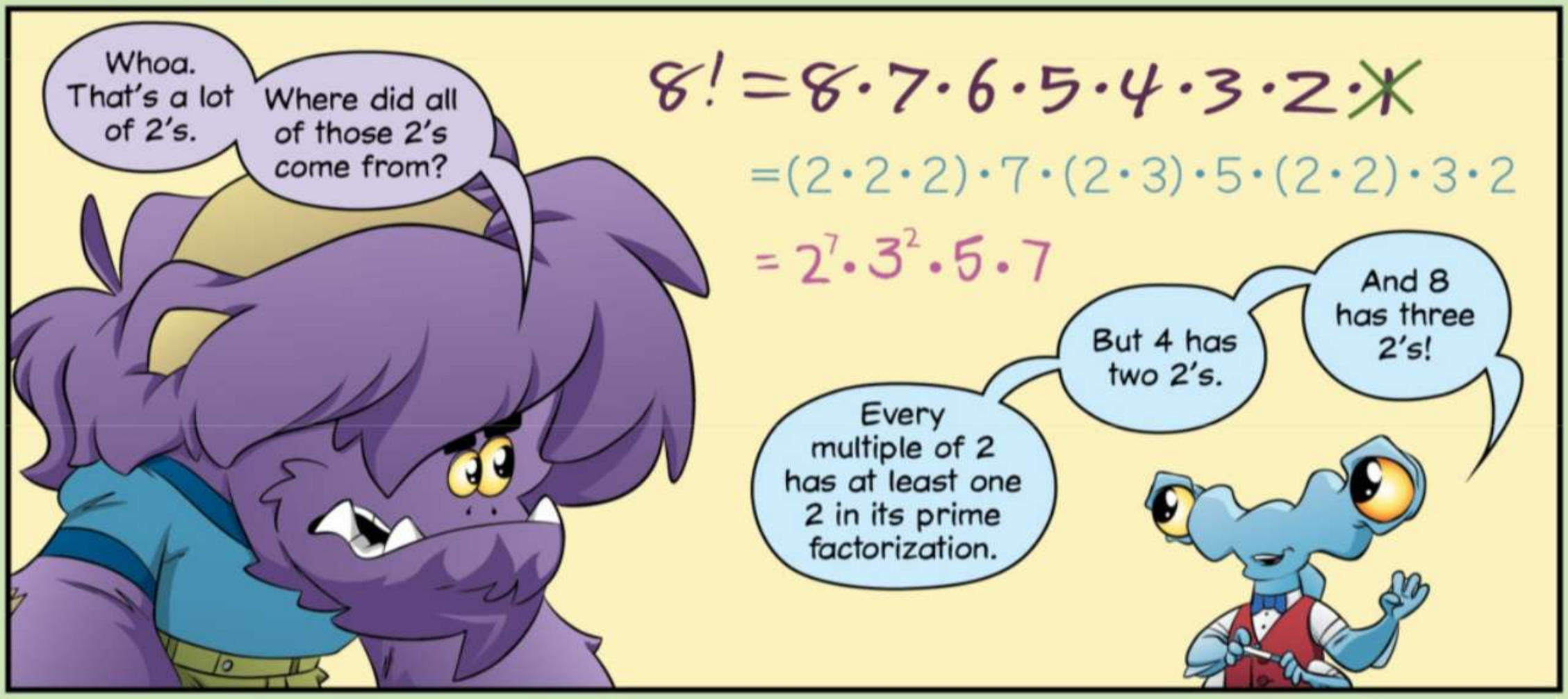
$$(7 \cdot 2 \cdot 2 \cdot 3) \cdot 3 \cdot 3 = 84 \cdot 9 = 756$$
$$7 \cdot (2 \cdot 2 \cdot 3 \cdot 3 \cdot 3) = 7 \cdot 108 = 756$$

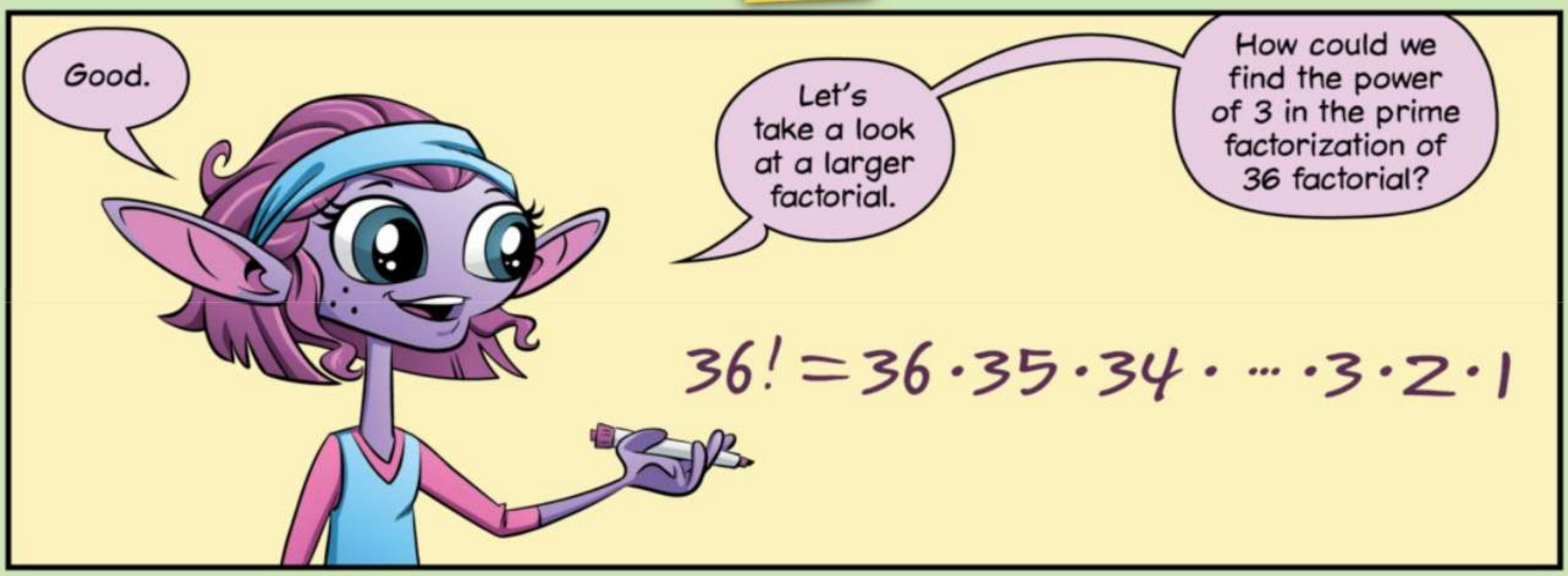
If we remove any of these prime factors, the product won't be a multiple of both 84 and 108. so, 756 is the **LCM** of 84 & 108.

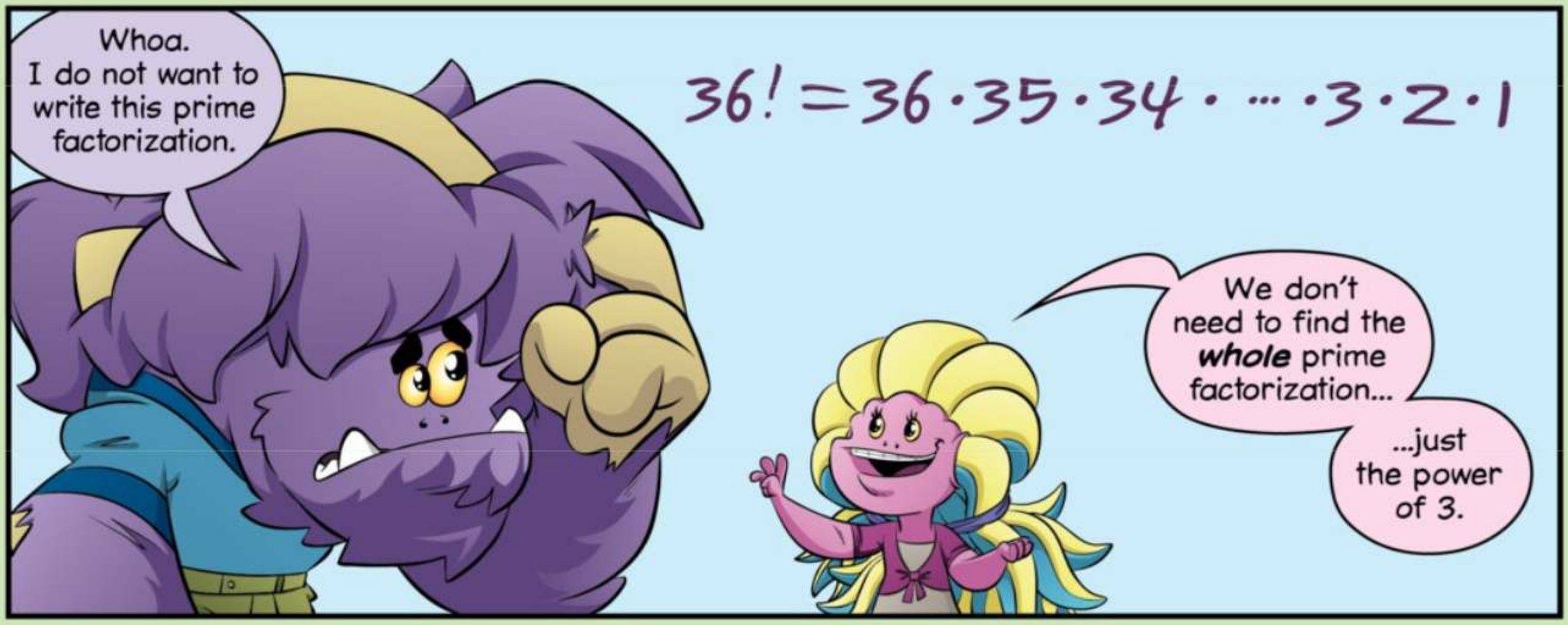
MATH TEAM
Factorials
Today's problems involve *factorial* notation.
4!
Oh, right! I remember factorials.
FOUR!
The exclamation point does *not* mean that we shout the number.
Awwww.
WE FIRST LEARNED ABOUT FACTORIALS IN THE COUNTING CHAPTER OF BEAST ACADEMY 4B.
A number followed by an exclamation point represents the product of the number and all of the positive integers below it.
For example, 4! means 4·3·2·1.
4! = 4·3·2·1
To begin, try writing the prime factorization of 8 factorial.
8! = 8·7·6·5·4·3·2·1
Try it.

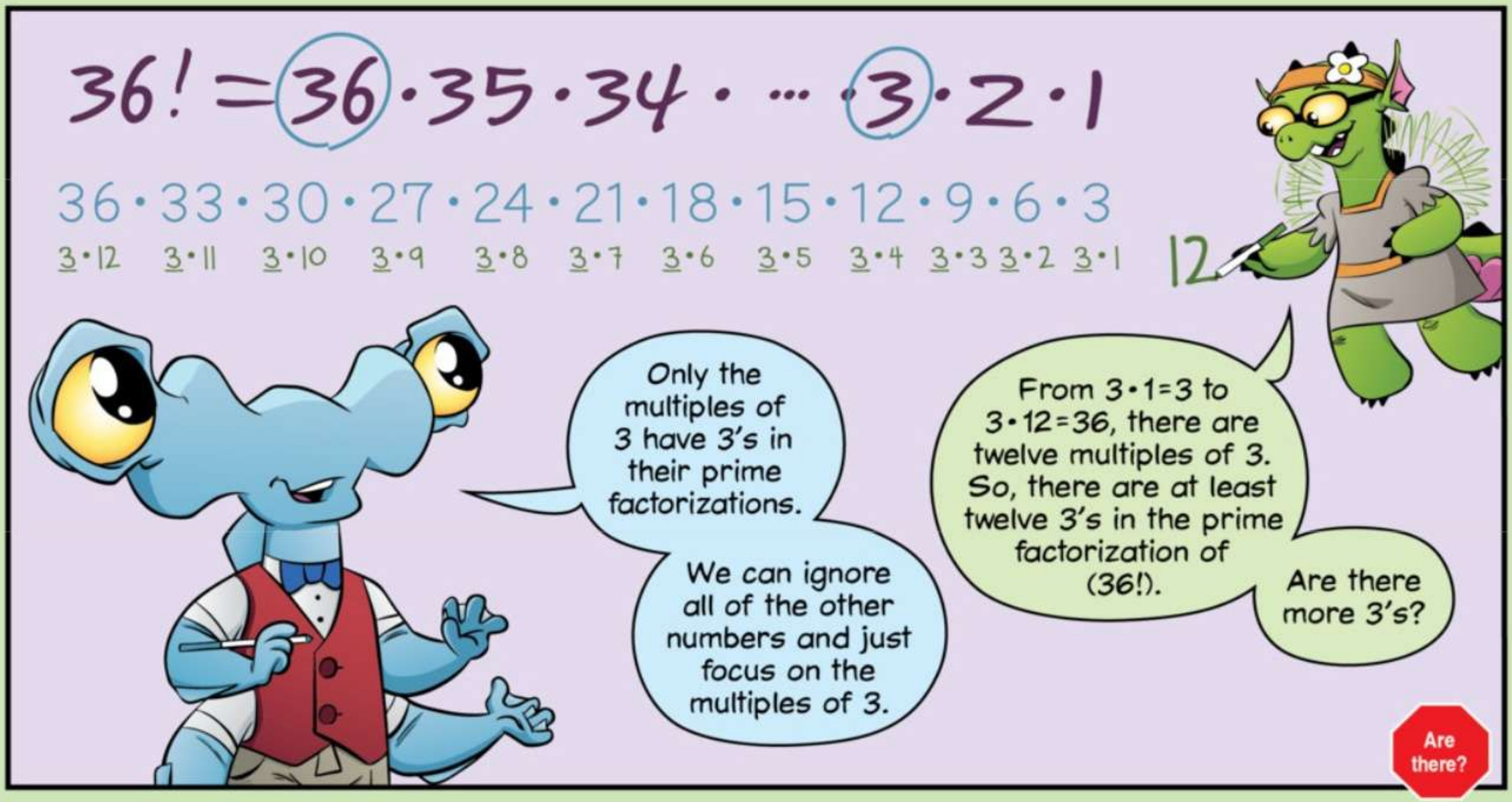

We can write each composite factor as a product of primes.
We can ignore the 1, since 1 doesn't have any prime factors.
8! = 8·7·6·5·4·3·2·1
=(2·2·2)·7·(2·3)·5·(2·2)·3·2

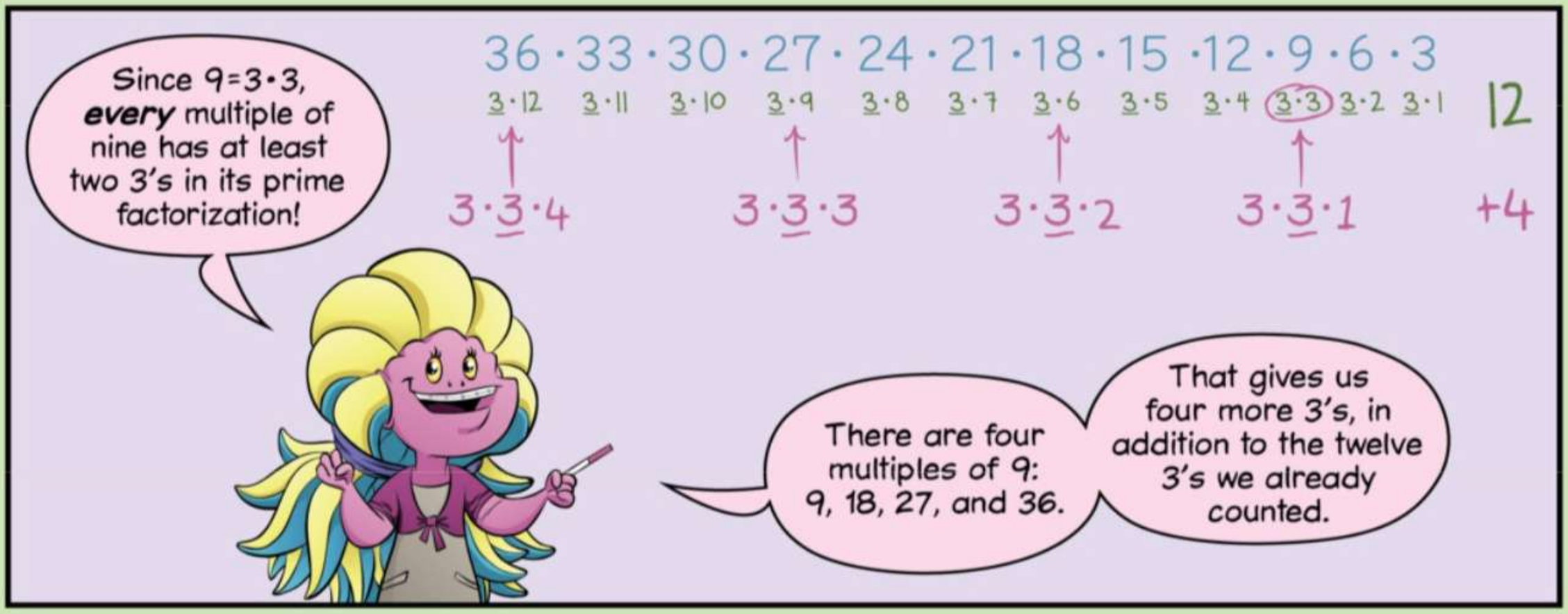

Then, we just count all the prime factors and write the prime factorization.
8! = 8·7·6·5·4·3·2·1
=(2·2·2)·7·(2·3)·5·(2·2)·3·2
= 2^7·3^2·5·7
There are seven 2's, two 3's, one 5, and one 7.
So, 8! =2^7·3^2·5·7.

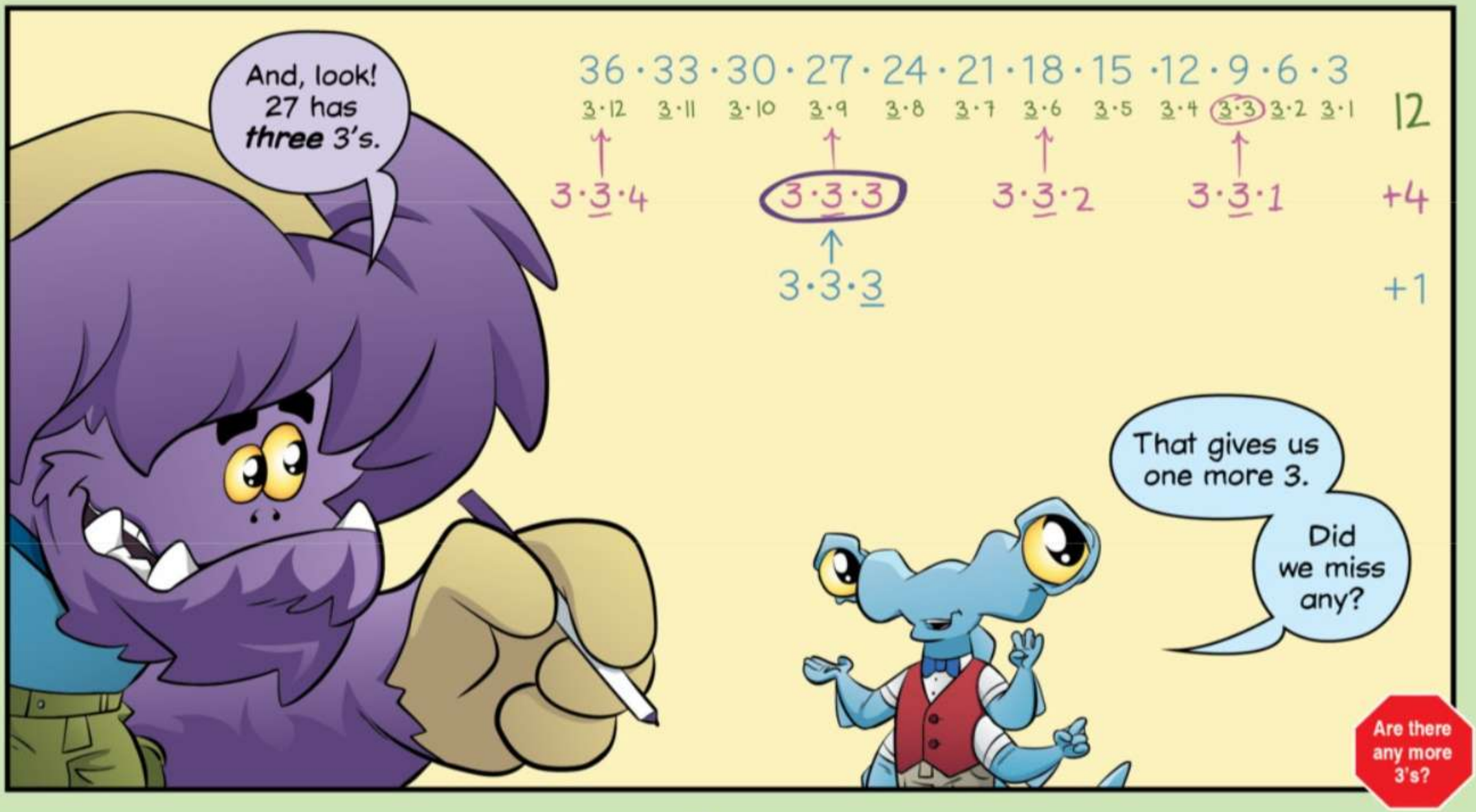

Whoa. That's a lot of 2's.
Where did all of those 2's come from?
8! = 8·7·6·5·4·3·2·1
=(2·2·2)·7·(2·3)·5·(2·2)·3·2
= 2^7·3^2·5·7
Every multiple of 2 has at least one 2 in its prime factorization.
But 4 has two 2's.
And 8 has three 2's!


Good.
Let's take a look at a larger factorial.
How could we find the power of 3 in the prime factorization of 36 factorial?
$36! = 36 \cdot 35 \cdot 34 \cdot \cdots \cdot 3 \cdot 2 \cdot 1$


Whoa. I do not want to write this prime factorization.
$36! = 36 \cdot 35 \cdot 34 \cdot \cdots \cdot 3 \cdot 2 \cdot 1$
We don't need to find the **whole** prime factorization...
...just the power of 3.


$36! = 36 \cdot 35 \cdot 34 \cdot \cdots \cdot 3 \cdot 2 \cdot 1$
36 • 33 • 30 • 27 • 24 • 21 • 18 • 15 • 12 • 9 • 6 • 3
3•12 3•11 3•10 3•9 3•8 3•7 3•6 3•5 3•4 3•3 3•2 3•1
12
Only the multiples of 3 have 3's in their prime factorizations.
We can ignore all of the other numbers and just focus on the multiples of 3.
From 3•1=3 to 3•12=36, there are twelve multiples of 3. So, there are at least twelve 3's in the prime factorization of (36!).
Are there more 3's?
Are there?





Are there any more 3's?

THE NUMBER 500! HAS 1,135 DIGITS, INCLUDING ALL OF THE ZEROS AT THE END!

How many zeros are at the end of (500!)?

What kinds of numbers have zeros at the end?

Multiples of 10 end in at least one zero.

Multiples of $10 \cdot 10 = 100$ end in at least two zeros.

Multiples of $10 \cdot 10 \cdot 10 = 1{,}000$ end in at least three zeros.

Every time you multiply by another 10, you get another zero.

$$398 \cdot 10 = 3{,}980$$

$$19 \cdot 10 \cdot 10 = 1{,}900$$

$$643 \cdot 10 \cdot 10 \cdot 10 = 643{,}000$$

THE ZEROS AT THE END OF A NUMBER ARE CALLED ***TRAILING ZEROS***. FOR EXAMPLE, 643,000 HAS THREE TRAILING ZEROS.

So, we need to see how many 10's are in the prime factorization of 500 factorial.

Grogg! There are no 10's in the ***prime*** factorization of (500!).

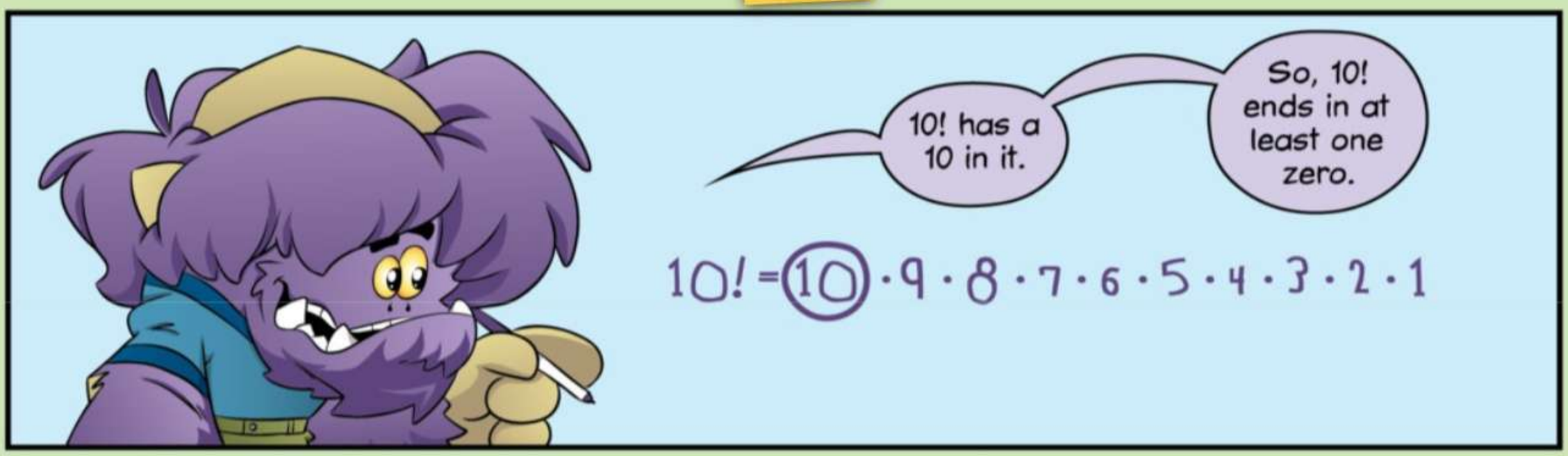
10 is not prime.

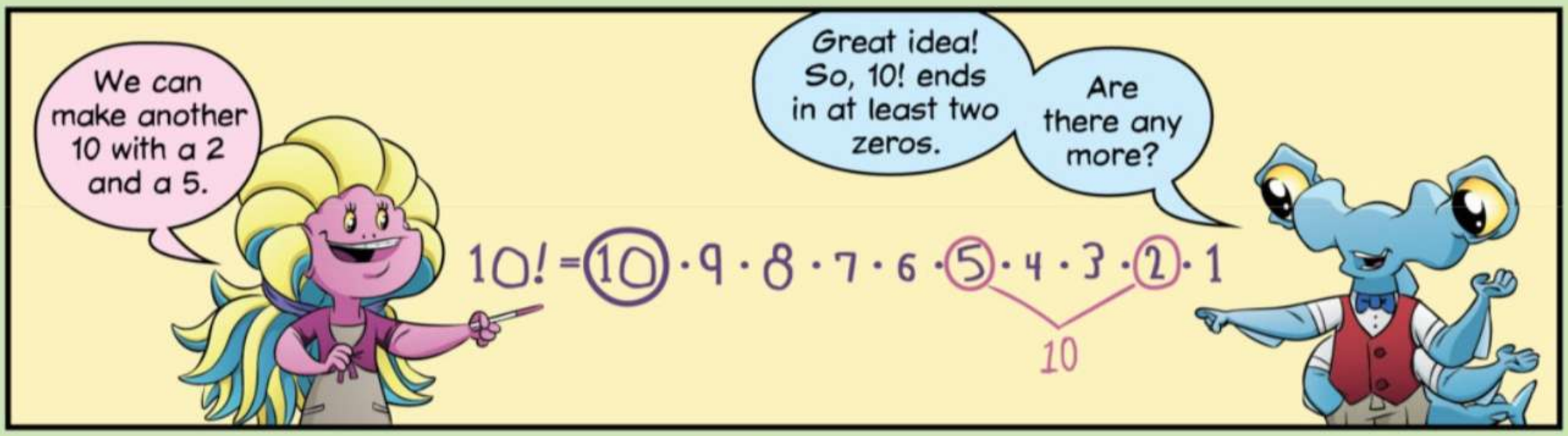
Oh, right. Then, how can we find out how many 10's are in 500 factorial?

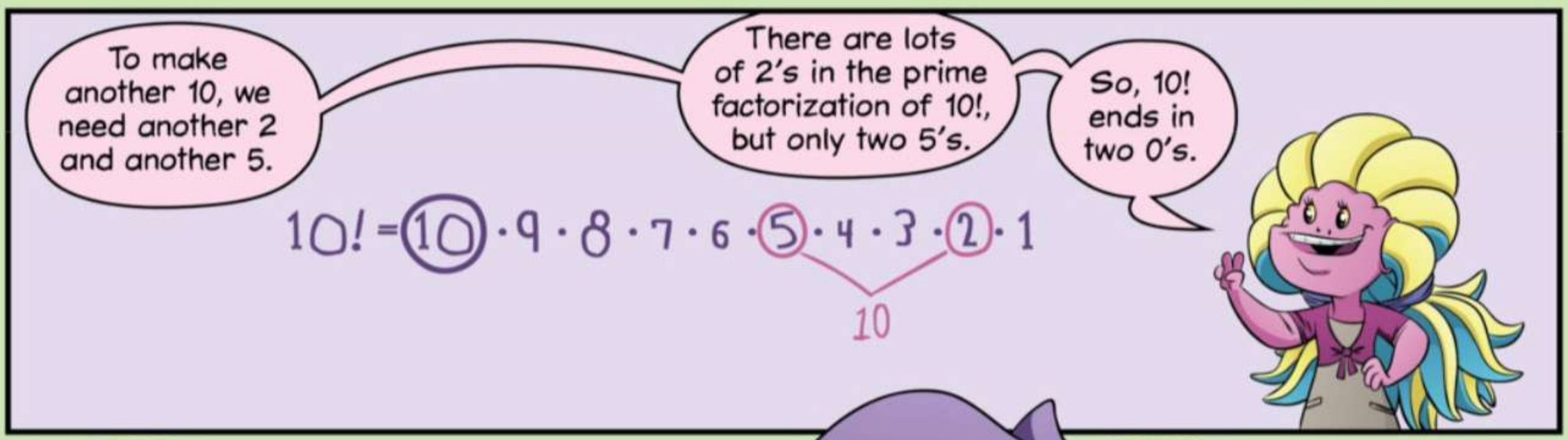
Try starting with an easier number, like 10 factorial.

How many zeros are at the end of 10 factorial?

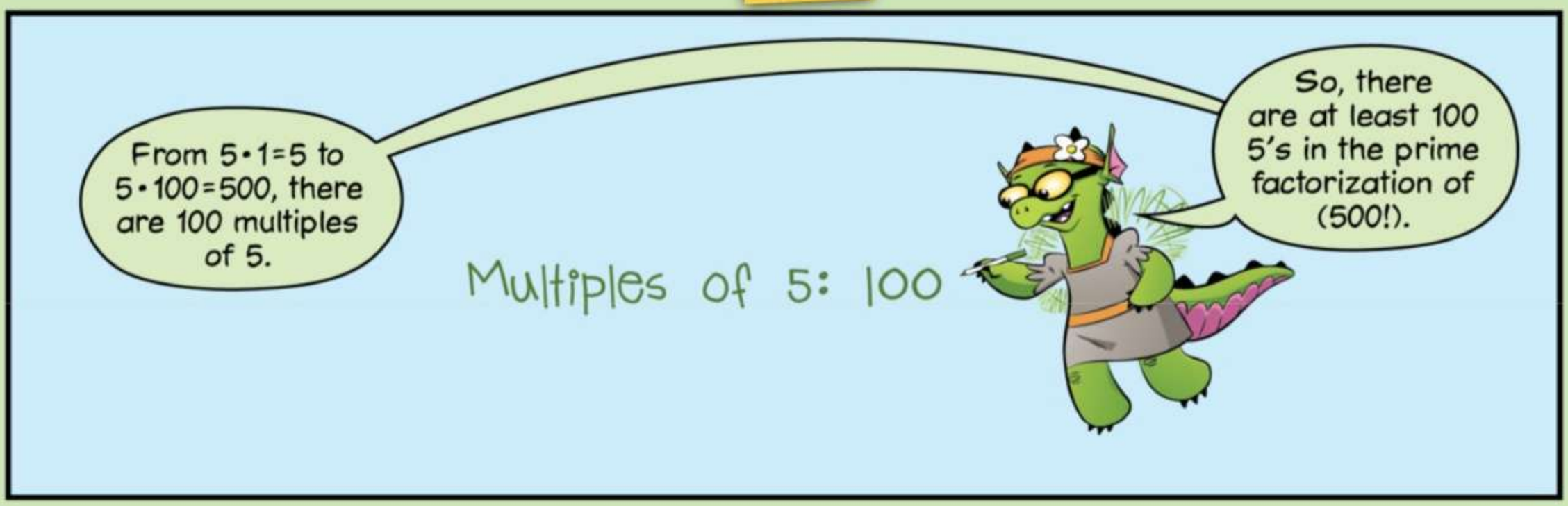
Try it.

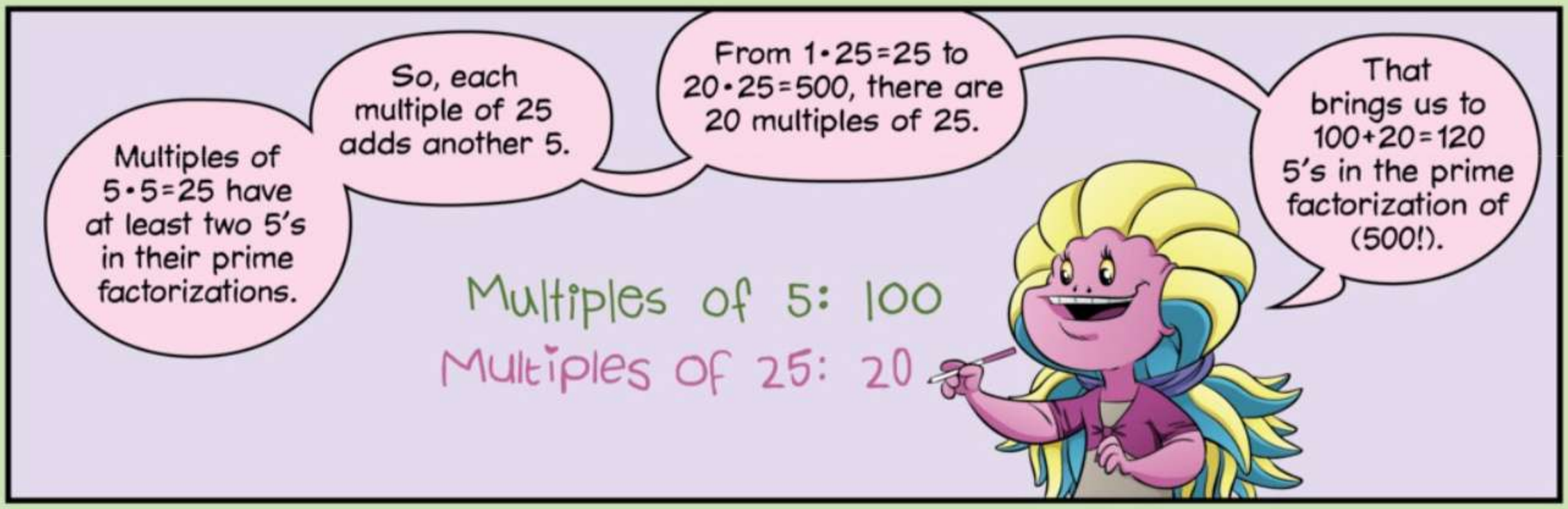

10! has a 10 in it.
So, 10! ends in at least one zero.
10! = 10 · 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1

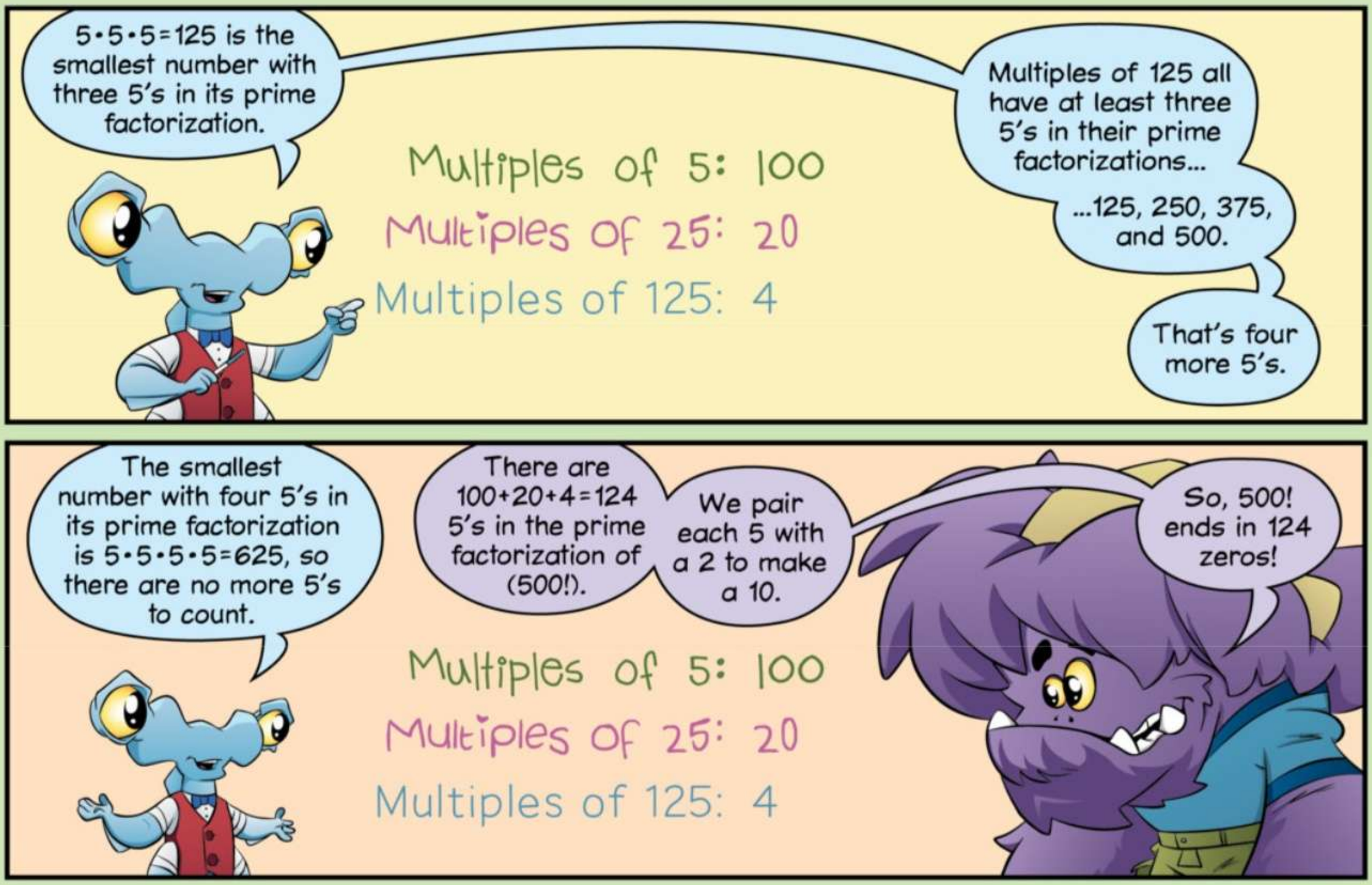

We can make another 10 with a 2 and a 5.
Great idea! So, 10! ends in at least two zeros.
Are there any more?
10! = 10 · 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1
10

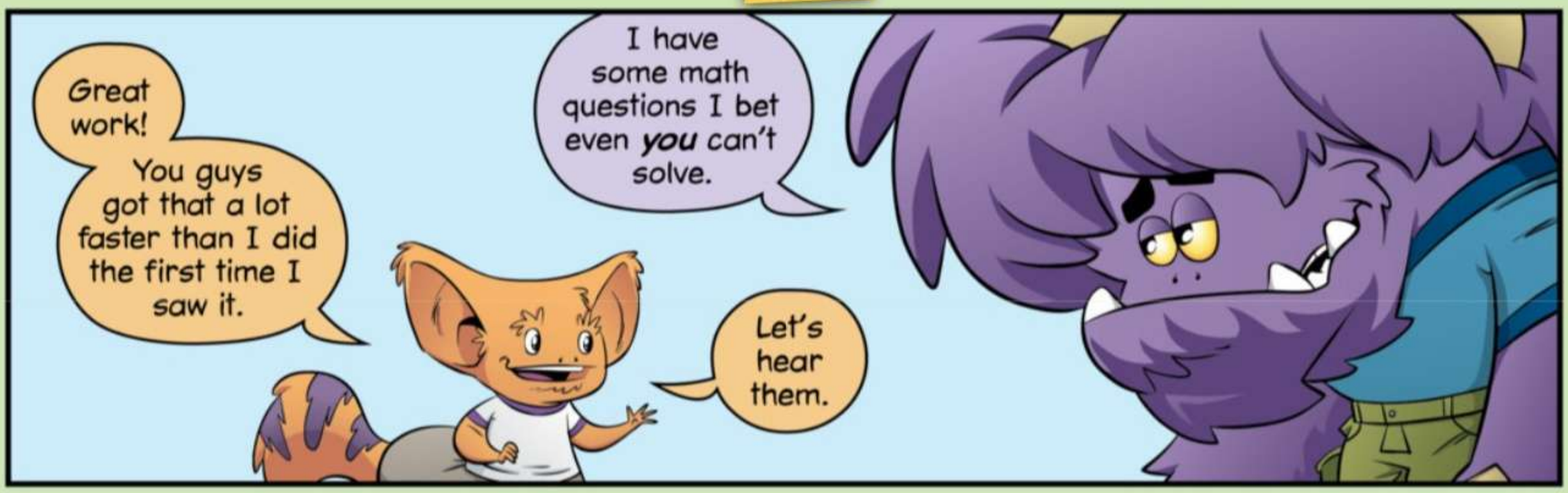

To make another 10, we need another 2 and another 5.
There are lots of 2's in the prime factorization of 10!, but only two 5's.
So, 10! ends in two 0's.
10! = 10 · 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1
10

I see. To count how many zeros are at the end of a factorial, we count how many 5's are in its prime factorization.
There will always be more 2's than 5's, and we can pair every 5 with a 2 to make a 10.
Great! Let's count the 5's in (500!).
How many 5's are in the prime factorization of (500!)?

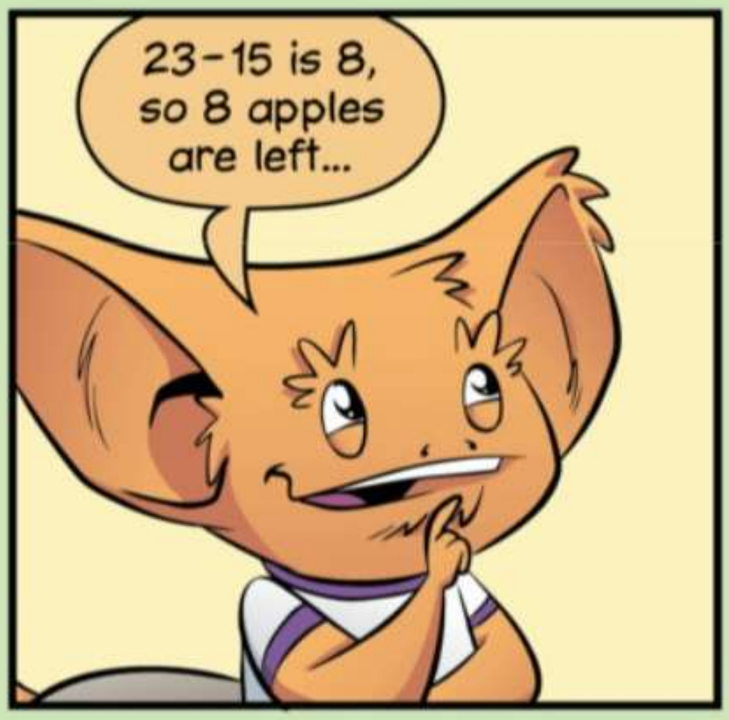

From 5•1=5 to 5•100=500, there are 100 multiples of 5.
So, there are at least 100 5's in the prime factorization of (500!).
Multiples of 5: 100


Multiples of 5•5=25 have at least two 5's in their prime factorizations.
So, each multiple of 25 adds another 5.
From 1•25=25 to 20•25=500, there are 20 multiples of 25.
That brings us to 100+20=120 5's in the prime factorization of (500!).
Multiples of 5: 100
Multiples of 25: 20


5•5•5=125 is the smallest number with three 5's in its prime factorization.
Multiples of 125 all have at least three 5's in their prime factorizations...
...125, 250, 375, and 500.
That's four more 5's.
Multiples of 5: 100
Multiples of 25: 20
Multiples of 125: 4
The smallest number with four 5's in its prime factorization is 5•5•5•5=625, so there are no more 5's to count.
There are 100+20+4=124 5's in the prime factorization of (500!).
We pair each 5 with a 2 to make a 10.
So, 500! ends in 124 zeros!
Multiples of 5: 100
Multiples of 25: 20
Multiples of 125: 4


Great work!
You guys got that a lot faster than I did the first time I saw it.
I have some math questions I bet even *you* can't solve.
Let's hear them.

If there are 23 apples, and you take away 15, how many apples do you have?


23−15 is 8, so 8 apples are left...

...but, *I* have *15*.

How many cubic feet of dirt are in a hole that is 15-by-17-by-29 feet?

Hmmm... 15 • 17 • 29 =7,395...

...but since it's a *hole*, there are *0* cubic feet of dirt in it.

OK, but you'll never get this one. I have two coins worth a total of 30 cents. One of the coins is not a nickel. What coins do I have?

There's only one way to make 30 cents with two coins.
A quarter and a nickel are worth 30 cents...

...and *one* of the coins is not a nickel.
Man, this guy is good!