Contents: Chapter 11

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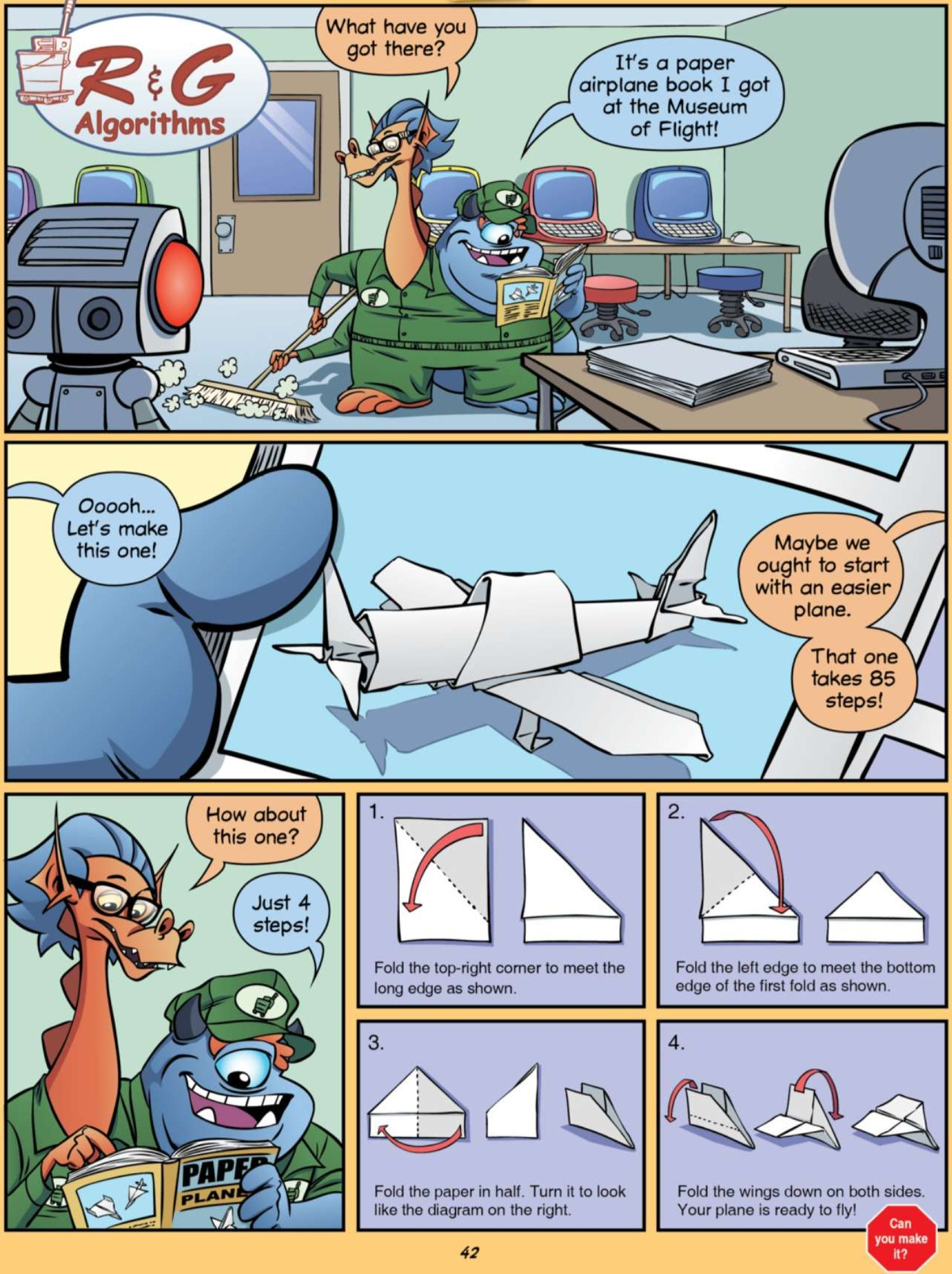
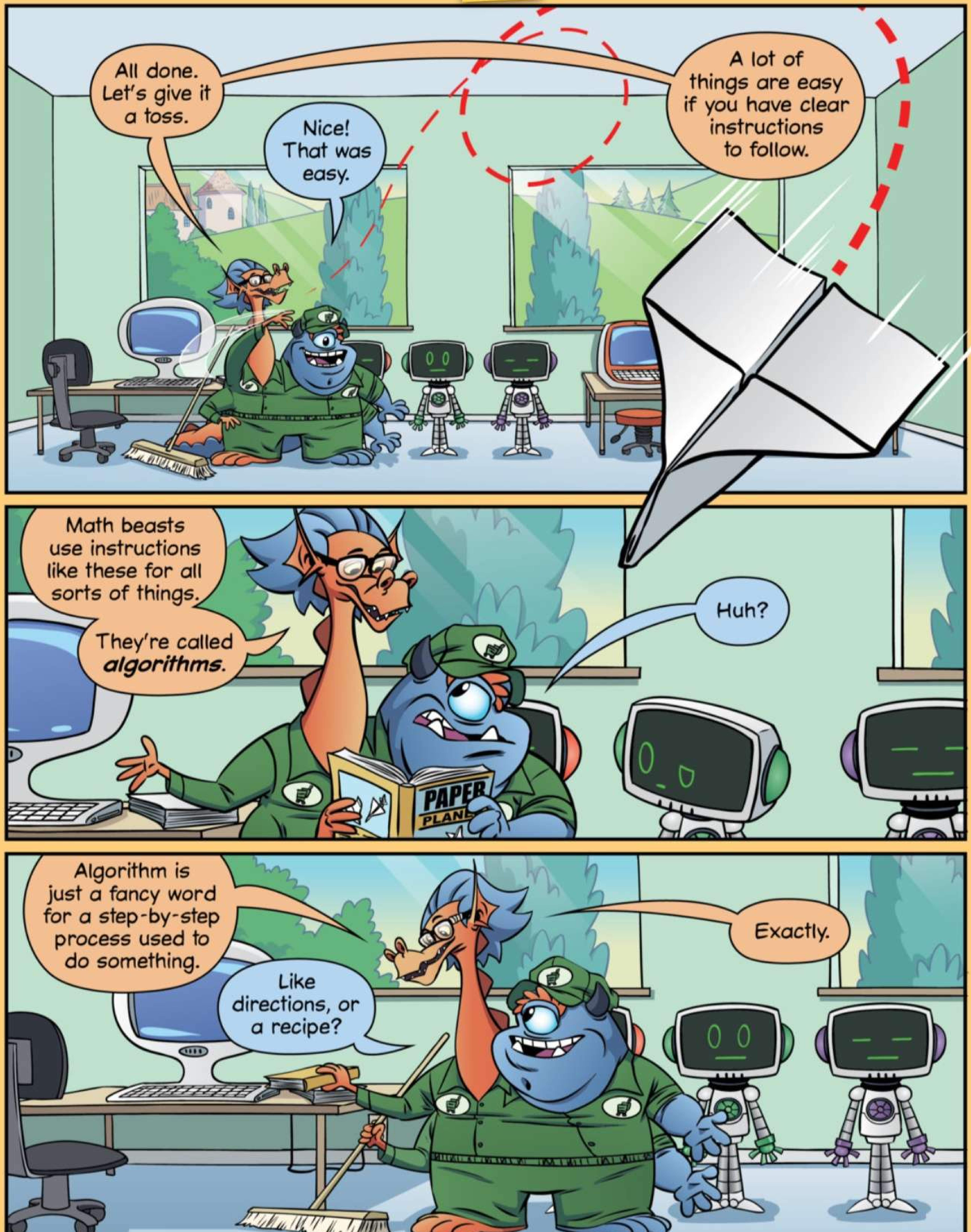
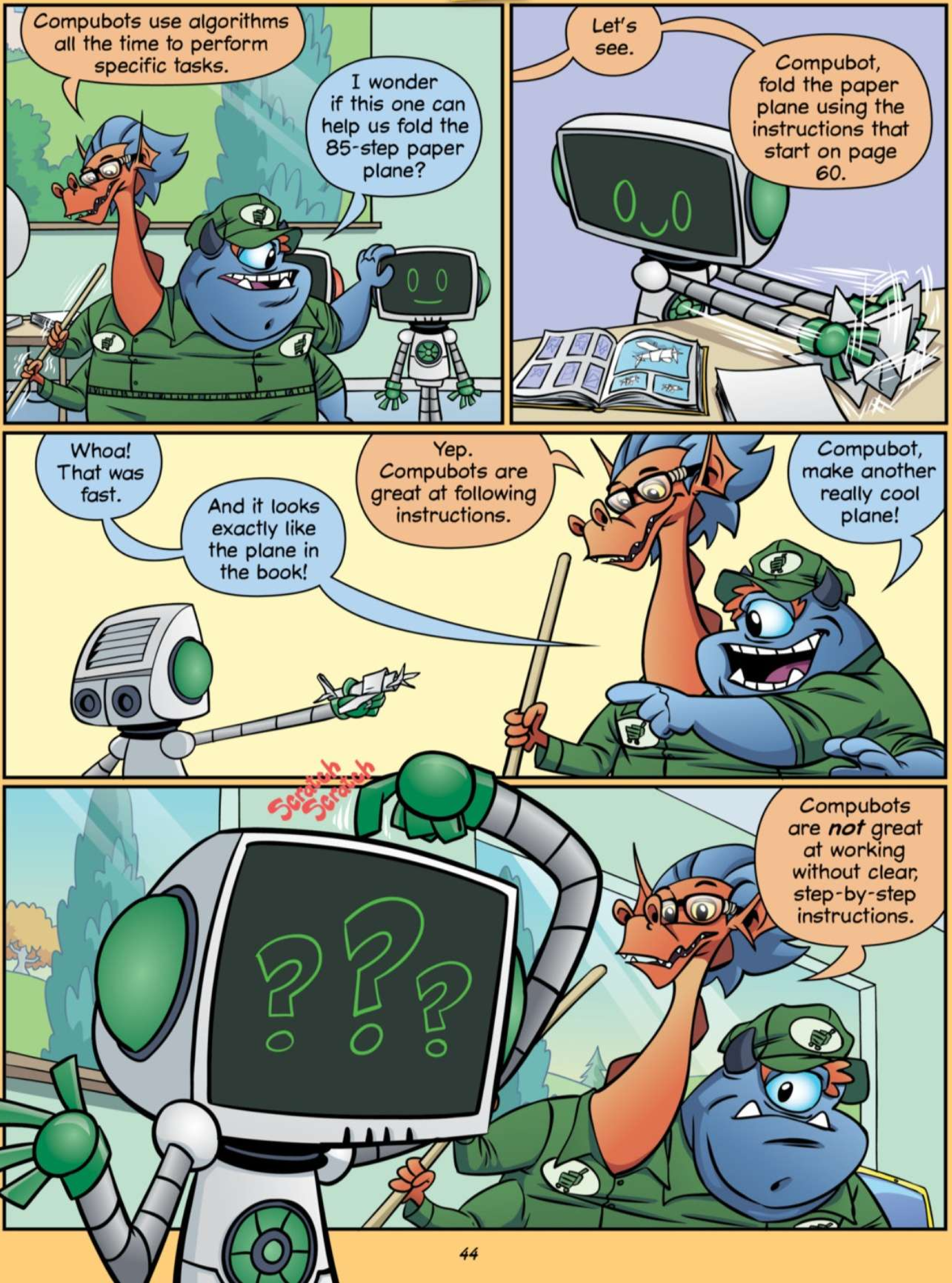
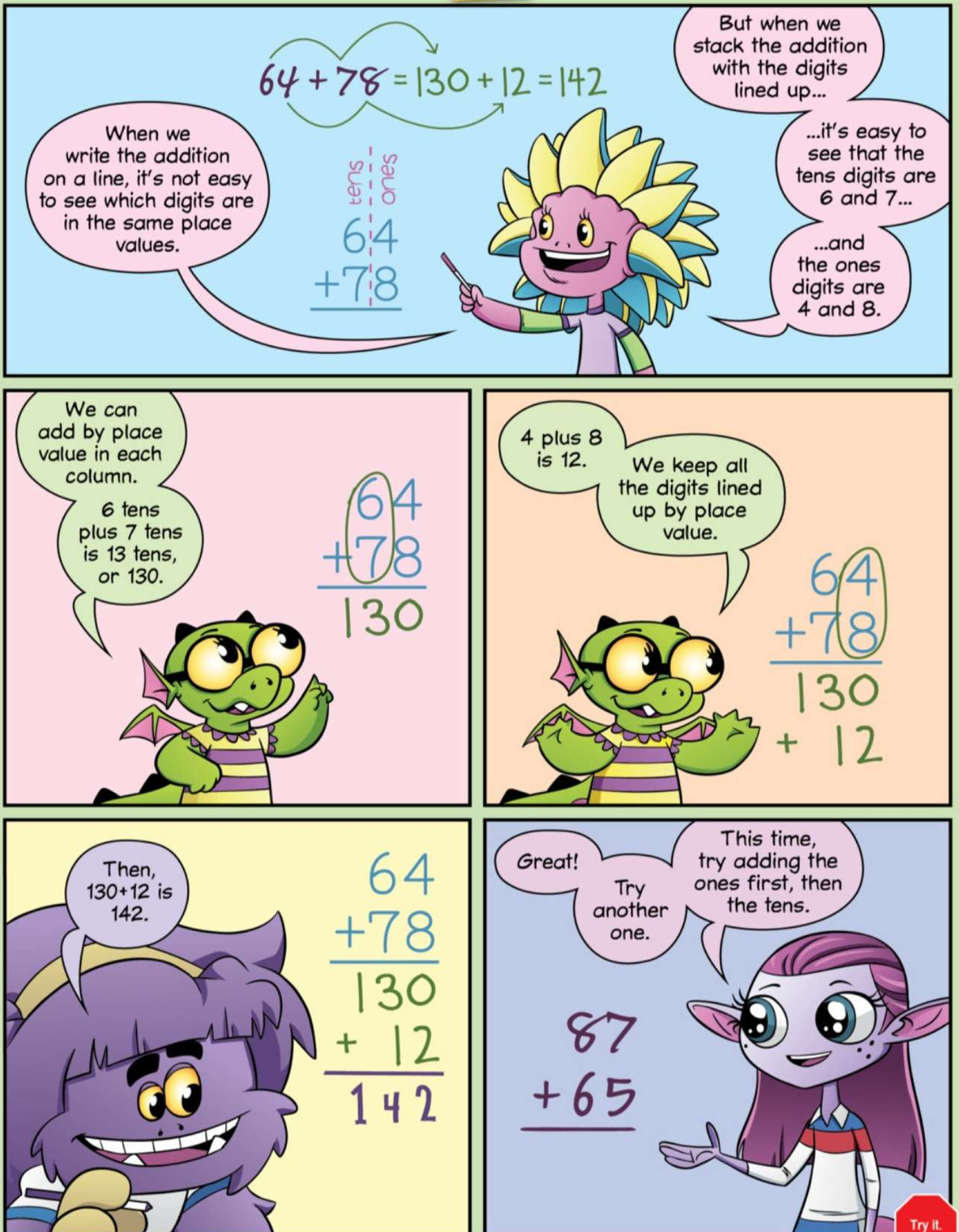
Chapter 11:
Algorithms

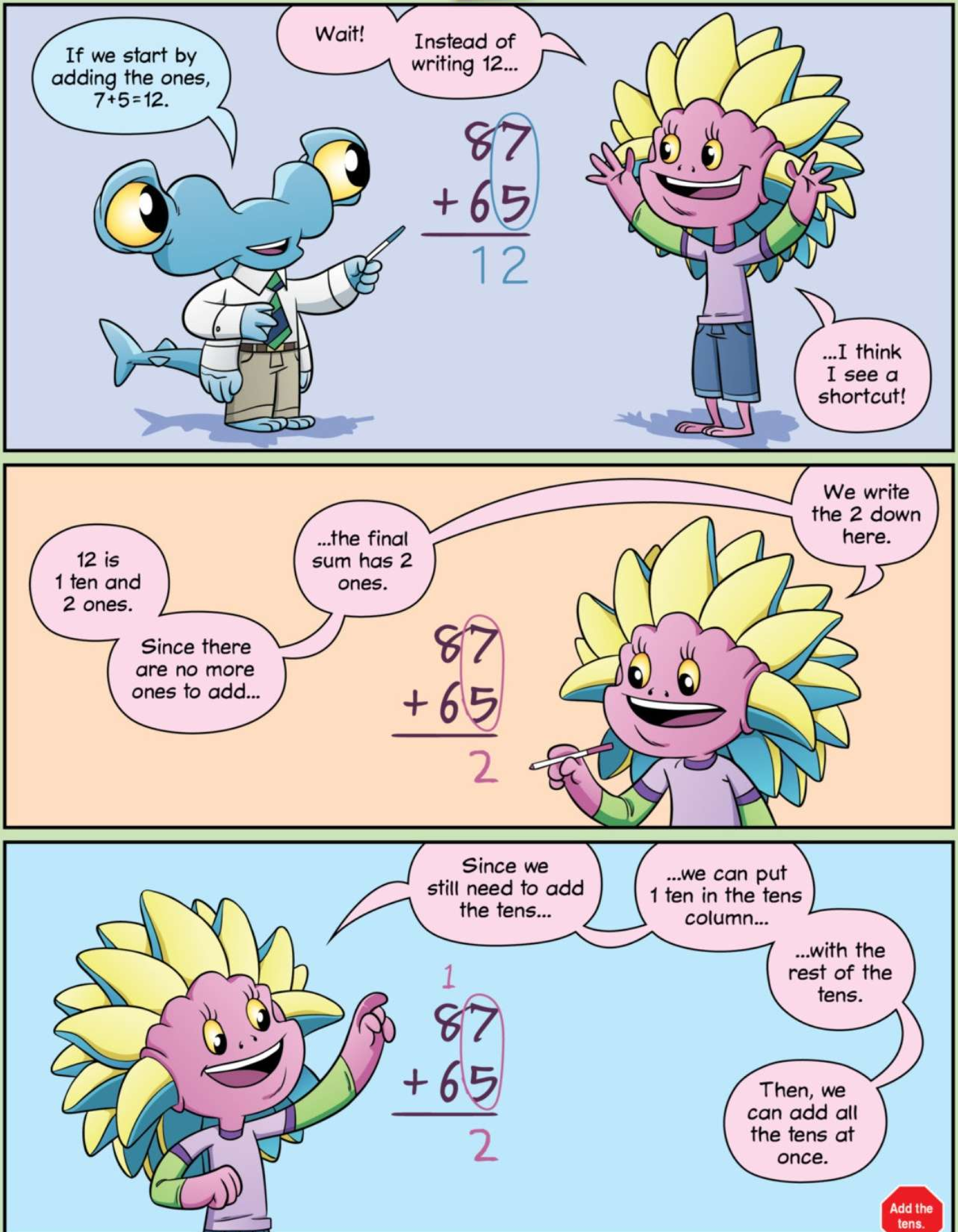

R&G Algorithms
What have you got there?
It's a paper airplane book I got at the Museum of Flight!
Ooooh... Let's make this one!
Maybe we ought to start with an easier plane.
That one takes 85 steps!
How about this one?
Just 4 steps!
PAPER PLANE
1.
Fold the top-right corner to meet the long edge as shown.
2.
Fold the left edge to meet the bottom edge of the first fold as shown.
3.
Fold the paper in half. Turn it to look like the diagram on the right.
4.
Fold the wings down on both sides. Your plane is ready to fly!
Can you make it?


All done. Let's give it a toss.
Nice! That was easy.
A lot of things are easy if you have clear instructions to follow.
Math beasts use instructions like these for all sorts of things.
They're called ***algorithms***.
Huh?
PAPER PLANE
Algorithm is just a fancy word for a step-by-step process used to do something.
Like directions, or a recipe?
Exactly.

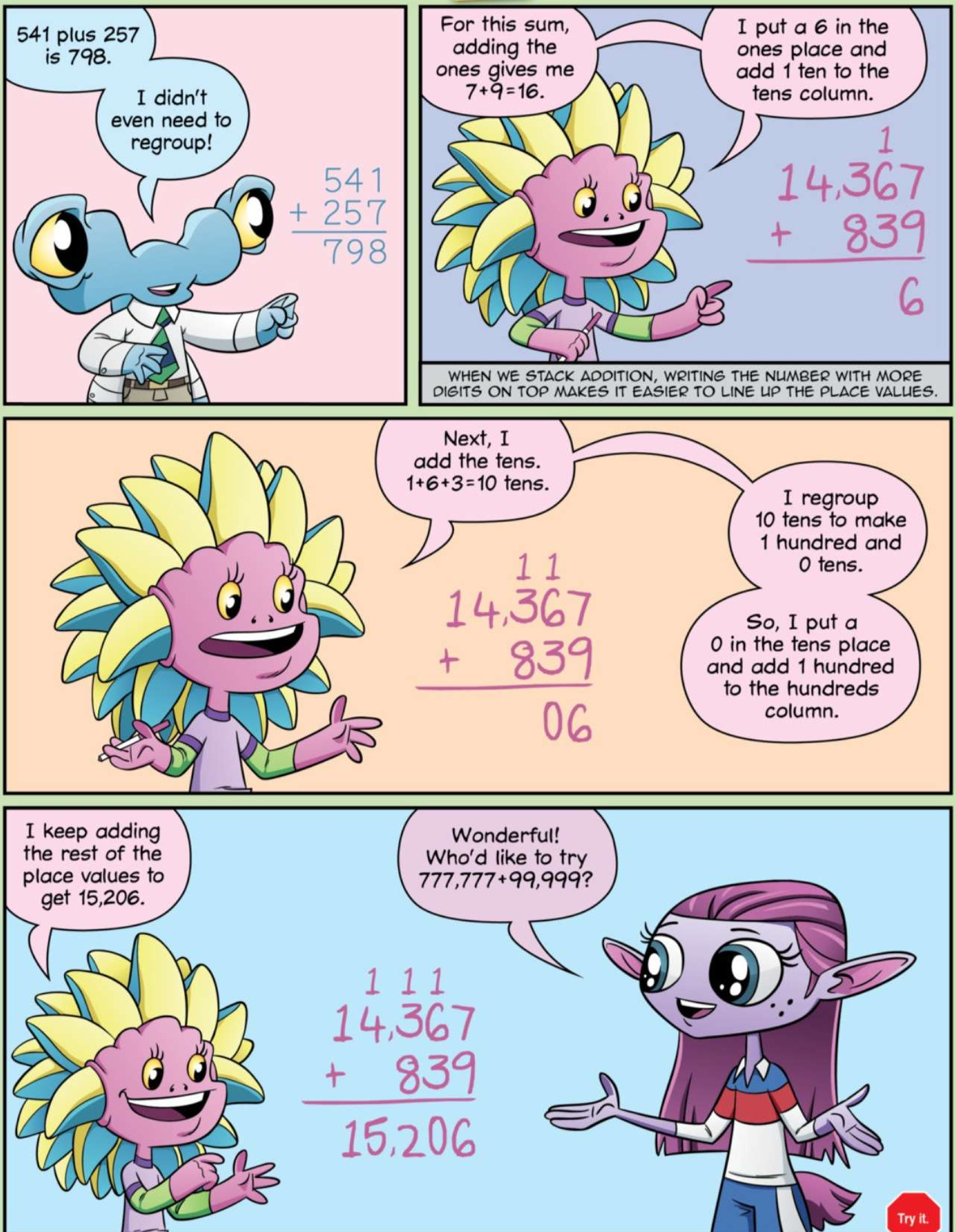

Compubots use algorithms all the time to perform specific tasks.
I wonder if this one can help us fold the 85-step paper plane?
Let's see.
Compubot, fold the paper plane using the instructions that start on page 60.
Whoa! That was fast.
And it looks exactly like the plane in the book!
Yep. Compubots are great at following instructions.
Compubot, make another really cool plane!
Scratch Scratch
Compubots are *not* great at working without clear, step-by-step instructions.


MATH TEAM
Stacking
How do we add numbers using place value?
We add the tens and the ones separately, then add the results.
64 + 78 = 130 + 12 = 142
I found a better way to keep the place values organized.
How do you do it, Alex?
Instead of writing addition in a line...
...I like to stack the numbers on top of each other, like this.
64
+78
That's a great idea. Can anyone else see why that's helpful?
Why is this helpful?


64 + 78 = 130 + 12 = 142
When we write the addition on a line, it's not easy to see which digits are in the same place values.
tens
ones
64
+78
But when we stack the addition with the digits lined up...
...it's easy to see that the tens digits are 6 and 7...
...and the ones digits are 4 and 8.
We can add by place value in each column.
6 tens plus 7 tens is 13 tens, or 130.
64
+78
130
4 plus 8 is 12.
We keep all the digits lined up by place value.
64
+78
130
+ 12
Then, 130+12 is 142.
64
+78
130
+ 12
142
Great!
Try another one.
This time, try adding the ones first, then the tens.
87
+65
Try it.


If we start by adding the ones, 7+5=12.
Wait!
Instead of writing 12...
87
+65
12
...I think I see a shortcut!
12 is 1 ten and 2 ones.
Since there are no more ones to add...
...the final sum has 2 ones.
We write the 2 down here.
87
+65
2
Since we still need to add the tens...
...we can put 1 ten in the tens column...
...with the rest of the tens.
Then, we can add all the tens at once.
1
87
+65
2
Add the tens.

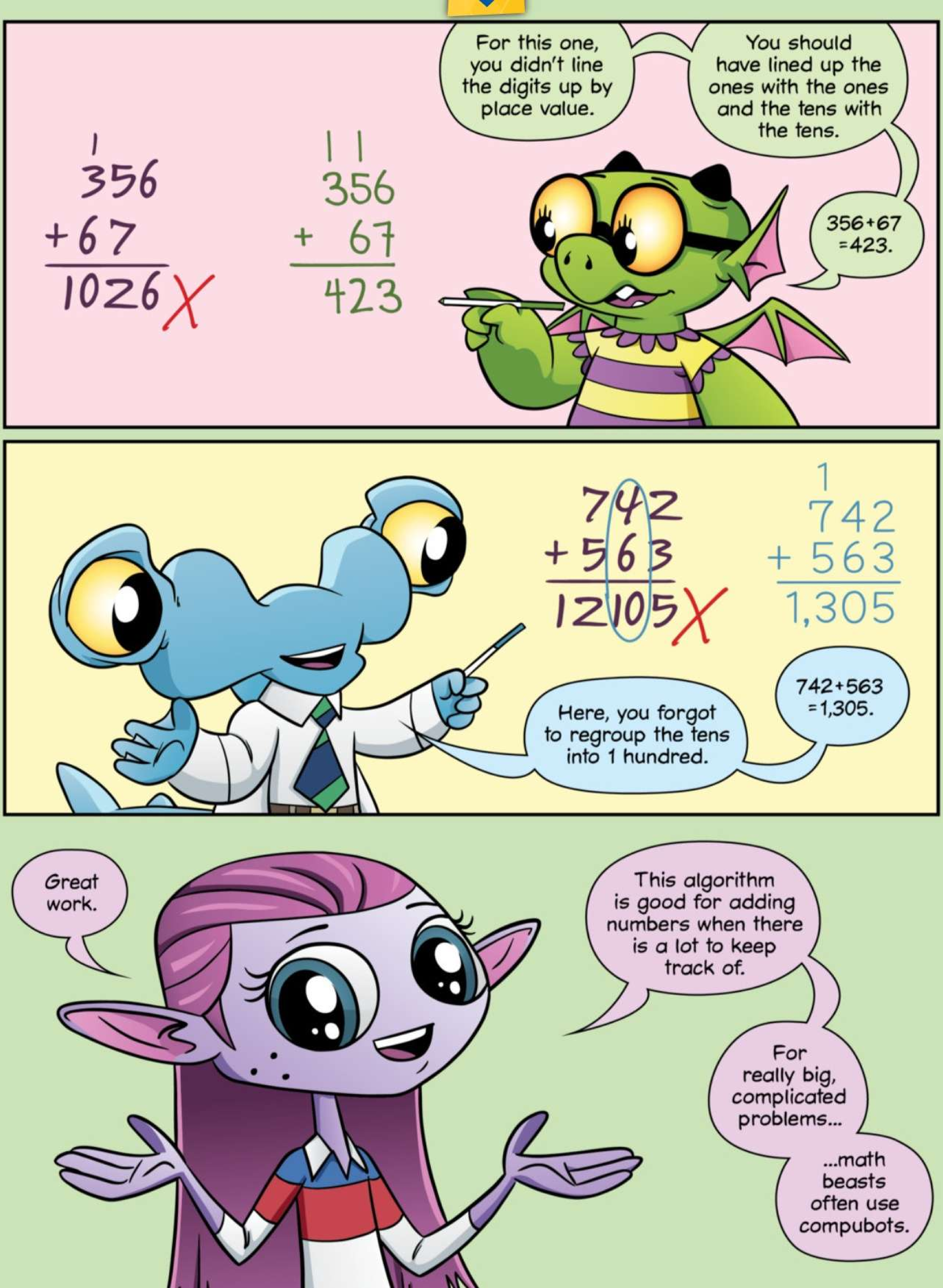

We have 1+8+6=15 tens. That's 1 hundred and 5 tens.
Since there are no more tens or hundreds to add...
...we write a 5 in the tens place, and a 1 in the hundreds place.
1
87
+65
152
Fantastic!
This is a popular addition algorithm.
We stack the numbers and line up the place values.
Then, we add the digits in each place value from right to left...
...regrouping as we go.
Try a few more sums using this algorithm.
541 + 257
839 + 14,367
777,777 + 99,999
Try all three.



541 plus 257 is 798.
I didn't even need to regroup!
541
+ 257
798
For this sum, adding the ones gives me 7+9=16.
I put a 6 in the ones place and add 1 ten to the tens column.
1
14,367
+ 839
6
WHEN WE STACK ADDITION, WRITING THE NUMBER WITH MORE DIGITS ON TOP MAKES IT EASIER TO LINE UP THE PLACE VALUES.
Next, I add the tens. 1+6+3=10 tens.
I regroup 10 tens to make 1 hundred and 0 tens.
So, I put a 0 in the tens place and add 1 hundred to the hundreds column.
1 1
14,367
+ 839
06
I keep adding the rest of the place values to get 15,206.
Wonderful! Who'd like to try 777,777+99,999?
1 1 1
14,367
+ 839
15,206
Try it.

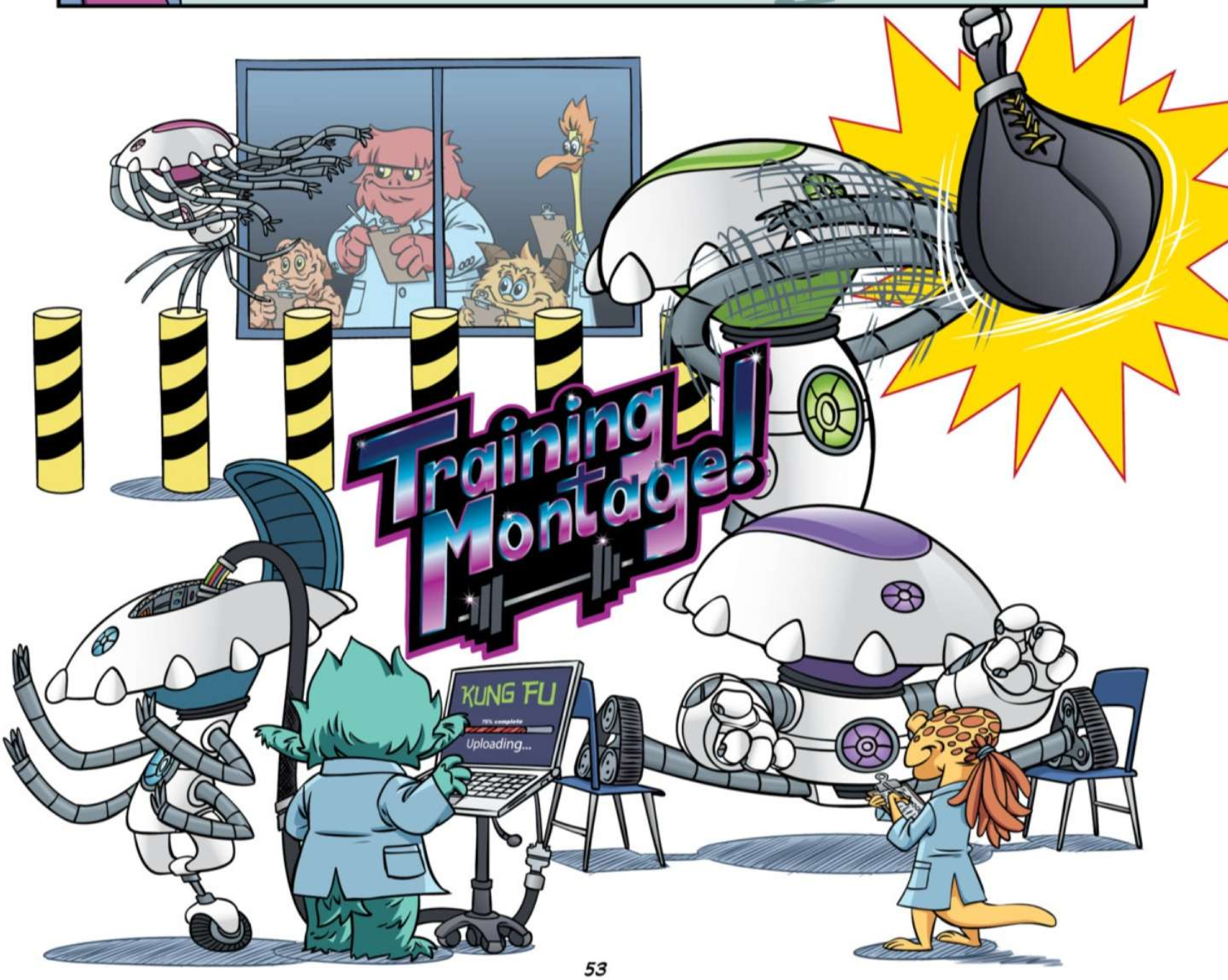

This is great! Why didn't you teach us this before!?
Stacking makes keeping track of the addition a lot easier.
111 11
777,777
+ 99,999
877,776
The algorithm isn't very good for helping you learn how to add.
Besides, most of the time, you don't need it.
You can do most addition using the strategies we've learned in the past year.
For example, you already forgot that there is a better way to add 777,777+99,999!
777,777 + 99,999
What's the better way?

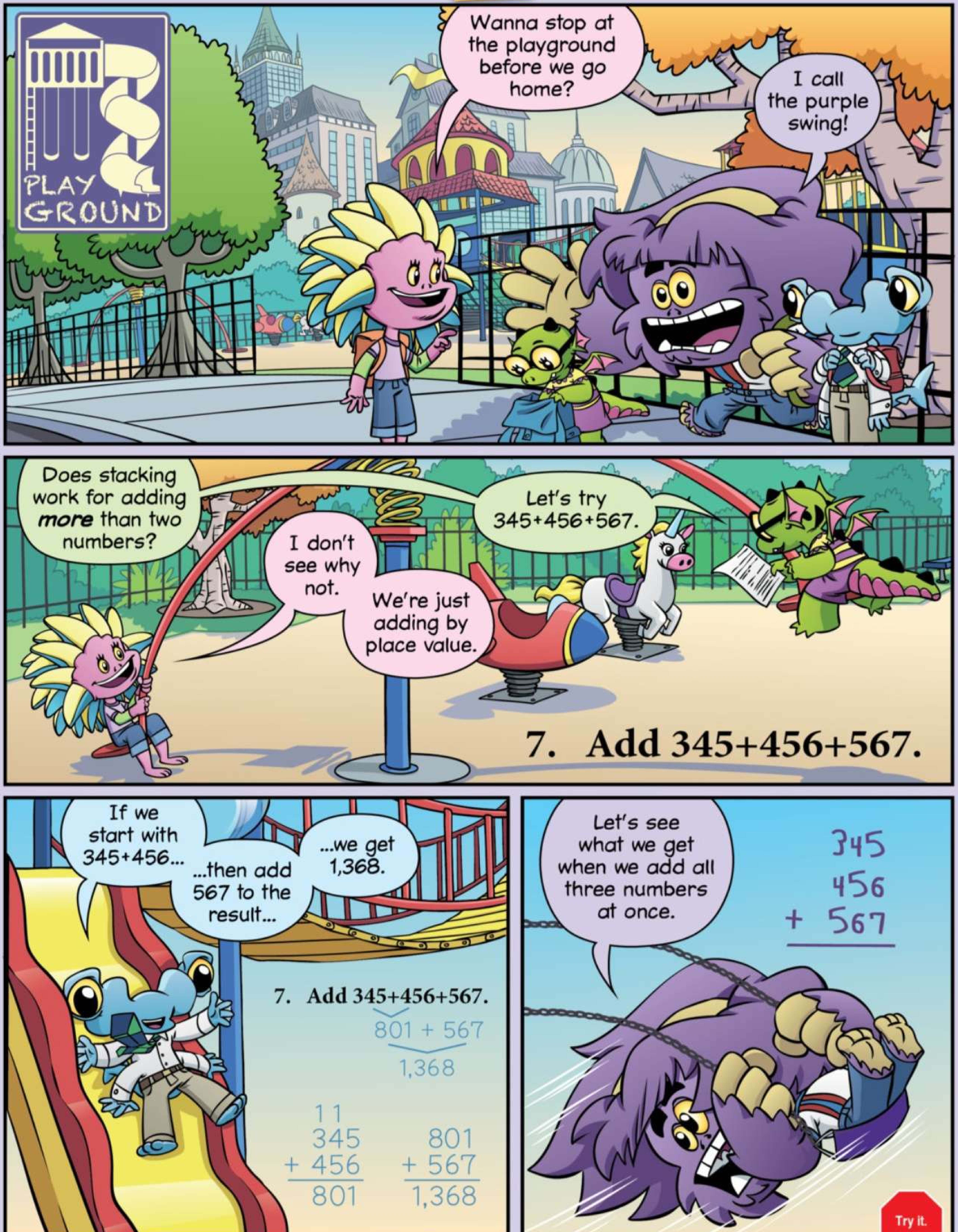

Oh, yeah.
99,999 is just one less than 100,000.
To add 99,999, we can add 100,000...
...then subtract 1.
777,777 + 99,999 = 777,777 + 100,000 - 1
777,777 + 99,999 = 777,777 + 100,000 - 1
= 877,776
That's 877,776!
Good.
Stacking addition isn't always helpful.
It's also really easy to mess up if you don't understand what's going on.
Can you find the mistakes I made in these sums?
1
356
+67
1026 X
742
+563
12105 X
Can you find the errors?

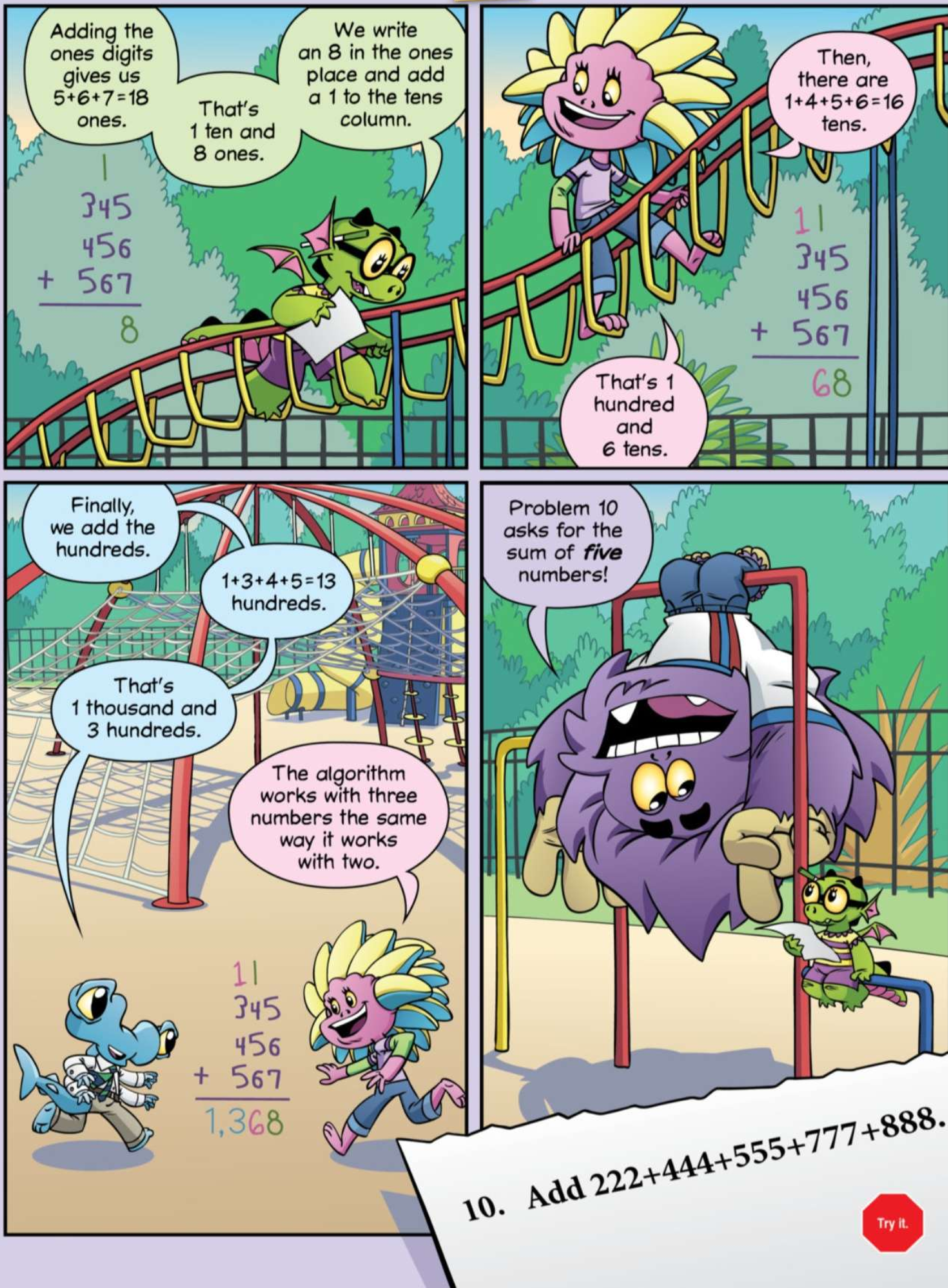


1
356
+67
1026
11
356
+ 67
423
For this one, you didn't line the digits up by place value.
You should have lined up the ones with the ones and the tens with the tens.
356+67 =423.
742
+563
12105
1
742
+ 563
1,305
Here, you forgot to regroup the tens into 1 hundred.
742+563 =1,305.
Great work.
This algorithm is good for adding numbers when there is a lot to keep track of.
For really big, complicated problems...
...math beasts often use compubots.

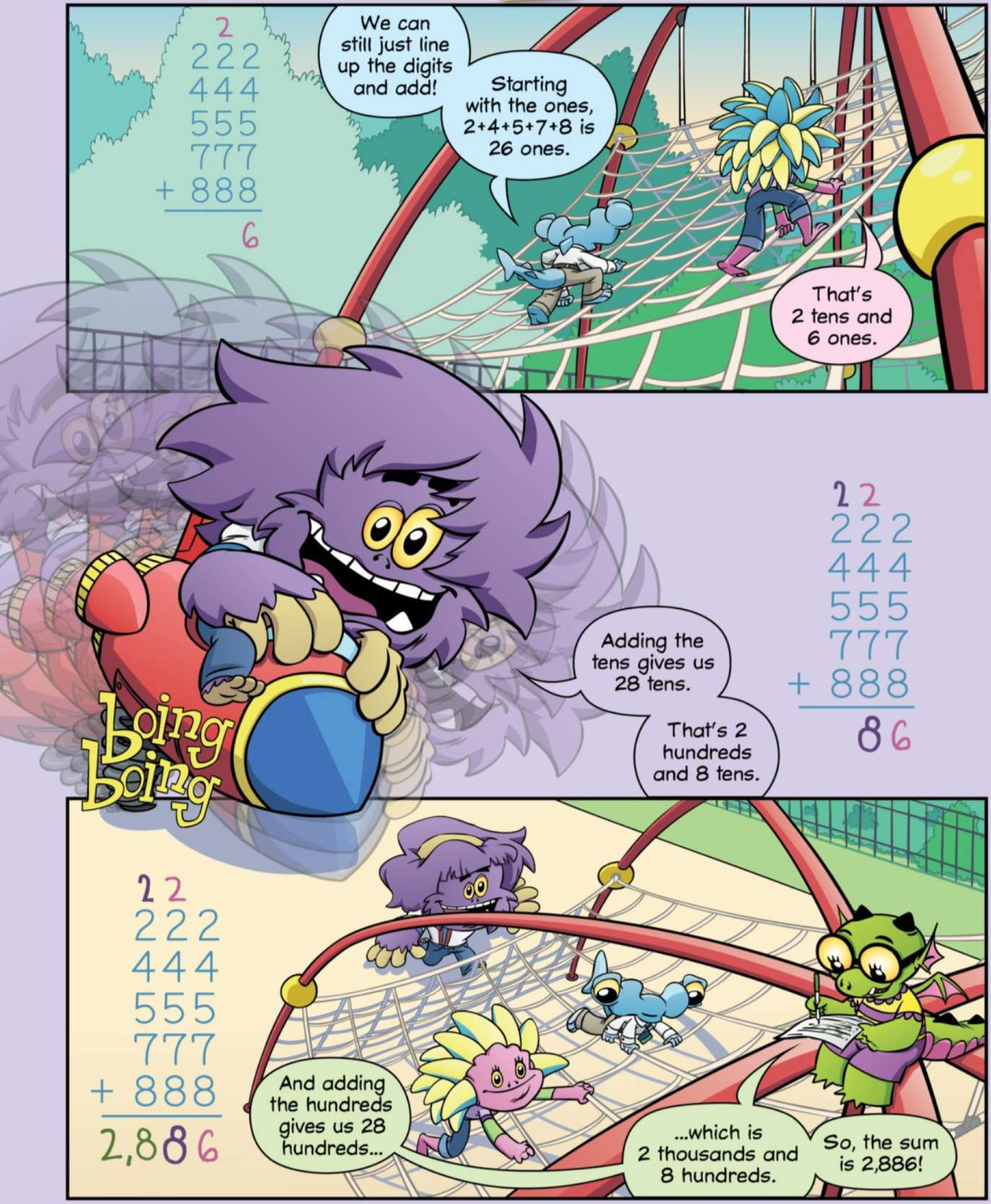

Speaking of bots...
...when is our next Math Meet?
You'll get to test your skills against the bots right before the end of school!
The students in the robotics lab are making sure they're ready for *any* challenge.


Training Montage!
KUNG FU
Uploading...


PLAY GROUND
Wanna stop at the playground before we go home?
I call the purple swing!
Does stacking work for adding *more* than two numbers?
Let's try 345+456+567.
I don't see why not.
We're just adding by place value.
7. Add 345+456+567.
If we start with 345+456...
...then add 567 to the result...
...we get 1,368.
7. Add 345+456+567.
801 + 567
1,368
11
345
+ 456
801
801
+ 567
1,368
Let's see what we get when we add all three numbers at once.
345
456
+ 567
Try it.

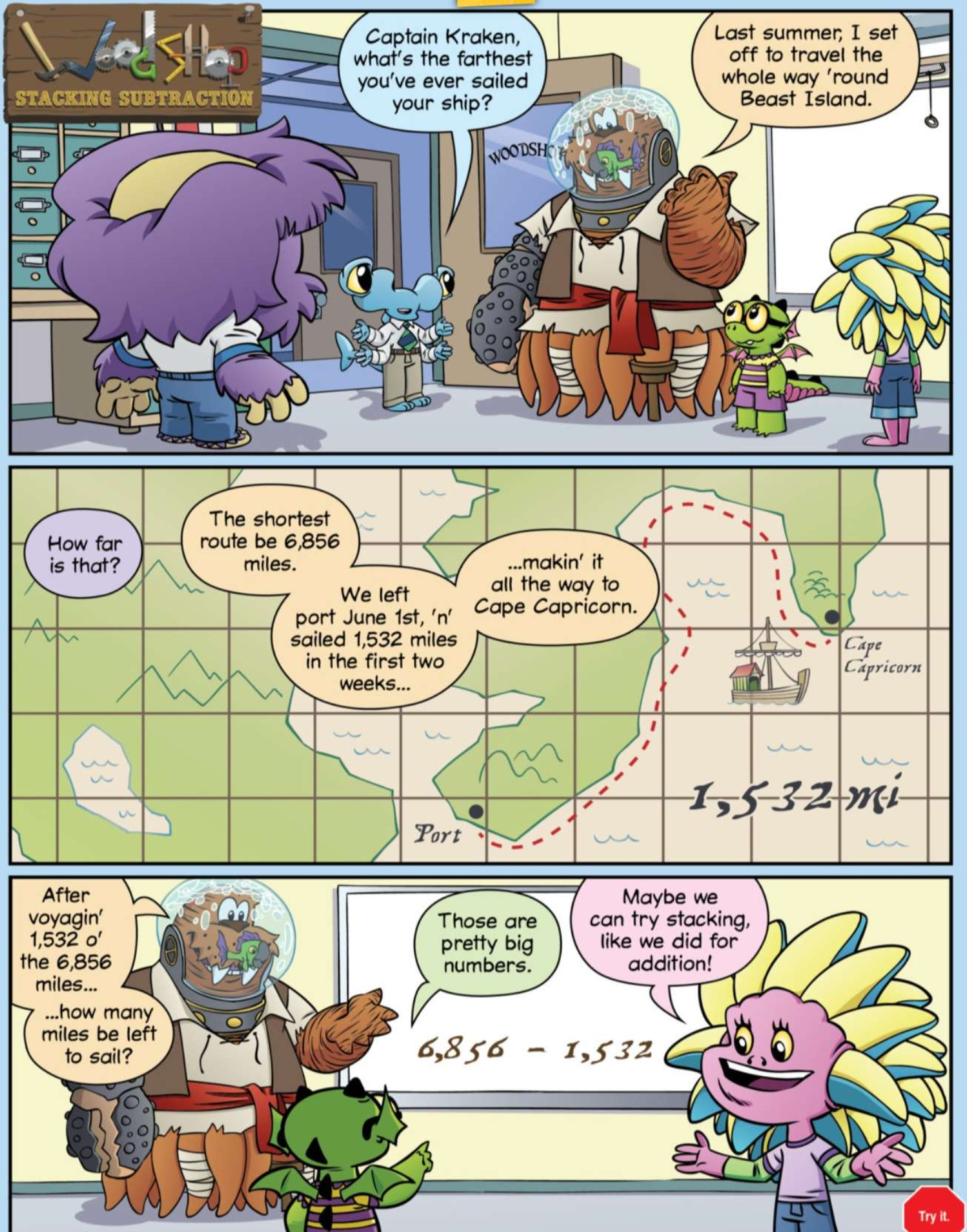


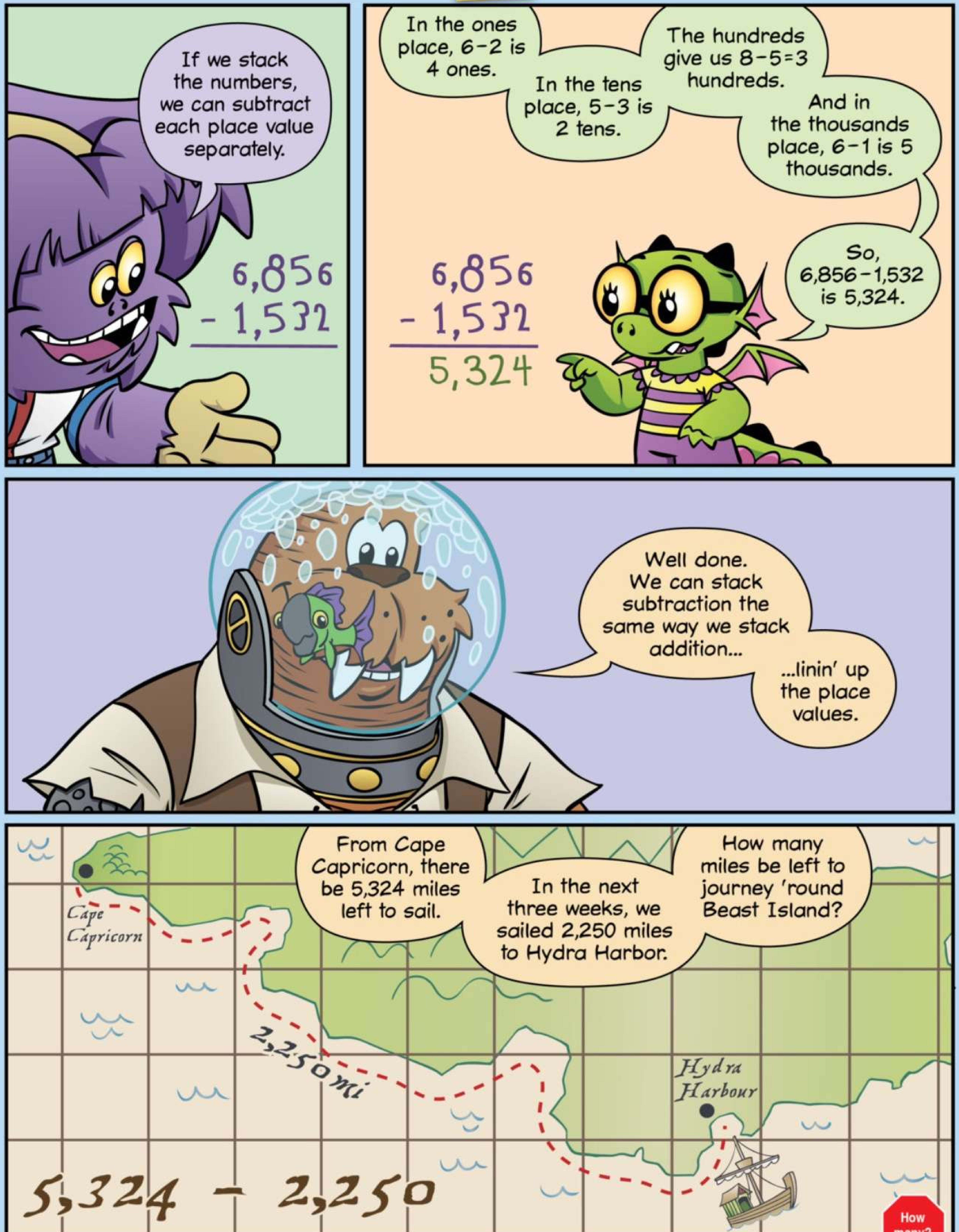
Adding the ones digits gives us 5+6+7=18 ones.
That's 1 ten and 8 ones.
We write an 8 in the ones place and add a 1 to the tens column.
1
345
456
+ 567
8
Then, there are 1+4+5+6=16 tens.
That's 1 hundred and 6 tens.
11
345
456
+ 567
68
Finally, we add the hundreds.
1+3+4+5=13 hundreds.
That's 1 thousand and 3 hundreds.
The algorithm works with three numbers the same way it works with two.
11
345
456
+ 567
1,368
Problem 10 asks for the sum of *five* numbers!
10. Add 222+444+555+777+888.
Try it.

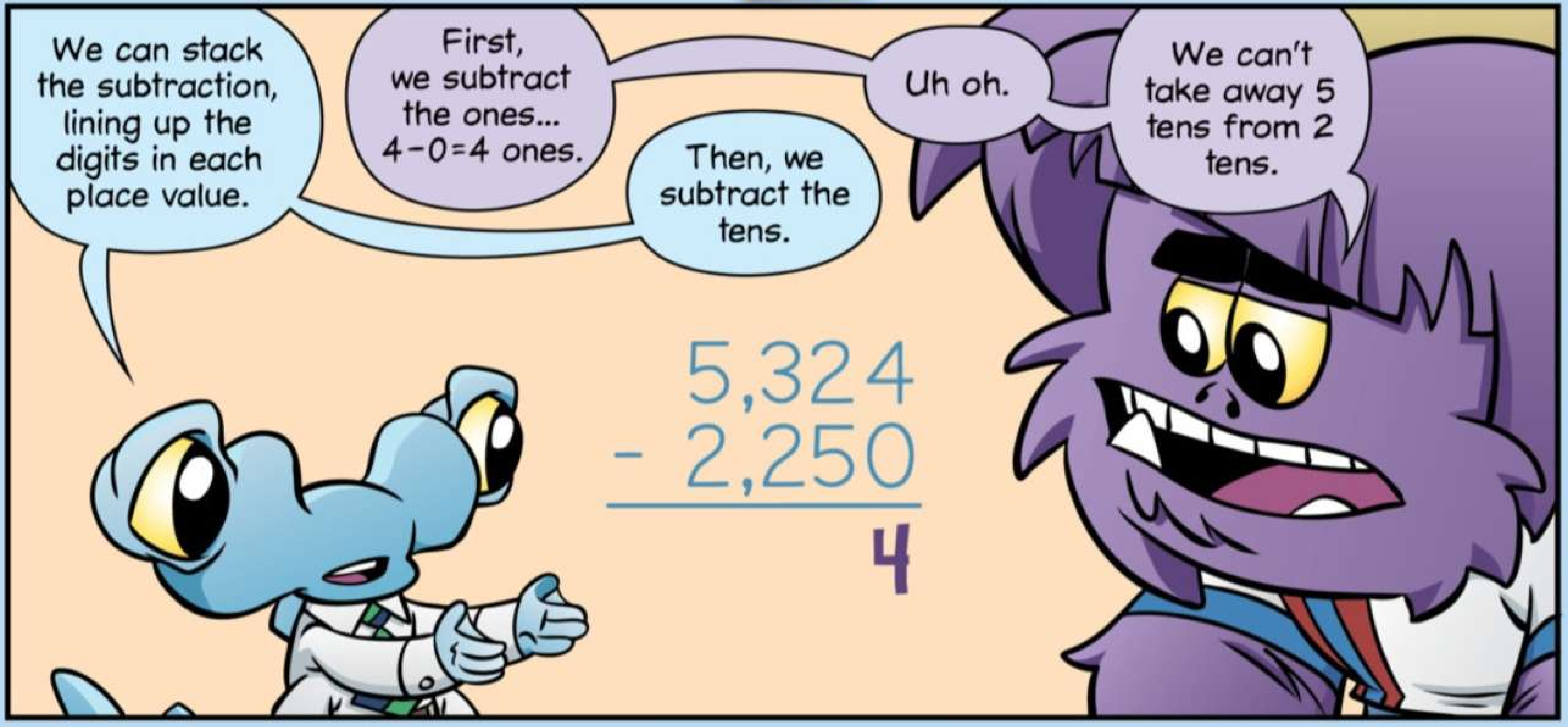


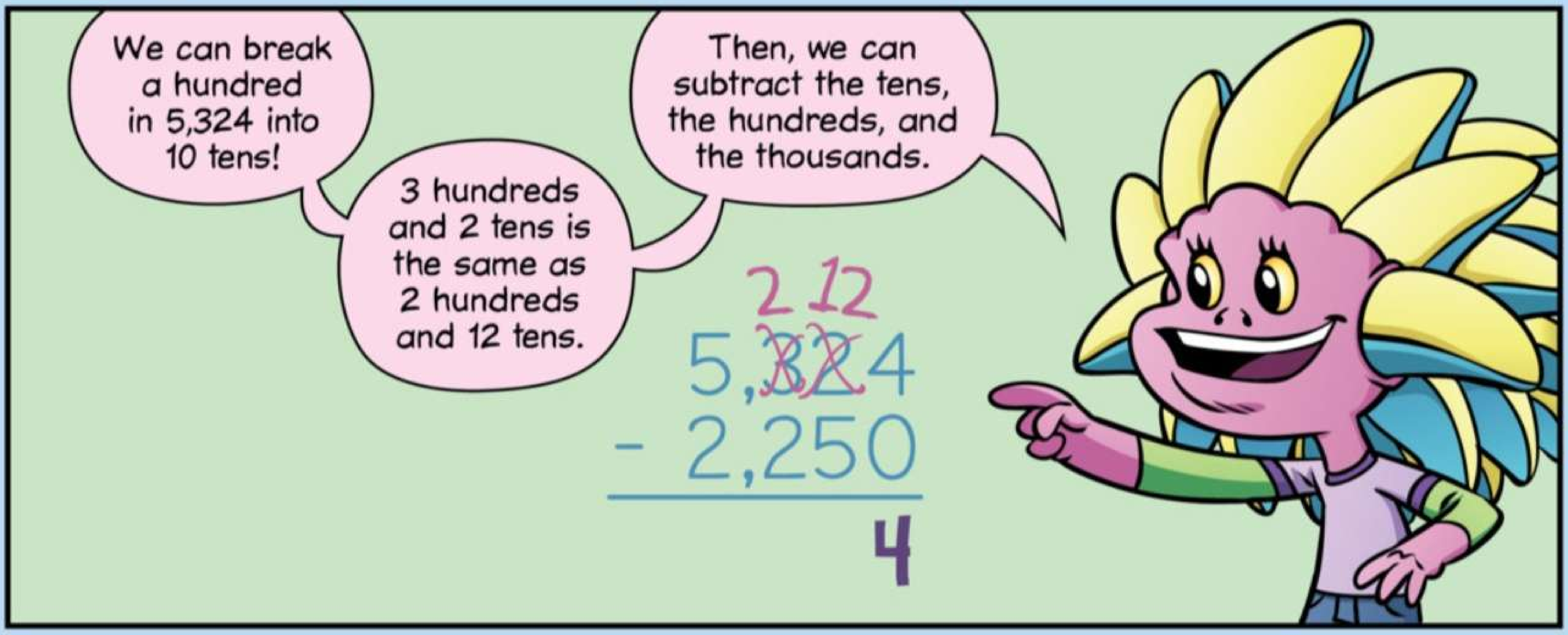
2
222
444
555
777
+ 888
6
We can still just line up the digits and add!
Starting with the ones, 2+4+5+7+8 is 26 ones.
That's 2 tens and 6 ones.
2 2
222
444
555
777
+ 888
86
Adding the tens gives us 28 tens.
That's 2 hundreds and 8 tens.
boing boing
2 2
222
444
555
777
+ 888
2,886
And adding the hundreds gives us 28 hundreds...
...which is 2 thousands and 8 hundreds.
So, the sum is 2,886!

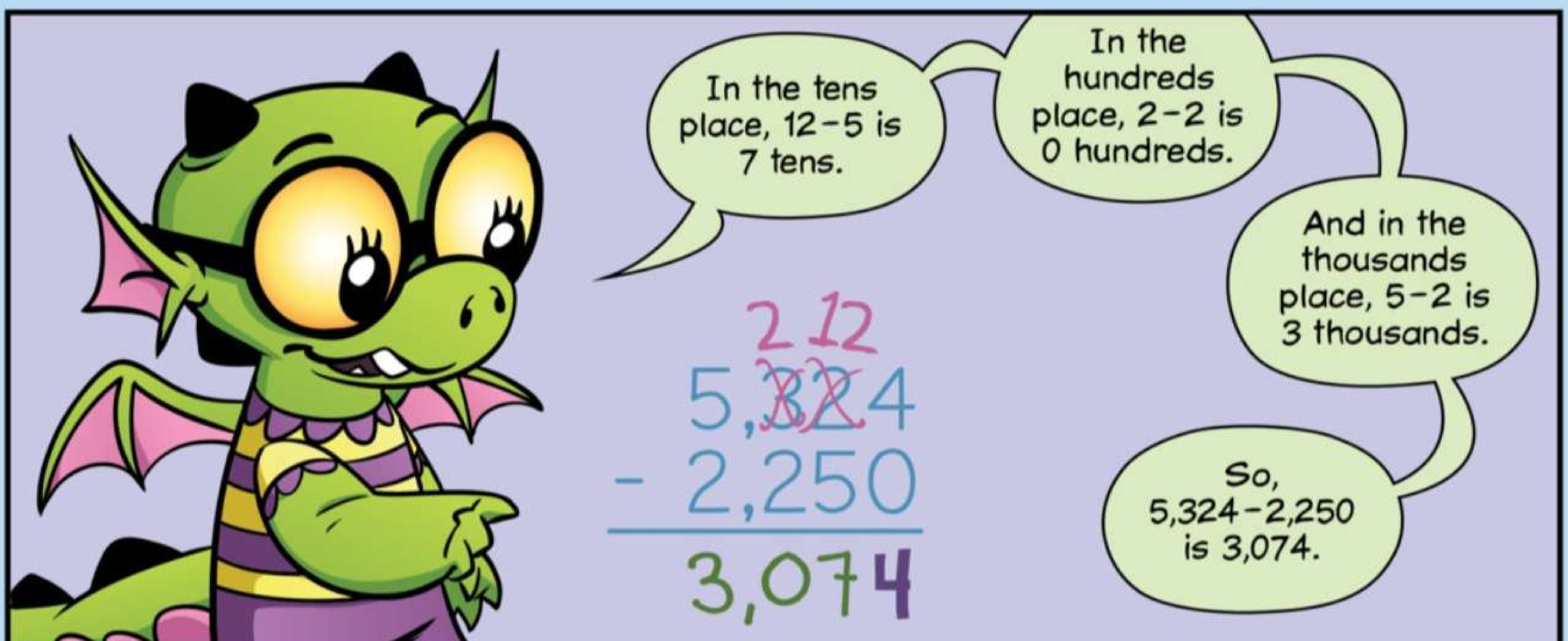

Whoa, Lizzie! Did you climb all the way up here without even using your hands!?
How did you *do* that?
I flew!
That's sooooo cool!
I finally got the hang of it!
Awesome, Lizzie!
Way to go!

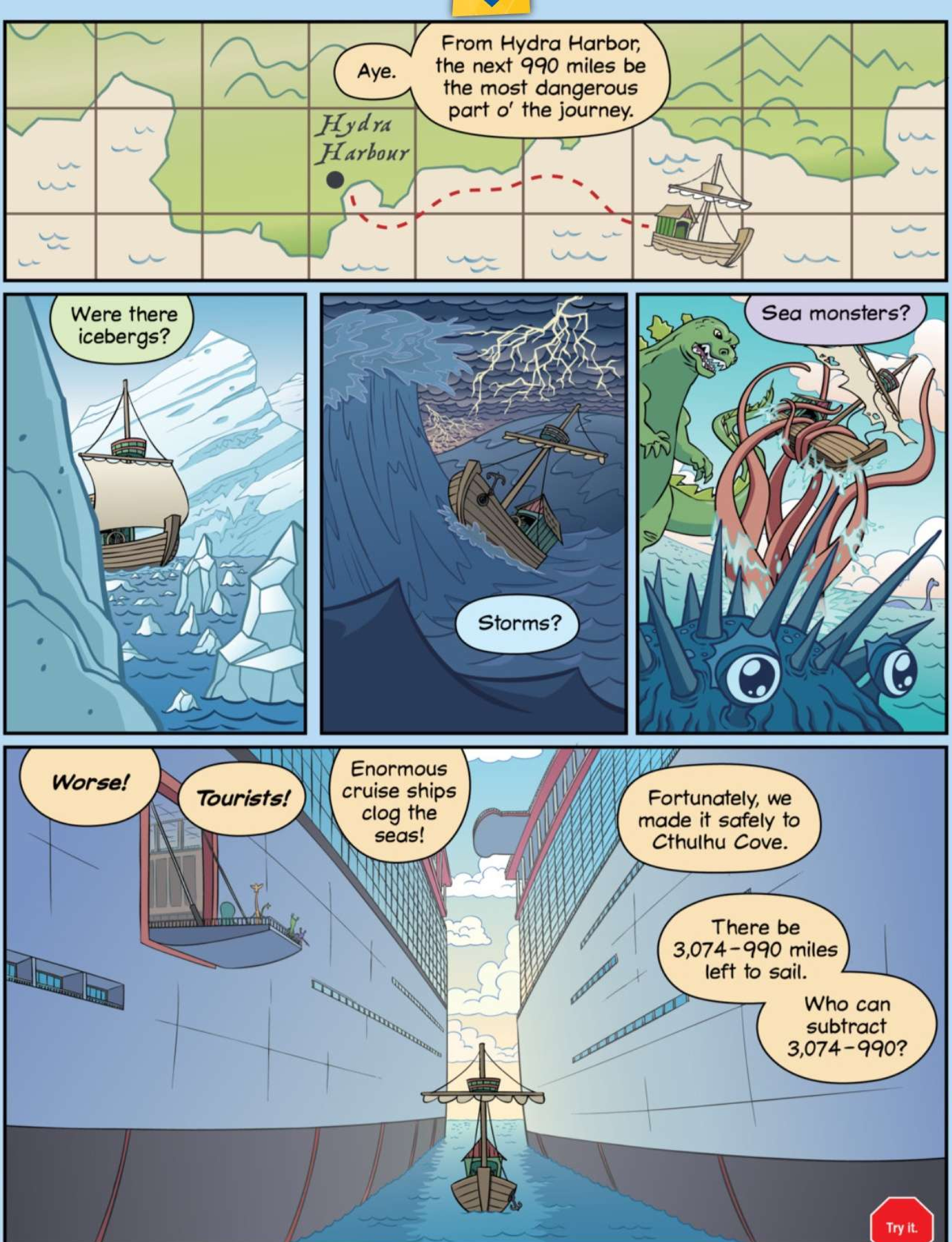

STACKING SUBTRACTION
Captain Kraken, what's the farthest you've ever sailed your ship?
Last summer, I set off to travel the whole way 'round Beast Island.
WOODSHOP
How far is that?
The shortest route be 6,856 miles.
We left port June 1st, 'n' sailed 1,532 miles in the first two weeks...
...makin' it all the way to Cape Capricorn.
Cape Capricorn
Port
1,532 mi
After voyagin' 1,532 o' the 6,856 miles...
...how many miles be left to sail?
Those are pretty big numbers.
Maybe we can try stacking, like we did for addition!
6,856 − 1,532
Try it.

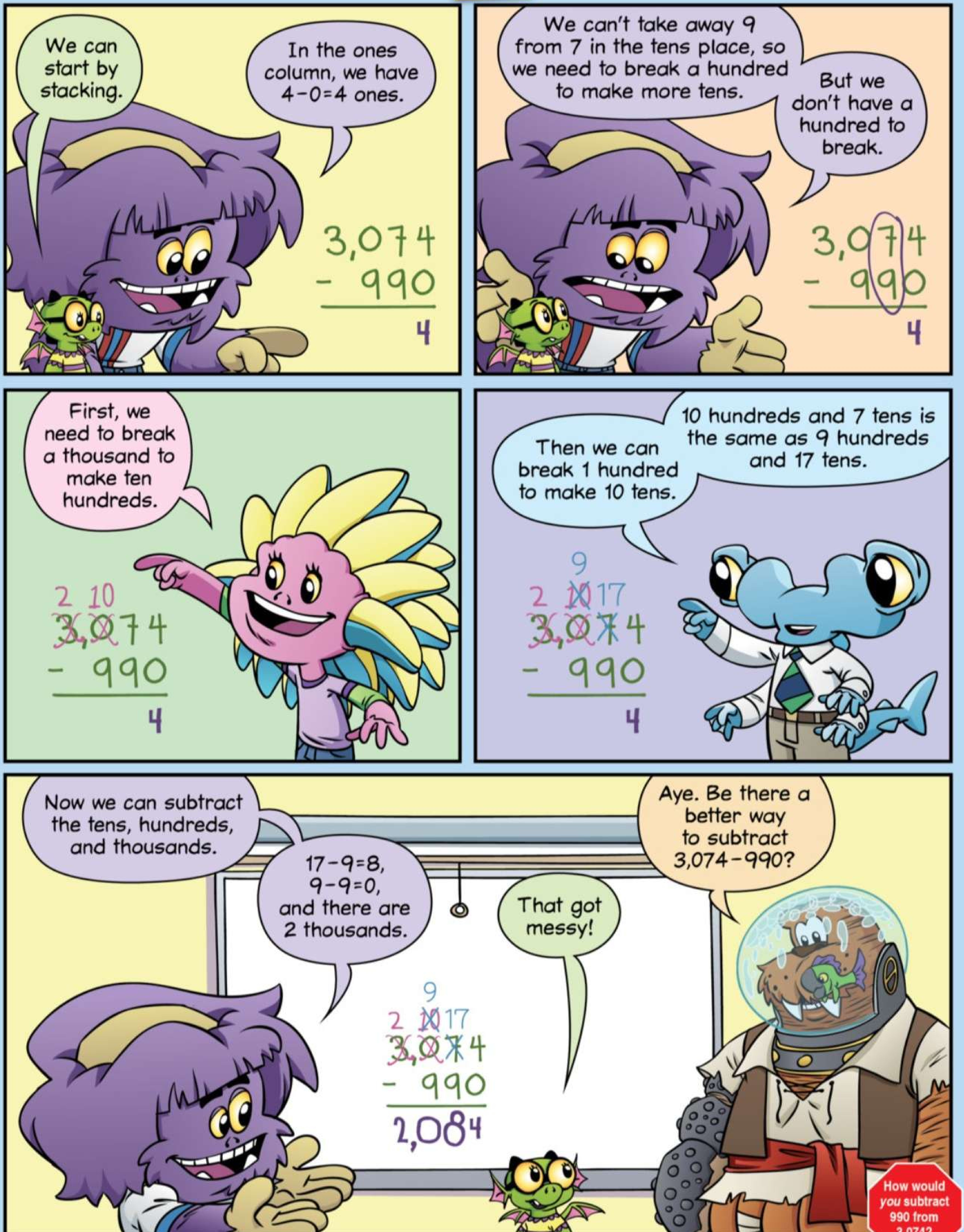

If we stack the numbers, we can subtract each place value separately.
6,856
- 1,532
In the ones place, 6-2 is 4 ones.
In the tens place, 5-3 is 2 tens.
The hundreds give us 8-5=3 hundreds.
And in the thousands place, 6-1 is 5 thousands.
So, 6,856-1,532 is 5,324.
6,856
- 1,532
5,324
Well done. We can stack subtraction the same way we stack addition...
...linin' up the place values.
From Cape Capricorn, there be 5,324 miles left to sail.
In the next three weeks, we sailed 2,250 miles to Hydra Harbor.
How many miles be left to journey 'round Beast Island?
Cape Capricorn
2,250 mi
Hydra Harbour
5,324 - 2,250
How many?


We can stack the subtraction, lining up the digits in each place value.
First, we subtract the ones... 4-0=4 ones.
Then, we subtract the tens.
Uh oh.
We can't take away 5 tens from 2 tens.
5,324
- 2,250
4

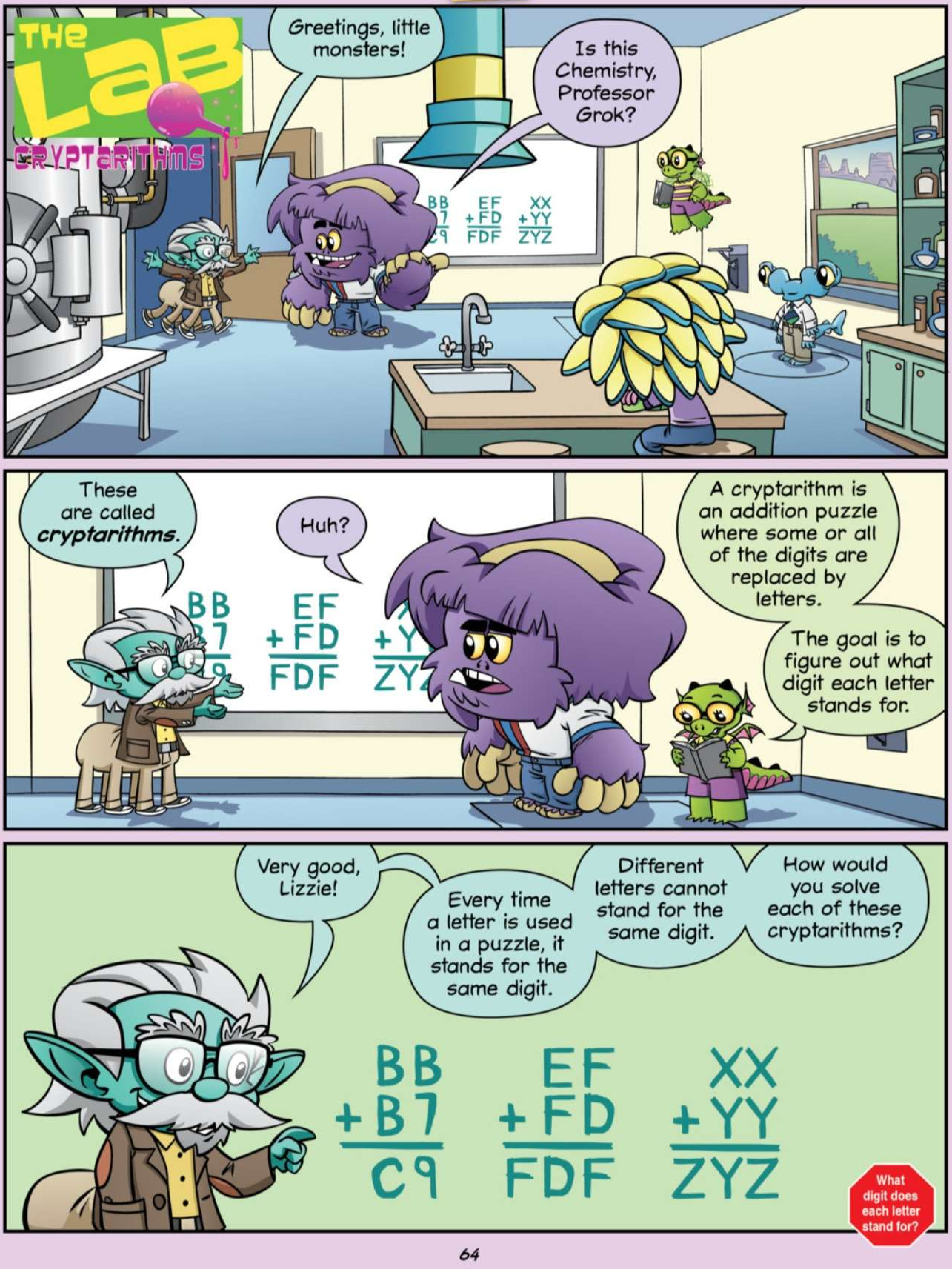

We can break a hundred in 5,324 into 10 tens!
3 hundreds and 2 tens is the same as 2 hundreds and 12 tens.
Then, we can subtract the tens, the hundreds, and the thousands.
2 12
5,~~32~~4
- 2,250
4

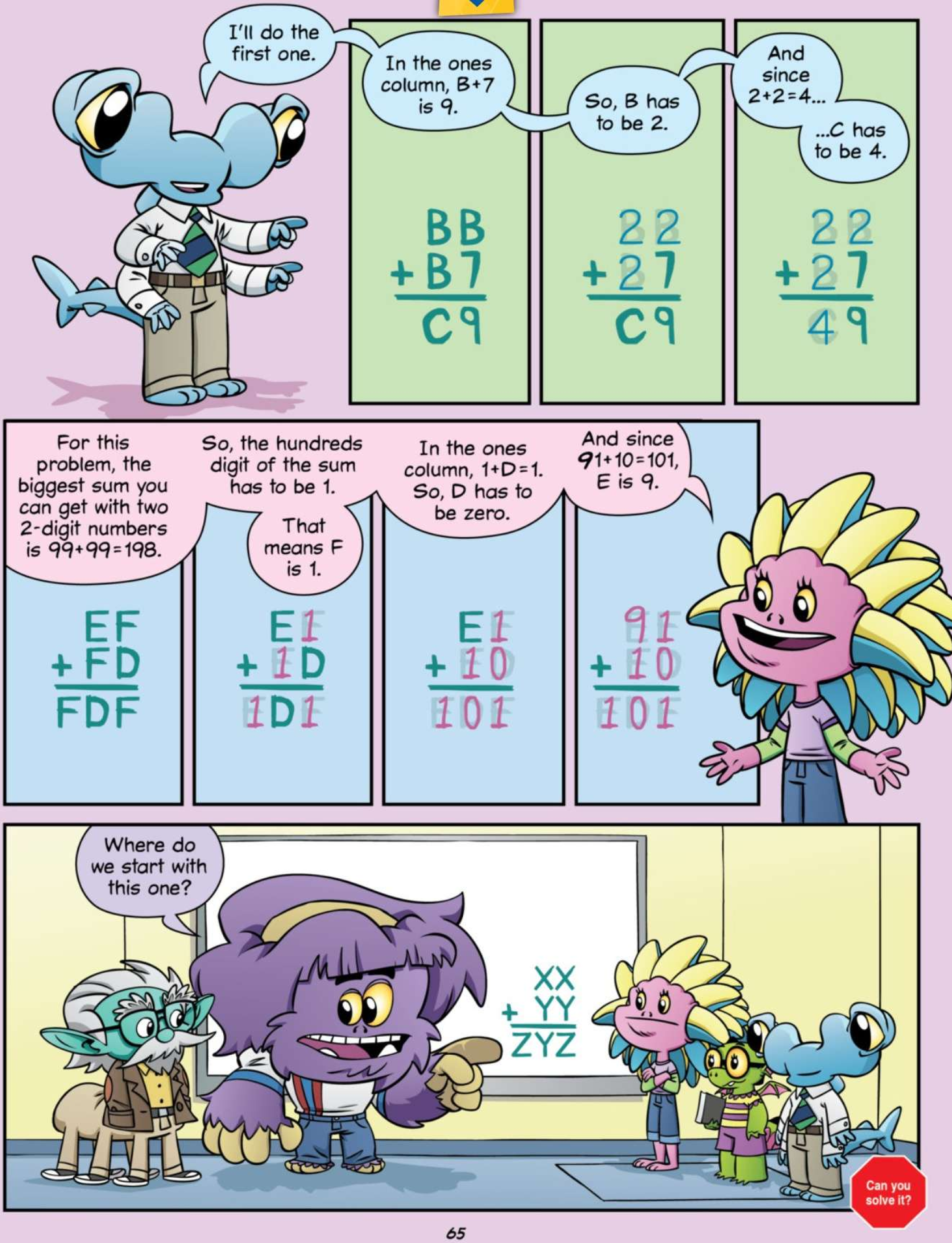

In the tens place, 12-5 is 7 tens.
In the hundreds place, 2-2 is 0 hundreds.
And in the thousands place, 5-2 is 3 thousands.
So, 5,324-2,250 is 3,074.
2 12
5,~~32~~4
- 2,250
3,074


Aye.
From Hydra Harbor, the next 990 miles be the most dangerous part o' the journey.
Hydra Harbour
Were there icebergs?
Storms?
Sea monsters?
Worse!
Tourists!
Enormous cruise ships clog the seas!
Fortunately, we made it safely to Cthulhu Cove.
There be 3,074−990 miles left to sail.
Who can subtract 3,074−990?
Try it.

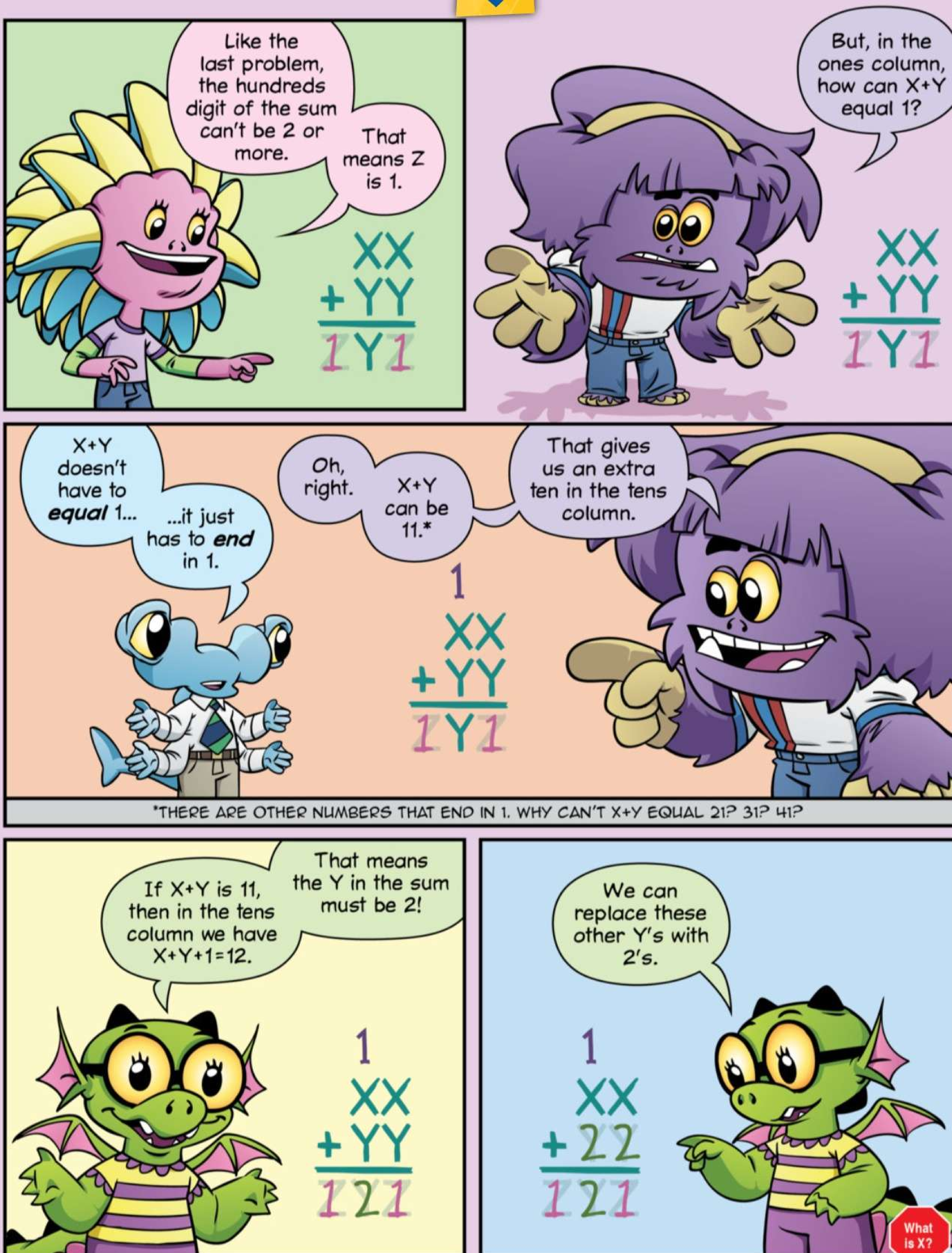

We can start by stacking.
In the ones column, we have 4−0=4 ones.
3,074
− 990
4
We can't take away 9 from 7 in the tens place, so we need to break a hundred to make more tens.
But we don't have a hundred to break.
First, we need to break a thousand to make ten hundreds.
Then we can break 1 hundred to make 10 tens.
10 hundreds and 7 tens is the same as 9 hundreds and 17 tens.
Now we can subtract the tens, hundreds, and thousands.
17−9=8,
9−9=0,
and there are 2 thousands.
That got messy!
Aye. Be there a better way to subtract 3,074−990?
2,084
How would *you* subtract 990 from 3,074?

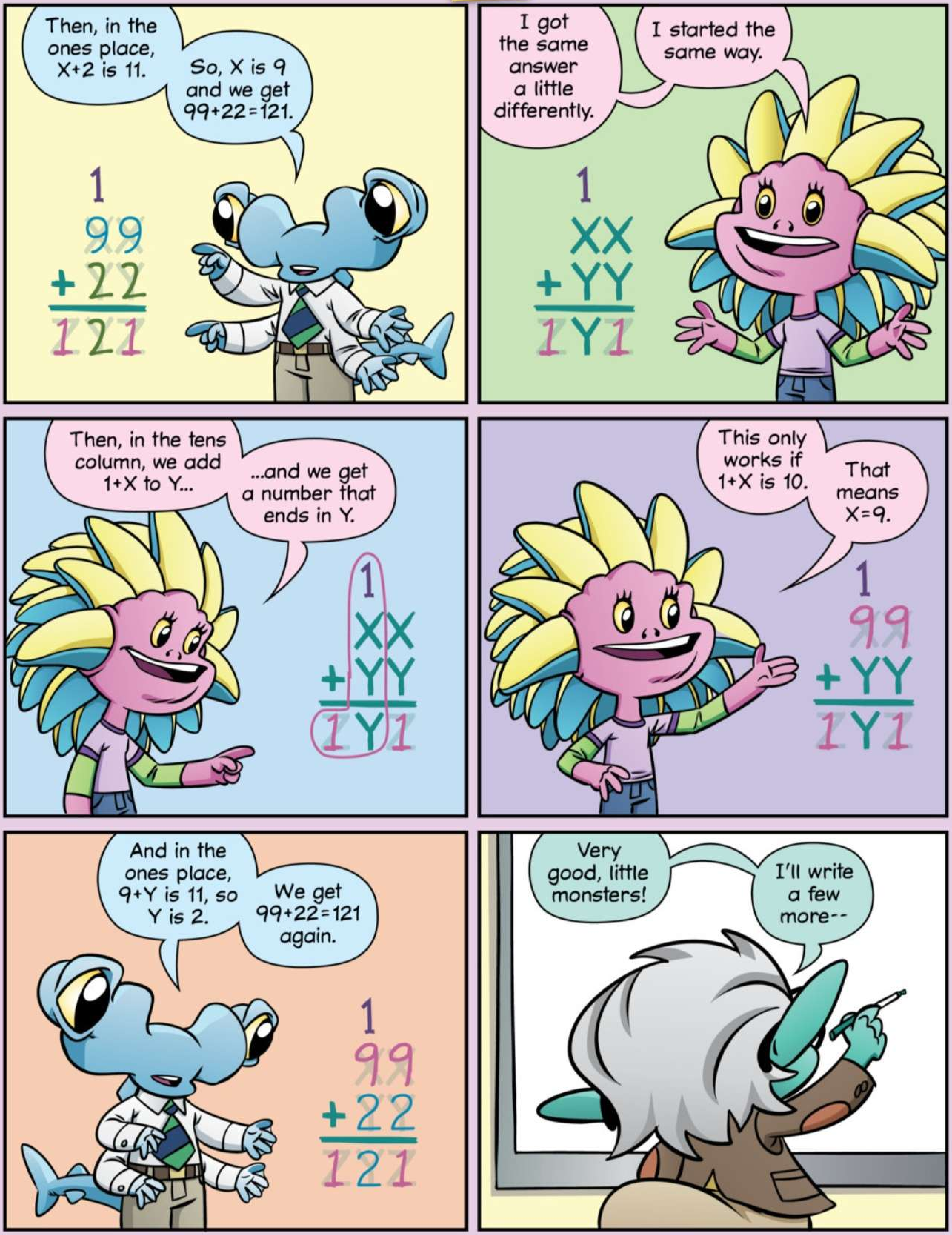
Oh, yeah!
990 is 10 less than 1,000.
We could have just subtracted 1,000...
...then added 10.
3,074 - 990 = 3,074 - 1,000 + 10
= 2,074 + 10
= 2,084
Aye.
'Tis always good to look for other strategies before stackin' subtraction.
'Tis also good to look for other strategies before sailin' 'round Beast Island!
With calm winds, we had to get creative for the last part o' the journey!


THE LAB
CRYPTARITHMS
Greetings, little monsters!
Is this Chemistry, Professor Grok?
These are called *cryptarithms.*
Huh?
A cryptarithm is an addition puzzle where some or all of the digits are replaced by letters.
The goal is to figure out what digit each letter stands for.
Very good, Lizzie!
Every time a letter is used in a puzzle, it stands for the same digit.
Different letters cannot stand for the same digit.
How would you solve each of these cryptarithms?
BB + B7 = C9
EF + FD = FDF
XX + YY = ZYZ
What digit does each letter stand for?

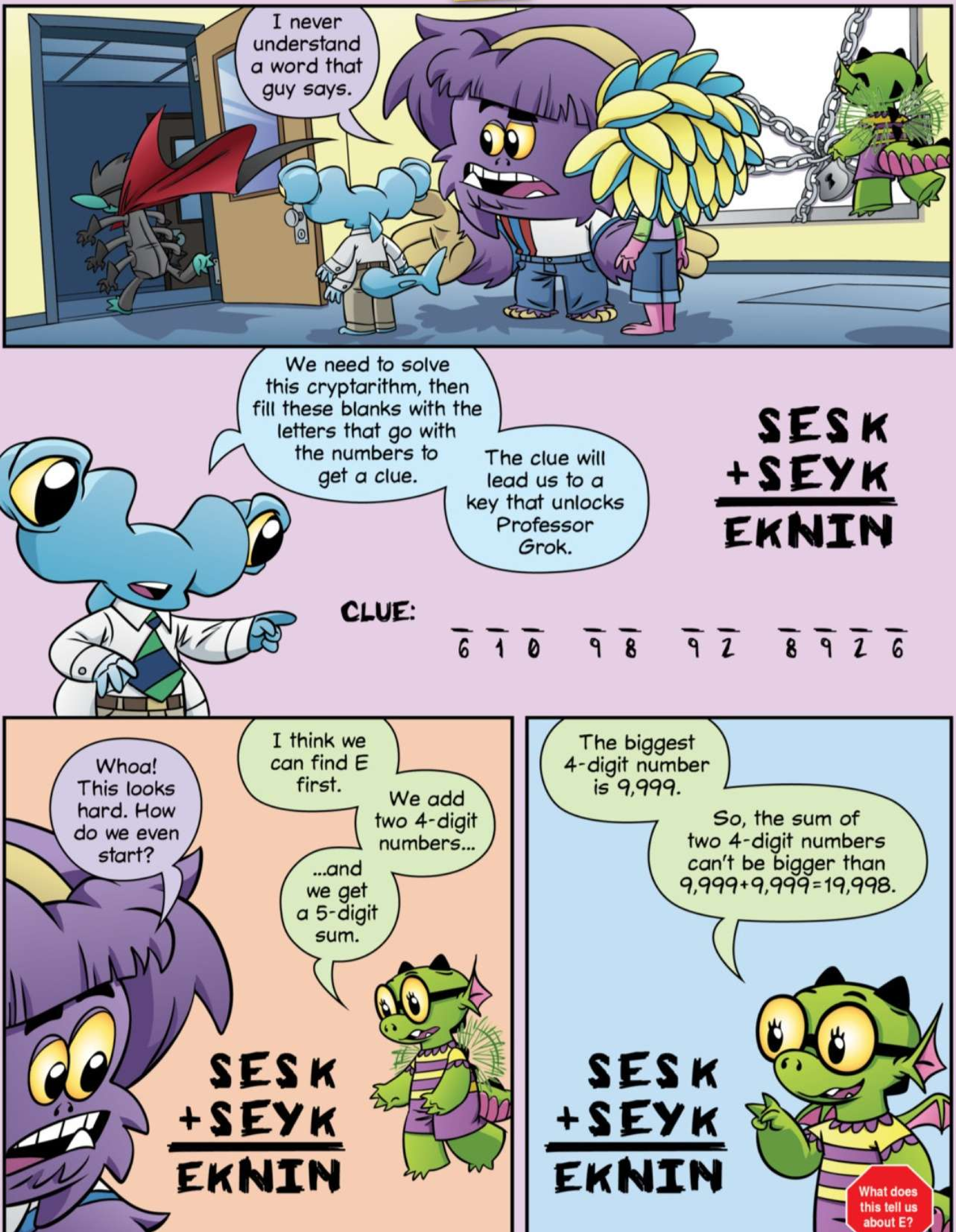


I'll do the first one.
In the ones column, B+7 is 9.
BB + B7 = C9
So, B has to be 2.
22 + 27 = C9
And since 2+2=4...
...C has to be 4.
22 + 27 = 49
For this problem, the biggest sum you can get with two 2-digit numbers is 99+99=198.
EF + FD = FDF
So, the hundreds digit of the sum has to be 1.
That means F is 1.
E1 + 1D = 1D1
In the ones column, 1+D=1. So, D has to be zero.
E1 + 10 = 101
And since 91+10=101, E is 9.
91 + 10 = 101
Where do we start with this one?
XX + YY = ZYZ
Can you solve it?


Like the last problem, the hundreds digit of the sum can't be 2 or more.
That means Z is 1.
XX
+YY
1Y1
But, in the ones column, how can X+Y equal 1?
XX
+YY
1Y1
X+Y doesn't have to ***equal*** 1...
...it just has to ***end*** in 1.
Oh, right.
X+Y can be 11.*
That gives us an extra ten in the tens column.
1
XX
+YY
1Y1
*THERE ARE OTHER NUMBERS THAT END IN 1. WHY CAN'T X+Y EQUAL 21? 31? 41?
If X+Y is 11, then in the tens column we have X+Y+1=12.
That means the Y in the sum must be 2!
1
XX
+YY
121
We can replace these other Y's with 2's.
1
XX
+22
121
What is X?

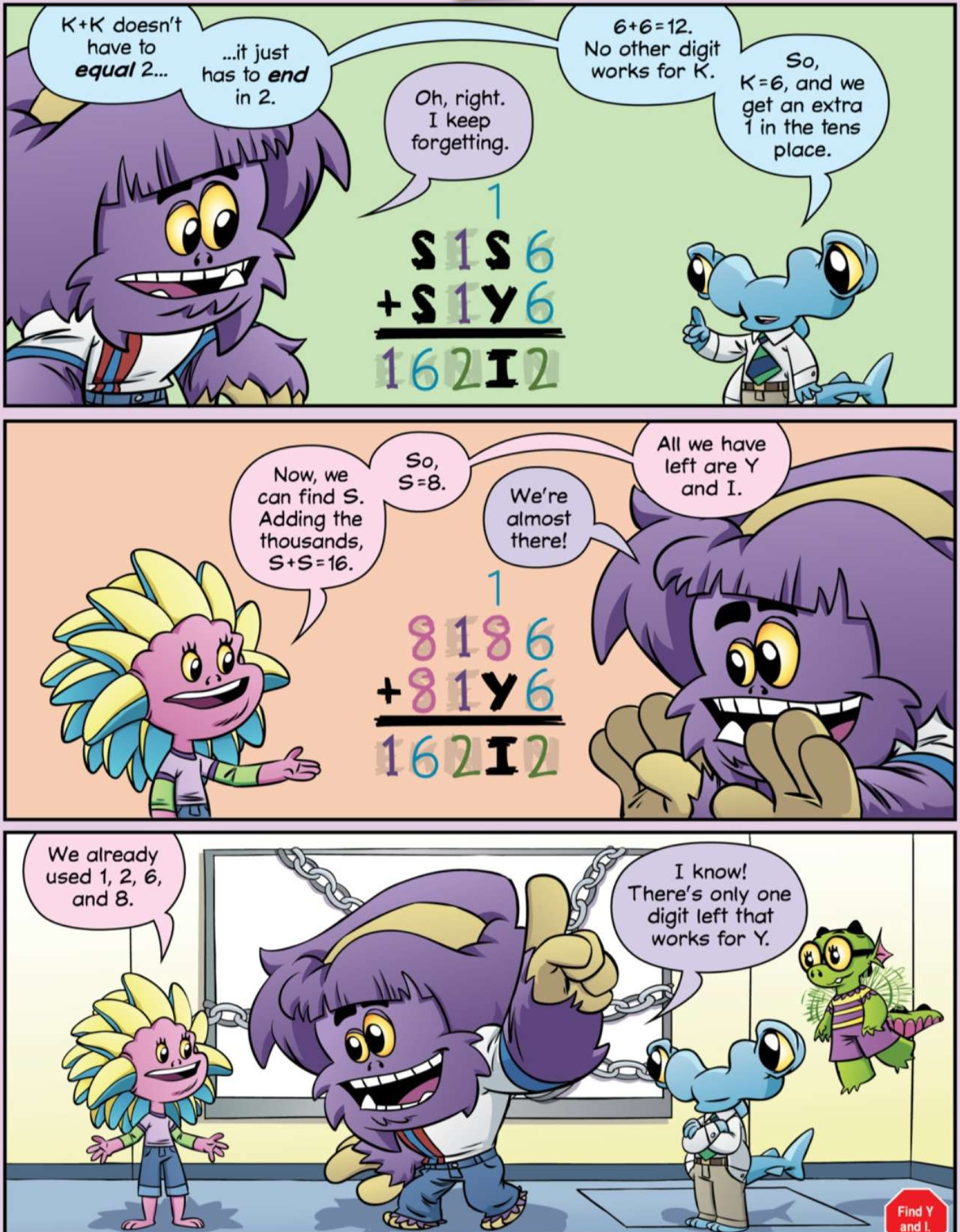

Then, in the ones place, X+2 is 11.
So, X is 9 and we get 99+22=121.
1
99
+22
121
I got the same answer a little differently.
I started the same way.
1
XX
+YY
1Y1
Then, in the tens column, we add 1+X to Y...
...and we get a number that ends in Y.
1
XX
+YY
1Y1
This only works if 1+X is 10.
That means X=9.
1
99
+YY
1Y1
And in the ones place, 9+Y is 11, so Y is 2.
We get 99+22=121 again.
1
99
+22
121
Very good, little monsters!
I'll write a few more--

ThWooo
Whooo
Whooo
Whooo
Whooo
Wooooosh
Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something much more diabolically difficult!
Decrypting double-digit addition dilemmas is undeniably dull.
My cryptarithm includes ***additional digits!***
Your beloved beast is barricaded behind this board.
Complete my clever cryptarithm to uncover the clue to the key's location...
...or your preeminent professor will face...
...Interminable Imprisonment!
SESK
+SEYK
EKNIN
CLUE:
6 1 0 9 8 9 2 8 9 2 6
Can you solve it? Try starting with E.

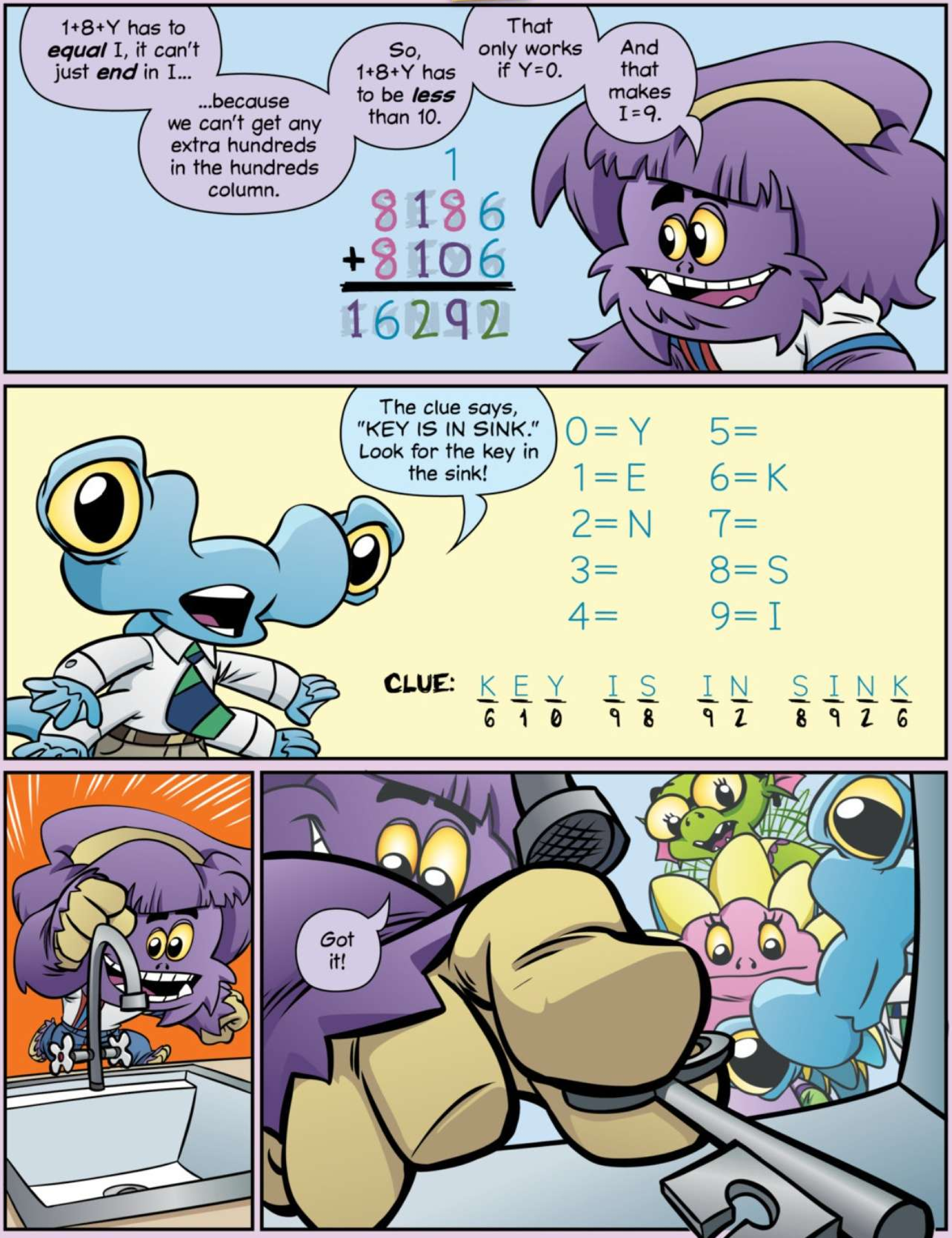


I never understand a word that guy says.
We need to solve this cryptarithm, then fill these blanks with the letters that go with the numbers to get a clue.
The clue will lead us to a key that unlocks Professor Grok.
SESK
+SEYK
EKNIN
CLUE:
6 1 0 9 8 9 2 8 9 2 6
Whoa! This looks hard. How do we even start?
I think we can find E first.
We add two 4-digit numbers...
...and we get a 5-digit sum.
SESK
+SEYK
EKNIN
The biggest 4-digit number is 9,999.
So, the sum of two 4-digit numbers can't be bigger than 9,999+9,999=19,998.
SESK
+SEYK
EKNIN
What does this tell us about E?


S 1 S K
+S 1 Y K
1 K N I N
I see. So, the E in EKNIN must be a 1.
And these other two E's are 1's, too.
Then, in the hundreds column, we have 1+1=N.
Does that mean N=2?
S 1 S K
+S 1 Y K
1 K N I N
It could be. Or, N could be 3 if we get an extra hundred when we add the tens.
But N can't be 3.
In the ones column, K+K=N. So, N has to be even.*
That means N has to be 2.
S 1 S K
+S 1 Y K
1 K 2 I 2
*DOUBLING A WHOLE NUMBER ALWAYS GIVES AN EVEN RESULT. REVIEW ODDS AND EVENS IN BEAST ACADEMY 2C.
How can K+K=2?
We already used 1 for E...
...so K can't be 1.
What is K?



K+K doesn't have to **equal** 2...
...it just has to **end** in 2.
6+6=12. No other digit works for K.
Oh, right. I keep forgetting.
So, K=6, and we get an extra 1 in the tens place.
1
S 1 S 6
+S 1 Y 6
1 6 2 I 2
Now, we can find S. Adding the thousands, S+S=16.
So, S=8.
All we have left are Y and I.
We're almost there!
1
8 1 8 6
+8 1 Y 6
1 6 2 I 2
We already used 1, 2, 6, and 8.
I know! There's only one digit left that works for Y.
Find Y and I.



1+8+Y has to ***equal*** I, it can't just ***end*** in I...
...because we can't get any extra hundreds in the hundreds column.
So, 1+8+Y has to be ***less*** than 10.
That only works if Y=0.
And that makes I=9.
1
8186
+8106
16292
The clue says, "KEY IS IN SINK." Look for the key in the sink!
0=Y 5=
1=E 6=K
2=N 7=
3= 8=S
4= 9=I
CLUE: K E Y I S I N S I N K
6 1 0 9 8 9 2 8 9 2 6
Got it!


KLiK
Incredible work, little monsters!
You've rescued me again!
ORE
GUMBALL MADNESS
11,051,978
Mr. CH
SUPER Digger
SUPER Digger