Contents: Chapter 9

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The sum of two whole numbers is odd. How many of those numbers are odd?

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Is the sum of all of the whole numbers from 1 to 50 odd or even?

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Is the number of times you've passed through your bedroom door odd or even?

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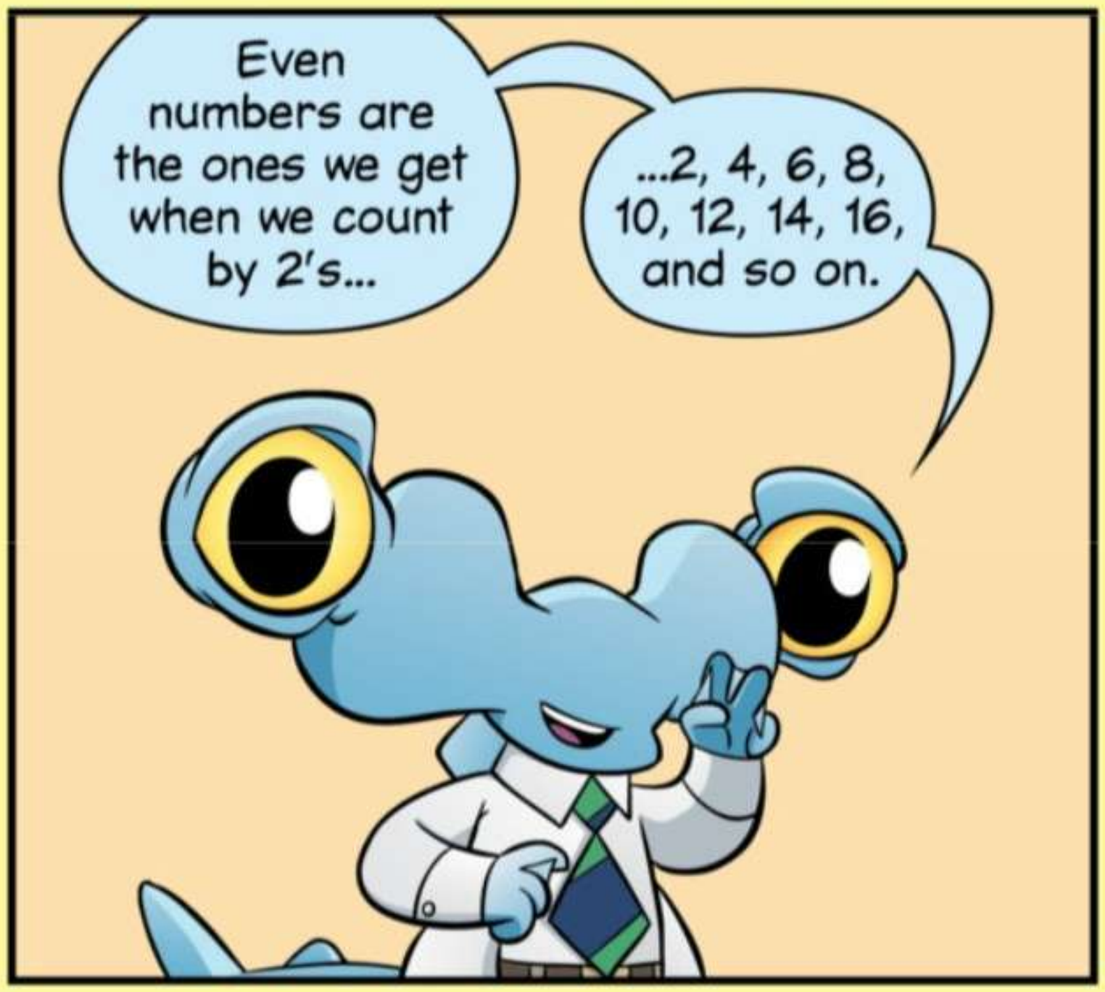
Starting with 5 coins on heads, can you turn over two coins at a time until there are five tails?

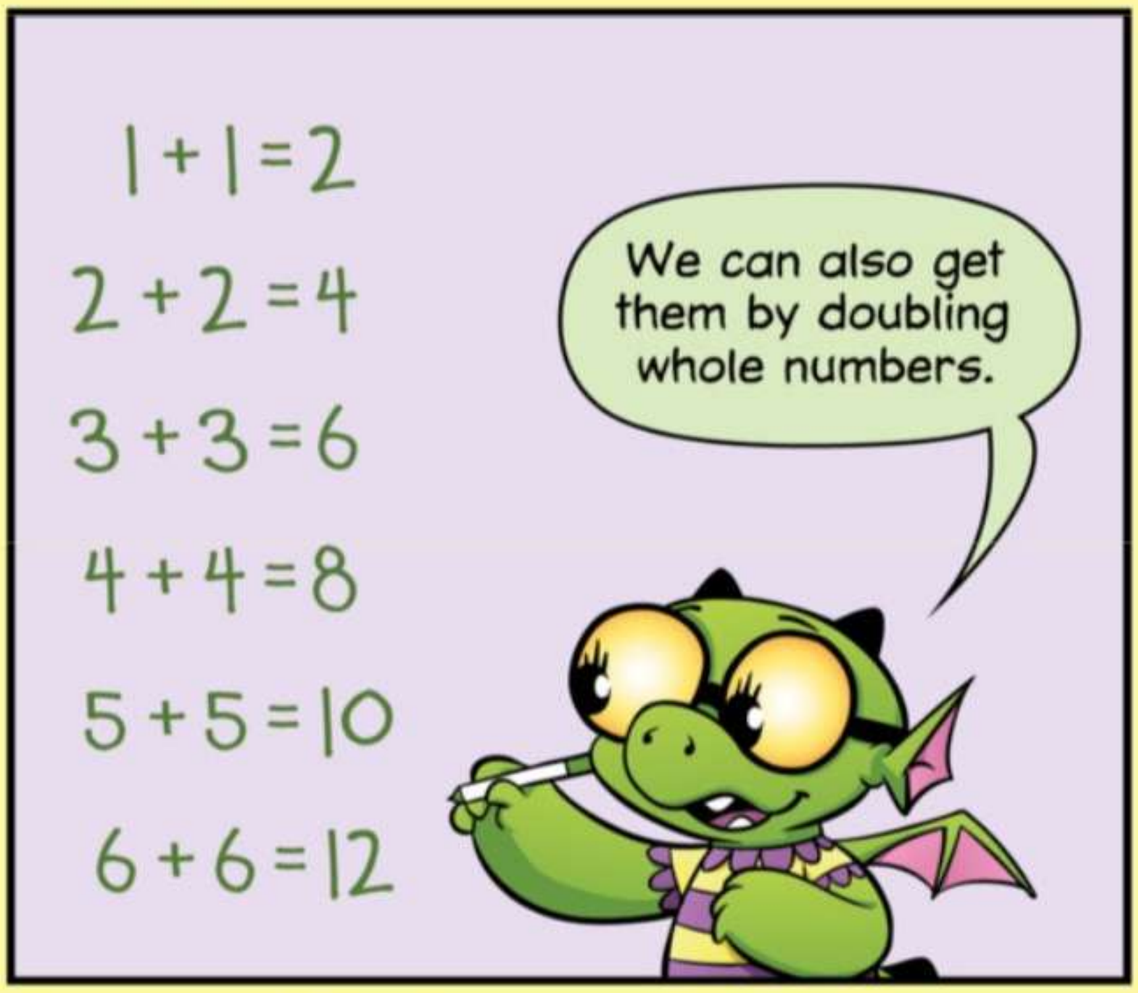
Checkerboards 96

What do odd and even numbers have to do with a checkerboard?

Chapter 9:
Odds & Evens

Ms. Q.
Odd & Even
What are even numbers?


Even numbers are the ones we get when we count by 2's...
...2, 4, 6, 8, 10, 12, 14, 16, and so on.

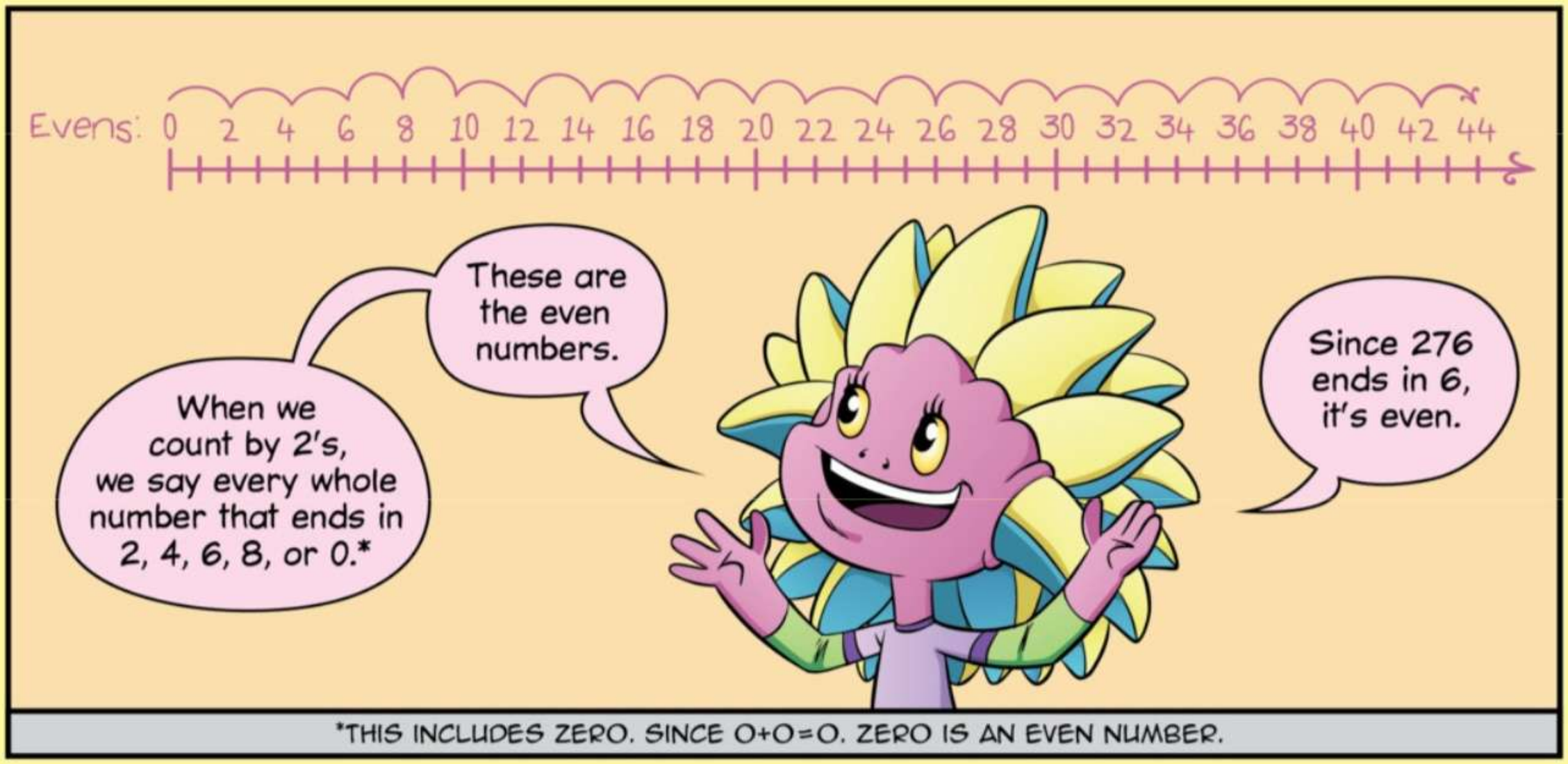

1 + 1 = 2
2 + 2 = 4
3 + 3 = 6
4 + 4 = 8
5 + 5 = 10
6 + 6 = 12
We can also get them by doubling whole numbers.

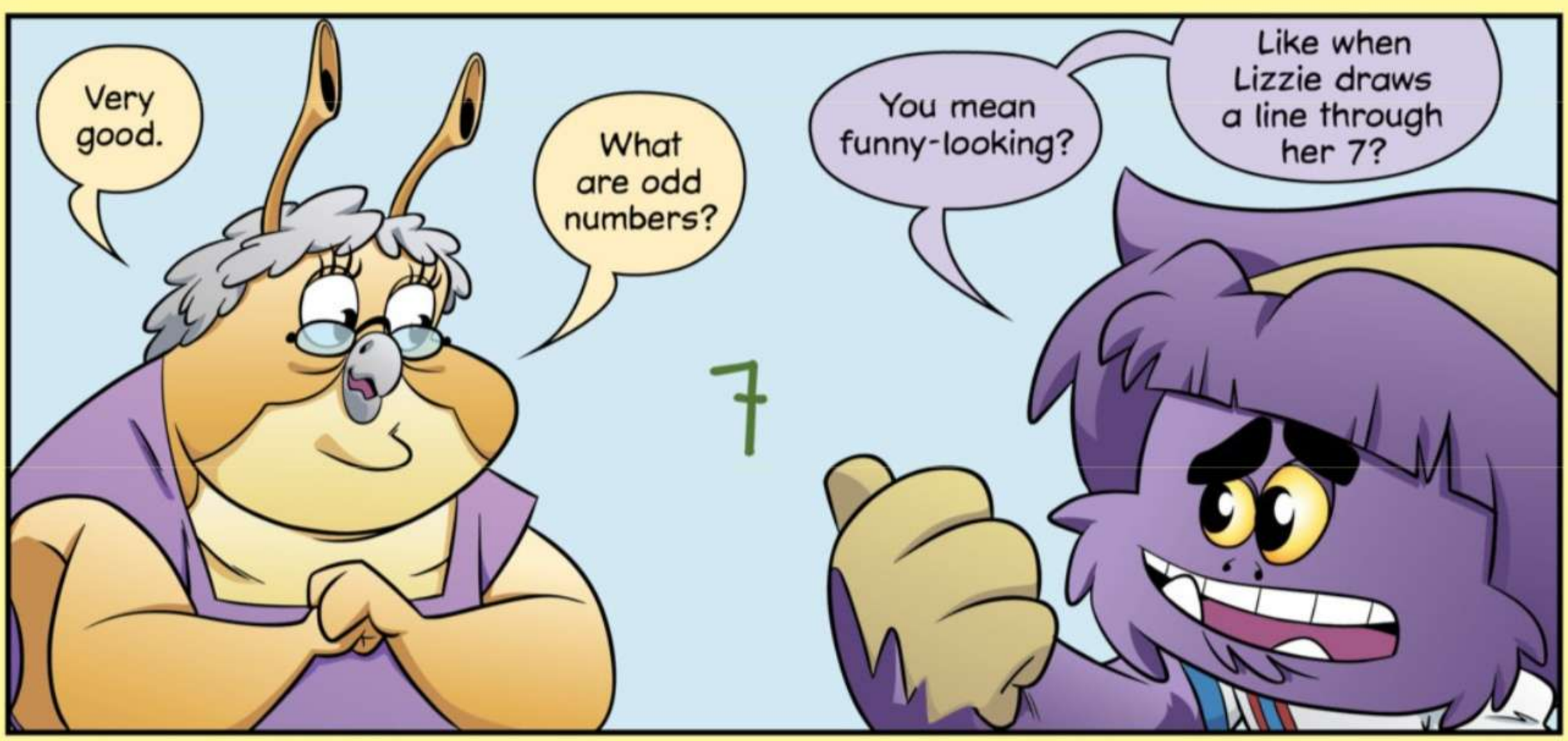
We can split every even number into groups of 2...
...or into 2 equal groups.
12
2
+2
+2
+2
+2
+2
12
6+6

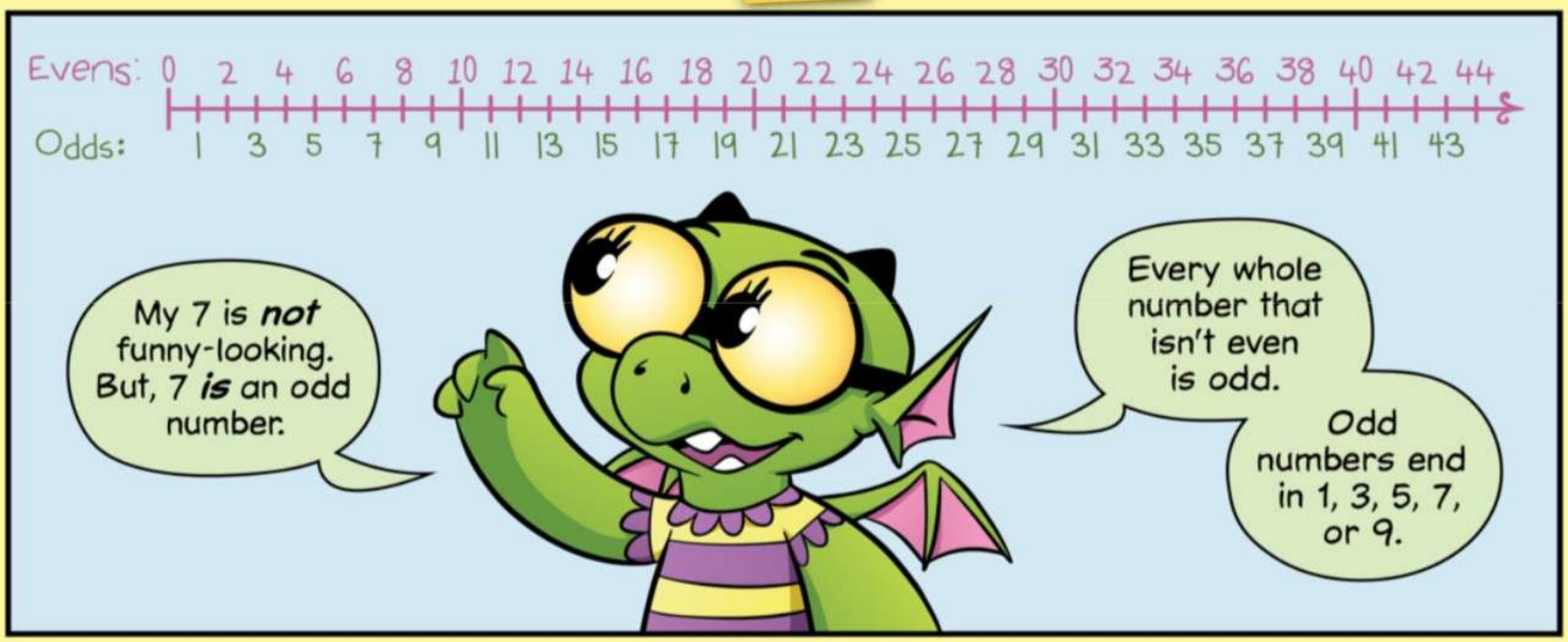
Good. Is 276 even?
276
Is it?

Yep.
276

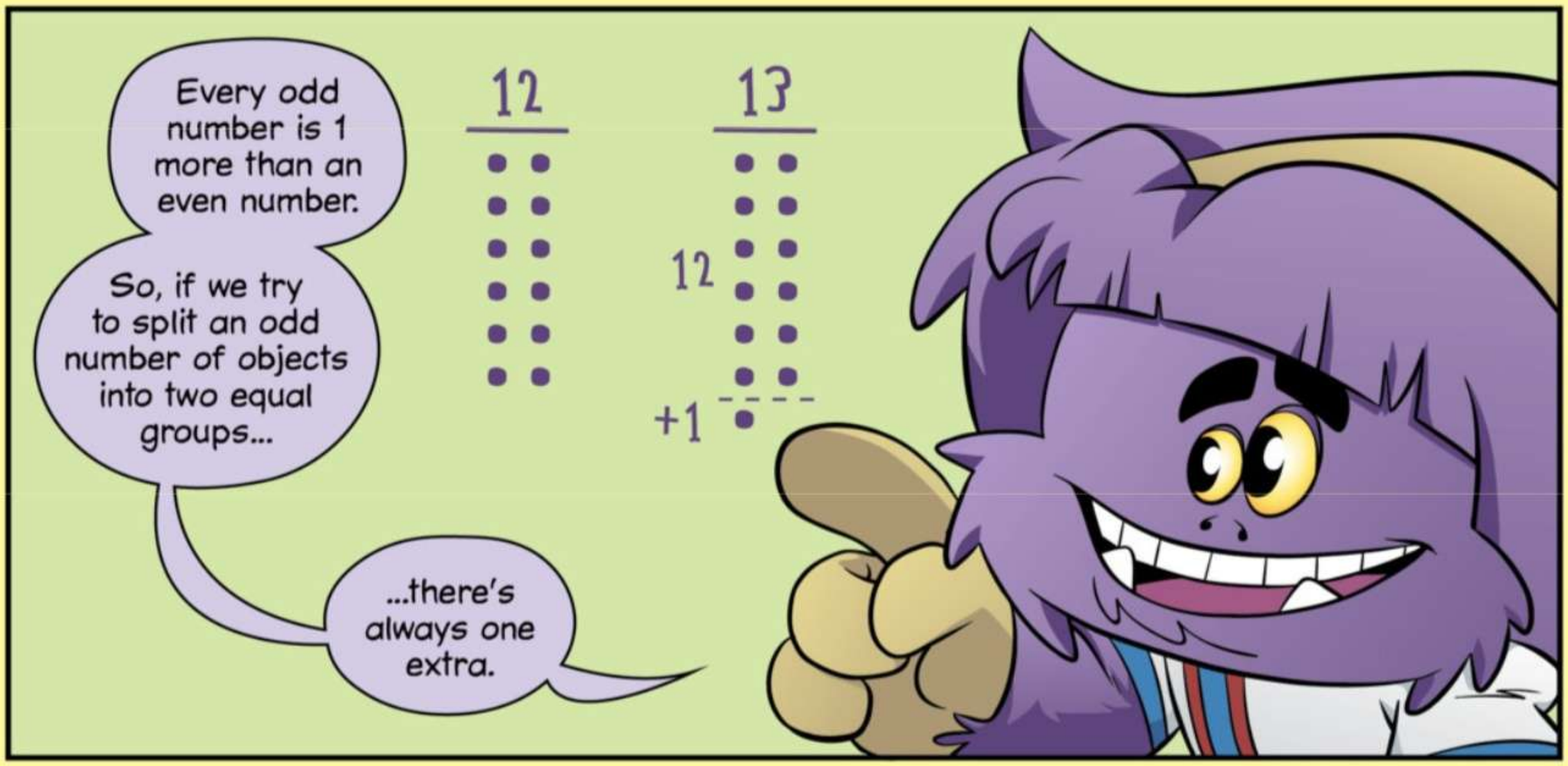
How can you tell?


Evens: 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44
When we count by 2's, we say every whole number that ends in 2, 4, 6, 8, or 0.*
These are the even numbers.
Since 276 ends in 6, it's even.
*THIS INCLUDES ZERO. SINCE 0+0=0. ZERO IS AN EVEN NUMBER.

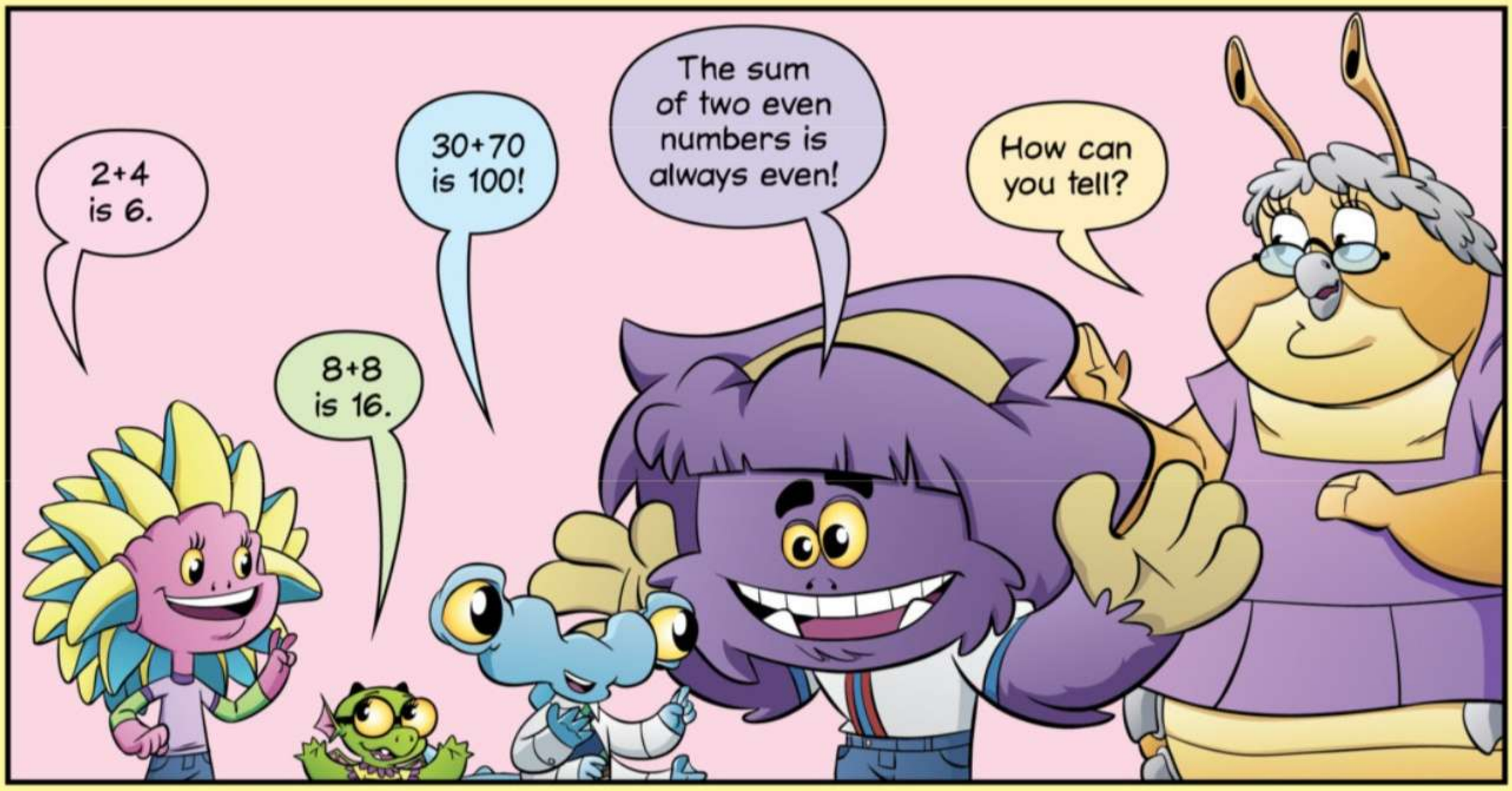

Very good.
What are odd numbers?
7
You mean funny-looking?
Like when Lizzie draws a line through her 7?

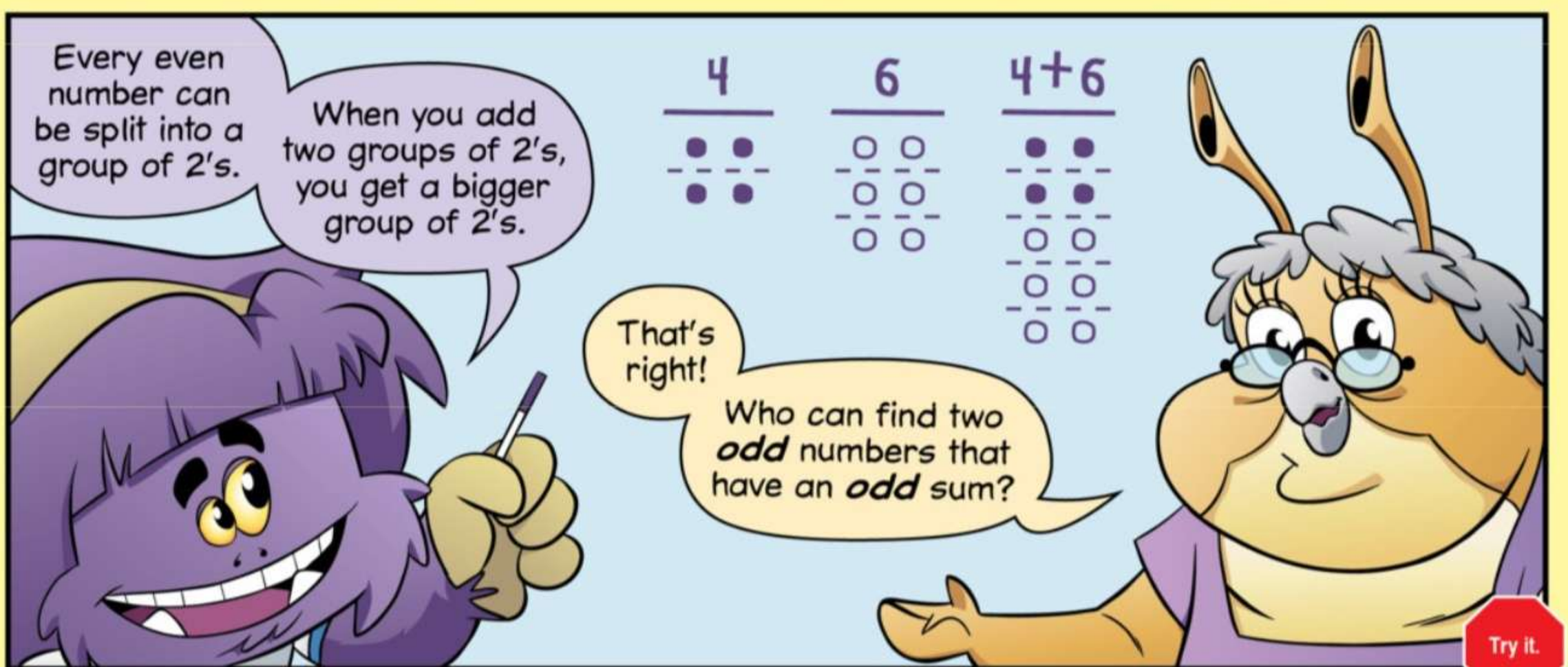

Evens: 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44
Odds: 1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43
My 7 is *not* funny-looking. But, 7 *is* an odd number.
Every whole number that isn't even is odd.
Odd numbers end in 1, 3, 5, 7, or 9.

That's right.
Can you split an *odd* number of objects into two equal groups?

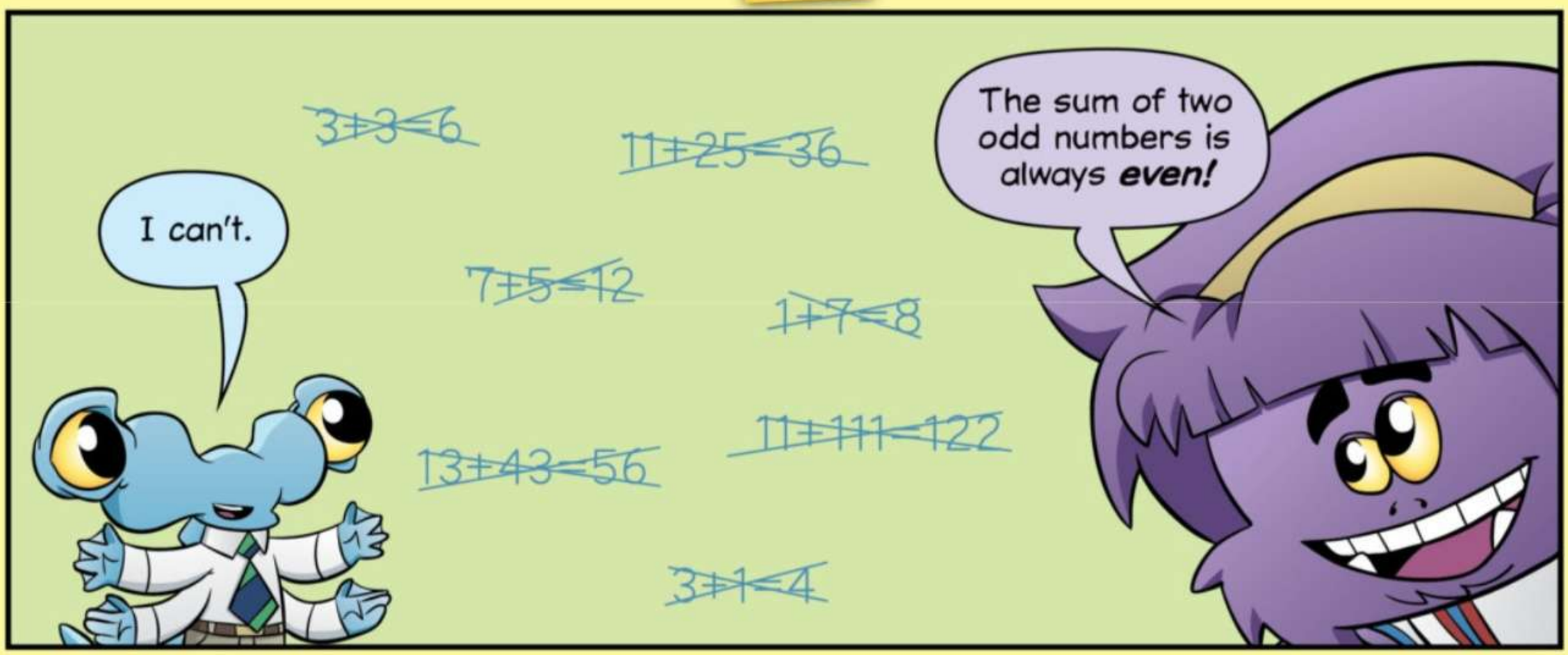

Every odd number is 1 more than an even number.
So, if we try to split an odd number of objects into two equal groups...
...there's always one extra.
12
13
12
+1

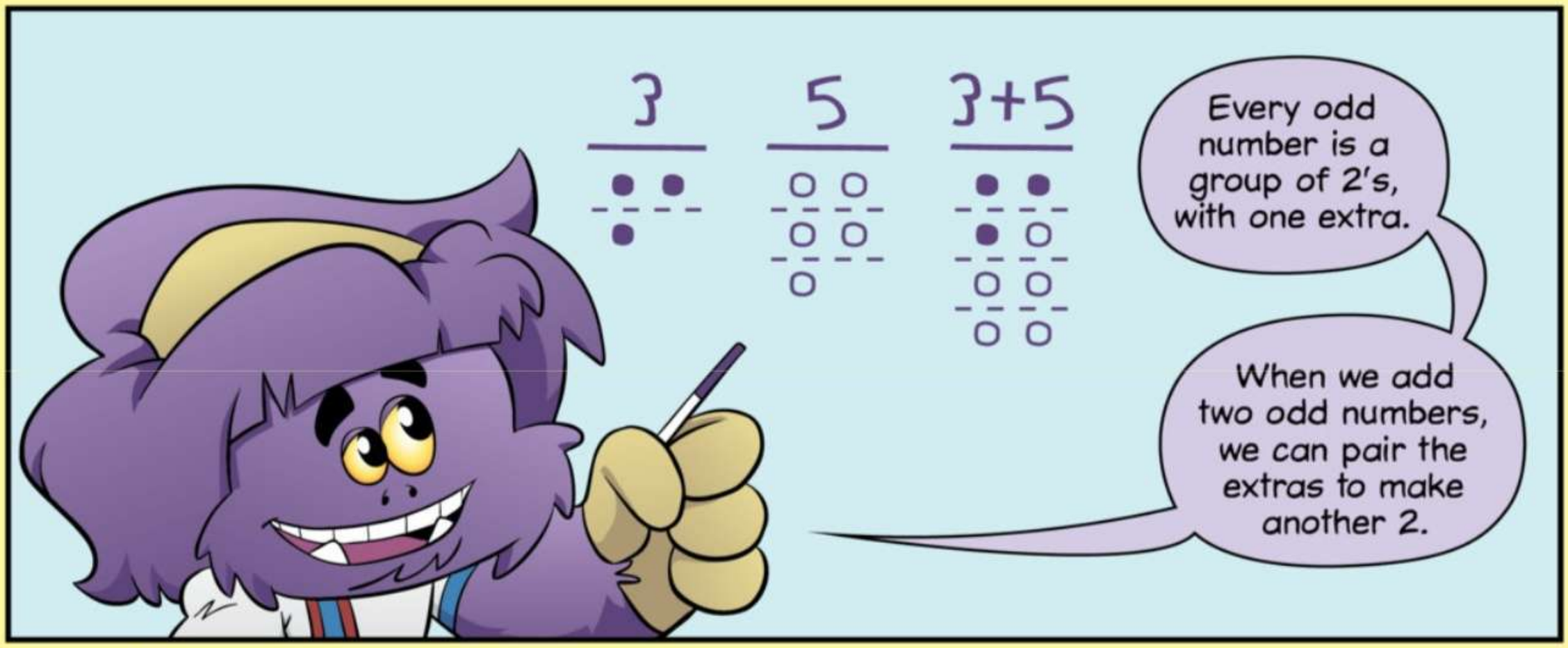
Very good. Let's explore what happens when we add odd and even numbers.
Who can find two even numbers that have an even sum?


2+4 is 6.
8+8 is 16.
30+70 is 100!
The sum of two even numbers is always even!
How can you tell?


Every even number can be split into a group of 2's.
When you add two groups of 2's, you get a bigger group of 2's.
4
6
4+6
That's right!
Who can find two odd numbers that have an odd sum?

Try it.





Great job, Grogg.

When we add two numbers that are both odd, or both even...

...the sum is always even.

What about an even number plus an odd number?

even+even=even

odd+odd=even

even+odd=

Is even+odd odd or even?

Adding a group of 2's to a group of 2's with one extra...

4 5 4+5

...gives us a bigger group of 2's, and there is still one extra.

So, even+odd is always odd.

That's right.

And since we can change the order of numbers in an addition problem...

...odd plus even is also odd.

Did we miss anything?

even + even = even

odd + odd = even

even + odd = odd

odd + even = odd

What's that, Lizzie?

There's something else you can add to "even" to make an odd number.

The letter "s".

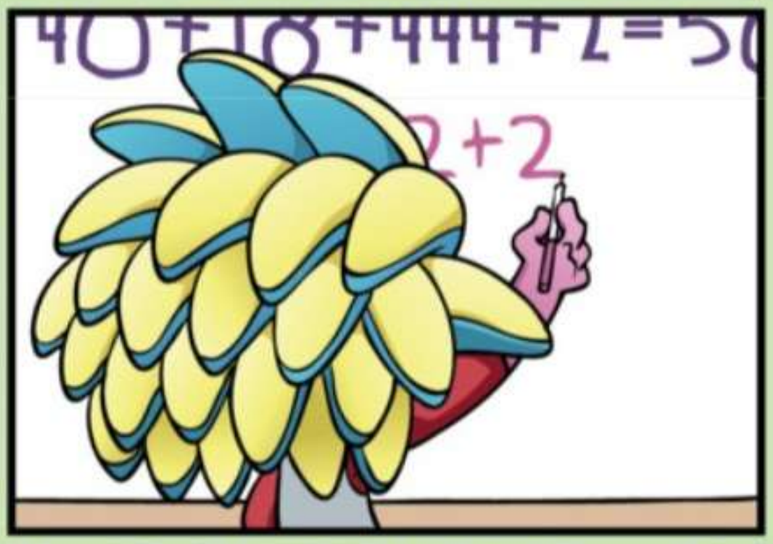
seven

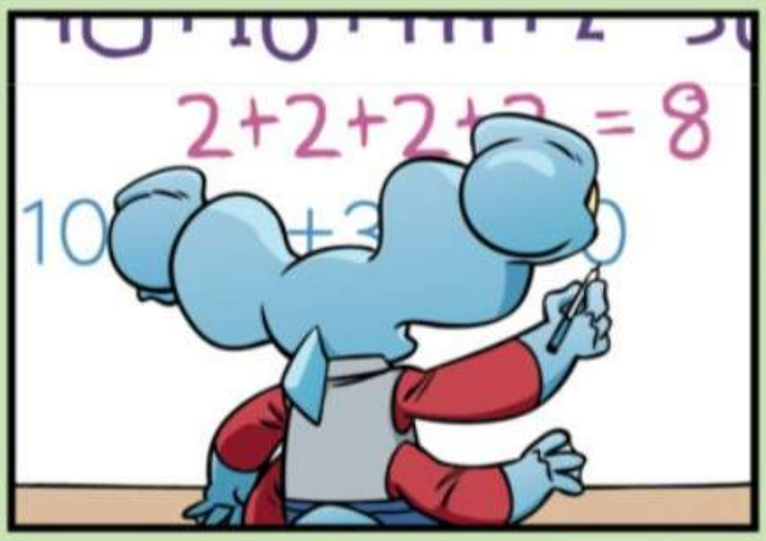
MATH TEAM
More Than Two

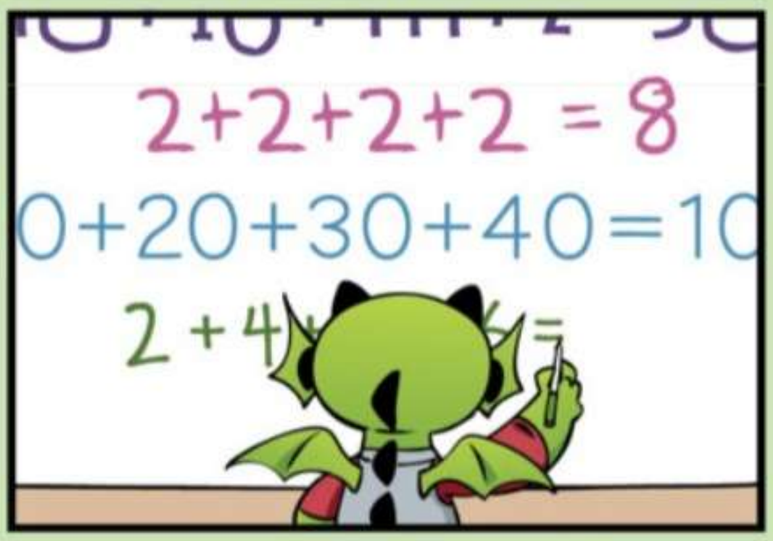
Today, we're going to explore what happens when we add more than two odds or two evens.*

*WE SOMETIMES CALL ODD NUMBERS AND EVEN NUMBERS "ODDS" AND "EVENS" FOR SHORT.

First, write four even numbers and add them.







All our sums are even.

Yep.

Will four even numbers ***always*** have an even sum?

40+18+444+2=504
2+2+2+2 = 8
10+20+30+40=100
2+4+8+16=30

Is the sum of four even numbers always even?

When you add four even numbers, the first two numbers have an even sum.

Adding the third even number gives us another even sum.

Same for the fourth.

And if you keep adding even numbers, the sum will always be even!

even + even + even + even

even + even + even

even + even

even

Every even number can be split into a group of 2's.

When you add groups of 2's, you get a bigger group of 2's.

So, no matter how many even numbers you add, you'll always get an even sum.

4+6+8

That's right.

Adding ***any number*** of evens always gives an even sum.

Next, add 4 ***odd*** numbers.

Do you think you'll all get odd sums, even sums, or some of both?

4+6+8

Try it. What do you think?

All our sums are even again.

That's true. Will four odds ***always*** have an even sum?

37+7+333+5=382

1+1+1+1 = 4

5+15+25+35=80

5+11+55+15 = 222

When we add four odd numbers, the first two numbers have an even sum.

Then, when we add the third odd number, we get an odd sum.

Finally, odd+odd is even.

odd+odd+odd+odd

even+odd+odd

odd+odd

even

You could also add the odd numbers in pairs, like this.

Perfect. The sum of 4 odds is always even.

Does adding more than two odds ***always*** give an even sum?

odd+odd+odd+odd

even + even

even

Does it?

When does adding odds give an even sum?

Not always.

3+5 = 8 (even)
3+5+7 = 15 (odd)
3+5+7+9 = 24 (even)
3+5+7+9+11 = 35 (odd)
3+5+7+9+11+13 = 48 (even)

When you add an ***odd*** number of odds, the sum is odd.

But when you add an ***even*** number of odds, the sum is even.

Huh?

DON'T SPEND TIME TRYING TO MEMORIZE ALL OF THE RULES IN THIS CHAPTER. ONCE YOU UNDERSTAND THE BASICS OF ODDS AND EVENS, YOU CAN COME UP WITH THESE RULES AND MORE ON YOUR OWN.

odd+odd+odd+odd+odd+odd+odd+odd

even + even + even + even

even

If you add an even number of odds...

...you can add them all in pairs.

Each pair has an even sum.

Since adding even numbers always gives an even sum...

...the sum is even.

odd+odd+odd+odd+odd+odd+odd+odd + odd

even + even + even + even + odd

even + odd

Odd

But, if you try to add an ***odd*** number of odds in pairs...

...there is always one extra.

So, you end up with even+odd, which is odd!

Well done.

An ***even*** number of odds always has an even sum.

But an ***odd*** number of odds always has an odd sum.

Next, write four ***consecutive*** numbers and add them.

CONSECUTIVE NUMBERS COME ONE AFTER ANOTHER. FOR EXAMPLE, 31, 32, 33, AND 34 ARE CONSECUTIVE.

All of our sums are even again!

Is the sum of four consecutive numbers ***always*** even?

222+223+224+225=894

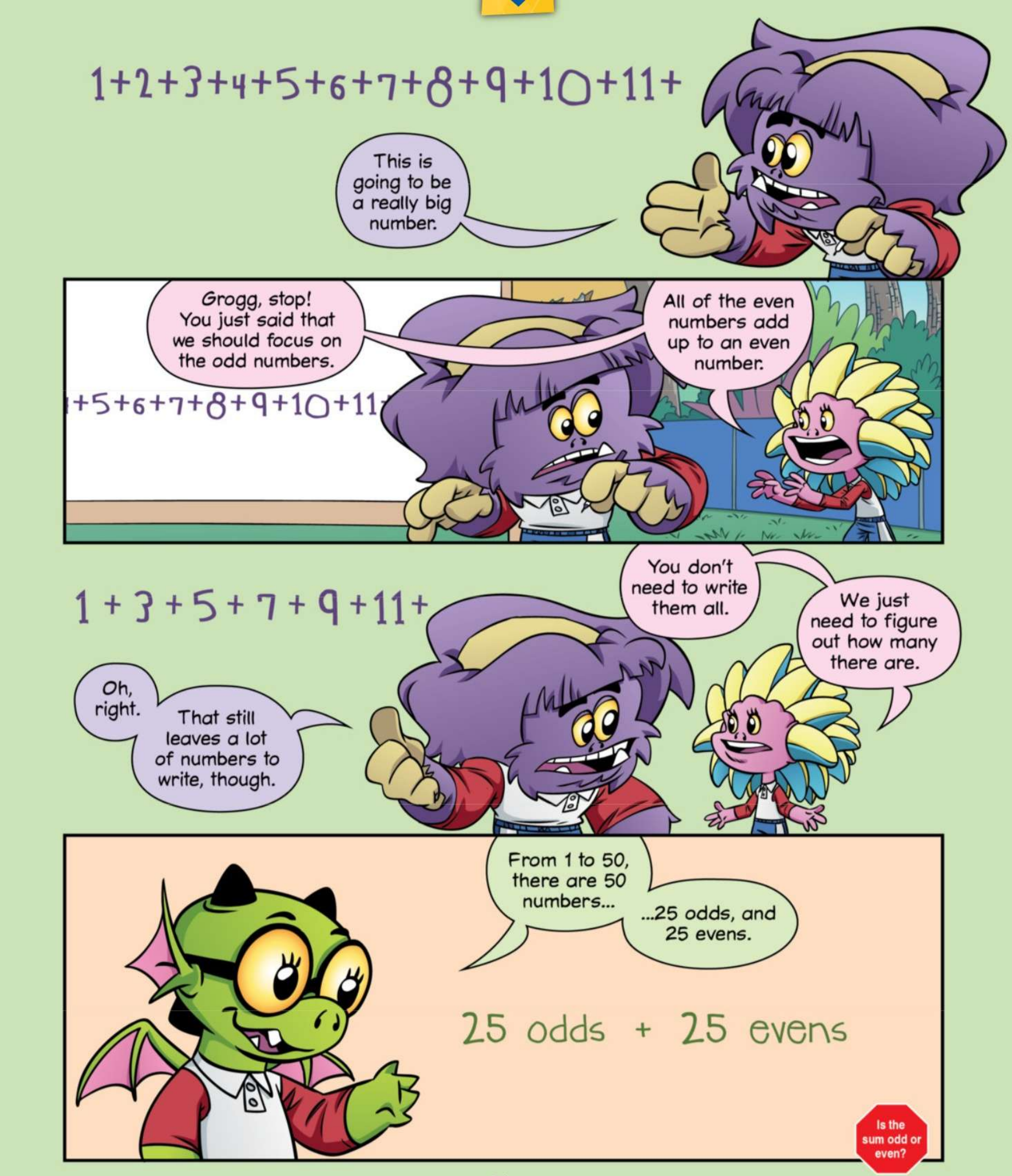
1+2+3+4 = 10

10+11+12+13=46

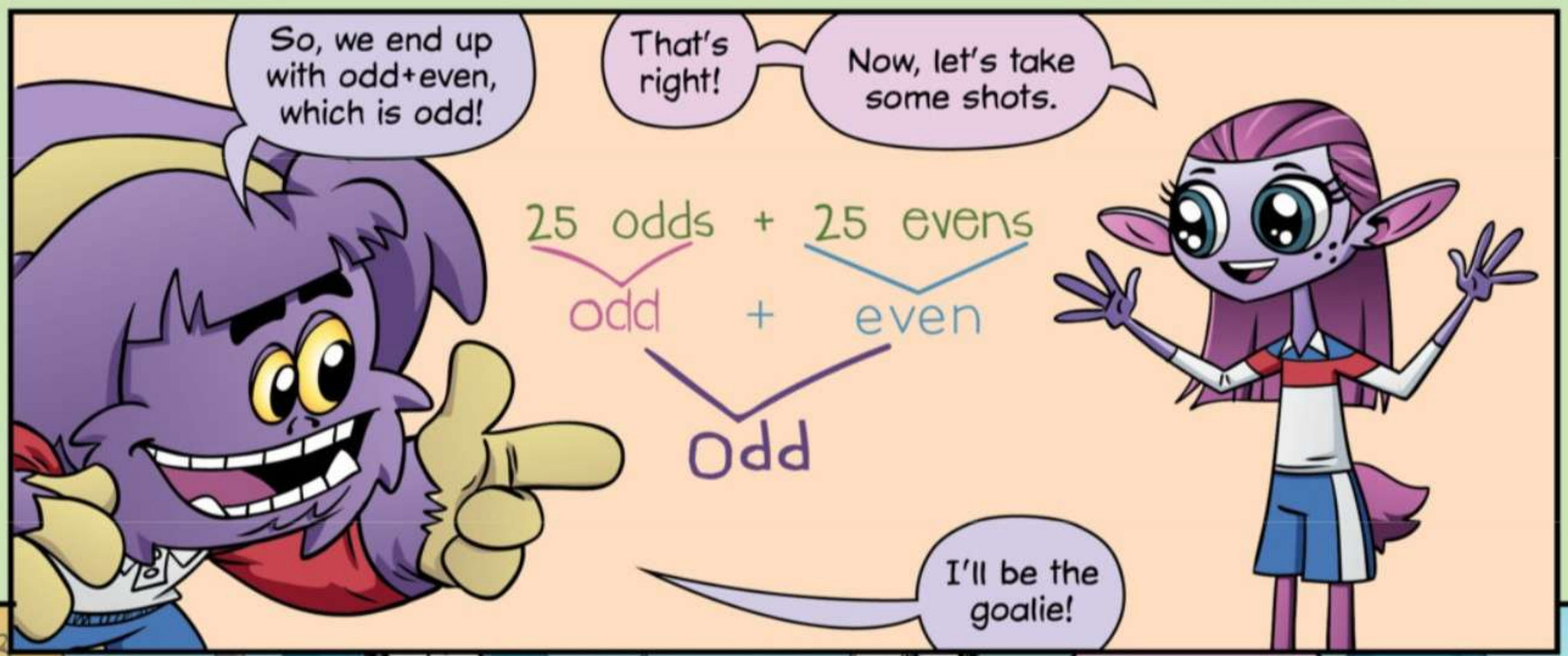
4+5+6+7 = 22

Is it?

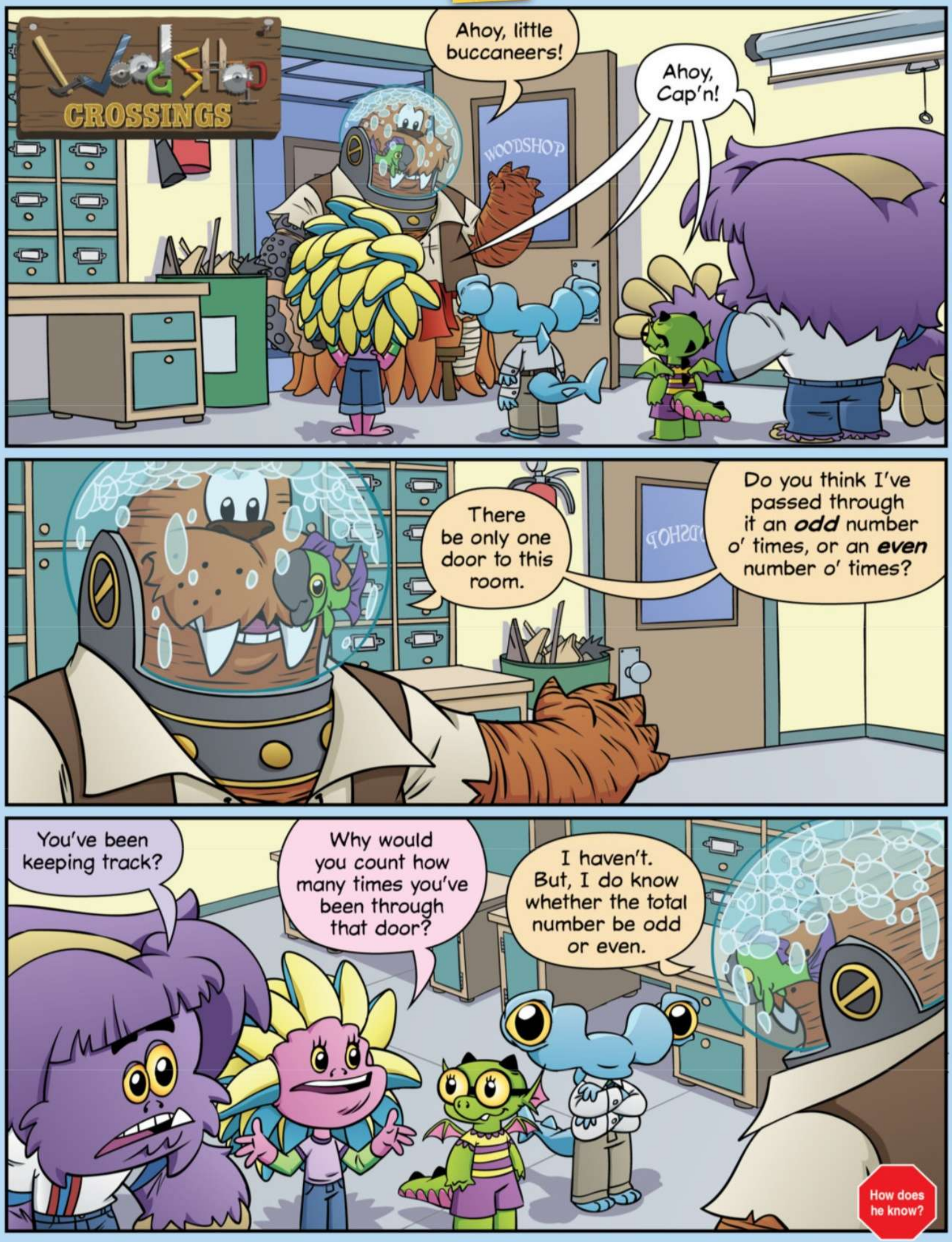
odd+even+odd+even or even+odd+even+odd
When you count, the numbers go odd, even, odd, even, and so on.
So, four numbers in a row always have two odds and two evens.
We can add the two odd numbers to get an even number.
Then, all we have left are evens.
So, the whole sum is even.
odd+even+odd+even
even+even+even
even
I see.
Even numbers always have an even sum.
So, it makes sense to focus on the odd numbers, since their sum can be odd or even.
Great! Let's finish with a hard one.
Is the sum of all the whole numbers from 1 to 50 odd or even?
Which is it?

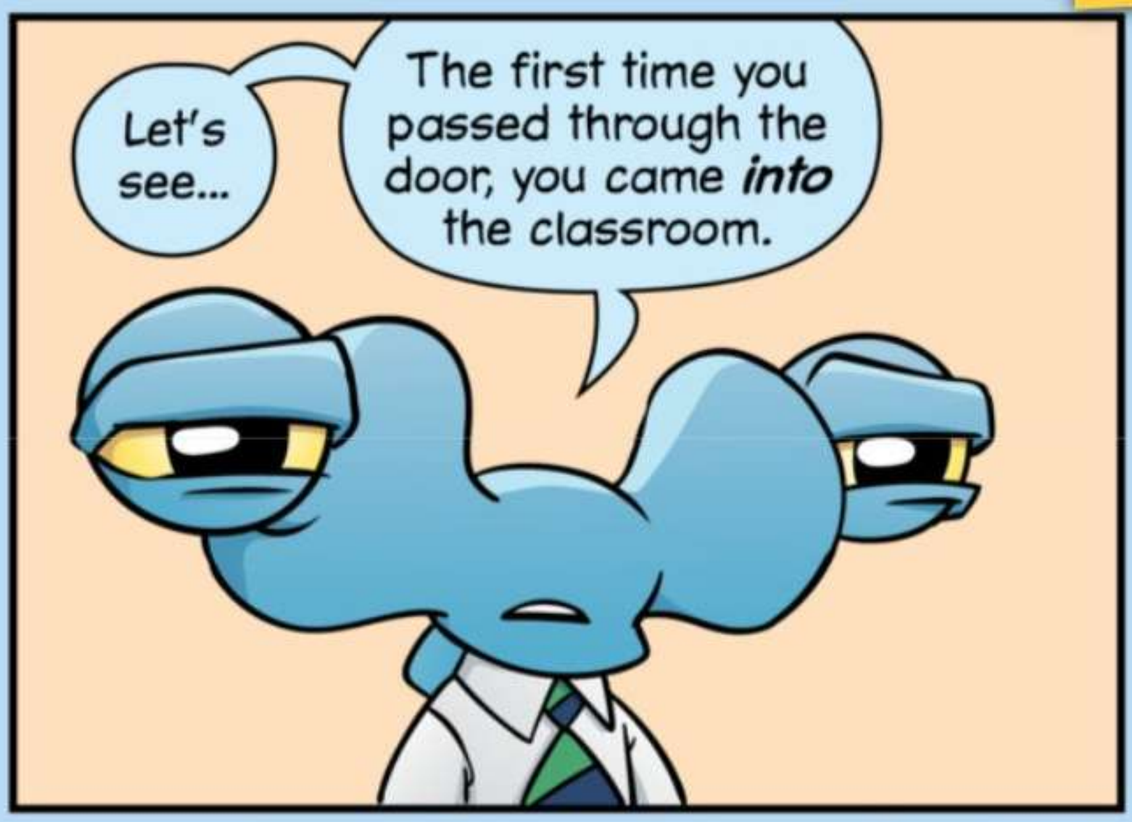

1+2+3+4+5+6+7+8+9+10+11+
This is going to be a really big number.
Grogg, stop! You just said that we should focus on the odd numbers.
+5+6+7+8+9+10+11+
All of the even numbers add up to an even number.
1 + 3 + 5 + 7 + 9 + 11 +
Oh, right.
That still leaves a lot of numbers to write, though.
You don't need to write them all.
We just need to figure out how many there are.
From 1 to 50, there are 50 numbers...
...25 odds, and 25 evens.
25 odds + 25 evens
Is the sum odd or even?

An *odd* number of odds always have an odd sum.
And the even numbers have an even sum.
25 odds + 25 evens
odd + even


So, we end up with odd+even, which is odd!
That's right!
Now, let's take some shots.
25 odds + 25 evens
odd + even
Odd
I'll be the goalie!

No fair! His head blocks the whole goal!
Kick!
Bonk!
Bonk!
Bonk!
Kick!
Kick!


WOODSHOP
CROSSINGS
Ahoy, little buccaneers!
Ahoy, Cap'n!
WOODSHOP
There be only one door to this room.
Do you think I've passed through it an **odd** number o' times, or an **even** number o' times?
You've been keeping track?
Why would you count how many times you've been through that door?
I haven't. But, I do know whether the total number be odd or even.
How does he know?


Let's see...
The first time you passed through the door, you came *into* the classroom.

And the second time you passed through the door...
...you went out.

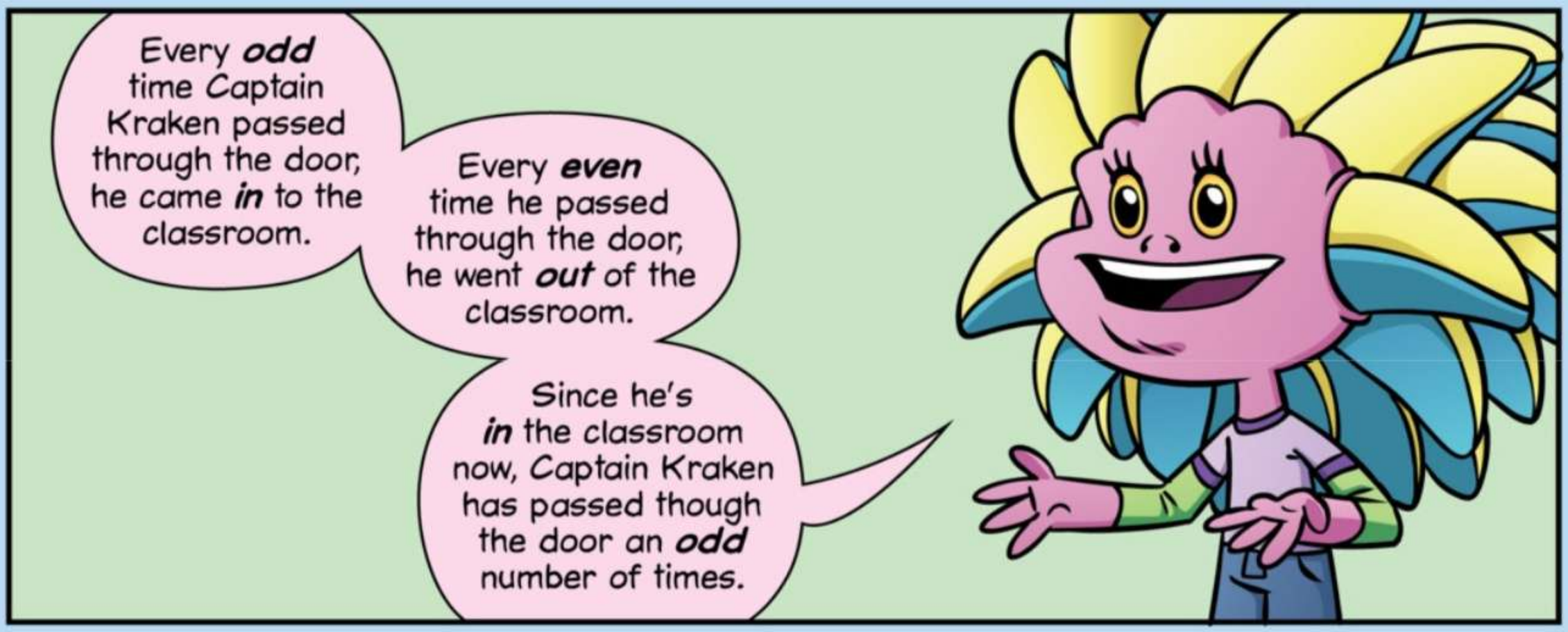
The 3rd time,
you came in...

...the 4th time, you went out...

...the 5th time, you came in...
KICK

...the 6th time, you went out...

...the 7th time, you came in--
Grogg!
Sliiiiiiide


Every *odd* time Captain Kraken passed through the door, he came *in* to the classroom.
Every *even* time he passed through the door, he went *out* of the classroom.
Since he's *in* the classroom now, Captain Kraken has passed though the door an *odd* number of times.

Aye, I've been through this door an odd number o' times.
All of us have passed through the door an odd number o' times!
Otherwise, we wouldn't be in the room!
Unless we came in through the window.

Well done. A similar idea helped me uncover a long-lost treasure.
Take a look at this map o' the shore lines o' the Squiggly Isles.
The treasure be marked with an X.
The first key to findin' the treasure be figurin' if the treasure be buried on land or at sea.

You can't even tell which part of the map is land and which part is water!
Aye, 'tis a terrible map.
Here's a clue! This boat must be in the water.
Is the treasure on land or at sea?

Since these are shore lines...

...if the boat is in the water...

...then the other side of the shore line is land.

Every time you cross a shore line, you switch...

...land, water, land, water, and so on.

Starting at the boat, the 1st time you cross a shore line you end up on land.

The 2nd time you cross a line, you end up in water.

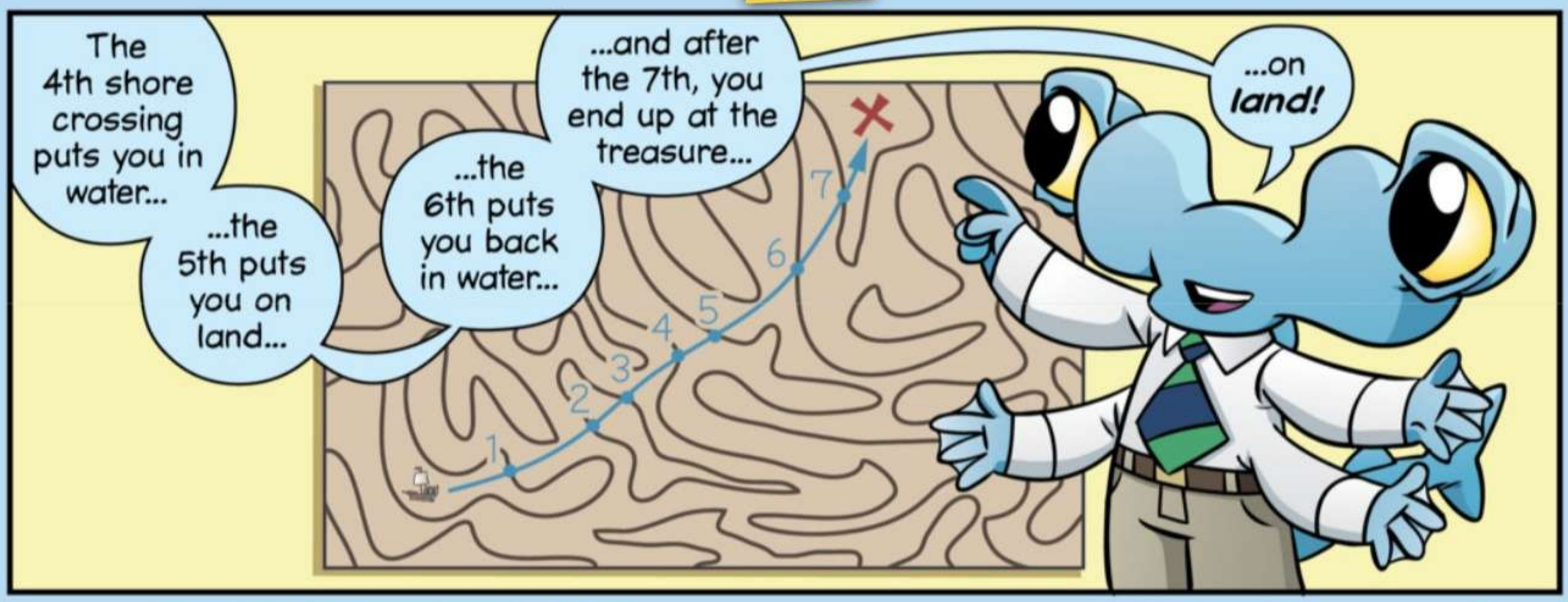
The 3rd time you cross a line, you end up on land again.

1

2

3

How many shore lines does Alex's path cross?

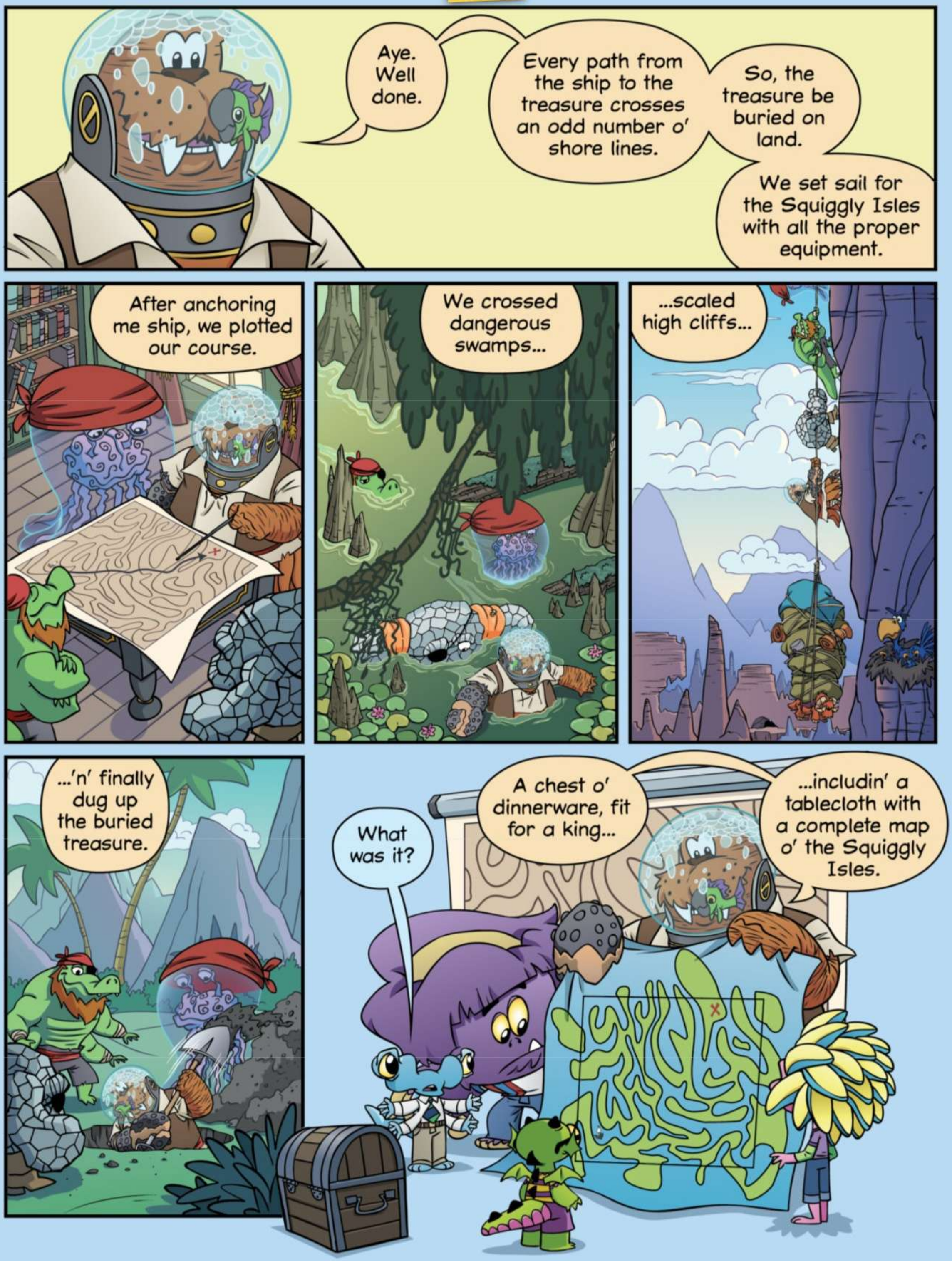

The 4th shore crossing puts you in water...
...the 5th puts you on land...
...the 6th puts you back in water...
...and after the 7th, you end up at the treasure...
1
2
3
4
5
6
7
...on land!

Starting in water, every time you cross 2 shore lines, you end up back in water.
So, if you cross an **even** number of shore lines, you always end up in the water.
And if you cross an **odd** number of shore lines, you always end up on land.

Cool!
If you go this way, you cross 7 shore lines, too.

This way, you only cross 5 shore lines.

And if you go this way, you'll cross 11!


Aye. Well done.
Every path from the ship to the treasure crosses an odd number o' shore lines.
So, the treasure be buried on land.
We set sail for the Squiggly Isles with all the proper equipment.
After anchoring me ship, we plotted our course.
We crossed dangerous swamps...
...scaled high cliffs...
...'n' finally dug up the buried treasure.
What was it?
A chest o' dinnerware, fit for a king...
...includin' a tablecloth with a complete map o' the Squiggly Isles.

R&G Coins

There is a lot of new stuff to install in this science lab.

Professor Grok has asked for a lot of strange equipment!

Sure.

Want to try a puzzle while we wait for the next delivery?

Great!

Start with five coins, all on heads.

You can turn over any ***two coins*** at a time.

For example, you can turn over the first coin and the fourth coin like this.

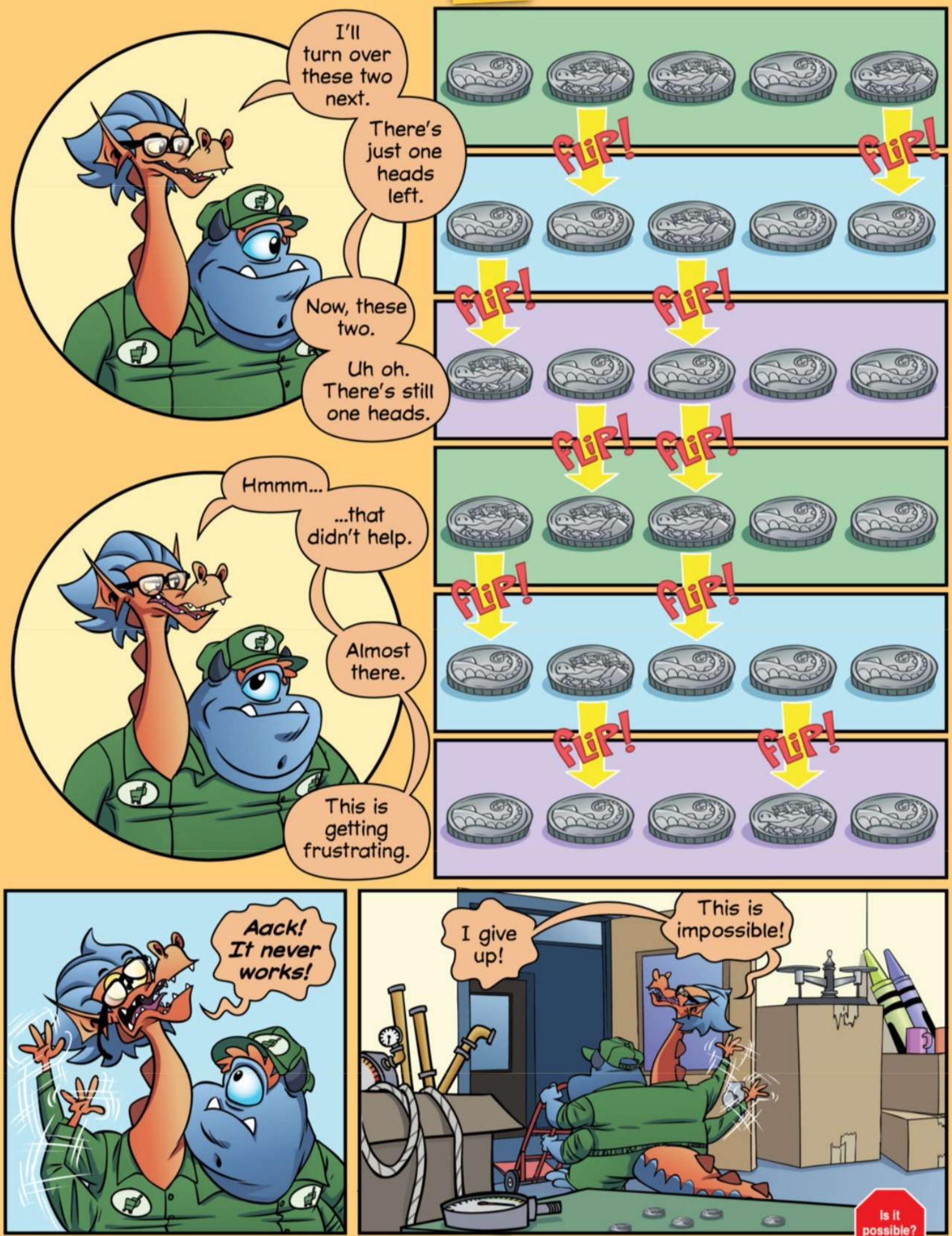
The goal is to get all of the coins on tails.*

FLIP!

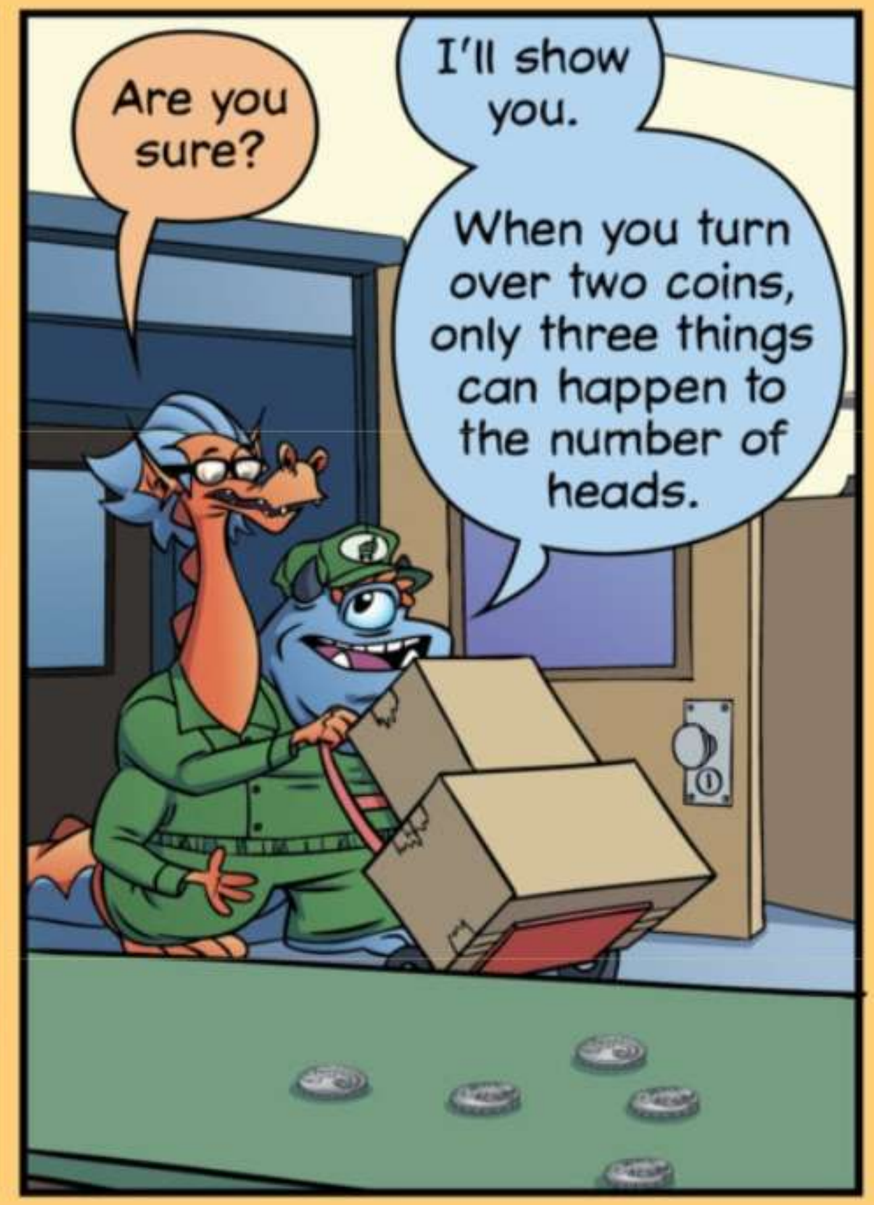
FLIP!

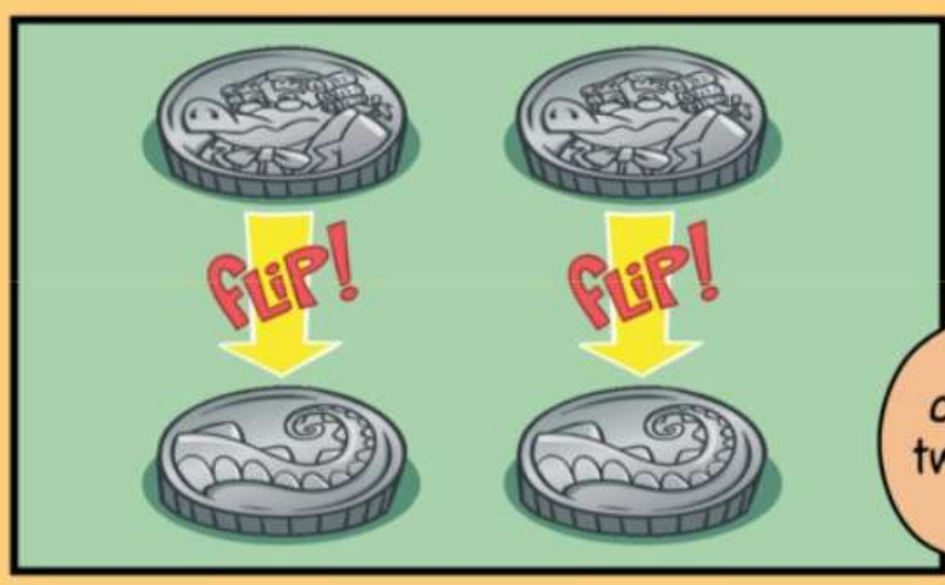
Get 5 coins and try it.

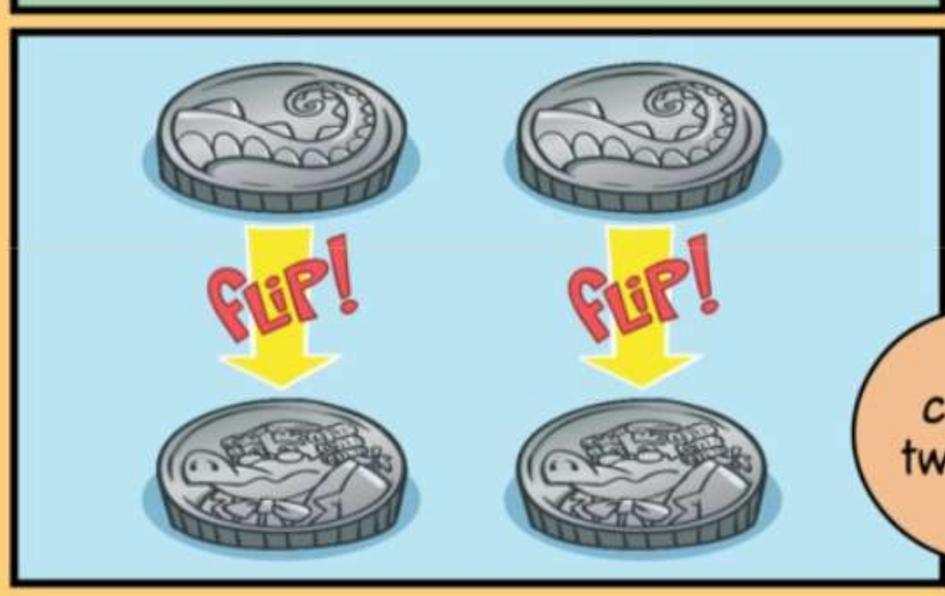
*THE HEADS SIDE OF A COIN HAS A HEAD ON IT. THE TAILS SIDE IS THE OTHER SIDE.

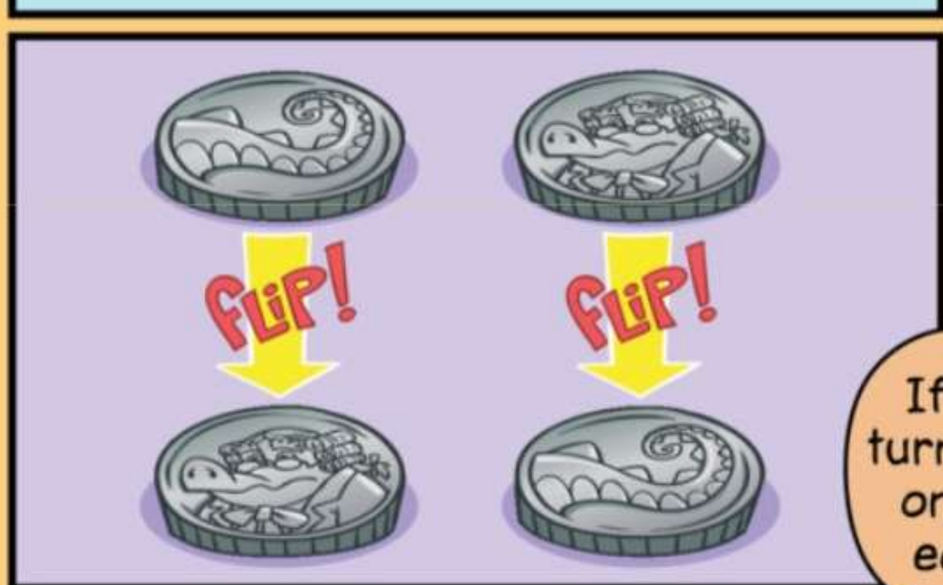

I'll turn over these two next.
There's just one heads left.
Now, these two.
Uh oh. There's still one heads.
FLIP!
Hmmm...
...that didn't help.
Almost there.
This is getting frustrating.
Aack! It never works!
I give up!
This is impossible!
Is it possible?

That's right!
It *is* impossible!
Yep!
What!? You had me trying to solve an impossible puzzle!?
If you start with 5 coins on heads and turn over 2 at a time...
...you can never get all tails.


Are you sure?
I'll show you.
When you turn over two coins, only three things can happen to the number of heads.


FLIP!
FLIP!


FLIP!
FLIP!


FLIP!
FLIP!

You can decrease the number of heads by 2.
By changing two heads to tails.
You can increase the number of heads by 2.
By changing two tails to heads.
Or, the number of heads can stay the same.
If you turn over one of each.

Since I can only change the number of heads by 2, I can only get 5, 3, or 1 heads.
That's right. There will always be an odd number of heads.
So, no matter what, there will always be at least 1 heads.

I have a coin trick to show you.
Watch me pull a quarter out of your ear!
Now *that's* impossible!

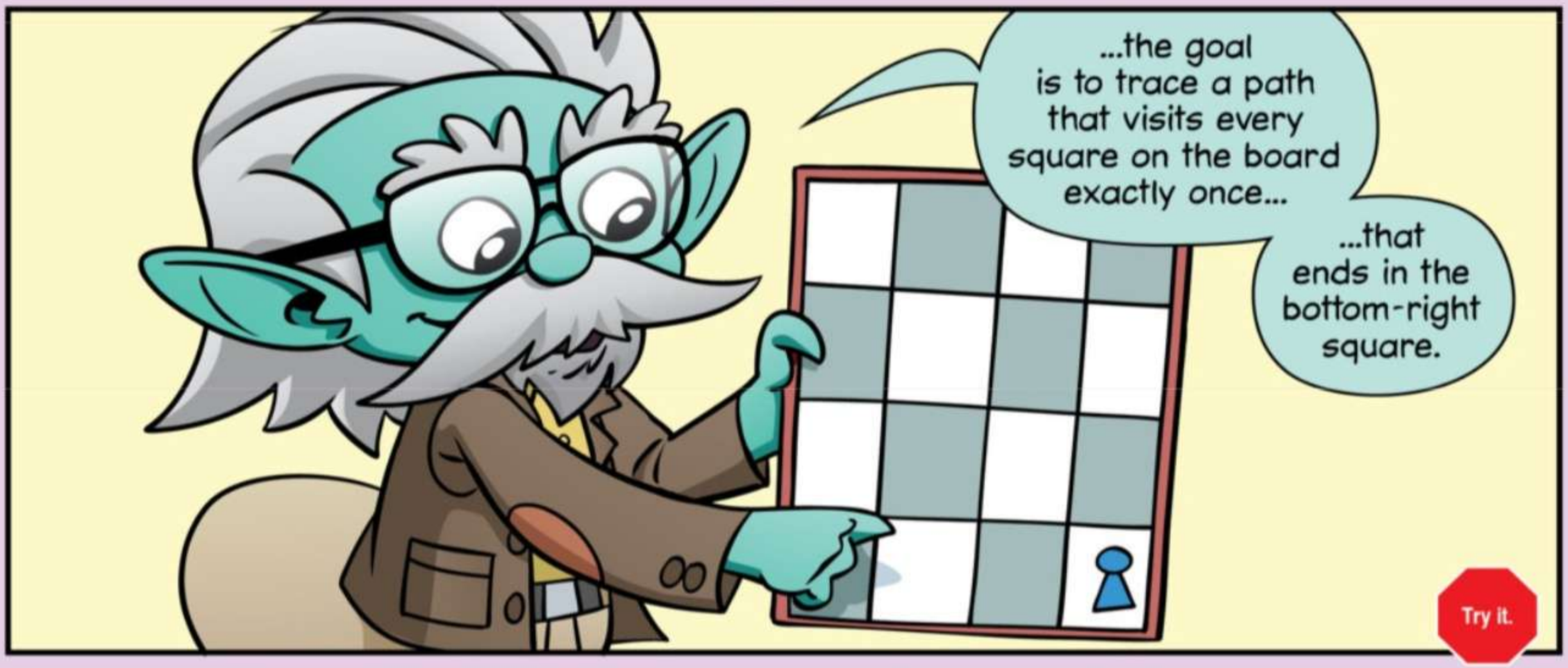
Nothing up my sleeves.
There's no way this is gonna work.

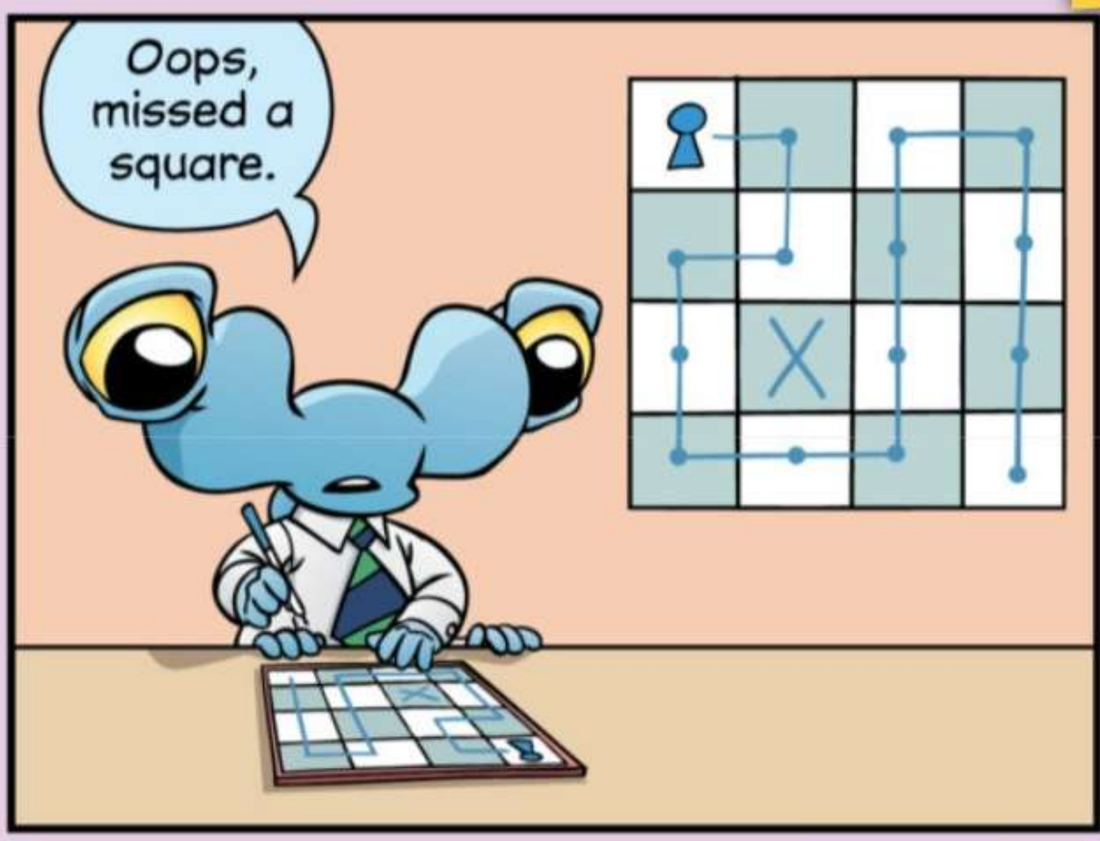
Prepare to be amazed!
I don't have any ears!

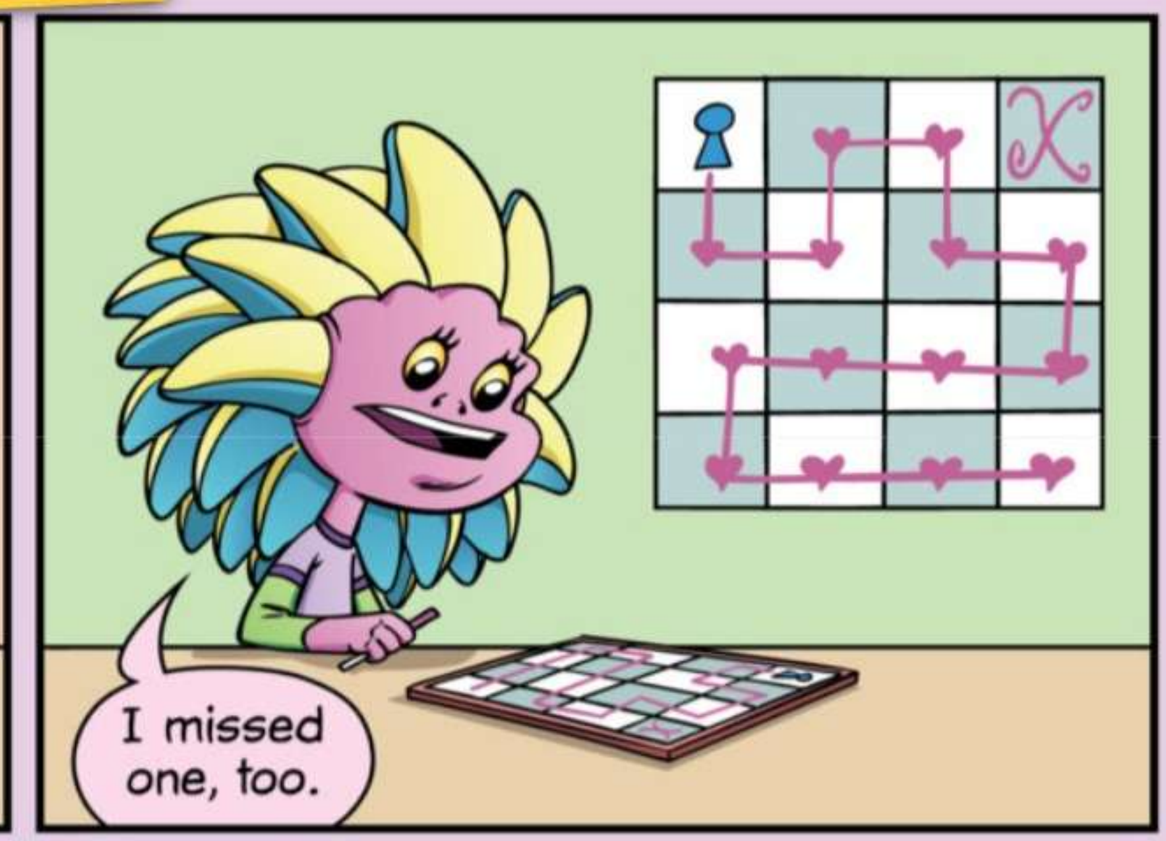
THE LAB
CHECKERBOARDS
Greetings, little monsters! Welcome to our new science lab.
Whoa. This place is amazing.
Don't touch that, please.
What does this lever do?
What's in all of these tiny bottles?
Is this real hippopotamoose drool?
Careful, now.
Little monsters are such curious creatures!
That's what makes you such natural scientists!

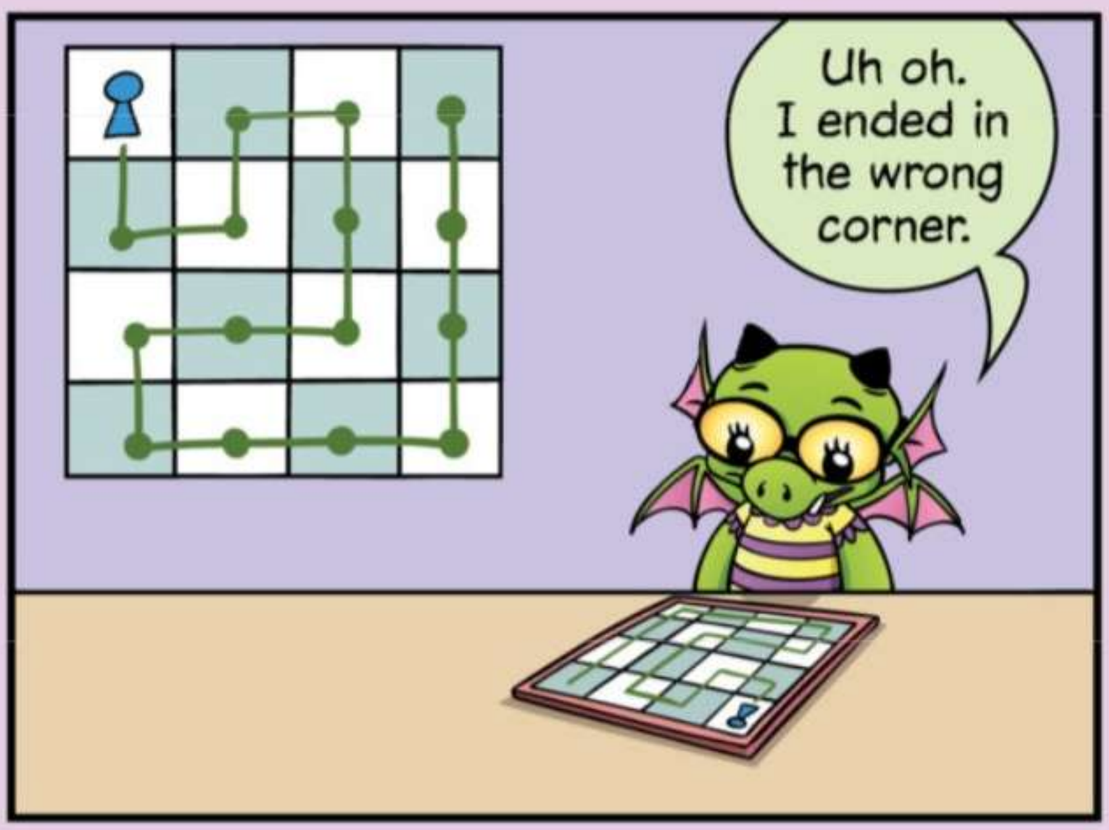
Every good scientist is a wide-eyed explorer.

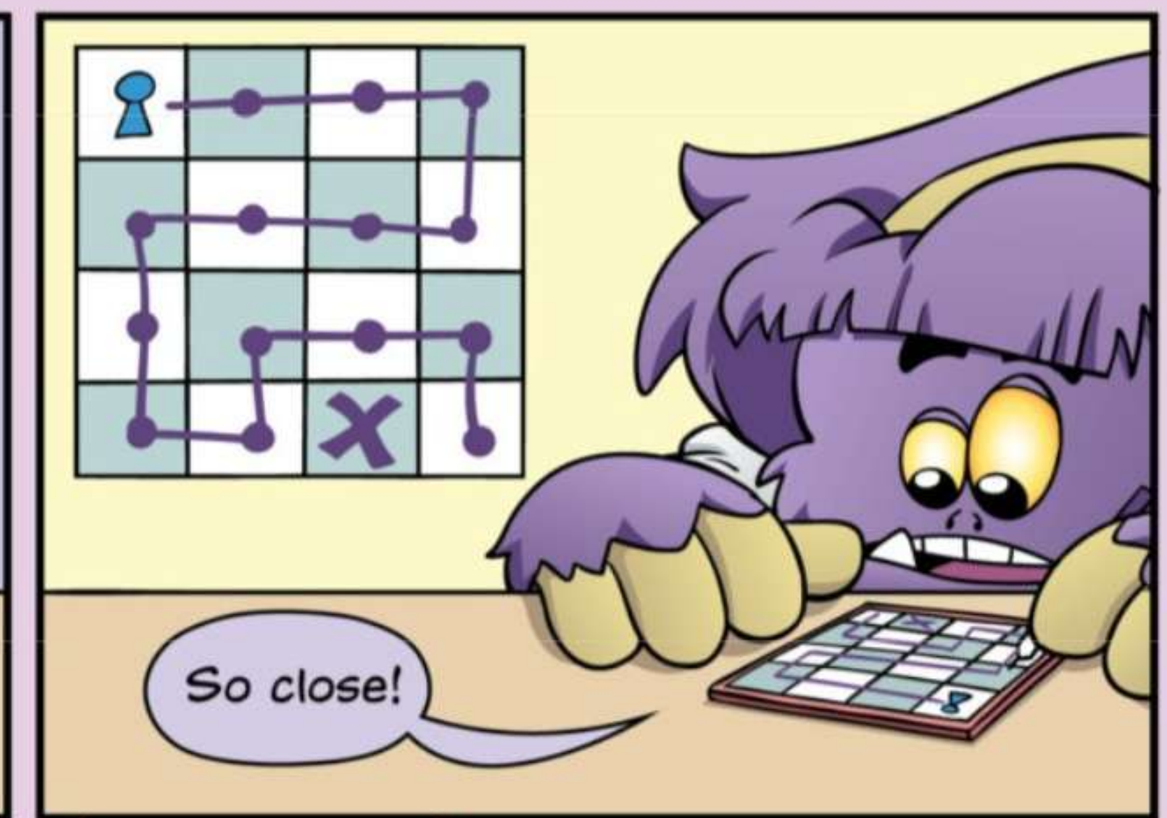
Today, we will be exploring paths on a checkerboard.




Oops, missed a square.


I missed one, too.


Uh oh. I ended in the wrong corner.


So close!

I don't think this is possible, Professor Grok.
Everything I try fails.
Interesting!
Squeek Squeek

Try numbering the squares as you visit them.
Start with a 1 in the top-left square.
What do you notice about the odd-numbered squares?
Try it on your paths. What do you notice?

We can only move from a light square to a dark one...

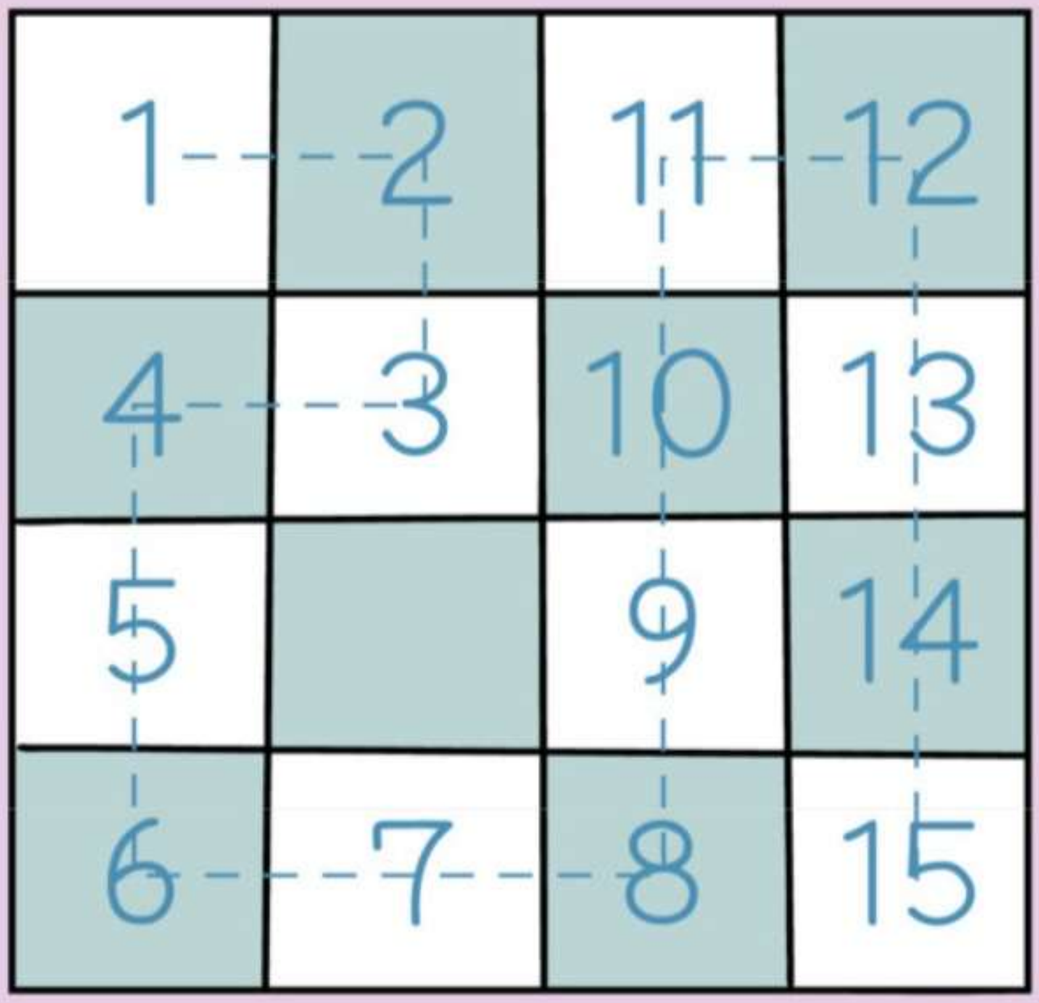
...or from a dark square to a light one.

So, we always switch between light and dark with every move.

So, when we number the squares...

...every ***odd*** square is light...

...and every ***even*** square is dark.



And since there are 16 squares, we can't finish in the bottom-right corner!

Why can't you finish in the bottom-right corner?

The last square you land on is number 16, which is even.

Since every even square is dark...

...we can only end on a ***dark*** square.

The bottom-right square isn't dark, so it's impossible to end on it!

Brilliant!

If you start on a light square and visit all 16 squares once, you will always end on a dark square.

I'll be right back, I just need to fetch a few supplies...

Wham!

Fujomp!

Thunk!

Professor Grok?

Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something much more diabolically difficult!

Behind one of these three doors is your inspirational instructor.

The threshold through which your teacher is trapped is the only opening that can be attained by traversing every tile...

...as in the previous puzzle.

Picking the proper portal will liberate your laboratory leader.

But opening the wrong one will reveal...

...a Colossal Collection of Creepy-Crawly Critters!

That guy uses big words.

What did he say?

Professor Grok is trapped in one of those rooms.

He's in the only room we can reach by crossing every tile once, starting here.

Like the puzzle Professor Grok gave us, only bigger.

And creepier.

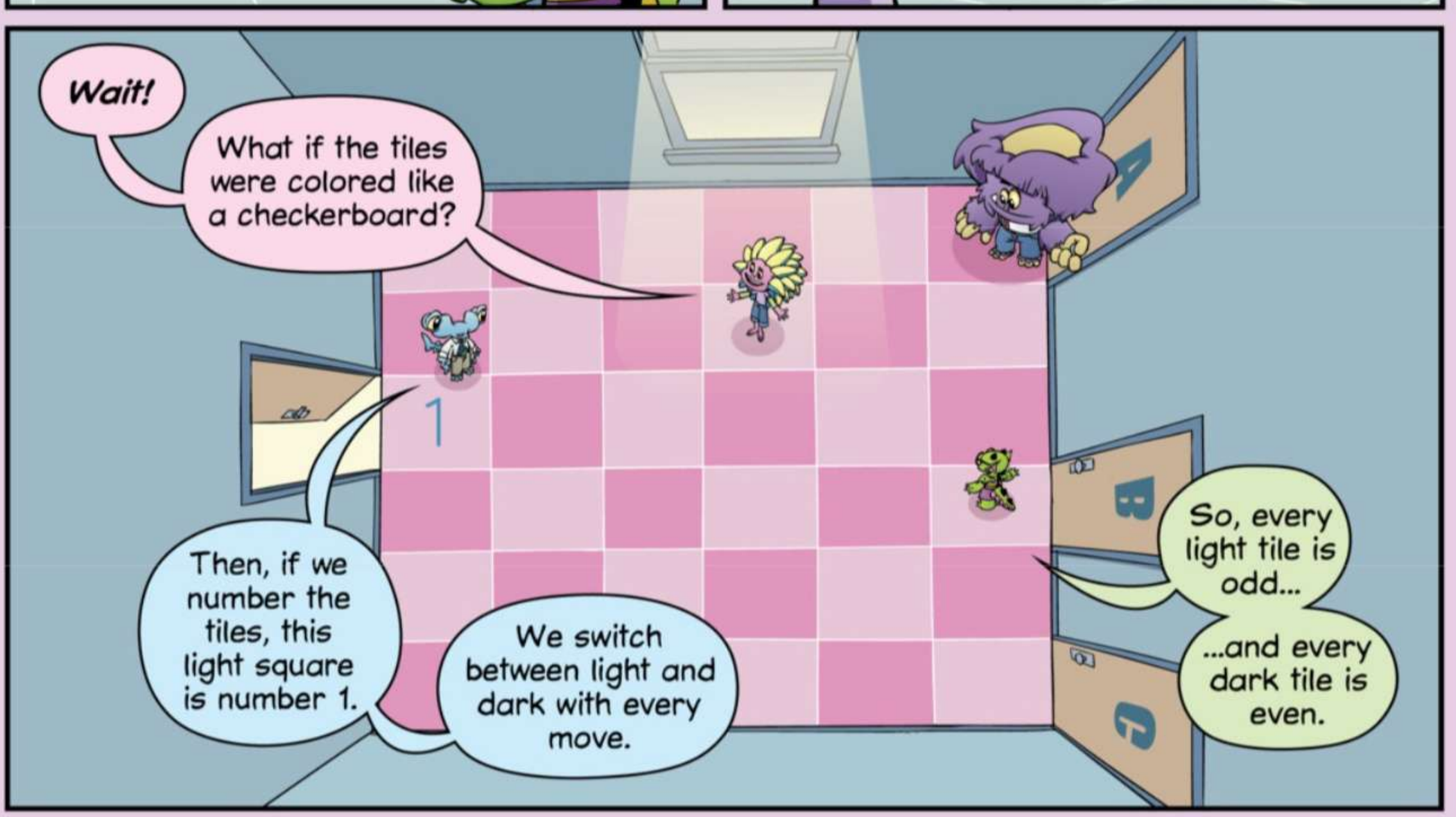
Which door is the Professor behind?

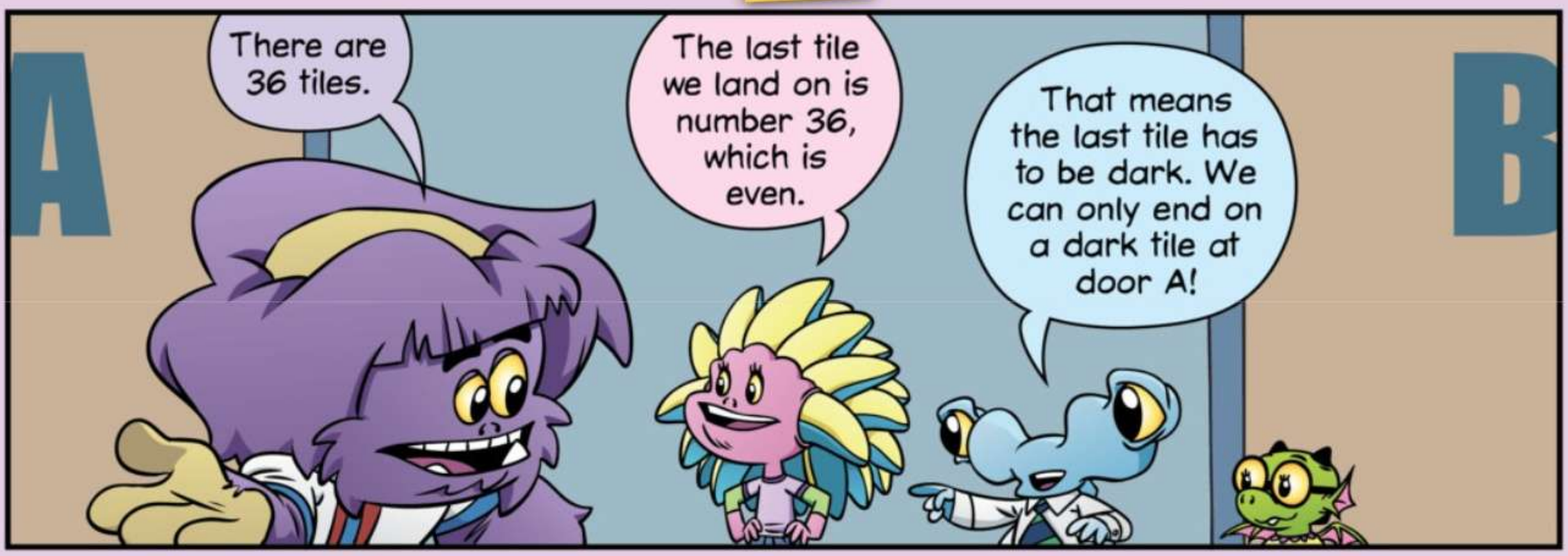
There are four of us.
It shouldn't take long for one of us to find a way to reach a door by crossing every tile.

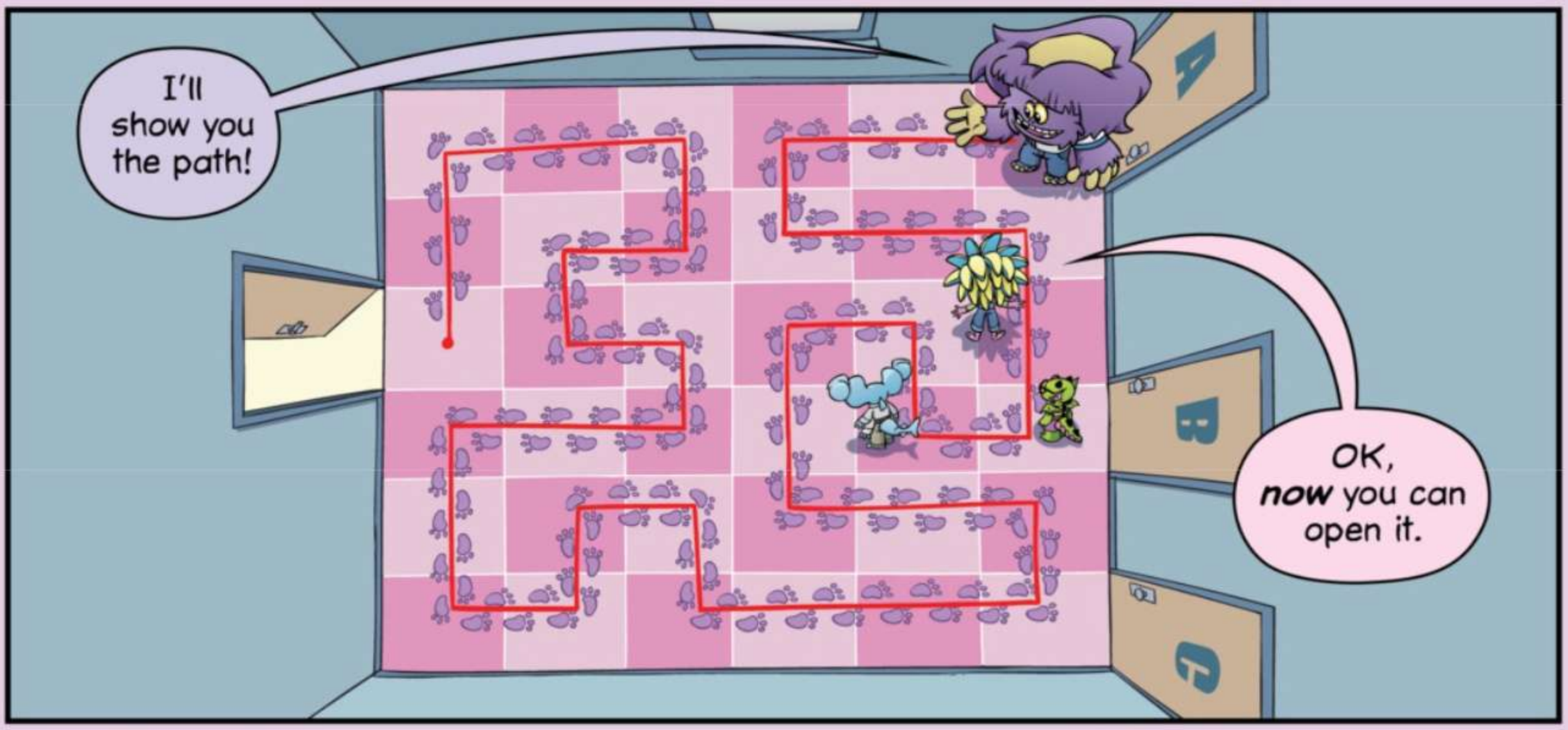
Found it!
I think it's this door.
I think I found a way to get to this door.
A
B

Are you sure you crossed every tile?
Pretty sure.
A

If only one door is possible, then two of the doors are impossible.
You can't both be right.
Only one way to know for sure.
Let's open it!
A


Wait!
What if the tiles were colored like a checkerboard?
1
Then, if we number the tiles, this light square is number 1.
We switch between light and dark with every move.
A
B
So, every light tile is odd...
...and every dark tile is even.


A
B
There are 36 tiles.
The last tile we land on is number 36, which is even.
That means the last tile has to be dark. We can only end on a dark tile at door A!


I'll show you the path!
A
B
OK, now you can open it.

Wonderful rescue, little monsters!
A