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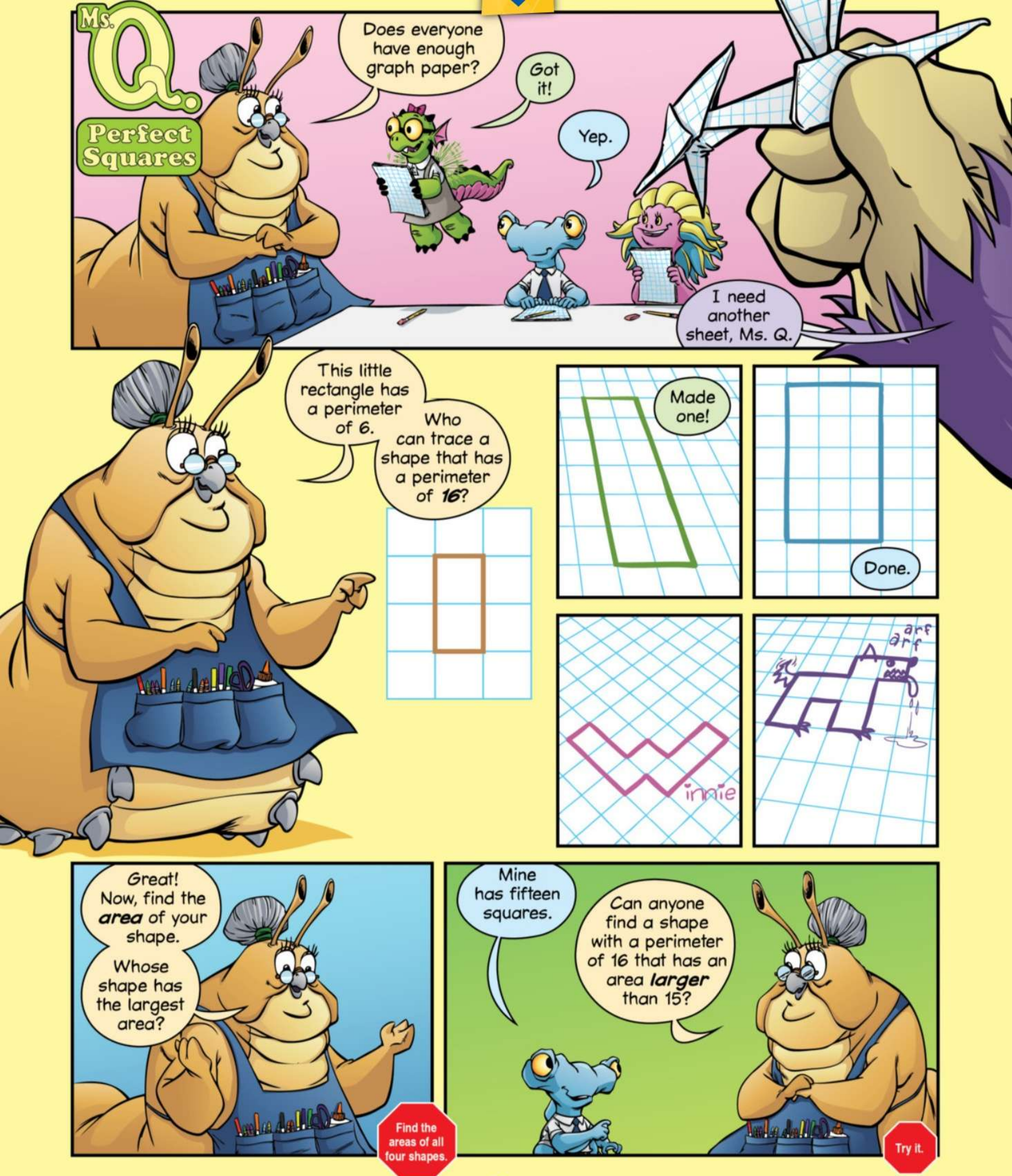
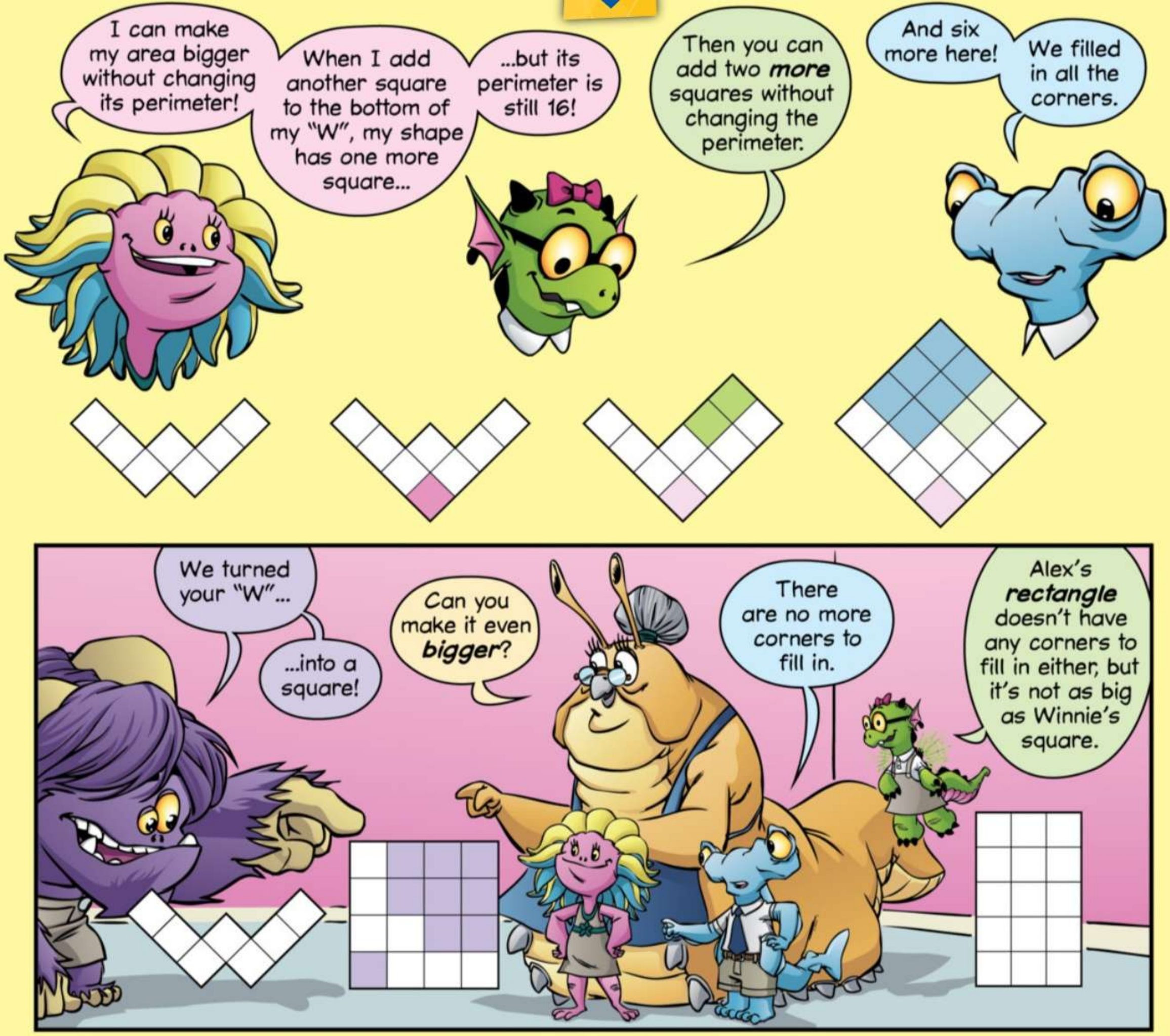
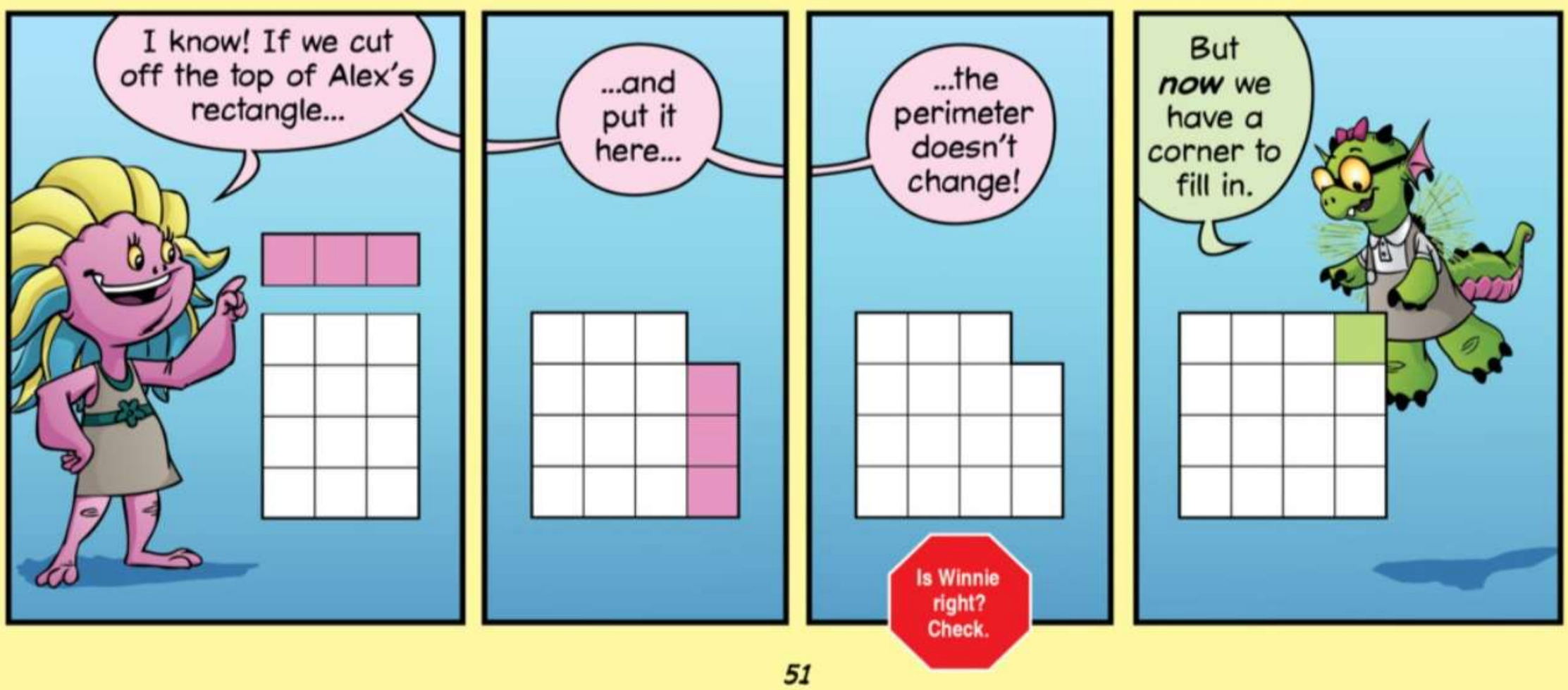
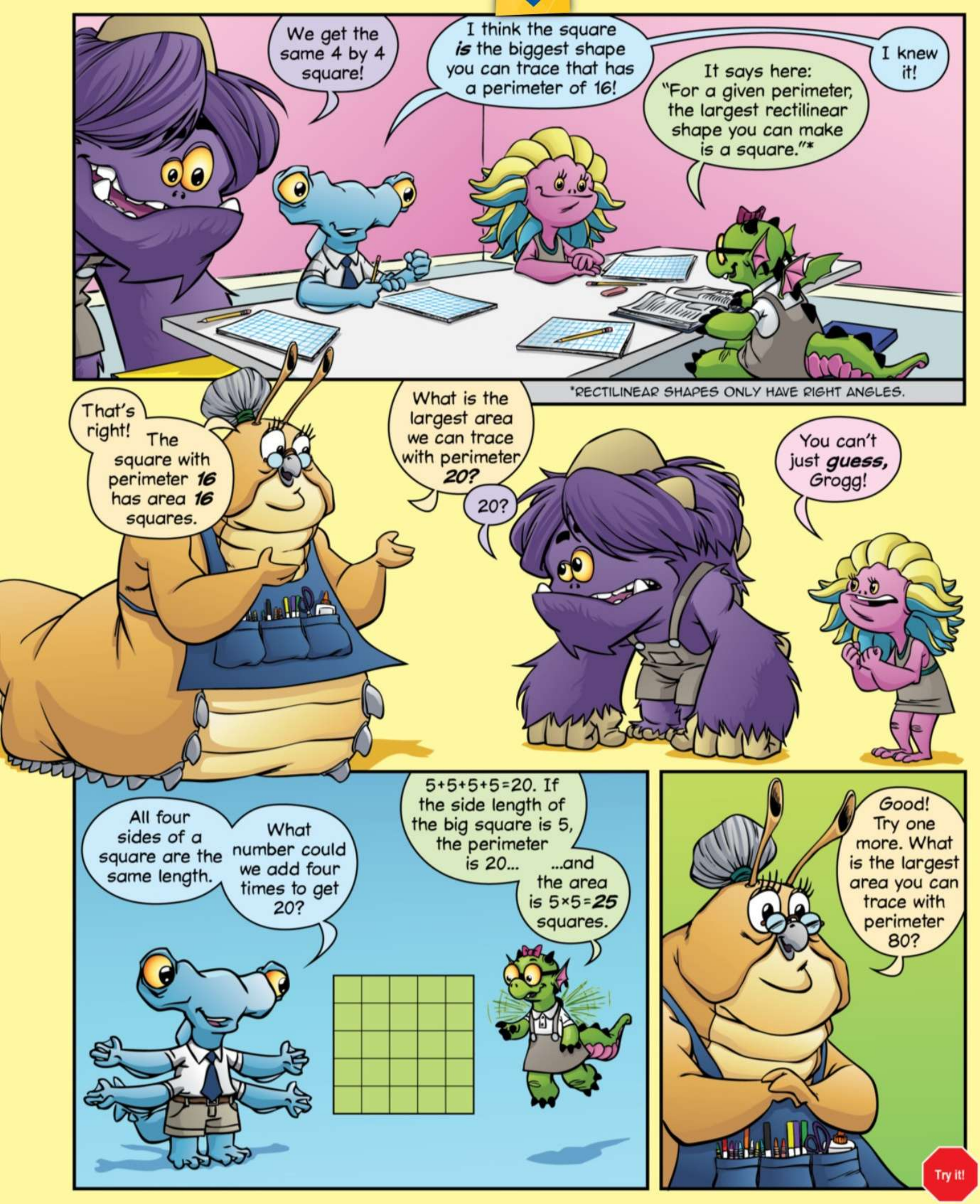
How many squares did it take to dissect Alex's 89×89 square?

Chapter 5:
Perfect Squares


Ms. Q.
Perfect Squares
Does everyone have enough graph paper?
Got it!
Yep.
I need another sheet, Ms. Q.
This little rectangle has a perimeter of 6.
Who can trace a shape that has a perimeter of 16?
Made one!
Done.
innie
arf arf
Great! Now, find the area of your shape.
Whose shape has the largest area?
Find the areas of all four shapes.
Mine has fifteen squares.
Can anyone find a shape with a perimeter of 16 that has an area larger than 15?
Try it.


I can make my area bigger without changing its perimeter!
When I add another square to the bottom of my "W", my shape has one more square...
...but its perimeter is still 16!
Then you can add two **more** squares without changing the perimeter.
And six more here!
We filled in all the corners.
We turned your "W"...
...into a square!
Can you make it even **bigger**?
There are no more corners to fill in.
Alex's **rectangle** doesn't have any corners to fill in either, but it's not as big as Winnie's square.


I know! If we cut off the top of Alex's rectangle...
...and put it here...
...the perimeter doesn't change!
But **now** we have a corner to fill in.
Is Winnie right? Check.


We get the same 4 by 4 square!
I think the square *is* the biggest shape you can trace that has a perimeter of 16!
It says here: "For a given perimeter, the largest rectilinear shape you can make is a square."*
I knew it!
*RECTILINEAR SHAPES ONLY HAVE RIGHT ANGLES.
That's right! The square with perimeter *16* has area *16* squares.
What is the largest area we can trace with perimeter *20?*
20?
You can't just *guess,* Grogg!
All four sides of a square are the same length.
What number could we add four times to get 20?
5+5+5+5=20. If the side length of the big square is 5, the perimeter is 20...
...and the area is 5×5=*25* squares.
Good! Try one more. What is the largest area you can trace with perimeter 80?
Try it!

Groggasaurus!
RAWR!

We just make another square! All four sides are the same...
...and they have to add up to 80.
Four times what is 80?
Solve it.

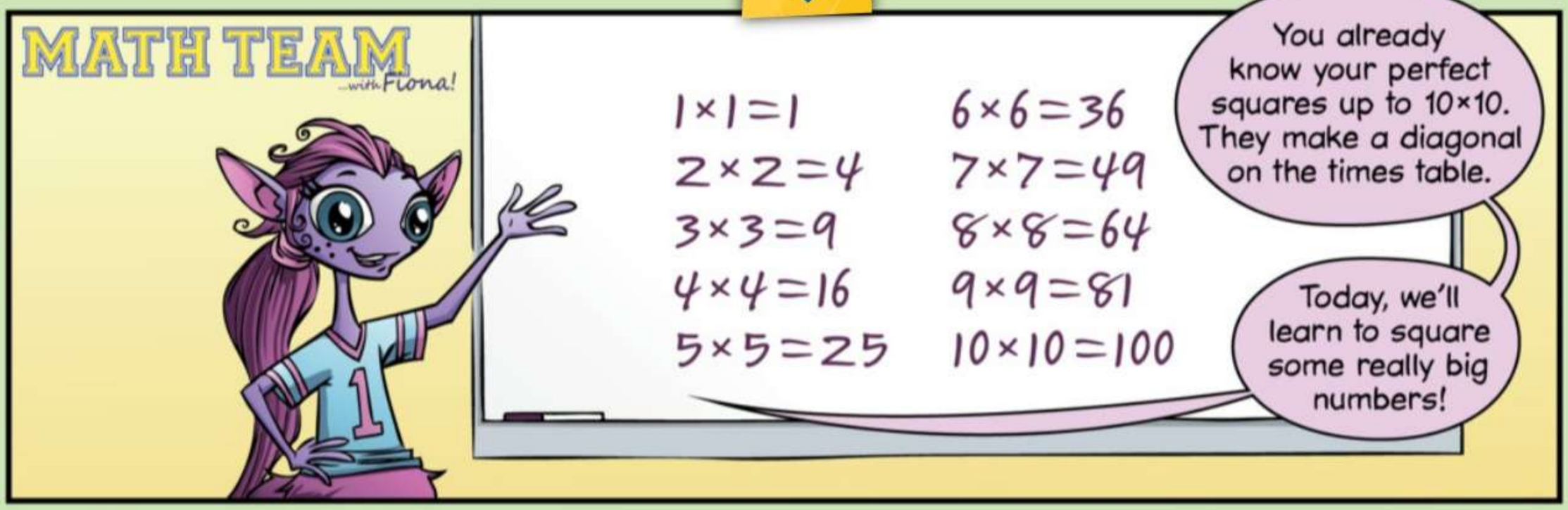
4×2=8, so 4×20=80.
All four sides are 20, so the area is 20×20.
That's 2×2 with two extra zeros...
...400!

There is a special word for multiplying a number by itself...
...like 5×5, or 20×20. Can you guess what it is?

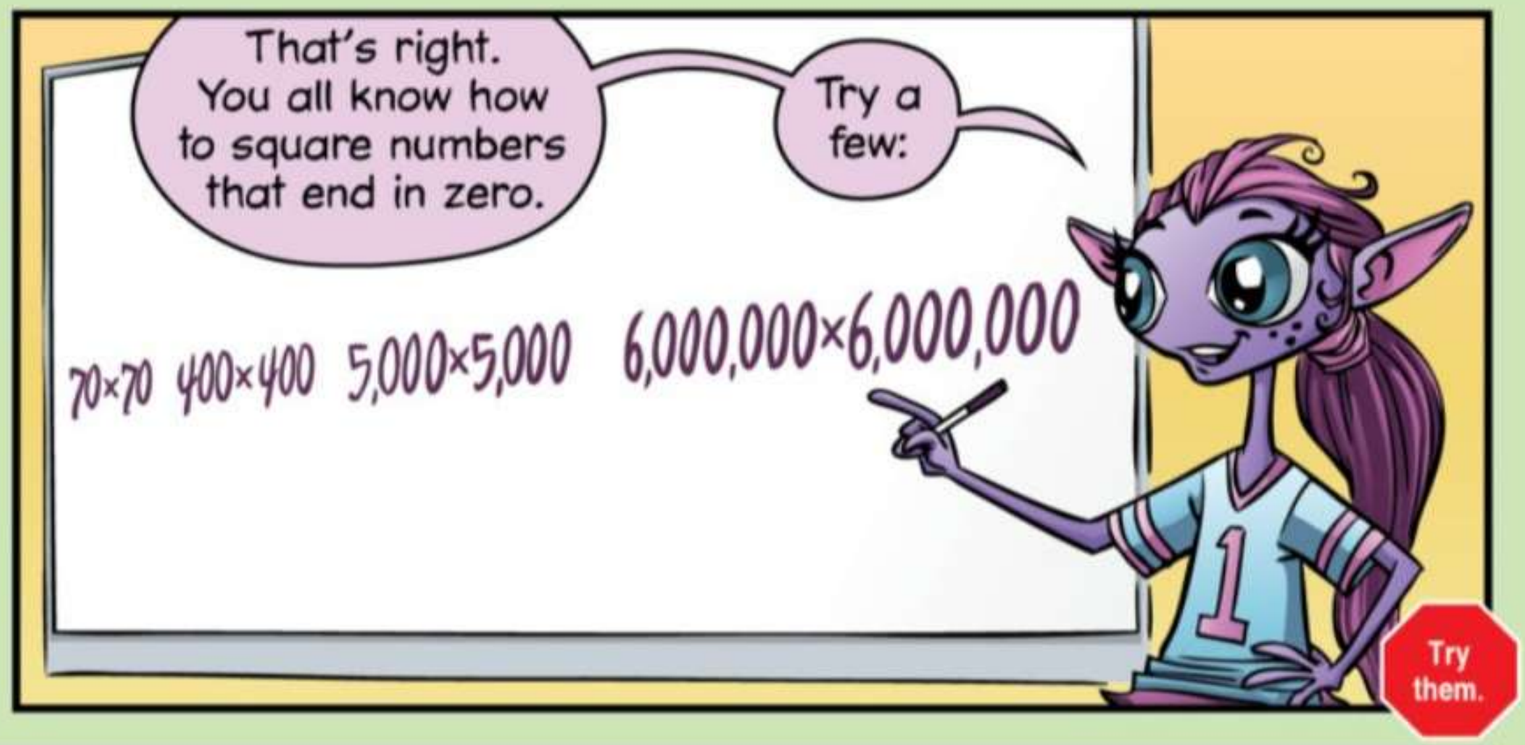
We're **squaring** the numbers!
That makes sense!
SQUARING A NUMBER MEANS MULTIPLYING IT BY ITSELF.

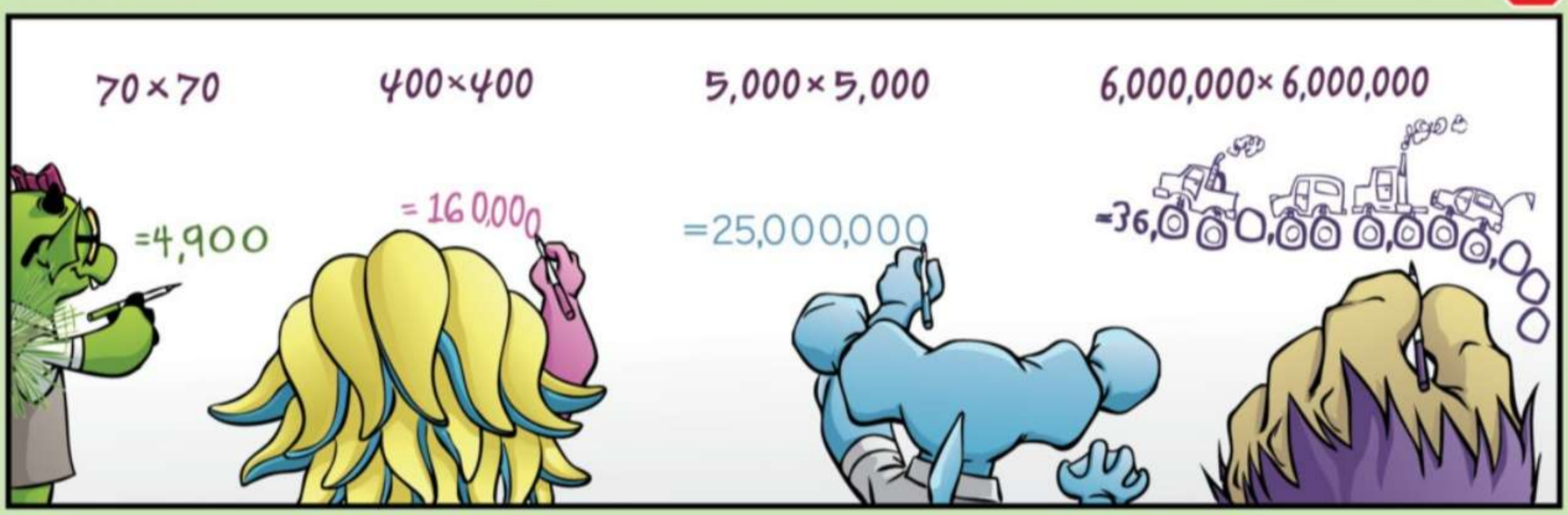
When you find the area of a square, you multiply a number by itself...
...and when you multiply a number by itself, you get a **perfect square.**
That's why we call numbers like 16 and 25 perfect squares!
4×4=16, and 5×5=25.

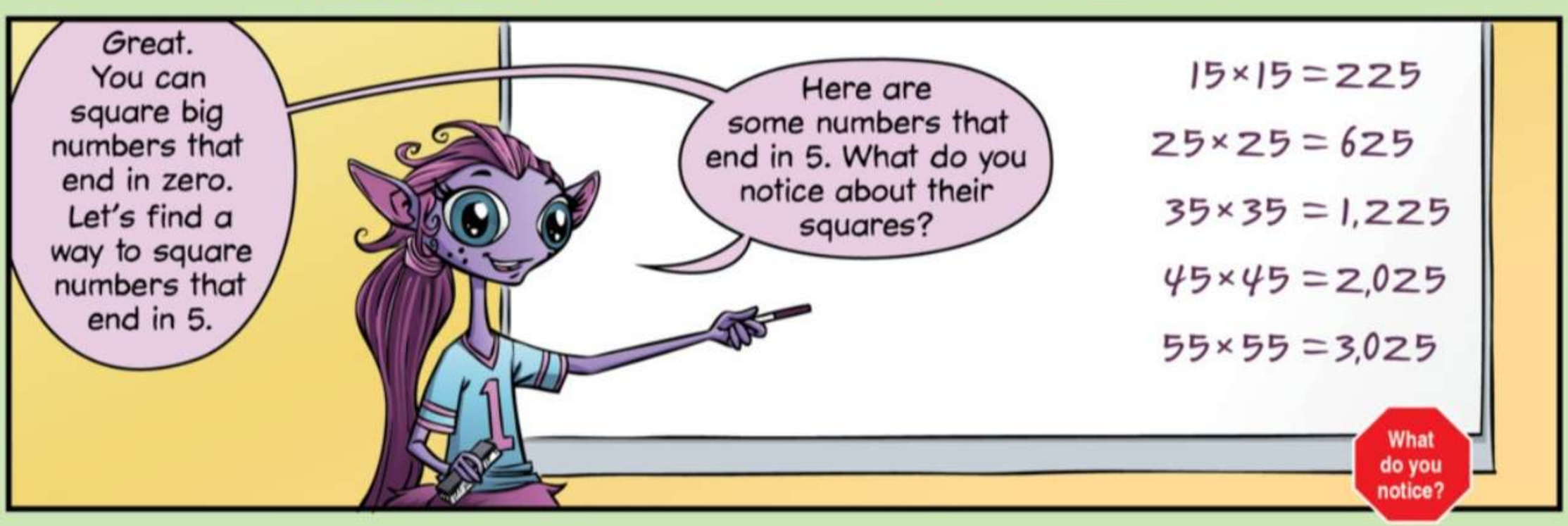
A perfect square number of squares can make a square!


MATH TEAM
...with Fiona!
1×1=1
2×2=4
3×3=9
4×4=16
5×5=25
6×6=36
7×7=49
8×8=64
9×9=81
10×10=100
You already know your perfect squares up to 10×10. They make a diagonal on the times table.
Today, we'll learn to square some really big numbers!

Like 90!
90×90 is 9×9, but with two zeros at the end...
...8,100!


That's right. You all know how to square numbers that end in zero.
Try a few:
70×70 400×400 5,000×5,000 6,000,000×6,000,000
Try them.

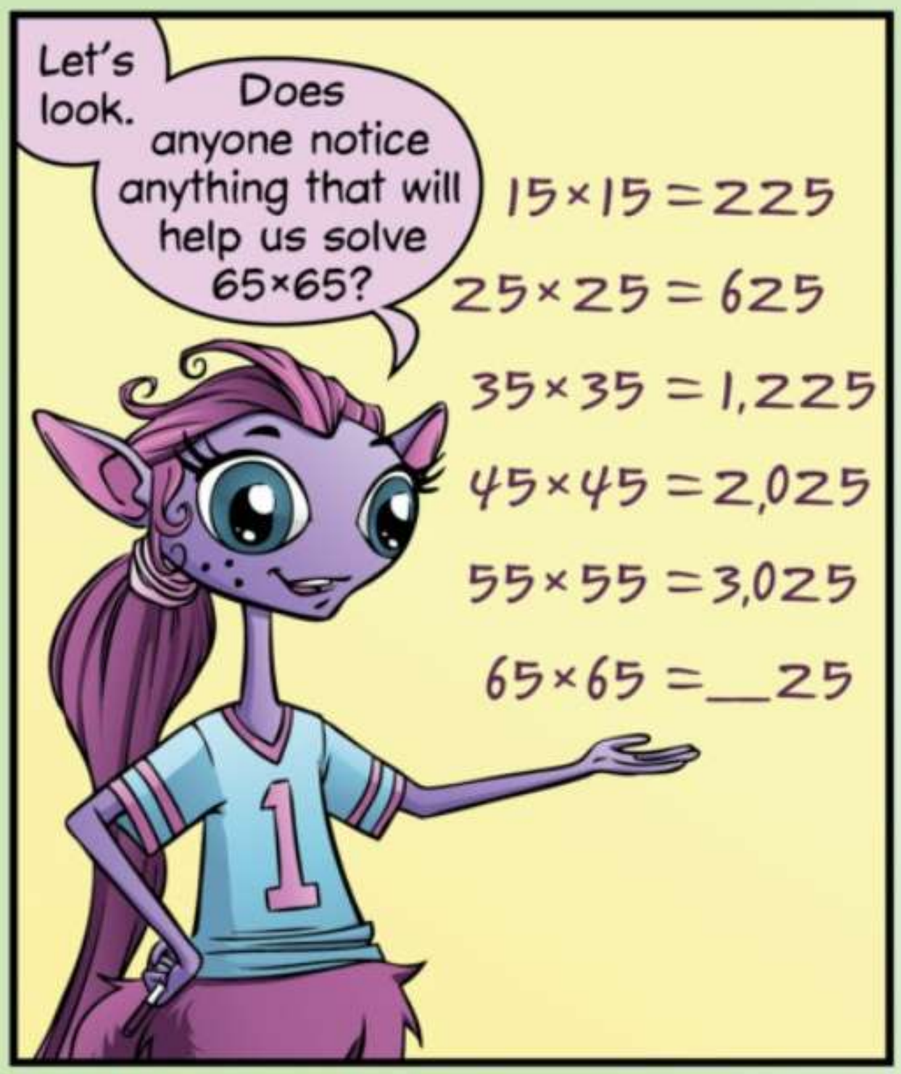

70×70
=4,900
400×400
=160,000
5,000×5,000
=25,000,000
6,000,000×6,000,000
=36,000,000,000,000

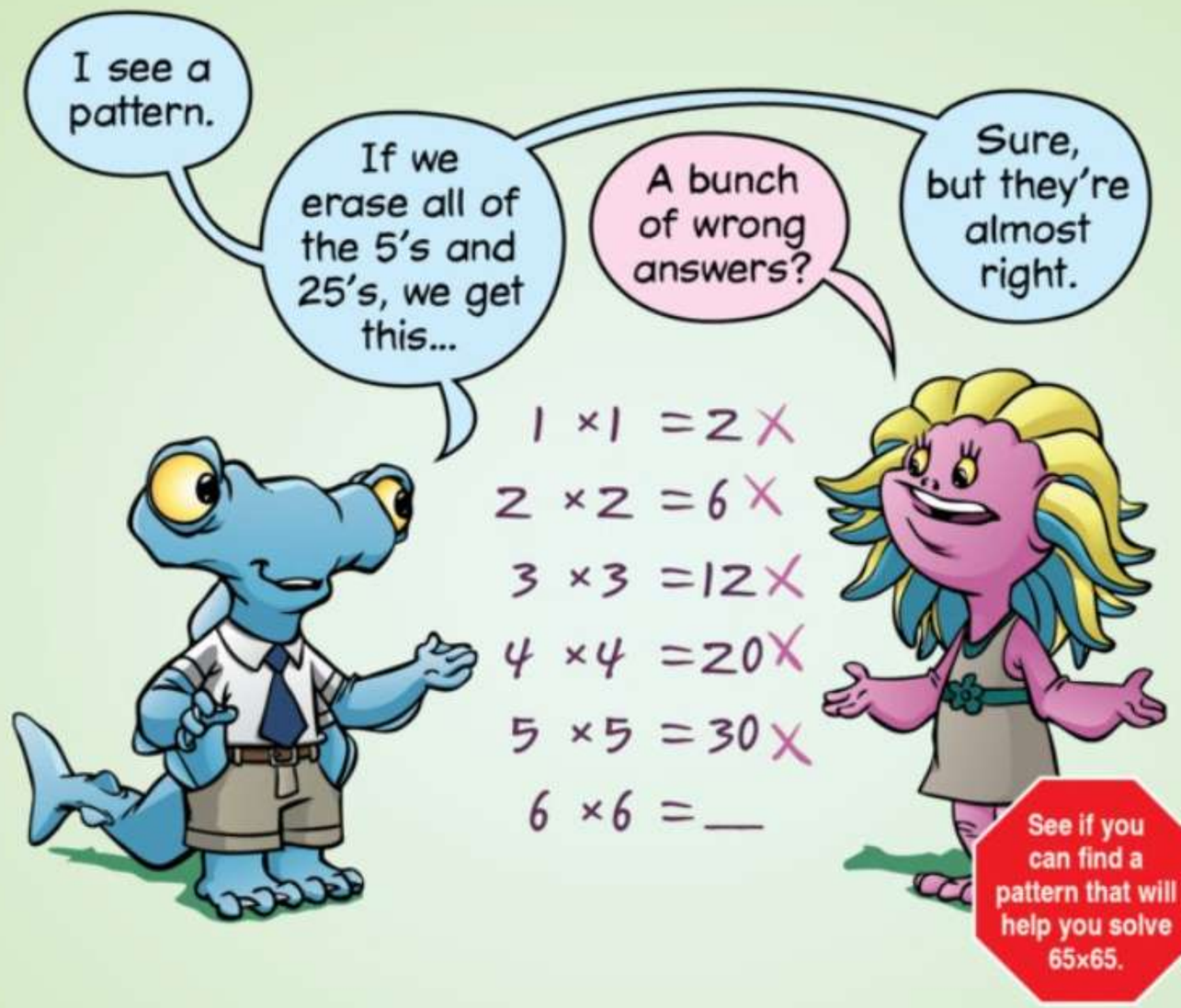

Great. You can square big numbers that end in zero. Let's find a way to square numbers that end in 5.
Here are some numbers that end in 5. What do you notice about their squares?
15×15=225
25×25=625
35×35=1,225
45×45=2,025
55×55=3,025
What do you notice?

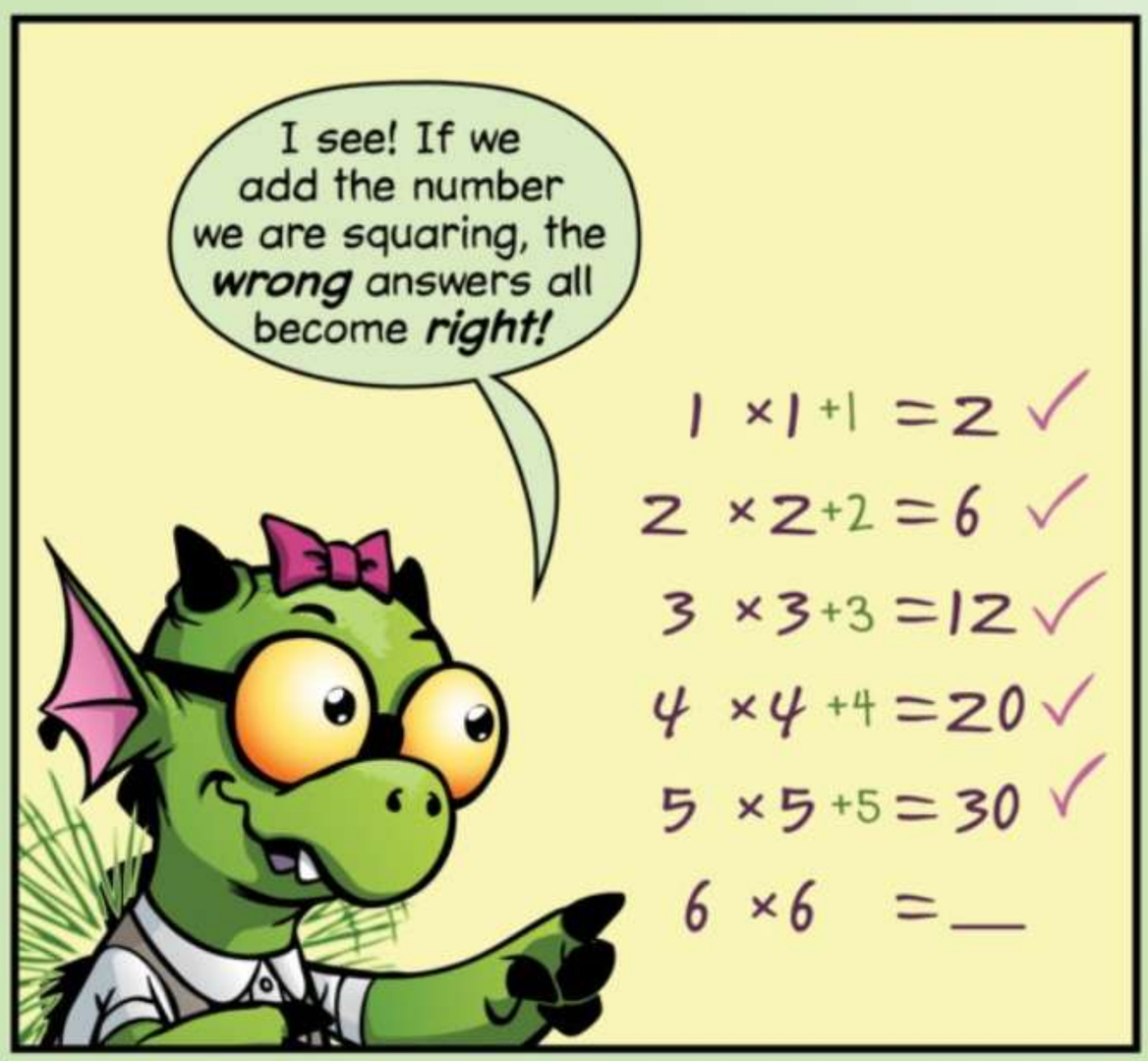
They all end in 25!

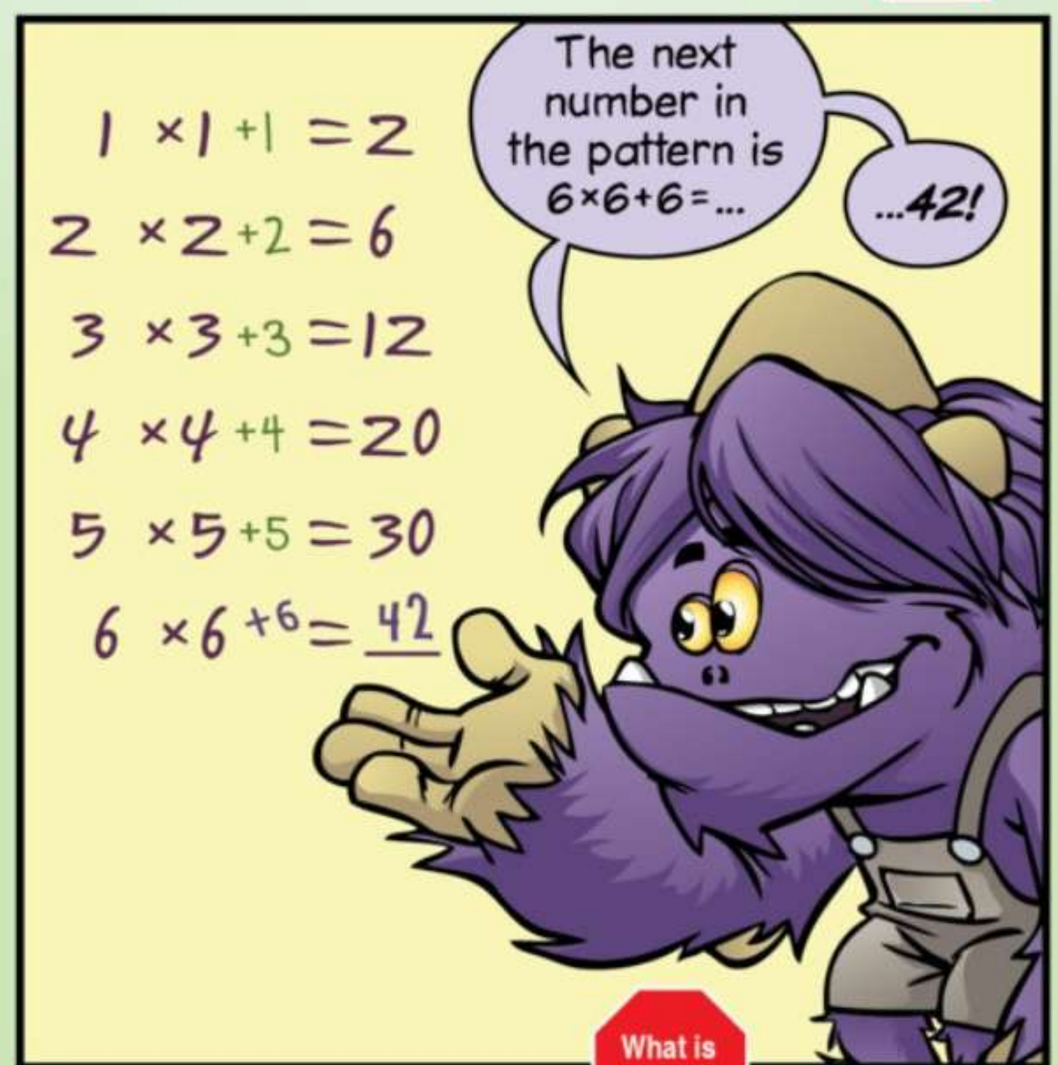
Right. If a number ends in 5, its square always ends in 25.

Great, but how do you get the part that comes **before** the 25?


Let's look.
Does anyone notice anything that will help us solve 65×65?
15×15=225
25×25=625
35×35=1,225
45×45=2,025
55×55=3,025
65×65=__25


I see a pattern.
If we erase all of the 5's and 25's, we get this...
A bunch of wrong answers?
Sure, but they're almost right.
1 ×1 =2 X
2 ×2 =6 X
3 ×3 =12 X
4 ×4 =20 X
5 ×5 =30 X
6 ×6 =__
See if you can find a pattern that will help you solve 65x65.


I see! If we add the number we are squaring, the **wrong** answers all become **right!**
1 ×1+1 =2 ✓
2 ×2+2 =6 ✓
3 ×3+3 =12 ✓
4 ×4+4 =20 ✓
5 ×5+5 =30 ✓
6 ×6 =__


1 ×1+1 =2
2 ×2+2 =6
3 ×3+3 =12
4 ×4+4 =20
5 ×5+5 =30
6 ×6+6 =42
The next number in the pattern is 6×6+6=...
...42!
What is 65x65?

Instead of 6×6+6, we can just multiply 6×7. They're the same.

Following the pattern, the number that goes in the blank is 6×7=42.

15×15 = 225
25×25 = 625
35×35 = 1,225
45×45 = 2,025
55×55 = 3,025
65×65 = 42 25

Ahhh... So for 65×65, we multiply 6×7=42 and put a 25 on the end.

65×65 =4,225.

Why does *that* work?

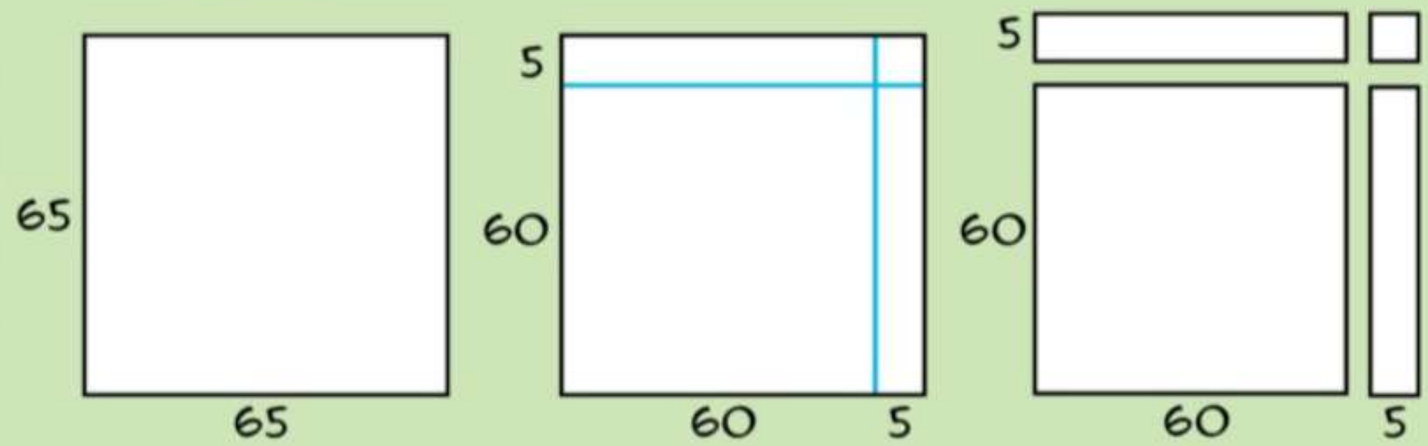
Here's Professor Mumbles. He can explain.

Elttil ih sretsnom!

Evif-ytxis eht si derauqs 60+5 sa emas 60+5 semit...

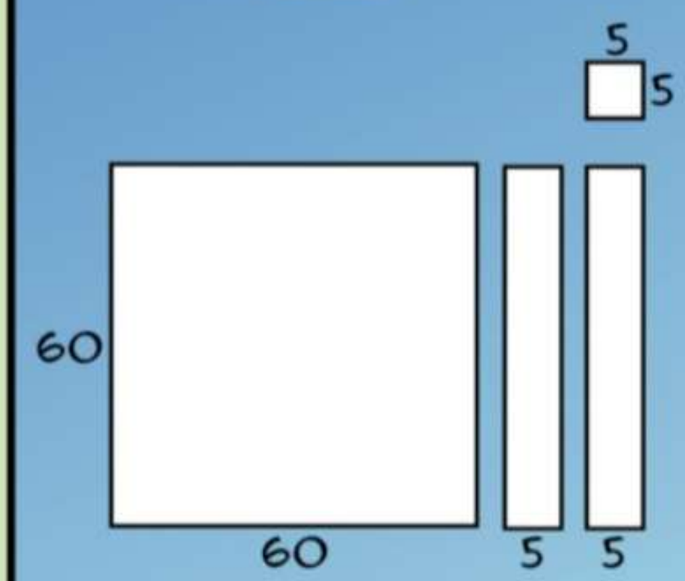
...ew os eht tilps nac pu erauqs siht ekil...

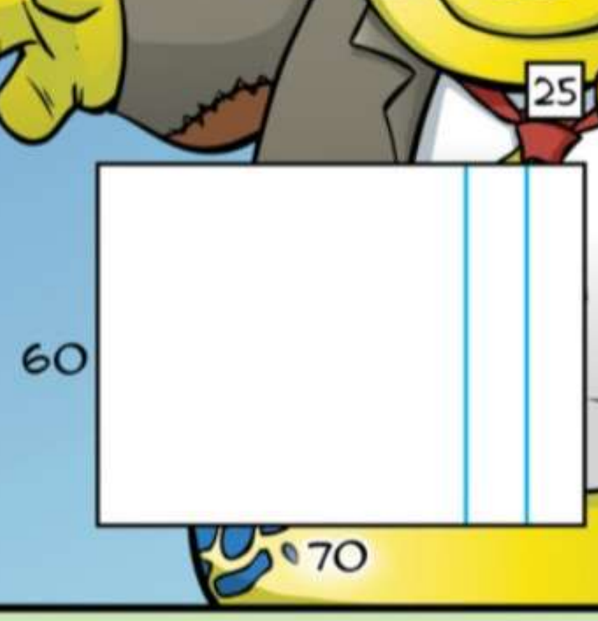
...otni ruof seceip!



Aha! 65×65= 60×70+25.

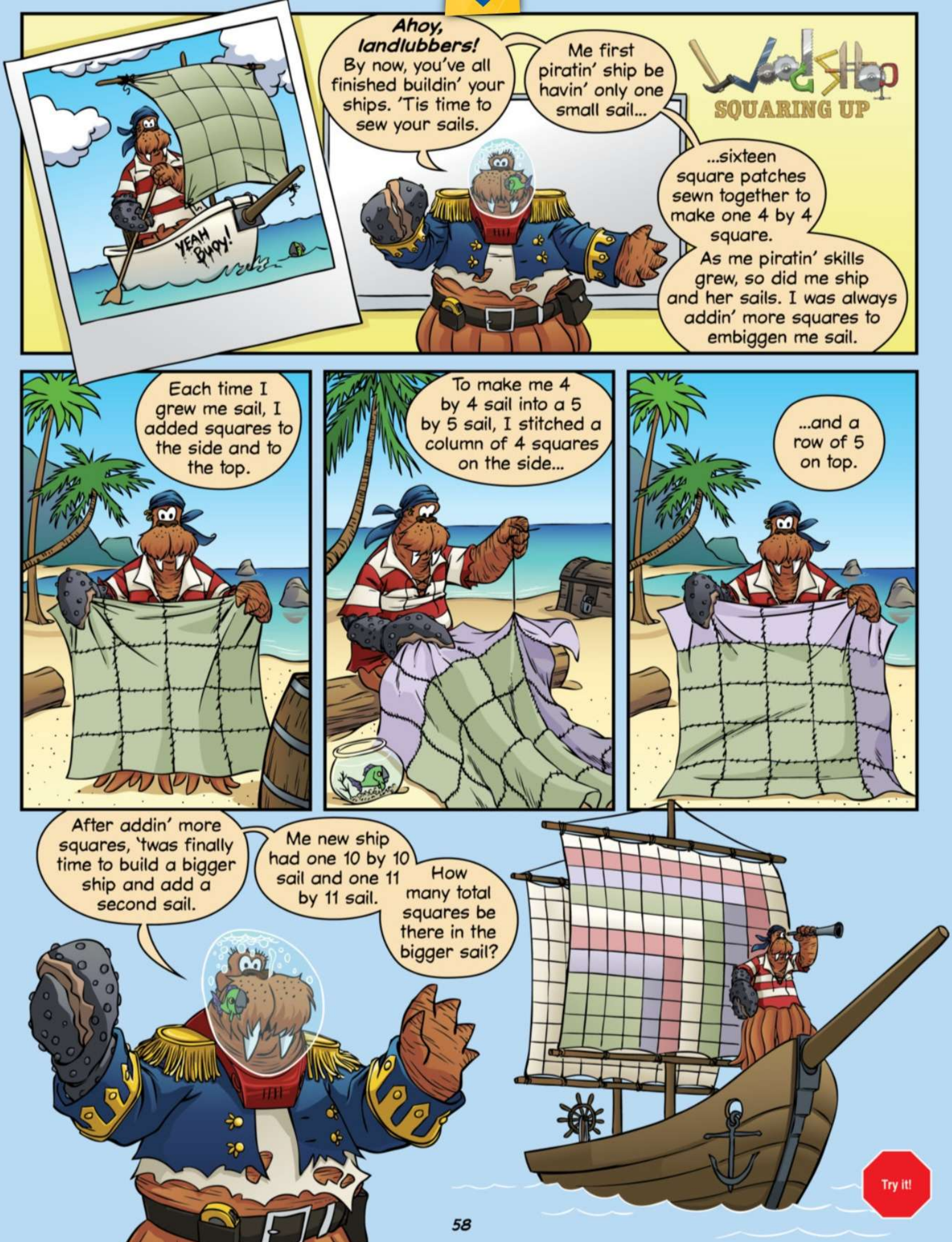
Elttil a tsuj htiw gnignarraer...

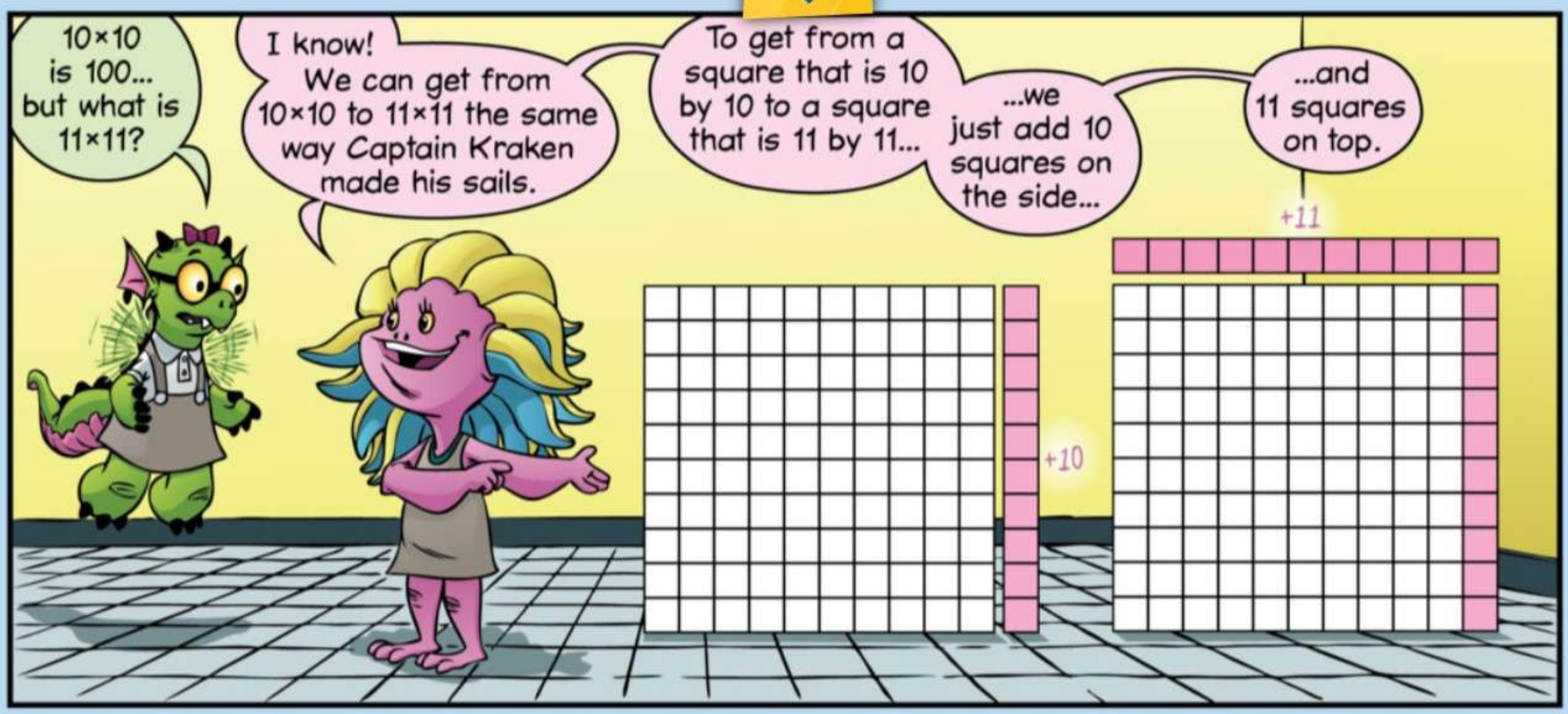




Got it! To square 65, we can multiply 60×70, then add 25. 60×70=4,200, and 4,200+25=4,225, so 65×65=***4,225!***

Great!
How would you square 75?
To square 75, we can multiply 70×80 and then add 25.
70×80=5,600, and 5,600+25=5,625.
So, 75×75 = 5,625!
75
75
=
70
80
5
5
How about 95 squared?
To square 95, we multiply 90×100 then add 25. That's 9,025.
Right! Ready for a really hard one?
Square 1,005.
To square 95, we multiplied 90×100 and added 25.
To square 1,005, we multiply 1,000×1,010 and then add 25.
To multiply 1,000×1,010 we put three zeros after 1,010...
1,010,000!
Then we add 25!
1,010,025!
Great job, Grogg!


YEAH BUOY!
Ahoy, landlubbers! By now, you've all finished buildin' your ships. 'Tis time to sew your sails.
Me first piratin' ship be havin' only one small sail...
...sixteen square patches sewn together to make one 4 by 4 square.
As me piratin' skills grew, so did me ship and her sails. I was always addin' more squares to embiggen me sail.
SQUARING UP
Each time I grew me sail, I added squares to the side and to the top.
To make me 4 by 4 sail into a 5 by 5 sail, I stitched a column of 4 squares on the side...
...and a row of 5 on top.
After addin' more squares, 'twas finally time to build a bigger ship and add a second sail.
Me new ship had one 10 by 10 sail and one 11 by 11 sail.
How many total squares be there in the bigger sail?
Try it!


10×10 is 100... but what is 11×11?
I know! We can get from 10×10 to 11×11 the same way Captain Kraken made his sails.
To get from a square that is 10 by 10 to a square that is 11 by 11...
...we just add 10 squares on the side...
...and 11 squares on top.
+11
+10

So 11×11 is just 10×10+10+11.

That makes 100+10+11=121 squares in the larger sail!
Aye! 121 stitched squares.

She be a sleek 'n nimble ship, able to turn on a dime.
Was it a fast ship?
Fast? She made the Kessel Run in less than 12 parsecs!
Alas, she was a puny vessel compared to...

...the ***Angela Viathan***, the largest ship to ever sail the seven seas.

She had four masts, each holding six square sails. The smallest sail atop each mast was 40 squares wide and tall!

So the ***smallest*** sail was made from 40×40=***1,600*** fabric squares!

Each o' the two sails beneath it be one square wider 'n taller.

41 by 41?

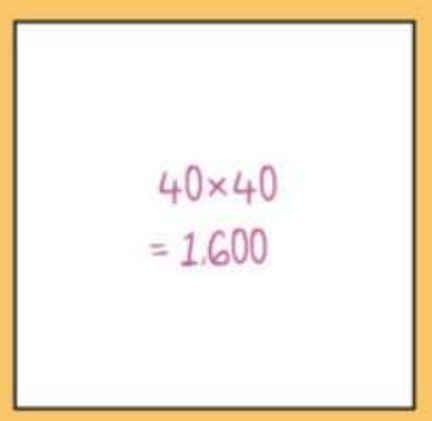
Aye. What be the number o' fabric squares in a 41 by 41 sail?

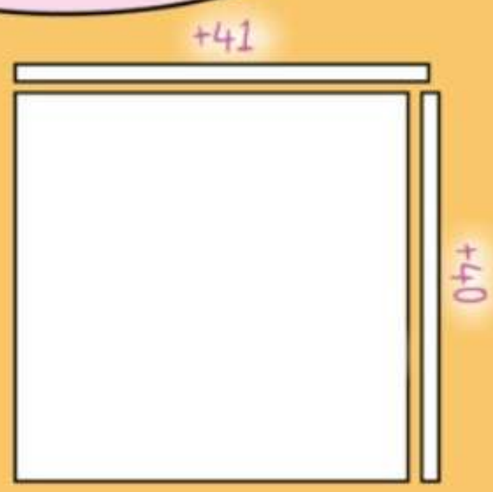
Can you find the number of squares?

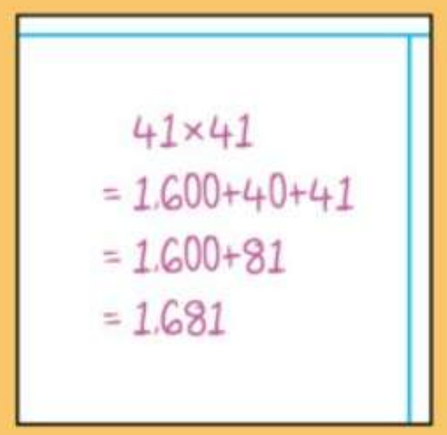
We already know 40 squared is 40×40=1,600.

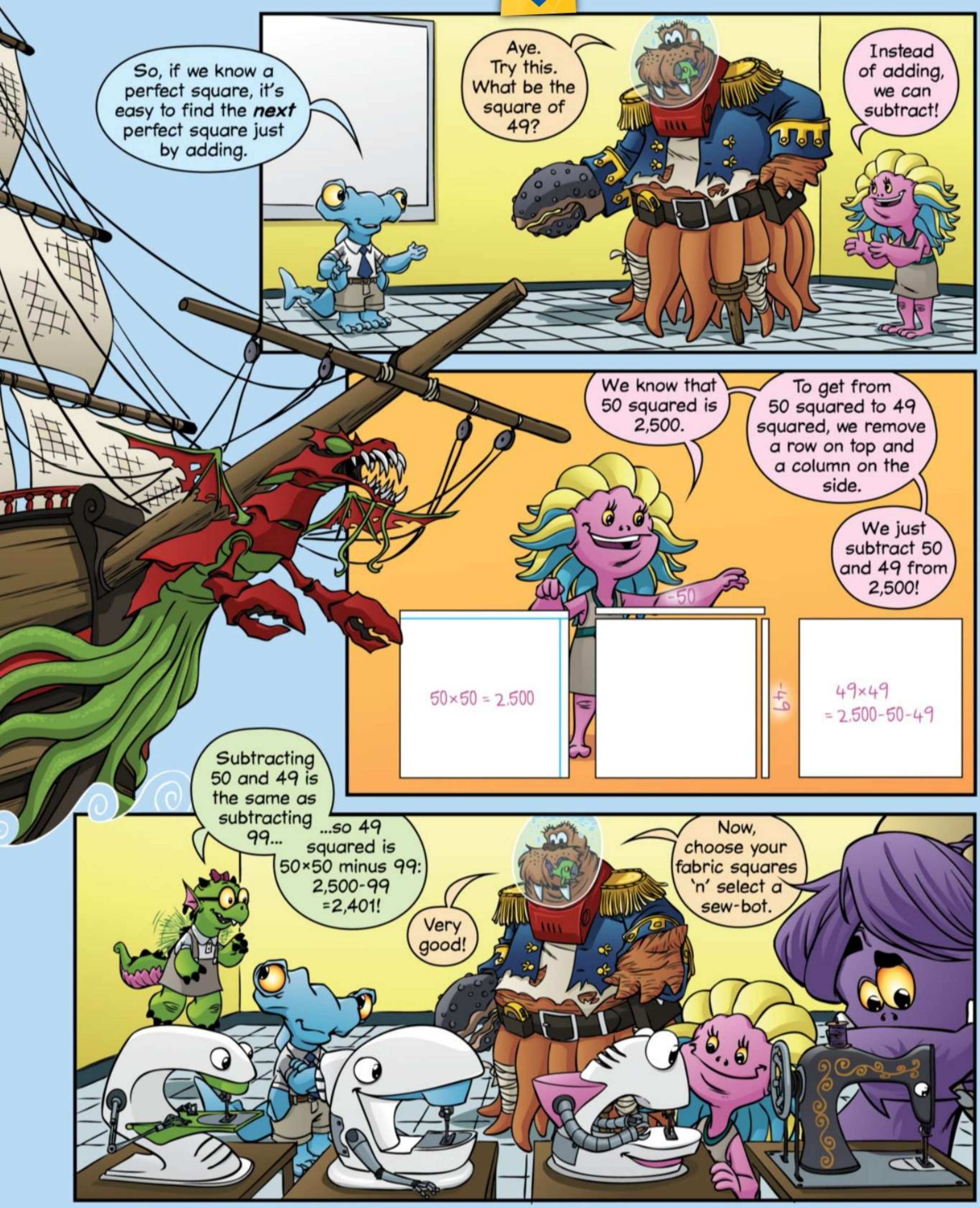
To get from a 40×40 square to a 41×41 square, we just add 40 squares on the side and 41 on top...

...41 squared is 1,681!

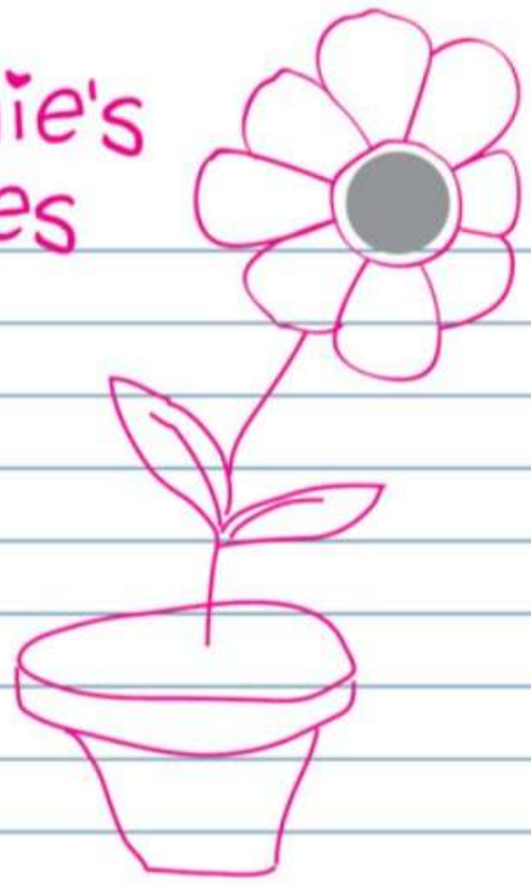




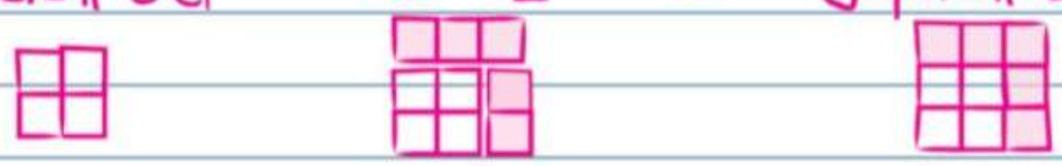



So, if we know a perfect square, it's easy to find the *next* perfect square just by adding.
Aye. Try this. What be the square of 49?
Instead of adding, we can subtract!
We know that 50 squared is 2,500.
To get from 50 squared to 49 squared, we remove a row on top and a column on the side.
We just subtract 50 and 49 from 2,500!
-50
-49
50×50 = 2,500
49×49
= 2,500-50-49
Subtracting 50 and 49 is the same as subtracting 99...
...so 49 squared is 50×50 minus 99: 2,500-99 =2,401!
Very good!
Now, choose your fabric squares 'n' select a sew-bot.

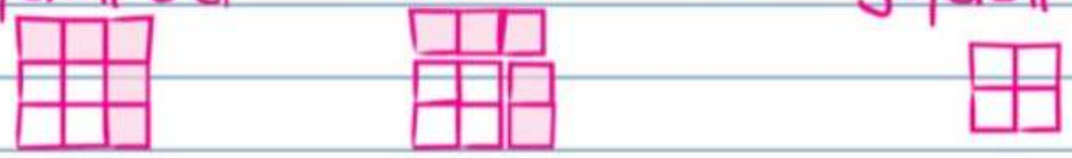
Winnie's Notes



2 squared +3 +2 = 3 squared



3 squared -3 -2 = 2 squared



Shortcut: 3^2 means 3 squared (3×3)

3^2 - 2^2 = 3+2

$3^2 - 2^2 = 3 + 2 = 5$

$4^2 - 3^2 = 4 + 3 = 7$

$5^2 - 4^2 = 5 + 4 = 9$ $\quad 9 = 3 \times 3 = 3^2$

$5^2 - 4^2 = 3^2$

$6^2 - 5^2 = 6 + 5 = 11$

$7^2 - 6^2 = 7 + 6 = 13$

$8^2 - 7^2 = 8 + 7 = 15$

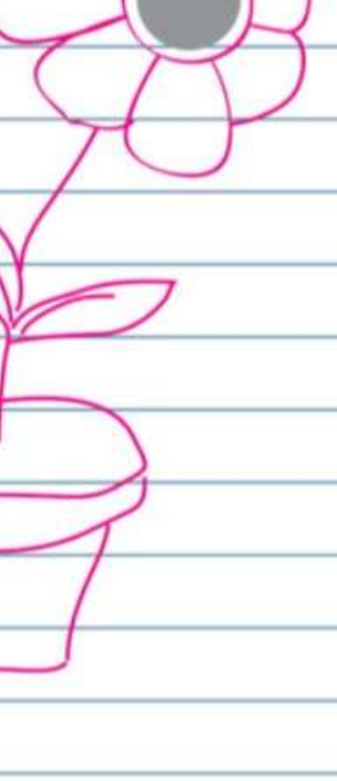
$9^2 - 8^2 = 9 + 8 = 17$

$10^2 - 9^2 = 10 + 9 = 19$

$11^2 - 10^2 = 11 + 10 = 21$

$12^2 - 11^2 = 12 + 11 = 23$

$13^2 - 12^2 = 13 + 12 = 25$ $\quad 13^2 - 12^2 = 5^2$



$25^2 - 24^2 = 25 + 24 = 49$ $\quad 25^2 - 24^2 = 7^2$

Odd perfect squares!!!

$41^2 - 40^2 = 41 + 40 = 81$ $\quad 41^2 - 40^2 = 9^2$

$??^2 - ??^2 = 11^2$

$11^2 = 10^2 + 10 + 11$

$= 121$

$61^2 - 60^2 = 61 + 60 = 121$ $\quad 61^2 - 60^2 = 11^2$!

$??^2 - ??^2 = 13^2$?

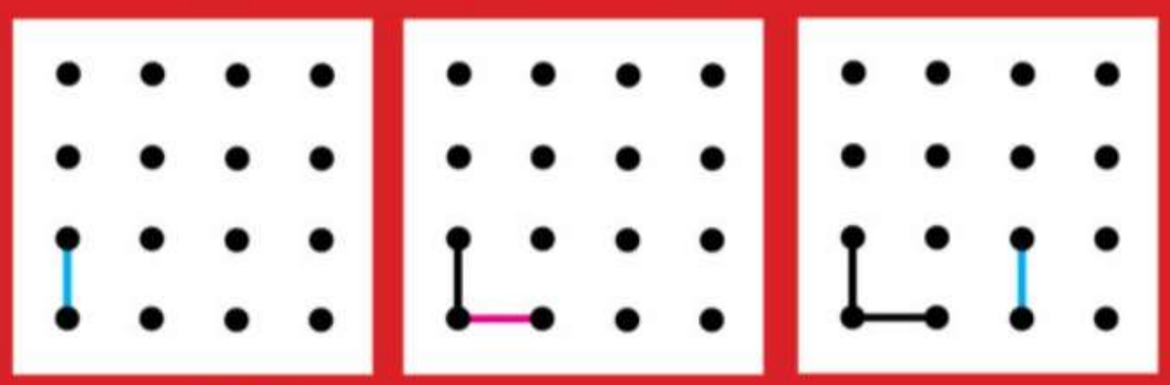
RECESS

Dots and Boxes

Instructions:

Start with a grid of dots like the one below. The grid may be any size.

The Beginning: Players take turns connecting a neighboring pair of dots with a horizontal or vertical line. Below are the first three moves in a sample game between Alex and Winnie.

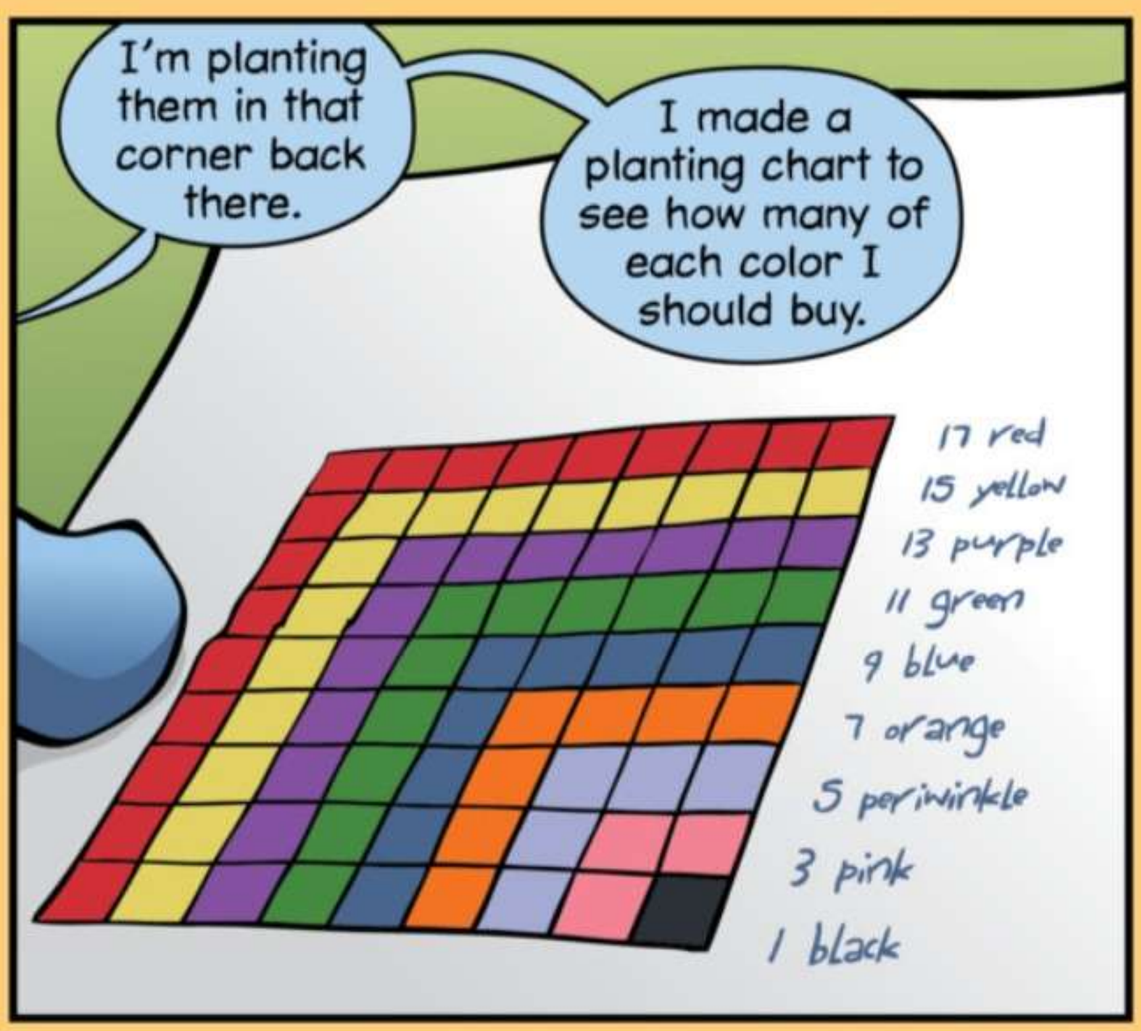


The Middle: When a player completes the fourth side of a small square, he or she "wins" the square and places his or her initial inside it, then takes another turn as shown below.

The End: The game ends when all the squares have been completed. The player with the most squares wins.

Winnie wins: 5 to 4.

Find a partner and play!

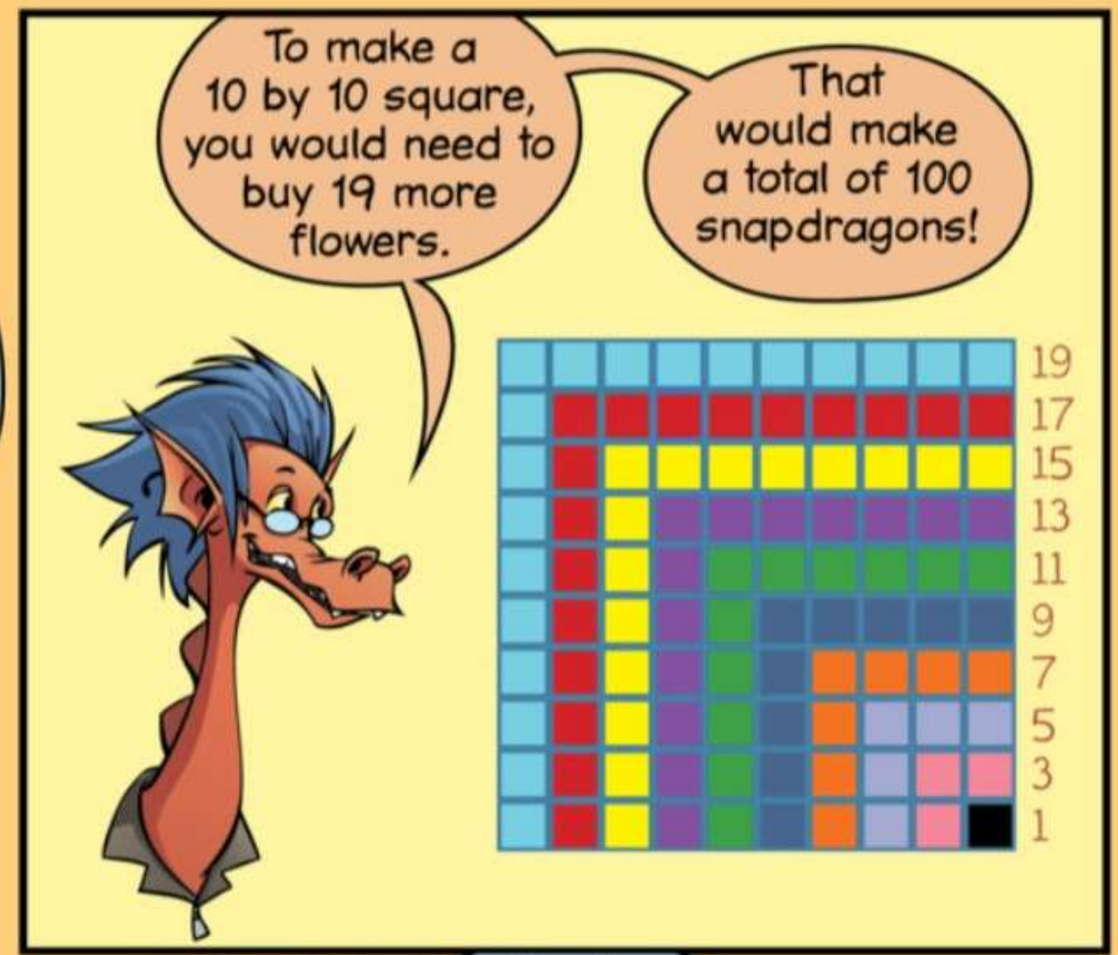


Do you know how he counted so fast?

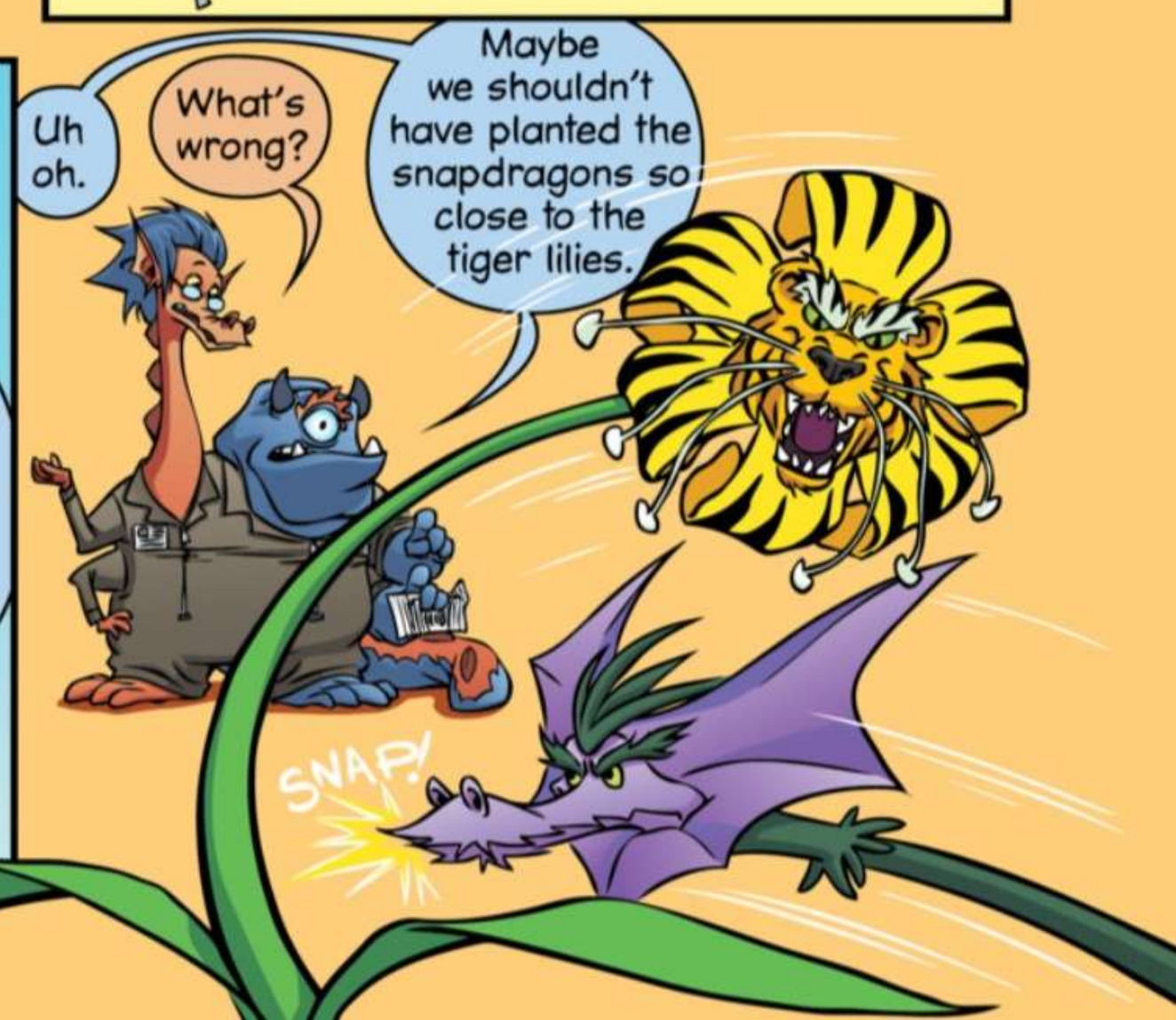
I didn't! I **squared** 9.
!?!
Look at your chart again.

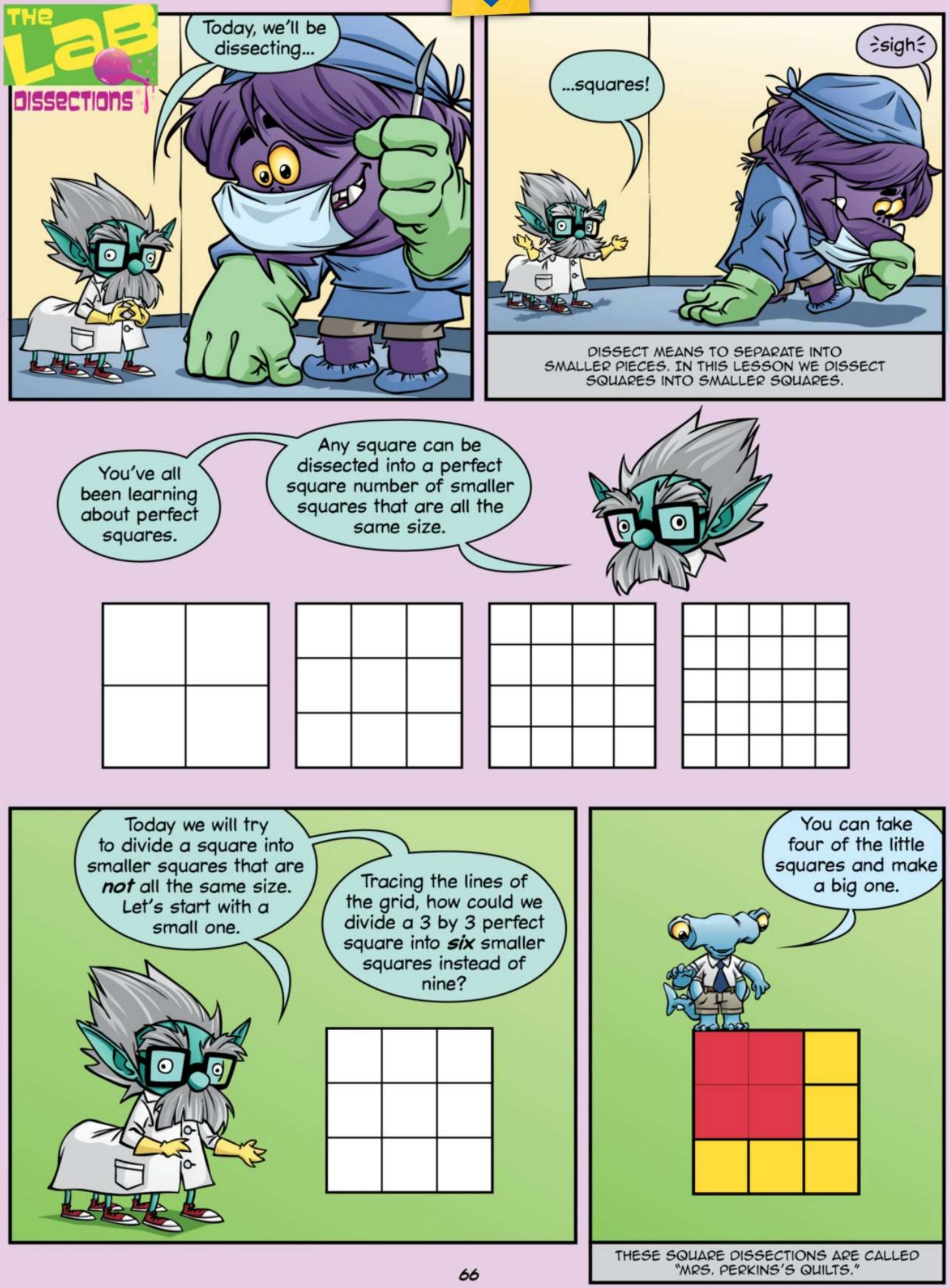
I see, it's a **square!** Nine rows of nine makes 9×9=81 snapdragons.

So, to add the first 9 **odd** numbers, you can just square 9.
To add the first 10 odd numbers, can you just square 10?


To make a 10 by 10 square, you would need to buy 19 more flowers.
That would make a total of 100 snapdragons!
19
17
15
13
11
9
7
5
3
1

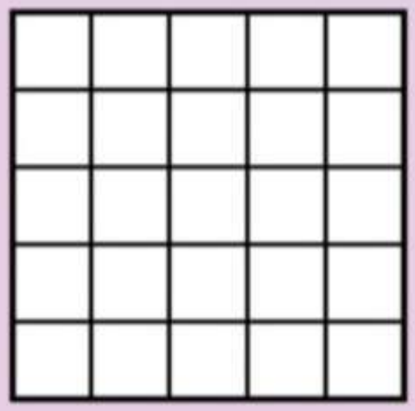
The first 5 odd numbers add up to 25, the first 6 odds add up to 36...
...and if we added the first 100 odd numbers, we would get 100 squared, which is 10,000!

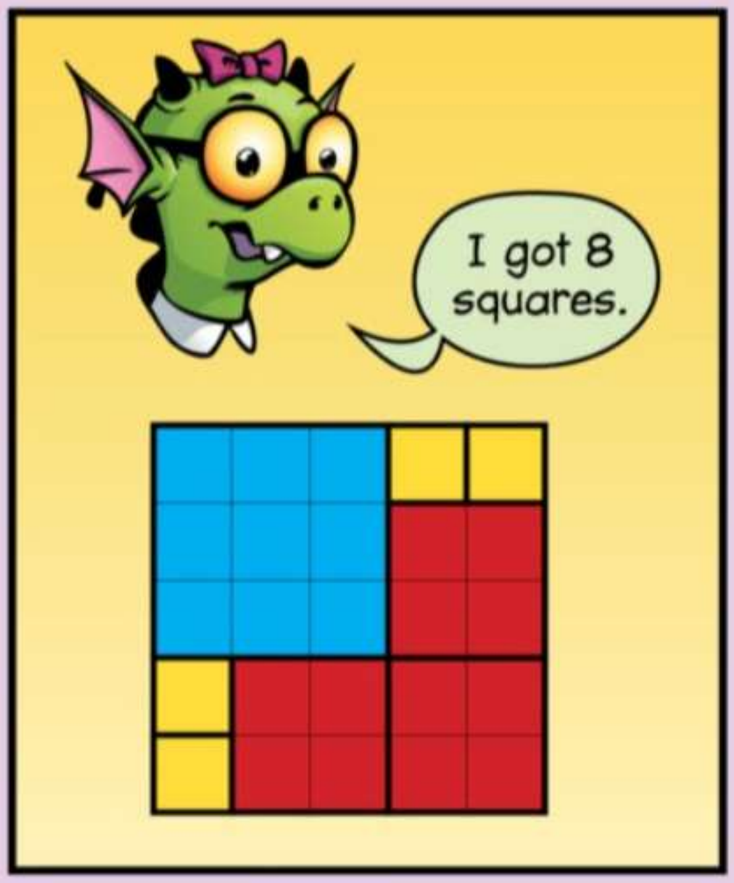

Uh oh.
What's wrong?
Maybe we shouldn't have planted the snapdragons so close to the tiger lilies.
SNAP!

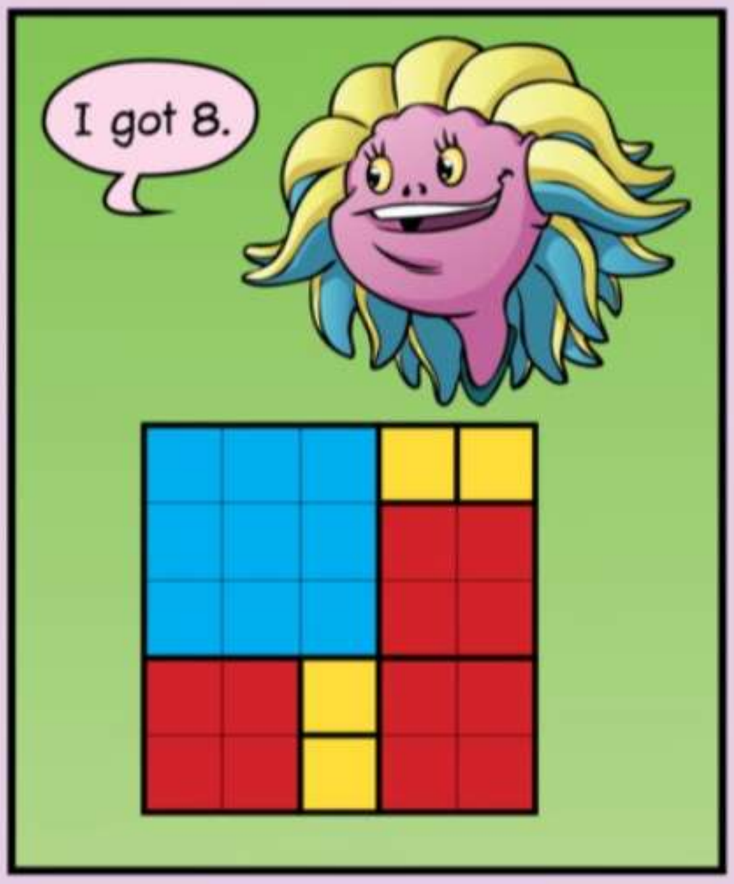

THE LAB
DISSECTIONS
Today, we'll be dissecting...
...squares!
sigh
DISSECT MEANS TO SEPARATE INTO SMALLER PIECES. IN THIS LESSON WE DISSECT SQUARES INTO SMALLER SQUARES.
You've all been learning about perfect squares.
Any square can be dissected into a perfect square number of smaller squares that are all the same size.
Today we will try to divide a square into smaller squares that are *not* all the same size. Let's start with a small one.
Tracing the lines of the grid, how could we divide a 3 by 3 perfect square into *six* smaller squares instead of nine?
You can take four of the little squares and make a big one.
THESE SQUARE DISSECTIONS ARE CALLED "MRS. PERKINS'S QUILTS."

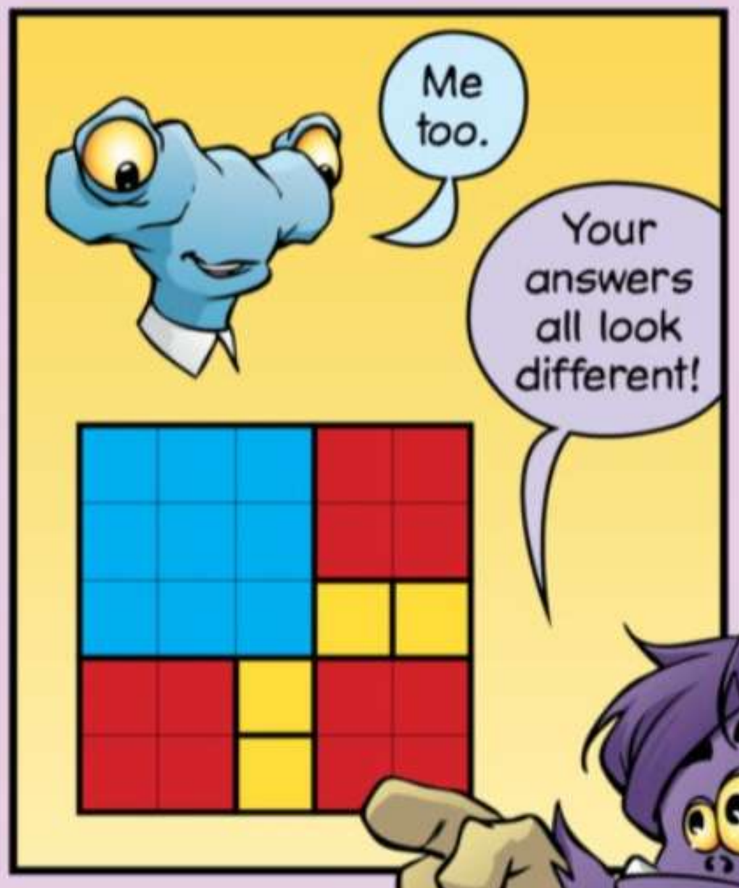
That's right!

Next, try dissecting a 5 by 5 square into smaller squares. What is the smallest number of squares you can make?





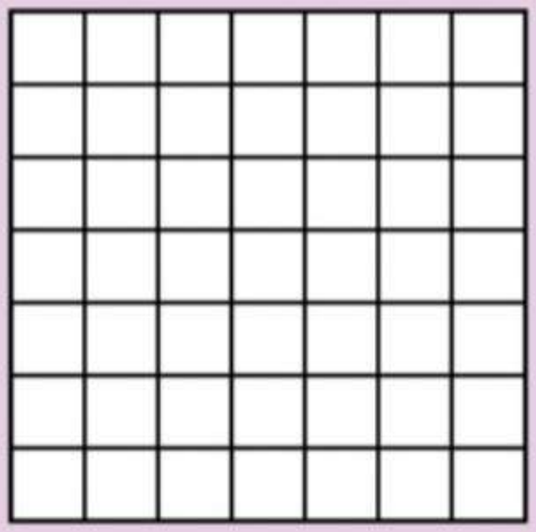




You're all right! One 3×3 square, three 2×2 squares, and four 1×1 squares makes 8 squares all together, and there are lots of ways to rearrange them...

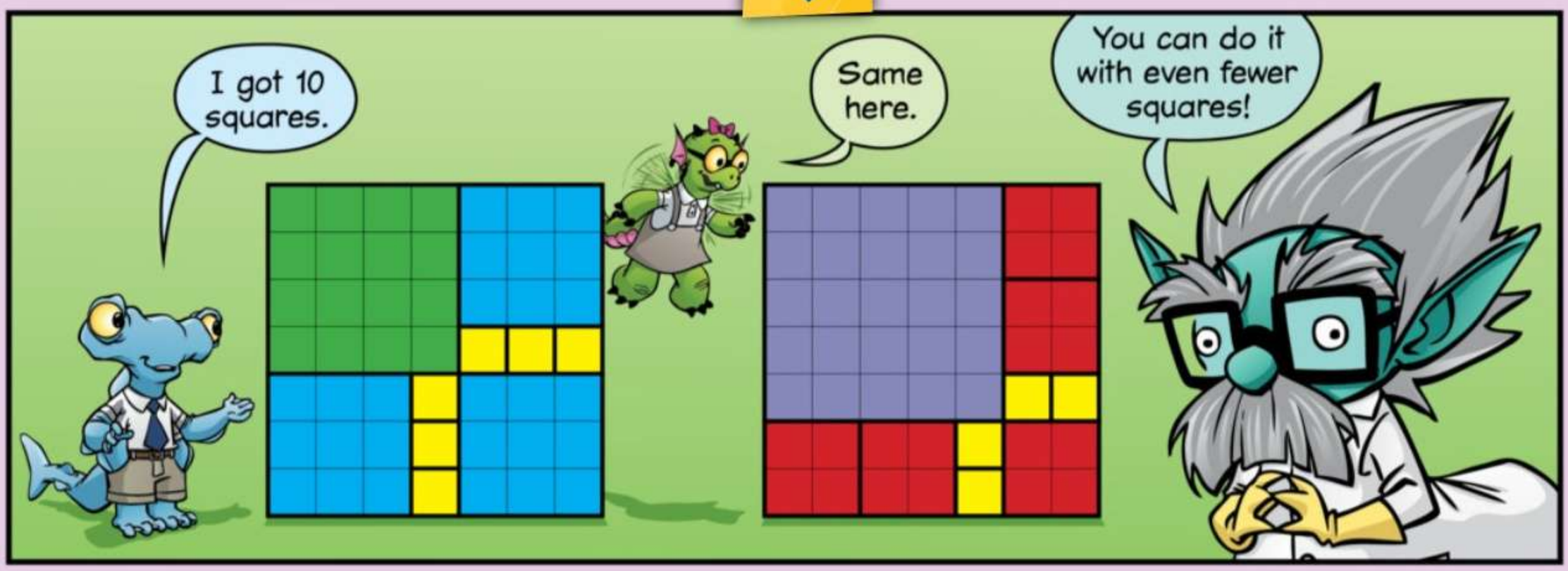
...but you cannot dissect the 5 by 5 square into fewer than 8 smaller squares.

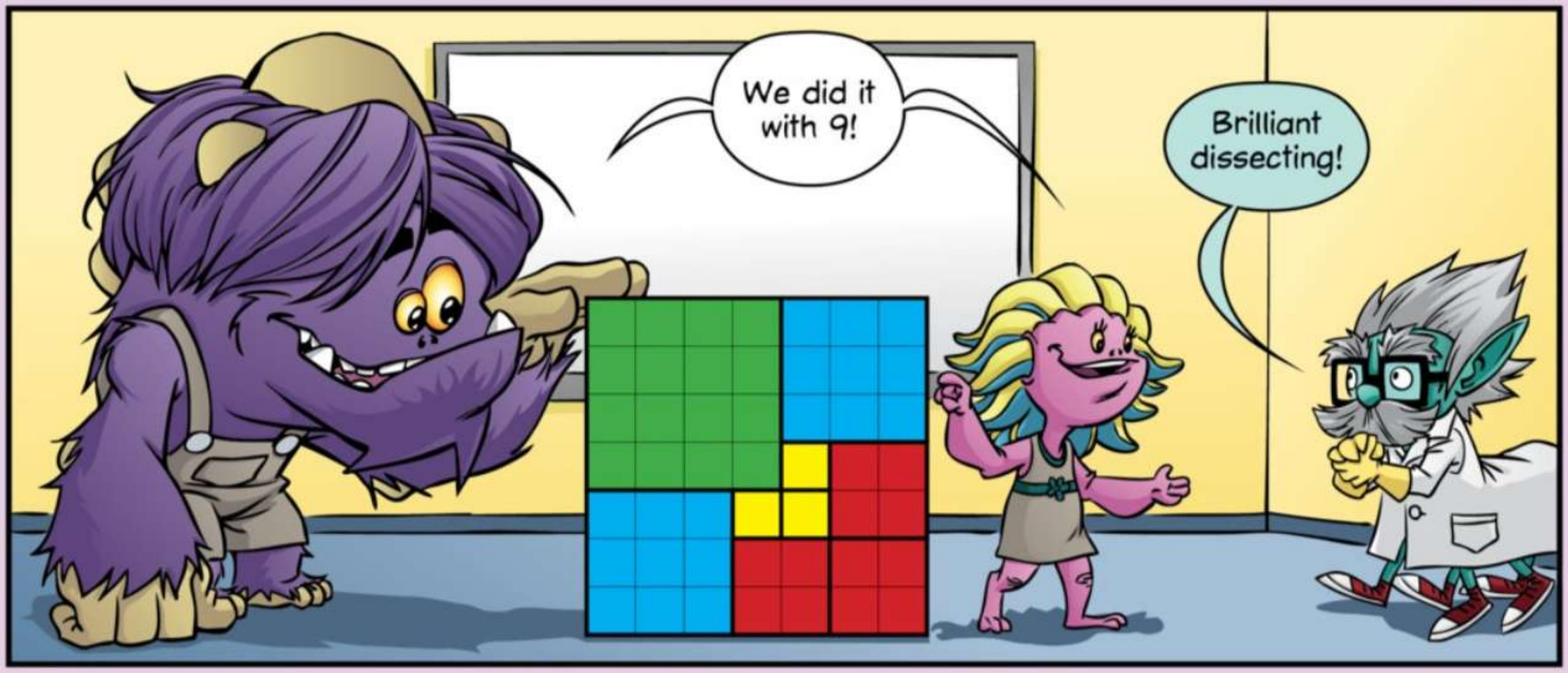
Next, try dissecting the 7 by 7 square into as few squares as possible.



What is the smallest number of squares you can use for the 7 by 7 square?

Print more blank squares at BeastAcademy.com


I got 10 squares.
Same here.
You can do it with even fewer squares!


We did it with 9!
Brilliant dissecting!

9 is the smallest number of squares into which you can dissect the 7 by 7 square.
I'm going to need to make more copies for our next dissection. I'll be right back!
IT'S A TRAP!

Bwah Ha Ha! Professor Grok is gone. I've abducted your educator. It's time for some...

...Diabolical Dissections!

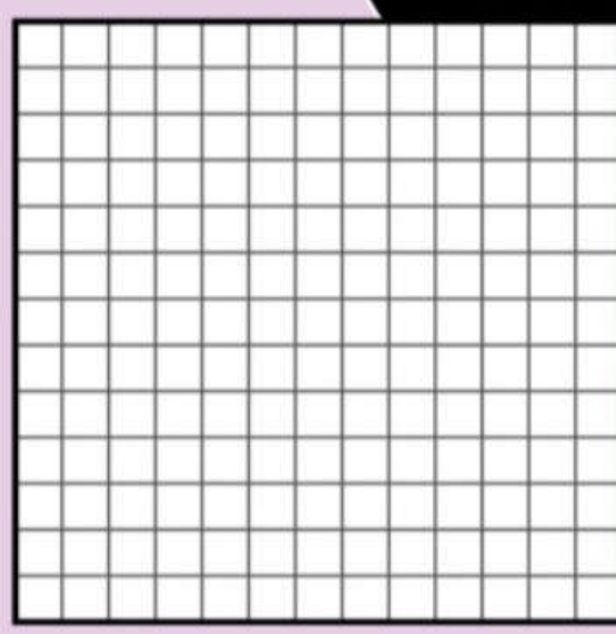
Dissecting a five by five square is perfectly pedestrian. Slicing the seven by seven square is certainly simple. But the devious dissection I demand is...

...Devastatingly Difficult!!!

A tricky tracing of this thirteen by thirteen square separates it into eleven squares.

Slip a sheet with the solution into the reading room and I will release your renowned rhetorician. Fail to do so, and I will...

...Demand His Dissection!!!



We need to dissect a 13×13 square into only ***eleven*** smaller squares?!

Is that even possible?

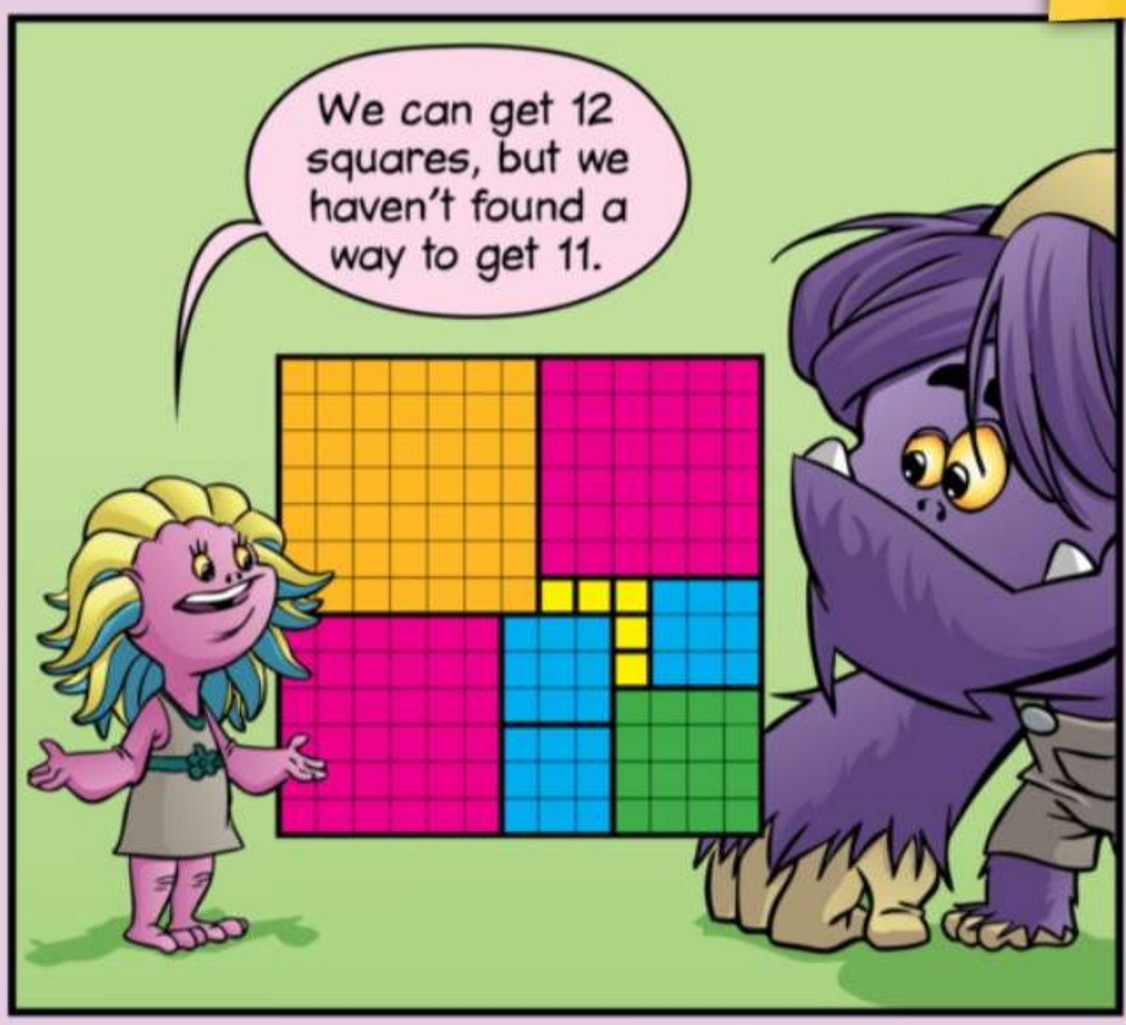
On the easier problems, we always had a big square in the top left corner.

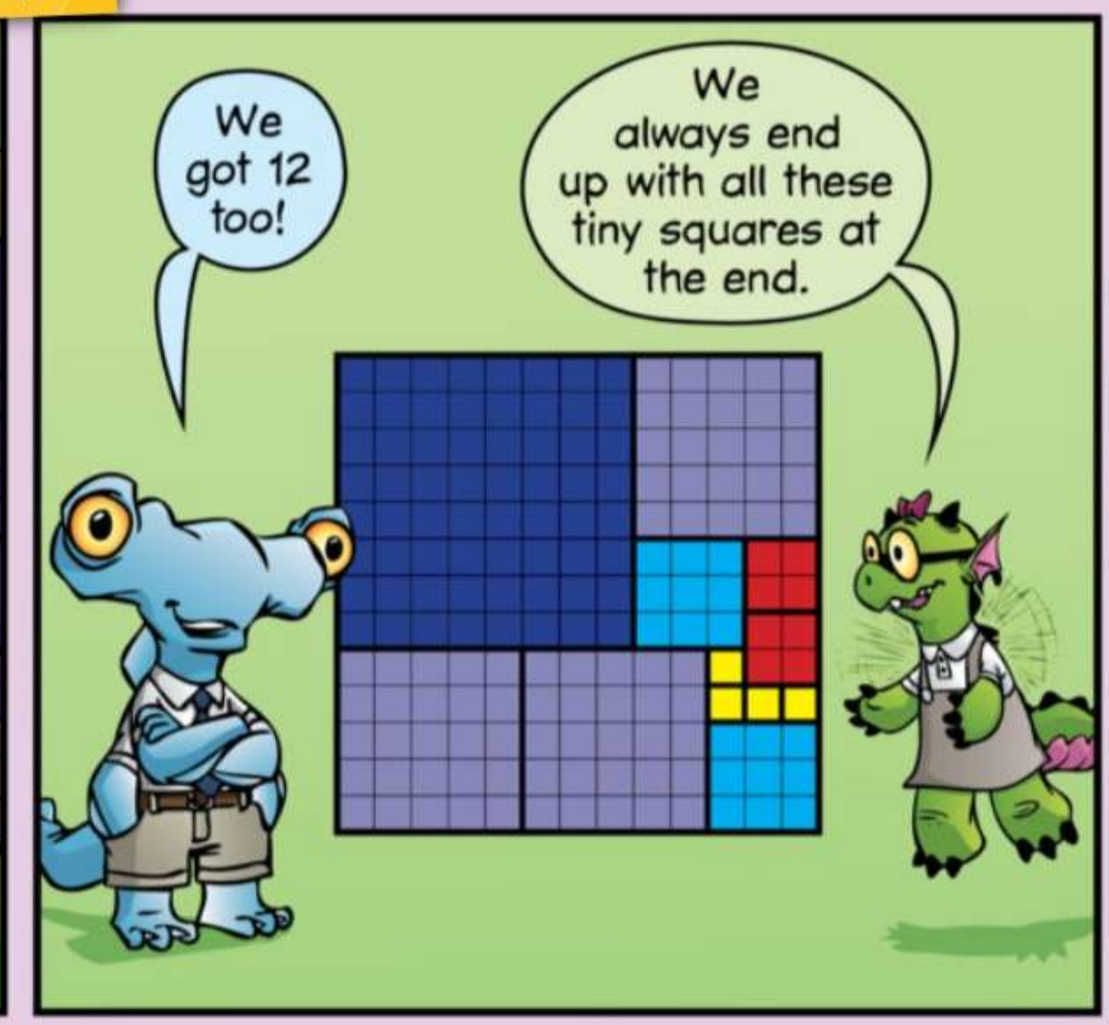
We should try that.

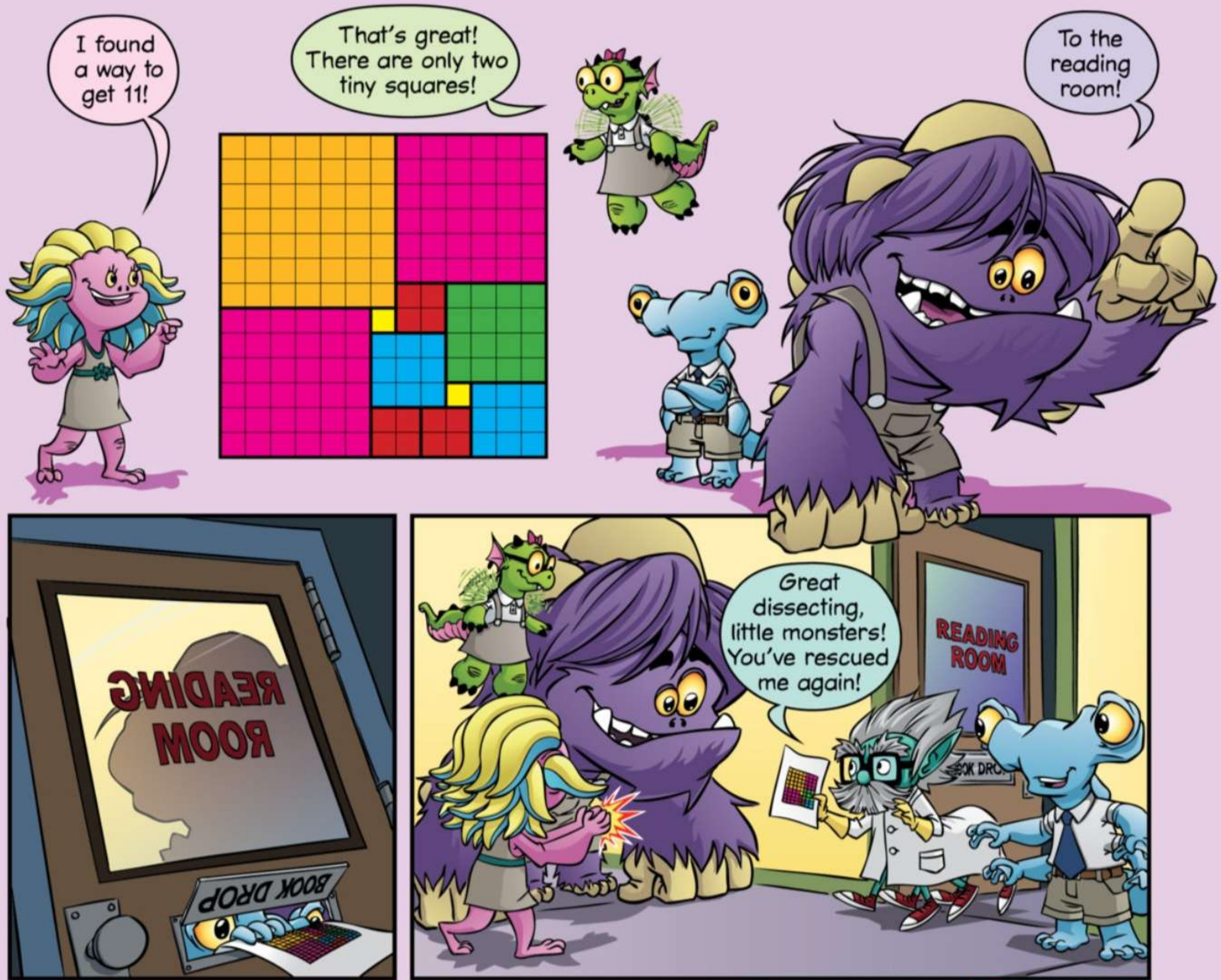
Lizzie and I will start with a 7 by 7 square in the top left corner. Winnie and Grogg, you start with an 8 by 8 square.

Try to find a way to dissect the 13 by 13 square into 11 smaller squares.

Print more blank squares at BeastAcademy.com







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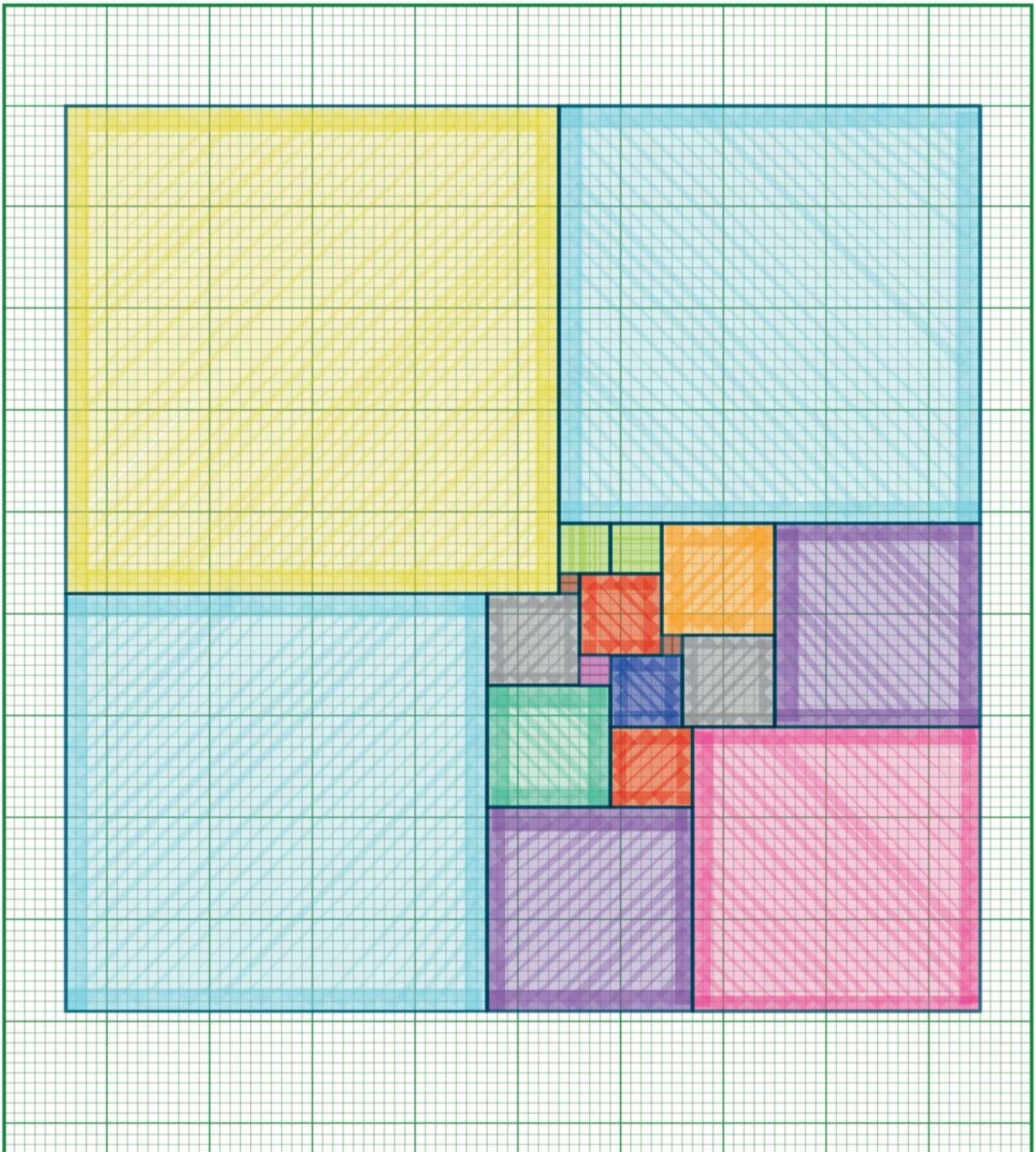
TITLE	89×89 Square Dissection		
NAME	Alex	DATE	12/5