Contents: Chapter 12

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Chapter 12:
Area

R & G
Measuring Area

This morning at the hardware store, I asked the monster in flooring about putting new tile in our kitchen.

He asked me how many ***square feet*** I had.

Remember? You were there!

I told him...

FLOOR TILE

...we've got ***two*** feet, but neither one of them is ***square!***

Right, but...

Then, I asked the monster in gardening about planting grass on our lawn.

She wanted to know how many ***square yards*** we have!

I told her, we've only got ***one*** yard, and it's ***not*** square.

It's really more of an L-shape.

Square feet and **square yards** are units of **area!**

If we want to know how much space something takes up, we can divide it into squares.

Huh?

Sure, but what does that have to do with feet?

A **foot** is a unit of length. It's about this long.

A **square foot** is the **area** of a square with sides that are one foot long.

Each of these tiles covers one square foot.

So, the monster in flooring wasn't insulting me?

Nope. He just wanted to know how big our kitchen is.

Let's find out! I'll start counting the tiles.

But...

What is the best way to find the area of the kitchen?

Some of the tiles are under the refrigerator. Could you help me lift this?

There's an easier way.

You're right!

We should use the dolly.

Not an easier way to move the refrigerator!

An easier way to count the number of square feet in our kitchen!

The kitchen is 12 feet long...

...and 20 feet wide.

12 feet

20 feet

We can split it into squares.

Square *feet!*

Since the kitchen is 12 feet long and 20 feet wide...

...we can fit 12 rows of 20 square feet on the floor.

That makes 12×20=240 square feet!

Right!

The area of our kitchen is 240 square feet.

Let's measure the lawn.

Since the lawn is bigger than the kitchen, we'll use a bigger unit to measure its area...

...the square yard.

A ***square yard*** is the area of a square with sides that are one yard long.

One yard is three feet.

1 yard (3 feet)

1 square yard

1 yard (3 feet)

Our lawn is L-shaped.

Here are the measurements I got in yards.

6

3

4

porch

8

Lawn

5

10 yards

We can split our lawn into two rectangles.

6

8

4

5

The big rectangle is 6 yards wide and 8 yards long.

6×8=48 square yards.

And this little part is 4×5=20 square yards.

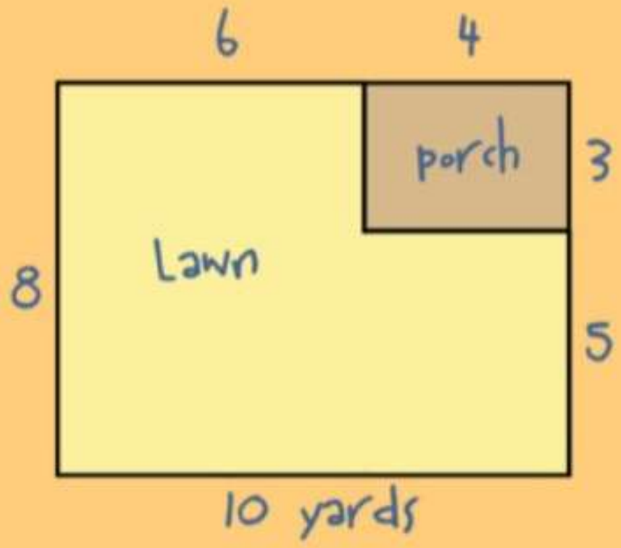
That makes 48+20=68 square yards.

You found the area of our lawn by adding two smaller areas.

You could also find the area by subtracting.

Huh?

This big rectangle is 8×10=80 square yards.



To find the area of our lawn, we have to subtract a 3×4=12 square-yard porch.

80−12=68 square yards. It's the same answer we got when we added!

Let's go back to the hardware store.

Great, but I changed my mind about planting grass.

I'm thinking more of a Zen rock garden might be nice.

This *is* nice!

Ms. Q.

Square Units

What is area?

"Area is the amount of space a shape takes up on the plane."

Not *that* kind of plane!

Plane is just another word for a flat surface...

...like the top of this table.

That's right! How do we measure area?

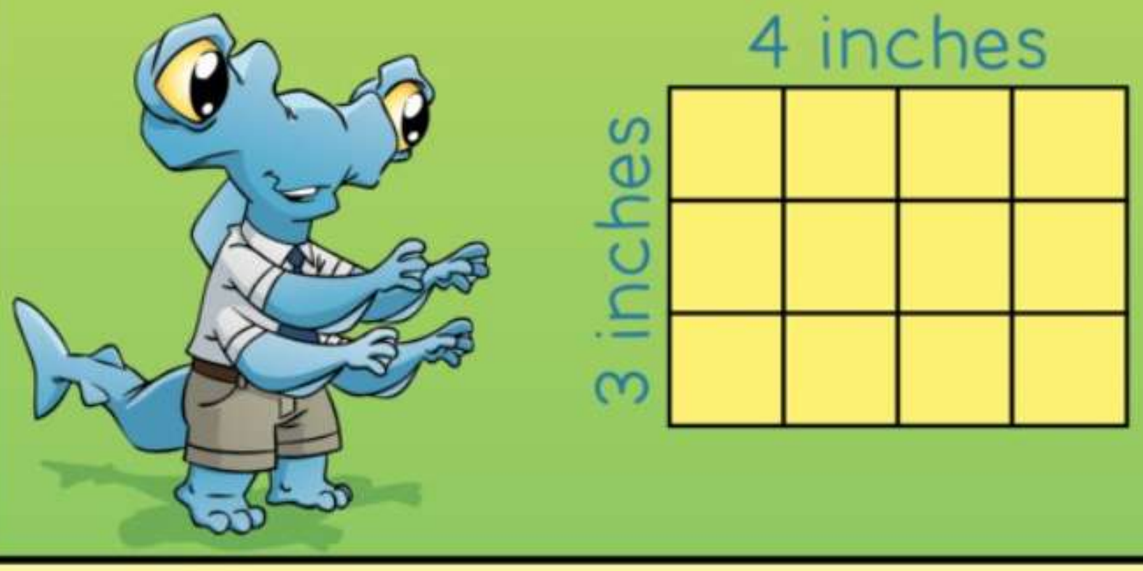
With squares!

We can split a rectangle into small squares.

Area is the number of squares that it takes to fill the shape.

This rectangle has an area of 3×4=12 squares.

How big are the squares?



How big?

Each little square is 1 inch tall and 1 inch wide.

They're ***square inches!***

What is a square inch?

A square inch is a unit of area, equal in size to a square with sides that are one inch long.

0 1 2 3 4

One square inch is an area this big.

Good.

A square that is 1 inch tall and 1 inch wide has an area of one square inch.

Any rectangle that covers the same amount of space as 12 of these squares has an area of 12 square inches.

What other units are used for area?

So this rectangle has an area of 12 ***square inches!***

4 inches

3 inches

A square mile is an area the size of a square that is one mile by one mile.

1 mile

A square centimeter is an area the size of a square that is one centimeter by one centimeter.

Every unit of length has its own unit of area.

There are square inches, square feet, square yards, square miles...

...and metric units like square centimeters, square meters, and square kilometers.

So a square that is one foot by one foot has an area of one square foot!

Right!

Is a one foot by one foot square the only shape that has an area of one square foot?

Grogg's square has an area of one square foot.

If we cut it in half...

...we can rearrange the pieces to make a rectangle.

Since the new rectangle is made from the same two pieces that made the square...

...it takes up the same amount of space as the square.

So, it has the same ***area*** as the square!

So, the rectangle has an area of one square foot!

If we cut a square foot into ***lots*** of pieces...

...we can rearrange them to make a new shape.

Since the new shape uses all of the same pieces as the square, it has the same area as the square...

...one square foot!

GRRR

THIS IS AN EXAMPLE OF A ***TANGRAM*** PUZZLE. A SQUARE IS CUT AS SHOWN INTO SEVEN PIECES THAT CAN BE REARRANGED TO FORM A NEW SHAPE. CUT OUT YOUR OWN SET OF TANGRAM PIECES AT BEASTACADEMY.COM.

Any shape that takes up the same amount of space as a one foot by one foot square has an area of one square foot.

And any shape that covers the same area as two 1 ft by 1 ft squares has an area of ***two*** square feet!

2 feet

1 foot

Like PolyGrogg!

What!?

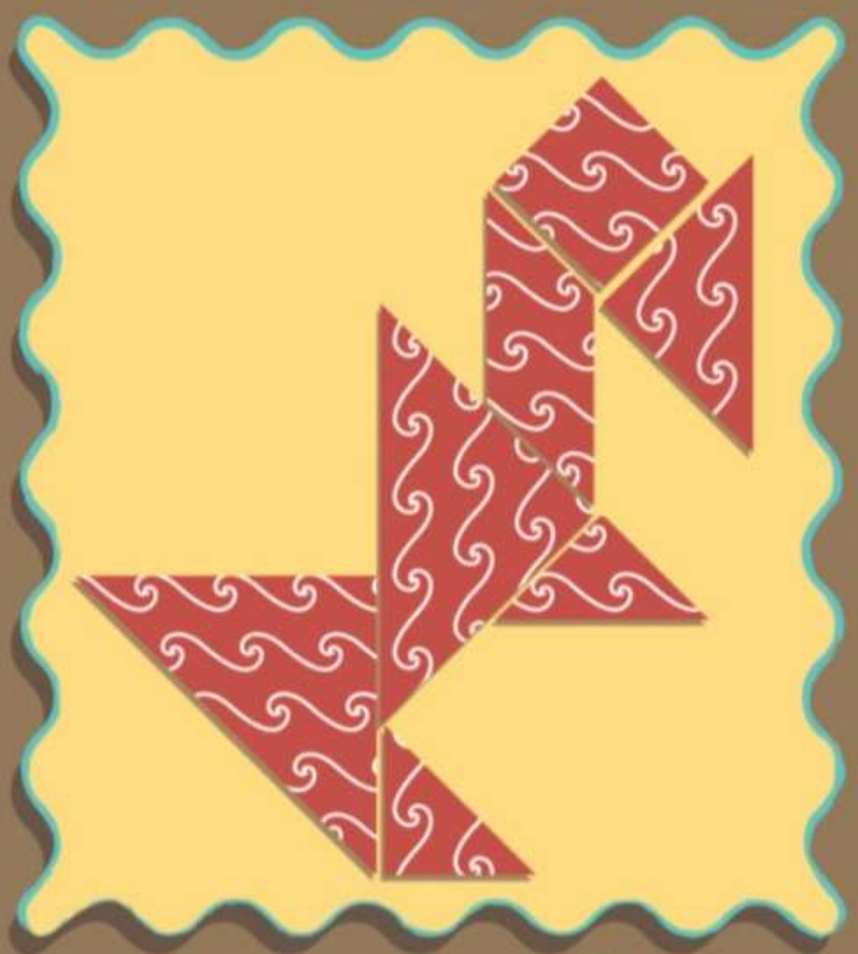
It's me, but in polygon form!

I made him from the pieces of two one foot by one foot squares!

So, PolyGrogg has an area of two square feet!

PolyGrogg! What will you little monsters think of next?

Tangrams



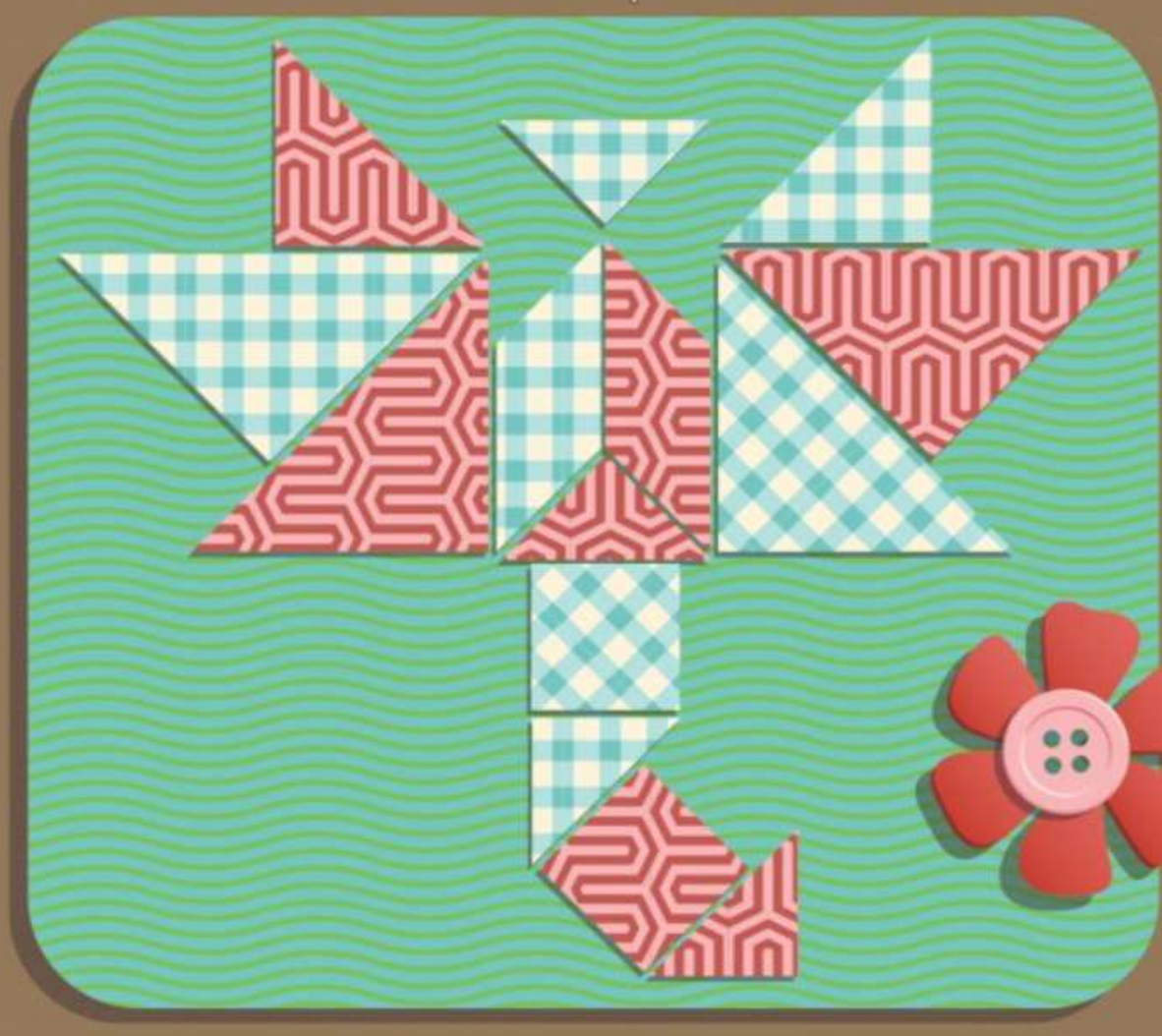
"Among all the kinds of serpents, there is none comparable to the Dragon."

Tangrams are an ancient puzzle form in which a square is cut into 7 pieces called tans that can be arranged to make new shapes.

TANS

by Lizzie

Tans can be arranged to make a shape that looks like a monster, a building, a letter, or even a dragon. Each of these dragons was made with one or two sets of tans. I made dragons because I am a dragon and I think dragons are awesome.



WORKSHOP

RIGHT TRIANGLES

BEAST ACADEMY

Today, we'll be talkin' more about me favorite shapes.

Triangles!

YEAH Buoy!

To begin, we'll be findin' the area of some of me first triangular sails.

Let's start with this one.

'Tis a right triangle, 2 feet wide and 4 feet tall.

How could we be findin' its area?

The area of this part of the sail is two square feet.

There are extra parts here and here.

How can we include the area of these pieces?

I know!

If we cut off the top of the sail...

...and put it down here...

...we make a square.

This triangle has the same area as a 2 ft by 2 ft square!

4 square feet!

Aye. The area of this triangle be 4 square feet.

I got 4 square feet a different way.

Shiver me timbers! How'd you do it, lad?

If we attach two right triangles, we can make a rectangle.

The rectangle has an area of 8 square feet...

...and each triangle is half the area of the rectangle!

Half of 8 is 4, so this triangle has an area of 4 square feet.

4 feet

2 feet

Good thinkin'.

Let's try another. What be the area of this right triangle?

6 feet

5 feet

We can put two right triangles together to make a rectangle.

The rectangle has an area of 6×5=30 square feet.

6 feet

5 feet

Since there are two triangles, each triangle has an area of 30÷2=15 square feet.

6 feet

5 feet

Aye. To find the area of a right triangle, 'tis easiest to think of it as half of a rectangle.

To find the area of a rectangle, we multiply its width by its height.

So, to find the area of a right triangle...

...we multiply its width by its height, then divide by 2.

height

width

Area of a rectangle
= width × height

height

width

Area of a right triangle
= width × height ÷ 2

Very good. These be called ***formulas.***

A formula is an equation that we can use for a particular type of problem...

...like findin' the area of a right triangle.

We can shorten formulas by usin' variables, like A for area, w for width, and h for height.

Area of a rectangle
= width × height

$$A = w \times h$$

Area of a right triangle
= width × height ÷ 2

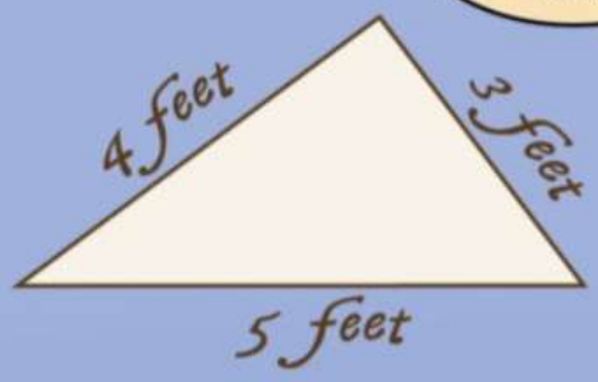
$$A = w \times h \div 2$$

Try this tricky triangle area problem.

Here be a triangle with sides of length 3, 4, and 5 feet.

This be a ***right*** triangle.*

Try findin' the area of this right triangle.



Try it.

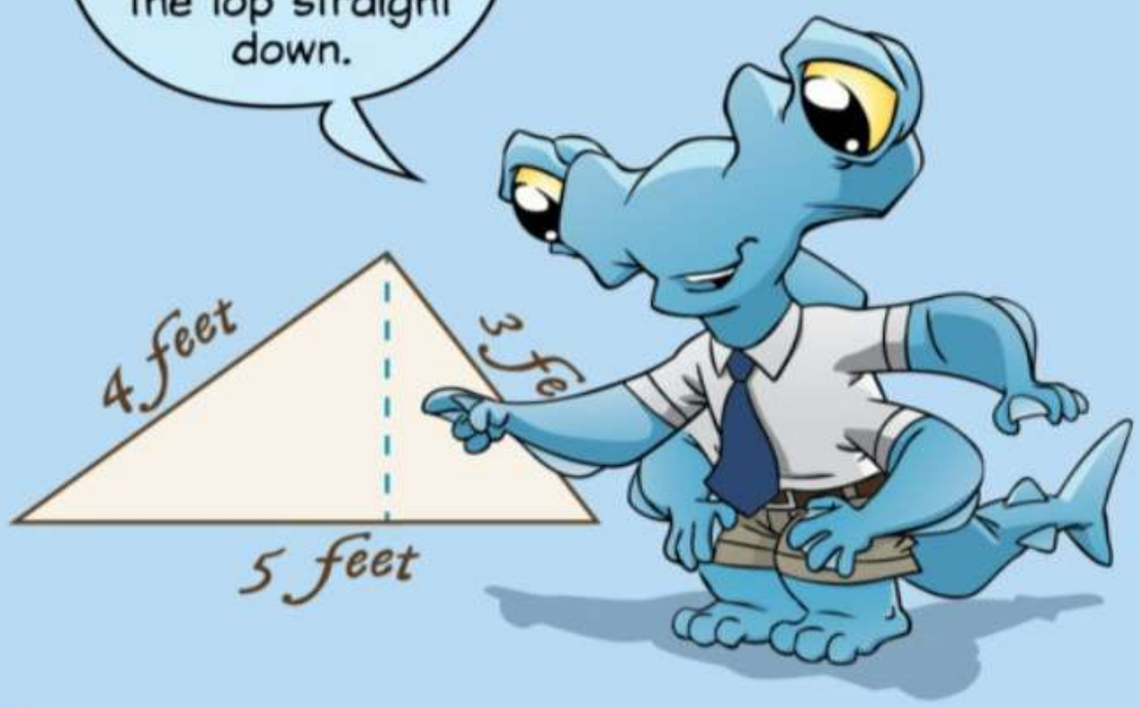
THREE SIDE LENGTHS DO NOT ALWAYS MAKE A RIGHT TRIANGLE. FOR EXAMPLE, A TRIANGLE WITH SIDE LENGTHS 2, 3, AND 4 IS OBTUSE. A TRIANGLE WITH SIDE LENGTHS 4, 5, AND 6 IS ACUTE.

We could try using the formula.

The triangle is five feet wide. But, how ***tall*** is it?

I don't think either of these slanty sides gives us its height.

To find its height, we need to measure from the top straight down.



Maybe if we look at it this way.

Or...

...we could flip the ***triangle*** over!

Turned this way, the triangle is 3 feet wide and 4 feet tall.

5 feet

4 feet

3 feet

So, its area is $3 \times 4 \div 2 = 12 \div 2 = 6$ square feet.

It's half the area of a 3 by 4 rectangle.

Aye, kids.

Sometimes lookin' at a thing in a different way can help you solve a problem.

Pirates Attttt Us

Captain

See, I've found me keys!

MATH TEAM

Triangle Area

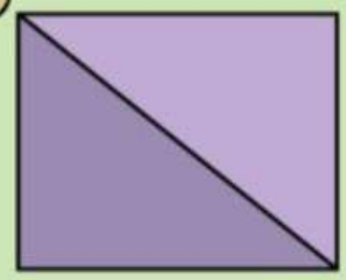
Are you all ready for the last math meet of the school year?

Captain Kraken just taught us the formula for finding the area of any right triangle!

Great! What's the formula?

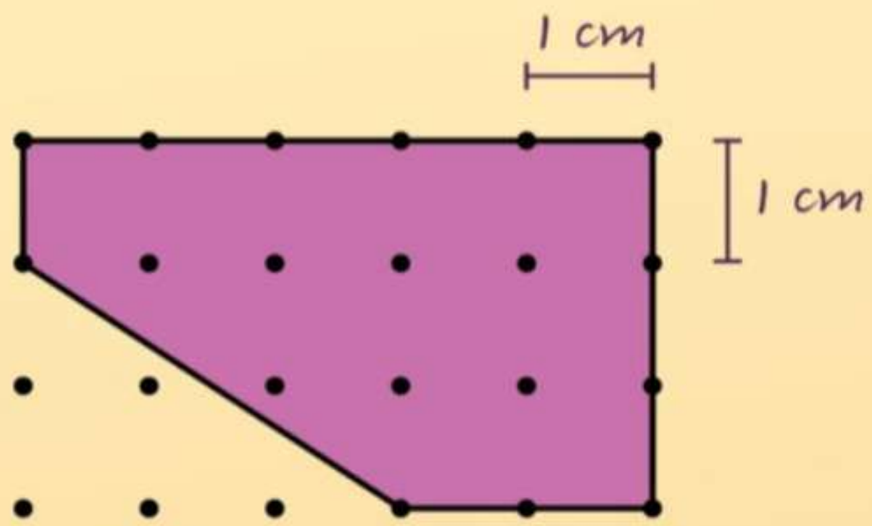
To find the area of a right triangle, we multiply its width by its height and divide by 2.

It works because two right triangles can be put together to make a rectangle!



Awesome. Let's see if we can use what you know about right triangles to find the area of some other shapes.

This shape is drawn on a dot grid. The dots are spaced 1 centimeter apart. Can you find the area of the shape?

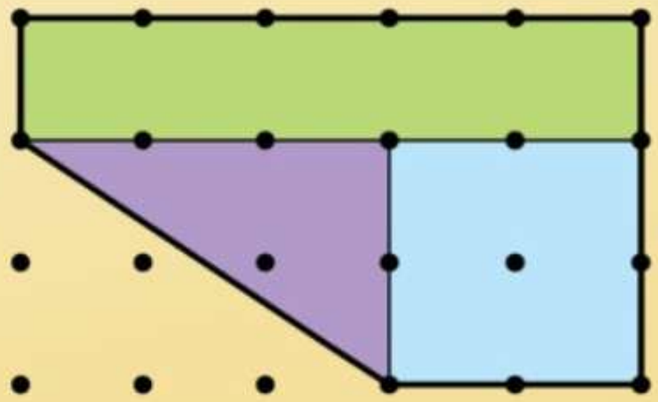


Can you find the area of the pentagon?

We can split the shape into three pieces.

The area of the top piece is 5×1=5 square centimeters.

The area of this corner piece is 2×2=4 square centimeters.

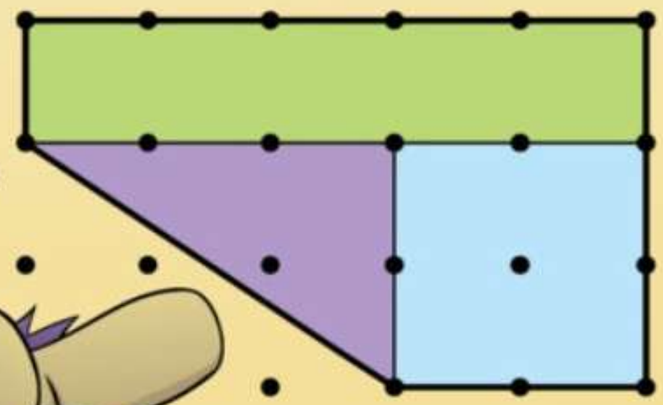


And this triangle has an area of (3×2)÷2=3 square centimeters.

That's a total of 5+4+3=12 square centimeters.

That's right!

I got the same answer in a different way.



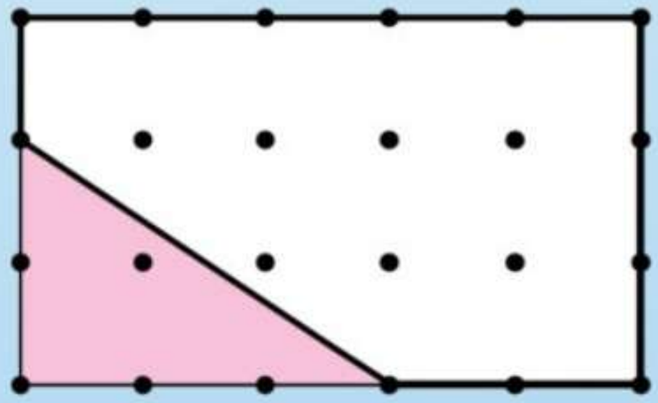
The shape looks like a rectangle that's missing a corner.

So, I found the area of the whole rectangle and subtracted the area of the missing corner.

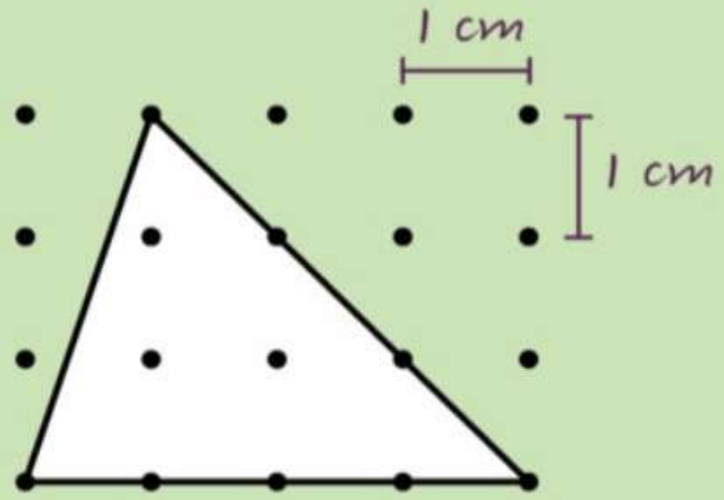
The whole rectangle has an area of 5×3=15 square centimeters.

The missing triangle has an area of (3×2)÷2=3 square centimeters.

So, the shape has an area of 15−3=12 square centimeters.

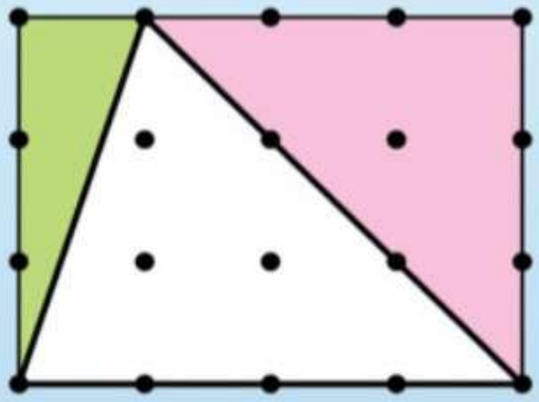


Great job! Next, let's use what we know about ***right*** triangles to find the area of this ***acute*** triangle.



We could find the areas of these two pieces and add them.

Or, find the area of the rectangle and subtract these two pieces.

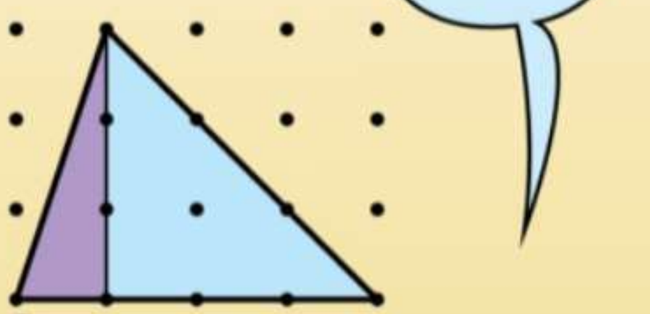


This piece has an area of $1\times3\div2$.

What's $3\div2$?

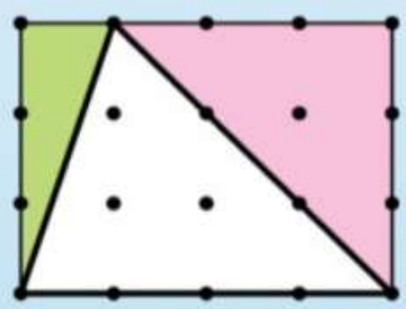
This piece has an area of $3\times3\div2$.

$9\div2$ is a fraction.



We have the same problem over here. I think that the areas of the pieces are fractions.

Can you find a way to compute the area of the triangle without using fractions?



Can you find the area of the triangle?

Hmmm...

My piece has the same area as Grogg's.

And my piece has the same area as Alex's.

The area of the pieces ***inside*** the triangle...

...is the same as the area of the pieces ***outside*** the triangle.

So, the area of the triangle is half the area of the rectangle!

The area of the rectangle is 3×4=12, so the area of the triangle is 12÷2=6 square centimeters.

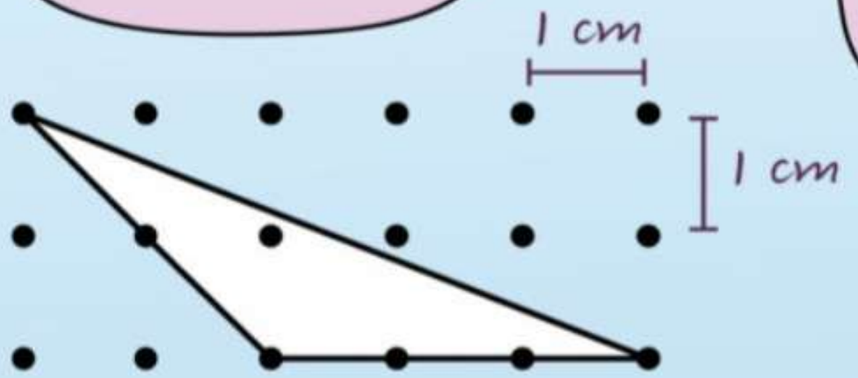
The formula we use to find the area of right triangles works for acute triangles, too!

$A = W \times h \div 2$

That's right!

You can find the area of any acute triangle by multiplying its width by its height and dividing by 2.

Does the same formula work for this obtuse triangle?

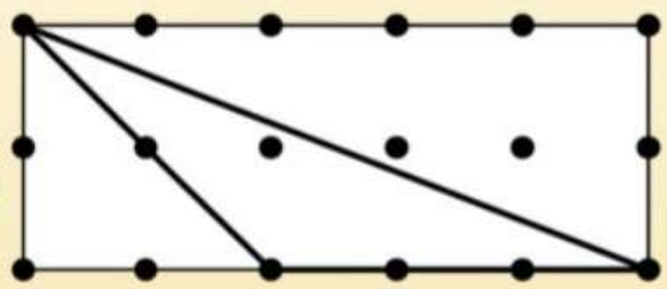


Does it?

Nope.

This triangle is definitely smaller than half the area of the rectangle around it.

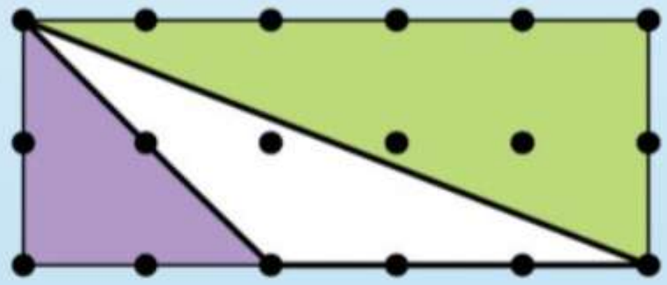
The area of the rectangle is 5×2=10 square centimeters.



The small piece has an area of 2×2÷2=2 square centimeters.

This big piece has an area of 5×2÷2=5 square centimeters.

So the triangle has an area of 10−5−2=3 square centimeters.



Nice work!

The formula for the area of an obtuse triangle looks like this:

Area of an obtuse triangle
= length of base × height ÷ 2

$A = b \times h \div 2$

What's the base?

The base is the side we measure the height from.

The base of this triangle is 3 centimeters long.

Its height is 2 centimeters, so its area is $3 \times 2 \div 2 = 3$ square centimeters.

2 cm

3 cm

$$A = b \times h \div 2$$
$$= 3 \times 2 \div 2$$

$$A = b \times h \div 2$$

h

b b b

This formula works for all triangles...

...since the ***width*** of a right triangle or an acute triangle is the same as the length of its ***base***.

And now...

Do your best!

Welcome to the final math meet of the academic year.

Today's math meet will include six questions, covering topics you've learned all year. The first five questions are each worth one point, and the final question is worth two. The team with the most points wins the meet. Let's begin!

Question 1:
What is the side length of a regular octagon whose perimeter is 296 centimeters?

Try it.

A regular octagon has eight equal sides. We need to divide 296 by 8...

BZZZT

37 centimeters.

Correct!

Whoa.

That was fast.

Question 2: Subtract 1,000×1,000 - 999×999.

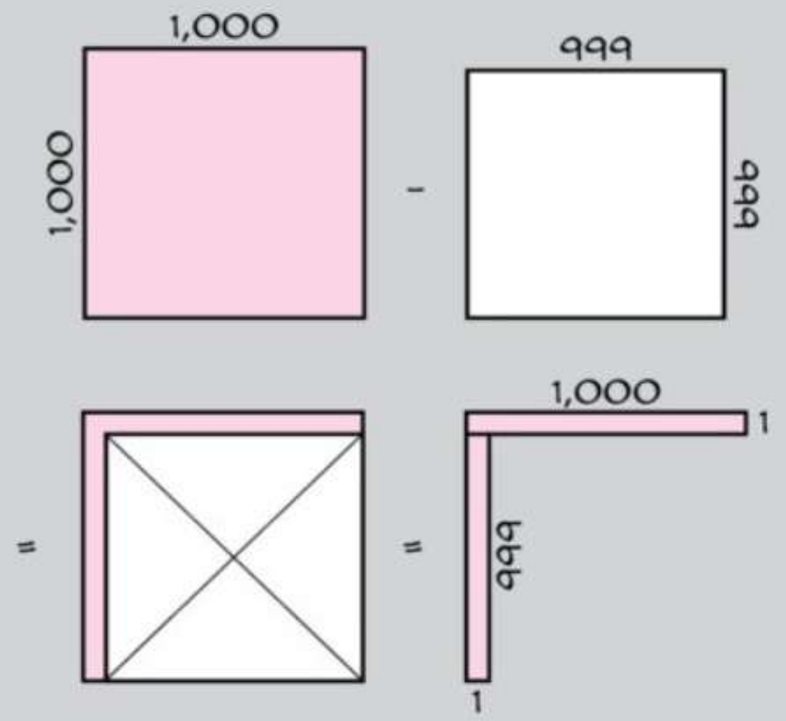
Try it.

Ding!

1,999.

Correct! Little Monsters tie the score.

We're fast, too!



1,000×1,000 - 999×999 = 1,000+999.

Area = 360 sq ft

Question 3:
This rectangle has an area of 360 square feet. What is the area of the rectangle in square yards?

Try it.

There are three feet in a yard.

So we just divide 360÷3 to figure out how many square yards equal 360 square feet.

Wait. I don't think that's right.

1 yard is 3 feet...

1 yard

1 ft 1 ft 1 ft

...but 1 ***square*** yard is 3 feet tall and 3 feet wide.

So, a square yard is ***nine*** square feet!

3 feet

3 feet

Since there are 9 square feet in a square yard, we divide 360 by 9!

40×9 is 360...

...so 360÷9 is--

BZZZT

40 square yards.

Correct! The Bots retake the lead.

Question 4:
How many fractions with denominator 15 are greater than $\frac{1}{4}$ but less than 1?

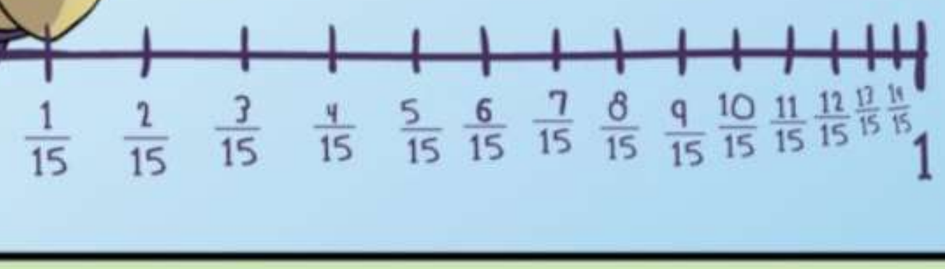
Try it.

Let's use a number line.

We need to find all of the fractions with denominator 15 that are between $\frac{1}{4}$ and 1.

Marking a number line with fifteenths is hard...

...I can't get the spacing equal.



We should compare $\frac{1}{4}$ to some fractions that have denominator 15.

We can compare $\frac{1}{4}$ to $\frac{3}{15}$.

$\frac{3}{15}$ is equal to $\frac{1}{5}$...

$$\frac{3}{15} \overset{\div 3}{=} \frac{1}{5}$$

And since fourths are bigger than fifths...

...$\frac{1}{4}$ is bigger than $\frac{1}{5}$.

So, $\frac{1}{4}$ is ***more*** than $\frac{3}{15}$.

$$\frac{1}{4} > \frac{1}{5}$$

$$\frac{1}{4} > \frac{3}{15}$$

We can also compare $\frac{1}{4}$ to $\frac{5}{15}$.

$\frac{5}{15}$ is equal to $\frac{1}{3}$.

$$\frac{5}{15} \overset{\div 5}{=} \frac{1}{3}$$

Since $\frac{1}{4}$ is less than $\frac{1}{3}$...

...$\frac{1}{4}$ is ***less*** than $\frac{5}{15}$.

$$\frac{1}{4} < \frac{1}{3}$$

$$\frac{1}{4} < \frac{5}{15}$$

So, $\frac{1}{4}$ is between $\frac{3}{15}$ and $\frac{5}{15}$.

What about $\frac{4}{15}$?

Is $\frac{1}{4}$ more or less than $\frac{4}{15}$?

$$\frac{3}{15} < \frac{1}{4} < \frac{5}{15}$$

$\frac{1}{4}$ equals $\frac{4}{16}$.

Since $\frac{4}{16}$ is less than $\frac{4}{15}$...

...$\frac{1}{4}$ is ***less*** than $\frac{4}{15}$.

$$\frac{1}{4} \overset{\times 4}{=} \frac{4}{16}$$

$$\frac{4}{16} < \frac{4}{15}$$

$$\frac{1}{4} < \frac{4}{15}$$

So, $\frac{1}{4}$ is between $\frac{3}{15}$ and $\frac{4}{15}$.

So, all the fractions from $\frac{4}{15}$ to $\frac{14}{15}$ are between $\frac{1}{4}$ and 1!

That makes a total of...

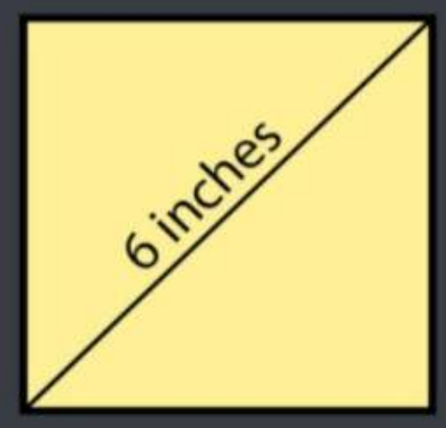
$$\frac{3}{15} < \frac{1}{4} < \frac{4}{15}$$

11 fractions.

11 is correct! There are 11 fractions with denominator 15 between $\frac{1}{4}$ and 1.

Whoa! That was smart. The Bots converted all of the fractions to sixtieths.

The score is now Bots: 3, Little Monsters: 1. The Little Monsters need to answer the last two questions correctly to win the meet.



Question 5: What is the area of a square that has a diagonal length of 6 inches as shown?

Try it.

To find the area of a square, we need to know its side length.

But all we know is the length of its diagonal.

Maybe we can find the square's area by splitting it into shapes and rearranging them.

Like PolyGrogg!

6 inches

If we cut the square along the diagonal...

...we get two right triangles.

6 inches

6 inches

We can put them together to make one big right triangle.

6 inches

6 inches

Together, the area of the two triangles is half the area of a 6 inch by 6 inch square!

6 inches

6 inches

6×6 is 36, and half of 36 is...

...18 square inches!

Correct!
The score is now 2 to 3. The last question will determine the winner of the meet.

Perimeter = 34

Perimeter = 26

Perimeter = __

Question 6:
When two identical rectangles are attached along their short sides, the perimeter of the rectangle they make is 34. When the same two rectangles are joined along their long sides, the perimeter of the rectangle they make is 26.

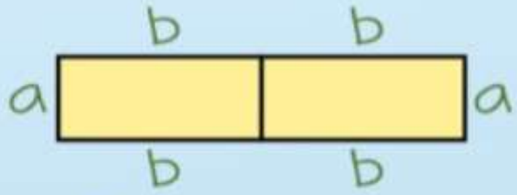
What is the perimeter of one of the two congruent rectangles?

The bots look stumped. Let's think this through.

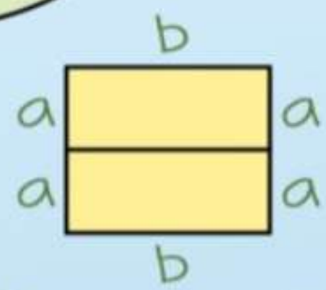
I'll start guessing and checking while you guys figure it out.

Will it help to use variables?

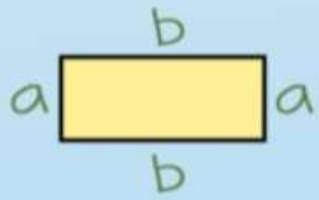
We can label the length of the short side a and the length of the long side b.



Perimeter=34



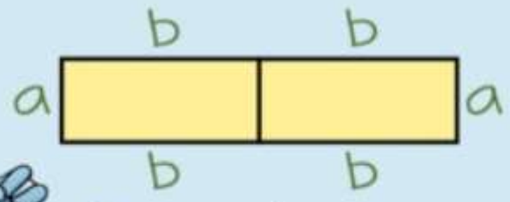
Perimeter=26



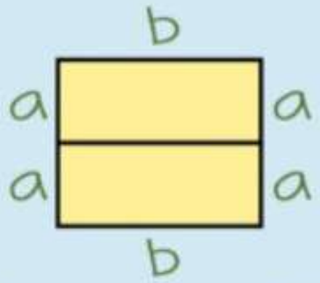
Perimeter=

What is the perimeter of the small rectangle?

So, two short sides plus four long sides is 34...

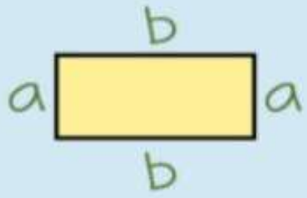


Perimeter=34



Perimeter=26

...and four short sides plus two long sides equals 26.



Perimeter=

We can write these two equations.

$$a+a + b+b+b+b = 34$$

$$a+a+a+a + b+b = 26$$

If we ***add*** the two perimeters, we get ***six*** long sides and ***six*** short sides...

...with a total perimeter of 60!

$$a+a + b+b+b+b = 34$$

$$a+a+a+a + b+b = 26$$

$$6\times a + 6\times b = 60$$

If ***six*** short sides and ***six*** long sides add up to 60, then ***one*** short side and ***one*** long side add up to 10.

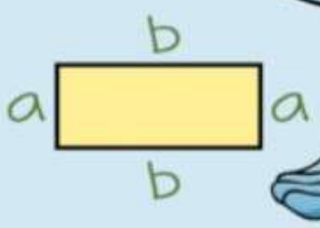
$$a+a + b+b+b+b = 34$$

$$a+a+a+a + b+b = 26$$

$$6\times a + 6\times b = 60$$

$$1\times a + 1\times b = 10$$

Since each small rectangle has ***two*** short sides and ***two*** long sides, its perimeter must be...



20 is...

...20!

Correct!

I got it, it's 20!

Right again!

The Little Monsters win the meet! Congratulations to the team of Alex, Winnie, Lizzie and Grogg!