Contents: Chapter 2

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Chapter 2:
Multiplication

Ms. Q

One Part at a Time

How do we multiply 7×123?

One part at a time.

To multiply 7×123, we split 123 into three parts: 100+20+3.

7×123

$= 7 \times (100+20+3)$

Then, we multiply each part by 7.

7×123

$= 7 \times (100+20+3)$

$= 7 \times 100 + 7 \times 20 + 7 \times 3$

We find all three products, then we add them.

7×123 =861!

7×123

$= 7 \times (100+20+3)$

$= 7 \times 100 + 7 \times 20 + 7 \times 3$

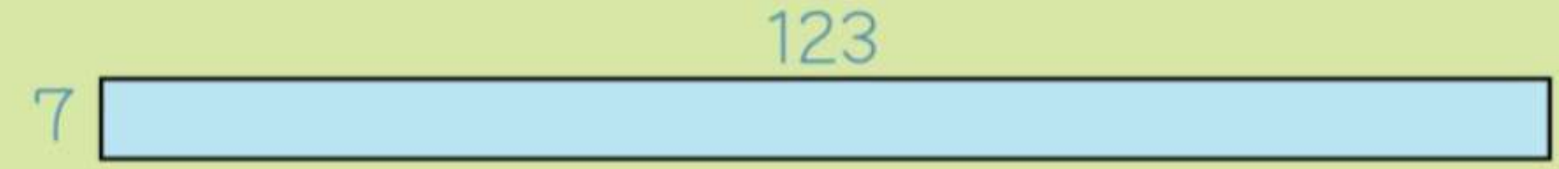
$= 700 + 140 + 21$

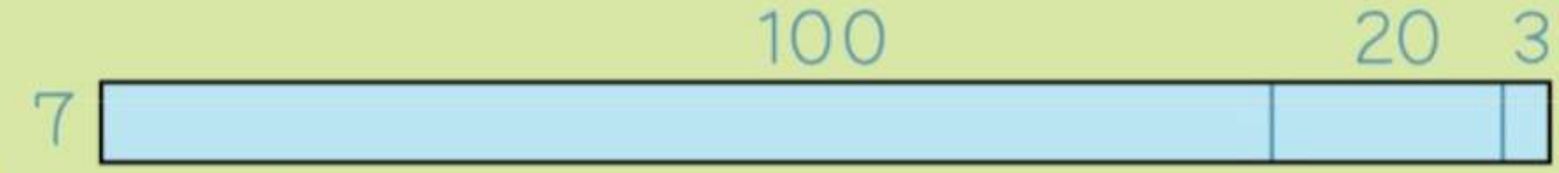
$= 861$

Multiplying one part at a time is called using the distributive property!

It's like finding the area of a big rectangle by splitting it into smaller pieces.

We find the area of each piece, then add the pieces together to find the total area.





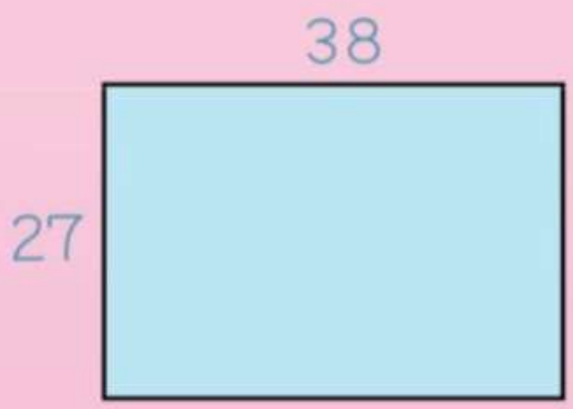
$$7\times123 = 7\times100 + 7\times20 + 7\times3$$
$$= 700 + 140 + 21$$
$$= 861$$

Today we are going to learn how to use the distributive property to multiply two 2-digit numbers.

That's right!

How could you use the distributive property to multiply 27×38?

One part at a time!



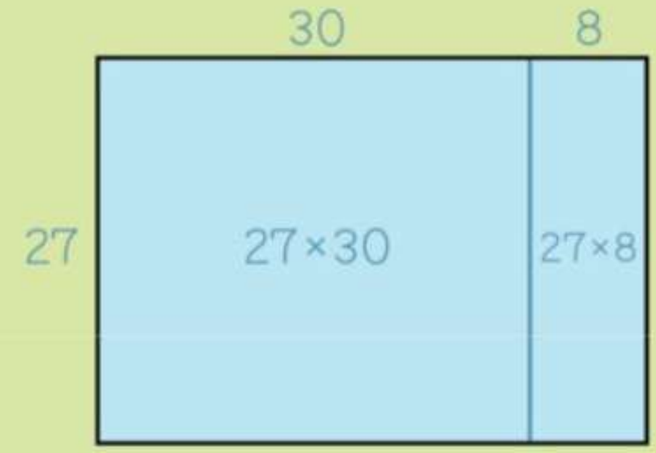
To multiply 27×38, we can split 38 into two parts: $30+8$.

$$27\times38$$
$$= 27\times(30+8)$$

Then, we multiply each part by 27.

$$27\times38$$
$$= 27\times(30+8)$$
$$= 27\times30 + 27\times8$$

It's like finding the area of a 27 by 38 rectangle by splitting it into smaller pieces.



27×38

= 27×(30+8)

= 27×30 + 27×8

Great, but now we need to solve ***two*** multiplication problems.

How do we multiply 27×30 and 27×8?

We use the distributive property again!

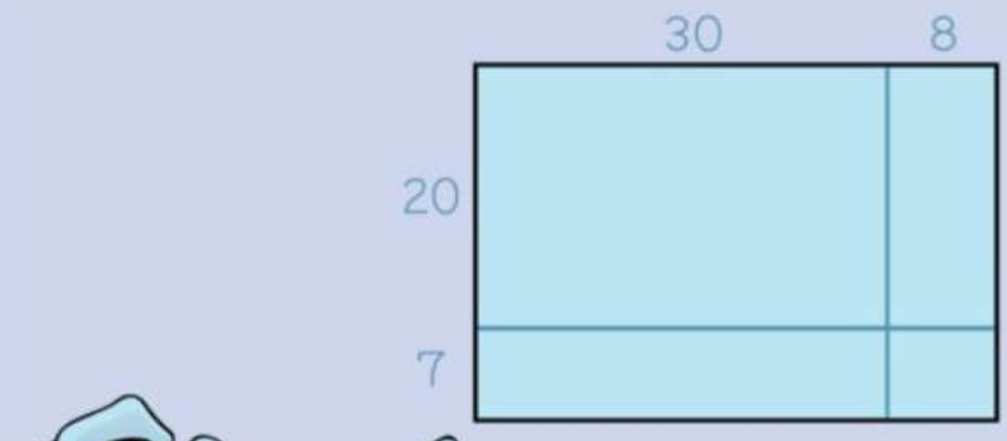
To multiply 27×30, and 27×8, we can split 27 into 20+7.

27×38

= 27×(30+8)

= 27×30 + 27×8

=(20+7)×30+(20+7)×8



We can do that on our area diagram, too!

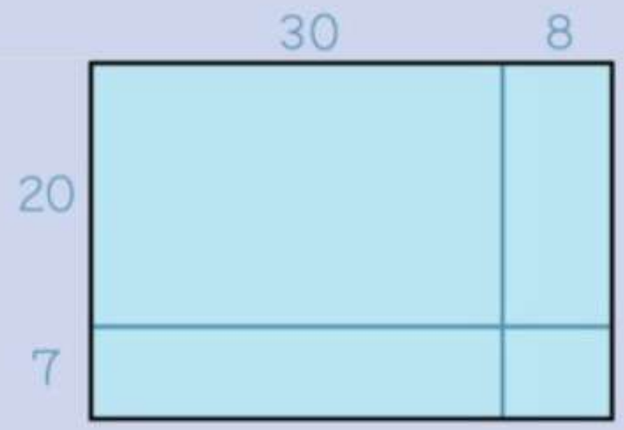
Now we have to distribute the 30 ***and*** the 8.

27×38

= 27×(30+8)

= 27×30 + 27×8

=(20+7)×30+(20+7)×8



It's the same as finding the areas of all four parts of our rectangle!

We distribute the 30 to (20+7)...

...and the 8 to (20+7).

$$27\times38$$
$$=27\times(30+8)$$
$$=27\times30+27\times8$$
$$=(20+7)\times30+(20+7)\times8$$
$$=20\times30+7\times30+20\times8+7\times8$$

We're finding the area of all four of these pieces.

	30	8
20	20×30	20×8
7	7×30	7×8

Then, we compute all of these products and add them to get...

...1,026!

$$27\times38$$
$$=27\times(30+8)$$
$$=27\times30+27\times8$$
$$=(20+7)\times30+(20+7)\times8$$
$$=20\times30+7\times30+20\times8+7\times8$$
$$=600+210+160+56$$
$$=1026$$

It's the same as adding these four areas together!

	30	8
20	$20\times30=600$	$20\times8=160$
7	$7\times30=210$	$7\times8=56$

$$\begin{array}{r} 600 \\ 210 \\ 160 \\ +\ 56 \\ \hline 1026 \end{array}$$

Uh-oh. I think you broke Grogg.

30
8
20
7
Grogg, stop being so dramatic! It's not ***that*** complicated.
Here, look at Alex's diagram.
First, I split each number into tens and ones.
We can write the math like this.
27×38
= (20+7) × (30+8)
To find the area of each piece, I multiplied these two numbers...
...by these two numbers.
It's the same as distributing the 20 to (30+8)...
...and the 7 to (30+8).

We get these four products.

	30	8
20	20×30	20×8
7	7×30	7×8

$$27\times38$$
$$=(20+7)\times(30+8)$$
$$=20\times30+20\times8+7\times30+7\times8$$

And the same four products here!

And to finish, we compute the products and add them!

	30	8
20	20×30 =600	20×8 =160
7	7×30 =210	7×8 =56

$$27\times38$$
$$=(20+7)\times(30+8)$$
$$=20\times30+20\times8+7\times30+7\times8$$
$$=600+160+210+56$$
$$=1{,}026$$

$$\begin{array}{r} 600 \\ 210 \\ 160 \\ +\ 56 \\ \hline 1026 \end{array}$$

Marvelous!

We are using the distributive property to multiply two-digit numbers.

Who would like to try 53×46 without drawing a rectangle?

Compute 53×46.

53×46

= (50+3) × (40+6)

First, we split the numbers.

Then, we distribute the 50.

50×40 =2,000.

53×46

= (50+3) × (40+6)

2000
300

And 50×6 =300.

If we line up all of our products, it makes them easier to add.

Next, we distribute the 3:

3×40 =120...

53×46

= (50+3) × (40+6)

2000
300
120
18

...and 3×6 =18.

Then we add all four products to get the total!

53×46 =2,438!

53×46

= (50+3) × (40+6)

2000
300
120
+ 18
2438

Grogg, you're back.

It's hard to keep a good monster down.

MATH TEAM
The Algorithm

Today, I'm going to teach you how to multiply using an ***algorithm.***

I've got rhythm!

Not rhythm, ***algorithm.***

"An algorithm is a set of steps used for solving a problem."

6 × 458

Today's algorithm is for multiplying any two numbers.

Let's start with an example.

Multiply 6×458.

We split 458 into three parts, multiply, then add the parts back together.

6 × 458
= 6 × (400+50+8)
= 6×400 + 6×50 + 6×8
= 2400 + 300 + 48

2400
300
+ 48
2748

6×458 =2,748.

Perfect!

Multiplying requires a ***lot*** of writing.

The multiplication algorithm we'll learn today uses all of those steps, but it takes a lot less writing.

To begin, we write 6×458 like this:

The number with more digits is usually placed on top.

It's important to remember that 458 stands for 400+50+8.

$$\begin{array}{r} 458 \\ \times \quad 6 \\ \hline \end{array}$$

Using the distributive property, we compute three products: 6×400, 6×50, and 6×8.

With our algorithm, we compute the same three products.

We start with 6×8.

$$6 \times 458 = 6 \times (400 + 50 + 8)$$

$$\begin{array}{r} 458 \\ \times \quad 6 \\ \hline \end{array}$$

6×8 =48.

Good. We write 48 down here.

$$\begin{array}{r} 458 \\ \times \quad 6 \\ \hline 48 \end{array}$$

The 5 in 458 stands for 5 tens, or 50.

We multiply 6×50 next.

$$\begin{array}{r} 458 \\ \times \quad 6 \\ \hline 48 \end{array}$$

6×50 =300.

Right. We write 300 here. Line up the digits with the 48 above it.

$$\begin{array}{r} 458 \\ \times \quad 6 \\ \hline 48 \\ 300 \end{array}$$

DIGITS FOR EACH PLACE VALUE SHOULD LINE UP.

Finally, we multiply 6×400.

458
× 6
48
300

REMEMBER: THE 4 IN 458 STANDS FOR 4 HUNDREDS, OR 400.

6×400 =2,400.

We write the 2,400 under the 300. Then, we add 48+300+2,400.

458
× 6
48
300
+ 2400

48+300+2,400 is 2,748.

458
× 6
48
300
+ 2400
2748

It's the same answer we got before, but we didn't have to write as much.

$$
\begin{aligned}
&6 \times 458 \\
&= 6 \times (400+50+8) \\
&= 6\times400 + 6\times50 + 6\times8 \\
&= 2400 + 300 + 48
\end{aligned}
$$

2400
300
+ 48
2748

458
× 6
48
300
+ 2400
2748

Let's try another. How do we use the distributive property to multiply 97×62?

$$97\times62$$
$$=(90+7)\times(60+2)$$
$$=90\times60+90\times2+7\times60+7\times2$$
$$=5400+180+420+14$$

$$\begin{array}{r} 5400 \\ 180 \\ 420 \\ +\quad 14 \\ \hline 6014 \end{array}$$

We split 97 into 90+7 and we split 62 into 60+2.

Then, we multiply...

...and add the products.

97×62 =6,014

It takes even more writing than 458×6!

How does the algorithm work for 97×62?

I'll show you!

We begin by stacking the numbers again.

$$\begin{array}{r} 97 \\ \times\ 62 \\ \hline \end{array}$$

We start with the 2: 2×7=14.

$$\begin{array}{r} 97 \\ \times\ 62 \\ \hline 14 \end{array}$$

Then, 2×90 =180.

$$\begin{array}{r} 97 \\ \times\ 62 \\ \hline 14 \\ 180 \end{array}$$

Can you guess what comes next?

Then, we multiply 60×7...

97
× 62
14
180
420

...and finally, 60×90.

97
× 62
14
180
420
5400

Add it all up, and you've got your answer...

...97×62 =6,014.

97
× 62
14
180
420
+5400
6014

97×62
=(90+7) × (60+2)
=90×60 + 90×2 + 7×60 + 7×2
= 5400 + 180 + 420 + 14

5400
180
420
+ 14
6014

We're still using the distributive property.

But it's less work if we use the algorithm.

Did somebody say ***rhythm?***

I've got rhythm!

I've got music.

I've got my math.

Snap

Snap

Who could ask for anything more?

THE LAB

THE UNITS DIGIT

After finding the product of two numbers, there are many ways to check your answer.

Estimation is a good way to make sure your answer makes sense.

For example, which of these five choices gives the product of 48×72?

48×72=

A. 1,342
B. 1,234
C. 3,456
D. 3,564
E. 8,765

48×72 is close to 50×70=3,500.

That means C and D are the only reasonable answer choices.

48×72=

~~A. 1,342~~
~~B. 1,234~~
C. 3,456
D. 3,564
~~E. 8,765~~

Very good. Is 48×72 equal to 3,456 or 3,564?

I know! 48×72 has to end in a 6. The answer must be 3,456!

Excellent observation! How do you know that 48×72 must have a 6 in the ones place?

Can you see why 48×72 must end in a 6?

When we multiply 48×72, we split it into four products.

I see.

Three of the products are multiples of 10.

So these three products all have a zero in the ones place.

$$\begin{array}{r} 48 \\ \times\ 72 \\ \hline 16 \\ 80 \leftarrow 2\times40 \\ 560 \leftarrow 70\times8 \\ +\ 2800 \leftarrow 70\times40 \end{array}$$

But for ***this*** product, we are multiplying the ones digits of both numbers. 2×8=16.

Then, when we add the ones digits, we get 6+0+0+0=6.

Right. To find the ones digit of 48×72, we only need to multiply 8×2.

$$\begin{array}{r} 48 \\ \times\ 72 \\ \hline 16 \leftarrow 2\times8 \\ 80 \\ 560 \\ +\ 2800 \\ \hline 3456 \end{array}$$

The ones digit of 48×72 is the same as the ones digit of 8×2!

Very good! To find the ***units digit*** of a product, you can multiply the units digits of the numbers you are multiplying.

Try this one.

What is the units digit of 67×89?

$$\begin{array}{r} 67 \\ \times\ 89 \\ \hline \end{array}$$

Try it!

THE NUMBER IN THE ONES PLACE OF A NUMBER IS ITS ONES DIGIT, AND IS ALSO CALLED ITS ***UNITS DIGIT***.

Since 7×9 =63...

...7×9 ends in a 3.

So 67×89 must end in a 3.

Since we are only looking for the units digit of 67×89, we don't need to finish the multiplication to know that 67×89 ends in a 3.

$$\begin{array}{r} 67 \\ \times\ 89 \\ \hline 63 \\ ?0 \\ ?0 \\ +\quad ?0 \\ \hline ???3 \end{array}$$

Extraordinary! 67×89 ends in a 3.

I see. No matter how big the numbers are, it's easy to compute the units digit of their product.

For example, if you want to know the units digit of 6,543×4,567, you just multiply 3×7.

Since 3×7=21 ends in a 1, we know that 6,543×4,567 ends in a 1.

Excellent!

Uh-oh, look at the time! I need to step out for just a moment, little monsters.

I'll be right back.

Donk

Brompf

Skritch

Bwah Hah Hah! Professor Grok is gone! I've abducted your educator! It's time for something much more diabolically difficult!

Units digit discovery for the product of a pair of numbers presumes ***paltry proficiency.***

But uncovering the units digit for a product of ninety-nine numbers requires ***remarkable resolve.***

The digit that will determine the destiny of your professor is in the ones place of the product of ***ninety-nine 99's!***

When the hour hand on the academy clock approaches the proper digit, it will point you to your prolific professor's place of imprisonment.

Find Professor Grok before the hour hand is aimed accordingly, or his ***time*** as your ***teacher*** will be ***terminated!***

That guy is weird.

Yep.

Grogg, focus! We've got to find Professor Grok!

How can we find the units digit of the product of ninety-nine 99's?

Try it.

It would take us forever to multiply just ***five*** 99's...

...how are we ever going to multiply ***ninety-nine*** 99's?

For 67×89, we only needed to multiply 7×9 to find the units digit.

We don't actually need to multiply ninety-nine 99's to find the units digit...

...we only need to multiply the units digits.

$$99 \times 99 \times 99 \times 99 \times 99 \times \cdots$$

Maybe if we find the units digit for a product of two 99's, then three 99's, then four 99's...

...we can keep going until we find a pattern.

Since 9×9=81, the units digit of 99×99 is 1.

When we multiply ***two*** 99's, the units digit is 1.

What if we multiply ***three*** 99's?

$$\underbrace{99 \times 99}_{\text{ends in 1}} \times 99 \times 99 \times 99 \times \cdots$$

Can you find a pattern?

LOOKING FOR A PATTERN IS A GREAT WAY TO START A DIFFICULT-LOOKING PROBLEM.

We only need to multiply the units digits: 9×9×9.

9×9=81, and we can multiply 81×9 to get...

$$\begin{array}{r} 81 \\ \times\ \ 9 \\ \hline 9 \\ +\ 720 \\ \hline 729 \end{array}$$

...729!

When we multiply three 99's, the units digit is 9!

Great, but you didn't need to multiply 81×9 to find the units digit!

Oh, right! The units digit of 81×9 is just 1×9=9!

So, 99×99 ends in 1, and 99×99×99 ends in 9.

	units digit
99×99	1
99×99×99	9
99×99×99×99	

What if we multiply *four* 99's?

If we multiply a number that ends in 9 by another number that ends in 9, we always get a number that ends in 1.

That's right, because 9×9=81 ends in 1.

Since 99×99×99 ends in 9...

...if we multiply by another number that ends in 9, we'll get a number that ends in 1!

99×99×99×99

ends in 9 ends in 9

When we multiply four 99's, the units digit is 1.

	units digit
99×99	1
99×99×99	9
99×99×99×99	1
99×99×99×99×99	

Next is five 99's.

Four 99's give us a product that ends in 1. If we multiply by another 99, we get a number that ends in 9!

Because 1×9=9!

What is the units digit of the product of ninety-nine 99's?

There's a pattern!

Every time we multiply a number that ends in 1 by a number that ends in 9...

...the product ends in 9.

	units digit
99×99	1
99×99×99	9
99×99×99×99	1
99×99×99×99×99	9

And when we multiply a number that ends in 9 by another number that ends in 9...

...the product ends in 1.

So, if we multiply by more 99's, the units digit will just go back and forth between 1 and 9!

The product of ***five*** 99's ends in a 9...

...so the product of ***six*** 99's ends in a 1...

...and the product of ***seven*** 99's ends in a 9.

If the number of 99's we multiply is ***even***, the product ends in 1...

...but if we multiply an ***odd*** number of 99's, the product ends in 9!

So, the product of ninety-nine 99's ends in ***9!***

To the academy clock!

When the hour hand reaches the 9, it will be pointing directly at the bell tower!

That's where Professor Grok must be!

Follow me. I know the way!

Great work, little monsters!

R&G
1's and 0's

Watch this!

You're not going to try to yank the tablecloth again, are you?

Huh?

Remember what happened last time?

No, it's nothing like that! I'm going to solve a multiplication problem!

I can multiply a 3-digit number by a 2-digit number!

Alright, let's see it.

Watch me multiply 456×91!

456
× 91

It's all about staying organized.

You start by multiplying the 1 in 91 by each digit in 456.

The 5 stands for 50, and the 4 stands for 400.

456
× 91
6
50
400

1×6=6.
1×50=50.
1×400=400.

That gives you these three ***partial products.***

456
× 91
6
50
400

THE PRODUCTS YOU GET IN EACH STEP ARE CALLED *PARTIAL PRODUCTS.*

Next, the 9 in 91 stands for nine tens, or 90.

So, you multiply 90×6...

...then 90×50...

...and finally 90×400.

456
× 91
6
50
400
540
4500
36000

Then, you add everything to get...

456
× 91
6
50
400
540
4500
+ 36000
41496

...41,496!

That's good, but I think I see a shortcut.

Huh?

When you multiply the 1 by 456, you don't need to multiply it one part at a time!

Oh, right!

Wait, what?

To multiply 1×456, you don't need to find three partial products.

456
× 91
456

You already know that 1×456=456!

I see!

Then, I can multiply the 90.

That *is* shorter!

456
× 91
456
540
4500
+ 36000
41496

Let's try another.

What do you get when you multiply 876×14?

Hmmm... First, I multiply the 4.

876
× 14
24
280
3200

That gives me these three partial products.

876
× 14
24
280
3200

Then, I multiply the 1, which stands for 10.

I multiply 10×6, 10×70, and 10×800.

Right, but you--

Wait, I know!

876
× 14
24
280
3200

I can multiply 10×876 all at once!

10×876 =8,760.

876
× 14
24
280
3200
8760

So, 876×14 =12,264.

That's right!

When you multiply by 1, or by 10, or by 100, you don't need to multiply one part at a time.

876
× 14
24
280
3200
+ 8760
12264

Try this. What is 203×104?

Try it.

203
× 104
12

I start with 4×3.

Next comes 4×0.

The 0 stands for zero tens... which is still zero.

That's easy. 4×0=0.

203
× 104
12

I don't even need to write anything down...

...since adding zero doesn't change a sum.

203
× 104
12
800

Next comes 4×200. That's 800.

203
× 104
12
800

Now we move on to this 0.

That's easy again! Zero times anything is zero!

I can just move on to the 1.

It's in the hundreds place, so it stands for 100.

I can multiply 100 by 203 all at once! I just put two zeroes after 203!

203
× 104
12
800
20300

100×203 =20,300!

Once I add these three products, I get...

203
× 104
12
800
+ 20300
21112

203×104 =21,112!

That wasn't too bad!

Multiplying large numbers is easier when the numbers have 0's and 1's as digits.

There aren't as many partial products to compute.

444
× 101
444
+ 44400
44844

808
× 120
160
16000
+ 80800
96960

Look at this one.

10101
× 1010
101010
+ 10101000
10202010

Cool. I'll try 10,001×11,111.

Try it.

10001
× 11111
10001
100010
1000100
10001000
+ 100010000
111121111

I did it with the numbers flipped.

11111
× 10001
11111
+ 111110000
111121111

Whoa!

Your way was less work.

Flipping the numbers can make the multiplication easier.

FLIPPING THE NUMBERS YOU ARE MULTIPLYING IS ALSO A GOOD WAY TO CHECK YOUR ANSWER!

Maybe flipping my hands will make my tablecloth trick easier!

Nope.