Contents: Chapter 4

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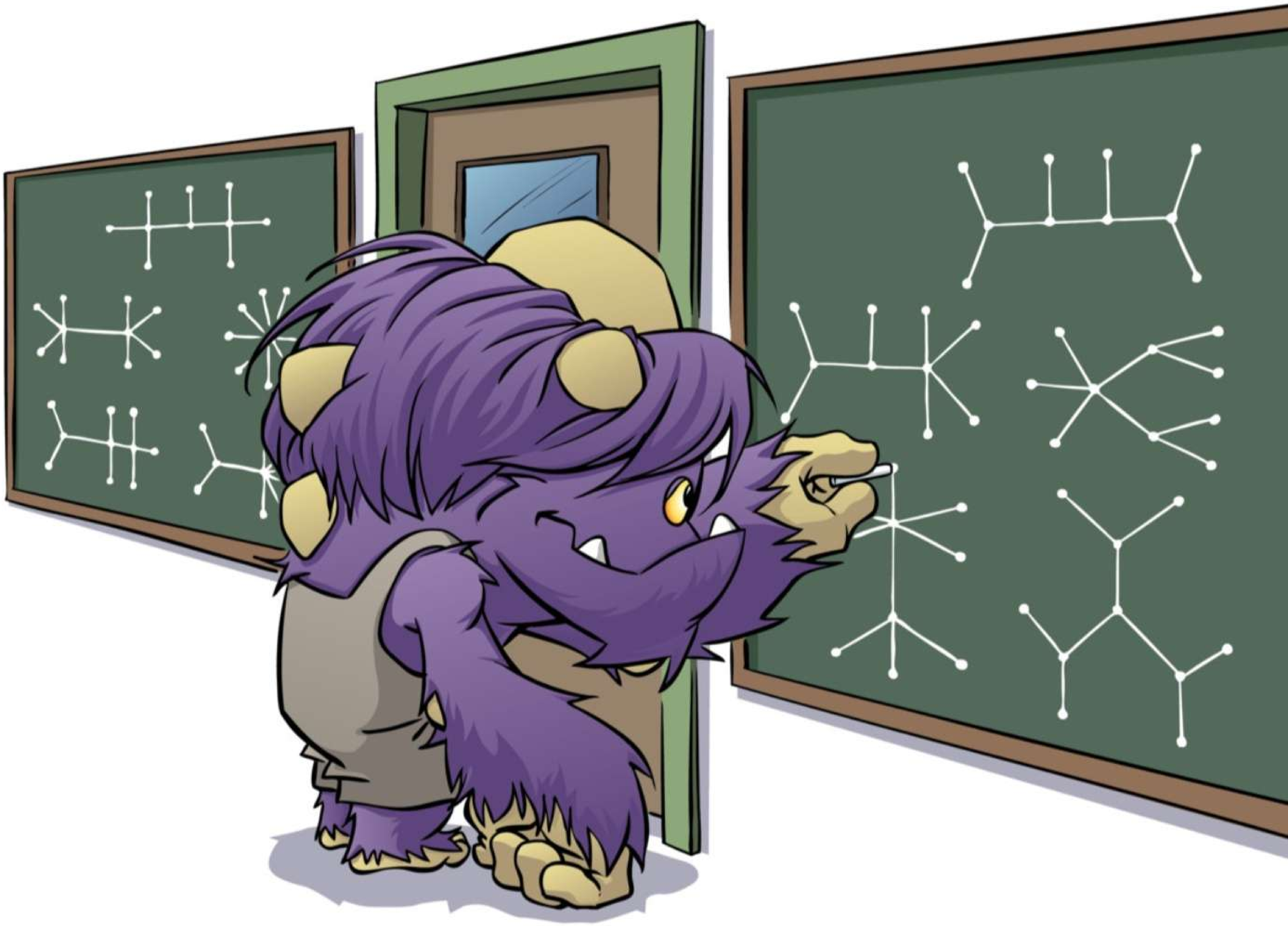
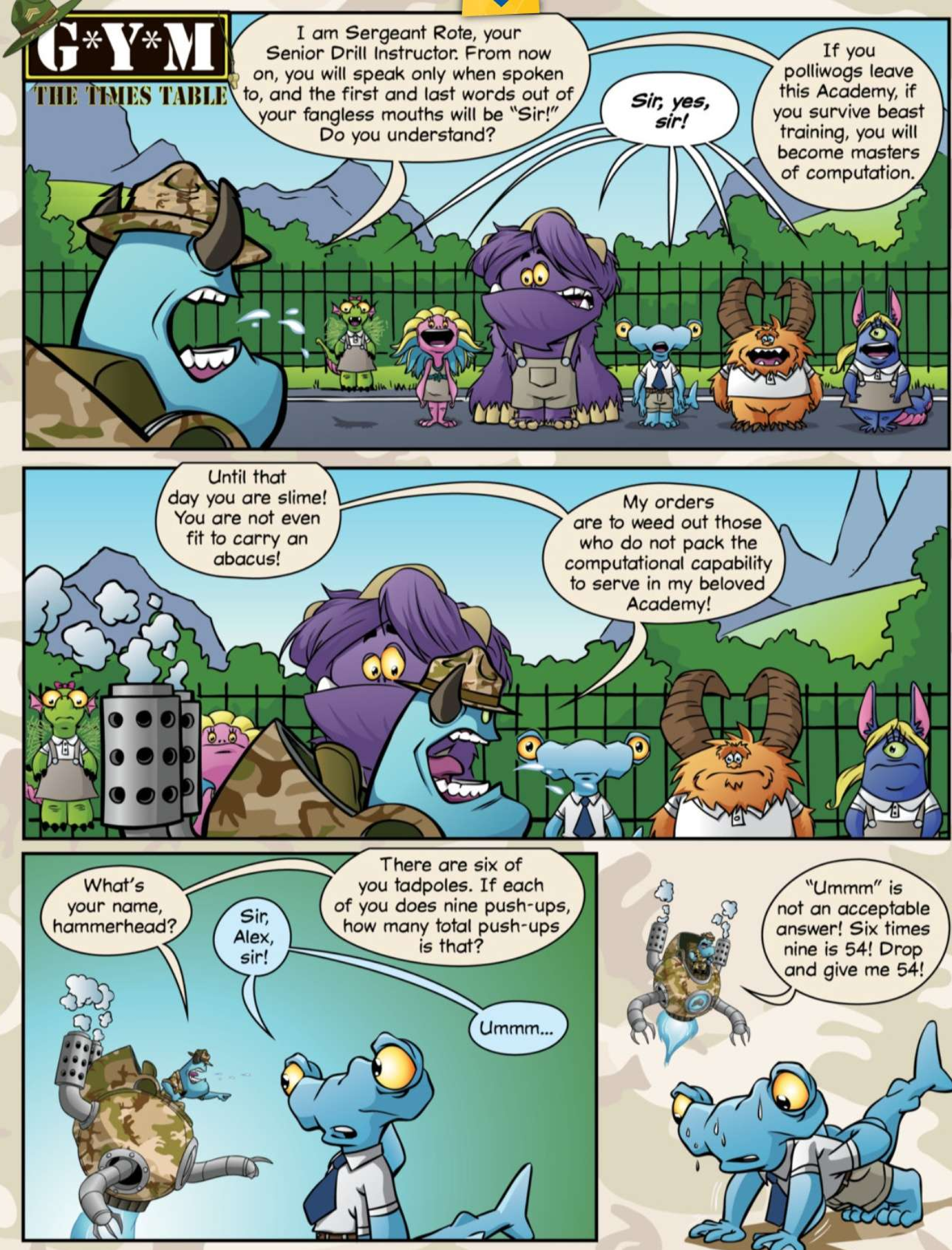
Chapter 4:
Multiplication

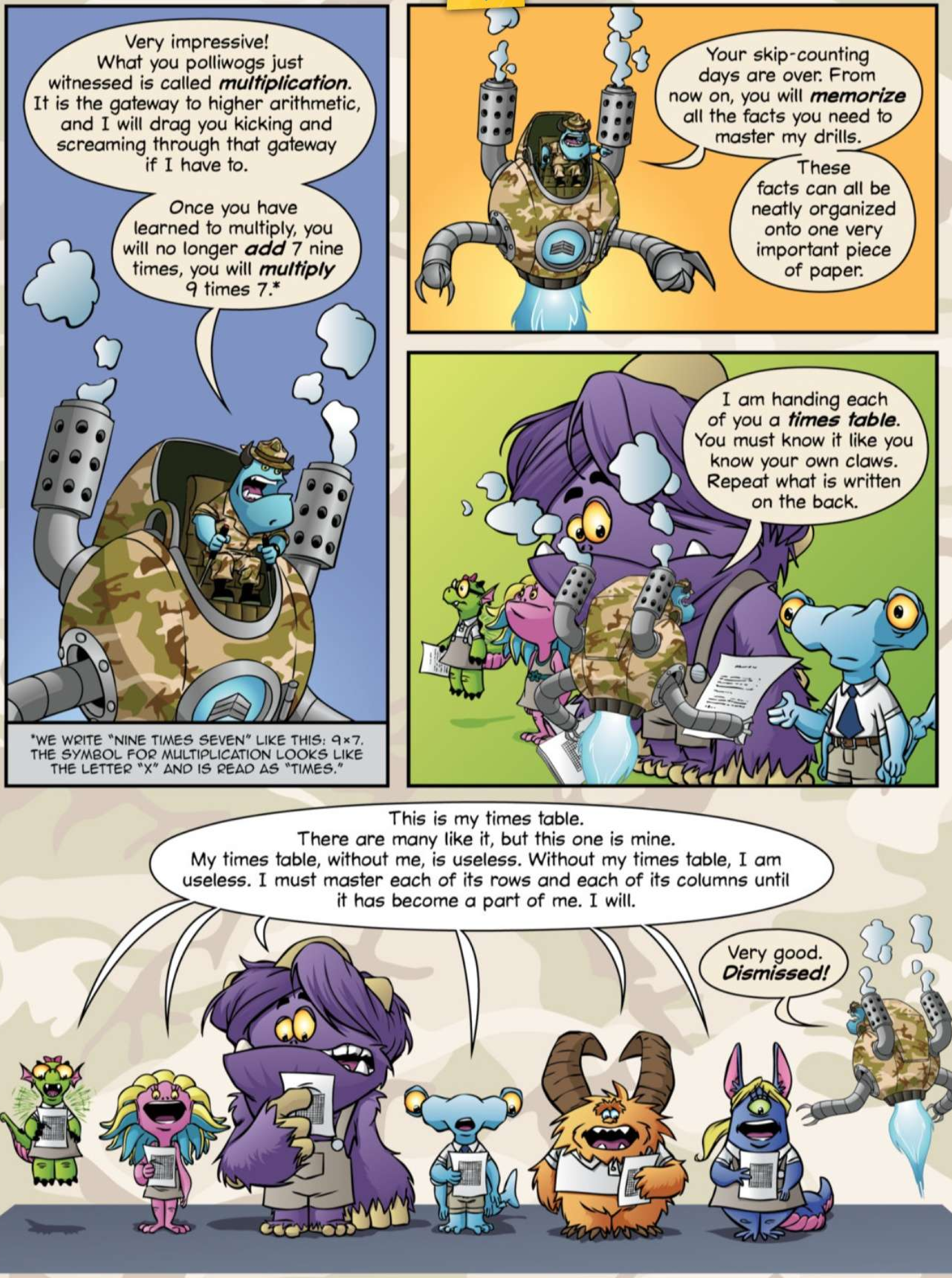



R&G
St. Kitts
I heard a clever riddle today.
Does it involve an albatross sandwich?
Nope.
Then I'm interested.
"As I was going to St. Kitts, I met a beast with seven kids. The seven kids had seven sacks. The seven sacks had seven cats. The seven cats had seven kittens.
Kittens, cats, sacks, and kids, How many were going to St. Kitts?"
Give it a try.
Sounds like a lot of counting.
To start, you've got the beast and his seven kids.
Go on.
Each kid had seven sacks, so we have 7+7+7+7+7+7+7 sacks. That makes...
7
14
21
28
35
42
49
sacks.
Each of the 49 sacks has 7 cats. That's a ***lot*** of skip-counting! Here goes:
7
14
21
28...

...315
322
329
336
343 cats!
Are you sure you counted all 49 sacks?
Nope.
I sort of lost track the second time I got to your foot.
Maybe it would help if you told me the problem again.
Sure. "As I was going to St. Kitts, I met a beast with seven--
Got it!
You were the only one going *to* St. Kitts! The monster, his kids, the cats and the kittens were all going the other direction. So the answer is ***one.***
Good!
Makes you wonder though.
Yeah, seven is a ***lot*** of kids.
Maybe he has seven heads.
But why would anyone have so many cats?
That ***is*** strange, but what I meant was, how many were coming ***from*** St. Kitts?
I could never skip-count that high.
Probably not, but you could use ***multiplication.***
What's that?
Shhh. Here comes Sergeant Rote!
Maybe he'll explain.

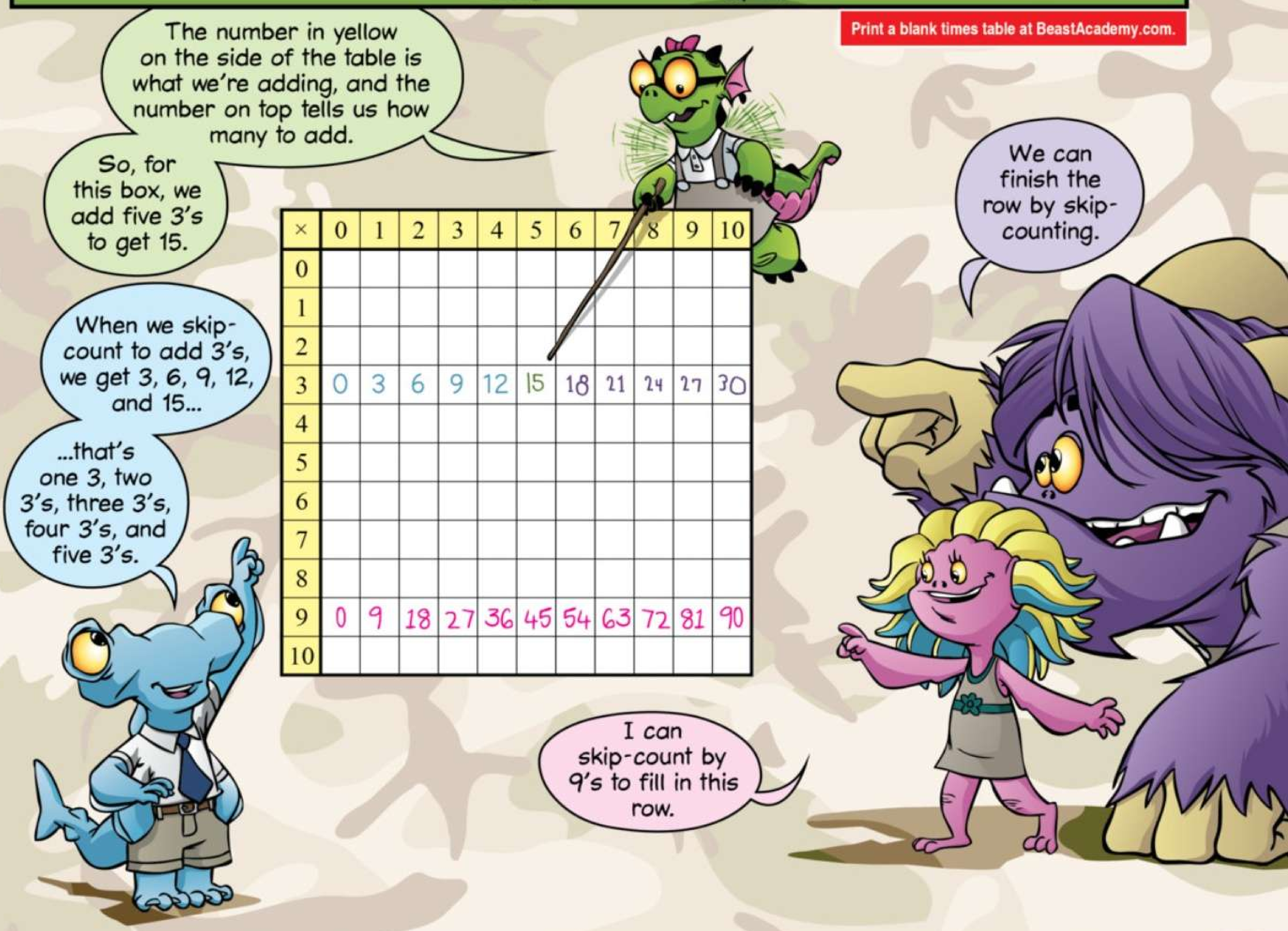

G*Y*M
THE TIMES TABLE
I am Sergeant Rote, your Senior Drill Instructor. From now on, you will speak only when spoken to, and the first and last words out of your fangless mouths will be "Sir!" Do you understand?
Sir, yes, sir!
If you polliwogs leave this Academy, if you survive beast training, you will become masters of computation.
Until that day you are slime! You are not even fit to carry an abacus!
My orders are to weed out those who do not pack the computational capability to serve in my beloved Academy!
What's your name, hammerhead?
Sir, Alex, sir!
There are six of you tadpoles. If each of you does nine push-ups, how many total push-ups is that?
Ummm...
"Ummm" is not an acceptable answer! Six times nine is 54! Drop and give me 54!


How about you, furball? What's your name?
Sir, Grogg, sir!
If I have 6 cages, and each cage holds 8 octapugs, how many octapugs is that?
8, 16, 24, 32, 40, 48...
Sir, **48**, sir!
Good, now drop and give me 48! There'll be no mumbling or finger-counting in this Academy!
The chomp monster has eight rows of nine teeth. How many teeth is that all together?
Sir, eight times nine is 72, sir!
Good work, tinkerbell! What's 7 times 6?
Sir, 42, sir!
8 times 4?
Sir, 32, sir!
7 times 7?
Sir, 49, sir!



Very impressive! What you polliwogs just witnessed is called ***multiplication***. It is the gateway to higher arithmetic, and I will drag you kicking and screaming through that gateway if I have to.
Once you have learned to multiply, you will no longer ***add*** 7 nine times, you will ***multiply*** 9 times 7.*
*WE WRITE "NINE TIMES SEVEN" LIKE THIS: 9×7. THE SYMBOL FOR MULTIPLICATION LOOKS LIKE THE LETTER "X" AND IS READ AS "TIMES."
Your skip-counting days are over. From now on, you will ***memorize*** all the facts you need to master my drills.
These facts can all be neatly organized onto one very important piece of paper.
I am handing each of you a ***times table***. You must know it like you know your own claws. Repeat what is written on the back.
This is my times table. There are many like it, but this one is mine. My times table, without me, is useless. Without my times table, I am useless. I must master each of its rows and each of its columns until it has become a part of me. I will.
Very good. ***Dismissed!***

Print a blank times table at BeastAcademy.com.



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9	0	9	18	27	36	45	54	63	72	81	90
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×	0	1	2	3	4	5	6	7	8	9	10
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1											
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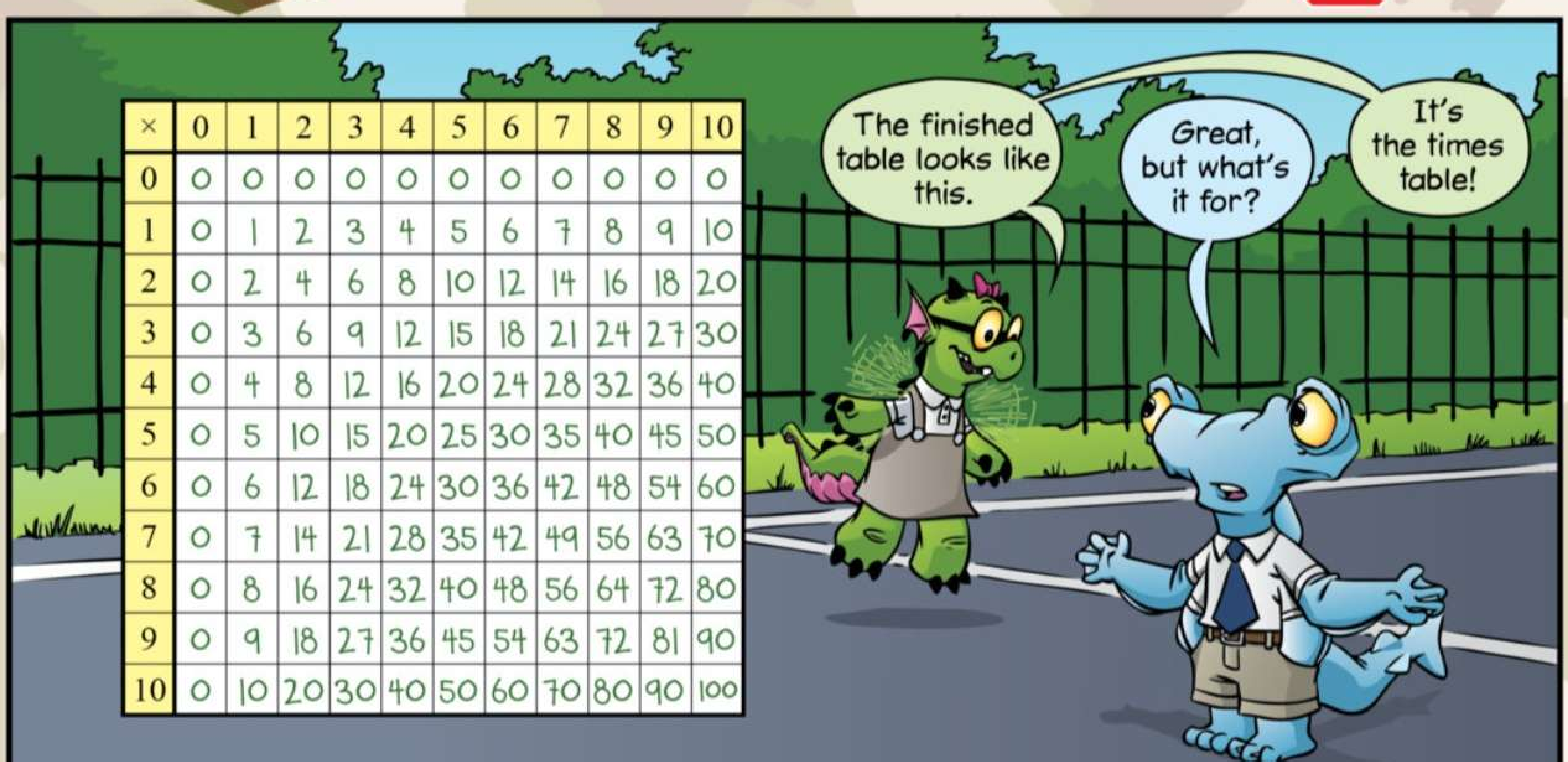

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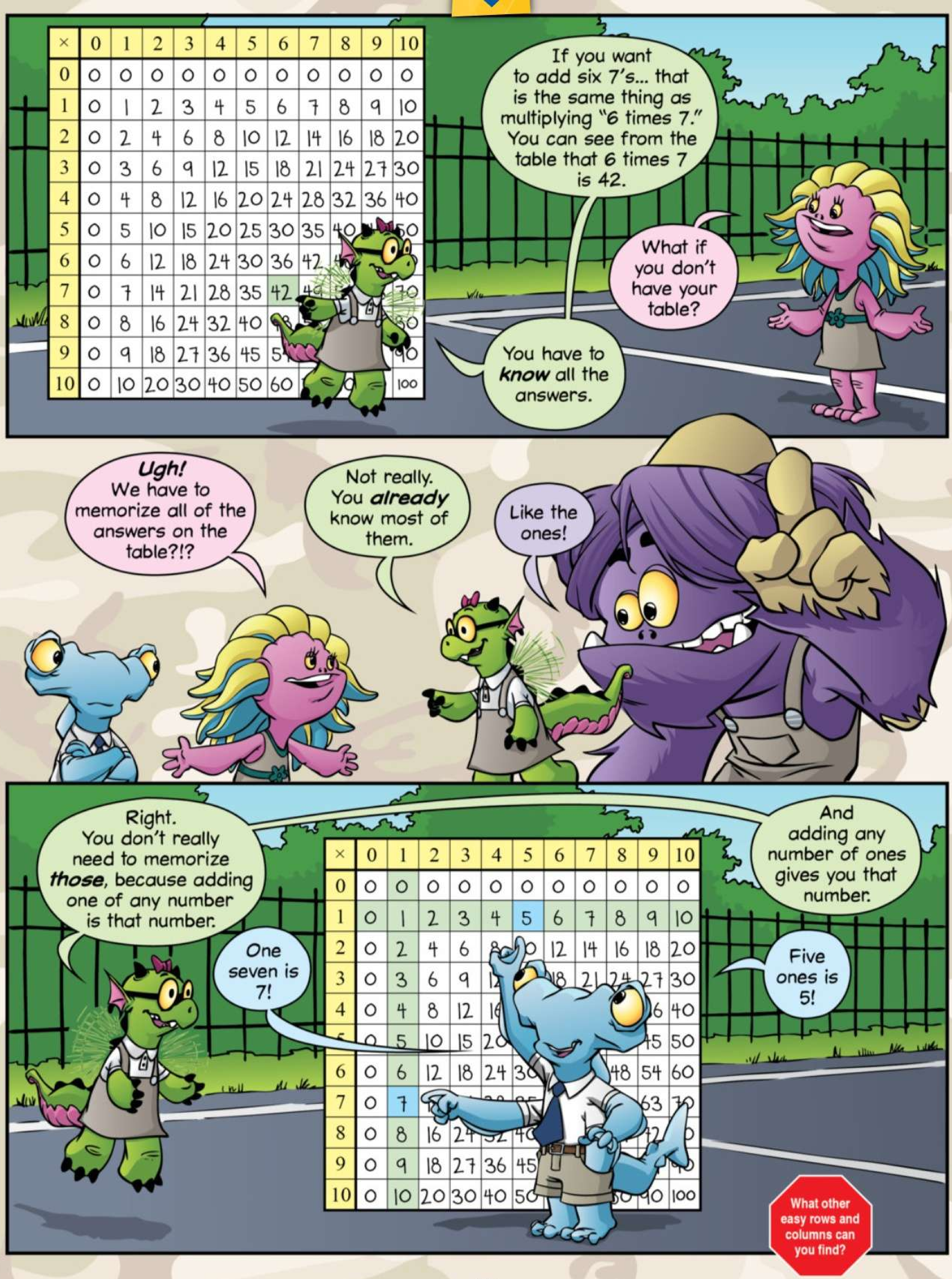
If you want to add six 7's... that is the same thing as multiplying "6 times 7." You can see from the table that 6 times 7 is 42.
What if you don't have your table?
You have to **know** all the answers.
Ugh! We have to memorize all of the answers on the table?!?
Not really. You ***already*** know most of them.
Like the ones!
Right. You don't really need to memorize ***those***, because adding one of any number is that number.
And adding any number of ones gives you that number.
One seven is 7!
Five ones is 5!
What other easy rows and columns can you find?

×	0	1	2	3	4	5	6	7	8	9	10
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	0	5	10	15	20	25	30	35	40	45	50
	0	6	12	18	24	30	36	42	48	54	60
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8	0	8	16	24	32	40	48	56	64	72	80
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×	0	1	2	3	4	5	6	7	8	9	10
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	0	4	8	12	16	20	24	28	32	36	40
	0	5	10	15	20	25	30	35	40	45	50
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8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

×	0	1	2	3	4	5	6	7	8	9	10
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4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
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8	0	8	16	24	32	40	48	56	64	72	80
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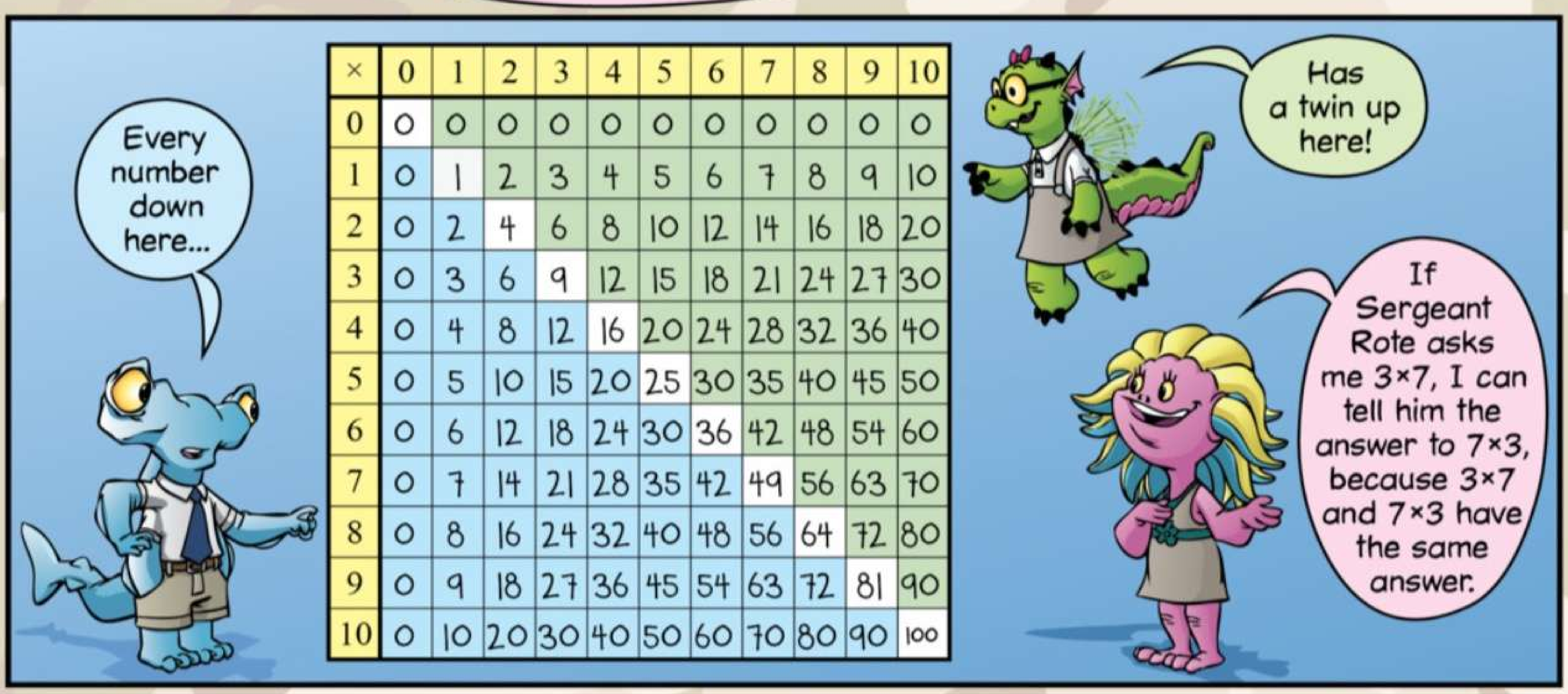

×	0	1	2	3	4	5	6	7	8	9	10
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3	0	3	6	9	12	15	18	21	24	27	30
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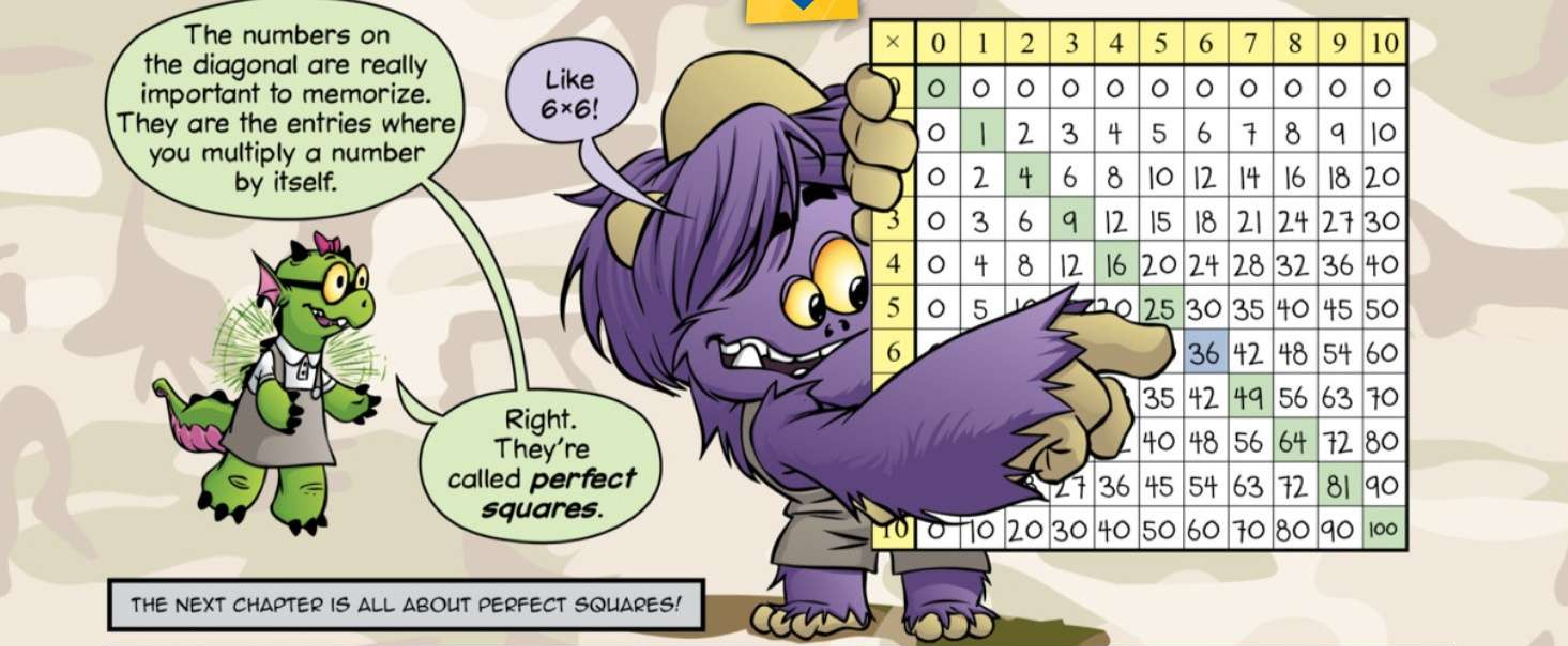
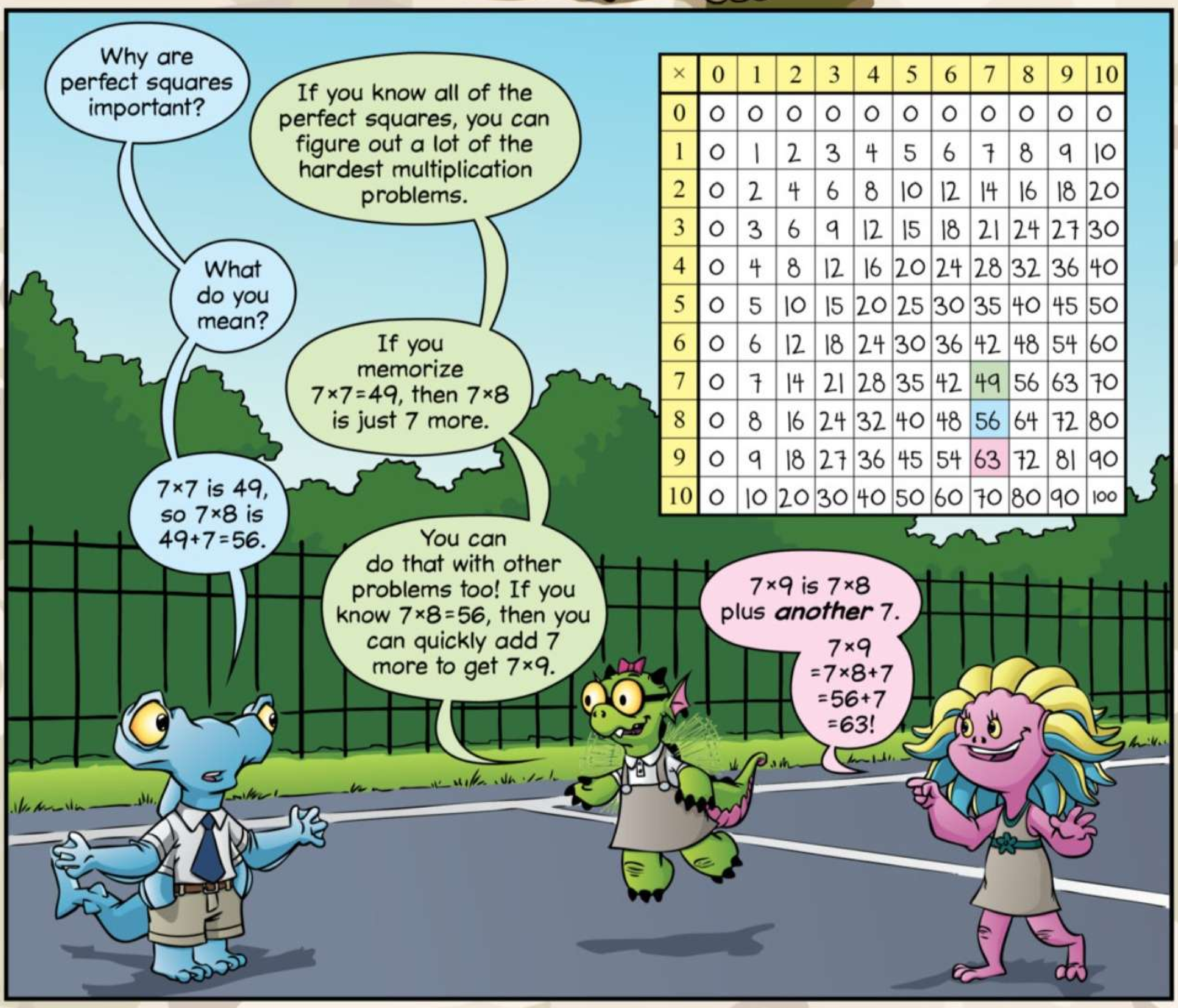


The numbers on the diagonal are really important to memorize. They are the entries where you multiply a number by itself.
Like 6×6!
Right. They're called ***perfect squares.***
THE NEXT CHAPTER IS ALL ABOUT PERFECT SQUARES!



Why are perfect squares important?
If you know all of the perfect squares, you can figure out a lot of the hardest multiplication problems.
What do you mean?
If you memorize 7×7=49, then 7×8 is just 7 more.
7×7 is 49, so 7×8 is 49+7=56.
You can do that with other problems too! If you know 7×8=56, then you can quickly add 7 more to get 7×9.
7×9 is 7×8 plus ***another*** 7.
7×9 =7×8+7 =56+7 =63!

×	0	1	2	3	4	5	6	7	8	9	10
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1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
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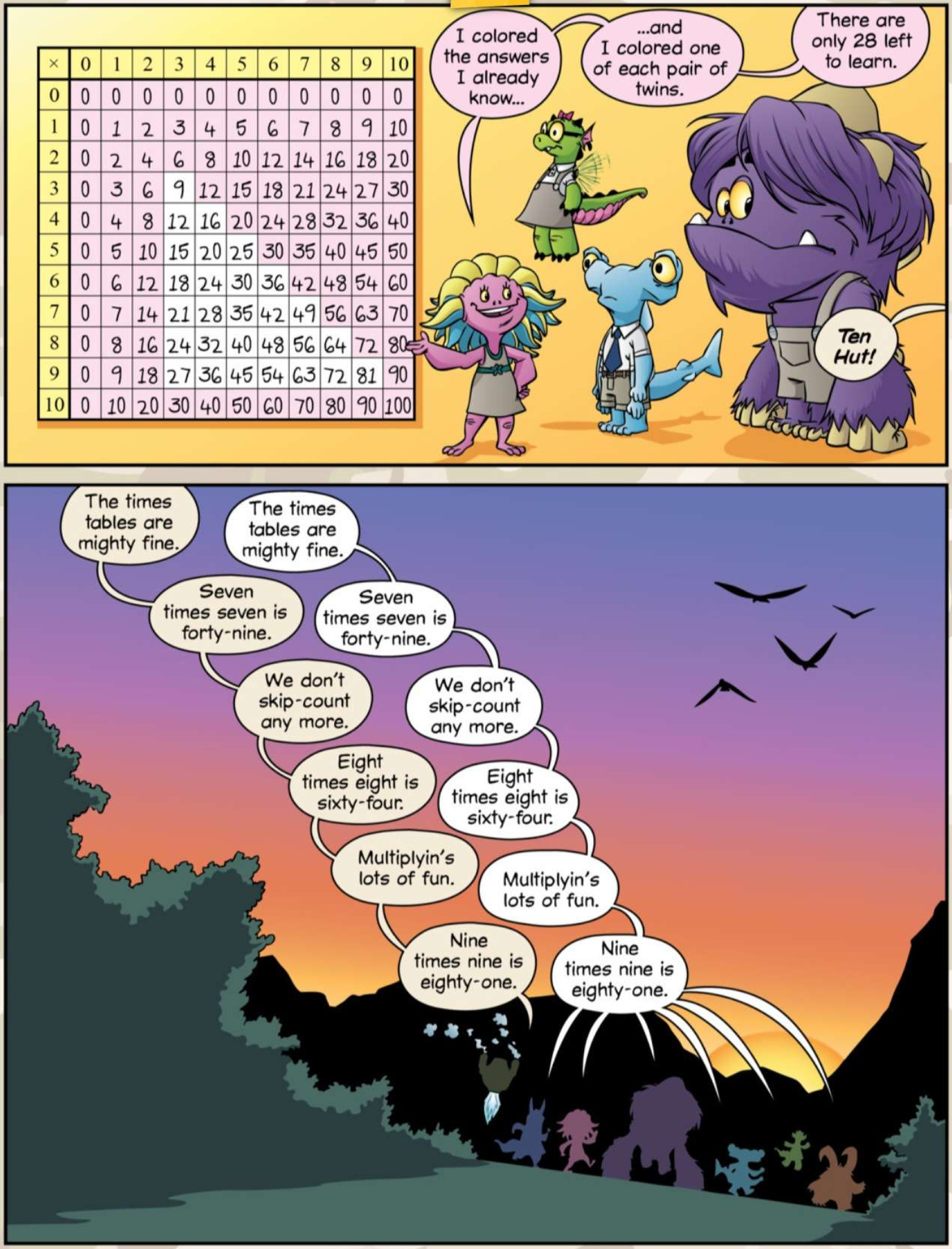
×	0	1	2	3	4	5	6	7	8	9	10
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3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
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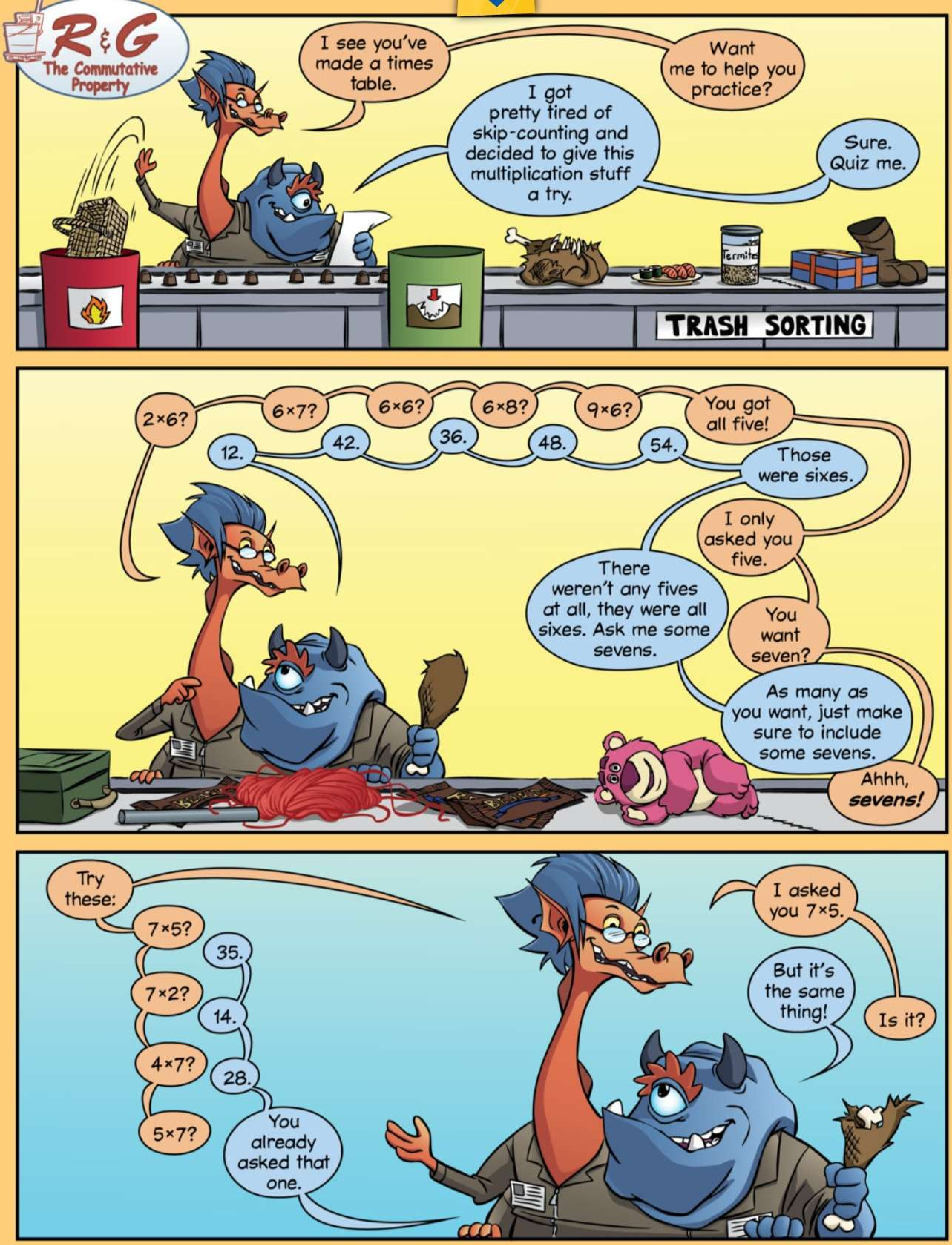
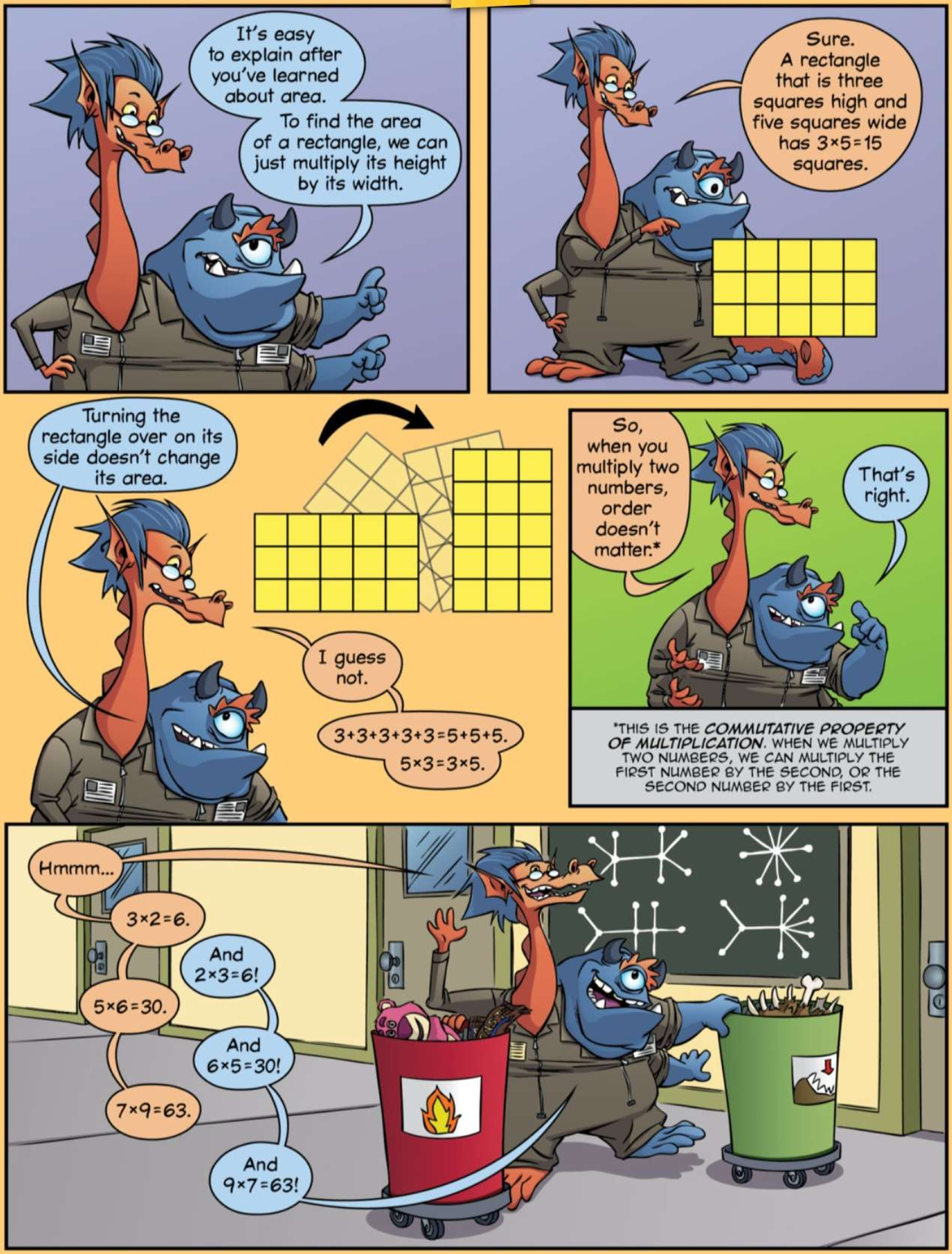
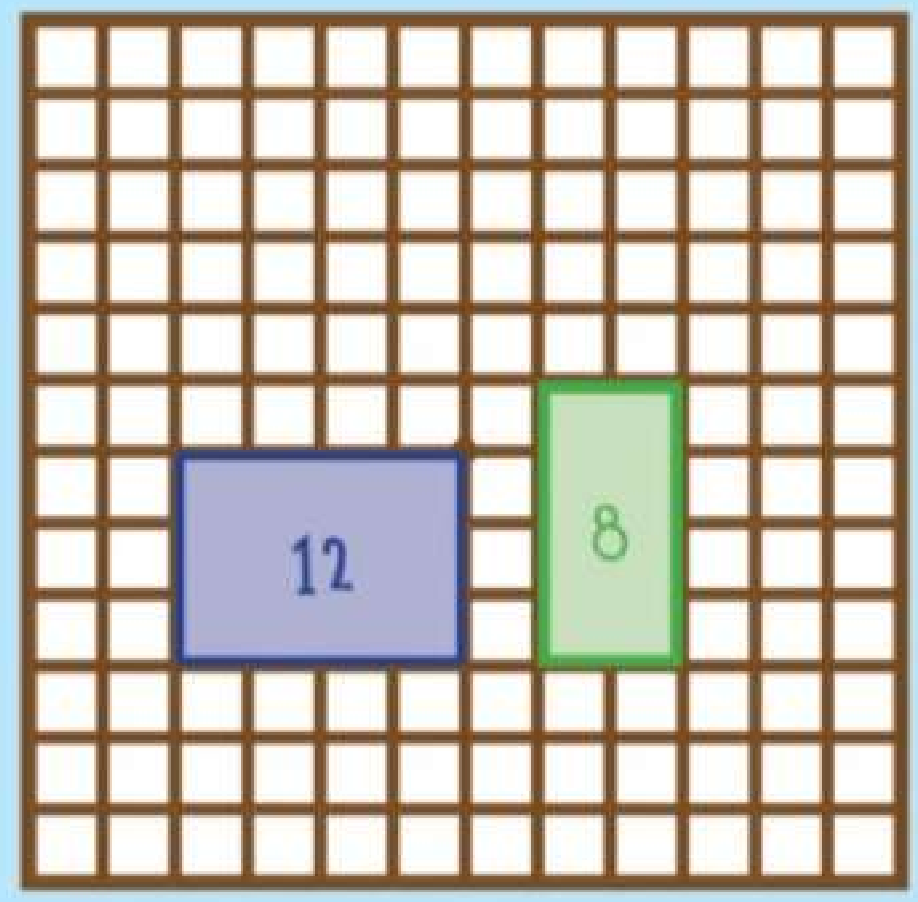

R&G
The Commutative Property
I see you've made a times table.
Want me to help you practice?
I got pretty tired of skip-counting and decided to give this multiplication stuff a try.
Sure. Quiz me.
Termite
TRASH SORTING
2×6?
12.
6×7?
42.
6×6?
36.
6×8?
48.
9×6?
54.
You got all five!
Those were sixes.
I only asked you five.
There weren't any fives at all, they were all sixes. Ask me some sevens.
You want seven?
As many as you want, just make sure to include some sevens.
Ahhh, *sevens!*
Try these:
7×5?
35.
7×2?
14.
4×7?
28.
5×7?
You already asked that one.
I asked you 7×5.
But it's the same thing!
Is it?


It's easy to explain after you've learned about area.
To find the area of a rectangle, we can just multiply its height by its width.
Sure. A rectangle that is three squares high and five squares wide has 3×5=15 squares.
Turning the rectangle over on its side doesn't change its area.
I guess not.
3+3+3+3+3=5+5+5.
5×3=3×5.
So, when you multiply two numbers, order doesn't matter.*
That's right.
*THIS IS THE COMMUTATIVE PROPERTY OF MULTIPLICATION. WHEN WE MULTIPLY TWO NUMBERS, WE CAN MULTIPLY THE FIRST NUMBER BY THE SECOND, OR THE SECOND NUMBER BY THE FIRST.
Hmmm...
3×2=6.
And 2×3=6!
5×6=30.
And 6×5=30!
7×9=63.
And 9×7=63!

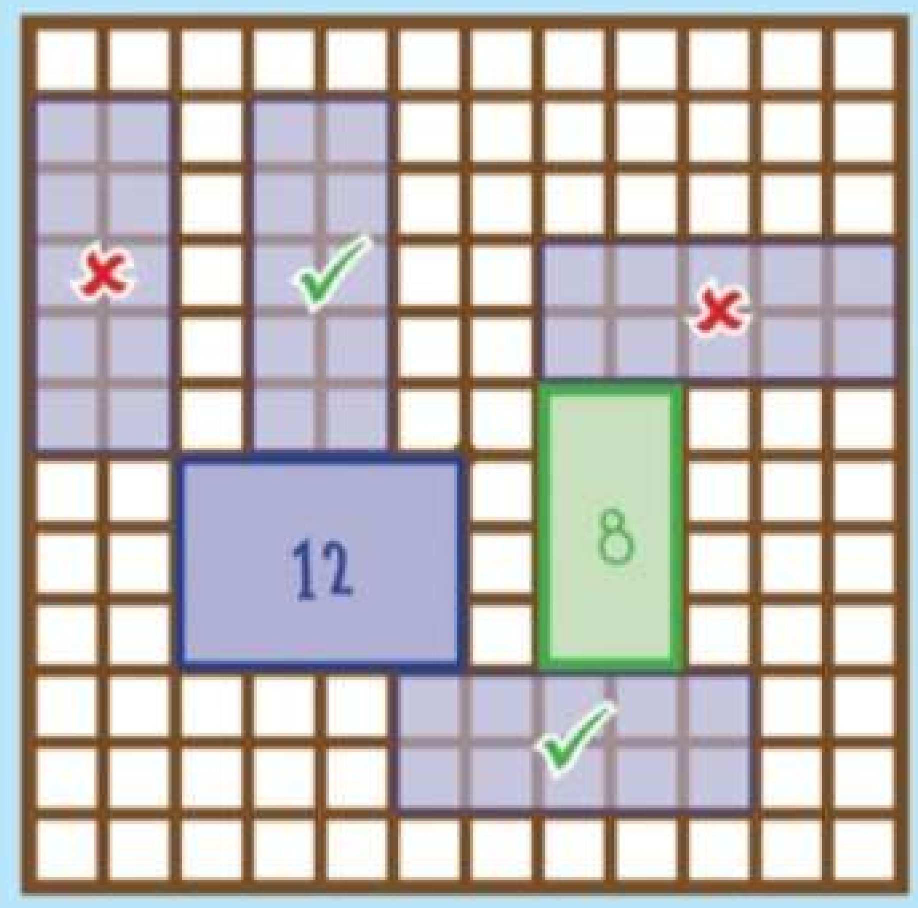
RECESS

BLOCK BLOB



The Beginning: Grogg and Lizzie are playing a game of Block Blob. Grogg rolls a pair of standard dice. The numbers Grogg rolls give him the length and width of a rectangle which he must trace on the 12×12 game board. Grogg's first rectangle must have one of its corners on the dot at the center of the board. Grogg writes the area of his rectangle inside of it. His rectangle is shown in purple.

Lizzie rolls next and uses a different color to trace her rectangle anywhere on the game board, but not overlapping Grogg's rectangle. Lizzie writes the area of her rectangle. Her rectangle is shown in green.



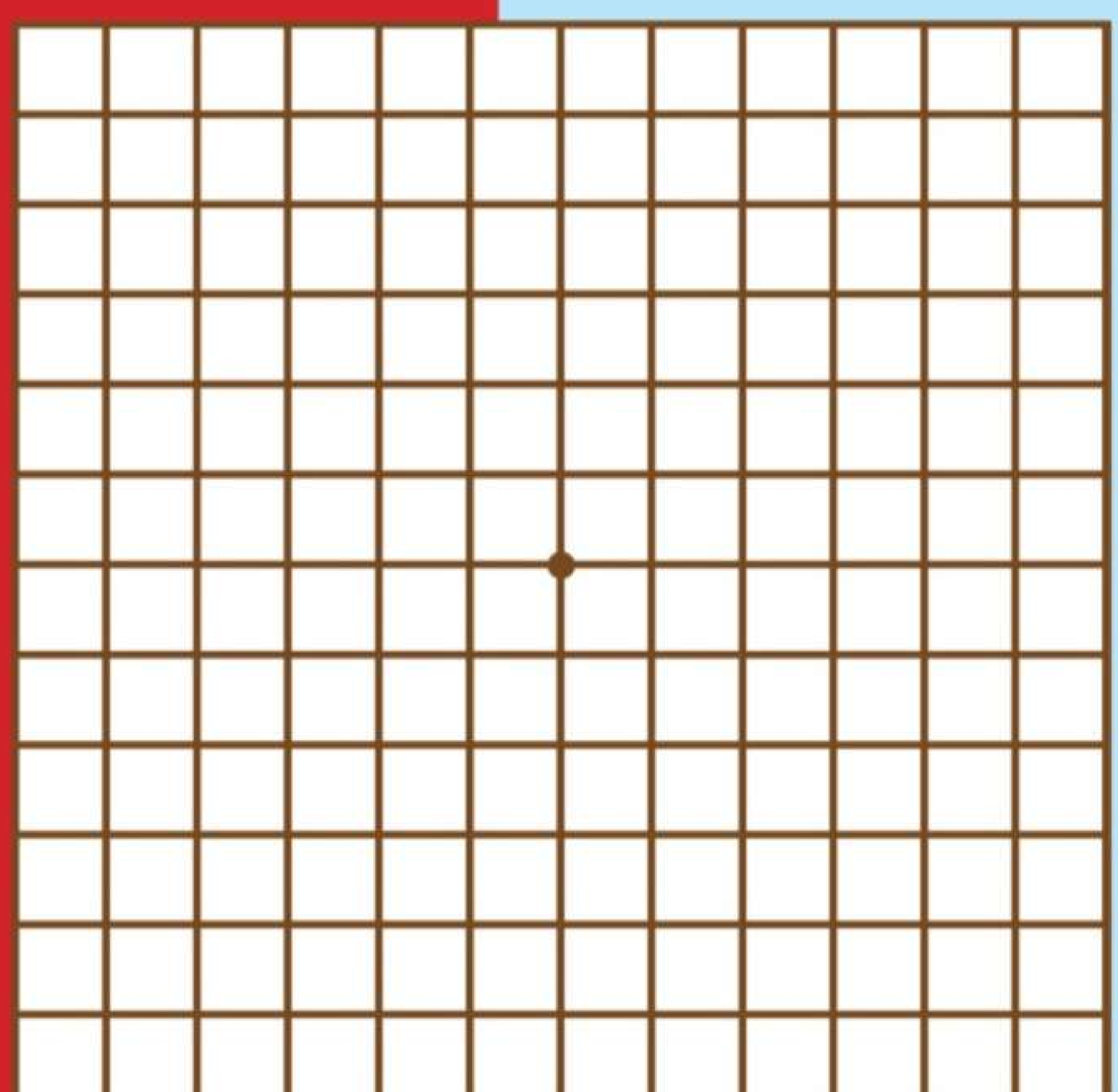
The Middle: Grogg and Lizzie continue taking turns rolling the dice and tracing new rectangles on the board. When a player traces a new rectangle, it must touch a side (not just a corner) of any rectangle that he or she has already drawn. The attached purple rectangles form Grogg's Block Blob. The green ones form Lizzie's Block Blob.

For example, if Grogg rolls a 2 and a 5 on his second roll, he can place his rectangle as marked with the green ✓'s, but not as marked with the red ✗'s.

The End: When a player rolls a rectangle that cannot be placed on the board, he or she must pass. The game ends when there are four passes in a row (two for each player).

When the game ends, each player adds the areas of all the rectangles in his or her Block Blob. The player whose Block Blob has the largest area wins.

Find a partner and play!



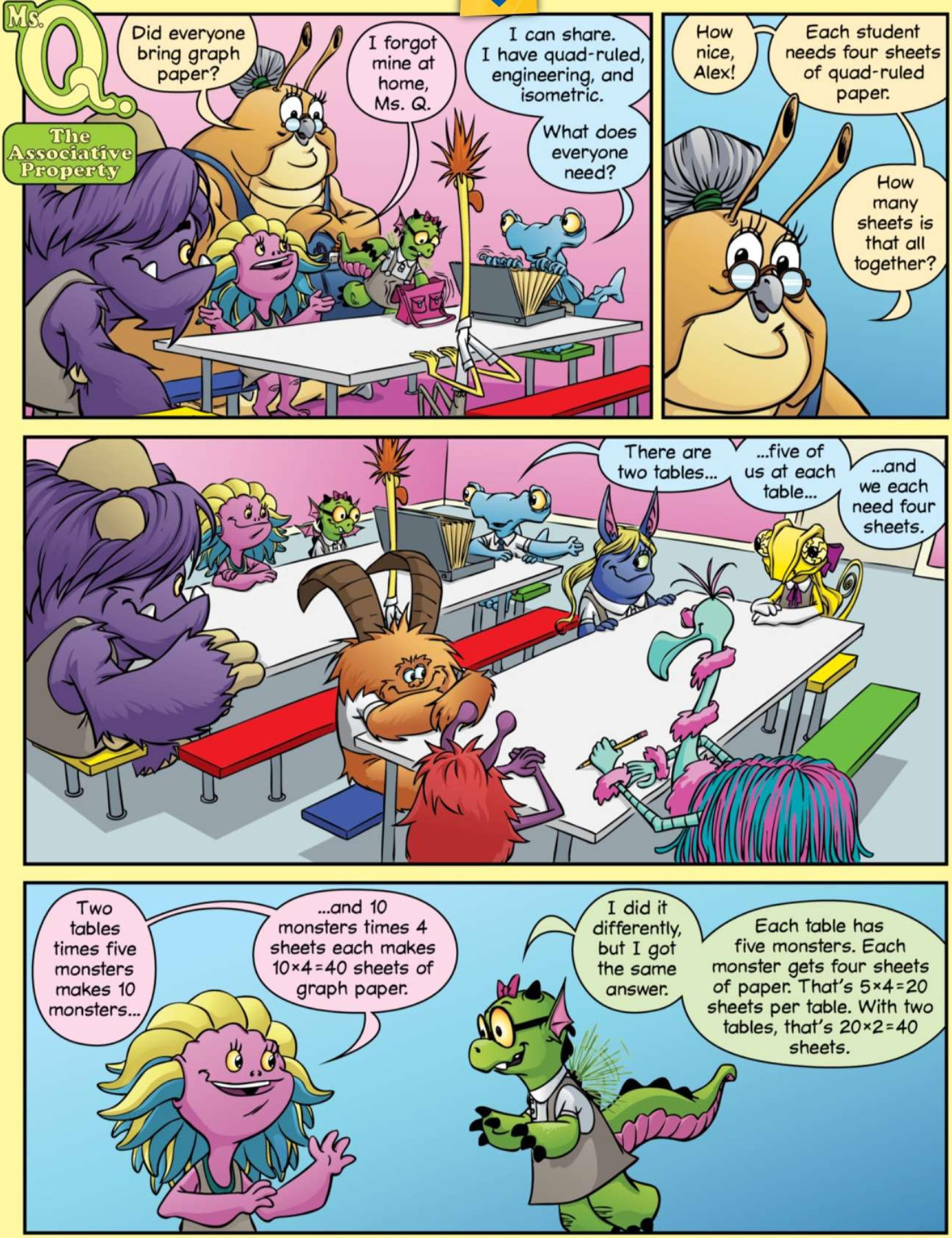
Print as many game boards as you want at BeastAcademy.com

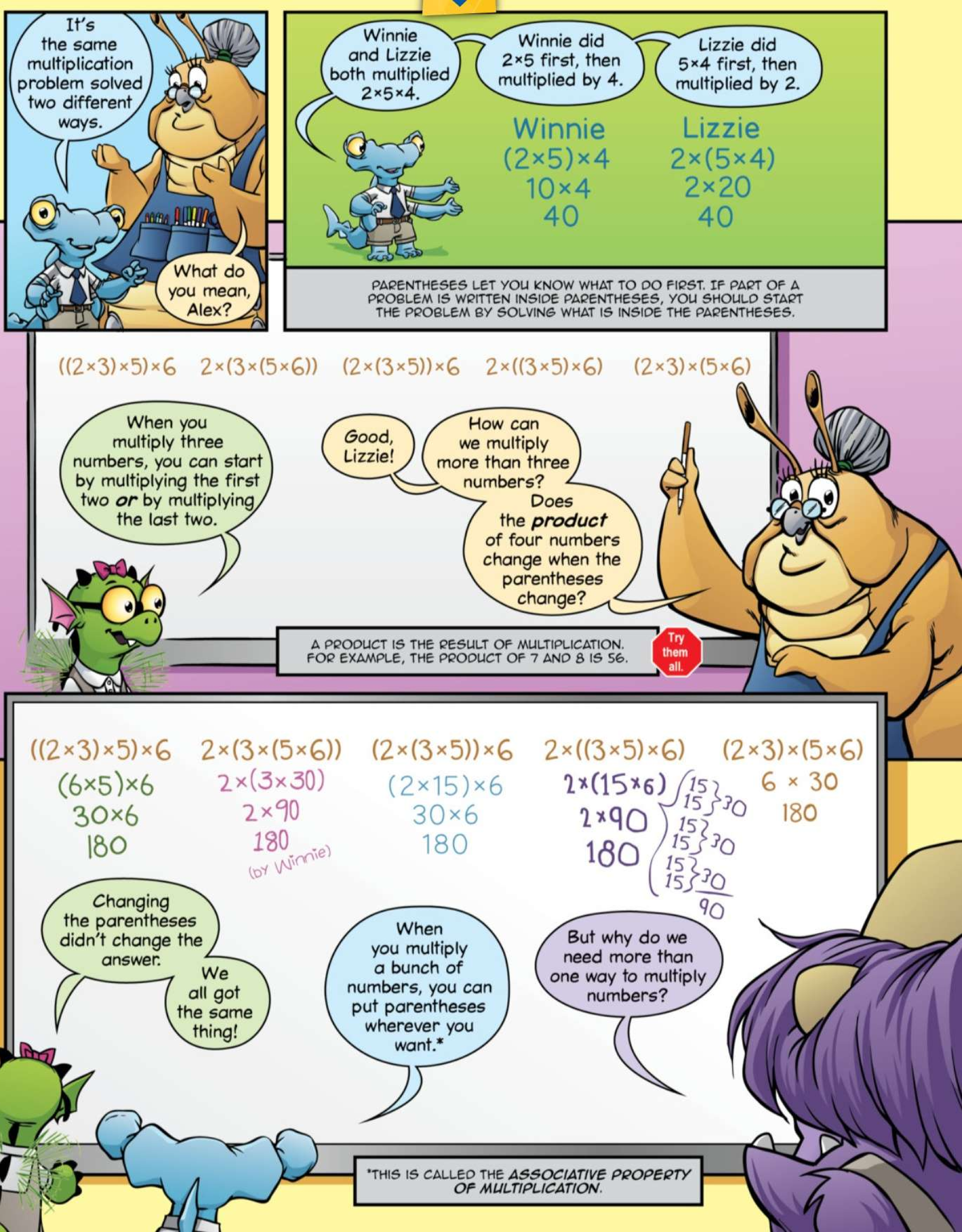

I've been trying to multiply some bigger numbers.
That makes things more difficult.
Not for me...
...but it does use more paper.
8×6=48
Actually, some very large numbers are easy to multiply.
Example?
You know 3×10=30, and 8×10=80.
Sure, 30 is three 10's, and 80 is eight 10's.
Right. Now try 13×10.
13 tens is 130.
I see. You just put a *zero* after the *13!*
Right, so to multiply 222×10...
...since 2,220 is 222 tens, 222×10=2,220! That *is* a pretty big number. It gives me an idea. If we multiply by another 10, we get another zero.
Uh huh...

Well, 34×10×10 is the same as 34×100...
That's thirty-four hundred! We get **two** zeros after the 34!
Right, and 34×1,000 is 34,000.
Of course, thirty-four thousand.
When you multiply a number by 10, you place a zero at the end of the number, and when you multiply by 1,000, you place **three** zeros at the end of the number.
And when you multiply a number by 1,000,000,000,000 you would place 12 zeros!
MIND THE LEDGE
It's the same as multiplying by 10 a bunch of times.
12 times in a row!
I wonder... ...if we multiply 9×10, we get 90. What about 9×20?
Since 20 is two tens, 9×20 is just 9 times two tens...
...9×2×10!
That gives us 18×10.
And 18×10 is 180!
So to multiply 9×20, you just multiply the 9 by the 2 and put a zero at the end.
So 7×80 is 560...
...I just did 7×8 and put on a zero.

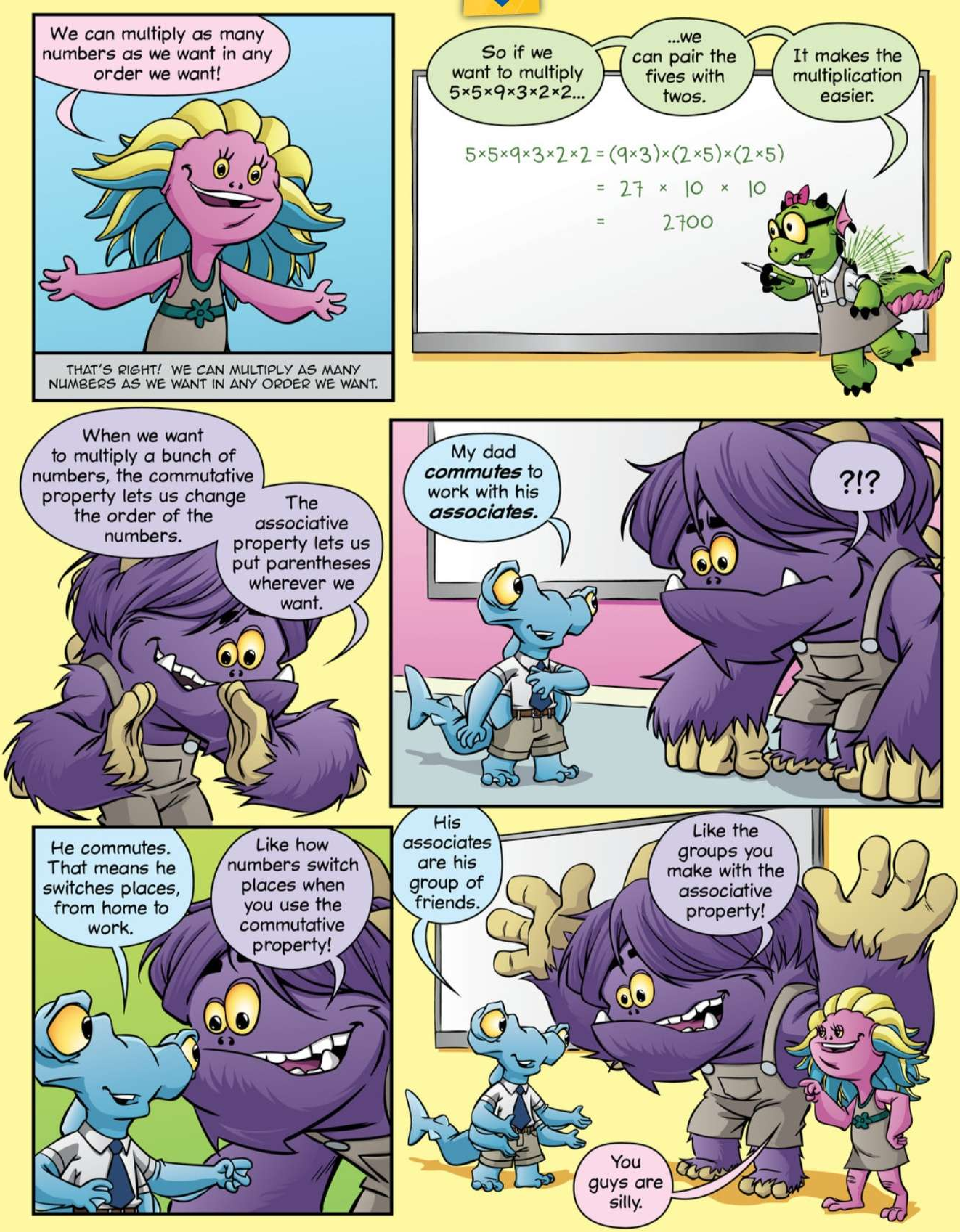

5×90 is 450!
70×4 is 280!
20×6 is 120!
90×3 is 270!
Hmmm... What about 90×30?
If 90×3 is 270, then 90×30 just needs another zero.
90×30 is 2,700.
It's 9×3 with *two* zeros at the end!
Right, because 90×30 is just 90×3...
...times 10!
XING
And 90×300 has one more zero.
27,000!
And 900,000 times 30,000,000,000?
900,000 has five zeros, and 30,000,000,000 has ten, so we get a 27 with 5+10=*15* zeros on the end... 27,000,000,000,000,000!

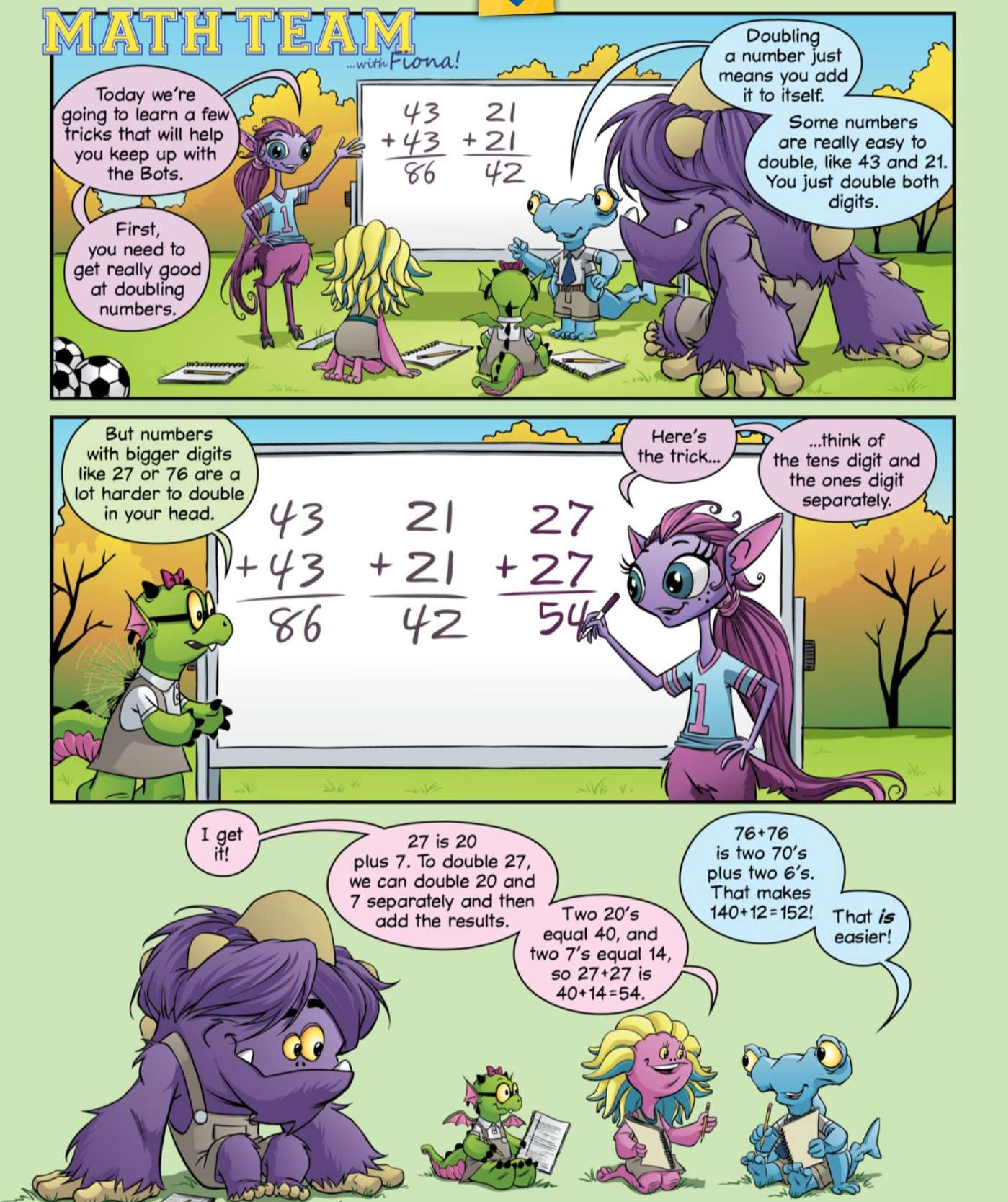

So, when we multiply two numbers that have zeros at the end, we can ignore the zeros and put them back *after* we multiply.
Right! To multiply 50×3, we just multiply 5×3 and put a zero on the end...
...50×3 =150.
DANGE
And if there are zeros at the ends of *both* numbers, we count them all up and tack them on.
4,000×600 is 4×6 with *five* zeros at the end... 2,400,000.
Here is one you might find tricky... what is 500×200?
That would be 100,000.
You've got four zeros in the problem and five in the answer. How did *that* happen?
Aren't you clever.
It comes from the 10 you get when you multiply 5×2. You put four *more* zeros after the 10.
How come I never get to drive?

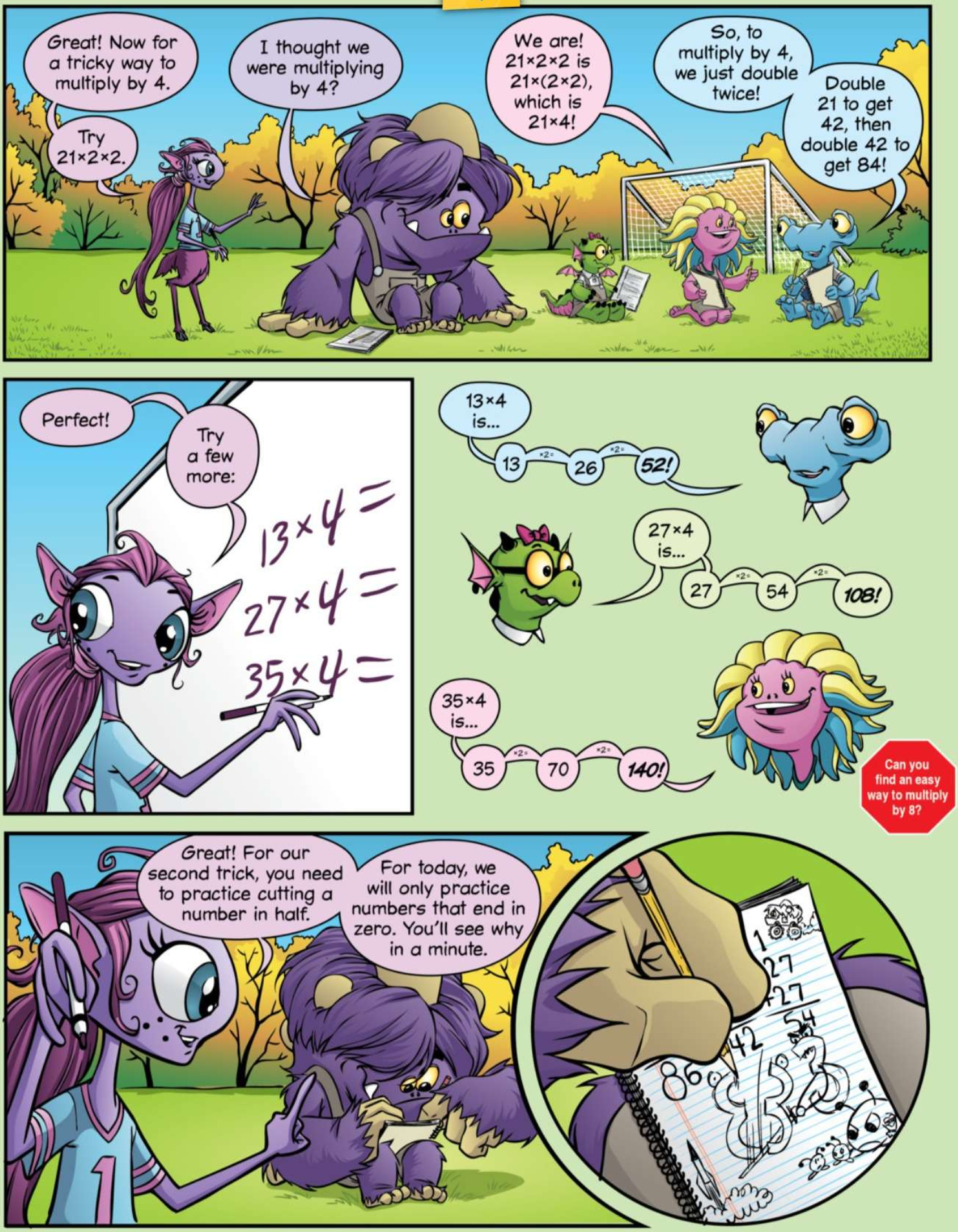

Ms. Q.
The Associative Property
Did everyone bring graph paper?
I forgot mine at home, Ms. Q.
I can share. I have quad-ruled, engineering, and isometric.
What does everyone need?
How nice, Alex!
Each student needs four sheets of quad-ruled paper.
How many sheets is that all together?
There are two tables...
...five of us at each table...
...and we each need four sheets.
Two tables times five monsters makes 10 monsters...
...and 10 monsters times 4 sheets each makes 10×4=40 sheets of graph paper.
I did it differently, but I got the same answer.
Each table has five monsters. Each monster gets four sheets of paper. That's 5×4=20 sheets per table. With two tables, that's 20×2=40 sheets.

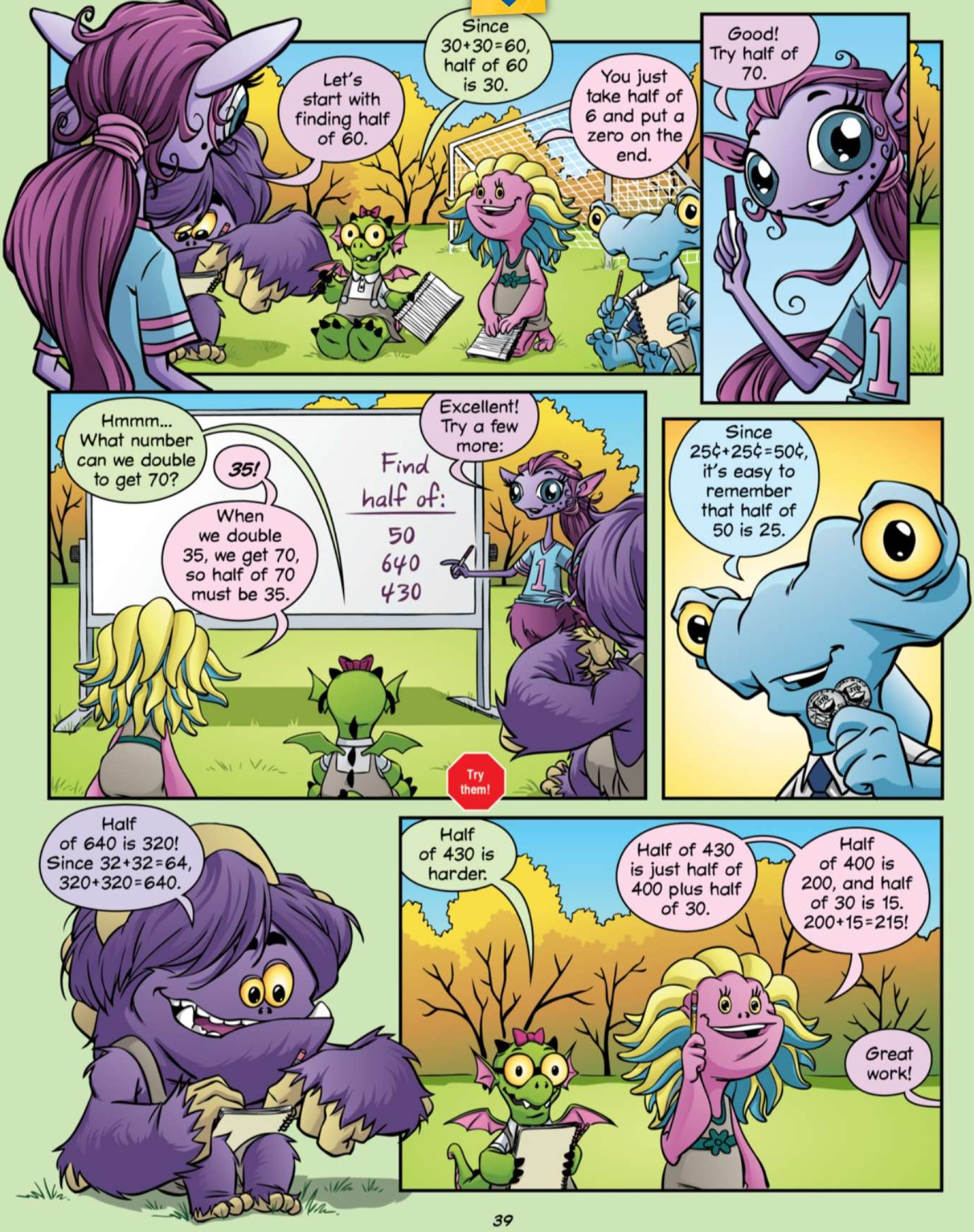

It's the same multiplication problem solved two different ways.
What do you mean, Alex?
Winnie and Lizzie both multiplied 2×5×4.
Winnie did 2×5 first, then multiplied by 4.
Lizzie did 5×4 first, then multiplied by 2.
Winnie
(2×5)×4
10×4
40
Lizzie
2×(5×4)
2×20
40
PARENTHESES LET YOU KNOW WHAT TO DO FIRST. IF PART OF A PROBLEM IS WRITTEN INSIDE PARENTHESES, YOU SHOULD START THE PROBLEM BY SOLVING WHAT IS INSIDE THE PARENTHESES.
((2×3)×5)×6 2×(3×(5×6)) (2×(3×5))×6 2×((3×5)×6) (2×3)×(5×6)
When you multiply three numbers, you can start by multiplying the first two *or* by multiplying the last two.
Good, Lizzie!
How can we multiply more than three numbers? Does the ***product*** of four numbers change when the parentheses change?
A PRODUCT IS THE RESULT OF MULTIPLICATION. FOR EXAMPLE, THE PRODUCT OF 7 AND 8 IS 56.
Try them all.
((2×3)×5)×6
(6×5)×6
30×6
180
2×(3×(5×6))
2×(3×30)
2×90
180
(by Winnie)
(2×(3×5))×6
(2×15)×6
30×6
180
2×((3×5)×6)
2×(15×6)
2×90
180
15 15 30
15 15 30
15 15 30
90
(2×3)×(5×6)
6 × 30
180
Changing the parentheses didn't change the answer.
We all got the same thing!
When you multiply a bunch of numbers, you can put parentheses wherever you want.*
But why do we need more than one way to multiply numbers?
*THIS IS CALLED THE *ASSOCIATIVE PROPERTY OF MULTIPLICATION*.


Try this. How would you multiply 9×2×5?
9×2 is 18, and 18×5 is...
...ummm...
To solve 9×2×5, you ***could*** multiply 2×5 first...
...9×2×5 is 9×(2×5), which is 9×10.
90!
It's easier to multiply the 2×5 first because 2×5=10.
To multiply a number by ten, you just put a zero at the end.
2×7×5×3
That's right!
How could you make this problem easier to solve?
We want to get the 2 and the 5 together.
We can rearrange the numbers like this...
...and group them like this!
2×7×5×3
2×5×7×3
(2×5)×(7×3)
10×21
210
10×21 is 210!

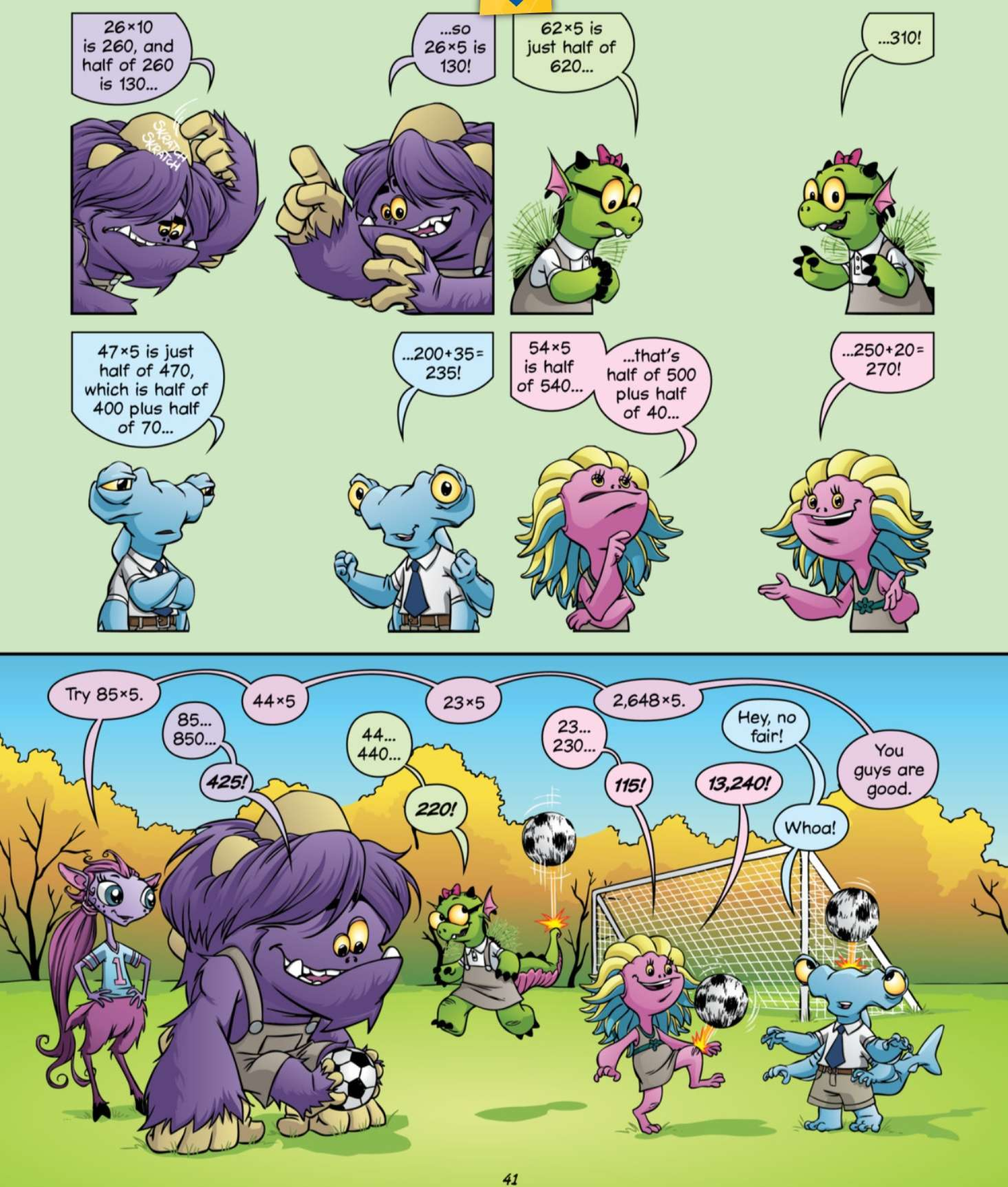

We can multiply as many numbers as we want in any order we want!
THAT'S RIGHT! WE CAN MULTIPLY AS MANY NUMBERS AS WE WANT IN ANY ORDER WE WANT.
So if we want to multiply 5×5×9×3×2×2...
...we can pair the fives with twos.
It makes the multiplication easier.
5×5×9×3×2×2 = (9×3)×(2×5)×(2×5)
= 27 × 10 × 10
= 2700
When we want to multiply a bunch of numbers, the commutative property lets us change the order of the numbers.
The associative property lets us put parentheses wherever we want.
My dad **commutes** to work with his ***associates.***
?!?
He commutes. That means he switches places, from home to work.
Like how numbers switch places when you use the commutative property!
His associates are his group of friends.
Like the groups you make with the associative property!
You guys are silly.


MATH TEAM
...with Fiona!
Today we're going to learn a few tricks that will help you keep up with the Bots.
First, you need to get really good at doubling numbers.
43 + 43 = 86   21 + 21 = 42
Doubling a number just means you add it to itself.
Some numbers are really easy to double, like 43 and 21. You just double both digits.
But numbers with bigger digits like 27 or 76 are a lot harder to double in your head.
43 + 43 = 86   21 + 21 = 42   27 + 27 = 54
Here's the trick...
...think of the tens digit and the ones digit separately.
I get it!
27 is 20 plus 7. To double 27, we can double 20 and 7 separately and then add the results.
Two 20's equal 40, and two 7's equal 14, so 27+27 is 40+14=54.
76+76 is two 70's plus two 6's. That makes 140+12=152!
That *is* easier!


Great! Now for a tricky way to multiply by 4.
Try 21×2×2.
I thought we were multiplying by 4?
We are! 21×2×2 is 21×(2×2), which is 21×4!
So, to multiply by 4, we just double twice!
Double 21 to get 42, then double 42 to get 84!
Perfect!
Try a few more:
13×4=
27×4=
35×4=
13×4 is...
13 ×2= 26 ×2= 52!
27×4 is...
27 ×2= 54 ×2= 108!
35×4 is...
35 ×2= 70 ×2= 140!
Can you find an easy way to multiply by 8?
Great! For our second trick, you need to practice cutting a number in half.
For today, we will only practice numbers that end in zero. You'll see why in a minute.


Let's start with finding half of 60.
Since 30+30=60, half of 60 is 30.
You just take half of 6 and put a zero on the end.
Good! Try half of 70.
Hmmm... What number can we double to get 70?
35!
When we double 35, we get 70, so half of 70 must be 35.
Excellent! Try a few more:
Find half of:
50
640
430
Try them!
Since 25¢+25¢=50¢, it's easy to remember that half of 50 is 25.
Half of 640 is 320! Since 32+32=64, 320+320=640.
Half of 430 is harder.
Half of 430 is just half of 400 plus half of 30.
Half of 400 is 200, and half of 30 is 15. 200+15=215!
Great work!


This next trick gives us an easy way to multiply large numbers by 5.
Try this.
Take a number, multiply it by ten, then find half of the result.
9×10 is 90, and half of 90 is 45.
It's the same as 9 times 5.
Multiplying by 10 then finding half is the same as multiplying by five!
10
5
5
9
=
+
Why does *that* work?
9×10 is the same as 9×5 plus 9×5...
...so half of 9×10 is 9×5!
5
5
9
To multiply by 5, we can multiply by 10 and take half!
That's right. Try a few more.
26×5=
62×5=
47×5=
54×5=
Try to solve all four of these in your head.



26×10 is 260, and half of 260 is 130...
SKRATCH SKRATCH
...so 26×5 is 130!
62×5 is just half of 620...
...310!
47×5 is just half of 470, which is half of 400 plus half of 70...
...200+35= 235!
54×5 is half of 540...
...that's half of 500 plus half of 40...
...250+20= 270!
Try 85×5.
85... 850...
425!
44×5
44... 440...
220!
23×5
23... 230...
115!
2,648×5.
13,240!
Hey, no fair!
Whoa!
You guys are good.

Winnie's Notes

Doubling Practice:

5		3	7	9
1	10	3	7	9
2	20	6	14	18
4	40	12	28	36
8	80	24	~~46~~ 56	72
16	160	48	112	144
32		96	224	288
64		~~182~~ 192	448	_6
128			896	_2
~~246~~ 256			~~16~~92	
512			17	
1024				
2048				
~~4086~~ 4096				

They match!

Look at the ones digits!
2 4 8 6 2 4 8 6 2 4 8 6

the ones digits repeat again!
2 4 8 6

more repeats!
←12→ ←24→ ←48→ ←96→ ←92→

Multiplying by 4:

×4 is the same as ×2×2:

45	43	57	108	92
×2 90 ×4	×2 86 ×4	114 ×4	216 ×4	184 ×4
×2 180	×2 172	228	432	368

75×4 = 300   33×4 = 132   83×4 = 332

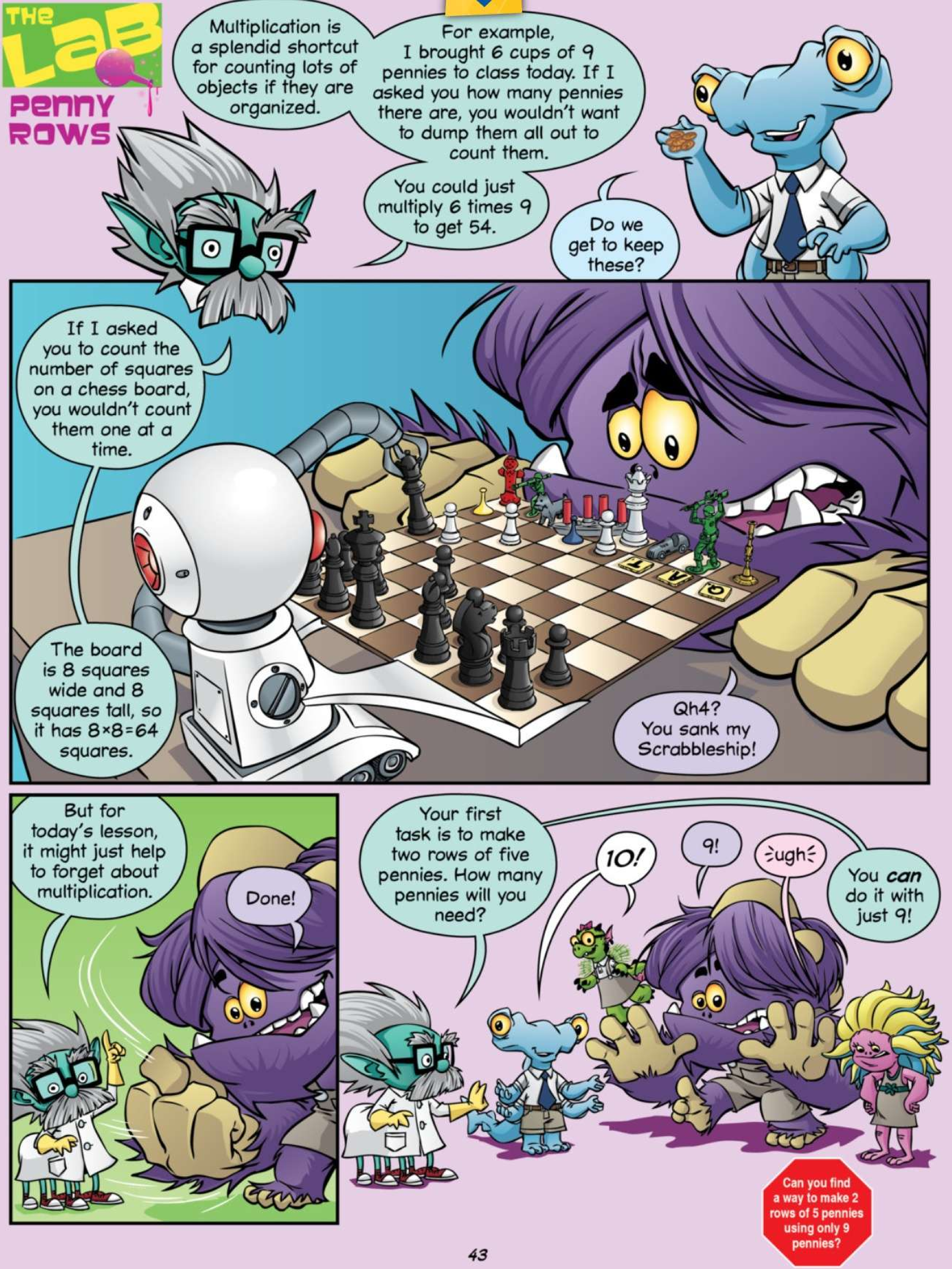
58×4 = 232   61×4 = 244   925×4 = ~~1850~~ 3700

Multiplying ~~Timesing~~ by 5:

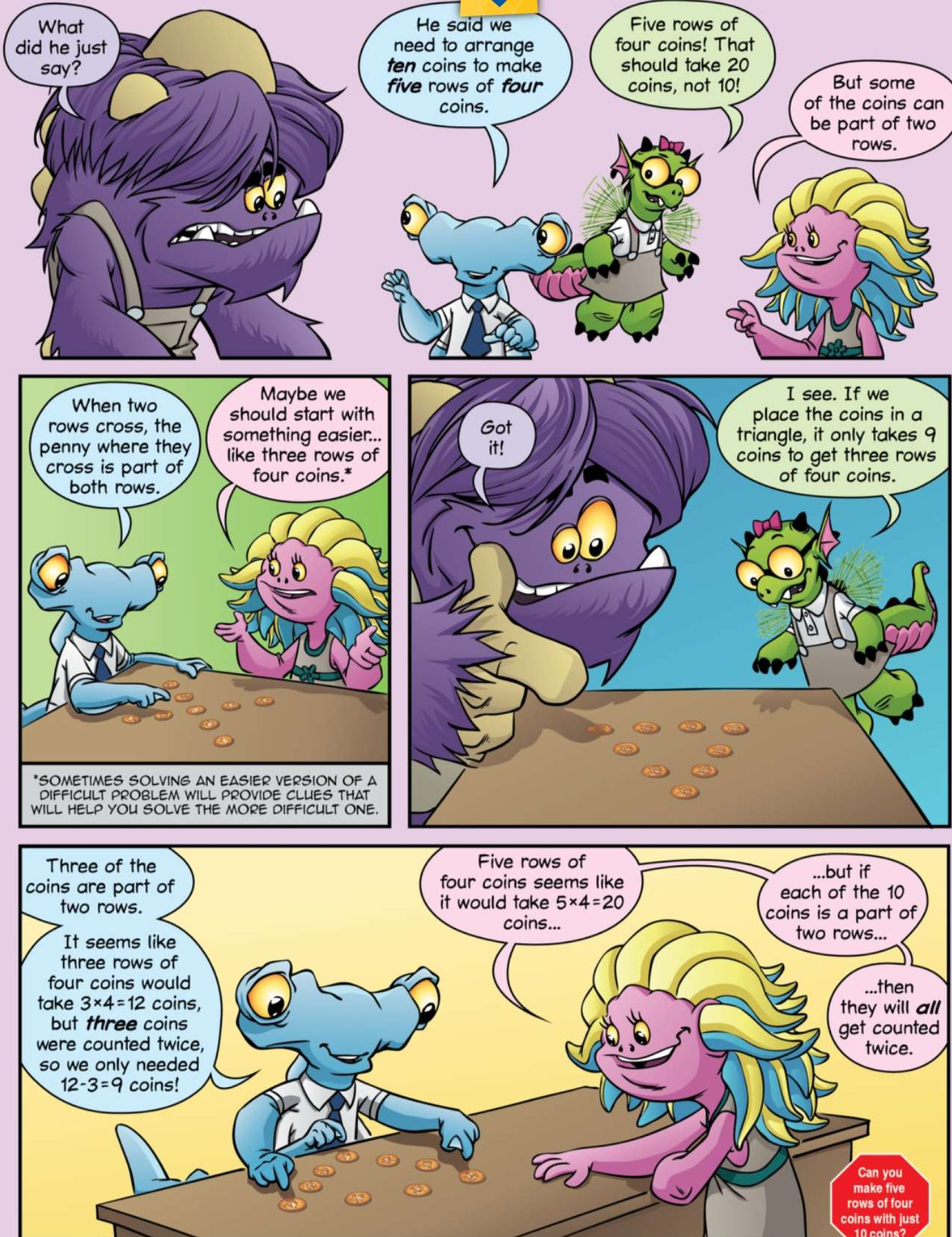
To do ×5, just do ×10 then find half.

16	72	45	861	3416
×10 160 ×5	720 ×5	450 ×5	8610 ×5	34160 ×5
half 80	360	225	4305	17080

24681012141618 × 5
= 123405060708090


THE LAB
PENNY ROWS
Multiplication is a splendid shortcut for counting lots of objects if they are organized.
For example, I brought 6 cups of 9 pennies to class today. If I asked you how many pennies there are, you wouldn't want to dump them all out to count them.
You could just multiply 6 times 9 to get 54.
Do we get to keep these?
If I asked you to count the number of squares on a chess board, you wouldn't count them one at a time.
The board is 8 squares wide and 8 squares tall, so it has 8×8=64 squares.
Qh4? You sank my Scrabbleship!
But for today's lesson, it might just help to forget about multiplication.
Done!
Your first task is to make two rows of five pennies. How many pennies will you need?
10!
9!
ugh
You *can* do it with just 9!
Can you find a way to make 2 rows of 5 pennies using only 9 pennies?


How?!
Like this!
One penny can be in two different rows!
That's brilliant, Grogg!
Well done! I'll be right back with more pennies and more puzzles!
PHRUMP
Bwah Ha Ha! Professor Grok is gone. I've abducted your educator. It's time for something much more diabolically difficult...
...Practically Impossible Penny Rows!
It doesn't take a brilliant beast to create a clever coin composition with just *two* rows. Making more rows can mystify the most masterful monsters!
Only completing a most confounding coin composition can help you crack the code, creating a clue for the location of your illustrious educator.
Ten coins can be composed to create five rows of four coins. The solution suggests a simple symbol that marks the site of your schoolteacher.
Think fast, because...
...Endangerment is Imminent!!!



What did he just say?
He said we need to arrange ***ten*** coins to make ***five*** rows of ***four*** coins.
Five rows of four coins! That should take 20 coins, not 10!
But some of the coins can be part of two rows.
When two rows cross, the penny where they cross is part of both rows.
Maybe we should start with something easier... like three rows of four coins.*
*SOMETIMES SOLVING AN EASIER VERSION OF A DIFFICULT PROBLEM WILL PROVIDE CLUES THAT WILL HELP YOU SOLVE THE MORE DIFFICULT ONE.
Got it!
I see. If we place the coins in a triangle, it only takes 9 coins to get three rows of four coins.
Three of the coins are part of two rows.
It seems like three rows of four coins would take 3×4=12 coins, but ***three*** coins were counted twice, so we only needed 12-3=9 coins!
Five rows of four coins seems like it would take 5×4=20 coins...
...but if each of the 10 coins is a part of two rows...
...then they will ***all*** get counted twice.
Can you make five rows of four coins with just 10 coins?


When two rows cross, the penny where they cross gets counted twice...
...so every penny has to be where two rows cross.
How is that possible?!
Grogg! Are you doodling again?! Why don't you ever--
Wait!
Grogg, you did it again!
?!?
Look at Grogg's star!
If we put pennies where all the lines cross, we get 5 rows...
...with 4 pennies in each row!
I know where Professor Grok is!
To the observatory!
Great work, little monsters! I thought I'd be locked in here for weeks!

Grogg

More ways to make five rows of four coins with 10 coins.