Contents: Chapter 8

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Chapter 8:
Ratios & Rates

MATH TEAM
Ratios

Ah. So, for every 2 ***quarts*** of soda, there are 5 ***quarts*** of juice!

That's right, Grogg!

A ratio doesn't tell us how much of each quantity there is, just how the quantities compare.

How much juice would we add to ***four*** quarts of soda to get a soda-to-juice ratio of 2 to 5?

For every ***2*** quarts of soda, we need ***5*** quarts of juice.

So, if we use ***4*** quarts of soda, we need ***10*** quarts of juice.

If we double the amount of soda...

...we need to double the amount of juice to keep the ratio the same.

That's right. Now, each of these pitchers holds 84 ounces.

How many ounces of soda and juice should we mix in each 84-ounce pitcher to get a soda-to-juice ratio of 2 to 5?

How many?

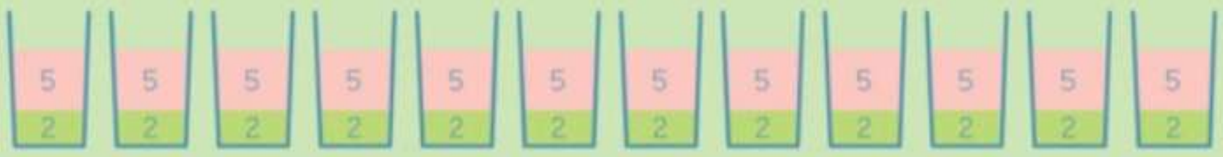
For every 2 ounces of soda, there are 5 ounces of juice.

So, for every 7 ounces of punch, 2 of the ounces are soda and 5 of the ounces are juice.

An 84-ounce pitcher will fill 84÷7=12 7-ounce servings.

With 2 ounces of soda and 5 ounces of juice in each serving...

...that makes 12•2=24 ounces of soda and 12•5=60 ounces of juice.



12•5=60 fl oz
12•2=24 fl oz

I came up with the same answer a different way.

If we divide the punch into equal parts, then for every 2 parts soda, there are 5 parts juice.

That's 7 parts total.

If we divide 84 ounces into 7 parts, each part will be 84÷7=12 ounces.

2 of those parts are soda, for a total of 2•12=24 ounces of soda.

5 of those parts are juice, for a total of 5•12=60 ounces of juice.

2•12=24 fl oz

5•12=60 fl oz

Both ways make sense.

In a punch that is 2 parts soda and 5 parts juice...

...2 of the 7 parts are soda...

...and 5 of the 7 parts are juice.

5 parts juice

2 parts soda

So, the punch is $\frac{2}{7}$ soda and $\frac{5}{7}$ juice.

To fill an 84-ounce pitcher, we need $\frac{2}{7} \cdot 84 = 24$ ounces of soda...

...and $\frac{5}{7} \cdot 84 = 60$ ounces of juice.

$\frac{5}{7}$ juice

$\frac{2}{7}$ soda

60 fl oz

24 fl oz

Great job, everyone.

When working with ratios, it is often useful to consider how the parts relate to the whole.

In a punch mixed with soda and juice in a 2 to 5 ratio...

...the ratio of soda to punch is 2 to 7...

...and the ratio of juice to punch is 5 to 7.

Or, like Grogg said, the punch is $\frac{2}{7}$ soda and $\frac{5}{7}$ juice.

There are a few different ways to write ratios.

The most common ratio notation uses a colon.

For example, the ratio of 24 ounces of soda to 60 ounces of juice is written as 24:60, which is read "24 to 60."

$$\text{soda} : \text{juice} = 24:60$$

We can also use fractions to work with ratios.

For example, if the ratio of soda to juice is 24 to 60, then the amount of soda divided by the amount of juice is $\frac{24}{60}$.

$$\frac{\text{soda}}{\text{juice}} = \frac{24}{60}$$

Like fractions, ratios are almost always written in simplest form.

We simplify ratios the same way we simplify fractions.

$$\frac{\text{soda}}{\text{juice}} = \frac{24}{60} = \frac{2}{5}$$

$\div 12$ $\div 12$

$$\text{soda} : \text{juice}$$
$$= 24 : 60$$
$$= 2 : 5$$

$\frac{24}{60}$ simplifies to $\frac{2}{5}$. So, a 24:60 ratio of soda to juice simplifies to 2:5.

In addition to punch, we'll also have customized pencils and erasers at our table.

We have 40 pencils and 72 erasers.

That's a 5:9 ratio of pencils to erasers!

Right, Grogg.

pencils : erasers
= 40 : 72
= 5 : 9

We've also got 36 keychains and 90 lanyards.

That's a 2:5 ratio!

Yep. What's the ratio of erasers to lanyards?

keychains : lanyards
= 36 : 90
= 2 : 5

We have 72 erasers, and 90 lanyards, so the ratio is 72:90. That simplifies to 4:5.

Perfect. Last of all, we have fruit...

...razbananas and dragonfruits.

erasers : lanyards
= 72 : 90
= 4 : 5

We have 63 dragonfruits.

How many razbananas do we need so that the ratio of razbananas to dragonfruits is 2:3?

How many?

To make the ratio of razbananas to dragonfruits 2:3, we need 2 razbananas for every 3 dragonfruits.

We can split 63 dragonfruits into piles of 3 to make 63÷3=21 piles.

Then, for every pile of 3 dragonfruits, we need 2 razbananas.

That makes 21•2=42 razbananas.

Or, we can think of it another way.

We can split 63 dragonfruits into 3 bowls of 63÷3=21 dragonfruits.

Since we have 3 bowls of 21 dragonfruits, we need 2 bowls of 21 razbananas.

That's 2•21=42 razbananas.

I don't think we can fit any more fruit on this table...

...it's maxed out.

Speaking of Max...

...where is he? I haven't seen him all day.

He's helping set up the Aerospace Club display.

Practice: Pages 37-47

Cool. Let's go check it out!

Whoa. We're gonna need more lanyards.

MOON CHEESE

Ms. Q.

Proportions

How can we tell which of these ratios are equivalent?

8 : 14
12 : 20
21 : 35
12 : 21

None of these are in simplest form. Maybe simplifying them will help.

8 : 14
12 : 20
21 : 35
12 : 21

8:14 and 12:21 both simplify to 4:7.

12:20 and 21:35 both simplify to 3:5.

8 : 14 = 4 : 7
12 : 20 = 3 : 5
21 : 35 = 3 : 5
12 : 21 = 4 : 7

So, we have 8:14=12:21, and 12:20=21:35.

8 : 14 = 4 : 7
12 : 20 = 3 : 5
21 : 35 = 3 : 5
12 : 21 = 4 : 7

8:14 = 12:21 12:20 = 21:35

$\frac{8}{14} = \frac{12}{21}$ $\frac{12}{20} = \frac{21}{35}$

As fractions, $\frac{8}{14} = \frac{12}{21}$, and $\frac{12}{20} = \frac{21}{35}$.

Do any of you know what we call an equation in which two ratios are equal?

Two equal ratios form a ***proportion.***

$5:9 = x:36$

$\frac{5}{9} = \frac{x}{36}$

That's right. We usually express proportions with fractions.

What is the value of x in this equation?

We can write $\frac{5}{9}$ with a denominator of 36 by multiplying the numerator and denominator by 4.

$\frac{5}{9} = \frac{20}{36}$.

So, $x = 5 \cdot 4 = 20$.

$\frac{5}{9} \overset{\cdot 4}{=} \frac{x}{36} \overset{\cdot 4}{}$

$x = 20$

Very good. Let's try a few more.

What is the missing value in each of these equations?

$\frac{a}{12} = \frac{2}{3}$

$\frac{15}{20} = \frac{b}{8}$

$\frac{3}{5} = \frac{7}{d}$

Try all three.

$\frac{8}{12}=\frac{2}{3}$, so $a=8$.

$\frac{a}{12}=\frac{2}{3}$ (·4 above, ·4 below)

$\frac{8}{12}=\frac{2}{3}$

We can simplify $\frac{15}{20}$ to $\frac{3}{4}$, and since $\frac{3}{4}=\frac{6}{8}$, we have $b=6$.

$\frac{15}{20}=\frac{b}{8}$

$\frac{15}{20}=\frac{3}{4}=\frac{6}{8}$ (÷5, ·2 above; ÷5, ·2 below)

So, $b=6$

I did something different. I solved $\frac{15}{20}=\frac{b}{8}$ like any other equation.

To isolate the variable, we can multiply both sides of the equation by 8.

$\frac{15}{20}\cdot 8=\frac{b}{8}\cdot 8$

YOU CAN REVIEW SOLVING EQUATIONS LIKE THIS ONE IN CHAPTER 3 OF BEAST ACADEMY 5A.

On the left side, we get $\frac{15}{20}\cdot 8=\frac{120}{20}=6$.

So, $b=6$.

$\frac{15}{20}\cdot 8=\frac{b}{8}\cdot 8$

$\frac{120}{20}=b$

$6=b$

I got $b=6$ a ***third*** way.

How, Grogg?

I wanted to get rid of the fractions first, so I multiplied both sides by 40.

$$\frac{15}{20} \cdot 40 = \frac{b}{8} \cdot 40$$

Why 40?

The LCM of 20 and 8 is 40, so multiplying both sides by 40 cancels the denominators on both sides.

$$\frac{15}{\cancel{20}} \cdot \cancel{40}^{2} = \frac{b}{\cancel{8}} \cdot \cancel{40}^{5}$$

$$30 = 5b$$

We get $30=5b$.

$$\frac{15}{\cancel{20}} \cdot \cancel{40}^{2} = \frac{b}{\cancel{8}} \cdot \cancel{40}^{5}$$

$$30 = 5b$$

$$6 = b$$

Dividing both sides by 5 gives us $b=6$.

All fine methods, little monsters!

Well done!

How can each method be used to solve ***this*** equation?

$$\frac{3}{5} = \frac{7}{d}$$

How would *you* solve for *d*?

We can't simplify $\frac{3}{5}$, and there isn't an integer we can multiply 3 by to get 7.

$$\frac{3}{5} = \frac{7}{d}$$

But, we can multiply 3 by $\frac{7}{3}$ to get 7.

Since $5 \cdot \frac{7}{3} = \frac{35}{3}$, we have $d = \frac{35}{3} = 11\frac{2}{3}$.

$$\frac{3}{5} \xrightarrow{\cdot\frac{7}{3}} \frac{7}{11\frac{2}{3}}$$

To solve $\frac{3}{5} = \frac{7}{d}$, we need to isolate d.

But, d is in the denominator.

$$\frac{3}{5} = \frac{7}{d}$$

However, if we multiply both sides by d, we remove d from the denominator!

$$\frac{3}{5} \cdot d = \frac{7}{d} \cdot d$$

Then, we multiply both sides by 5...

...and divide both sides by 3.

$d = \frac{35}{3}$, which equals $11\frac{2}{3}$.

$$\frac{3}{5} \cdot d = \frac{7}{d} \cdot d$$

$$\frac{3d}{5} = 7$$

$$3d = 35$$

$$d = \frac{35}{3}$$

$$d = 11\frac{2}{3}$$

Or, we can get rid of both denominators in one step.

If we multiply both sides by $5d$, the 5's cancel on the left, and the d's cancel on the right.

$$\frac{3}{5} \cdot 5d = \frac{7}{d} \cdot 5d$$

$$3 \cdot d = 7 \cdot 5$$

Dividing both sides by 3 gives us $d = \frac{35}{3} = 11\frac{2}{3}$.

$$\frac{3}{5} \cdot 5d = \frac{7}{d} \cdot 5d$$

$$3 \cdot d = 7 \cdot 5$$

$$d = \frac{35}{3}$$

$$d = 11\frac{2}{3}$$

Well done, little monsters.

With so many different ways to solve a proportion, how will you decide which to choose?

Some methods work better than others, depending on the situation.

My dad always says that you can never have too many tools.

You never know when you'll need a box wrench...

...or a clamp...

...or a brush...

...or a cheese cuber...

...or a pair of snub-nosed pliers...

...or a $\frac{3}{4}$-inch axle disk brake adapter...

...or a horn-nosed flonk norble...

I don't think this is what your dad meant, Grogg.

Urrgh! Now you're just making up words.

Practice: Pages 48-57

RATES

Map Room

What are these, Captain Kraken?

These be the fastest recorded times for sailin' between various points on Beast Island.

Are any of these *your* records?

Aye, a few.

Whoa! Captain Conch sailed 51 miles in just 3 hours!?

Captain Crustacean sailed 90 miles from Cape Grindylow to Port Hydra in just 5 hours!

We call him Cap'n Crusty, for short.

Beast Island

Beast Bay

Serpent Strait

Wait...

...which of those is faster?

'Tis an excellent inquiry.

How can we be comparin' the speeds o' Captain Conch 'n' Captain Crusty?

Conch:
51 miles in 3 hours

Crusty:
90 miles in 5 hours

Who sailed faster?

Conch:
51 miles in 3 hours
255 miles in 15 hours

Crusty:
90 miles in 5 hours
270 miles in 15 hours

What if the two captains sailed for the ***same*** amount of time? Since 3 and 5 are both factors of 15, we can figure out how far each captain could sail in 15 hours.

Conch sailed 51 miles in 3 hours. At that speed, he could go 51•5=255 miles in 3•5=15 hours.

Crusty sailed 90 miles in 5 hours. At that speed, he could go 90•3=270 miles in 5•3=15 hours.

Crusty was faster.

Conch:
51 miles in 3 hours = 17 miles per hour

Crusty:
90 miles in 5 hours = 18 miles per hour

Instead of figuring out how far each Captain could sail in ***15*** hours, I figured out how far each could go in ***1*** hour.

For Conch to travel 51 miles in 3 hours, he had to travel 51÷3=17 miles per hour.

For Crusty to travel 90 miles in 5 hours, he had to travel 90÷5=18 miles per hour.

"PER" MEANS "FOR EACH" OR "FOR EVERY."

Excellent figurin', little monsters. ***Speed*** be the ratio of distance to time.

We often express speed as the distance traveled for one unit o' time...

...for example, as the number o' miles traveled per hour.

We can abbreviate "miles per hour" as mph.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{90 \text{ mi}}{5 \text{ hr}}$$

$$= \frac{18 \text{ mi}}{1 \text{ hr}}$$

$$= 18 \text{ mph}$$

Other units can be used to measure speed.

Some things happen very slowly.

For example, the horn o' a spiralac grows just $\frac{3}{4}$ inches per year, starting at birth.

How long be the horn o' a 40-year-old spiralac?

If the horn grows $\frac{3}{4}$ inches each year, then in 40 years, it will be $\frac{3}{4} \cdot 40 = 30$ inches long.

Aye. In fact, we can find the age of a spiralac by the length o' its horn.

What be the age o' a spiralac with a 60-inch horn?

Trying a similar problem without fractions can help us figure out what to compute.

If the horn grows 2 inches per year, it will take $60 \div 2 = 30$ years to grow 60 inches.

So, if the horn grows $\frac{3}{4}$ inches per year, it will take $60 \div \frac{3}{4}$ years to grow 60 inches.

$60 \div \frac{3}{4} = 60 \cdot \frac{4}{3} = 80$ years.

$$60 \div \frac{3}{4} = 60 \cdot \frac{4}{3} = \frac{240}{3} = 80$$

Speed be a special type o' ratio, called a ***rate***.

If ye' be usin' the word ***per***, then you're probably usin' a rate.

A rate describes how much o' one quantity there be for one unit of another quantity.

Rate

For example, the time I faced a mighty spiralac, me heart be thumpin' 200 beats ***per*** minute.

Its enormous feet be havin' 5 razor-sharp talons ***per*** toe!

Its nostrils could spray powerful jets of water like a firehose...

...30 gallons ***per*** second!

What treasure were you after?

Spiralacs shed their soft, fluffy feathers.

While me crew distracted her, I was able to gather a dozen sacks full o' the downy plumage.

The largest sack o' feathers weighed just $4\frac{1}{2}$ pounds 'n' sold for $720!

The smallest sack weighed only $2\frac{1}{5}$ pounds 'n' sold for $330!

Which sack be worth more per pound?

What was the cost per pound of each sack?

To find out what a sack of feathers is worth in dollars per pound...

...we divide its price in dollars by its weight in pounds.

$$\frac{720 \text{ dollars}}{4\frac{1}{2} \text{ pounds}}$$

A $720 sack that weighs $4\frac{1}{2}$ pounds is worth $720 \div 4\frac{1}{2}$ dollars per pound.

To divide 720 by $4\frac{1}{2}$, we convert $4\frac{1}{2}$ to a fraction... $4\frac{1}{2} = \frac{9}{2}$.

$$\frac{720 \text{ dollars}}{4\frac{1}{2} \text{ pounds}} = 720 \div 4\frac{1}{2}$$

$$= 720 \div \frac{9}{2}$$

Then, we multiply $720 \cdot \frac{2}{9}$.

The big sack sold for $160 per pound.

$$\frac{720 \text{ dollars}}{4\frac{1}{2} \text{ pounds}} = 720 \div 4\frac{1}{2}$$

$$= 720 \div \frac{9}{2}$$

$$= 720 \cdot \frac{2}{9}$$

$$= \frac{{}^{80}\cancel{720} \cdot 2}{\cancel{9}_{1}}$$

$$= 160 \text{ dollars per pound}$$

A $330 sack that weighs $2\frac{1}{5}$ pounds is worth $330 \div 2\frac{1}{5}$ dollars per pound.

So, the small sack sold for $150 per pound.

The large sack was worth $10 more per pound!

$$330 \div 2\frac{1}{5}$$

$$= 330 \div \frac{11}{5}$$

$$= 330 \cdot \frac{5}{11}$$

$$= \frac{{}^{30}\cancel{330} \cdot 5}{\cancel{11}_{1}}$$

$$= 150 \text{ dollars per pound}$$

WE OFTEN WRITE "160 DOLLARS PER POUND" AS $160/LB.

Aye.

'Tis easy to compare two rates when both be in dollars per pound.

Were there any sacks of feathers that were even ***more*** valuable than those two?

Sadly, I'll never know.

All o' the other sacks be lost at sea.

The frightful creature chased us through Serpent Strait across Beast Bay.

Beast Bay

Serpent Strait

*not to scale

We tossed all non-essentials overboard to lighten the ship 'n' outrun the surprisingly swift spiralac...

...includin' ten o' the twelve sacks o' feathers.

'Twas a sad day.

But, we did set a speed record for the trip 'cross Beast Bay!

Beast Bay

Serpent Strait

Practice: Pages 58-63

R&G Unit Conversions

What a perfect day for a trip to the farmer's market!

What's on our list?

First, we need some plumangos for our Custodial Arts table at the Club Carnival.

How many do you think we need?

Plumangos 4 for $1

Five pounds ought to be plenty.

Hmmm... This bag of plumangos weighs 96 ounces. Is that enough?

Well, since there are 16 ounces in a pound, five pounds equals 5 • 16 = 80 ounces.

So, 96 ounces is more than enough.

4 for $1

To convert from pounds to ounces, we just multiply by 16.

Right, because one pound is 16 ounces.

So, to convert from ounces to pounds, can we divide by 16?

CIABATTA BOOM

How many pounds is 96 ounces?

Sure. A 96-ounce bag weighs $\frac{96}{16}$ = 6 pounds.

But for more complicated unit conversions, we usually use ***conversion factors***.

Huh?

A conversion factor is a fraction that equals 1.

Like $\frac{8}{8}$?

Not exactly. In a conversion factor, the units in the numerator and the denominator are different.

But... How can the fraction equal 1?

The numerator and denominator have to be equal.

For example, since 1 pound equals 16 ounces, $\frac{1\text{ lb}}{16\text{ oz}}$ equals 1.

We can use this conversion factor to convert any number of ounces to pounds.

$\frac{1\text{ lb}}{16\text{ oz}} = 1$

Conversion Factor

How?

How could you use a conversion factor to convert 96 ounces to pounds?

We multiply!

You can multiply anything by 1 without changing its value.

$$96\,\text{oz} \cdot \frac{1\,\text{lb}}{16\,\text{oz}}$$

So, when we multiply 96 ounces by $\frac{1\,\text{lb}}{16\,\text{oz}}$, we aren't changing the weight...

...just the units.

$$96\,\text{oz} \cdot \frac{1\,\text{lb}}{16\,\text{oz}}$$

$$= \frac{96\,\text{oz} \cdot 1\,\text{lb}}{16\,\text{oz}}$$

$$= \frac{96\,\text{lb}}{16}$$

$$= 6\,\text{lb}$$

Units cancel the same way common factors do.

I see. Ounces cancel in the numerator and the denominator, and we're left with pounds.

Yep. $96\,\text{oz} \cdot \frac{1\,\text{lb}}{16\,\text{oz}} = \frac{96\,\text{lb}}{16} = 6\,\text{lb}$.

What else do we need to buy?

Captain Kraken asked me to pick up 7 kilograms of melonberries for the Swashbucklin' Club.

Kilograms? Each of these cartons holds 20 ***ounces*** of melonberries.

We'll need to convert.

How?

We can use conversion factors!

There are 1,000 grams in a kilogram...

...and there are 28 grams in an ounce.*

How many ounces are in 7 kilograms?

ACTUALLY, 1 OZ = 28.3495 GRAMS, BUT 28 GRAMS IS CLOSE ENOUGH FOR MELONBERRY PURCHASES.

First, we can convert from kilograms to grams...

...then from grams to ounces.

$kg \rightarrow g \rightarrow oz$

$\frac{1{,}000\,g}{1\,kg} = \frac{1\,kg}{1{,}000\,g} = 1$

Since there are 1,000 grams in a kilogram, both of these fractions equal 1.

How do we know which one to multiply by?

Remember... We want the kilogram units to cancel.

So, we multiply by $\frac{1{,}000\,g}{1\,kg}$ to convert kilograms to grams.

$7\,\cancel{kg} \cdot \frac{1{,}000\,g}{1\,\cancel{kg}} = 7{,}000\,g$

That gives us 7,000 grams.

Now, we need to convert from grams to ounces.

For the gram units to cancel, we need to multiply by $\frac{1\,oz}{28\,g}$.

$\cancel{250}\ \cancel{1{,}000}\ \cancel{7{,}000\,g} \cdot \frac{1\,oz}{\cancel{28\,g}\ \cancel{4}\ 1} = 250\,oz$

Right. We can cancel the grams...

...and a factor of 7...

...and a factor of 4.

7,000 grams is 250 ounces.

We need 250 ounces of melonberries.

There are 20 ounces of berries in each carton.

So, we need $250 \div 20 = 12\frac{1}{2}$ cartons.

There are 20 ounces per carton, so we multiply by $\frac{1 \text{ carton}}{20 \text{ oz}}$ to convert from ounces to cartons.

Right. If we want, we can even use a conversion factor to convert ounces to cartons!

I see. The ounces cancel, along with a factor of 10.

Then, $\frac{25}{2} = 12\frac{1}{2}$ cartons.

$$\cancel{250}^{25}\ \cancel{\text{oz}} \cdot \frac{1 \text{ carton}}{\cancel{20}_{2}\ \cancel{\text{oz}}} = \frac{25 \text{ cartons}}{2} = 12\tfrac{1}{2} \text{ cartons}$$

By the way, we don't need to convert from kilograms to cartons one step at a time.

We can convert from kilograms...

...to grams...

...to ounces...

...to cartons...

$$7 \text{ kg} \cdot \frac{1{,}000 \text{ g}}{1 \text{ kg}} \cdot \frac{1 \text{ oz}}{28 \text{ g}} \cdot \frac{1 \text{ carton}}{20 \text{ oz}}$$

...all in one step!

The units all cancel, except for cartons!

$$7\,\text{kg} \cdot \frac{1{,}000\,\text{g}}{1\,\text{kg}} \cdot \frac{1\,\text{oz}}{28\,\text{g}} \cdot \frac{1\text{ carton}}{20\,\text{oz}} = \frac{7 \cdot 1{,}000}{28 \cdot 20}\text{ cartons}$$

Then, we simplify.

$$\frac{\overset{1}{\cancel{7}} \cdot \overset{50}{\cancel{1{,}000}}}{\underset{4}{\cancel{28}} \cdot \underset{1}{\cancel{20}}}\text{ cartons}$$

$$= \frac{50}{4}\text{ cartons}$$

$$= 12\tfrac{1}{2}\text{ cartons}$$

Since we can't buy half a carton, I guess we'll need 13 cartons of melonberries.

Do we need anything else?

Oooh... Free-range vegebeasts!

SALE
Brocolemur
Rutabeagle
Cauliflounder

Please DO NOT feed or eat the vegebeasts

FREE-RANGE VEGEBEASTS

SWIPE

Practice: Pages 64-73