Contents: Chapter 10

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Chapter 10:
Fractions

R&G
One-Eighth

What are you doing?

I'm measuring my prize watermelon!

I'm entering it in the contest at the fair.

That's the smallest watermelon I've ever seen!

It's so tiny, I can't even measure it!

What do you mean?

It's not even one inch long!

You can still measure it!

How?!

You put the zero at one end of your watermelon, then see where the other end is on your ruler.

It reaches this little mark here.

0

1

But there are no ***numbers*** between zero and one on my ruler.

They're not written on your ruler, but there ***are*** numbers between zero and one.

???

Your watermelon is longer than zero inches, but shorter than one inch.

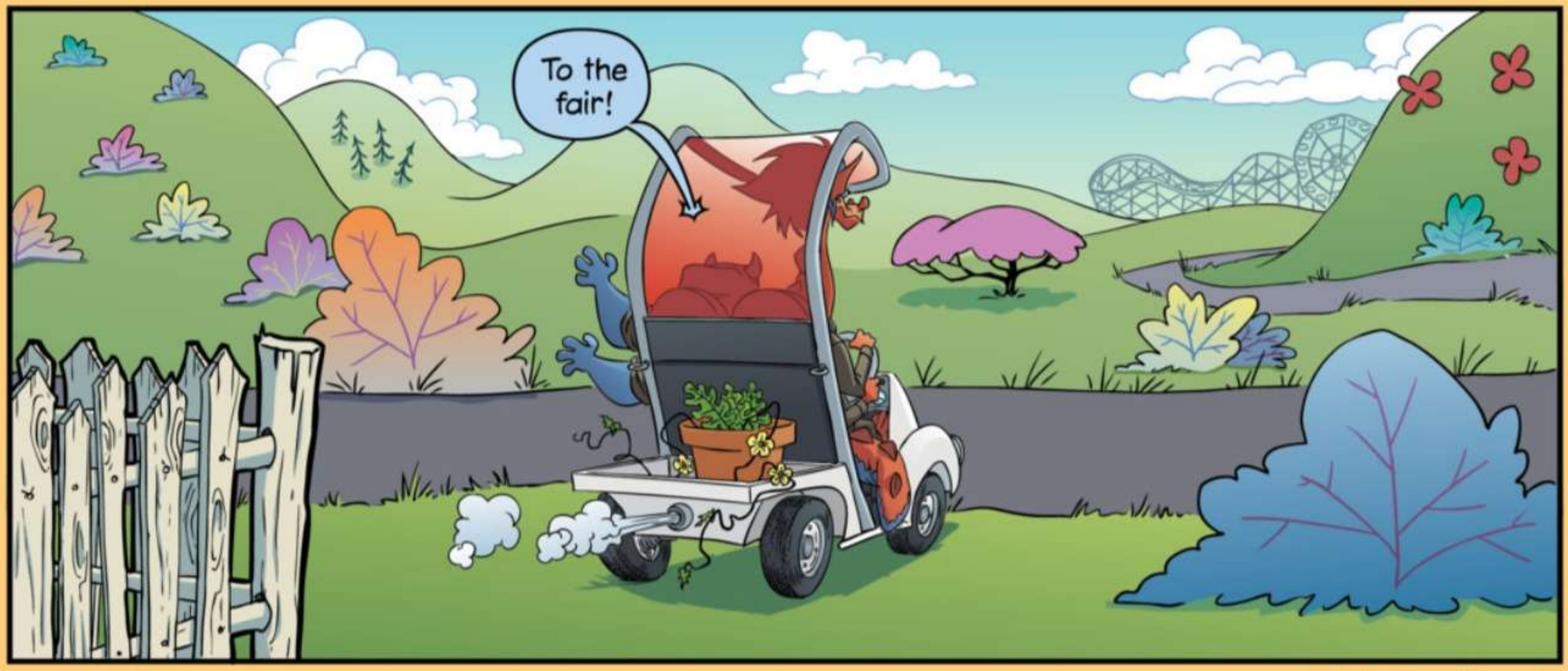
It's only ***part*** of an inch long.

Right, so you need a number to describe ***what*** part of an inch.

Since there are eight of them, the pieces are called "eighths."

My watermelon is as long as one piece!

That's right! It's one-eighth inch long.



To be continued...

Ms. Q.

Unit Fractions

What is a fraction?

It's a number.

It's a division.

It's both!

Right! A fraction is a number that is the result of division.

How would we write 1÷3 as a fraction?

The 1 goes on top. It's the number being divided.

It's called the numerator!

The 3 goes on the bottom. It's the number you are dividing by.

It's called the denominator!

$$1 \div 3 = \frac{1}{3}$$

THE FRACTION $\frac{1}{3}$ IS CALLED "ONE-THIRD" AND CAN ALSO BE WRITTEN 1/3.

Great!

Since a fraction is a number, we can put it on the number line.

Where does $\frac{1}{3}$ belong on the number line?

Where is $\frac{1}{3}$ on the number line?

Let's start with an easier problem.

We can put $\frac{6}{3}$ on the number line.

$\frac{6}{3}$ is $6 \div 3$.

To divide 6 by 3 on the number line, we can split the number line from 0 to 6 into three equal pieces.

Since $6 \div 3 = 2$, each piece has a length of 2.

So, the piece that starts at zero ends at 2.

0 1 2 3 4 5 6 7

SOMETIMES, SOLVING A PROBLEM YOU ALREADY UNDERSTAND CAN HELP YOU FIGURE OUT A SIMILAR PROBLEM THAT YOU HAVEN'T SEEN BEFORE.

To put $\frac{1}{3}$ on the number line, we need to divide 1 by 3.

To divide 1 by 3 on the number line, we can split the number line from 0 to 1 into three equal pieces.

Since $1 \div 3 = \frac{1}{3}$, each piece has a length of $\frac{1}{3}$.

0 $\frac{1}{3}$ 1

The piece that starts at zero must end at $\frac{1}{3}$!

Wonderful! Let's try another.

Where is $\frac{1}{5}$ on the number line?

THE FRACTION $\frac{1}{5}$ IS READ "ONE-FIFTH." $\frac{1}{6}$ IS READ "ONE-SIXTH," $\frac{1}{7}$ IS "ONE-SEVENTH," AND SO ON.

Where is $\frac{1}{5}$ on the number line?

When we found $\frac{1}{3}$, we split the number line between 0 and 1 into three equal pieces.

So, to find $\frac{1}{5}$, we can split the number line between 0 and 1 into ***five*** equal pieces.

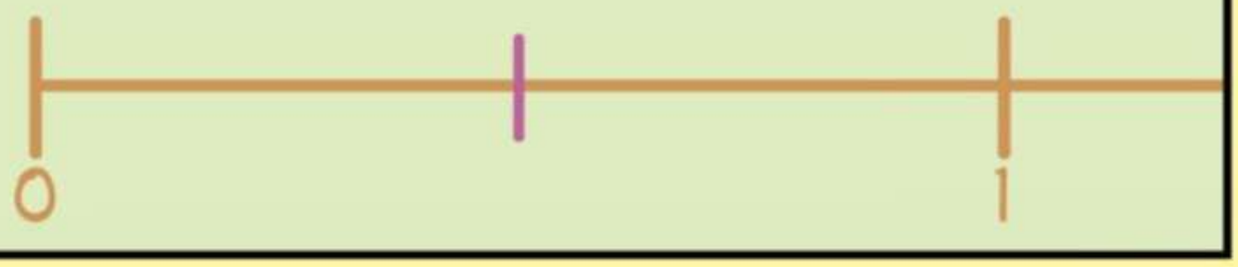
The first piece begins at zero and ends at $\frac{1}{5}$!

Excellent. Try one more. Locate $\frac{1}{16}$ on the number line.

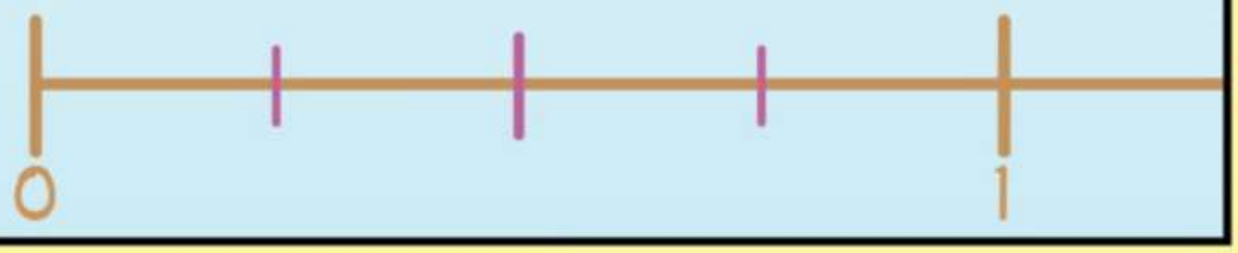
We need to split the number line between 0 and 1 into ***sixteen*** equal pieces.

That's hard!

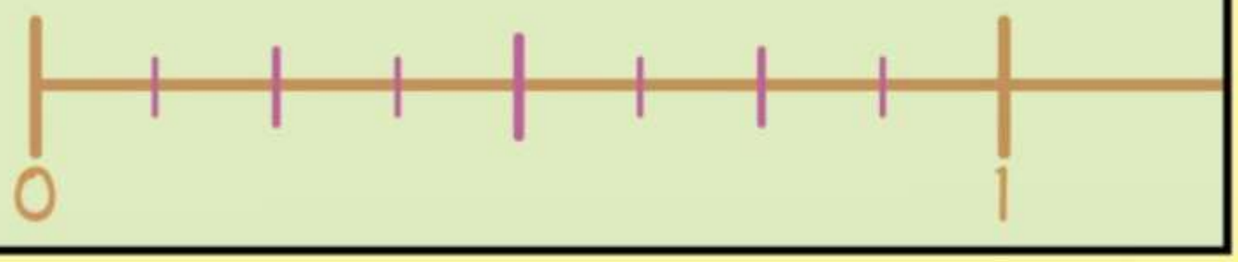
If we add one line exactly between 0 and 1, we get two equal pieces.



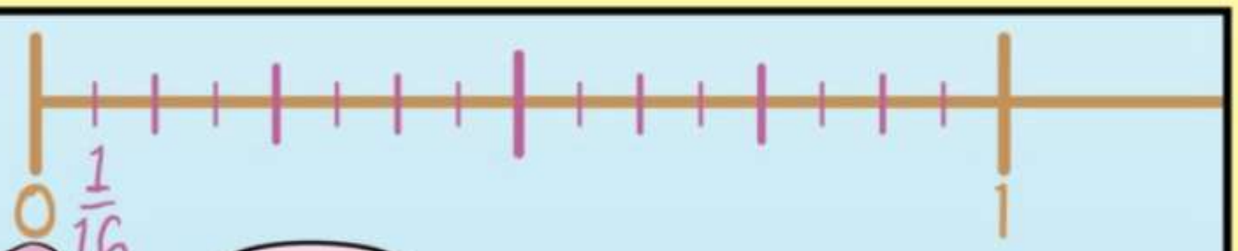
If we put a line in the middle of each of those, we get 2×2=4 pieces.



Splitting each of those down the middle gives us 4×2=8 pieces.



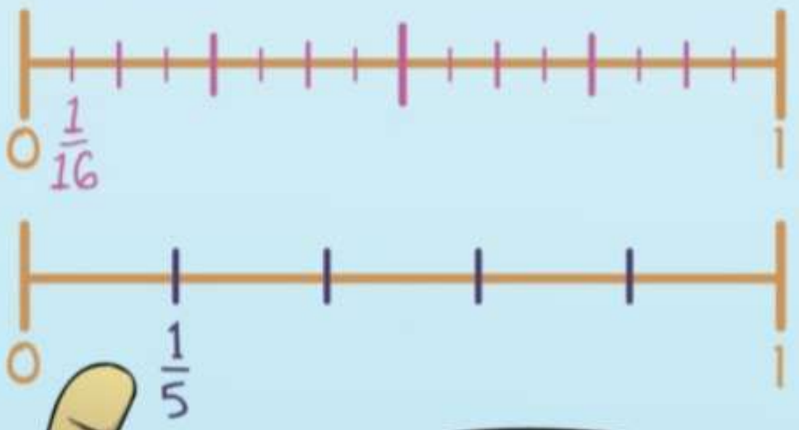
And if we do it one more time, we get 8×2=16 equal pieces.



$\frac{1}{16}$ is right here.

Which fraction is larger: $\frac{1}{5}$, or $\frac{1}{16}$?

$\frac{1}{16}$ is tiny! It's really close to zero.



$\frac{1}{5}$ is farther from zero, so it is bigger than $\frac{1}{16}$!

Perfect! $\frac{1}{5}$ ***is*** greater than $\frac{1}{16}$!

Let's compare two more fractions.

Which is larger: $\frac{1}{35}$ or $\frac{1}{36}$?

We could try putting them on the number line...

...but splitting the line between 0 and 1 into 35 or 36 equal pieces won't be easy.

Which fraction is larger?

When we compared $\frac{1}{5}$ to $\frac{1}{16}$, the fraction with the smaller denominator was the larger number.

That's because when we placed $\frac{1}{5}$ on the number line, we only divided the number line between zero and one into ***five*** equal pieces.

But when we placed $\frac{1}{16}$ on the number line, we divided it into ***sixteen*** pieces.

Each piece had to be smaller!

The more pieces there were, the smaller the pieces had to be!

We can think about $\frac{1}{35}$ and $\frac{1}{36}$ the same way!

If we divide the number line between 0 and 1 into 35 pieces, each piece will be a little bigger than if we divide it into 36 pieces.

Because the more pieces you divide something into, the smaller the pieces have to be!

THAT'S RIGHT! THE MORE EQUAL PIECES YOU DIVIDE SOMETHING INTO, THE SMALLER THE PIECES HAVE TO BE.

So, $\frac{1}{35}$ is a little bigger than $\frac{1}{36}$!

Marvelous work.

The fractions we've been comparing all have 1 in the numerator.

How can we easily compare any two fractions with numerator 1?

A FRACTION WITH 1 IN THE NUMERATOR IS CALLED A *UNIT FRACTION*.

How do we compare two unit fractions?

The larger the denominator, the smaller the fraction.

So $\frac{1}{\text{Grogg}}$ is ***smaller*** than $\frac{1}{\text{Lizzie}}$!

You can't divide by "Grogg!"

Of course you can, I'm the denominator!

Hasta la vista, baby.

I'll be back!

MATH TEAM
Whole-Number Fractions

We learned about a new set of numbers today!

They're called fractions!

They're numbers between zero and one.

Great!

But not every fraction is between zero and one.

Some fractions are bigger than 1.

And some fractions are even ***equal*** to 1!

You just blew my mind.

Remember, a fraction is a division.

So, to find a fraction that equals 1, we need to find a division that equals 1.

Like 5÷5!

$\frac{5}{5} = 5 \div 5 = 1$

Exactly!

So, any fraction with the same numerator and denominator equals 1.*

Because when you divide a number by itself, the answer is always one.

Great work. All these fractions are equal to 1.

$\frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = 1$

*EXCEPT $\frac{0}{0}$, WHICH DOESN'T MAKE ANY SENSE BECAUSE WE CAN'T DIVIDE BY ZERO.

Who can think of a fraction that is equal to 2?

Find one.

All we need is a division that equals 2.

$4 \div 2$ is 2, so $\frac{4}{2}=2$!

$4 \div 2 = \frac{4}{2} = 2$

$6 \div 3$ is 2, so $\frac{6}{3}=2$!

$6 \div 3 = \frac{6}{3} = 2$

And $14 \div 7$ is 2, so $\frac{14}{7}=2$!

$14 \div 7 = \frac{14}{7} = 2$

And since $2 \div 1$ is 2, $\frac{2}{1}$ must be 2!

Because when you divide a number by 1, you get the same number!

$\frac{2}{1}=2$

$\frac{5}{1}=5$

Very good. You can write any whole number as a fraction just by putting it over 1.

$\frac{13}{1}=13$

Next, try 3. Let's make a list of fractions that equal 3.

I'll start. Three over one is three.

$\frac{3}{1}$

Nine thirds is three.

$\frac{9}{3}$

Fifteen over five is three.

$\frac{15}{5}$

And thirty tenths equals three.

$\frac{30}{10}$

What if the denominator is nine?

What value of n will make this fraction equal to 3?

$\frac{n}{9}$

Try it.

For a fraction to equal 3, the numerator has to be three times the denominator.

Since $9 \times 3 = 27$, $\frac{27}{9} = 3$.

That's right!

$\times 3 \left(\frac{n}{9} \quad n = 27 \right.$

There are lots of fractions that equal 3.

We can list fractions with whole numbers on the bottom by putting the multiples of 3 on top.

$\times 3 \left(\frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} = \frac{15}{5} = \frac{18}{6} = \frac{21}{7} = \frac{24}{8} = \frac{27}{9} \right.$

We could go on and on forever.

The numbers on top and bottom keep getting bigger, but the fraction still equals 3!

We're counting by 1's on the bottom and skip-counting by 3's on top.

And all the fractions equal 3!

$= \frac{24}{8} = \frac{27}{9} = \frac{30}{10} = \frac{33}{11} = \frac{36}{12}$

Exactly!

And if we write fractions that equal five?

The numbers on top and bottom get bigger, but the fractions stay the same.

$\frac{5}{1} = \frac{10}{2} = \frac{15}{3} = \frac{20}{4} = \frac{25}{5} = \frac{30}{6} =$

Just because the numbers on the top and bottom of a fraction are big doesn't mean that the fraction is big.

$\frac{999}{999}$ only ***seems*** like a big number.

But it equals 1!

R&G
Equal Fractions

Here we are!

Beast Island's largest smallest fruit and vegetable competition.

FAIR

1ft Corn Dog

Cotton Candy

Candyapples

POPCOR

Funnel cakes

Uh-oh!

What's wrong?

I think your watermelon grew.

Maybe it only *looks* bigger.

Let's measure it.

It was one-eighth inch long when we left...

...and now it's one-fourth inch.

Looks like two-eighths to me.

0

1

That's right, but most of the time we call two-eighths one-fourth.

???

Two-eighths and one-fourth are equal!

ONE-FOURTH IS WRITTEN $\frac{1}{4}$, AND IS SOMETIMES CALLED ONE-QUARTER.

Can you explain why $\frac{2}{8}$ and $\frac{1}{4}$ are equal?

Look at your ruler.

The lines divide each inch into eight equal pieces.

Eighths!

Exactly.

My watermelon is as long as two of these pieces, so it is ***two-eighths*** of an inch long.

0

$\frac{1}{8}$

$\frac{2}{8}$

1

Right, but if we erase all of the smallest lines on the ruler...

...the lines that are left divide each inch into just ***four*** pieces.

0

1

Fourths!

And since my watermelon is as long as one of these pieces, it's ***one-fourth*** of an inch long!

0

$\frac{1}{4}$

1

Yep. So, $\frac{2}{8}=\frac{1}{4}$.

I get it! $\frac{2}{8}$ and $\frac{1}{4}$ are different ways to write the same number.

That's right.

Every fraction can be written lots of ways.

Let's look at $\frac{1}{2}$.

Where is one-half on your ruler?

Find $\frac{1}{2}$ on the ruler.

TWO FRACTIONS THAT ARE DIFFERENT WAYS TO WRITE THE SAME NUMBER ARE CALLED ***EQUIVALENT.***

The tall line in the middle splits the inch into two equal pieces.

Two halves!

So this line marks one-half inch.

0

$\frac{1}{2}$

1

Right, but if we include these two medium lines...

...they split each of the two halves into two equal pieces.

That makes 2×2=4 pieces.

0

$\frac{1}{2}$

1

So, there are four fourths between 0 and 1!

Two of those fourths make one half.

So, $\frac{2}{4}=\frac{1}{2}$!

0

$\frac{2}{4}$

$\frac{1}{2}$

1

These little lines split each of the fourths into two equal pieces.

That makes 4×2=8 pieces.

Eight eighths!

0

$\frac{4}{8}$

$\frac{2}{4}$

$\frac{1}{2}$

1

And it takes four eighths to make one half.

$\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$!

$\frac{3}{6}=\frac{1}{2}$ too!

?

How do you know?

How can you show that $\frac{3}{6}$ and $\frac{1}{2}$ are equal?

Instead of splitting each half into **two** equal pieces, we can split each half into **three** equal pieces.

That makes 2×3=6 sixths.

0 $\frac{1}{2}$ 1

And it takes three of those sixths to equal one-half.

So $\frac{3}{6}=\frac{1}{2}$!

0 $\frac{3}{6}$ $\frac{1}{2}$ 1

I bet we can do other fractions too.

If we start with three pieces, we can label $\frac{1}{3}$.

0 $\frac{1}{3}$ 1

And if we split each of those three pieces into two pieces...

...$\frac{2}{6}=\frac{1}{3}$!

0 $\frac{2}{6}$ $\frac{1}{3}$ 1

Two-eighths equals one-fourth.

That's right.

Two-sixths equals one-third.

Yep. And two-fourths equals one-half.

And two halves make a whole.

Like us!

You complete me.

Ms. Q

Mixed Numbers

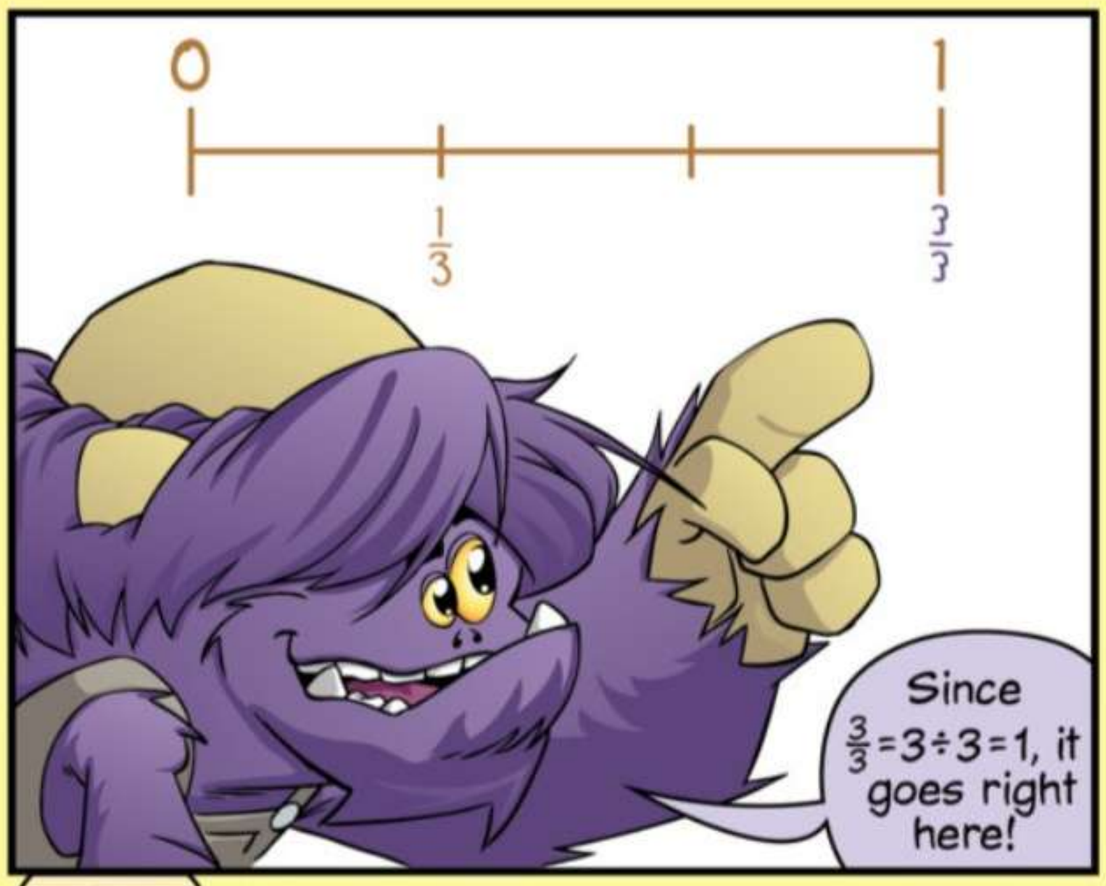
Last class, we began by locating $\frac{1}{3}$ on the number line.

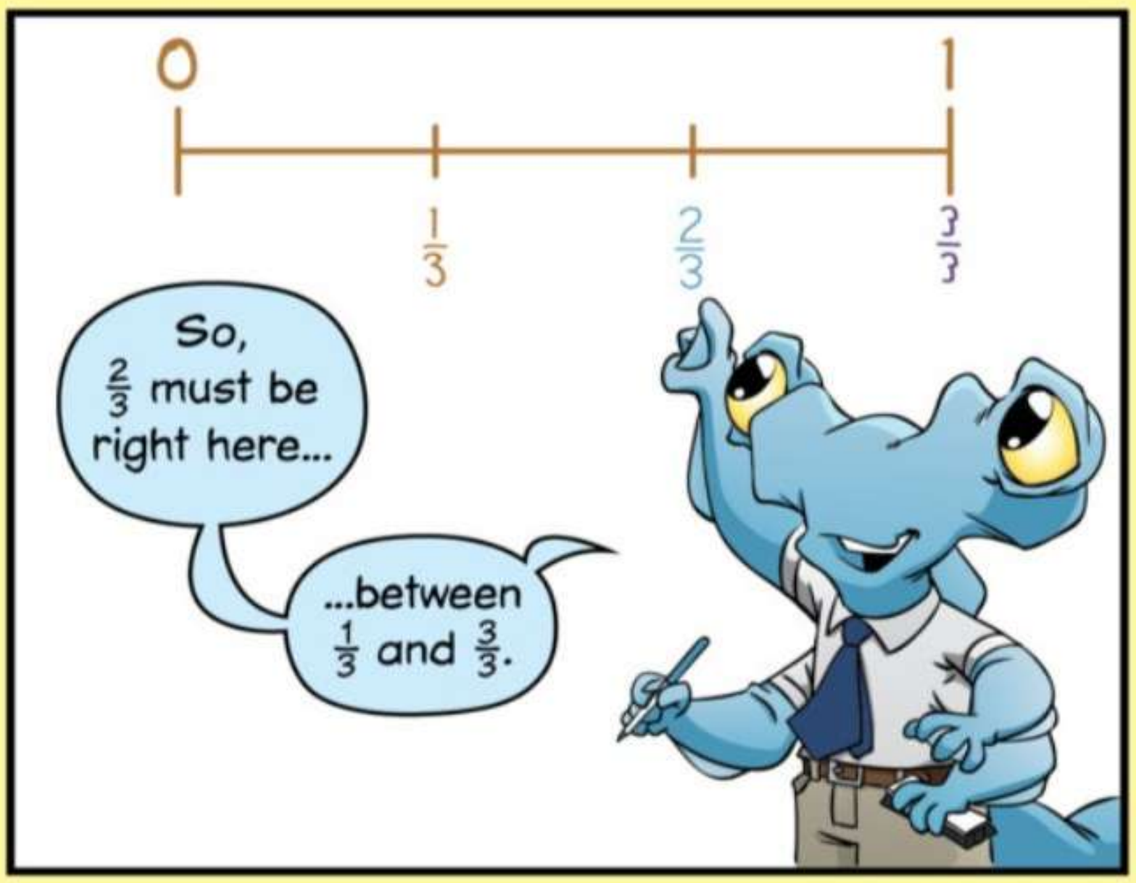
Who can locate $\frac{2}{3}$ on the number line?

That's a good start!

Where is $\frac{3}{3}$ located on the number line?

I know where $\frac{3}{3}$ goes!



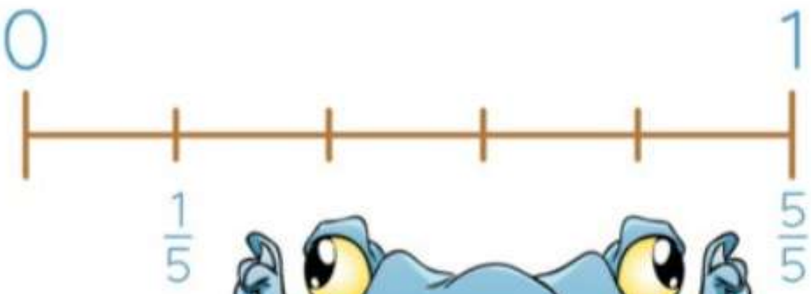


You got it!

The number exactly between one-third and three-thirds is two-thirds.

Where is $\frac{3}{5}$ on the number line?

We can start with the fractions we know.



One-fifth and five-fifths go here and here.

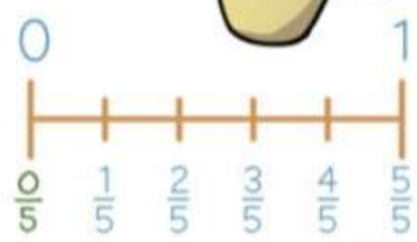
Zero divided by anything is zero.*

So, $\frac{0}{5}=0$.

And these must be $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$!

$\frac{3}{5}$ is right between $\frac{2}{5}$ and $\frac{4}{5}$!

$\frac{3}{5}$ is a little closer to one than it is to zero.

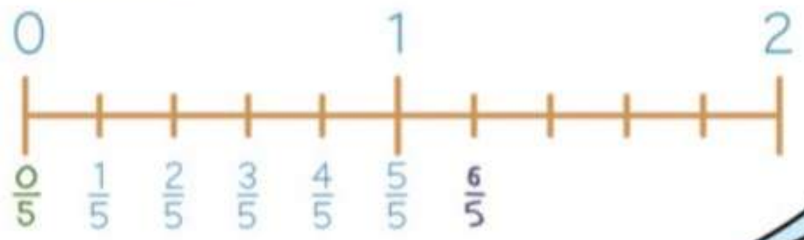


*EXCEPT $\frac{0}{0}$, WHICH DOESN'T MAKE ANY SENSE.

Excellent observations!

Where is $\frac{6}{5}$ on the number line?

Six-fifths is just one fifth past five-fifths!



It's a little more than one.

That's right. Is there an easy way to tell if a fraction is greater than one?

Very good. Let's try another.

Where is $\frac{13}{5}$ on the number line?

If a fraction has the same number on top and bottom, it ***equals*** one.

If the top number is ***bigger*** than the bottom number, then the fraction is ***greater*** than 1.

But if the numerator is ***smaller*** than the denominator, the fraction is ***less*** than 1.

$\frac{20}{20}=1$ $\frac{21}{20}>1$ $\frac{19}{20}<1$

REMEMBER: WHEN USING < AND >, THE ARROW POINTS TO THE SMALLER NUMBER (THE OPEN SIDE "EATS" THE BIGGER NUMBER). FOR EXAMPLE, 1<2 AND 5>4.

Try it.

We can keep counting by fifths past $\frac{6}{5}$...

...all the way up the number line.

$\frac{13}{5}$ is between 2 and 3.

Perfect! Let's try a bigger one. Where is $\frac{51}{5}$ on the number line?

We're gonna need a bigger board!

Instead of drawing the entire number line, can you figure out which whole number $\frac{51}{5}$ is closest to?

Where is $\frac{51}{5}$ on the number line?

Since $\frac{50}{5} = 50 \div 5 = 10$, we know that $\frac{51}{5}$ is a little bit more than 10.

$\frac{51}{5}$ goes right here! Between 10 and 11.

Wonderful!

You can figure out where a fraction belongs on the number line by finding the whole numbers the fraction is between.

What two whole numbers is $\frac{15}{4}$ between?

Solve it.

$\frac{4}{4}=1$. $\frac{8}{4}=2$. $\frac{12}{4}=3$. And $\frac{16}{4}=4$.

1 2 3 4

$\frac{4}{4}$ $\frac{8}{4}$ $\frac{12}{4}$ $\frac{13}{4}$ $\frac{14}{4}$ $\frac{15}{4}$ $\frac{16}{4}$

$\frac{15}{4}$ is between $\frac{12}{4}$ and $\frac{16}{4}$, so it is between 3 and 4.

$\frac{15}{4}$ is three fourths more than 3!

Is $\frac{15}{4}$ equal to $3+\frac{3}{4}$?

If we start at 3 and count up three more fourths, we get to $\frac{15}{4}$.

So $\frac{15}{4}=3+\frac{3}{4}$!

3 4

$\frac{12}{4}$ $\frac{13}{4}$ $\frac{14}{4}$ $\frac{15}{4}$ $\frac{16}{4}$

Right! Since $\frac{15}{4}$ is three fourths more than 3, we can write $\frac{15}{4}$ as $3\frac{3}{4}$.

$3\frac{3}{4}$ is called a ***mixed number***.

How could you write $\frac{13}{6}$ as a mixed number?

$3\frac{3}{4}$ IS READ "THREE AND THREE-FOURTHS" AND MEANS THE SAME THING AS $3+\frac{3}{4}$.

$\frac{12}{6}$ equals 2.

We need one more sixth to get from $\frac{12}{6}$ to $\frac{13}{6}$.

2 3

$\frac{12}{6}$ $\frac{13}{6}$

So $\frac{13}{6}=2\frac{1}{6}$!

Superb! When a fraction's numerator is greater than its denominator, we can write the fraction as a mixed number.

Why are mixed numbers useful?

SOME PEOPLE CALL A FRACTION WITH A NUMERATOR THAT IS LARGER THAN ITS DENOMINATOR "IMPROPER." SINCE THERE'S NOTHING WRONG WITH "IMPROPER" FRACTIONS, WE PREFER JUST TO CALL THEM FRACTIONS.

It's easier to tell how big the fraction is when it's written as a mixed number.

If a recipe calls for $\frac{7}{3}$ cups of flour, it's easier if it says $2\frac{1}{3}$ cups.

If you are $8\frac{1}{2}$ years old...

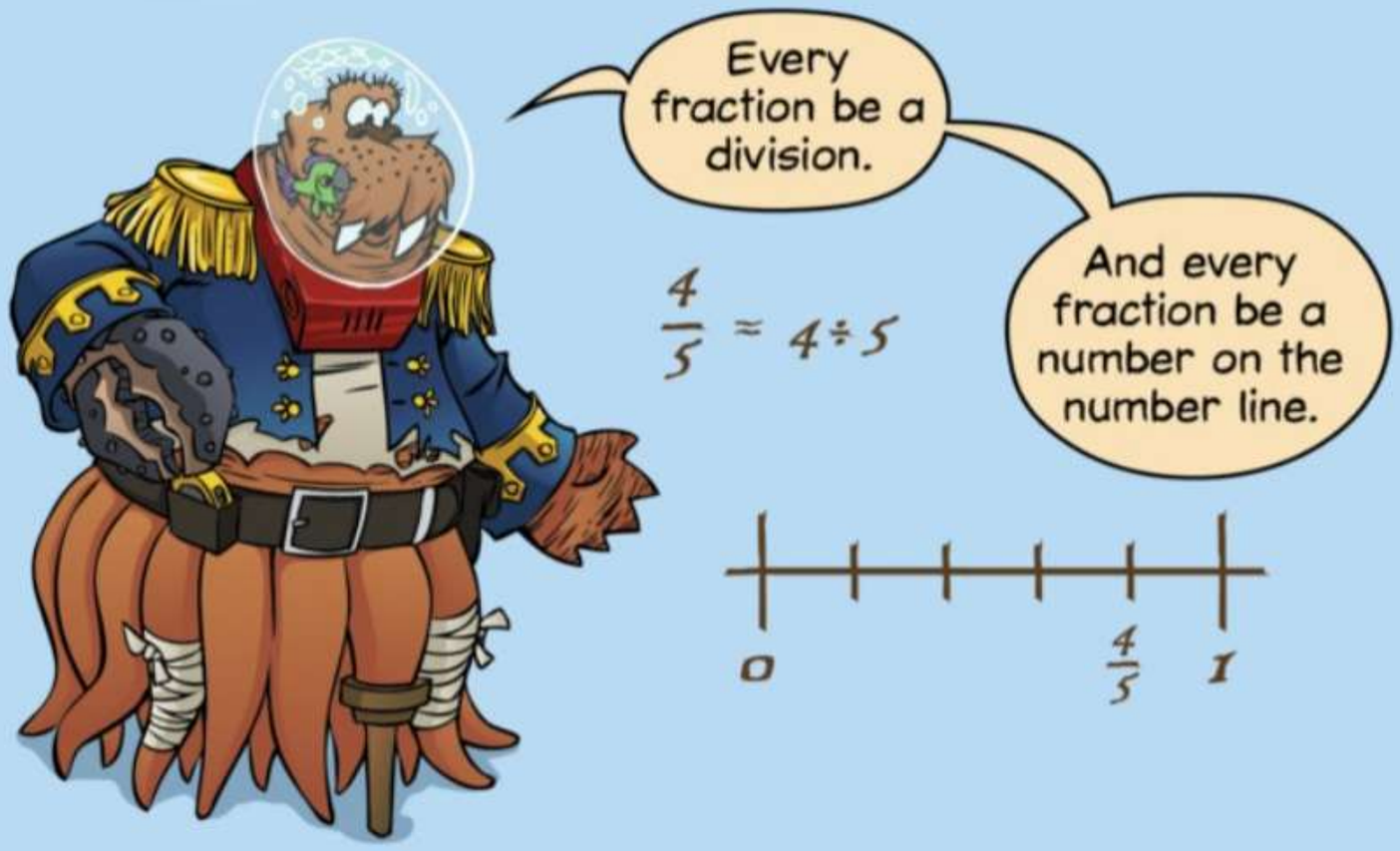
...it's confusing if you say that you are $\frac{17}{2}$ years old.

17 quarters equals 4 dollars and one quarter: $\frac{17}{4}=4\frac{1}{4}$.

And the thirteen half-meatballs in my pocket...

...make $6\frac{1}{2}$ meatballs! Want one?

Grogg!



REMEMBER, $\frac{1}{2}$ IS A NUMBER. SO, "$\frac{1}{2}$ OF AN APPLE" DESCRIBES AN AMOUNT IN THE SAME WAY THE NUMBER 3 IS USED TO DESCRIBE "3 APPLES."

With nine pirates in me crew, we be dividin' the treasure nine ways.

Each o' the nine parts be $\frac{1}{9}$ of the whole.

'Tis usually a simple task.

But some treasures be more difficult to divide than others.

Like what?

Like a priceless paintin'.

How do you divide a ***painting*** between nine pirates?

By cuttin' it into nine equal rectangles!

Each pirate be gettin' $\frac{1}{9}$ of the whole paintin'.

That's ridiculous!

Aye. 'Twas a poor choice.

Here be my part o' the paintin'...

...$\frac{1}{9}$ of the Mona Beasta.

Other treasures be presentin' different difficulties.

On one voyage, we found two platinum discs.

'Tis no easy task, dividin' two discs equally among nine pirates.

How could the discs be cut so that each pirate be gettin' an equal share?

How could it be done?

If you cut one disc into four pieces...

...and the other disc into five pieces...

...that makes 9 pieces, but they're not all the ***same***.

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$

But if you cut both discs into nine pieces, then each pirate could get two pieces.

Each piece is one-ninth of a disc, so each pirate would get two-ninths of a disc.

That makes sense! We divided 2 discs between 9 pirates, and each pirate got two-ninths of a disc!

$2 \div 9 = \frac{2}{9}$!

So, if you divide ***three*** discs between 9 pirates, each pirate would get $3 \div 9 = \frac{3}{9}$ of a disc!

Aye, $\frac{3}{9}$. But there be a way to divide three discs without cuttin' each into nine pieces.

How could it be done?

We can cut each disc into ***three*** equal pieces.

Each pirate gets $\frac{1}{3}$ of a disc.

But, you said $\frac{3}{9}$ was correct.

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

Which is correct, $\frac{1}{3}$ or $\frac{3}{9}$?

Both ***arrrr!***

I know! $\frac{1}{3}$ ***equals*** $\frac{3}{9}$!

If each third is cut into three pieces, then there are nine total pieces.

Ninths!

Right, but it takes three ninths to make one third!

$\frac{3}{9} = \frac{1}{3}$!

Excellent figurin', little buccaneers!

Let's try one more.

Here be the most troublin' treasure I ever divided.

Each o' these three cheese wheels be a different size.

How could we be dividin' this cheese equally among nine pirates?

How would you do it?

You *can't* just give each pirate one-third of a wheel.

$\frac{1}{3}$ of a ***small*** cheese wheel is less than $\frac{1}{3}$ of a ***big*** cheese wheel.

To compare two fractions, they have to be part of the same whole.

One-ninth of the Mona Beasta is not the same as one-ninth of a cheese wheel!

Aye! So, how can we be dividin' the cheese?

You would need to cut all three wheels into 9 wedges.

Every pirate would get $\frac{1}{9}$ of each wheel.

Aye. Tis' the best way for each pirate to receive an equal share.

Here be me three cheese wedges.

Who be hankerin' fer a hunk o' cheese?