

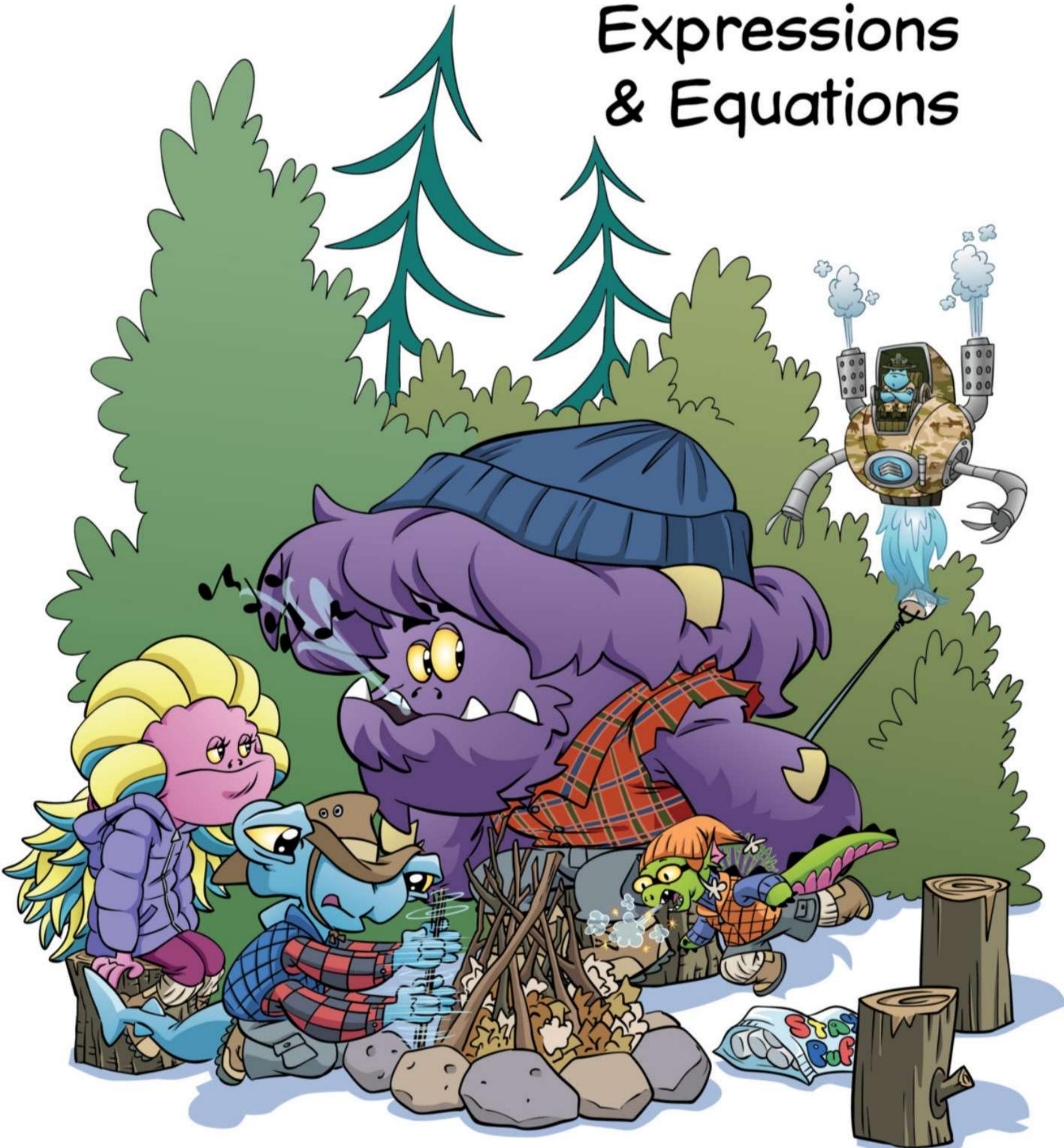
# Contents: Chapter 3

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# Chapter 3:

## Expressions & Equations



# G\*Y\*M NOTATION

Ten-hut!

Until now, you little monsters have only been using the most basic math symbols.

Today, we take off our training wheels and learn to write like true math beasts!

Hammerhead,  
how would you write  
"three times  $x$ "?

Sir, like  
this, sir?

That  
multiplication  
symbol looks  
just like an  $x$ !

$3 \times x$

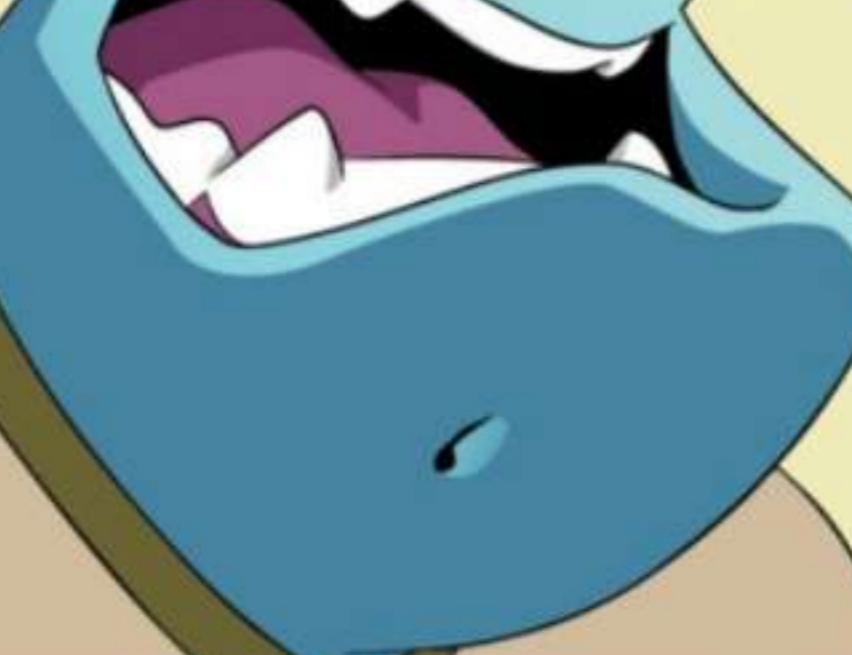
Starting today, you will  
replace the rudimentary  
multiplication symbol with  
a dot.

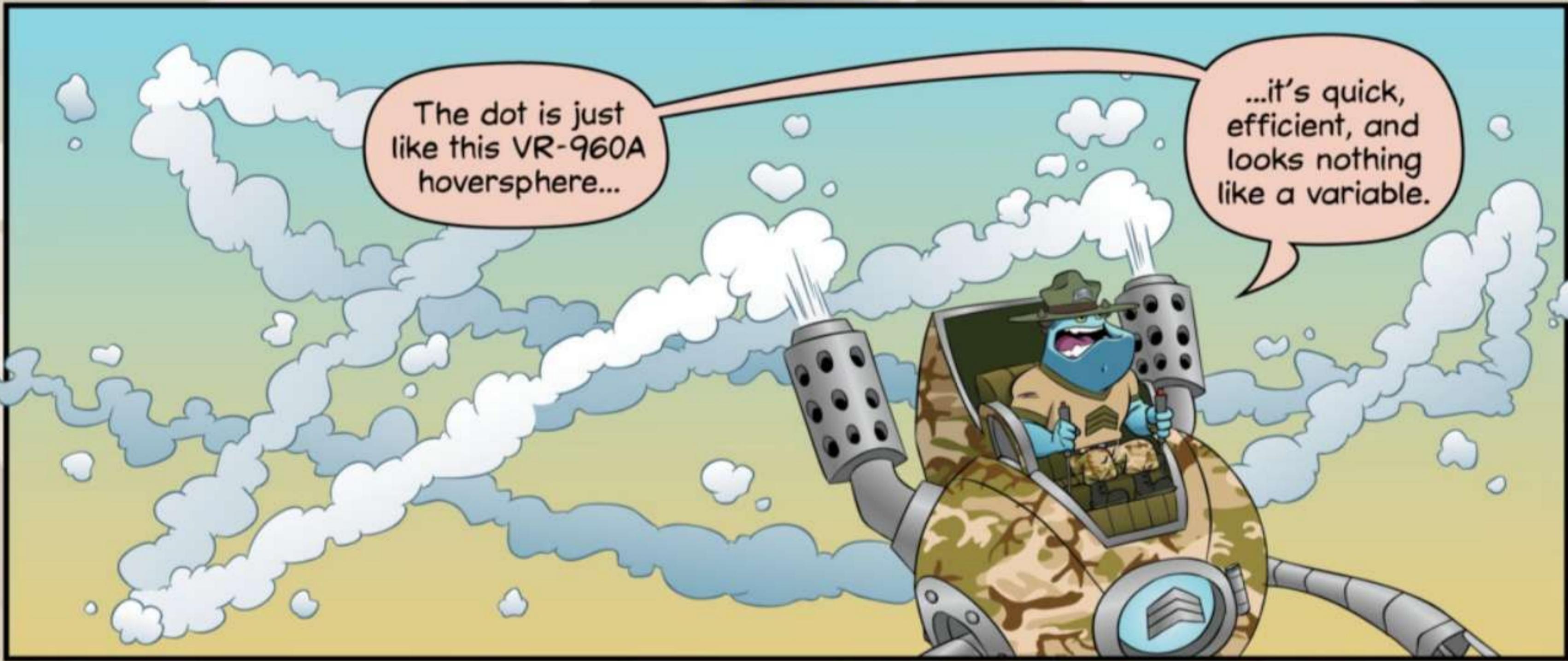
A  
dot?

In the heat  
of mathematical  
computation,  
we can't afford  
that kind of  
confusion!

$$\begin{aligned}2 \cdot 5 &= 10 \\7 \cdot 6 &= 42 \\8 \cdot 4 &= 32 \\3 \cdot 9 &= 27\end{aligned}$$

That's  
right,  
Grape  
Ape.





Sometimes, we ditch the dot entirely.

$3 \cdot x = 3x$

The product of a number and a variable is written without a dot.

"Three times  $x$ " is written "3x."

WHEN WE WRITE THE PRODUCT OF A NUMBER AND A VARIABLE, THE NUMBER IS WRITTEN BEFORE THE VARIABLE. ( $3x$ , NOT  $x3$ .)

$4 \cdot (-5) = 4(-5) = -20$

$9 \cdot (-7) = 9(-7) = -63$

For most products, the dot is unnecessary.

We can use parentheses to represent multiplication.

THE EXPRESSIONS  $9 \times (-7)$ ,  $9 \cdot (-7)$ ,  $9(-7)$ , AND  $(9)(-7)$  ALL MEAN "9 TIMES NEGATIVE 7."

These expressions may look weird and unfamiliar to you now, but with practice you will learn to use dots and parentheses to write multiplication.

Time to drill!

Evaluate these four expressions.

$-9 \cdot 2$

$-7(-6)$

$8(6-9)$

$(3+4)(1-5)$

Try all four.

The dot between -9 and 2 means we multiply -9 times 2.

$$\begin{aligned}-9 \cdot 2 \\ = -18\end{aligned}$$



$$\begin{aligned}-9 \cdot 2 \\ = -18.\end{aligned}$$

Here, we multiply -7 times -6. The product of two negatives is positive.

So, -7 times -6 is 42.

$$\begin{aligned}-7(-6) \\ = 42\end{aligned}$$



Since  $6-9$  is in parentheses, we subtract first.  
 $6-9=-3$ .

Then, 8 times -3 is -24.

$$\begin{aligned}8(6-9) \\ = 8(-3) \\ = -24\end{aligned}$$



And for this last one, we evaluate what's in both pairs of parentheses.

$$\begin{aligned}(3+4)(1-5) \\ = (7)(-4) \\ = -28\end{aligned}$$



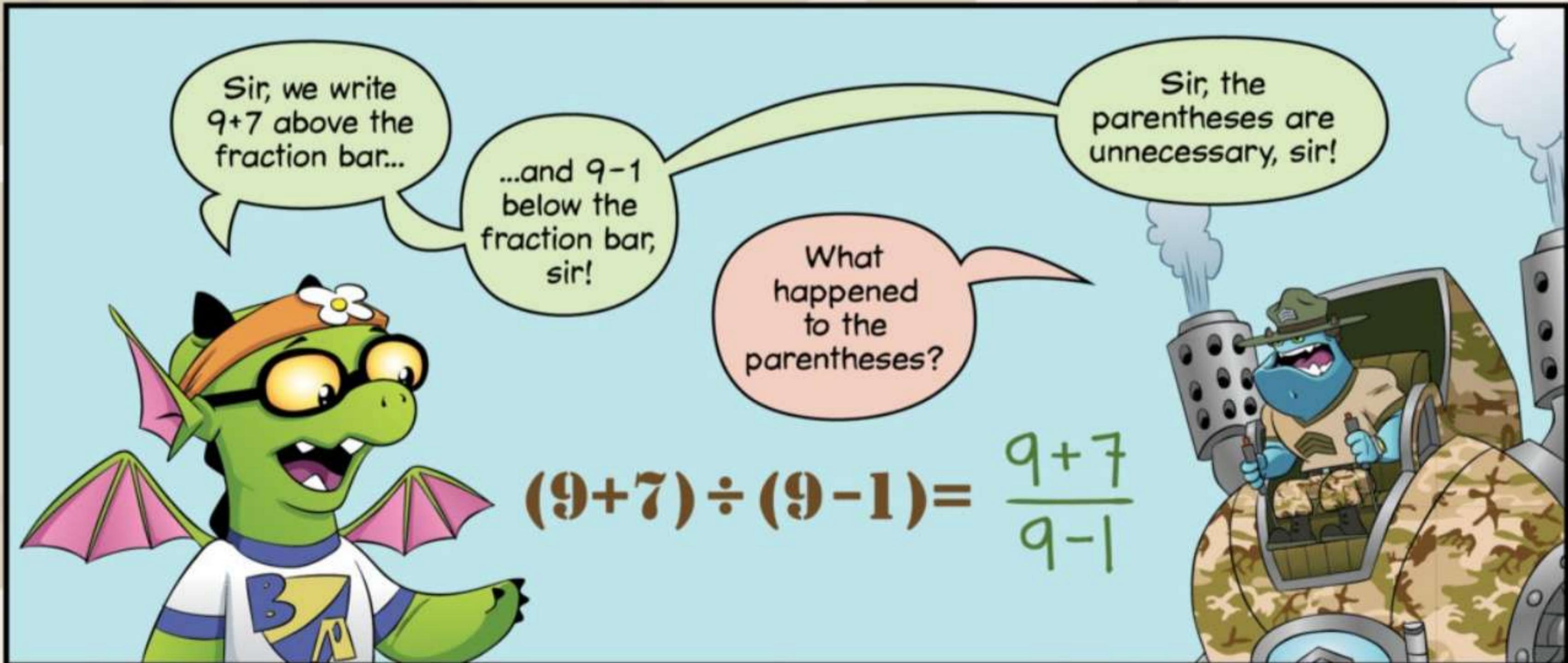
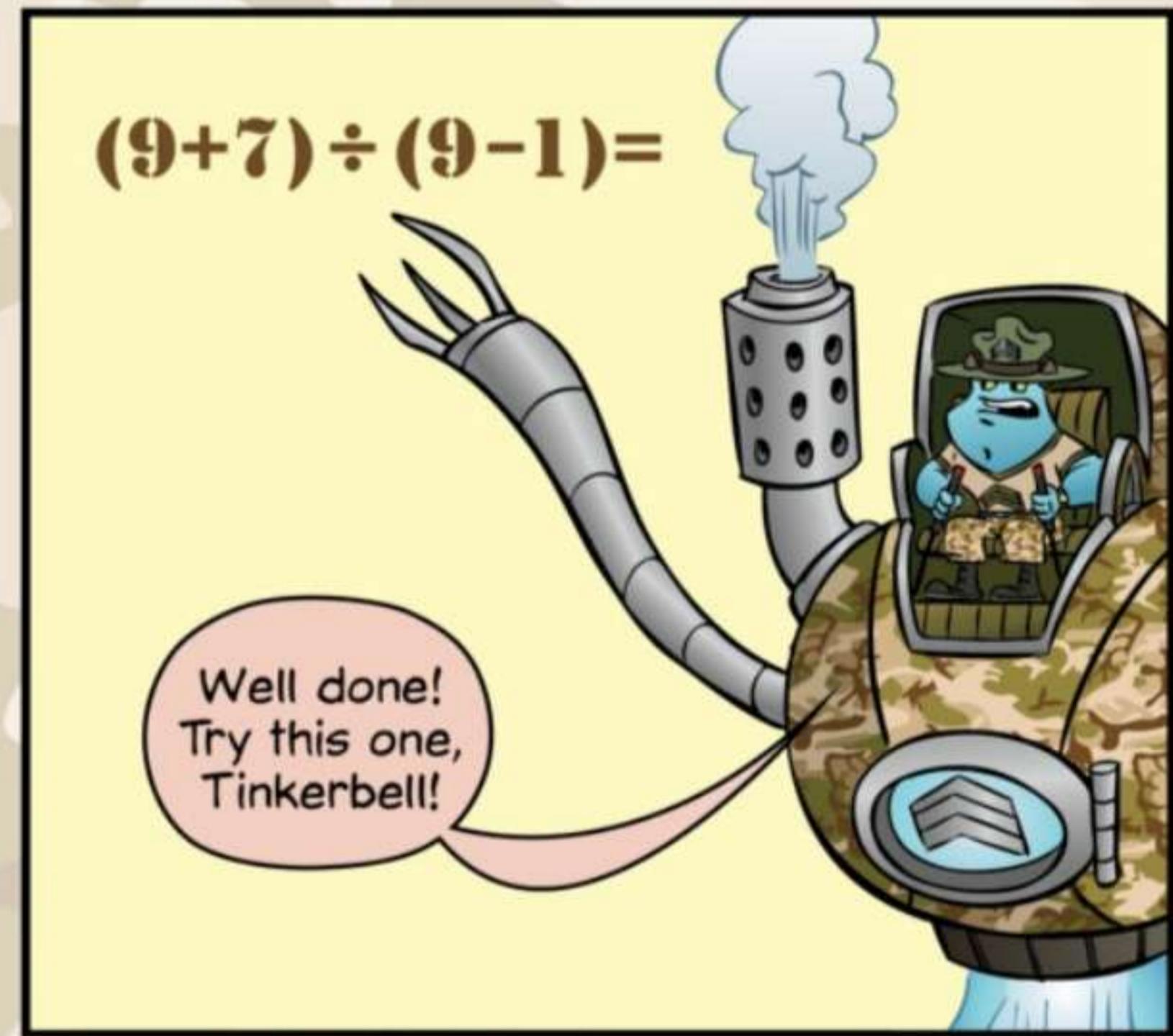
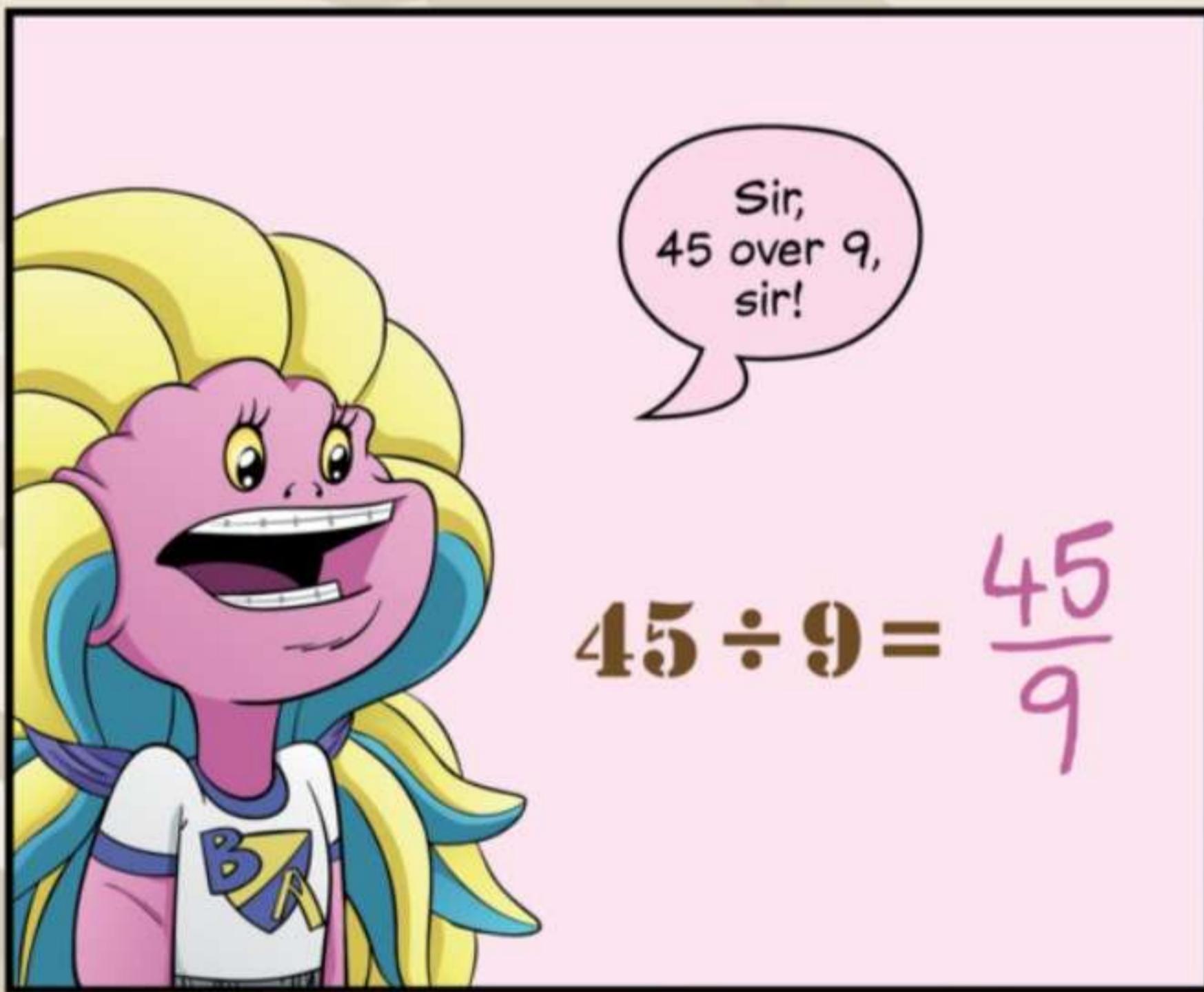
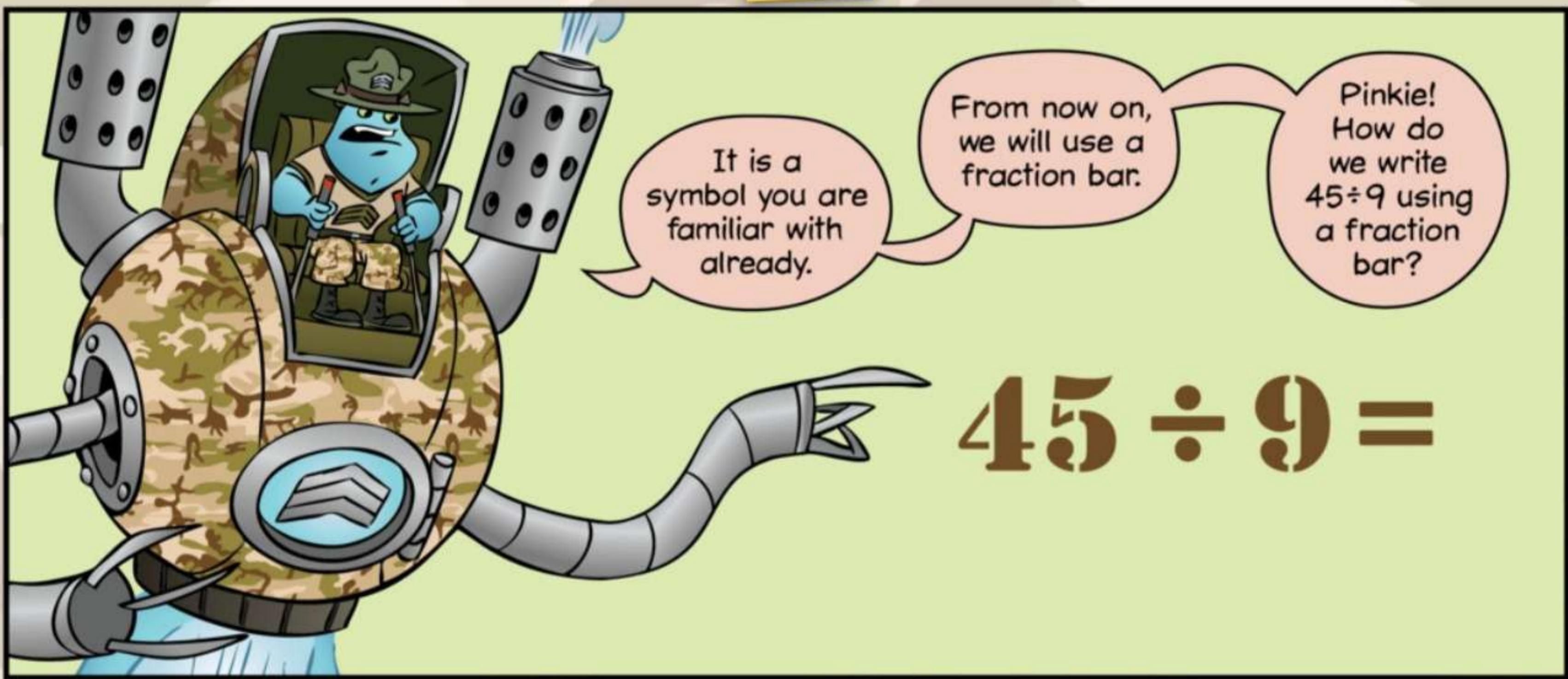
Then, we multiply  $(7)(-4) = -28$ .

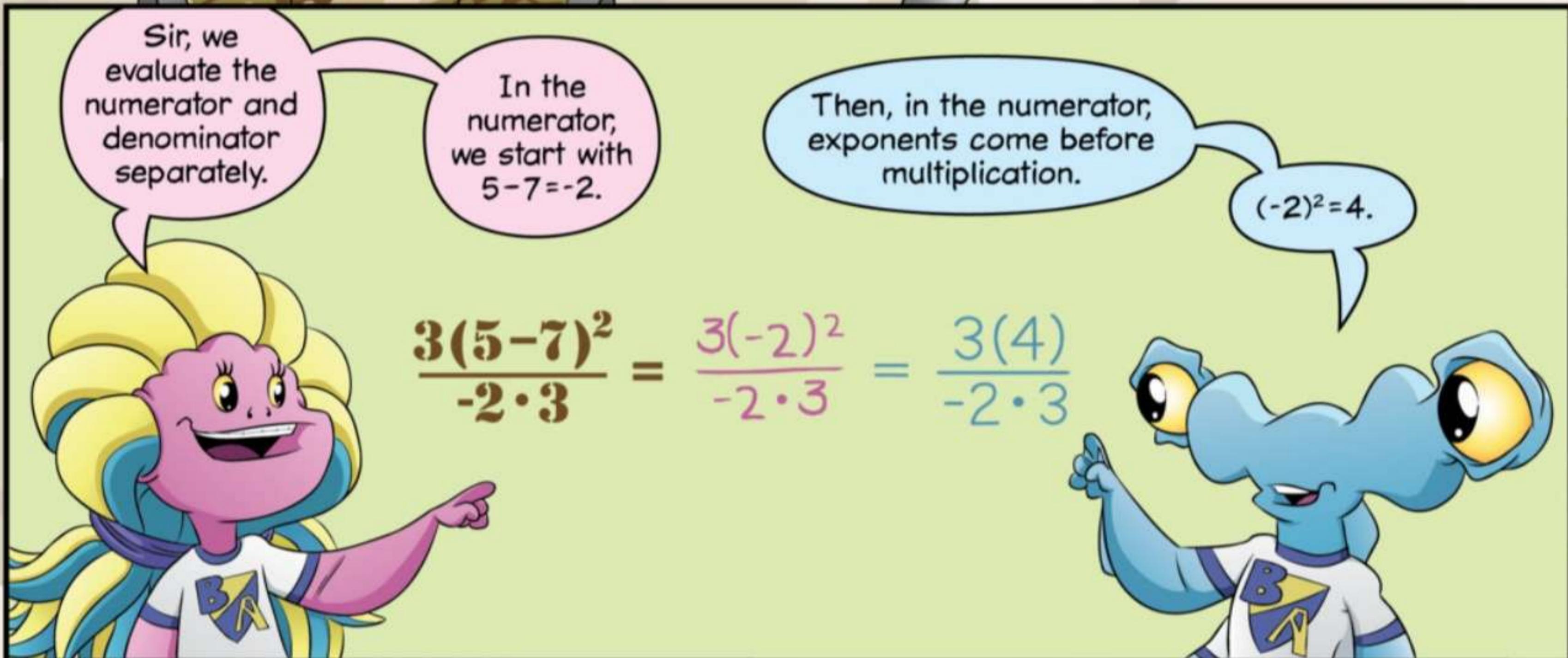
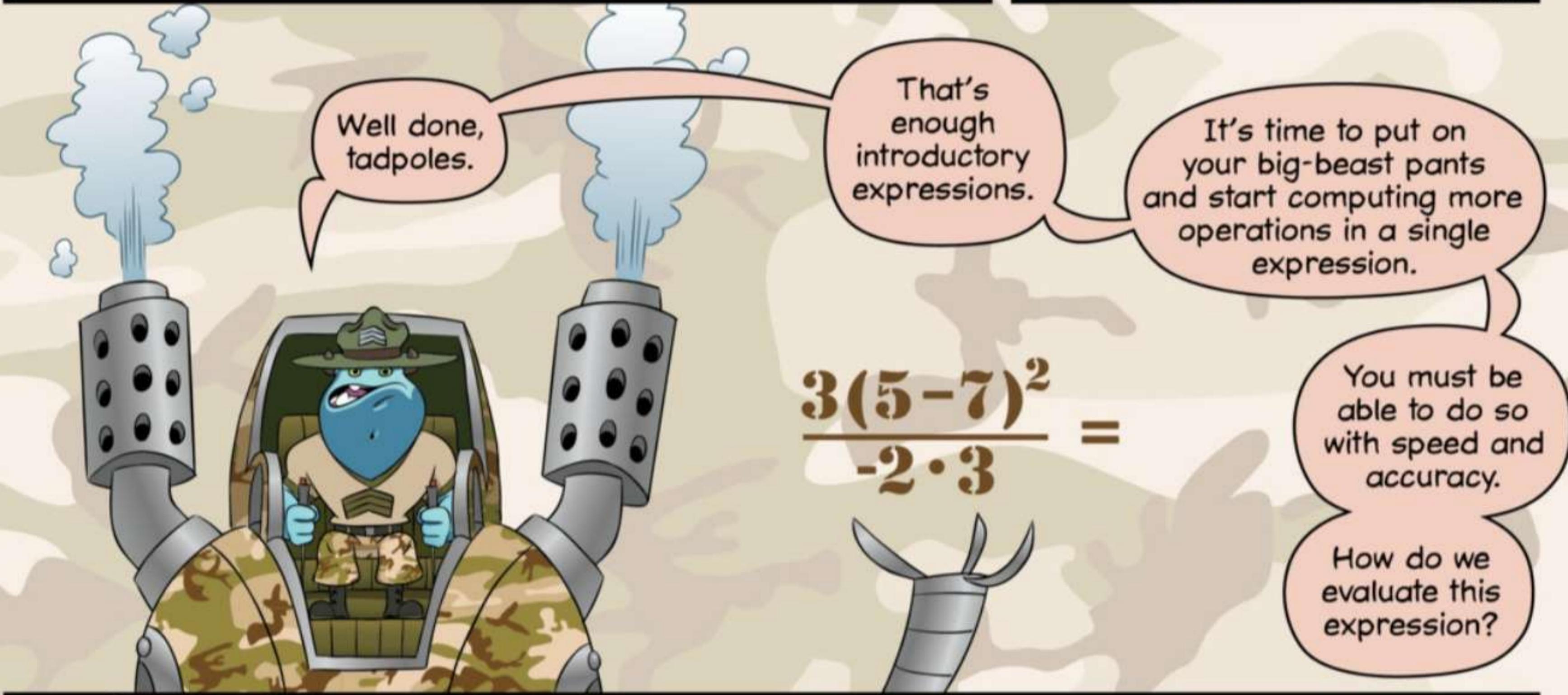
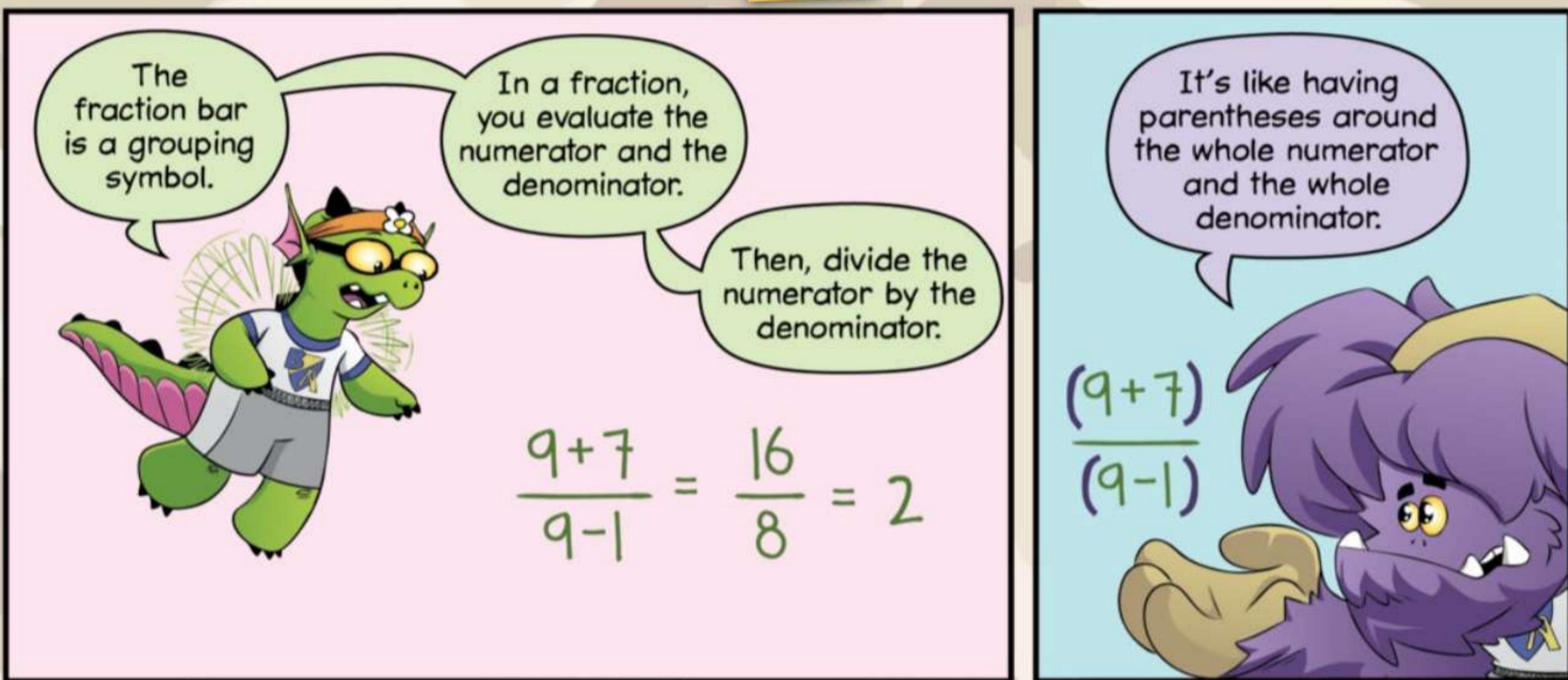
Good work, polliwogs.

Next, not only will your multiplication expressions look different from now on...  
...we will also phase out this symbol for division.

Sir, what symbol will replace the division symbol, sir?!







Now, we can compute the numerator and the denominator.

3 times 4 is 12, and -2 times 3 is -6.

$$\frac{3(5-7)^2}{-2 \cdot 3} = \frac{3(-2)^2}{-2 \cdot 3} = \frac{3(4)}{-2 \cdot 3} = \frac{12}{-6} = -2$$

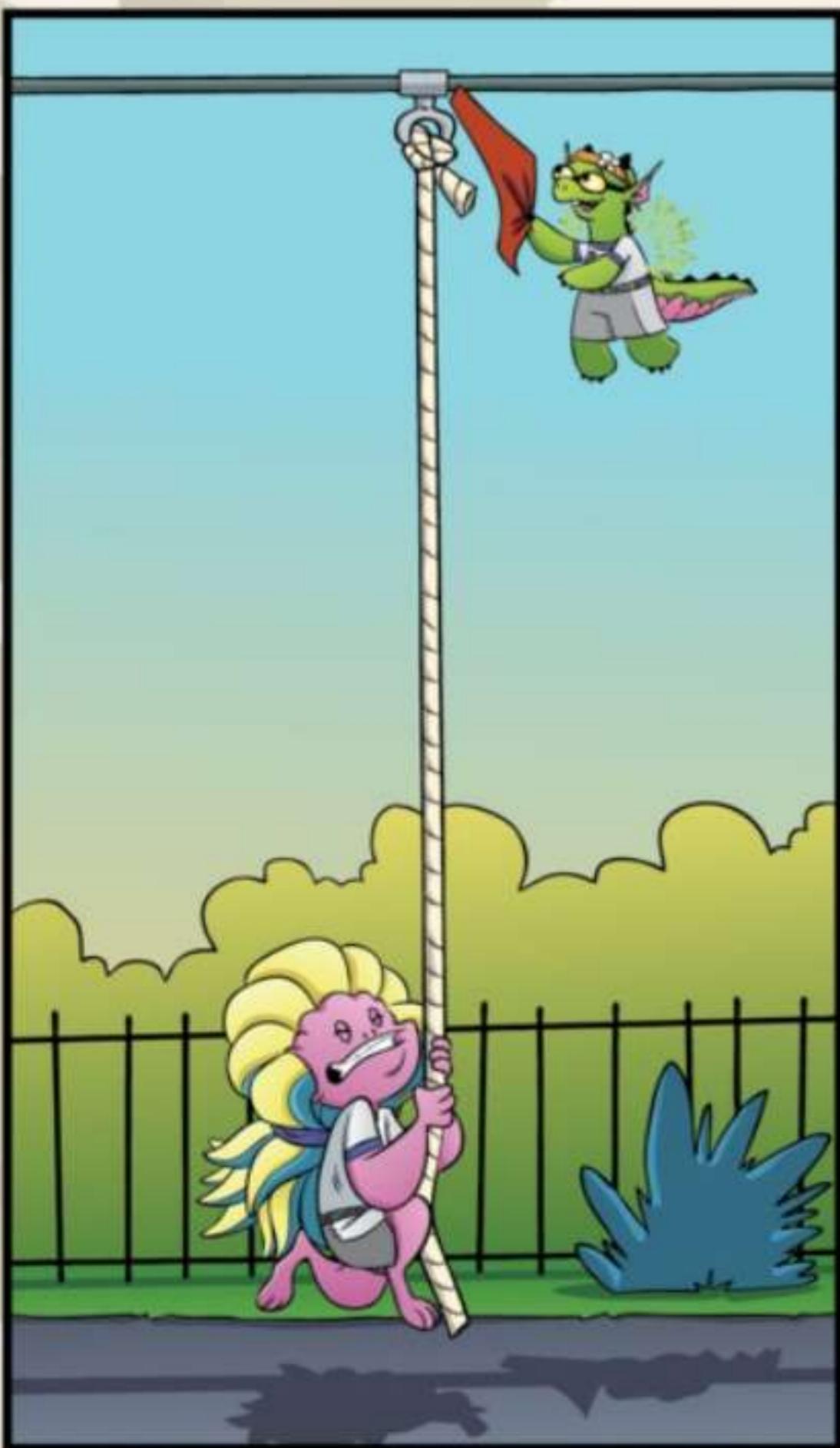
And to finish, we divide 12 by -6 to get -2, sir!

That's right.

Your math skills are in peak condition.

But, your physical skills need work before this weekend's outdoor retreat.

It's time for physical fitness drills!



**Ms.  
Q.**  
Simplifying

What is  
the value of  
 $15x$  when  $x$   
is 6?

$15x$  means  
15 times  $x$ .

If  $x=6$ ,  
then we have  
 $15x=15 \cdot 6$ .

So, when  $x$   
is 6,  
 $15x$  is 90.

$$\begin{aligned}15x &= 15 \cdot 6 \\&= 90\end{aligned}$$

Very  
good.

What is  
the value of  
 $7n+3n$ , when  $n$   
is 27?

First, we replace the  
 $n$ 's with 27's and find the  
products:  $7n=7(27)$  and  
 $3n=3(27)$ .

So, we have  
 $189+81$ , which  
is 270.

$$\begin{aligned}7n + 3n &= 7(27) + 3(27) \\&= 189 + 81 \\&= 270\end{aligned}$$

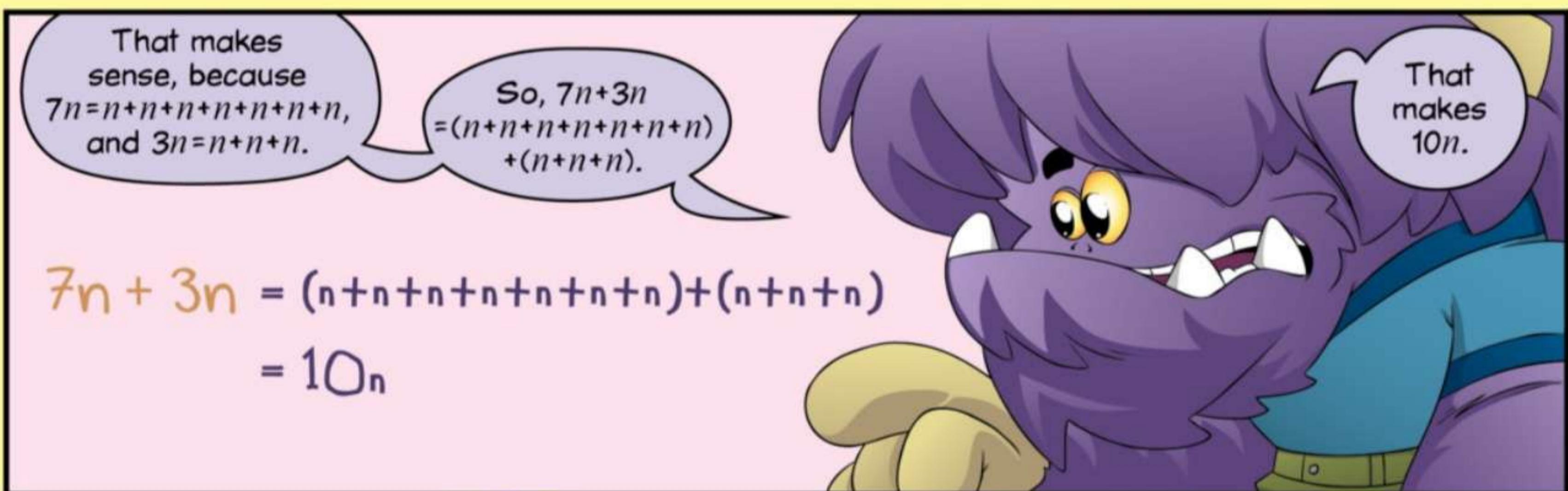
I think  
I know a  
better way  
to compute  
 $7n+3n$ .

How,  
Winnie?

We can  
**simplify** the  
expression by  
adding  $7n+3n$ .

How do  
we add  
 $7n+3n$ ?





We can factor out the  $a$  in  $5a - 3a$  to get  $(5-3)a$ .

So,  
 $5a - 3a = 2a$ .



$$\begin{aligned} 5a - 3a &= (5-3)a \\ &= 2a \end{aligned}$$

Of course!

If I have 5 somethings, and you take 3 somethings away...

...I'm left with 2 somethings.

For the second problem, it's important to see that  $x$  can be written as  $1x$ .

We can factor out the  $x$ .



$$\begin{aligned} 1x + 2x + 3x &= (1+2+3)x \\ &= 6x \end{aligned}$$

So,  
 $1x + 2x + 3x = (1+2+3)x = 6x$ .



Elementary!  $1+2+3$  of anything gives us 6 of that thing. Adding  $x$ 's is no different.

So,  
 $1x + 2x + 3x = 6x$ .

For this problem, we have  $j$ 's and  $k$ 's.

We can't factor  $j$  or  $k$  from the whole expression.

$$2j + 3k + 5j - k$$



We can simplify the  $j$ 's and  $k$ 's separately.



Great idea. Let's do some rearranging.

We can put the  $j$ 's together and the  $k$ 's together...

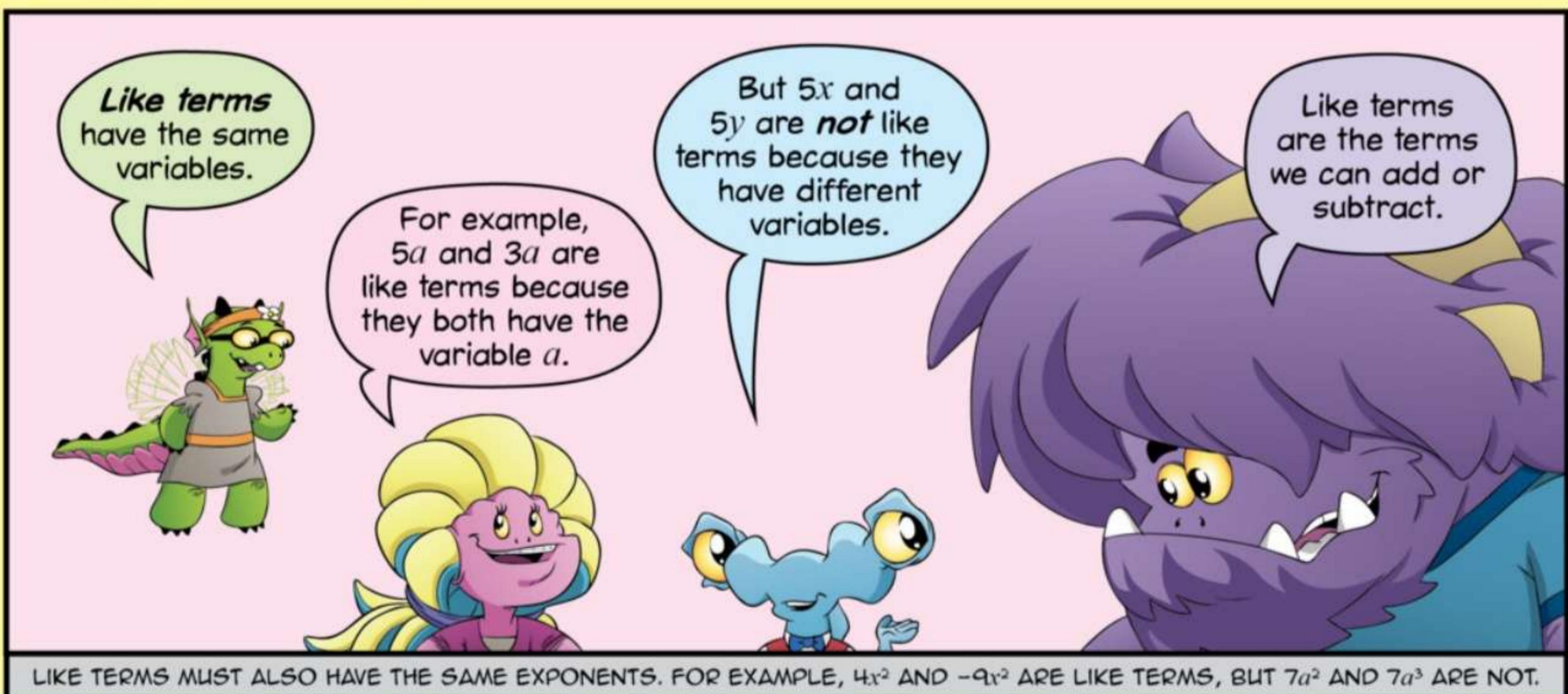
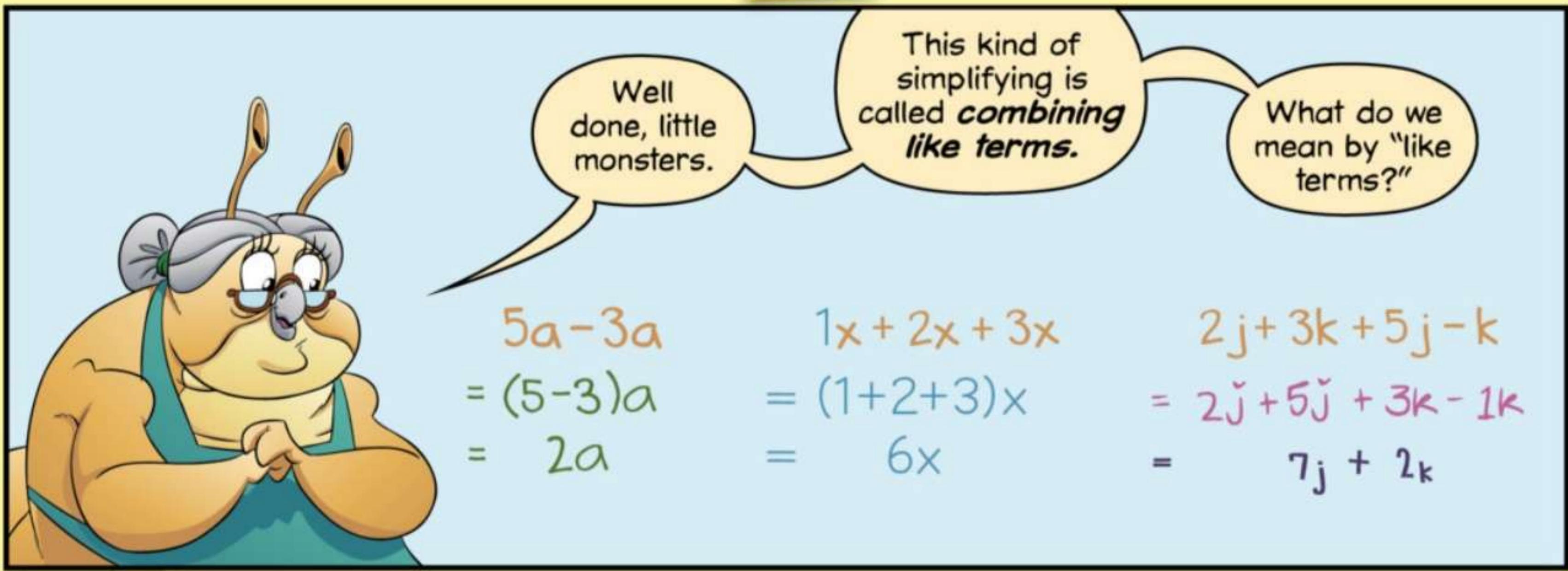
...and write a 1 in front of the  $k$ , if we want.

Then,  
 $2j + 5j = 7j$ , and  
 $3k - 1k = 2k$ .

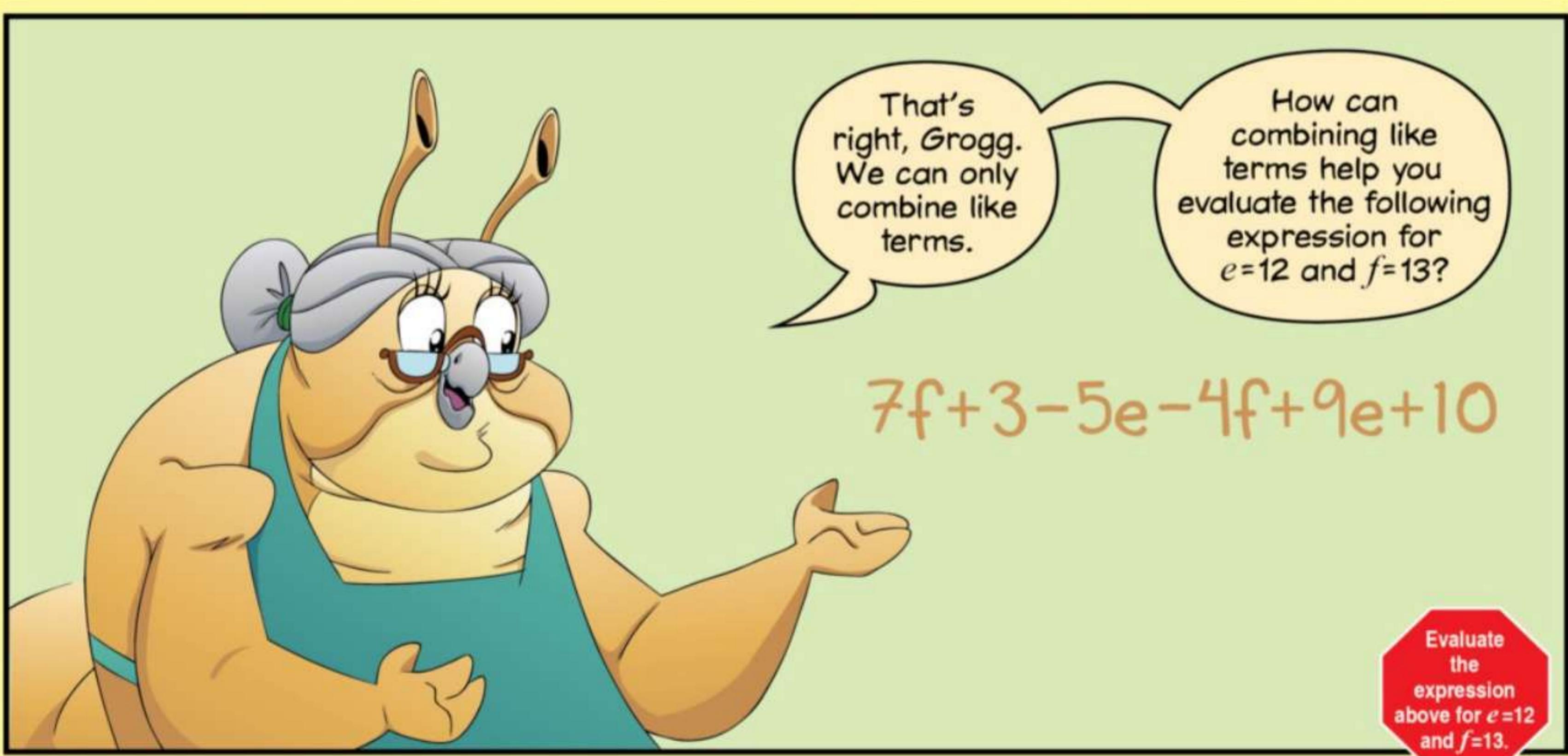
So, we get  
 $7j + 2k$ .

$$\begin{aligned} 2j + 3k + 5j - k &= 2\cancel{j} + 5\cancel{j} + 3k - 1k \\ &= 7j + 2k \end{aligned}$$





LIKE TERMS MUST ALSO HAVE THE SAME EXPONENTS. FOR EXAMPLE,  $4x^2$  AND  $-9x^2$  ARE LIKE TERMS, BUT  $7a^2$  AND  $7a^3$  ARE NOT.



First, we put the  $e$ 's together and the  $f$ 's together.

We can change all the subtraction to addition, then rearrange.

That way, the like terms are together.

$$\begin{aligned} & 7f + 3 - 5e - 4f + 9e + 10 \\ = & 7f + 3 + (-5e) + (-4f) + 9e + 10 \\ = & \underbrace{7f + (-4f)}_{3f} + \underbrace{(-5e) + 9e}_{4e} + \underbrace{3 + 10}_{13} \end{aligned}$$

Then we add the  $e$ 's, the  $f$ 's, and the numbers separately.



$$\begin{aligned} & 7f + 3 - 5e - 4f + 9e + 10 \\ = & 7f + 3 + (-5e) + (-4f) + 9e + 10 \\ = & \underbrace{7f + (-4f)}_{3f} + \underbrace{(-5e) + 9e}_{4e} + \underbrace{3 + 10}_{13} \\ = & 3f + 4e + 13 \end{aligned}$$

Then, since  $e=12$  and  $f=13$ , we have  $3(13)+4(12)+13$ .

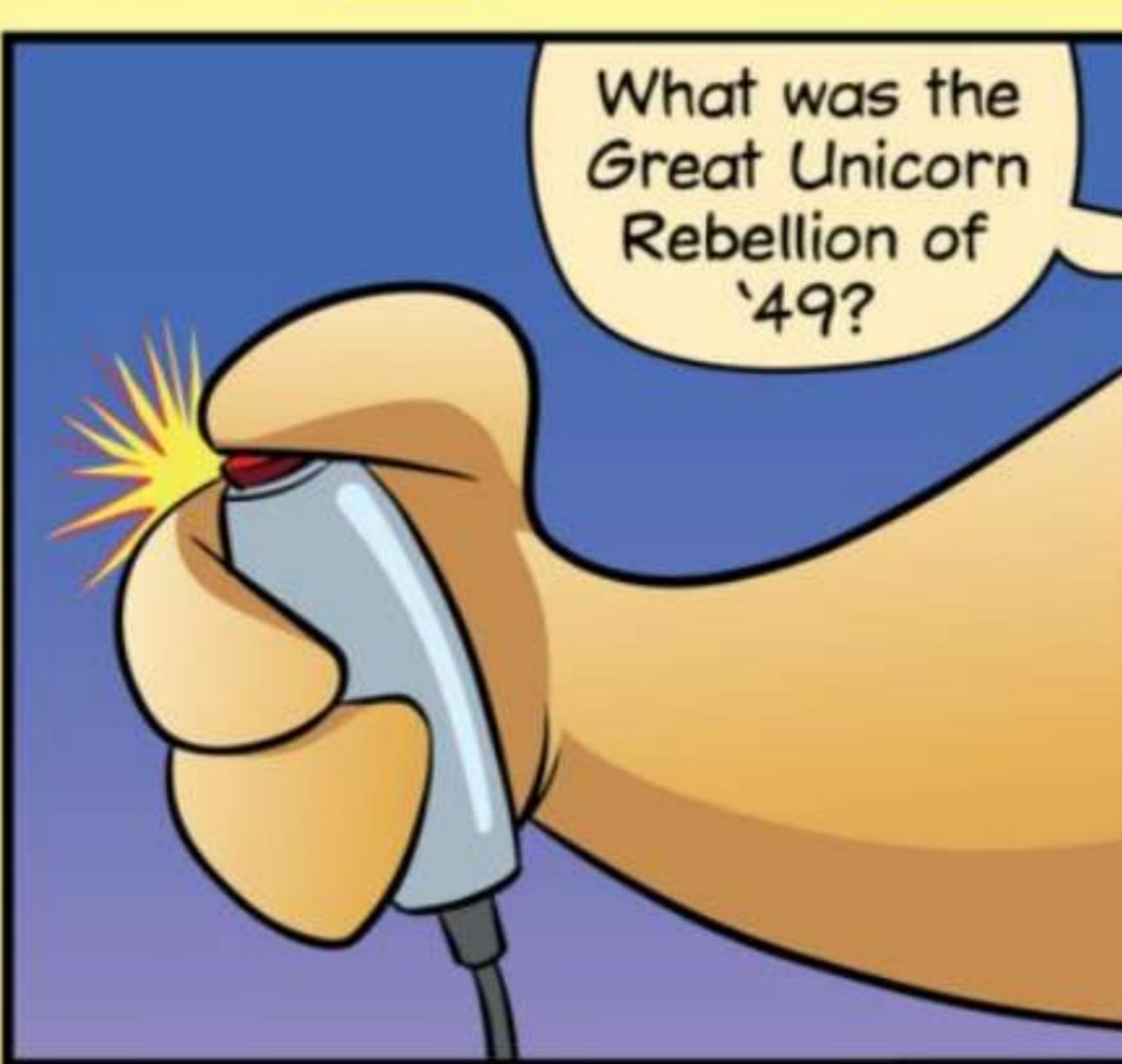


$$\begin{aligned} & 7f + 3 - 5e - 4f + 9e + 10 \\ = & 7f + 3 + (-5e) + (-4f) + 9e + 10 \\ = & \underbrace{7f + (-4f)}_{3f} + \underbrace{(-5e) + 9e}_{4e} + \underbrace{3 + 10}_{13} \\ = & 3f + 4e + 13 \\ = & 3(13) + 4(12) + 13 \end{aligned}$$

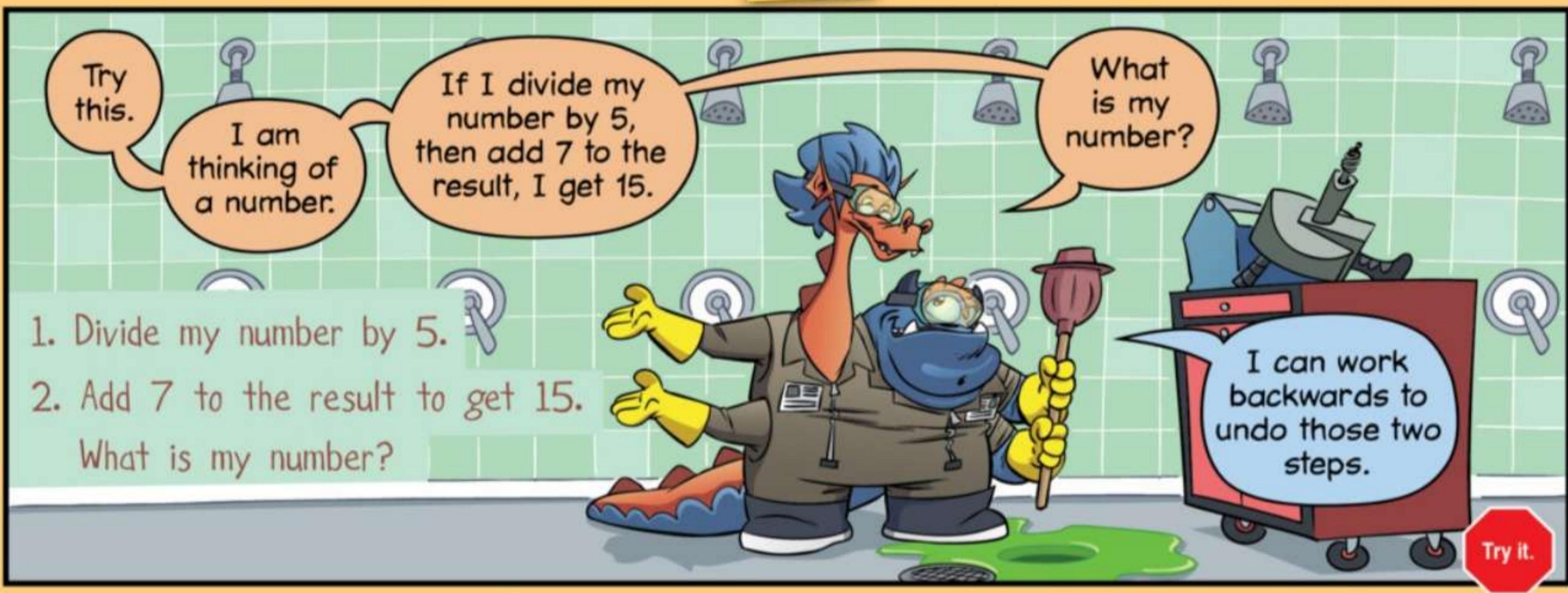
That gives us  $39+48+13$ , which is 100!



$$\begin{aligned} & 7f + 3 - 5e - 4f + 9e + 10 \\ = & 7f + 3 + (-5e) + (-4f) + 9e + 10 \\ = & \underbrace{7f + (-4f)}_{3f} + \underbrace{(-5e) + 9e}_{4e} + \underbrace{3 + 10}_{13} \\ = & 3f + 4e + 13 \\ = & 3(13) + 4(12) + 13 \\ = & 39 + 48 + 13 \\ = & 100 \end{aligned}$$







2. Add 7 to the result to get 15.  
 $\square + 7 = 15$   
 $15 - 7 = \square 8$

To find the number that you added 7 to to get 15, we subtract 7 from 15.  
 $15 - 7 = 8$ .

1. Divide my number by 5.  
 $\square \div 5 = 8$

That means that when you divided your number by 5, you got 8.

$\square \div 5 = 8$   
 $8 \times 5 = \square 40$

So, we can multiply 8 by 5 to find your number!

$8 \times 5 = 40$ .

Perfect! You figured out my number.

That's how math beasts solve equations... by undoing operations.\*

Really?

Yep. I'll show you. Start by turning these sentences into an equation.

1. Divide my number by 5.  
2. Add 7 to the result to get 15.

What is my number?

Write an equation. Use  $x$  to represent the unknown number.

\*MATH OPERATIONS ARE THE THINGS WE DO TO NUMBERS, LIKE ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.



Let's see...

I'll use  $x$  to represent your number.

Dividing your number by 5 gives us  $\frac{x}{5}$ .

1. Divide my number by 5.  $\rightarrow \frac{x}{5}$

2. Add 7 to the result to get 15.

What is my number?

Then, we add 7 to the result.

That gives us  $\frac{x}{5} + 7$ .

1. Divide my number by 5.  $\rightarrow \frac{x}{5}$

2. Add 7 to the result to get 15.  $\rightarrow \frac{x}{5} + 7$

What is my number?

Dividing your number by 5 and adding 7, we get 15.

So,  $\frac{x}{5} + 7 = 15$ .

1. Divide my number by 5.  $\rightarrow \frac{x}{5}$

2. Add 7 to the result to get 15.  $\rightarrow \frac{x}{5} + 7 = 15$

What is my number?

But, how do we figure out what  $x$  is?

The same way you figured out my number before...

...by working backwards!

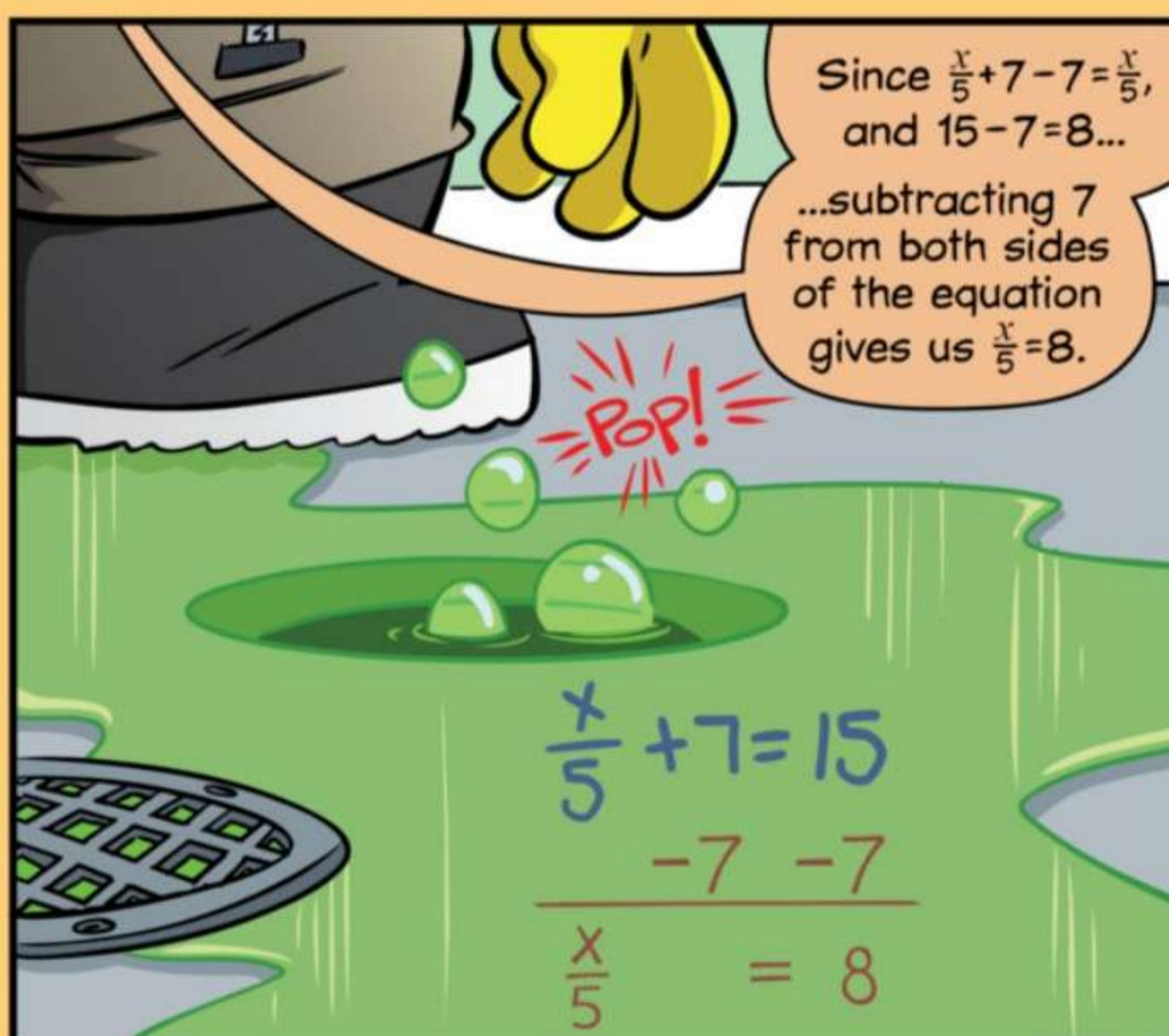
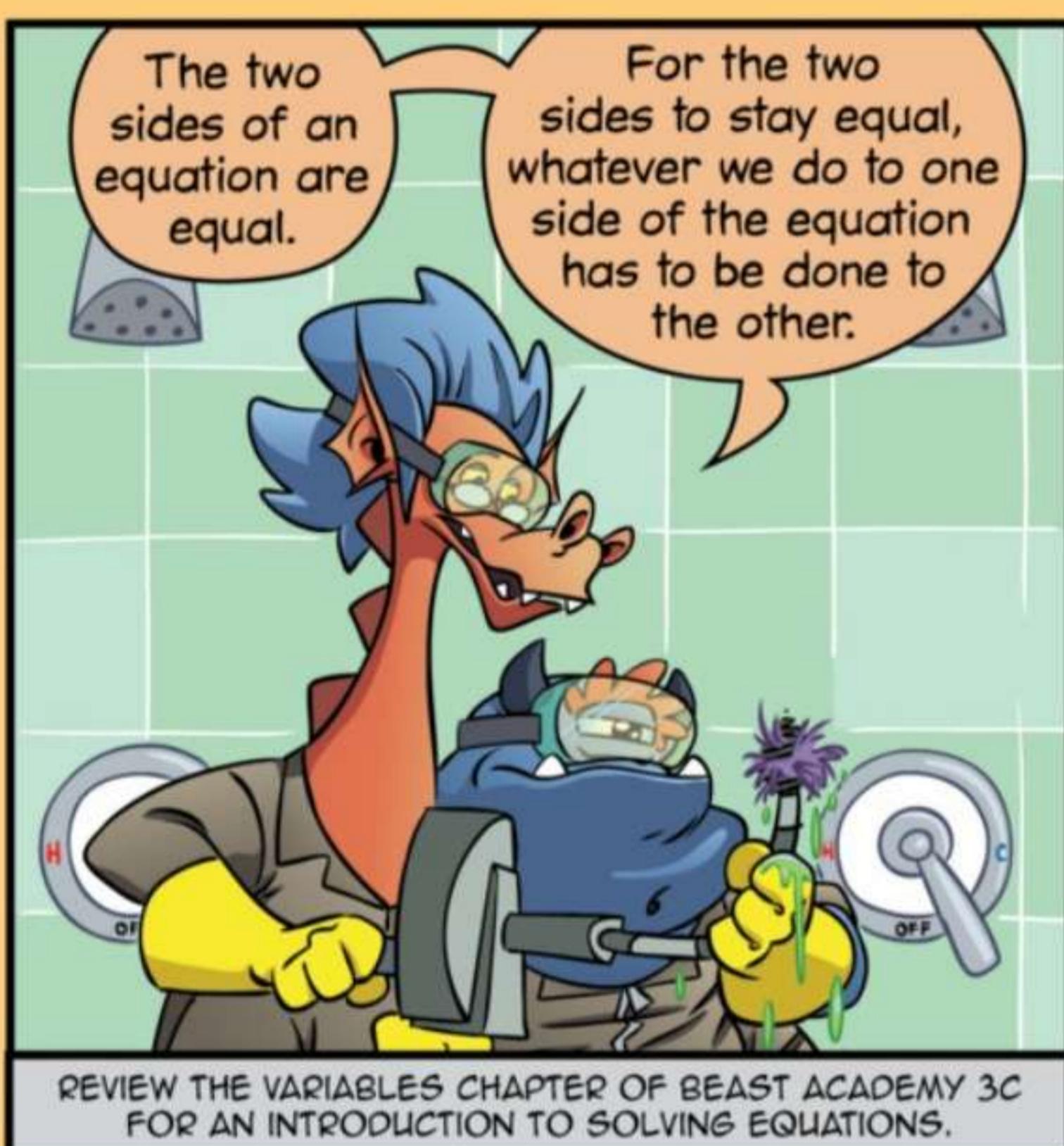
Our goal is to *isolate the variable*.

Huh?

That just means we try to get  $x$  by itself on one side of the equation.

$$\frac{x}{5} + 7 = 15$$

How could you solve this equation for  $x$ ?



Division and multiplication are opposites, too!

To undo **dividing** by 5, we **multiply** both sides of the equation by 5. That gives us  $x = 40$ .

That's the same answer I got before.

$$\begin{aligned}\frac{x}{5} + 7 &= 15 \\ -7 \quad -7 \\ \frac{x}{5} \cdot 5 &= 8 \cdot 5 \\ x &= 40\end{aligned}$$

How could you check to make sure your answer is correct?



"PLUGGING IN" THE SOLUTION TO AN EQUATION MEANS REPLACING THE VARIABLE WITH THE SOLUTION.

$$\frac{40}{5} + 7$$

equals  
8 + 7 ...

...which is 15!

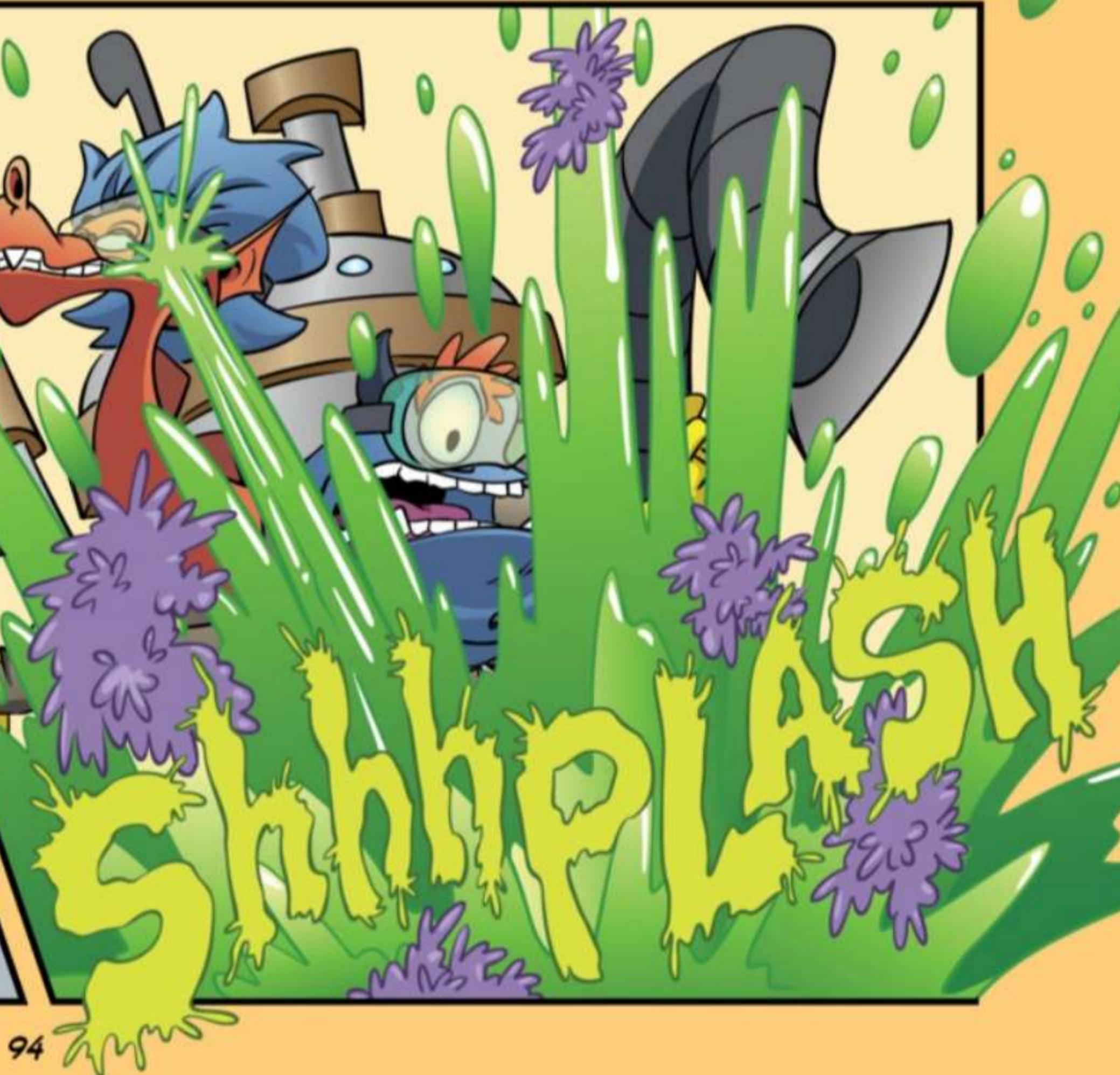
My answer works.

$$\begin{aligned}\frac{x}{5} + 7 &= 15 \\ \frac{40}{5} + 7 &= 15 \\ 8 + 7 &= 15 \\ 15 &= 15 \checkmark\end{aligned}$$

Plugging stuff in is a great way to make sure it works.

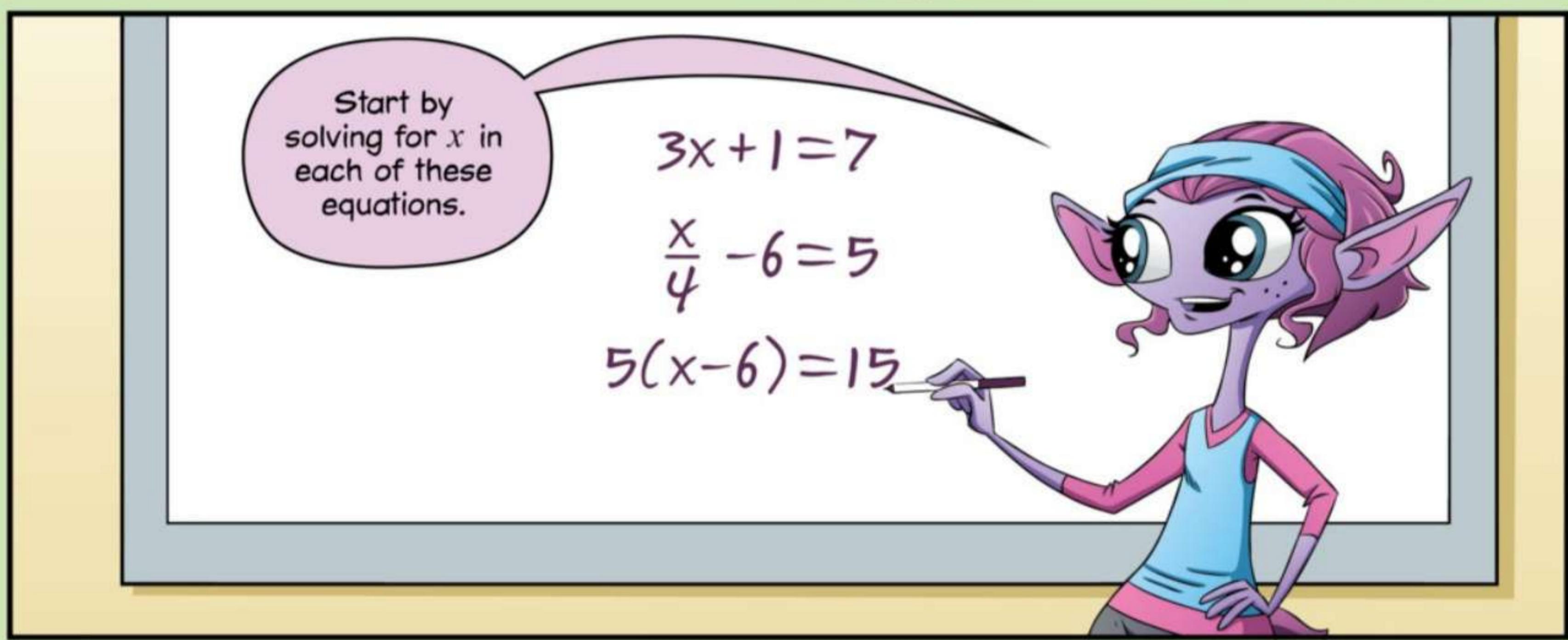
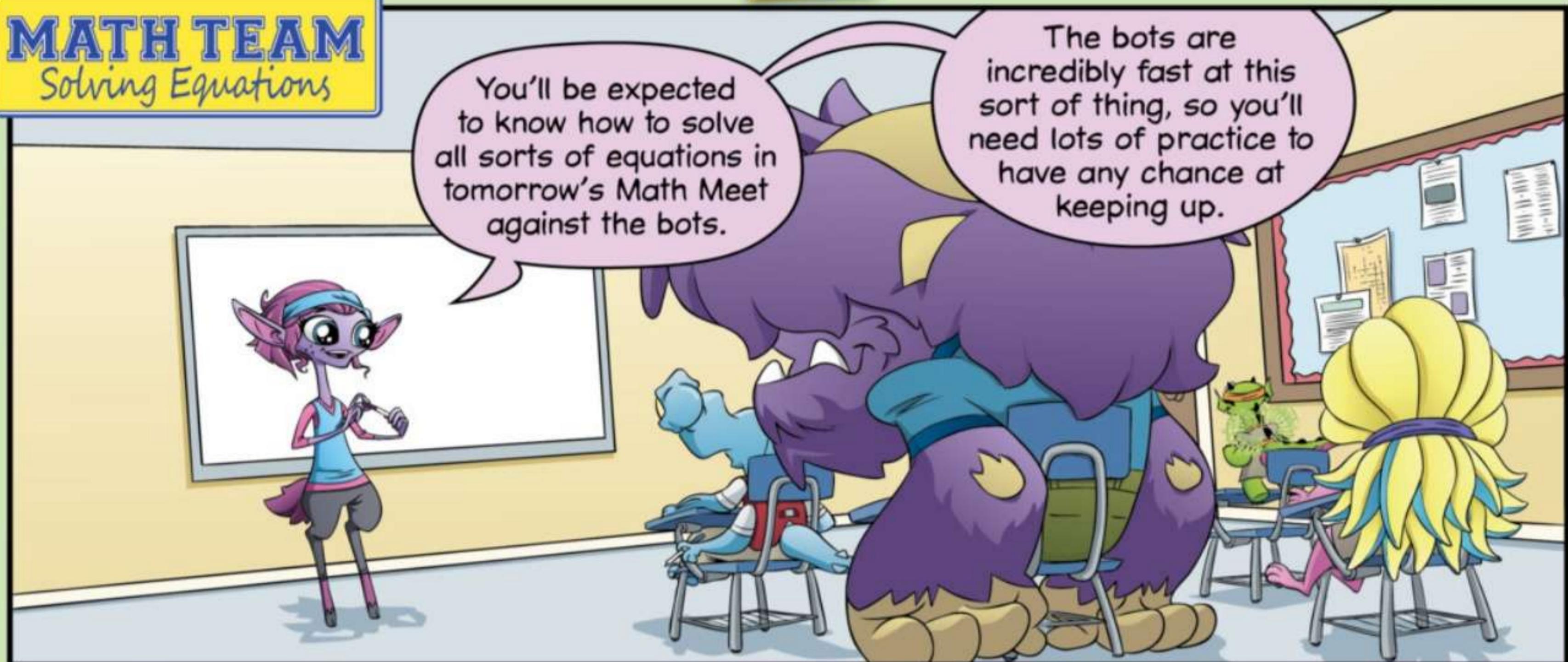
ON  
OFF  
POWER

Wait, don't--



# MATH TEAM

## Solving Equations



To solve  $3x + 1 = 7$ , first we subtract 1 from both sides of the equation.

$$3x + 1 = 7$$

$$\frac{-1 \quad -1}{3x \quad \quad \quad} = \frac{6}{}$$

We get  $3x = 6$ .

Then, we divide both sides by 3 to get  $x = 2$ .

$$3x + 1 = 7$$

$$\frac{-1 \quad -1}{\cancel{3x} \quad \quad \quad} = \frac{\cancel{6}}{3}$$

$$x = 2$$

We can check our work by plugging in 2 for  $x$ .

$$3x + 1 = 7$$

$$3(2) + 1 = 7$$

$$6 + 1 = 7$$

$$7 = 7 \checkmark$$

**Solve the other two equations.**

We solve this equation by adding 6 to both sides...

$$\begin{array}{rcl} \frac{x}{4} - 6 & = & 5 \\ +6 & & +6 \\ \hline \frac{x}{4} & = & 11 \end{array}$$



$$\begin{array}{rcl} \frac{x}{4} - 6 & = & 5 \\ +6 & & +6 \\ \hline \frac{x}{4} \cdot 4 & = & 11 \cdot 4 \end{array}$$

...then multiplying both sides by 4.

$$\begin{array}{rcl} \frac{x}{4} - 6 & = & 5 \\ +6 & & +6 \\ \hline \frac{x}{4} \cdot 4 & = & 11 \cdot 4 \\ x & = & 44 \end{array}$$



So,  $x = 44$ .

We can start this problem by distributing the 5.

$$\begin{array}{l} 5(x-6) = 15 \\ 5x - 30 = 15 \end{array}$$



Then, we add 30 to both sides...

$$\begin{array}{rcl} 5(x-6) & = & 15 \\ 5x - 30 & = & 15 \\ +30 & & +30 \\ \hline 5x & = & 45 \end{array}$$



...and finally, we divide both sides by 5.

$$\begin{array}{rcl} 5(x-6) & = & 15 \\ 5x - 30 & = & 15 \\ +30 & & +30 \\ \hline 5x & = & 45 \\ \frac{5x}{5} & = & \frac{45}{5} \\ x & = & 9 \end{array}$$



Neat.

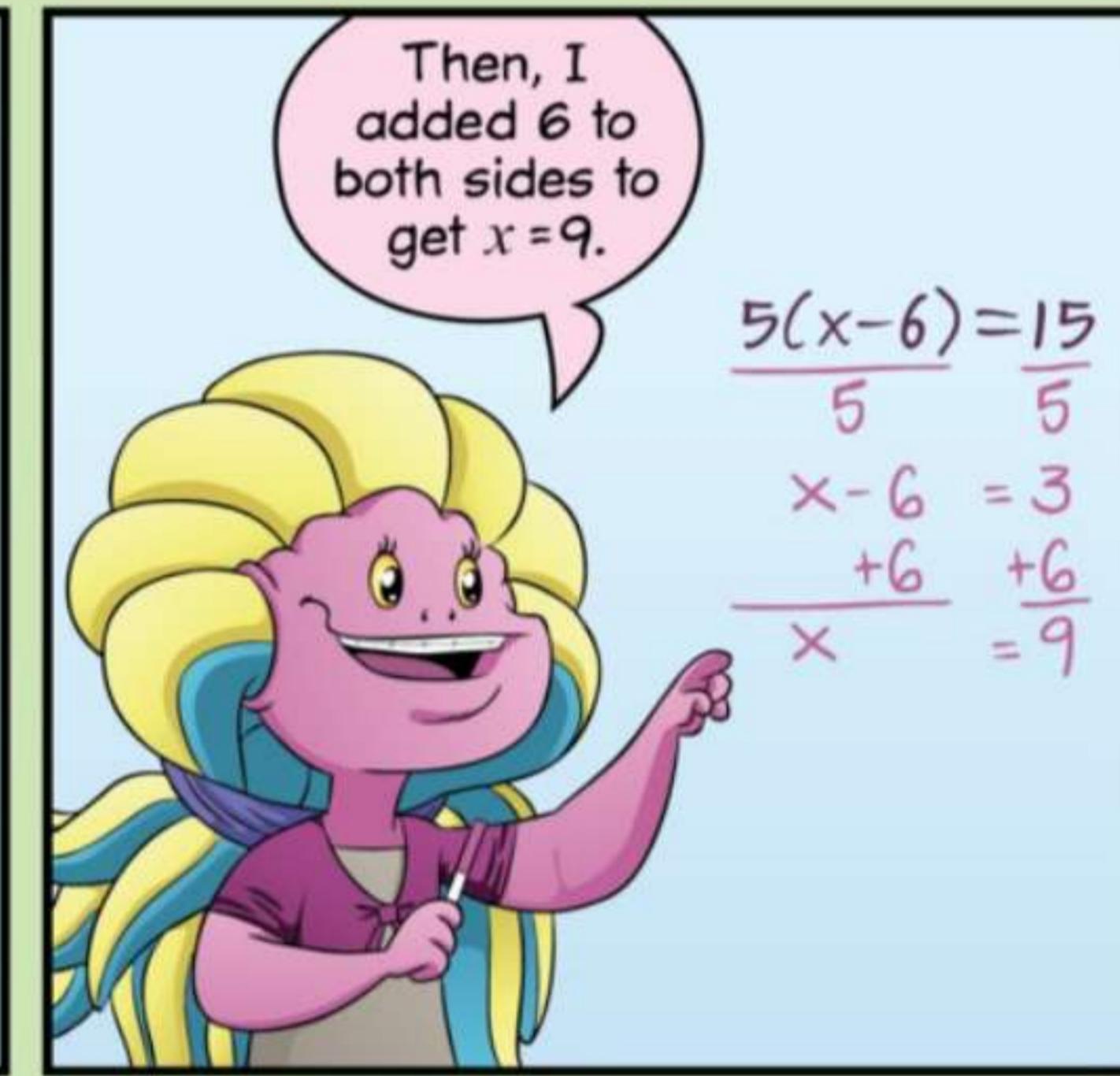
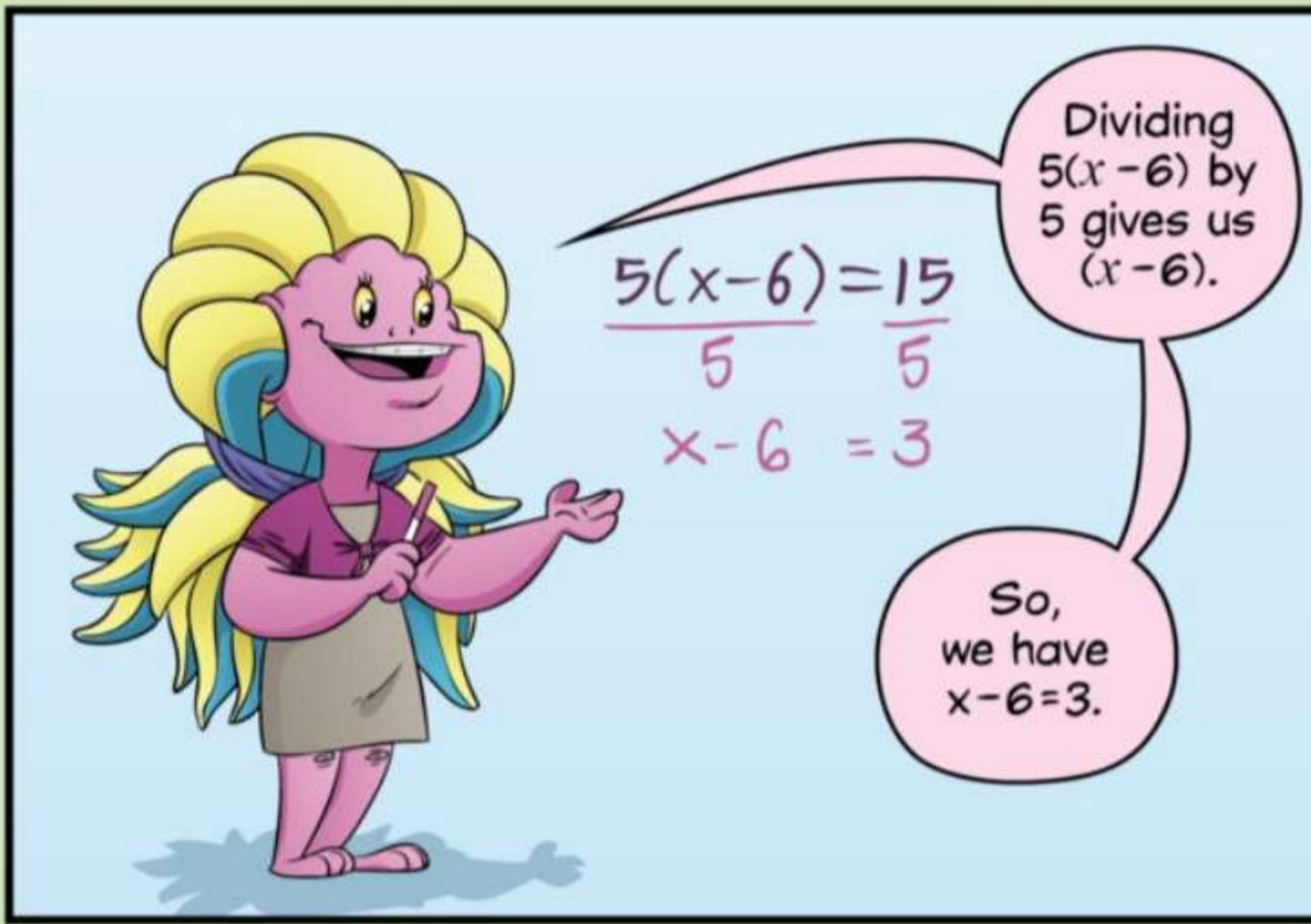
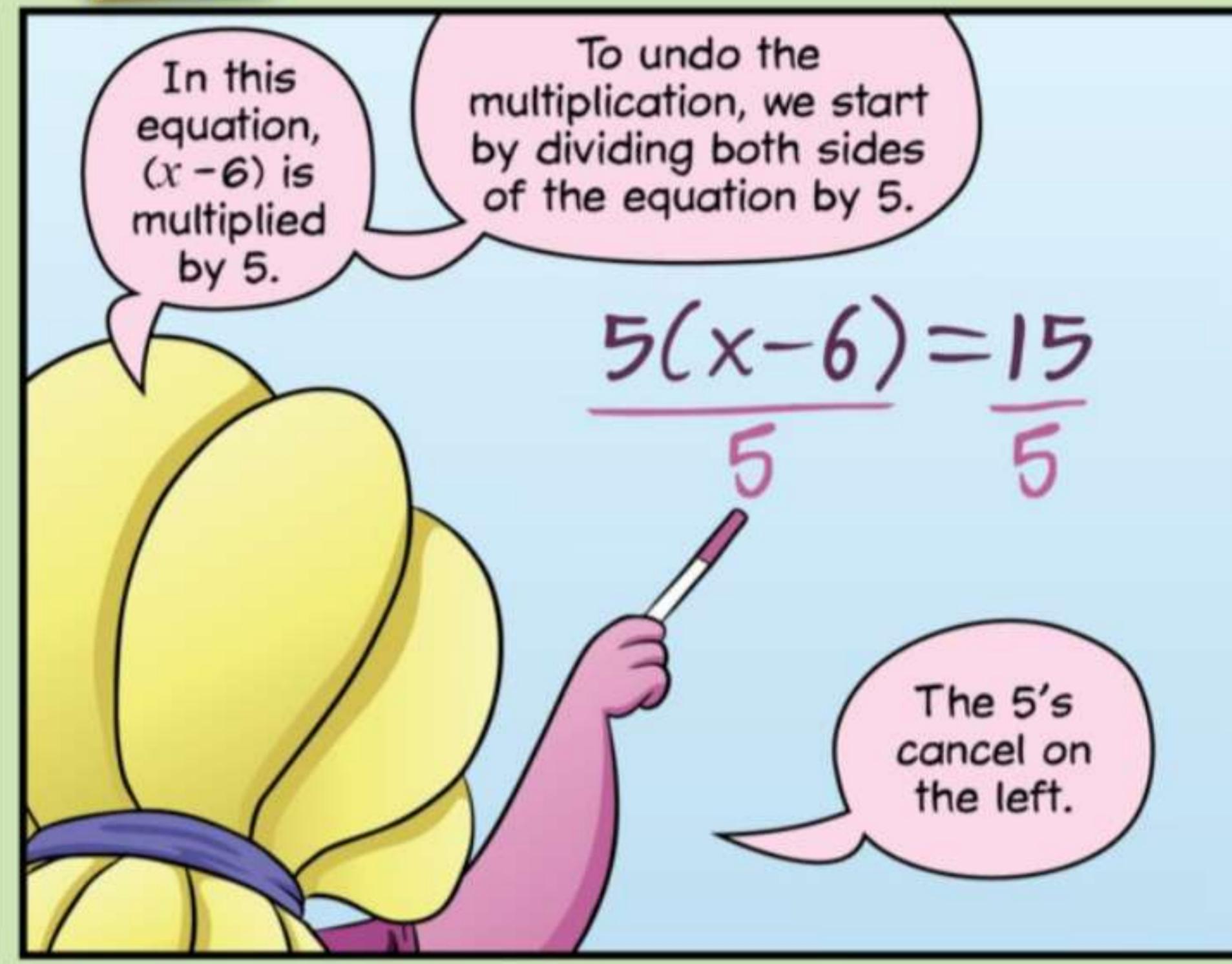
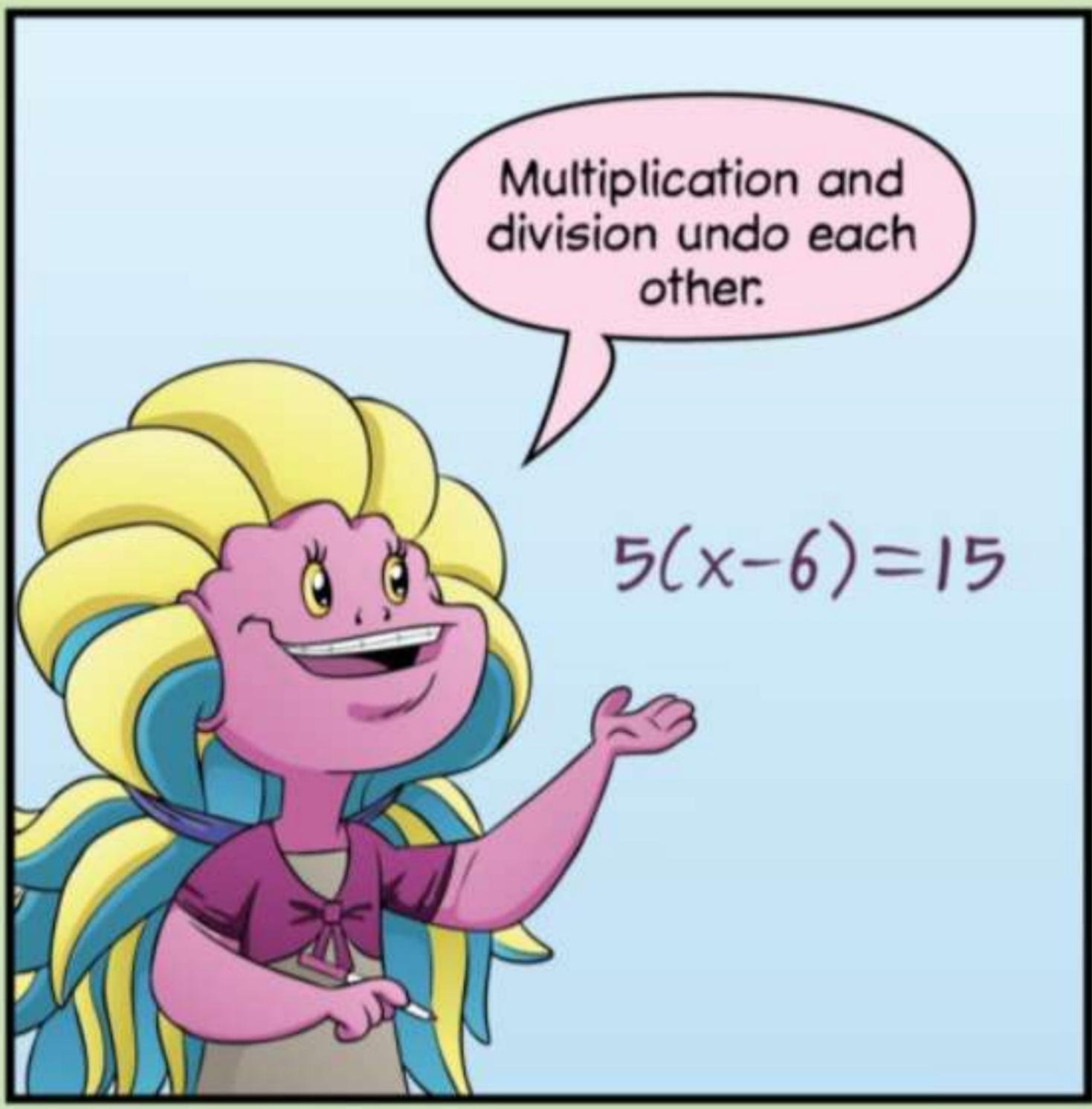
What, Winnie?

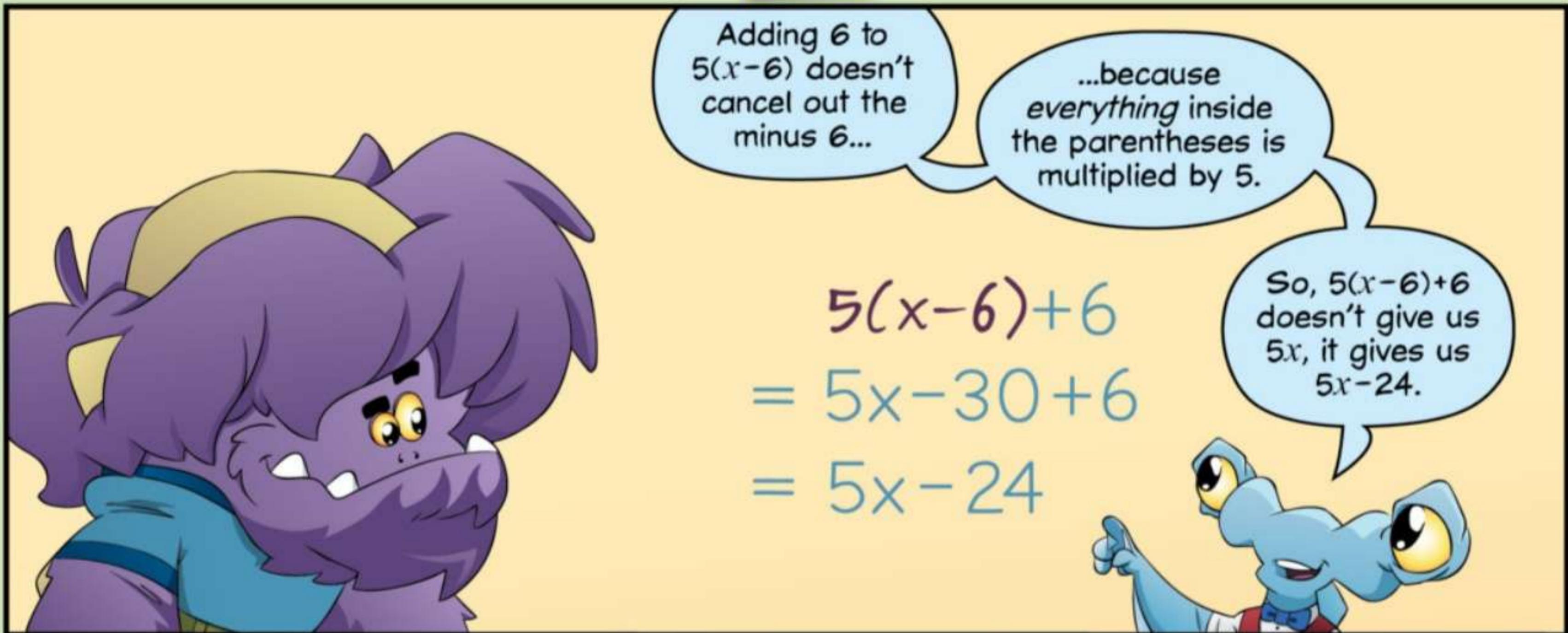
I got the same answer without distributing the 5.

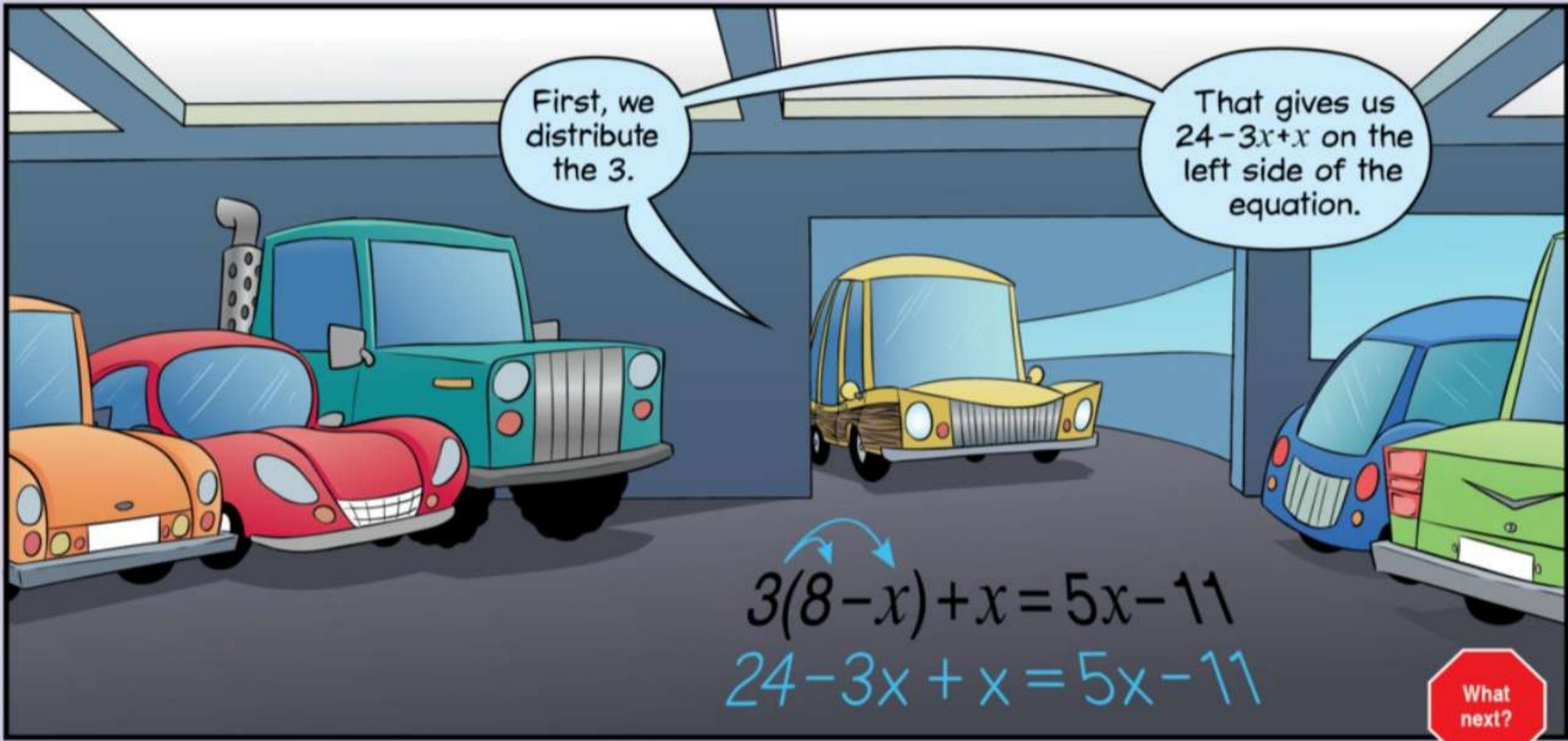
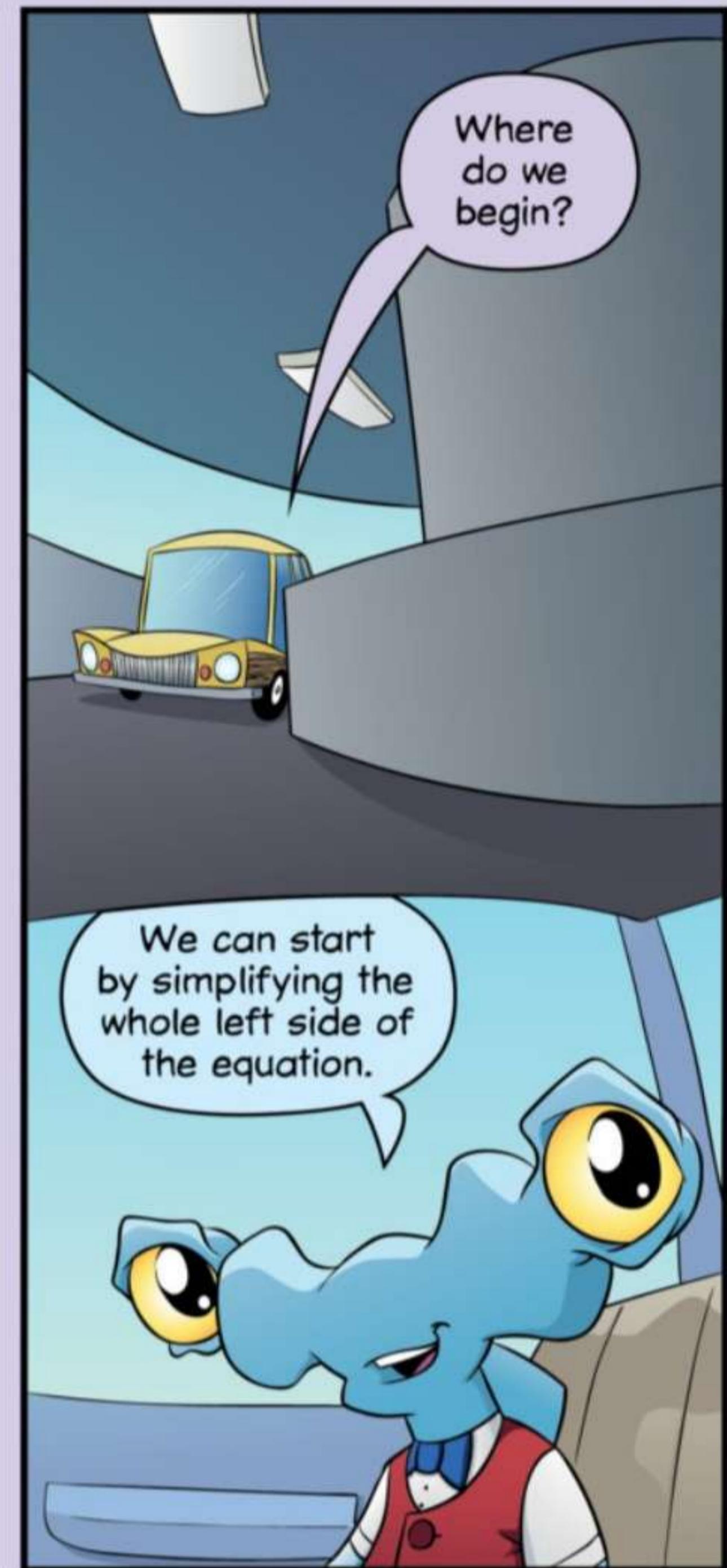
How?

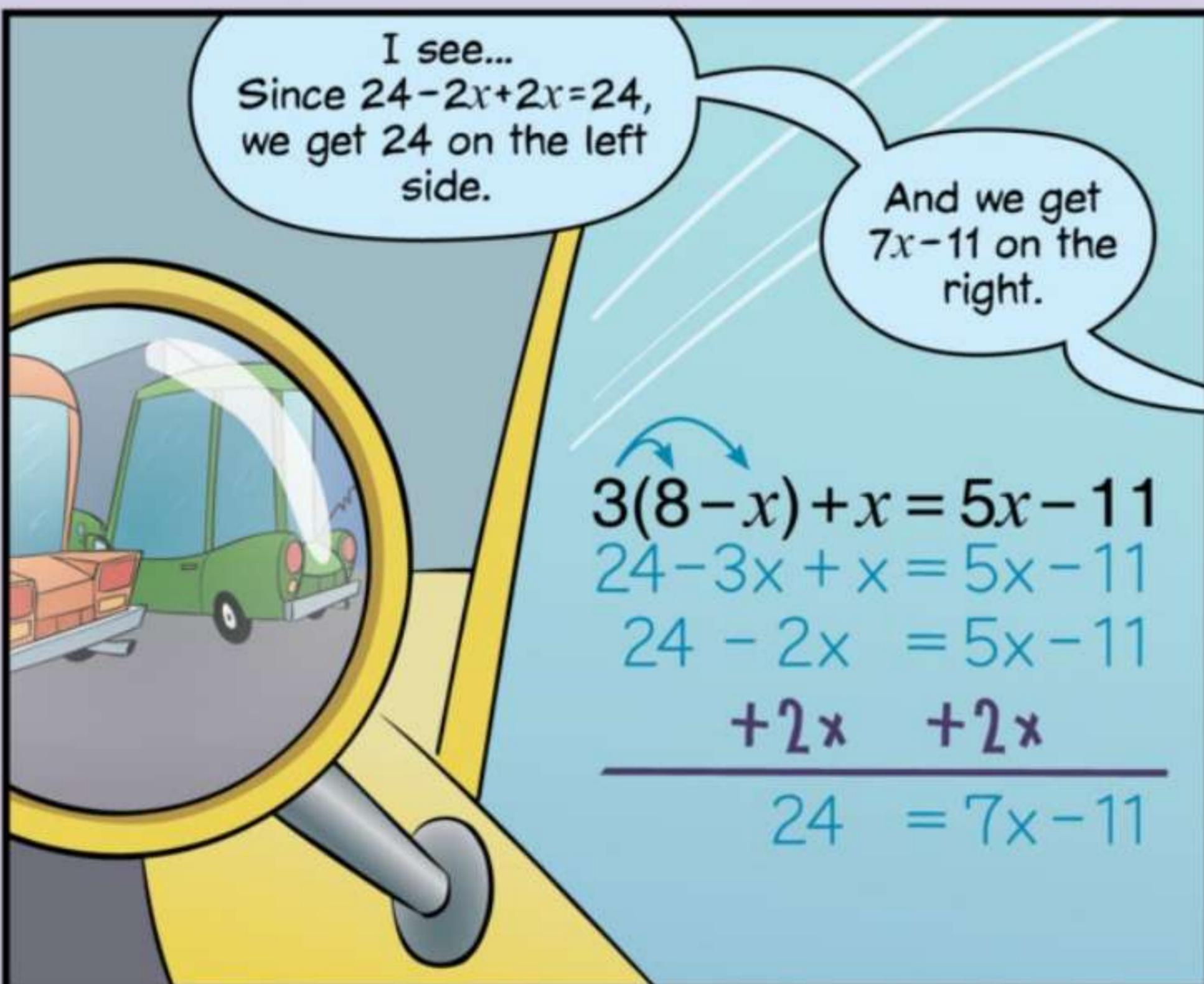
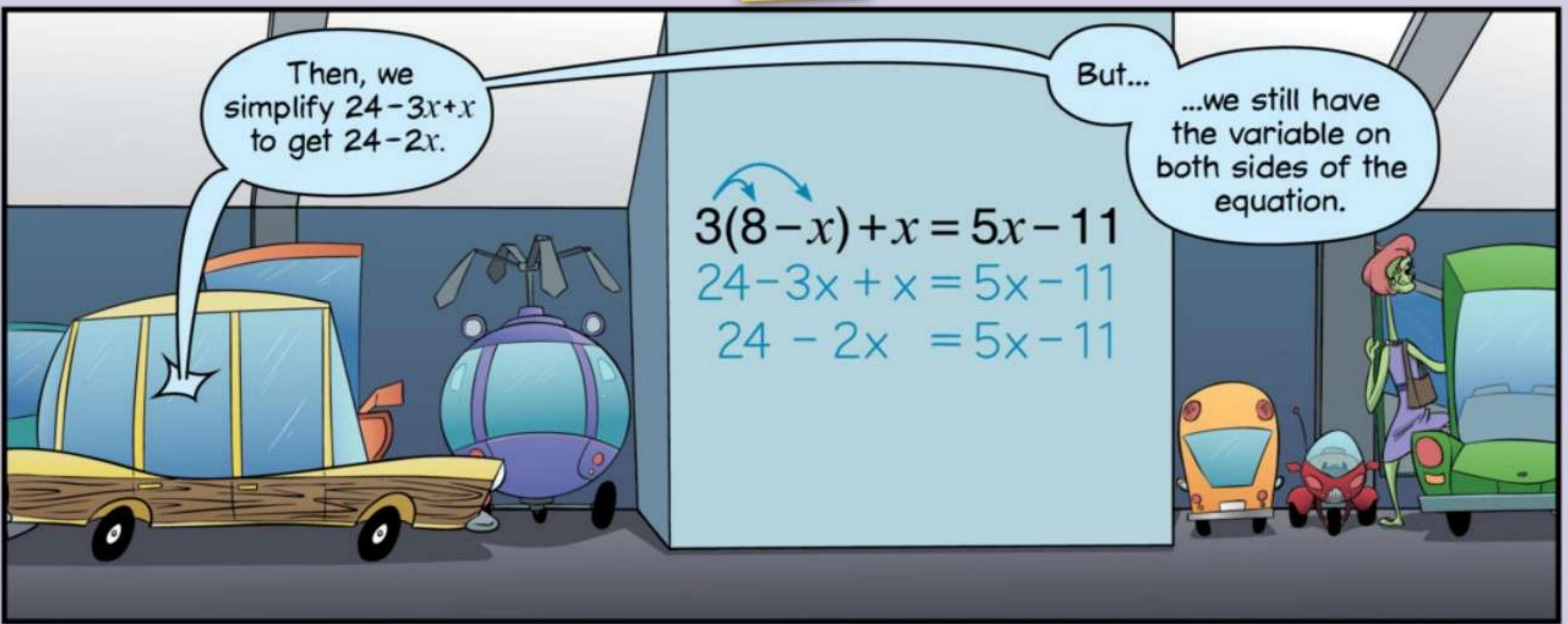


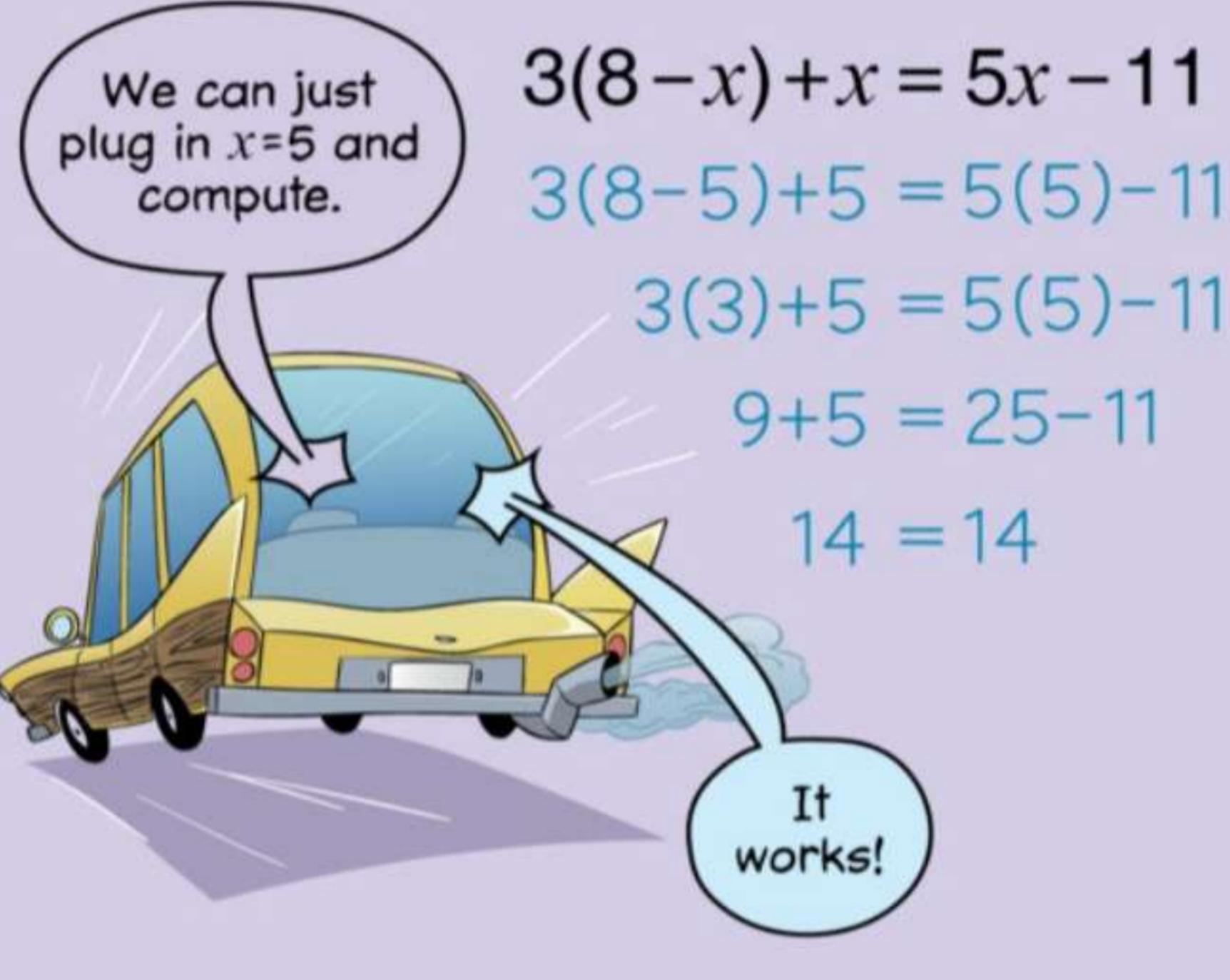
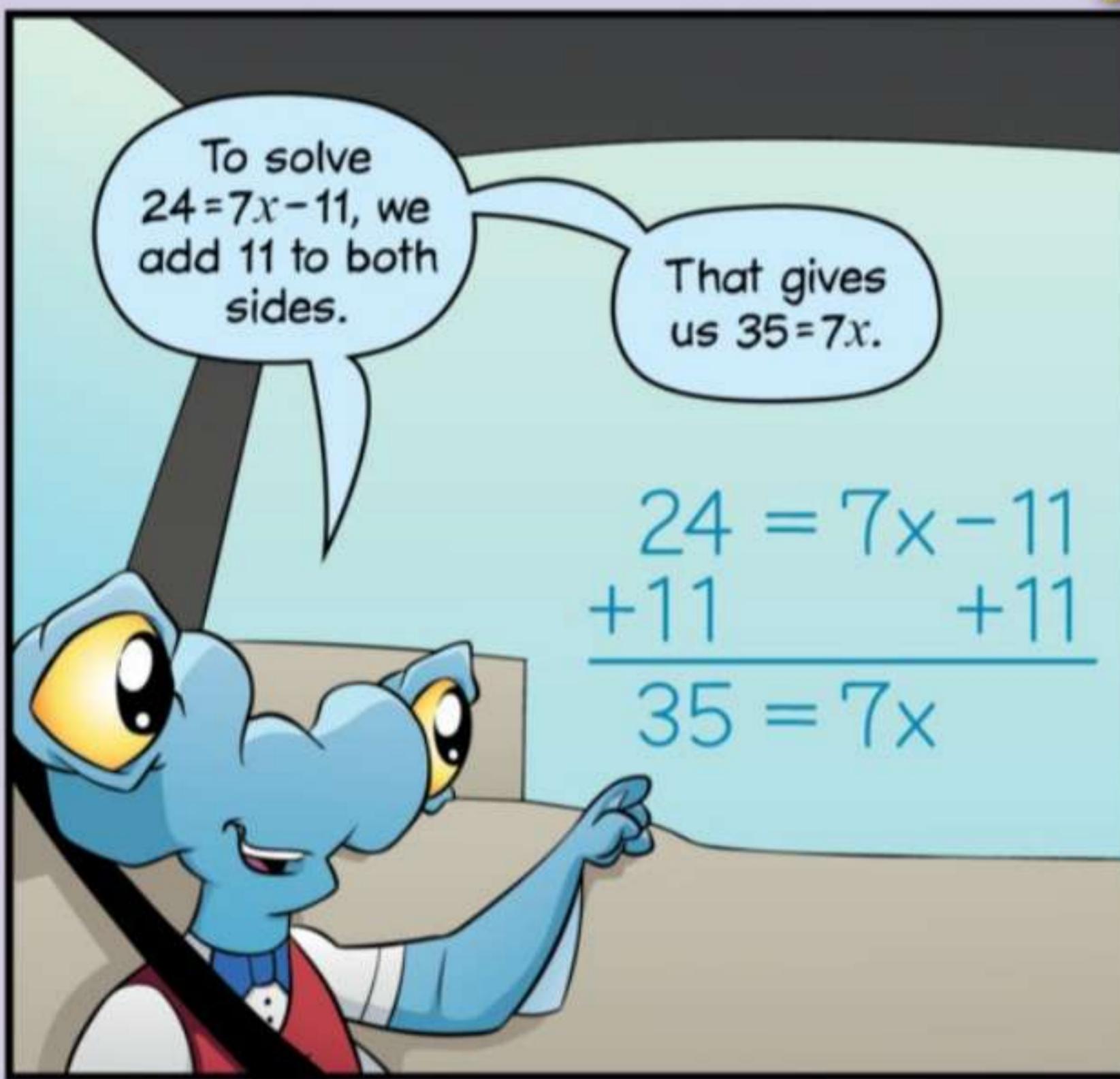
How else could you solve  $5(x-6)=15$ ?











# MATH TEAM

## Math Meet

The Math Meet is about to begin.

Most of the questions will require you to solve equations, which is going to make it tough against the bots.

And now...



Buzz in right away if you know an answer.

Good luck!

...please welcome the Little Monsters of Beast Academy and their opponents... the Bots.



Today's Math Meet will consist of 6 questions. The first five are each worth one point, and the final question is worth two. The team with the most points wins the meet. If everyone is ready for the first question, let's begin.

Question 1:  
Solve for  $x$  in the equation:  
 $\frac{x+7}{3} = 17$ .

ding! BZZZT!





*Oh no!*  
I subtracted  
7 first.

Then, I  
multiplied  
by 3.

I should have  
multiplied both  
sides by 3, **then**  
subtracted.

That's ok. None of  
us could have beaten  
the Bots to that  
answer.

$$\frac{x+7}{3} - 7 = 17 - 7$$

$$\cancel{\frac{x}{3} \cdot 3 = 10 \cdot 3}$$

$$\cancel{x = 30}$$

$$\frac{x+7}{3} \cdot 3 = 17 \cdot 3$$

$$x+7 = 51$$

$$\frac{-7}{x} = \frac{-7}{44} \checkmark$$

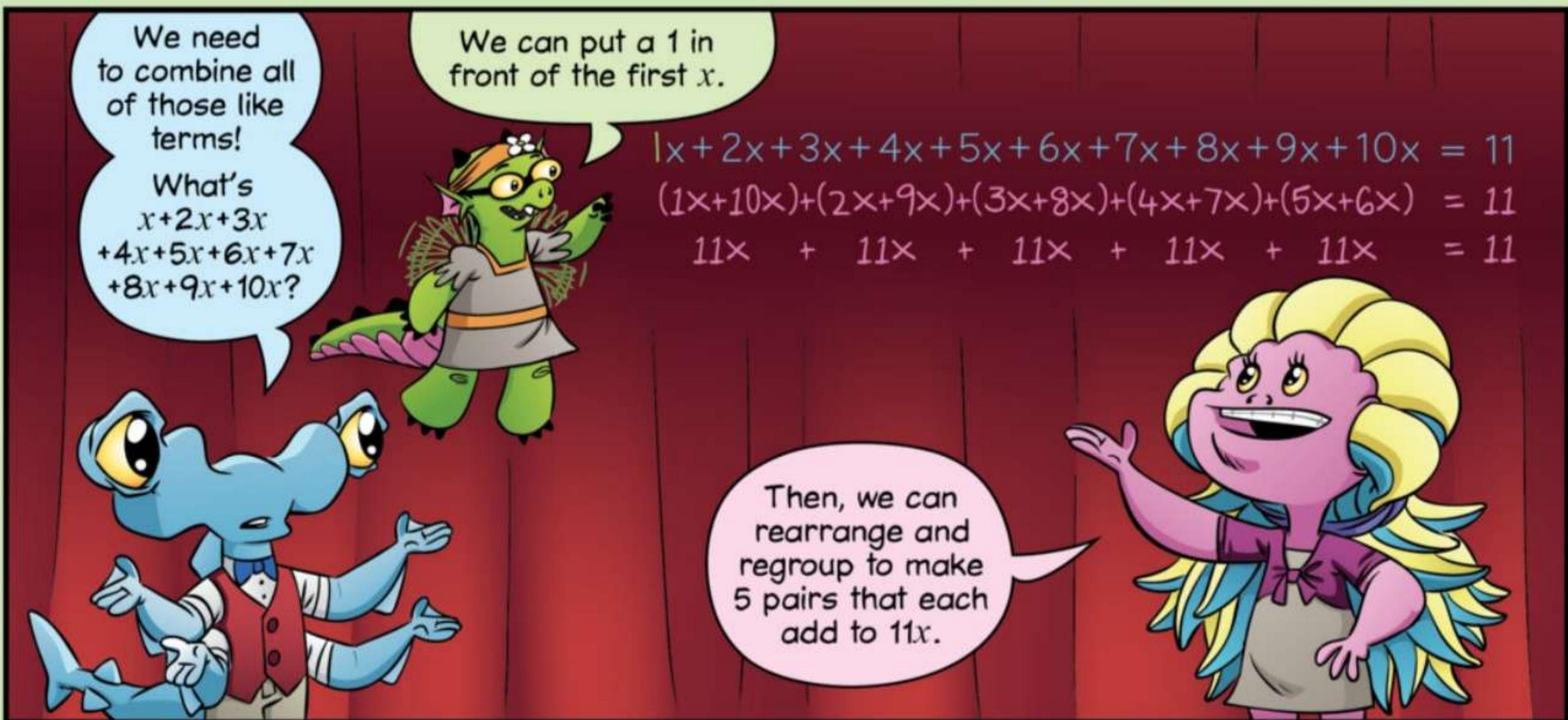
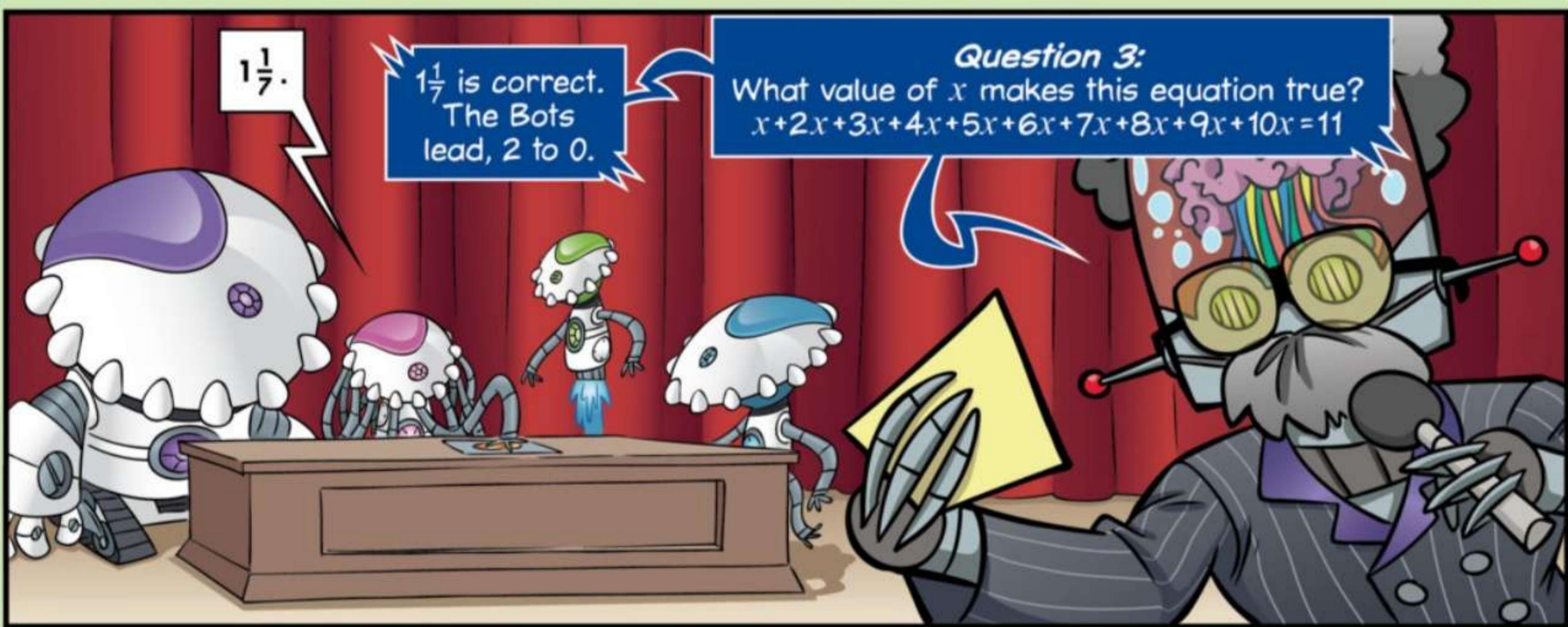
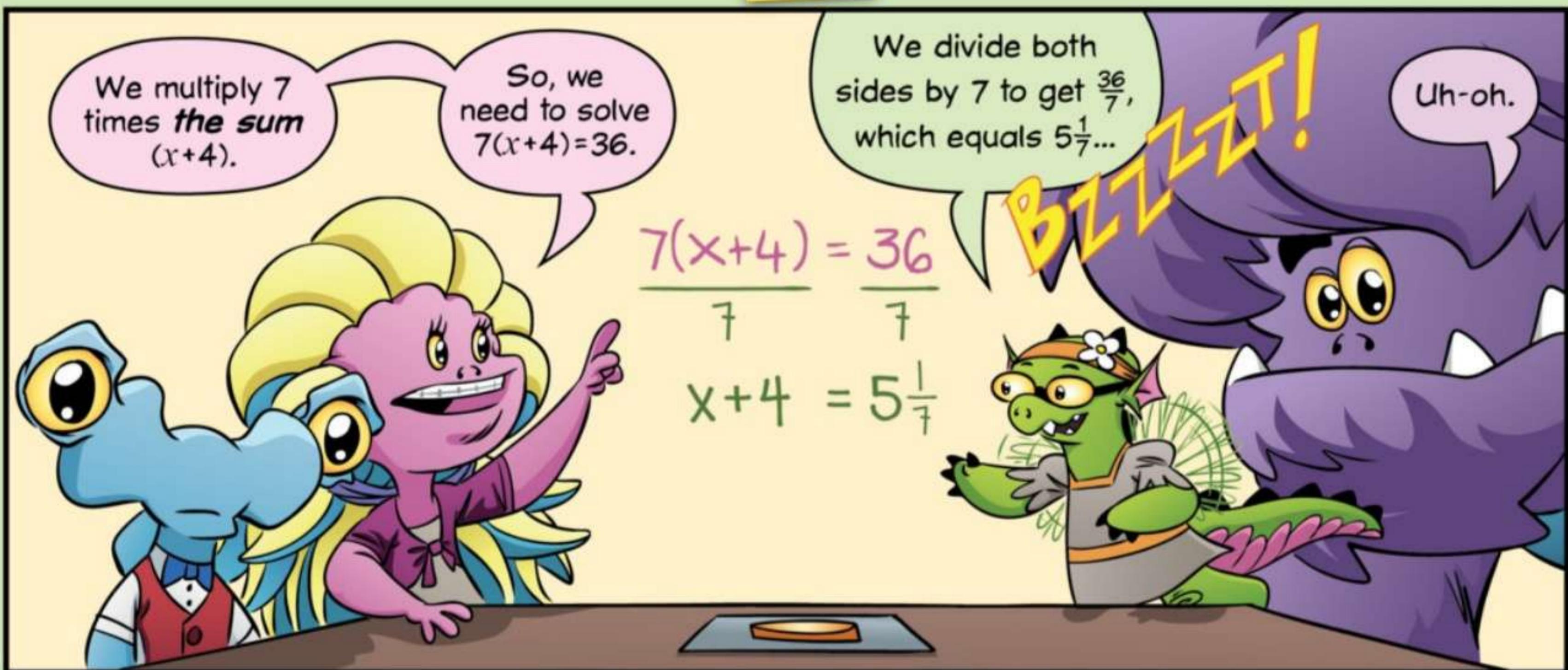
BECAUSE ALL OF  $x+7$  IS DIVIDED BY 3, WE CANNOT SUBTRACT 7 FROM  $\frac{x+7}{3}$  BY SIMPLY SUBTRACTING 7 FROM THE NUMERATOR.  
 $\frac{x+7}{3} - 7$  IS NOT EQUAL TO  $\frac{x}{3}$ .

**Question 2:**  
Seven times the sum of  $x$  and 4 is 36. Express  $x$  as a mixed number in simplest form.

It looks like the Bots are arguing. Let's write an equation.

Is the equation  $7x + 4 = 36$ , or  $7(x + 4) = 36$ ?

Which equation is correct?



$$1x + 2x + 3x + 4x + 5x + 6x + 7x + 8x + 9x + 10x = 11$$

$$(1x+10x)+(2x+9x)+(3x+8x)+(4x+7x)+(5x+6x) = 11$$

$$11x + 11x + 11x + 11x + 11x = 11$$

$$\frac{55x}{55}$$

$$= \frac{11}{55}$$

That gives us  $55x=11$ , so we divide both sides by 55 and simplify to get--

BZZZIT!

Oh, come on!

$\frac{1}{5}$ .

$\frac{1}{5}$  is correct.  
The Bots lead, 3 to 0.

The Little Monsters will need to get all of the remaining questions correct to win the Math Meet.

Question 4:  
The equation  
 $|-2x+5|=19$ --

BZZZIT!

Urrrgh!

-7.

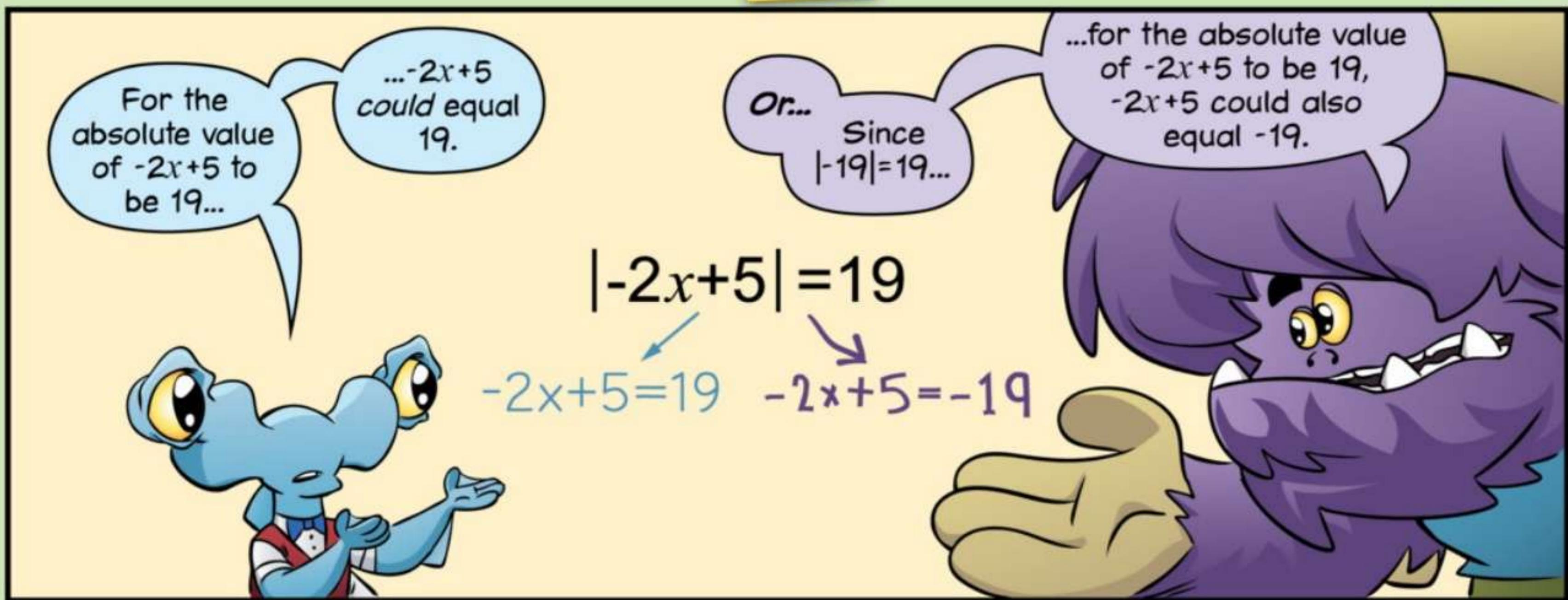
I'm sorry, -7 is incorrect.  
I will finish reading the question for the Little Monsters who will then have 3 minutes to answer.

The equation  
 $|-2x+5|=19$  has two solutions.  
Find their sum.

We've got to get this one.

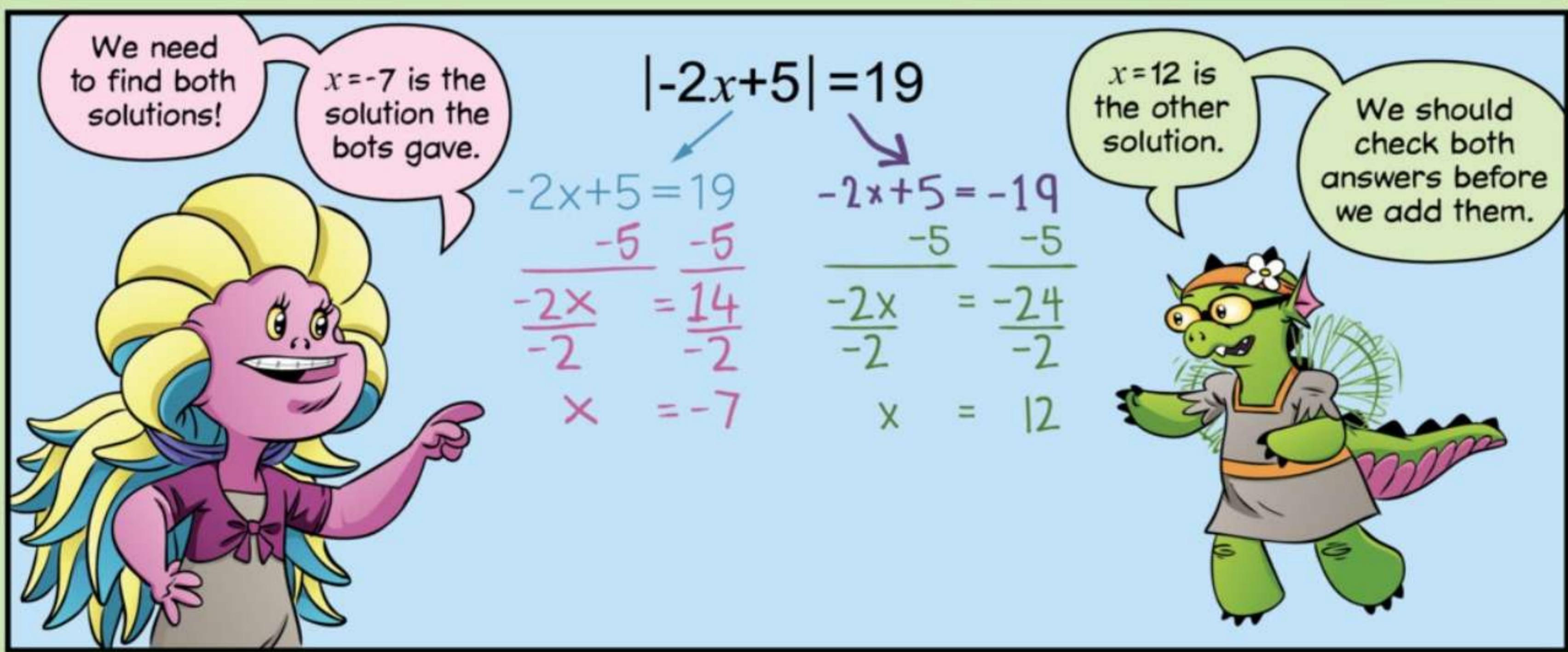
Try it.

THE TALL BARS IN THE EXPRESSION  $|-2x+5|$  INDICATE ABSOLUTE VALUE, WHICH IS A NUMBER'S DISTANCE FROM ZERO. FOR EXAMPLE,  $|-7|=7$  AND  $|3|=3$ . LEARN MORE ABOUT ABSOLUTE VALUE IN THE INTEGERS CHAPTER OF BEAST ACADEMY 4C.



$$|-2x+5| = 19$$

$-2x+5=19$        $-2x+5=-19$



$$|-2x+5| = 19$$

$-2x+5=19$        $-2x+5=-19$

$$\begin{array}{r} -5 \\ \hline -2x \\ -2 \end{array} = \begin{array}{r} -5 \\ \hline 14 \\ -2 \end{array}$$

$$x = -7$$

$$\begin{array}{r} -5 \\ \hline -2x \\ -2 \end{array} = \begin{array}{r} -5 \\ \hline -24 \\ -2 \end{array}$$

$$x = 12$$



$$|-2x+5| = 19$$

$|-2(-7)+5| = 19$        $|-2(12)+5| = 19$

$$|14+5| = 19$$

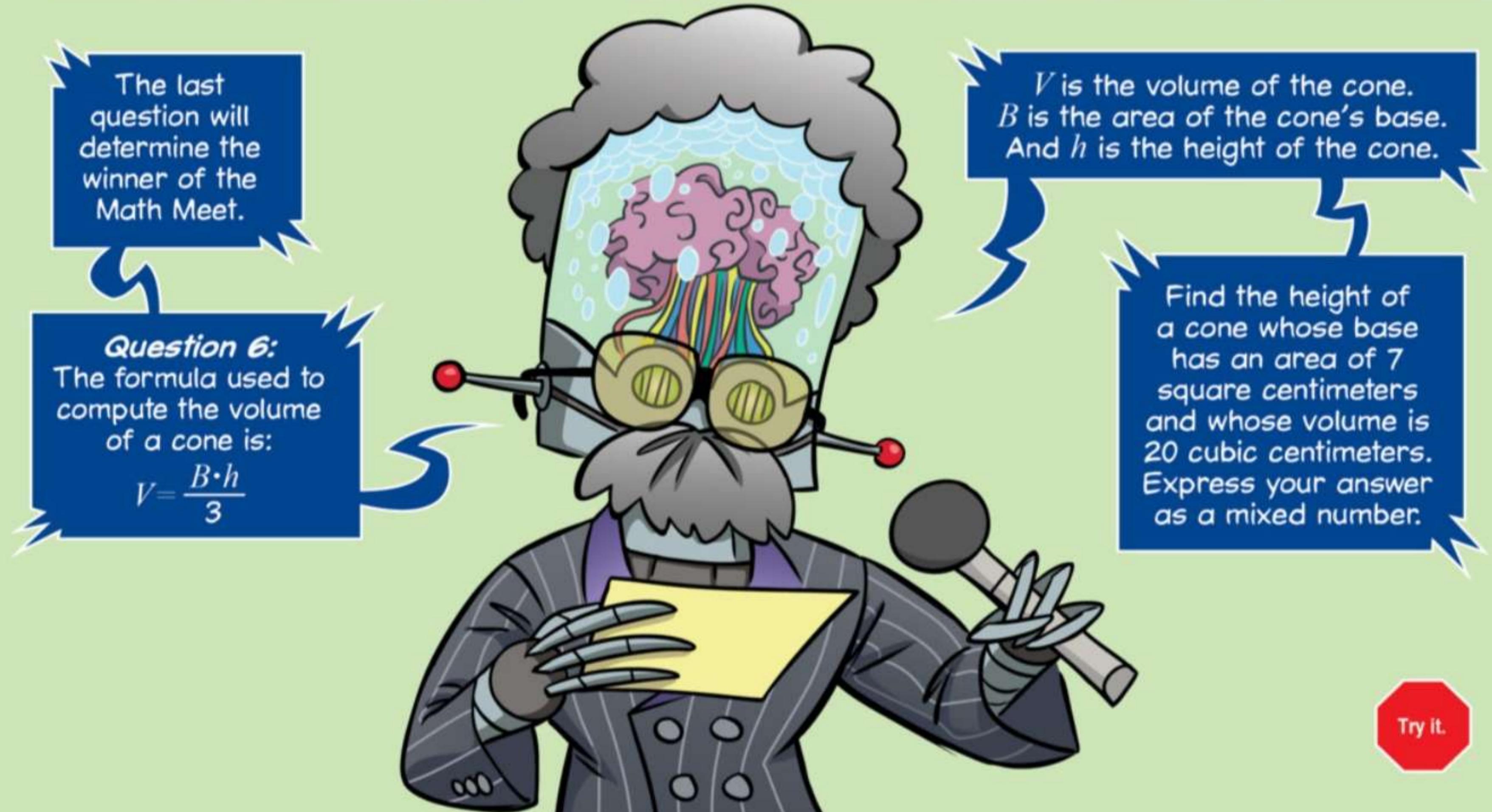
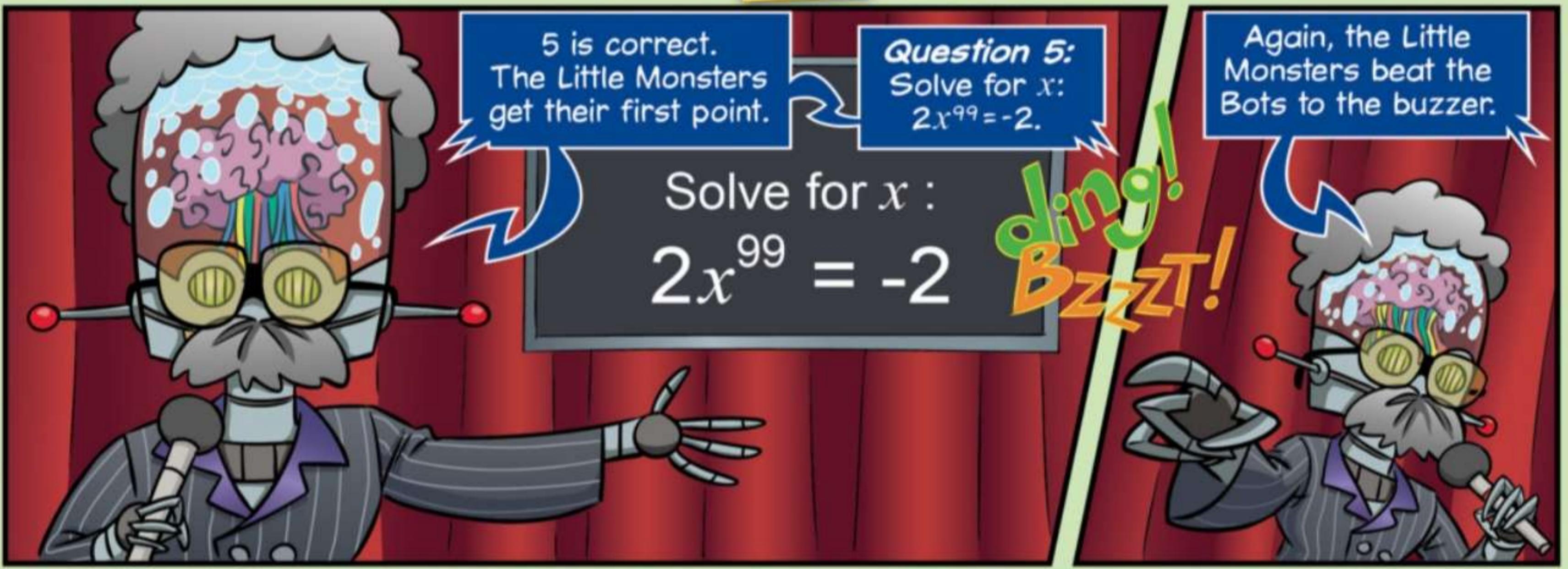
$$|-24+5| = 19$$

$$|19| = 19$$

$$|-19| = 19$$

$$19 = 19$$

$$19 = 19$$





We start by plugging the values we know into the formula.

The base area is  $7 \text{ cm}^2$ .

$$V = \frac{B \cdot h}{3}$$

$$20 = \frac{7 \cdot h}{3}$$

And the volume is 20 cubic centimeters.

We multiply both sides by 3 to get  $60 = 7 \cdot h$ .

$$V = \frac{B \cdot h}{3}$$

$$20 \cdot 3 = \frac{7 \cdot h}{3} \cdot 3$$

$$60 = 7 \cdot h$$

Then, we divide both sides by 7.

$$h = \frac{60}{7}$$

$$V = \frac{B \cdot h}{3}$$

$$20 \cdot 3 = \frac{7 \cdot h}{3} \cdot 3$$

$$\frac{60}{7} = \frac{7 \cdot h}{7}$$

$$\frac{60}{7} = h$$

We convert to a mixed number...

**ding!**

