Indices Rules- Practice practice practice

Power of 2	Value	Power of 2	Value	Power of 3	Value
2^0	1	2^7	128	3^0	1
2^1	2	2^8	256	3^1	3
2^2	4	2^9	512	3^2	9
2^3	8	2^{10}	1024	3^3	27
2^4	16	2^{11}	2048	3^4	81
2^5	32	2^{12}	4096	3^5	243
2^6	64			3^6	729

1. *The product* $\times$ $+$ *Rule says to multiply powers add the indices.*

Due to popular demand we are keeping the $\times$ symbol for the Christmas test — but be prepared to let it go afterwards.

In algebra, multiplication is usually implied: ab, $3x$, and m^4m^3 are all products even though no $\times$ is written.

And in arithmetic the dot notation $2 \cdot 3 \cdot 5 = 30$ gradually replaces the old $\times$ because it avoids confusion with the letter x.

$$a^p \times a^q = a^{p+q} \qquad \text{Example: } m^4 \times m^3 = m^{4+3} = m^7$$

a) $2^2 \times 2^3$	$\boxed{2^5}$	b) $b^4 \times b^6$	$\boxed{b^{10}}$	c) $7^5 \times 7^7$	$\boxed{7^{12}}$
d) $m^3 \times m^6$	$\boxed{m^9}$	e) $2^8 \times 2^2$	$\boxed{2^{10}}$	f) $y^9 \times y^4$	$\boxed{y^{13}}$
g) $p^{11} \times p^3$	$\boxed{p^{14}}$	h) $3^6 \times 3^5$	$\boxed{3^{11}}$	i) $k^7 \times k^{10}$	$\boxed{k^{17}}$
j) $w^{12} \times w$	$\boxed{w^{13}}$	k) $\pi^{11} \times \pi^{89}$	$\boxed{\pi^{100}}$	l) $\theta^6 \times \theta^4$	$\boxed{\theta^{10}}$

2. The Quotient $\div$ $-$ Rule says to divide powers we subtract the indices. *Write each expression as a single power.*

a) $2^2 \times 2^3$	$\boxed{2^5}$	*b)* $b^4 \times b^6$	$\boxed{b^{10}}$	*c)* $7^5 \times 7^7$	$\boxed{7^{12}}$
d) $m^3 \times m^6$	$\boxed{m^9}$	*e)* $2^8 \times 2^2$	$\boxed{2^{10}}$	*f)* $y^9 \times y^4$	$\boxed{y^{13}}$
g) $p^{11} \times p^3$	$\boxed{p^{14}}$	*h)* $3^6 \times 3^5$	$\boxed{3^{11}}$	*i)* $k^7 \times k^{10}$	$\boxed{k^{17}}$
j) $w^{12} \times w$	$\boxed{w^{13}}$	*k)* $\pi^{11} \times \pi^{89}$	$\boxed{\pi^{100}}$	*l)* $\theta^6 \times \theta^4$	$\boxed{\theta^{10}}$

3. *The Quotient Rule says to divide powers we subtract the indices.*

$$\frac{a^p}{a^q} = a^{p-q} \qquad \textit{Example: } \frac{m^7}{m^3} = m^{7-3} = m^4$$

Write each expression as a single power:

a) $a^9 \div a^3$	$\boxed{a^6}$	b) $b^7 \div b^2$	$\boxed{b^5}$	c) $c^{12} \div c^5$	$\boxed{c^7}$
d) $m^{10} \div m^4$	$\boxed{m^6}$	e) $x^8 \div x$	$\boxed{x^7}$	f) $y^{11} \div y^6$	$\boxed{y^5}$
g) $p^{15} \div p^9$	$\boxed{p^6}$	h) $q^{13} \div q^7$	$\boxed{q^6}$	i) $k^{14} \div k^3$	$\boxed{k^{11}}$
j) $w^{20} \div w^5$	$\boxed{w^{15}}$	k) $t^9 \div t^2$	$\boxed{t^7}$	l) $r^{18} \div r^{10}$	$\boxed{r^8}$

4. *The Power of a Power Rule says that to take a power of a power, we multiply the indices.*

$$(a^p)^q = a^{pq} \qquad \textit{Example: } (m^4)^3 = m^{4\times 3} = m^{12}$$

Write each expression as a single power:

a) $\left(a^2\right)^5$	$\boxed{a^{10}}$	b) $\left(a^5\right)^2$	$\boxed{a^{10}}$	c) $\left(b^7\right)^2$	$\boxed{b^{14}}$
d) $\left(m^5\right)^3$	$\boxed{m^{15}}$	e) $\left(t^3\right)^{17}$	$\boxed{t^{51}}$	f) $\left(y^7\right)^{13}$	$\boxed{y^{91}}$
g) $\left(p^{11}\right)^3$	$\boxed{p^{33}}$	h) $\left(q^6\right)^5$	$\boxed{q^{30}}$	i) $\left(k^7\right)^{10}$	$\boxed{k^{70}}$

The process of ***powering a power*** is *commutative in its final outcome. After all,*

$$(a^p)^q \text{ and } (a^q)^p \quad \text{both simplify to} \quad a^{pq} = a^{qp}.$$

Some numbers show this especially clearly. For example, $2^6 = 64$ *is both a perfect square and a perfect cube:*

$$\sqrt{64} = 8, \qquad \sqrt[3]{64} = 4.$$

Likewise, $3^6 = 729$ *and* $5^6 = 15625$ *have the same property. Numbers of the form* n^6 *are sometimes called* ***squbes*** *because they are both squares* ($\sqrt{n^6} \in N$) *and cubes* ($\sqrt[3]{n^6} \in N$).

5. *The zero index rule takes a while to trust. Anything (except 0) raised to the power 0 is defined to be 1. It feels strange at first but becomes routine quickly.*

$$a^0 = 1$$

Evaluate each:

a) 5^0	$\boxed{1}$	b) x^0	$\boxed{1}$	c) 7^0	$\boxed{1}$	d) p^0	$\boxed{1}$
e) π^0	$\boxed{1}$	f) θ^0	$\boxed{1}$				

6. *Negative powers are counter intuitive so it is a good strategy to convert them to positive powers as fast as possible. A negative index in the numerator simply means the base belongs on the bottom of a fraction, with the power made positive.*

$$a^{-p} = \frac{1}{a^p}$$

Rewrite each with a positive index:

a) a^{-3}	$\boxed{\frac{1}{a^3}}$	b) b^{-5}	$\boxed{\frac{1}{b^5}}$	c) x^{-7}	$\boxed{\frac{1}{x^7}}$	d) m^{-2}	$\boxed{\frac{1}{m^2}}$
e) t^{-9}	$\boxed{\frac{1}{t^9}}$	f) y^{-4}	$\boxed{\frac{1}{y^4}}$	g) p^{-6}	$\boxed{\frac{1}{p^6}}$	h) q^{-8}	$\boxed{\frac{1}{q^8}}$

7. *When the negative power is already in the denominator, it just comes up and becomes positive. One familiar sounding phrase covers both situations — change floors, change signs.*

$$\frac{1}{a^{-p}} = a^p$$

Rewrite each with a positive power and no fraction:

a) $\frac{1}{a^{-3}}$	$\boxed{a^3}$	b) $\frac{1}{b^{-4}}$	$\boxed{b^4}$	c) $\frac{1}{x^{-7}}$	$\boxed{x^7}$
d) $\frac{1}{m^{-1}}$	$\boxed{m^1}$	e) $\frac{1}{t^{-9}}$	$\boxed{t^9}$	f) $\frac{1}{y^{-5}}$	$\boxed{y^5}$
g) $\frac{1}{p^{-6}}$	$\boxed{p^6}$	h) $\frac{1}{q^{-8}}$	$\boxed{q^8}$		

8. *When a product is inside a bracket, the power applies to the whole product. To see why, look at the square of a product:*

$$(ab)^2 = (ab)(ab).$$

Remove the brackets and rearrange the factors:

$$(ab)(ab) = abab = aa\,bb = a^2b^2.$$

Exactly the same pattern holds for any power.

$$(ab)^p = a^pb^p$$

Expand each:

a) $(ab)^3$	$\boxed{a^3b^3}$	b) $(5b)^4$	$\boxed{5^4b^4}$	c) $(10q)^2$	$\boxed{10^2q^2}$
d) $(t7)^5$	$\boxed{t^57^5}$	e) $(rs)^7$	$\boxed{r^7s^7}$	f) $(2r)^6$	$\boxed{2^6r^6}$

Soundbite: The power of a product equals the product of the powers.

9. *Division takes priority due to the bracket even though it is normally of lower priority than powers, the fraction must be dealt with first. That means the outside power applies to the whole fraction.*

The power simply repeats the fraction as many times as needed, and we use the usual rule: multiply the tops together and the bottoms together.

For example,

$$\left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{2 \cdot 2}{5 \cdot 5} = \frac{4}{25}.$$

The same structure works for all powers:

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}.$$

Conclusion: *the power of a fraction equals the fraction of the powers.*

Expand each:

a) $\left(\frac{a}{b}\right)^3$	$\boxed{\frac{a^3}{b^3}}$	*b)* $\left(\frac{3}{5}\right)^2$	$\boxed{\frac{3^2}{5^2}}$	*c)* $\left(\frac{p}{q}\right)^6$	$\boxed{\frac{p^6}{q^6}}$
d) $\left(\frac{t}{y}\right)^5$	$\boxed{\frac{t^5}{y^5}}$	*e)* $\left(\frac{r}{s}\right)^7$	$\boxed{\frac{r^7}{s^7}}$	*f)* $\left(\frac{k}{m}\right)^2$	$\boxed{\frac{k^2}{m^2}}$

A Note on Priority of Operations — Brackets Are King

Most of the rules in this chapter really come down to one idea: ***priority of operations****. With no brackets, the order is fixed:* ***powers first****, then* ***multiplication and division****, and finally* ***addition and subtraction****. That is the natural hierarchy.*

But once a bracket *appears, everything changes. A bracket is the ultimate authority — whatever is inside it becomes the new* first job. *If you want something to overpower everything else, you simply put brackets on it.*

Because of that: - multiplication can distribute over addition inside a bracket, - and now, powering can distribute over a product or over a fraction — but only because the bracket forces it to.

Without the bracket, these rules simply do not hold.

With brackets: $(ab)^3 = a^3b^3$

Without brackets: $ab^3 \neq a^3b^3$

Non–examples that show why brackets matter:

$$\frac{3^2}{5} = \frac{9}{5} \qquad \textit{but} \qquad \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$2x^2 \quad \textit{means} \quad 2(x^2) = 2x^2$$

$$(2x)^2 \quad \textit{means} \quad (2x)(2x) = 4x^2$$

The bracket completely changes the meaning.

Difference of two squares — another bracket story:

$$9x^2 - 49 = (3x)^2 - 7^2$$

The only way this becomes a recognisable difference of two squares is by seeing the entire expressions $(3x)$ *and* 7 *as squared quantities — again, brackets controlling priority.*

The big message: *Powering is a high–priority operation — but a bracket is even more powerful. If a product or a fraction is in brackets, the bracket forces us to deal with it first, and only then does the power distribute. That is why:*

$$(ab)^p = a^p b^p, \qquad \left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}.$$

Everything in this chapter is really one theme: ***brackets decide the order — and when the order changes, new rules appear.***