2024.M30

2024L003A2EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2024

Mathematics

Paper 2

Higher Level

Monday 10 June Morning 9:30 - 12:00

300 marks

Examination Number

Date of Birth

For example, 3rd February 2005 is entered as 03 02 05

Centre Stamp

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer **any five** questions from Section A.

Answer **any three** questions from Section B.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here: [ ]

Section A — Concepts and Skills — 150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

A group of 22 students was tested to see how far, in metres, each of them could swim without stopping for a rest. The results of the testing are shown in the ordered stem and leaf plot below. Four of the entries have been replaced with the letters a, b, c, and d.

2	b	2	7			
3	a	4	4	5	8	
4	0	1	d	5	6	9
5	2	3	7	7	8	
6	1	8	c			

Key: $2|7 = 27$ metres

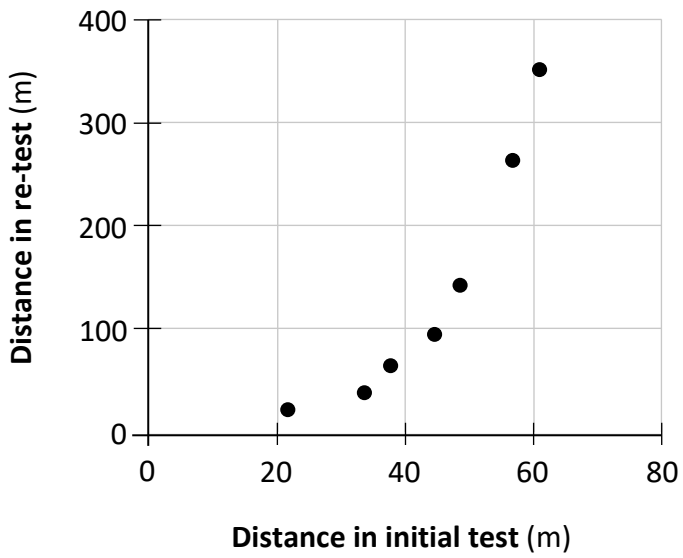
(a) **(i)** The **mode** of the data is 34 metres. Use this to write down the value of a.

(ii) The **range** of the data is 49 metres. Use this to find the value of b and the value of c.

(iii) The **median** of the data is $43{\cdot}5$ metres. Use this to find the value of d.

Seven of the 22 students took swimming lessons.
They were re-tested after the lessons, to see how far they could now swim without stopping.
The table and graph below show the results of the initial test and of the re-test for these students.

Distance in initial test (m)	**Distance in re-test** (m)
22	25
34	40
38	65
45	96
49	142
57	262
61	350



(b) How would you best describe how the results changed for these students, from the initial test to the re-test?

(c) The swimming coach worked out r, the correlation coefficient between the distance in the initial test and the distance in the re-test, for these seven students.
Find the value of r, correct to 4 decimal places.

$r =$

Question 2 **(30 marks)**

(a) The table below shows the prizes, in euro, that a player can win in a game, as well as the probability of winning each prize. The player wins at most one prize each time that she plays. Some of the prizes are given in terms of $x \in \mathbb{R}$.

Prize (€)	None	2	$x - 10$	x
Probability	30%	40%	28%	2%

It costs €10 to play the game once.

The game is **fair** – that is, the expected value of the winnings, taking the cost into account, is €0.

Work out the value of x.

(b) A and B are **mutually exclusive** events.

$P(A) = 0{\cdot}1$ and $P(B) = 0{\cdot}4$.

Write down the value of each of the following:

$P(A \cap B) =$ $P(A \cup B) =$

(c) C and D are two other events, with universal set U.
$P(C) = 0{\cdot}5$ and $P(D) = 0{\cdot}7$.

Find the **maximum** value of $P[(C \cup D)']$.

Note: $(C \cup D)'$ is the complement of the set $C \cup D$ in the set U.

(d) E is the event that it will be raining tomorrow morning.
F is the event that I will wear a coat going outside tomorrow morning.

Explain why it would **not** be reasonable to assume that E and F are independent events.

Question 3 **(30 marks)**

(a) $ABCD$ is a parallelogram.

$|AB| = 10$ cm, $|BC| = 13$ cm, and $|\angle ABC| = 110°$.

Find the area of $ABCD$, correct to the nearest cm^2.

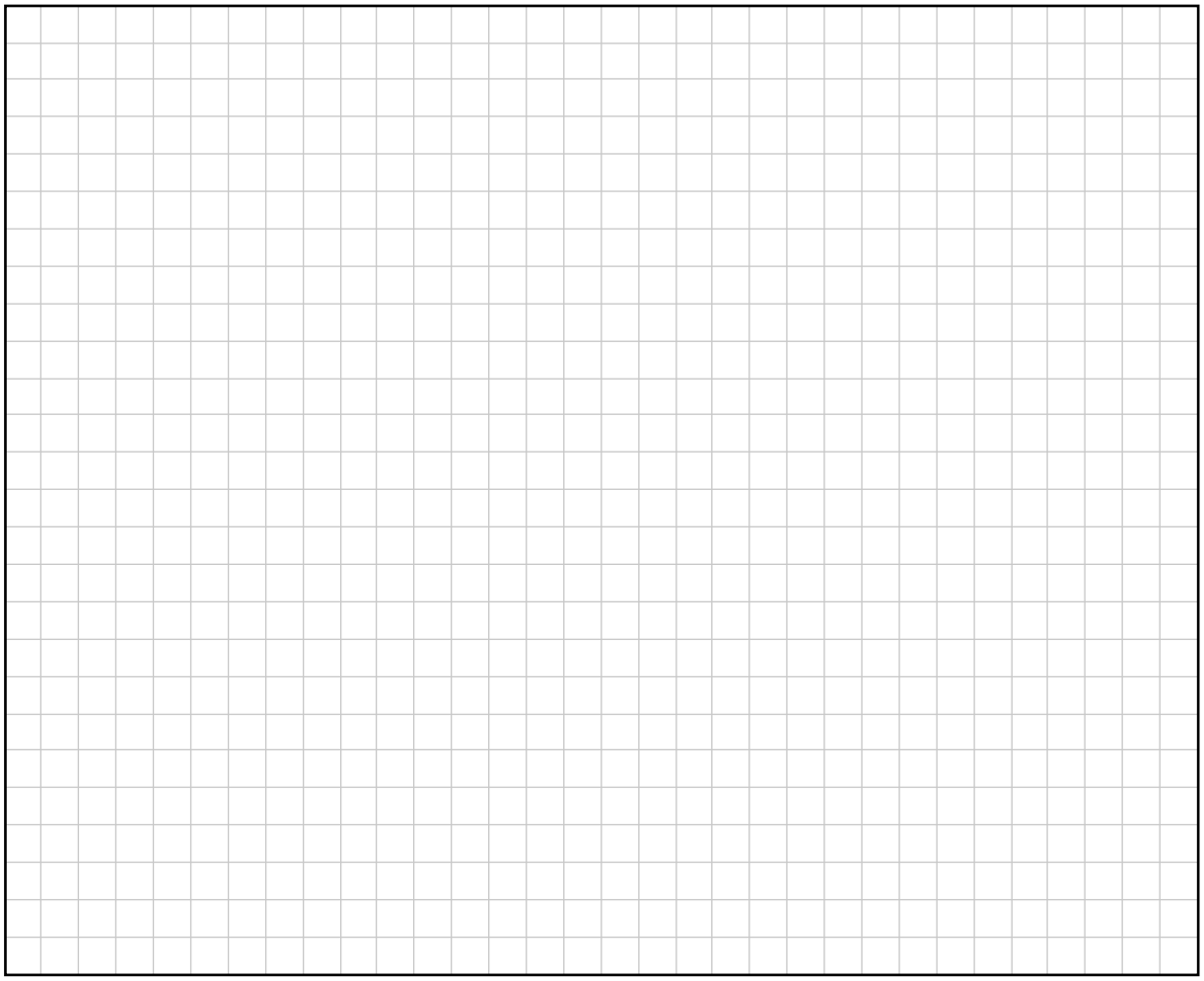
(b) X is an angle, with $0° \leq X \leq 360°$, and

$$\cos(2X) = \frac{\sqrt{3}}{2}$$

Find **all** the possible values of X.

(c) KLM is a triangle where $|MK| = 15\sqrt{3}$ cm, $|ML| = 45$ cm, and $|\angle KLM| = 25°$.
θ is the angle $\angle LKM$.

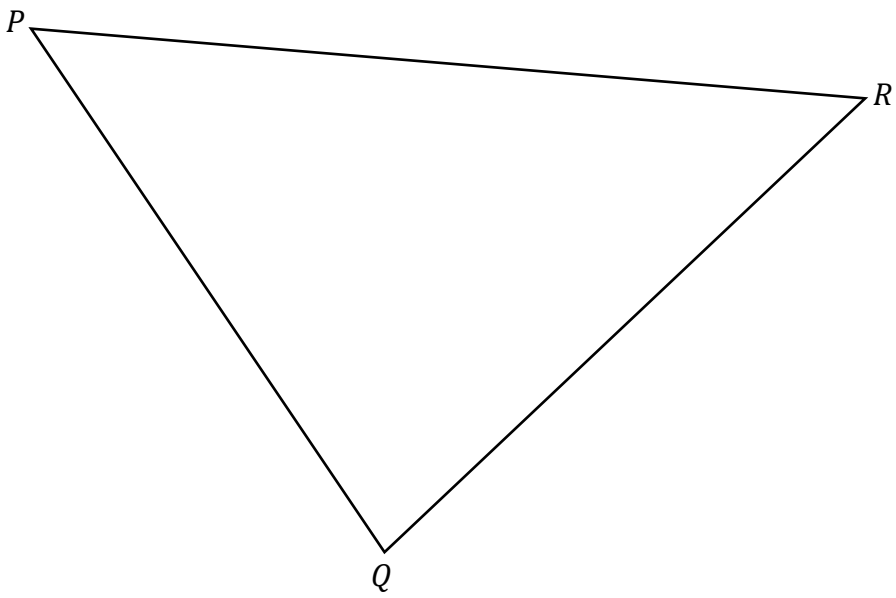
Work out the **two** possible values of θ, for $0° < \theta < 180°$.
Give each answer correct to the nearest degree.



Question 4 **(30 marks)**

(a) Construct the **centroid** of the triangle PQR below.

Show all your construction lines clearly.
Where measurement is used, show all relevant measurements and calculations clearly.



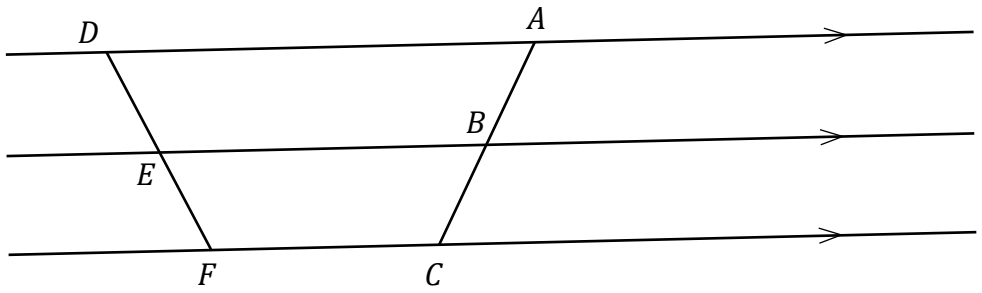
(b) The lines AD, BE, and CF in the diagram below are parallel. $E \in DF$ and $B \in AC$.

$|AB| = |BC|$.

Prove that $|DE| = |EF|$.

(That is, prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.)

Give a reason for each statement that you make in your proof, as appropriate.



Construction:

Proof:

Question 5 **(30 marks)**

(a) The circle s has equation:

$$x^2 + y^2 + 4x - 6y + 5 = 0$$

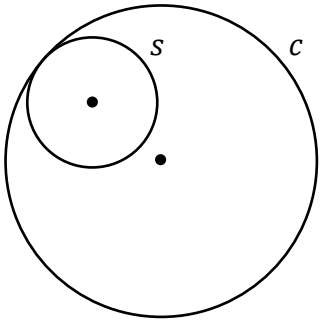
(i) Write down the centre and radius of the circle s.

Centre $= ($, $)$ Radius $=$ _____

(ii) The circle c has equation:

$$(x - 2)^2 + (y + 1)^2 = 72$$

Show that the circles s and c touch internally.

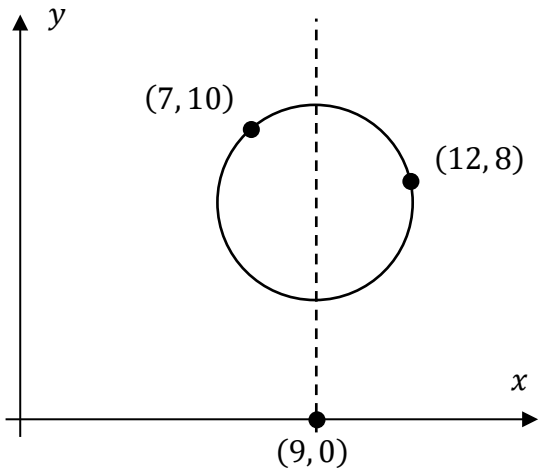


(b) Another circle has its centre on the vertical line through the point $(9, 0)$.

The points $(7, 10)$ and $(12, 8)$ are on this circle.

Find the equation of this circle.

Note that your answer may contain non-integer values.



Question 6 **(30 marks)**

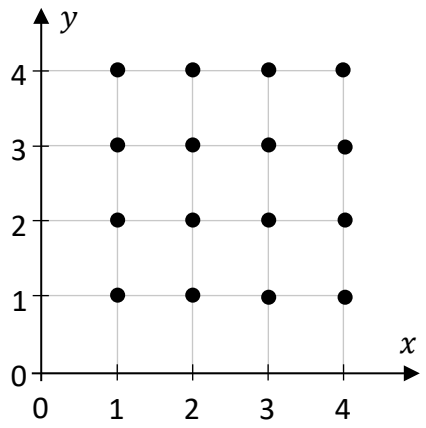
(a) $[AB]$ is a line segment.
The point $C\ (6, 11)$ divides the line segment $[AB]$ internally in the ratio $1:3$.

A is the point $(1, 13)$.
Find the co-ordinates of the point B.

(b) Find the perpendicular distance from the point $(5, -2)$ to the line:

$$y = \frac{4}{3}\,x - 11$$

(c) In the co-ordinate diagram below, 16 points are marked with a dot (●).
These are all of the points of the form (m, n), where $m, n \in \mathbb{N}$ and $m, n \leq 4$.



A pair of these points is picked at random.

(i) How many different pairs of points can be picked from these 16 points?

(ii) The two points that are picked are joined with a straight line.
Find the **probability** that this line is horizontal.

Section B — Contexts and Applications — 150 marks

Answer **any three** questions from this section.

Question 7 **(50 marks)**

PK Hotels is a hotel chain in Europe.

(a) The ages of the people who stayed in a PK Hotel in 2023 are roughly normally distributed, with a mean age of $48{\cdot}2$ years and a standard deviation of $10{\cdot}6$ years.

(i) One person is picked at random from the people who stayed in a PK Hotel in 2023. Find the probability that this person is less than 50 years old.

(ii) Exactly 10% of people who stayed in a PK Hotel in 2023 are at least A years old. Find the value of A, correct to the nearest whole number.

(b) During their most recent stay, $\frac{1}{5}$ of PK Hotel customers used the pool.

Use this to answer parts **(b)(i)** and **(b)(ii)**.

(i) 6 of the PK Hotel customers are picked at random. Find the probability that exactly 2 of them used the pool.

(ii) n of the PK Hotel customers are picked at random, where $n \in \mathbb{N}$.

The probability that **none** of them used the pool, correct to 4 decimal places, is $0{\cdot}0047$. Work out the value of n.

(c) PK Hotels are testing a new booking system.

45% of people who log on to the PK Hotels website are shown the old booking system; the other 55% are shown the new booking system.
People are assigned the booking system (old or new) at random.

One third of people who see the old booking system end up booking a room.
Two fifths of people who see the new booking system end up booking a room.

One person is selected at random from those who booked a room through the PK Hotels website. Find the probability that this person used the **new** booking system.
Give your answer as a percentage, correct to the nearest percent.

This question continues on the next page

(d) In 2020, PK Hotels were rated the best hotel chain in Europe by 75% of their customers.

In 2024, PK Hotels carried out a survey of a random sample of 1000 of their customers to see if this percentage had changed. Of these, 765 rated PK Hotels the best hotel chain in Europe.

Carry out a hypothesis test at the 5% level of significance to see if this shows a change in the percentage of their customers who rate PK Hotels the best chain in Europe.

State your null hypothesis and your alternative hypothesis, state your conclusion, and give a reason for your conclusion.

Null Hypothesis:

Alternative Hypothesis:

Calculations:

Conclusion:

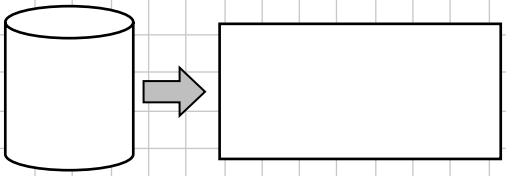
Reason for your conclusion:

Question 8 **(50 marks)**

Tommy makes ornaments from metal and glass.

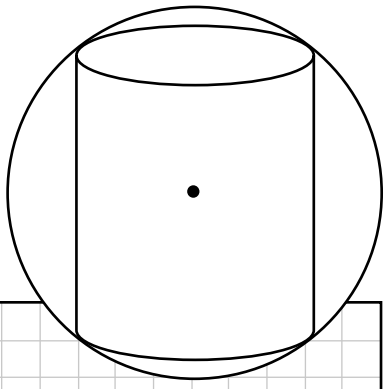
(a) He makes an open metal cylinder with a height of 15 cm and a radius of 5 cm.
The **net** of this cylinder is a rectangle.

Find the dimensions of this rectangle.
Give your answers in cm, correct to 1 decimal place where appropriate.



Dimensions: _____ by _____

(b) Tommy makes another cylinder with a height of 22 cm and a diameter of 12 cm.



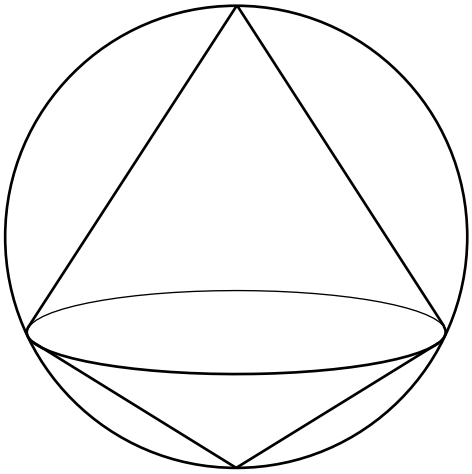
This cylinder fits exactly inside a glass sphere.
The top and bottom edges of the cylinder touch the sphere.

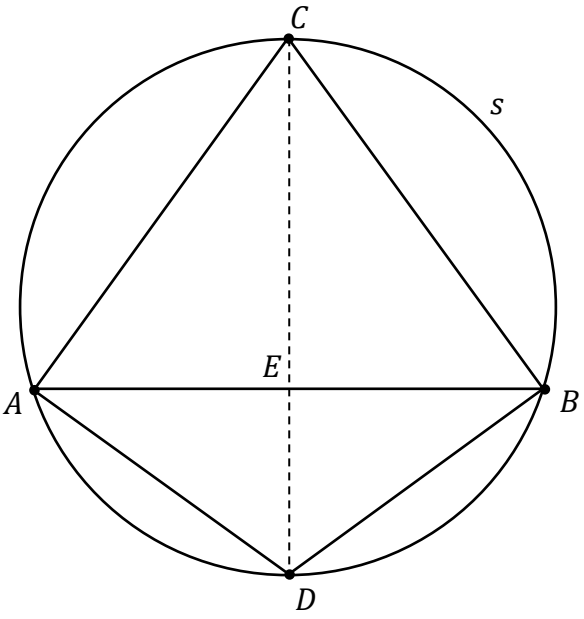
Find the **volume** of the **sphere**, in cm^3, correct to 1 decimal place. Use the Theorem of Pythagoras in your solution.

This question continues on the next page.

(c) Another ornament is made of two cones inscribed in a sphere.
The top cone is upright; the bottom cone is inverted. The cones have the same base.

A vertical cross-section of the ornament, taken through the centre of the sphere, shows the cones as two triangles, ABC and ADB, with a common side $[AB]$. ABC is the top cone. The points A, B, C, and D all lie on the circle s, which represents the cross-section of the sphere. The lines AB and CD intersect at the point E.





(i) The diagram is symmetrical about the line DC. State why $|\angle CBD| = 90°$.

(ii) Hence, or otherwise, **prove** that the triangles BCE and DBE are **similar**.
Give a reason for each statement that you make, where appropriate.

There is more space for work on the next page.

(iii) The top cone has a radius of r and a height of h; that is, $|EB| = r$ and $|EC| = h$. The sphere, represented by s, has a radius of 10 cm.

Use the similar triangles BCE and DBE to show that:

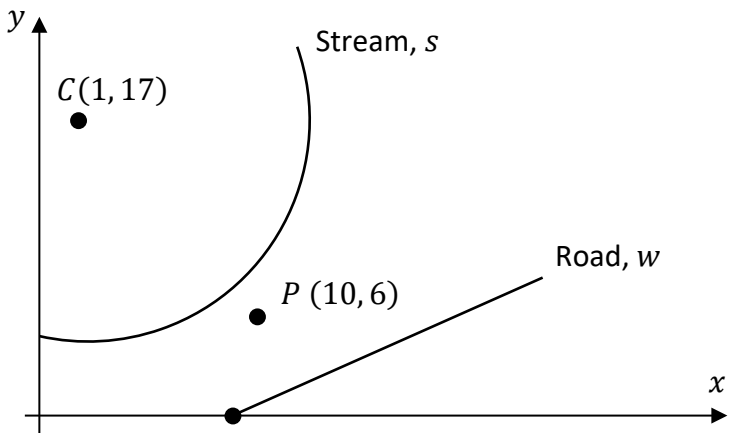
$$r^2 = 20h - h^2$$

(iv) Hence, write the volume of the top cone in terms of h and π, **and** find the value of h that gives the **maximum volume** for the top cone.

$h =$ _____

Question 9 **(50 marks)**

Ameena, Petro, and Fiadh are taking part in an adventure race.
The co-ordinate diagram below shows part of the course for this race.
Each unit on the diagram represents 100 metres.



(a) The arc in the diagram represents part of a stream.
The arc is part of a circle s with centre $C(1, 17)$ and radius 12.

(i) Write down the equation of the circle s.

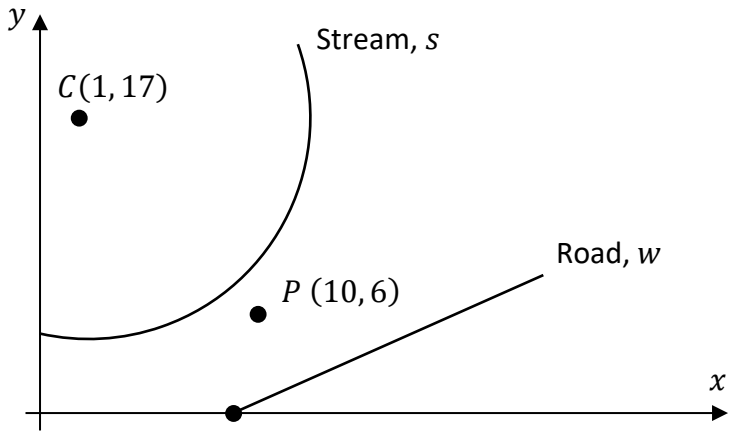
(ii) Ameena is at the point $(a, 8)$ on the stream (circle s), where $a \in \mathbb{R}$, $a > 0$.
Work out the value of a. Give your answer in surd form.

(iii) Petro is at the point $P(10, 6)$.
Work out the shortest distance from the point P to the stream (circle s).

Give your answer correct to the nearest metre.
Remember that each unit on the diagram represents 100 metres.

This question continues on the next page.

The original diagram is shown again below.



(b) There is a straight path, l, that is not shown on the diagram.
l is parallel to the y-axis, and is a **tangent** to the stream s in the first quadrant.

Write down the equation of this path l. Remember that the radius of s is 12.

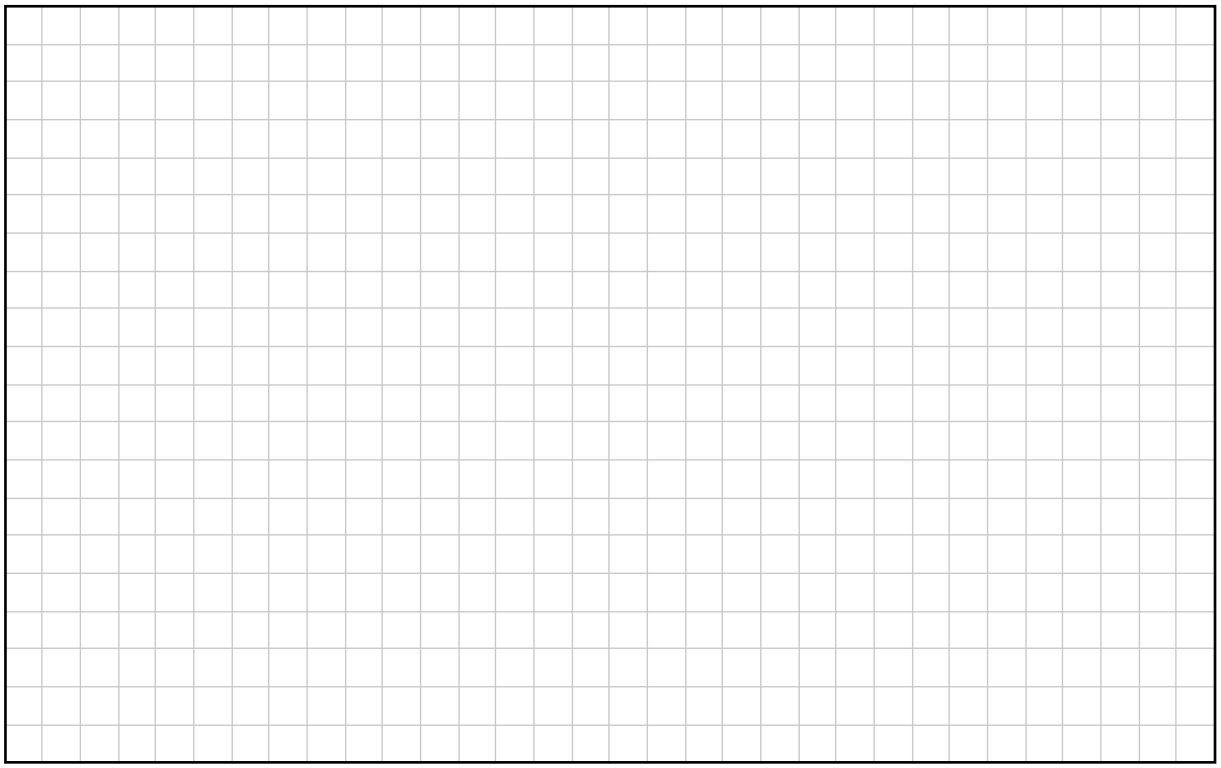
The line segment w represents a road, where w has the equation:

$$x - 3y = 9$$

for $0 \leq y \leq 8$.

(c) Find the co-ordinates of the point on the road w that is closest to the point $P(10, 6)$.
It might be useful to find the equation of the line through P that is perpendicular to w.

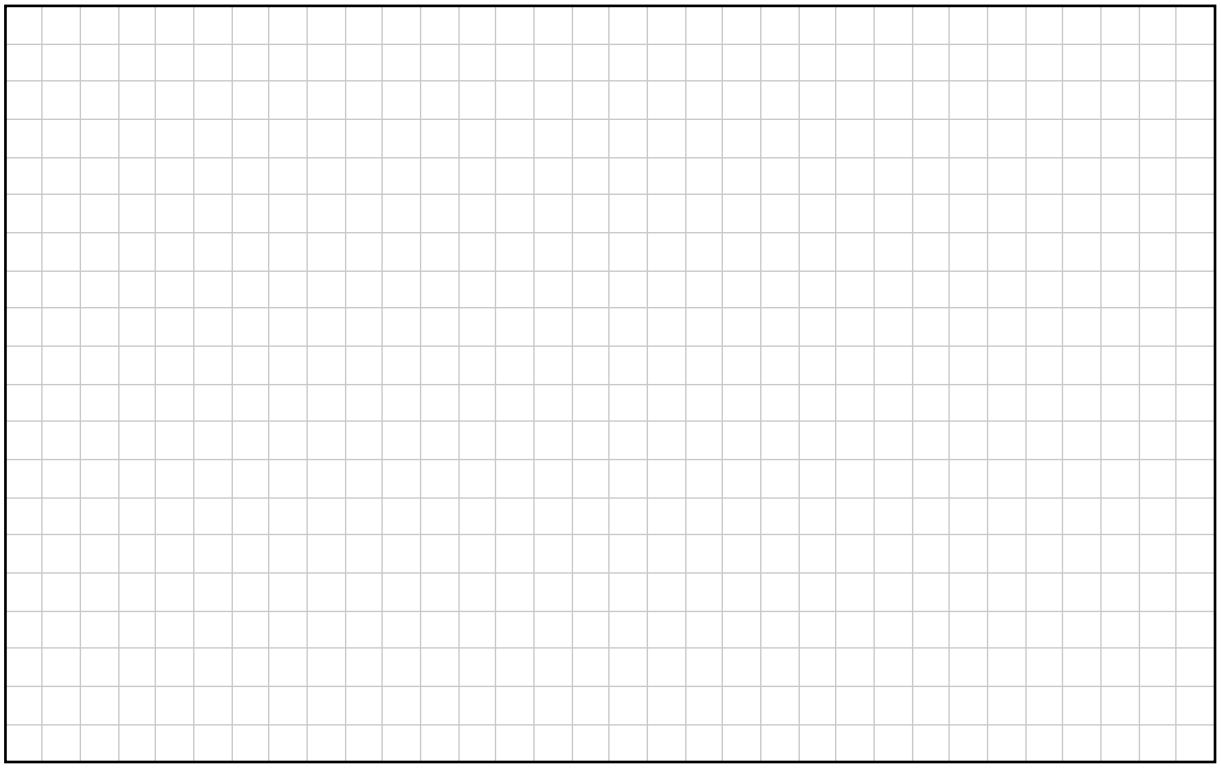
There is more space for work on the next page.



(d) Fiadh is at the point $F(9, 0)$ on the road w.

She travels 1200 m along the road w away from the point F, in the first quadrant, and then stops.

Work out the co-ordinates of the point at which she stops. Give each value correct to 1 decimal place. Remember that each unit on the diagram represents 100 metres.



Question 10 **(50 marks)**

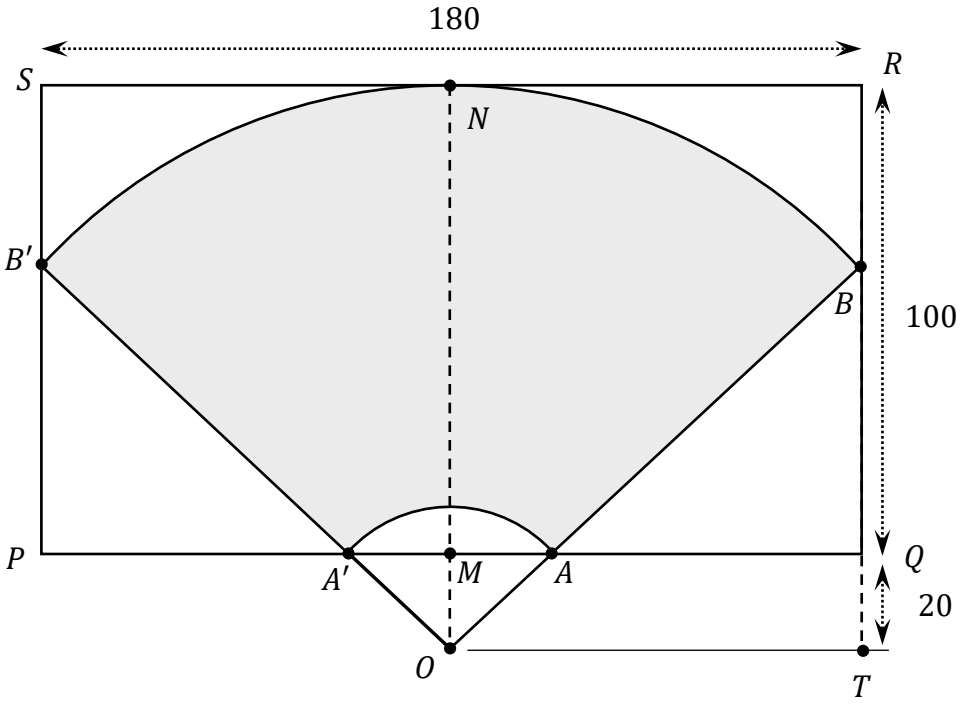
A company makes windscreen wipers.

In this question, the rectangle $PQRS$ has a width of 180 cm and a height of 100 cm.
M is the midpoint of $[PQ]$, N is the midpoint of $[SR]$, and $O \in NM$. All lengths are given in cm.

(a) In the diagram below, the line segment $[AB]$ shows a type of wiper blade.
$[AB]$ rotates around the point O, where $O \in AB$, until it reaches the position $[A'B']$.

The region that it cleans is $ABB'A'$, which is the sector OBB' with the sector OAA' removed.

A, A', B, and B' lie on the rectangle $PQRS$, and N lies on the arc from B to B'.
The line segment $[RQ]$ is extended 20 cm to T, as shown. $\angle OTR$ is a right angle.



(i) Show that $|OB| = 120$ cm.

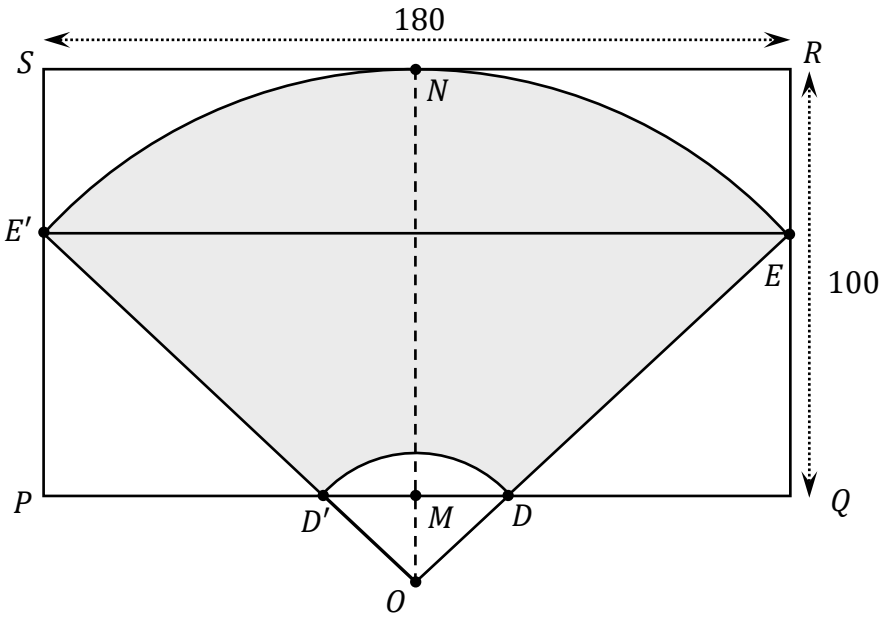
(ii) Hence, show that $|\angle BOT| = 41{\cdot}4°$, correct to 1 decimal place.

(iii) Hence, work out the area of $ABB'A'$.

Remember that $ABB'A'$ is the sector OBB' with the sector OAA' removed.
Give your answer correct to the nearest cm^2.

This question continues on the next page.

(b) Moving the point O along the line NM changes the size of the wiper blade and the region that it cleans. In the diagram below, $[DE]$ rotates about O, where $O \in DE$, until it reaches $[D'E']$. As in part **(a)**, E and E' lie on the rectangle $PQRS$.



Here, $|\angle E'OE| = 105°$.

By setting $|OE| = x$, use the **triangle** OEE' and the **Cosine Rule** to find the value of $|OE|$. Give your answer in cm, correct to 1 decimal place.

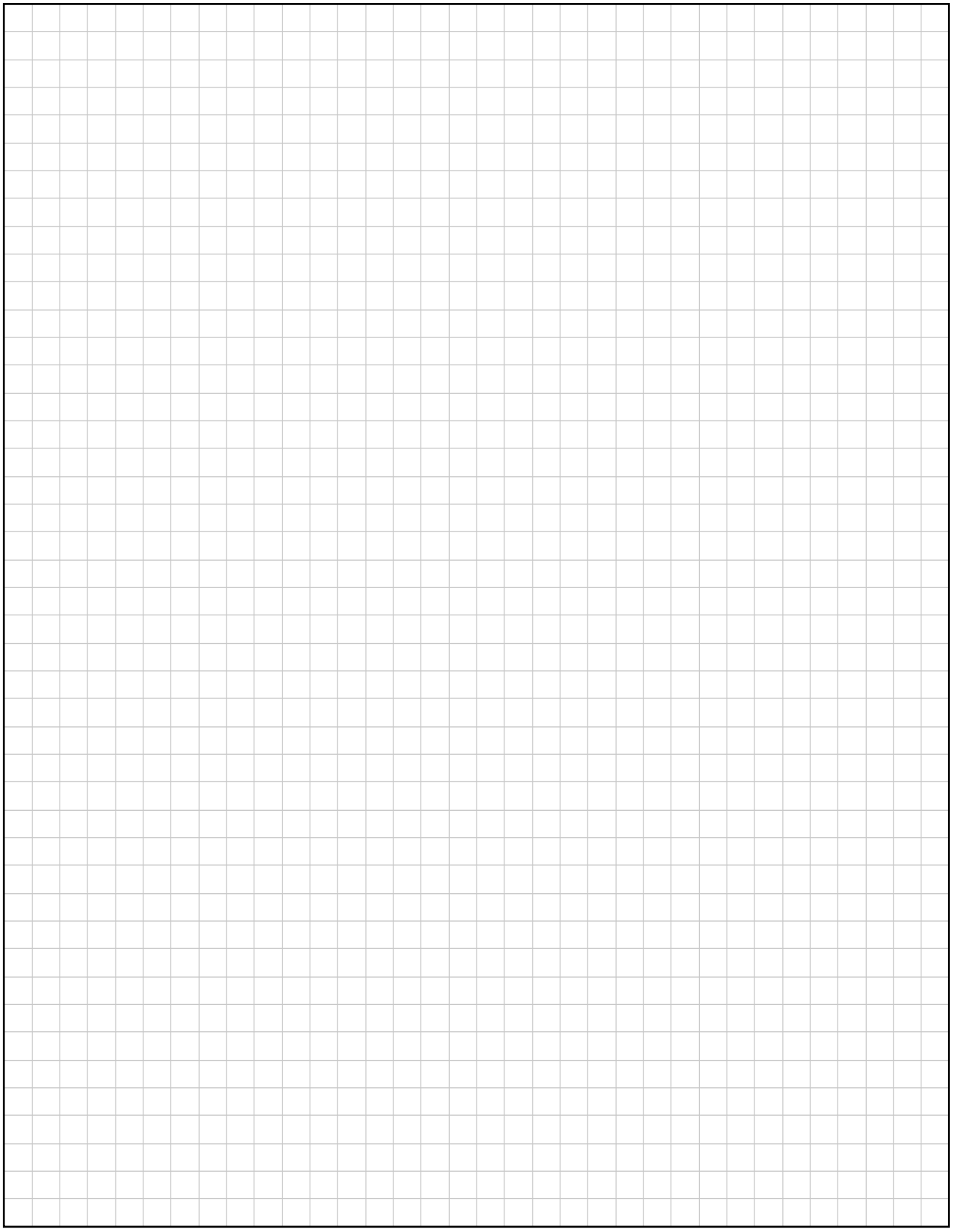
(c) Mattie is driving home. On the way, she passes five traffic lights.
Each traffic light is either red (**R**), green (**G**), or orange (**O**) when she arrives at it.
One day, she notes the pattern made by the colour of each traffic light when she arrives at it.
For example, this pattern could be **R R O G R**.

(i) How many different patterns could the five traffic lights make?

(ii) How many different patterns could the five traffic lights make, if the first light is red **and** the fifth light is **not** red?

(iii) How many different patterns could the five traffic lights make if no two consecutive lights are the same colour?

Page for extra work.
Label any extra work clearly with the question number and part.



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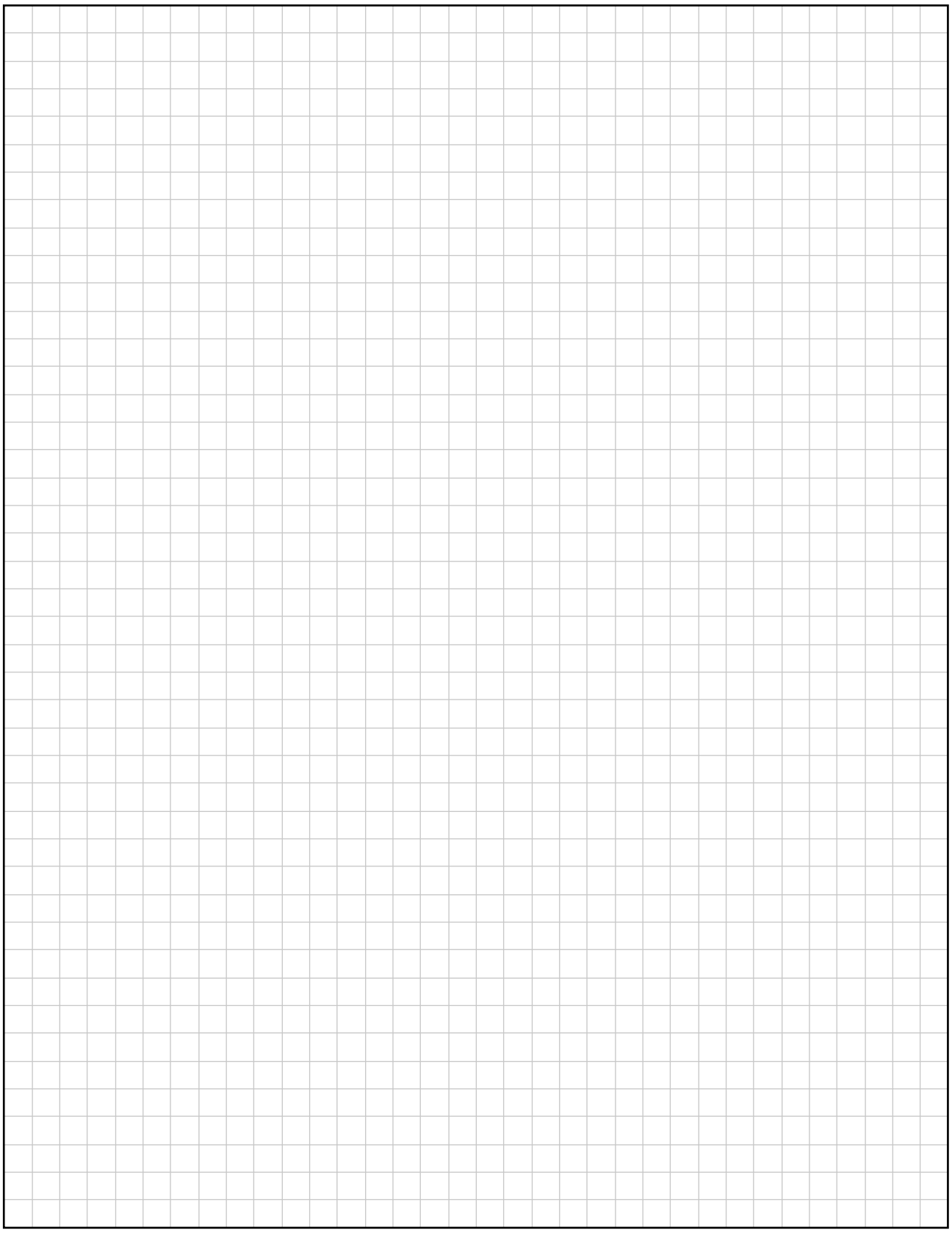

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Leaving Certificate – Higher Level

Mathematics Paper 2

Monday 10 June

Morning 9:30 - 12:00