L.116

2024-L116-1-EL-01/32

Pre-Leaving Certificate Examination, 2024

Mathematics

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

CANDIDATE DETAILS

DAY and MONTH of BIRTH ✎ | | | / | | | For example, 3rd February is entered as 03/02

NAME ✎

SCHOOL ✎

TEACHER ✎

School Stamp

For Examiner only		
Question		Mark
A.	1.	
	2.	
	3.	
	4.	
	5.	
	6.	
B.	7.	
	8.	
	9.	
	10.	
Total		
%		

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- any **five** questions from Section A – Concepts and Skills
- any **three** questions from Section B – Contexts and Applications.

Write your Name and Individual Details in the grid on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

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Section A — Concepts and Skills — 150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

(a) Jim got his car serviced recently in a local garage.
An extract from the bill is shown below. Some of the details are missing.

Item	Details	Cost (€)
1	Paintwork	65·00
2	Bulbs	12·00
3	Steering	25·00
4	Lights set	25·00
5	Brake pads	49·00
6	Labour	
	Sub-total	326·00
	VAT	
	Total (including VAT)	370·01

(i) Work out how much was charged for labour **and** VAT.

(ii) Work out the percentage rate of VAT applied to the bill.

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(b) The speedometer in Jim's car shows the speed of the car in **miles per hour**.
Jim drives at an average speed of 55 miles per hour from his home in Dublin to Cork.
The total distance travelled is 245 km.

(i) The conversion rate between kilometres and miles is 1 km = 0·62137119 miles.
Work out Jim's average speed in **km/hour**.
Give your answer correct to 1 decimal place.

(ii) Jim leaves his home at 07:10 a.m. and stops for 25 minutes along the route for a break.
At what time does he arrive at his destination?
Give your answer correct to the nearest minute.

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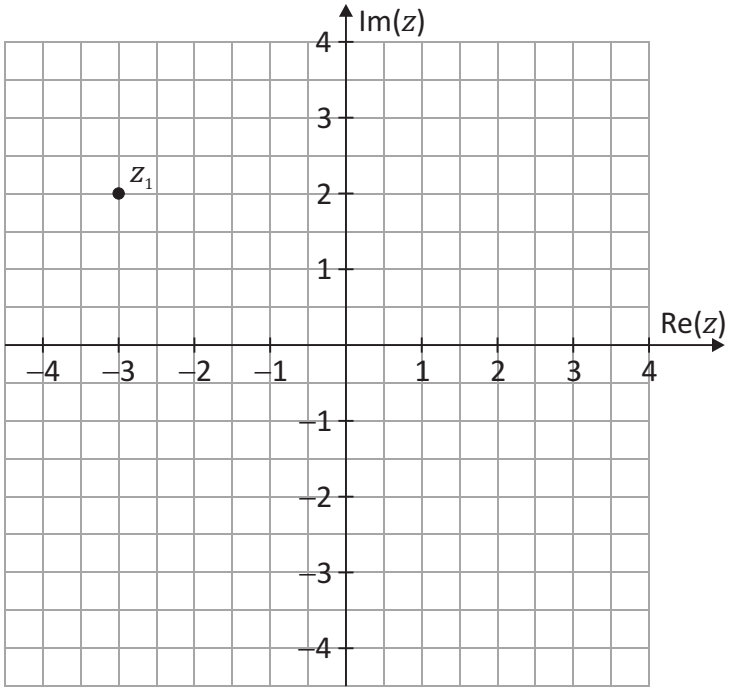
Question 2 **(30 marks)**

(a) The complex number z_1 is shown on the Argand diagram below.



(i) Write down z_1 in the form $a + bi$, where $a, b \in \mathbb{Z}$ and $i^2 = -1$.

(ii) Let $z_2 = iz_1$.
Find z_2 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

(iii) **Plot** and **label** z_2 on the Argand diagram above.
What can you observe from the relative positions of z_1 and z_2 on the diagram?

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(b) z_3 and z_4 are two other complex numbers.
$z_3 = 3 - 4i$ and $z_4 = 12 + 5i$, where $i^2 = -1$.

$\overline{z}_3$ and $\overline{z}_4$ are the complex conjugates of z_3 and z_4, respectively.

(i) Show that $z_3\overline{z}_4 + \overline{z}_3 z_4$ is a real number.

(ii) Investigate if $|z_3| + |z_4| = |z_3 + z_4|$.

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Question 3 **(30 marks)**

(a) **(i)** Solve for x:

$$2(4-3x)+12 \;=\; 7x-5(2x-7), \text{ where } x \in \mathbb{R}.$$

(ii) Verify your answer to **part (a)(i)** above.

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(b) Solve the simultaneous equations:

$$x + 2y = 2$$
$$x^2 - 4y^2 = 4.$$

Question 4 **(30 marks)**

The function f is defined as $f(x) = x^3 - 6x^2 + 9x - 2$, where $x \in \mathbb{R}$.

(a) Find the co-ordinates of the point at which the graph of $f(x)$ cuts the y-axis.

(b) **(i)** Find $f'(x)$, the derivative of $f(x)$.

(ii) Hence, find the co-ordinates of the local maximum turning point and the local minimum turning point of $f(x)$.

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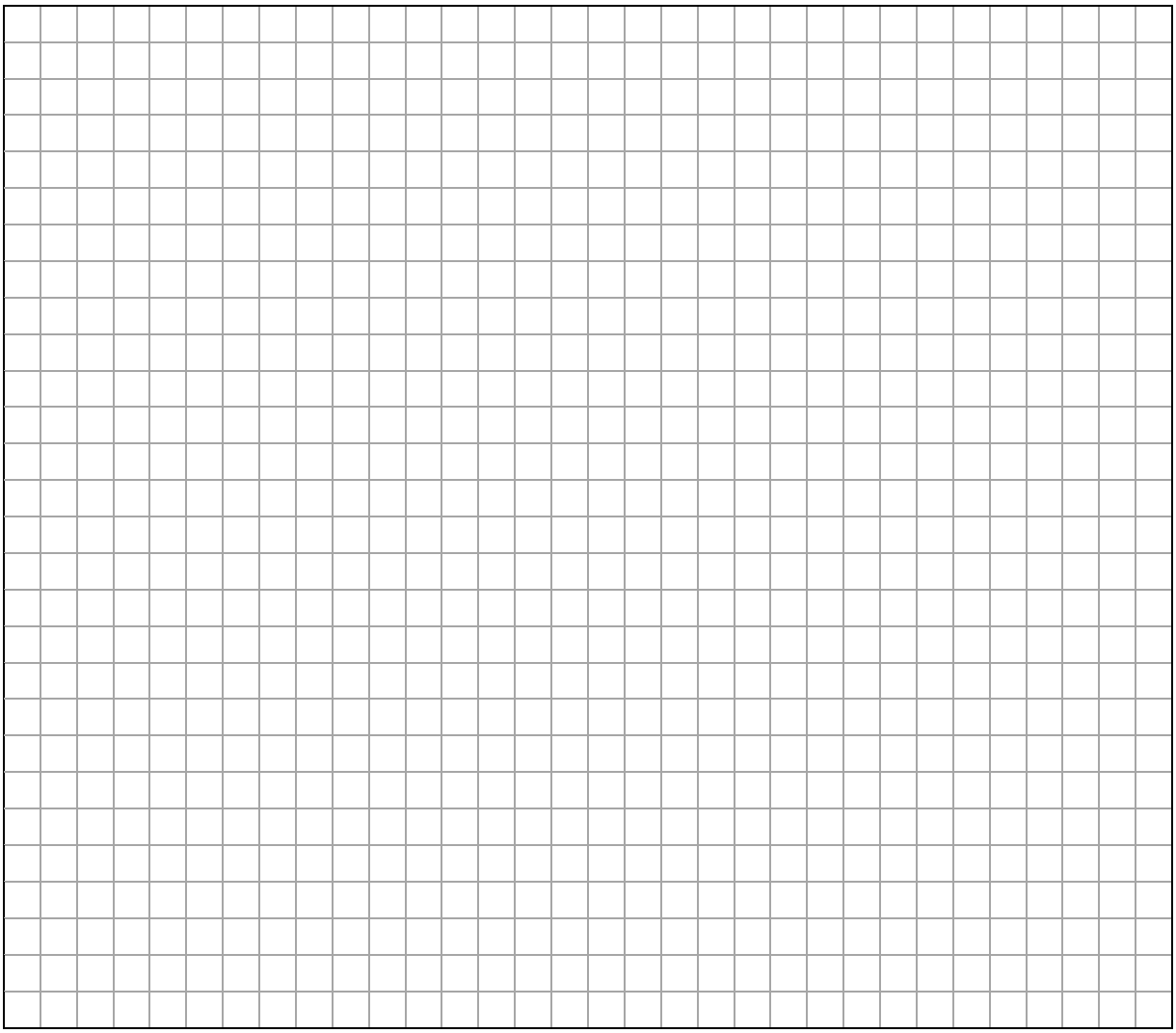
(c) Use your answer from **part (b)(i)** to find the equation of the tangent to the graph of $f(x)$ at the point $(-1, 18)$.



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Question 5 **(30 marks)**

(a) **(i)** Write $\sqrt{125}$ as a power of 5.

(ii) Hence, solve for x:

$$5^{2x-1} = \sqrt{125}.$$

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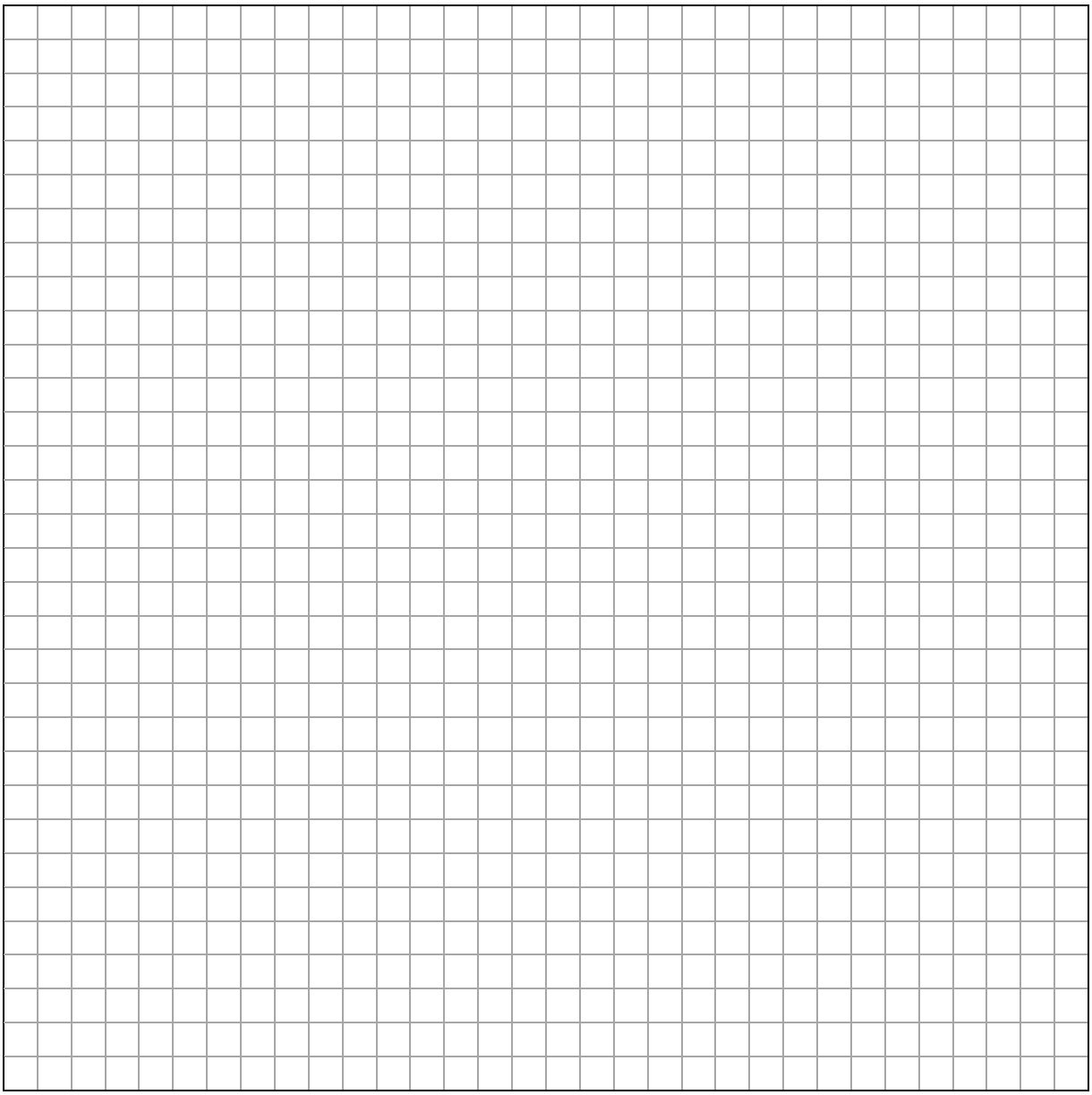
(b) Find the two values of x for which $2x^2 + 5x - 10 = 0$.
Give your answers correct to 1 decimal place.



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Question 6 **(30 marks)**

(a) The first three terms of an arithmetic sequence are 6, 13, 20.

(i) Show that S_n, the sum of the first n terms of the sequence, is equal to $\frac{7n^2+5n}{2}$.

(ii) Work out S_{10}, the sum of the first 10 terms of the sequence.

(iii) Find the minimum value of n for which the sum of the first n terms of the sequence is greater than 750.

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(b) The first five terms of **another** sequence are shown in the table below.

Term	T_1	T_2	T_3	T_4	T_5
Number	3	6	11	18	27

(i) Show that the sequence is quadratic.

(ii) The nth term in the sequence is given by the expression, for $n \in \mathbb{N}$:

$$T_n = n^2 + bn + c, \text{ where } b, c \in \mathbb{Z}.$$

Find the value of b and the value of c.

$b =$ $c =$

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Section B — Contexts and Applications — 150 marks

Answer **any three** questions from this section.

Question 7 **(50 marks)**

A bowling ball is rolled in a straight line along a standard bowling lane, which is 18·3 m in length from the foul line to the front skittle (pin). The distance of the ball, $s(t)$, in metres, from the foul line after the player releases it can be partially modelled by the function:

$$s(t) = 9t - t^2 - 0{\cdot}2,$$

where t is the time, in seconds, after the ball is released and $0 \leq t \leq 4{\cdot}5$, where $t \in \mathbb{R}$.

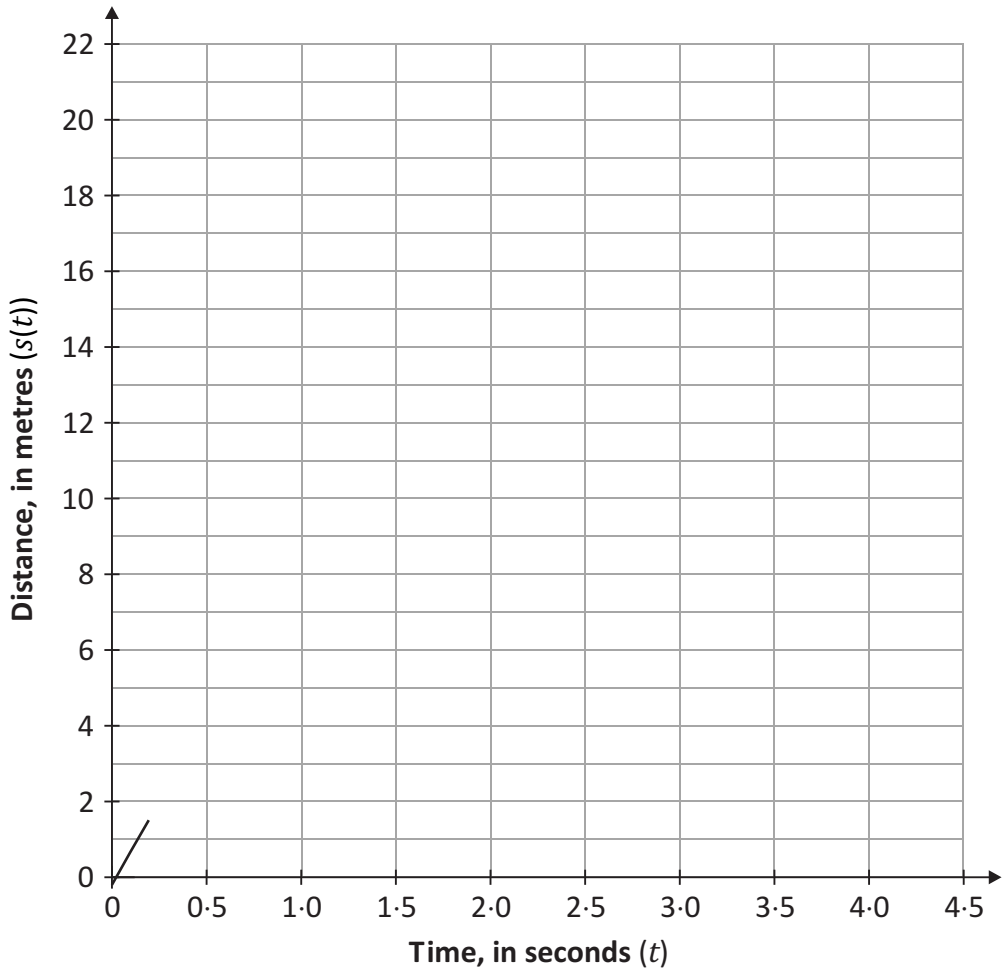
[Note: The foul line is the line on the bowling lane before which the player must release the ball; otherwise the throw is disallowed.]

(a) **(i)** How far behind the foul line does the player release the ball (when $t = 0$)?

(ii) Complete the table below to show the values of $s(t)$ for the given values of t. Give each value of $s(t)$ correct to 2 decimal places.

t	0	0·5	1	1·5	2	2·5	3	3·5	4	4·5
$s(t)$		4·05			13·80					

(iii) Complete the graph of the function $s(t)$ on the axes below for $0 \leq t \leq 4{\cdot}5$, where $t \in \mathbb{R}$.



(b) **(i)** Use your graph to estimate how long it takes the ball to reach the front skittle (pin).

(ii) Explain what the graph of the function $s(t)$ below the horizontal axis represents.

This question continues on the next page.

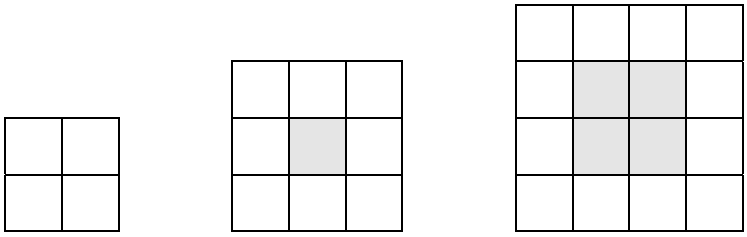
(c) **(i)** Use calculus to find, in terms of t, the rate at which the distance of the bowling ball from the foul line is changing after t seconds.

(ii) Use your answer from **part (c)(i)** to find the speed of the bowling ball when it reaches the front skittle (pin).
Give your answer in metres per second, correct to 1 decimal place.

(d) Find $h''(t)$, the second derivative of $h(t)$.
Explain the significance of your answer.

Question 8 **(50 marks)**

The first three patterns in a sequence of patterns containing grey and white tiles are shown below.



Pattern 1 Pattern 2 Pattern 3

(a) **(i)** Based on the patterns shown, complete the table to show the number of **white tiles** in each of the first five patterns of the sequence.

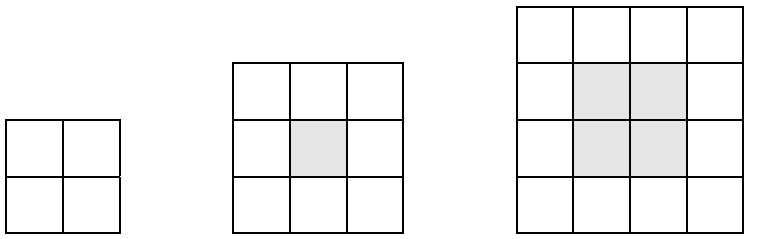
Pattern number (n)	**Number of white tiles**
1	4
2	
3	
4	
5	

(ii) Find a formula, in n, for the number of **white tiles** in pattern n of the sequence (W_n).

(iii) Work out the number of **white tiles** in pattern 15 of the sequence.

This question continues on the next page.

(b) **(i)** Draw the next pattern in the sequence.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

(ii) Based on the pattern shown, complete the table to show the number of **grey tiles** in each of the first five patterns of the sequence.

Pattern number (n)	**Number of grey tiles**
1	0
2	1
3	
4	
5	

(iii) Show that the number of **grey tiles** in pattern n of the sequence is given by the formula $G_n = (n-1)^2$.

(iv) Work out the number of **grey tiles** in pattern 15 of the sequence.

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(c) **(i)** Using your answers from **parts (a)(ii)** and **(b)(iii)**, or otherwise, find a formula, in n, for the total number of tiles **(white and grey)** in pattern n of the sequence (T_n).

(ii) Hence, or otherwise, find the number of white tiles **and** the number of grey tiles in the pattern containing 289 tiles.

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Question 9 **(50 marks)**

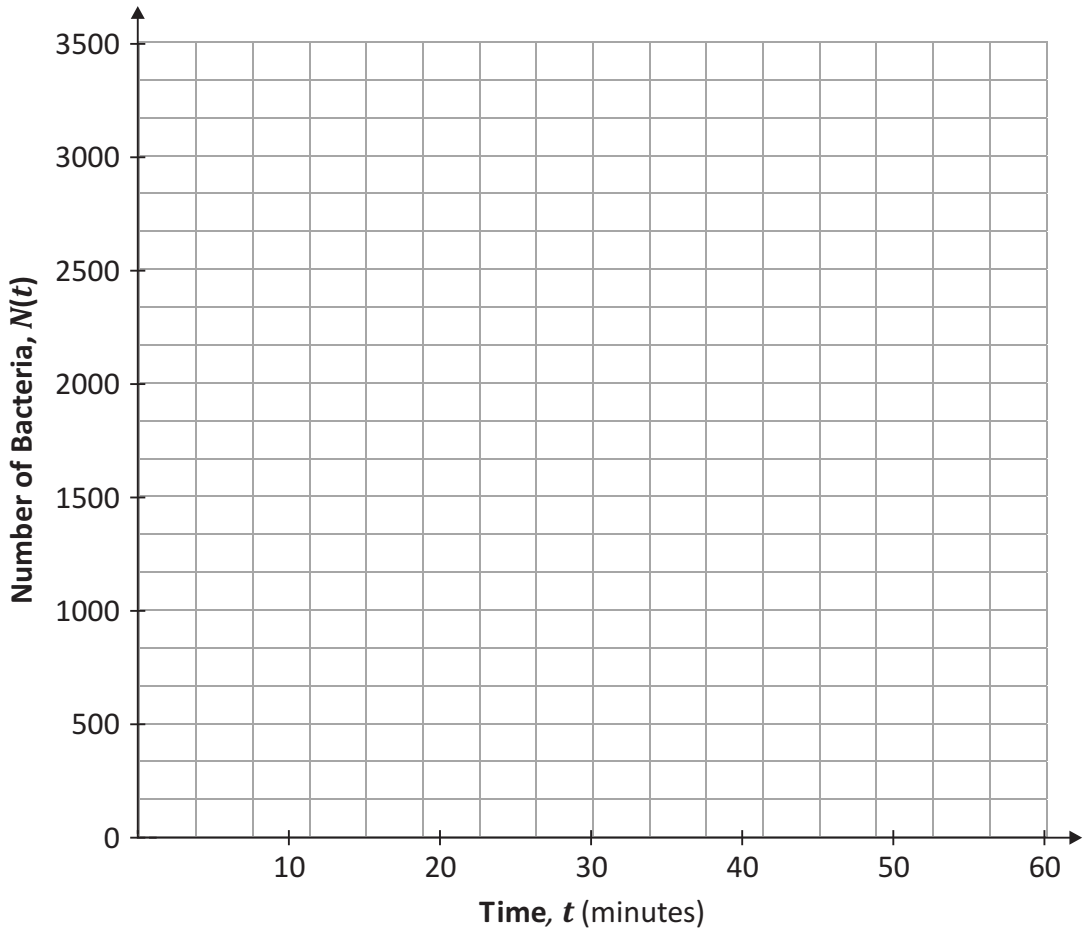
The growth of a certain bacterium is observed under different laboratory conditions.

(a) Under a particular set of conditions, the number of bacteria, $N(t)$, present in a petri dish sample **doubles** every 10 minutes.

(i) Complete the table below to show the values of $N(t)$ for the given values of t.

Time, t (minutes)	0	10	20	30	40	50	60
Number of bacteria, $N(t)$	50						

(ii) Draw the graph of $N(t)$ on the axes below, for $0 \leq t \leq 60$, where $t \in \mathbb{R}$.



(iii) What name is given to this type of function (i.e. the number of bacteria $N(t)$ present in the sample over time)?

(iv) Use your graph to estimate the number of bacteria present in the sample after 55 minutes.

(v) Use your graph to estimate the time it takes for the number of bacteria present in the sample to exceed 1000.

(vi) The number of bacteria present in the sample can be approximated using the function:

$$N(t) = 50(2)^{\frac{t}{10}}$$

where t is the time, measured in minutes, from when the observation began.

Work out the number of bacteria present in the sample after 2 hours (i.e. 120 minutes).

This question continues on the next page.

(b) Under a **different** set of conditions, the number of bacteria in a petri dish sample **doubles** every 40 minutes. The sample initially contains 3200 bacteria. The number of bacteria, $P(t)$, present in this sample can be approximated using the function:

$$P(t) = a(b)^{\frac{t}{k}}, \text{ where } a, b, k \in \mathbb{N} \text{ and } t \in \mathbb{R},$$

and where t is the time, measured in minutes, from when the observation began.

(i) Explain why the value of k is 40.

(ii) Work out the value of a and the value of b.

$a =$ $b =$

(iii) Work out the number of bacteria present in the sample after 9 hours.
Give your answer in the form $c \times 10^n$, where $1 \leq c < 10$, $n \in \mathbb{N}$.
Give the value of c correct to three significant figures.

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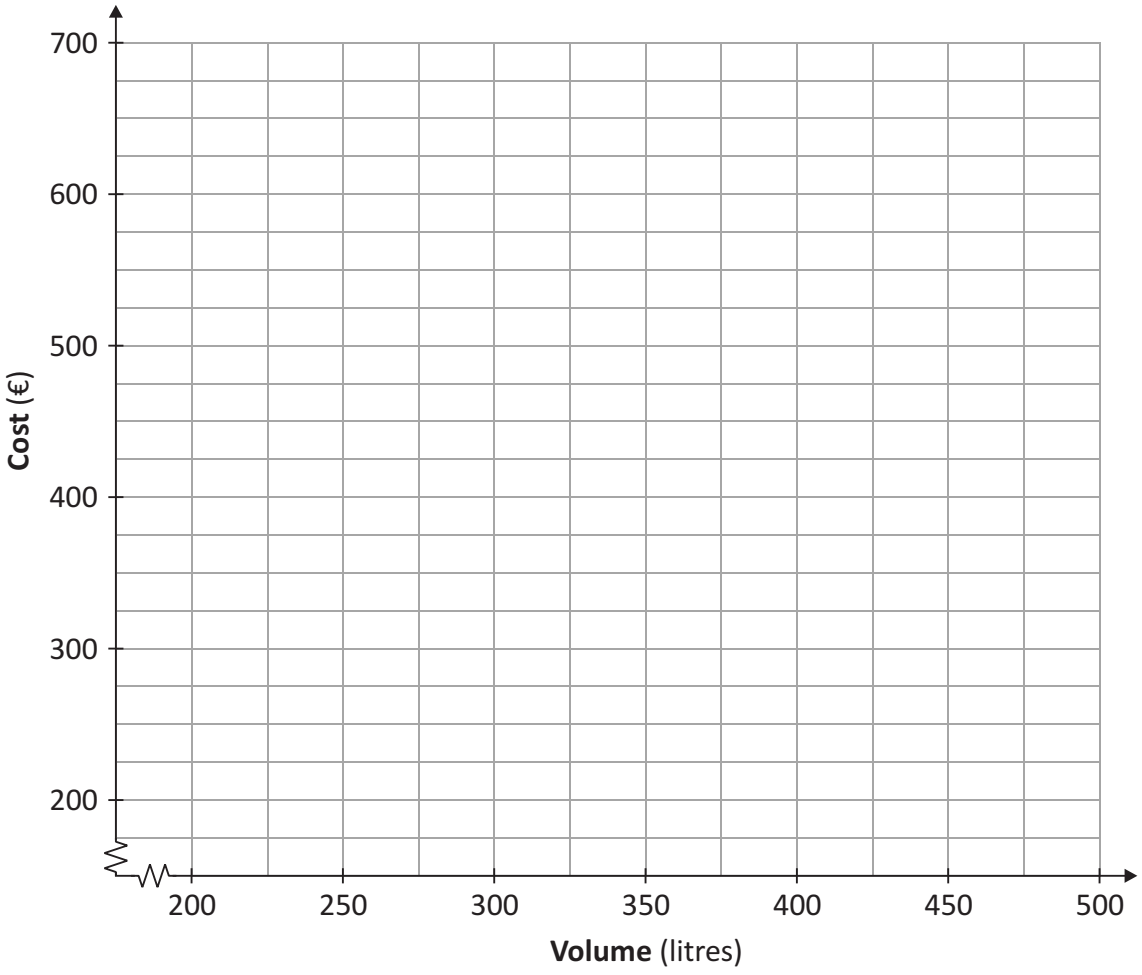
Question 10 **(50 marks)**

Company X delivers heating oil (kerosene) to homes in Galway.
The cost depends on the quantity of heating oil required.
The company has a minimum order value of €275.

(a) **(i)** Complete the table below to show the volume of heating oil delivered to homes at various price points.

Cost, C (€)	**275**	**325**	**375**	**425**	**475**	**525**	**575**	**625**
Volume, V (litres)	220	260	300					

(ii) On the axes below, draw a graph to show the relationship between cost, C, and volume, V, for home heating oil delivered by Company X. Label your graph clearly.



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(b) **(i)** Write down a formula which gives the cost of home heating oil for any given volume delivered by Company X.
State clearly the meaning of any letters used in your formula.

(ii) Use your answer from **part (b)(i)** to find the cost of 550 litres of home heating oil delivered by Company X.

(c) Company Y also delivers heating oil (kerosene) to homes in Galway.
It charges €1·05 per litre of heating oil **plus** a delivery charge of €65.
The company has no minimum order value.

(i) Write down a formula which gives the cost of home heating oil for any given volume delivered by Company Y.
State clearly the meaning of any letters used in your formula.

(ii) Use your answer from **part (c)(i)** to show that the cost of 300 litres of home heating oil delivered by Company Y is €380.

This question continues on the next page.

(iii) Complete the table below to show the cost of home heating oil delivered by Company Y. Where necessary, give the cost correct to the nearest cent.

Volume, V (litres)	**200**	**250**	**300**	**350**	**400**	**450**	**500**
Cost, C (€)			380				

(iv) On the same axes on page 26, draw a graph to show the relationship between cost, C, and volume, V, for home heating oil delivered by Company Y. Label your graph clearly.

(d) **(i)** Use your graphs to find the volume of home heating oil that would cost the same from both companies.

(ii) Verify your answer algebraically.

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Label any extra work clearly with the question number and part.

Q. & part	

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Q. & part	

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