



Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Junior Certificate Examination 2014

# Mathematics (Project Maths – Phase 2)

Paper 1

Higher Level

Friday 6 June – Afternoon, 2:00 to 4:30

300 marks

Examination number
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Centre stamp
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Running total	
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For examiner			
Question	Mark	Question	Mark
1		11	
2		12	
3		13	
4		14	
5			
6			
7			
8			
9			
10		Total	

Grade
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## **Instructions**

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Question 14 carries a total of 50 marks.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

## Question 1

**(Suggested maximum time: 5 minutes)**

- (a)** Place the following numbers in order, starting with the smallest:

3  
2

1·4

2

- (b) Which one of the following is **not** a rational number? Explain your answer.

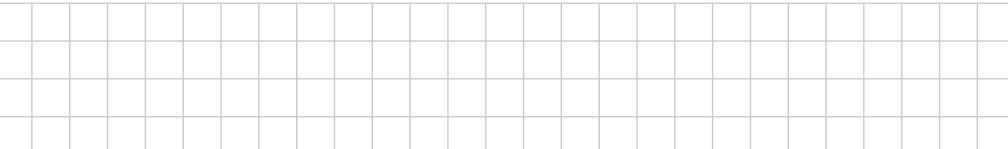
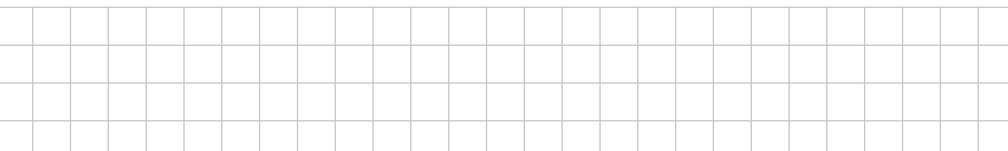
3  $\frac{1}{7}$

3:142

22  
7

π

- (c) (i) Find the values of  $\frac{4n^2+1}{13}$ , where  $n \in \{17, 19, 21\}$ .

$n$	$\frac{4n^2 + 1}{13}$
17	
19	
21	

- (ii) State which **one** of your answers is a natural number, and explain your choice.

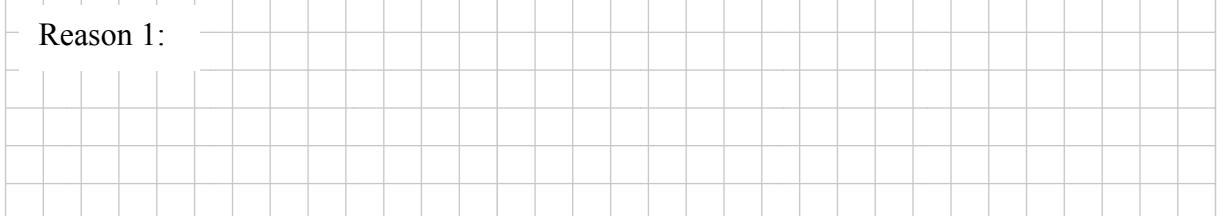
**Question 2****(Suggested maximum time: 5 minutes)**

- (a) John thinks that he has a method for finding **all** prime numbers.  
He says that if he uses the formulas in the table below, he will generate the prime numbers.  
He also says that these formulas will generate **only** the prime numbers.
- (i) Complete the table.

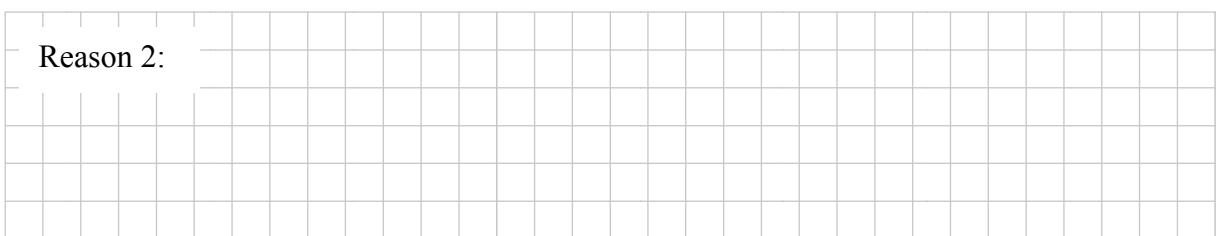
$p$	$6p + 1$	$6p + 5$
0	1	5
1		
2		
3		
4		
5		

- (ii) Give two reasons why his method is not fully correct.

Reason 1:



Reason 2:



- (b) The Swiss mathematician and physicist, Euler, first noticed (in 1772) that the expression  $n^2 - n + 41$  gives a prime number for all positive integer values of  $n$  less than 41.

Explain why it does not give a prime number for  $n=41$ .



**Question 3****(Suggested maximum time: 10 minutes)**

- (a) The sets  $A$ ,  $B$ , and  $C$  are as follows:

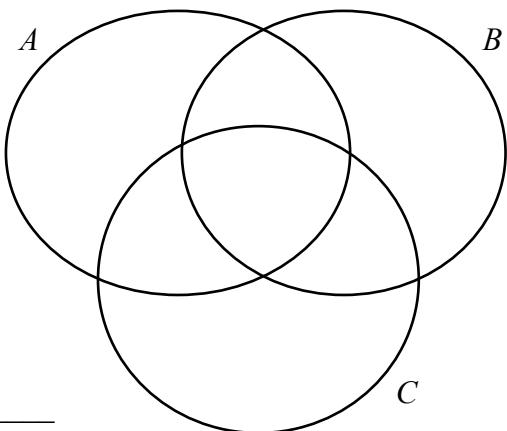
$$A = \{2, 3, 4, 5, 6\}, B = \{2, 4, 6, 8, 10\}, \text{ and } C = \{1, 4, 8, 12, 14\}.$$

- (i) Complete the Venn diagram.  
(ii) List the elements of each of the following sets:

$$A \cap B = \underline{\hspace{10cm}}$$

$$B \setminus (A \cap C) = \underline{\hspace{10cm}}$$

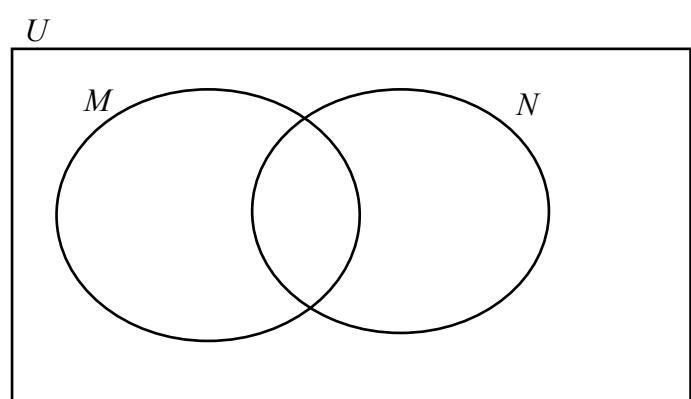
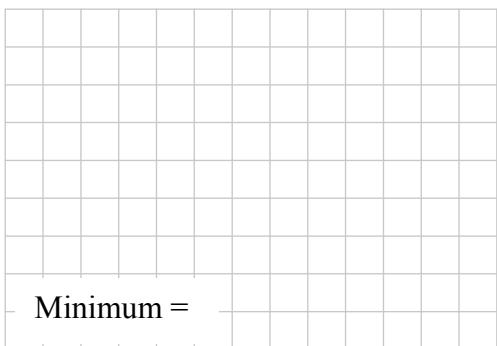
$$(B \setminus A) \cup (B \setminus C) = \underline{\hspace{10cm}}$$



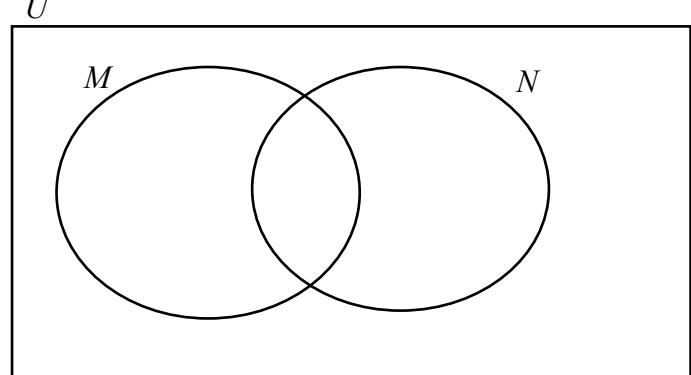
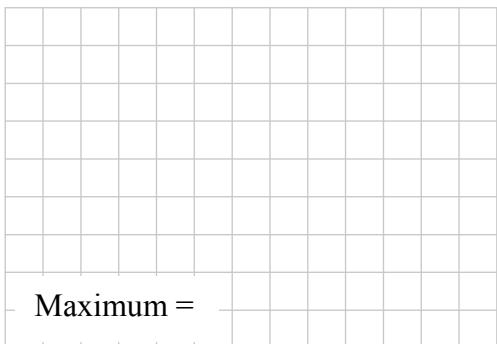
- (iii) Write down a null set, in terms of  $A$ ,  $B$ , and  $C$ .  $\underline{\hspace{10cm}}$

- (b) In a table quiz, 100 questions were asked. Team  $M$  answered 72 questions correctly. Team  $N$  answered 38 questions correctly.

- (i) Find, with the aid of the Venn diagram, the minimum number of questions answered correctly by both teams.



- (ii) Find, with the aid of the Venn diagram, the maximum number of questions answered correctly by both teams.



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**Question 4****(Suggested maximum time: 10 minutes)**

- (a) Factorise fully  $9a^2 - 6ab + 12ac - 8bc$ .

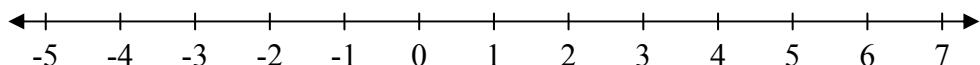
- (b) Factorise  $9x^2 - 16y^2$ .

- (c) Use factors to simplify the following:  $\frac{2x^2 + 4x}{2x^2 + x - 6}$ .

**Question 5****(Suggested maximum time: 5 minutes)**

Solve the following inequality and show the solution on the number line.

$$-17 \leq 1 - 3x < 13, \quad x \in \mathbb{Z}$$

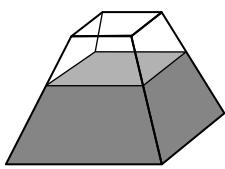
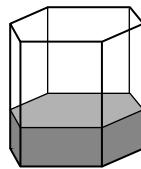
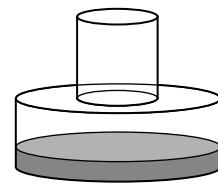
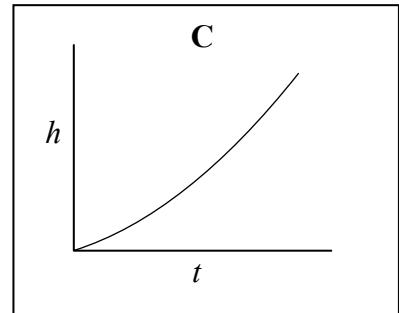
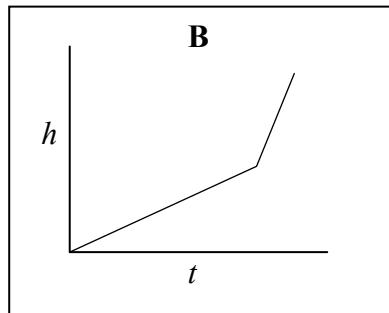
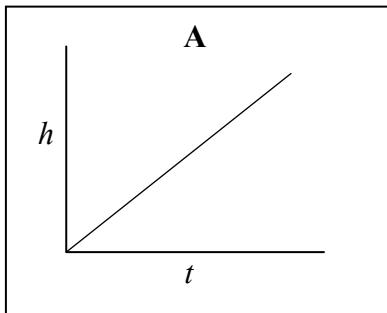


**Question 6**

(Suggested maximum time: 10 minutes)

Below are three containers, labelled **1**, **2**, and **3**.

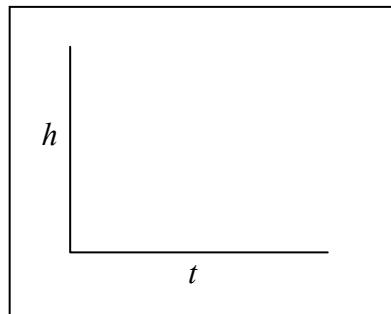
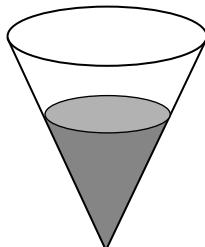
Water is poured into each container at a constant rate, until it is full.

**1****2****3**The three graphs, **A**, **B**, and **C**, show the height of the water,  $h$ , in the containers after time  $t$ .

- (a) Write **A**, **B**, and **C** in the table below to match each container to its corresponding graph.

Container	1	2	3
Graph			

- (b) Another container is shown below. Water is also poured into this container at a constant rate until it is full. Sketch the graph you would expect to get when plotting height ( $h$ ) against time ( $t$ ) for this container.



**Question 7****(Suggested maximum time: 10 minutes)**

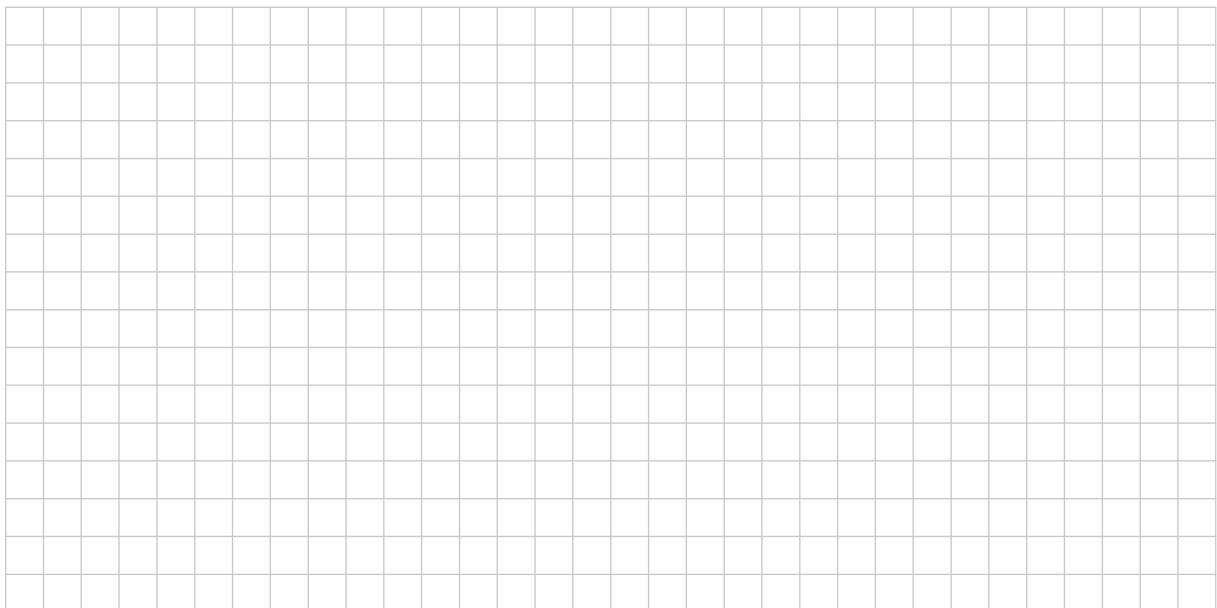
Last year Elena had a gross income of €36 960.

She had to pay Universal Social Charge (USC) and income tax on her gross income.

The rates and bands of USC are as follows.

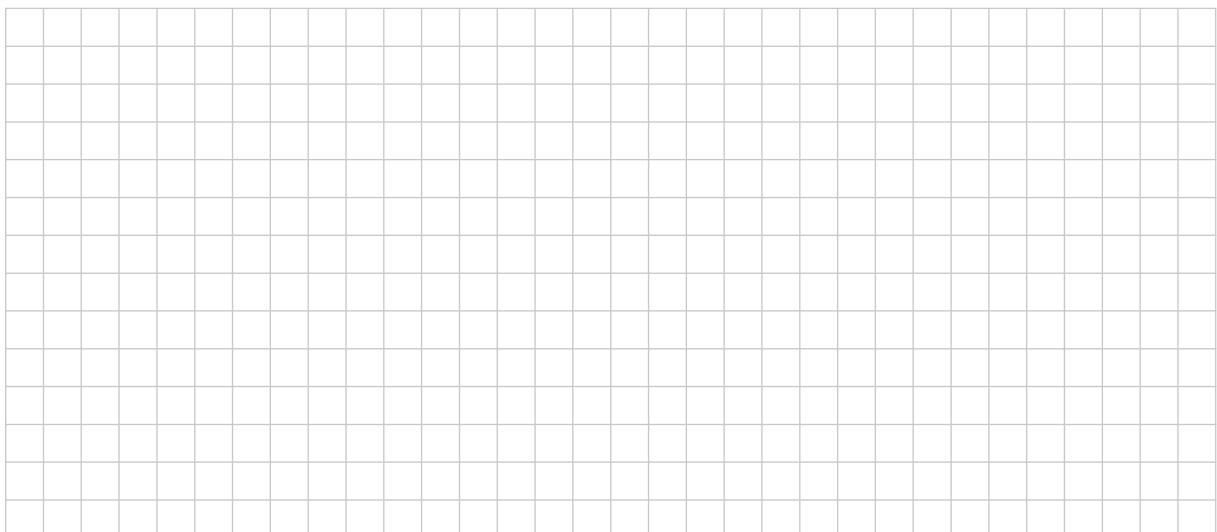
Income band	Rate of USC
Up to €10 036	2%
Between €10 036 and €16 016	4%
Above €16 016	7%

- (i)** Find the amount of USC that was deducted from Elena's gross income last year.



- (ii)** The standard rate of income tax was 20% and the higher rate was 41%.  
The standard rate cut-off point was €32 800.  
Elena paid a total of €4965·60 income tax last year.

Find Elena's tax credits for the year.



- (iii) Find Elena's total deduction (USC and income tax) as a percentage of her gross income. Give your answer correct to the nearest percent.

## Question 8

**(Suggested maximum time: 10 minutes)**

The table shows the height, in metres, of a ball at various times after being kicked into the air.

- (i) Is the pattern of heights in the table linear, quadratic, or exponential? Explain your answer.

<b>Time (seconds)</b>	0	0.5	1	1.5	2	2.5	3
<b>Height (metres)</b>	0.3	3.4	5.7	7.2	7.9	7.8	6.9

- (ii) Estimate the height of the ball after 3.5 seconds.

- (iii) Estimate the total time the ball spends in the air. Justify your answer.

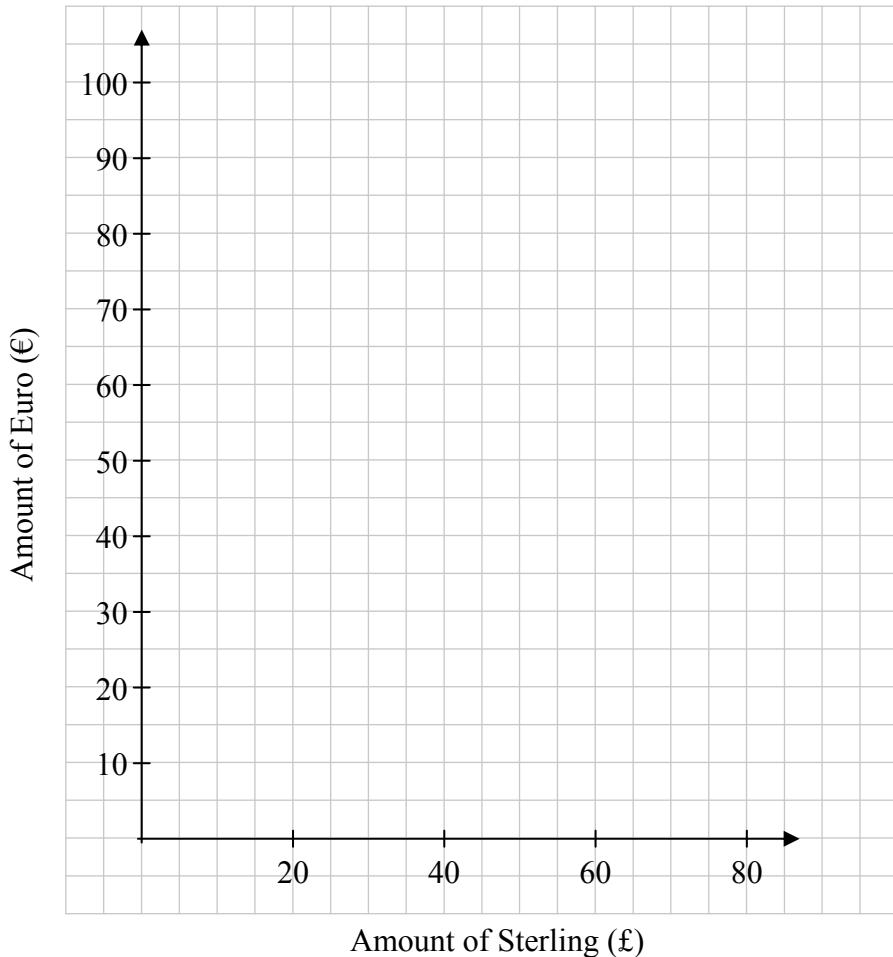
**Question 9****(Suggested maximum time: 15 minutes)**

Jack and Sarah are going on a school tour to England. They investigate how much different amounts of sterling (£) will cost them in euro (€). They each go to a different bank.

Their results are shown in the table below.

Amount of sterling (£)	Cost in euro (€) for Jack	Cost in euro (€) for Sarah
20	33	24
40	56	48
60	79	72
80	102	96

- (i) On the grid below, draw graphs to show how much the sterling will cost Jack and Sarah, for up to £80.



- (ii) Using the table, or your graph, find the slope (rate of change) of Jack's graph. Explain what this value means. Refer to both euro and sterling in your explanation.

Slope:

Explanation:

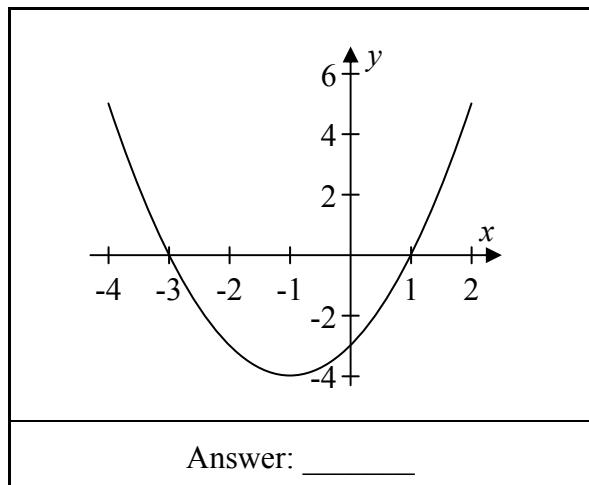
- (iii) Write down a formula to represent what Jack must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

(iv) Write down a formula to represent what Sarah must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

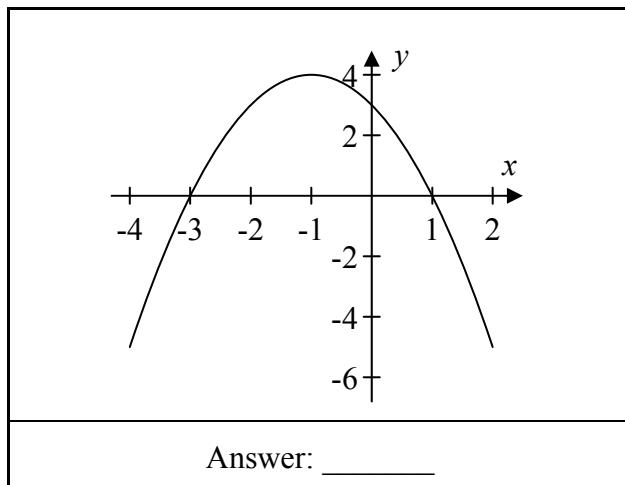
(v) Using your formulas from (iii) and (iv), or otherwise, find the amount of sterling Jack and Sarah could buy that would cost them the same amount each in euro.

**Question 10****(Suggested maximum time: 10 minutes)**

- (a) The graphs of the functions  $f(x) = x^2 + 2x - 3$  and  $g(x) = -x^2 - 2x + 3$  are shown below.  
Identify each graph by writing  $f(x)$  or  $g(x)$  in the space provided below the graph.

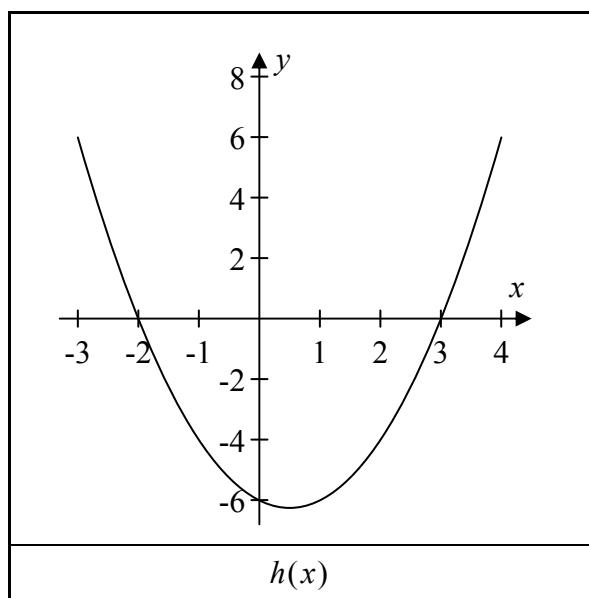
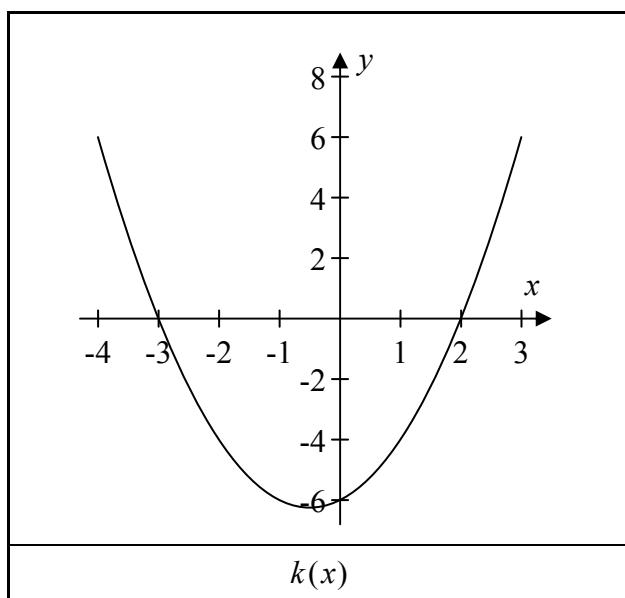


Answer: \_\_\_\_\_



Answer: \_\_\_\_\_

- (b) The graphs of the functions  $y = h(x)$  and  $y = k(x)$  are shown below.

 $h(x)$  $k(x)$ 

Write down the roots of each function.

Hence, or otherwise, write down an equation for each function.

Roots of  $h(x)$ :

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Equation:  $h(x) =$

---

Roots of  $k(x)$ :

---

Equation:  $k(x) =$

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## Question 11

**(Suggested maximum time: 10 minutes)**

$x$  is a real number.

One new number is formed by increasing  $x$  by 1.

A second new number is formed by decreasing  $x$  by 2.

- (i) Write down each of these new numbers, in terms of  $x$ .

Increase  $x$  by 1:

Decrease  $x$  by 2:

- (ii) The product of these two new numbers is 1.  
Use this information to write an equation in  $x$ .

- (iii) Solve this equation to find the two possible values of  $x$ . Give each of your answers correct to 3 decimal places.

## Question 12

**(Suggested maximum time: 15 minutes)**

- (a)** Simplify  $(6x-3)(2x-1)$ .

- (b)** Simplify  $(3x^3 - 2x^2 - 3x + 2) \div (x - 1)$ .

- (c) (i)** Solve the simultaneous equations:

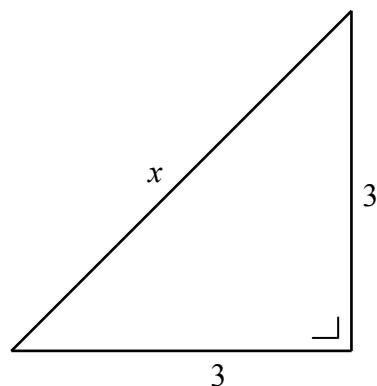
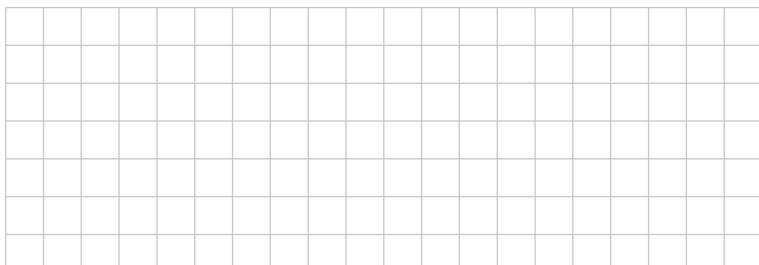
$$2x - 3y = 18$$

$$5x + 9y = -10.$$

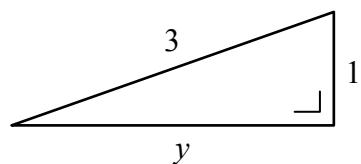
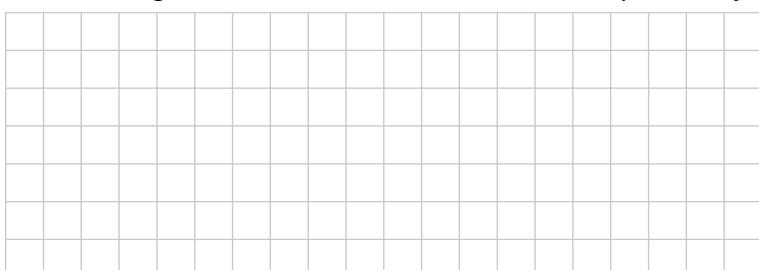
- (ii)** Verify your answer to (c)(i).

**Question 13****(Suggested maximum time: 5 minutes)**

- (i) Use the diagram on the right to calculate the value of  $x$ . Give your answer in surd form.

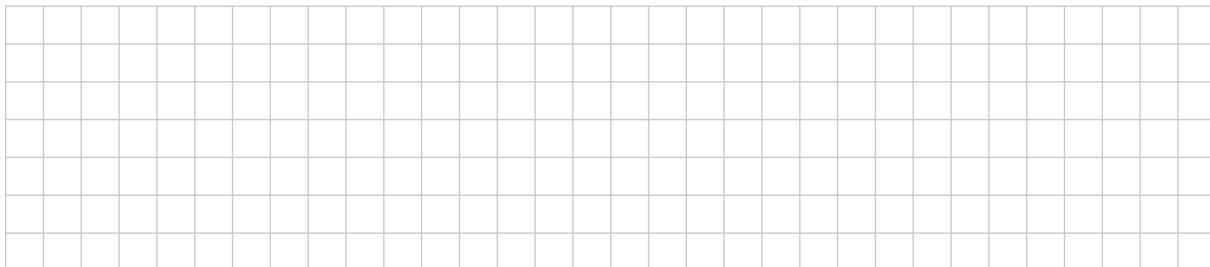
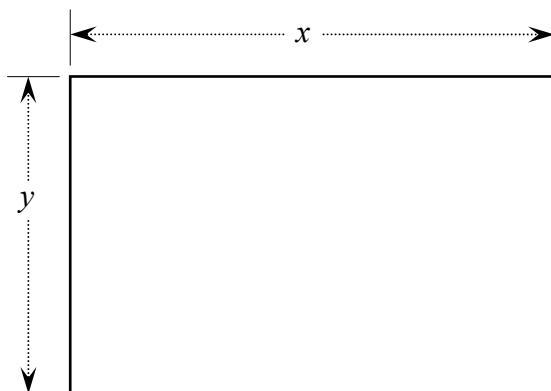


- (ii) Use the diagram below to calculate the value of  $y$ . Give your answer in surd form.



- (iii) A rectangle with sides of length  $x$  and  $y$  is drawn using the values of  $x$  and  $y$  from parts (i) and (ii), as shown below.

Write the **perimeter** of this rectangle in the form  $a\sqrt{2}$ , where  $a \in \mathbb{N}$ .



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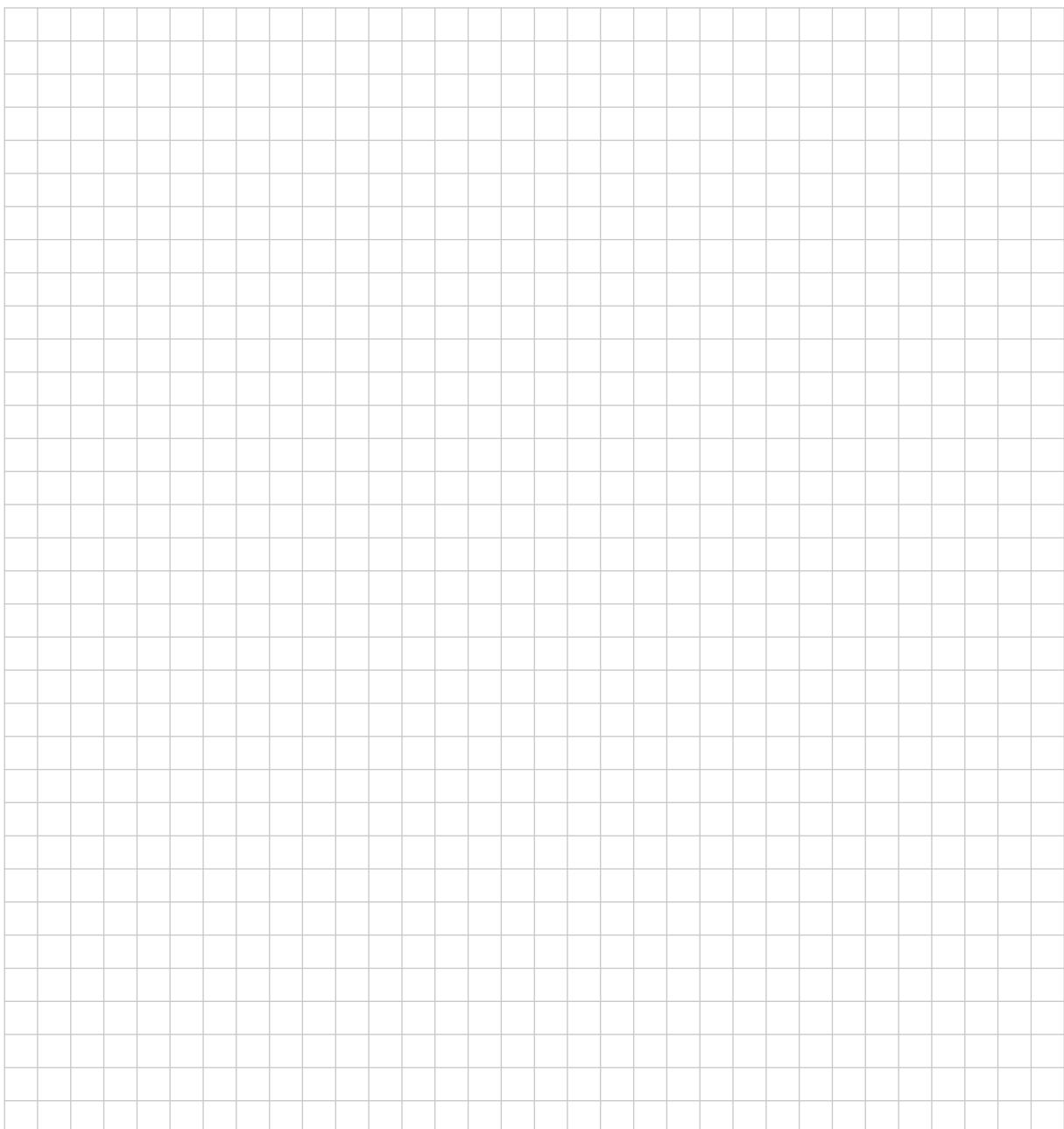
**Question 14****(Suggested maximum time: 20 minutes)**

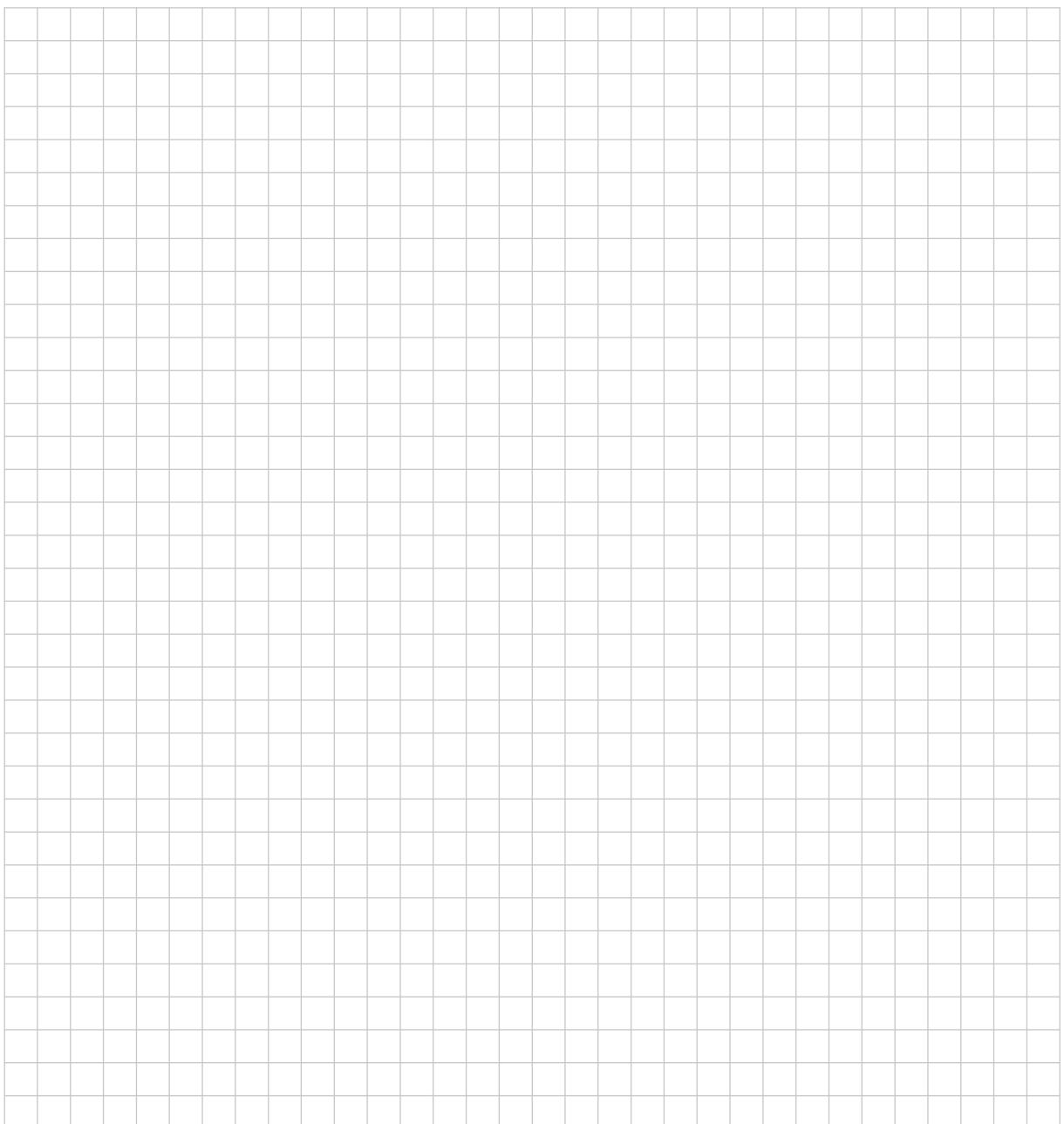
- (i)  $g$  is the function  $g : x \mapsto x - 1$ , where  $x \in \mathbb{R}$ . Find the value of each of the following.

$g(3)$
$g(-2)$

- (ii)  $f$  is the function  $f : x \mapsto 2x^2 - x - 6$ , where  $x \in \mathbb{R}$ .

Using the same axes and scales, draw the graphs of the functions  $y = f(x)$  and  $y = g(x)$  in the domain  $-2 \leq x \leq 3$ .



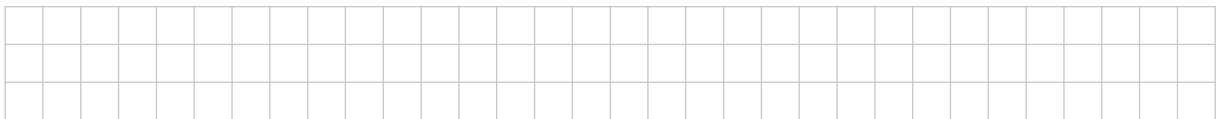


Use your graphs from (ii) to estimate:

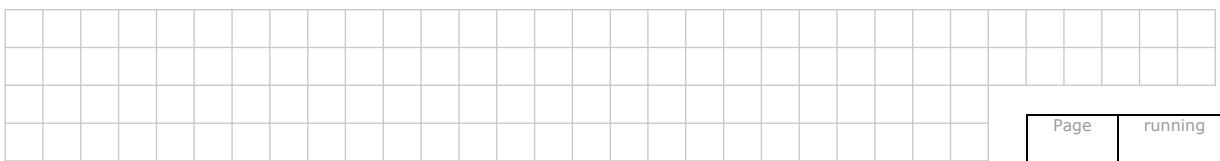
- (iii) the minimum value of  $f(x)$



- (iv) the range of values of  $x$  for which  $f(x) < 0$

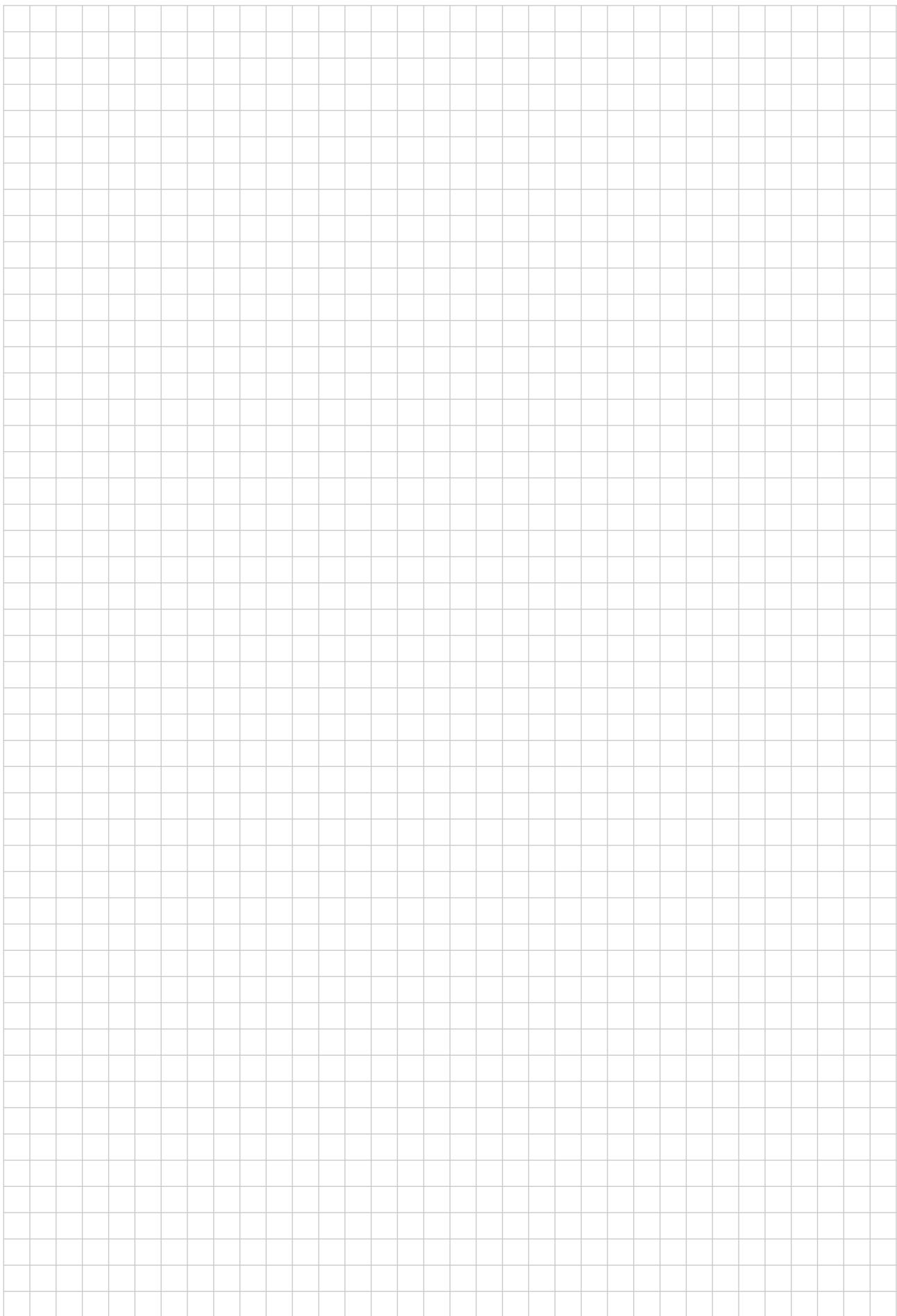


- (v) the range of values of  $x$  for which  $g(x) \geq 0$ .

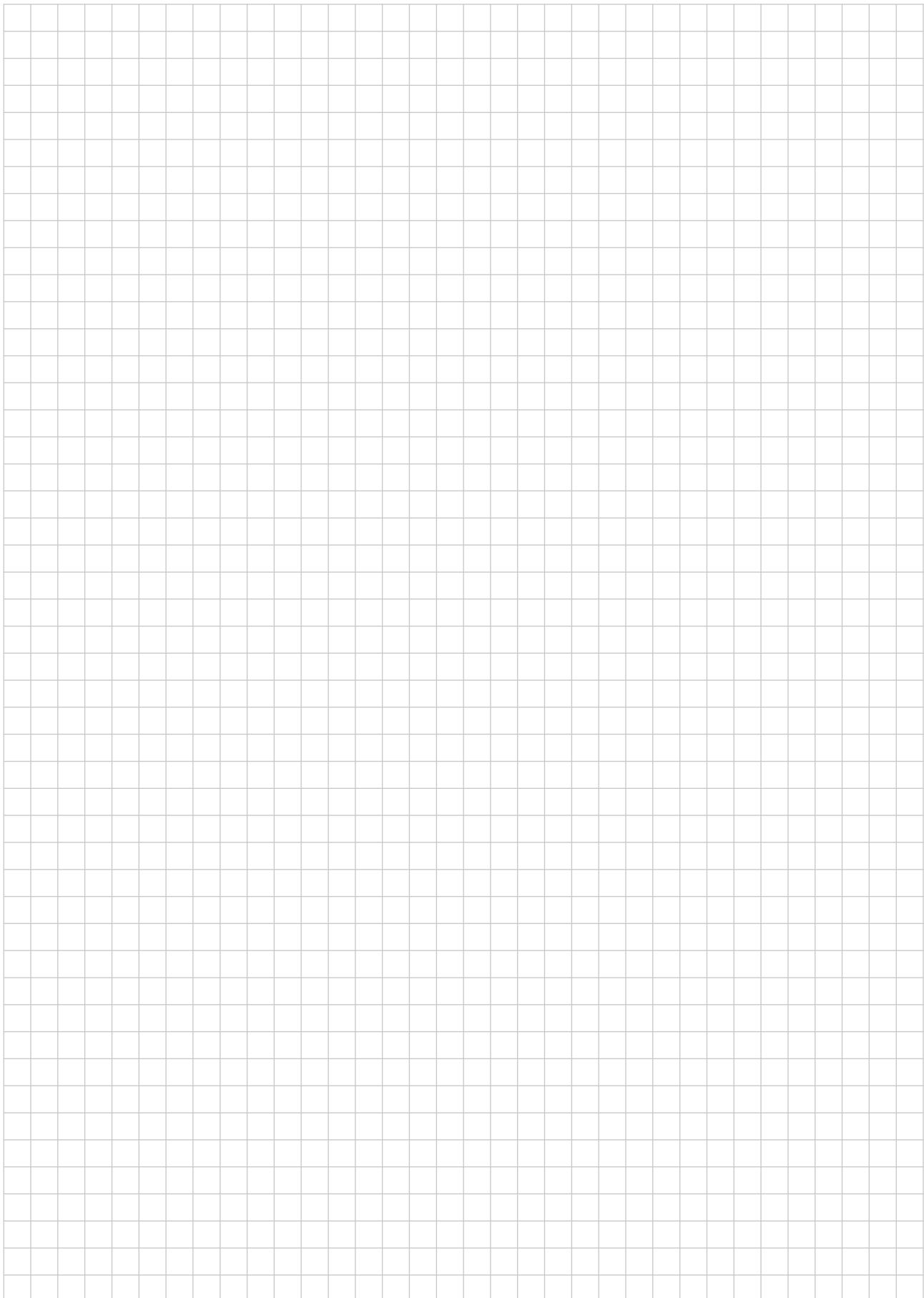


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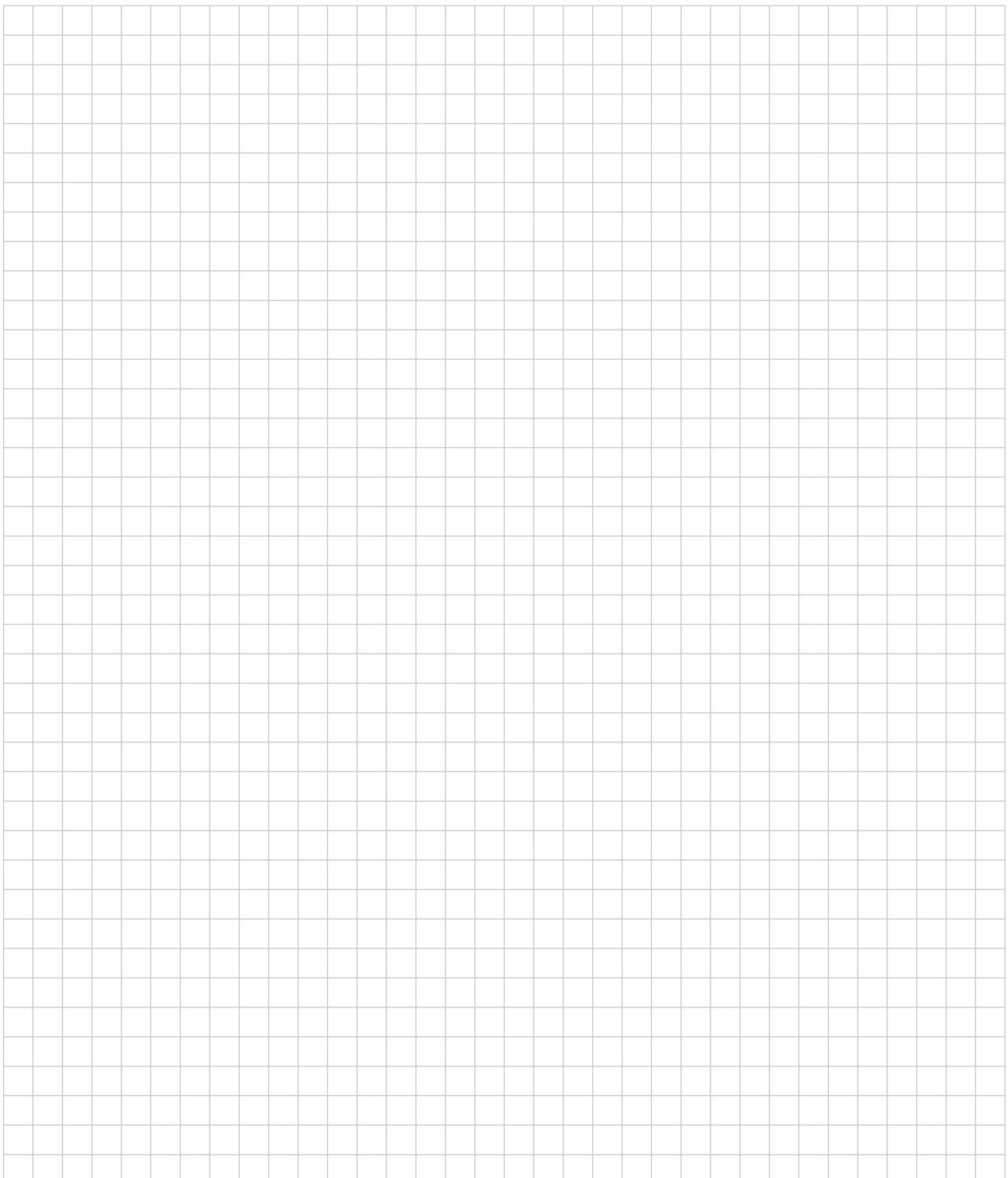
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Junior Certificate 2014 – Higher Level

## **Mathematics (Project Maths – Phase 2) – Paper 1**

Friday 6 June

Afternoon, 2:00 to 4:30