



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2016
Mathematics

Paper 1

Ordinary Level

Friday 10 June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

Emma works part time after school at the local takeaway. She is paid a rate per hour and also receives €2 for each delivery she makes.

- (a) One day, she works for 2 hours, makes 5 deliveries and is paid a total of €28.
Find her hourly rate of pay.

- (b) One week, she works for h hours and makes d deliveries.
Write a formula in h and d for the wage (w) she receives.

$w =$

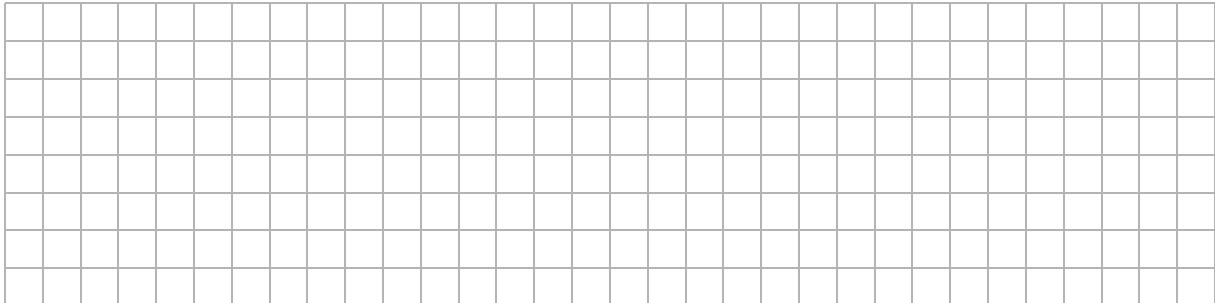
- (c) Another week, she works for 6 hours and makes 12 deliveries. She also works 5 hours on a Sunday, at time and a half, and makes some deliveries. In total, she receives €161.50 for that week. Find how many deliveries she makes on the Sunday.

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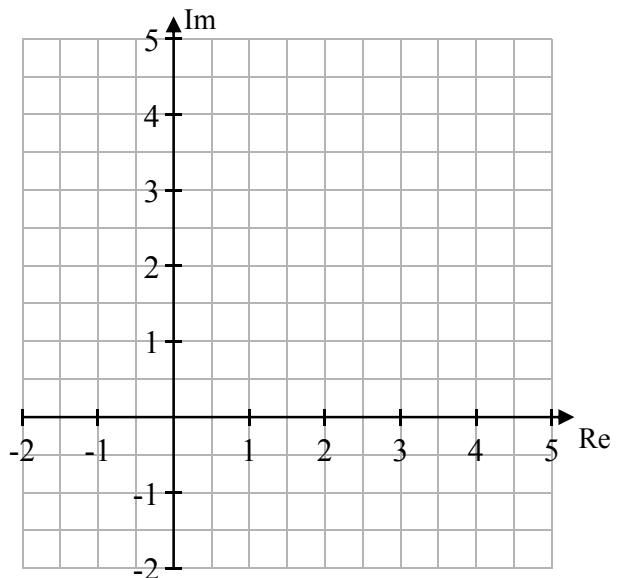
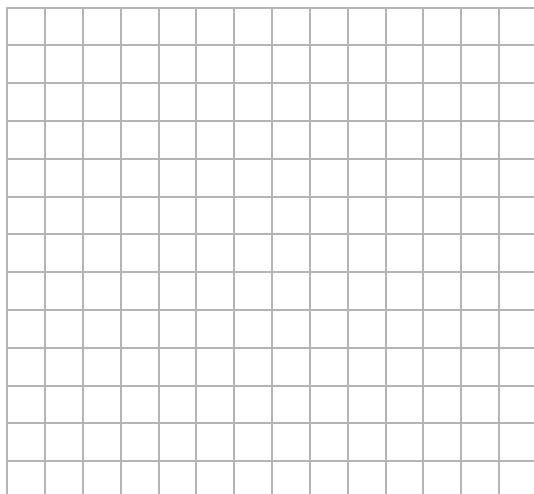
Question 2**(25 marks)**

$z_1 = 1 + 3i$ and $z_2 = 2 - i$, where $i^2 = -1$, are two complex numbers.

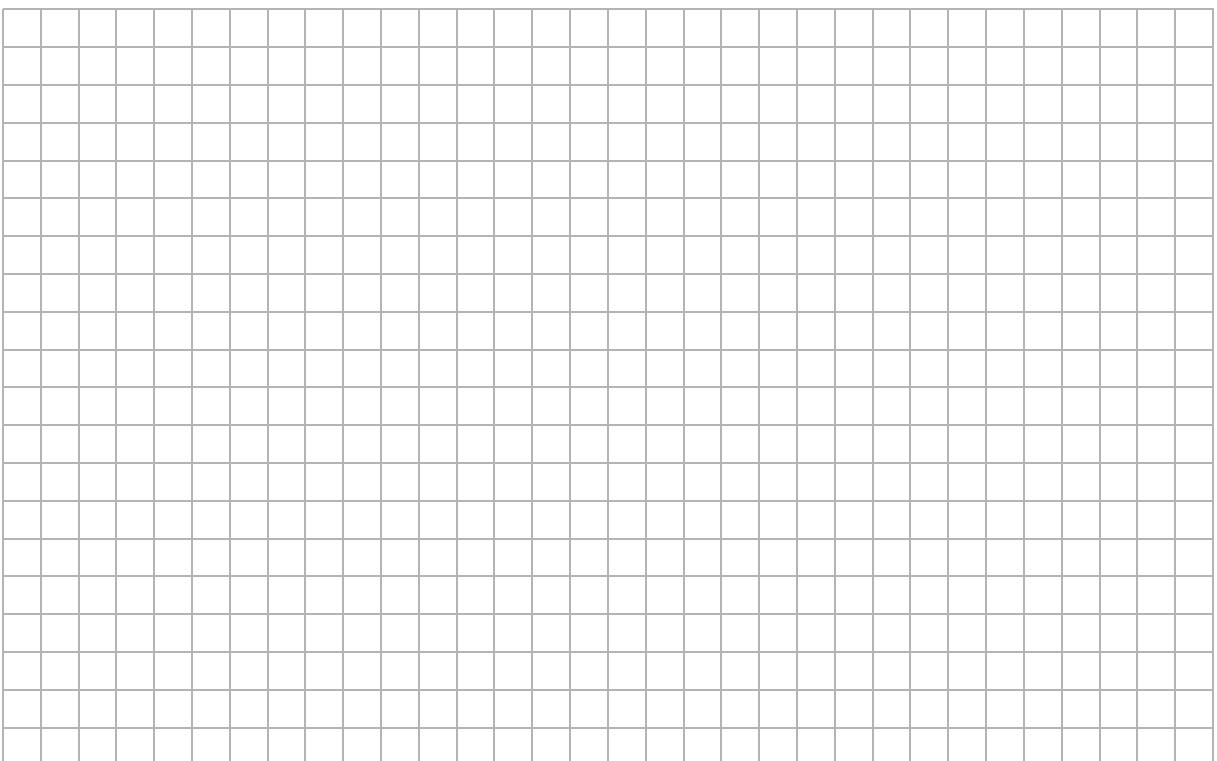
- (a) Let $z_3 = z_1 + 2z_2$. Find z_3 in the form $a + bi$ where $a, b \in \mathbb{Z}$.



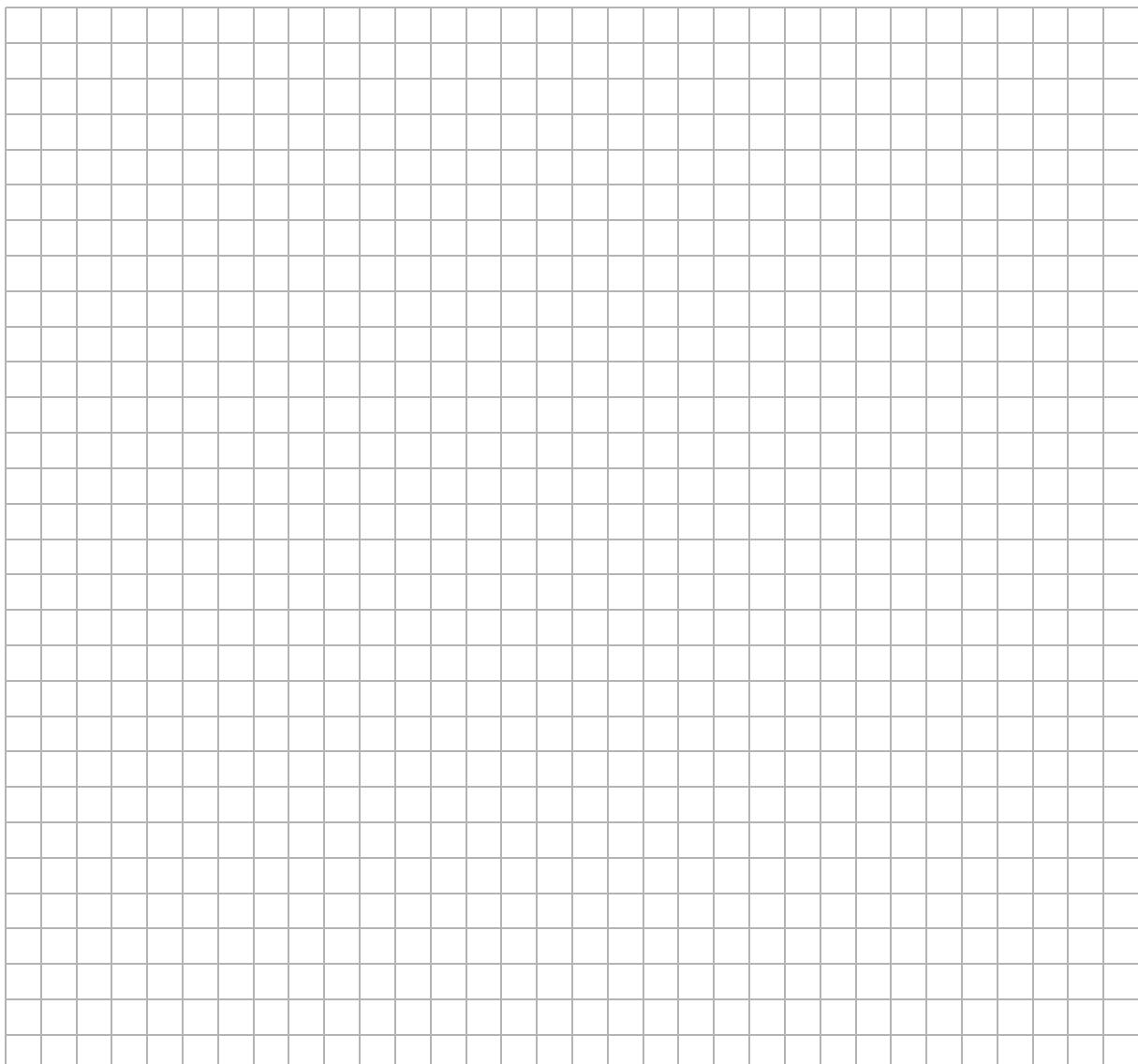
- (b) Plot z_1 , z_2 , and z_3 on the given Argand diagram and label each point clearly.



- (c) Investigate if $|z_2 - z_3| = |z_1 + z_2|$.



- (d) Find the complex number w , such that $w = \frac{z_1}{z_2}$.
Give your answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

A large grid of squares, approximately 20 columns by 25 rows, intended for考生 to work out their answers.

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Question 3**(25 marks)**

- (a)** Solve for x :

$$3(x - 7) + 5(x - 4) = 15, \text{ where } x \in \mathbb{R}.$$

- (b)** Solve the equations below to find the value of a and the value of b :

$$\begin{aligned}4a + 3b &= -3 \\5a &= 25 + 2b.\end{aligned}$$

- (c)** List all the values of x that satisfy the inequality $2(2x - 3) + 6x < 25$, where $x \in \mathbb{N}$.

Question 4

(25 marks)

The function $f: x \mapsto x^3 + x^2 - 2x + 7$ is defined for $x \in \mathbb{R}$.

- (a) Find the coordinates of the point at which the graph of f cuts the y -axis.

- (b) Verify, using algebra, that the point $A(1, 7)$ is on the graph of f .

- (c) (i) Find $f'(x)$, the derivative of $f(x)$.

Hence find the slope of the tangent to the graph of f when $x = 1$.

$f'(x) =$	
Slope =	

- (ii) Hence, find the equation of the tangent to the graph of f at the point $A(1, 7)$.

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Question 5

(25 marks)

The first five numbers in a pattern of numbers are given in the table below.

Term	Number
U_1	13
U_2	15
U_3	19
U_4	25
U_5	33
U_6	
U_7	
U_8	

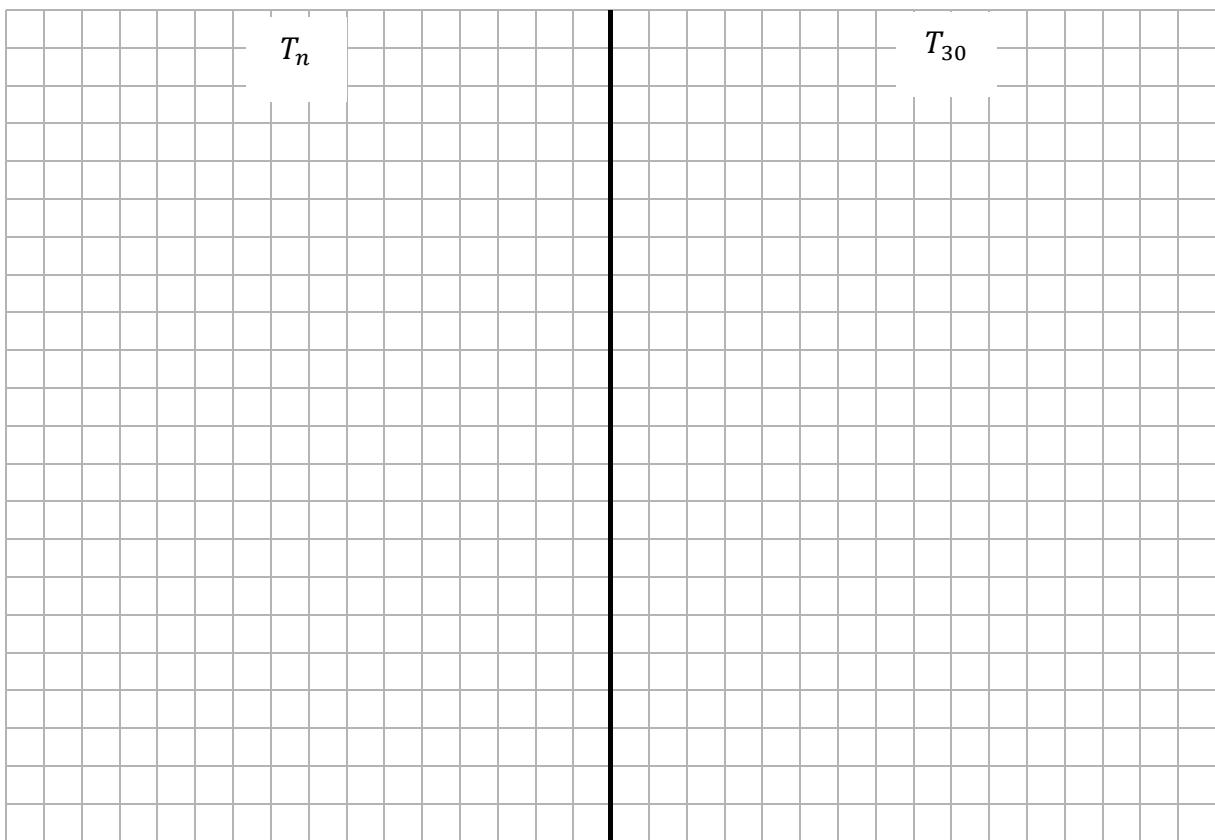
- (a) (i) Follow the pattern in the table above to write the next three numbers into the table.

- (ii) Use the data in the table to show that the pattern is quadratic.

- (b) $U_n = n^2 + bn + c$ where $b, c \in \mathbb{Z}$. Find the value of b and the value of c .

- (c) The table below shows the first five terms of an arithmetic sequence.
Find an expression for T_n , the n^{th} term of the sequence.
Hence, or otherwise, find the value of T_{30} , the 30th term.

Term	Number
T_1	12
T_2	14
T_3	16
T_4	18
T_5	20



Question 6**(25 marks)**

Fiona earns a gross wage of €1550 every fortnight. She pays income tax, a Universal Social Charge (USC), and Pay Related Social Insurance (PRSI) on this wage.

- (a) Each fortnight, Fiona pays income tax at the rate of 20% on the first €1300 she earns and 40% on the remainder. She has tax credits of €126 per fortnight.
Find how much income tax she pays per fortnight.

- (b) Each fortnight, Fiona also pays USC on her gross wage. The rates are:
1% on the first €462 she earns, 3% on the next €214, and 5.5% on the balance.
Find the total amount of USC she pays each fortnight.

- (c) (i) Fiona pays PRSI amounting to €18 each fortnight.
Find the sum of her fortnightly deductions.

- (ii) Write the sum of her fortnightly deductions as a percentage of her gross wage. Give your answer correct to one decimal place.

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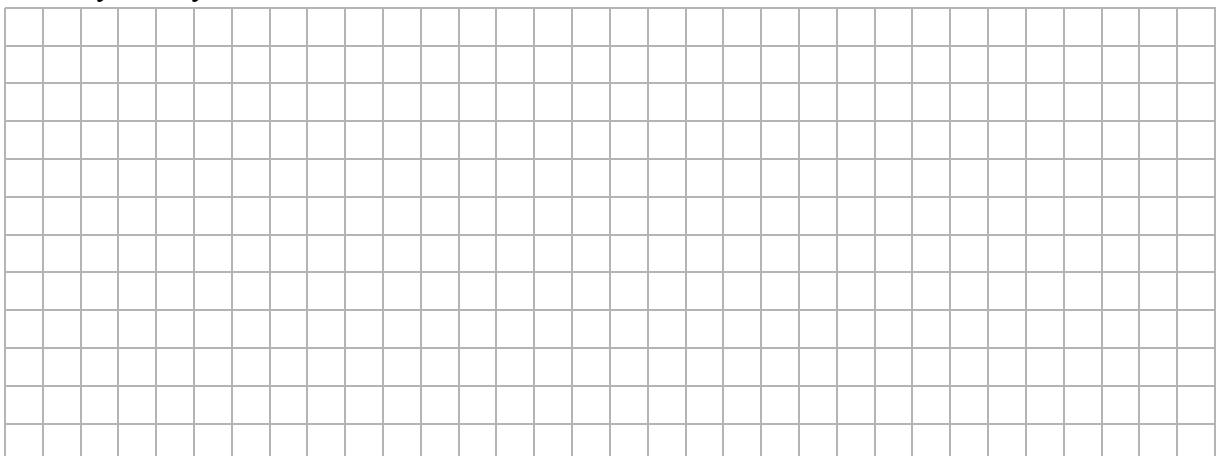
Answer **all three** questions from this section.

Question 7**(55 marks)**

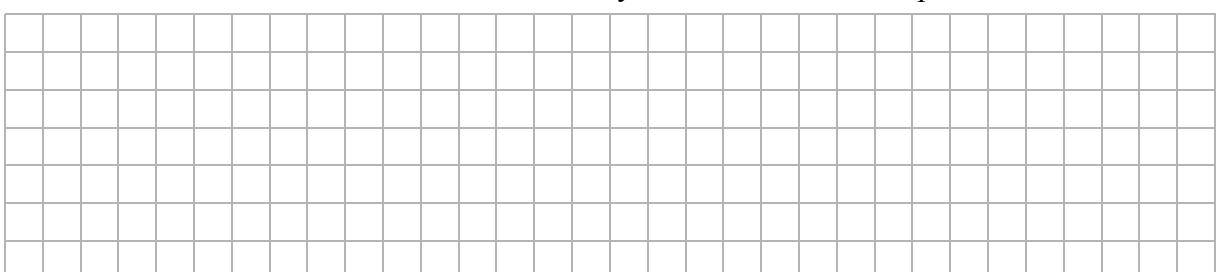
Paul has €8000 that he wants to invest for a maximum of 3 years. His local bank is offering him two options, Option 1 and Option 2, as shown in the table below.

Option 1	Option 2
2% interest in Year 1	
3% interest in Year 2	3.7% compound interest per year, for 3 years.
5% interest in Year 3	
Money can be taken out at the end of Year 1 or Year 2 without penalty.	Money may not be taken out until the end of Year 3.

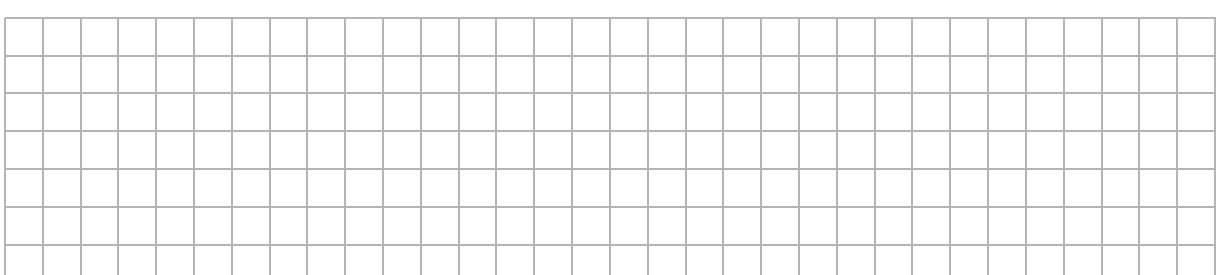
- (a) Find the value of the investment at the end of 3 years if Paul chooses Option 1 and does not take any money out.



- (b) Find the value of the investment at the end of 3 years if Paul chooses Option 2.



- (c) Give one issue, other than the rate of interest earned, that Paul might take into account when deciding between Option 1 and Option 2.



- (d) Paul would like his investment of €8000 to amount to €9000 after 3 years. What annual rate of compound interest would be required for this to happen? Give your answer as a percentage.

- (e) Another bank offers to invest his €8000 in a special fund. The bank has found that, in the short term, the amount of money in this fund follows the formula

$$v = 8000 + 36t - 1.2t^2$$

where v is the value of the fund, in euro, and t is the time, **in months**, since the €8000 was invested.

- (i) Find the value of the money in the fund after 12 months.

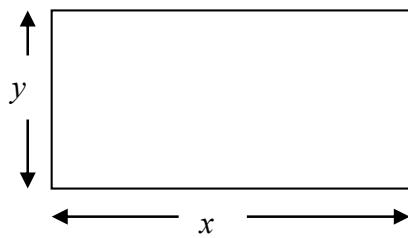
- (ii) Mary invests €8000 for 1 year at r % per annum. After 1 year her investment is worth the same amount as the answer to part (e)(i) above.
Find the value of r , correct to 2 decimal places.

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Question 8**(50 marks)**

Kieran has 21 metres of fencing. He wants to enclose a vegetable garden in a rectangular shape as shown.

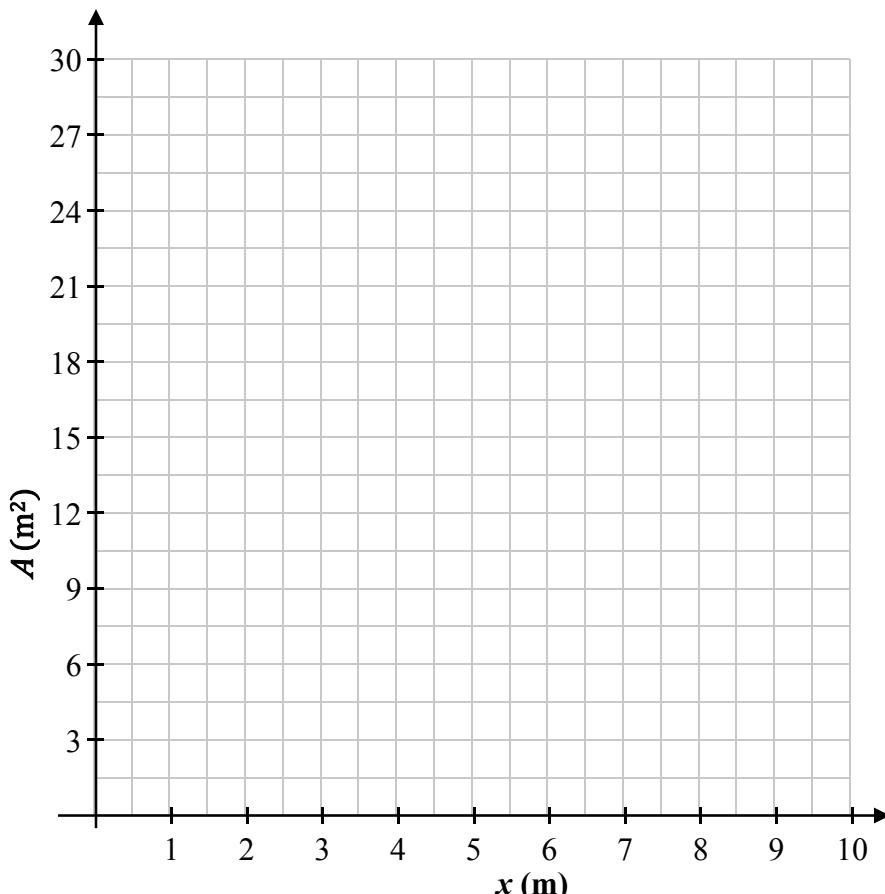
- (a) By writing an expression for the perimeter of the vegetable garden in terms of x (length in metres) and y (width in metres), show that $y = 10.5 - x$.



- (b) (i) Complete the table below to show the values of y and A (the area of the garden) for each given value of x .

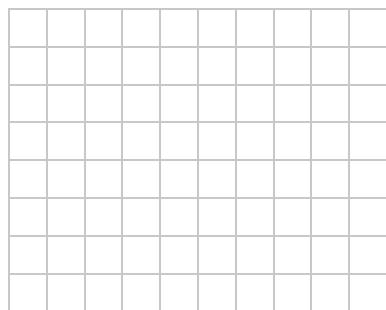
x (m)	0	1	2	3	4	5	6	7	8	9	10
y (m)					6.5						
A (m^2)					26						

- (ii) Use the values of x and A from the table to plot the graph of A on the grid below.

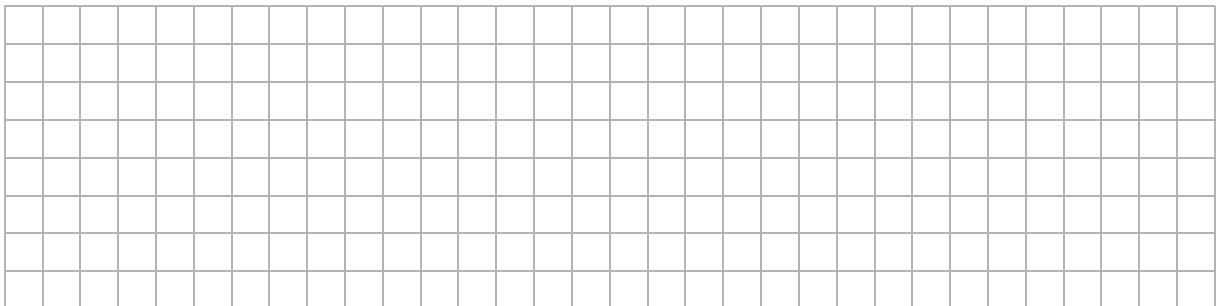


- (c) Use your graph to estimate the maximum value of A and write the corresponding length and width.

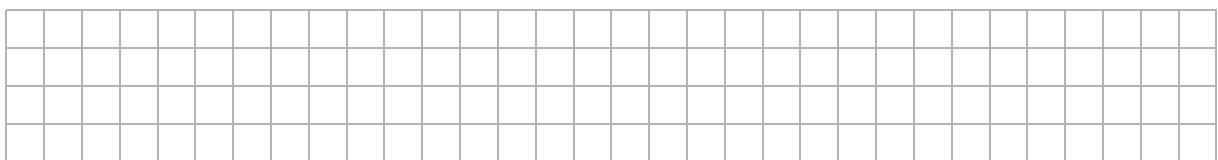
A: Maximum area (m^2)	
Length (m)	
Width (m)	



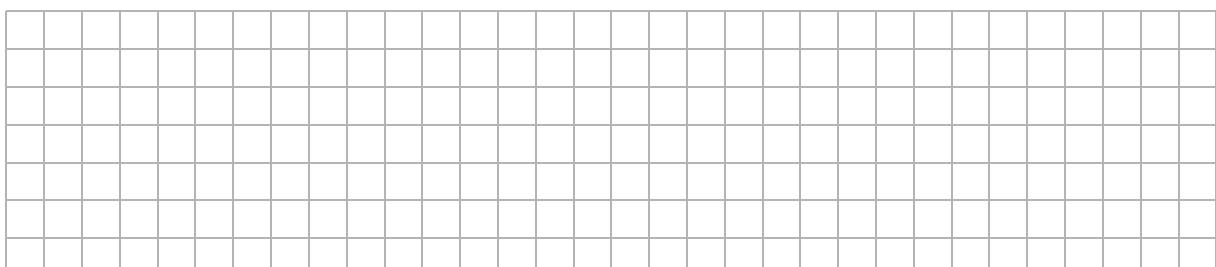
- (d) (i) Show that the area of the rectangle can be written as $A = 10 \cdot 5x - x^2$.



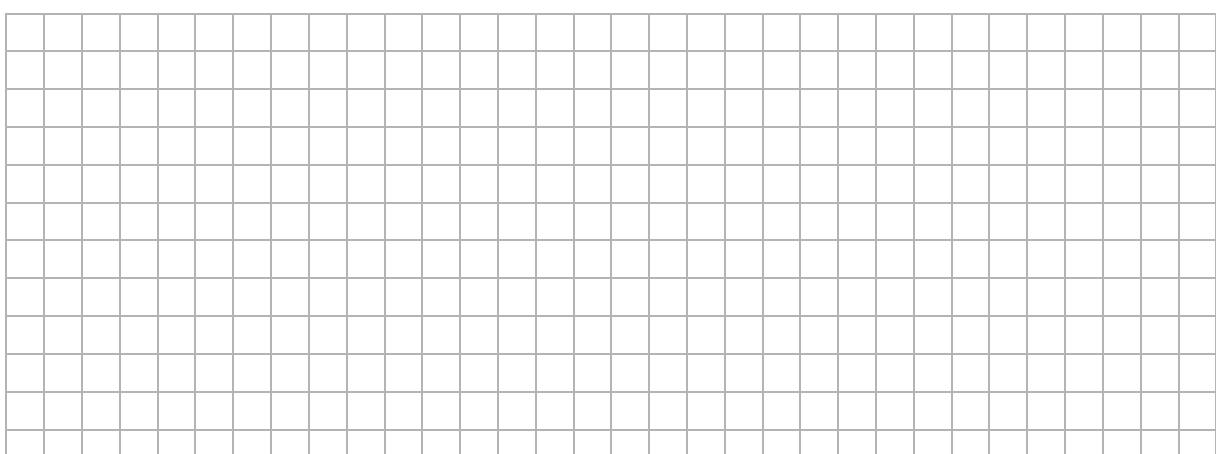
(ii) Find $\frac{dA}{dx}$.



- (iii) Hence, find the value of x which will give the maximum area.



- (iv) Find this maximum area.



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Question 9**(45 marks)**

Company A uses the following formula to charge a customer for a job:

$$A(h) = 30 + 9.5h,$$

where $A(h)$ is the cost of the job, in euro, and h is the length of time that the job takes in hours.

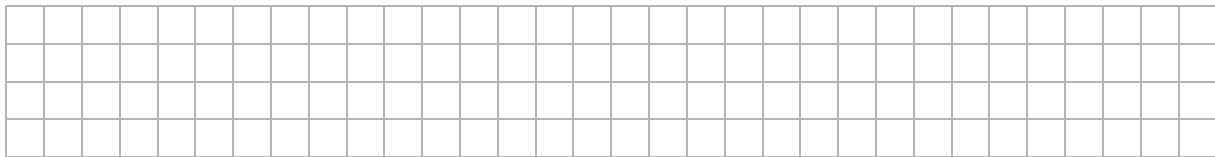
Company B uses the following formula to charge a customer for the same job:

$$B(h) = 10(1.74)^h$$

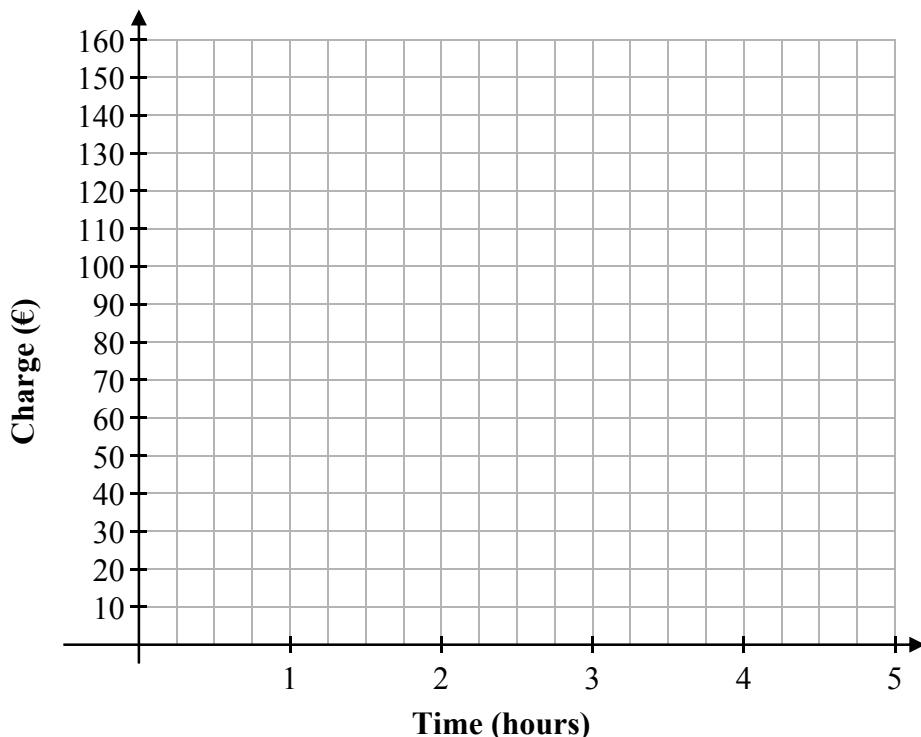
where $B(h)$ is the cost of the job, in euro, and h is again the length of time that the job takes in hours.

- (a) (i) Complete the table below to show what Company A charges and what Company B charges for jobs that take up to 5 hours. Where necessary give the charge correct to the nearest cent.

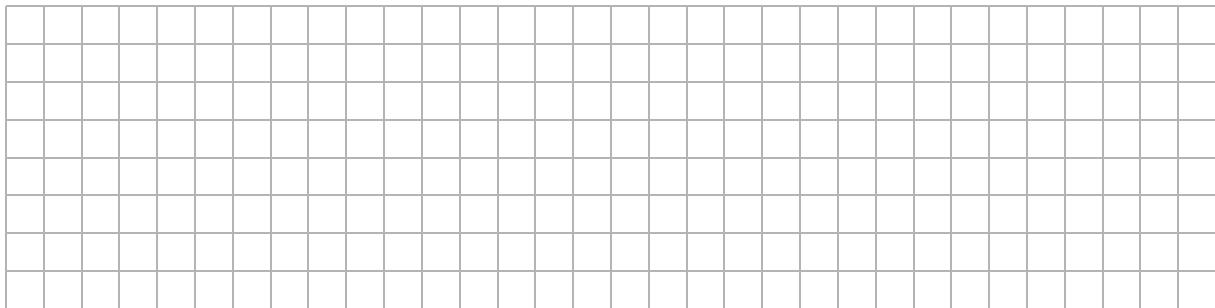
Time (hours)	0	1	2	3	4	5
$A(h)$ (€)					68	
$B(h)$ (€)					91.66	



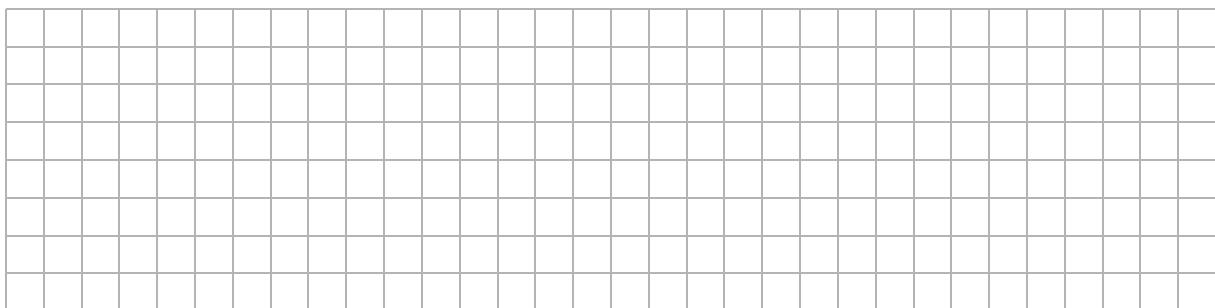
- (ii) On the grid below draw separate graphs to show the charge for Company A and the charge for Company B. Label each graph clearly.



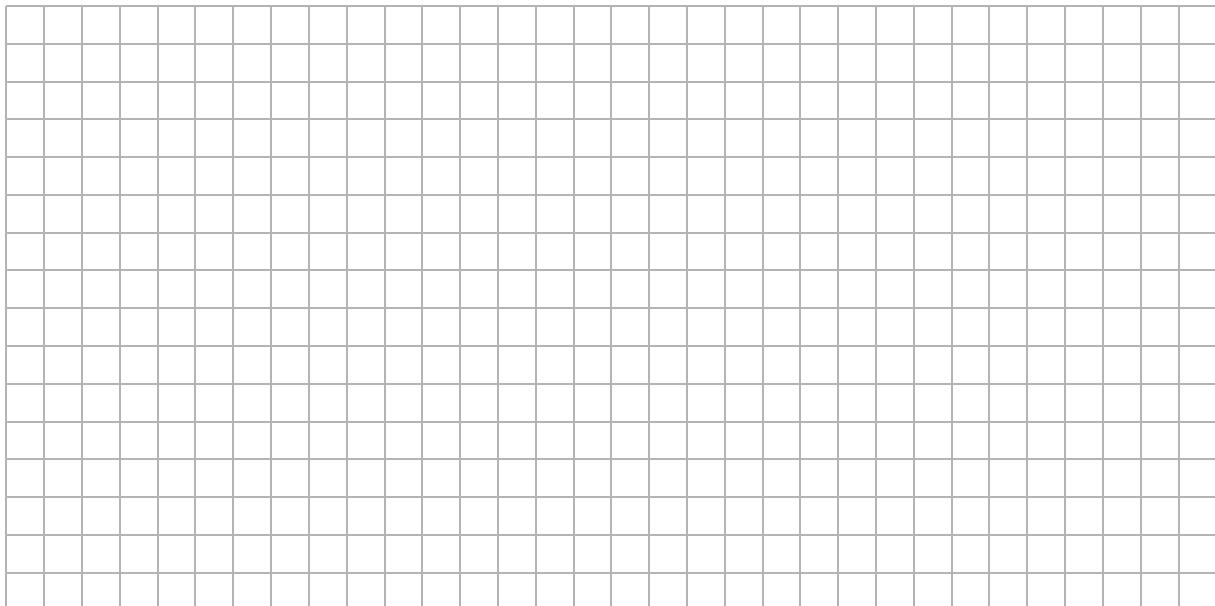
- (b) Which company would charge least for a job that takes $2\frac{1}{2}$ hours to complete?
Give a reason for your answer.



- (c) Use your graphs to estimate the value of h for which the charge is the same for both companies.

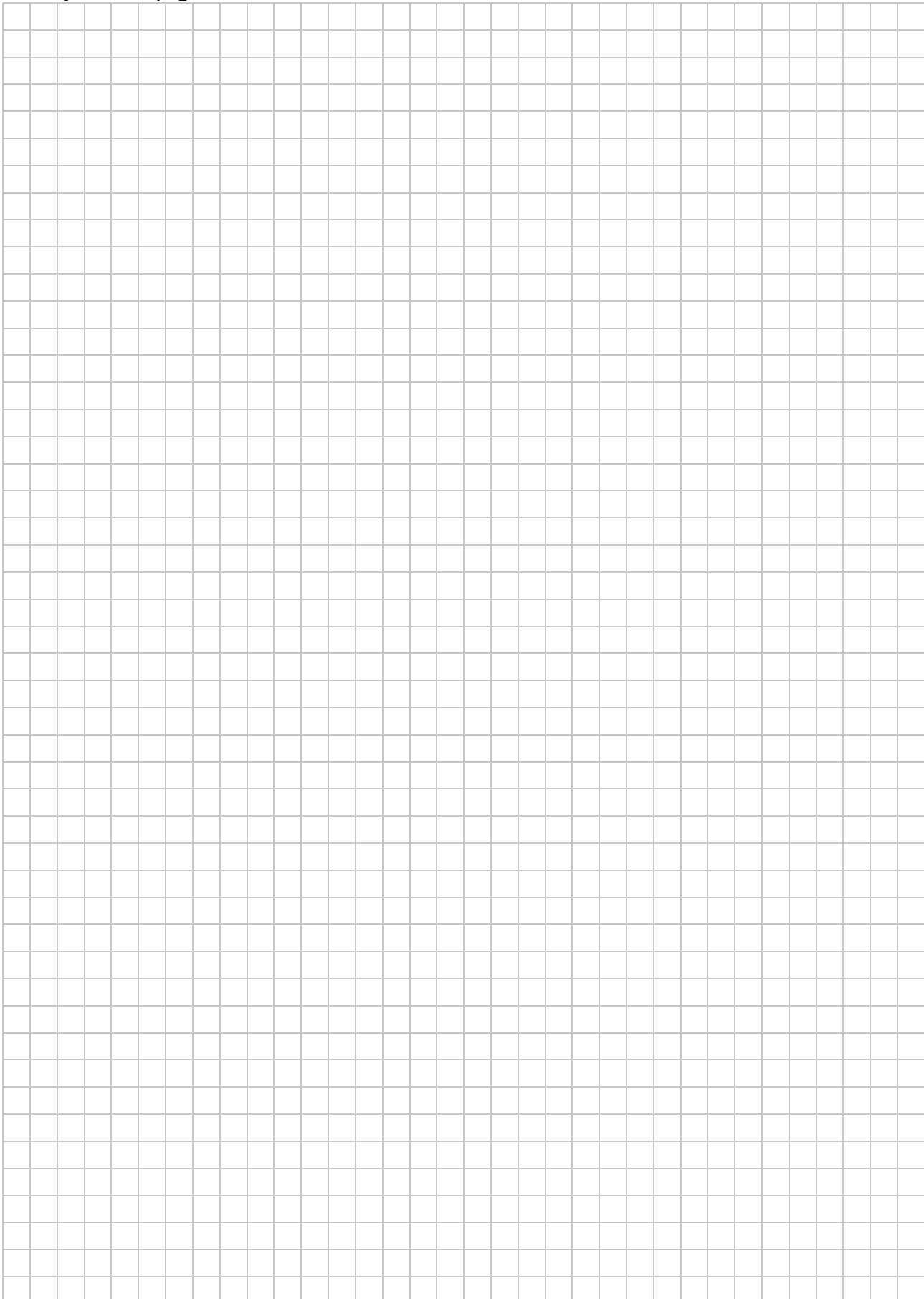


- (d) Find the difference in cost for a job that takes 6 hours to complete.

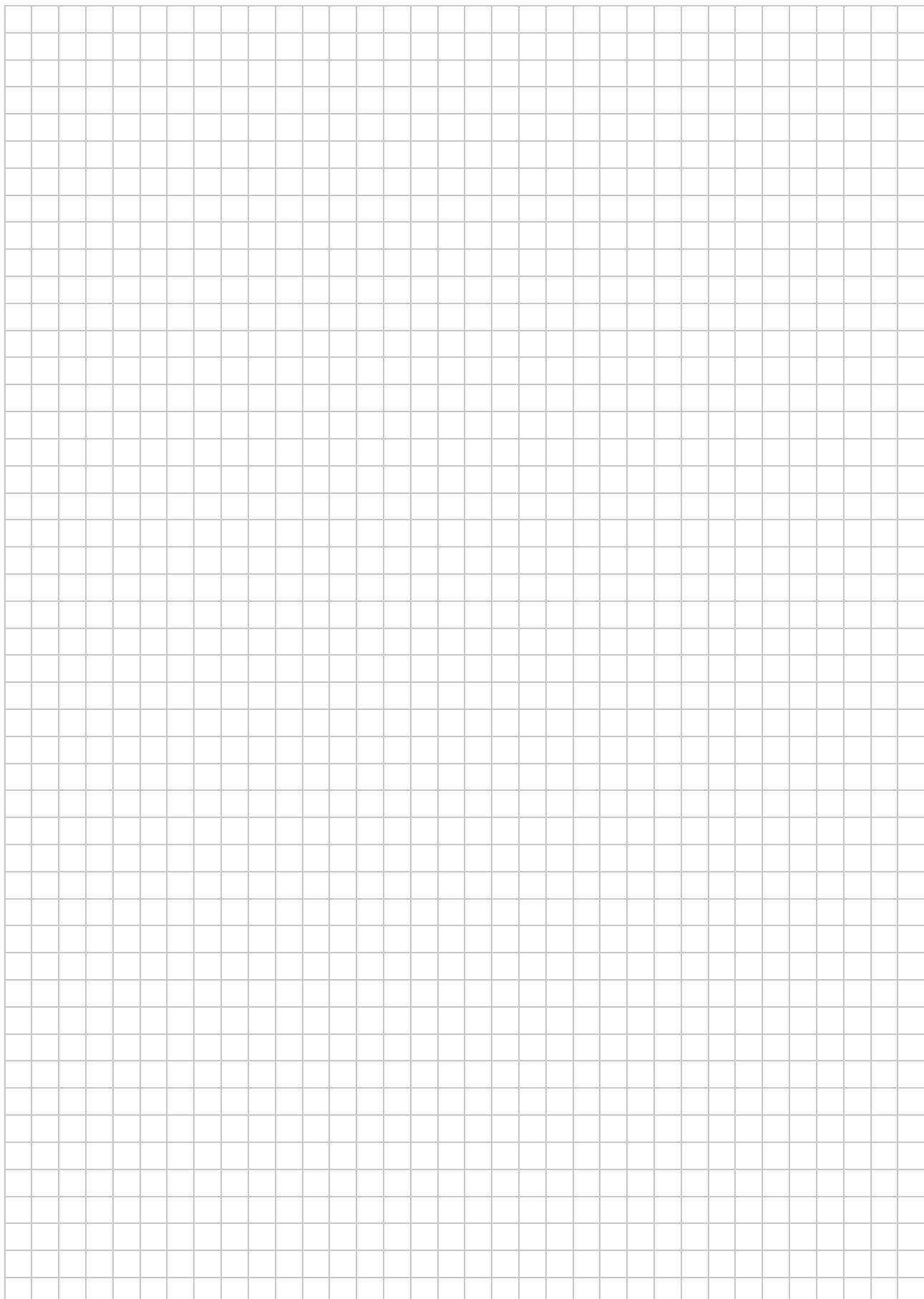


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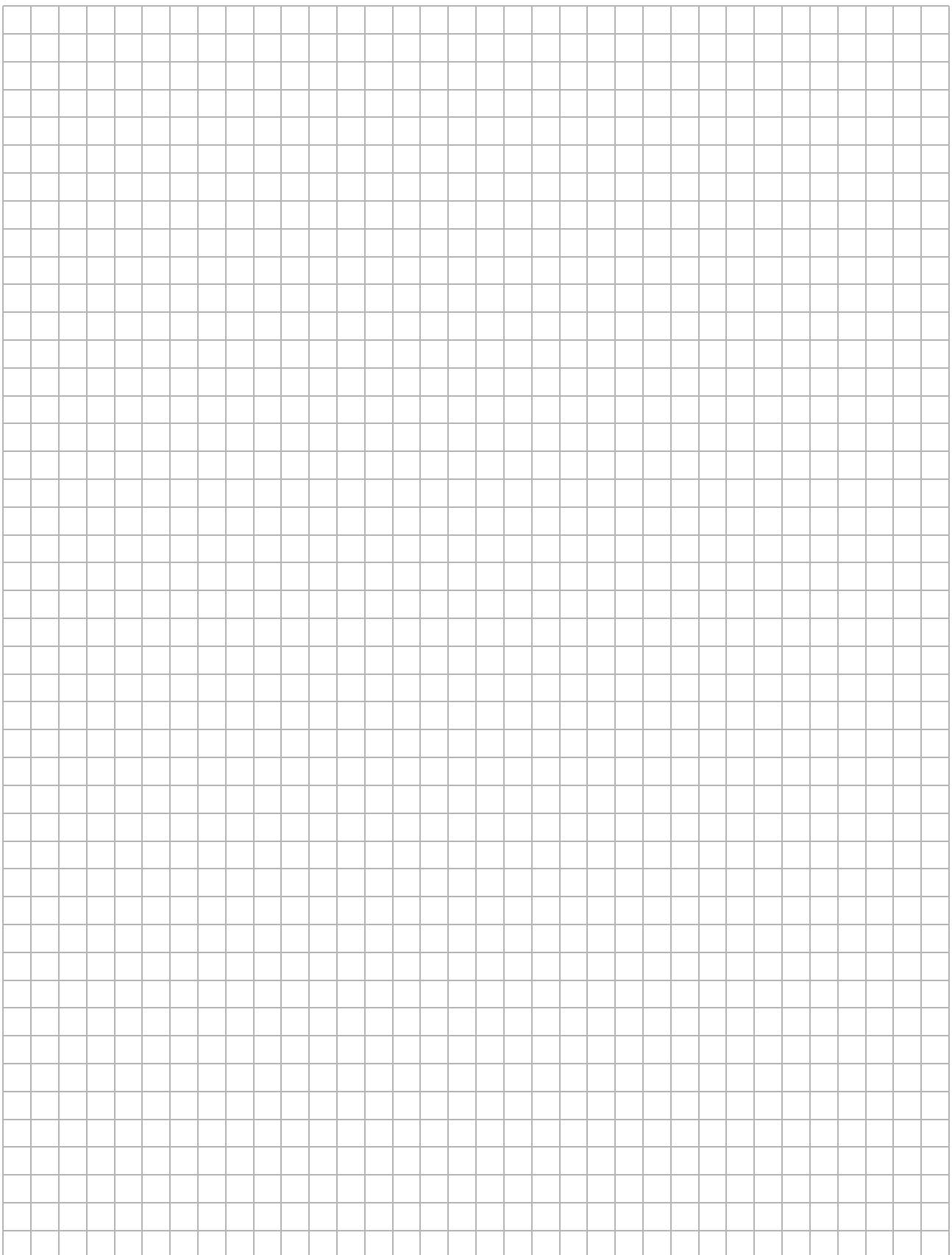
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Mathematics – Paper 1

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