# Indices Rules- Practice practice practice

1. *The product* $\times$ $+$ *Rule says to multiply powers add the indices.*

Due to popular demand we are keeping the $\times$ symbol for the Christmas test — but be prepared to let it go afterwards.

In algebra, multiplication is usually implied: $ab$, $3x$, and $m^4m^3$ are all products even though no $\times$ is written.

And in arithmetic the dot notation $2 \cdot 3 \cdot 5 = 30$ gradually replaces the old $\times$ because it avoids confusion with the letter $x$.

$$a^p \times a^q = a^{p+q} \qquad \text{Example: } m^4 \times m^3 = m^{4+3} = m^7$$

| | | | | | |
|---|---|---|---|---|---|
| a) $2^2 \times 2^3$ | $\boxed{2^5}$ | b) $b^4 \times b^6$ | $\boxed{b^{10}}$ | c) $7^5 \times 7^7$ | $\boxed{7^{12}}$ |
| d) $m^3 \times m^6$ | $\boxed{m^9}$ | e) $2^8 \times 2^2$ | $\boxed{2^{10}}$ | f) $y^9 \times y^4$ | $\boxed{y^{13}}$ |
| g) $p^{11} \times p^3$ | $\boxed{p^{14}}$ | h) $3^6 \times 3^5$ | $\boxed{3^{11}}$ | i) $k^7 \times k^{10}$ | $\boxed{k^{17}}$ |
| j) $w^{12} \times w$ | $\boxed{w^{13}}$ | k) $\pi^{11} \times \pi^{89}$ | $\boxed{\pi^{100}}$ | l) $\theta^6 \times \theta^4$ | $\boxed{\theta^{10}}$ |

2. The Quotient $\div$ $-$ Rule says to divide powers we subtract the indices. *Write each expression as a single power.*

| | | | | | |
|---|---|---|---|---|---|
| *a)* $2^2 \times 2^3$ | $\boxed{2^5}$ | *b)* $b^4 \times b^6$ | $\boxed{b^{10}}$ | *c)* $7^5 \times 7^7$ | $\boxed{7^{12}}$ |
| *d)* $m^3 \times m^6$ | $\boxed{m^9}$ | *e)* $2^8 \times 2^2$ | $\boxed{2^{10}}$ | *f)* $y^9 \times y^4$ | $\boxed{y^{13}}$ |
| *g)* $p^{11} \times p^3$ | $\boxed{p^{14}}$ | *h)* $3^6 \times 3^5$ | $\boxed{3^{11}}$ | *i)* $k^7 \times k^{10}$ | $\boxed{k^{17}}$ |
| *j)* $w^{12} \times w$ | $\boxed{w^{13}}$ | *k)* $\pi^{11} \times \pi^{89}$ | $\boxed{\pi^{100}}$ | *l)* $\theta^6 \times \theta^4$ | $\boxed{\theta^{10}}$ |

*3. The Quotient* $\div$ $-$ *Rule says to divide powers we subtract the indices.*

$$\frac{a^p}{a^q} = a^{p-q} \qquad \textit{Example: } \frac{m^7}{m^3} = m^{7-3} = m^4$$

*Write each expression as a single power:*

| | | | | | |
|---|---|---|---|---|---|
| *a)* $a^9 \div a^3$ | $\boxed{a^6}$ | *b)* $b^7 \div b^2$ | $\boxed{b^5}$ | *c)* $c^{12} \div c^5$ | $\boxed{c^7}$ |
| *d)* $m^{10} \div m^4$ | $\boxed{m^6}$ | *e)* $x^8 \div x$ | $\boxed{x^7}$ | *f)* $y^{11} \div y^6$ | $\boxed{y^5}$ |
| *g)* $p^{15} \div p^9$ | $\boxed{p^6}$ | *h)* $q^{13} \div q^7$ | $\boxed{q^6}$ | *i)* $k^{14} \div k^3$ | $\boxed{k^{11}}$ |
| *j)* $w^{20} \div w^5$ | $\boxed{w^{15}}$ | *k)* $t^9 \div t^2$ | $\boxed{t^7}$ | *l)* $r^{18} \div r^{10}$ | $\boxed{r^8}$ |

4. *The Power of a Power* ■▫ *Rule says that to take a power of a power, we multiply the indices.*

$$(a^p)^q = a^{pq} \qquad \textit{Example: } (m^4)^3 = m^{4\times 3} = m^{12}$$

*Write each expression as a single power:*

| | | | | | |
|---|---|---|---|---|---|
| a) $\left(a^2\right)^5$ | $\boxed{a^{10}}$ | b) $\left(x^3\right)^4$ | $\boxed{x^{12}}$ | c) $\left(b^7\right)^2$ | $\boxed{b^{14}}$ |
| d) $\left(m^5\right)^3$ | $\boxed{m^{15}}$ | e) $\left(t^9\right)^2$ | $\boxed{t^{18}}$ | f) $\left(y^4\right)^6$ | $\boxed{y^{24}}$ |
| g) $\left(p^{11}\right)^3$ | $\boxed{p^{33}}$ | h) $\left(q^6\right)^5$ | $\boxed{q^{30}}$ | i) $\left(k^7\right)^{10}$ | $\boxed{k^{70}}$ |
| j) $\left(w^3\right)^8$ | $\boxed{w^{24}}$ | k) $\left(r^{12}\right)^2$ | $\boxed{r^{24}}$ | l) $\left(\theta^5\right)^7$ | $\boxed{\theta^{35}}$ |

5. *The zero index rule takes a while to trust. Anything (except 0) raised to the power 0 is defined to be 1. It feels strange at first but becomes routine quickly.*

$$a^0 = 1$$

*Evaluate each:*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| a) $m^0$ | $\boxed{1}$ | b) $x^0$ | $\boxed{1}$ | c) $7^0$ | $\boxed{1}$ | d) $p^0$ | $\boxed{1}$ |
| e) $\pi^0$ | $\boxed{1}$ | f) $\theta^0$ | $\boxed{1}$ | | | | |

6. *Negative powers take a little getting used to. A negative index in the numerator simply means the base belongs on the bottom of a fraction, with the power made positive.*

$$a^{-p} = \frac{1}{a^p}$$

*Rewrite each with a positive index:*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| a) $a^{-3}$ | $\boxed{\frac{1}{a^3}}$ | b) $b^{-5}$ | $\boxed{\frac{1}{b^5}}$ | c) $x^{-7}$ | $\boxed{\frac{1}{x^7}}$ | d) $m^{-2}$ | $\boxed{\frac{1}{m^2}}$ |
| e) $t^{-9}$ | $\boxed{\frac{1}{t^9}}$ | f) $y^{-4}$ | $\boxed{\frac{1}{y^4}}$ | g) $p^{-6}$ | $\boxed{\frac{1}{p^6}}$ | h) $q^{-8}$ | $\boxed{\frac{1}{q^8}}$ |

7. *When the negative power is already in the denominator, it just comes up and becomes positive. One familiar sounding phrase covers both situations — change floors, change signs.*

$$\frac{1}{a^{-p}} = a^p$$

*Rewrite each with a positive power and no fraction:*

*a)* $\dfrac{1}{a^{-3}}$ $\boxed{a^3}$ *b)* $\dfrac{1}{b^{-4}}$ $\boxed{b^4}$ *c)* $\dfrac{1}{x^{-7}}$ $\boxed{x^7}$

*d)* $\dfrac{1}{m^{-1}}$ $\boxed{m^1}$ *e)* $\dfrac{1}{t^{-9}}$ $\boxed{t^9}$ *f)* $\dfrac{1}{y^{-5}}$ $\boxed{y^5}$

*g)* $\dfrac{1}{p^{-6}}$ $\boxed{p^6}$ *h)* $\dfrac{1}{q^{-8}}$ $\boxed{q^8}$

8. *Before using the power–of–a–product rule, recall how ordinary distribution works. A factor in front of a bracket simply multiplies everything inside the bracket:*

$$3x(4x-5) = 3x\cdot 4x \;-\; 3x\cdot 5$$

*With two brackets, the same idea continues: each term in the first bracket multiplies each term in the second bracket. In both case we say that multiplication distributes over the addition that is happening inside the brackert/*

$$(a+b)(c+d) = ac \;+\; ad \;+\; bc \;+\; bd$$

*We often phrase it as: "the power of a product equals the product of the powers." The single outside power simply copies itself onto each factor.*

*Expand each:*

*a)* $(ab)^3$ $\boxed{a^3b^3}$ *b)* $(5b)^4$ $\boxed{5^4b^4}$

*c)* $(pq)^2$ $\boxed{p^2q^2}$ *d)* $(t7)^5$ $\boxed{t^57^5}$

*e)* $(rs)^7$ $\boxed{r^7s^7}$ *f)* $(25)^6$ $\boxed{2^65^6}$

*g)* $(uv)^9$ $\boxed{u^9v^9}$ *h)* $(cd)^8$ $\boxed{c^8d^8}$

9. *This rule is the "same story" with a fraction instead of a product.* ***Powering distributes across a quotient*** *just as it distributes across a product.*

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

*We read it as: "the power of a quotient equals the quotient of the powers," or, more informally, "a power outside a fraction pushes itself onto the top and the bottom." This is the powering–over–a–reciprocal idea.*

*Expand each:*

*a)* $\left(\dfrac{a}{b}\right)^3$ $\boxed{\dfrac{a^3}{b^3}}$ *b)* $\left(\dfrac{3}{5}\right)^2$ $\boxed{\dfrac{3^2}{5^2}}$ *c)* $\left(\dfrac{p}{q}\right)^6$ $\boxed{\dfrac{p^6}{q^6}}$

*d)* $\left(\dfrac{t}{y}\right)^5$ $\boxed{\dfrac{t^5}{y^5}}$ *e)* $\left(\dfrac{r}{s}\right)^7$ $\boxed{\dfrac{r^7}{s^7}}$ *f)* $\left(\dfrac{k}{m}\right)^2$ $\boxed{\dfrac{k^2}{m^2}}$

*g)* $\left(\dfrac{u}{v}\right)^9$ $\boxed{\dfrac{u^9}{v^9}}$ *h)* $\left(\dfrac{c}{d}\right)^8$ $\boxed{\dfrac{c^8}{d^8}}$ *i)* $\left(\dfrac{h}{k}\right)^{11}$ $\boxed{\dfrac{h^{11}}{k^{11}}}$

*j)* $\left(\dfrac{w}{z}\right)^3$ $\boxed{\dfrac{w^3}{z^3}}$ *k)* $\left(\dfrac{\theta}{\phi}\right)^4$ $\boxed{\dfrac{\theta^4}{\phi^4}}$ *l)* $\left(\dfrac{a}{c}\right)^{10}$ $\boxed{\dfrac{a^{10}}{c^{10}}}$

**A note on priority of operations.** Much of this topic is really about understanding how operations line up in order of priority. Without brackets, powers come first, then multiplication and division, then addition and subtraction. But once a bracket appears, the normal priority is reversed — everything inside the bracket becomes the new "first task". That is why multiplication distributes over addition in a bracket, and why a single power outside a bracket distributes over each factor in a product or a quotient. It is the same idea expressed in different ways.