L.16

NAME ☞	
SCHOOL ☞	
TEACHER ☞	

Pre-Leaving Certificate Examination, 2017

Mathematics

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

School stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) **(i)** Simplify:

$$5(4x + 3) - 3(2 - 4x).$$

(ii) Solve for x:

$$\frac{5x + 1}{6} + \frac{1 - x}{2} = -\frac{1}{3}, \text{ where } x \in \mathbb{R}.$$

(iii) Verify your answer to part **(ii)** above.

(b) Solve the equation $x^2 + 5x - 7 = 0$ and give your answers correct to two decimal places.

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Question 2 **(25 marks)**

(a) Sophie is going on holiday to America and wants to exchange €500 for US dollars. She checks online and finds the exchange rate on that day to be \$1 = €0·88.

(i) Find, correct to the nearest cent, the amount that Sophie can expect to receive in dollars.

(ii) Sophie goes into her local bank and exchanges her euro for US dollars. The exchange rate for the transaction is €1 = \$1·0990. Find the amount that Sophie receives in dollars.

(iii) Suggest a reason why there is a difference between the two amounts.

Reason:

(b) Graham wishes to invest €3000 for 3 years. His local bank offers him two options, Option 1 and Option 2, as shown in the table below.

Option 1	**Option 2**
2·75% compound interest per year, for 3 years	5% interest after 2 years
	3% compound interest per year thereafter
No penalty applies if money withdrawn at the end of any year	Penalty applies if money withdrawn within the first 2 years

(i) Find, correct to the nearest cent, the value of Graham's investment at the end of 3 years under both options.

Option 1:

Option 2:

(ii) Which option would you recommend? Give a reason for your answer.

Option:

Reason:

Question 3 **(25 marks)**

$z_1 = 3 + 2i$ and $z_2 = -1 + 2i$ are two complex numbers, where $i^2 = -1$.

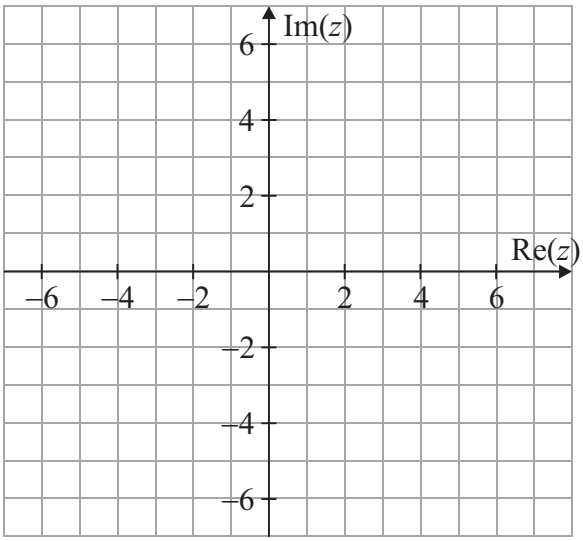
(a) **(i)** Evaluate each of the following complex numbers in the form $a + bi$, where $a, b \in \mathbb{Z}$.

$2z_1 =$

$z_1 + z_2 =$

$iz_1 =$

(ii) Plot each of your answers in part **(i)** on the given Argand diagram and label each point clearly.



(iii) Using your answers to parts **(i)** and **(ii)** above, or otherwise, explain what happens under each of the transformations given in the table below.

Transformation	Evaluation	Explanation
$z = 2z_1$		
$z = z_1 + z_2$		
$z = iz_1$		

(b) Find the complex number z_3 such that $z_3 = \frac{z_1}{z_2}$.

Give your answer in the form $a + bi$, where $a, b \in \mathbb{Z}$.

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Question 4 **(25 marks)**

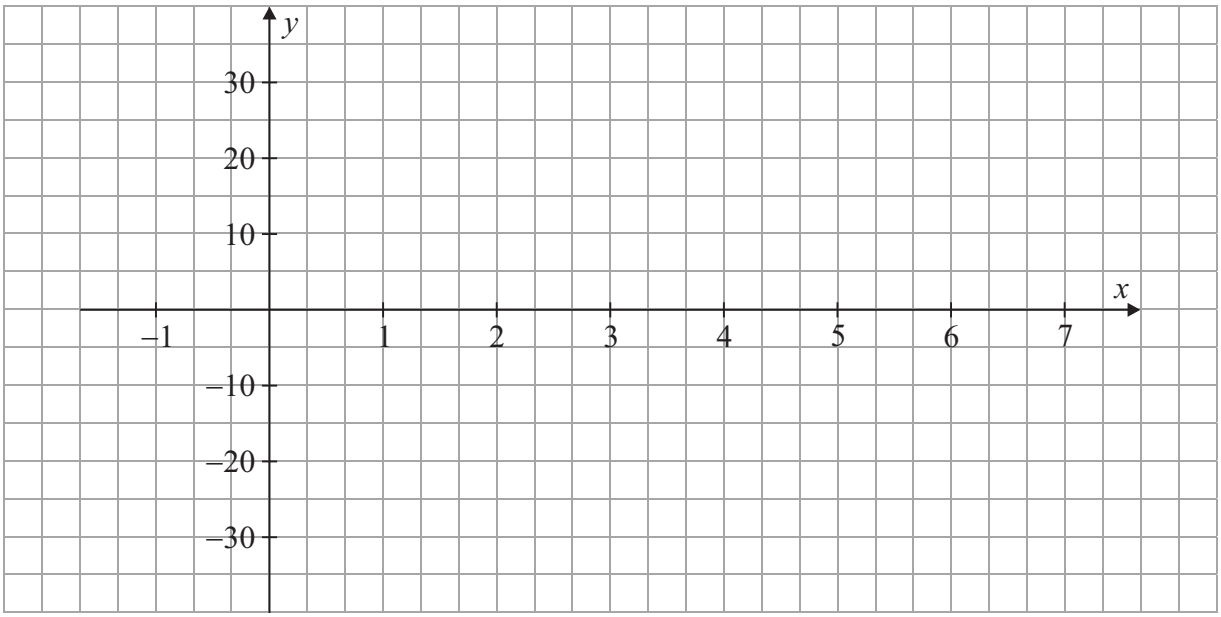
The function $f : x \mapsto x^3 - 9x^2 + 15x + 2$ is defined for $x \in \mathbb{R}$.

(a) **(i)** Find the co-ordinates of the point where the graph of f cuts the y-axis.

(ii) Verify that the graph of f cuts the x-axis between $x = 2$ and $x = 2{\cdot}5$. Explain your answer.

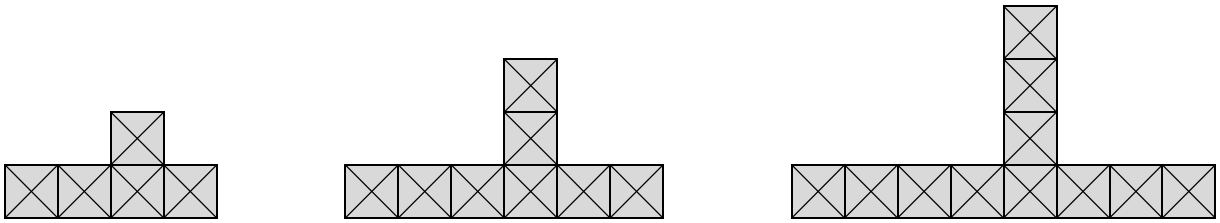
(b) Find $f'(x)$, the derivative of $f(x)$.
Hence, find the co-ordinates of the local maximum turning point and local minimum turning point of f.

(c) Hence, sketch the graph of f on the axes below and indicate clearly both turning points.



Question 5 **(25 marks)**

The first three patterns in a sequence of patterns formed by arranging square tiles are shown below.



(a) Draw the fourth pattern in the sequence.

(b) **(i)** Complete the table below.

Pattern Number	**1**	**2**	**3**	**4**	**5**	**6**
Number of Tiles	5	8	11			

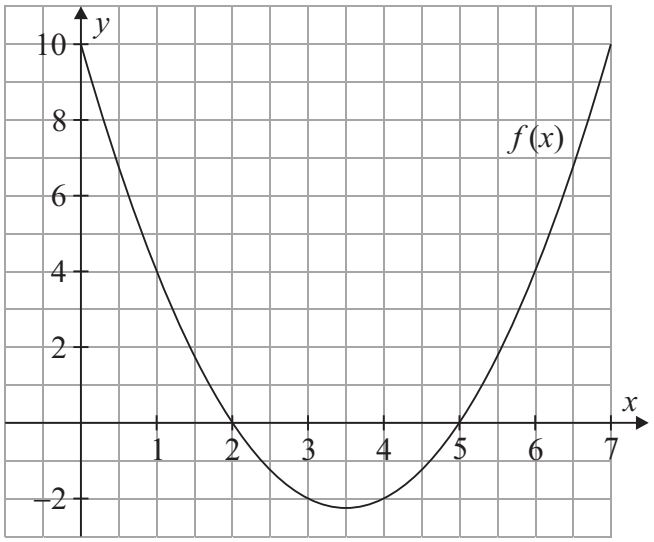
(ii) Find, in terms of n, a formula for the number of tiles in the nth pattern in the sequence.

(iii) Using your formula, or otherwise, find the number of tiles in the 40th pattern.

(iv) Find the total number of tiles used in the first 40 patterns in the sequence.

Question 6 **(25 marks)**

The diagram shows the graph of the function $f(x) = x^2 - 7x + 10$ in the domain $0 \le x \le 7, x \in \mathbb{R}$.



(a) **(i)** The function $g(x)$ is defined as

$g : x \mapsto 5 - x$, where $x \in \mathbb{R}$.

Draw the graph of $g(x)$ on the diagram.

(ii) Use the graphs to find the two values of x for which $g(x) = f(x)$.

(b) Use algebra to solve $g(x) = f(x)$.

(c) Explain why the tangents to the curve $y = f(x)$, at the points where the graphs of $f(x)$ and $g(x)$ intersect, are not parallel to each other.

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Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

The height of a diver after jumping off a platform into a diving pool is given by the formula:

$$h(t) = -5t^2 + 5t + 10$$

where h is the height of the diver above the water line (the surface) of the diving pool in metres and t is the time in seconds after she jumps.

(a) **(i)** Find the height of the diving platform.

(ii) Find the length of time it takes before the diver enters the water.

(b) **(i)** Find $\frac{dh}{dt}$.

(ii) Hence, find the rate of change of height at the instant that the diver enters the water.

(c) **(i)** Find the length of time it takes the diver to reach the maximum height above the water line of the diving pool.

(ii) Hence, find this maximum height.

(d) The diver performs a different dive in competition. She reaches a maximum height of 0·75 m above the diving platform after 0·5 s. After 1 second, she is at the same height again as her initial dive position. The height of the diver above the water line after t seconds is given by the formula $g(t) = at^2 + bt + 10$, where $a, b \in \mathbb{Q}$.

Find two expressions for the position of the diver in terms of a and b and hence, find the value of a and the value of b.

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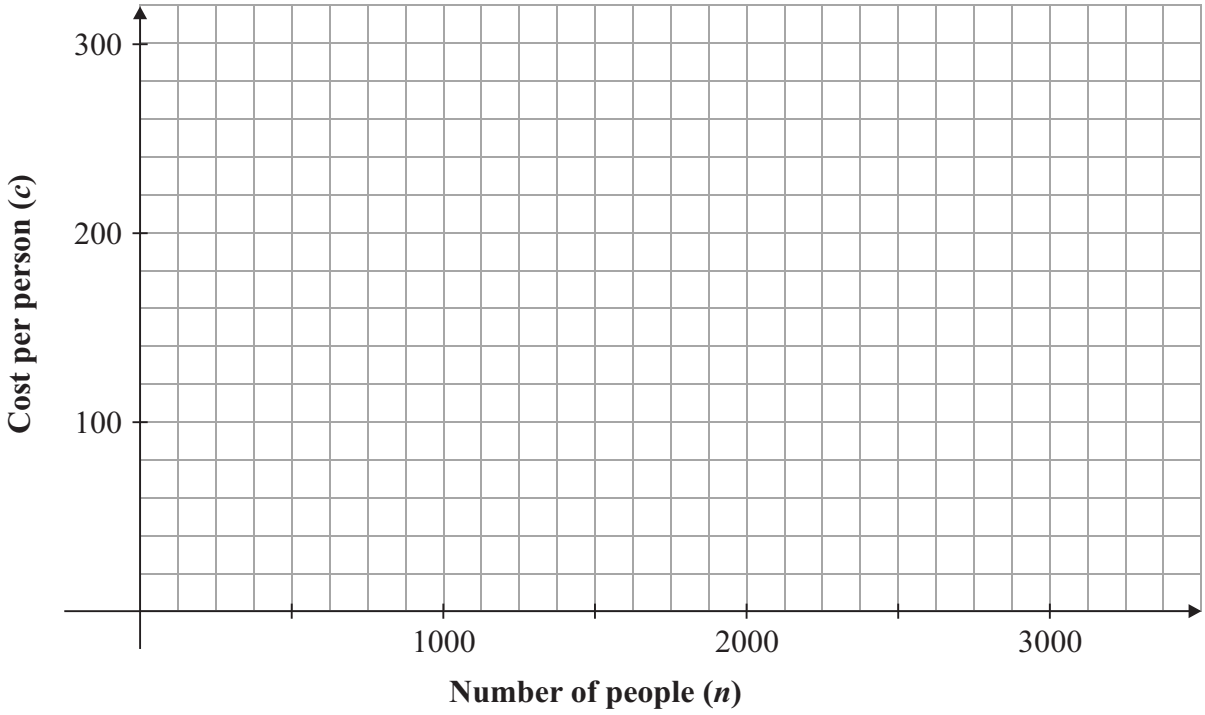
Question 8 **(50 marks)**

(a) The cost of hiring a well-known artist for a community music festival is €120 000. The cost will be shared equally by all the people who come to the event. The organisers are unsure of the number of people (n) who will attend.

(i) Complete the table below to show the cost per person (c) for each given value of n.

Number of people (n)	500	1000	1500	2000	2500	3000
Cost per person (c)				60	48	40

(ii) On the grid below, draw a graph to show the relationship between n and c.



(iii) Use your graph to estimate the minimum number of people required to attend the festival so that the cost per person is €50 or less.

(iv) Write a formula in n for the cost per person who attends the festival.

(v) Verify your answer to part **(iii)** using your formula from part **(iv)**.

(b) The organisers have sold 2000 tickets for the festival. They have only 3810 m^2 available for parking of both cars and buses. The organisers plan to arrange buses to bring people to the event and plan to sell tickets for car parking spaces in advance.

The space allotted for each car is 12 m^2 while the space allotted for each bus is 45 m^2. It is assumed that the average occupancy per car is 2 and the average occupancy per bus is 53.

(i) Write down an expression for the total parking space available in terms of x (number of cars) and y (number of buses).

(ii) Write down an expression for the number of people who have purchased tickets for the event in terms of x and y.

(iii) Hence, find the number of car parking spaces and bus parking spaces required in order to accomodate all ticket-holders at the event.

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Question 9 **(50 marks)**

(a) Bob works as a computer engineer. He pays income tax, a universal social charge (USC), and pay-related social insurance (PRSI) on his gross salary. His gross monthly salary is €4250.

(i) The standard rate of income tax is 20% and the higher rate is 40%.
Bob has tax credits of €3300 per annum and a standard rate cut-off point of €32 800 per annum. How much income tax does Bob pay per month, correct to the nearest cent?

(ii) Bob also pays USC on his gross salary every month. He pays USC at the rates of 0·5% on the first €1001 per month, 2·5% on earnings between €1001 and €1564 per month and 5% on the balance.
Calculate the total amount of USC that Bob pays per month, correct to the nearest cent.

(iii) Bob also pays PRSI of €109 per month.
How much can Bob expect his annual take-home pay to be?

(b) **(i)** Bob wants to find out how much money he regularly spends. He prepares a spreadsheet of all his regular financial outgoings, as shown in the table below.

Complete the table to find the total cost of Bob's regular financial outgoings each month, correct to the nearest cent.

Description		Unit Cost	Usage	Cost (€)
Electricity	Standing Charge			14·50
	Price per unit	15·33 cent / kWh	66 kWh	
Broadband + TV				60·00
Rent				800·00
Gas	Standing Charge			6·50
	Price per unit	4·589 cent / kWh	300 kWh	
Bus Ticket		€132·00		132·00
Mobile Phone		€45·00		45·00
Total				

(ii) How much disposable income does Bob have left each month after he pays all his regular outgoings?

(iii) Find this disposable income as a percentage of his net income.
Give your answer correct to one decimal place.

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(c) At the start of January, Bob owed €1700 on his credit card. Interest is charged monthly at an annual equivalent rate (AER) of 18% of the amount owed.

(i) Using the formula $(1 + r)^{12} = 1 + i$, where r is the monthly rate and i is the annual rate of interest, find the rate of interest charged monthly which is equivalent to an AER of 18%, correct to three decimal places.

(ii) Find the amount that Bob will owe on his credit card after 9 months if he does not make any further purchases or repayments over that period of time. Give your answer correct to the nearest euro.

You may use this page for extra work.

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Mathematics – Paper 1

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