



NAME 

SCHOOL 

TEACHER 

Pre-Leaving Certificate Examination, 2019

Mathematics

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

School stamp

Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1

(25 marks)

The table below shows the population of Ireland according to the most recent censuses.

Census Year	2006	2011	2016
Population	4 239 848	4 588 252	4 761 865

- (a) What is the population of Ireland according to the most recent census? Write this number in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{N}$, correct to two significant figures.

[illegible]

- (b)** Show that the average **annual** increase in population between 2011 and 2016 was 0.757%, correct to three decimal places.

[illegible]

- (c) Find the expected increase in the population of Ireland by the time of the next census in 2021, assuming the population grows at the same rate **year-on-year**.

[illegible]

Question 2

(25 marks)

- (a)** $z_1 = 2 - 3i$ is a complex number, where $i^2 = -1$.

- (i) Let $z_2 = iz_1$ and $z_3 = iz_2$.

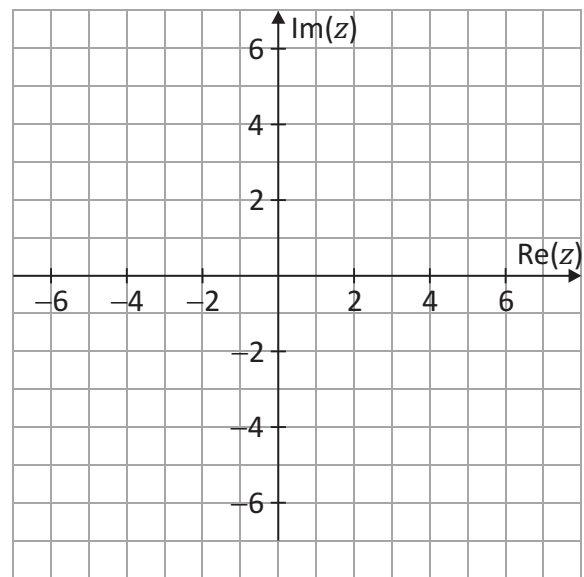
Find z_2 and z_3 , in the form $a + bi$, where $a, b \in \mathbb{R}$.

[illegible]

- (ii)** Plot z_1 , z_2 and z_3 on the Argand diagram. Label each point clearly.

[illegible]

- (iii)** Use your diagram to describe what happens when a complex number is multiplied by i .

[illegible]

- (b)** Let $w = 2 - 4i$, where $i^2 = -1$.
Find the real number k such that

$$k(w\overline{w}) = 15,$$

where \overline{w} is the complex conjugate of w .

[illegible]

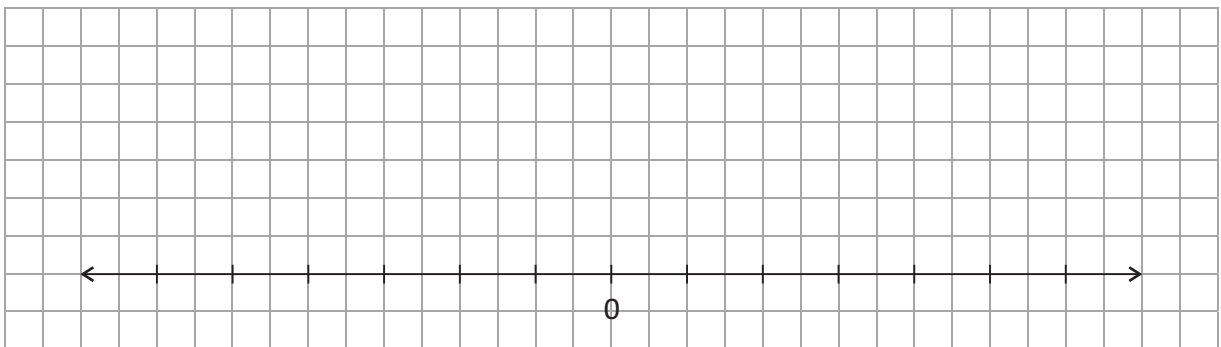
Question 3

(25 marks)

- (a) (i)** Solve for x :

$$3(5x + 2) - 14x = 8 - 13(5 - x).$$

- (ii)** Hence, or otherwise, solve the inequality $3(5x + 2) - 14x \geq 8 - 13(5 - x)$, where $x \in \mathbb{N}$, and show the solution set on the number line below.



- (b) (i)** Write $\frac{3}{x+3} - \frac{4}{2x-1}$ as a single fraction.

[illegible]

- (ii) Hence show that $\frac{3}{x+3} - \frac{4}{2x-1} = \frac{3}{2}$ has no real solutions.

[illegible]

Question 4

(25 marks)

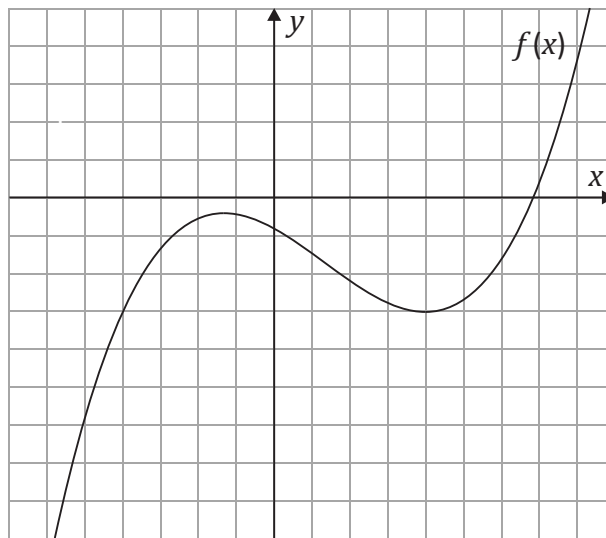
The function $f \mapsto x^3 - 2x^2 - 4x - 3$ is defined for $x \in \mathbb{R}$.

- (a) (i)** Find $f(-1)$ and $f(3)$.

- (ii) Find the co-ordinates of the point at which the graph of f cuts the y -axis.

[illegible]

- (b)** The diagram below shows the graph of the function $f(x) = x^3 - 2x^2 - 4x - 3$, where $x \in \mathbb{R}$.



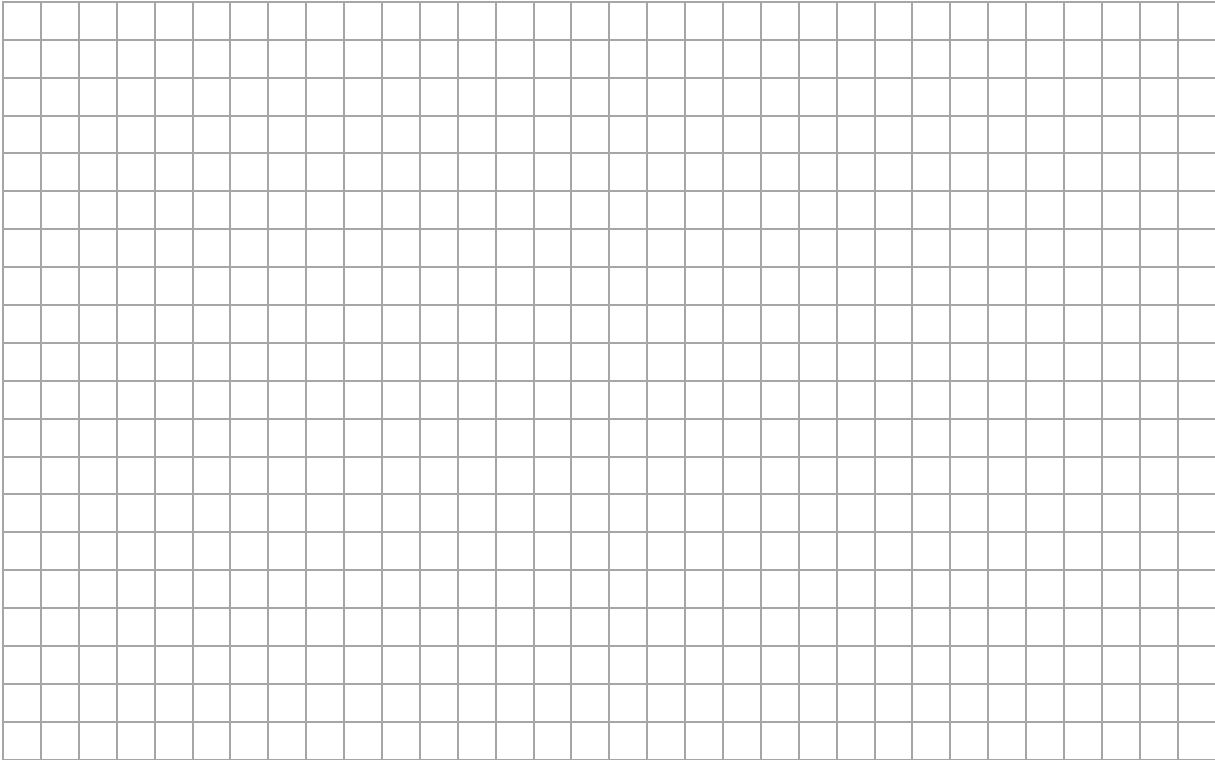
- (i) Mark on the diagram the range of values of x for which $f(x)$ is decreasing.

[illegible]

- (ii) On the same diagram, sketch the graph of the function $f'(x)$, the derivative of $f(x)$.

[illegible]

(c) Find $f'(x)$, the derivative of $f(x)$.
Hence find the co-ordinates of the local minimum turning point of the function $f(x)$.



(25 marks)

(a) (i) Find the amount of money paid into the syndicate each week.

[illegible][illegible][illegible][illegible]

(25 marks)

(a) The standard rate of income tax is 20% and the higher rate is 40%.
Mia has tax credits of €3302 per annum and a standard rate cut-off point of €35 308.

- [illegible]

- [illegible]

- | Rate (%) | Amount (€) |
|----------|------------|
| 0.5 | |
| 2 | |
| x | |

[illegible]

- [illegible]

Answer **all three** questions from this section.

Question 7

(50 marks)

E. coli is a bacterium that exists naturally in our bodies. However, some strains can cause sickness if contaminated food or polluted water is consumed.

A microbiologist wishes to examine the growth of *E. coli* bacteria under different sets of conditions. Three samples of meat are prepared in Petri dishes and examined under a microscope every hour to observe the numbers of bacteria present.

- (a) The **first** sample of meat was fully cooked, allowed to cool and then stored in the fridge. The numbers of bacteria observed in the sample form an **arithmetic** sequence.

- (i)** Complete the table below to show the growth rate of bacteria over the first 6 hours.

After n hours	1	2	3	4	5	6
Number of bacteria	2000	4000	6000			

- (ii) The number of bacteria present after n hours is given by the formula $T_n = pn + q$, where $p, q \in \mathbb{Z}$. Find the value of p and the value of q .

[illegible]

- (iii)** Using your formula, or otherwise, find the expected number of bacteria after 12 hours.

[illegible]

- (b) The **second** sample of meat was partially cooked and then stored in the fridge. The numbers of bacteria observed form a **quadratic** sequence with a second difference of 2000.

- (i)** Complete the table below to show the growth rate of bacteria over the first 6 hours.

After n hours	1	2	3	4	5	6
Number of bacteria	2000	4000				

[illegible]

(50 marks)

where h is the height of the distress flare in metres above sea-level and t is the time in seconds after it is fired.

$$h(t) = 25t - t^2 + 650$$

where h is the height of the distress flare in metres above sea-level and t is the time in seconds after it is fired.

- [illegible]

- [illegible]

- [illegible]

-
- A full-page sheet of graph paper featuring a uniform grid of thin, light gray lines on a white background. The grid consists of small squares covering the entire area.

- (d)** Find $h''(t)$, the second derivative of $h(t)$.
Hence explain the significance of your answer.

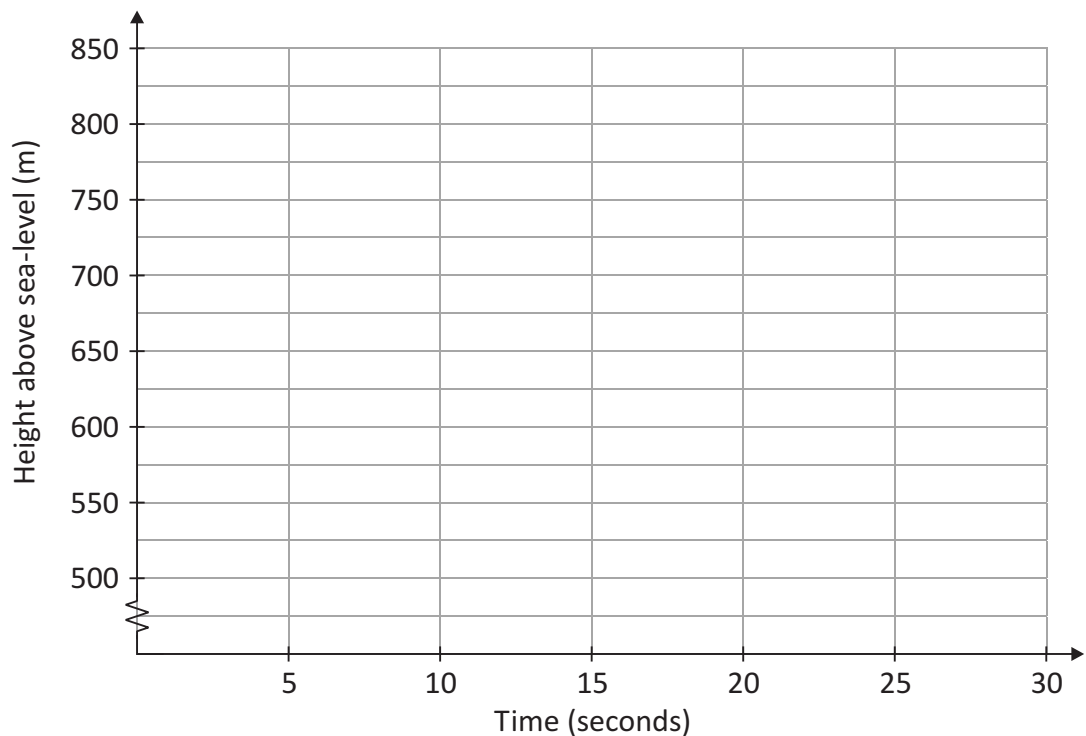
[illegible]

- (e) (i) Use the height function, $h(t)$, to complete the table, showing the estimated height of the distress flare above sea-level over time.

t (seconds)	0	5	10	15	20	25	30
$h(t)$ (metres)			800				

[illegible]

- (ii) Use the data in the table to draw the graph of the function $h(t)$ on the axes below for $0 \leq t \leq 30$, where $t \in \mathbb{R}$.



- (iii)** Use your graph to estimate the time interval for which the height of the distress flare is at least 75 m above the position of the climber.

[illegible]

Question 9

(50 marks)

When thunderstorms occur, people often try to establish how far away they are from where lightning strikes the ground.

One technique, called the “flash-to-bang” method, involves counting the number of seconds, t , that pass between the flash of lightning being seen and the clap of thunder being heard and then dividing that number by 5. The result denotes how far away, in miles, from where lightning strikes.



- (a) (i) Write down a formula to estimate the distance, in miles, from where lightning strikes. State clearly the meaning of any letters used in your formula.

[illegible]

- (ii) Use your formula to find how far away does lightning strike if there is a 20-second gap between the flash of lightning and the clap of thunder.

[illegible]

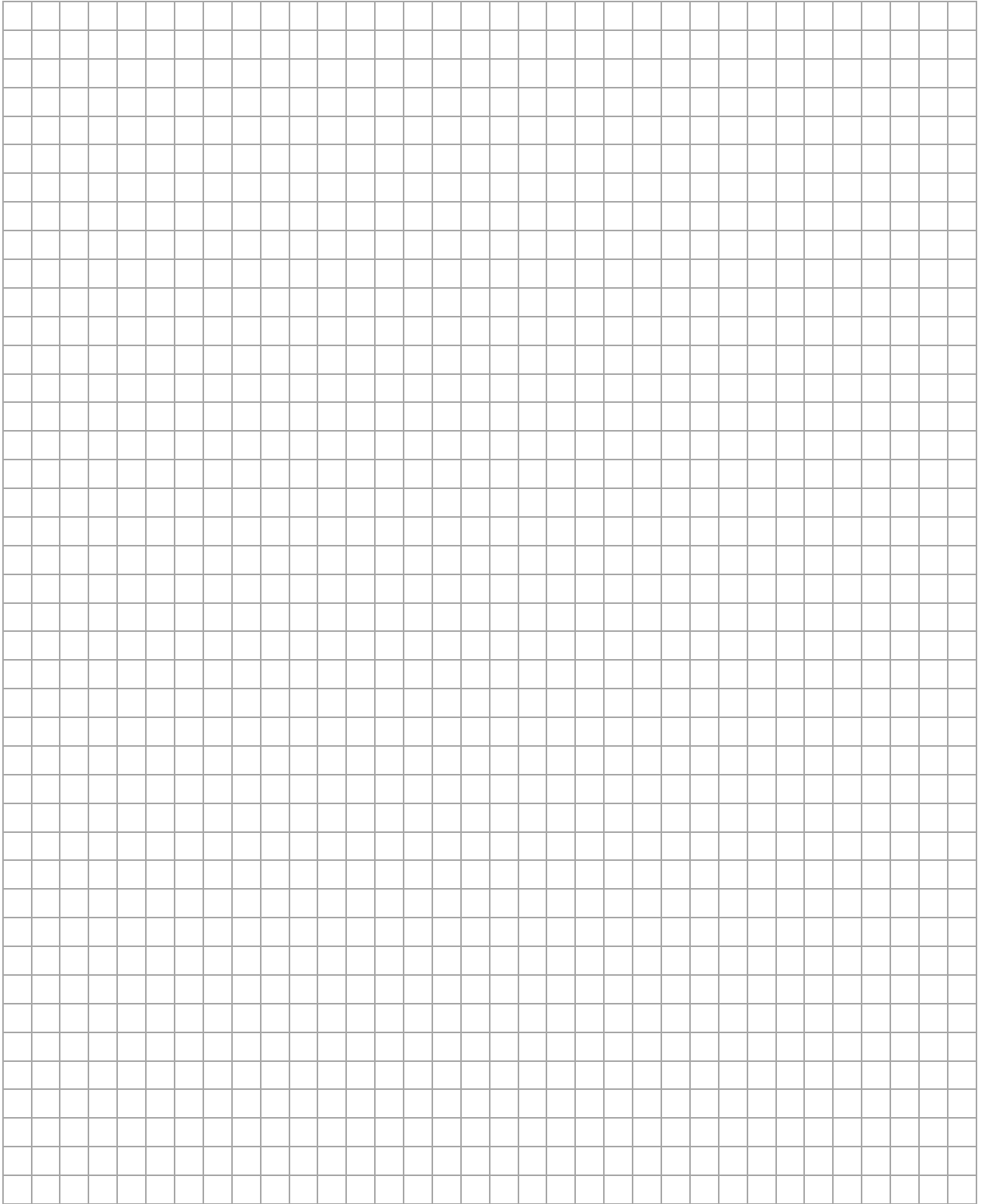
- (b)** In Ireland we usually use the metric system. The speed of sound is approximately 340 metres per second depending on air temperature.

- (i) Find the number of seconds that would pass between the flash of lightning and the clap of thunder for the lightning strike to be 1 kilometre away. Give your answer correct to the nearest whole number.
[Hint: How long does it take for the sound of thunder to travel 1 kilometre?]

[illegible]

- (ii) Hence, or otherwise, write down a formula to estimate the distance, in kilometres, from where lightning strikes.

[illegible]



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