Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2021

Mathematics

Paper 1

Ordinary Level

Friday 11 June   Afternoon 2:00 – 4:30

220 marks

Examination Number

Day and Month of Birth

For example, 3rd February
is entered as 0302

Centre Stamp

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Instructions

There are **two** sections in this examination paper.

| Section A | Concepts and Skills | 120 marks | 6 questions |
| Section B | Contexts and Applications | 100 marks | 4 questions |

Answer questions as follows:
- any four questions from Section A – Concepts and Skills
- any two questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **any four** questions from this section.

Question 1   **(30 marks)**

(a) A television costs €380 before VAT at 21% has been added.
Find the cost of the television after VAT has been added.

(b) When VAT at 21% is included, the price of a laptop increases by €130·20.
Find the total cost of the laptop including VAT.

(c) A printer is priced at €290·40 including VAT at 21%.
Find how much VAT is included in the price of this printer.

(d) On September 1, 2020 the Standard Rate of VAT in Ireland was reduced from 23% to 21%.
A company bought 30 computers in September, all at the same price.
The company calculated that it saved €336 due to the reduction in the VAT rate.
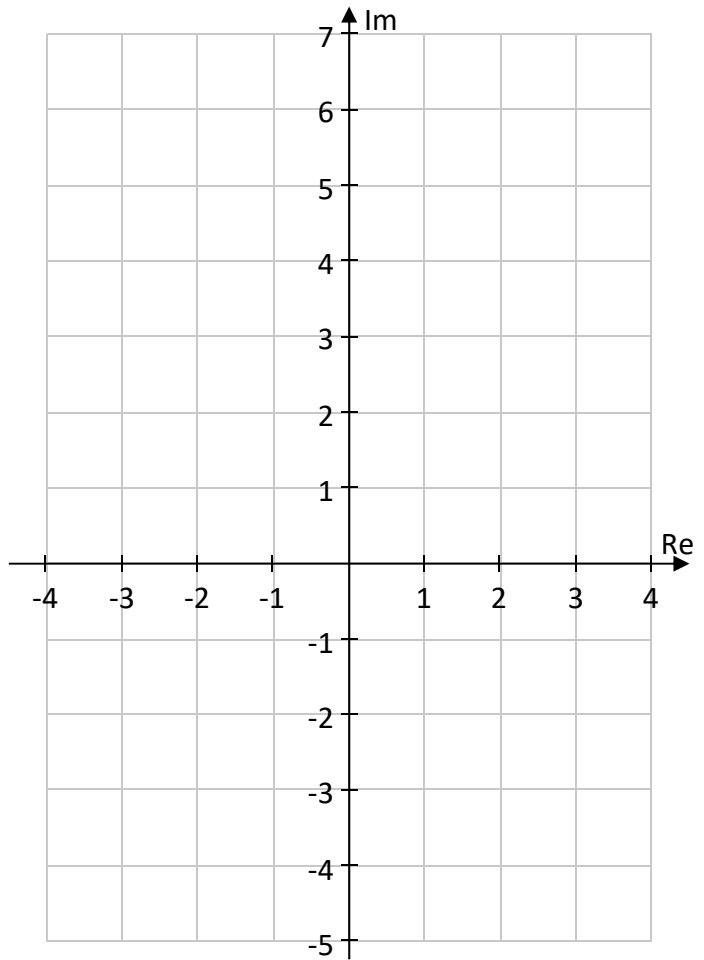Find the price of one computer before VAT had been added.
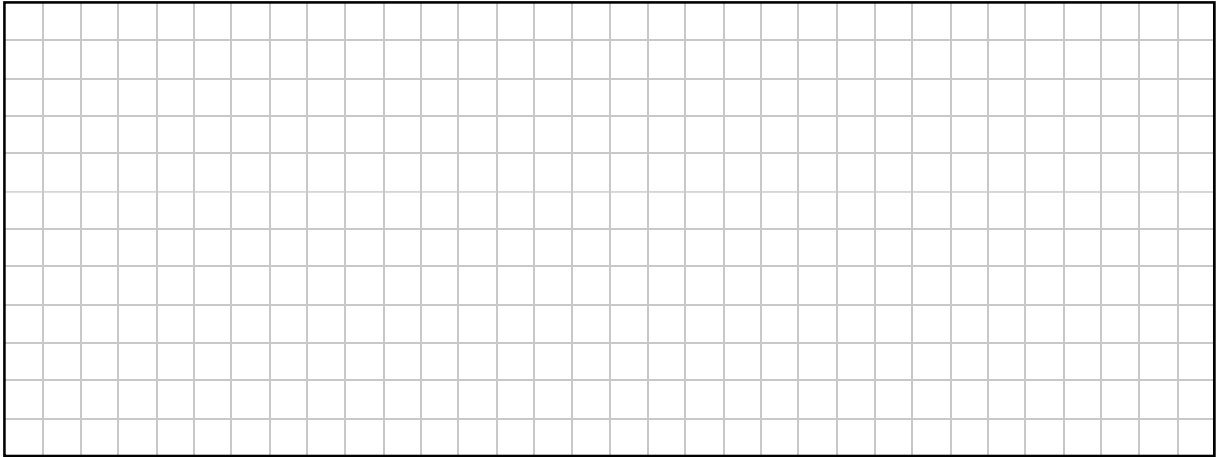
Question 2 (30 marks)
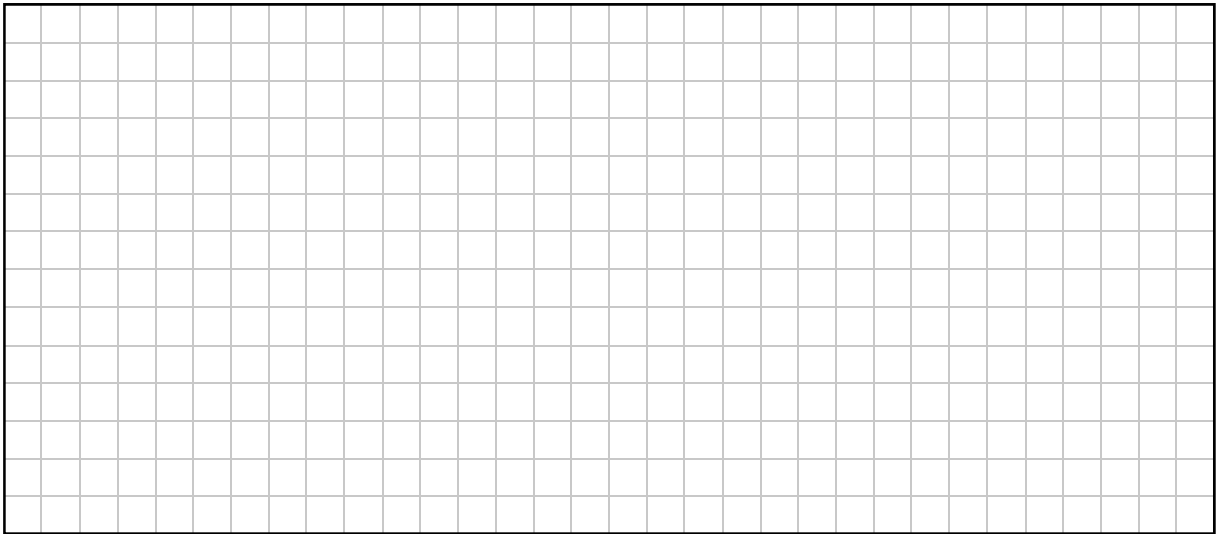
$z_1 = -3 + 4i$ and $z_2 = 4 + 3i$,
where $i^2 = -1$.

(a) Plot and label z_1, z_2, and $z_1 + z_2$ on the Argand Diagram.

$z_1 + z_2 =$

(b) $z_3 = \dfrac{z_1}{z_2}$. Find z_3 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

(c) Find $|\bar{z}_1 - z_2|$, where $\bar{z}_1$ is the complex conjugate of z_1.

Give your answer in the form $p\sqrt{q}$, where p and $q \in \mathbb{N}$.
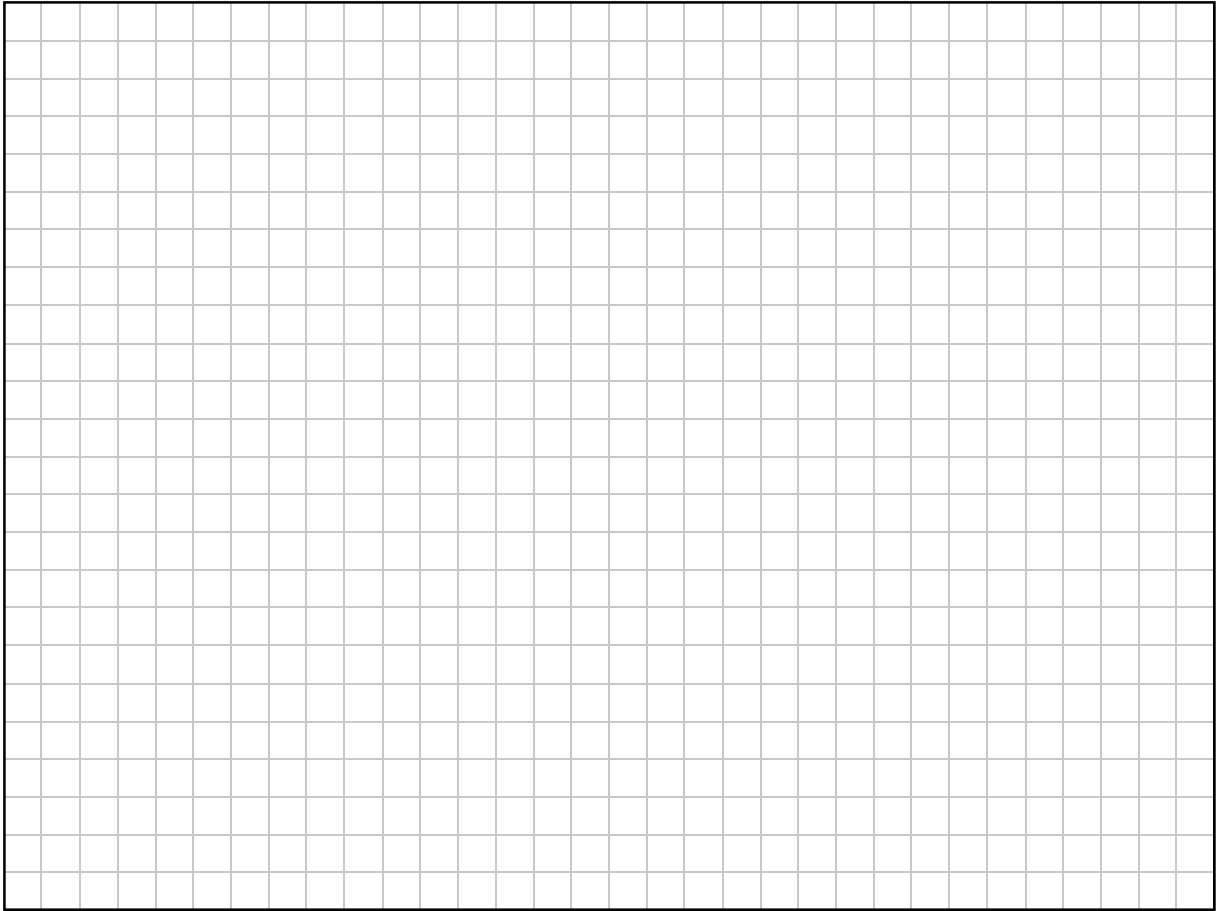
Question 3 (30 marks)

(a) Show that $x = 4$ is a solution of the equation $x^2 - 2x - 8 = 0$.

(b) The equation $x^2 + ax + b = 0$, where $a, b \in \mathbb{Z}$, has solutions $x = 5$ and $x = -2$.
Find the value of a and the value of b.

$a =$ $b =$

(c) Find the solutions of the equation $5x^2 - 2x - 9 = 0$, where $x \in \mathbb{R}$.
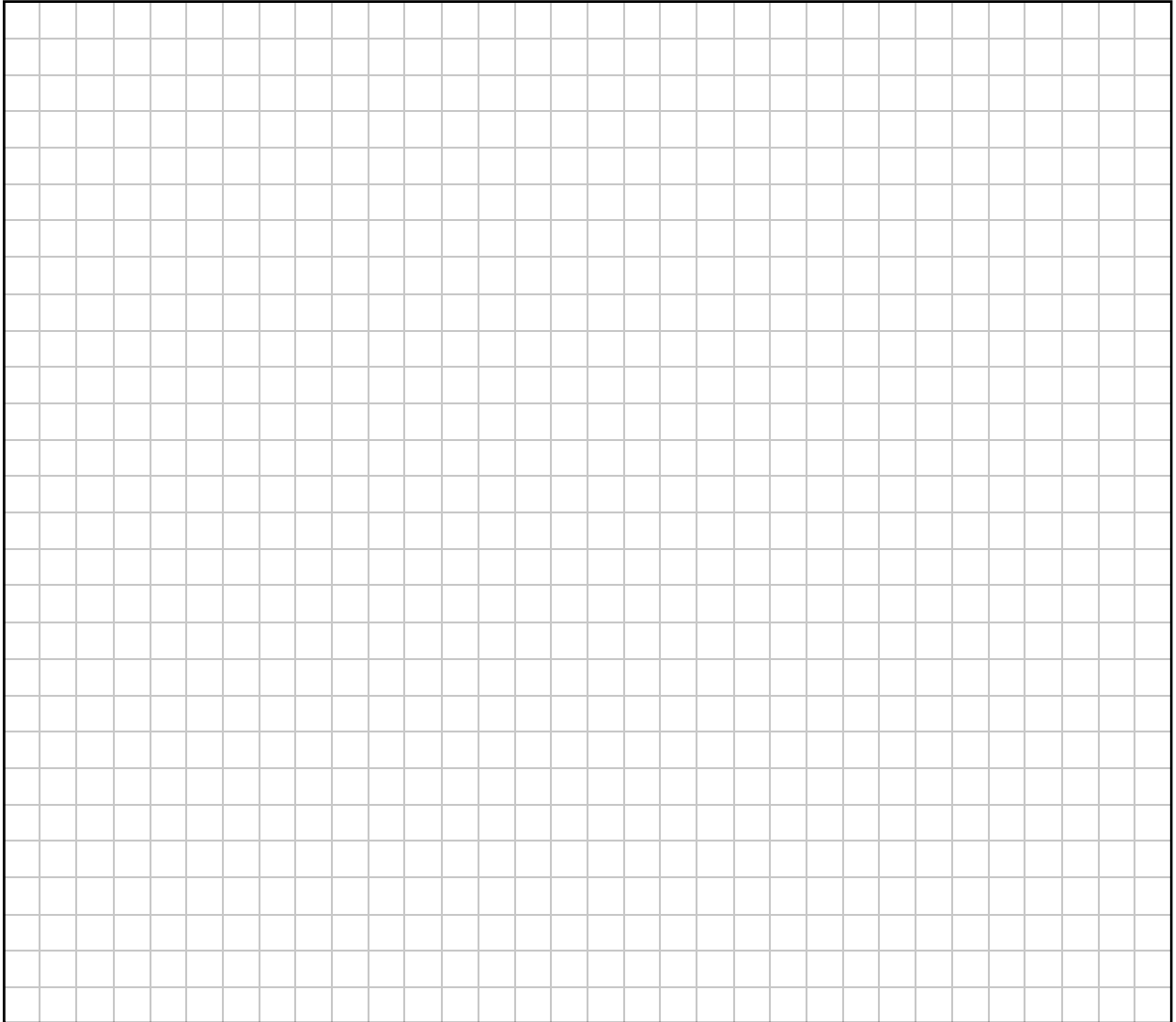Give each answer correct to 2 decimal places.

Question 4 **(30 marks)**

(a) Solve the equation:

$$4(2x + 3) - 7 = 3(x - 5), \text{ where } x \in \mathbb{R}.$$
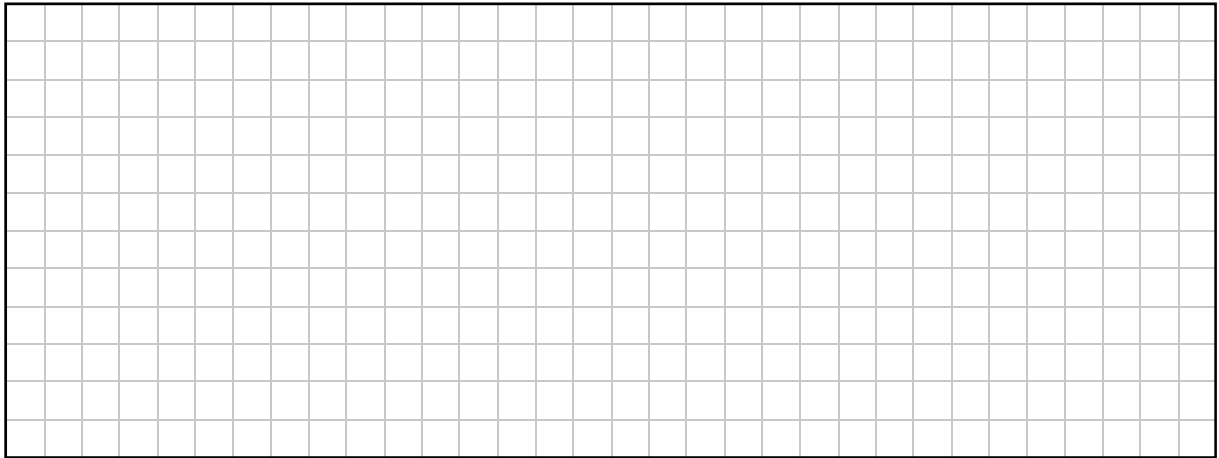
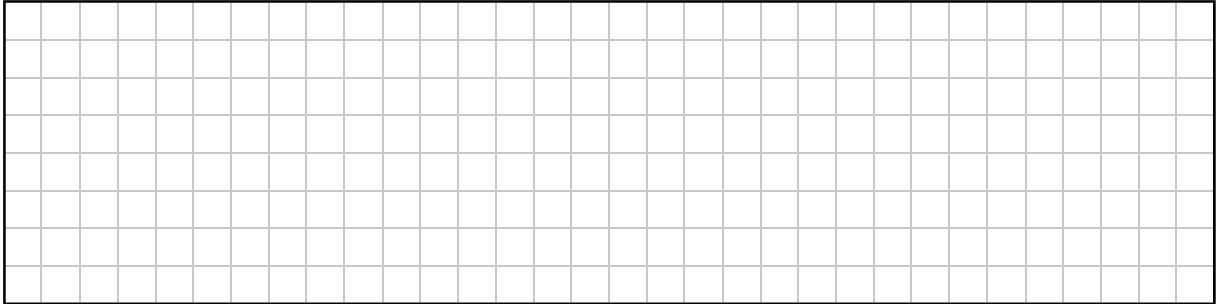**(b)** Solve the simultaneous equations:
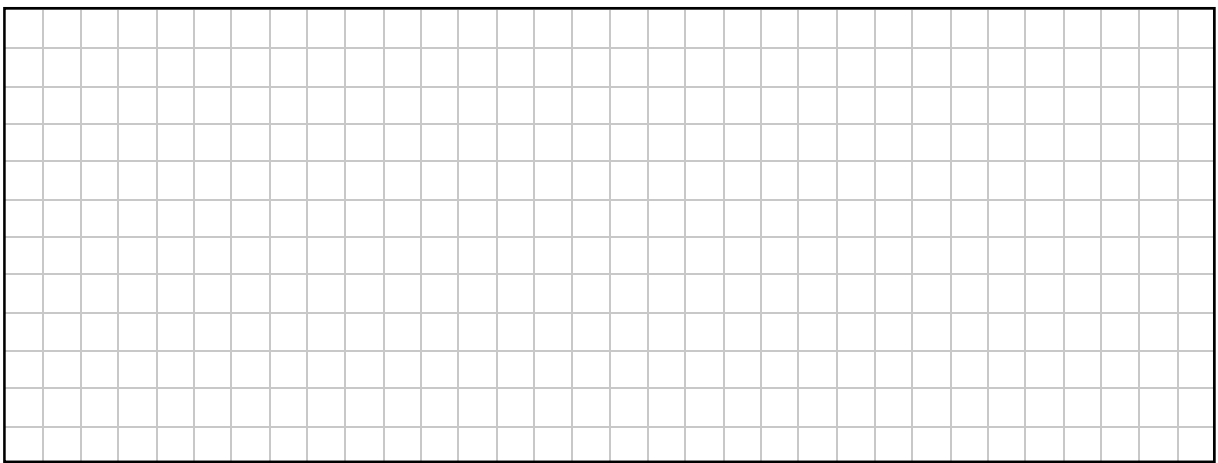
$$2x - y = 7$$
$$x^2 + y^2 = 49.$$

Question 5 (30 marks)

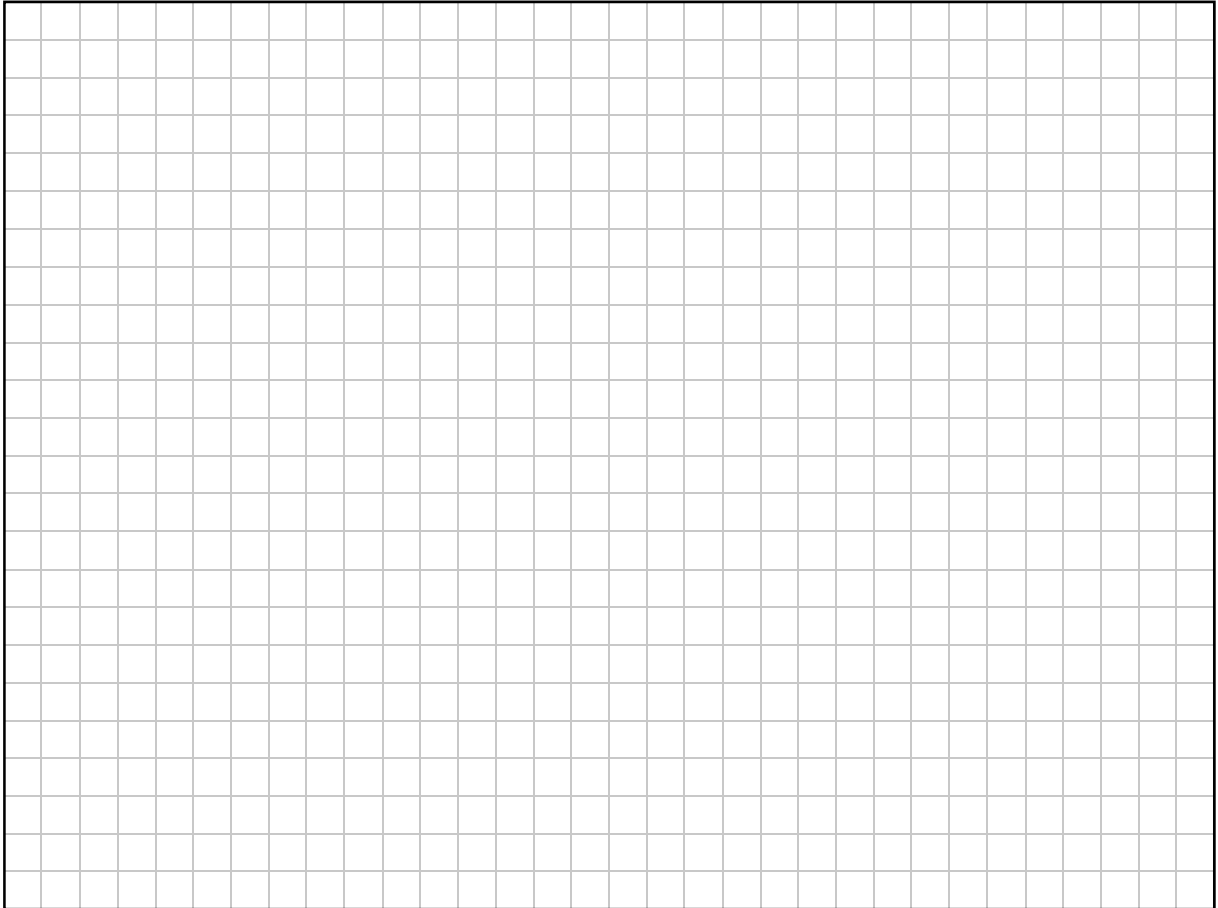The function, f is defined as $f(x) = 3x^2 - 6x + 7$, where $x \in \mathbb{R}$.

(a) Find $f(0{\cdot}67)$, correct to 2 decimal places.

(b) Find the value of x when $f(x) = 4$.

(c) Use calculus to find the co-ordinates of the local minimum point of f.

Question 6 **(30 marks)**

(a) The first three terms of an arithmetic sequence are $-5, \ k, \ 1$.

(i) Find k and hence or otherwise show that the common difference is 3.

$k =$

Common difference:

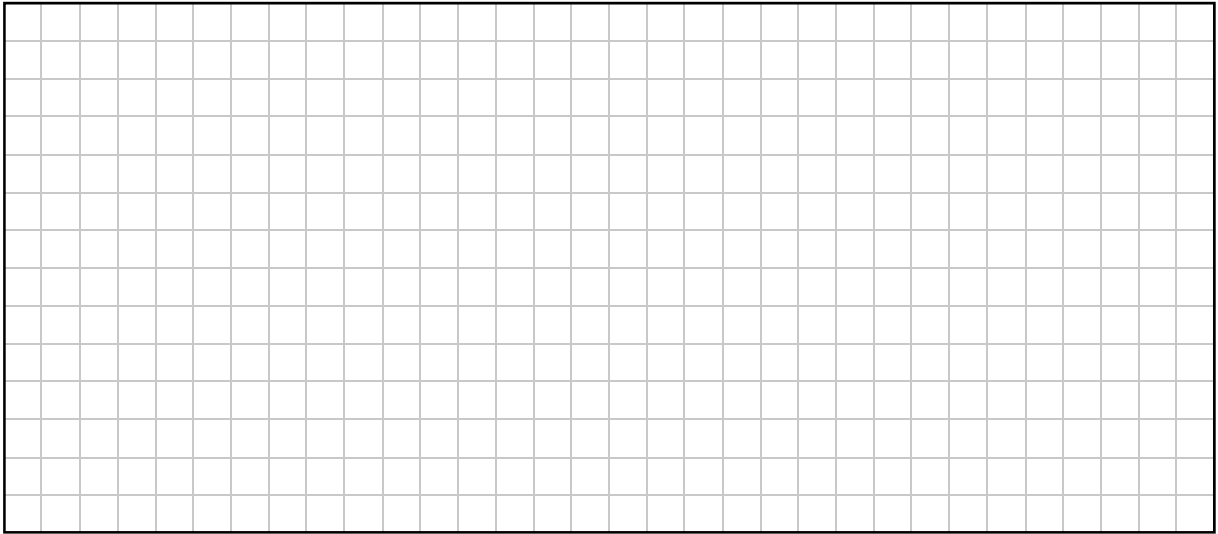**(ii)** Find T_{10}, the 10th term in the sequence.

(iii) Find which term in the sequence has a value of 247.

(b) The first three terms of a different arithmetic sequence are 4, 9, 14.
Find S_{50}, the sum of the first 50 terms of the sequence.

Answer **any two** questions from this section.

Question 7 (50 marks)

The rates and thresholds of the Universal Social Charge (USC) in Ireland (excluding the top rate) during 2020 are given in the table below.

Standard rates and thresholds for USC 2020

Annual Income	Rate
First €12 012	0·5%
Next €8472	2%
Next €49 560	4·5%
Balance	(Top Rate)

Source: *www.revenue.ie*

(a) At what level of annual income does a worker start paying the top rate of USC?

(b) How much USC will a worker have paid in total if they pay the maximum amount due at each of the first three rates?
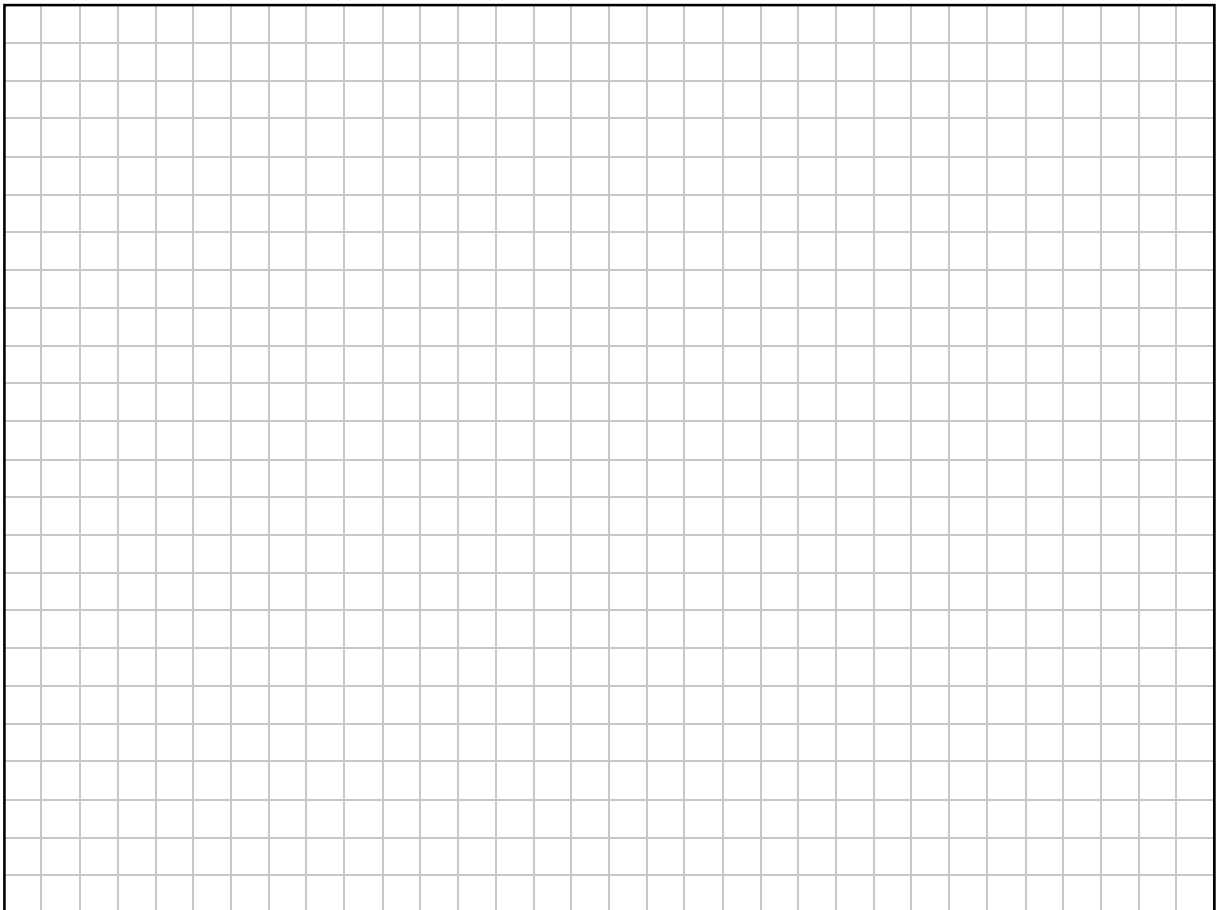
(c) John's annual income is €54 800. Find the amount of USC he will pay.

**(d)** Mary pays €1602·72 in USC in 2020. Find her annual income.
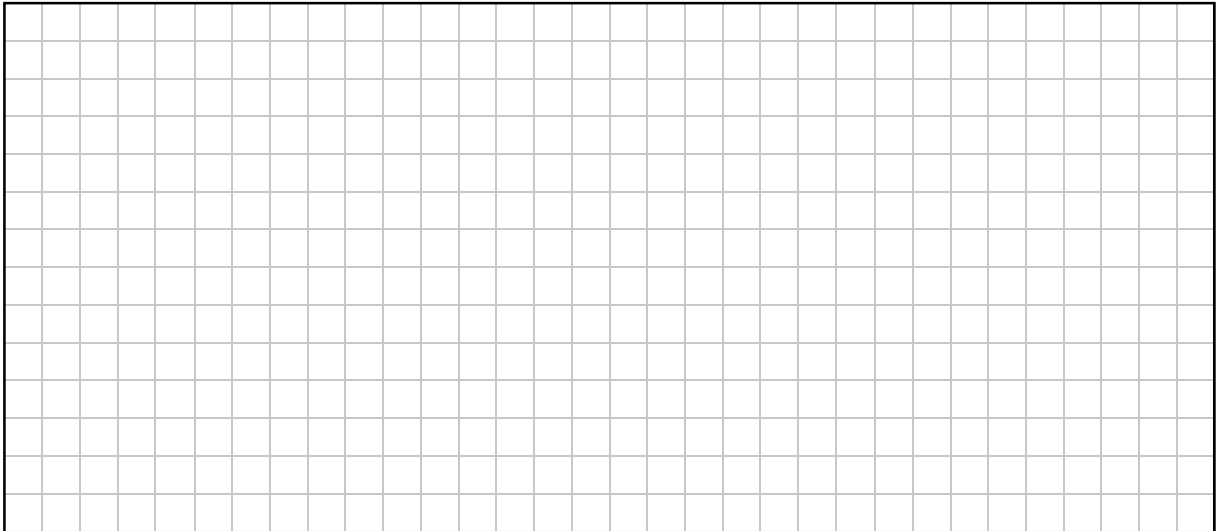
(e) Patrick has an annual income of €83 000 and pays a total of €3496·18 in USC.
Find the top rate of USC.

Question 8 (50 marks)

A square sheet of cardboard, of side 10 units, is used to make an open box.
Squares of side x units, where $x \in \mathbb{R}$, are removed from each corner of the cardboard and it is then folded along the dotted lines, as shown in the diagram below, in order to create the box.
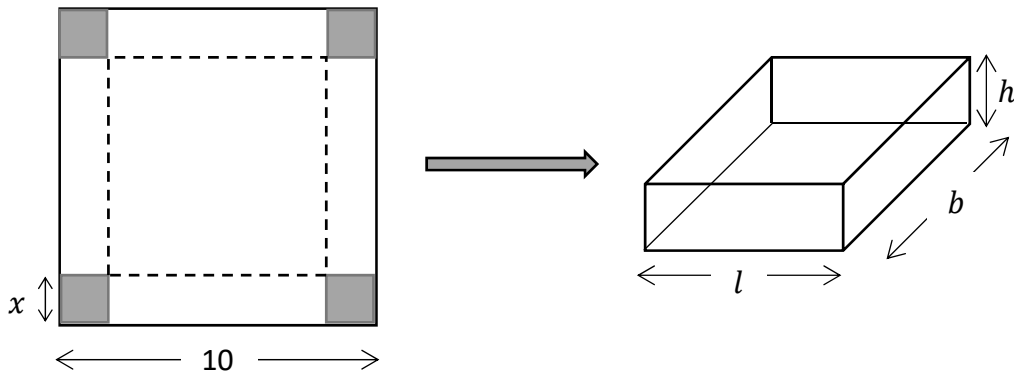
(a) The length (l), breadth (b) and height (h) of the box are shown in the diagram above.
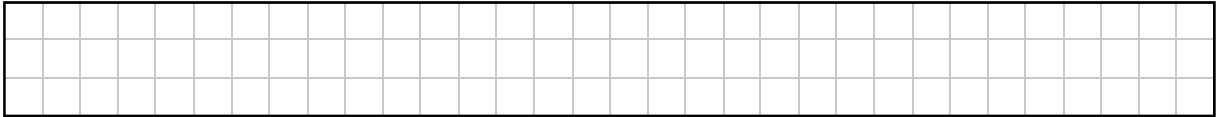Write l, b, and h in terms of x.

$$l =$$

$$b =$$

$$h =$$

(b) Show that the volume of the box can be written as
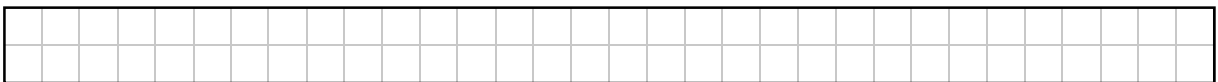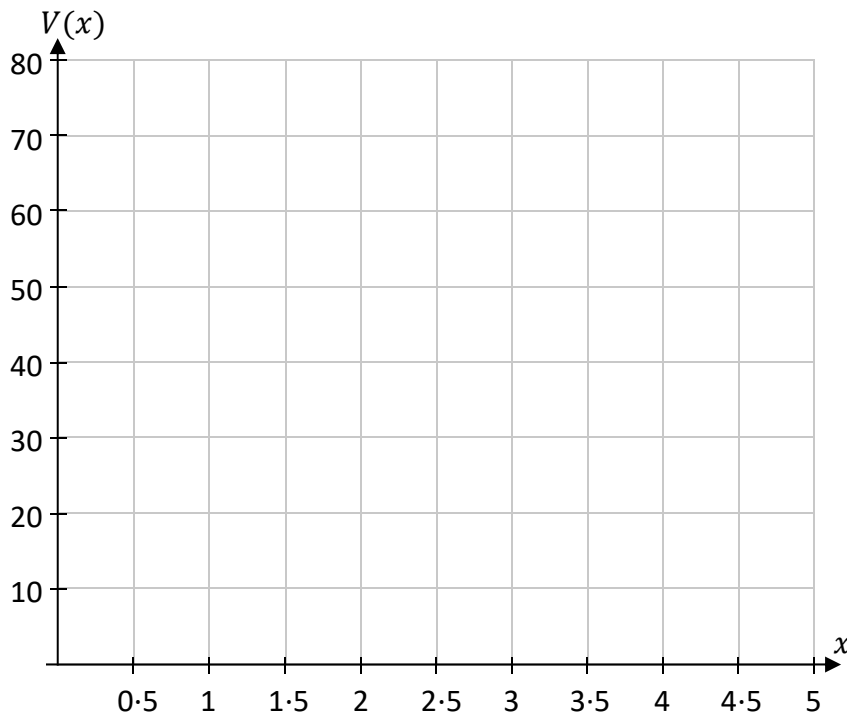
$$V(x) = 4x^3 - 40x^2 + 100x.$$

(c) Explain why a box of height 6 units cannot be made from the sheet of cardboard.

(d) Complete the table below to show the values of $V(x) = 4x^3 - 40x^2 + 100x$, where $x \in \mathbb{R}$, for the given values of x in the domain $0 \leq x \leq 5$.

x	0	0·5	1	1·5	2	2·5	3	3·5	4	4·5	5
$V(x)$		40·5								4·5	

(e) Draw the graph of the function $V(x)$ on the grid below.

(f) Use your graph to estimate each of the following values.
In each case show your work on the graph above.
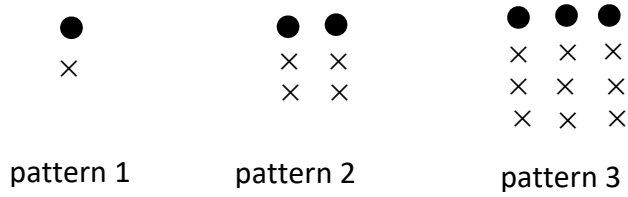
(i) The maximum volume of the box.

(ii) The values of x which will create a box which has a volume of 30 units cubed.
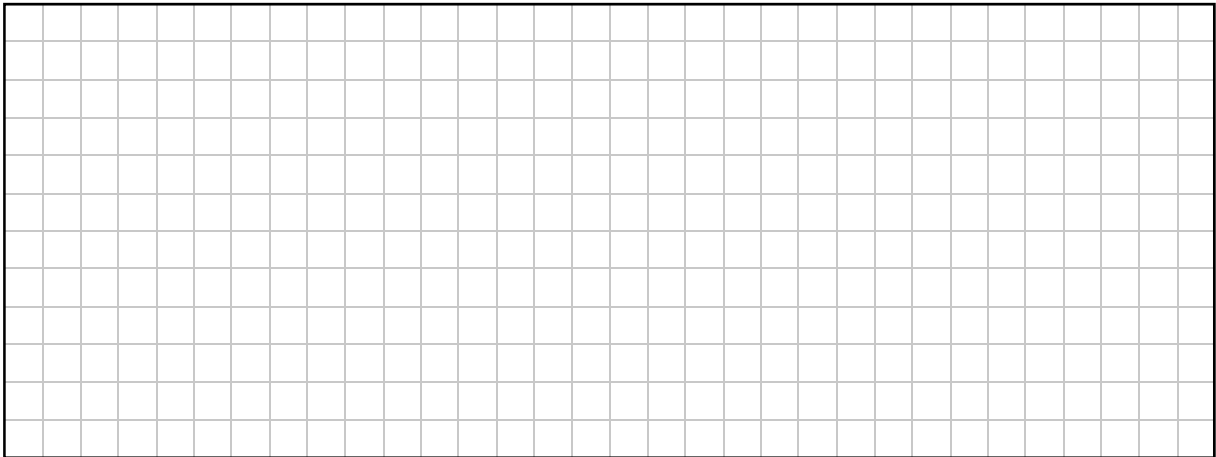
(iii) The volume of the box when x is 2·8 units.
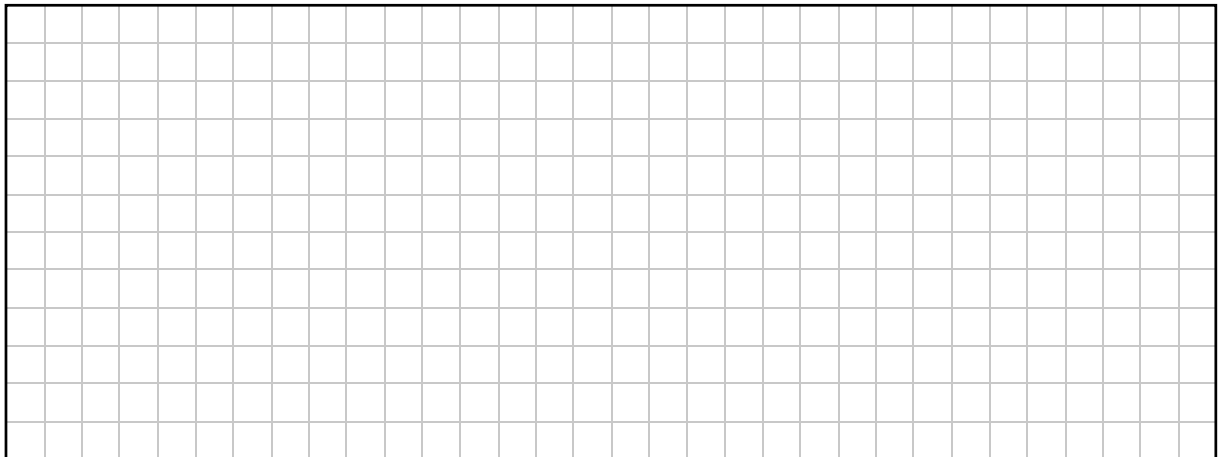
Question 9 (50 marks)

(a) The first three patterns in a sequence of patterns containing dots and crosses are shown below.

●
×

pattern 1

● ●
× ×
× ×

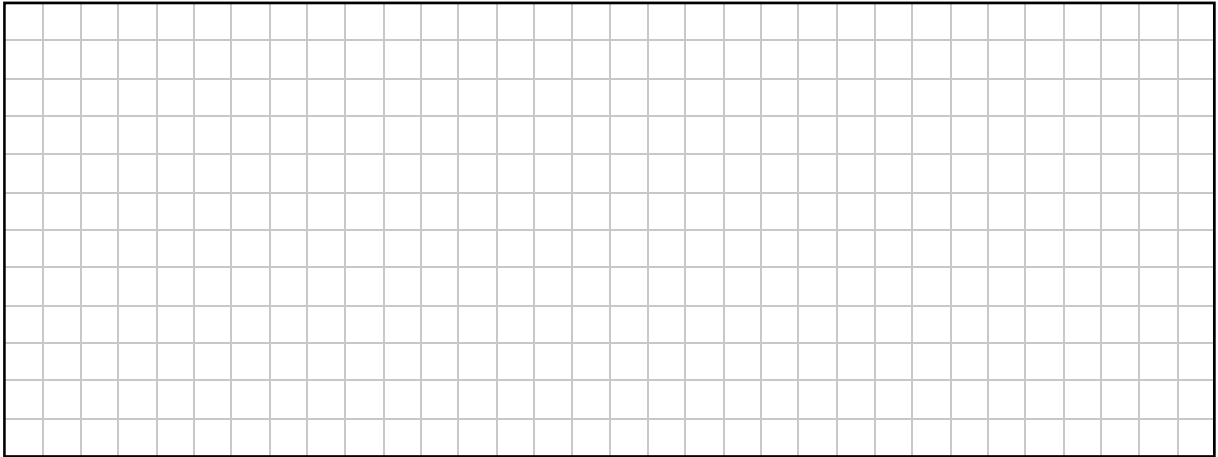pattern 2

● ● ●
× × ×
× × ×
× × ×

pattern 3

(i) Draw the fourth pattern in the sequence into the box below.

(ii) Find a formula, in n, for the number of **dots** in pattern n of the sequence (T_n).
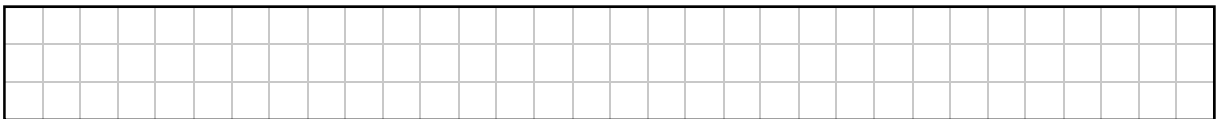
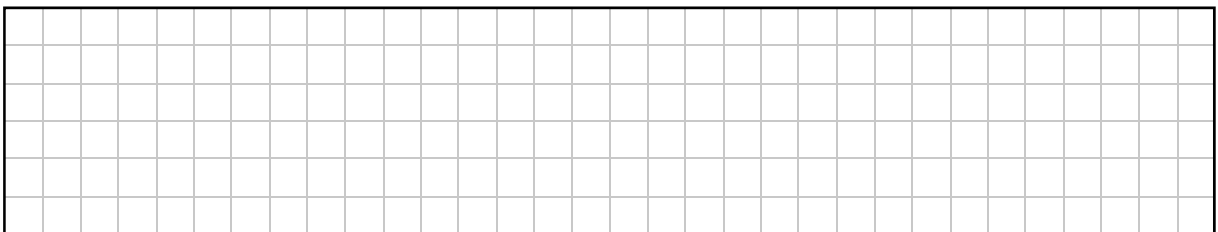**(iii)** Find the total number of **dots** in the first 20 patterns of the sequence.

(iv) The table shows the number of **crosses** for the first two patterns. Complete the table, and hence find a formula for the number of crosses in pattern n of the sequence.

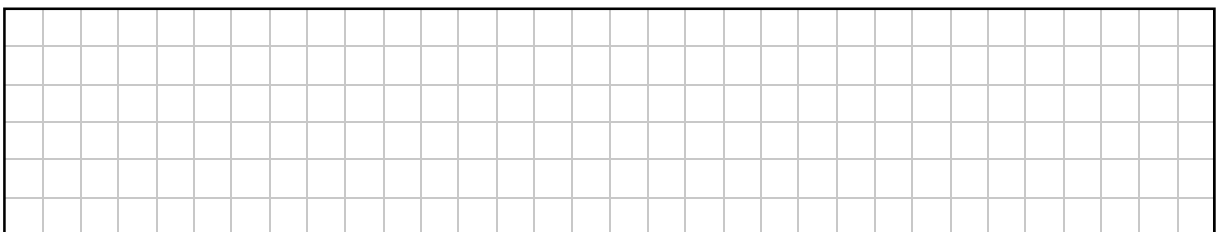| Pattern	1	2	3	4	5	6
Number of crosses	1	4				

Number of crosses in pattern $n = $ _____

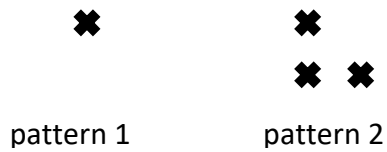**(v)** Find the number of **crosses** in pattern 12 of the sequence.

(vi) Find the number of shapes (**dots and crosses**) in pattern 10 of the sequence.

This question continues on the next page

(b) The first two patterns in a sequence of patterns of crosses are shown below.

✖

✖
✖ ✖

pattern 1   pattern 2

The number of crosses in pattern n is T_n.
The general term describing T_n can be written in the form:

$$T_n = \frac{n^2}{2} + bn + c, \text{ where } b, c \in \mathbb{R}.$$

(i) Use substitution for n to write T_1 and T_2 in terms of b, c, and a number.

$T_1 =$

$T_2 =$

(ii) Hence, or otherwise, find the value of b and the value of c.

Question 10 (50 marks)

(a) The water behind a dam is normally released at a rate of 250 000 litres per second.

(i) Find how long it takes to release 1 million cubic metres ($1\,000\,000$ m³) of water.
**Note** 1 m³ = 1000 litres.
Give your answer correct to the nearest minute.

(ii) Due to heavy rainfall, the operators of the dam decide to increase the flow by 10% for 24 hours. Find how many m³ of water were released in that 24 hour period.
Give your answer in the form $a \times 10^n$, where $1 \leq a < 10$, and $n \in \mathbb{N}$.
Give the value of a correct to 3 significant figures.

This question continues on the next page

(b) **(i)** John walks around a circular trail of radius 0·5 km at a steady speed of 6 km/h.
How long will it take him to complete 3 full circuits of the trail?
Give your answer correct to the nearest minute.
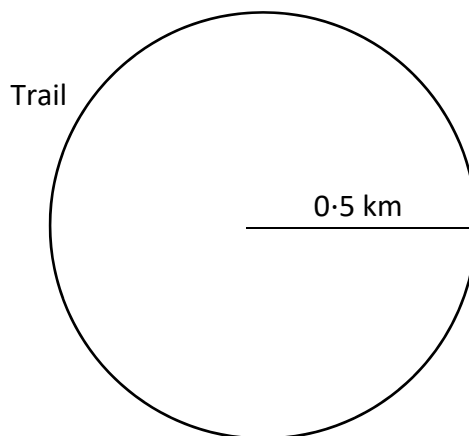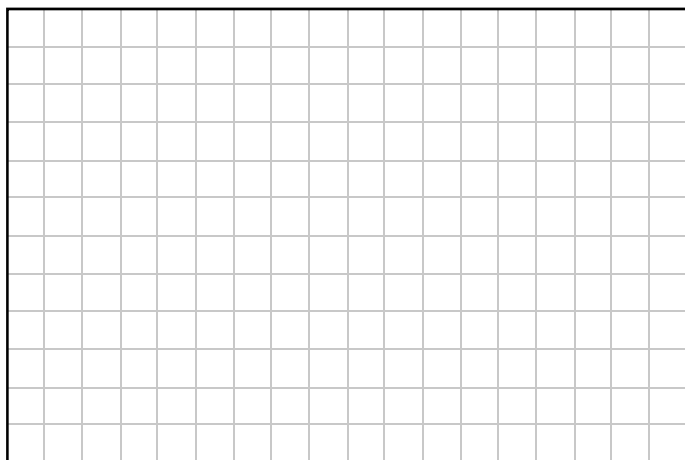
(ii) Mary decides to walk every day over a 5 day period.
She walks a distance of 3 km on day one.
She increases the length of her walk by 15% each day for the next four days.
Her average speed on day 5 is 4 km/h.
Find how long it will take her to complete her walk on day 5.
Give your answer in minutes, correct to the nearest minute.
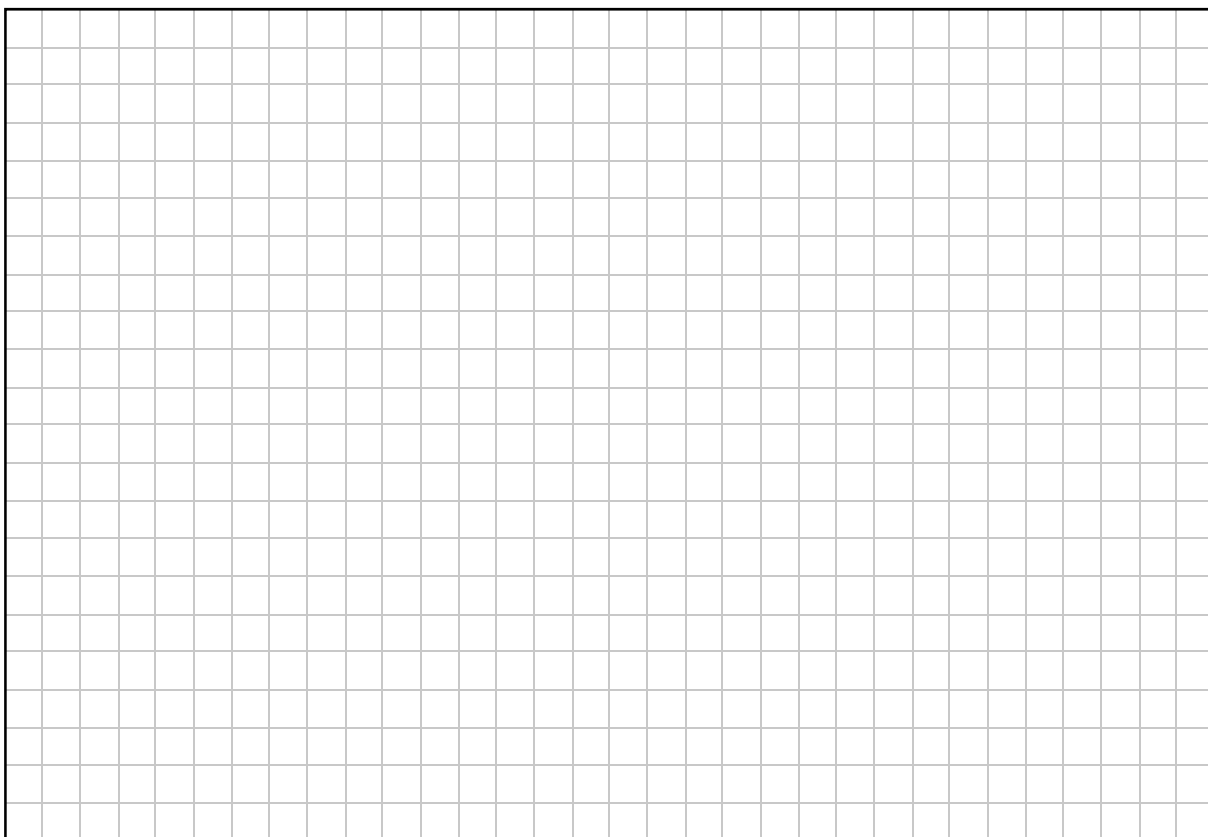
(iii) One day, during John's walk he meets Mary at point P on the trail.
Mary is walking in the opposite direction at a steady speed of 4 km/h.
John continues walking at 6 km/h.
How far will he travel until he meets Mary again?
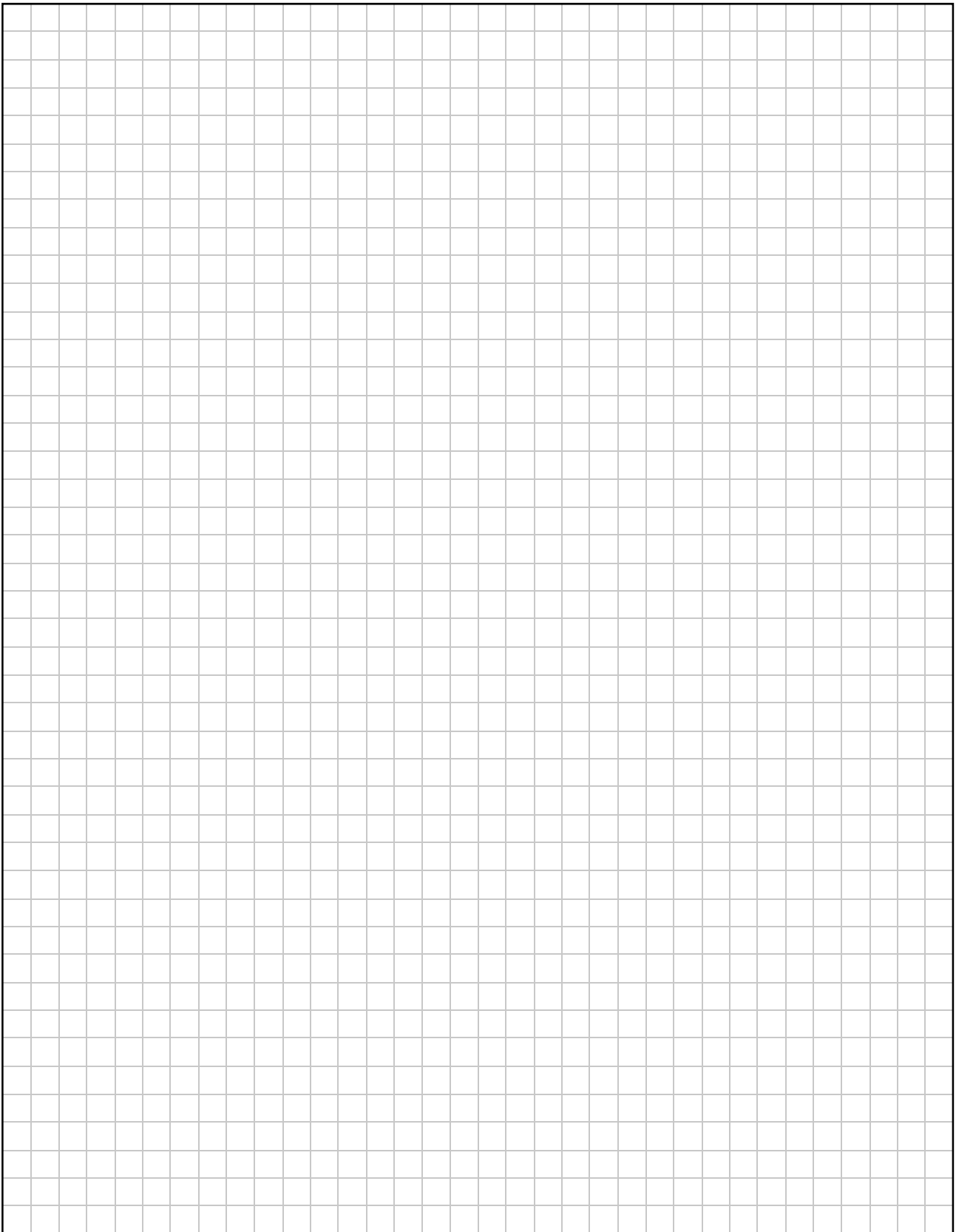Give your answer correct to the nearest metre.
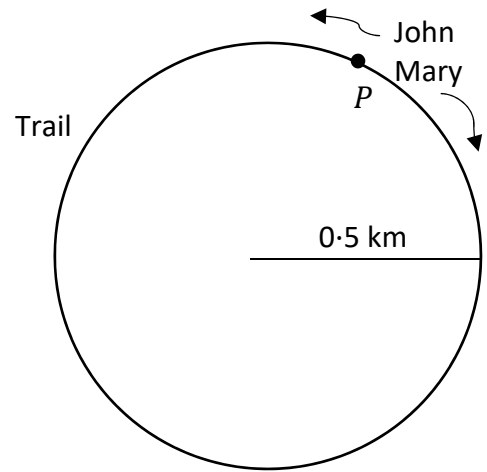
John

Mary

P

Trail

0·5 km

Page for extra work.
Label any extra work clearly with the question number and part.
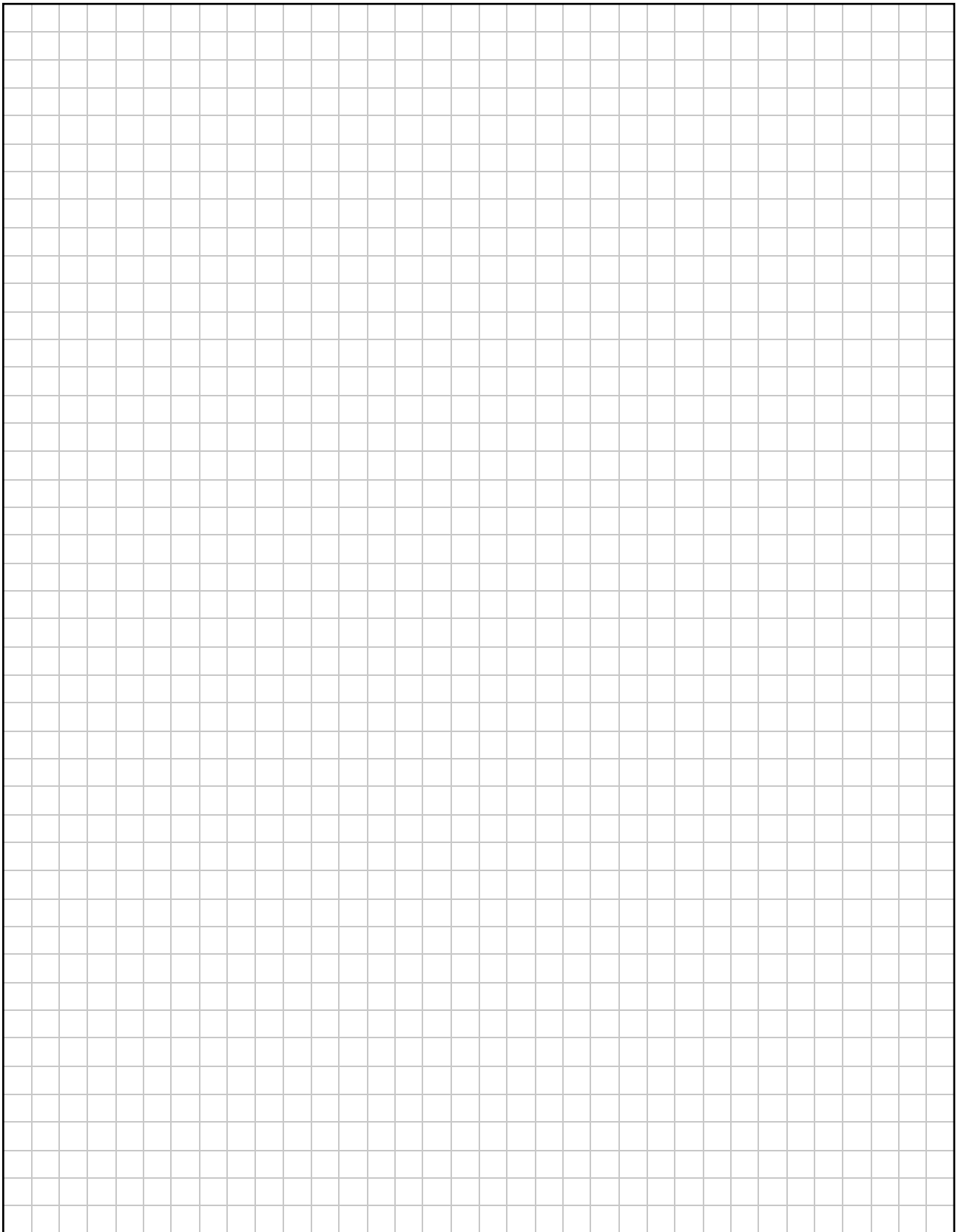
Page for extra work.
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