



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Cycle Final Examination 2023

Mathematics

Higher Level

Friday 9 June Afternoon 1:30 - 3:30

270 marks

Examination Number

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Day and Month of Birth

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For example, 3rd February is
entered as 0302

Centre Stamp

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Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1

(Suggested maximum time: 5 minutes)

Xena's car has a screen which shows the temperature in degrees Celsius, correct to the nearest whole number.

When Xena was going to work one day, the temperature on the screen was 18°C .

- (a) Write down **three** possible values that the **actual** temperature could have been when Xena was going to work.



| | | |
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Let t represent the **actual** temperature at a given time.

- (b) Fill in the missing values in the inequality in t below to show the range of possible values of t when Xena was going to work.

| | | |
|--|------------|--|
| | $\leq t <$ | |
|--|------------|--|

- (c) If you were to graph the **actual** temperature, t , over the course of a day, would it be more appropriate to graph $t \in \mathbb{N}$, $t \in \mathbb{Z}$, or $t \in \mathbb{R}$? Give a reason for your answer.

Answer:

 $t \in \mathbb{N}$ $t \in \mathbb{Z}$ $t \in \mathbb{R}$ (Tick (\checkmark) **one** box only)

Reason:

| |
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Question 2

(Suggested maximum time: 10 minutes)

A company sells t-shirts on a website.

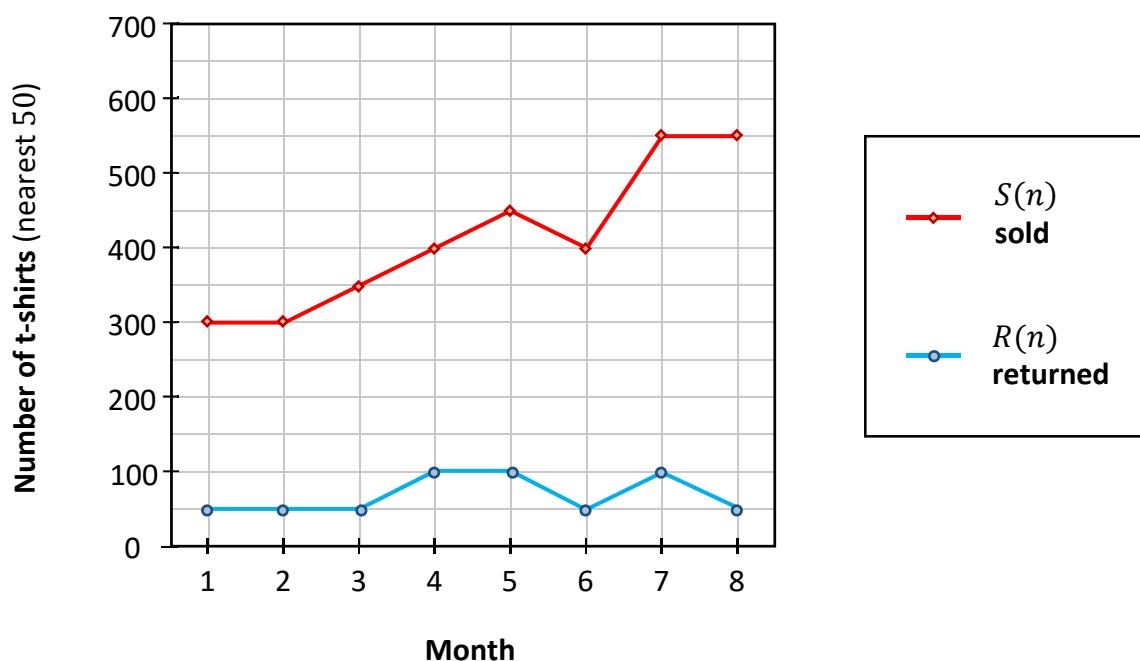
Sometimes, when people buy a t-shirt, they return it again to get their money back.

The diagram below shows the following information for an 8-month period:

- $S(n)$, the number of t-shirts **sold** on the website in month n , and
- $R(n)$, the number of t-shirts sold in month n that were **returned**,



for $n \in \mathbb{N}$ and $n \leq 8$. All values are rounded to the nearest 50.



Use the information in the diagram to answer the following questions.

- (a) (i) How many t-shirts were **sold** in Month 1?

Answer =

- (ii) How many t-shirts were **returned** in Month 2?

Answer =

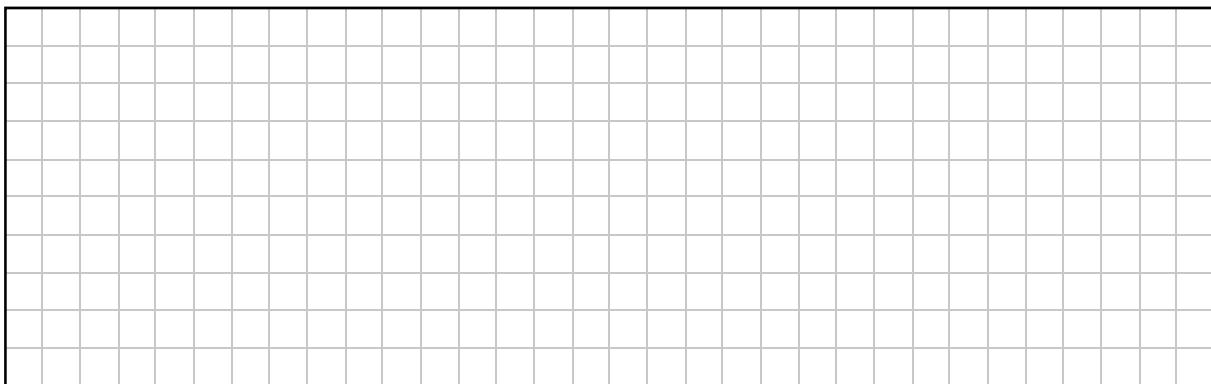
- (b) Which month had the **greatest increase** in the number of t-shirts **sold**, compared to the previous month?

Answer =

- (c) Complete the table below to show the number of t-shirts that were **not returned** each month. For example, 350 t-shirts were not returned in Month 6.

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------|---|---|---|---|---|-----|---|---|
| Number not returned | | | | | | 350 | | |

- (d) On the given diagram, **draw** a graph to show the number of t-shirts that were **not returned** each month for the 8 months. Use the same axes and scales.



- (e) Which month had the greater **percentage** increase in the number of t-shirts **sold**, compared to the previous month: Month 3 or Month 5? Justify your answer.

Month with the greater percentage increase:

(Tick (\checkmark) **one** box only)

Month 3

Month 5

Justification:

Question 3

(Suggested maximum time: 10 minutes)

The students in a Transition Year mini-company make and sell candles.

They buy a solid rectangular block of wax measuring 35 cm by 45 cm by 16 cm.



- (a) Work out the **volume** of the block of wax.

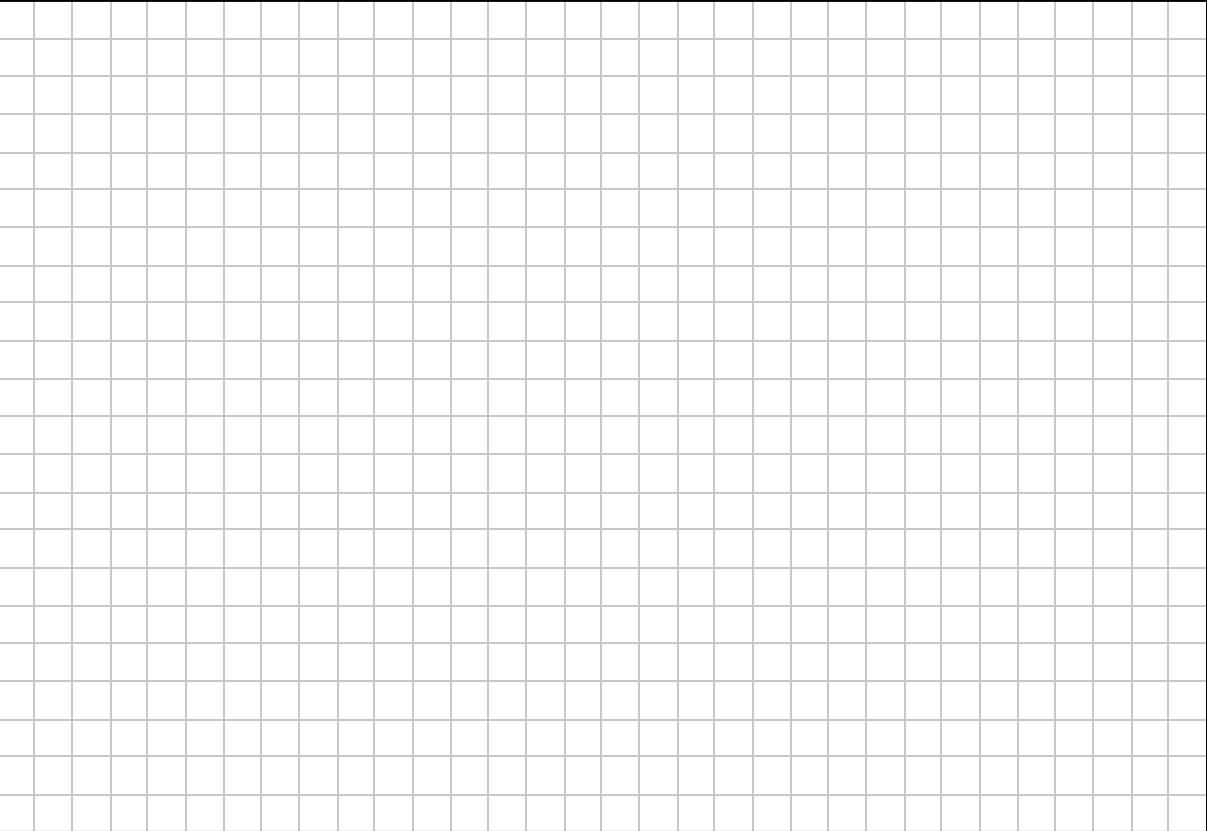
- (b)** The students are going to use this wax to make candles in the shape of cylinders.

Each candle will have a radius of r cm and a height of 9 cm.
Work out the **volume** of each candle, in terms of r and π .



- (c) The students make 100 candles from the block of wax.
In making the candles, 10% of the wax in the block is wasted.

Use this information to work out the value of r , the **radius** of each candle.
Give your answer correct to 1 decimal place.

A large rectangular grid consisting of 20 columns and 25 rows of small squares, intended for working out the value of r .

Question 4**(Suggested maximum time: 10 minutes)**

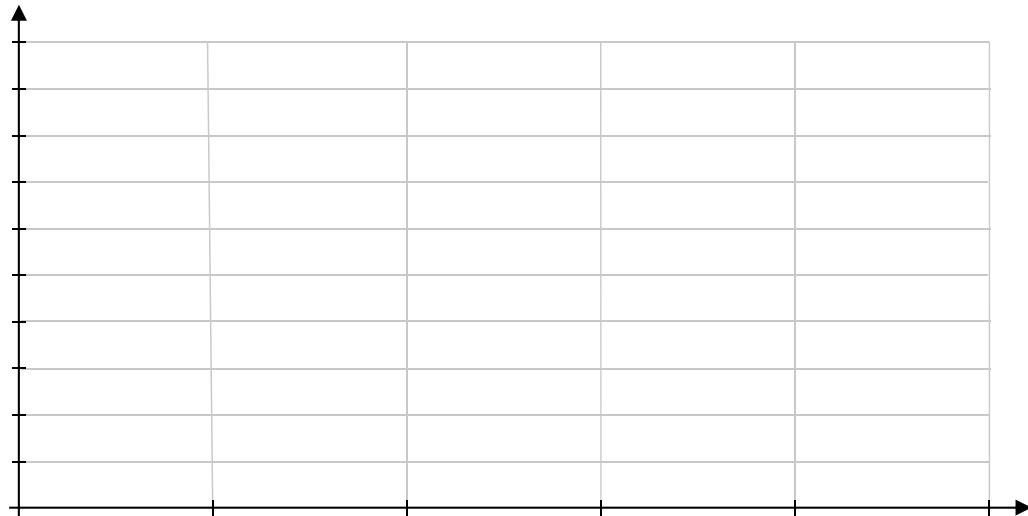
The ages of children in a crèche were recorded.

The data are shown in the table below.

| Age (years) | 0 – 1 | 1 – 2 | 2 – 3 | 3 – 4 | 4 – 5 |
|--------------------|-------|-------|-------|-------|-------|
| Number of children | 6 | 10 | 19 | 3 | 5 |

[Note: 2 – 3 means “2 years or more, but less than 3 years”, etc.]

- (a) Represent the data on a **histogram**. Label the axes clearly.



- (b) Work out the **percentage** of children in the crèche who are aged 1 – 2 years.
Give your answer correct to the nearest percent.

A large rectangular grid for working out calculations. It consists of 10 columns and 10 rows of small squares, totaling 100 squares.

- (c) Use mid-interval values to estimate the **mean** age of the children in the crèche. Give your answer in years, correct to 1 decimal place.

- (d) The table below shows the **maximum** number of children that **one** staff member in the crèche can mind, depending on the age of the children. So, for example, one staff member can mind up to 3 children aged 0 – 1 years, **or** up to 5 children aged 1 – 2 years, and so on.

| Age (years) | Maximum number of children that one staff member can mind |
|--------------------|--|
| 0 – 1 | 3 |
| 1 – 2 | 5 |
| 2 – 3 | 6 |
| 3 – 6 | 8 |

Work out the **least** number of staff members that are needed to mind all of the children in the crèche.

Question 5

(Suggested maximum time: 5 minutes)

- (a) The table below shows the outcome when the numbers a and b are multiplied together, for $a, b \in \mathbb{N}$.

Write “odd” or “even” in each box to complete each of the mathematical statements correctly. The first one is done for you.

| | <i>a</i> | \times | <i>b</i> | = | <i>c</i> |
|----|----------|----------|----------|---|----------|
| 1. | even | \times | even | = | even |
| 2. | odd | \times | odd | = | |
| 3. | odd | \times | even | = | |

Ruairí rolls a regular, unbiased, six-sided die three times.

He writes down the number on each roll of the die.

- (b)** Complete the sample space below, showing the different possible combinations of odd numbers (**O**) and even numbers (**E**) that Ruairí could roll. Two possible outcomes have been filled in for you.



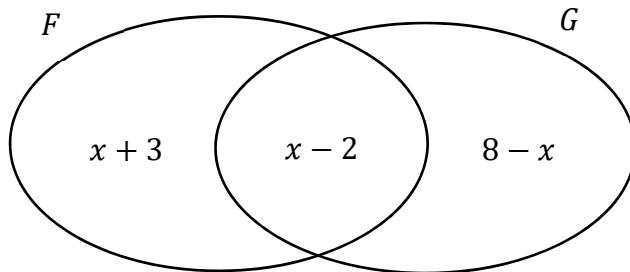
| | | | |
|-----|-----|--|--|
| EEE | EEO | | |
| | | | |

- (c) Each outcome in the table is equally likely.
Ruairí **multiplies** together the three numbers that he rolls.
Find the **probability** that Ruairí will get an **even number** as his answer.

Question 6

(Suggested maximum time: 5 minutes)

The Venn diagram below shows the number of elements in three regions in the sets F and G , where $x \in \mathbb{N}$.



- (a) Write down the value of $\#(F \setminus G)$, in terms of x .

Answer =

- (b) From the Venn diagram, find the **greatest** possible value of x . Justify your answer.

| | |
|----------------|--|
| Answer: | |
| Justification: | |

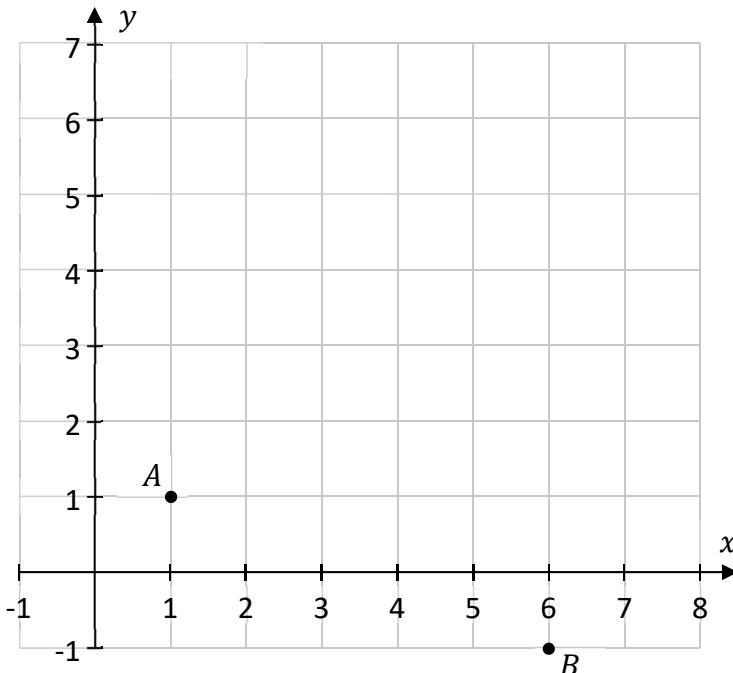
Question 7

(Suggested maximum time: 10 minutes)

A, B, C, and D are four points in the co-ordinate plane.

- (a) (i) The points *A* and *B* are shown on the co-ordinate diagram below.

Plot and label the points *C*(8, 4) and *D*(3, 6) on the same co-ordinate diagram.



- (ii) Write the co-ordinates of the points *A* and *B* in the spaces below.

The co-ordinates of the points *C* and *D* are already given.

$$A = \boxed{(\quad, \quad)}$$

$$B = \boxed{(\quad, \quad)}$$

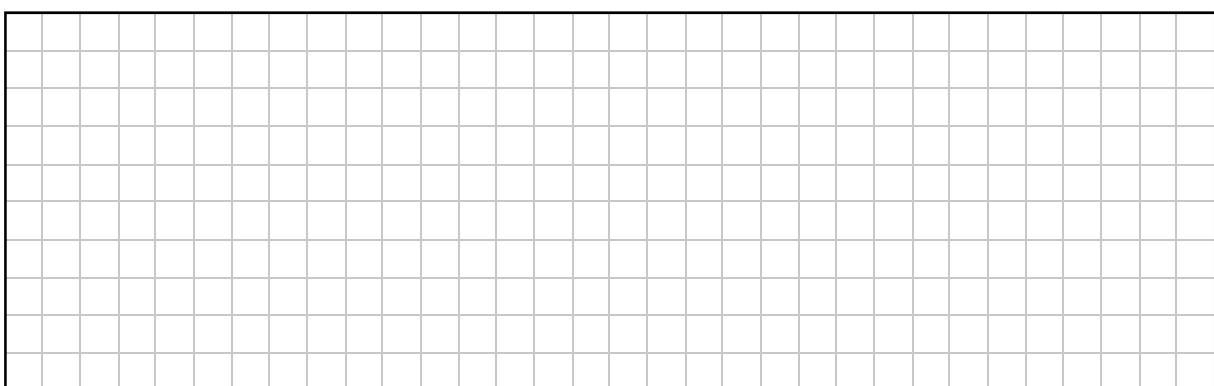
$$C = (8, 4)$$

$$D = (3, 6)$$

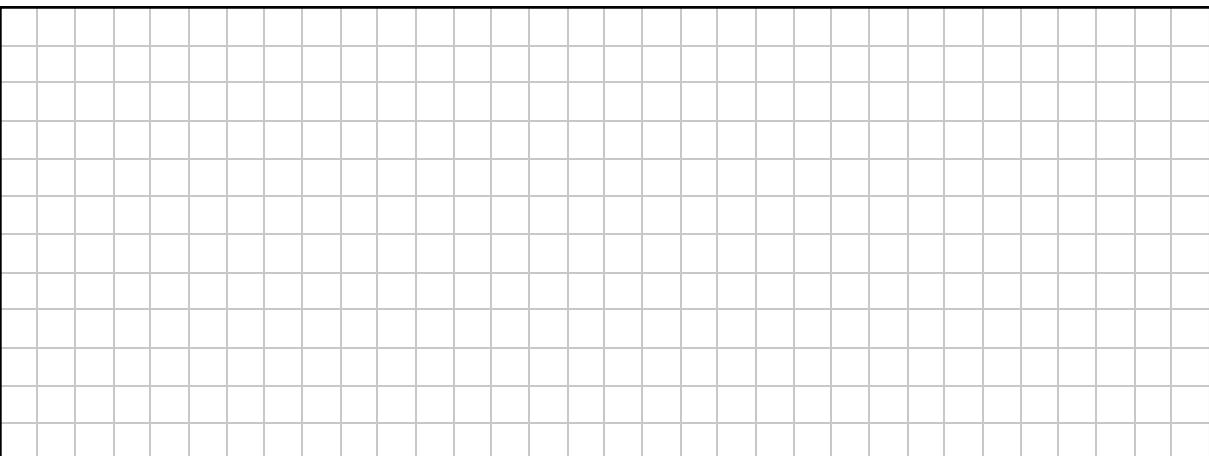
- (b) Paulina wants to prove that *ABCD* is actually a **square**.

Answer (b)(i) and (b)(ii) below to show that *ABCD* is a square.

- (i) Show that $|AB| = |BC|$.



- (ii) Without measuring the diagram, show that AB is **perpendicular** to BC .



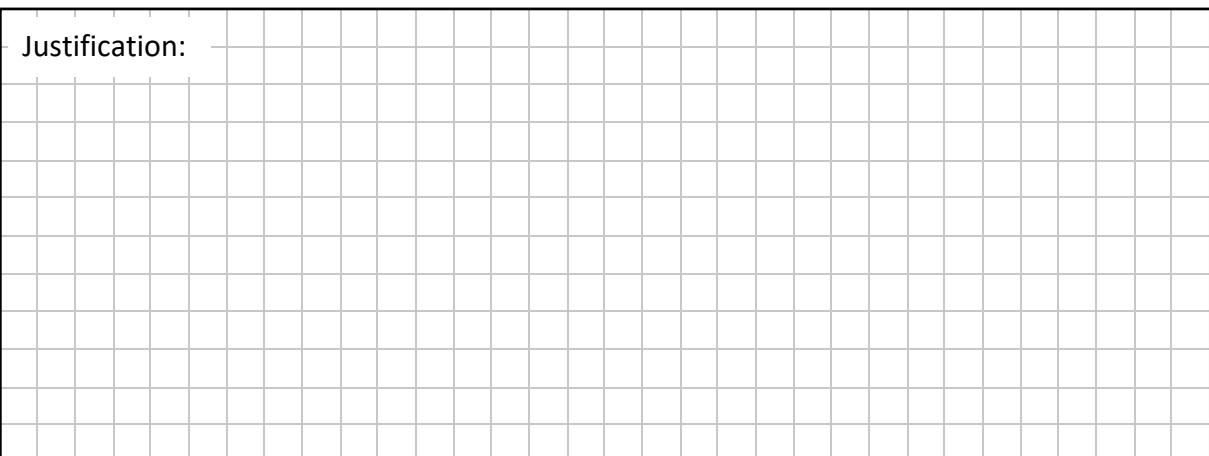
- (c) Ciarán makes the following statement:

Ciarán's statement: "Every parallelogram is a square."

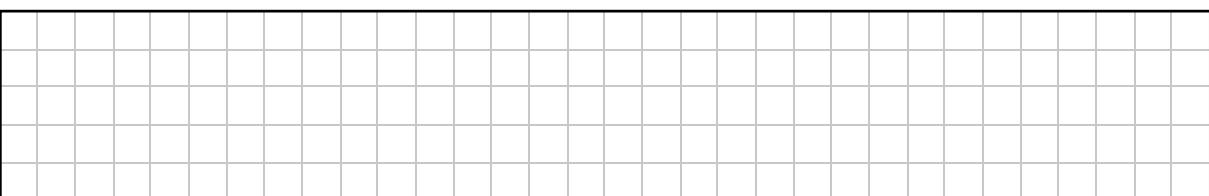
- (i) Is Ciarán's statement true or false? Justify your answer.

Ciarán's statement is: true false
(Tick (\checkmark) **one** box only)

Justification:



- (ii) Write down the **converse** of Ciarán's statement.



Question 8

(Suggested maximum time: 10 minutes)

- (a) Convert 7.5 kilometres to metres.

$$7.5 \text{ km} = \boxed{} \text{ m}$$

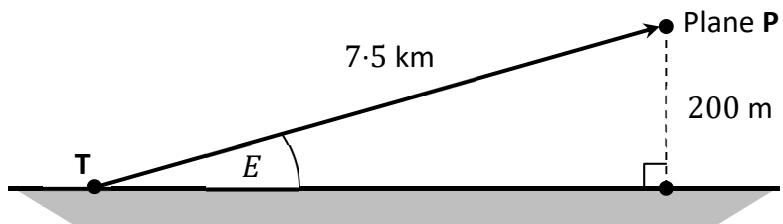
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- (b) A plane, P, took off from an airport.
It left the ground at the point T.



After a few minutes, it was 7.5 km from T, and it was 200 m directly above level ground, as shown in the diagram below (not to scale).

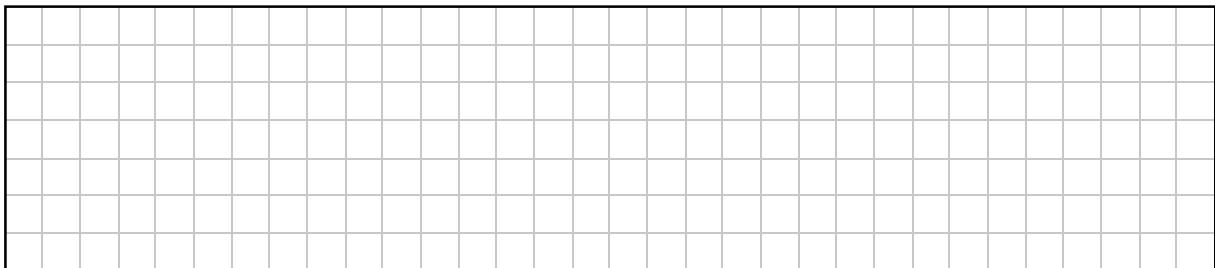
Use trigonometry to work out the size of the angle marked E in the diagram.
Give your answer in degrees, correct to 1 decimal place.



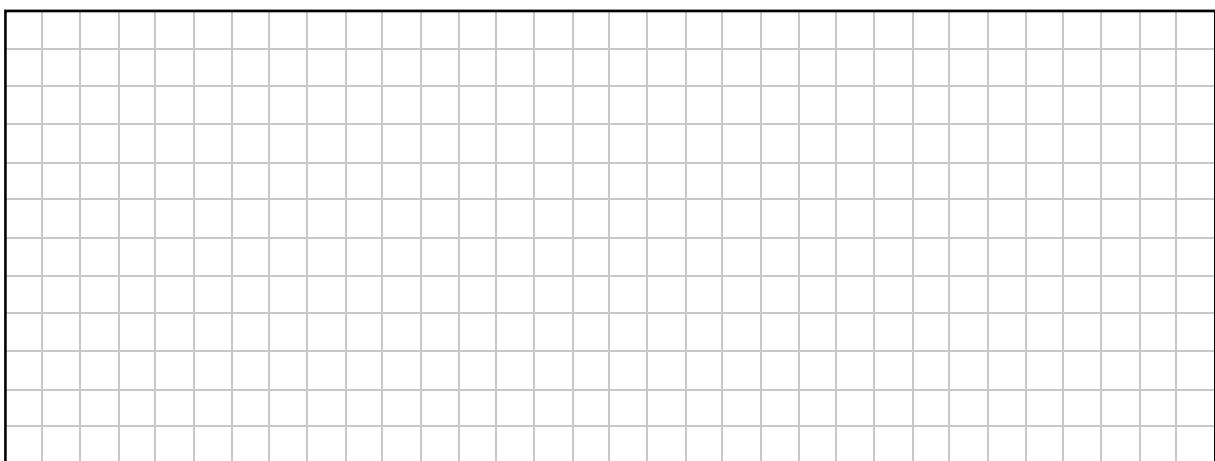
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(c) (i) Write 20 minutes as a fraction of an **hour**. Give your answer in simplest form.

$$20 \text{ minutes} = \frac{\boxed{}}{\boxed{}} \text{ hour}$$

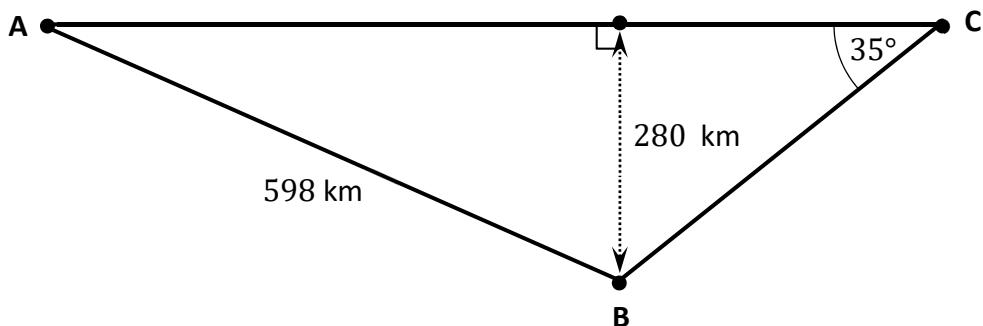
A large rectangular grid consisting of 20 columns and 10 rows of small squares, intended for working space.

(ii) During part of its flight, plane P travelled 598 km in two hours and twenty minutes.
Work out the average **speed** of plane P during that time, in km per hour.
Give your answer correct to 1 decimal place.

A large rectangular grid consisting of 20 columns and 10 rows of small squares, intended for working space.

This question continues on the next page

- (d) In the diagram below, each of the points **A**, **B**, and **C** represents an airport.
The plane travelled 598 km from Airport **A** to Airport **B**, and then travelled on to Airport **C**.



Work out the distance $|AC|$, correct to the nearest km.

Question 9**(Suggested maximum time: 5 minutes)**

- (a) Multiply out and simplify $4x(5x + 4) - 3(x - 2)$.

- (b) Factorise fully $9 - 25y^2$.

- (c) Write $\frac{5^7 \times 25^3}{\sqrt{5}}$ in the form 5^p , where $p \in \mathbb{Q}$.

Question 10**(Suggested maximum time: 10 minutes)**

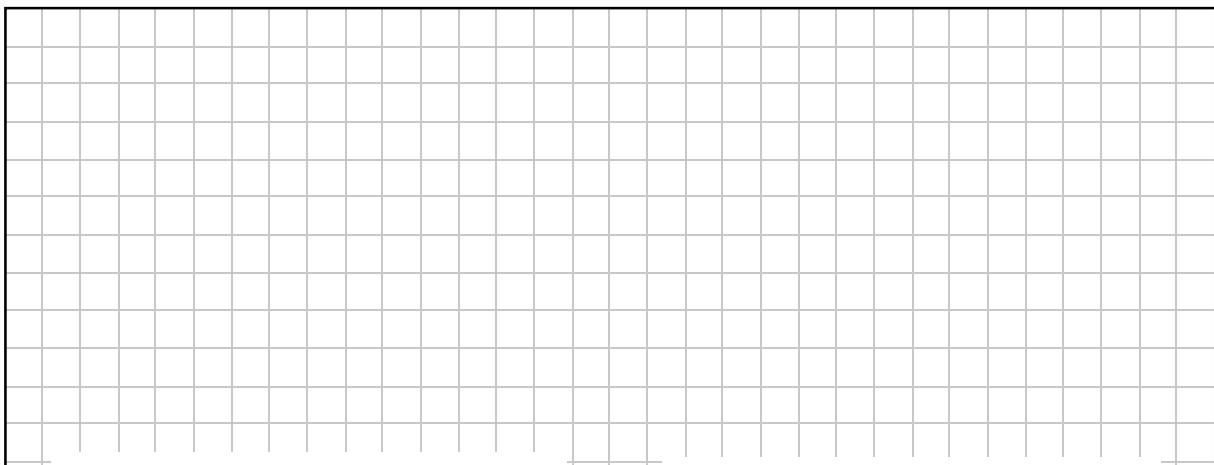
Michelle must pay Universal Social Charge (USC) on all of her gross income.

The income bands and rates of USC that Michelle must pay are given in the table below.

| Income Band | Rate of USC |
|---------------|-------------|
| Up to €12 012 | 1·5% |
| Next €5564 | 3·5% |
| Above €17 576 | 7% |

Michelle pays USC at all three rates.

- (a) Find the amount of USC that she must pay at each of the first two rates (1·5% and 3·5%).

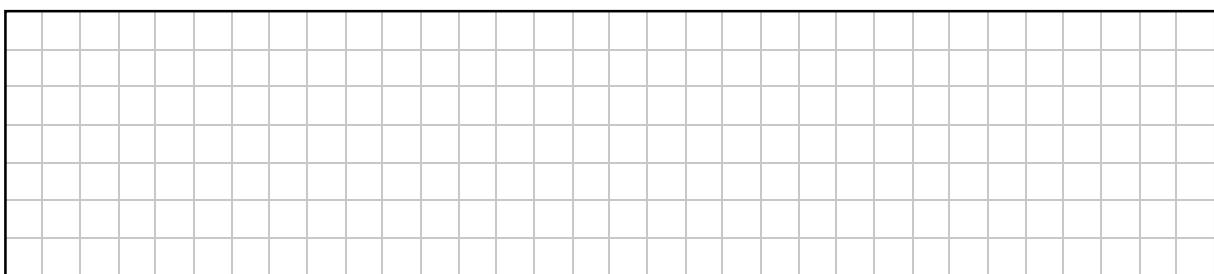


USC @ 1·5% = _____ USC @ 3·5% = _____

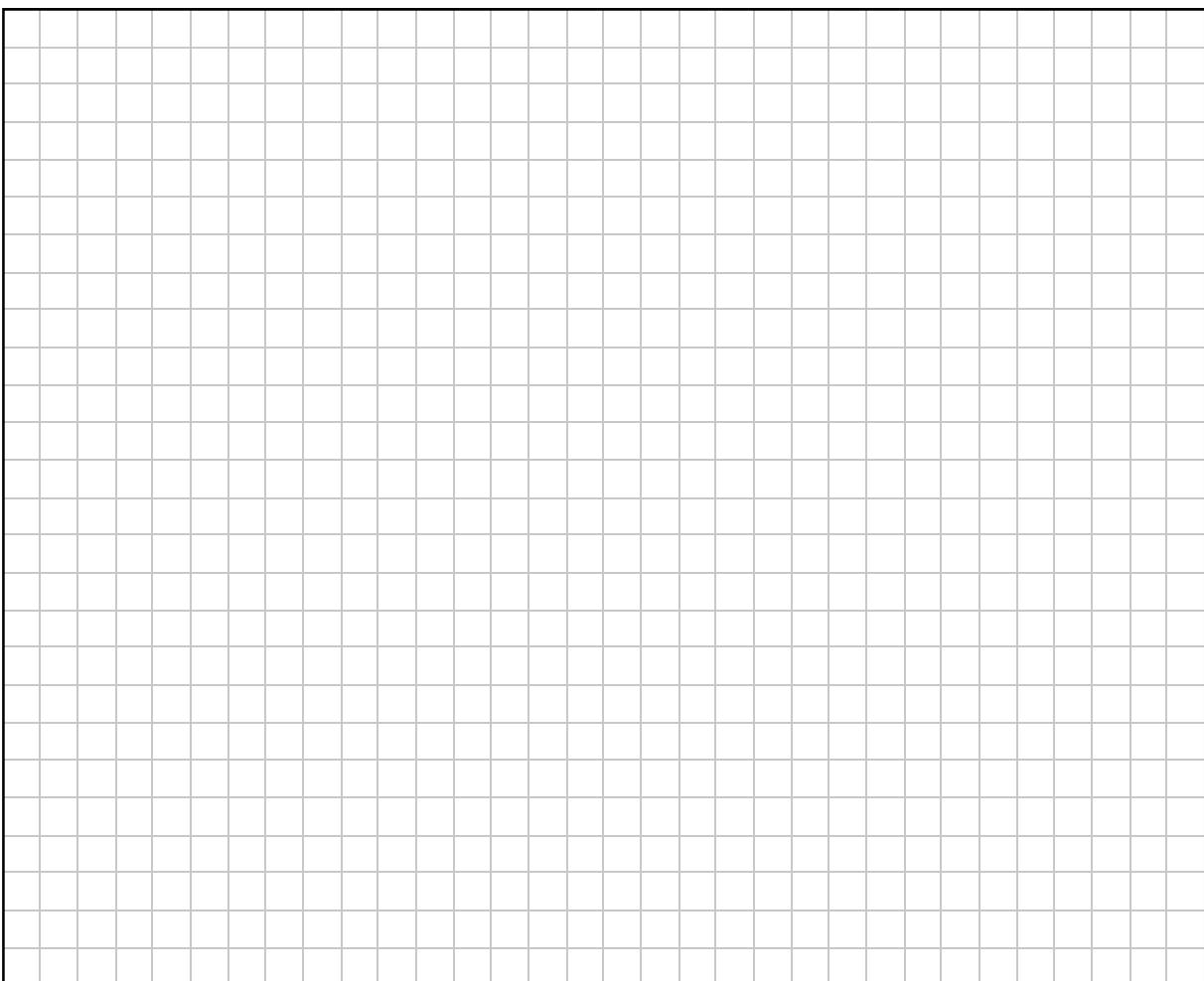
Michelle's gross income is x .

Michelle pays a total of 5% of her gross income in USC.

- (b) Write an expression, in terms of x , for the **total amount** of money that Michelle pays in USC.



- (c) Find the value of x , Michelle's gross income.

A large rectangular grid of squares, approximately 20 columns by 25 rows, enclosed in a black border. It is intended for students to show their working for part (c) of the question.

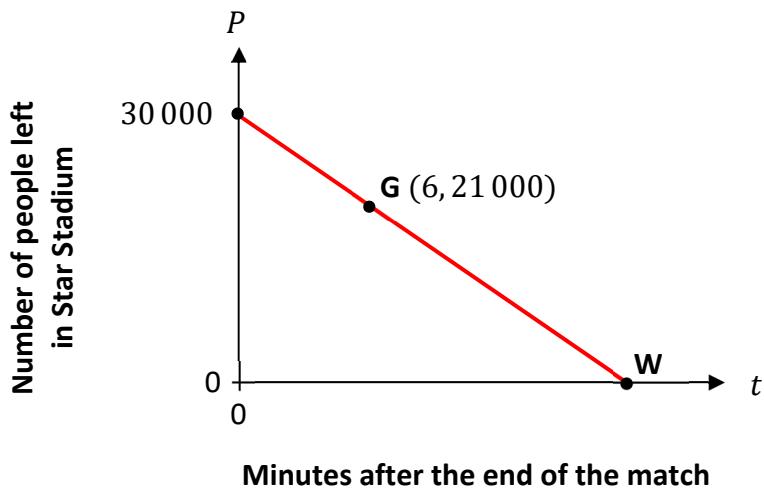
Question 11

(Suggested maximum time: 5 minutes)

30 000 people went to a football match in Star Stadium.

After the match, the 30 000 people left the stadium at a constant rate.

The graph below shows P , the number of people still in the stadium t minutes after the end of the match. Two points are marked on the graph: **G** and **W**.



- (a) The point **G** has co-ordinates (6, 21 000).
Explain what the 6 and the 21 000 mean, in terms of the number of people in the stadium.

- (b) It took 20 minutes for all of the 30 000 people to leave the stadium.
Use this fact to write down the co-ordinates of the point **W**, which is on the t -axis.

$$W = \boxed{(\quad, \quad)}$$

(c) The equation of the graph can be written as:

$$P = 30\,000 - 1500\,t$$

(i) How many people were in the stadium 12 minutes after the match had finished?

(ii) Explain what the 1500 in this equation means, in terms of the number of people in the stadium.

Question 12**(Suggested maximum time: 10 minutes)**

The diagram below shows a curved shape ABC .

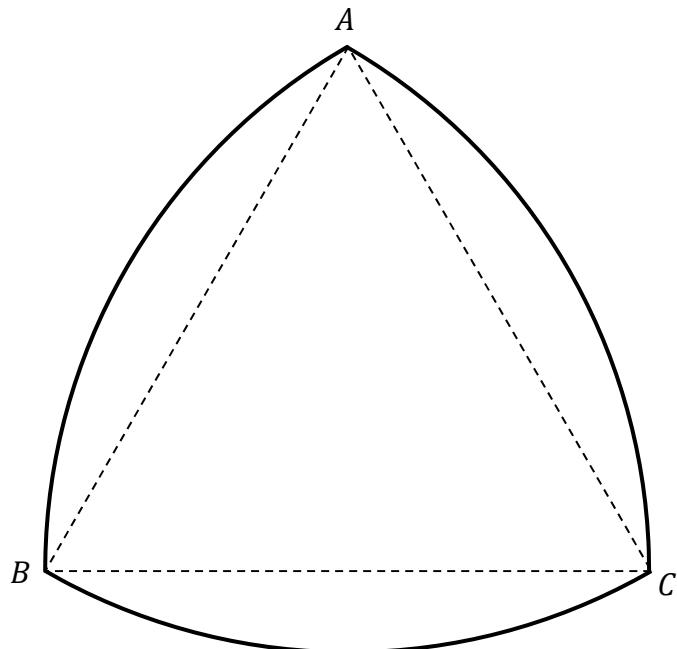
C is the centre of the circular arc from A to B .

A is the centre of the circular arc from B to C .

B is the centre of the circular arc from C to A .

The dotted triangle ABC is an equilateral triangle.

- (a) Using only a compass and straight edge, **construct one** of the axes of symmetry of the curved shape ABC . Show all of your construction lines clearly.



- (b) (i) Write down the size of the angle $\angle ACB$.

- (ii) The length of the straight-line segment $[BC]$ is 40 cm.

Find the length of the **circular** arc from B to C .

Give your answer in cm, correct to 2 decimal places.

Question 13

(Suggested maximum time: 10 minutes)

A company is making a large rectangular poster, as shown in the diagram on the right.

The top section will be a square image.

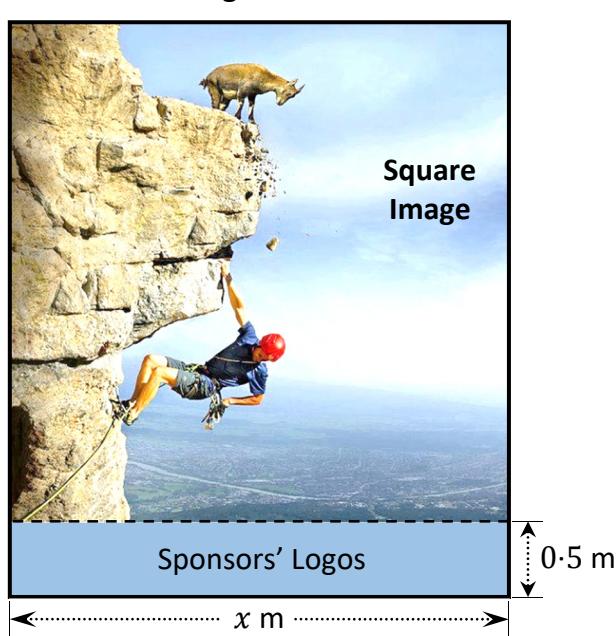
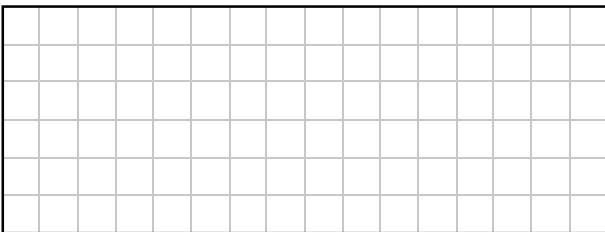
The bottom section will be a rectangle of height 0.5 m, which will have the sponsors' logos.

The width of the poster is x m, as shown.

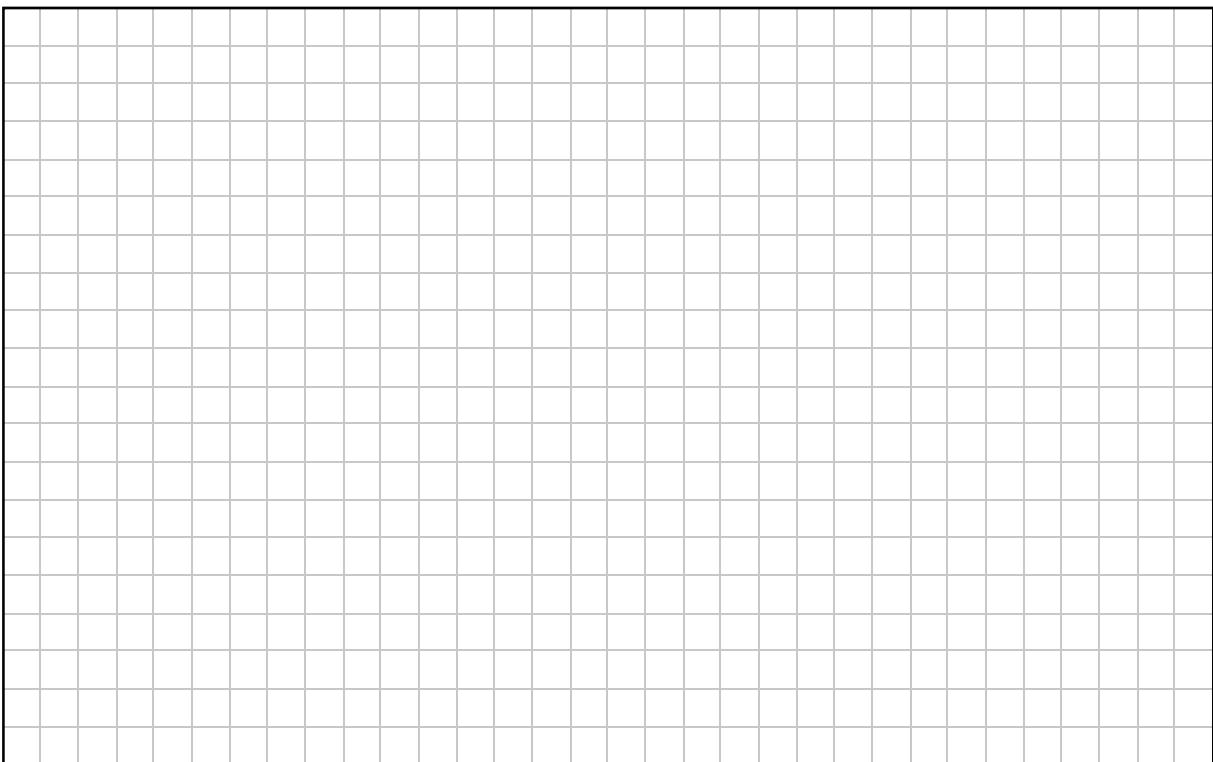
- (a) Write the **total height** of the poster, in terms of x .

Total height = m

- (b) The **total area** of the poster is 50 m². Show that $2x^2 + x - 100 = 0$.



- (c) Solve the equation $2x^2 + x - 100 = 0$ to find the value of x , the width of the poster. Give your answer in **metres**, correct to the nearest **centimetre**.



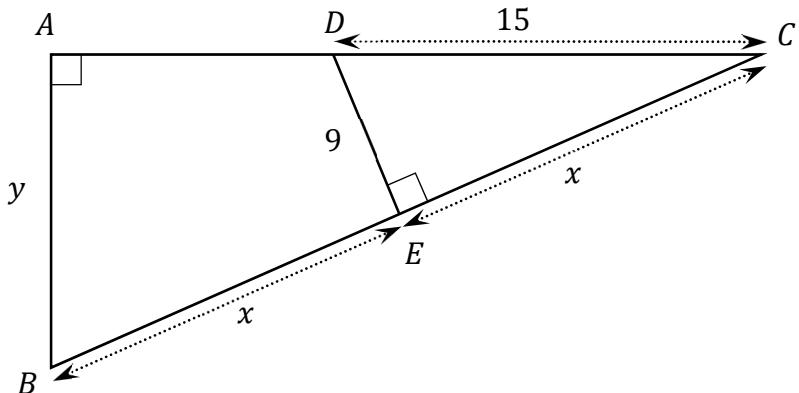
Question 14

(Suggested maximum time: 5 minutes)

The diagram below shows a triangle ABC (not to scale).

The points D and E lie on $[AC]$ and $[BC]$, respectively. $|\angle CAB| = |\angle DEC| = 90^\circ$.

The lengths of some of the sides are given in the diagram, with $x, y \in \mathbb{R}$.

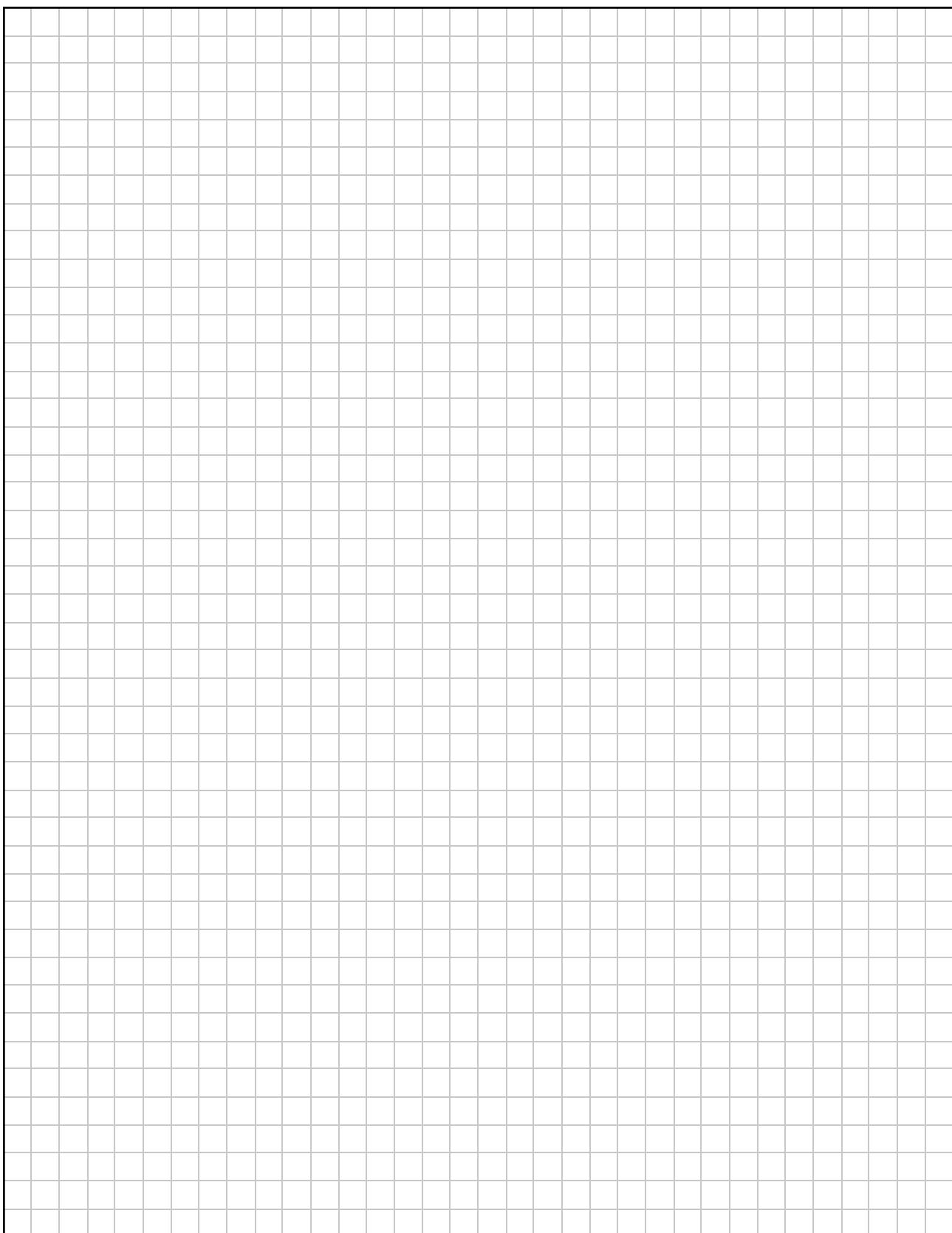


- (a) Work out the value of x .

- (b) Using similar triangles, work out the value of y .

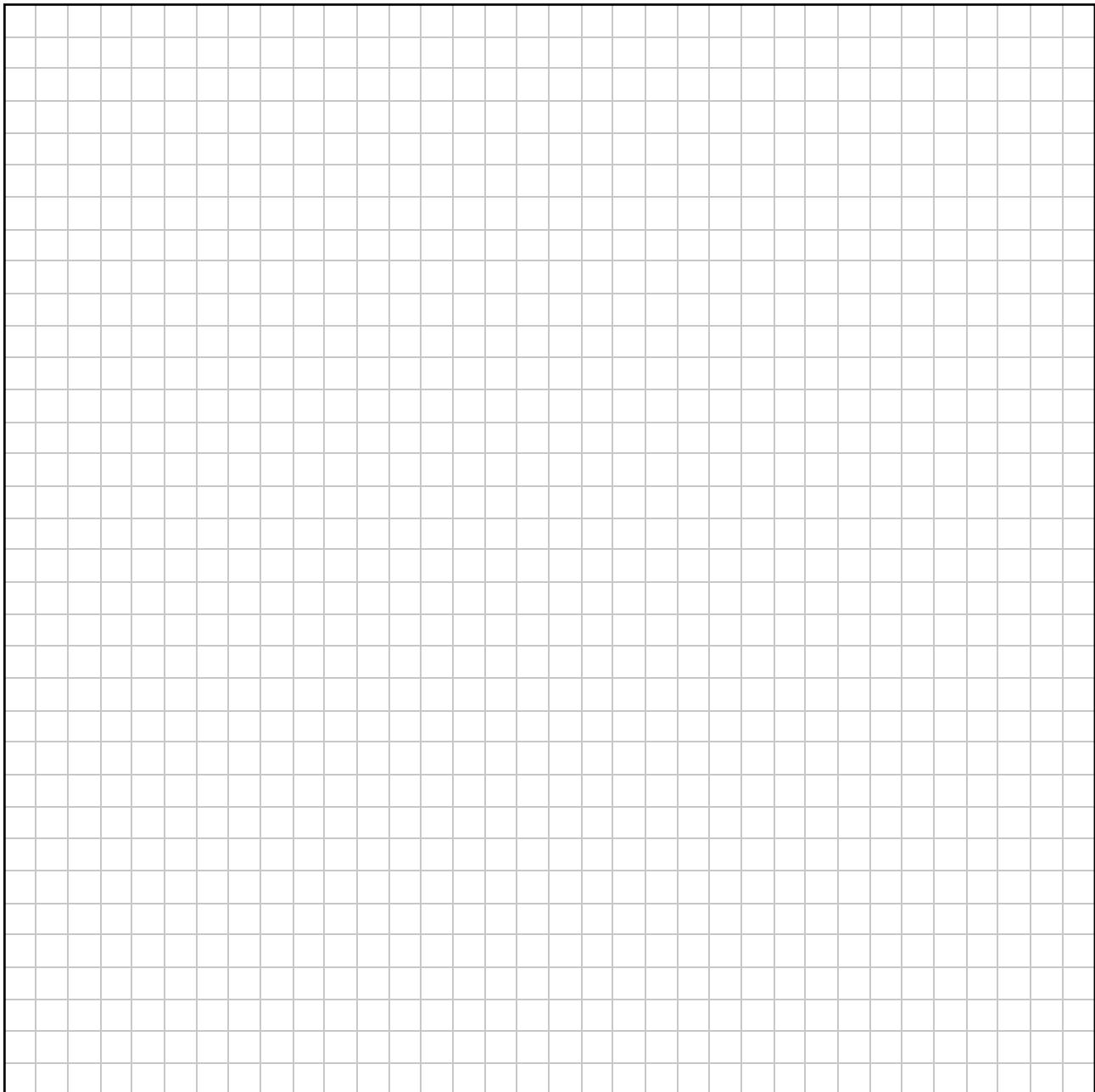
Page for extra work.

Label any extra work clearly with the question number and part.



Page for extra work.

Label any extra work clearly with the question number and part.



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Junior Cycle Final Examination – Higher Level

Mathematics

Friday 9 June

Afternoon 1:30 - 3:30