L.16

NAME ☞	
SCHOOL ☞	
TEACHER ☞	

Pre-Leaving Certificate Examination, 2018

Mathematics

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

School stamp

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total

Grade

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here: [ ]

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Joe earns a gross wage of €855 for a standard 38-hour working week. He pays income tax, universal social charge (USC) and pay-related social insurance (PRSI) on his gross wage.

(i) Joe pays income tax at the rate of 20% on the first €670 he earns and 40% on the balance. He has weekly tax credits of €63. How much income tax does Joe pay weekly?

(ii) Joe also pays USC and PRSI on his gross wage. USC amounts to €29·10 each week and he pays PRSI on his gross wage. His net weekly take-home pay is €646·70. Find the percentage rate at which Joe pays PRSI.

(b) Joe is paid "time and a half" for weekday overtime and weekend work. In addition to income tax and PRSI, he pays USC at the rate of 5% on his extra earnings.
Find the minimum number of hours that Joe must work above his standard working week in order to receive a net weekly take-home pay in excess of €800.

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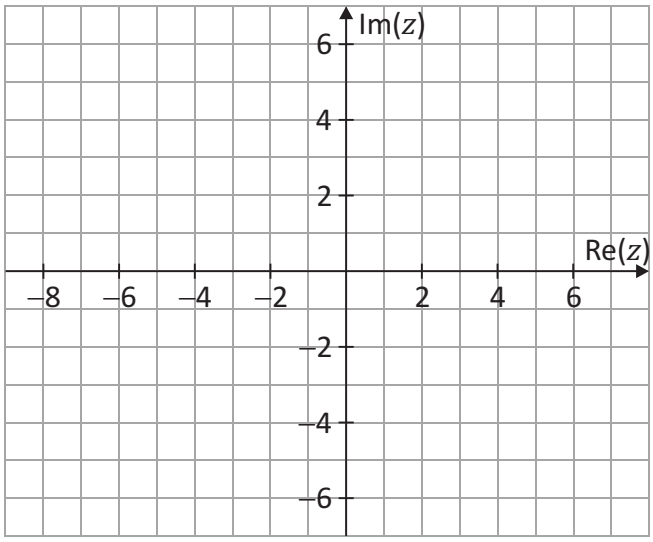
Question 2 **(25 marks)**

$z_1 = 3 + 4i$ is a complex number, where $i^2 = -1$.

(a) **(i)** Let $z_2 = 2iz_1$. Find z_2, in the form $a + bi$, where $a, b \in \mathbb{R}$.

(ii) Let $z_3 = \frac{z_1}{2i}$. Find z_3, in the form $a + bi$, where $a, b \in \mathbb{R}$.

(b) **(i)** Plot each of the points z_1, z_2 and z_3 on the given Argand diagram and label each point clearly.



(ii) Make one observation about the relative positions of the points you plotted on the diagram above.

Question 3 **(25 marks)**

(a) **(i)** Solve for x:

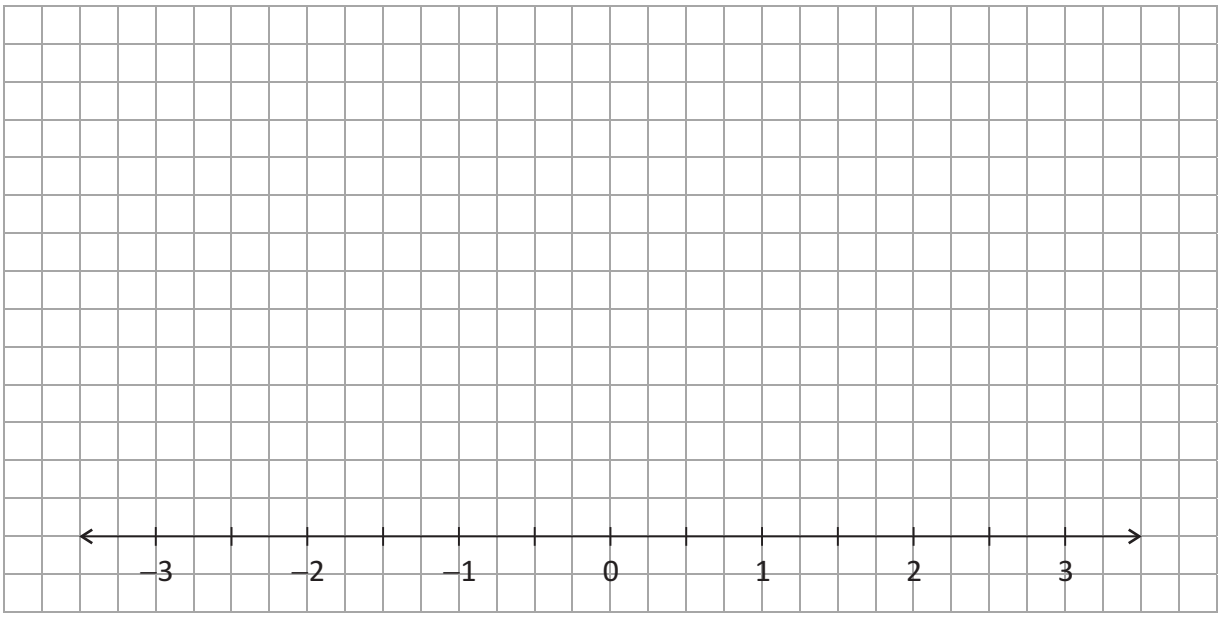
$$4(5 + 2x) - 5 = 5x - 3(1 - 2x), \text{ where } x \in \mathbb{R}.$$

(ii) Verify your answer to **part (i)** above.

(b) Solve the inequality:

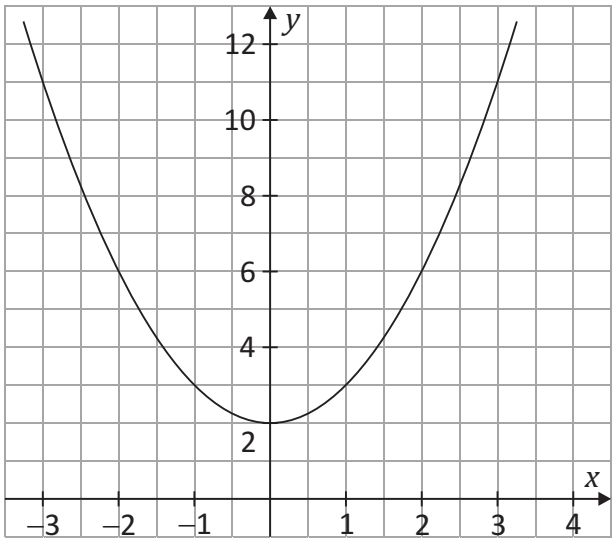
$$2(1 + 2x) - 8x \geq -7, \quad x \in \mathbb{Z},$$

and show the solution set on the number line below.



Question 4 **(25 marks)**

The diagram shows the graph of the function $f(x) = x^2 + 2$ in the domain $-3 \leq x \leq 4$, $x \in \mathbb{R}$.



(a) **(i)** On the same diagram, draw the graph of the function $g(x) = 2x + 5$, $x \in \mathbb{R}$. Hence, use the graphs to find the two values of x for which $g(x) = f(x)$.

(ii) Verify your answer to **part (i)** above by using algebra to solve $g(x) = f(x)$.

(b) **(i)** Find $f'(x)$, the derivative of $f(x)$.
Hence, find the value of x at which the tangent to the graph of $f(x)$ is parallel to $g(x)$.

(ii) Hence, find the equation of this tangent.

Question 5 **(25 marks)**

(a) Find the two values of x for which $2x^2 + 4x - 5 = 0$.
Give your answer correct to one decimal place.

(b) When two numbers are added together, the sum is equal to 46. When the smaller number is subtracted from the larger number, the result is equal to 12.
By writing two equations to represent this information, or otherwise, find the values of both numbers.

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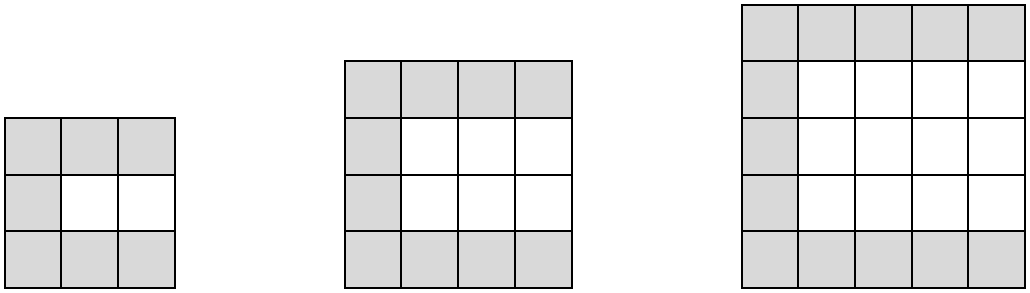
Question 6 **(25 marks)**

The first three patterns in a sequence of patterns of tiles are shown below.



Pattern 1 Pattern 2 Pattern 3

(a) Draw the next two patterns in the sequence.

(b) **(i)** Based on the patterns shown, complete the table below.

Pattern number (n)	Number of White Tiles	Number of Grey Tiles
1	2	7
2	6	10
3		
4		
5		

(ii) Show that the number of **white** tiles in each pattern forms a quadratic sequence.

(c) Assuming the pattern continues, the number of **white** tiles in the nth pattern of the sequence is given by the formula $W_n = n^2 + bn + c$, where $b, c \in \mathbb{Z}$.
Find the value of b and the value of c.

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Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

Fiona wishes to save up to buy a car. She joins a savings scheme in her local credit union. She plans to save €250 per month and to increase this amount by €5 every month.

(a) **(i)** Complete the table below to show Fiona's monthly contributions for the period shown.

Month (n)	1	2	3	4	12	24
Contribution (€)	250					

(ii) Show that Fiona's monthly contributions form an arithmetic sequence.

(iii) Find, in terms of n, a formula that gives Fiona's monthly contribution in the nth month of the scheme.

(iv) Using your formula, or otherwise, find in which month Fiona contributes €400.

(b) **(i)** Find, in terms of n, a formula that gives the sum of Fiona's total contributions from the first to the nth month of the scheme.

(ii) Fiona is guaranteed to receive a fixed return of €14 000 at the end of three years if she does not take any money out of the savings scheme until that time.

Find the percentage return that Fiona will receive on her savings. Give your answer correct to two decimal places.

(c) Another option is for Fiona to borrow €14 000 from a finance company to buy a car. The loan is to be repaid in equal monthly repayments over the term of the loan. Interest is charged monthly at an annual percentage rate (APR) of 13·5% on the amount **borrowed** for the entire term of the loan.

(i) Using the formula $(1 + r)^{12} = 1 + i$, where r is the monthly rate and i is the annual rate of interest, find the rate of interest charged monthly which corresponds to an APR of 13·5%, correct to two decimal places.

(ii) Fiona decides to borrow the money and repay the loan over 3½ years. Calculate, correct to the nearest euro, the amount of her monthly repayments.

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Question 8 **(50 marks)**

A company seeking new investors predicts that the projected value of their investment will follow the function:

$$v(t) = 5t^2 - 180t + 10\,000,$$

where v is the value of the investment, in euro, and t is the time, **in months**, after the investment is made.

(a) **(i)** How much are potential investors asked to invest initially?

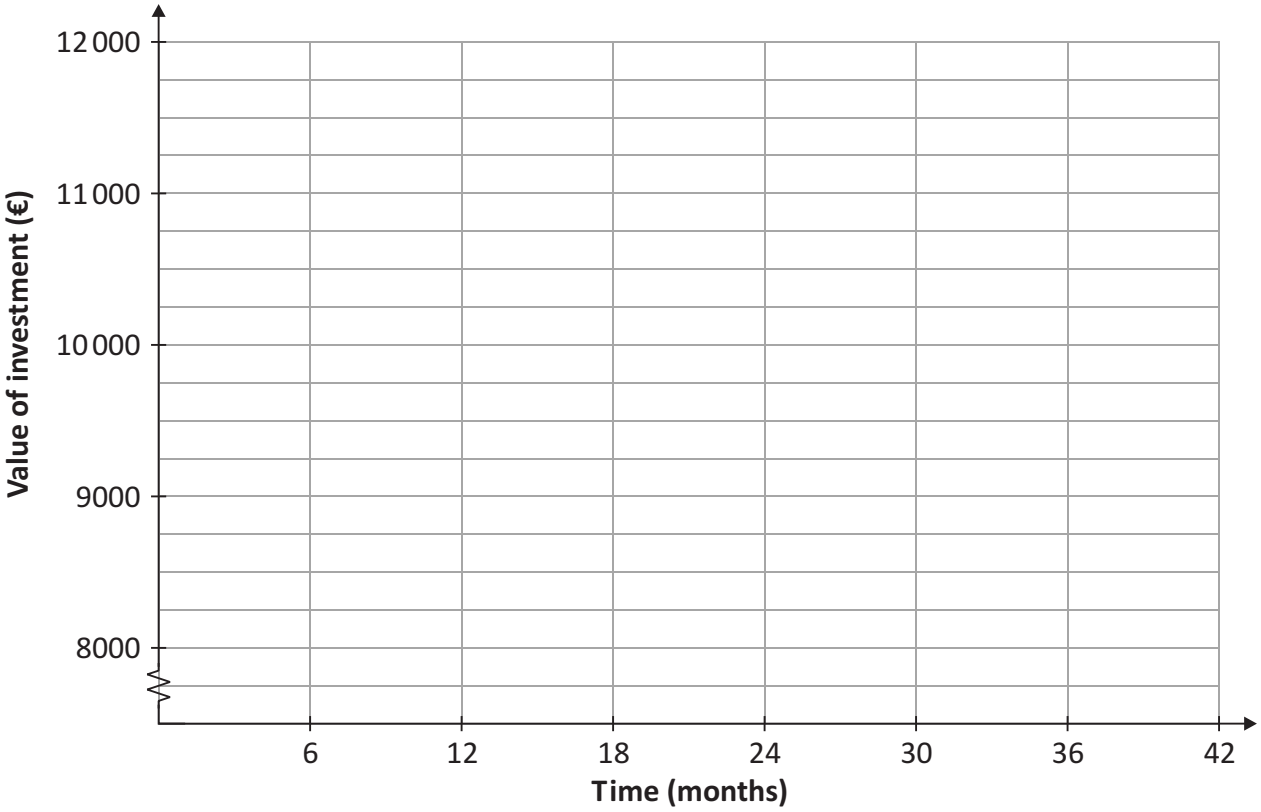
(ii) How long will it take before their investment is profitable?

(b) Use calculus to find how much the investment is worth when it reaches its minimum value.

(c) **(i)** Use the value function, $v(t)$, to complete the table, showing the projected value of the investment over time.

Time (t)	6	12	18	24	30	36	42
Value (€)	9100	8560					

(ii) Use the data in the table to draw a graph to represent the projected value of the investment over the period of time indicated. Label your graph clearly.



(d) The projected value of another investment opportunity is predicted to follow the function:

$$w(t) = 70t + 8000,$$

where w is the value, in euro, and t is the time, **in months**, after the investment is made.

(i) On the same axes above, draw a graph to show the projected value of this investment for $0 \leq t \leq 42$, $t \in \mathbb{R}$. Label your graph clearly.

(ii) Use your graphs to estimate the time interval for which the projected value of this investment is higher than that of the other investment.

(iii) Which of the above investment opportunities would you recommend?
Give a reason for your answer.

Option:

Reason:

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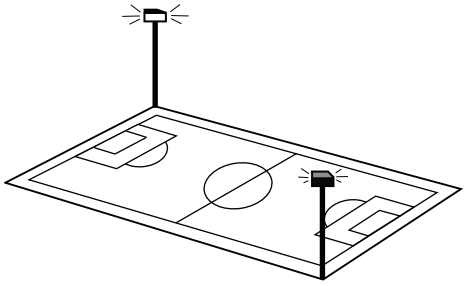
Question 9 **(50 marks)**

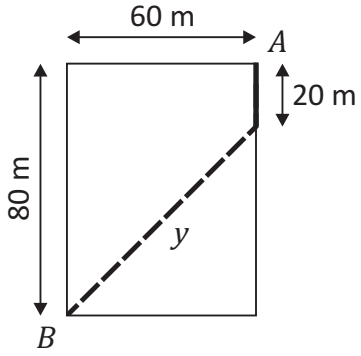
A club proposes to erect two floodlighting towers at opposite corners of its rectangular playing field of length 80 m and width 60 m. A power cable is required between the diagonal corners, A and B.



The cost of digging a trench to lay the power cable is €20 per m, while the cost of boring underground to install the cable is €40 per m.

The club does not wish to dig a trench across the playing field.

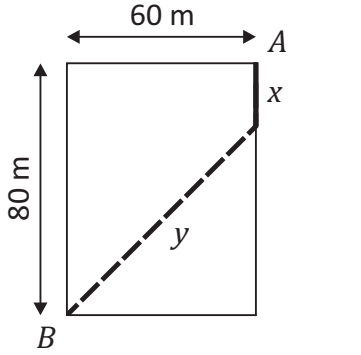
(a) One option is to dig a trench for 20 m from A along the edge of the playing field and then to bore underground from this point diagonally across the field to B, as shown.



(i) Use the Theorem of Pythagoras to find y, the distance required to be bored underground.
Give your answer correct to one decimal place.

(ii) Hence, find the **total** cost of this option.

(b) To investigate other options, a general formula for y (distance to be bored underground) is derived for different values of x (length of trench to be dug).



(i) Show that the general formula for y is $\sqrt{(80-x)^2+60^2}$.

(ii) Find the value of y for each value of x given in the table below.
Give your answers correct to one decimal place.

x (metres)	y (metres)	Total Cost (€)
10	92·2	3888
20		
30		
40		
50		
60		
70		
80		

(iii) Write a formula in x and y for the total cost of installing the power cable.
Hence, complete the table above to show the total cost for each option given.

(iv) Use the formula from **part (i)** to find the shortest route between A and B.
Hence, state whether this is the cheapest option. Justify your answer by calculation.

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Pre-Leaving Certificate, 2018 – Ordinary Level

Mathematics – Paper 1

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