L.16

2022-L016-1-EL-01/32

Pre-Leaving Certificate Examination, 2022

Mathematics

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

CANDIDATE DETAILS

EXAM ID

Optional 4 or 5-digit number
(only if provided by your school)

NAME

SCHOOL

TEACHER

4e6b4069-6072-49fb-a557-f5acf7e96109

2022-L016-1-EL-01/32

Do not write on this page

4e6b4069-6072-49fb-a557-f5acf7e96109

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer **any five** questions from Section A and **any three** questions from Section B.

Write your Exam ID, Name, School's Name and Teacher's Name in the grid on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

4e6b4069-6072-49fb-a557-f5acf7e96109

Section A — Concepts and Skills — 150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

Jim travels from Dublin to work in London every week.

(a) **(i)** In Dublin, Jim exchanges £3500 sterling for euro.
The exchange rate for the transaction is €1 = £0·87 sterling.
Find, correct to the nearest euro, the amount that he can expect to receive.

(ii) Jim is charged 1·25% commission on the transaction.
Find, correct to the nearest cent, the amount that he receives.

(b) In London, Jim exchanges £1500 sterling for euro.
He receives €1630 after 1·5% commission is charged on the transaction.
Find the exchange rate for the transaction **before** commission is charged.
Give your answer in the form £1 = €□·□□.

4e6b4069-6072-49fb-a557-f5acf7e96109

(c) Jim wishes to exchange £2000 sterling for Australian dollars. On a given day, the exchange rate for euro to sterling is €1 = £0·89 and for euro to Australian dollars is €1 = A$1·68.

Find the sterling to dollar exchange rate and write your answer in the form £1 = A$□·□□. Hence find the amount that Jim can expect to receive in Australian dollars if no commission is charged on the transaction.

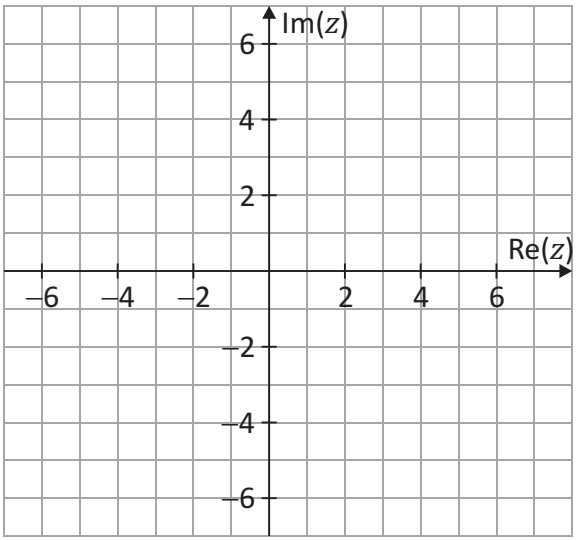
4e6b4069-6072-49fb-a557-f5acf7e96109

Question 2 **(30 marks)**

(a) $z_1 = -3 + i$ and $z_2 = 2 - 4i$ are complex numbers, where $i^2 = -1$.
$z_3 = z_2 - z_1$.

(i) Find the value of z_3, and **plot and label** z_1, z_2 and z_3 on the Argand Diagram.

$z_3 =$



(ii) From your diagram, is it possible to say that $|z_2| > |z_1|$?
Give a reason for your answer.

Answer:

Reason:

(iii) Verify algebraically that $|z_2| > |z_1|$.

(b) Find $\frac{2z_1 - z_2}{z_3}$ in the form $a + bi$, where $a, b \in \mathbb{R}$.

4e6b4069-6072-49fb-a557-f5acf7e96109

Question 3 **(30 marks)**

(a) Solve for x:

$$2(x+3) = 5(x-3).$$

(b) **(i)** Show that $\frac{2}{2x-1} - \frac{3}{x+3}$ can be written as $\frac{9-4x}{(2x-1)(x+3)}$.

4e6b4069-6072-49fb-a557-f5acf7e96109

(ii) Hence solve the equation:

$$\frac{2}{2x-1} - \frac{3}{x+3} = 1, \text{ where } x \neq \frac{1}{2}, -3 \text{ and } x \in \mathbb{R}.$$

Give each answer correct to two decimal places.

4e6b4069-6072-49fb-a557-f5acf7e96109

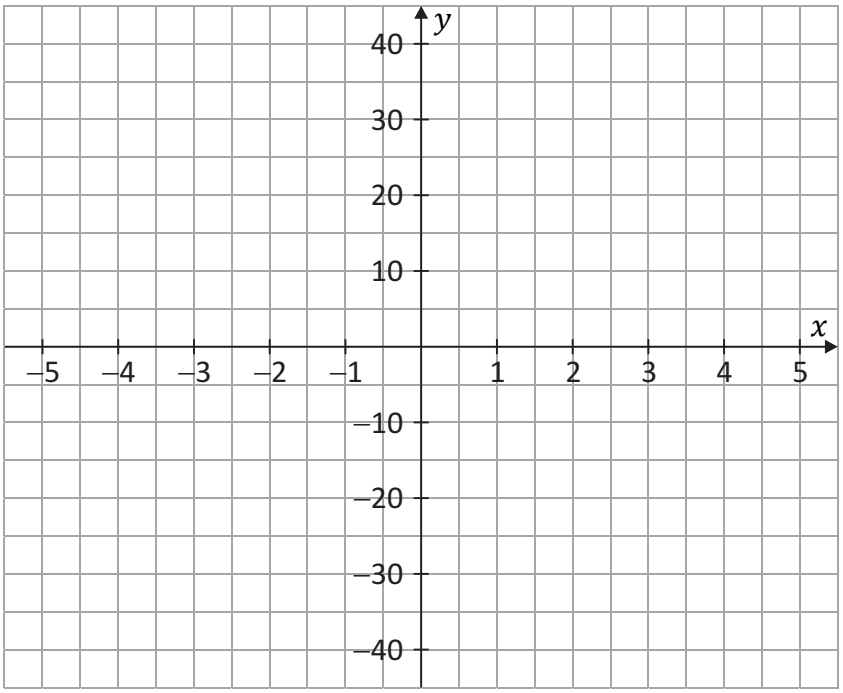
Question 4 **(30 marks)**

The function f is defined as $f(x) = 2x^3 + 3x^2 - 12x - 9$, where $x \in \mathbb{R}$.

(a) Find the co-ordinates of the point at which the graph of f cuts the y-axis.

(b) Find $f'(x)$, the derivative of $f(x)$.
Hence find the co-ordinates of the local maximum turning point and the local minimum turning point of $f(x)$.

(c) **(i)** Hence sketch the graph of $f(x)$ on the axes below.
Indicate clearly both turning points.



(ii) Use your graph to determine the range of values of x for which $f(x)$ is decreasing.

4e6b4069-6072-49fb-a557-f5acf7e96109

Question 5 **(30 marks)**

(a) Temperature can be converted from Celsius (°C) to Fahrenheit (°F) using the formula:

$$F = \frac{9C}{5} + 32$$

where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

(i) Find the difference in temperature between the freezing point of water (0 °C) and the boiling point of water (100 °C) in degrees Fahrenheit.

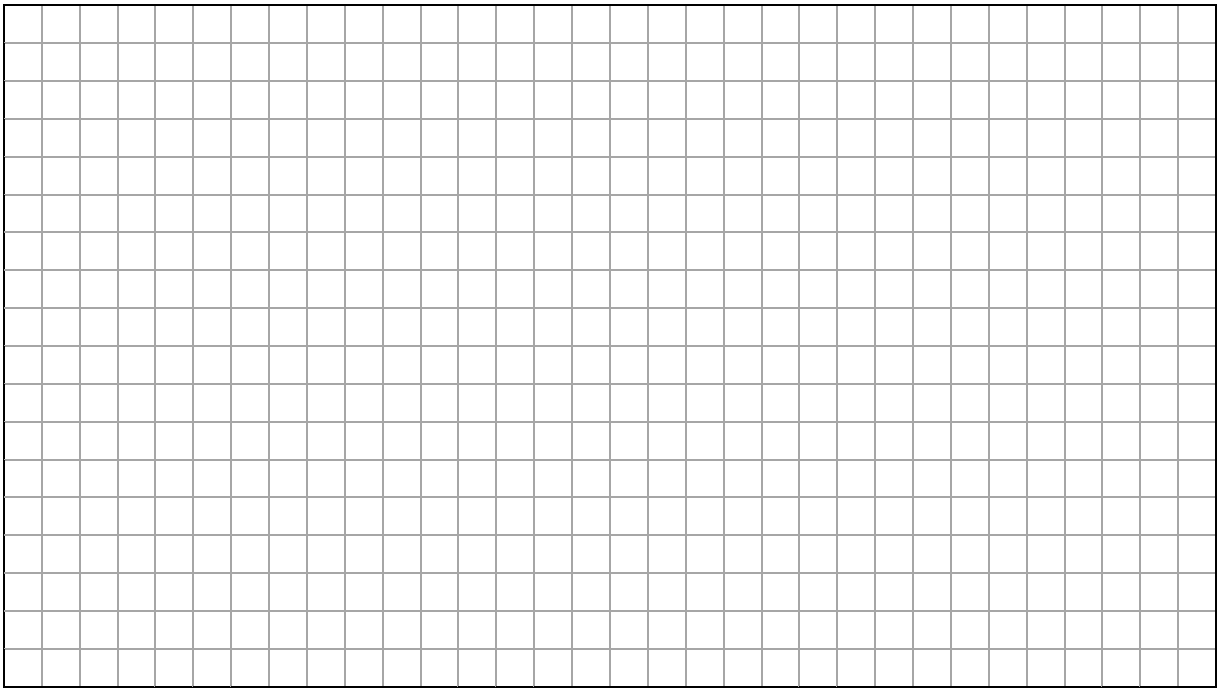
(ii) Rearrange the formula to express C in terms of F.

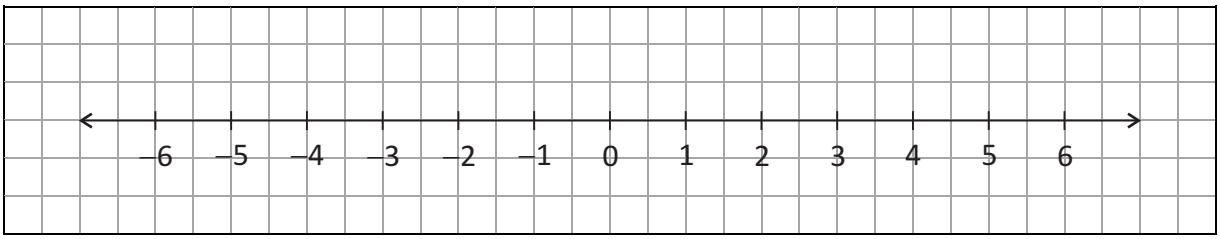
4e6b4069-6072-49fb-a557-f5acf7e96109

(b) Solve the inequality:

$$3(2-3x) \leq 24, \text{ where } x \in \mathbb{Z},$$

and show the solution set on the number line below.





4e6b4069-6072-49fb-a557-f5acf7e96109

Question 6 **(30 marks)**

Jane buys a new car which costs €35 000.

(a) The value of the car depreciates by 35% in the first year and by 12·5% each year after that.

(i) Find the value of Jane's car at the end of the first year.

(ii) Find the value of the car at the end of five years.
Give your answer correct to the nearest euro.

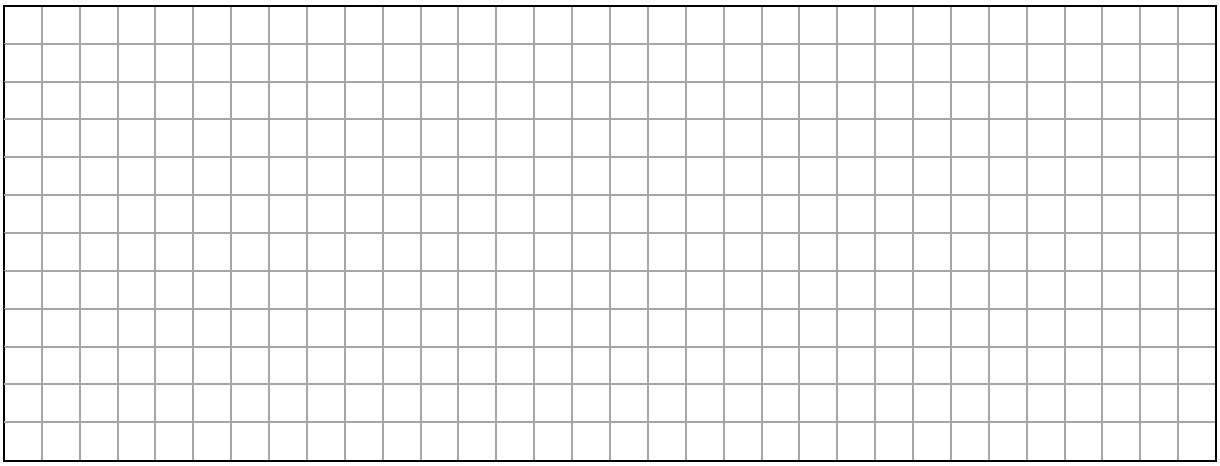
4e6b4069-6072-49fb-a557-f5acf7e96109

(b) Another method depreciates the value of the car by a fixed percentage **per month**.

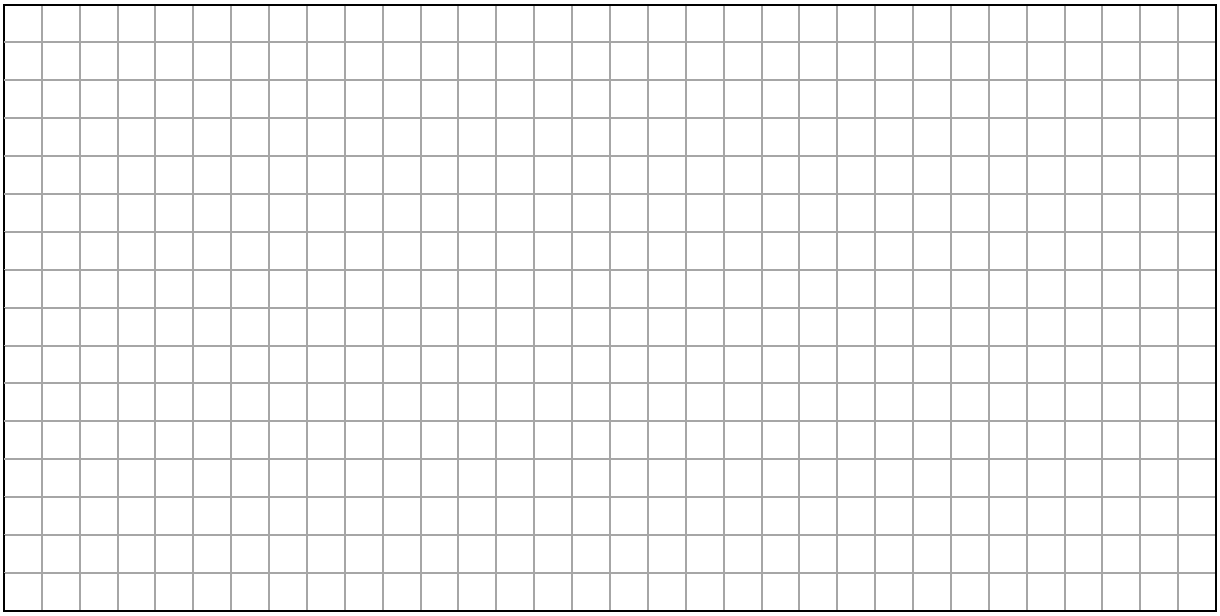
(i) Find the annual percentage rate (APR) of depreciation that corresponds to a rate of 1·5% per month, compounded monthly, using the formula:

$$(1 - r)^{12} = 1 - i,$$

where r is the monthly rate and i is the annual rate.
Give your answer correct to four significant figures.



(ii) Using the formula $F = P(1 - i)^t$, or otherwise, find the value of Jane's car at the end of five years if it depreciates at a fixed rate of 1·5% per month.
Give your answer correct to the nearest euro.

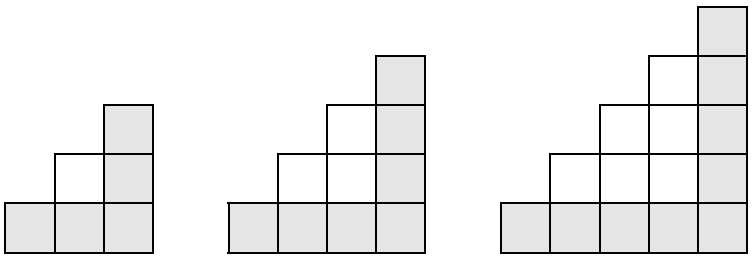


Section B — Contexts and Applications — 150 marks

Answer **any three** questions from this section.

Question 7 **(50 marks)**

The first three patterns in a sequence of patterns of grey and white tiles are shown.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

(a) **(i)** Draw the next pattern in the sequence onto the diagram above.

(ii) Based on the patterns shown, complete the table below to show the numbers of tiles in each of the first five patterns of the sequence.

Pattern (n)	Number of Grey Tiles	Number of White Tiles	Total Number of Tiles
1	5	1	
2			
3			
4			
5			

(b) **(i)** Assuming the pattern continues, the number of **grey** tiles in the nth pattern of the sequence is given by the formula $G_n = pn + q$, where $p, q \in \mathbb{Z}$. Find the value of p and the value of q.

(ii) How many **grey** tiles are in the 30th pattern of the sequence?

(c) **(i)** Show that the number of **white** tiles in each pattern forms a quadratic sequence.

This question continues on the next page.

(ii) The number of **white** tiles in the nth pattern of the sequence is given by the formula:

$$W_n = \frac{1}{2}n^2 + bn + c, \text{ where } b, c \in \mathbb{Q}.$$

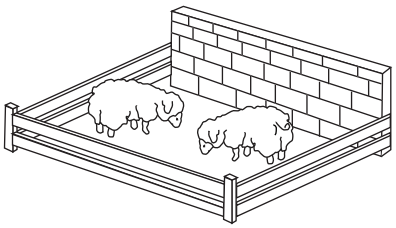
Find the value of b and the value of c.

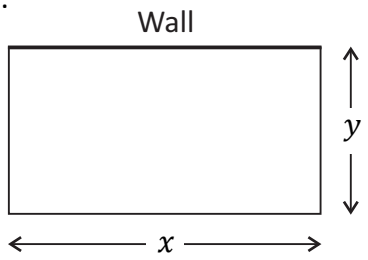
(iii) How many **white** tiles are in the 30th pattern of the sequence?

(d) Find the pattern in the sequence which has 253 tiles **in total**.

Question 8 **(50 marks)**

A farmer wishes to construct a rectangular pen for his sheep. He has 19 metres of fencing to cover three sides of the proposed pen, with the other side being a block wall.
The length of the proposed pen is x metres and the width is y metres.





(a) **(i)** Write the length of fencing in terms of x and y.

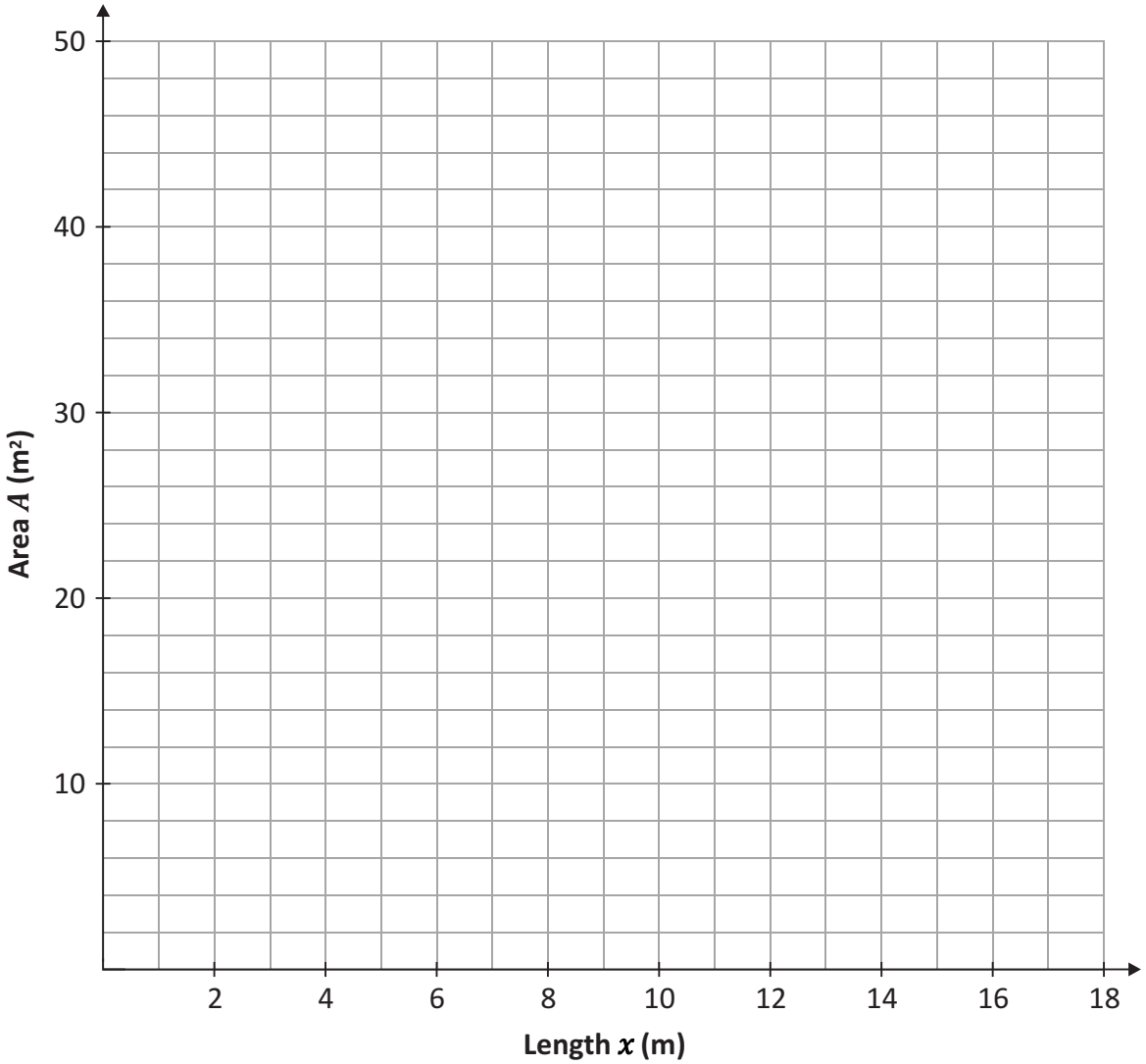
(ii) Hence show that the width of the proposed pen can be written as $y = 9{\cdot}5 - 0{\cdot}5x$.

(b) **(i)** Complete the table below to show the area of the proposed pen, A, for each given value of x.

Length x (m)	0	2	4	6	8	10	12	14	16	18
Width y (m)			7·5							
Area A (m²)			30							

This question continues on the next page.

(ii) On the grid below, draw the graph of A for $0 \leq x \leq 18$, where $x \in \mathbb{R}$.



(c) Use your graph to estimate:
(Show your work on the graph above)

(i) the maximum area of the rectangular pen and write down the corresponding length and width

A: Maximum Area (m²)	
Length x (m)	
Width y (m)	

(ii) the range of values of x for which the area of the rectangular pen is greater than 36 m².

(d) **(i)** Show that the area of the rectangular pen can be written as $A(x) = 9{\cdot}5x - 0{\cdot}5x^2$.

(ii) Find $A'(x)$, the derivative of $A(x) = 9{\cdot}5x - 0{\cdot}5x^2$.

(iii) Use your answer from **part (d)(ii)** to find the maximum area of the rectangular pen.

Question 9 **(50 marks)**

Two farmers, A and B, are beef producers. They both buy cattle and fatten them for a certain period of time before selling them to a meat factory.

(a) Farmer A uses the following function to estimate the expected weight of his animals:

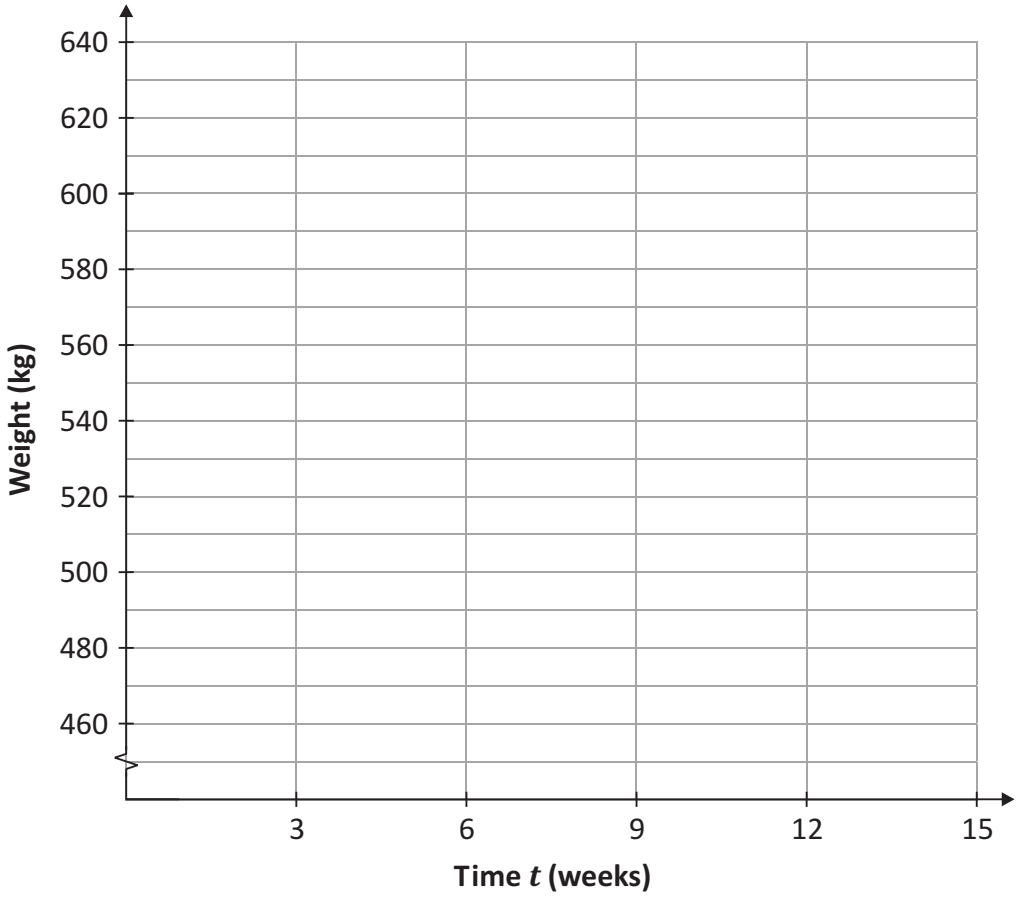
$$A(t) = 500 + 7{\cdot}9t,$$

where $A(t)$ is the weight, in kg, of the animal and t is the time, in weeks, from when the animal was purchased.

(i) The table below shows the expected weights of Farmer A's animals over time. Use the weight function $A(t)$ to complete the table.

Time t (weeks)	0	3	6	9	12	15
Weight $A(t)$ (kg)	500					

(ii) On the grid below, draw the graph of the function $A(t)$ for $0 \leq t \leq 15$, where $t \in \mathbb{R}$. Label your graph clearly.



(b) Farmer B prefers to buy lighter cattle and fatten them for the same period of time. She uses the following function to estimate the weight of her animals:

$$B(t) = 480(1{\cdot}018)^t,$$

where $B(t)$ is the weight, in kg, of the animal and t is again the time, in weeks, from when the animal was purchased.

(i) The table below shows the expected weights of Farmer B's animals over time. Use the weight function $B(t)$ to complete the table. Give your answers correct to one decimal place.

Time t (weeks)	0	3	6	9	12	15
Weight $B(t)$ (kg)	480					

(ii) Draw the graph of the function $B(t)$ for $0 \leq t \leq 15$, where $t \in \mathbb{R}$, on the grid in **part (a)(ii)** on the opposite page. Label your graph clearly.

(iii) Use your graphs to find the difference in the expected weights between the animals of Farmer A and Farmer B at the midpoint of the fattening process.

(iv) Verify your answer to **part (b)(iii)** above.

This question continues on the next page.

(c) The meat factory pays €3·55 per kg of weight for each animal and an extra 3·5% per kg for each kg of weight in excess of 600 kg.

(i) How much can both farmers expect to receive for each animal when they are sold after 15 weeks?
Give your answers correct to the nearest euro.

Farmer A:

Farmer B:

(ii) According to farming representative bodies, when consumers spend €10 on beef, retailers get €5·10, meat factories get €2·90 and farmers get €2.

Given that Farmer A is paid €3·60 per kg on average for each of his animals, how much each does the retailer and the meat factory receive?
Give your answers in euro per kg, correct to the nearest cent.

4e6b4069-6072-49fb-a557-f5acf7e96109

Question 10 **(50 marks)**

(a) Ed works as an IT technician. He has a time card for recording the hours he works every week. His time card for one week is shown below. Some information is missing.

Avon Laboratories **Time Card**

Employee:	Ed Carroll					Date:	17/01/22
Work ID:	54321					Week No:	3

Day:	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Start:	08:05	07:50	07:55	08:00	08:10	08:15	10:20
Finish:	17:15	17:20	17:30	16:50	17:40	13:45	12:35
Total Break(s):	–1:00	–1:00	–0:50	–0:45	–1:15	–	–
Total Daily Hours:	**8·17 hr**	**8·50 hr**	**hr**	**hr**	**8·25 hr**	**hr**	**2·25 hr**

Summary							
				Total Weekday Hours:	**hr**	Total Sat/Sun Hours:	**hr**
		Weekday Standard:	**39 hr**	Weekday Overtime:	**hr**		

(i) Complete the time card above to show the total daily hours, the total weekday hours, the total weekday overtime hours and the total Saturday/Sunday hours that Ed worked. Give each of your answers as a **decimal**, correct to two decimal places.

(ii) Ed is paid €17·10 per hour for standard weekday hours.
He is paid "time and a half" for weekday overtime and Saturday/Sunday work.
Calculate Ed's gross wage for the week.
Give your answer correct to the nearest cent.

This question continues on the next page.

(b) **(i)** Another week, Ed works h hours above his standard 39-hour week.
Write a formula in h for Ed's gross wage for that week.

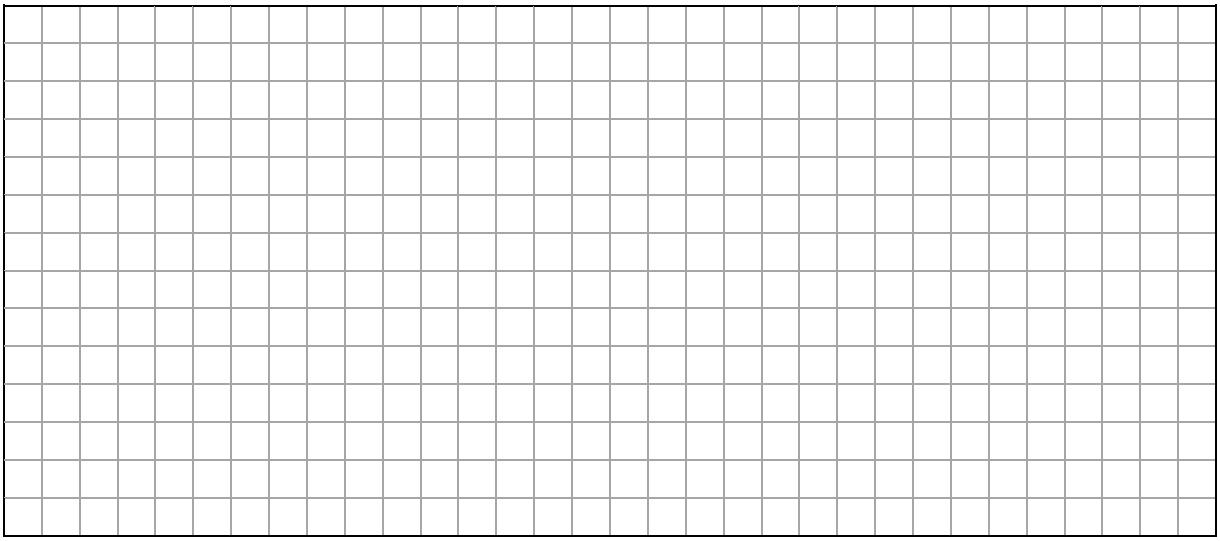
(ii) Hence find the minimum number of whole hours Ed must work above his standard week in order to receive a gross wage in excess of €850.

(c) Ed earns a gross wage of €875 this week.
He pays income tax, universal social charge (USC) and pay-related social insurance (PRSI) on his gross wage.

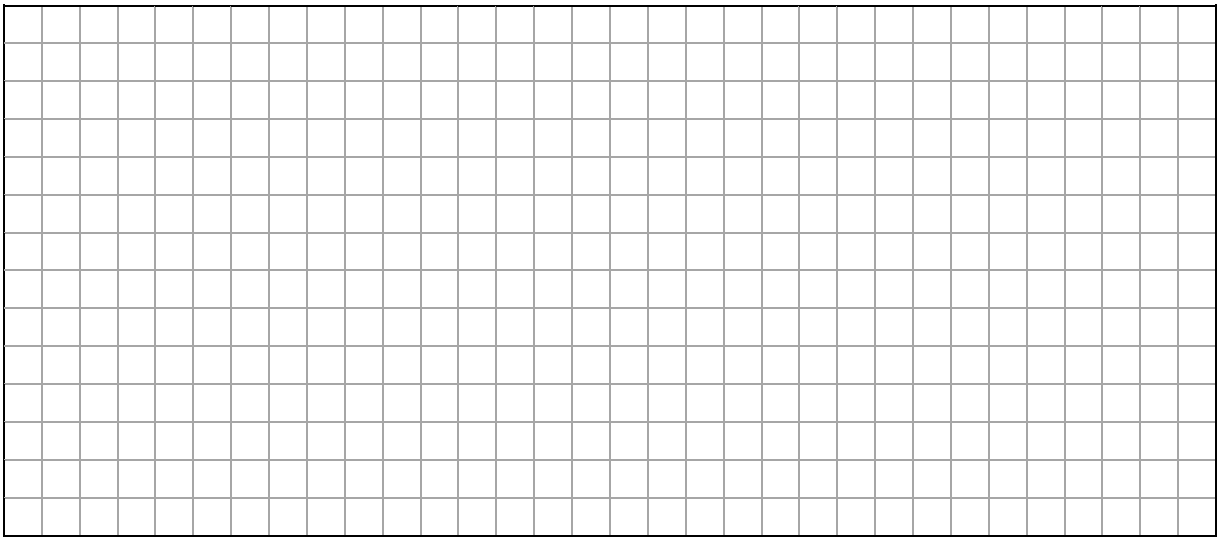
(i) Ed pays income tax at the rate of 20% on the first €679 he earns and 40% on the balance.
He has weekly tax credits of €63·46.
How much income tax does Ed pay?

4e6b4069-6072-49fb-a557-f5acf7e96109

(ii) Ed pays USC on his gross wage.
He pays 0·5% on the first €231 he earns, 2% on the next €163 and 4·5% on the balance.
Find the total amount of USC that Ed pays this week.



(iii) Ed also pays PRSI on his gross wage.
His net take-home pay is €663·20.
Find the percentage rate at which Ed pays PRSI.



You may use this page for extra work.
Label any extra work clearly with the question number and part.

Q. & part	

4e6b4069-6072-49fb-a557-f5acf7e96109

You may use this page for extra work.
Label any extra work clearly with the question number and part.

Q. & part	

4e6b4069-6072-49fb-a557-f5acf7e96109

You may use this page for extra work.
Label any extra work clearly with the question number and part.

Q. & part	

4e6b4069-6072-49fb-a557-f5acf7e96109

You may use this page for extra work.
Label any extra work clearly with the question number and part.

Q. & part	

4e6b4069-6072-49fb-a557-f5acf7e96109

Do not write on this page

Pre-Leaving Certificate Examination, 2022 – Ordinary Level

Mathematics – Paper 1

Time: 2 hours, 30 minutes

4e6b4069-6072-49fb-a557-f5acf7e96109

2022-L016-1-EL-32/32