NAME ☞

SCHOOL ☞

TEACHER ☞

Pre-Leaving Certificate Examination, 2019

Mathematics

Paper 2

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

School stamp

Running total

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Pre-Leaving Certificate Examination, 2019   2
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 2/20

Answer **all six** questions from this section.

Question 1 (25 marks)

(a) Ben encounters three sets of traffic lights on his way to school. He will only proceed if the lights are "green". The probability of each set of traffic lights being "green" is $\frac{1}{4}$.

(i) Find the probability of Ben being stopped at the first set of traffic lights.

(ii) Find the probability of Ben being stopped at two sets of traffic lights on a particular day.

(b) Fay takes a different route to school. She will only proceed if the traffic lights she encounters are "green". The probability of each set of traffic lights being "green" is $\frac{1}{5}$.
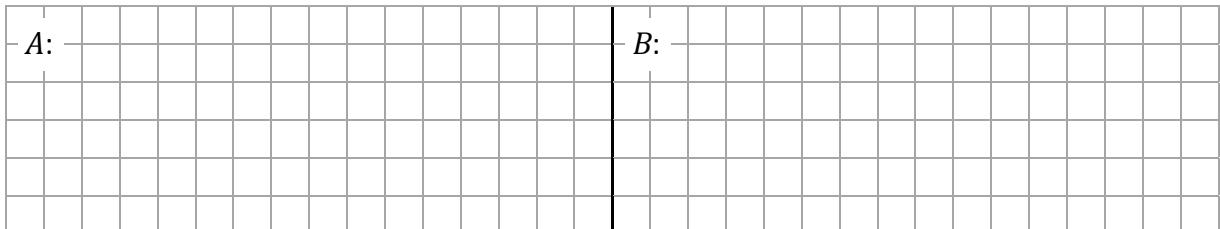
(i) Find the probability of Fay being stopped for the first time at the fifth set of traffic lights she encounters.

(ii) Find the probability of Fay being stopped at least once before she approaches the fifth set of traffic lights.
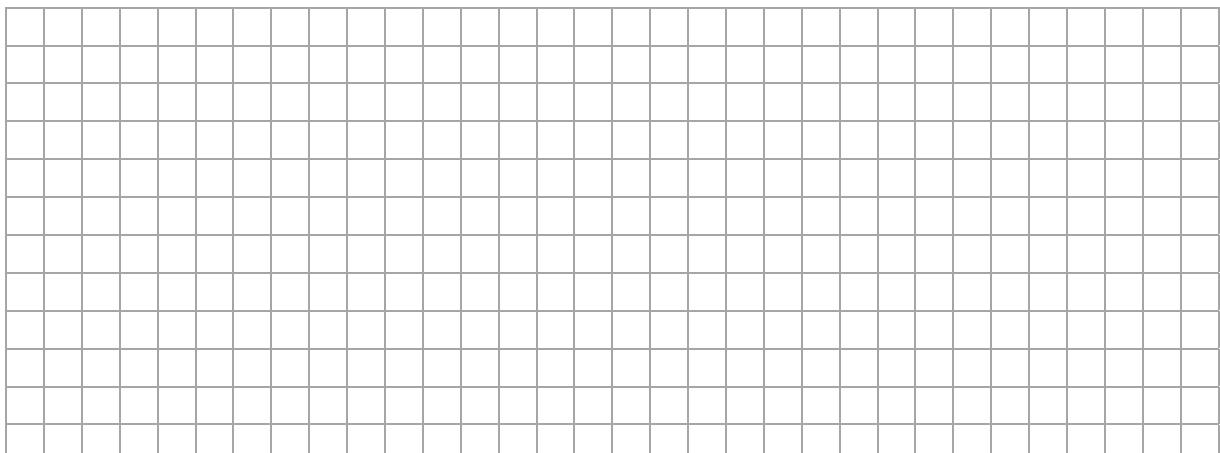
Question 2 (25 marks)

The line l has equation $x + y - 12 = 0$ and the line k has equation $3x - 7y + 14 = 0$.
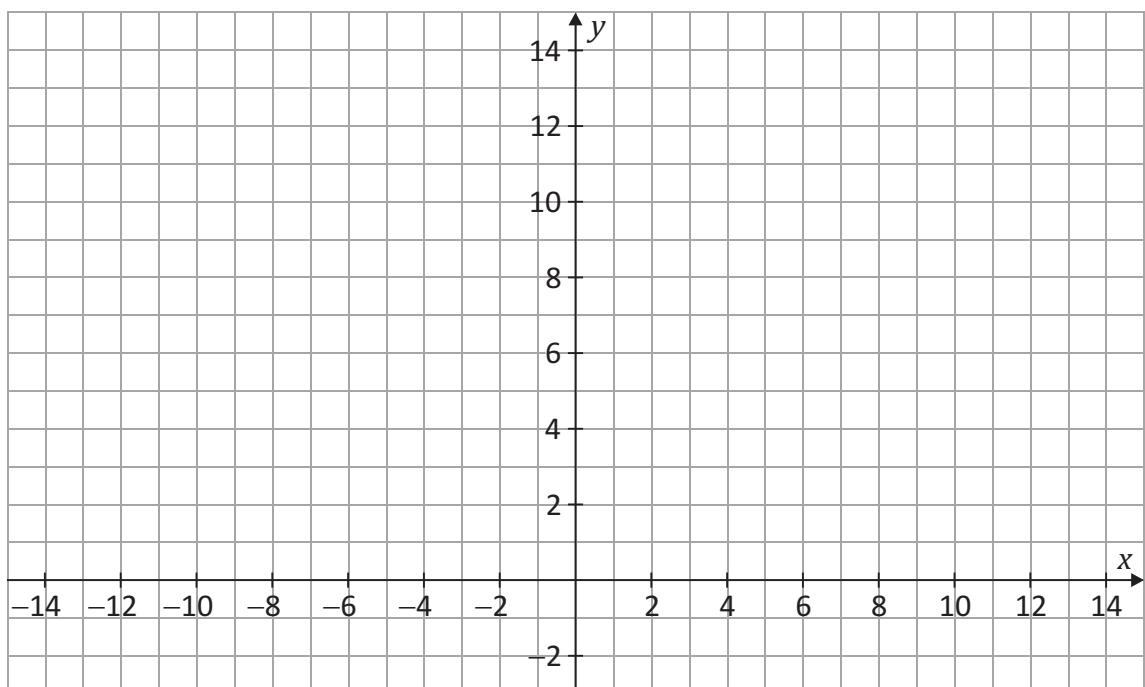
(a) **(i)** Line l cuts the y-axis at the point A and line k cuts the y-axis at the point B.
Write down the co-ordinates of A and B.

A: B:
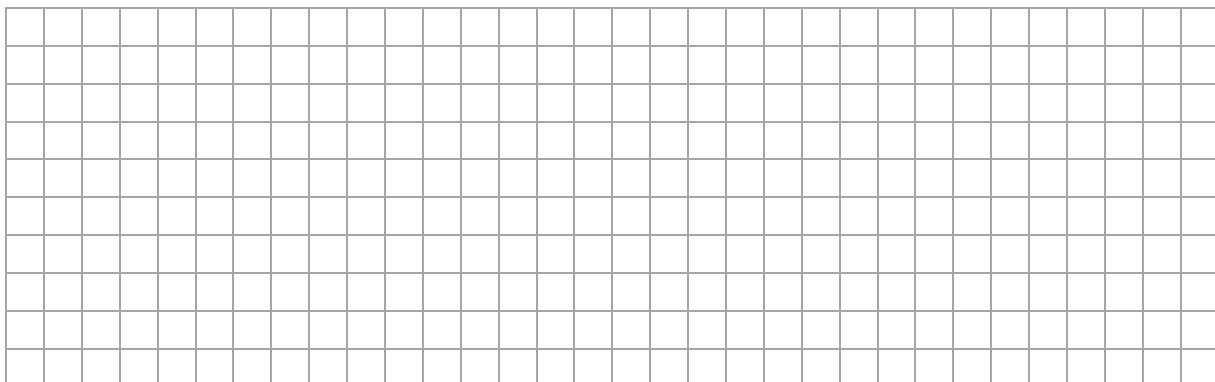
(ii) Show, using algebra, that the co-ordinates of C, the point of intersection of lines l and k, are $(7, 5)$.
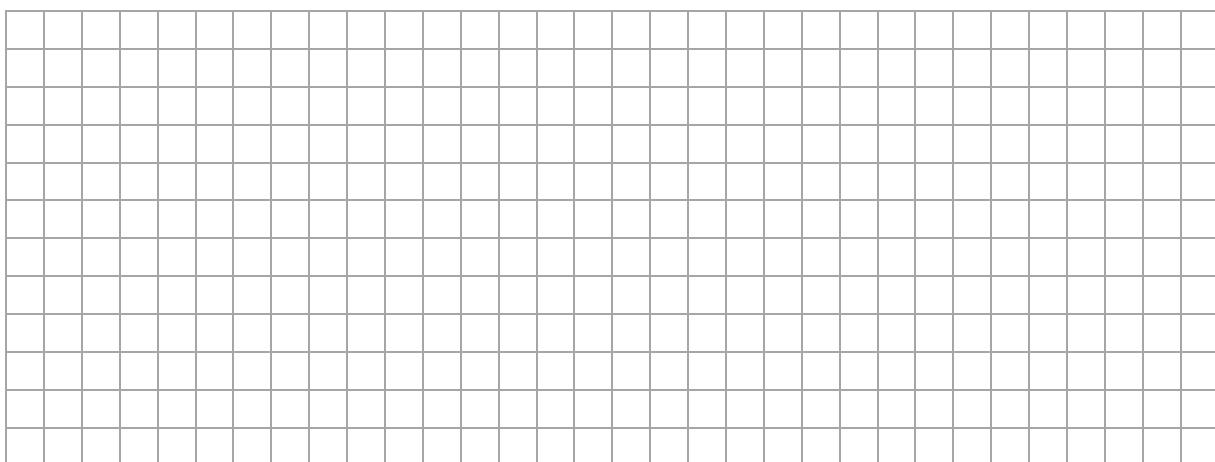
(iii) Draw the lines l and k on the co-ordinate diagram below.
Label the points A, B and C.

Pre-Leaving Certificate Examination, 2019   4
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 4/20

(b) **(i)** *ACBD* is a parallelogram in which [*AB*] and [*CD*] are diagonals.
Find the co-ordinates of the point *D*.

**(ii)** Hence, or otherwise, find the equation of the line *CD*.

Pre-Leaving Certificate Examination, 2019   5
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 5/20

| previous | page | running |
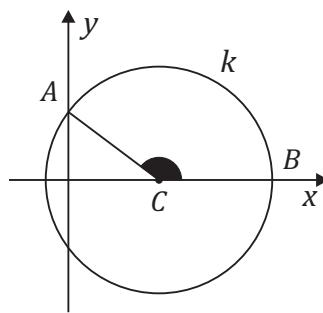
Question 3 **(25 marks)**

The circle k has centre $C(4, 0)$ and cuts the y-axis at $A(0, 3)$, as shown.

(a) **(i)** Find the radius-length of k.
Hence, write down the equation of k.

Radius-length of k:

Equation of k:

(ii) Circle k cuts the positive x-axis at the point B. Find the co-ordinates of B.

(b) Use trigonometry to find the measure of the obtuse angle ACB shown in the diagram.
Give your answer correct to the nearest degree.
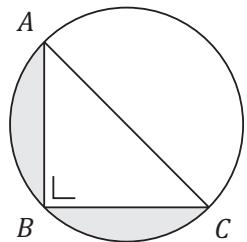
(c) Find the area of the circle enclosed in the first quadrant.
Give your answer in square units, correct to one decimal place.

Pre-Leaving Certificate Examination, 2019   6
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 6/20

Question 4 **(25 marks)**

The diagram shows a right-angled isosceles triangle ABC
with $|AB| = |BC| = 14$ units and $|\angle ABC| = 90°$.
A semi-circle is drawn with $[AC]$ as the diameter.

(a) **(i)** Find the radius of the semi-circle. Give your answer
in the form $p\sqrt{q}$, where $p, q \in \mathbb{N}$. Take $\pi = \dfrac{22}{7}$.

(ii) Hence show that the area of the shaded region is 56 square units.

(b) The area of the image of the shaded region under an enlargement is 448 square units.
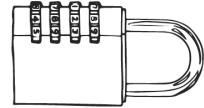
(i) Find the scale factor of the enlargement in the form $a\sqrt{2}$, where $a \in \mathbb{N}$.

(ii) Hence find the side lengths and hypotenuse of the image triangle of triangle ABC.

Pre-Leaving Certificate Examination, 2019   7
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

| previous | page | running |

2019.1 L.19 7/20

**Question 5** (25 marks)

(a) The code for a combination lock consists of a four-digit number which is formed from the digits 0 to 9, inclusive. A code cannot begin or end with 0 but digits may be repeated in any code.

(i) Write down the smallest possible four-digit code.

(ii) How many different codes are possible?

(iii) Find the number of four-digit codes in which no digit is repeated as a percentage of the total number of possible codes. Give your answer correct to one decimal place.

(b) Five members of a team, three boys and two girls, are seated in a row for a photograph.

(i) Find the number of different ways that the team can be arranged for the photograph if there is no restriction on the order of seating.
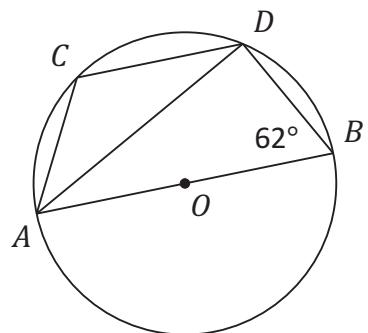
(ii) Find the number of different ways that the team can be arranged if the two girls must **not** be seated beside each other.

Pre-Leaving Certificate Examination, 2019
Mathematics, Paper 2 – Ordinary Level
8
4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 8/20

Question 6 **(25 marks)**

(a) In the diagram, $[AB]$ is a diameter of a circle with centre O.
CD is parallel to AB and $|\angle ABD| = 62°$.
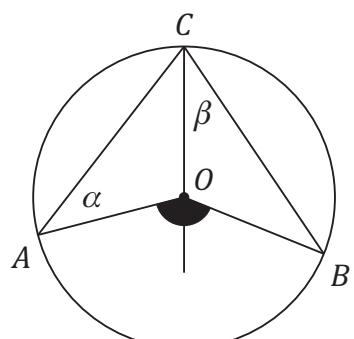
(i) Write down the value of $|\angle BDA|$.
Give a reason for your answer.

(ii) Hence find the value of $|\angle ADC|$.

(b) The points A, B and C lie on a circle with centre O,
as shown. $|\angle CAO| = \alpha$ and $|\angle BCO| = \beta$.

(i) Find $|\angle AOB|$, in terms of α and β.
Give a reason for each step you make in your answer.

(ii) Hence find $|\angle OAB|$, in terms of α and β.

4e6b4069-6072-49fb-a557-f5acf7e96109

previous	page	running
	62°	

2019.1 L.19 9/20

Answer **all three** questions from this section.

Question 7   **(60 marks)**

Table 1 below shows the numbers of tourists to Ireland, in thousands, by geographical region between 2005 and 2017. A tourist is defined as a visitor whose trip includes an overnight stay.

Table 1 – Numbers of tourists (in '000s) by geographical region							
Year	2005	2007	2009	2011	2013	2015	2017
Britain	3640	3776	3034	2799	2870	3346	3445
Mainland Europe	1903	2577	2323	2184	2346	2880	3256
North America	937	1071	920	904	1039	1294	1715
Rest of World	284	316	301	353	431	516	607
Total Overseas							9023
Northern Ireland	570	593	n/a	1416	1572	1492	1315
Domestic Trips	7173	7942	8340	7169	8413	9125	9626

n/a = not available

(a) Based on the data in the table, write down:

     (i) the geographical region from which most tourists to Ireland came _____

     (ii) the year with the greatest increase in tourists from Mainland Europe _____

     (iii) the lowest number of tourists from North America in a particular year _____

(b) **(i)** Use the data in Table 1 to complete Table 2 by finding the total numbers of overseas tourists to Ireland between 2005 and 2017.
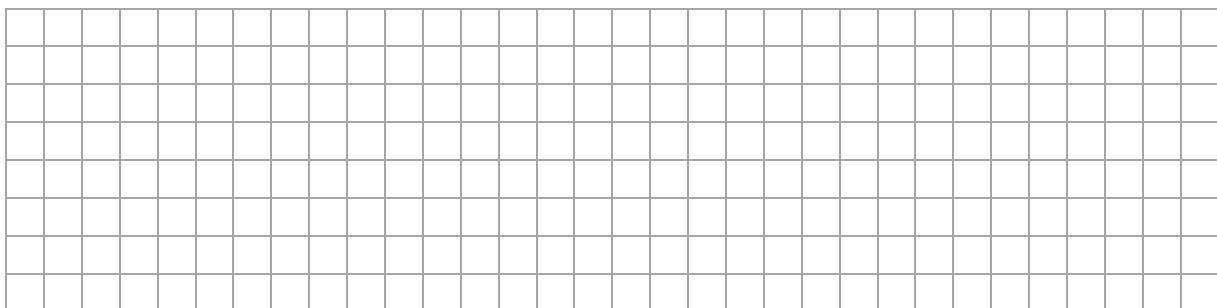
| Table 2 – Numbers of tourists (in '000s)							
Year	2005	2007	2009	2011	2013	2015	2017
Total Overseas							9023

     (ii) Based on the data in Table 2, write down the year in which the tourism sector in Ireland recovered following the financial crash of 2008 and give a reason for your answer.

Year:

Reason:

Pre-Leaving Certificate Examination, 2019   10
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109
2019.1 L.19   10/20
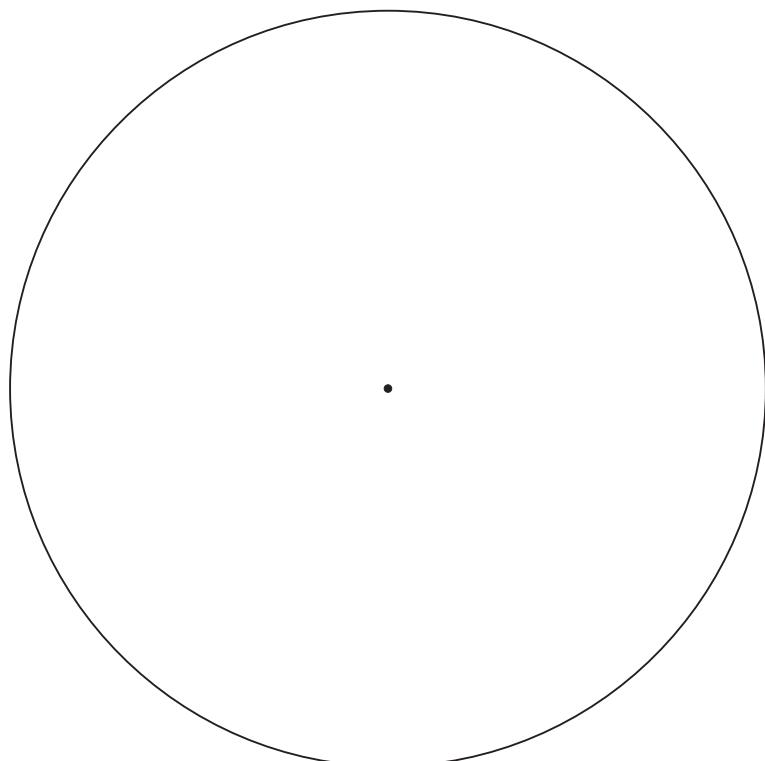
(c) A pie chart to represent the numbers of tourists from all geographical regions outside Ireland in 2017 is to be drawn.

    (i) Complete Table 3 to show the angle corresponding to each geographical region in the pie chart. Give your answers in degrees, correct to one decimal place.

Table 3 – Numbers of tourists		
Region	**Number ('000s)**	**Angle (degrees)**
Britain	3445	
Mainland Europe	3256	
North America	1715	
Rest of World	607	21·1°
Northern Ireland	1315	

    (ii) Draw a pie chart to represent the data in Table 3. Label the sector corresponding to each geographical region and write the size of the angle in each sector.

Pre-Leaving Certificate Examination, 2019
Mathematics, Paper 2 – Ordinary Level
11
4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 11/20

| previous | page | running |

(d) Tourism is measured not only by the number of tourists to Ireland but also by the revenue they generated in the Irish economy.

(i) Complete Table 4 to show the average spend by tourists from each geographical region in 2017. Give your answers correct to the nearest euro.

Table 4 – Total revenue generated by tourists in 2017			
Region	**Number ('000s)**	**Revenue (€m)**	**Average spend (€)**
Britain	3445	1043·5	
Mainland Europe	3256	1763·0	541
North America	1715	1525·5	
Rest of World	607	588·9	
Northern Ireland	1315	371·3	
Domestic Trips	9626	1879·1	

(ii) Fáilte Ireland is the state body responsible for promoting Ireland as a tourist destination. Based on the data in Table 1 (on page 10) and your answers to part **(d)(i)** above, in which geographical region would you most recommend undertaking a marketing campaign to promote tourism to Ireland? Give a reason for your answer.

Answer:

Reason:

(e) It is estimated that every €1 million generated in Ireland by tourism helps support 27 jobs and that every 1000 additional overseas tourists create 19 jobs in the tourism sector.

(i) Use this information and the data in Table 4 to estimate the total number of jobs that tourism supported in 2017. Give your answer correct to two significant figures.

Pre-Leaving Certificate Examination, 2019
12
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 12/20

(ii) In total, 9·023 million overseas tourists visited Ireland in 2017.
Find the number of extra jobs that would be created if this number exceeded 10 million.
Hence find the additional revenue generated by this increased level of tourism.
Give your answer in millions of euro, correct to the nearest whole number.

(f) A survey was carried out on behalf of Fáilte Ireland to investigate whether tourists to Ireland would recommend this country as a holiday destination.

(i) 750 overseas tourists were surveyed. Find the margin of error of the survey.
Give your answer as a percentage, correct to two decimal places.

(ii) In the survey, 582 of those tourists surveyed said that they would recommend Ireland as a holiday destination.
Use your answer to part **(f)(i)** above to create a 95% confidence interval for the percentage of overseas tourists who said that they would recommend Ireland.

(iii) After the survey, a media campaign claimed that 80% of overseas tourists would recommend Ireland as a holiday destination. Use your answer to part **(f)(ii)** above to conduct a hypothesis test, at the 5% level of significance, to test this claim.
State your null hypothesis, your alternative hypothesis and give your conclusion in the context of the question.

Pre-Leaving Certificate Examination, 2019 13
Mathematics, Paper 2 – Ordinary Level 4e6b4069-6072-49fb-a557-f5acf7e96109

| previous | page | running |

2019.1 L.19 13/20
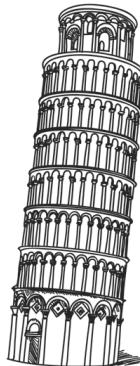
Question 8 (50 marks)

The Leaning Tower of Pisa is a freestanding bell tower in the Italian city of Pisa, known worldwide for its unintended tilt.

(a) To measure the slant height of the bell tower, h, Marco stood at the point A, which is on level ground 94 m from the centre of the base of the tower, C. From there, he observed the angle of elevation to the centre of the top of the tower, D, to be 29·88°. He then used the trigonometric tan ratio to find the height.
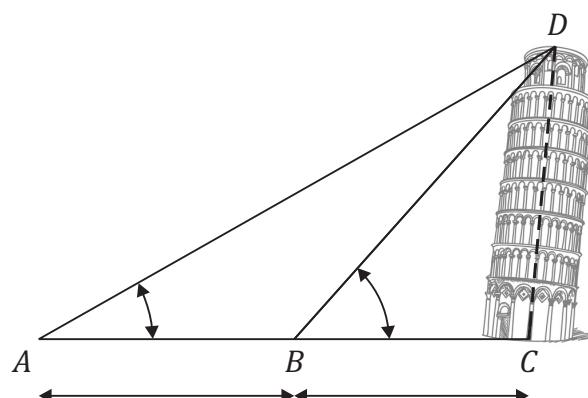
(i) Find $|CD|$, the slant height of the tower, according to Marco. Give your answer in metres, correct to two decimal places.

(ii) Explain why the slant height calculated by Marco is not completely accurate.
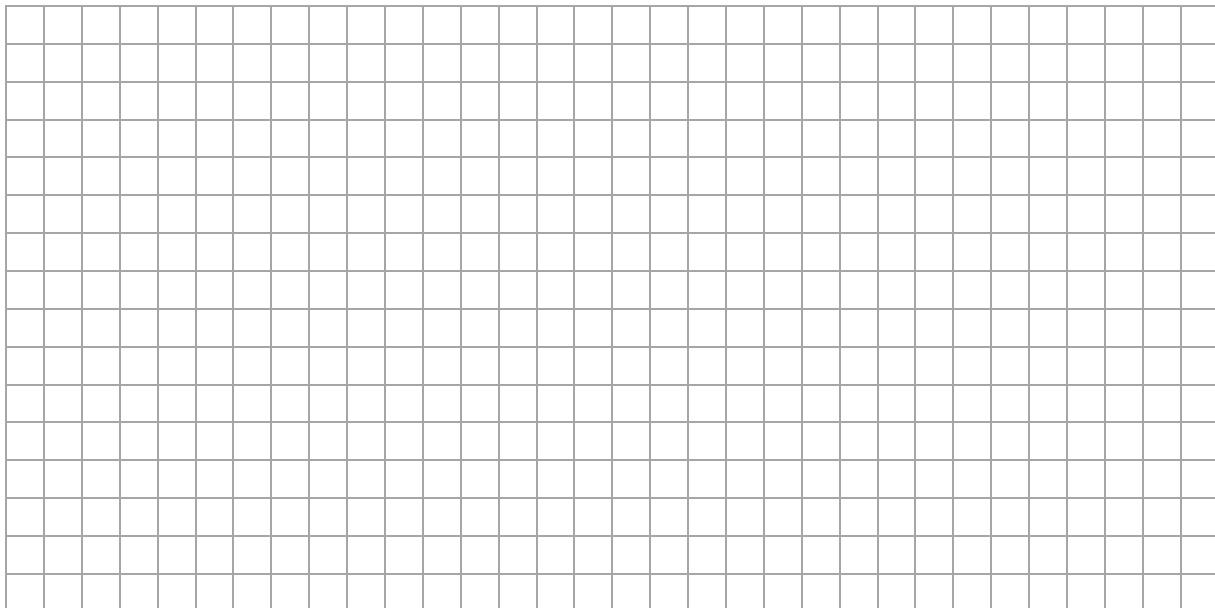
(b) Lucia realised Marco's error and decided to take a second set of measurements. From the point B, which is on level ground 45 m from C, she observed the angle of elevation to D to be 49°.

(i) On the diagram below, mark the two sets of measurements observed.
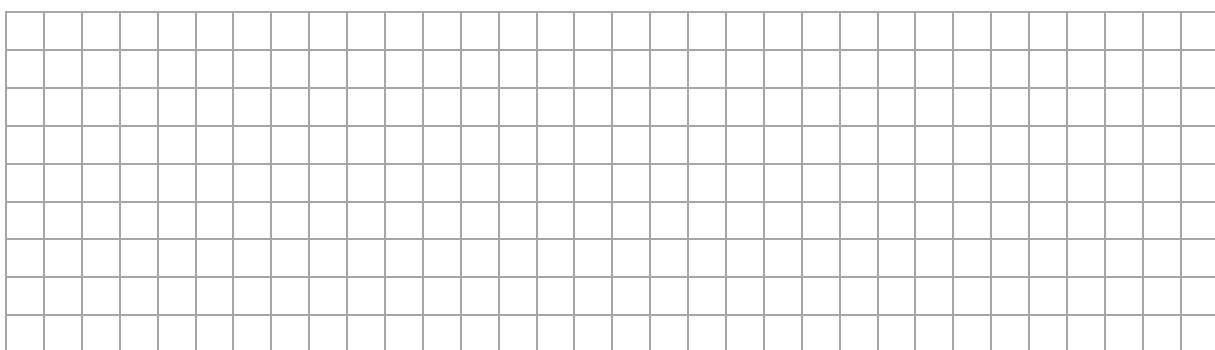
(ii) Use the Sine Rule to show that $|BD| = 74·53$ m, correct to two decimal places.

Pre-Leaving Certificate Examination, 2019
Mathematics, Paper 2 – Ordinary Level   14   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 14/20
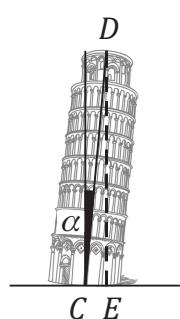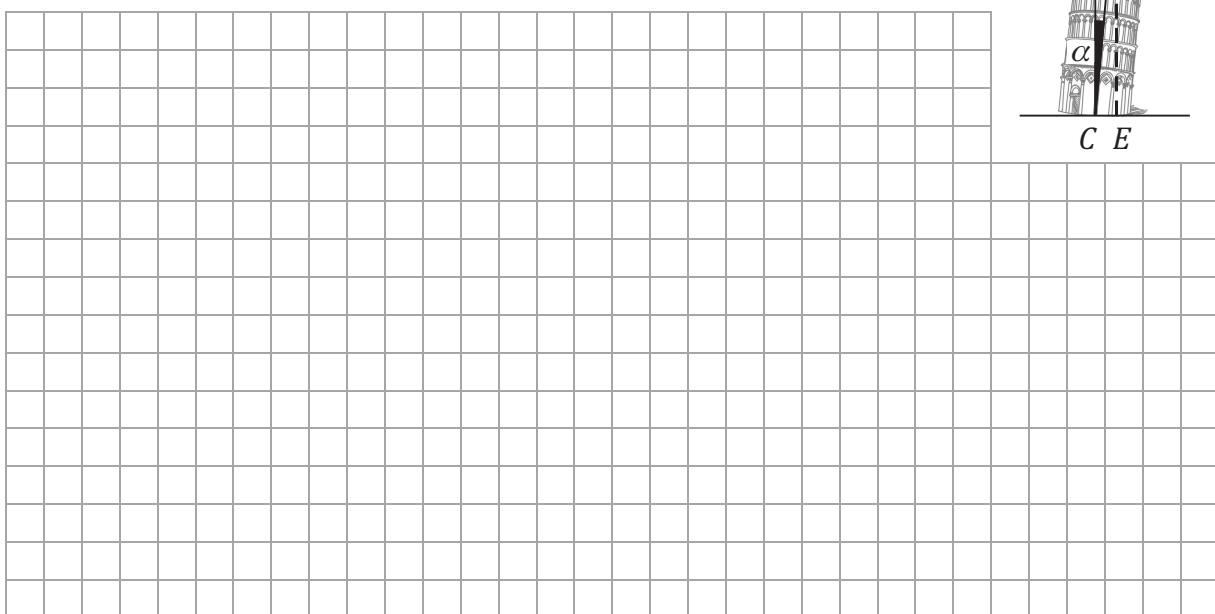
(iii) Hence find $|CD|$, the slant height of the tower, correct to one decimal place.

(iv) Find the percentage error in Marco's calculation of the slant height of the tower. Give your answer correct to two decimal places.

(c) Use trigonometry to find $|CE|$, the distance by which the top of the tower is off centre from the base.
Hence find α, the angle at which the tower tilts from the vertical axis.
Give your answers correct to two decimal places.

Pre-Leaving Certificate Examination, 2019
15
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

previous | page | running

2019.1 L.19 15/20

Question 9 (40 marks)

The reservoir tank in a water tower is in the shape of an inverted right cone.
The cone has a diameter of 24 m and a height of 9 m.
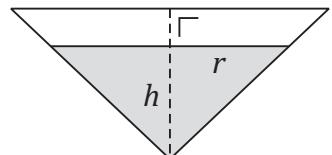
(a) **(i)** Write down the radius of the reservoir tank.

(ii) Find the capacity (volume) of the reservoir tank, in terms of π.

(b) Water used from the reservoir tank during the day is replaced at night. The volume of the tank is refilled to 90% of its capacity.

The diagram (not to scale) shows h, the depth of the water, and r, the surface radius of the water, in the tank after it is refilled.

(i) Using similar triangles, or otherwise, show that $r = \dfrac{4h}{3}$.

(ii) Hence find h, the depth of the water in the tank after it is refilled.
Give your answer in metres, correct to one decimal place.

Pre-Leaving Certificate Examination, 2019   16
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 16/20

(c)   The water tower automatically shuts down when the volume of water in the reservoir tank falls below 10% of its capacity. It restarts when it reaches 90% of its capacity.

For how long is the water tower not operational if water can only be pumped into the reservoir tank at the rate of 160 litres per second?
Give your answer in hours and minutes. [1000 litres = 1 m³]

(d)   Water charges are being re-introduced this year for excessive usage. Households using more water than a specified threshold will be charged for water usage above that amount. The threshold figure for all households has been set at 213 000 litres per year, with an extra allowance of 25 000 litres per person per year for households with 5 or more people.

(i)   Find the average amount of water that a 7-person household is allowed to use per day before water charges are applied. Give your answer correct to the nearest litre.

(ii)   Water usage above the threshold figure set for each household is expected to be charged at €3·70 plus VAT at 13·5% per 1000 litres.
Given that the same 7-person household continuously uses 950 litres of water per day, find what the household can expect to pay in water charges in a full year.
Give your answer correct to the nearest cent.

You may use this page for extra work.
Label any extra work clearly with the question number and part.

Pre-Leaving Certificate Examination, 2019   18
Mathematics, Paper 2 – Ordinary Level   4e6b4069-6072-49fb-a557-f5acf7e96109

2019.1 L.19 18/20

You may use this page for extra work.
Label any extra work clearly with the question number and part.

Pre-Leaving Certificate Examination, 2019
19
4e6b4069-6072-49fb-a557-f5acf7e96109
Mathematics, Paper 2 – Ordinary Level

2019.1 L.19 19/20

previous	page	running