Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION, 2006

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

THURSDAY, 8 JUNE – MORNING, 9:30 to 12:00

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Attempt **ALL** questions.

Each question carries 50 marks.

Graph paper may be obtained from the superintendent.

The symbol ✍ indicates that supporting work <u>must</u> be shown to obtain full marks.

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1. **(a)** The price of a DVD increases from €12·50 to €13·75.

✍ Express this increase as a percentage of the original price.

(b) **(i)** The time taken by Jack to travel from Derry to Waterford, a distance of 378 km, is 6 hours.

His return journey from Waterford to Derry, by the same route, takes an extra 45 mins.

✍ By how many km/h is his average speed slower on the return journey?

(ii) Jill has a gross income of €50 000.

Her total income tax payable amounts to €10 460.

The standard rate cut off point is €32 000.

The standard rate of tax is 20% and the higher rate is 42%.

✍ What are Jill's tax credits for the year?

(c) **(i)** ✍ By rounding to the nearest whole number, estimate the value of

$$\frac{\sqrt{42 \cdot 91 + 21 \cdot 3}}{17 \cdot 56 - 3 \cdot 7 \times 4 \cdot 2}.$$

Then, evaluate $\dfrac{\sqrt{42 \cdot 91 + 21 \cdot 3}}{17 \cdot 56 - 3 \cdot 7 \times 4 \cdot 2}$, correct to two decimal places.

(ii) A sum of money was invested at compound interest for two years.

The interest rate for each year was 5%.

After the two years the sum amounted to €5512·50.

✍ Calculate the original sum of money invested.

2. **(a)** Derek processed 390 passport applications during the month of July.

He processed 10% fewer applications during the month of August.

✎ How many applications did he process in August?

(b) 70 teenagers responded to a survey about holiday destinations.

30 had travelled to France,

26 had travelled to Spain

and 28 had travelled to Italy.

12 had travelled to both France and Spain,

8 had travelled to both Spain and Italy,

while x had travelled to France and Italy only.

4 teenagers had travelled to all three countries.

Twice as many had never travelled to any of these destinations as had travelled to France and Italy only.

(i) ✎ Represent the above information on a Venn diagram.

(ii) ✎ Find the number of teenagers who had travelled to France only.

(c) **(i)** A merchant buys tea for €3·29 per kg and then sells it at a profit of 60% of the cost price to a customer in England.

The exchange rate is £1 (sterling) = €1·46.

✎ Calculate the selling price of the tea in £ sterling, correct to two decimal places.

(ii) The exchange rate changes to £1 (sterling) = €1·50. The selling price, in sterling, remains the same.

✎ Calculate the merchant's percentage profit in this case, correct to the nearest whole number.

3. **(a)** **(i)** Simplify $\left(64\right)^{3/2}$.

(ii) Simplify $\left(64\right)^{2/3}$.

(b) Let the cost of a meal for an adult be €x and the cost of a meal for a child be €y.

The cost of a meal for 3 adults and 2 children amounts to €125.

The cost of a meal for 2 adults and 3 children amounts to €115.

(i) Write down two equations in x and y to represent this information.

(ii) ✍ Solve these equations to find the cost of an adult's meal

and the cost of a child's meal.

(c) **(i)** ✍ Express in its simplest form:

$$\frac{1}{2x-3} - \frac{1}{2x+3}.$$

(ii) ✍ Hence, or otherwise, solve the equation:

$$\frac{1}{2x-3} - \frac{1}{2x+3} = \frac{6}{7}, \quad x \in \mathbf{Z}.$$

4. **(a)** ✍ Express in its simplest form:

$$2x - [3 - (4 - 3x)] + 6.$$

(b) **(i)** ✍ Solve $x^2 - 4x - 8 = 0$, giving your answer in the form

$a \pm a\sqrt{b}$, where $a, b \in \mathbf{N}$.

(ii) Factorise $9x^2 - 16y^2$.

(iii) The length of one side of a rectangle is $x + 4$.

The area of the rectangle is $x^2 + 16x + 48$.

✍ Find an expression in x for the length of the other side.

(c) A farmer must feed bales of hay to his cattle for a total of 90 days.

He feeds the cattle 540 bales of hay over a number of days.

Their average consumption over this period is x bales per day.

(i) Write an expression in x for the number of days taken to consume the 540 bales.

If the average consumption is increased by 1 bale per day, then the cattle consume 300 bales in the remaining days.

(ii) Write an expression in x for the number of days taken to consume the 300 bales.

(iii) Using the above information, write an equation in x.

(iv) ✍ Solve this equation to find the value of x and the number of days taken to consume the first 540 bales.

5. **(a)** The temperature on Sunday is $x°$.

The temperature rose by $3°$ each day for the next two days.

The temperature then dropped by $4°$ each day for the next three days.

✍ Derive an expression in x for the temperature on the fifth day (i.e. Friday).

(b) Let f be the function $f : x \rightarrow 35x - 5x^2$.

Draw the graph of f for $0 \le x \le 7, \ x \in \mathbf{R}$.

(c) The formula for the height, y metres, of a ball above ground level, x seconds after it is fired vertically into the air, is given by:

$$y \ = \ 35x - 5x^2.$$

Use your graph from part **(b)** to estimate:

(i) ✍ the maximum height reached by the ball

(ii) ✍ the height of the ball after $5·5$ seconds.

On two occasions the ball is 20 metres above the ground.

(iii) ✍ Use your graph from part **(b)** to estimate the two times when this occurred.

6. **(a)** Given that:

$$v^2 = u^2 + 2as.$$

✍ Write s in terms of v, u and a.

(b) **(i)** ✍ Factorise $2l - kl + km - 2m$.

(ii) Factorise $6x^2 - 19x + 10$.

(iii) Factorise $17x - 5x^2$.

(c) Let f be the function $f : x \rightarrow 1 - 3x$ and g be the function $g : x \rightarrow 1 - x^2$.

(i) ✍ Find $f(-2)$ and $g(5)$.

(ii) ✍ Express $f(x + 1)$ in the form $ax + b$, a and $b \in \mathbf{Z}$.

(iii) ✍ Solve for x: $f(x + 1) = f(-2) + g(5)$.

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