

JUNIOR CERTIFICATE EXAMINATION, 1994

MATHEMATICS – HIGHER LEVEL – PAPER 1 (300 marks)

THURSDAY, 9 JUNE - MORNING, 9.30 to 12.00

Attempt QUESTION 1 (100 marks) and FOUR other questions (50 marks each).

Marks may be lost if all your work is not clearly shown.
Mathematics Tables may be obtained from the Superintendent.

1.

 - (i) An athlete ran 1500 metres in 4 minutes.
Find the average speed of the athlete in metres per second.

 - (ii) A bill of IR£62.92 includes V.A.T. at the rate of 21%. Calculate the amount of the V.A.T. in this bill.

 - (iii) Two solid cones of equal height have the lengths of the radii of their bases in the ratio 1:2.
Calculate the ratio of their volumes.

 - (iv) Solve for x : $\frac{1}{x} = 5 - 4x$.

 - (v) If $\frac{ap}{3} = \frac{p}{2} + c$, express p in terms of a and c .

 - (vi) If $f : x \rightarrow 5x - 1$, find $f(1)$.
Find an expression for f^{-1} in the form $f^{-1} : x \rightarrow ax + b$ and hence or otherwise find $f^{-1}(19)$.

 - (vii) Calculate the value of x if
 $3 + \log_5 x = \log_3 81$.

(viii) Find the value of x if $\log_x \left(\frac{1}{8}\right) = 3$.

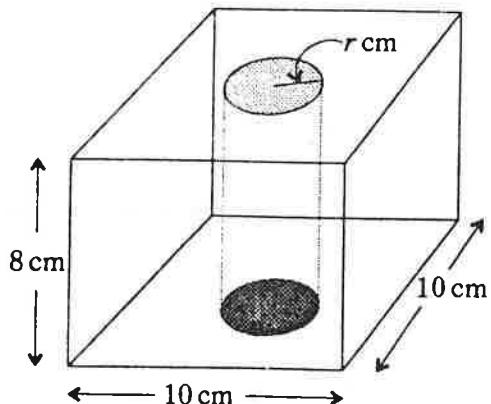
(ix) If $a^*b = (2a - 1)b$, write out a^*a .
Calculate the two values of a for which $a^*a = a^*3$.

(x) Show on the number line the range of values of $x \in \mathbb{R}$ for which $x^2 \leq 1$.

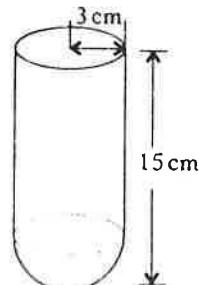
2. (a) (i) A solid rectangular block has a square base of side 10 cm and is 8 cm high. Calculate the volume of the block.

(ii) A cylindrical hole is drilled all the way through the block as shown, that is, the cylindrical hole is of height 8 cm and has radius of length r cm. The volume of the block remaining is 56% of the total volume of the block.

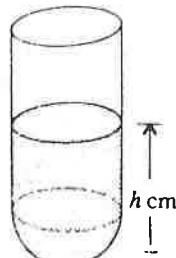
Calculate the radius length r of the cylindrical hole, taking $\pi = 22/7$. Give your answer correct to 2 places of decimals.



(b) (i) A container is in the shape of a cylinder on top of a hemisphere as shown. The cylinder has a radius of length 3 cm and the container has total height of 15 cm. Calculate the volume of the container in terms of π .



(ii) If half the volume of the container is filled with liquid, calculate the height, h , of liquid in the container.



3. (a) Factorize each of the following

(i) $3x^2 + 22x - 16$

(ii) $4a^2b - 3b - 6a + 2ab^2$

- (b) Solve the equation $4x^2 + 7x - 2 = 0$.

Hence, or otherwise, find the values of y for which

$$2 - 7(y^2 - 6) - 4(y^2 - 6)^2 = 0.$$

- (c) Solve for x :

$$\frac{x+3}{x+5} + \frac{x-2}{x+1} = 1.$$

4. The table below shows the distribution of the amount of money spent by 100 customers in a supermarket.

Amount of money (IR£)	0-5	5-10	10-15	15-20	20-30
Number of customers	18	22	28	20	12

Note: 0-5 means 0 or more but less than 5 etc.

Draw the histogram of this distribution.

Using mid-interval values, estimate the mean amount spent per customer.

Complete the cumulative frequency table:

Amount of money (IR£)	<5	<10	<15	<20	<30
Number of customers					

Draw the cumulative frequency curve (ogive) from this table, putting numbers of customers on the vertical axis.

Use this curve to estimate the maximum number of customers who could have spent IR£8 or more but not more than IR£17.

5. (a) $f : x \rightarrow 3x^2 - 1$ and $g : x \rightarrow 2x$ are two functions defined on \mathbb{R} .
Find expressions in x for

(i) $(f \circ g)(x)$
(ii) $(g \circ f)(x)$.

- (b) Graph the function $f : x \rightarrow 7 + 5x - 2x^2$ in the domain $-1 \leq x \leq 4$, $x \in \mathbb{R}$.

Use your graph, or otherwise, to solve $7 + 5x - 2x^2 = 0$.

$f(x)$ is the height in metres reached by a particle fired from level ground at the point where $x = -1$, the x axis representing the level ground. From the time of firing until it hits the ground again, the particle was in flight for exactly 4.5 seconds.

Use your graph to estimate

- (i) the maximum height reached by the particle
(ii) the height reached by the particle after 1.5 seconds of flight
(iii) the number of seconds the particle is 4 m or more above the ground.

6. (a) Solve the simultaneous equations:

$$\begin{aligned} 5x + y &= 19 \\ 2x - y &= 2y - 2x \end{aligned}$$

- (b) A person spent IR£7.20 in buying x identical pens.
If each pen had been 4p cheaper, then an extra 2 pens could have been purchased for the IR£7.20.

Find the value of x .