Strand 1: Statistics and Probability – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.1 Counting	– count the arrangements of n distinct objects ($n!$) – count the number of ways of arranging r objects from n distinct objects	– count the number of ways of selecting r objects from n distinct objects – compute binomial coefficients
1.2 Concepts of probability	– use set theory to discuss experiments, outcomes, sample spaces – discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams – calculate expected value and understand that this does not need to be one of the outcomes – recognise the role of expected value in decision making and explore the issue of fair games	– extend their understanding of the basic rules of probability (AND/OR, mutually exclusive) through the use of formulae – Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ – Multiplication Rule (Independent Events): $P(A \cap B) = P(A) \times P(B)$ – Multiplication Rule (General Case): $P(A \cap B) = P(A) \times P(B \mid A)$ – solve problems involving sampling, with or without replacement – appreciate that in general $P(A \mid B) \neq P(B \mid A)$ – examine the implications of $P(A \mid B) \neq P(B \mid A)$ in context
1.3 Outcomes of random processes	– find the probability that two independent events both occur – apply an understanding of Bernoulli trials* – solve problems involving up to 3 Bernoulli trials – calculate the probability that the 1^{st} success occurs on the n^{th} Bernoulli trial where n is specified	– solve problems involving calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required) – calculate the probability that the k^{th} success occurs on the n^{th} Bernoulli trial – use simulations to explore the variability of sample statistics from a known population, to construct sampling distributions and to draw conclusions about the sampling distribution of the mean – solve problems involving reading probabilities from the normal distribution tables
1.4 Statistical reasoning with an aim to becoming a statistically aware consumer	– discuss populations and samples – decide to what extent conclusions can be generalised – work with different types of bivariate data	

* A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: “success” or “failure”.

Strand 1: Statistics and Probability – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.5 Finding, collecting and organising data	– select a sample (Simple Random Sample) – recognise the importance of representativeness so as to avoid biased samples – discuss different types of studies: sample surveys, observational studies and designed experiments – design a plan and collect data on the basis of above knowledge	– recognise the importance of randomisation and the role of the control group in studies – recognise biases, limitations and ethical issues of each type of study – select a sample (stratified, cluster, quota – no formulae required, just definitions of these) – design a plan and collect data on the basis of above knowledge
1.6 Representing data graphically and numerically	**Graphical** – describe the sample (both univariate and bivariate data) by selecting appropriate graphical or numerical methods – explore the distribution of data, including concepts of symmetry and skewness – compare data sets using appropriate displays including back-to-back stem and leaf plots – determine the relationship between variables using scatterplots – recognise that correlation is a value from -1 to +1 and that it measures the extent of the linear relationship between two variables – match correlation coefficient values to appropriate scatterplots – understand that correlation does not imply causality **Numerical** – recognise standard deviation and interquartile range as measures of variability – use a calculator to calculate standard deviation – find quartiles and the interquartile range – use the interquartile range appropriately when analysing data – recognise the existence of outliers	**Graphical** – analyse plots of the data to explain differences in measures of centre and spread – draw the line of best fit by eye – make predictions based on the line of best fit – calculate the correlation coefficient by calculator **Numerical** – recognise the effect of outliers – use percentiles to assign relative standing

Strand 1: Statistics and Probability – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.7 Analysing, interpreting and drawing inferences from data	– recognise how sampling variability influences the use of sample information to make statements about the population – use appropriate tools to describe variability drawing inferences about the population from the sample – interpret the analysis and relate the interpretation to the original question – interpret a histogram in terms of distribution of data – make decisions based on the empirical rule – recognise the concept of a hypothesis test – calculate the margin of error ($\frac{1}{\sqrt{n}}$) for a population proportion* – conduct a hypothesis test on a population proportion using the margin of error	– build on the concept of margin of error and understand that increased confidence level implies wider intervals – construct 95% confidence intervals for the population mean from a large sample and for the population proportion, in both cases using z tables – use sampling distributions as the basis for informal inference – perform univariate large sample tests of the population mean (two-tailed z-test only) – use and interpret p-values

* The margin of error referred to here is the maximum value of the radius of the 95% confidence interval.

Strand 2: Geometry and Trigonometry – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
2.1 Synthetic geometry	– perform constructions **16-21** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies – investigate theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 and corollary 6 (see Geometry for Post-primary School Mathematics) and use them to solve problems	– perform construction **22** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems 11,12,13, concerning ratios (see Geometry for Post-primary School Mathematics), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle
2.2 Co-ordinate geometry	– use slopes to show that two lines are • parallel • perpendicular – recognise the fact that the relationship $ax + by + c = 0$ is linear – solve problems involving slopes of lines – calculate the area of a triangle – recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with centre (h, k) and radius r – solve problems involving a line and a circle with centre (0, 0)	– solve problems involving • the perpendicular distance from a point to a line • the angle between two lines – divide a line segment internally in a given ratio $m: n$ – recognise that $x^2+y^2+2gx+2fy+c = 0$ represents the relationship between the x and y co-ordinates of points on a circle with centre $(-g,-f)$ and radius r where $r = \sqrt{(g^2+f^2-c)}$ – solve problems involving a line and a circle
2.3 Trigonometry	– use of the theorem of Pythagoras to solve problems (2D only) – use trigonometry to calculate the area of a triangle – solve problems using the sine and cosine rules (2D) – define sin θ and cos θ for all values of θ – define tan θ – solve problems involving the area of a sector of a circle and the length of an arc – work with trigonometric ratios in surd form	– use trigonometry to solve problems in 3D – graph the trigonometric functions sine, cosine, tangent – graph trigonometric functions of type • $f(\boldsymbol{\theta}) = a+b\text{Sin } c\boldsymbol{\theta}$ • $g(\boldsymbol{\theta}) = a+b\text{Cos } c\boldsymbol{\theta}$ for $a,b,c \in \mathbf{R}$ – solve trigonometric equations such as Sin $n\boldsymbol{\theta}=0$ and Cos $n\boldsymbol{\theta}= \frac{1}{2}$ giving all solutions – use the radian measure of angles – derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix) – apply the trigonometric formulae 1-24 (see appendix)
2.4 Transformation geometry, enlargements	– investigate enlargements and their effect on area, paying attention to • centre of enlargement • scale factor k where $0<k<1$, $k>1$ $k \in \mathbf{Q}$ – solve problems involving enlargements	

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems	– recognise irrational numbers and appreciate that **R** ≠ **Q** – work with irrational numbers – revisit the operations of addition, multiplication, subtraction and division in the following domains: • **N** of natural numbers • **Z** of integers • **Q** of rational numbers • **R** of real numbers and represent these numbers on a number line – investigate the operations of addition, multiplication, subtraction and division with complex numbers **C** in rectangular form a+ib – illustrate complex numbers on an Argand diagram – interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate – develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place-value understanding – consolidate their understanding of factors, multiples, prime numbers in **N** – express numbers in terms of their prime factors – appreciate the order of operations, including brackets – express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form	– geometrically construct √2 and √3 – prove that √2 is not rational – calculate conjugates of sums and products of complex numbers – verify and justify formulae from number patterns – investigate geometric sequences and series – prove by induction • simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series • simple inequalities such as $n! > 2^n$, $2^n \geq n^2$ $(n \geq 4)$ $(1+x)^n \geq 1+nx$ $(x > -1)$ • factorisation results such as 3 is a factor of $4^n - 1$ – apply the rules for sums, products, quotients of limits – find by inspection the limits of sequences such as $\lim_{n\to\infty} \frac{n}{n+1}$; $\lim_{n\to\infty} r^n$, $\lvert r \rvert < 1$ – solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment – derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems (continued)	– appreciate that processes can generate sequences of numbers or objects – investigate patterns among these sequences – use patterns to continue the sequence – generalise and explain patterns and relationships in algebraic form – recognise whether a sequence is arithmetic, geometric or neither – find the sum to n terms of an arithmetic series	
3.2 Indices	– solve problems using the rules for indices (where $a, b \in \mathbf{R}$; $p, q \in \mathbf{Q}$; $a^p, a^q \in \mathbf{Q}$; $a, b \neq \mathbf{0}$): • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $a^0 = 1$ • $(a^p)^q = a^{pq}$ • $a^{\frac{1}{q}} = \sqrt[q]{a}\quad q \in \mathbf{Z},\ q \neq 0, a > 0$ • $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p\ p, q \in \mathbf{Z},\ q \neq 0, a > 0$ • $a^{-p} = \frac{1}{a^p}$ • $(ab)^p = a^p b^p$ • $(\frac{a}{b})^p = \frac{a^p}{b^p}$	– solve problems using the rules of logarithms • $\log_a(xy) = \log_a x + \log_a y$ • $\log_a(\frac{x}{y}) = \log_a x - \log_a y$ • $\log_a x^q = q\log_a x$ • $\log_a a = 1$ and $\log_a 1 = 0$ • $\log_a x = \frac{\log_b x}{\log_b a}$

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.3 Arithmetic	– check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – accumulate error (by addition or subtraction only) – make and justify estimates and approximations of calculations; calculate percentage error and tolerance – calculate average rates of change (with respect to time) – solve problems that involve • calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) • compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions) • costing: materials, labour and wastage • metric system; change of units; everyday imperial units (conversion factors provided for imperial units) – make estimates of measures in the physical world around them	– use *present value* when solving problems involving loan repayments and investments
3.4 Length, area and volume	– investigate the nets of prisms, cylinders and cones – solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs, and figures made from combinations of these – solve problems involving surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular-based prism (right angle, isosceles and equilateral), sphere, hemisphere, and solids made from combinations of these – use the trapezoidal rule to approximate area	

Strand 4: Algebra – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
4.1 Expressions	– evaluate expressions given the value of the variables – expand and re-group expressions – factorise expressions of order 2 – add and subtract expressions of the form • $(ax+by+c) \pm\ldots\pm (dx+ey+f)$ • $(ax^2+bx+c) \pm\ldots\pm (dx^2+ex+f)$ where $a,b,c,d,e,f \in \mathbf{Z}$ • $\dfrac{a}{bx+c} \pm \dfrac{p}{qx+r}$ where $a,b,c,p,q,r \in \mathbf{Z}$ – use the associative and distributive properties to simplify expressions of the form • $a(bx \pm cy \pm d) \pm\ldots\pm e(fx \pm gy \pm h)$ where $a, b, c, d, e, f, g, h \in \mathbf{Z}$ • $(x \pm y)(w \pm z)$ – rearrange formulae	– perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds – apply the binomial theorem

Strand 4: Algebra – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
4.2 Solving equations	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x) = g(x)$, with $f(x) = ax+b$, $g(x) = cx+d$ where $a, b, c, d \in \mathbf{Q}$ • $f(x) = g(x)$ with $f(x) = \frac{a}{bx+c} \pm \frac{p}{qx+r}$; $g(x) = \frac{e}{f}$ where $a, b, c, e, f, p, q, r \in \mathbf{Z}$ • $f(x) = k$ with $f(x) = ax^2 + bx + c$ (and not necessarily factorisable) where $a, b, c \in \mathbf{Q}$ and interpret the results – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to • simultaneous linear equations with two unknowns and interpret the results • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is ± 1 in the linear equation) and interpret the results – form quadratic equations given whole number roots	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$ with $f(x) = \frac{ax+b}{ex+f} \pm \frac{cx+d}{qx+r}$; $g(x) = k$ where $a, b, c, d, e, f, q, r \in \mathbf{Z}$ – use the Factor Theorem for polynomials – select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to • cubic equations with at least one integer root • simultaneous linear equations with three unknowns • one linear equation and one equation of order 2 with two unknowns and interpret the results
4.3 Inequalities	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$, • $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and $a, b, k \in \mathbf{Q}$	– use notation $\mid x \mid$ – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$; • $g(x) < k$, $g(x) > k$, with $g(x) = ax^2+bx+c$ or $g(x) = \frac{ax+b}{cx+d}$ and $a, b, c, d, k \in \mathbf{Q}$, $x \in \mathbf{R}$ • $\mid x - a \mid < b$, $\mid x - a \mid > b$ and combinations of these, with $a, b, \in \mathbf{Q}$, $x \in \mathbf{R}$
4.4 Complex Numbers	See strand 3, section 3.1	– use the Conjugate Root Theorem to find the roots of polynomials – work with complex numbers in rectangular and polar form to solve quadratic and other equations including those in the form $z^n = a$, where $n \in \mathbf{Z}$ and $z = r(\text{Cos}\ \theta + i\text{Sin}\ \theta)$ – use De Moivre's Theorem – prove De Moivre's Theorem by induction for $n \in \mathbf{N}$ – use applications such as n^{th} roots of unity, $n \in \mathbf{N}$, and identities such as $\text{Cos}\ 3\theta = 4\ \text{Cos}^3\ \theta - 3\ \text{Cos}\ \theta$

Strand 5: Functions – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
5.1 Functions	– recognise that a function assigns a unique output to a given input – form composite functions – graph functions of the form • $ax+b$ where $a,b \in \mathbf{Q}$, $x \in \mathbf{R}$ • ax^2+bx+c where $a, b, c \in \mathbf{Z}$, $x \in \mathbf{R}$ • ax^3+bx^2+cx+d where $a,b,c,d \in \mathbf{Z}$, $x \in \mathbf{R}$ • ab^x where $a \in \mathbf{N}$, $b, x \in \mathbf{R}$ – interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions – use graphical methods to find approximate solutions to • $f(x) = 0$ • $f(x) = k$ • $f(x) = g(x)$ where $f(x)$ and $g(x)$ are of the above form, or where graphs of $f(x)$ and $g(x)$ are provided – investigate the concept of the limit of a function	– recognise surjective, injective and bijective functions – find the inverse of a bijective function – given a graph of a function sketch the graph of its inverse – express quadratic functions in complete square form – use the complete square form of a quadratic function to • find the roots and turning points • sketch the function – graph functions of the form • ax^2+bx+c where $a,b,c \in \mathbf{Q}$, $x \in \mathbf{R}$ • ab^x where $a, b \in \mathbf{R}$ • logarithmic • exponential • trigonometric – interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions – informally explore limits and continuity of functions
5.2 Calculus	– find first and second derivatives of linear, quadratic and cubic functions by rule – associate derivatives with slopes and tangent lines – apply differentiation to • rates of change • maxima and minima • curve sketching	– differentiate linear and quadratic functions from first principles – differentiate the following functions • polynomial • exponential • trigonometric • rational powers • inverse functions • logarithms – find the derivatives of sums, differences, products, quotients and compositions of functions of the above form – apply the differentiation of above functions to solve problems – use differentiation to find the slope of a tangent to a circle – recognise integration as the reverse process of differentiation – use integration to find the average value of a function over an interval – integrate sums, differences and constant multiples of functions of the form • x^a where $a \in \mathbf{Q}$ • a^x where $a \in \mathbf{R}$, $a>0$ • Sin ax where $a \in \mathbf{R}$ • Cos ax where $a \in \mathbf{R}$ – determine areas of plane regions bounded by polynomial and exponential curves