L.16

2020-L016-1-EL-01/28

Pre-Leaving Certificate Examination, 2020

Mathematics

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

CANDIDATE DETAILS

EXAM ID

Optional 4 or 5-digit number
(only if provided by your school)

NAME

SCHOOL

TEACHER

2020-L016-1-EL-01/28

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your Exam ID, Name, School's Name and Teacher's Name in the grid on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

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Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

Tom and Anne are planning a working holiday to Australia. They intend to save €120 per month and to increase this amount by €10 per month until they go. The amount of money they save each month can be represented by an arithmetic sequence.

(a) **(i)** Complete the table below to show the amount of money that Tom and Anne intend to save each month for the period shown.

Month	1	2	3	4	6	12
Monthly savings (€)	120	130				

(ii) Find the **total** amount of money that they intend to save over 18 months.

(iii) Tom and Anne decide they will travel to Australia after they have saved €4300. For how many months do they intend to save?

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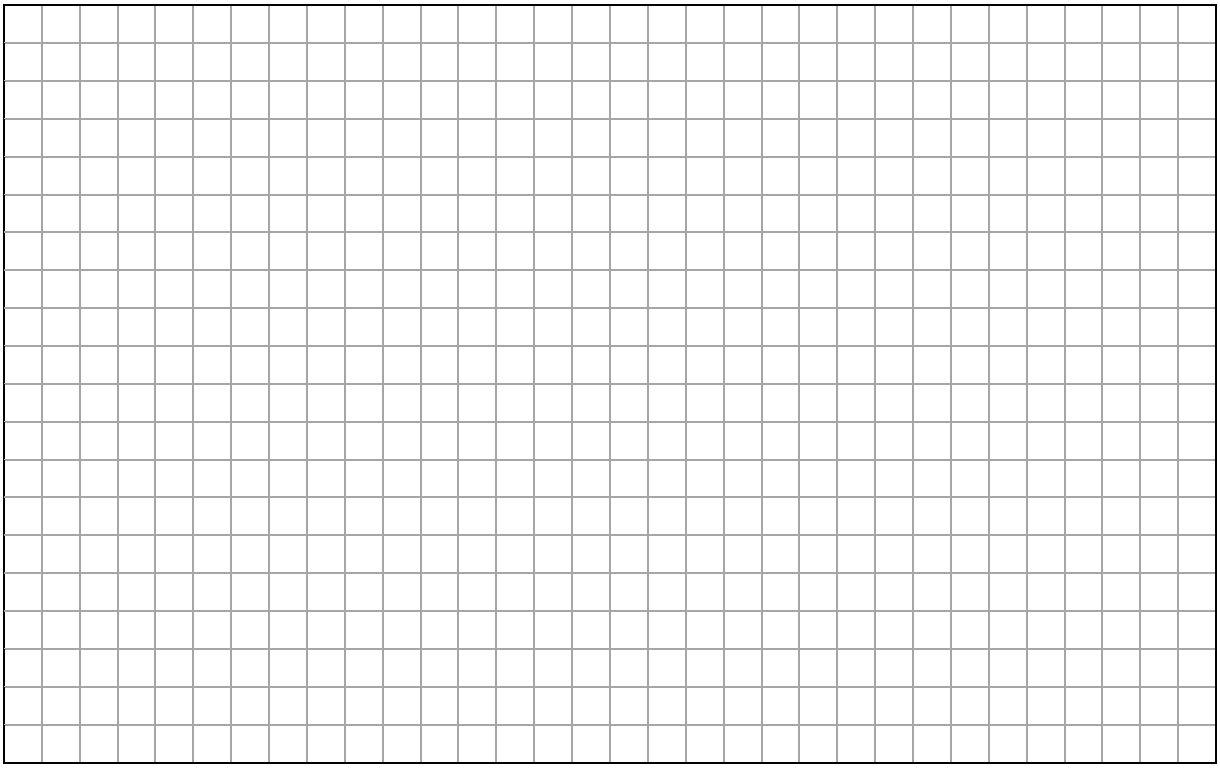
(b) As part of their visa requirements, Tom and Anne must have savings of 5000 Australian dollars **per person**. They intend to borrow the remainder of the money required from their parents.

Given that the exchange rate is €1 = A$1·61, how much do they intend to borrow?
Give your answer correct to the nearest euro.



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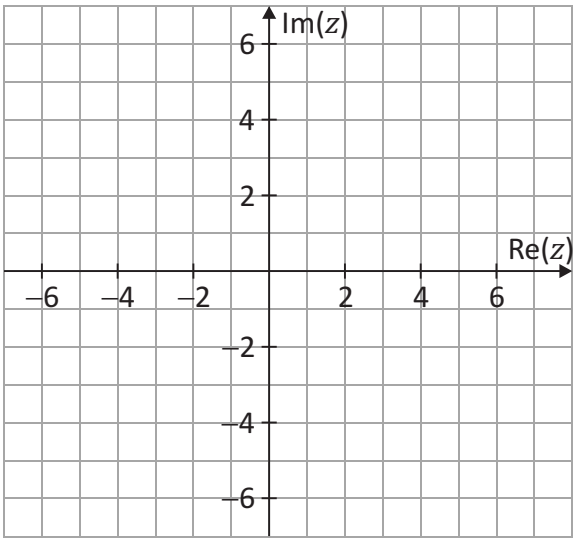
Question 2 **(25 marks)**

(a) $z_1 = -3 + 2i$ is a complex number, where $i^2 = -1$.

(i) Let $z_2 = 2iz_1$.
Find z_2 in the form $a + bi$, where $a, b \in \mathbb{R}$.

(ii) Plot z_1 and z_2 on the Argand Diagram.
Label each point clearly.



(iii) Show that $2|z_1| = |z_2|$.
What does your answer tell you about the points you plotted in **part (a)(ii)**?

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(b) Find $\frac{z_1}{z_2}$ in the form $a + bi$, where $a, b \in \mathbb{R}$.

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Question 3 **(25 marks)**

(a) Solve for x:

$$2x - 6(3 - x) = 2(5x - 1) - 10.$$

(b) Solve the simultaneous equations:

$$-6x + 5y = 26$$
$$3x - 2y = -8.$$

(c) Solve the equation:

$$\frac{1}{x} - \frac{3(x+1)}{x-1} = -1, \text{ where } x \neq 0, 1 \text{ and } x \in \mathbb{R}.$$

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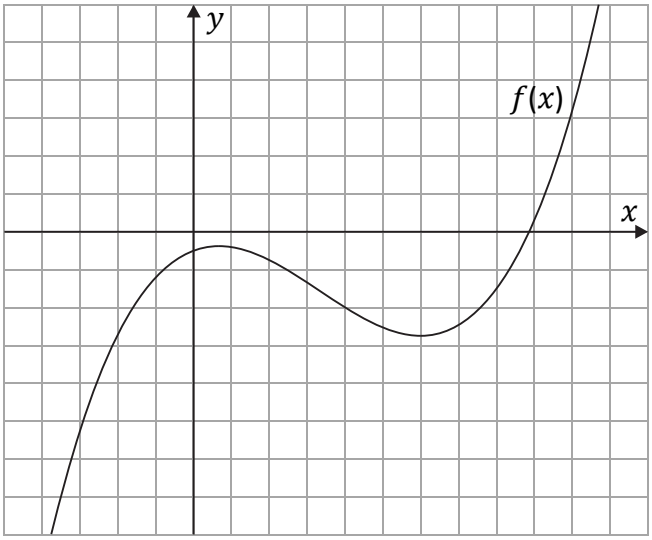
Question 4 **(25 marks)**

The diagram below shows the graph of the function $f(x) = x^3 - 5x^2 + 3x - 2$, where $x \in \mathbb{R}$.



(a) **(i)** Find the co-ordinates of the point at which the graph of f cuts the y-axis.

(ii) Find $f(4)$ and $f(4{\cdot}5)$.
Hence explain the significance of your answers.

$f(4) =$ $f(4{\cdot}5) =$

Explanation:

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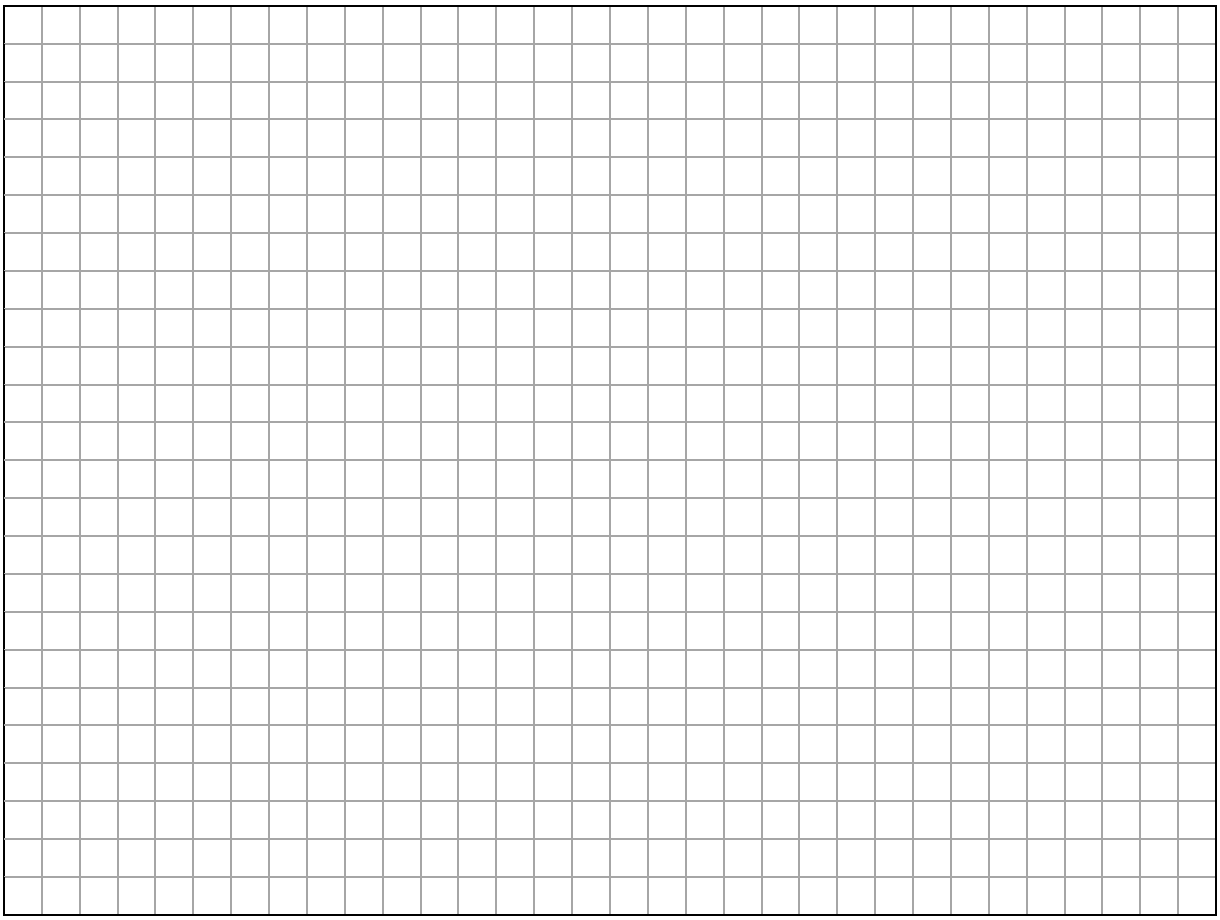
(b) Find $f'(x)$, the derivative of $f(x)$.
Hence find the co-ordinates of the local minimum turning point of the function $f(x)$.



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Question 5 **(25 marks)**

The speed of light emitted from the sun is 299 792 458 metres per second.

(a) **(i)** Write this number in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{N}$, correct to four significant figures.

(ii) Find the percentage error in using your answer to **part (a)(i)** as an approximation for the speed of light instead of the actual figure.
Give your answer correct to four decimal places.

(b) Find the speed of light emitted from the sun in **kilometres per hour**.
Give your answer correct to the nearest whole number.

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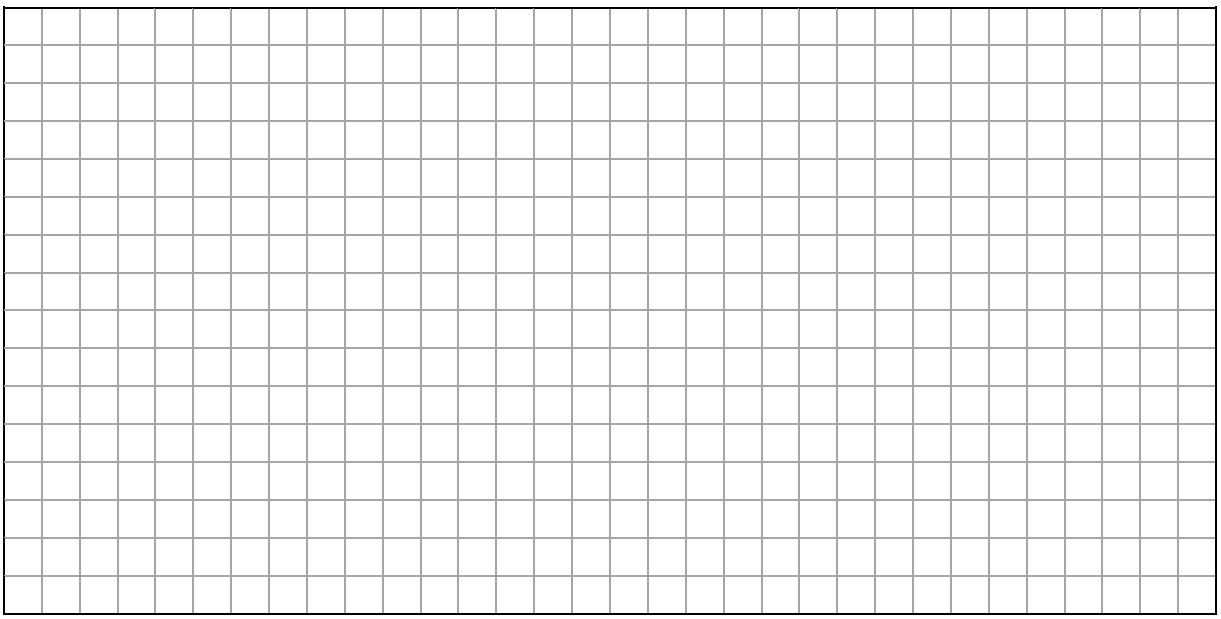
(c) It takes 8 minutes and 19 seconds for light emitted from the sun to reach Earth.
Find the mean distance, in kilometres, between Earth and the sun.
Give your answer correct to two significant figures.



Question 6 **(25 marks)**

A sum of €25 000 is invested in a government bond which pays interest, compounded monthly, that corresponds to an annual equivalent rate (AER) of 3·5%.

(a) Find the value of the investment at the end of five years.
Give your answer correct to the nearest euro.

(b) Small changes in interest rates can have a huge impact on savings as well as borrowings.

(i) Given that $(1 + r)^{12} = 1 + i$, where r is the monthly rate and i is the annual rate of interest, show that the rate of interest which, compounded monthly, corresponds to an AER of 3% is 0·2466%, correct to four decimal places.

(ii) Using the formula $F = P(1 + i)^t$, show that the value of the same amount invested in a different bond, which pays interest, compounded monthly, corresponding to an AER of 3% will **only** exceed the value of the investment in **part (a)** after 70 months.

Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 **(55 marks)**

The number of people exiting a concert venue can be modelled using the function:

$$P(t) = 27t - t^2 + 160, \quad t \in \mathbb{R},$$

where $P(t)$ is the number of people and t is the time, in minutes, after the concert finishes.

(a) **(i)** How many people are expected to exit immediately after a concert finishes ($t = 0$)?

(ii) How long does it take all people to exit the venue after a concert finishes?

(b) **(i)** The table below shows some of the data representing the number of people expected to exit the venue at 5-minute intervals after a concert finishes.
Use the function $P(t)$ to complete the table.

Time, t	0	5	10	15	20	25	30
Number of people, $P(t)$				340			

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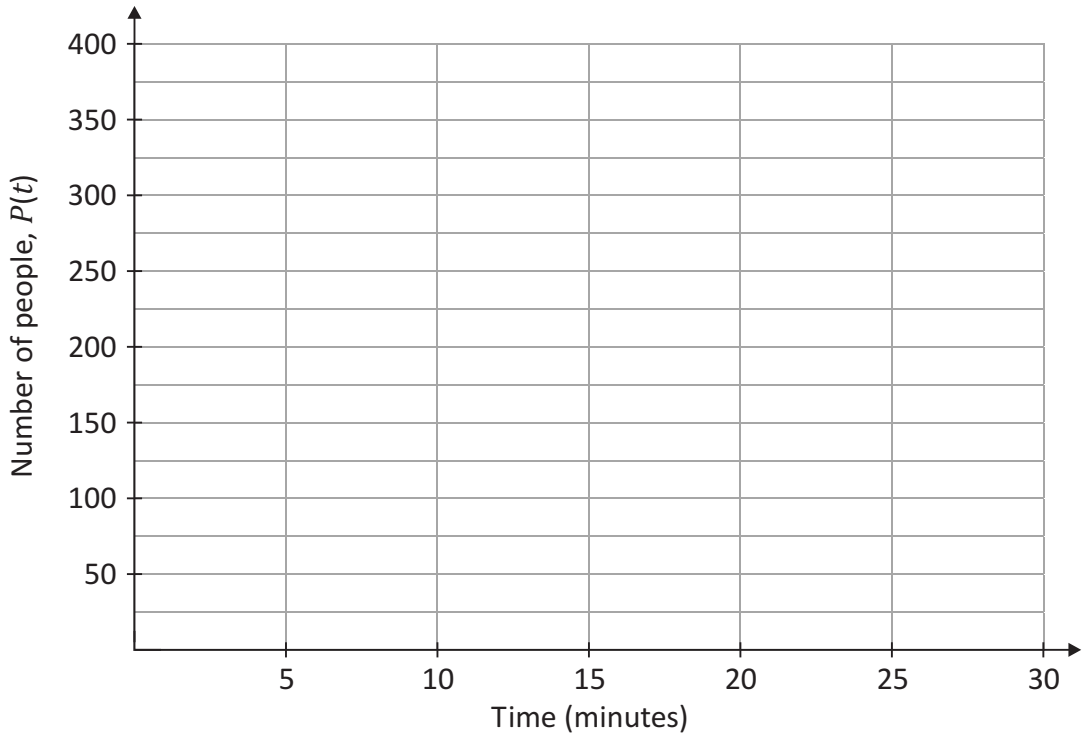
(ii) Draw the graph of the function $P(t)$ on the grid below for $0 \leq t \leq 30$, where $t \in \mathbb{R}$.



(iii) Use your graph to estimate the time interval for which the number of people expected to exit the venue will exceed 250 per minute.
Show your work on the graph above.

(iv) Explain what the area between the curve and the horizontal axis represents.

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(c) **(i)** Find $P'(t)$, the derivative of $P(t) = 27t - t^2 + 160$.

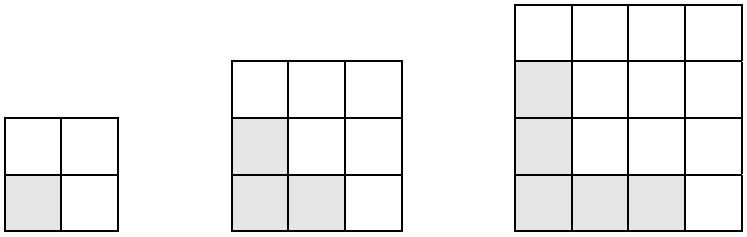
(ii) Use your answer from **part (c)(i)** to find the maximum number of people per minute expected to exit the venue.

(d) Additional stewards are required at the exits when the number of people exiting the venue exceeds 250 per minute.

Use algebra to find the time interval for which additional stewards will be required.
Give your answer correct to one decimal place.

Question 8 **(55 marks)**

The first three stages in a pattern of grey and white tiles are shown below.



Stage 1 Stage 2 Stage 3

(a) Based on the pattern shown, complete the table below to show the number of **grey** tiles in each stage of the pattern.

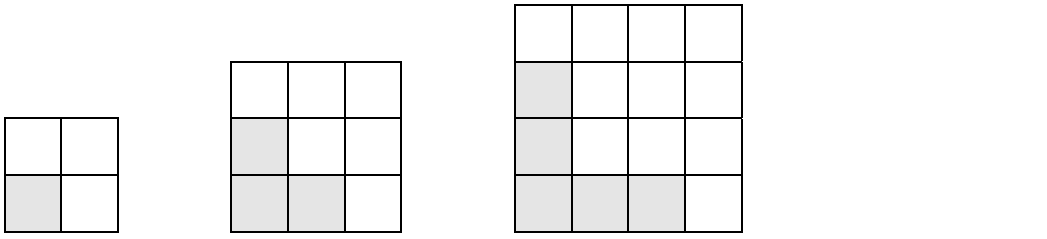
Stage (n)	**Number of Grey Tiles**
1	1
2	
3	
4	
5	

(b) **(i)** Assuming the pattern continues, the number of **grey** tiles in the nth stage of the pattern is given by the formula $G_n = pn + q$, where $p, q \in \mathbb{Z}$.
Find the value of p and the value of q.

(ii) How many **grey** tiles are in the 40th stage of the pattern?

(c) Show that the total number of **grey** tiles in the first n stages of the pattern is equal to n^2.

(d) **(i)** Draw the next stage in the pattern.



Stage 1 Stage 2 Stage 3 Stage 4

(ii) Based on the pattern shown, complete the table below to show the number of **white** tiles in each stage of the pattern.

Stage (n)	**Number of White Tiles**
1	3
2	
3	
4	
5	

(iii) Show that the number of **white** tiles in each stage forms a quadratic sequence.

(e) **(i)** The number of **white** tiles in the nth stage of the pattern is given by the formula:

$$W_n = n^2 + bn + c, \quad b, c \in \mathbb{Z}.$$

Find the value of b and the value of c.

(ii) How many **white** tiles are in the 40th stage of the pattern?

(f) The **total** number of tiles in the first n stages of the pattern is given by the formula:

$$S_n = \frac{n}{6}\{2n^2 + 9n + 13\}.$$

Find the total number of **white** tiles in the first 40 stages of the pattern.

Question 9 **(40 marks)**

A company depreciates the value of its assets by a **fixed percentage** each year. For a certain period of time, the value of a particular piece of equipment can be modelled using the function:

$$V(t) = 20\,000(0{\cdot}875)^t,$$

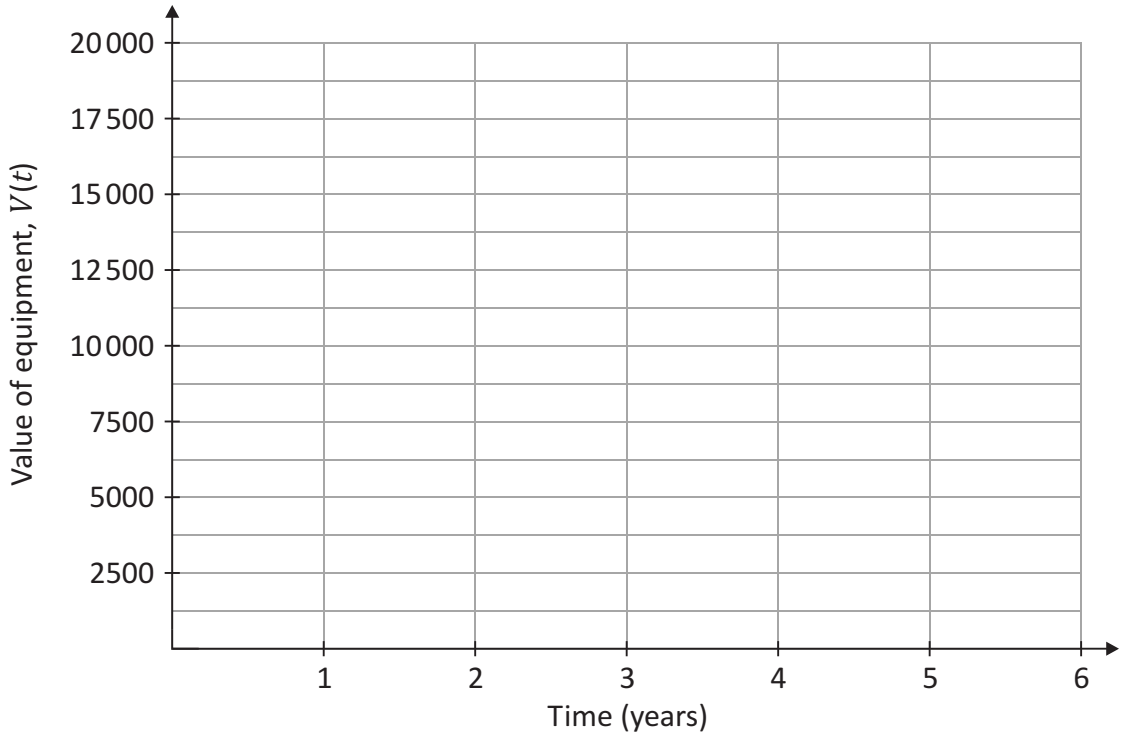
where $V(t)$ is the value of the equipment, in euro, and t is the time, in years, from when the equipment was purchased.

(a) Use the value function, $V(t)$, to show that the value of the equipment after 4 years is €11 724, correct to the nearest euro.

(b) **(i)** Complete the table to show the value of the equipment after each of the first six years. Where necessary, give the value correct to the nearest euro.

Time, t	0	1	2	3	4	5	6
Value, $V(t)$					11 724		

(ii) Draw the graph of the function $V(t)$ on the grid below for $0 \leq t \leq 6$, where $t \in \mathbb{R}$.



(iii) Use your graph to estimate the length of time it takes for the equipment to depreciate by 50% of its purchase price.

(iv) Using your graph, or otherwise, find the **fixed percentage** per year by which the company depreciates this piece of equipment.

(c) A different method is to depreciate the value of assets by a **fixed amount** each year. Using this method, the value of the same piece of equipment can be modelled using the function:

$$W(t) = 20\,000(1 - 0{\cdot}125t),$$

where $W(t)$ is the value of the equipment, in euro, and t is again the time, in years, from when the equipment was purchased.

Use a graphic, numeric or algebraic method to find the difference between the values of the equipment after 5 years using the two methods of depreciation.

You may use this page for extra work.
Label any extra work clearly with the question number and part.

Q. & part	

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