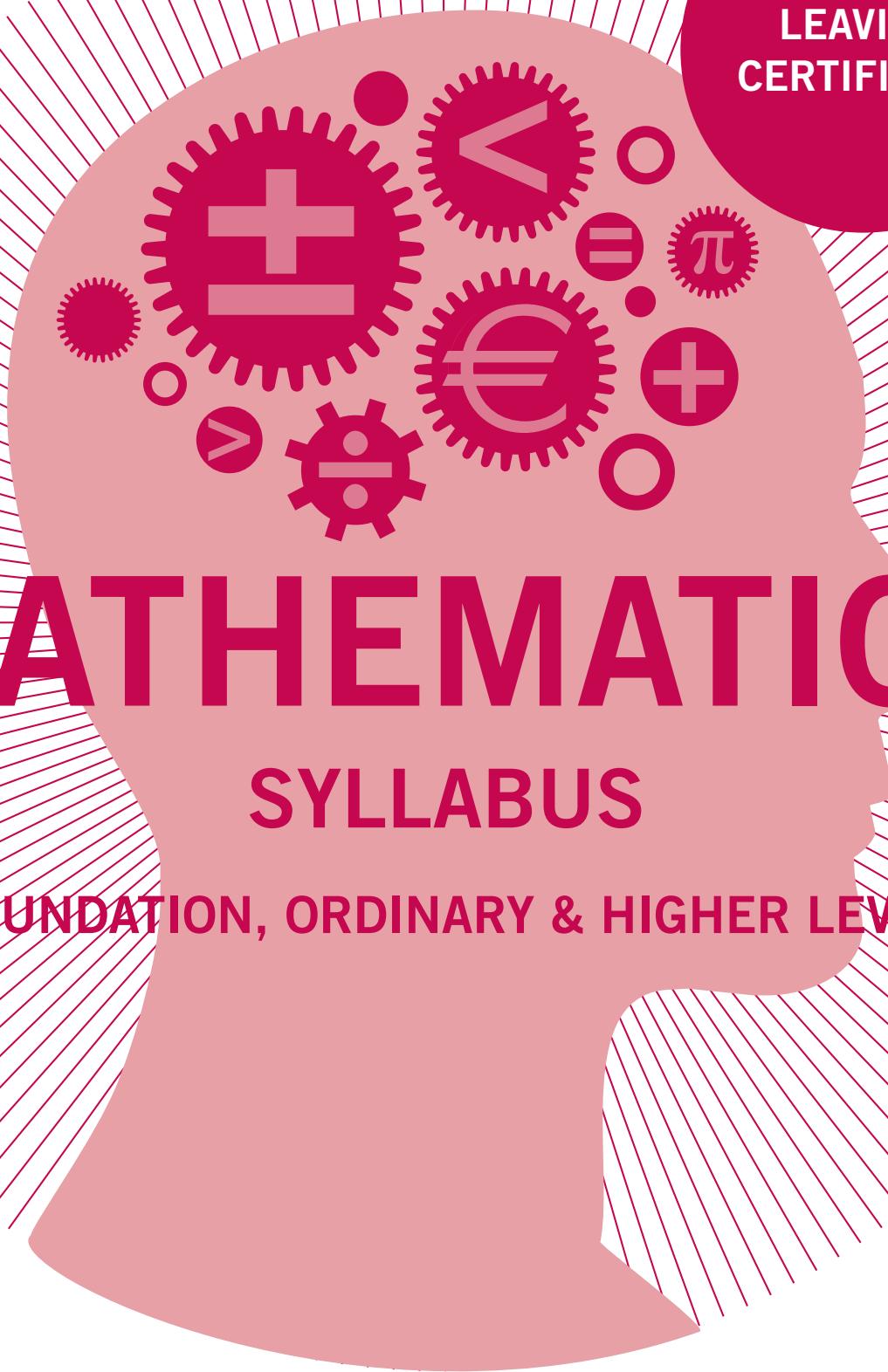An Roinn Oideachais
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# MATHEMATICS

## SYLLABUS

### FOUNDATION, ORDINARY & HIGHER LEVEL

**For examination from 2015**

# Strand 1: Statistics and Probability
## – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **1.1 Counting** | – count the arrangements of n distinct objects ($n!$)<br>– count the number of ways of arranging r objects from $n$ distinct objects | – count the number of ways of selecting $r$ objects from $n$ distinct objects<br>– compute binomial coefficients |
| **1.2 Concepts of probability** | – use set theory to discuss experiments, outcomes, sample spaces<br>– discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams<br>– calculate expected value and understand that this does not need to be one of the outcomes<br>– recognise the role of expected value in decision making and explore the issue of fair games | – extend their understanding of the basic rules of probability (AND/OR, mutually exclusive) through the use of formulae<br>– Addition Rule:<br>$P(A \cup B) = P(A) + P(B) - P(A \cap B)$<br>– Multiplication Rule (Independent Events):<br>$P(A \cap B) = P(A) \times P(B)$<br>– Multiplication Rule (General Case):<br>$P(A \cap B) = P(A) \times P(B \mid A)$<br>– solve problems involving sampling, with or without replacement<br>– appreciate that in general $P(A \mid B) \neq P(B \mid A)$<br>– examine the implications of $P(A \mid B) \neq P(B \mid A)$ in context |
| **1.3 Outcomes of random processes** | – find the probability that two independent events both occur<br>– apply an understanding of Bernoulli trials∗<br>– solve problems involving up to 3 Bernoulli trials<br>– calculate the probability that the $1^{st}$ success occurs on the $n^{th}$ Bernoulli trial where $n$ is specified | – solve problems involving calculating the probability of $k$ successes in $n$ repeated Bernoulli trials (normal approximation not required)<br>– calculate the probability that the $k^{th}$ success occurs on the $n^{th}$ Bernoulli trial<br>– use simulations to explore the variability of sample statistics from a known population, to construct sampling distributions and to draw conclusions about the sampling distribution of the mean<br>– solve problems involving reading probabilities from the normal distribution tables |
| **1.4 Statistical reasoning with an aim to becoming a statistically aware consumer** | – discuss populations and samples<br>– decide to what extent conclusions can be generalised<br>– work with different types of bivariate data | |

∗ A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure".

# Strand 1: Statistics and Probability
## – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **1.5 Finding, collecting and organising data** | – select a sample (Simple Random Sample)<br>– recognise the importance of representativeness so as to avoid biased samples<br>– discuss different types of studies: sample surveys, observational studies and designed experiments<br>– design a plan and collect data on the basis of above knowledge | – recognise the importance of randomisation and the role of the control group in studies<br>– recognise biases, limitations and ethical issues of each type of study<br>– select a sample (stratified, cluster, quota – no formulae required, just definitions of these)<br>– design a plan and collect data on the basis of above knowledge |
| **1.6 Representing data graphically and numerically** | **Graphical**<br>– describe the sample (both univariate and bivariate data) by selecting appropriate graphical or numerical methods<br>– explore the distribution of data, including concepts of symmetry and skewness<br>– compare data sets using appropriate displays including back-to-back stem and leaf plots<br>– determine the relationship between variables using scatterplots<br>– recognise that correlation is a value from -1 to +1 and that it measures the extent of the linear relationship between two variables<br>– match correlation coefficient values to appropriate scatterplots<br>– understand that correlation does not imply causality<br><br>**Numerical**<br>– recognise standard deviation and interquartile range as measures of variability<br>– use a calculator to calculate standard deviation<br>– find quartiles and the interquartile range<br>– use the interquartile range appropriately when analysing data<br>– recognise the existence of outliers | **Graphical**<br>– analyse plots of the data to explain differences in measures of centre and spread<br>– draw the line of best fit by eye<br>– make predictions based on the line of best fit<br>– calculate the correlation coefficient by calculator<br><br><br><br><br><br>**Numerical**<br>– recognise the effect of outliers<br>– use percentiles to assign relative standing |

# Strand 1: Statistics and Probability
## – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
| --- | --- | --- |
| **1.7 Analysing, interpreting and drawing inferences from data** | – recognise how sampling variability influences the use of sample information to make statements about the population<br>– use appropriate tools to describe variability drawing inferences about the population from the sample<br>– interpret the analysis and relate the interpretation to the original question<br>– interpret a histogram in terms of distribution of data<br>– make decisions based on the empirical rule<br>– recognise the concept of a hypothesis test<br>– calculate the margin of error ($\frac{1}{\sqrt{n}}$) for a population proportion*<br>– conduct a hypothesis test on a population proportion using the margin of error | – build on the concept of margin of error and understand that increased confidence level implies wider intervals<br>– construct 95% confidence intervals for the population mean from a large sample and for the population proportion, in both cases using z tables<br>– use sampling distributions as the basis for informal inference<br>– perform univariate large sample tests of the population mean (two-tailed z-test only)<br>– use and interpret p-values |

* The margin of error referred to here is the maximum value of the radius of the 95% confidence interval.

# Strand 2: Geometry and Trigonometry
## – Foundation level

| Topic | Description of topic<br>*Students learn about* | Learning outcomes<br>*Students should be able to* |
|---|---|---|
| **2.1 Synthetic geometry** | Constructions and how to apply these in real-life situations.<br>Dynamic geometry software.<br>The instruments that are used to perform constructions with precision. | – revisit constructions 4,5,10,13 and 15 in real-life contexts<br>– draw a circle of given radius<br>– use the instruments: straight edge, compass, ruler, protractor and set square appropriately when drawing geometric diagrams |
| **2.2 Co-ordinate geometry** | Co-ordinating the plane.<br>Linear relationships in real-life contexts and representing these relationships in tabular and graphical form.<br>Equivalence of the slope of the graph and the rate of change of the relationship.<br>Comparing linear relationships in real-life contexts, paying particular attention to the significance of the start value and the rate of change.<br>The significance of the point of intersection of two linear relationships. | – select and use suitable strategies (graphic, numeric, mental) for finding solutions to real-life problems involving up to two linear relationships |
| **2.3 Trigonometry** | Right-angled triangles.<br><br>Trigonometric ratios. | – apply the result of the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances<br>– use trigonometric ratios to solve real world problems involving angles |
| **2.4 Transformation geometry, enlargements** | Translations, central symmetry, axial symmetry and rotations.<br><br>Enlargements. | – locate axes of symmetry in simple shapes<br>– recognise images of points and objects under translation, central symmetry, axial symmetry and rotation<br>– investigate enlargements and their effect on area, paying attention to<br>  • centre of enlargement<br>  • scale factor $k$<br>    where $0<k<1$, $k>1$ $k \in \mathbf{Q}$<br>– solve problems involving enlargements |

# Strand 2: Geometry and Trigonometry
## – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **2.1 Synthetic geometry** | – perform constructions **16-21** (see *Geometry for Post-primary School Mathematics*)<br>– use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies<br>– investigate theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 and corollary 6 (see Geometry for Post-primary School Mathematics) and use them to solve problems | – perform construction **22** (see *Geometry for Post-primary School Mathematics*)<br>– use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction<br>– prove theorems 11,12,13, concerning ratios (see Geometry for Post-primary School Mathematics), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle |
| **2.2 Co-ordinate geometry** | – use slopes to show that two lines are<br>  • parallel<br>  • perpendicular<br>– recognise the fact that the relationship $ax + by + c = 0$ is linear<br>– solve problems involving slopes of lines<br>– calculate the area of a triangle<br>– recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the $x$ and $y$ co-ordinates of points on a circle with centre $(h, k)$ and radius $r$<br>– solve problems involving a line and a circle with centre $(0, 0)$ | – solve problems involving<br>  • the perpendicular distance from a point to a line<br>  • the angle between two lines<br>– divide a line segment internally in a given ratio $m: n$<br>– recognise that $x^2+y^2 +2gx+2fy+c = 0$ represents the relationship between the x and y co-ordinates of points on a circle with centre $(-g,-f)$ and radius $r$ where $r = \sqrt{(g^2+f^2 -c)}$<br>– solve problems involving a line and a circle |
| **2.3 Trigonometry** | – use of the theorem of Pythagoras to solve problems (2D only)<br>– use trigonometry to calculate the area of a triangle<br>– solve problems using the sine and cosine rules (2D)<br>– define $\sin \theta$ and $\cos \theta$ for all values of $\theta$<br>– define $\tan \theta$<br>– solve problems involving the area of a sector of a circle and the length of an arc<br>– work with trigonometric ratios in surd form | – use trigonometry to solve problems in 3D<br>– graph the trigonometric functions sine, cosine, tangent<br>– graph trigonometric functions of type<br>  • $f(\theta)= a+b\mathrm{Sin}\ c\theta$<br>  • $g(\theta) = a+b\mathrm{Cos}\ c\theta$<br>  for $a,b,c \in \mathbf{R}$<br>– solve trigonometric equations such as $\mathrm{Sin}\ n\theta=0$ and $\mathrm{Cos}\ n\theta= ½$ giving all solutions<br>– use the radian measure of angles<br>– derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix)<br>– apply the trigonometric formulae 1-24 (see appendix) |
| **2.4 Transformation geometry, enlargements** | – investigate enlargements and their effect on area, paying attention to<br>  • centre of enlargement<br>  • scale factor $k$<br>  where $0<k<1$, $k>1$ $k \in \mathbf{Q}$<br>– solve problems involving enlargements | |

# Strand 3: Number – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **3.1 Number systems** | – recognise irrational numbers and appreciate that **R** ≠ **Q**<br>– work with irrational numbers<br>– revisit the operations of addition, multiplication, subtraction and division in the following domains:<br>  • **N** of natural numbers<br>  • **Z** of integers<br>  • **Q** of rational numbers<br>  • **R** of real numbers<br>  and represent these numbers on a number line<br>– investigate the operations of addition, multiplication, subtraction and division with complex numbers **C** in rectangular form a+ib<br>– illustrate complex numbers on an Argand diagram<br>– interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate<br>– develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place-value understanding<br>– consolidate their understanding of factors, multiples, prime numbers in **N**<br>– express numbers in terms of their prime factors<br>– appreciate  the order of operations, including brackets<br>– express non-zero positive rational numbers in the form<br>$a \times 10^n$, where $n \in$ **Z** and $1 \le a < 10$ and perform arithmetic operations on numbers in this form | – geometrically construct $\sqrt{2}$ and $\sqrt{3}$<br>– prove that $\sqrt{2}$ is not rational<br>– calculate conjugates of sums and products of complex numbers<br>– verify and justify formulae from number patterns<br>– investigate geometric sequences and series<br>– prove by induction<br>  • simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series<br>  • simple inequalities such as $n! > 2^n$, $2^n \ge n^2$ $(n \ge 4)$ $(1+x)^n \ge 1+nx$ $(x > -1)$<br>  • factorisation results such as 3 is a factor of $4^n - 1$<br>– apply the rules for sums, products, quotients of limits<br>– find by inspection the limits of sequences such as<br>$\lim_{n \to \infty} \dfrac{n}{n+1}$; $\lim_{n \to \infty} r^n$, $|r| < 1$<br>– solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment<br>– derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums |

# Strand 3: Number – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **3.1 Number systems (continued)** | – appreciate that processes can generate sequences of numbers or objects<br>– investigate patterns among these sequences<br>– use patterns to continue the sequence<br>– generalise and explain patterns and relationships in algebraic form<br>– recognise whether a sequence is arithmetic, geometric or neither<br>– find the sum to $n$ terms of an arithmetic series | |
| **3.2 Indices** | – solve problems using the rules for indices (where $a, b \in \mathbf{R}$; $p, q \in \mathbf{Q}$; $a^p, a^q \in \mathbf{Q}$; $a, b \neq \mathbf{0}$):<br><br>• $a^p a^q = a^{p+q}$<br><br>• $\dfrac{a^p}{a^q} = a^{p-q}$<br><br>• $a^0 = 1$<br><br>• $(a^p)^q = a^{pq}$<br><br>• $a^{\frac{1}{q}} = \sqrt[q]{a}$  $q \in \mathbf{Z}, q \neq 0, a > 0$<br><br>• $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$ $p, q \in \mathbf{Z}, q \neq 0, a > 0$<br><br>• $a^{-p} = \dfrac{1}{a^p}$<br><br>• $(ab)^p = a^p b^p$<br><br>• $\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$ | – solve problems using the rules of logarithms<br><br>• $\log_a(xy) = \log_a x + \log_a y$<br><br>• $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$<br><br>• $\log_a x^q = q\log_a x$<br><br>• $\log_a a = 1$ and $\log_a 1 = 0$<br><br>• $\log_a x = \dfrac{\log_b x}{\log_b a}$ |

# Strand 3: Number – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **3.3 Arithmetic** | – check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result<br>– accumulate error (by addition or subtraction only)<br>– make and justify estimates and approximations of calculations; calculate percentage error and tolerance<br>– calculate average rates of change (with respect to time)<br>– solve problems that involve<br>  • calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price)<br>  • compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions)<br>  • costing: materials, labour and wastage<br>  • metric system; change of units; everyday imperial units (conversion factors provided for imperial units)<br>– make estimates of measures in the physical world around them | – use *present value* when solving problems involving loan repayments and investments |
| **3.4 Length, area and volume** | – investigate the nets of prisms, cylinders and cones<br>– solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs, and figures made from combinations of these<br>– solve problems involving surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular-based prism (right angle, isosceles and equilateral), sphere, hemisphere, and solids made from combinations of these<br>– use the trapezoidal rule to approximate area | |

# Strand 4: Algebra – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **4.1 Expressions** | – evaluate expressions given the value of the variables<br>– expand and re-group expressions<br>– factorise expressions of order 2<br>– add and subtract expressions of the form<br> • $(ax+by+c) \pm \ldots \pm (dx+ey+f)$<br> • $(ax^2+bx+c) \pm \ldots \pm (dx^2+ex+f)$<br> where $a,b,c,d,e,f \in \mathbf{Z}$<br> • $\dfrac{a}{bx+c} \pm \dfrac{p}{qx+r}$<br> where $a,b,c,p,q,r \in \mathbf{Z}$<br>– use the associative and distributive properties to simplify expressions of the form<br> • $a(bx \pm cy \pm d) \pm \ldots \pm e(fx \pm gy \pm h)$<br> where $a, b, c, d, e, f, g, h \in \mathbf{Z}$<br> • $(x \pm y)(w \pm z)$<br>– rearrange formulae | – perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds<br>– apply the binomial theorem |

# Strand 4: Algebra – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **4.2 Solving equations** | – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form:<br>• $f(x) = g(x)$, with $f(x) = ax+b$, $g(x) = cx+d$ where $a, b, c, d \in \mathbf{Q}$<br>• $f(x) = g(x)$ with $f(x) = \dfrac{a}{bx+c} \pm \dfrac{p}{qx+r}$; $g(x) = \dfrac{e}{f}$ where $a, b, c, e, f, p, q, r \in \mathbf{Z}$<br>• $f(x) = k$ with $f(x) = ax^2 + bx + c$ (and not necessarily factorisable) where $a, b, c \in \mathbf{Q}$ and interpret the results<br>– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to<br>• simultaneous linear equations with two unknowns and interpret the results<br>• one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of $x$ or the coefficient of $y$ is $\pm 1$ in the linear equation) and interpret the results<br>– form quadratic equations given whole number roots | – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form:<br>$f(x) = g(x)$<br>with $f(x) = \dfrac{ax+b}{ex+f} \pm \dfrac{cx+d}{qx+r}$; $g(x) = k$ where $a, b, c, d, e, f, q, r \in \mathbf{Z}$<br>– use the Factor Theorem for polynomials<br>– select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to<br>• cubic equations with at least one integer root<br>• simultaneous linear equations with three unknowns<br>• one linear equation and one equation of order 2 with two unknowns<br>and interpret the results |
| **4.3 Inequalities** | – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form:<br>• $g(x) \le k$, $g(x) \ge k$,<br>• $g(x) < k$, $g(x) > k$,<br>where $g(x) = ax + b$ and $a, b, k \in \mathbf{Q}$ | – use notation $\lvert x \rvert$<br>– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form:<br>• $g(x) \le k$, $g(x) \ge k$;<br>• $g(x) < k$, $g(x) > k$,<br>with $g(x) = ax^2+bx+c$ or $g(x) = \dfrac{ax+b}{cx+d}$<br>and $a, b, c, d, k \in \mathbf{Q}$, $x \in \mathbf{R}$<br>• $\lvert x - a \rvert < b, \lvert x - a \rvert > b$ and combinations of these, with $a, b, \in \mathbf{Q}$, $x \in \mathbf{R}$ |
| **4.4 Complex Numbers** | See strand 3, section 3.1 | – use the Conjugate Root Theorem to find the roots of polynomials<br>– work with complex numbers in rectangular and polar form to solve quadratic and other equations including those in the form $z^n = a$, where $n \in \mathbf{Z}$ and $z = r(\cos \theta + i\sin \theta)$<br>– use De Moivre's Theorem<br>– prove De Moivre's Theorem by induction for $n \in \mathbf{N}$<br>– use applications such as $n^{\text{th}}$ roots of unity, $n \in \mathbf{N}$, and identities such as $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$ |

# Strand 5: Functions – Ordinary level and Higher level

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **5.1 Functions** | – recognise that a function assigns a unique output to a given input<br>– form composite functions<br>– graph functions of the form<br>  • $ax+b$ where $a,b \in \mathbf{Q}$, $x \in \mathbf{R}$<br>  • $ax^2+bx + c$<br>    where $a, b, c \in \mathbf{Z}$, $x \in \mathbf{R}$<br>  • $ax^3+bx^2 + cx+d$<br>    where $a,b,c,d \in \mathbf{Z}$, $x \in \mathbf{R}$<br>  • $ab^x$ where $a \in \mathbf{N}$, $b, x \in \mathbf{R}$<br>– interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions<br>– use graphical methods to find approximate solutions to<br>  • $f(x) = 0$<br>  • $f(x) = k$<br>  • $f(x) = g(x)$<br>  where $f(x)$ and $g(x)$ are of the above form, or where graphs of $f(x)$ and $g(x)$ are provided<br>– investigate the concept of the limit of a function | – recognise surjective, injective and bijective functions<br>– find the inverse of a bijective function<br>– given a graph of a function sketch the graph of its inverse<br>– express quadratic functions in complete square form<br>– use the complete square form of a quadratic function to<br>  • find the roots and turning points<br>  • sketch the function<br>– graph functions of the form<br>  • $ax^2+bx + c$ where $a,b,c \in \mathbf{Q}$, $x \in \mathbf{R}$<br>  • $ab^x$ where $a, b \in \mathbf{R}$<br>  • logarithmic<br>  • exponential<br>  • trigonometric<br>– interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions<br>– informally explore limits and continuity of functions |
| **5.2 Calculus** | – find first and second derivatives of linear, quadratic and cubic functions by rule<br>– associate derivatives with slopes and tangent lines<br>– apply differentiation to<br>  • rates of change<br>  • maxima and minima<br>  • curve sketching | – differentiate linear and quadratic functions from first principles<br>– differentiate the following functions<br>  • polynomial<br>  • exponential<br>  • trigonometric<br>  • rational powers<br>  • inverse functions<br>  • logarithms<br>– find the derivatives of sums, differences, products, quotients and compositions of functions of the above form<br>– apply the differentiation of above functions to solve problems<br>– use differentiation to find the slope of a tangent to a circle<br>– recognise integration as the reverse process of differentiation<br>– use integration to find the average value of a function over an interval<br>– integrate sums, differences and constant multiples of functions of the form<br>  • $x^a$ where $a \in \mathbf{Q}$<br>  • $a^x$ where $a \in \mathbf{R}$, $a>0$<br>  • Sin $ax$ where $a \in \mathbf{R}$<br>  • Cos $ax$ where $a \in \mathbf{R}$<br>– determine areas of plane regions bounded by polynomial and exponential curves |