Scattered field

$$f_p^{(in)}(\vec{r_p}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{plem}\, j_l(k_p r_p)\, Y_l^m(\theta_p)$$

$$f_p^{(out)}(\vec{r_p}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} b_{plem}\, h_l^{(1)}(k_0 r_p)\, Y_l^m(\theta_p) .$$

où $\begin{cases} k_p = k_0\sqrt{1+\frac{V_p}{E}} \\ k_0 = \sqrt{\frac{2m_e E}{\hbar^2}} \end{cases}$

Relations de continuité

$$\left(\sum_{q=1}^{N} f_q^{(out)} + f^{(i)}\right)\Big|_{r_p=a_p} = \left(f_p^{(in)}\right)\Big|_{r_p=a_p}$$

$\Leftrightarrow$ $\left(f_p^{(in)}\right)\Big|_{r_p=a_p} - \left(\sum_{q=1}^{N} f_q^{(out)}\right)\Big|_{r_p=a_p} = \left(f^{(i)}\right)\Big|_{r_p=a_p}$. $\quad \psi_{em}^{(out)}(\vec{r'})$

$\Leftrightarrow$ $\sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{plem}\, j_l(k_p a_p)\, Y_l^m(\theta_p) - \left(\sum_{q=1}^{N}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} b_{plem}\, h_l^{(1)}(k_0 r_q)\right)\Big|_{r_p=a_p} Y_l^m(\theta_p) = e^{j k_0 d_p} C_{em}\, j_l(k_0 a_p)\, Y_l^m(\theta_p)$

$\Leftrightarrow$ $\sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{plem}\, j_l(k_p a_p)\, Y_l^m(\theta_p) - \sum_{l=0}^{\infty}\sum_{m=-l}^{l} b_{plem}\, h_l^{(1)}(k_0 a_p)\, Y_l^m(\theta_p)$

$$- \sum_{q\neq p}^{N}\sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu}\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'} a_{\nu\mu l'm'}(k_0(a_q-a_p))\, b_{qi\nu}\, j_\nu(k_0 a_q)\, Y_{l'}^{m'}(\theta_p) = e^{j k_0 d_p} C_{lm}\, j_l(k_0 a_p)\, Y_l^m(\theta_p)$$

$\Leftrightarrow$ $\int_\Omega \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{plem}\, j_l(k_p a_p)\, Y_l^m(\theta_p)\, Y_l^{m*}(\theta_p)\, d\Omega - \int_\Omega \sum_{l=0}^{\infty}\sum_{m=-l}^{l} b_{plem}\, h_l^{(1)}(k_0 a_p) Y_l^m(\theta_p)\, Y_l^{m*}(\theta_p)\, d\Omega$

$$- \int_\Omega \sum_{q\neq p}^{N}\sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu}\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'} a_{\nu\mu l'm'}(k_0(a_q-a_p))\, b_{qi\nu}\, j_\nu(k_0 a_q)\, Y_{l'}^{m'}(\theta_p)\, Y_l^{m*}(\theta_p)\, d\Omega = \int_\Omega e^{j k_0 d_p} C_{em}\, j_l(k_0 a_p)\, Y_l^m Y_l^{m*} d\Omega$$

$\Leftrightarrow$ $\boxed{a_{plem}\, j_l(k_p a_p) - b_{plem}\, h_l^{(1)}(k_0 a_p) - \sum_{q\neq p}^{N}\sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu} a_{\nu\mu em}(k_0(a_q-a_p))\, b_{qi\nu}\, j_\nu(k_0 a_q) = C_{em}\, j_l(k_0 a_p) e^{j k_0 d_p}}$

$$\partial_{r_p}\left(\sum_{q=1}^{N} f_q^{(out)} + f^{(i)}\right)\Big|_{r_p=a_p} = \partial_{r_p}\left(f_p^{(in)}\right)\Big|_{r_p=a_p}$$

$$\boxed{a_{plem}\, \partial_p j_l(k_p a_p) - b_{plem}\, \partial_p h_l^{(1)}(k_0 a_p) - \sum_{q\neq p}^{N}\sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu} a_{\nu\mu em}(k_0(a_q-a_p))\, b_{qi\nu}\, \partial_p j_\nu(k_0 a_q) = e^{j k_0 d_p} C_{em}\, \partial_p j_l(k_0 a_p)}$$

Système linéaire

$$(\mathbb{P} - \mathbb{T})\, A = \mathbb{L}$$

$$\mathbb{P} = \begin{bmatrix} j_l(k_p a_p) \;(m) & & -h_l^{(1)}(k_0 a_p) \;(m) \\ & j_l(k_p a_p) & \\ \partial_p j_l(k_p a_p) \;(m) & & -\partial_p h_l^{(1)}(k_0 a_p) \;(m) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{plem} \\ b_{plem} \end{bmatrix} \qquad \mathbb{L} = \begin{bmatrix} e^{j k_0 d_p} C_{em}\, j_l(k_0 a_p) \\ e^{j k_0 d_p} C_{em}\, \partial_p j_l(k_0 a_p) \end{bmatrix}$$

$$\mathbb{T} = \begin{bmatrix} 0 & & \\ & T_{j1} & \\ & & 0 \end{bmatrix}$$

$$T_{j1} = j_\nu(k_0 a_p) \sum_{q}\sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu} a_{\nu\mu lm}(k_0(a_q-a_p))$$

$$T_{j2} = \partial_p j_\nu(k_0 a_p) \sum_{q}\sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu} a_{\nu\mu lm}(k_0(a_q-a_p))$$