Numerical Project

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1 Exercise 3

All of my analytical work was uploaded with the textbook problems and is labeled by exercise. In order to visualize the fields, I used the fields function. This function takes in values for m, n, x, y, z, t, and , and outputs values for B_z, E_x, E_y, B_x, and B_y. x, y, and z can either be single number values or arrays. We choose which two axes we want to see, and set the third axis to a single slice value. The output will be an array of the field values across the arrays of the two axes we're looking at. We can then plot all three directions of the fields as a function of the two axes as field lines and a color map. For example, if we want to look at x and y, we would choose one value of z and look at that slice. We would blot Bx and By as a 2D vector field and Bz would be represented through the color map. I'm first going to look at the mode m=0 and n=1. We'll look at slices z=0.5, x=0.5, and y=0.5 for each mode. Figure 1 is a visualization of the magnetic field in the xy plane. You can tell by the fact that the vector field is only vertical that B_x is zero everywhere, which is what we would expect based on the field equations when m=0. B_y is also constant with respect to x. B_z is dependant on y but not x. Figure 2 is a visualization of the magnetic field in the xz plane. Again, B_x is zero everywhere since the vector field is only in the z directions. B_y is only dependant on z. Figure 3 is a visualization of the magnetic field in the yz plane. The colormap is blue everywhere because it is zero everywhere, which is what we would expect. The field curls around the x direction periodically, which is also what we would expect. Next, we can look at the m=0, n=2 mode to see what differences exist when we double n. Figure

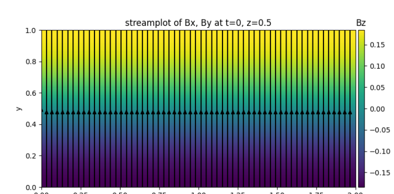

Figure 1: Magnetic field in the xy plane in the 01 mode

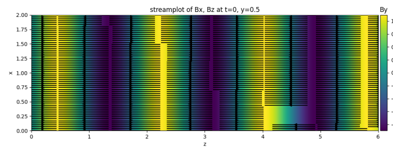

Figure 2: Magnetic field in the xz plane in the 01 mode

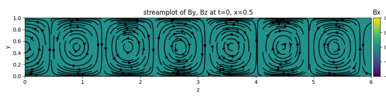

Figure 3: Magnetic field in the yz plane in the 01 mode

4 is a visualization of the magnetic field in the xy plane. The difference between this field and the same slice in the 01 mode seems to be kind of like the different modes of a standing wave. The 01 goes from negative to zero to positive, like half a period of a sinusoidal function. The 02 mode goes from negative to zero to positive to zero to negative, or like a full period of a sinusoidal function. This makes sense, since increasing n essentially just increases the frequency of the oscillation of the waves in the y direction. Figure 5 is a visualization of the magnetic field in the xz plane. We can see from this slice that the frequency of oscillation in the z direction has also increased. This is because the oscillation in z is dependant on k which is dependant on n, so increasing n increases the frequency of oscillation in the z direction. Figure 6 is a visualization of the magnetic field in the yz plane. The increased number of spirals reflects the increased frequency in both the y and z directions. Now we can look at a mode that has non zero values in both x and y. First, looking at the m=3, n=1 mode, Figure 7 shows a visualization of the magnetic field in the xy plane. Now B_z oscillates in two directions, and both B_x and B_y are both nonzero, which is expected. The field in the xy plane flows from peak to trough in the z field. Figure 8 is a visualization of the magnetic field in the xz plane. The field in the xz plane curls around the y direction, and B_y oscillates in both the x and

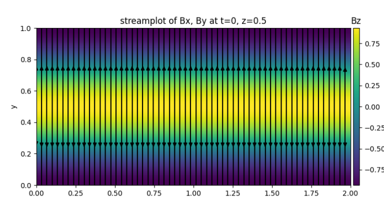

Figure 4: Magnetic field in the xy plane in the 02 mode

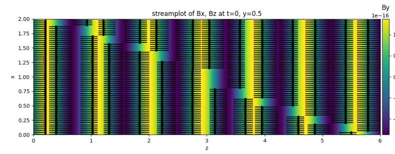

Figure 5: Magnetic field in the xz plane in the 02 mode

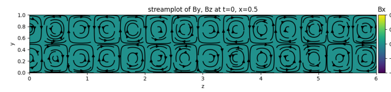

Figure 6: Magnetic field in the yz plane in the 02 mode

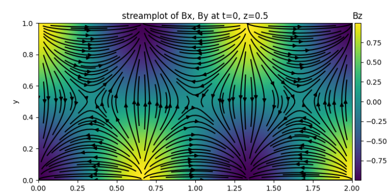

Figure 7: Magnetic field in the xy plane in the 31 mode

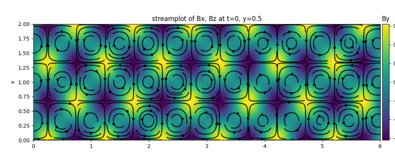

Figure 8: Magnetic field in the xz plane in the 31 mode

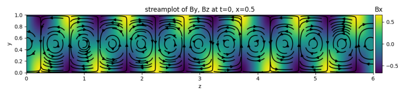

Figure 9: Magnetic field in the yz plane in the 31 mode

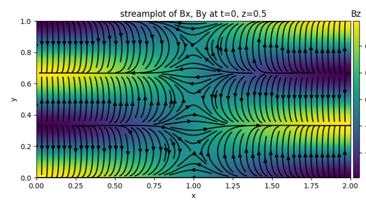

Figure 10: Magnetic field in the xy plane in the 13 mode

z directions. Figure 9 is a visualization of the magnetic field in the yz plane. Something of note that we can see clearly when comparing the yz and xz slices is that in the y direction we see half a period whereas in the x direction we see a period and a half. This makes sense if we think of standing waves, where the number of periods is generally 0.5 times the index of the mode. We can confirm this by swapping m and n and looking now at the m=1, n=3 mode. Figure 10 shows a visualization of the magnetic field in the xy plane. We can see that as we expected, there is now more oscillation in the y direction than the x direction. With this mode, we can also see more clearly that the field in the xy plane is flowing from regions of positive B_z to regions of negative B_z Figure 11 is a visualization of the magnetic field in the xz plane. When comparing this to the 31 mode, we can see that z oscillation has a higher frequency (Which makes sense because k should be larger in this mode than in the previous one). We can also see that there is only half a period in the x direction, which is expected. Figure 12 is a visualization of the magnetic field in the yz plane. As expected, we now see 1.5 periods in the y direction. Sticking with the m=1, n=3 mode, we can also look at the electric field. We know that there isn't a component of the electric field in the z direction, so we can just look at the 2D vector field. The electric field curls around the z direction, which is not true for the B field.

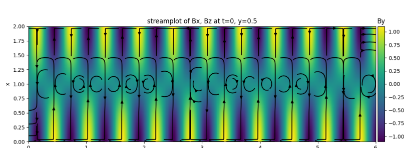

Figure 11: Magnetic field in the xz plane in the 13 mode

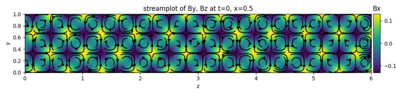

Figure 12: Magnetic field in the yz plane in the 13 mode

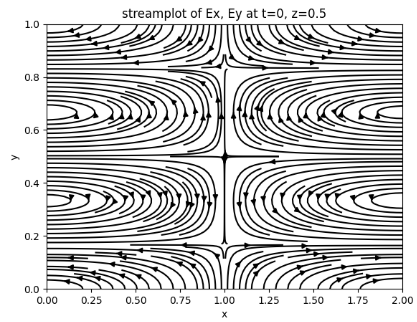

Figure 13: E field in the xy plane for the 13 mode

2 Exercise 4

My work finding the the longitudinal electric field, the cutoff frequencies, and the wave and group velocities is with the textbook problems. In order to visualize the TM waves, I used the same code that was used in exercise three to visualize the general TE waves. The only difference is that I redefined the fields function, replacing the TE field equations with the TM field equations. In the following figures I show still slices of the electric field in the m=1, n=1 mode, which is the lowest TM mode possible. We can see that the electric field flows along the wave guide in the z direction while it oscillates in the x and y directions. We can switch the color map to show the magnitude of the magnetic field. This lets us see that the boundary conditions are actually being applied, as we see that the electric field is zero at the boundaries, whereas the magnetic field must

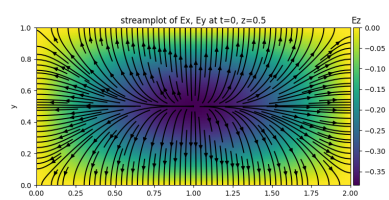

Figure 14: Electric field in the xy plane for the 11 TM mode

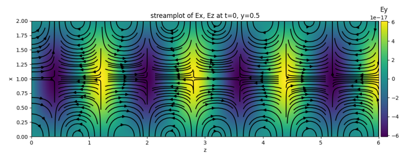

Figure 15: Electric field in the xz plane for the 11 TM mode

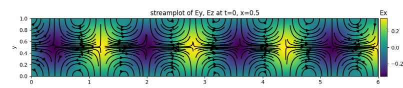

Figure 16: Electric field in the yz plane for the 11 TM mode

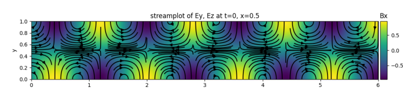

Figure 17: Magnetic field in the X direction at x=0.5 for the 11 TM mode

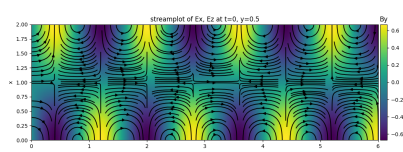

Figure 18: Magnetic field in the y direction at y=0.5 for the 11 TM mode

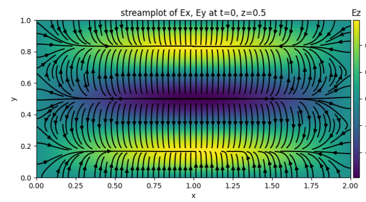

Figure 19: Electric field in xy plane for 13 TM mode

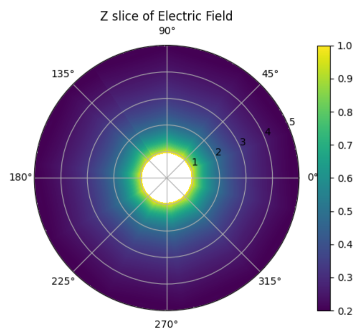

Figure 20: Electric Field in the s-phi plane at z-0

be at a local maxima or minima at the boundaries, which we see is in fact the case. Animations of these slices in time were also included in the submission as separate files. Now, comparing the E fields at a higher mode, Similar to before, increasing the mode results in more periods of oscillation being present in the wave guide.

3 Bonus Problem

In order to visualize the form of TEM waves in a coaxial transmission cable, I'm just going to use a colormap. The book shows that both the electric and magnetic fields only have components in one direction. The electric field is in the s direction and the magnetic field is in the ϕ direction. As a result, it isn't particularly informative to draw the vector field. Instead, I will just plot the magnitude of the field as a function of two variables as a color map. Also, the form of E and B are identical except for the constant value in front, so I will only show electric field here. The magnetic field magnitudes will look the exact same on a colormap so it's not worth showing twice. It is important to remember that the fields point in different directions, though. We can see that the field is constant with respect to angle, decreases in magnitude as we get farther from the center cable, and oscillates with z. I have also submitted animations of these slices in time, where it shows the waves moving in the z direction.

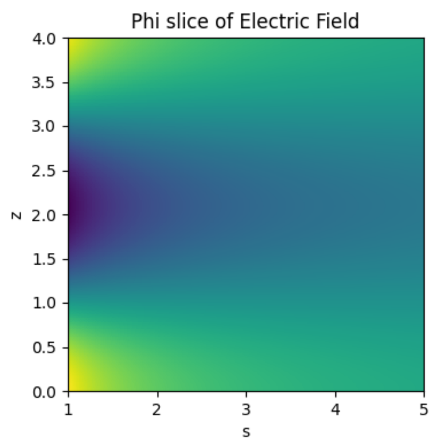

Figure 21: Electric Field in the s-z plane at phi=0