Mathematical Overview of DriveWise Route Planning Algorithm

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September 15, 2024

1 Introduction

DriveWise is a route-planning algorithm that focuses on minimizing accidents and optimizing the safety of routes. The algorithm uses historical accident data and various mathematical models to create safer driving routes, especially for new drivers. This document provides a mathematical overview of the methods and equations used in the development of the algorithm, including accident density mapping, turn penalties, and the Haversine distance formula.

2 Accident Density Calculation

To map the accidents to the nearest edges on the graph, we utilize the `BallTree` algorithm, which efficiently handles spatial queries using the Haversine distance as the metric. The edge coordinates are represented by their midpoints, which are calculated as follows:

$$\text{Midpoint}(u, v) = \frac{\mathbf{u} + \mathbf{v}}{2}$$

where $\mathbf{u}$ and $\mathbf{v}$ are the latitude and longitude coordinates of the nodes u and v.

Given an accident at location (x_i, y_i), the Haversine distance d to the midpoint of the edge is calculated as:

$$d = 2r \cdot \arcsin\left(\sqrt{\sin^2\left(\frac{\Delta\phi}{2}\right) + \cos(\phi_1)\cos(\phi_2)\sin^2\left(\frac{\Delta\lambda}{2}\right)}\right)$$

where:

- r is the radius of the Earth (approximated as 6,371,000 meters),
- $\Delta\phi = \phi_2 - \phi_1$ is the difference in latitudes,
- $\Delta\lambda = \lambda_2 - \lambda_1$ is the difference in longitudes,

- ϕ_1 and ϕ_2 are the latitudes of the two points (in radians),
- λ_1 and λ_2 are the longitudes of the two points (in radians).

This distance is used to determine whether an accident point falls within a certain buffer distance (e.g., 50 meters) from an edge.

3 Turn Angle Calculation

We also consider the angle of turns along a route to penalize sharp turns, as sharper turns are more difficult for inexperienced drivers. The angle between two edges $(u \to v)$ and $(v \to w)$ is calculated using the following formula for the dot product:

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{w}}{\|\mathbf{u}\|\|\mathbf{w}\|}$$

where:

- $\mathbf{u} = \mathbf{v} - \mathbf{u}$ is the vector representing the first edge,
- $\mathbf{w} = \mathbf{w} - \mathbf{v}$ is the vector representing the second edge,
- $\|\mathbf{u}\|$ and $\|\mathbf{w}\|$ are the magnitudes (Euclidean norms) of the respective vectors.

The turn angle θ is then given by:

$$\theta = \arccos\left(\frac{\mathbf{u} \cdot \mathbf{w}}{\|\mathbf{u}\|\|\mathbf{w}\|}\right)$$

If this angle exceeds a threshold (e.g., 45 degrees), we apply a fixed penalty to the route:

$$\text{Turn Penalty} = \text{Fixed Penalty Value}$$

4 Weight Adjustment Based on Accident Density and Turns

To determine the safest route, we assign weights to each edge in the graph based on the accident density and the turn penalties. The composite weight for each edge is computed as:

$$w_{uv} = \text{Base Weight} + \text{Accident Weight} + \text{Turn Penalty}$$

The base weight is proportional to the length of the edge, and the accident weight is based on the accident count for that edge:

$$\text{Accident Weight} = k_{\text{accident}} \cdot \text{accident\_count}$$

where k_{accident} is a scaling factor and accident_count is the number of accidents associated with that edge.

5 Routing Algorithm

We use the A* algorithm to find the shortest path between the origin and destination, taking into account the adjusted weights. The heuristic function $h(u, v)$ used in the A* algorithm is the Haversine distance between the nodes u and v:

$$h(u, v) = 2r \cdot \arcsin\left(\sqrt{\sin^2\left(\frac{\Delta\phi}{2}\right) + \cos(\phi_u)\cos(\phi_v)\sin^2\left(\frac{\Delta\lambda}{2}\right)}\right)$$

where ϕ_u, ϕ_v are the latitudes of the nodes u and v, and $\Delta\phi, \Delta\lambda$ are their differences in latitude and longitude, respectively.

6 Skillfulness Metric

The *Skillfulness Metric* adapts route difficulty based on the driver's experience. The weights of the edges are adjusted by a factor α_{skill}, which scales based on the driver's experience:

$$w_{uv} = \text{Base Weight} + \alpha_{\text{skill}} \cdot (\text{Accident Weight} + \text{Turn Penalty})$$

For inexperienced drivers (skill level near 0), the accident and turn penalties are heavily weighted, ensuring safer routes. For experienced drivers (skill level near 1), the algorithm returns routes similar to standard map services.

The DriveWise algorithm integrates historical accident data, turn angle penalties, and the Haversine distance to optimize routes based on safety, especially for new drivers. This approach allows us to dynamically adjust routes based on the driver's skill level and the inherent safety of different roads.