f.

$$H = F \Lambda F^*$$

$$H = \bar{H} = \bar{F} \bar{\Lambda} \overline{F^*} = \bar{F}^T \bar{\Lambda} \bar{\bar{F}}^T = F^* \bar{\Lambda} \bar{\bar{F}} = F^* \bar{\Lambda} F$$

H is real

F is symetric

F is symetric

i. $DFT^k$

$k=2$:

$$DFT^2 = \frac{1}{\sqrt{m}}\begin{pmatrix} w^{00} & w^{10} & \cdots & w^{(m-1)0} \\ w^{01} & w^{11} & & w^{(m-1)1} \\ \vdots & & & \vdots \\ w^{0(m-1)} & w^{1(m-1)} & & w^{(m-1)^2} \end{pmatrix}^* \frac{1}{\sqrt{m}}\begin{pmatrix} w^{00} & w^{10} & \cdots & w^{(m-1)0} \\ \vdots & & & \vdots \\ w^{0(m-1)} & & \cdots & w^{(m-1)^2} \end{pmatrix} =$$

$$\Rightarrow DFT^2_{l,j} = \frac{1}{m}\left(w^{0l^*} w^{j\cdot 0^*} + w^{1l^*} w^{j\cdot 1^*} + \dots + w^{ml^*} w^{j\cdot (m)^*}\right) =$$

$$= \frac{1}{m}\sum_{k=0}^{m-1} (w^{kl})^* (w^{jk})^* = \frac{1}{m}\sum_{k=0}^{m-1} e^{-\frac{i2\pi kl}{m}} e^{-\frac{i2\pi kj}{m}} =$$

$$= \frac{1}{m}\sum_{k=0}^{m-1} e^{-\frac{i2\pi k}{m}(l+j)} = \begin{cases} 1, & l+j=0 \mod m \\ 0, & \text{else} \end{cases}$$

Therefore, the result is:

$$\begin{array}{c|c} & \begin{matrix} 0 & 1 & \cdots & m-2 & m-1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ \vdots \\ m-2 \\ m-1 \end{matrix} & \begin{pmatrix} 1 & 0 & & & 0 \\ 0 & 0 & & 0 & 1 \\ & & 0 & 1 & 0 \\ & & 1 & 0 & \\ 0 & 1 & & & 0 \end{pmatrix} \end{array} = Ac$$

$k=4$:

$$DFT^4 = DFT^2\, DFT^2 = Ac^2 = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & 1 \\ & & \iddots & \\ 0 & 1 & & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & 1 \\ \vdots & & \iddots & \\ 0 & 1 & \cdots & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \\ 0 & & \ddots & \\ \vdots & & & \\ 0 & & & 1 \end{pmatrix} = I$$

$k=3$:

$$DFT^3 = DFT^4\, DFT^* = DFT^*$$

In conclusion:

$$DFT^k = \begin{cases} DFT & k=1 \mod 4 \\ Ac & k=2 \mod 4 \\ DFT^* & k=3 \mod 4 \\ I & k=0 \mod 4 \end{cases}$$