1. **Numerical and Practical Bit allocation for Two-Dimensional Signals**
   1. From differentiating and integrating we get that:

Because

* + 1. Value-range =
  1. Below is the approximated signal.

**Graphical user interface, application

Description automatically generated**

* 1. From numerically computing the vertical derivative energy, horizontal derivative energy and the value range:

**Text

Description automatically generated**

The results computed analytically are , in that order.

The numerical computed results deviate slightly. A higher resolution would generate a smaller deviation.

* 1. Implemented the formulas in code.
  2. The obtained results are displayed in the table below:

|  |  |  |
| --- | --- | --- |
| Bit-budget | 5000 | 50000 |
|  | 20.73, 72.54, 3.32 | 53.53, 187.32, 4.9855 |
|  | 21, 79, 3 | 54, 185, 5 |

* 1. Implemented in code.
  2. The results obtained by the search procedure are displayed in the table below:

|  |  |  |
| --- | --- | --- |
| Bit-budget | 5000 | 50000 |
|  | 20.69, 72.58, 3.32 | 53.49, 187.42, 4.9866 |
|  | 21, 79, 3 | 54, 185, 5 |

The “real” results differ slightly. But the rounded results are identical. This is because we round the results to the best feasible solution.

The reconstructed images obtained in the experiment are:

A picture containing tiled, tile, tub

Description automatically generated

A picture containing text

Description automatically generated

The images for both experiments are the same because the parameters are the same.

* + 1. From the same analysis we get that:
    2. The approximated image:

Table

Description automatically generated

* + 1. The numerical calculated results are:
       1. Value-range = 5000
       2. horizontal derivative energy: 3020708099.8682528
       3. vertical derivative energy: 246727724.06950286

By no surprise, the results are the same only that the horizontal and vertical derivatives energy are swapped (actually the results differ slightly because we used a lower resolution when sampling )

* + 1. The obtained results are displayed in the table below:

|  |  |  |
| --- | --- | --- |
| Bit-budget | 5000 | 50000 |
|  | 72.54, 20.73, 3.32 | 187.32, 53.53, 4.9855 |
|  | 79, 21, 3 | 185, 54, 5 |

* + 1. The results obtained by the search procedure are displayed in the table below:

|  |  |  |
| --- | --- | --- |
| Bit-budget | 5000 | 50000 |
|  | 72.58, 20.69, 3.32 | 187.42, 53.49, 4.9866 |
|  | 79, 21, 3 | 185, 54, 5 |

As expected, these are the same results where are swapped.

The reconstructed images obtained in the experiment are:

A picture containing diagram

Description automatically generated

A picture containing table

Description automatically generated

1. **Hadamard, Hadamard-Walsh, and Haar matrices**
   1. Implemented in code.