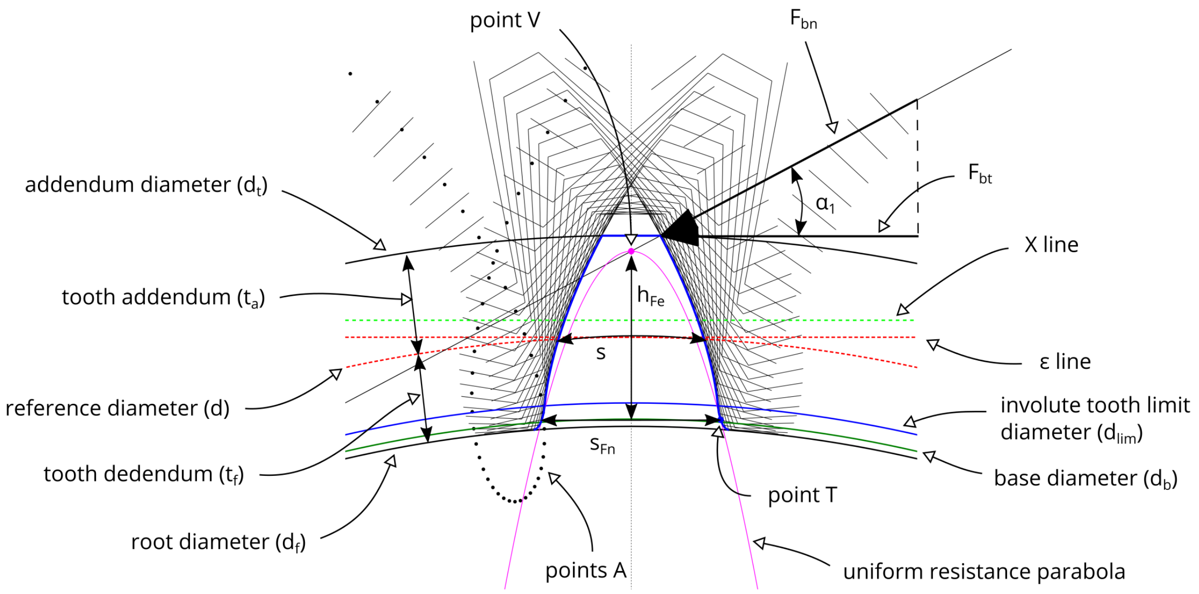
Figure #1 contains a visual representation of the terms related to **spur gears** and to the **tooth root stress calculation**.

#1 - Detail of a spur gear



In figure #2 there is the schema used to generate the **involute profile of the tooth of the gear**.

#2 - Involute schema to create the gear

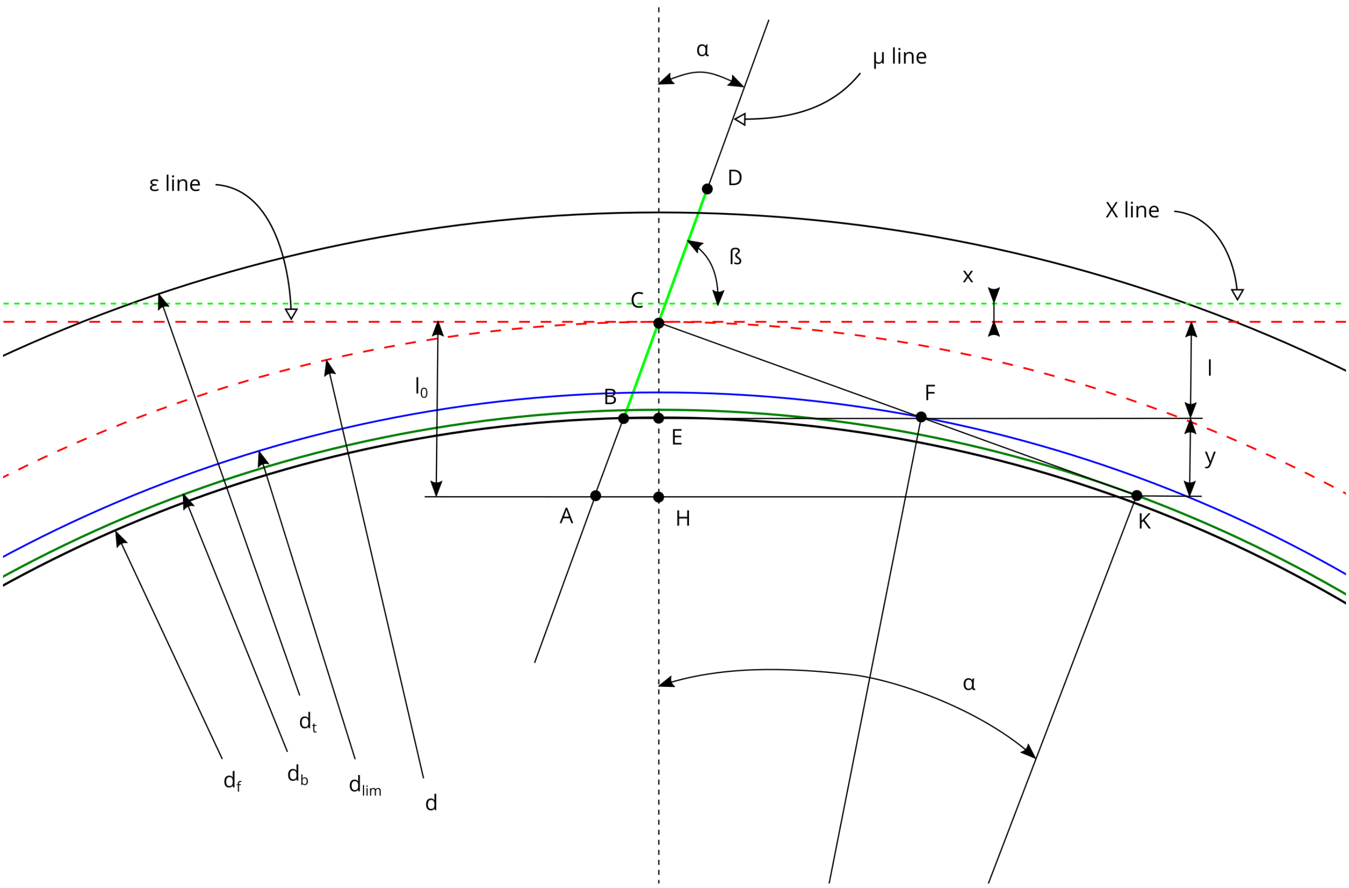
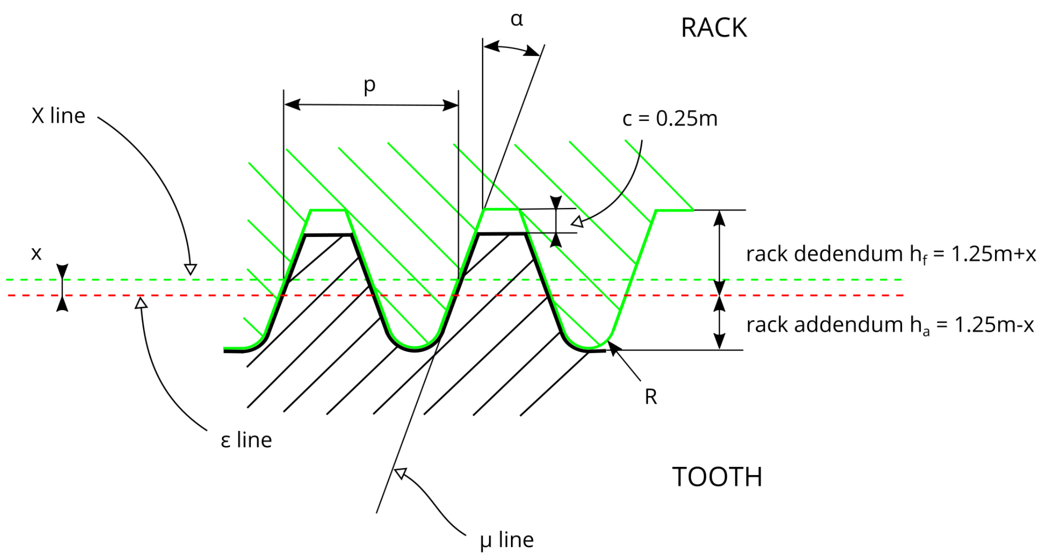


Figure #3 represents the **rack cutter** used to generate the gear, as defined in ISO 53:1998.

#3 - Rack tooth profile



In the following table there are a numerical example and some basic **formulas related to standard spur gears** valid if R/m = 0 and x/m = 0.

Element	Formula	Example
number of teeth	z z	30
module	m m	5 mm
pressure angle	α α	20°
rack shift coefficient	x/m x/m	0
coefficient of fillet radius of the rack cutter	R/m R/m	0

static nominal torque	C C	250 Nm
face width	b b	10 mm
	$l_0 = \frac{d}{2} \cdot \sin^2(\alpha)$ l0=d2·sin2(α)	8.77 mm
	$\frac{y}{d/2} = \frac{2.5}{z} - \sin^2(\alpha)$ yd/2=2.5z−sin2(α)	-0.0336
	$l = l_0 + y$ l=l0+y	6.25 mm
pitch	$p = m \cdot \pi$ p=m·π	15.71 mm
reference diameter	$d = m \cdot z$ d=m·z	150 mm
base diameter	$d_b = d \cdot \cos(\alpha)$ db=d·cos(α)	140.95 mm
involute tooth limit diameter	$d_{lim} = 2 \cdot \sqrt{(r - l)^2 + \left(\frac{l}{\tan(\alpha)}\right)^2}$ dlim=2·(r−l)2+(ltan(α))2	141.72 mm
root diameter	$d_f = d - 2 \cdot l$ df=d−2·l	137.5 mm
addendum diameter	$d_t = d + 2 \cdot m$ dt=d+2·m	160 mm

tooth addendum	$t_a = m$ ta=m	5 mm
tooth dedendum	$t_f = 1.25 \cdot m$ tf=1.25·m	6.25 mm
circular reference tooth thickness	$s = \frac{m \cdot \pi}{2}$ s=m·π2	7.85 mm
	$z_{min} = 1.25 \cdot \frac{2}{\sin^2(\alpha)}$ zmin=1.25·2sin2(α)	22
rack addendum	$h_a = 1.25 \cdot m$ ha=1.25·m	6.25 mm
rack dedendum	$h_f = 1.25 \cdot m$ hf=1.25·m	6.25 mm
nominal load, normal to the line of contact	$F_{bn} = \frac{C}{d/2 \cdot \cos(\alpha)}$ Fbn=Cd/2·cos(α)	3547.26 N
	α_1 α1	26.92°
nominal transverse load in plane of action	$F_{bt} = F_{bn} \cdot \cos(\alpha_1)$ Fbt=Fbn·cos(α1)	3162.85 N
tooth root chord at the critical section	s_{Fn} sFn	9.74 mm
bending moment arm relevant to load application at the tooth tip	h_{Fe} hFe	9.4 mm

tooth form factor - Lewis method	$Y_L = \frac{{s_{Fn}}^2}{6 \cdot h_{Fe} \cdot m}$ YL=sFn26·hFe·m	0.3361
tooth root bending stress at point T	$\sigma_f = \frac{F_{bt}}{Y_L \cdot b \cdot m}$ σf=FbtYL·b·m	188.21 N/mm^2

Gear ratio

The gear ratio ττ of a gear train is the ratio of the angular velocity of the input gear to the angular velocity of the output gear:

$$\tau = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

τ=ω1ω2=d2d1=z2z1

where

ω_1ω1 is the angular velocity of the input gear e ω_2ω2 is the angular velocity of the output gear;
d_1d1 is the reference diameter of the input gear e d_2d2 is the reference diameter of the output gear;
z_1z1 is the number of teeth of the input gear e z_2z2 is the number of teeth of the output gear.

Center distance

For a pinion and a wheel without correction (x/m = 0) or in case of complementary correction (e.g. the pinion with a positive correction x/m = +0.5 and the wheel with a negative correction x/m = -0.5), the center distance ii is calculated with the formula:

$$i = \frac{d_1}{2} + \frac{d_2}{2} = \frac{m \cdot (z_1 + z_2)}{2}$$

i=d12+d22=m·(z1+z2)2

In case $x_1 + x_2 \neq 0$ x1+x2≠0, the center distance i'i' is different from ii and may be calculated solving the following formulas:

$$inv(\alpha') = \frac{2 \cdot (x_1 + x_2) \cdot \tan(\alpha)}{m \cdot (z_1 + z_2)} + inv(\alpha)$$

inv(α')=2·(x1+x2)·tan(α)m·(z1+z2)+inv(α)

$$i' = i \cdot \frac{\cos(\alpha)}{\cos(\alpha')}$$

i'=i·cos(α)cos(α')

where α'α' is the working pressure angle, different from the pressure angle αα of the rack cutter.

Clearance

The pinion-wheel clearance cc depends from the value of $(x_1 + x_2)$ (x1+x2) and may be calculated with the formula

$$c = m \cdot [0.25 - \frac{x_1 + x_2}{m} + \frac{z_1 + z_2}{2} \cdot (\frac{\cos(\alpha)}{\cos(\alpha')} - 1)]$$

c=m·[0.25−x1+x2m+z1+z22·(cos(α)cos(α')−1)]

For gears with $x_1 + x_2 = 0$ x1+x2=0, the clearance is equal to 0.25m (type A basic rack tooth profile - ISO 53:1998).