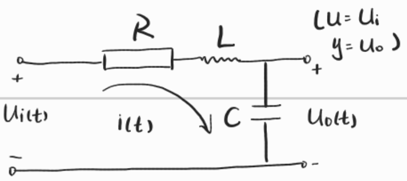
8.2. 线性定常系统状态空间表达式的建立

一、机理建模



$$\begin{cases} R\,i(t) + L\dfrac{di(t)}{dt} + U_o(t) = U_i(t) \\ i(t) = C\dfrac{du_o(t)}{dt} \end{cases}$$

取 $\begin{cases} x_1(t) = U_o(t) \\ x_2(t) = i(t) \end{cases}$

不能直接 $x_1 + x_2$

注意单位

$$\begin{cases} x_2(t) = C\dot{x}_1(t) \\ R x_2(t) + L\dot{x}_2(t) + x_1(t) = U_i \end{cases}$$

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C}x_2 \\ \dot{x}_2 = -\dfrac{1}{C}x_1 - \dfrac{R}{L}x_2 + \dfrac{1}{L}U_i \end{cases}$$

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

令 $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}x(t) \end{cases}$$

二、根据微分方程的传递函数建立状态空间表达式

1. 传递函数没有零点（不含输入的导数）

$$y^{(n)}(t) + a_1 y^{(n-1)}(t) + \cdots + a_{n-1}\dot{y}(t) + a_n y(t) = b_0 u(t)$$

$$G(s) = \frac{b_0}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

取 $\begin{cases} x_1(t) = y(t) \\ x_2(t) = \dot{x}_1(t) = \dot{y}(t) \\ \vdots \\ x_n(t) = \dot{x}_{n-1}(t) = y^{(n-1)}(t) \end{cases}$

$$\dot{x}_n(t) = y^{(n)}(t)$$

$$= -a_n x_1(t) - a_{n-1}x_2(t)$$

$$\cdots - a_1 x_n(t) + b_0 u(t)$$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & \cdots & & 1 \\ -a_n & -a_{n-1} & \cdots & & -a_1 \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix}u(t) \\ y(t) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}x(t) \end{cases}$$

可观规范I型

例8.1 $\dddot{y}+5\ddot{y}+\dot{y}+2y=2u$

三阶导数 ⟶ 对应三维矩阵

$$\begin{cases}\dot{x}(t)=\begin{bmatrix}0&1&0\\0&0&1\\-2&-1&-5\end{bmatrix}x(t)+\begin{bmatrix}0\\0\\2\end{bmatrix}b(t)\\ y(t)=\begin{bmatrix}1&0&0\end{bmatrix}x(t)\end{cases}$$

2.传递函数有零点

方法一，

取 $x_1=y-h_0u$

$$\begin{cases}\dot{x}_1=x_2+h_1u\\ \dot{x}_2=x_3+h_2u\\ \quad\cdots\\ \dot{x}_{n-1}=x_n+h_{n-1}u\end{cases}\qquad\begin{cases}h_0=b_0\\ h_1=b_1-a_1b_0\\ h_2=(b_2-a_2b_0)-a_1h_1\\ \quad\vdots\end{cases}$$

$$\dot{x}_n=-a_nx_1-\cdots-a_1x_n+h_nu$$

方法二：

$$y^{(n)}+a_1y^{(n-1)}+\cdots+a_ny=b_0u^{(n)}+b_1u^{(n-1)}+\cdots+b_nu$$

① $b_0=0$

$$G(s)=\frac{b_1s^{n-1}+\cdots+b_n}{s^n+a_1s^{n-1}+\cdots+a_n}$$

$$U(s)\longrightarrow\boxed{\frac{1}{s^n+a_1s^{n-1}+\cdots+a_n}}\xrightarrow{Z(s)}\boxed{b_1s^{n-1}+\cdots+b_n}\longrightarrow Y(s)$$

$$Z(s)=\frac{1}{s^n+\cdots+a_n}U(s)$$

$$s^nZ(s)+a_1s^{n-1}Z(s)+\cdots+a_nZ(s)=U(s)$$

L^{-1}: $z^{(n)}(t)+a_1z^{(n-1)}(t)+\cdots+a_nz(t)=u(t)$

取 $x_1=z$

$x_2=\dot{x}_1=\dot{z}$

$\vdots$

$x_n=\dot{x}_{n-1}=z^{(n-1)}$

$\dot{x}_n=z^{(n)}(t)=-a_nx_1\cdots-a_1x_n+u$

令 $X=\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix}$

$b_1s^{n-1}Z(s)+\cdots+b_nZ(s)=Y(s)$

L^{-1}: $y=b_1z^{(n-1)}(t)+\cdots+b_nz(t)$

$=b_nx_1+\cdots+b_1x_n$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 1 \\ -a_n & -a_{n-1} & & -a_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = [\, b_n \;\; b_{n-1} \;\; \cdots \;\; b_1 \,] \, x(t) \end{cases}$$

② $b_0 \neq 0$

$$G(s) = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

$$\begin{array}{r} b_0 \\ s^n + a_1 s^{n-1} + \cdots + a_n \overline{\big) \, b_0 s^n + b_1 s^{n-1} + \cdots + b_n} \\ b_0 s^n + a_1 b_0 s^{n-1} + \cdots + a_n b_0 \\ \hline (b_1 - a_1 b_0) s^{n-1} + \cdots + (b_n - a_n b_0) \end{array}$$

$$G(s) = b_0 + \frac{(b_1 - a_1 b_0) s^{n-1} + \cdots + (b_n - a_n b_0)}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & & \cdots & -a_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = [\, b_n - a_n b_0 \;\; \cdots \;\; b_1 - a_1 b_0 \,] \, x(t) + b_0 u(t) \end{cases}$$

例 8.1 $\dddot{y} + 5\ddot{y} + \dot{y} + 2y = \dot{u} + 2u$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -1 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\, 2 \;\; 1 \;\; 0 \,] \, x(t)$$

三. 根据传递函数的实数极点建立状态空间表达式

1. 单极点

$$\frac{Y(s)}{U(s)} = \frac{C_1}{s - s_1} + \cdots + \frac{C_n}{s - s_n}$$

$$\begin{cases} x_1(s) = \dfrac{\cancel{1}\; C_1}{s - s_1} U(s) \\ x_2(s) = \dfrac{\cancel{1}\; C_2}{s - s_2} U(s) \\ \cdots \\ x_n(s) = \dfrac{\cancel{1}\; C_3}{s - s_n} U(s) \end{cases}$$

$s x_1(s) - s_1 x(s) = U(s)$

$$\begin{cases} \dot{x}_1(t) = s_1 x(t) + u(t) \\ \dot{x}_2(t) = s_2 x(t) + u(t) \\ \cdots \\ \dot{x}_n(t) = s_n x_n(t) + u(t) \end{cases}$$

$Y(s) = C_1 x_1(s) + C_2 x_2(s) + \cdots + C_n x_n(s)$

$y(t) = C_1 x_1(t) + C_2 x_2(t) + \cdots + C_n x_n(t)$

令 $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdots \\ x_n(t) \end{bmatrix}$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} s_1 & & \\ & s_2 & \\ & & \ddots \\ & & & s_n \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u(t) \\ y(t) = [\, C_1 \;\; C_2 \;\; \cdots \;\; C_n \,] \, x(t) \end{cases}$$

或 $$\begin{cases} \dot{x}(t) = \begin{bmatrix} s_1 & & \\ & s_2 & \\ & & \ddots \\ & & & s_n \end{bmatrix} x(t) + \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix} u(t) \\ y(t) = [\, 1 \;\; 1 \;\; \cdots \;\; 1 \,] \, x(t) \end{cases}$$

例 8.2 $$\frac{Y(s)}{U(s)} = \frac{2s+1}{s^3 + 7s^2 + 14s + 8} = \frac{-\frac{1}{3}}{s+1} + \frac{\frac{3}{2}}{s+2} - \frac{\frac{7}{6}}{s+4}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & & \\ & -2 & \\ & & -4 \end{bmatrix} x(t) + \begin{bmatrix} -\frac{1}{3} \\ \frac{3}{2} \\ -\frac{7}{6} \end{bmatrix} u(t)$$

2. 单重实极点

$$\frac{Y(s)}{U(s)} = \frac{\overset{s\text{多项式}}{*}}{(s-s_1)^n} = \frac{C_1}{(s-s_1)^n} + \frac{C_2}{(s-s_1)^{n-1}} + \cdots + \frac{C_n}{s-s_1}$$

$$\begin{cases} x_1(s) = \dfrac{1}{(s-s_1)^n}U(s) = \dfrac{1}{s-s_1}x_2(s) \\ x_2(s) = \dfrac{1}{(s-s_1)^{n-1}}U(s) = \dfrac{1}{s-s_1}x_3(s) \\ \vdots \\ x_{n-1}(s) = \dfrac{1}{(s-s_1)^2}U(s) = \dfrac{1}{s-s_1}x_n(s) \\ X_n(s) = \dfrac{1}{s-s_1}U(s) \end{cases}$$

$$sX_1(s) - s_1X_1(s) = X_2(s)$$

$$\begin{cases} \dot{X}_1(t) = s_1X_1(t) + X_2(t) \\ \dot{X}_2(t) = s_1X_2(t) + X_3(t) \\ \vdots \\ \dot{X}_{n-1}(t) = s_1X_{n-1}(t) + X_n(t) \\ \dot{X}_n(t) = s_1X_n(t) + U(t) \end{cases}$$

$$Y(s) = C_1X_1(s) + \ldots + C_nX_n(s)$$

$$L^{-1}:\ y(t) = C_1X_1(t) + \cdots + C_nX_n(t)$$

令 $X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_n(t) \end{bmatrix}$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} s_1 & 1 & & \\ & s_1 & 1 & \\ & & \ddots & 1 \\ & & & s_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} U(t) \\ y(t) = [C_1\ C_2\ \cdots\ C_n]\, x(t) \end{cases}$$

例 8.3 $\dfrac{Y(s)}{U(s)} = \dfrac{2s^2+5s+1}{(s-2)^3}$

$$= \frac{2(s-2)^2+13s-7}{(s-2)^3}$$

$$= \frac{2(s-2)^2+13(s-2)+19}{(s-2)^3}$$

eg. $\dfrac{Y(s)}{U(s)} = \dfrac{2s^2+5s+1}{(s-2)^3}$

$$\begin{cases} X_1(s) = \dfrac{1}{(s-2)^3}U(s) \\ X_2(s) = \dfrac{1}{(s-2)^2}U(s) \\ X_3(s) = \dfrac{1}{(s-2)}U(s) \end{cases} \Rightarrow \begin{cases} \dot{x}_1(t) = 2x_1(t) + x_2(t) \\ \dot{x}_2(t) = 2x_2(t) + x_3(t) \\ \dot{x}_3(t) = 2x_3(t) + u(t) \end{cases}$$

$$y(t) = 19x_1(t) + 13x_2(t) + 2x_3(t)$$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = [19,\ 13,\ 2]\, x(t) \end{cases}$$

8.3. 由状态空间表达式求传递函数

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \Rightarrow G(s) = ?$$

$$\begin{cases} sX(s) = AX(s) + BU(s) \\ Y(s) = CX(s) + DU(s) \end{cases}$$

$$(sI - A)X(s) = BU(s)$$

$$X(s) = (sI-A)^{-1}BU(s)$$

$$\therefore\ Y(s) = [C(sI-A)^{-1}B + D]\,U(s) \Rightarrow G(s) = C(sI-A)^{-1}B + D = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

8.4. 线性定常系统状态方程的解

一、齐次状态方程的解

$\dot{X} = Ax$ $\quad x(0) \neq 0$

$sX(s) - x(0) = Ax(s)$

$(sI - A)X(s) = X(0)$

$X(s) = (sI - A)^{-1}X(0)$

L^{-1}: $x(t) = L^{-1}[(sI-A)^{-1}]x(0)$

$(sI-A)\left(\frac{I}{s} + \frac{A}{s^2} + \cdots + \frac{A^k}{s^{k+1}} + \cdots\right)$

$= I + \frac{A}{s} + \cdots + \frac{A^k}{s^k} + \cdots - \frac{A}{s} - \cdots + \frac{A^{k+1}}{s^{k+1}} + \cdots = I$

$\therefore (sI-A)^{-1} = \frac{I}{s} + \frac{A}{s^2} + \cdots + \frac{A^k}{s^{k+1}} + \cdots\cdots$

$L^{-1}[(sI-A)^{-1}] = I + At + \frac{A^2t^2}{2!} + \cdots + \frac{1}{k!}A^k t^k + \cdots$

$L^{-1}[(sI-A)^{-1}] = e^{At}$ 矩阵指数

$x(t) = e^{At}x(0)$ —状态转移矩阵

例8.4 $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x$ ，$x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 求 $x(t)$

$(sI-A)^{-1} = \begin{bmatrix} s & -1 \\ +2 & s+3 \end{bmatrix}^{-1}$

$= \frac{1}{s(s+3)+2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix}$

$e^{At} = L^{-1}[(sI-A)^{-1}]$

$= \begin{bmatrix} 2e^{-t} - e^{-2t}, & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t}, & -e^{-t} + 2e^{-2t} \end{bmatrix}$

$x(t) = e^{At}x(0)$

$= \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$

$A^{-1} = \frac{\mathrm{adj}A}{|A|}$

求adjA时，若主对角线上，则直接去掉求行列式

若i行j列，则去掉i行j列的行列式 $(-1)^{i+j}|\quad|$

正则 ⟺ 非奇异（满秩，行列式不为0）

二、矩阵指数的性质

① $\frac{d}{dt}e^{At} = Ae^{At}$

② $e^{At}\big|_{t=0} = I_n$

③ $(e^{At})^{-1} = e^{-At}$

④ $e^{A(t_1+t_2)} = e^{At_1}e^{At_2}$

⑤ n为整数 $(e^{At})^n = e^{nAt}$

⑥ 若$AB=BA$，则 $e^{(A+B)t} = e^{At}e^{Bt}$

⑦ P非奇异，则 $e^{P^{-1}APt} = P^{-1}e^{At}P$

三、非齐次方程的解

$\dot{x} = Ax + Bu$

1、一般法

$e^{-At}(\dot{x} - Ax) = e^{-At}Bu(t)$

$\int_0^t \frac{d}{dt}[e^{-At}x(t)]dt = \int_0^t e^{-A\tau}Bu(\tau)d\tau$

2、拉氏变换法

$sX(s) - X(0) = Ax(s) + Bu(s)$

$(sI-A)X(s) = X(0) + Bu(s)$

$e^{-At}x(t)\Big|_0^t = e^{-At}x(t) - x(0) = \int_0^\infty e^{-A\tau}Bu(\tau)d\tau$

$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$　　　$X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s)$

若$t_0 \neq 0$，$x(t) = e^{At}x(t_0) + \int_{t_0}^t e^{A(t-\tau)}Bu(\tau)d\tau$　　L^{-1}: $x(t) = L^{-1}[(sI-A)^{-1}]x(0) + L^{-1}[(sI-A)^{-1}BU(s)]$

不能解

例8.5. $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$，$u(t) = 1(t)$　$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$　$e^{At} = \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix}$

$e^{At}x_0$ 已知

$$\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau = \int_0^t \begin{bmatrix} e^{-(t-\tau)} - e^{-2(t-\tau)} \\ -e^{-(t-\tau)} + 2e^{-2(t-\tau)} \end{bmatrix} d\tau = \begin{bmatrix} e^{-(t-\tau)} - \frac{1}{2}e^{-2(t-\tau)} \\ -e^{-(t-\tau)} + e^{-2(t-\tau)} \end{bmatrix}\Bigg|_0^t$$

$$= \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau = \begin{bmatrix} \frac{1}{2} + e^{-t} - \frac{1}{2}e^{-2t} \\ -e^{-t} + e^{-2t} \end{bmatrix}$$

8.7. 线性系统的可控性与可观测性

一、可控性及其判据

$$\begin{cases} \dot{x} = Ax + Bu & x \in R^n \\ y = Cx + Du & A \in R^{n\times n} \end{cases}$$

可控性矩阵

$$Q_k = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

充要条件：$\text{rank } Q_k = n$

例8.6. $\dot{x} = \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix}x + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u$

$AB = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$　$Q_k = [B \quad AB] = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$

$\text{rank } Q_k = 1 < 2$　不可控

例8.7. $\dot{x} = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}x + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}u$

$AB = \begin{bmatrix} 1 & -3 \\ -3 & 3 \end{bmatrix}$　$Q_k = \begin{bmatrix} 0 & 1 & 1 & -3 \\ 1 & -1 & -3 & 3 \end{bmatrix}$ 可控

二、可观测性及其判据

$$Q_g = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$\text{rank } Q_g = n$

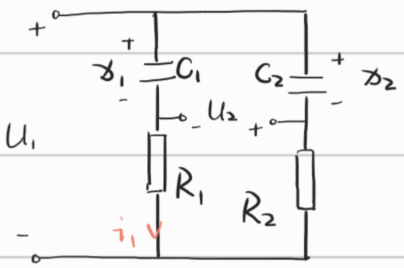
例8.8
$$\begin{cases} \dot{x} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \end{cases}$$

可观测

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$CA = [\,1$$

例8.9



不可控条件？ 不可观条件？

$$u_1 = x_1 + i_1 R_1 = x_2 + i_2 R_2$$

$$u_2 = (u_1 - x_2) - (u_1 - x_1) = x_1 - x_2$$

$$u_1 = C\frac{dx_1}{dt}R_1 + x_1$$

$$u_1 = C\frac{dx_2}{dt}R_2 + x_2$$

$$\begin{cases} \dot{x}_1 = -\frac{1}{C_1R_1}x_1 + \frac{1}{C_1R_1}u_1 \\ \dot{x}_2 = -\frac{1}{C_2R_2}x_2 + \frac{1}{C_2R_2}u_1 \end{cases}$$

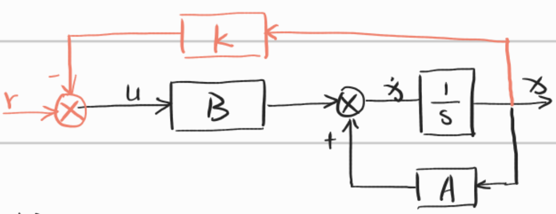
$$A = \begin{bmatrix} -\frac{1}{C_1R_1} & 0 \\ 0 & -\frac{1}{C_2R_2} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{C_1R_1} \\ \frac{1}{C_2R_2} \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

前提：可控

8.8 线性系统的状态反馈与极点配置

$$\dot{x} = Ax + Bu$$

引入 $u = r - kx$



$$\dot{x} = Ax + B(r - kx)$$
$$= (A - Bk)x + Br$$

$(A-Bk)$的特征根

$eig(A-Bk)$ = 闭环极点

例8.10
$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y = [100 \quad 0 \quad 0]\, x \end{cases}$$

解: $Q_k=[B\ \ AB\ \ A^2B]$

$$=\begin{bmatrix}0&0&1\\0&1&-3\\1&-3&7\end{bmatrix}$$

rank $Q_k=3$

∴可控

设 $k=[k_1\ \ k_2\ \ k_3]$

$$A-Bk=\begin{bmatrix}0&1&0\\0&0&1\\0&-2&-3\end{bmatrix}-\begin{bmatrix}0&0&0\\0&0&0\\k_1&k_2&k_3\end{bmatrix}$$

$$=\begin{bmatrix}0&1&0\\0&0&1\\-k_1&-2-k_2&-3-k_3\end{bmatrix}$$

$$|sI-(A-Bk)|=\begin{vmatrix}s&-1&0\\0&s&-1\\k_1&2+k_2&s+3+k_3\end{vmatrix}$$

$$=s[s(s+3+k_3)+2+k_2]+(-1)^{1+2}(-1)k_1$$

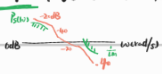
$$=s^3+(3+k_3)s^2+(2+k_2)s+k_1$$

$$=(s+5)(s+2+j2)(s+2-j2)$$

$$=s^3+9s^2+28s+40$$

$\begin{cases}k_1=40\\2+k_2=28\\3+k_3=9\end{cases}$ ∴ $\begin{cases}k_1=40\\k_2=26\\k_3=6\end{cases}$ $k=[40\ \ 26\ \ 6]$

系统与控制

- 绪论
  - 稳定好
  - 速度快
  - 精度高
- 古典
  - 模型
    - 传递函数 $G(s)=\frac{C(s)}{R(s)}$
    - 方框图化简
      - 平移
      - 梅森增益公式 $\phi(s)=\frac{\sum_{i=1}^{n}P_i\Delta_i}{\Delta}$，$\Delta=1-\sum L_i+\sum L_iL_j-\cdots$
        - ①两两互不接触
        - ②回路个数
      - 代数法
  - 时域
    - 二阶: $0<\xi<1$ $C(t)=1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\varphi)$, $\varphi=tg^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$?
      - $t_r=\frac{\pi-\varphi}{\omega_d}$, $t_p=\frac{\pi}{\omega_d}$, $t_s=\frac{3\sim4}{\xi\omega_n}$, $\sigma_p=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\times100\%$
      - $N=\frac{2\sqrt{1-\xi^2}}{\pi\xi}$ 或 $\frac{1.5\sqrt{1-\xi^2}}{\pi\xi}$
    - 稳定: Routh稳定判据
    - 稳态误差: $E_1(s)=\frac{E(s)}{H(s)}$ 终值定理 稳定? ν型
  - 频域 ($r(t)=A\sin\omega t$)
    - $G(j\omega)=G(s)|_{s=j\omega}$ 或 $G(j\omega)=\frac{C(j\omega)}{R(j\omega)}$ 几何表示
      - Bode图 (含前两图线)
      - Nyquist图 (大致走向)
      - Nichols图
    - Nyquist稳定判据: $\omega:(-\infty,\infty)$ 或 $[0,\infty)$
      - $N^+-N^-=\frac{1}{2}(P-Z)$
    - 初步设计 
    - PID: $G_c(s)=k_p+\frac{k_I}{s}+k_ds$
    - 校正: 超前
- 现代
  - 建立状态空间表达式 $\begin{cases}\dot{x}=Ax+Bu\\y=Cx+Du\end{cases}\Rightarrow G(s)=C(sI-A)^{-1}B+D$
  - $x(t)=e^{At}x(0)+\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$ 或 $x(t)=L^{-1}[(sI-A)^{-1}]x(0)+L^{-1}[(sI-A)^{-1}BU(s)]$
  - 能控, 能观: rank $[B\ \ AB\ \cdots\ A^{n-1}B]=n$　rank $\begin{bmatrix}C\\CA\\\vdots\\CA^{n-1}\end{bmatrix}=n$
  - 极点配置 $A=\begin{bmatrix}0&1&&\\\vdots&&\ddots&\\0&&&1\\\times&\times&\cdots&\times\end{bmatrix}$　$|sI-(A-Bk)|=(s-\lambda_1)\cdots(s-\lambda_n)$