ECE374 SP23 HW7

Contributors

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Problem 4

Can we solve the single-source longest-path problem by changing minimum to maximum in Dijkstra's algorithm? If so, prove your answer. If not, provide a counterexample. Assume that the graph only has positive edge weights.

Solution

Now our algorithm becomes

$\text{ModifiedDijkstra}(V, E, s)$

$\quad \textbf{for } v \in V$

$\quad\quad v.\text{dist} \leftarrow -\infty$

$\quad s.\text{dist} \leftarrow 0$

$\quad V' \leftarrow V$

$\quad \textbf{while } V' \neq \emptyset$

$\quad\quad v \leftarrow V'.\text{extractMax}() \qquad //\text{ node farthest from the visited region}$

$\quad\quad \textbf{for } u \in v.\text{neighbors}$

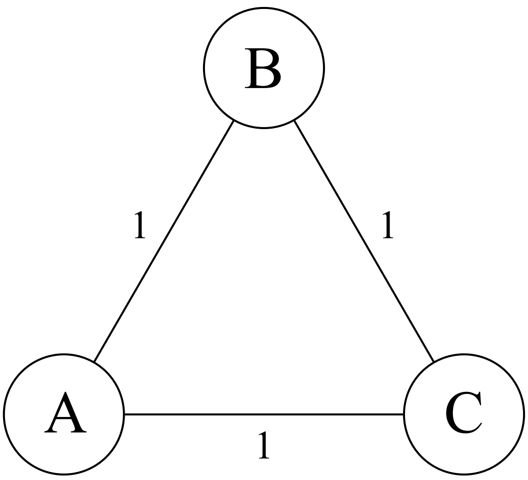
$\quad\quad\quad \textbf{if } u \in V' \qquad //\text{ neighbor unvisited}$

$\quad\quad\quad\quad e \leftarrow (v, u)$

$\quad\quad\quad\quad u.\text{dist} \leftarrow \max(u.\text{dist}, v.\text{dist} + e.\text{weight})$

$\quad \textbf{return } V$

But consider the following simple graph:



Step	Visited	B	C
0	A	$1, A$	$\cancel{1, A}$
1	AB	$[1, A]$	$2, B$
2	ABC		$[2, B]$

Once B is visited, its optimal (in this case, the longest) path is **fixed** at length 1. When C is visited, the desired path of $A \to C \to B$ will not be updated!

In the original Dijkstra's algorithm, since all the edges have positive weights and we are taking the shortest edge at each step, it is impossible to find an even shorter path that exits the visited region before entering it again. However, this is not the case here, and the "modified Dijkstra" is incorrect in looking for the single-source longest path.