ECE374 SP23 HW6

Contributors

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Problem 4

Plum blossom poles are a Kung Fu training technique, consisting of n large posts partially sunk into the ground, with each pole p_i at position (x_i, y_i). Students practice martial arts techniques by stepping from the top of one pole to the top of another pole. In order to keep balance, each step must be more than d meters but less than $2d$ meters.

Give an efficient algorithm to find a safe path from pole p_s to p_t if it exists.

Solution

Let the list of poles be $P \in \mathbb{R}^{n \times 2}$. We construct a graph $G = (V, E)$ to describe the map, where $V = \{p_1, p_2, \ldots, p_n\}$ are the poles and E is the set of all eligible moves.

$\text{FindSafePath}(n, P, d, s, t)$
    $G \leftarrow \text{emptyGraph}()$
    sort P by x_i
    for $i \leftarrow 1$ to n
        $G.\text{addVertex}(p_i)$
    for $i \leftarrow 1$ to n
        for $j \leftarrow i + 1$ to n
            if $x_j - x_i \geq 2d$
                break
            $\text{dist} \leftarrow \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
            if $\text{dist} \in (d, 2d)$
                $G.\text{addEdge}(p_i, p_j)$
    return $\text{DFS}(G, s, t)$

Time complexity.

- We choose DFS because the map is 2D. We may expect the graph to be sparse and contain lots of "chains."
- Constructing G takes $O(n^2)$ time, but in practice this is a loose upper bound since the inner loop breaks early.
- DFS takes $O(n + m)$ time, where $m = \#\text{Edges} \leq n^2$. Again, $m \approx n$ in our sparse case.
- Sorting P takes $O(n \log n)$ time. Total complexity is $O(n^2)$ but can go as low as $O(n)$ for sorted input and sparse graphs.