ECE374 SP23 HW3

Contributors

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Problem 2

For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either prove that the language is regular (by constructing a DFA or regular expression) or prove that the language is not regular (using fooling sets). Recall that Σ^+ denotes the set of all nonempty strings over Σ.

(a) $L_{2a} = \{0^n 1^n w \mid w \in \Sigma^* \text{ and } n \geq 0\}$

(b) $L_{2b} = \{w 0^n w \mid w \in \Sigma^* \text{ and } n > 0\}$

(c) $L_{2c} = \{xwwy \mid w, x, y \in \Sigma^+\}$

(d) $L_{2d} = \{xwwx \mid w, x \in \Sigma^+\}$

Solution

(a) Regular.

The regular expression is $(0+1)^*$.

Note that for $n \geq 0$, the language $0^n 1^n (0+1)^*$ is a subset of $(0+1)^*$.

(b) Non-regular. Let the fooling set be

$$F = \{0^n 1^n \mid n \geq 0\}$$

Let $a, b \in F, a = 0^i 1^i, b = 0^j 1^j$, where $i \neq j$. Also let $c = 0^{i+1} 1^i$.

(c) Non-regular. Let the fooling set be

$$F = \{0^{n+1} 1^n \mid n > 0\}$$

Let $a, b \in F, a = 0^{i+1} 1^i, b = 0^{j+1} 1^j$, where $i \neq j$. Also let $c = 1^i 0^{i+1}$.

(d) Non-regular. Let the fooling set be

$$F = \{0^n 1^n \mid n > 0\}$$

Let $a, b \in F, a = 0^i 1^i, b = 0^j 1^j$, where $i \neq j$. Also let $c = 1^i 0^i$.

In each case, $ac \in A$ and $bc \notin A$. Considering that F is an infinite set and each of its elements belongs to a distinct state, the corresponding language is non-regular.