ECE374 SP23 HW2

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Problem 2

Let

$$\Sigma = \{\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$$

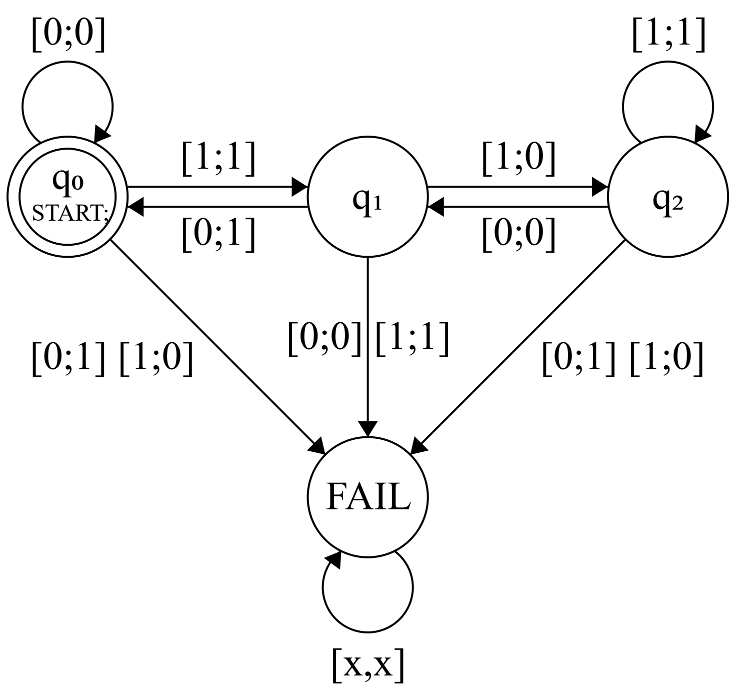
Consider each row to be a binary number and let

$$C = \{w \in \Sigma^* \mid \text{the bottom row of } w \text{ is three times the top row}\}$$

Show that C is regular.

Solution

We construct a DFA that accepts C to show that the language is regular.



Intuition

Important patterns include:

- Standalone 1

$$\begin{bmatrix}0\\1\end{bmatrix} \begin{bmatrix}1\\1\end{bmatrix}$$

- Two 1's

$$\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

- Three or more consequent 1's

$$\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\quad\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\quad\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

Rigid Formulation

Another way to think about the condition that "the bottom row is 3 times the top row" is that

$$\text{Bottom} = \text{shiftLeft}(\text{Top}, 1) + \text{Top}$$

- The DFA reads the characters of w from right to left as input.
- Each input pair can be viewed operands of a **full adder**.
  - First operand (A) = Digit in the top row
  - Second operand (B) = Digit in the *last input*'s top row
  - Sum (S) = Digit in the bottom row
- To judge if a string is in the language, we check if the sum S (bottom row) is valid under the sum of A, B and C_{in}.

The states can then be formulated as follows:

State	Last Input's Top Row B	Carry In C_{in}
q_0	0	0
q_{1a}	1	0
q_{1b}	0	1
q_2	1	1

The transition table can be calculated as follows:

This State	Last Input $\begin{bmatrix}B\\?\end{bmatrix}$	Carry In C_{in}	This Input $\begin{bmatrix}A\\S\end{bmatrix}$	Next State	Carry Out C_{out}
q_0	$\begin{bmatrix}0\\?\end{bmatrix}$	0	$\begin{bmatrix}0\\0\end{bmatrix}$	q_0	0
q_0	$\begin{bmatrix}0\\?\end{bmatrix}$	0	$\begin{bmatrix}1\\1\end{bmatrix}$	q_{1a}	0
q_{1a}	$\begin{bmatrix}1\\?\end{bmatrix}$	0	$\begin{bmatrix}0\\1\end{bmatrix}$	q_0	0
q_{1a}	$\begin{bmatrix}1\\?\end{bmatrix}$	0	$\begin{bmatrix}1\\0\end{bmatrix}$	q_2	1

This State	Last Input $\begin{bmatrix} B \\ ? \end{bmatrix}$	Carry In C_{in}	This Input $\begin{bmatrix} A \\ S \end{bmatrix}$	Next State	Carry Out C_{out}
q_{1b}	$\begin{bmatrix} 0 \\ ? \end{bmatrix}$	1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	q_0	0
q_{1b}	$\begin{bmatrix} 0 \\ ? \end{bmatrix}$	1	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	q_2	1
q_2	$\begin{bmatrix} 1 \\ ? \end{bmatrix}$	1	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	q_{1b}	1
q_2	$\begin{bmatrix} 1 \\ ? \end{bmatrix}$	1	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	q_2	1

Note that q_{1a} and q_{1b} transition to the same next states upon the same inputs and are thus equivalent.

Unlisted state-input pairs are invalid and therefore transition to the FAIL state.