

ECE374 SP23 HW2

Contributors

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Problem 1

(a) Describe the DFA that describes the following languages ($\Sigma = \{0, 1\}$). Formally define the DFAs and make sure their definitions are unique w.r.t. the other languages in part (a).

1. L_1 contains all strings where the substring 01 appears an odd number of times.
2. L_2 contains all strings where $\#(1, w)$ is divisible by three.
3. L_3 contains all strings where the binary value of w is divisible by seven.

(b) Let L denote the set of all strings $w \in \{0, 1\}^*$ that are in **at most two** of the languages in part (a). Formally describe a DFA with input alphabet $\Sigma = \{0, 1\}$ that accepts the language L .

Solution

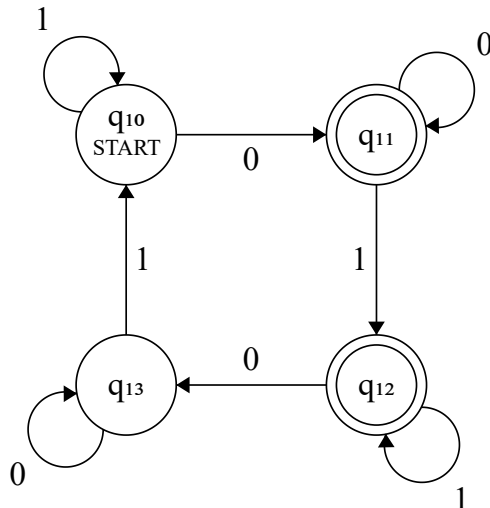
(a) Formal Definitions of the three DFAs

1. $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$, where

$$Q_1 = \{q_{10}, q_{11}, q_{12}, q_{13}\} \quad s_1 = q_{10} \quad A_1 = \{q_{11}, q_{12}\}$$

and δ_1 is described by the table below:

	q_{10}	q_{11}	q_{12}	q_{13}
0	q_{11}	q_{11}	q_{13}	q_{13}
1	q_{10}	q_{12}	q_{12}	q_{10}



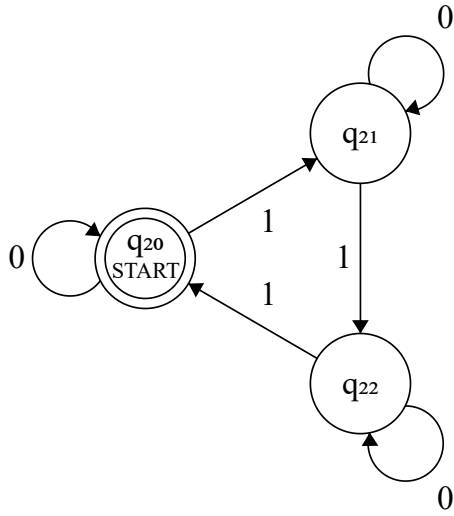
2. $M = (Q_2, \Sigma, \delta_2, s_2, A_2)$, where

$$Q_2 = \{q_{20}, q_{21}, q_{22}\} \quad s_2 = q_{20} \quad A_2 = \{q_{20}\}$$

and δ_2 is described by the table below:

	q_{20}	q_{21}	q_{22}
0	q_{20}	q_{21}	q_{22}

	q_{20}	q_{21}	q_{22}
1	q_{21}	q_{22}	q_{20}

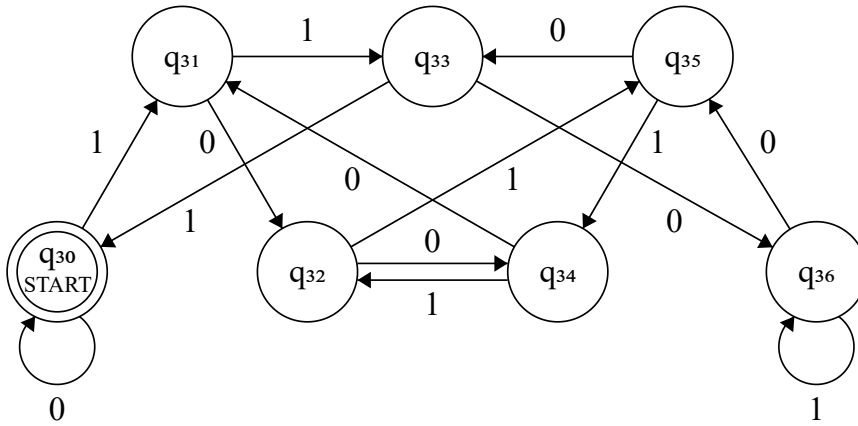


3. $M = (Q_3, \Sigma, \delta_3, s_3, A_3)$, where

$$Q_3 = \{q_{30}, q_{31}, q_{32}, q_{33}, q_{34}, q_{35}, q_{36}\} \quad s_3 = q_{30} \quad A_3 = \{q_{30}\}$$

and δ_3 is described by the table below:

	q_{30}	q_{31}	q_{32}	q_{33}	q_{34}	q_{35}	q_{36}
0	q_{30}	q_{32}	q_{34}	q_{36}	q_{31}	q_{33}	q_{35}
1	q_{31}	q_{33}	q_{35}	q_{30}	q_{32}	q_{34}	q_{36}



(b) Putting it together

The combined DFA should be the product of the three DFAs above.

$M = (Q, \Sigma, \delta, s, A)$, where

- $Q = Q_1 \times Q_2 \times Q_3 = \{(q_i, q_j, q_k) \mid q_i \in Q_1, q_j \in Q_2, q_k \in Q_3\}$
- $\delta((q_i, q_j, q_k), a) = (\delta_1(q_i, a), \delta_2(q_j, a), \delta_3(q_k, a))$
- $s = (s_1, s_2, s_3) = (q_{10}, q_{20}, q_{30})$
- $A = Q - A_1 \times A_2 \times A_3$