ECE 374 B ✧ Spring 2023
Homework 6

- Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.
- **Submit your solutions electronically on the course Gradescope site as PDF files.** please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
- For algorithmic problems, **we will be grading clarity and conciseness of solutions** (in addition to correctness) which admittedly is a subjective measure, but it is necessary. Being able to communicate your code is a far more important skill than being able to code itself.
- Solutions to a dynamic programming problem have (at minimum) three things:
  - A recurrence relation
  - A *brief* description of what your recurrence function represents and what each case represents.
  - A *brief* description of the memory element/storage and how it's filled in.
- Last minute tips:
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. **Largest Square of 1's** You are given a $n \times n$ bitonic array A and the goal is to find the set of elements within that array that form a square filled with only 1's.

$$
\begin{array}{cc}
 & j \rightarrow \\
i \downarrow & \begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

Figure 1: Example: The output is the side-length of the largest square of 1's (4 in the case of the graph above, yes there can be multiple squares of the greatest size).

2. Suppose you are given n poker chips stacked in two stacks, where the edges of all chips can be seen. Each chip is one of three colors. A turn consists of choosing a color and removing all chips of that color from the tops of the stacks. The goal is to minimize the number of turns until the chips are gone. For example, consider the stacks (RRGG,GBBB). Playing red, green, and then blue suffices to clear the stacks in three moves. Give an O(n2) dynamic programming algorithm to find the best strategy for a given pair of chip piles.

3. The traditional world chess championship is a match of 24 games. The current champion retains the title in case the match is a tie. Each game ends in a win, loss, or draw (tie) where wins count as 1, losses as 0, and draws as 1/2. The players take turns playing white and black. White plays first and so has an advantage. The champion plays white in the first game. The champ has probabilities ww, wd, and wl of winning, drawing, and losing playing white, and has probabilities bw, bd, and bl of winning, drawing, and losing playing black.

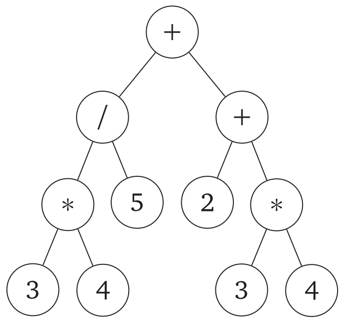
   (a) Write a recurrence for the probability that the champion retains the title. Assume that there are g games left to play in the match and that the champion needs to get i points (which may be a multiple of 1/2).

   (b) Based on your recurrence, give a dynamic programming algorithm to calculate the champion's probability of retaining the title.

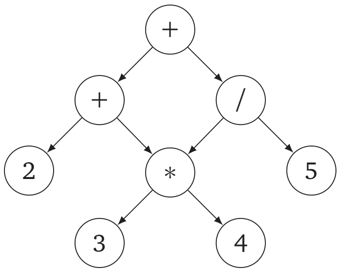
   (c) Analyze its running time for an n game match.

4. Plum blossom poles are a Kung Fu training technique, consisting of n large posts partially sunk into the ground, with each pole pi at position (xi, yi). Students practice martial arts techniques by stepping from the top of one pole to the top of another pole. In order to keep balance, each step must be more than d meters but less than 2d meters. Give an efficient algorithm to find a safe path from pole ps to pt if it exists.

5. Expressions as Graphs:



(a) Tree representation



(b) DAG representation

Figure 2: Representations of the arithmetic expression: $2 + 3 * 4 + (3 * 4)/5$.

(a) Suppose an arithmetic expression is given as a tree. Each leaf is an integer and each internal node is one of the standard arithmetical operations (+, -, *, /). For example, the expression 2+3*4+(3*4)/5 is represented by the tree in the figure above. Give an O(n) algorithm for evaluating such an expression, where there are n nodes in the tree.

(b) Suppose an arithmetic expression is given as a DAG (directed acyclic graph) with common subexpressions removed. Each leaf is an integer and each internal node is one of the standard arithmetical operations (+,-, *, /). For example, the expression 2+3*4+(3*4)/5 is represented by the DAG in the figure above. Give an O(n+m) algorithm for evaluating such a DAG, where there are n nodes and m edges in the DAG. (Hint: modify an algorithm for the tree case to achieve the desired efficiency.)