ECE374 SP23 HW4

Contributors

Zhirong Chen (zhirong4)

Ziyuan Chen (ziyuanc3)

Problem 1

Solve the following recurrence relations. For parts (a) and (b), give an exact solution. For parts (c) and (d), give an asymptotic one. In both cases, justify your solution.

(a) $A(n) = A(n-1) + 2n + 1; A(0) = 0$

(b) $B(n) = B(n-1) + n(n-1) - 1; B(0) = 0$

(c) $C(n) = C\left(\frac{n}{2}\right) + C\left(\frac{n}{3}\right) + C\left(\frac{n}{6}\right) + n$

(d) $D(n) = D\left(\frac{n}{2}\right) + D\left(\frac{n}{3}\right) + D\left(\frac{n}{6}\right) + n^2$

Solution

(a) $n^2 + 2n$

$$\begin{aligned} A(n) &= A(0) + \sum_{k=1}^{n}(2k+1) \\ &= 0 + 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 \\ &= 2 \times \frac{n(n+1)}{2} + n \\ &= n^2 + 2n \end{aligned}$$

(b) $\frac{1}{3}n^3 - \frac{4}{3}n$

$$\begin{aligned} B(n) &= B(0) + \sum_{k=1}^{n}(k(k-1)-1) \\ &= 0 + \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 \\ &= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} - n \\ &= \frac{n(n^2-4)}{3} \end{aligned}$$

(c) $O(n \log n)$

```
          C(n)           ··· n
        /   |   \
C(n/2) C(n/3) C(n/6)  ··· m
    .        .        .
    .        .        .
    .        .        .
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$m = \frac{n}{2} + \frac{n}{3} + \frac{n}{6} = n$

$C(n) = n \times \text{depthOfTree} = O(n \log n)$

(d) $O(n^2)$

```
          C(n)           ··· n ^ 2
        /   |   \
C(n/2) C(n/3) C(n/6)  ··· m
    .        .        .
    .        .        .
    .        .        .
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$m = \left(\frac{n}{2}\right)^2 + \left(\frac{n}{3}\right)^2 + \left(\frac{n}{6}\right)^2 = \frac{7}{18}n^2$

$C(n) = n^2 \times (1 + \frac{7}{18} + \left(\frac{7}{18}\right)^2 + \cdots + \left(\frac{7}{18}\right)^{\log n}) = O(n^2)$