ECE374 SP23 HW7

Contributors

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Problem 3

Your job is to arrange n schildren in a straight line, facing front. You are given a list of m statements of the form "i hates j," which indicates that i cannot be put somewhere behind j.

(a) Give an algorithm that orders the line (or says that it is not possible) in $O(m+n)$ time.

(b) Suppose instead you want to arrange the children in rows such that if i hates j, then i must be in a lower numbered row than j. Give an efficient algorithm to find the minimum number of rows needed, if possible.

Solution

(a)

First we construct a graph based on the relation of hatred, where

- each node represents a student, and
- an edge $u \to v$ represents that the student u hates another student v.

Then we do a topological sort on the graph. The node rank gives a order of the students.

- Note that the algorithm fails if a cycle is detected in the topological sorting procedure. A legal arrangement is not possible in this case.

(b)

Starting from the "source" with topological rank 0, we perform a BFS on the graph and find the ***maximum depth*** (or the longest path from the source). The algorithm reports failure when backward edges (i.e., cycles) are detected.

Alternatively, we can use Kahn's algorithm to find the topological order. The number of rows is the length of the longest path.

Pseudocode.

- Construct 2 queues, $\mathrm{current}$ and next
- Add all vertices with no incoming edges to $\mathrm{current}$
- Initialize an empty list to store the rows
- While $\mathrm{current}$ and next is not empty
    - construct a new row
    - while $\mathrm{current}$ is not empty

- Dequeue a vertex w from $\mathrm{current}$ and add it to row
- For each vertex x adjacent to w
  - Remove the edge $x \to w$
  - If w has no incoming edges, add it to next
- Swap $\mathrm{current}$ and next
- Add row to the list of rows

If all edges have been removed from the graph, the topological order is successfully found, and the rows are constructed. Having remaining edges indicates a cycle, in which case a topological order is not possible.