

# ECE374 SP23 HW3

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## Contributors

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## Problem 3

Describe the context-free grammar that describes each of the following languages:

- (a) All strings in  $\{0, 1\}^*$  whose length is divisible by 5.
- (b)  $L_{3b} = \{0^i 1^j 2^{i+j} \mid i, j \geq 0\}$
- (c)  $L_{3c} = \{0^i 1^j 2^k \mid i = j \text{ or } j = k\}$
- (d)  $L_{3d} = \{w \in \{0, 1\}^* \mid \#(01, w) = \#(10, w)\}$

## Solution

In each case, the context-free grammar is defined as

$$G = (V, T, P, S)$$

- (a) Recursively append groups of 5 bits.

- $V = \{S, A\}$
- $T = \{0, 1\}$
- $P = \begin{cases} S \rightarrow \epsilon \mid AS \\ A \rightarrow (0 + 1)^5 \end{cases}$

- (b) Symmetrically grow the "outer shell" before growing the "inner shell."

- $V = \{S, A\}$
- $T = \{0, 1, 2\}$
- $P = \begin{cases} S \rightarrow A \mid 0S2 \\ A \rightarrow \epsilon \mid 1A2 \end{cases}$

- (c) There are 2 cases.

- $V = \{S, A, B, C, D\}$
- $T = \{0, 1, 2\}$
- $P = \begin{cases} S \rightarrow A \mid B \\ A \rightarrow A2 \mid C \\ B \rightarrow 0B \mid D \\ C \rightarrow \epsilon \mid 0C1 \\ D \rightarrow \epsilon \mid 1D2 \end{cases}$

(d) # rising edge = # falling edge. Edges are created in pairs.

- $V = \{S, A, B, C, D\}$
- $T = \{0, 1\}$
- $P = \begin{cases} S \rightarrow \epsilon \mid A \mid B \\ A \rightarrow C \mid CBC \\ B \rightarrow D \mid DAD \\ C \rightarrow 0 \mid 0C \\ D \rightarrow 1 \mid 1D \end{cases}$