ECE374 SP23 HW3

Contributors

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Problem 1

A *finite-state transducer* (FST) gives an output based on the transition instead of the current state. It is defined by a 5-tuple:

$$(\Sigma, \Gamma, Q, \delta, s)$$

The output alphabet of a FST_{AR} consists of two signals, namely accept and reject ($\Gamma = \{\mathrm{A}, \mathrm{R}\}$). We say that $L(FST_{\mathrm{AR}})$ represents the language consisting of all strings that end with an accept (A) output signal.

Prove that $L(FST_{\mathrm{AR}})$ represents the class of regular languages.

Solution

Proof. We prove that a language is regular (i.e., can be represented by a DFA) *iff* it can be represented by a FST_{AR}.

If. For any given FST_{AR}

$$M_{T1} = (\Sigma_1, \Gamma, Q_{T1}, \delta_{T1}, s_{T1})$$

, we construct a DFA

$$M_{D1} = (\Sigma_1, Q_{D1}, \delta_{D1}, s_{D1}, A_{D1})$$

such that $L(M_{D1}) = L(M_{T1})$, where

- $Q_{D1} = Q_{T1} \times \Gamma$
- $\delta_{D1}((q, b), a) = \delta_{T1}(q, a), \quad \forall q \in Q_{T1}, a \in \Sigma_1, b \in \Gamma$
- $s_{D1} = (s_{T1}, \mathrm{R})$
- $A_{D1} = \{(q, \mathrm{A}) \mid q \in Q_{T1}\}$

Here $\delta_{T1} : Q_{T1} \times \Sigma_1 \to Q_{T1} \times \Gamma$ and $\delta_{D1} : Q_{D1} \times \Sigma_1 \to Q_{D1}$.

The core idea is to explicitly encode the last output signal in the states. There is a clear boundary between the *next-state logic* and *output logic*; thus, the expression of δ_{D1} is unrelated to b.

We have proven that any language representable by a FST_{AR} is regular.

Only if. For any given regular language L represented by a DFA

$$M_{D2} = (\Sigma_2, Q_2, \delta_{D2}, s_2, A_{D2})$$

, we construct a FST_{AR}

$$M_{T2} = (\Sigma_2, \Gamma, Q_2, \delta_{T2}, s_2)$$

such that $L(M_{T2}) = L(M_{D2})$, where

- $\delta_{T2}(q, a) = \begin{cases} (\delta_{D2}(q, a), \mathrm{A}) & \text{if } \delta_{D2}(q, a) \in A_{D2}, \\ (\delta_{D2}(q, a), \mathrm{R}) & \text{if } \delta_{D2}(q, a) \notin A_{D2}, \end{cases} \quad \forall q \in Q_2, a \in \Sigma_2$

The idea is to encode the status of the next state in the arcs pointing to it.

We have proven that any regular language can be represented by a FST_{AR}. **Q.E.D.**