ECE374 SP23 HW5

Contributors

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Problem 2

In lecture, we defined the recurrence of the longest increasing subsequence (LIS) problem as

$$LIS_{LEC}(i,j) = \begin{cases} 0 & i = 0 \\ LIS_{LEC}(i-1,j) & A[i] \geq A[j] \\ \max\begin{Bmatrix} LIS_{LEC}(i-1,j) \\ 1 + LIS_{LEC}(i-1,i) \end{Bmatrix} & A[i] < A[j] \end{cases}$$

But when we worked out the problem in lab, it looks like

$$LIS_{LAB}(i,j) = \begin{cases} 0 & i > n \\ LIS_{LAB}(i+1,j) & i \leq n \text{ and } A[j] \geq A[i] \\ \max\begin{Bmatrix} LIS_{LAB}(i+1,j) \\ 1 + LIS_{LAB}(i+1,i) \end{Bmatrix} & i \leq n \text{ and } A[j] < A[i] \end{cases}$$

Is one of them wrong? If not, what's the difference? Give a simple, short, English description of each recurrence. No long proofs for correctness are necessary.

Solution

Both LIS recurrences are correct and yield identical results.

The first recurrence starts the candidate LIS $A[1 \ldots i]$ from the **beginning** of the array and scans forward, selectively including numbers less than $A[j]$. $LIS(i,j)$ represents the length of LIS in $A[1 \ldots i]$ among numbers **less** than $A[j]$.

On the contrary, the second recurrence starts from the **end** of the array and scans backward. $LIS(i,j)$ represents the length of LIS in $A[i \ldots n]$ among numbers **greater** than $A[j]$.