ECE374 SP23 HW2

Contributors

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Problem 5

Given an arbitrary regular language L on some alphabet Σ, prove that it is closed under the following operation:

$$\text{cycle}(L) := \{xy \mid x, y \in \Sigma^*, yx \in L\}$$

Solution

Proof: Suppose that L can be represented by a **DFA** $M = (Q, \Sigma, \delta, s, A)$. We construct an **NFA** M' that accepts $\text{cycle}(L)$ as follows:

$$M' = (Q', \Sigma, \delta', s', A')$$

, where

- $Q' = Q \times Q \times \{\text{pre}, \text{post}\} \cup \{\text{start}\}$
- $s' = \text{start}$
- $A' = A \times A \times \{\text{post}\}$
- δ' is defined as follows:
  - $\delta'(\text{start}, \epsilon) = \{(q, q, \text{pre}) \mid q \in Q\}$
  - For all $q_i, q_j \in Q, a \in \Sigma$, $\delta'((q_i, q_j, \text{pre}), a) = \{(q_i, \delta(q_j, a), \text{pre}), (q_i, s, \text{post})\}$
  - For all $q_i, q_j \in Q, a \in \Sigma$, $\delta'((q_i, q_j, \text{post}), a) = \{(q_i, \delta(q_j, a), \text{post})\}$

Informally described, M' has $|Q|$ "branches" that explore all possibilities of the cycled string's position compared to the original string by allowing the machine to start at *any* of the $|Q|$ states. Each branch has $|Q| \times 2$ states which use the $\{\text{pre}, \text{post}\}$ flags to track whether the string has "cycled back."

Starting with a pre flag, each transition can either **(1)** simply move to the DFA's next state as if performing a typical pattern match or **(2)** cycle back and "restart" the branch by setting the flag to post. With the presence of the flags, we can ensure that the string is cycled back *exactly once*. Finally, note that each branch has a different accept state corresponding to the branch-specific starting state.