ECE374 SP23 HW5

Contributors

Zhirong Chen (zhirong4)

Ziyuan Chen (ziyuanc3)

Problem 3

Given an $n \times m$ grid filled with non-negative numbers, find a path from top left $(1, 1)$ to bottom right (n, m) that minimizes the sum of all numbers along its path. You can only move either down $(++i)$ or right $(++j)$ at any point in time. What is the running time of your algorithm?

Solution

Recurrence function. We denote the grid as G and the DP matrix as M. The DP recurrence is

$$M(i,j) = \begin{cases} G(i,j) & i = 1 \text{ and } j = 1 \quad // \text{ upper-left corner} \\ M(i-1,j) + G(i,j) & i > 1 \text{ and } j = 1 \quad // \text{ left edge} \\ M(i,j-1) + G(i,j) & i = 1 \text{ and } j > 1 \quad // \text{ upper edge} \\ \min \begin{Bmatrix} M(i-1,j) + G(i,j) \\ M(i,j-1) + G(i,j) \end{Bmatrix} & i > 1 \text{ and } j > 1 \quad // \text{ interior} \end{cases}$$

Memoization. $M(i,j)$ represents the minimum sum of all numbers along the optimal path from $(1,1)$ to (i,j). The matrix should be filled top-to-bottom, left-to-right. A grid can only be filled after its left and upper neighbors are filled.

The optimal path is obtained by backtracking from the bottom-right corner (n,m) to the top-left corner $(1,1)$, choosing the neighbor with the minimum sum at each step. Time complexity is $O(nm)$.