Dielectric Resonator Antennas—A Review and General Design Relations for Resonant Frequency and Bandwidth

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ABSTRACT

Open dielectric resonators (DRs) offer attractive features as antenna elements. These include their small size, mechanical simplicity, high radiation efficiency due to no inherent conductor loss, relatively large bandwidth, simple coupling schemes to nearly all commonly used transmission lines, and the advantage of obtaining different radiation characteristics using different modes of the resonator. In this article, we give a comprehensive review of the modes and the radiation characteristics of DRs of different shapes, such as cylindrical, cylindrical ring, spherical, and rectangular. Further, accurate closed form expressions are derived for the resonant frequencies, radiation Q-factors, and the inside fields of a cylindrical DR. These design expressions are valid over a wide range of DR parameters. Finally, the techniques used to feed DR antennas are discussed.

1. INTRODUCTION

Dielectric resonators fabricated out of low loss materials ($\tan\delta \approx 10^{-4}$, or less) and high relative permittivity ($\epsilon_r \approx$ 20–100) are widely used in shielded microwave circuits such as in filters and oscillators. In these applications, a dielectric resonator can exhibit a very high unloaded Q-factor given by

$$Q_u \approx \frac{1}{\tan\delta}. \tag{1}$$

On the other hand, if a dielectric resonator is placed in an open environment, the Q-factors of the lowest order modes of the resonator are reduced significantly ($Q_u \approx 10 - 100$), since power is now lost in the radiated fields. This fact makes dielectric resonators useful as antenna elements. Dielectric resonator (DR) antennas offer following attractive features:

- The dimensions of a DR antenna are of the order of $\lambda_0/\sqrt{\epsilon_r}$, where λ_0 is the free-space wavelength and ϵ_r is the dielectric constant of the resonator material. Thus, by choosing a high value of ϵ_r ($\epsilon_r \approx$ 10–100), the size of the DR antenna can be significantly reduced.
- There is no inherent conductor loss in dielectric resonators. This leads to high radiation efficiency of the antenna. This feature is especially attractive for millimeter (mm)-wave antennas, where the loss in metal fabricated antennas can be high.
- DRs offer simple coupling schemes to nearly all transmission lines used at microwave and mm-wave frequencies. This makes them suit-

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able for integration into different planar technologies. The coupling between a DR and the planar transmission line can be easily controlled by varying the position of the DR with respect to the line. The performance of DR antennas can therefore be easily optimized *experimentally*.

- The operating bandwidth of a DR antenna can be varied over a wide range by suitably choosing resonator parameters. For example, the bandwidth of the lower order modes of a DR antenna can be easily varied from a fraction of a percent to about 10% or more by the suitable choice of the dielectric constant of the resonator material.
- Each mode of a DR antenna has a unique internal and associated external field distribution. Therefore, different radiation characteristics can be obtained by exciting different modes of a DR antenna.

It was realized by Richtmyer as early as 1939 that open DRs radiate into free-space[1] [1]. The radiation Q-factors of isolated spherical dielectric resonators were reported by Gastine et al. in 1967 [2]. The possibility of constructing very small antennas using dielectric resonators was first considered by Sager and Tisi [3]. In 1975, Van Bladel reported a rigorous asymptotic theory for evaluating the modes of dielectric resonators of *arbitrary* shape but high permittivity [4,5]. In this studies, Van Bladel derived the general nature of the internal and radiated fields of dielectric resonators. Later, he and his group presented numerical results for the resonant frequencies, fields and radiation Q-factors of the lower order axisymmetric modes of a cylindrical ring dielectric resonator [6–9]. A small experimental array of rectangular dielectric resonators excited by a dielectric guide was reported by Birand and Gelsthorpe [10]. However, the first systematic theoretical and experimental study on a specific dielectric resonator antenna configuration was reported by Long et al. in 1983 [11]. Since then, more work on the same subject has been reported [11–29]. The most popular shape studied for practical antenna applications has been the cylindrical, although other shapes such as spherical [13,20,23], rectangular [12,27], and cylindrical ring [28] have also been studied. Since each shape has many resonant modes, use of different modes has also been reported.

The design of DR antennas requires information on basic quantities such as the radiated fields, resonant frequency, bandwidth (or, the radiation Q-factor), and the field distribution inside the resonator. Unfortunately, no comprehensive information of this form is available in the literature which can be used by an antenna engineer. The literature survey shows that most of the work reported so far on DR *antennas* is quite preliminary, devoted mainly to demonstrate their feasibility. On the other hand, significant information has been generated on DRs during the last decade due to the impetus provided by their popularity in microwave circuits. Most of the reported work on DRs relates to their use as circuit elements in shielded circuits, and therefore, radiated fields are generally not considered. Nevertheless, some information is available in the literature which is useful in understanding the radiation characteristics of the different modes of resonators of different shapes. The purpose of this article is to collate this scattered information and to provide new simple formulas for the design of dielectric resonators. The formulas given for the resonant frequency and bandwidth are valid over a wide range of resonator parameters. Fairly accurate expressions to compute the fields inside the resonator are also provided, and resonator shapes such as cylindrical, ring (cylindrical ring), spherical, and rectangular have been considered. In addition, different schemes for excitation of dielectric resonator antennas are also discussed.

In this article, we discuss isolated dielectric resonators and dielectric resonators directly placed on a metallic plane only. The considered isolated resonator shapes have one or more planes of symmetry. It is well known that the plane of symmetry behaves as an electirc wall for some modes and as a magnetic wall for the other modes. The modes for which the plane of symmetry behaves as an electric wall are of special practical interest, because for these modes, one can use one half of the structure with a metallic wall placed at the plane of symmetry. In practice, a metallic plane

[1]It was pointed out by one of the reviewers that the phenomenon of resonance in dielectric spheres was first predicted by P. Debye (P. Debye, "Der lichtdruck auf kugeln von beliebigem material," *Ann. d. Physik*, vol. 30, 1909, pp. 57–136. We could not refer to the original paper as it is in German. Stratton has given some account of Debye's work in his book (*Electromagnetc Theory*, McGraw-Hill, New York, 1941). From Stratton's account, it appears that the resonances in dielectric spheres were first predicted by Debye. However, it was probably concluded by Debye that these modes do not radiate. Referring to Debye's work, Stratton states: "On the other hand, if the sphere is a perfect dielectric, the characteristic values are real: *there is no damping, whence it appears that these modes do not radiate*" (p.560).

is required to provide a mechanical support for the antenna and its feed structure. The feed circuitry can be placed on the other side of the antenna without interfering with the radiated fields. Since the resonant characteristics of such a structure can be derived from those of an equivalent isolated resonator, it is sufficient to study the modes of an isolated resonator.

2. CYLINDRICAL RESONATORS

2.1. Modes and Mode Nomenclature

Figure 1(a) shows an isolated cylindrical dielectric resonator. The modes of a cylindrical resonator can be divided into three distinct types: TE (TE to z), TM (TM to z), and hybrid. The fields for the TE and TM modes are axisymmetric and thus have no azimuthal variation. On the other hand, the fields of the hybrid modes are azimuthally dependent. The hybrid modes can be further subdivided into two groups; HE and EH [30]. For HE modes, the H_z component is quite small compared to the E_z component. The other field components for HE modes can thus be derived from a knowledge of the E_z component only. The reverse is true for EH modes.

To denote the variation of fields along the azimuthal, radial, and z-direction inside the resonator, the mode indices are added as subscripts to each family of modes. The TE, TM, HE, and EH modes are classified as $TE_{0mp+\delta}$, $TM_{0mp+\delta}$, $HE_{nmp+\delta}$, and $EH_{nmp+\delta}$ modes, respectively. The first index denotes the azimuthal variation of the fields. The azimuthal variation is of the form $\cos n\phi$ or $\sin n\phi$. The index m ($m = 1, 2 \ldots$) denotes the order of variation of the field along the radial direction and the index $p + \delta$ ($p = 0, 1, 2 \ldots$) denotes the order of variation of fields along the z-direction. The nomenclature as described above is due to Kobayashi et al. [30,31] and has been used by other workers (e.g., ref. 32). This nomenclature is in turn based on the mode nomenclature of cylindrical dielectric waveguides as first given by Snitzer [33].

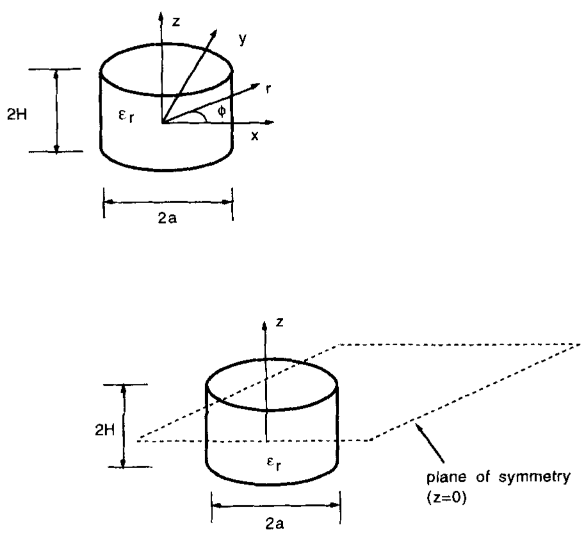


Figure 1. (a, b) Isolated cylindrical DRs.

It may be remarked that for the hybrid modes, the mode nomenclature as described above becomes inconsistent in some cases when a dielectric resonator is placed in a metallic enclosure. Different nomenclatures have therefore been used in the literature. The most notable among them is perhaps that of Kajfez [34–36]. Another scheme that has been used in the nomenclature of modes of cylindrical DRs is to denote only the family of the mode and two indices [37]. In this scheme, the first index denotes the azimuthal variation of the fields, and the second index is used to denote the order of the mode in ascending order of the resonant frequency. This nomenclature thus does not give an idea of the variation of fields in the radial and z-directions. It may be pointed out that the mode nomenclature as used by Kobayashi et al. [30] is consistent for isolated dielectric resonators which are of interest for antenna applications. Further, this nomenclature provides very useful information on the nature of the radiation of the different modes, as discussed in the next section.

2.2. Radiation Characteristics of Different Modes

An interesting feature of isolated DR antennas is that, in general, the different modes of a DR radiate like electric and magnetic multipoles such as a dipole, quadrupole, octupole, etc. The radiation pattern of a regular shape DR antenna can be predicted quite accurately without any extensive computations. For example, it is well known that the $TE_{01\delta}$ mode of an isolated cylindrical resonator radiates like a magnetic dipole oriented along its axis as shown in Figure 2 (e.g., ref. 38). The $TE_{011+\delta}$ radiates like an axial magnetic quadrupole [9]. Similarly, the $TM_{01\delta}$ and $TM_{011+\delta}$ modes radiate like an axial electric dipole and a quadrupole, respectively. The above-stated nature of the radiated fields of the TE and TM modes is independent of the dielectric constant of the resonator

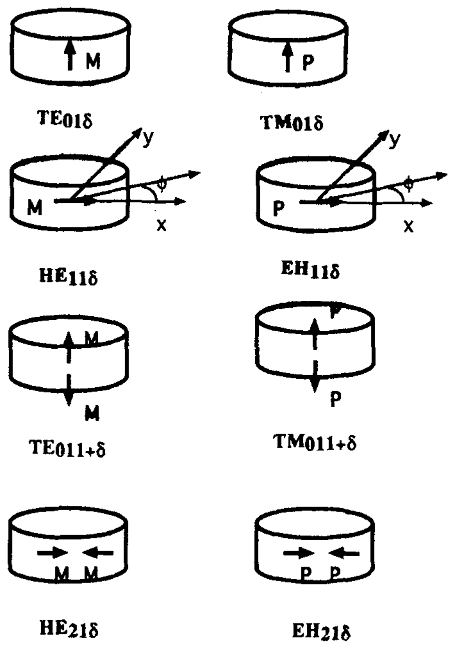


Figure 2. Nature of radiation of different modes of an isolated cylindrical DR. **M** denotes a magnetic dipole, and **P** denotes an electric dipole.

material [6–8]. It is also seen from the above discussion that the index denoting the variation of fields in the z-direction inside the resonator can be very useful for predicting the far-field pattern in the elevation plane.

The field distribution of hybrid modes of a cylindrical DR has been studied in detail, by Kobayashi et al. [30,31]. These results show that the lowest order $HE_{11\delta}$ mode radiates like a magnetic dipole oriented along the transverse (horizontal) direction as shown in Figure 2, where it is assumed that the dielectric constant of the resonator material is high ($\epsilon_r \geqslant 10$). This appears to be in agreement with the results of far-fields computed using rigorous theory [19,21]. These results confirm that the far fields of the $HE_{11\delta}$ mode are identical to that of a magnetic dipole, and are nearly independent of the value of ϵ_r and the aspect ratio (H/a) of the resonator. Since the hybrids modes are degenerate, there exists a multipole corresponding to each degenerate hybrid mode. For example, for the $HE_{11\delta}$ mode, whose E_z component varies as $\cos \phi$ inside the resonator, the radiating magnetic dipole corresponding to this mode is oriented in the radial direction along $\phi = \pi/2$, since such a dipole will radiate a maximum along the $\phi = 0$ direction. The $HE_{21\delta}$ mode radiates like a magnetic quadrupole oriented along the transverse direction as shown in Figure 2. On the other hand, the $EH_{11\delta}$ and $EH_{21\delta}$ modes radiate like an electric dipole and electric quadrupole, respectively, oriented along the transverse direction.

It is appropriate here to discuss some of the conclusions of the asymptotic theory of Van Bladel [4,5], which are quite significant. According to this theory, the modes of an *arbitrary*-shaped dielectric resonator are of the "nonconfined" type. The dominant term that contributes to the radiation from nonconfined modes is the magnetic-dipole term. The other terms, in order of diminishing contribution, that contribute to radiation, are the electric dipole and higher-order magnetic and electric multipole terms. Therefore, it is expected that the dominant nonconfined mode of an *arbitrary* shaped dielectric resonator of high permittivity will radiate like a magnetic dipole. This is a very *interesting* result indeed. The $TE_{01\delta}$ and $HE_{11\delta}$ modes which are the lowest nonconfined modes of a cylindrical resonator radiate like magnetic dipoles as mentioned earlier. The lowest order modes of a spherical and rectangular DR are also known to radiate like a magnetic dipole. For the higher-order modes, the first or the first few leading terms that contribute to radiation may vanish depending upon the field configuration of the higher-order mode. In that case, the dominant term that contributes to the radiation is "some" higher-order multipole term. From the asymptotic theory, one can expect that different modes of a dielectric resonator of sufficiently high permittivity radiate like dipole or higher-order multipole terms. In addition, Van Bladel has concluded that a resonator shape, which is axisymmetric, supports "confined" modes in addition to nonconfined modes. For confined modes, the dominant term that contributes to radiation is the electric dipole term; since for these modes, the magnetic dipole term is always zero. The $TM_{01\delta}$ mode is the dominant confined mode of an isolated cylindrical resonator, and is known to radiate like an electric-dipole.

The salient features of different modes of a cylindrical dielectric resonator are listed in Table I. Also shown in Table I is the corresponding mode nomenclature as used by Kajfez [34–36]. For different modes of an isolated cylindrical DR, the plane of symmetry $z = 0$, as shown in Figure 1(b), behaves as an electric wall or a magnetic wall which is also listed in Table I.

2.2.1. Directivity of Different Modes. The nature of radiation of different modes of an isolated

TABLE I. Salient Features of Modes of a Cylindrical Dielectric Resonator Shown in Figure 1(a)

Mode	Plane of Symmetry ($z = 0$)	Fields Inside Resonator	Nomenclature, Ref. 34–36	Far Fields	Orientation of Multipole
$TE_{01\delta}$	Mag. wall	$H_z = J_0(hr)\cos(\beta z)$ $E_z = 0$	$TE_{01\delta}$	Magnetic dipole	Axial (vertical)
$TE_{011+\delta}$	Elec. wall	$H_z = J_0(hr)\sin(\beta z)$ $E_z = 0$	$TE_{011+\delta}$	Magnetic quadrupole	Axial (vertical)
$TM_{01\delta}$	Elec. wall	$E_z = J_0(hr)\cos(\beta z)$ $H_z = 0$	$TM_{01\delta}$	Electric dipole	Axial (vertical)
$TM_{011+\delta}$	Mag. wall	$E_z = J_0(hr)\sin(\beta z)$ $E_z = 0$	$TM_{011+\delta}$	Electric quadrupole	Axial (vertical)
$HE_{11\delta}$	Elec. wall	$E_z = J_1(hr)\cos(\beta z)\begin{matrix}\cos\phi\\ \sin\phi\end{matrix}$ $H_z \approx 0$	$HEM_{11\delta}$	Magnetic dipole	Transverse (horizontal)
$HE_{21\delta}$	Elec. wall	$E_z = J_2(hr)\cos(\beta z)\begin{matrix}\cos 2\phi\\ \sin 2\phi\end{matrix}$ $H_z \approx 0$	$HEM_{21\delta}$	Magnetic quadrupole	Transverse (horizontal)
$EH_{11\delta}$	Mag. wall	$H_z = J_1(hr)\cos(\beta z)\begin{matrix}\cos\phi\\ \sin\phi\end{matrix}$ $E_z \approx 0$	$HEM_{12\delta}$	Electric dipole	Transverse (horizontal)
$EH_{21\delta}$	Mag. wall	$H_z = J_2(hr)\cos(\beta z)\begin{matrix}\cos 2\phi\\ \sin 2\phi\end{matrix}$ $E_z \approx 0$	$HEM_{22\delta}$	Electric quadrupole	Transverse (horizontal)

cylindrical resonator is shown in Figure 2. If the dielectric constant of the resonator is very high, the size of the resonator becomes very small compared to the wavelength. In that case, the different modes radiate like corresponding point multipoles, and their directivity becomes same as that of the corresponding multipole (For example, modes which radiate like an electric or magnetic dipole have a directivity of about 1.7 dB.) In practice, the dielectric constant of the resonator will have a moderate value, in which case the size of the resonator cannot be neglected in comparison to the wavelength, especially for higher-order modes. In that case, the directivity of a mode will differ from that of the corresponding point multipole. This effect will still be small for modes which radiate like a dipole (it is known that gain of a point dipole is not very different from that of a half-wave–long dipole), but can be substantial for the modes which radiate like higher-order multipoles. It may be remarked here that if the resonator is placed on a large ground plane, its directivity increases by 3 dB compared to that of an isolated resonator since the power is radiated only in the half-space.

2.3. Resonant Frequencies of Isolated Cylindrical DRs

The determination of resonant frequencies of isolated dielectric resonators requires use of rigorous numerical methods (e.g., refs. 6–9,32,35,39). The numerical methods are, however, of limited interest to designers because of their complexity. One is interested in computing the value of normalized wavenumber k_0a for a given value of ϵ_r and aspect ratio (H/a) of the resonator, where $k_0 = 2\pi f_0/c$ denotes the free-space wavenumber corresponding to the resonant frequency f_0, and c is the velocity of light in free space. If the value of ϵ_r is very high ($\epsilon_r \gg 100$), the value of the normalized wavenumber varies with ϵ_r [4], as

$$k_0 a \propto \frac{1}{\sqrt{\epsilon_r}} \tag{2}$$

for a given aspect ratio (H/a) of the resonator.

For high values of ϵ_r, one, therefore, needs to determine the value of normalized wavenumber as a function of aspect ratio (H/a) of the resonator for a single value of ϵ_r only. For other values of ϵ_r, the resonant frequency can then be determined using eq. (2). However, in the case of DR antennas, the ϵ_r of the material used may not be very high in which case the formula of eq. (2) does not hold exactly. Therefore, strictly speaking, if the value of ϵ_r is not very high, computations are required for each different value of ϵ_r. By comparing the results from rigorous methods available in the literature for different values of ϵ_r, it was found that the following empirical relation can be used as a good approximation to describe the dependence of normalized wavenumber as a function of ϵ_r, even for moderate values of ϵ_r,

$$k_0 a \propto \frac{1}{\sqrt{\epsilon_r + X}} \tag{3}$$

where the value of X is found empirically by comparing the numerical results of numerical methods. Its value is quite small (of the order of unity), and is assumed to depend only on the mode. It may be noted that for a large value of ϵ_r, $X \ll \epsilon_r$, and in that case eq. (3) reduces to eq. (2).

A few numerical values of $k_0 a$ for discrete values of ϵ_r and H/a computed using rigorous methods are available in the literature for different modes. Using these results, closed-form expressions were obtained for different modes as discussed below. The range of aspect ratio in which each expression is valid is also given. The accuracy of the different expressions for different values of ϵ_r is discussed in the next section.

***HE$_{11\delta}$* Mode.** For this mode, expressions for the normalized wavenumber $k_0 a$ are available in the literature for values of $\epsilon_r = 38$ and 20 [19,21]. These expressions are based on the numerical results of a rigorous method [35] and are believed to be quite accurate. Using these results, it was determined that a suitable value of X for this mode is nearly 2. The expression given in ref. 19 for the resonant wavenumber of the $HE_{11\delta}$ mode for a value of $\epsilon_r = 38$ is

$$k_0 a_{(\epsilon_r = 38)} = 0.27 + 0.36\left(\frac{a}{2H}\right) + 0.02\left(\frac{a}{2H}\right)^2 \tag{4}$$

By using eqs. (3) and (4) and substituting the value of $X = 2$, we obtain the following general formula

$$k_0 a = \frac{2\pi f_o a}{c} = \frac{6.324}{\sqrt{\epsilon_r + 2}}\left[0.27 + 0.36\left(\frac{a}{2H}\right) + 0.02\left(\frac{a}{2H}\right)^2\right] \tag{5}$$

where c is the velocity of light in free-space. The above expression is valid in the range $0.4 \leq a/H \leq 6$.

***TE$_{01\delta}$* Mode.** Accurate results for the resonant frequency of the $TE_{01\delta}$ mode for discrete values of aspect ratio and valid for any value of $\epsilon_r (\epsilon_r \gtrsim 25)$ have been given by DeSmedt [8]. From these results, the following formula was obtained:

$$k_0 a = \frac{2.327}{\sqrt{\epsilon_r + 1}}\left[1.0 + 0.2123\left(\frac{a}{H}\right) - 0.00898\left(\frac{a}{H}\right)^2\right] \tag{6}$$

The above formula is valid in the range $0.33 \leq a/H \leq 5$.

***TE$_{011+\delta}$* Mode.** Accurate results for the resonant frequency of the $TE_{011+\delta}$ mode have also been given by DeSmedt [8]. Using these results, the following formula was obtained for the normalized wavenumber of the $TE_{011+\delta}$ mode:

$$k_0 a = \frac{2.208}{\sqrt{\epsilon_r + 1}}\left[1.0 + 0.7013\left(\frac{a}{H}\right) - 0.002713\left(\frac{a}{H}\right)^2\right] \tag{7}$$

The above formula is valid in the range $0.33 \leq a/H \leq 5$.

***TM$_{01\delta}$* Mode.** For the $TM_{01\delta}$ mode, the "magnetic wall" method (MWM) gives the true value of resonant frequency, if the value of ϵ_r is very high ($\epsilon_r \gtrsim 100$) [6]. For lower values of ϵ_r, a relation of the type given by eq. (3) can be employed. The value of X in this case was found to be nearly 2 by comparing the numerical results available in the literature [32]. This led to the following expression for the resonant wavenumber:

$$k_0 a = \frac{\sqrt{3.83^2 + \left(\frac{\pi a}{2H}\right)^2}}{\sqrt{\epsilon_r + 2}} \tag{8}$$

The above expression is valid at least in the range $0.33 \leq a/H \leq 5$.

2.4. Bandwidth of Cylindrical DRs

The impedance bandwidth of an antenna is defined as the frequency bandwidth in which the input VSWR of the antenna is less than a specified value S. The impedance bandwidth of a resonant antenna, which is completely matched to a transmission line at its "resonant frequency," is related to the total unloaded Q-factor (Q_u) of the resonator by the relation [40]:

$$BW = \frac{S-1}{Q_u\sqrt{S}} \tag{9}$$

For a DR antenna which has negligible dielectric and conductor loss compared to its radiated power, the total unloaded Q-factor (Q_u) is related to the radiation Q-factor (Q_{rad}) by the following relation,

$$Q_u \simeq Q_{rad} \tag{10}$$

Rigorous numerical methods are required to compute the value of the radiation Q-factor of a cylindrical DR. For a given mode, the value of the radiation Q-factor depends on the aspect ratio and dielectric constant of the resonator. Van Bladel [4] has shown that for resonators of very high permittivity, Q_{rad} varies with ϵ_r as

$$Q_{rad} \propto (\epsilon_r)^P \tag{11}$$

where:

P = 1.5, for modes that radiate like a magnetic dipole;
P = 2.5, for modes that radiate like an electric dipole; and
P = 2.5, for modes that radiate like a magnetic quadrupole.

Eq. (11) and the values of P given above are quite general, and are valid independent of the shape of the resonator. However, they are valid only for values of ϵ_r which are very high ($\epsilon_r \gtrsim 100$). By comparing the available numerical results in the literature, it was found that the Q-factors of different modes follow to a good approximation, the relation of eq. (11) even for moderate values of ϵ_r, although the values of P were somewhat different from those given above. It was also found that the value of index P is nearly independent of the aspect ratio of the resonator. These observations enable us to derive closed-form expressions for the Q-factors, which are also valid over a large range of ϵ_r values. For the $TE_{011+\delta}$ mode, the value of P was found by comparing the theoretical results available in the literature [8,9]. For the other modes, the value of P was found by comparing the measured results of the radiation Q on two samples of the same aspect ratio, but different permittivity [42]. The values of P for the different modes are given in Table II. Further, closed form expressions for the Q-factors of different modes were obtained as discussed below.

TABLE II. The Value of Index P for Various Modes of a Cylindrical Resonator

Mode	P
$TE_{01\delta}$	1.27
$HE_{11\delta}$	1.30
$HE_{21\delta}$	2.49
$EH_{11\delta}$	2.71
$TE_{011+\delta}$	2.38

$HE_{11\delta}$ Mode. For this mode, the following expression for the radiation Q-factor is available in the literature for a value of $\epsilon_r = 38$ [19]

$$Q_{rad(\epsilon_r=38)} = 1.14\frac{a}{H}\{1 + 100e^{-2.05[0.5a/H - 0.0125(a/H)^2]}\} \tag{12}$$

The above expression is based on curve fitting the numerical results of a rigorous numerical method [35], and is therefore believed to be quite accurate. Further, using the above equation, the general relation of eq. (11) and the value of P given in Table II, the following general relation for radiation Q-factor can be obtained:

$$Q_{rad} = 0.01007(\epsilon_r)^{1.30}\frac{a}{H}\{1 + 100e^{-2.05[0.5a/H - 0.0125(a/H)^2]}\} \tag{13}$$

The above equation is valid in the range $0.4 \le a/H \le 6$.

$TE_{01\delta}$ Mode. Accurate results for the radiation Q-factor of the $TE_{01\delta}$ mode for discrete values of aspect ratio and valid for any value of $\epsilon_r(\epsilon_r \gtrsim 25)$ are available in the literature [8]. Using these results, the relation of Eq. (11) and the value of P given in Table II, the following formula for the radiation Q-factor of the $TE_{01\delta}$ mode was obtained:

$$Q_{rad} = 0.078192(\epsilon_r)^{1.27}\cdot\left[1.0 + 17.31\left(\frac{H}{a}\right) - 21.57\left(\frac{H}{a}\right)^2 + 10.86\left(\frac{H}{a}\right)^3 - 1.98\left(\frac{H}{a}\right)^4\right] \tag{14}$$

The above equation is valid in the range $0.5 \le a/H \le 5$.

$TE_{011+\delta}$ Mode. Accurate results for the radiation Q-factor of the $TE_{011+\delta}$ mode for discrete values of aspect ratio, and valid for any value of ϵ_r ($\epsilon_r \gtrsim 25$) are available in the literature [9]. Using these results, the following formula for the radiation Q-factor of the $TE_{011+\delta}$ mode was obtained:

$$Q_{rad} = .03628(\epsilon_r)^{2.38}\left[-1.0 + 7.81\left(\frac{H}{a}\right) - 5.858\left(\frac{H}{a}\right)^2 + 1.277\left(\frac{H}{a}\right)^3\right] \quad (15)$$

The above equation is valid in the range $0.5 \le a/H \le 5$.

2.5. Results and Accuracy of Closed-Form Expressions

The normalized resonant frequencies of the lower-order modes of an isolated cylindrical dielectric resonator are plotted in Figure 3(a) for a value of $\epsilon_r = 20$. It is seen that for $H/a \gtrsim 1$, the $TE_{01\delta}$ mode, whch radiates like an axial magnetic dipole, is the lowest order mode of the structure. The next-higher-order mode is the $HE_{11\delta}$ mode which radiates like a transverse (horizontal) magnetic dipole. The next two higher order modes are $TM_{01\delta}$ and $TE_{011+\delta}$ modes, which radiate like an axial electric dipole and an axial mgnetic quadrupole, respectively. If the resonator is placed with its flat side on a ground plane, the $HE_{11\delta}$ mode becomes the dominant mode of the structure. This will be discussed further in a later section. In Figure 3(b), the radiation Q-factors of the $TE_{01\delta}$ and $TE_{011+\delta}$ modes have been plotted for a value of $\epsilon_r = 20$. It is seen that for the $TE_{01\delta}$ mode, the radiation Q-factor is quite small, thus making this mode suitable for relatively large bandwidth antenna applications. The resonant frequencies and Q-factors of the $HE_{11\delta}$ mode are plotted in Figure 3(c). This mode also has a low value of radiation Q-factor and is therefore also suitable for relatively large bandwidth antenna applications. Also, shown in figure are the results of the expressions given in ref. 21 which have been obtained by directly curve-fitting the results of a numerical method for a value of $\epsilon_r = 22$. It is seen from Figure 3(c) that the two sets of results are in quite good agreement over the complete range of aspect ratios. However, the expressions given in ref. 21 are valid over a very restricted range of ϵ_r ($20 \le \epsilon_r \le 24$), whereas the expressions given in this article are valid over a very large range of ϵ_r as discussed below.

For values of $\epsilon_r \ge 20$, the closed form expressions given in this article were found to be accurate to within about 2% for the resonant frequencies. For the radiation Q-factors, the given expressions have an accuracy of about 10–15% for values of ϵ_r in the range $20 \le \epsilon_r \le 88$. Because of a lack of

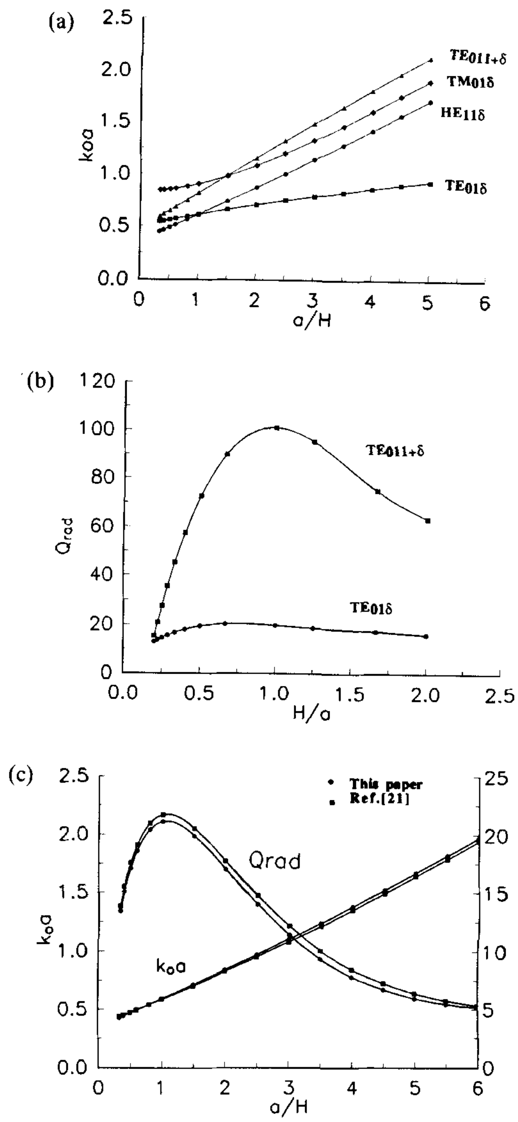


Figure 3. (a) Resonant wavenumbers of different modes of an isolated cylindrical dielectric resonator shown in Figure 1(a); $\epsilon_r = 20$. The results for the $HE_{11\delta}$, $TM_{01\delta}$, $TE_{011+\delta}$ modes are also valid for the structure shown in Figure 4 (see equivalence as shown in Fig. 5). (b) Radiation Q-factors of $TE_{01\delta}$ and $TE_{011+\delta}$ modes of an isolated cylindrical resonator shown in Figure 1(a); $\epsilon_r = 20$. The results of the $TE_{011+\delta}$ modes are also valid for the $TE_{01\delta}$ mode of the structure shown in Figure 4 (see equivalence as shown in Fig. 5). (c) Resonant wavenumber and radiation Q-factor of $HE_{11\delta}$ mode of an isolated cylindrical resonator shown in Figure 1(a); $\epsilon_r = 22$. The results are also valid for the structure shown in Figure 4 (see equivalence as shown in Fig. 5).

reported data, the accuracy of the expressions could not be checked for all modes for the lower values of ϵ_r. However, it was found that the values of the resonant frequency and Q-factor found using the above expressions matched quite well with the experimental results for the $TE_{01\delta}$ mode for a value of $\epsilon_r = 10.8$ [26]. In Table III, the accuracy of the formulas presented above is demonstrated for some typical resonator parameters for which the results of rigorous numerical methods are available in the literature.

A mode chart for the resonant frequencies and the radiation Q-factors of some more higher order modes (such as $HE_{21\delta}$, $EH_{21\delta}$, etc.) of an isolated cylindrical resonator is given in ref. 39. However, this chart gives information for a specific value of ϵ_r ($\epsilon_r = 38$) only.

2.6. Fields Inside the Resonator

A knowledge of the fields inside the DR is required to design coupling schemes to the DR. The expressions for the E_z and H_z components inside the dielectric resonator are listed in Table I for each mode. Once the E_z and H_z components are known, the other field components E_ϕ, E_r, H_ϕ, and H_r can be determined [41]. The expressions given in Table I for the field components are based on the "dielectric waveguide model" (DWM) method of analysis of dielectric resonators [43]. This method assumes that the fields in the radial direction inside the DR vary in a similar fashion as in an infinite dielectric waveguide, having the same radius and dielectric constant as those of the resonator. Further, the fields have a standing wave pattern in the axial direction similar to that of an infinite radial dielectric slab guide of the same height and the dielectric constant as those of the resonator. The DWM method predicts fairly accurately the resonant frequencies of the various modes (typical error is 5–6%) [44] and the field distribution *inside* the resonator [36, Fig. 4] [45]. The radial wavenumber h and the axial-wavenumber β of a mode are related by the separation equation,

$$h^2 + \beta^2 = \epsilon_r k_o^2 \tag{16}$$

where k_o denotes the free-space wavenumber corresponding to the resonant frequency of the mode. Equations for the wavenumbers h and β for different modes are given below:

***$HE_{11\delta}$* Mode.** According to the DWM method, the axial wavenumber β of the $HE_{11\delta}$ mode of a cylindrical DR is the same as that of the TM_0 mode of a dielectric slab guide of the same height and dielectric constant as those of the resonator. The characteristic equation for the TM_0 mode in a dielectric slabguide is given by [41]

$$\beta H = \tan^{-1}\left[\frac{\epsilon_r\sqrt{(\epsilon_r - 1)k_o^2 - \beta^2}}{\beta}\right] \tag{17}$$

If the value of ϵ_r is high, then the condition

$$\frac{\epsilon_r\sqrt{(\epsilon_r - 1)k_o^2 - \beta^2}}{\beta} >> 1$$

TABLE III. Resonant Frequencies and Radiation Q-Factors of an Isolated Cylindrical Dielectric Resonator—Comparison of Results of Closed-Form Expressions with Those of Other Methods

	Resonator Parameters		Resonant Frequency and Radiation Q-Factor				
Mode	ϵ_r	a/H	k_oa (This study)	Q_{rad} (This study)	k_oa (Ref.)	Q_{rad} (Ref.)	Ref.
$TE_{01\delta}$	79.7	2.283	0.373	107.3	0.375	118.9	[42]
$HE_{11\delta}$	79.7	2.283	0.495	81.8	0.491	79.1	[42]
$TM_{01\delta}$	79.7	2.283	0.581		0.583		[42]
$TE_{01\delta}$	35	1.0	0.467	40.1	0.467	39.3	[32]
$HE_{11\delta}$	35	1.0	0.473	38.7	0.467	39.2	[32]
$TM_{01\delta}$	35	1.0	0.680		0.669		[32]
$TE_{01\delta}$	38	2.283	0.536	41.9	0.533	46.4	[36]
$HE_{11\delta}$	38	2.283	0.707	31.2	0.696	30.7	[36]
$TM_{01\delta}$	38	2.283	0.829		0.827		[36]
$TE_{01\delta}$	25	5.0	0.838	17.2	0.843	17.3	[8]
$TE_{01 1+\delta}$	25	5.0	1.922	26.0	1.880	30.9	[8]
$TE_{01 1+\delta}$	19.5	1.056	0.847	95.0	0.853	110.0	[32]
$TM_{01\delta}$	19.5	1.056	0.900		0.890		[32]

is usually satisfied and the solution of eq.(17) reduces to

$$\beta \simeq \frac{\pi}{2H} \tag{18}$$

It may be noted that the above equation results directly if it is assumed that the flat ends of the dielectric resonator behave as perfect magnetic walls.

Once the value of β is known, the value of wavenumber h can be computed using eq. (16), since the resonant frequency of the $HE_{11\delta}$ mode is known from eq. (5).

***$TE_{01\delta}$* Mode.** The wavenumbers for the $TE_{01\delta}$ mode can be determined using the "effective dielectric constant" EDC method [44]. The EDC method is similar to the DWM method, but gives an improved approximation for the wavenumbers inside the resonator. The values of the wavenumbers computed using this method lead to a field distribution that matches very well with that predicted by the rigorous methods [45]. Results for the radial wavenumber computed using the EDC method were curve-fitted which led to the following expression:

$$ha = \frac{\sqrt{\epsilon_r}}{\sqrt{\epsilon_r + 1}} \left\{ 2.3620 + 0.2379 \left(\frac{a}{H}\right) - 0.0591 \left(\frac{a}{H}\right)^2 + 0.00533 \left(\frac{a}{H}\right)^3 \right\} \tag{19}$$

The above expression is valid in the range $1 \le a/H \le 5$.

Once the wavenumber h is known, the value of the wavenumber β can be determined using eq. (16).

***$TE_{011+\delta}$* Mode.** Wavenumbers for the $TE_{011+\delta}$ mode can also be determined using the EDC method [46]. Values of the radial wavenumber computed using this method were curve-fitted, which led to the following expression:

$$ha = \frac{\sqrt{\epsilon_r}}{\sqrt{\epsilon_r + 1}} \left\{ 3.6686 - 3.518 \left(\frac{H}{a}\right) + 5.975 \left(\frac{H}{a}\right)^2 - 5.317 \left(\frac{H}{a}\right)^3 + 1.845 \left(\frac{H}{a}\right)^4 \right\} \tag{20}$$

The above expression is valid in the range $1 \le a/H \le 5$.

***$TM_{01\delta}$* Mode.** The wavenumbers for the $TM_{01\delta}$ mode can be determined quite accurately using

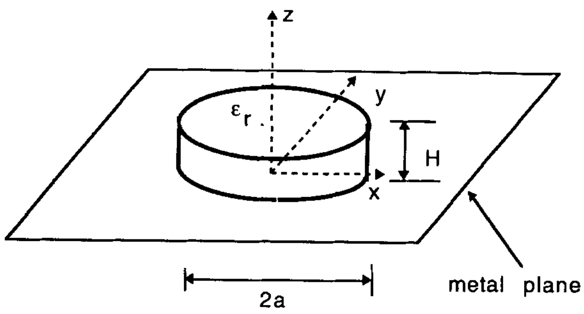


Figure 4. A cylindrical DR placed on a metallic plane.

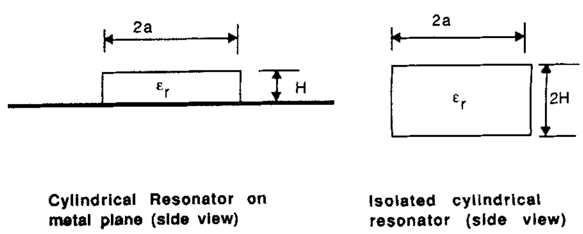
the following expressions for arbitrary aspect ratio of the resonator:

$$h = \frac{3.83}{a} \sqrt{\frac{\epsilon_r}{\epsilon_r + 2}} \tag{21}$$

$$\beta = \frac{\pi}{2H} \sqrt{\frac{\epsilon_r}{\epsilon_r + 2}} \tag{22}$$

It may be noted that for very high values of ϵ_r, the values of the wavenumbers computed by the above expression reduce to those given by the "magnetic wall model" (MWM) method [6]; i.e., $h = 3.83/a$ and $\beta = \pi/2H$.

2.6.1. Field Plots. The field configuration inside the dielectric resonator for different modes has been plotted in ref. 36. These field plots which are computed using a rigorous theory are very useful in understanding the modes of a cylindrical DR and to design coupling schemes for various modes.



$TE_{01\delta}$	≡	$TE_{011+\delta}$
$HE_{11\delta}$	≡	$HE_{11\delta}$
$EH_{11\delta}$	≡	$EH_{111+\delta}$
$TM_{01\delta}$	≡	$TM_{01\delta}$
$HE_{21\delta}$	≡	$HE_{21\delta}$
$EH_{21\delta}$	≡	$EH_{211+\delta}$

Figure 5. Equivalence between the modes of an isolated cylindrical DR of height "2H" and a cylindrical DR of height "H" placed on a metallic plane.

2.7. Cylindrical Resonator on a Metallic Plane

Figure 4 shows a cylindrical dielectric resonator placed on a metallic plane. This structure is of practical interest because the metallic plane acts as a mechanical support for the antenna and the feed structure. The structure is equivalent to an isolated resonator of double the height as shown in Figure 1(a) for those modes of the isolated resonator for which the plane of symmetry ($z = 0$) is an electric wall. These modes of an isolated resonator are listed in Table I. Using this table, the equivalent relations as shown in Figure 5 can be written between the modes of an isolated resonator and a resonator placed on a metallic plane. For example, the lowest order TE mode of the structure shown in Figure 4 (therefore called the $TE_{01\delta}$ mode) is equivalent to the $TE_{011+\delta}$ mode of an isolated resonator of double the height. The lowest order TE mode of the structure shown in Figure 4, therefore, radiates like a magnetic quadrupole (like a vertical magnetic dipole placed above a metallic plane).

For the $TE_{01\delta}$ mode of an isolated resonator as shown in Figure 1(a), the nonzero field components are E_ϕ, H_r, and H_z which are circularly symmetric. For this mode, an electric wall can be placed in any ϕ = *constant rz* plane without disturbing the field distribution. Therefore, a half-split dielectric resonator placed on a metal plane as shown in Figure 6(a) is equivalent to an isolated resonator of double the size as shown in Figure 6(b) [25]. Since an isolated cylindrical DR in its $TE_{01\delta}$ mode radiates like a magnetic dipole oriented along its axis, the resonator configuration shown in Figure 6(a) also radiates like a horizontal magnetic dipole. The $HE_{11\delta}$ mode of a cylindrical DR in the configuration shown in Figure 4 also radiates like a horizontal magnetic dipole as already discussed. Both the $TE_{01\delta}$ mode of the configuration shown in Figure 6(a) and the $HE_{11\delta}$ mode of the configuration shown in Figure 4 can be used to obtain a horizontal magnetic dipole radiation characteristics. However, each configuration offers certain unique features. For the $TE_{01\delta}$ mode of the configuration shown in Figure 6(a), the orientation of the radiating magnetic dipole is along the axis of the resonator, and is, therefore, uniquely defined. On the other hand, for the $HE_{11\delta}$ mode of the configuration shown in Figure 4, the orientation of the radiating magnetic dipole is not uniquely defined, and depends on the feed point. In other words, since the $HE_{11\delta}$ modes are degenerate, care must be taken in practice to excite the desired mode only. However, degenerate modes provide an advantage that circular polarization can be obtained by properly exciting both the degenerate modes in proper phase and amplitude.

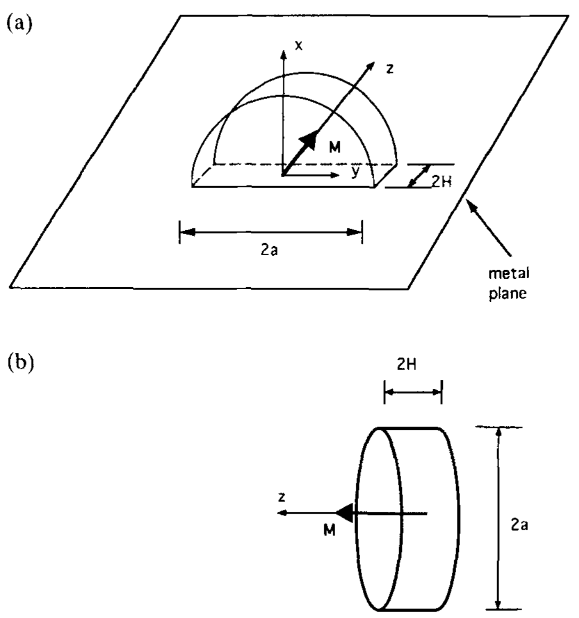


Figure 6. (a) A half-split cylindrical DR placed on a metallic plane and excited in the $TE_{01\delta}$ mode; (b) equivalent isolated cylindrical DR.

2.7.1. Practical Applications of Different Modes. As already remarked, both the $TE_{01\delta}$ mode of the configuration shown in Figure 6(a) and the $HE_{11\delta}$ mode of the configuration shown in Figure 4 radiate a maximum along the broad-side similar to the radiation from a resonant slot cut in a horizontal metallic plane. It is known that the bandwidth of slot antennas is quite small and it becomes quite difficult to match their impedance to 50-Ω lines (such as microstrip, coaxial line, etc.). Further, since it is usually required to radiate power in a half-space only, cavity-backed slots are used in practice, which makes the construction of the antenna more cumbersome. On the other hand, the DR antennas placed on a ground plane are mechanically simple structures, their bandwidth can be made quite large, and their matching to 50-Ω circuits usually offers no practical difficulties. The DR antenna configurations that radiate like magnetic dipoles can therefore be used to replace slot radiators. In the $TM_{01\delta}$ mode, a cylindrical resonator placed on a ground plane has an azimuthally symmetric pattern and radiates strongly along low elevation angles. This mode can be used in applications where an electric monopole pattern is desired. On the other hand,

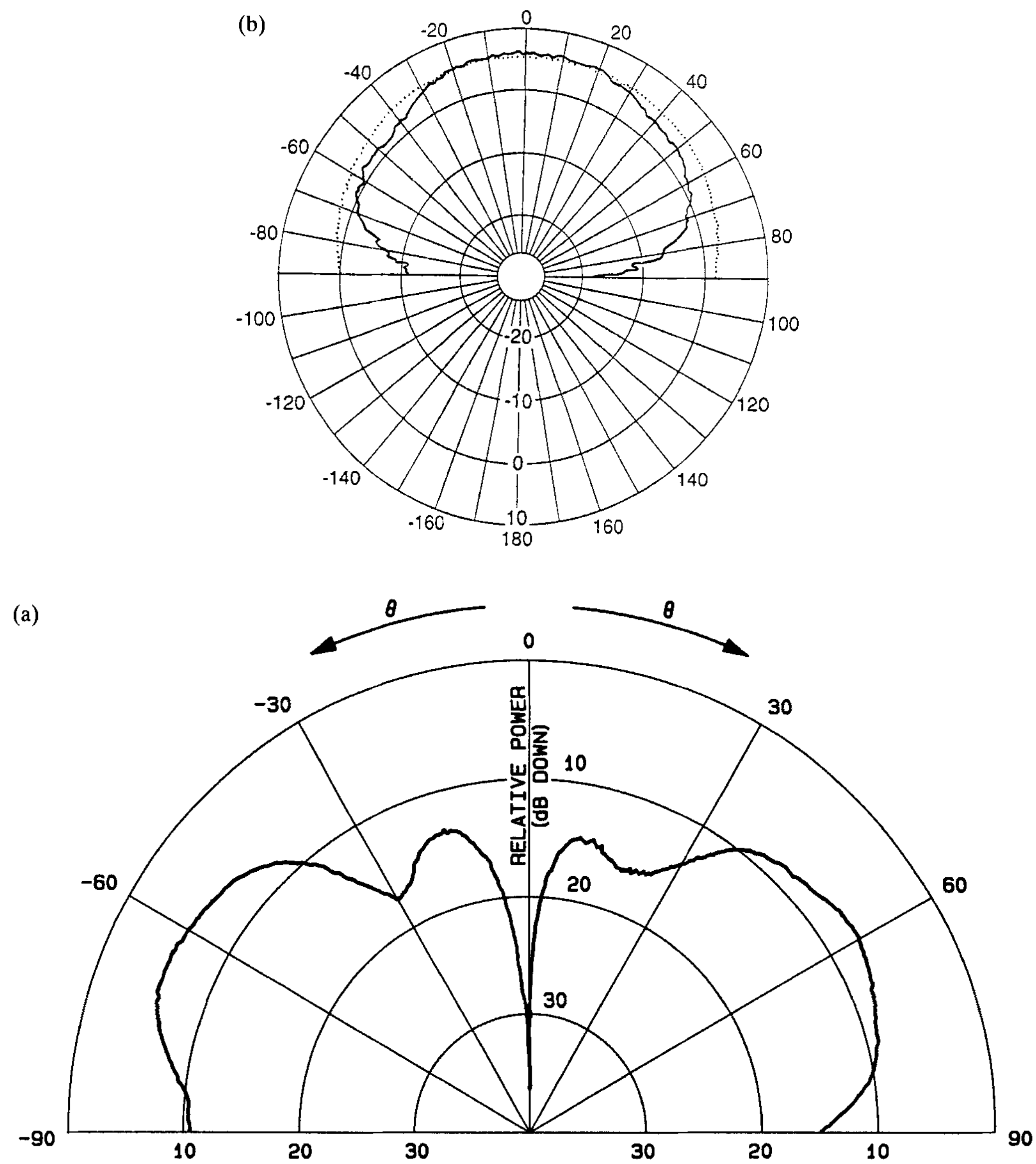


Figure 7. (a) Measured radiation characteristics of a half-split DR antenna configuration shown in Figure 6(a). (———) $H(xz)$ plane; (---------) $E(xy)$ plane. DR is excited in the $TE_{01\delta}$ mode using a microstrip-slot coupling scheme, as shown in Figure 11. Reproduced from ref. 26. © IEEE. (b) Measured elevation plane radiation characteristics of a ring DR antenna placed on a metallic plane in the configuration shown in Figure 3. DR is excited in the $TM_{01\delta}$ mode using a probe coupling scheme as shown in Figure 10(a). Reproduced from ref. 28. © IEE.

in the $TE_{01\delta}$ mode of the configuration shown in Figure 4, which is equivalent to the $TE_{011+\delta}$ mode of an isolated resonator, the antenna radiates a maximum along an angle away from the broadside in the elevation plane. This pattern is also required in many practical applications, especially in communication with satellites from locations of different latitudes on the earth.

2.7.2. Measured Radiation Characteristics. The measured radiation characteristics of the half-split DR antenna configuration shown in Figure 6(a), in which the DR is excited in the $TE_{01\delta}$ mode, is shown in Figure 7(a). It is seen that the measured radiation characteristics are similar to those of a horizontal magnetic dipole for all except small elevation angles. The discrepancy observed at small elevation angles between the measured results and those due to an ideal horizontal magnetic dipole is attributed to the effect of the finite size ground plane. The antenna was fabricated out of dielectric material having ϵ_r = 10.8, and excited by means

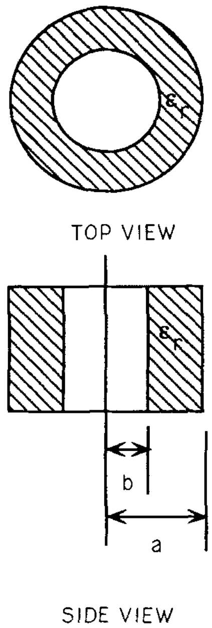


Figure 8. Isolated cylindrical ring DR antenna.

of a microstrip-slot coupling scheme as shown in Figure 11. The impedance bandwidth (VSWR $\leq$ 2) of this antenna was measured to be more than 10%. In Figure 7(b), the measured elevation plane radiation characteristics in the upper half plane of a cylindrical ring DR antenna placed on a metallic plane[2] and excited in the $TM_{01\delta}$ mode is shown. It is seen that, as expected, the measured radiation pattern in the upper half plane is similar to that of an electric dipole placed on a finite ground plane. The value of ϵ_r of the resonator material was 36.2, and the resonator was excited by means of a conducting probe as shown in Figure 10(a), with the probe placed along the axis of the ring resonator. The impedance bandwidth of this antenna (VSWR $\leq$ 2) was measured to be more than 5%.

2.8. Cylindrical Ring DRs

If a cylindrical dielectric plug of radius b is removed from near the axis of a cylindrical DR, the resulting structure as shown in Figure 8 is called a cylindrical ring DR or, more simply, a ring DR. A ring DR supports the same type of modes as a cylindrical DR. However, the resonant frequencies of a ring DR are greater than those of the corresponding cylindrical DR. The radiation Q-factors of a ring DR are smaller to those of the corresponding cylindrical DR. Therefore, ring resonators offer a larger impedance bandwidth than the cylindrical DRs. The improvement in bandwidth offered by ring resonators over cylindrical resonators may be quite substantial for some modes. For example, for the $TM_{01\delta}$ mode, the value of radiation Q-factor for a ring resonator as shown in Figure 8 and having a value of b/a = 0.25 is nearly one half of the corresponding cylindrical DR (b/a = 0) [6]. Numerical results for the resonant frequencies and the Q-factors of the $TE_{01\delta}$, $TE_{011+\delta}$, and $TM_{01\delta}$ modes of a ring DR are given in refs. 6 and 8.

[2]As shown in Figure 4, with the cylindrical resonator replaced by a cylindrical ring resonator.

3. SPHERICAL DRs

A dielectric resonator of spherical shape is of special interest because this is the only shape for which an analytical solution is possible. A spherical DR can support TE_{nmr} and TM_{nmr} modes ($m \leq n$). The indices n, m, and r denote the order of the variation of the fields in the elevation, azimuth, and radial directions, respectively. For the TE modes, the radial components of the electric and the magnetic field inside the resonator are of the following form [2]:

$$E_r \equiv 0 \tag{23}$$

and

$$H_r = \frac{n(n+1)}{r^2} \sqrt{kr} J_{n+1/2}(kr) P_n^m(\cos\theta) \begin{matrix}\sin \\ \cos\end{matrix}\, m\phi \tag{24}$$

where $P_n^m(\cos\theta)$ is the associated Legendre function of first order n, m, and $J_{n+1/2}$ is the first kind Bessel function of the order $n + \frac{1}{2}$. Further $k = \sqrt{\epsilon_r} k_0$, where k_0 denotes the free-space wavenumber. The other field components can be derived from knowledge of the E_r and H_r components.

For the TM modes

$$H_r \equiv 0 \tag{25}$$

and E_r has the same form as given by the right side of Eq. (24).

Computation of resonant frequency and radiation Q-factor of spherical dielectric resonators involves solution of transcendental equations which involve Bessel and Hankel functions of fractional orders. Since the fields outside the resonator are of the radiative type, the resonant frequencies are complex.

The spherical resonator has only one dimensional parameter, i.e., its radius. Therefore, it becomes easier to generate "universal" curves for the design of spherical DRs. Values of the normalized wavenumber $\sqrt{\epsilon_r}k_0a$ and Q_{rad}, where a denotes the radius of the resonator, have been computed and plotted in ref. 2 for various modes for ϵ_r in the range 1–100. It may be noted that spherical DR modes can support "internal" and "external" TE and TM modes. The modes of interest for practical applications are the "internal" modes, because for "external" modes, the values of the radiation Q-factor are always less than 1.

For given values of indices n and r, all the TE_{nmr} modes with different values of $m \leq n$ have the same resonant frequency. The same is true of TM_{nmr} modes. The modes of spherical DRs are, therefore, degenerate in m. Further, each TE_{nmr} and TM_{nmr} mode can have azimuthal variation of the type sin $m\phi$ or cos $m\phi$. The degeneracy of modes of spherical DRs should be considered when used in practical applications.

The field plots for some of the axisymmetric modes of a spherical DR are also given in ref. 2. One concludes from these plots that the TE_{101}, TE_{201}, and TM_{101} modes of a spherical DR radiate like an axial magnetic dipole, magnetic quadrupole, and electric dipole, respectively. On the other hand, the two TE_{111} radiate like horizontal magnetic dipoles oriented radially along $\phi = 0$ and $\phi = \pi/2$ directions, respectively. Similarly, the TM_{111} modes radiate like horizontal electric dipoles.

3.1. Hemispherical DR on a Metallic Plane

A practical antenna configuration of interest is a hemispherical DR placed on a metallic plane. The modes of this structure can be derived on the basis of the modes of an isolated spherical DR. For example, the TE_{111} modes of a hemispherical DR placed on a ground plane are equivalent to the TE_{111} modes of an isolated spherical DR. These modes radiate like a horizontal magnetic dipole. The TM_{101} mode of a hemispherical DR placed on a metallic plane is equivalent to the TM_{101} mode of an isolated spherical DR, and radiates like a vertical electric monopole. Similarly, in the case of the TE_{201} mode of a spherical DR, which radiates like a vertical magnetic quadrupole, a metallic plane can be placed in the midplane of the resonator perpendicular to the orientation of the quadrupole without disturbing the resonant frequency and radiation characteristics.

4. RECTANGULAR DRs

Cylindrical and spherical dielectric resonators always support degenerate modes which may be unwanted. Figure 9 shows a rectangular dielectric resonator. Rectangular resonators offer an advantage over cylindrical and spherical dielectric resonators in that the resonant frequencies of the different modes can be chosen to be different from each other by properly choosing the three dimensions of the resonator. Rectangular resonators, however, have been less frequently used in practice so far. One of the reasons for this is that modes of a rectangular dielectric resonator have not been well understood. No rigorous technique has been developed so far for evaluating the modes of a rectangular resonator. Approximate methods for the analysis of rectangular resonators do exist. According to Okaya and Barash [47], the modes of a rectangular resonator can be divided into two families: TE and TM. One comes to the same conclusion when the "dielectric waveguide model" (DWM) method is used to analyze rectangular resonators [48]. The presence of the lower-order TM modes was not confirmed experimentally [48]. Therefore, in this section we discuss the lower-order TE modes only. For the structure shown in Figure 9, TE^x, TE^y, and TE^z modes are possible. The lowest order TE mode for the structure is the TE^z_{111} mode, where it is assumed that the smallest dimension of the resonator lies in the z-direction (The same mode has been called $\mathrm{H}_{11\delta}$ in ref. 47.) This mode radiates like a magnetic dipole oriented along the z-direction and is located at the center of the resonator [47]. Similarly, the TE^x_{111} and TE^y_{111} modes radiate like x- and y-directed magnetic dipoles. For the TE^x_{111}, TE^y_{111}, and the TE^z_{111} modes, the resonant frequencies computed by the DWM method [48] are very close to those computed using the Okaya and Barash model [47]. Since the approach used in ref. 47 leads to formulas that are simpler, these are described here only briefly.

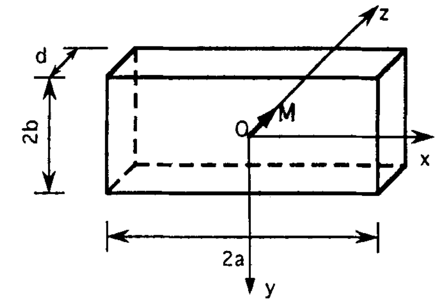


Figure 9. Isolated rectangular DR antenna.

***TE^z_{111}* Mode.** For this mode, the z-component of the magnetic field is assumed to be of the following form:

$$H_z = A\cos(k_x x)\cos(k_y y)\cos(k_z z) \quad (26)$$

$$E_z = 0 \quad (27)$$

where A is an arbitrary constant, and k_x, k_y, and k_z denote, respectively, the wavenumbers in the x-, y-, and the z-directions inside the resonator. The other field components can be determined from the H_z component. The wavenumbers k_x, k_y, and k_z satisfy the following separation equation:

$$k_x^2 + k_y^2 + k_z^2 = \epsilon_r k_0^2 \quad (28)$$

where ϵ_r is the dielectric constant of the resonator, and k_0 denotes the free-space wavenumber corresponding to the resonant frequency. The equations for k_x, k_y, and k_z are given by

$$k_x = \frac{\pi}{2a} \quad (29)$$

$$k_y = \frac{\pi}{2b} \quad (30)$$

and

$$k_z \tan\left(\frac{k_z d}{2}\right) = k_{z0} \quad (31)$$

where

$$k_{z0} = \sqrt{(\epsilon_r - 1)k_0^2 - k_z^2} \quad (32)$$

Eq. (31) can also be written as

$$d = \frac{2}{k_z}\tan^{-1}\frac{k_{z0}}{k_z} \quad (33)$$

For a given resonant frequency f_0, and resonator parameters ϵ_r, a, and b, the wavenumber k_z can be computed using eqs. (29), (30), and (28) in order. The value of d can then be computed using eqs. (32), and (33). Alternatively, for given resonator parameters ϵ_r, a, b, and d, the resonant frequency can be found by using the above procedure in an iterative manner, starting with an initial value of resonant frequency.

TE^x_{111} and TE^y_{111} Modes. The resonant frequencies and the fields for the TE^x_{111} and TE^y_{111} modes can be computed in a similar manner as that used for the TE^z_{111} mode.

4.1. Rectangular DR on a Metallic Plane

For the TE^z_{111} mode of the structure shown in Figure 9, the xz plane defined by $y = 0$ and the yz plane defined by $x = 0$, act as electric walls. Therefore, one can place a metallic plane in either of these planes and use one half of the structure without disturbing the field distribution or other resonant characteristics. Similarly, for the TE^y_{111} mode, one can place a metallic plane in the plane $x = 0$, or $z = 0$ without disturbing the field distribution or other resonant characteristics. For the TE^x_{111} mode, one can place a metallic plane in the plane $z = 0$, or $y = 0$.

5. EXCITATION SCHEMES FOR DR ANTENNAS

Various modes of a DR antenna can be easily excited using different structures such as a conducting probe, microstrip line, microstrip-slot, coplanar waveguide, etc. Figure 10(a) shows a side view of a probe coupled cylindrical DR placed on a metallic plane. To excite the $HE_{11\delta}$ mode [11], the probe is kept away from the axis, because the value of the E_z component for this mode is zero at the axis, as shown in Table I. A similar scheme of excitation with the probe placed collinear with

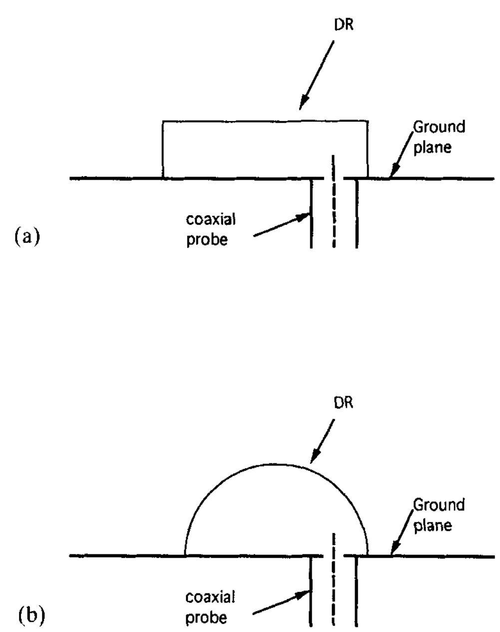


Figure 10. (a) Probe excitation scheme for a DR antenna placed on a metallic plane. (b) Probe excitation scheme for a half-split cylindrical DR antenna placed on a metallic plane.

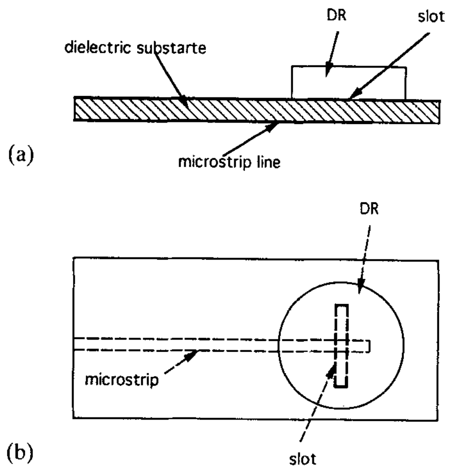


Figure 11. (a) Side view, and (b) top view of microstrip-slot excitation scheme for a DR antenna.

the resonator axis has been used to excite the $TM_{01\delta}$ mode of a cylindrical ring resonator [28]. Figure 10(b) shows the side view of a half-split cylindrical DR fed by a probe conductor to excite the $TE_{01\delta}$ mode [25]. Probe excitation has also been used for exciting hemispherical and rectangular DRs placed on a metallic plane [12,13].

Another scheme of excitation which is potentially very useful for integrating DR antennas with MICs is the mircostrip-slot coupling scheme shown in Figure 11 [22]. In this scheme, a DR is placed above a slot etched in the ground plane of a microstrip transmission line. The fields of the microstrip line "leak" through the slot to excite the DR. This scheme has also been used to excite half-split cylindrical [26], rectangular [27], and spherical DRs [29].

An excitation scheme in which a DR placed on a dielectric substrate is excited by a microstrip line has also been used [14]. Although this provides a very convenient coupling scheme, this excitation scheme may also generate surface waveguide modes in the dielectric substate. The excitation of DRs using coplanar lines [15] also appears to be quite promising, especially in coupling DRs to MMICs.

6. CONCLUSIONS

In this article, a review and design of DR antennas is presented. It is shown that sufficient information exists in the literature on the radiation characteristics of the lower-order modes of DRs of different shapes, such as cyindrical, ring, spherical, and rectangular. New simple design relations have been given for cylindrical DRs. Results show that by proper choice of the dielectric constant of the resonator and its aspect ratio, many lower-order modes of DRs of different shapes can be used for antenna applications.

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BIOGRAPHIES

Rajesh K. Mongia was born in New Delhi, India, in 1960. He obtained his BSc degree in Electrical Engineering from Delhi College of Engineering, University of Delhi, in 1981, and the PhD degree in Electrical Engineering from the Indian Institute of Technology (IIT), Delhi, India, in 1989. From 1981 to 1989, he worked in the Microwave Group of the Centre for Applied Research in Electronics (CARE) at IIT, Delhi. During his stay at CARE, he worked on various projects in the area of microwave and millimeter-wave circuits including the development of ferrite phase shifters for phased array radar. From 1990–1991, he was a research associate at the joint engineering college of Florida A&M University and Florida State University, Talahassee, Florida, where he worked on open resonators in the W-band frequency range. From November 1991 to July 1993, he was a postdoctoral fellow at the University of Ottawa, Canada, where he was involved in the theoretical and experimental study of dielectric resonators. He is presently a NSERC postdoctoral fellow at the Communications Research Centre, Ottawa, where he is working on the use of dielectric resonators as antennas.

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