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A review on physics-informed data-driven remaining useful life prediction: Challenges and opportunities

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ABSTRACT

Remaining useful life (RUL) prediction, known as 'prognostics', has long been recognized as one of the key technologies in prognostics and health management (PHM) to maintain the safety and reliability of the system, and reduce the operating and management costs. Particularly, thanks to great advances in sensing and condition monitoring techniques, data-driven RUL prediction has attracted much attention and various data-driven RUL prediction methods have been reported. Despite the extensive studies on data-driven RUL prediction methods, the successful applications of such methods depend heavily on the volume and quality of the data, and purely data-driven methods possibly generate physically infeasible/inconsistent RUL prediction results and have the limited generalizability and interpretability. It is noted that there is an increasing consensus that embedding the physics or the domain knowledge into the data-driven methods and developing physics-informed data-driven methods will hold promise to improve the interpretability and efficiency of the RUL prediction results and lower the requirement of the volume and quality of the data. In this context, physics-informed data-driven RUL prediction has become an emerging topic in the prognostics field. However, there has not been a systematic review particularly focused on this emerging topic. To fill this gap, this paper reviews recent developments of physics-informed data-driven RUL prediction methods. In this review, current methods fallen into this type are broadly divided into three categories, i.e. physical model and data fusion methods, stochastic degradation model based methods, and physics-informed machine learning (PIML) based methods. Particularly, this review is centered on the PIML based methods since the fast development of such methods have been witnessed in the past five years. Through discussing the pros and cons of existing methods, we provide discussions on challenges and possible opportunities to steer the future development of physics-informed data-driven RUL prediction methods.

1. Introduction

With significant advances of the fourth industrial revolution and the continuous improvement of modern manufacturing technology, much large-scale and complex equipment such as aerospace systems, industrial robots, wind power systems, high-speed trains increasingly show an increasing trend with the characteristics of automation, integration, and intelligence. However, the performance of these systems will degrade with the evolution of the operating time during their lifecycle due to the influence of various internal and external factors, including mechanical wear, fatigue, creep damage, varying loads, dynamic operating environments, large

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disturbances, external shocks, etc. Such performance degradation will impair the health of the system and lead to the final failure of the system, and even cause catastrophic accidents and significant economic loss, if no actions are taken. Therefore, techniques enabling the active health management are vital for ensuring the operating reliability and safety of such critical systems [1].

Driven by the desire to ensuring the operating reliability and safety of critical systems, prognostics and health management (PHM) has emerged and been proved to be a systematic troubleshooting technique devoted to actively preventing the unexpected failure through three key ingredients such as diagnostics, prognostics, and system health management [2]. Here, prognostics is the process of monitoring the system health and predicting its remaining useful life (RUL) by assessing the degradation extent from its expected health state during intended usage conditions. Based on such prognostic information, health management decisions related to safety, condition-based maintenance, spare parts ordering and life extension can be actively made to improve the working availability, and ultimately decrease the operating risks and costs for the concerned systems [3].

In the following, we first discuss the general development trend of the RUL prediction and primary principle of existing RUL prediction methods, and then, the difference of this paper from existing review papers on the RUL prediction is clarified. Finally, we present the motivation and the associated contribution of this paper.

1.1. Development trend of the RUL prediction

In PHM, RUL prediction, known as 'prognostics', has long been recognized as one of the key technologies in the PHM to maintain the safety and reliability of the system, and reduce the operating and management costs [4]. In recent years, RUL prediction has become a research hotspot and attracted more and more attention from both academic and industrial fields. In the academic research facet, the Web of Science database is an internationally recognized database reflecting the development of scientific research and includes most of research papers in various fields. Therefore, we select the Web of Science database to analyze the published papers in the RUL prediction field. Specifically, to avoid the impact of the database updates, all publications collection was completed in

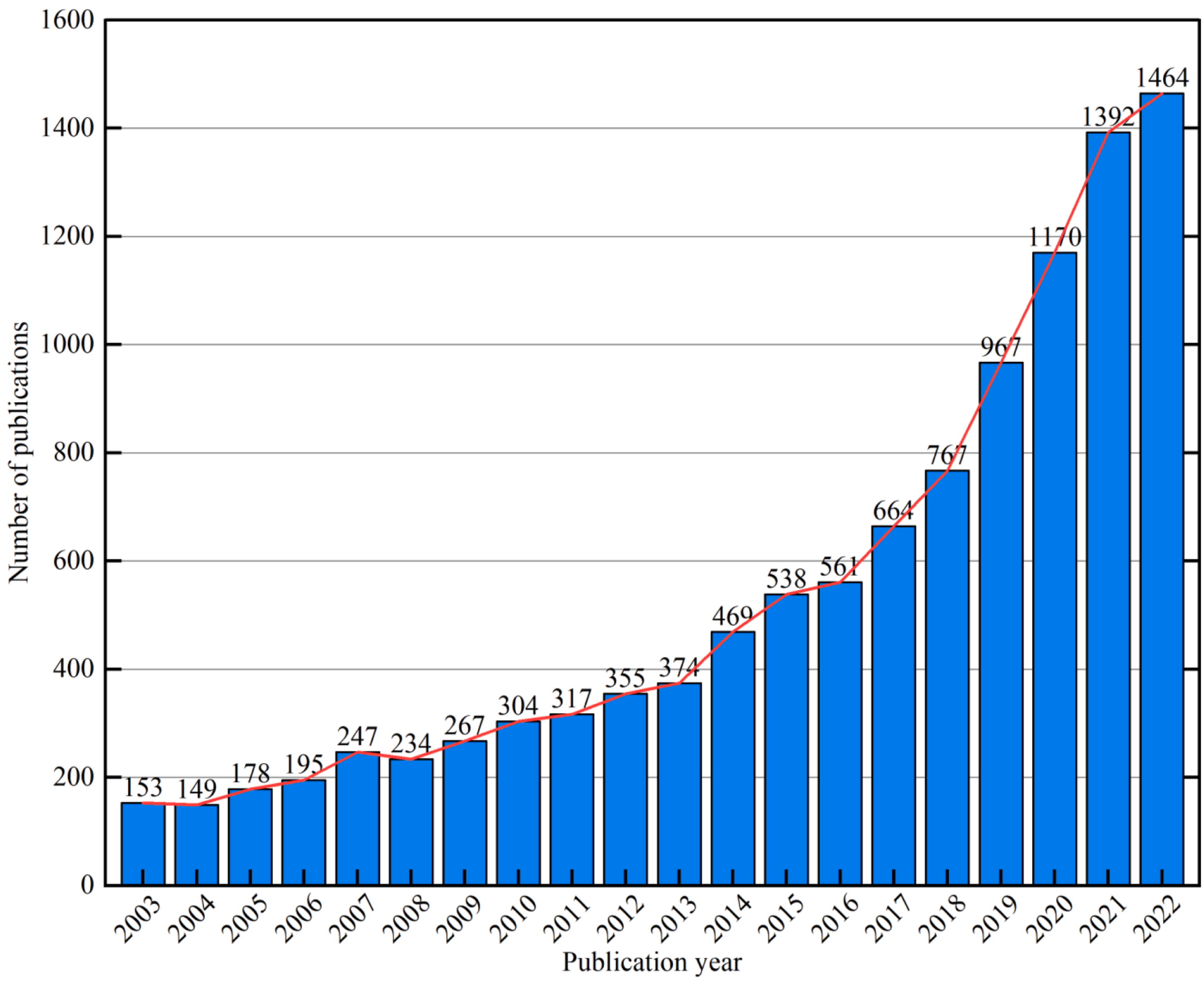

Fig. 1. The number of publications on the subject of the RUL prediction in Web of Science.

December 2022, and to ensure the authority and accuracy of the data, this paper uses the Web of Science Core Collection as the primary data source, encompassing publications from 2003 to 2022. To ensure that the retrieved literature is closely related to this study, defined as "articles not related with the RUL prediction were excluded by checking titles and abstracts", the specific inclusion criteria is [Topic = ("remaining useful life" OR "residual useful life" OR "RUL")] AND [Publication type = (Article) OR (Review) OR (Proceeding paper)] AND [Language = (English)] AND [Year published = ("2003–2022")]. According to the above criteria, this paper counts the number of publications on the subject of the RUL prediction in the past 20 years, as shown in Fig. 1.

It is observed from Fig. 1 that the total number of publications on the RUL prediction per year has an obviously growing trend since 2003 and particularly shows an almost exponentially increasing trend since 2013. At the meanwhile, in the industrial application facet, RUL prediction techniques have been applied to a number of important fields, including but not limited to aerospace systems [5–7], industrial robots [8,9], wind power systems [10–12], high-speed trains [13,14], etc. These advancements in academic and industrial fields reflect the increasing importance and popularity of the RUL prediction topic.

1.2. Existing methods for the RUL prediction

In general, the RUL of a system refers to the time interval from the current time t to the performance degradation failure time T, which is unknown in advance [15], i.e., $T - t|T > t$. Therefore, the main task of the RUL prediction is to predict the remaining time $T - t$ conditional on the information related to the system's degradation when it is still functioning (i.e. $T > t$). Accurate prediction of the RUL of the degrading system is of great theoretical significance and practical application value for scheduling timely maintenance and spare parts replenishment so that the safe and reliable operation of the system and the operating cost saving can be ensured. To achieve the accurate RUL prediction, a great number of prognosis methods have been developed in literature, which can be broadly divided into three categories, i.e. physics-based methods, data-driven methods, and hybrid methods [16,17].

Physics-based methods realize the RUL prediction by analyzing the physical failure mechanisms of systems and modeling their physical models based on the first principle to characterize the associated degradation processes. Classical models fallen into this category mainly include crack-growth models [18], bearing dynamic models [19], wear models [20], electrochemical models [21], etc. Generally, if the physical failure mechanisms or the domain knowledge can be fully understood and the parameters in the associated physical models can be estimated with the help of the measurement data, physics-based methods can achieve accurate, generalizable and interpretable predictions of the RUL. However, in most cases, particularly for complex engineering systems, their physical failure mechanisms and the domain knowledge are incomplete and even hardly available [22]. In this circumstance, the established physical models are inevitably forced to perform several simplifications or make a number of and assumptions. As such, the associated physical models are necessarily approximations of the reality, which will not only introduce bias and degrade the performance of the RUL prediction but also render the failure physics difficult to understand and analyze, particularly due to the large heterogeneity in the underlying degradation processes of different systems even belonging to the same type. All these aspects restrict the widespread applications of physics-based methods for the RUL prediction.

With advances in sensing and condition monitoring technologies, the monitoring data related to the degradation process of the system can be obtained more easily and economically, and these degradation monitoring data are promising to provide the abundant information about the system's life. Therefore, if such degradation data can be appropriately modeled, the RUL of the concerned system can be predicted accordingly. This provides an economical and feasible way to alleviate the limitations in physics-based methods with high requirements on the completeness of physical knowledge and in traditional life data fitting methods with sufficient failure time data, which are scarcely available in practice due to unaffordable costs or time-consuming testing process. Driven by

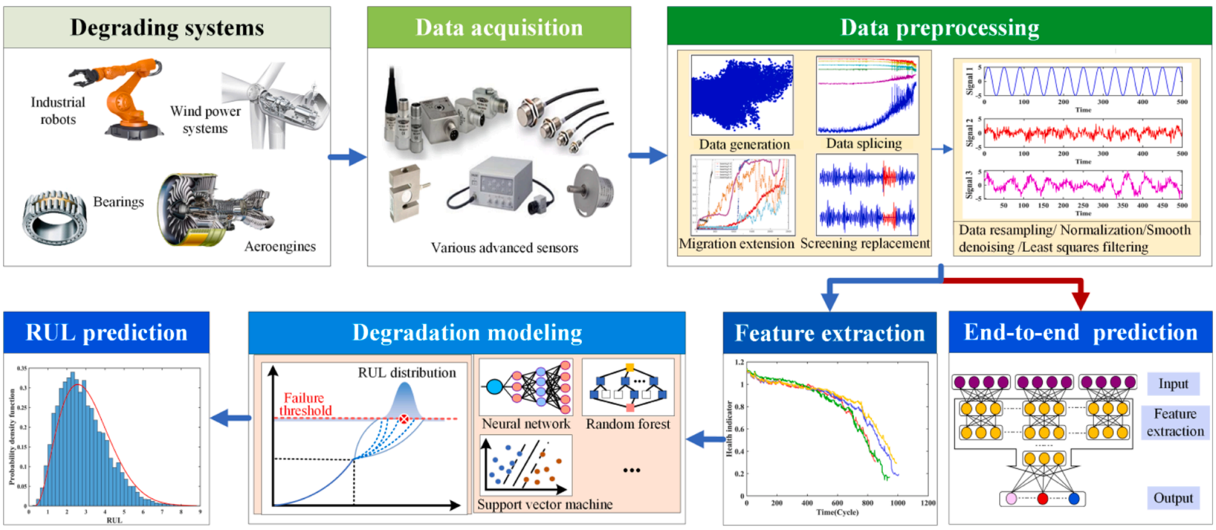

Fig. 2. Flowchart of the data-driven RUL prediction.

the monitoring data, the past decades have witnessed extensive studies on developing various data-drive RUL prediction methods [23]. The general process of the data-driven RUL prediction is illustrated in Fig. 2. In this process, the degrading system is first monitored by various sensors to acquire sensory signals to collects the condition monitoring data, and then the monitoring data are preprocessed by appropriate signal processing algorithms such as normalization and smoothing. With the preprocessed data, the degradation feature will be extracted to characterize the degradation progression of the concerned system. Finally, by modeling the degradation progression of the system, the RUL can be predicted according to the time of the degradation processing crossing the pre-defined failure threshold or failure labels.

At present, data-driven RUL prediction methods mainly include statistical data-driven methods and machine learning (ML) based methods. The basic idea of statistical data-driven methods is to utilize stochastic models to fit the evolving processes of the degradation indicators and then extrapolate the degradation process to the defined failure threshold so as to achieve the RUL prediction, in which the model parameters are estimated based on the monitoring data [24]. The primary advantage of such methods lies in the capability of providing a visual form of the degradation progression and quantifying the prognosis uncertainty by obtaining the probabilistic distribution of the RUL, which is a basic requirement for the RUL prediction since the RUL prediction corresponds to the prediction of the future failure event and has the uncertainty in nature. Additional advantage is that the model parameters have the interpretability to certain extent such as the parameters reflecting the degradation speed and time-varying dynamics, and this explainable mechanism will facilitate the understanding of the degradation failure process and the predictive results. Nevertheless, the stochastic degradation model selection and the justification of statistical assumptions made for degradation modeling are important aspects affecting the prediction performance of such methods. More importantly, the success of statistical data-driven methods depends heavily on the availability of the degradation indicator with an increasing or decreasing trend. In other words, the quality of such degradation indicator has the inherent impact on the prognosis accuracy of statistical data-driven methods, but the capability of this kind of methods extracting the desirable degradation indicator from the monitoring data is relatively limited.

With advances in Industry 4.0 [25], Cyber-Physical System (CPS) [26], and Internet of Things (IoT) [27], the increased availability of the condition monitoring data of stochastic degrading systems has become the reality in practice and thus promotes studies on the RUL prediction into the big data era. In the context of the big data, ML based RUL prediction, particularly deep learning (DL), has become a hotspot topic in the fields of prognostics, and a significant volume of studies have been conducted to develop various ML based RUL prediction methods [28]. Generally, ML based RUL prediction methods are achieved by establishing the mapping relationship from the monitoring data to the RUL information with ML models such as neural networks. Especially, as a representative, DL method has been proved to be a powerful tool to handle high-dimensional yet nonlinear monitoring data and extract the deep degradation feature automatically from massive data or achieve the end-to-end prognosis so as to lower the artificial intervene [29]. The main merit of such methods lies in the strong flexibility and universality in the model fitting when the underlying failure physics or the domain knowledge cannot be fully mastered. Therefore, it is not surprising to observe the explosive growth of ML-based or more precisely DL-based RUL prediction studies in the past five years. However, it is well known that ML based methods operate in a black-box manner and suffer from the lack of transparency and interpretability [30]. Despite the relative simplicity of each component in a ML model, the structure of the whole model is still so intricate that it may not be fully understood by its designer and user. As a result, an accuracy-interpretability trade-off is frequently adopted in practice. In addition, the performance of ML based methods depends heavily on the volume and quality of the monitoring data, and if there are no representative data with labels, the prognosis performance of such methods cannot be guaranteed. In particular, it is easy to observe that ML models look deceptively good on training/testing datasets (even after cross-validation), but do not perform well outside the available labeled data or even generate physically infeasible/inconsistent RUL prediction results. Thus, how to improve the generalizability of ML based method to unseen data is still a very challenging issue. Last but not least, in most of ML based RUL prediction studies, although the satisfactory prediction results in the accuracy can be realized in some applications, it is quietly difficult to provide the probability distribution of the predicted RUL due to the deterministic structure of ML models. As such, the capability of quantifying the prognosis uncertainty by ML based methods is

Table 1
Summary of advantages and disadvantages about the RUL prediction methods.

Methods	Advantages	Disadvantages
Physics-based methods [22]	• Interpretable results • Without requirement of mass data • High generalization abilityHigh accuracy and credibility	• Difficult to obtain accurate physical models of complex systems • Depend heavily on the completeness of physics or domain knowledge
Statistical data-driven methods [24]	• Low requirement on physical knowledge • Interpretability in the model parameters • Effectively quantify the prognosis uncertainty by outputting the probabilistic distribution • Provide a visual form of the degradation progression	• Lack of the ability handling massive data • Require the degradation indicator exhibiting an increasing or decreasing trend • Overwhelmed by the degradation model selection • Difficult to justify the statistical assumptions made for modeling
Machine learning based methods [28]	• Ability to process high-dimensional data • Independent of physical knowledge • Have the ability achieving end-to-end prognosis • High accuracy in the model fitting	• Black-box operation with the lack of interpretability • Affected by the volume and quality of the data • Low generalization to unseen data • Difficult to characterize the prognosis uncertainty
Hybrid methods [31]	• Integrate advantages of different prognosis methods	• No general combination structure • Complex implementation process

limited though the uncertainty quantification of the RUL prediction is a critical issue assisting the assessment of the failure risk in the PHM applications.

From the above general discussions, it is observed that purely physics-based methods and purely data-driven methods have their individual strengths and weaknesses in different aspects including the model establishment, model assumptions, data requirement, generalizability, interpretability, etc. In this case, a natural idea is to combine these two types of methods to leverage the merits of both physics-based methods and data-driven methods to improve the RUL prediction performance. As a result, hybrid methods that combine physics-based and data-driven methods have been developed for the RUL prediction. Liao and Köttig [31] provided a comprehensive review on hybrid methods for the failure prognosis and illustrates four categories of hybrid methods using various combinations of physics-based data-driven and methods. The main advantage of hybrid methods is that predicted RUL is robust against possible impacts such as model assumptions and data selection policies as such methods integrate advantages of different prognostic methods. Nevertheless, it is noted that no general combination structure and complex implementation process are two frequently encountered problems when applying hybrid methods for the RUL prediction. In addition, most of hybrid methods are implemented in shallow manners simply combining different methods and consequently the prognosis performance will be limited by the performance of single methods used in such combinations.

Together with the above discussions, it is noted that the advantages and disadvantages coexist in current physics-based methods, data-driven methods, and hybrid methods when applied to the RUL prediction. Table 1 summarizes advantages and disadvantages of four categories of existing RUL prediction methods.

1.3. Difference from existing review papers on the RUL prediction

In the past decade, many scholars have reviewed various methods for the RUL prediction, and Table 2 lists some representative reviews on the RUL prediction topic with different methodological focuses. Specifically, Si et al. [32] systematically reviewed statistical data-driven RUL prediction methods relying on available monitoring data and statistical models. Liao and Köttig [31] provided a comprehensive review on studies of the RUL prediction driven by hybrid methods, and illustrated the potential benefit of hybrid methods by the applications to a battery degradation case. Cubillo et al. [33] summarized rotating machinery's common failure modes and degradation mechanisms, and mainly discussed the physics-based RUL prediction models and their applications to gears and bearings. Zhang et al. [34] focused on the modeling progress of Wiener process-based degradation data analysis and RUL prediction methods as well as their applications in the field of PHM. Lei et al. [35] systematically reviewed the whole prognostics process for machineries from data acquisition to the RUL prediction. Guo et al. [36] reviewed advances and applications of prognostics modeling methods for engineering systems, and discussed the advantages and limitations of data-driven, physics-based, and hybrid methods based on whether the failure physics knowledge was incorporated into the RUL prediction. Kordestani et al. [37] provided a survey of studies on fault prognosis by data-driven, physics-based, and hybrid methods, and highlighted the associated significant application domains. Wang et al. [38] compared different adaptive mathematical models on deep learning algorithms for the Li-ion battery RUL prediction. Ferreira and Gonçalves [39] proposed an explicit general process analysis framework for the ML based RUL prediction process and discussed the advantages and drawbacks of the most relevant ML methods. Guo et al. [40] systematically reviewed advances of "grey box" lifetime modeling through different combination approaches of physics-based models and data-driven models for battery lifetime prediction. Wang et al. [41] provided a comprehensive overview of data-driven approaches and the associated modeling process, and discussed the combination of physical and data-driven models and their applications in fatigue life prediction of metal materials.

Although not exhaustive to include all review papers appearing in literature, the review articles in Table 2 definitely reflect the importance, prosperity and fast development of the RUL prediction field. However, it is noted that the existing reviews on the RUL prediction focus mainly on data-driven methods without the emphasis on the integration of physical knowledge. In addition, except [31,40,41], most reviews partially discussed advances in hybrid methods from the perspective of combining data-driven and physics-based methods and the discussions on the integration of physical knowledge are not sufficient. Although Liao and Köttig [31] focused on the systematic discussions of hybrid methods, they did not cover the recent research progress of hybrid methods in recent years because the review [31] was published more than nine years ago. Recent advances in lifetime prediction through different

Table 2
Summary of the typical review papers about the RUL prediction.

References	Publication year	Mainly discussed methods	Whether to integrate physical knowledge
Si et al. [32]	2011	Statistical data-driven methods	Partially
Liao and Köttig [31]	2014	Hybrid approaches	Partially
Cubillo et al. [33]	2016	Physics-based methods	Yes
Zhang et al. [34]	2018	Statistical data-driven methods	Partially
Lei et al. [35]	2018	Data-driven, physics-based, hybrid method	Partially
Guo et al. [36]	2019	Data-driven, physics-based, hybrid method	Partially
Kordestani et al. [37]	2021	Data-driven, physics-based, hybrid method	Partially
Wang et al. [38]	2021	ML based methods	No
Ferreira and Gonçalves [39]	2022	ML based methods	No
Guo et al. [40]	2022	Hybrid methods	Partially
Wang et al. [41]	2023	Hybrid methods	Partially

combination approaches of physics-based models and data-driven models are included in the latest review [40,41], but the involved methods are limited to applications of batteries or metal materials and thus do not have the generality to other applications. Last but not least, even if hybrid methods have been frequently discussed in the existing reviews, such methods are basically implemented by simply combining different methods such that the prognosis performance will be limited by the performance of single methods. As a result, the physical knowledge is integrated in shallow manners rather than the deep integration manner that embeds the physical knowledge to guide the implementation of data-driven methods.

1.4. Motivation and contribution of this review paper

By analyzing the studies on the RUL prediction to date, despite significant advances, opportunities and challenges still coexist. Particularly, known as Germany's Industry 4.0, America's Industrial Internet, Made in China 2025, and Japan's Super Intelligent Society 5.0, the new industrial revolution has provided development opportunities for advanced technologies targeted for the RUL prediction and proposed knowledge-centered intelligent technology driven by the big data [42]. In this context, the desire for maintaining the safe and reliable operation of industrial systems has provided opportunities for developing advanced technologies targeted for the RUL prediction. In the context of the new industrial revolution, massive monitoring data will be collected when the performance degradation occurs in the engineering systems. However, the monitoring data in practice generally show characteristics of multi-source heterogeneity [43], low-value density [44], incompleteness [45], and low data quality. As such, it is difficult to accurately and efficiently predict the RUL of the concerned system using purely data-driven methods. Actually, the evolution of the system from the functioning state to the failure usually follows some physical laws or chemical mechanisms and some physical knowledge or the domain knowledge may be available in practice. In the cases that either the data or the knowledge is available, some new avenues for the modeling and prediction tasks can be designed. For example, based on the analysis and mining of the big data, problem modeling and prediction can be made to obtain the knowledge contained in the data. In addition, the physical or domain knowledge can be used to assist the data modeling and prognosis so as to form a closed-loop mechanism to achieve the deep integration of the data and knowledge, as shown in Fig. 3.

It is noted that, although the essential idea proposed in Fig. 3 is similar to the cross-industry standard process for data mining (CRISP-DM) methodology [46] in terms of acquiring knowledge from data, the main difference here is that each step in Fig. 3 is irreversible and indispensable in the context of prognostics, forming a closed loop in general for data and knowledge integration, but the order of each stage in the CRISP-DM methodology is not fixed, depending on whether or not the output of a specific task in the previous stage is a necessary input for the next stage. Another difference is that, besides showing that knowledge can be obtained from data based on data mining analysis, Fig. 3 also emphasizes the importance of using physical knowledge to help data modeling and prognosis achieve deep integration of data and knowledge, which is absent in the CRISP-DM methodology. In this new paradigm as illustrated by Fig. 3, introducing physical knowledge can effectively alleviate the problems caused by purely data-driven models. By supporting data modeling with professional mechanisms, the data required for the model training are hopefully to be reduced, and the reliability, robustness and interpretability of the prediction results will be improved. In addition, unlike the shallow combination in hybrid methods, embedding physical knowledge into the data-driven modeling to realize the deep integration will hold promise to steer the established predictive models towards generating physically consistent results. Therefore, there is an increasing consensus that embedding the physics or the domain knowledge into the data-driven methods and developing physics-informed data-driven methods will hold promise to improve the interpretability and efficiency of the RUL prediction results and lower the requirement of the volume and quality of the data. In this context, physics-informed data-driven RUL prediction has become an emerging topic in the prognostics field and a sharply increasing trend on studies of physics-informed data-driven RUL prediction have been observed in the

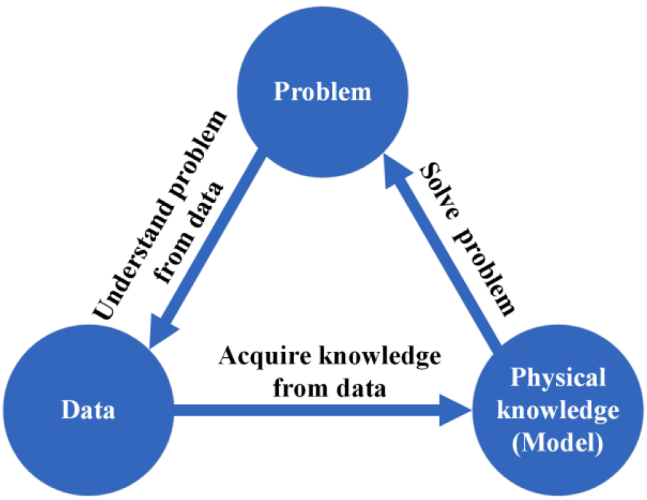

Fig. 3. Deep integration mechanism between the data and knowledge.

past three years. Recently, Xu et al. [47] provided a review of the state-of-the-art of physics-informed ML methods in reliability and system safety applications and noted the fast development of such methods in the context of the reliability and system safety evaluation. However, rare attention is paid to the applications of physics-informed ML techniques in the RUL prediction task.

From the above discussions and analysis, it is concluded that the topic of physics-informed data-driven RUL prediction will be a new hotspot in the future. However, there has not been a systematic review particularly focused on this emerging topic, at least not well-touched. To fill this gap, this paper reviews recent developments of physics-informed data-driven RUL prediction methods so as to serve as a stimulus for developing new physics-informed data-driven methods for prognostics research. This consists of the main contribution of this paper. Considering that the works on this topic have been referred to by many names including "physics-guided", "physics-informed", "physics-aware", "physics-embedded", "physics-inherited", etc, we will use the term "physics-informed" throughout this review in order to be consistent. Specifically, according to the integration mechanism between the physics or the domain knowledge and the data, current methods fallen into this type are broadly divided into three categories, i.e. physical model and data fusion methods, stochastic degradation model based methods, and physics-informed machine learning (PIML) based methods. Particularly, this review is centered on the PIML based methods since the fast development of such methods have been witnessed in the past five years. Through discussing the pros and cons of existing methods, we provide comprehensive discussions on the possible opportunities and challenges to steer the future development of physics-informed data-driven RUL prediction methods.

The reminder of this paper is structured as follows. Section 2 provides an overview of three types of physics-informed data-driven RUL prediction methods discussed in this paper. In Section 3, the research status of physical model and data fusion methods are briefly reviewed. Section 4 discusses advances and development trend of stochastic degradation model based RUL prediction methods. Recent advances in RUL prediction methods based on PIML are detailed in a comprehensive way in Section 5. Following the state of the art, Section 6 discusses current challenges and future opportunities in developing physics-informed data-driven RUL prediction methods. This paper is concluded in Section 7.

2. Overview of physics-informed data-driven RUL prediction

In this part, we attempt to provide an overview of physics-informed data-driven RUL prediction methods discussed in this paper. Broadly, according to the adopted models characterizing the degradation phenomenon and the integration mechanism between physics or the domain knowledge and the data, there are three ideas to achieve physics-informed data-driven RUL prediction. The first is to build physical models based on the underlying failure mechanism and calibrate the model parameters through fusing the monitoring data. The second is to build an interpretable stochastic degradation model based on the domain knowledge and historical monitoring data. The third is to integrate physical or domain knowledge into ML methods and establish the mapping relationships between the monitoring data and RUL to achieve prognosis. Therefore, this paper divides existing physics-informed data-driven RUL prediction methods into three categories such as physical model and data fusion methods, stochastic degradation model based methods, and physics-informed machine learning (PIML) based methods. The detailed classification of physics-informed data-driven RUL prediction methods is shown in Fig. 4.

Following the classification shown in Fig. 4, bibliometric analysis is conducted on the topic of physics-informed data-driven RUL prediction, mainly based on the search results in the database of Web of Science. To be more complete, considering that the Web of Science database only includes the published papers and accepted papers with early access, we additionally include the online available papers related with the physics-informed data-driven RUL prediction here, not necessarily published in peer-reviewed journals or conferences, such as the papers searched from Google Scholar and arXiv. Fig. 5 shows a pie chart of publications related to each of the three physics-informed data-driven RUL prediction methods involved in this paper.

Differing from the results in Fig. 1 that counts over 10,000 papers related to the whole field of the RUL prediction, Fig. 5 counts 106

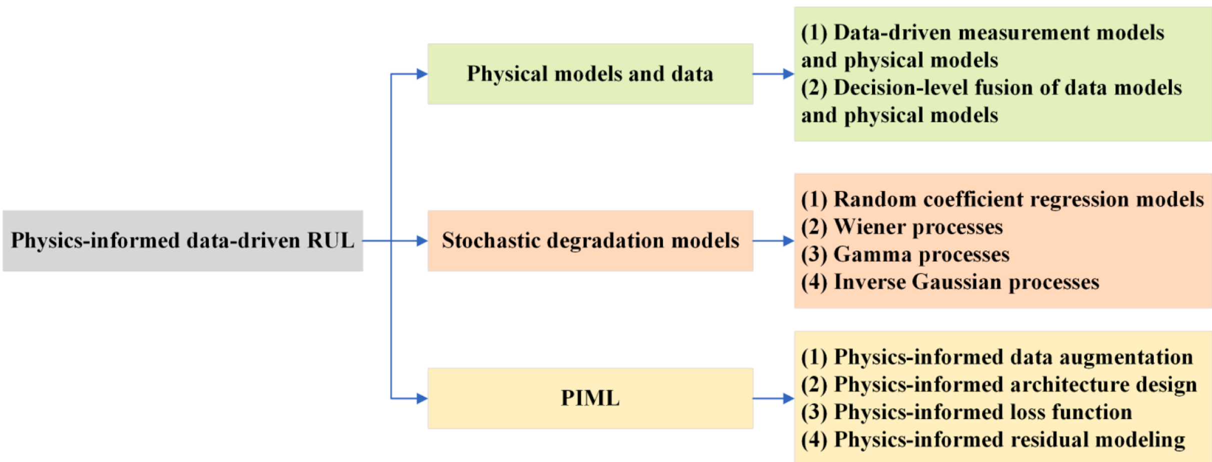

Fig. 4. Classification of physics-informed data-driven RUL prediction methods.

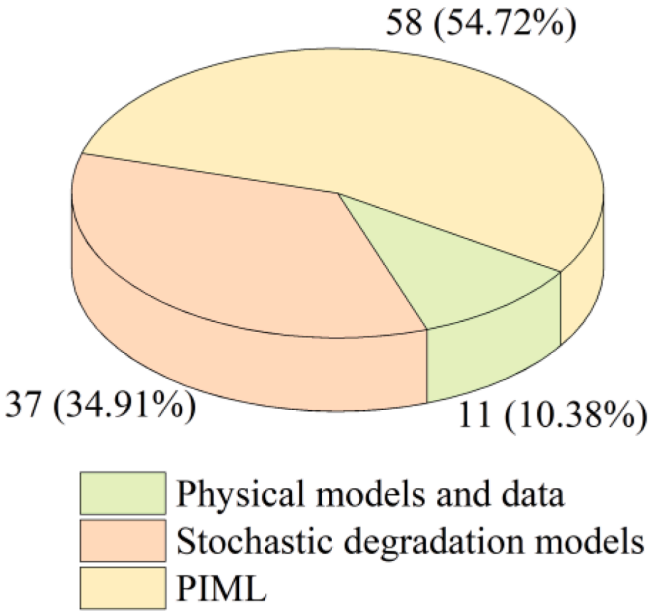

Fig. 5. Publications related to three kinds of physics-informed data-driven RUL prediction methods involved in this paper.

papers closely related to studies on physics-informed data-driven RUL prediction. It is noted however that an exhaustive reference collection on physics-informed data-driven RUL prediction methods remains elusive and this paper may not include all publications in recent years. One of main reasons is that the topic on physics-informed data-driven RUL prediction has attracted increasing attention and more research papers with this topic are on the way. Nevertheless, the statistical results in Fig. 5 still indicate the approximate proportion of each category and hold promise to reflect the development trend of physics-informed data-driven RUL prediction. We hope that perspectives and discussions in this paper can serve as a stimulus for developing new physics-informed data-driven methods for prognostics research.

In the following subsections, the brief overview on three categories is respectively provided to have a look at the general idea of each category and the associated reason why we consider such methods to be physics-informed data-driven methods.

2.1. Physical model and data fusion methods

In physical model and data fusion methods, physical models are established based on the underlying failure mechanism of the concerned stochastic degrading system and the monitoring data are integrated to estimate the parameters of physical models. With the fusion of physical models and the monitoring data, the RUL for the in-service system can be predicted by updating the physical model parameters with its real-time monitoring data. According to the different hybrid mechanism between the physical model and the monitoring data, the RUL prediction methods fallen into this category can be further divided into two groups: combination of data-driven measurement models and physical models, and decision-level fusion of data models and physical models. Common physical models to characterize the degradation-to-failure process include the Paris model [48] and Forman model [49] of fatigue crack growth, the empirical aging model [50] of battery capacity degradation, the dynamic model of bearing failure [51], etc. The common parameter updating methods include the Bayesian method [52], the Kalman filtering [53], the particle filtering [54], etc. Thanks in part to the utilization of physical models to represent the degradation process, physical model and data fusion methods for the RUL prediction can have the natural advantage of integrating the physical knowledge. At the meanwhile, the monitoring data of the in-service system can also be used to calibrate the physical models. Therefore, physical model and data fusion methods belong to one of the basic methods of physics-informed data-driven methods.

2.2. Stochastic degradation model based methods

In engineering practice, dynamic environments and various uncertain factors induce changes in the physics of failure. As such, accurate physical models for the degrading systems are difficult to be available due to the limited or incomplete knowledge and the large heterogeneity in the underlying degradation processes. In such circumstance, applying stochastic models to characterize the degradation processes, known as stochastic degradation models, will provide flexibility with respect to describing the failure-generating mechanisms and characteristics of the uncertain operating environments. With stochastic degradation models whose parameters are estimated based on the monitoring data, the RUL can be predicted by solving the probability distribution of the characterized degradation processes hitting the predefined failure threshold. From the physical degradation phenomenon perspective, resulted from the integrated actions of internal stresses and the external environments, the performance degradation of the system is inevitable and the monitoring data of the associated degradation variable will exhibit an increasing or a decreasing trend. This can be considered as the domain knowledge induced from the physical phenomenon. Inspired by such domain knowledge, various stochastic models have been developed to describe such degradation phenomenon, including random-effect regression models [55], Wiener process models [56], Gamma process models [57], Markov chain models [58], stochastic filter models [59], inverse Gaussian process models [60], and so on. In addition, it is proved that a necessary property for a stochastic model to be able to describe a physical degradation process is infinite divisibility in mathematics [61]. Representative models with such property include Wiener process models, Gamma process models, inverse Gaussian process models, etc. For examples, due to excellent mathematical properties and physical interpretability, Wiener processes can well describe the non-monotone dynamic characteristics of the degradation process and have been widely used in the RUL prediction of rolling bearings, inertial gyro, and laser devices. Unlike Wiener processes, Gamma processes and inverse Gaussian processes depict strictly monotone degradation processes, which are usually used to model the degradation of fatigue crack growth, corrosion wear, and other irreversible processes. Here, Gamma process is a pure jump process, and the degradation trajectory is discontinuous. Therefore, such model can be used to describe the slow degradation process caused by continuous small shocks and the severe damage caused by large shocks. The random shock attainment process can be approximated by Poisson processes. In a Poisson process, each shock causes minor and random damage to the system. Thus, the compound Poisson processes can be used to describe the impacts of random shocks and the associated damage of each shock on the system's degradation. Ye and Chen [62] proved that inverse Gaussian processes are the limiting distribution of compound Poisson processes. This provides a clear physical explanation for inverse Gaussian degradation modeling for the system operating in random environments. From the above general discussions of stochastic degradation model based methods, it is noted that physical models are replaced with stochastic degradation models to represent the failure process. However, the common shared by stochastic degradation model based methods is that the adopted stochastic degradation models are inspired by the domain knowledge induced from the physical phenomenon. In addition, the selection of stochastic degradation model is based on the characters of the degradation monitoring data and physical properties of models. Based on the selected stochastic degradation model, the model parameters can be estimated based on the historical degradation data and updated in line with the real-time monitoring data of the in-service system.

Together with the above discussions, it is noted the stochastic degradation process of the system has the following characteristics: (1) The degradation process of the system is gradually increasing or decreasing over the operating time, and the degradation rate and degradation fluctuation are often time-dependent; (2) physical or chemical properties of the system performance degradation characteristics could lead to the convex or concave nonlinear changing trend of the degradation path; (3) the initial degradation value, degradation rate, and degradation fluctuation, are generally random due to the involved monitoring mechanism and various uncertain factors; (4) environmental conditions and stresses could directly accelerate or inhibit the degradation process of the system, resulting in the complicated characteristics of the degradation process, including multiple stages, the thick tail characteristics of the degradation amount, etc. The above characteristics exist objectively and are largely arising from the changes of the intended physical mechanisms of the concerned system. Therefore, developing stochastic degradation models that can describe the above general characters of the degrading systems naturally has physical interpretability to some extent, i.e. physics informed. In sum, it is observed that stochastic degradation model based methods utilize the domain knowledge and have the data-driven nature in conducting the RUL prediction. Therefore, we consider stochastic degradation model based methods as another category of physics-informed data-driven methods in this review paper.

2.3. PIML based methods

In recent years, due to fast feed-forward characteristics and powerful data-driven capability, ML methods have been widely used to predict the RUL prediction of complex system. In general, ML based RUL prediction methods conduct the RUL prediction through establishing the mapping relationship from the monitoring data to the RUL information with ML models. However, traditional ML based methods ignore the underlying degradation mechanisms of systems, and the associated predictions suffer from the lack of transparency and interpretability due to the fact without considering domain-specific knowledge into the modeling process. To solve the problems in traditional ML based methods, embedding physical or domain knowledge into the development of ML models has received much attention in the research community, i.e. PIML. Although the idea of PIML was proposed not long ago, it has been rapidly developed in many scientific fields due to its potential advantages, including fluid mechanics [63], biomedicine [64], fracture mechanics [65], power systems [66], geophysics [67], and so on. Generally, ML methods consist of four key components such as data, architecture design, optimization, and initialization. Therefore, according to the ways of physical knowledge integrated into the above four components, the core ideas in PIML methods mainly include physics-informed data augmentation, physics-informed architecture design, physics-informed initialization, and physics-informed loss function. In addition, ML models can also be used to predict the errors generated by physical models and the prediction errors model based on ML will be combined with physical models to improve the final prediction performance. This implementation process for integrating physical knowledge is termed as physics-informed residual modeling in this paper. Based on the above ideas, PIML methods utilize physical knowledge to guide the data augmentation, architecture design, function optimization, and model evaluation of ML so as to leverage the advantages of purely data-driven methods and physical knowledge. To do so, PIML methods hold promise to own many merits including improved the interpretability, physically consistent result, better generalizability, improved optimization efficiency, lowered requirement on the volume and quality of the training data, etc. At present, PIML has become an increasingly popular paradigm in the ML field due to its potential superiority, and there is an irresistible research trends to integrate physics knowledge into ML methods to develop PIML based RUL prediction methods. Therefore, we consider PIML based methods as an emerging yet important category of physics-informed data-driven methods for the RUL prediction in this paper.

In line with the above classification and discussions, we will review the state of the art of the methods in each category of physics-informed data-driven methods for the RUL prediction in the following three sections. It is noted however that the first two categories have been partially considered in many existing reviews as summarized in Table 2 even if not conducted from the physics-informed viewpoint. Therefore, these two categories are briefly discussed with emphasis on the most recent advances in this paper. In contrast, since there is no existing review tailored to PIML based RUL prediction methods, advances in PIML based RUL prediction methods will be detailed in this paper.

3. Physical model and data fusion methods for RUL prediction

3.1. General physical models

In this part, we first introduce some physical models used for different kinds of systems constructed based on the associated physics-governing equations. Broadly, physical models used for the RUL prediction mainly include Paris model describing crack growth, electrochemical model of battery aging, dynamic model of bearing failure, and so on. The associated formulations for such representative physical models are provided in the following.

- Fatigue crack growth model

The fatigue crack growth model refers to the physical model in which the fatigue crack appears and gradually expands in the material under repeated loads. The law of fatigue crack growth has various forms. One of the well-known models to represent the fatigue crack growth is the Paris model. In general, the crack propagation law under the Paris model can be formulated in the form as follows [48]

$$\frac{da}{dN} = f(\sigma, a, C) \tag{1}$$

where a is the crack length, N is the number of cycles of load applied, σ is the stress range, and C is the material constant.

- Battery aging model

Lithium battery degradation can be divided into the calendar aging and cyclic aging. When cells are idle, calendar aging occurs as a side effect due to the thermodynamic instability of the material. Taking the calendar aging as an example, three factors affecting the calendar aging are the temperature, state-of-charge and time, as shown in the following equation [68]

$$Q_{\mathrm{loss}} = f(\mathrm{SoC}, T)(t)^z \tag{2}$$

where Q_{loss} is the capacity loss, SoC is the state-of-charge, T is the temperature, $f(\mathrm{SoC}, T)$ is the function describing SoC and T dependence of the calendar aging, t is the time and z is the fitting parameter.

- Bearing dynamic model

Dynamic modeling and analysis can effectively describe the motion state of faulty bearings and reveal the associated underlying fault mechanism, providing theoretical support for bearing RUL prediction research. In practice, the dynamics of bearing system is very complex and difficult to accurately model, particularly in the case of the occurrence of the bearing degradation. Taking the simplest single-degree-of-freedom bearing system as an example, its dynamic model is basically described by the following equation [51]

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{F\} \tag{3}$$

where y is the displacement of the vibration responses, $\{F\}$ is the excitation force matrix, $[M]$, $[C]$, and $[K]$ are respectively matrices of the system mass, damping coefficient and stiffness.

In practice, wear degradation influences the dynamic responses of the bearing system, including but not limited to the mass, damping coefficient, stiffness, and excitation force. For example, the removed material from the bearing surface will influence the mass by reducing or changing the uniform distribution. The surface defect disturbs the uniformity of the lubrication film and will lead to some changes in damping ratio. When a dent or a defect occurs, it will leave a space on the surface, which will change the stiffness. The defect also introduces an excitation force when the rolling element passes through the edges (i.e., impact areas) of the defected surface. According to the dynamic model in Eq. (3), the deformation index reflecting the degenerate state can be derived [69].

The above models are just some examples of physical models used for representing the degradation mechanisms of systems in different contexts. It is noted that there are many other physical models reported in literature, and the detailed discussions on physical models in different contexts can be found in some reviews including for battery from the perspective of its electrochemical behavior [40], for machineries [70], for metal materials [71], etc. With physical models, the monitoring data or measurements of associated variables can be used to identify unknown model parameters. As such, with the fusion of physical models and the monitoring data, the RUL for the concerned system can be predicted according to the definition of the failure. In the following, some developments on physical model and data fusion methods for the RUL prediction are discussed from two perspectives: combination of data-driven measurement models and physical models, and decision-level fusion of data models and physical models.

3.2. Combination of data-driven measurement models and physical models

The combination of data-driven measurement models and physical models is a typical avenue to develop physical model and data fusion based RUL prediction methods. Following this avenue, a data-driven measurement model for the degradation state described by the physical model is constructed based on the monitoring data. To do so, a state-space model can be formed through the combination of the state equation formulated from the physical model and the measurement equation resulted by the data-driven description. As such, the degradation state and unknown model parameters can be estimated by the Kalman filter, particle filter, and other filtering algorithms. With the estimated degradation state and model parameters, the RUL of the system can be predicted through the extrapolation of the degradation state characterized by the physical model to the predefined failure threshold. Cheng et al. [72] and Pecht et al. [73] effectively combined the physical model and data-driven methods to realize dynamic RUL prediction, in which the monitoring parameters were determined through the failure mode, mechanism, and impact analysis and a regression model was proposed to calculate residuals to detect abnormal data and estimate the model parameters, and finally the RUL was predicted through the physical failure model. Liao and F. Köttig [74] introduced the data-driven measurement model and predictive future measurement model into the physics-based particle filter framework, which improved the prediction accuracy of Li-ion batteries' RUL. Lei et al. [70] proposed a novel RUL prediction method for machinery by combining the physical model and data-driven measurement model. This method first applied the Paris–Erdogan model to describe the semi crack length as the degradation state. Then, a data-driven health indicator named weighted minimum quantization error was constructed by fusing mutual information from multiple features and properly correlates to the degradation processes of machinery. As such, by the combining the Paris–Erdogan model as the state and the constructed health indicator as the measurement, the RUL of machinery was predicted using a particle filter-based algorithm whole model parameters were initialized by the maximum-likelihood estimation method. Wang et al. [75] used the Frechet distance to select the optimal parameters of the exponential degradation model induced from the physical principle, effectively alleviating the problem of the model parameter initialization. It is found that the bearing degradation path can be predicted more accurately by combining the Frechet distance and exponential degradation model. Han et al. [76] developed an improved semi-empirical model of battery capacity degradation considering internal resistance and temperature. After the wavelet packet denoising was carried out on the monitoring data, the genetic algorithm was used to identify initial model parameters. Then, the particle filter framework was designed to update model parameters for the online RUL prediction. Based on the wear mechanism, Zeng et al. [77] established the basic wear model of wheel diameter and flange thickness and trained the wear model with the wear data. With the physical wear model identified by the wear measurements, the RUL distribution was obtained and the associated RUL prediction is output with the forms of the point estimation and interval estimation. Considering the issue of insufficient monitoring data and its impact on the prediction accuracy, Pan et al. [78] developed a data augmentation method by integrating degradation mechanisms and monitoring data to improve the oil condition prediction accuracy under small samples. In the specific implementation process, a degradation model considering the degradation mechanisms was first established, whose parameters were estimated with the monitoring data. Then, the augmented data was obtained with a particle filtering method and used for prediction. In these works with the combining manner, the degradation state represented by the physical model is considered to be hidden and not directly yet accurately measured, and the model parameters and the hidden degradation state are estimated from the monitoring data. Despite the utilization of the monitoring data in calibrating the physical model, the performance of the RUL prediction is largely depending on the accuracy of the physical model since the prediction is achieved by extrapolating the current degradation state to the failure threshold. In addition, combining data-driven measurement models and physical models for prognosis is often achieved in the framework of the state-space modeling, but the innovative ways of blending the physical model with the data-driven measurement model are desirable to be further explored to improve the RUL prediction performance and lower the dependency on the accuracy of the physical model.

3.3. Decision-level fusion of data-driven models and physical models

Another important avenue to develop physical model and data fusion based RUL prediction methods is to respectively conduct the RUL prediction based on the physical model and the data-driven model and then integrate the RUL prediction results from physics-based and data-driven models by some fusion methods, known as decision-level fusion of data-driven models and physical models. Along the line of decision-level fusion, Goebel and Eklund [79] first considered the system's physical characteristics to simulate the fault propagation and then utilized the experimental data and component damage level under known conditions to estimate the conditional fault propagation rate. Finally, the outputs of these two prediction methods were integrated by the kernel-based time regression algorithm. Wen et al. [80] proposed a data-level and decision-level hybrid fusion method to predict the RUL. In this study, the genetic programming was first used to integrate physical sensor sources into composite health indicators, and an explicit nonlinear data-level fusion model was obtained. Then, under the framework of the reliability theory, the RUL prediction based on various physical sensors and composite health indicators was integrated by a decision-level fusion method. Goebel et al. [81] established an empirical model to estimate the conditional fault propagation rate by using experimental data and component damage level and introduced Dempster-Shafer (DS) evidence combination methods to achieve the decision-level fusion. The experimental results indicated that the fused method can produce more accurate and reliable results than either method alone. It is noted that the implementation process of the decision-level fusion avenue is relatively simple and independent, and the advantages of various data-driven and physics-based methods can be integrated to improve the robustness and accuracy of the RUL prediction results. However, the forms of decision-level fusion to realize the ensemble of different prognosis methods are relatively diverse and there are many different fusion methods, including the arithmetic average, weighted average, geometric average, kernel regression, DS evidence combination, etc. Therefore, the design and selection of the decision-level fusion mechanism are context-dependent and challenging. In addition, the performance of the final prediction will be restricted by the single prediction method used for the fusion though the robustness of the RUL prediction can be improved to some extent.

It is observed from the existing studies that the idea behind physical model and data fusion methods for the RUL prediction is direct and natural to achieve physics-informed RUL prediction and the prediction performance and interpretability can be improved due to the inherited advantages of both physics-based and data-driven methods. Hence, the prerequisite for successfully applying physical model and data fusion methods is to obtain accurate and reliable physical models and real-time monitoring data. However, accurate and reliable physical models are often difficult to be available or costly yet time-consuming to master the failure mechanism for modeling. This is particularly true for the vital and complex systems. Therefore, more efforts are needed for complex system to conduct physicochemical and experimental analysis so that reliable physics-based models can be effectively constructed. In addition, since physical model and data fusion methods are generally realized by combining a variety of algorithms, the associated computational complexity is usually high. Therefore, developing a more reasonable fusion framework will be helpful to improve the computational efficiency. Last but not least, for the multi-parameter highly nonlinear physical models, the large number of parameters included in the model will lead to the problems of poor parameter identification and over-fitting, which may affect the accuracy and efficiency of the prediction. Toward this challenge, an alternate way to develop physical model and data fusion methods is to use data-driven methods to predict the intermediate parameters in physical models that are difficult or inaccurate to estimate rather than the simple combination or shallow fusion between data-driven methods and physical models. By feeding data-driven results into physical models, such deeply integrated methods will not only exhibit the better prognosis performance but also amend the deficiencies in existing physical models such as model simplifications or assumptions.

4. Stochastic degradation model based methods for RUL prediction

In this section, we first provide the general principle and key components of stochastic degradation model based methods for the RUL prediction, and then discuss recent advances in some representatives of stochastic degradation models when applied to the RUL prediction. With these discussions, merits and limitations of existing works are elicited to trigger new research directions in the future.

4.1. Basics of stochastic degradation model based RUL prediction methods

As discussed in Section 2.2, dynamic environments and various uncertain external/internal factors induce changes in the physics of failure and the degradation processes of the practical systems will exhibit the large heterogeneity. Due to advantages in reflecting the uncertainty and randomness of the degradation process and providing the probability distribution of the RUL to quantify the prediction uncertainty, significant advances have been witnessed in stochastic degradation model based RUL prediction methods since this kind of methods can provide a natural description of the random failure-generating mechanisms of practical systems and the impacts of uncertain operating environments. The basic principle of stochastic degradation model based RUL prediction methods is illustrated in Fig. 6. The principle indicated by Fig. 6 is that, based on the monitoring data of degrading systems, the RUL of the system can be predicted based on stochastic models by fitting the evolution process of the system performance degradation variable and extrapolating it to the pre-defined failure threshold [32]. During this implementation process, the stochastic degradation models to depict the performance degradation can be determined according to the characteristics of the degradation data and the domain knowledge, including the tendency, monotonicity, robustness, etc.

Following the above basic principle, current stochastic degradation model based methods for the RUL prediction generally consist of three key components as discussed in the following. The first component is the stochastic degradation modeling. The performance deterioration of degrading systems will be inevitable due to mutual effects of various random factors including aging, loads, and

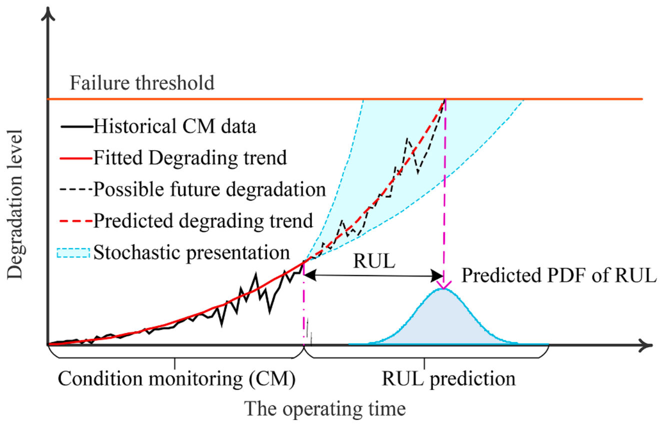

Fig. 6. Illustration of the basic principle of stochastic degradation model based methods for the RUL prediction.

varying environments. The deterioration process is accumulated over the operating time and will lead to the final failures of these systems. Therefore, the degradation variable of the system will randomly evolve during the system operating process. As such, adopting stochastic models to characterize such randomly evolving process is a natural choice. At present, despite many variants on stochastic models used for degradation modeling, they can be generally described as

$$Z(t) = z_0 + f(t;\boldsymbol{\theta}) + \varepsilon(t) \tag{4}$$

where $Z(t)$ is the degradation variable of the system reflecting the degradation state at the operating time t, z_0 is the initial degradation, $f(t;\boldsymbol{\theta})$ is a time-dependent function with parameter vector $\boldsymbol{\theta}$ to model the time-varying trend of the degradation process, and $\varepsilon(t)$ is random term to model the temporal uncertainty or randomness of the degradation process. According to the modeling principles for the degradation trend $f(t;\boldsymbol{\theta})$, stochastic degradation models can be divided into being parametric, semi-parametric, and nonparametric models. Based on the functional form of $f(t;\boldsymbol{\theta})$, stochastic degradation models include linear models and nonlinear models. Besides the degradation trend modeling, modeling the random term $\varepsilon(t)$ is another important aspect in prognostics since the degradation process of the system has the inherent randomness due to the impacts of various uncertain factors.

The second component in stochastic degradation model based methods is the parameter estimating of stochastic degradation models. Because the adopted stochastic model is selected according to the statistical characters or the domain knowledge of the concerned systems, the model parameters are unknown. In this case, to perform the prognostics, the model parameters of the used stochastic models should be first estimated based on the monitoring degradation data. The widely used methods for parameter estimation include the maximum likelihood estimation method, the Bayesian method, the expectation maximum algorithm, etc. The third component in such kind of methods is to solve the probabilistic distribution of the RUL. Based on the stochastic degradation modeling and the associated parameter estimating, to solve the probabilistic distribution of the RUL is the key task for prognostics. It is noted that the solution to the RUL distribution is closely related with the definition of the degradation-to-failure. Until now, there are several different failure definitions such as the threshold hitting time [82], the first hitting time [32], the last exiting time [83], and the mean arrival time [84]. As such, the probabilistic distribution of the RUL can be solved according to the used failure definition associated with the degradation process characterized by the stochastic model. The difference between the solutions to the RUL distribution derived by different failure definitions can be found in [83,84].

In current studies on stochastic degradation model based methods, the implementation processes of the first two components are basically fixed or seldom changed. In contrast, there are significant variants on stochastic models for characterizing the degradation processes of systems. Particularly, there are many variants in the random term $\varepsilon(t)$ leading to different stochastic degradation models used for the RUL prediction. According to difference in modeling $\varepsilon(t)$, stochastic degradation model based methods mainly include random-effect regression models, Gamma processes, inverse Gaussian processes, Wiener processes, and recently developed Beta processes, Tweedie exponential dispersion process, Student-t processes, etc. In contrast with significant advances in random-effect regression models, Gamma processes, inverse Gaussian processes and Wiener processes, the applications of Beta processes [85], Tweedie exponential dispersion process [86] and Student-t processes [87] in prognostics is relatively limited. In addition, it is worth noting that the development of these models have been partially discussed in existing reviews listed in Table 1 though most of them were published three year ago. Therefore, we mainly provide some recent advances in representative models such as random coefficient regression models, Wiener processes, Gamma processes and inverse Gaussian processes in the following.

4.2. Random coefficient regression models

Random coefficient regression models have been widely used in degradation modeling and life prediction. The most commonly used random coefficient regression model can be generally formulated as follows [82]:

$$X(t_{ij}) = h(t_{ij};\phi,\theta) + \varepsilon \tag{5}$$

where $X(t_{ij})$ is degradation of the ith system at the jth monitoring time, ϕ is a fixed parameter characterizing the system's common degradation feature, θ is the random-effect coefficient describing the individual variability between different systems of the same type, and ε is the noise item frequently assumed to be normally distributed random variable with zero mean and constant variance.

The advantage of random coefficient regression models is that they can simultaneously model the commonly shared degradation feature in the population and the variability existing in different individuals. Since the pioneering work [82], there are extensive applications of such models in the RUL prediction as discussed in [32]. Recently, Yan et al. [55] used an iterative updating random coefficient regression model to depict the degradation trend of bearing health indicators and combined with the adaptive method of the key point detection based on Munsell transform to improve the prediction performance of the RUL. Aiming at the imperfect prior information, Wang et al. [88] proposed a RUL prediction method based on a nonlinear random coefficient regression model by considering the fusion of the failure time data. In addition, a multi-state health model was proposed by Zhao et al. [89] to predict the bearing failure, in which the regression method was used to detect the health transition points and the exponential random coefficient model with the Bayesian updating process was used to derive the failure time distribution. For the machines with complicated structure, Chen et al. [90] respectively applied two well-developed expectancy regression models (i.e. linear and exponential) to calculate and update the RUL distribution of the system. From these advances, it is found that current focus of random coefficient regression models is mainly placed on the applications or improvements in the model formulations according to the specific requirements of the concerned systems. In the future, the issue of how to inherit the general idea of simultaneously modeling the commonality and individuality in the degradation process and develop more flexible model to characterize the temporal variability in the degradation process deserves more efforts and attention.

4.3. Wiener processes

The Wiener process is a kind of stochastic processes driven by the Brownian motion (BM) with the time-varying drift. It is noted here that BM with the drift was originally used to represent the progression of random walk of small particles in fluids and air in physics. Due to the drifting effect, such random walk has a trend and thus the model driven by the BM is appropriate for modeling a dynamic process with an increasing or decreasing trend. Formally, the stochastic process $\{X(t), t\geqslant 0\}$ is called the Wiener process if the stochastic process $\{X(t), t\geqslant 0\}$ satisfies the following three conditions: 1) $X(t)$ is an independent incremental process; 2) $\forall u, t > 0, X(u+t) - X(u) \sim N(\lambda(u+t) - \lambda(u), \sigma^2 t)$, where $\lambda(u+t)$ is the drift function; 3) $X(t)$ is a continuous function of time t. It is observed that Wiener process is a type of stochastic process with Gaussian noise, and thus it is a non-monotonic process. In addition, the variance of the noise is a function of time, and therefore the cumulative degradation is infinitely divisible, as required by any physical degradation process [61].

Because of its excellent mathematical properties, clarity in concept and similarity to physical degradation processes, it can well describe the non-monotone degradation characteristics of the system and has been widely applied in degradation modeling and RUL prediction. Zhang et al. [34] specifically reviewed advances in Wiener process-based degradation data analysis, RUL prediction, and their applications in PHM, in which the nonlinearity, multi-source variability, covariates, and multivariable involved in the degradation process were emphasized. However, it is noted that the advances discussed in this review are based on the publications before 2018. In the recent years, there are some new developments reported in literature. To overcome the limitations of logarithmic transformation and time-scale transformation in handling nonlinear degradation data, some new transformation techniques have been developed to convert nonlinear data into approximately linear data so as to facilitate the degradation modeling and RUL prediction [91,92]. For example, Si et al. [91] presented a nonlinear stochastic degradation modelling and RUL prediction method from a Box-Cox transformation (BCT) perspective. In this work, the BCT was first used to transform the nonlinear degradation data into nearly linear data and then the Wiener process with random drift was utilized to model the evolving process of the transformed data for prognostics. From extensive experimental results and comparative analysis, it is found that the BCT can include many existing transformation techniques as special cases and have the powerful ability to handling the nonlinear data and improve the early prognosis performance. As for the case that the nonlinear degradation process is difficult to transform into a linear one, Cao et al. [93] established a stochastic degradation model based on the nonlinear Wiener process, estimated the parameters of the degradation model by combining off-line estimation and real-time updating, and deduced the RUL distribution of the system. Lu et al. [94] used the Wiener process with random drift parameters to simulate the non-uniform degradation process, and by using the real-time degradation data of the unit, the posterior distribution of the drift parameters was dynamically updated, and the RUL distribution of the concerned system was further updated in line with the online monitoring data.

To overcome the challenge of tracking both dynamics and multi-source variability in the degradation process, a universal time-varying Wiener process was proposed by Wang et al. [95]. In this work, a state-space model was constructed to consider both nonlinear and three-source variability and an implicit transformation model was introduced to describe the evolution of model parameters over time. The degradation process of the practical system depends largely on the external operating environments, including temperature, workloads, etc. The impact of dynamic environments on the degradation process is an important aspect considered for the RUL prediction in recent years. For the prognosis issue under dynamic environments, Zhang et al. [96] proposed a nonlinear Wiener process model with stochastic time-varying covariates. In this study, the Ornstein-Uhlenbeck process was used to model dynamic covariates, and it was associated with a time-varying degradation rate through the exponential covariate-effect function. Based on the previous work [96], Zhang et al. [97] fully considered the uncertainty of measurable and unmeasurable covariate factors in the degradation process and realized the RUL prediction with Wiener process considering measurable and unobservable external impacts.

In addition to the above two directions, the systems with multi-stage degradation character have attracted much attention in recent years. For the multi-stage degradation process, the idea of degradation angle was first proposed by Wang et al. [98] to accurately identify the change points between the different stages so that different drift functions of the Wiener process model could be used to match different degradation stages to improve the prognosis performance. Based on the constructed model, the multi-stage RUL distribution was obtained according to the definition of the degradation angle and the concept of the first hitting time. In [99], two multi-stage nonlinear Wiener degradation models were proposed to conduct the study of the RUL prediction, i.e. the nonlinear degradation model with time-scale transformation and purely time-varying drift–diffusion process. In this work, the state transition probability at the change point of the degradation stages was fully considered and the analytical expression of the RUL distribution was obtained based on the concept of the first hitting time. It is observed from existing studies that the RUL prediction under Wiener process based models is an active research direction and application of such methods in prognosis will be further promoted by their interpretability and excellent properties in both the modeling flexibility and mathematic convenience. One developing direction for such methods is to construct more sophisticated models by designing physics- or domain knowledge-informed drift and diffusion functions so as to achieve the customized modeling for specific application context. Additional challenge of such methods lies in finding the exact solution to the RUL distribution under the nonlinear degradation case resulted by the non-monotonic nature of Wiener processes.

4.4. Gamma processes

In engineering practice, some performance degradation processes are monotonic and evolving only in one direction such as wear processes and fatigue crack growth processes. In these cases, applying monotonic stochastic processes to model such degradation progressions is a natural choice, and the Gamma process is an example for such stochastic processes. Different from Wiener processes, the Gamma process is a kind of stochastic process with non-negative increments and thus can be used to describe the strictly monotone degradation processes. Generally, the Gamma process, denoted as $\{X(t), t\geqslant 0\}$, has the following characteristics [100]: 1) $X(0) = 0$; 2)$\forall u, t > 0, X(u+t) - X(u) \sim Ga(\alpha(u+t) - \alpha(u), \beta)$,$\alpha(t) > 0$ is the shape function, $\beta(t) > 0$ is that scale parameter, and $Ga(\cdot)$ is the Gamma distribution; 3)$\forall n\geqslant 1, 0\leqslant t_0 < t_1 < \cdots t_n < \infty$, $X(0), X(t_1) - X(t_0), \cdots X(t_n) - X(t_{n-1})$ are independent of each other.

Application of Gamma processes in the degradation data analysis and life prediction has the long history and many existing reviews in Table 2 have discussed the previous advances more or less. Recently, Zhao et al. [101] proposed a framework based on the Gamma state space model for tracking implicit monotone health state changes. In this study, the temporal uncertainty, measurement uncertainty, and individual heterogeneity were jointly incorporated into the degradation model to predict the RUL of power devices accurately. To solve the aging problem of rechargeable batteries, a two-stage Gamma process model with fixed change points was proposed by Lin et al. [57], which was used to simulate the voltage-discharge curve of battery cyclic aging under constant current conditions. Based on the Gamma process, Zhang et al. [102] obtained the key empirical parameter information of the time series of the degradation process and the RUL distribution under different degradation conditions. Wang et al. [103] proposed a method of dynamic RUL prediction and optimal maintenance time determination based on the Gamma process model. Because the pre-processed data showed a nonlinear increasing degradation trend with the jump point, the Gamma process with nonlinear time transformation was used as the prediction model of the. The experimental results indicated that the efficiency of the online RUL prediction can be significantly improved. To solve the randomness of the system degradation trend, Esfahani et al. [104] established an accelerated degradation model based on the physical model and the Gamma process model and applied the constructed model to predict the RUL of the turbofan engine. In [105], Hazra et al. presented a mixed Gamma model with stationary increments to represent the degradation process of the tube wall and proposed a non-likelihood approximate Bayesian calculation method to effectively estimate the parameters of the mixed Gamma degradation model from noise data. It is noted that, despite long history and advances of Gamma processes in prognostics, it is difficult to obtain the analytical form of the RUL distribution due to the complex expression of Gamma distribution, and at the meanwhile, the model parameter updating with the online monitoring data is challenging to implement. Therefore, it is not surprising to observe that the recent focus of Gamma process based on RUL prediction is mainly on developing advanced parameter estimation and updating methods to make the predictions exactly reflect the current degradation state of the concerned system.

4.5. Inverse Gaussian processes

The inverse Gaussian process is another kind of monotonic stochastic processes. Therefore, similar to the Gamma process, the inverse Gaussian process is suitable for describing some systems with monotone degradation characteristics. The mathematical definition of the inverse Gaussian process $\{X(t), t\geqslant 0\}$ is described as follows [106]: 1) $X(t)$ is an independent incremental process;2) $\forall u$, $t > 0, X(u+t) - X(u) \sim IG(\mu[\Lambda(u+t) - \Lambda(t)], \eta[\Lambda(u+t) - \Lambda(t)]^2)$, $\Lambda(t)$ is the monotone increasing function of time t, μ and η are the process parameters that control the degradation rate and degradation volatility, and $IG(a, b)$ is the inverse Gaussian distribution with the parameters a and b.

Although the history of applying the inverse Gaussian process to degradation modeling and RUL prediction is not too long as opposed to the Gamma process, it has been found that the inverse Gaussian process has the important potential in modeling monotone degradation processes after the earliest work in [106]. Wang and Xu [106] presented the inverse Gaussian process model and studied the maximum likelihood estimation of a class of inverse Gaussian process models for degradation data with both the subject-to-subject heterogeneity and covariate information. Simulations and laser data are provided to conduct for the goodness-of-fit tests and life prediction. Based on the seminal work [106], many applications and improvements in inverse Gaussian processes have been reported in literature. In recent years, Sun et al. [60] proposed an improved inverse Gaussian process model considering random effects and measurement errors to describe wear degradation and the results show that the proposed model can improve the prediction accuracy of the wear rate of hydraulic piston pumps,. In Ref. [107], the inverse Gaussian process with random effects was used to characterize the degradation process of the system. In this work, the expectation–maximization algorithm was first used to initialize the model parameters, and then, the Bayesian method was used to update the random parameters in the degradation model so that the predicted RUL could be updated in real time according to the newly available degradation data. Huang et al. [108] proposed a cutting tools wear degradation model based on the inverse Gaussian process with varying drift coefficient to describe the influence of the individual heterogeneity in the cutting tools wear process under a dynamic working environment. Hu et al. [109] established a RUL prediction model of wind turbine bearings based on the inverse Gaussian process according to the failure principle that the first temperature monitoring value exceeds the first warning threshold. In contrast with significant advances in Gamma processes, the development of inverse Gaussian processes have large space and opportunities since it is found in many applications that inverse Gaussian processes and Gamma process exhibit the similar modeling fitting and prediction performance. In addition, the issues of applying inverse Gaussian processes to the RUL prediction for systems with multi-stage degradation or random operating environments deserve in-depth studies in the future.

In this section, the basics of stochastic degradation model based methods for the RUL prediction and advances in four representative models are discussed. It is noted that there are other models such as Markov model [110] and stochastic filter model [111], whose basic idea is similar to the above models, so they are not elaborated. It is observed from the above discussions that great advances have been made on stochastic degradation model based methods and such methods are still in the stage of fast development. Nevertheless, there are some directions deserving more attention in the future. The first is the RUL prediction with the stochastic degradation model calibration. Current studies generally adopt the stochastic model to characterize the evolving progression of the degradation variable. In this circumstance, the appropriate functional form of $f(t;\boldsymbol{\theta})$ should be determined in advances. Then, the model parameters are estimated or updated by the degradation data of the concerned system to perform the model calibration. However, selecting the functional form of $f(t;\boldsymbol{\theta})$ is itself a challenging problem. More importantly, when the selected functional form of the degradation model is inappropriate, it is difficult and ineffective to calibrate the degradation model simply by updating the model parameters, and the prediction accuracy will be thus affected. Hence, how to achieve simultaneous calibration of the functional form and parameters of the degradation model is an important direction holding promise to improve the prognosis accuracy and avoid the difficulty of selecting $f(t;\boldsymbol{\theta})$. The second is the RUL prediction for systems with multiple degradation variables. An important potential prerequisite in most of existing studies is that the health state of the concerned system can be simply reflected by a single performance degradation variable. The univariate hypothesis provides great convenience for the RUL prediction. However, this assumption is not practical for complex systems whose health state is often codetermined by multiple variables related with the system performance and the health state can rarely be exactly described by a single variable. In this case, considering multiple performance variables is a must in prognostics. Thus, new framework for the RUL prediction with multiple degradation variables should be developed in the future. The third is to further develop physics-informed stochastic degradation models to improve the interpretability of the model and predictive results. The current methods based on stochastic degradation models mainly predict the RUL from objective physical phenomena by considering the monotonicity and tendency. Thus, the physical explanation is insufficient, and other physical properties such as robustness, sensitivity and correlation need to be further integrated into the degradation modeling. Last but not least, current methods are suitable for failure characteristics analysis of small sample data and lack powerful data processing ability. Extracting degradation characteristics from massive data through deep learning and realizing data-model linkage between deep learning and stochastic degradation model is a promising direction toward the RUL prediction in the big data era.

5. PIML based methods for RUL prediction

5.1. Overview of PIML paradigm

Due to the powerful ability of handling high-dimensional yet nonlinear data and capturing the hidden representation from massive data, ML models have been widely used in extensive applications as well as the prognosis field. As typical ML models, neural networks (NN) have been widely used in the field of the RUL prediction [112–116]. It is well known that NN can map any nonlinear function and has obvious advantages in solving the problems related with complex practical systems. The basic process is to input the monitoring data into the NN model for training, output the prediction results, and combine the real value with the loss function to reverse optimize the structural parameters of the NN, which realizes the nonlinear mapping relationship between the input data and the output results. The main merit of such methods is that such methods are purely data-driven and thus has the strong flexibility and universality in the

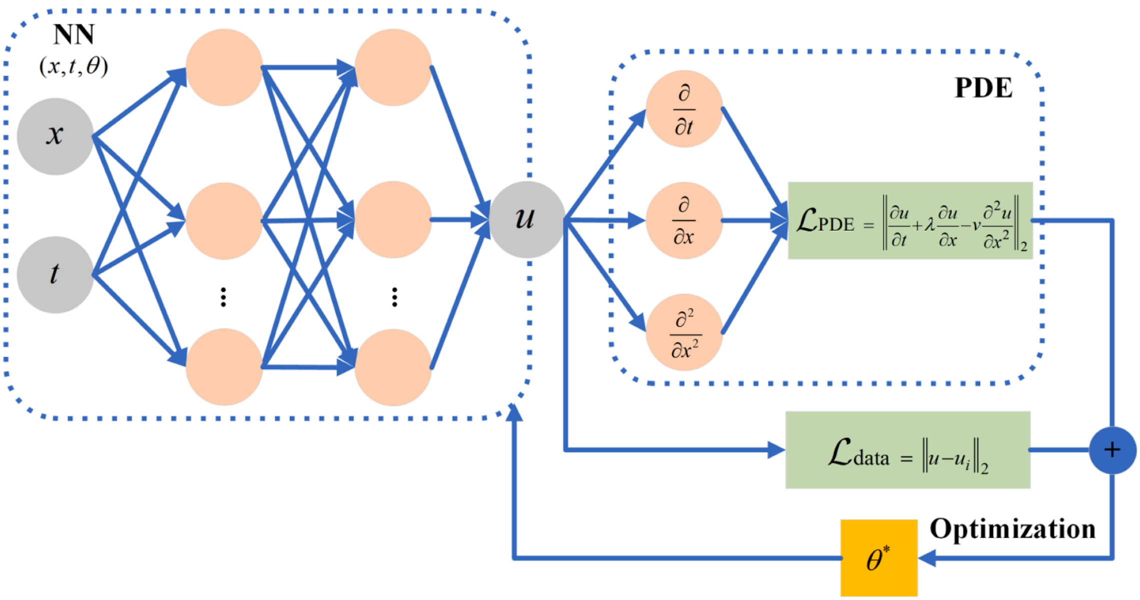

Fig. 7. PINN network structure for solving viscous Burgers' equation.

model fitting even when the underlying physical knowledge is not available. Despite their powerful ability and extensive applications, most ML methods are unable to extract interpretable information and operate in a black-box manner suffering the lack of transparency and interpretability. In addition, purely data- driven nature will cause that these models may fit observations very well, but predictions may be physically inconsistent or implausible, owing to extrapolation outside the available labeled data that may lead to poor generalization performance. Therefore, there is a pressing need for 'teaching' ML models by integrating physical or domain knowledge about physical governing equation or physical constraints [117]. As such, physics-informed learning is advocated to improve the performance of the learning algorithms. Karpatne et al. [118] first formally conceptualized the paradigm of theory-guided data science for knowledge discovery and highlight some of the promising avenues of novel studies for integrating scientific theories into data science models for different research themes with illustrative examples from different disciplines. It is observed that physics-informed learning can be considered as a specific realization of the paradigm of theory-guided data science.

Inspired by the fact that NN can approximate arbitrary nonlinear functions automatically and the desire of theory-guided data science for knowledge discovery discussed in [118], Karniadakis et al. [119] first proposed "Physic-informed Neural Networks (PINN)" for solving forward and inverse problems of partial differential equations (PDE). Specifically, the basic idea of PINN in this work is to integrate the differential formal constraints in the PDE into the loss function of NN so as to obtain NN with physical model constraints. Compared with traditional NN, PINN can approximate the observed data and automatically satisfy the physical properties of the PDE such as conservation, invariance and symmetry. In this review, we introduce PINN's design idea by taking solving the viscous Burgers equation forward problem as an example. Here, the viscous Burgers equation [120] is generally formulated as

$$\frac{\partial u}{\partial t}+\lambda\frac{\partial u}{\partial x}=v\frac{\partial^2 u}{\partial x^2} \tag{6}$$

Specifically, the PINN network structure used for solving viscous Burgers' equation is illustrated in Fig. 7.

As shown in Fig. 7, PINN considered building a NN to approximate the solution of the PDE (6). To do so, $u(x,t)$ is defined as the surrogate of the PDE solution, and $\frac{\partial u}{\partial t}+\lambda\frac{\partial u}{\partial x}-v\frac{\partial^2 u}{\partial x^2}$ is defined as the PDE residual. The loss function includes a data-driven loss $\mathscr{L}_{\mathrm{data}}$ of the data measurements of u from the initial and boundary conditions and a physics-driven loss $\mathscr{L}_{\mathrm{PDE}}$ of PDE:

$$Loss=\omega_{\mathrm{data}}\mathscr{L}_{\mathrm{data}}+\omega_{\mathrm{PDE}}\mathscr{L}_{\mathrm{PDE}} \tag{7}$$

with

$$\mathscr{L}\mathrm{data}=\frac{1}{N_{\mathrm{data}}}\sum_{i=1}^{N_{\mathrm{data}}}\left(u(x_i,t_i)-u_i\right)^2 \tag{8}$$

$$\mathscr{L}\mathrm{PDE}=\frac{1}{N_{\mathrm{PDE}}}\sum_{j=1}^{N_{\mathrm{PDE}}}\left(\frac{\partial u}{\partial t}+\lambda\frac{\partial u}{\partial x}-v\frac{\partial^2 u}{\partial x^2}\right)^2\Big|_{(x_j,t_j)} \tag{9}$$

where $\{(x_i,t_i)\}$ and $\{(x_j,t_j)\}$ are two sets of the training data obtained through initial state and boundary state, u_i are values of u at (x_i, t_i), N_{PDE} is the number of the data, and ω_{data} and ω_{PDE} are the weights used to balance the interplay between the data-driven and physics-driven loss terms.

After inputting the time and space data, the function u is first approximated by the fully connected NN, and then the residual constraints of the PDE and initial boundary value are obtained by using the automatic differential technology and are put into the loss function as the regular term. Finally, the weight parameters of the NN and the physical parameters of the PDE are obtained by using some optimization algorithms such as the gradient descent method. In the above implementation process, the addition of physics-driven loss $\mathscr{L}_{\mathrm{PDE}}$ aims to ensure consistency with physical laws of the trained NN and this is one of the most common ways to make ML models consistent with physical laws. The merits of steering ML models towards physically consistent outputs are multiple facets. The first is that there is no requirement of the observation data in computing the physics-based loss and thus optimizing $\mathscr{L}_{\mathrm{PDE}}$ allows including unlabeled data in training. Second, the integration of physical constraints can shrink the possible search space of parameters. This provides possibility to improve the optimization efficiency and learn with less labeled data while making the outputs maintain the consistency with physical laws after training. Third, ML models which follow desired physical properties are more likely to be generalizable to unseen scenarios in the training, and can be improve the generalizability of the trained ML models.

Motivated by the above merits, PIML have become an emerging topic in the ML field, and been quickly developed and applied in the past five years. The above mainly illustrates the case of constructing physics-informed loss function with NN in response to the leading desire that incorporating the physical knowledge yields more interpretable ML models that remain robust in the presence of out-of-sample scenarios and can provide accurate and physically consistent predictions. There are many other ways of embedding physics in the ML models to develop PIMLs, including physics-informed data augmentation, physics-informed architecture design, physics-informed initialization, residual modeling, hybrid physics-ML models, etc. The detailed advances of PIML with different ways embedding physics can be found in some recent comprehensive and excellent reviews such as [117,121,122].

Although significant advances of PIML have been made in recent years, we mainly focus on the studies applying PIML to the RUL prediction in this paper. According to the whole process of the RUL prediction modeling and the ways implementing PIML, the existing RUL prediction methods based on PIML can be roughly divided into three categories: physics-informed data enhancement [122], physics-informed architecture design [121], and physics-informed loss function [47]. In addition, PIML based RUL prediction can be achieved by predicting the error of the physical model by using the ML model and correcting the predicted results, and thus we consider this embedding way of physics as physics-informed residual modeling [40]. Fig. 8 outlines the general framework of the PIML method used for the RUL prediction, which mainly includes the four ways of embedding physics mentioned above.

It is noted that, although the ML method mainly concerned in this section is the NN, the above physics-informed way can also be appropriately applied to other ML methods such as the Gaussian process regression (GPR) model [123]. In the following subsections, we separately discuss the advances of each category of the PIML based RUL prediction methods.

5.2. Physics-informed data augmentation

RUL prediction based on ML models generally requires large amounts of high-quality training data. However, the experiment cost of vital systems (such as weapons, aerospace, etc.) is expensive and the testing process is time-consuming. In this case, the performance of the ML models depends heavily on the size and quality of the degradation monitoring data. By integrating the physical models to achieve the data augmentation, the ML model's dependence on the data can be reduced and the types of data can be enriched to effectively improve the accuracy and convergence of the RUL prediction. At present, there are three common methods for physics-informed data augmentation, as shown in Fig. 9.

One of the methods to realize physics-informed data augmentation is to learn from the idea of transfer learning [124]. The main idea is to use the simulation data generated by the physical model to pre-train the ML model to initialize the ML model parameters, and then use the experimental monitoring data to update the ML model, as shown in Fig. 9(a). With this idea, Kohtz et al. [125] presented a PIML model for battery state of health prognostics using partial charging segments. In this work, they applied the finite element model to generate the simulation data to train the GPR model, and the solid electrolyte interface thickness was predicted from the data of the part charging section. Based on the battery instance data and the finite element simulation results, the physics-informed multi-fidelity model was established to predict the battery's health state. The results indicated that the proposed PIML model provided an accurate and fast estimation for the health state. Shi et al. [126] established a physical model based on calendar aging and cyclic aging of lithium batteries, which was combined with the long and short-term memory (LSTM) network to learn the mapping relationship between the degradation trend obtained from the physical model and real-time monitoring data. After the degradation pattern of a battery was estimated by the physics-informed LSTM model, another LSTM network was used to predict lithium batteries' degradation trend and RUL in the future cycle by learning the degradation trend estimated by the physics-informed LSTM model. In [127], considering the effect of the two-stage degradation, time–frequency health indicators with physical significance were constructed based on the Hilbert spectrum and used as input signals of ML models to predict the RUL of aircraft cooling units.

Another method to realize physics-informed data augmentation is to combine the output of the physical model and the monitoring data into an enhanced data set for the training of the ML model [128], as shown in Fig. 9(b). Arias Chao et al. [129] inferred unobservable model parameters related to the health of complex safety–critical system's components by solving the calibration problems based on physical models. These parameters were combined with sensor readings and used as inputs to a deep neural network (DNN) to generate a data-driven prediction model with physically-enhanced features, which was experimentally demonstrated to be superior to a purely data-driven approach. Thelen et al. [130] used the simulation data and early degradation data from the physics-based half-cell model to train the ML model. By expanding the observation space of the training data, the model could interpolate between low and high degradation data points and estimate the late capacity and degradation parameters more accurately. Based on the expansion of previous work [130], Thelen et al. [131] trained the estimator model with the simulated data and then corrected the prediction bias of

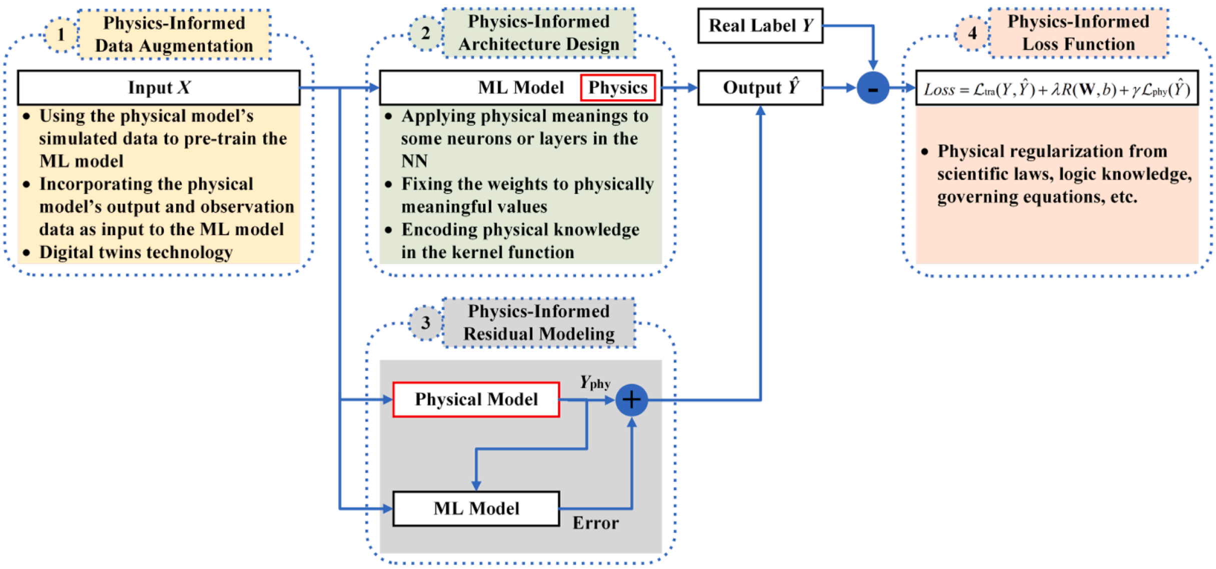

Fig. 8. General framework of the PIML based RUL prediction methods.

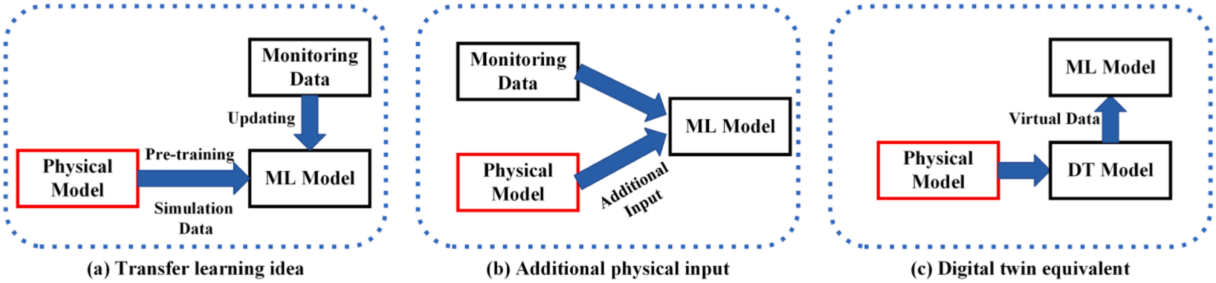

Fig. 9. Physics- informed data augmentation schematic.

the estimator model with the aging data of early life experiments. Experimental results and comparative analysis further showed that physics-based simulation data could improve the model's prediction accuracy.

The third way for physics-informed data augmentation is to construct the digital twin (DT) equivalent. DT technology [132] can build a high-fidelity digital twin model for physical entity and generate virtual data sets, which can be used as a means of physics-informed data augmentation, as shown in Fig. 9 (c). Meraghni et al. [133] proposed a data-driven DT technique to integrate the physical knowledge of proton exchange membrane fuel cell systems and to predict health status online using a deep transfer learning model based on stacked denoising autoencoders. In Ref. [134], DT was combined with the bearing life prediction. The twin data set framework was constructed using sensitive features. The missing data set was supplemented by the integrated learning Cat-Boost method to form a complete digital twin data set used to train macro and micro attention Bi-LSTM models so as to obtain the final RUL prediction results. A new DT-driven framework was developed by Wang et al. [135], in which the crack tracking model established by the extended finite element method generated sufficient and effective training data for the high-precision approximate model constructed by the radial basis function neural network. Then, the dynamic Bayesian network model connected the virtual model and physical model for the RUL prediction. The validation results indicated that the uncertainty of model parameters can be significantly reduced while the accuracy of structural fatigue life prediction was improved.

Besides the above mentioned physics-informed data augmentation methods, there are some avenues to achieve physics-informed data augmentation and enhancement such as the physics-informed data preprocessing and physics-informed data generation. For example, Xiong et al. [136] proposed an adaptive deep learning based RUL prediction framework with failure modes (FM) recognition in preprocessing the data, in which the physics-informed FM classifier with deep convolutional neural networks was developed to improve the interpretability and the accuracy of the RUL prediction. To alleviate the limitation in the availability of representative time-to-failure trajectories on the DL-based RUL prediction, a hybrid framework combining the controlled physics-informed data generation approach with a DL-based prediction model is proposed for prognostics with the generated physically plausible synthetic data in [137]. In this work, a controlled physics-informed generative adversarial network (GAN) is developed to generate synthetic degradation data by considering five basic physics constraints controlling the generator. A physics-informed loss function with penalty was designed as the regularization term to ensure that the changing trend of system health state recorded in the synthetic data was consistent with the underlying physical laws. Then, the generated synthetic data is used as input of the DL-based prediction model to obtain the RUL predictions.

Based on the literature analysis, it is found that physics-informed data augmentation can pre-train ML models based on physical knowledge to avoid the problems of slow convergence and local optimum caused by the random initialization of ML model parameters, lower the requirement of the volume and quality of the monitoring data, and reduce the complexity of the model training. However, for large complex systems, the computational cost of physical models is heavy, and the advantages of this method will be affected to so extent. Even in the case that the physical knowledge is available, synthesizing high-quality simulation data is still challenging. When the quality of the simulation data based on physical model synthesis is low, it will bring too much noise if it is directly used for training the ML model. Therefore, it is necessary to filter the simulation data and reserve high-quality data for model training or fine-tuning the model in the subsequent stage. The simulation data generated by the physical model may have different degrees of credibility or have a large gap with the real data monitored by the sensors. The simplest and most direct method is to study the weight distribution of these two data sources by considering the difference of their credibility. Another potential direction to narrow the gap is to develop advanced physics-informed generative data augmentation techniques since such techniques have been proved to be powerful to enrich the data quantity and diversity [137,138]. In addition, data augmentation supported by the advanced intelligent techniques such as the DT can help to improve the reliability of the generated synthetic data for developing PIML based RUL prediction methods. Therefore, constructing highly reliable DT models of complex systems that can evolve in real-time deserve more studies in the future.

5.3. Physics-informed architecture design

Although physics-informed data augmentation discussed in the previous section provides physically meaningful data to train the ML model, the ML structure is still a black box that is not interpretable. Thus, to improve the interpretability of the ML models, one avenue is to use architectural properties to implicitly encode desired physical properties. The modularity and flexibility of NN facilitate the modification of its network structure. Combining physical knowledge to design the network structures and node connections that capture physics-based dependencies among variables and make them subject to physical constraints can produce a more general and physically consistent model and render the traditional black-box structure more interpretable. In the PIML based RUL prediction, existing methods with physics-informed architecture design can be roughly divided into two categories, as illustrated in Fig. 10.

The first category to achieve physics-informed architecture design is to assign physical meaning to neurons or layers, as shown in Fig. 10 (a). Nascimento and Viana [139] proposed a cumulative damage modeling method based on a physics-informed recurrent neural network (RNN). The information of the system's physical model was embedded into RNN by modifying the RNN cells. The stress intensity was modeled by using a multi-layer perceptron in the recurrent cell and fed into the Paris law layer. The results showed that the method could simulate fatigue crack propagation accurately and the grease state seriously affects the fatigue life of wind turbine main bearings. Nevertheless, the complexity of its degradation mechanism and the uncertainty of its mass change make modeling very difficult. Based on the method proposed in [139], Yucesan and Viana [140–142] modeled fatigue life through physics-informed RNN combined with a purely data-driven method to model lubricant degradation. The physics-informed layer was used to model the cumulative damage of bearing fatigue while the NN layer was used to model the unknown degradation process of grease. Similarly, Nascimento et al. [143] and Kim et al. [144] adopted the method proposed by Nascimento and Viana [139] to model the capacity degradation process in the charge–discharge cycle and embedded the known discharge equation into the RNN cell architecture. The difference is that Nascimento et al. [143] estimated the PDF of the weight and deviation of the RNN cell by variational inference, whereas Kim et al. [144] further quantified the prediction uncertainty using the Monte Carlo dropout method. Yan et al. [145] combined physics-based signal processing technology with the artificial network for machine degradation modeling. The first three hidden layers of the network were composed of the Hilbert transform, square envelopment, and Fourier transform to map the time-domain signal to the frequency-domain signal with rich fault information. The fourth hidden layer was composed of the full connection layer and knowledge-guided loss function was designed to extract health indicators. The proposed network was fully transparent and interpretable, and the generated health indicators generally had better monotonicity trend and early detection capability. Nguyen et al. [146] proposed a fuzzy generative adversarial network (GAN) embedded in physics. The physical model was added to the aggregation module of the fuzzy logic model to improve the generative network architecture and reduce the error of the prediction evaluation index. The results showed that the combination of fuzzy logic and a physics-based model helped improve the prediction ability of fuzzy GAN. In [147], Hajiha et al. developed a physics-regularized data-driven approach for the RUL prediction of complex engineered systems with multiple hidden and dependent health states. The developed framework included a data layer to capture the statistically-correlated temporal dynamics of hidden system states and a physics layer to impose regularizations among observed system operating parameters and system health states through system working principles and governing physics. To do so, the proposed method realized the integration of engineering domain knowledge and sensor data streams to improve the interpretability and accuracy of the predictions.

The second category to achieve physics-informed architecture design is to constrain the feasible solution space of network structural parameters based on the physical knowledge, i.e. to integrate physical knowledge into the NN by imposing appropriate constraints on weights and offset terms, as shown in Fig. 10 (b). Chen and Liu [148] proposed a probabilistic physics-guided NN to estimate the probabilistic fatigue S-N curve, in which the stress level S, stress ratio R, and the logarithm of fatigue life were inputs while mean and variance were outputs. According to the physical knowledge that the standard deviation of the fatigue life is negatively correlated with stress and the curvature of the fatigue curve is positively correlated with stress, the interpretable stress-life relationship was obtained by imposing physical constraints on structural parameters such as network weight and bias. Aiming at the problem with missing fatigue data of Ti-64, based on the previous work [148], Chen and Liu [149], in addition to the pure mechanical analysis of materials, also regarded process parameters (such as scanning speed, laser power, heating temperature, etc.) as control factors, and

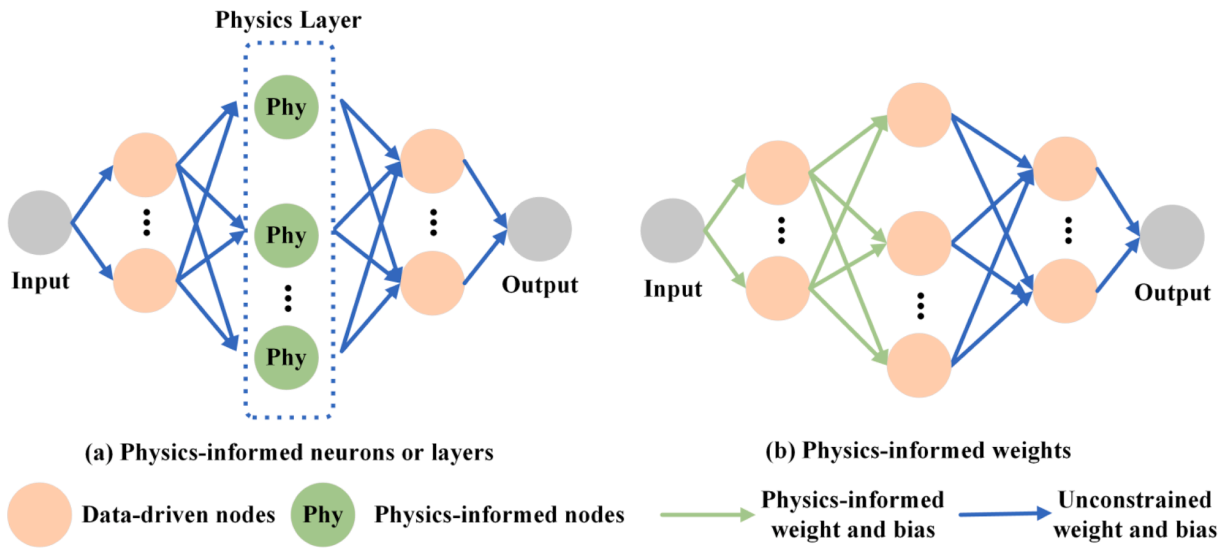

Fig. 10. Physics-informed architecture design.

adopted the sensitivity analysis method to determine the influence of process parameters on fatigue properties. In addition, the physical knowledge that the mean value increases with the decrease of stress ratio was added to the probabilistic physics-guided NN to constrain the feasible solution space of the network parameters so as to obtain more reliable life prediction.

Unlike the NN structure, GPR is a probabilistic model with characteristics of non-parametric modeling and strong generalization ability. It can quantify the uncertainty of prediction results and has attracted wide attention in the RUL prediction. The way to embed physics into the GPR is to encode physics equations into the mean or covariance functions. Pfingstl and Zimmermann [150] used the basis functions based on physical information to derive the mean function and covariance function, and verified the applicability and validity of fatigue crack growth, laser degradation, and milling machine wear data. The use of physical basis functions further improved the accuracy and reduced the computational burdens of training. Similarly, Richardson et al. [123] embedded the empirical degradation equation into the GPR mean function, established three-output GPR, and predicted the future degradation trend of batteries. In addition, Zhang et al. [151] also designed the time-varying physical kernel function to simulate the time-varying change of the degradation and quantify the uncertainty, and learned the parameters of the physical mean function and kernel function based on the monitoring data to capture the information not embedded in the existing physical knowledge, and successfully applied it to the performance prediction of the heating, ventilation, and air-conditioning (HVAC) system.

At present, the design of physics-informed architecture is basically context-dependent. There is no general way of physics-informed architecture design applied to various cases. In other words, the model architecture designed for a concerned system's RUL prediction may fail when applied to other systems. As a result, the issue of how to automatically design high-performance network architecture according to different tasks and requirements to solve the RUL prediction problem of different systems in practice and improve the approximation effect of the ML model to actual physical process are potential research directions in the future. One possible way to achieve automatic architecture design of the PIML is to apply neural architecture search technology [152], which is a method of automated architecture design. With such technology, physical knowledge is limited to the predefined search space such that a potential excellent architecture is found according to the search strategy and the performance of this architecture is quickly verified by the evaluation method. After the iterative search and verification process, the optimal architecture with physical consistency will be hopefully be found.

5.4. Physics-informed loss function

The training of PIML is often completed by minimizing the loss function in the back-propagation process. The most direct way to integrate physical information with the ML models is to add physical information as constraints to the loss function during the process of training NN. The physics-informed loss function can be generally expressed as

$$Loss = \mathscr{L}\mathrm{tra}(Y, \widehat{Y}) + \lambda R(\boldsymbol{W}, b) + \gamma \mathscr{L}\mathrm{phy}(\widehat{Y}) \tag{10}$$

where $\mathscr{L}\mathrm{tra}(Y, \widehat{Y})$ is the mean squared error or cross entropy between input and output in the loss function, λ is the hyperparameter to measure the complexity loss of the model, $\boldsymbol{W}$ and b are the weight and bias of the NN for example, respectively. The first two losses are standard losses for the ML model and represent the loss term of the data-driven part. $\mathscr{L}\mathrm{phy}(\widehat{Y})$ is the physical regularization term based on $\widehat{Y}$ and represents the loss term of the physics-driven part, and γ is the hyperparameter to measure the weight of the physical model.

As discussed in Section 5.1, embedding the physical information into the loss function will bring multiple benefits, including lowering the requirement of labeled data, shrinking the search space in optimizing the ML models, and improving the interpretability and generalizability to unseen scenarios. With the physics-informed loss, Wang et al. [153] proposed a PINN model for machining tool wear prediction, which preserved physical consistency and eliminated physical inconsistencies by introducing the ReLU function considering physical law into the loss function. In addition to monotone constraints, another way to embed knowledge is to add upper and lower bounds to the results of the NN through approximate constraints. In [154], the Weibull-based loss function was used as an approximate constraint to integrate the reliability engineering knowledge represented by Weibull distribution into the ML model. In [155], a semi-empirical law constraint of linear elastic fracture mechanics model was considered, and a loss function with a loss value of 0 at the mean of normal distribution and a loss value of approximately 1 at the point far from the mean was constructed. The results showed that the fatigue life of metal materials can be accurately predicted. Shen et al. [156] integrated a threshold model and deep convolutional neural network (CNN) model for bearing fault detection. The threshold model evaluated the health grade based on the known physics of bearing failure. The CNN model automatically extracted features from the input data to predict the future health levels. The physical knowledge in the threshold model was embedded into the training process of the CNN model through the loss function. When the prediction of the health level of the two models was inconsistent, the penalty was added to the loss function to make the model have physical significance. Russell and Wang [157] used the domain knowledge of the monitoring signal periodicity to study the physical information loss terms of the combination of autocorrelation and two fast Fourier transform (FFT) measurements, which improved the ability of deep convolutional autoencoder to compress and reconstruct fault signals and was of great significance for efficient transmission and analysis of big data in industrial status monitoring. Sun et al. [158] proposed a physics-informed doubly fed cross-residual DNN for magnetic flux leakage defect detection. The quantization theory of magnetic flux leakage defect based on physics was studied and integrated into the loss function during the NN training. Sun et al. [159] integrated the logic theory of ultrasonic nondestructive testing into the NN training. It proposed feedback and feed-forward loss functions for evaluating variables of different forms to quantify microcrack defect accurately. In [160], a multi-input NN based on LSTM was used to model and predict the degradation of the electro-hydraulic actuator system, and the failure mechanism was introduced into the loss function. Besides the physical knowledge, the domain knowledge can also be embedded into the ML models. For example, Zhang et al. [161] constructed physics-based deep neural network under physics-informed feature engineering and physics-informed loss function to predict creep-fatigue life by adding some constraints acquired in the steel domain, and experimental results on creep-fatigue data of 316 austenitic stainless steel indicated that the prediction accuracy of the developed method was significantly higher than empirical model and purely data-driven method. Considering the domain knowledge in the prognosis practice that the late prediction of the RUL will increase the operating failure risk of the system, the measure for the prognosis performance should distinguish between the late predictions and the early predictions, and should penalize the late predictions more. With embedding this domain knowledge with the loss based on the mean squared error of the RUL predictions, He et al. [162] proposed a systematic method based on the ARMA regression and graph convolutional network model to predict the RUL with multisensory data under dynamic operating conditions and multiple failure modes. In the model training, the physics equations of balancing for economy and security in preventive maintenance policies was introduced in the loss function to impose a higher penalty on delayed predictions so as to make the proposed method be feasibly applied in high-risk situations.

In addition, differential formal constraints in partial differential equations describing the system's dynamics can also be incorporated into the loss function design of the NN to obtain NN with physical model constraints [119], as shown in Fig. 7. Xu et al. [163] designed a physics-informed dynamic depth autoencoder model which used the state equation and capacity equation of lithium batteries as penalty terms to adjust loss functions. In this way, the current challenges in the accuracy and interpretability of data-driven health state prediction will be hopefully solved. Zhou et al. [164] proposed a system reliability evaluation method based on physics-informed GAN and deduced the system state probability according to the forward Kolmogorov equation to determine the system reliability. The uncertainty of the system reliability was quantified by embedding the forward Kolmogorov equation constraint into the loss function of GAN. In [165], PINN was used to predict the crack growth of quasi-brittle materials under complex loads, the loss function was constructed by the energy variational principle, and boundary conditions were enforced in the loss function by the penalty function method, which eliminated the dependence on the label data. When the physical degradation equation was unknown, Cofre-Martel et al. [166] mapped the monitoring data and time variable into the implicit variable related to system degradation, introduced the derivative of the RUL with respect to the operating time and latent variable in the form of the PDE so as to find the unknown PDE, and then introduced this term into the loss function to train DNN model for the RUL prediction. Recently, Wen et al. [167] developed a model fusion scheme based on PINN for prognostics of Li-ion-batteries. In the implementation process, a semi-empirical semi-physical PDE was established to model the degradation dynamics of Li-ion-batteries. In the case that there is little physical knowledge, the data-driven deep hidden physics model was used to discover the underlying governing dynamic models. The uncovered dynamics information was then fused with that mined by the surrogate neural network in the PINN framework. The physics-informed loss function based on the PDE was designed by an uncertainty-based adaptive weighting method to train the PINN for the RUL prediction.

By the above discussions, it is found that the loss function is mostly expressed by the mean squared error in most of ML based applications. Therefore, it is necessary to add the regular terms to the expression of the loss function to improve the ability of outputting the physically consistent results. In addition, the weights of the physical model and ML model influences the prediction results in the physics-informed loss function. However, there is no uniform standard for determining the optimal weights, even after using methods such as cross-validation. Particularly, when multiple physical constraints are added to the loss function of the PIML model, the weights of the physical penalty term of each condition is usually calculated empirically according to the actual problem, and there is no unified and complete theoretical guidance. This difficulty will make the training process of the PIML model rather complicated. As such, a meaningful future research direction is to design some optimization algorithms customized for the loss function of the PIML to enhance its stability and convergence, and improve the efficiency of the PINN training process.

5.5. Physics-informed residual modeling

Physics-informed residual modeling is one of frequently adopted ways to solve the defects of physical models by using ML models to learn and predict the errors or residuals made by physics-based models. The primary idea behind physics-informed residual modeling is to learn biases of the physical model and use it to make corrections to the physical model's predictions. Fig. 11 illustrates the general principle of the physics-informed residual modeling to achieve the PIML.

With the idea of physics-informed residual modeling, Tu et al. [168,169] combined the electrochemical model and equivalent circuit model with the feed-forward NN to construct two hybrid models. In this work, NN was used to capture the residual and predicted voltage of a single particle model (SPM) of thermal effect, realizing the deep integration between physics-based model and ML based model. Similarly, an electrochemical-thermal-neural-network (ETNN) coupling model was established by Feng et al. [170]. The

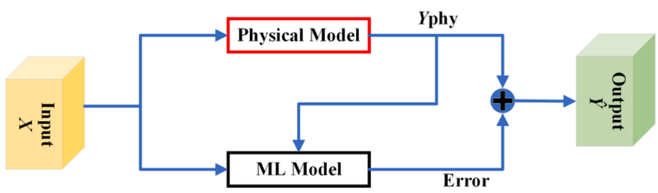

Fig. 11. Physics-informed residual modeling.

simplified SPM and lumped heat model were used as the sub-models of ETNN to predict the core temperature and provide an approximate terminal voltage. Then, NN was introduced to improve the performance of the sub-model. Yu et al. [171] integrated the physical equation of the aircraft dynamics system into the deep residual recurrent NN to minimize the residual function in the training process. The results showed that the physics-informed deep residual recurrent NN significantly improved the prediction performance, reduced the calculation cost, and had good extrapolation performance. It is observed that physics-informed residual modeling is a relatively simple and direct avenue to construct the PIML, and works like the previously discussed hybrid modeling process. Therefore, the physical model and the ML model are combined in a shallow way and the combination structure design is a challenging problem. In addition, residual modeling only models errors of the outputs of physics-based models, rather than actual monitoring quantities. Therefore, a key limitation of this kind of methods is that physical constraints such as those described in the previous section cannot be forced to be embedded in the modeling process. Therefore, extending physics-informed residual modeling to considering physical constraints is an important direction to improve the modeling ability and interpretability of the developed PIML as this residual modeling way.

Together with the discussions from Section 5.1 to Section 5.4, it is observed that, compared with the traditional ML based RUL prediction methods, the PIML based methods forms a new learning mode for the RUL prediction by integrating the physical guidance and data-driven learning, which can effectively improve the interpretability and generalizability of the ML based methods. At the meanwhile, there are still many challenges and opportunities in applying and developing such PIML based RUL prediction methods as discussed in the previous subsections, including the issues of how to initialize the model, design a more stable network structure and reasonable loss function, improve the adaptability and generalization of the model under different working conditions, etc. Combined with the above literature analysis, it is observed that PIML has not only emerged as a research frontier in the field of the RUL prediction since it was proposed in 2017, but also has aroused many industrial partners to extend the application contexts of PIML, including NVIDIA, NASA, the Argonne National Laboratory, and Siemens. For example, ANSYS and NVIDIA are accelerating the production of PIML algorithms [172]. NVIDIA has shown the great potential of PIML for AI-based simulation computing, applying it to the shape design of the own chip radiator, which can reduce the simulation time from months to days or even hours compared to traditional commercial numerical simulation solvers. Nevertheless, there is a profound gap between what is researched in academic literature and what is effectively applied in the real world for the PIML methods. This is particularly true for the PIML based RUL prediction studies. In practical engineering context, due to the differences in the system structure, operating conditions, and data quality, scenarios for PIML based RUL prediction are complex and diverse, making the practical application of PIML in the field of the RUL prediction lag behind other fields. Therefore, the PIML-based RUL prediction is still in the infancy with more focus on theoretical studies and thus has great development potential in the prognostics since the idea behind PIML and the associated existing studies have shown the great promise of PIML to enrich the field of the RUL prediction and provide theoretical support for prognostics applications. It can be anticipated that the studies of PIML based RUL prediction will be continuingly growing in the future as the quick development of PIML in multidisciplinary fields, but the important direction in the RUL prediction itself is to develop prognosis-oriented PIML rather than simply applying PIMLs presented in other fields to the RUL prediction problems.

6. Discussions on challenges and opportunities

Physics-informed data-driven RUL prediction is a frontier research direction driven by practical engineering requirements. To further quantitatively illustrate the advantages of physics-informed data-driven RUL prediction methods, we take the bridge deck rebar corrosion datasets, the Commercial Modular Aero-Propulsion System Simulation (C-MAPSS) datasets, and lithium battery aging datasets as representative case studies, and provide the following comparative results gathered from the publications regarding physics-informed data-driven methods and other methods using the same dataset, as shown in Tables 3–5, where the metrics for comparing different prognosis methods include the root mean squared error (RMSE), the Score value, and the mean absolute percent error (MAPE).

In Tables 3–5, the method highlighted in bold is the physics-informed data-driven method proposed in the corresponding references. It is noted that the smaller the prediction performance metrics (RMSE, Score values, and MAPE), the better the prediction performance. From the above three representative case studies, it can be intuitively concluded that, compared with physical models, stochastic degradation models, and ML models, the RUL prediction performance of the physics-informed data-driven method is better. These comparative results further indicate the necessity of developing physics-informed data-driven methods for prognostics. In the previous sections, we have reviewed the state of the art of physics-informed data-driven RUL prediction methods reported in recent years, and discussed the limitations and possible directions for each category, i.e. physical model and data fusion methods, stochastic degradation model based methods, and PIML based methods. It is observed that advances in the understanding of how to embedding

Table 3
Performance comparison of different prognosis methods in terms of RMSE [173].

Methods	Datasets			
	BS	EC	SS	MMFX
Gamma-gamma two-stage model	60.99	17.67	22.94	32.50
BP-ANN	40.81	36.35	31.79	33.29
Bi-LSTM	29.20	18.39	14.62	7.73
Bi-LSTM + gamma-gamma	**29.15**	**15.63**	**9.88**	**6.61**

Table 4
Performance comparison of different prognosis methods in terms of Score values [174].

Methods	Datasets			
	FD001	FD002	FD003	FD004
Wiener	1263	5342	1509	8369
CNN	412	3642	438	6274
CNN-Wiener	293	2665	313	4849
CBAM-CNN	372	3218	406	5841
CBAM-CNN-Wiener	**196**	**2200**	**228**	**3800**

Table 5
Performance comparison of different prognosis methods in terms of MAPE [126].

Methods	Datasets							
	Dataset 1 (2 A)			Dataset 2 (4 A load)			Dataset 3 (4 A square wave)	
	#6	#7	#18	#29	#30	#31	#25	#26
CCA	6.85 %	12.20 %	3.59 %	44.76 %	23.20 %	45.38 %	16.78 %	11.23 %
CNN	10.69 %	14.74 %	11.31 %	21.18 %	17.90 %	22.74 %	19.34 %	17.21 %
BiLSTM	5.33 %	8.54 %	4.12 %	16.51 %	14.39 %	19.88 %	14.84 %	12.10 %
PI-LSTM	**0.92 %**	**1.41 %**	**1.17 %**	**1.72 %**	**1.66 %**	**3.39 %**	**4.19 %**	**5.59 %**

physical or domain knowledge into data-driven models hold great promise to render them generalize better to unseen/out-of-sample cases while at the same time explaining underlying physics of the failure. Besides the challenges and opportunities discussed in previous sections tailored to each category, we present some challenges and opportunities for further developing physics-informed data-driven RUL prediction from a more general perspective in the following.

6.1. RUL prediction under imperfect physical models

Modern systems gradually show large-scale, complicated and diversified characteristics. In these scenarios, the constructed physical model of the concerned complicated system, if possible, is imperfect with high complexity. In addition, due to the complexity, nonlinearity, and high dimension of modern systems, the performance degradation variables that characterize the health state of the system are often interrelated and the relationship between the variables is difficult to be clearly understood. At the meanwhile, the system degradation-to-failure process is affected by complex factors such as varying operating conditions, environments, and loads. All these factors make it very difficult to construct the exact and perfect physical model. In contrast, data-driven methods can mine the relationship between variables automatically, so they can be used to guide the construction of physical models. For example, new combination features mined by convolutional neural networks can be used to improve the physical model [175]. In addition, the hidden rules in the data can also be explained by adding disturbance to the data-driven method [176], which helps analyze the relationship between various degeneration variables. When the physical model does not match the actual working conditions or there are uncertain factors such as noise interference, the physical model is not accurate enough for the RUL prediction. By modifying the physical model's key parameters, quantifying the physical model's difference, assisting in updating the physical model, or analyzing the uncertain factors in a data-driven way, the accuracy of the calculation results can be improved. In this case, the data-driven method can be used to simplify the construction of the physical model. One developing opportunity is that the physical model is used to represent deterministic, linear and representable parts while the data-driven model is applied to represent nonlinear and hardly explainable parts. To do so, the advantages of physics-based models and data-driven methods can be inherited while at the same time the underlying physics of the failure can be understood.

6.2. RUL prediction under massive data

Although the aforementioned stochastic model based RUL prediction methods have the natural superiority in quantifying the underlying prognosis uncertainty, the success of such methods depends on an important potential prerequisite that the degradation feature with the trend can be extracted from the monitoring data of the system. With the good degradation feature, stochastic degradation models can be effectively constructed and output the predicted RUL in probabilistic distribution forms to quantify the prognosis uncertainty. This is also known as the major advantage of statistical data-driven prognostic approaches. With great advances in Industry 4.0 and the Internet of Things, a large number of monitoring data can be obtained providing abundant information on the system's health state and the RUL. However, the ability of automatic processing the massive data and extracting the associated degradation feature is limited for the existing stochastic model based methods. In contrast, DL techniques have the powerful ability to process the massive data and extract the hidden feature in the data. Therefore, fusing deep learning and stochastic model based methods will hold great promise to pave the way on prognostics for the massive data cases. A possible avenue is to apply deep learning techniques to extract the degradation feature and then model the degradation progression of such feature with stochastic models. To do so, the capability of quantifying prognosis uncertainty and handling the massive data can be jointly achieved by such data-model linkage mechanism. It is noted that there are some preliminary attempts following this avenue and great potentials have been observed even, see for examples [173,177]. Fusion of deep learning and stochastic models is still in the infancy but it is an important ongoing direction with potentials to improve the prognosis performance under the massive data, where the challenge lies in how to establish the effective and explainable fusion mechanism.

6.3. RUL prediction based on PIML

As mentioned in Section 5, although the PIML method realizes the organic combination and mutual complement of physical knowledge and ML, which has attracted much attention in the field of the RUL prediction, there are still some problems in the data augmentation, network architecture design, loss function optimization and other aspects of this kind of methods. Firstly, the data quality determines the accuracy and robustness of the PIML training results. For non-balanced, locally missing, unmarked and other incomplete data, techniques such as the GAN [178] to enhance expanded data and unsupervised/semi-supervised learning [179] to reconstruct signals can be used to improve the data quality so as to improve the RUL prediction performance. Second, for complex large-scale nonlinear high-order physical equations, the PIML may converge to the local optimal solution or even collapse during the training process. It is necessary to design more stable network architectures and select appropriate activation functions and parameter updating algorithms to enhance its performance. Third, the loss function constructed by the physical equations has strong non-convex properties in general and thus how to construct a more reasonable loss function and optimizer to improve the convergence speed of the training process needs to be further explored. Fourth, PIML based methods can be combined with transfer learning and *meta*-learning [180] to realize the rapid transfer of the model to new tasks under different operating conditions, environmental loads, degradation stages and other changing conditions so as to improve the adaptability and generalization of the PIML based methods. Fifth, in current PIML based RUL predictions, the deterministic ML models are extensively adopted but they have the limited ability to quantify the prognosis uncertainty which is an inherent issues of the prognosis problem. Therefore, how to develop probabilistic PIML models for the RUL prediction is an underlying research direction in the future. Last, existing PIML based RUL prediction methods mainly embed the physical or domain knowledge into the ML models by physics-informed data enhancement, physics-informed architecture design, physics-informed loss function and physics-informed residual modeling as discussed in the previous section. Actually, according to the RUL prediction process illustrated in Fig. 2, embedding the physical or domain knowledge into the whole process of the RUL prediction, from the data acquisition, pre-processing, feature extraction, degradation modeling to the parameter estimation and optimization, will greatly enrich the PIML based RUL prediction studies and needs more attention in the future.

6.4. Benchmark datasets for validating physics-informed data-driven methods

Comprehensive benchmark datasets have been proved to be great boosters for the development of corresponding techniques. This is particularly true in the development process of purely data-driven RUL prediction methods. For example, there are many open-source run-to-failure datasets for prognosis, including the IMS bearing dataset [181], FEMTO bearing dataset [182], milling dataset [183], the NASA CMAPSS and N-CMAPSS aircraft engine datasets [184], batteries datasets by the CALCE of Maryland University and Massachusetts Institute of Technology [91], bearing datasets by Xi'an Jiaotong University [75], etc. All these publicly available datasets have substantially and positively promoted the fast development of purely data-driven RUL prediction studies by providing common platforms for performance benchmarking. However, the available datasets are basically intended for data-driven modeling and prediction. Due to the complexity and heterogeneity of problem settings, physics-informed data-driven RUL prediction still lacks comprehensive benchmark datasets for evaluating various methods of knowledge integration, which creates barriers in the development of such methods. The unavailability of such domain-specific datasets greatly increases the difficulty of fairly comparing different physics-informed RUL prediction methods. In this case, constructing comprehensive benchmark dataset for physics-informed prognosis is of great need for boosting its development. Such datasets targeted for physics-informed prognosis, in which the proper parameterized physical models or domain-specific knowledge are included, are desirable to be provided publicly and general enough to accommodate various physics-informed prognosis methods as well as purely data-driven and physics-based methods. It is noted that constructing such datasets are rather challenging but essential to develop and validate physics-informed prognosis methods for practical engineering applications.

6.5. Explainability and interpretability for the RUL prediction

With the development of artificial intelligence (AI) technologies in high-stakes decision-making areas such as healthcare, finance, law, and transportation [185], a growing number of academics, companies, authorities, and civil society organizations are increasingly focusing on the explainability and interpretability of AI techniques when applied to the practice. Arrieta et al. [186] provided a necessary clarification of the terms used in the code of ethics for explainable AI (XAI): explainability (explaining the decisions made) and interpretability (understanding the inner workings of the models) are not interchangeable. This is definitely true for the RUL prediction field when applying AI techniques. The RUL prediction serves the decision making and is often used in safety–critical areas such as high-speed trains, aerospace equipment, nuclear power equipment, and other major industrial equipment, so the risk is an important consideration in decision making based on the prognosis information. The ultimate goal of the RUL prediction is to provide information and insights for the end users to decide when and how to keep the concerned system healthy through managerial activities such as preventive replacement, maintenance scheduling, and spare parts ordering. As for the end users, the questions of when they can trust the RUL prediction made by the model and when they need extra caution while making decisions based on such prediction are frequently encountered. This is especially important when incorrect decisions can lead to severe financial losses or even life-threatening outcomes in safety–critical applications. Therefore, the explainability and interpretability of the RUL prediction results are two significant aspects deserving more attention in developing prognostic methods. At present, Black-box models such as neural networks can achieve high accuracy with the growing availability of large volumes of data but lack interpretability of the RUL prediction results to the end users, while white-box models such stochastic process models are easy to understand but lack the sufficient power handling the massive data for prognostics. Compared with the purely data-driven methods, the physics-informed data-driven methods can embed physical knowledge into the ML models, improving the interpretability within the models and the explainability in decisions made for practical applications, and reducing the occurrence of security accidents and costs caused by unexpected equipment failure. However, some hyperparameters of ML models (e.g., the number of layers in neural networks and the learning rate and batch size in optimization algorithms) cannot be well explained with physical knowledge and may cause ML models to be affected by the local minima [187]. In addition, as discussed in Sections 6.2 and 6.3, current physics-informed data-driven RUL prediction methods often adopt the deterministic ML models and thus have the limited ability to quantify the prognosis uncertainty. The lack of the powerful prognosis uncertainty quantification ability has prevented physics-informed data-driven RUL prediction methods from being practically deployable in real mission- and safety–critical decision making applications. Therefore, both explainability and interpretability of the physics-informed data-driven method should be deeply studied and further improved.

There are several ways to improve the explainability and interpretability of the physics-informed data-driven method. The first way is to select interpretable algorithms, such as decision trees and regression models, to avoid possible uncertainties caused by hyperparameters of the model. Secondly, XAI can be used to analyze correlations between hyperparameters and data, and based on such correlations, more interpretable physics-informed data-driven models can be designed for the RUL prediction. These two ways can be used to improve the interpretability of the inner working mechanisms of the physics-informed data-driven models. The third way is to integrate the uncertainty quantification techniques into the physics-informed data-driven model to enhance the transparency and trustworthiness of the predictive results. To do so, the physics-informed data-driven model with the uncertainty quantification ability can provide useful information regarding the confidence of the prognosis results. As such, the physics-informed data-driven RUL prediction method can quantify the prognosis uncertainty and communicate this uncertainty to end users for guiding the decision making applications in an easy-to-understand way. This way has the potential to improve the explainability associated with the decisions made. In sum, the RUL prediction results are ultimately oriented for the decision making while decision making in practical applications involves a complicated trade-off between risk and potential economic benefits. Therefore, RUL prediction methods need to be as trustworthy and reliable as possible. For this reason, improving explainability and interpretability of the RUL prediction results is a priority as it makes the prediction model more consistent with physics and enhances the predictive robustness, thus finally promoting the wide acceptance of the RUL prediction methods in real-world applications.

6.6. RUL prediction fusing physical degradation observation and condition monitoring data

Due to the complexity of the system and the high cost of directly monitoring the degradation state, some internal physical degradation observations, such as the growth of fatigue cracks or tool wear, are difficult to directly observe. In engineering practice, implicit or partially observable degradation scenario is frequently encountered. Still, condition monitoring data is relatively easy to obtain by various external sensors, including piezoelectric sensors, vibration sensors, acoustic emission sensors, etc. Through the analysis and processing of the condition monitoring data from sensors, important information (such as the degradation trend, abnormal conditions, etc.) about the internal physical degradation states can be extracted to evaluate and predict the health state and performance of the system. Following this line, how to establish the relationship between physical degradation observations and the condition monitoring data is critical for conducting the prognostics task. In other words, the key to the RUL prediction under implicit physical degradation scenario is to reasonably model the stochastic degradation process and determine the stochastic relationship between the implicit physical degradation process and the condition monitoring data. In this case, the state-space model is a very powerful tool, which can conveniently model the relationship between implicit physical degradation process and the condition monitoring data under a unified framework and realize real-time estimation and update according to the associated prediction equation and update equation [188,189]. If a few physical observations can be obtained in some cases, these physical observations can be considered to calibrate or correct the state-space model. In addition, if available, physical observations and condition monitoring data can be integrated at the data level, feature extraction level, or model construction level to better understand the internal state and performance of the system and improve the accuracy and reliability of the prediction. Although the principle fusing physical observations and condition monitoring data is feasible, there are some challenges remaining to be solved. First, the physical degradation observations may only be intermittently available, and the sensor's sampling output is continuous. In this case, the sampling intervals for the physical degradation observations and the condition monitoring data are different. How to design the data processing technique and fusion mechanism under the inconsistence of the sampling intervals deserves further studies. Another challenge that has not been fully explored is the prediction of intermittently failing systems. Intermittent fault refers to a fault that occurs randomly during the system's operation and then quickly disappears without any maintenance activities [190]. The degradation of such system manifests a gradual increase arising from the frequency or intensity of intermittent fault. It is noted that the intermittent faults might not be obviously visible in the sensory data and has the hidden nature. In this case, the physical degradation observations are hardly available and affected by the mechanism of intermittent fault occurrence, which brings another challenge for analyzing it. A possible avenue is to design the switching state-space model so as to characterize the impacts of the intermittent fault on the state equation, and then to uncover the relationship between the physical degradation affected by intermittent fault and the condition monitoring data from sensors through data-driven methods. In sum, developing new methods to reveal the relationship between the physical degradation and the condition monitoring data for prognostics task is a challenging but promising direction and has the urgent application desire in the RUL prediction field.

7. Conclusions

In this paper, we comprehensively and systematically reviews the latest research progress of physics-informed data-driven RUL prediction, aiming at shedding light on develop new framework to realize the deep integration of the physical knowledge and the monitoring data. To complete this task, by clarifying the idea of physics-informed data-driven RUL prediction, current relevant methods are divided into three categories such as physical model and data fusion methods, stochastic degradation model based methods, and PIML based methods. As for each category, the involved modeling principle and recent advances have been discussed with some suggestions on the future development. Particularly, the advances PIML based methods are discussed in details with extensive references. Through discussing the pros and cons of existing methods, possible opportunities and challenges in physics-informed data-driven RUL prediction methods are provided to steer and speed up the future development of advanced physics-informed data-driven techniques to improve the prognosis performance.

CRediT authorship contribution statement

Huiqin Li: Data curation, Methodology, Software, Writing – original draft. **Zhengxin Zhang:** Data curation, Methodology, Software, Writing – review & editing. **Tianmei Li:** Data curation, Supervision, Writing – review & editing. **Xiaosheng Si:** Conceptualization, Funding acquisition, Methodology, Project administration, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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