*Journal of Electrostatics*, 21 (1988) 135-144
Elsevier Science Publishers B.V., Amsterdam — Printed in The Netherlands


# MULTIPOLE INTERACTION BETWEEN DIELECTRIC PARTICLES

M. SANCHO, G. MARTÍNEZ and M. LLAMAS

*Departamento de Electricidad y Electrónica, Universidad Complutense de Madrid, Madrid, 28040 (Spain)*

(Received January 18, 1988; accepted in revised form April 14, 1988)

## Summary

A method for calculating the induced multipoles on interacting dielectric spheres in the presence of an external electric field is derived. Losses in the medium and the particles are included in this model. The results for the dipole moments and the interaction force between two dielectric spheres are strongly dependent on these losses. The model can be applied to dielectric particles as well as to bubbles in a liquid, and can be useful for studying dielectric properties of biological cells.

## 1. Introduction

The electrical interaction between cells in the presence of an external field has attracted considerable attention from investigators in biophysical science. Models of this interaction are useful in the interpretation of cell suspension permittivity measurements, dielectrophoretic (DEP) collection, or for studying phenomena like cell chaining, electrofusion, or emission of radiation by living cells by means of microdielectrophoresis. Jones has used the method of electrical images for calculating the dipole moment induced in a chain of conducting particles by an external field [1]. His method cannot be applied to dielectric particles. Sauer has developed a rigorous treatment for the problem of dielectric particles in an external field [2]. His results, though illuminating for analysing the limitations of the usual derivation of electrical forces on dielectric particles, cannot be easily applied, without very restrictive approximations, to the case of several particles.

In this paper we propose a multipole expansion approach for studying the electrical interaction between dielectric spheres, characterized by a complex permittivity, in a medium with losses. The model allows the calculation of the multipoles induced on each particle as well as the attractive forces between the different spheres. We shall restrict ourselves to the case of aligned particles, which is usually the case in the chaining of cells along field lines.

0304-3886/88/$03.50 

## 2. Theory

We take two polarised spheres under mutual influence (Fig. 1). We consider the first dielectric sphere as a system of fixed point charges, and we shall study the potential due to the charge distribution and the polarized second sphere. We have [3]

$$\phi_- = \sum_{k=o}^{\infty} A_k r^k P_k(\cos\theta), \quad r \leqslant R \tag{1}$$

$$\phi_+ = \phi_0 + \phi_1 = \sum_{k=o}^{\infty} a_k\, r^k P_k(\cos\theta) + \sum_{k=o}^{\infty} B_k r^{-(k+1)} P_k(\cos\theta), \quad r \geqslant R \tag{2}$$

where $\phi_0$ and $\phi_1$ are the contributions to the potential outside the sphere produced by the charge distribution and the polarization of the dielectric, respectively. $P_k$ is the Legendre polynomial of degree $k$.

If the dielectric sphere is characterized by a complex permittivity $\epsilon_1^*$ and the medium by $\epsilon_2^*$, the boundary conditions for $r=R$ and harmonic radiofrequency fields are [4]

$$\phi_- = \phi_+ \tag{3}$$

$$\epsilon_1^* \frac{\partial \phi_-}{\partial r} = \epsilon_2^* \frac{\partial \phi_+}{\partial r} \tag{4}$$

with $\epsilon_1^* = \epsilon_1' - \mathrm{j}\,\epsilon_1''$ and $\epsilon_2^* = \epsilon_2' - \mathrm{j}\,\epsilon_2''$.

A calculation of the coefficients from these boundary conditions leads to

$$A_k = \frac{2k+1}{\left(\dfrac{\epsilon_1^*}{\epsilon_2^*} + 1\right)k + 1}\, a_k \tag{5}$$

and

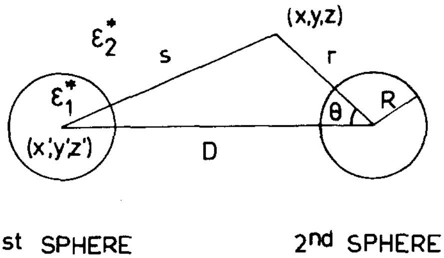


Fig. 1. Geometrical coordinates for the two polarized dielectric spheres in a lossy medium.

$$B_k = -\frac{\left(\frac{\epsilon_1^*}{\epsilon_2^*}-1\right)k}{\left(\frac{\epsilon_1^*}{\epsilon_2^*}+1\right)k+1} R^{2k+1}\, a_k \tag{6}$$

In the case that the first charge distribution is a point charge, the potential can be expressed as

$$\phi_0 = \frac{q}{4\pi\epsilon'_2 S} = \frac{q}{4\pi\epsilon'_2} \sum_{k=0}^{\infty} \frac{r^k}{D^{k+1}} P_k(\cos\theta) \tag{7}$$

for $r<D$, where $D$ is the inter-sphere spacing (Fig. 1).

If the distribution consists of an axial $n$-multipole $p^{(n)}$, directed along the $z$-axis, the potential is

$$\phi_0 = \frac{p^{(n)}}{4\pi\epsilon'_2 n!} \frac{\mathrm{d}^n}{\mathrm{d}z^n}\left(\frac{1}{S}\right) = \frac{p^{(n)}}{4\pi\epsilon'_2 n!} \sum_{k=0}^{\infty} \frac{(k+1)\ldots(k+n)}{D^{k+n+1}} r^k P_k(\cos\theta) \tag{8}$$

Then, in general, for the $n$-multipole we have

$$a_k = \frac{1}{4\pi\epsilon'_2 n!} \frac{(k+1)\ldots(k+n)}{D^{k+n+1}} p^{(n)} \tag{9}$$

Therefore the potential induced on the sphere by a set of multipoles $p_1^{(n)}$ is, from eqns. (6) and (2),

$$\phi_1 = \frac{1}{4\pi\epsilon'_2} \sum_{n=0}^{\infty} \frac{p_1^{(n)}}{n!} \sum_{k=0}^{\infty} \frac{(\epsilon_2^* - \epsilon_1^*)k}{\epsilon_1^* k + \epsilon_2^*(k+1)} \frac{R^{2k+1}}{r^{k+1}} \frac{(k+1)\ldots(k+n)}{D^{k+n+1}} P_k(\cos\theta) \tag{10}$$

This potential can be interpreted as produced by the induced multipoles,

$$\phi_1 = \frac{1}{4\pi\epsilon'_2} \sum_{k=0}^{\infty} \frac{p_2^{(k)}}{r^{k+1}} P_k(\cos\theta) \tag{11}$$

Equating both expressions, we obtain for the induced multipoles

$$p_2^{(k)} = \frac{\epsilon_2^* - \epsilon_1^*}{\epsilon_1^* k + \epsilon_2^*(k+1)} \frac{R^{2k+1}}{D^{k+1}} \sum_{n=0}^{\infty} p_1^{(n)} \frac{k(k+1)\ldots(k+n)}{n! D^n} \tag{12}$$

which coincides with the result used by Jones [5] in the derivation of DEP forces for non-conducting media and particles.

In our case we have two dielectric particles immersed in a uniform field $E_0$. The multipole moments will be the same for both spheres. Then

$$p^{(k)}=\frac{\epsilon_2^*-\epsilon_1^*}{\epsilon_1^*k+\epsilon_2^*(k+1)}\frac{R^{2k+1}}{D^{k+1}}\sum_{n=0}^{\infty}\frac{k(k+1)...(k+n)}{n!\,D^n}p^{(n)}$$

$$+4\pi\epsilon_2'\mathrm{R}^3\frac{\epsilon_1^*-\epsilon_2^*}{\epsilon_1^*+2\epsilon_2^*}\mathrm{E}_0\delta(\mathrm{k}-1) \qquad (13)$$

where $\delta(k-1)$ is a Kronecker delta, giving only a dipolar term for the contribution induced by the uniform field $E_0$.

Equation (13) gives a self consistent condition for the calculation of the successive multipole moments induced on both spheres. The complex character of these moments represents the fact that the $(k)$-polarization of the sphere has in-phase and out-of-phase components with respect to the external field. The calculation can easily be generalised to the case of three or more aligned spheres.

The force acting on the first sphere can now be obtained through the expression for the electric field produced by the second sphere and the external charges. In the case of a uniform electric field, the force on the $n$th multipole of the first sphere is

$$F_n=\frac{\mathrm{Re}(p^{(n)})}{n!}\frac{\partial^n\mathrm{Re}(E_1)}{\partial r^n}\bigg|_{\substack{r=D\\ \theta=0}}=-\frac{\mathrm{Re}(p^{(n)})}{n!}\frac{\partial^{n+1}\mathrm{Re}(\phi_1)}{\partial r^{n+1}}\bigg|_{\substack{r=D\\ \theta=0}} \qquad (14)$$

where Re means the real part of the corresponding complex magnitude. We obtain for the total force

$$F=\sum_{n=0}^{\infty}F_n=\frac{1}{4\pi\epsilon_2'}\sum_{k=0}^{\infty}\frac{|p^{(k)}|}{k!}\sum_{n=0}^{\infty}\frac{(n+k+1)!}{D^{k+n+2}}\cos(\varphi_k-\varphi_n) \qquad (15)$$

in terms of the modules of the multipole moments $p^{(n)}$, $p^{(k)}$ and their phase angles $\varphi_n$, $\varphi_k$ with respect to the external field.

## 3. Results

### *3.1 Medium and particles without losses*

We have studied the interaction between two dielectric particles in a uniform external field $E_0$. Figure 2 shows the dipole moment of one of the spheres as a function of the distance between them, for several relative permittivities of the particles. When the relative permittivity increases, the results tend to those obtained by Jones for conducting particles. For distances greater than twice the diameter, the moments are practically constant and coincide with the dipole moment of an isolated sphere.

In Fig. 3 the force between the spheres is given. The main contribution to this force is the dipole attraction, but the other terms can be important for values of $D/2R$ close to 1. For example, for spheres in contact and $\epsilon_1/\epsilon_2=20$,

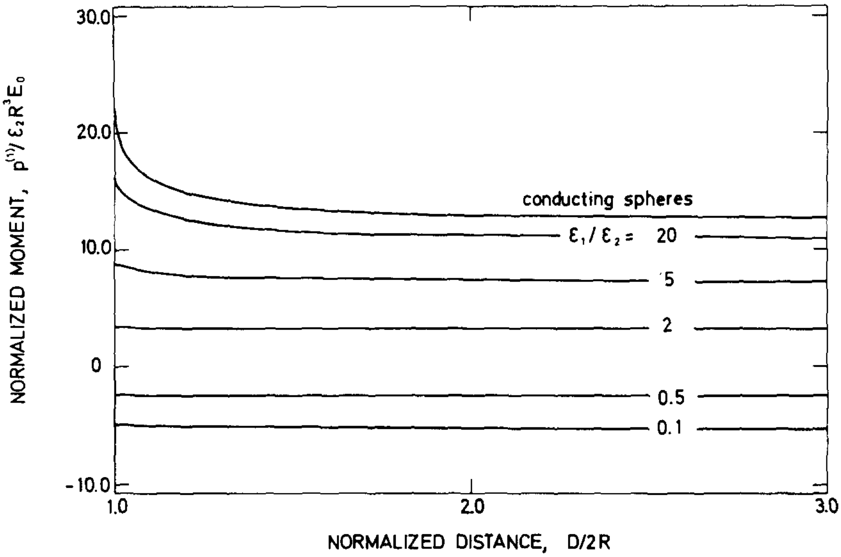


Fig. 2. Normalized dipole moment of one sphere versus normalized distance, for several relative permittivities. The limiting case of conducting particles using the image charges method [1] is also represented.

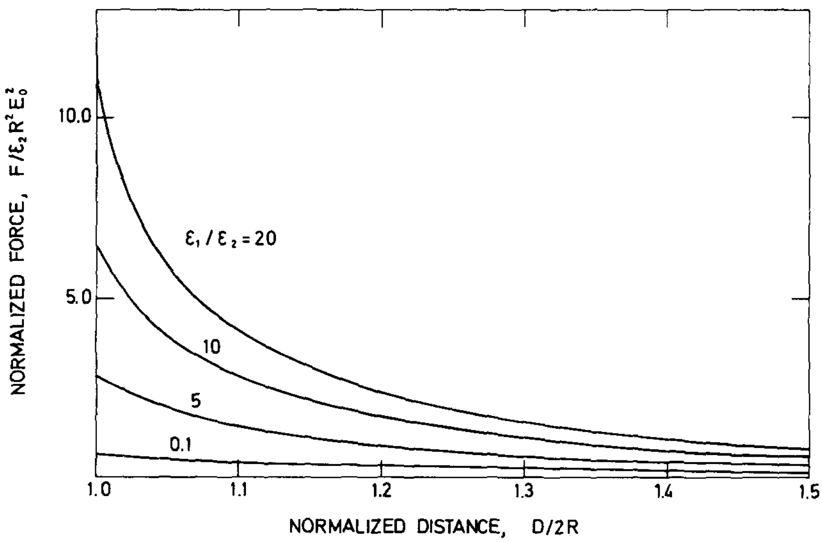


Fig. 3. Normalized attractive force between two dielectric spheres in a uniform field versus normalized distance, for several relative permittivities.

the non-dipolar terms account for 33% of the total force.

The calculations have been performed using up to 10 terms in eqn. (13), the results being practically unaffected by adding more terms in the summation.

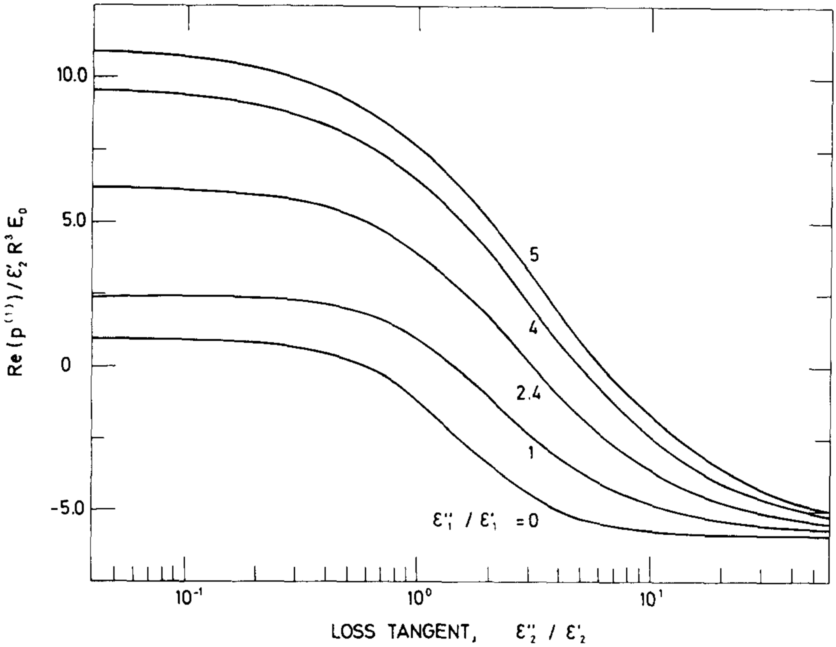


Fig. 4. Normalized real part of the complex moment of one dielectric sphere versus loss tangent of the medium, for $D/2R = 1.2$, $\epsilon_1' = 100$, and several loss tangents of the particles.

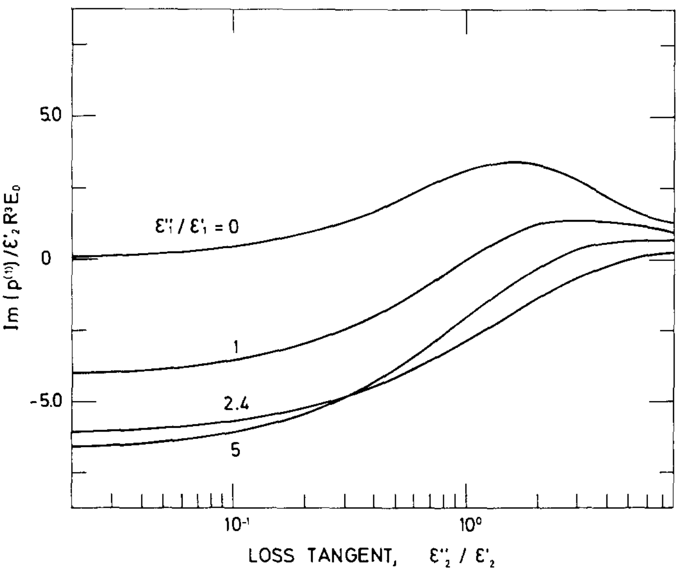


Fig. 5. Normalized imaginary part of the complex moment of one dielectric sphere versus loss tangent of the medium, for $D/2R = 1.2$, $\epsilon_1' = 100$, and several loss tangents of the particles.

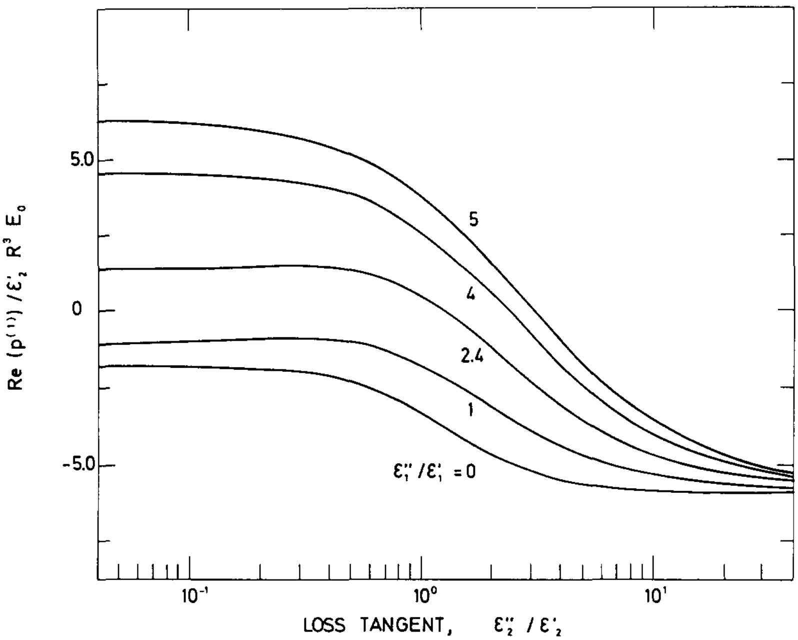


Fig. 6. Normalized real part of the complex moment of one dielectric sphere versus loss tangent of the medium, for $D/2R = 1.2$, $\epsilon_1' = 50$, and several loss tangents of the particles.

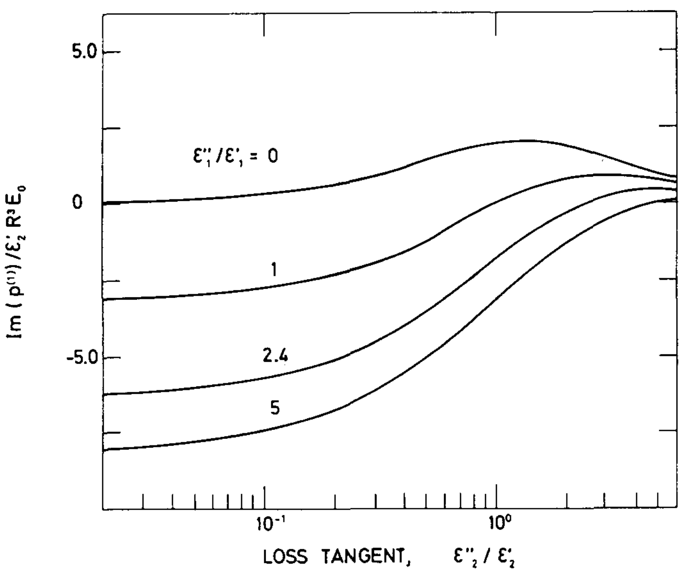


Fig. 7. Normalized imaginary part of the complex moment of one dielectric sphere versus loss tangent of the medium, for $D/2R = 1.2$, $\epsilon_1' = 50$, and several loss tangents of the particles.

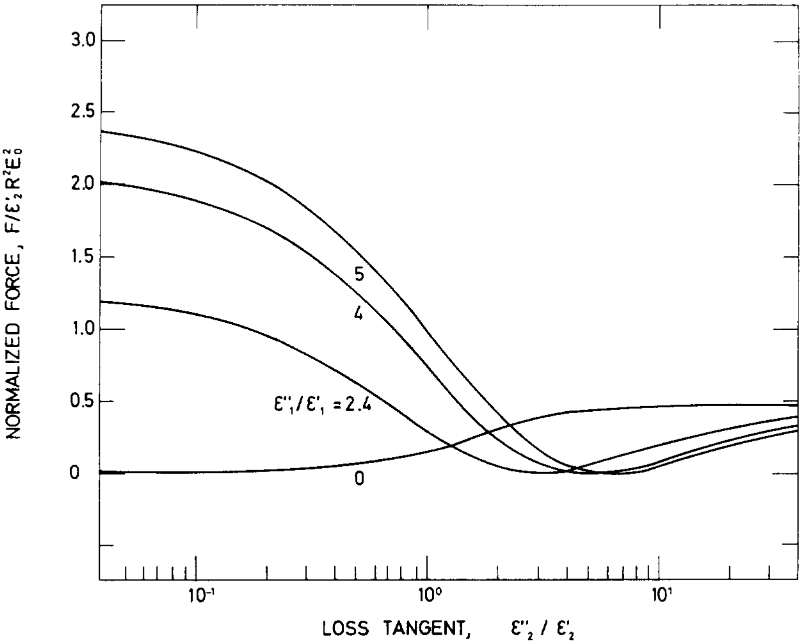


Fig. 8. Normalized attractive force between two dielectric spheres in a uniform field, versus loss tangent of the medium, for $D/2R = 1.2$, $\epsilon_1' = 100$, and several loss tangents of the particles.

### *3.2 Lossy medium and particles*

In this case eqn. (13) gives complex values for the multipole moments. In Figs. 4 and 5, the real and imaginary parts of the first-order moment are represented for a value of $\epsilon_1'$ greater than $\epsilon_2'$ and $D/2R = 1.2$. It can be seen that the real part diminishes and even becomes negative when the medium losses increase. The physical reason for this behaviour is the accumulation of charge on the particle surface produced by the continuity of the current at the boundary.

In Figs. 6 and 7, the corresponding values for a case with $\epsilon_1'$ less than $\epsilon_2'$ are given. Now the real part of the moment, which is negative for $\epsilon_1'' = \epsilon_2'' = 0$ due to the polarization charges, can be affected in magnitude and sign by changes in the losses $\epsilon_1''$ or $\epsilon_2''$.

Finally Figs. 8 and 9 show the force in both cases, $\epsilon_1' > \epsilon_2'$ and $\epsilon_1' < \epsilon_2'$ respectively. The force is always attractive, but a certain combination of the losses can make it zero.

We found that the asymptotic values of the curves in Figs. 4 to 9, for $\epsilon_2''/\epsilon_2' \to \infty$, are constant. Thus $\mathrm{Re}(p^{(1)})/\epsilon_2' R^3 E_0 \to 5.880$, $\mathrm{Im}(p^{(1)})/\epsilon_2' R^3 E_0 \to 0$ and $F/\epsilon_2' R^2 E_0^2 \to 0.475$ in both cases, corresponding to the situation of two interacting particles immersed in a highly conducting medium.

In all the cases we have verified that the results for a distance going to infin-

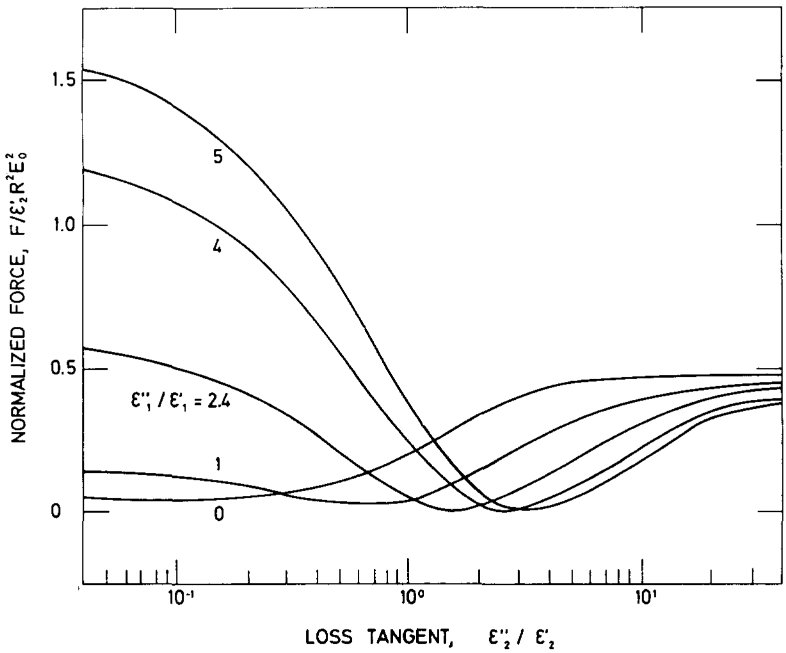


Fig. 9. Normalized attractive force between two dielectric spheres in a uniform field, versus loss tangent of the medium, for $D/2R = 1.2$, $\epsilon_1' = 50$, and several loss tangents of the particles.

ity coincide with the analytical solutions for an isolated sphere in a uniform field.

## 4. Conclusions

The simple method based on eqn. (13) gives the dipole moments and the interaction force between aligned dielectric spheres. It can be applied to the cases of particles with permittivities greater or lower than the surrounding medium and, in particular, may be useful for analyzing gas bubbles levitation experiments.

The method allows to study the influence of dielectric losses on the interaction between dielectric particles. This influence is important in the behaviour of biological cells in DEP experiments. The results show a strong dependence of dipole moments and forces on the losses. Losses in the medium can reduce the interaction force considerably. The calculations could easily be extended to chains of three or more cells as well as to more realistic models of cells using shelled spheres. It would also be desirable to include in the model the double layer effects at the interface.

Coherent dipole interactions between cells have been proposed [6], and several multipole field configurations produced by living cells have been measured using microdielectrophoresis [7]. The representation of the cell as a set of multipoles is very adequate for interpreting this type of cell interactions.

## Acknowledgment

One of the authors (M.S.) is grateful to Prof. Ronald Pethig for his support and encouragement during the first stage of this work.

## References

1 T.B. Jones, Dipole moments of conducting particle chains, J. Appl. Phys., 60 (1986) 2226-2230.
2 F.A. Sauer, Forces on suspended particles in the electromagnetic field, in: H. Frohlich and F. Kremer (Eds.), Coherent Excitations in Biological Systems, Springer Verlag, Berlin, F.R.G., 1983, pp. 134-143.
3 C.J.F. Böttcher, Theory of Electric Polarization, Elsevier, Amsterdam, The Netherlands, 1952, Chap. 4.
4 T.B. Jones and G.A. Kallio, Dielectrophoretic levitation of spheres and shells, J. Electrostatics, 6 (1979) 207-224.
5 T.B. Jones, Dielectrophoretic force in axisymmetric fields, J. Electrostatics, 18 (1986) 55-62.
6 H. Frohlich, Long-range coherence in biological systems, Riv. Nuovo Cimento, 7 (1977) 399-418.
7 H. Rivera, J.K. Pollock and H.A. Pohl, The ac field patterns about living cells, Cell Biophys., 7 (1985) 43-55.