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NOTES ON CRYSTAL OPTICS OF SUPERLATTICES

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Dielectric permeability of superlattices has been calculated with regard to effects of spatial dispersion and two-dimensional nature of excitons. Dispersion of polaritons in the region of exciton resonance is discussed. The expressions have been obtained for the dispersion of surface waves with the inclusion of retardation effects and for different orientations of the surface with respect to the superlattice axis. In the simplest case the expressions have been obtained for the nonlinear (macroscopic) polarizability tensor of the superlattice and the gyrotropy constant.

SUPERLATTICES are of great interest for the study of phenomena which arise at interfaces of media [1]. These artificial layered crystals are systems with "condensed" interfaces since the total area of interfaces in them is proportional to the volume. In these conditions specific surface and quasi-two-dimensional effects must make an important contribution to the bulk crystal optics. Some of them are analyzed in the present paper

1. First we consider the dielectric constant tensor which for superlattices consisting of optically isotoropic layers has the form characteristic for uniaxial crystals

$$\epsilon_{ij} = \epsilon_i \delta_{ij},$$

where $\epsilon_x = \epsilon_y = \epsilon_\perp(\omega)$, $\epsilon_z = \epsilon_\parallel(\omega)$ (the axis z is directed along the optical axis). If a superlattice is formed by alternating layers of thicknesses l_1 and l_2 and, accordingly, with dielectric constants $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$, then the values $\epsilon_\perp$ and $\epsilon_\parallel$ may be calculated quite elementarily. In fact, by definition, the value of the x-component of electric displacement vector averaged over the superlattice period is

$$\bar{D}_x = \frac{1}{l_1 + l_2}(\epsilon_1 E_x^{(1)} l_1 + \epsilon_2 E_x^{(2)} l_2), \tag{1}$$

where $E_x^{(1)}$ and $E_x^{(2)}$ are the x-components of the electric field strength in the layers 1 and 2. Since E_x is continuous at the interface of the layers and its variation inside each layer at $l_{1,2} \ll \lambda$ (λ is the wavelength $= 2\pi/k$) may be neglected, then $E_x^{(1)} = E_x^{(2)} = \bar{E}_x$, where $\bar{E}_x$ is the value of the electric field strength averaged over the superlattice period.

Thus, from (1) it follows that $\bar{D}_x = \epsilon_\perp \bar{E}_x$, where

$$\epsilon_\perp(\omega) = \frac{1}{l_1 + l_2}(\epsilon_1(\omega) l_1 + \epsilon_2(\omega) l_2). \tag{2}$$

Analogously, the average value of E_z is

$$\bar{E}_z = \frac{1}{l_1 + l_2}\left(\frac{D_z^{(1)}}{\epsilon_1} l_1 + \frac{D_z^{(2)}}{\epsilon_2} l_2\right) \tag{3}$$

Since D_z is continuous, then at $k(l_1 + l_2) \ll 1$ we have $D_z^{(1)} = D_z^{(2)} = \bar{D}_z$, so that $\bar{E}_z = \epsilon_\parallel^{-1} \bar{D}_z$, where

$$\epsilon_\parallel^{-1}(\omega) = \frac{1}{l_1 + l_2}\left(\frac{l_1}{\epsilon_1(\omega)} + \frac{l_2}{\epsilon_2(\omega)}\right). \tag{4}$$

Up to now we have not been taking into account the presence of transition regions near the boundaries of the layers, but their inclusion is equivalent to that of surface currents.

$$D_x = \epsilon(z) E_x + \nu \sum_n \delta(z - z_n) E_x \tag{5}$$

$$E_z = \epsilon^{-1}(z) D_z + \mu \sum_n \delta(z - z_n) D_z, \tag{6}$$

where $\epsilon(z) = \epsilon_1$ or ϵ_2, if z belongs, respectively, to the first or the second layer; μ, ν are the constants which characterize the transition regions near the interfaces.

With due regard to surface currents in the expressions (1) and (3) for $\bar{D} = 1/(l_1 + l_2)\int D(z)\mathrm{d}z$ and $\bar{E} = 1/(l_1 + l_2)\int E(z)\mathrm{d}z$ there appear contributions of δ-functions so that

$$\epsilon_\perp(\omega) = \frac{1}{l_1 + l_2}(\epsilon_1 l_1 + \epsilon_2 l_2 + 2\nu) \tag{7}$$

$$\epsilon_\parallel^{-1}(\omega) = \frac{1}{l_1 + l_2}\left(\frac{l_1}{\epsilon_1} + \frac{l_2}{\epsilon_2} + 2\mu\right). \tag{8}$$

The values μ and ν, the same as ϵ_1, ϵ_2, are functions of frequency They may have poles at the frequencies of surface excitations, which do not coincide with the frequencies of bulk excitations that manifest themselves as the peculiarities in $\epsilon_{1,2}(\omega)$ and $\epsilon_{1,2}^{-1}(\omega)$ In particular, $\mu(\omega)$ must have a pole at the frequency of plasma oscillations in the conducting regions which arise near the interfaces of two semiconductors due to band-bending.

If one knows the values of $\epsilon_\perp(\omega)$ and $\epsilon_\parallel(\omega)$ the dispersion of normal optical bulk and surface waves in superlattices may be determined in the conventional way Thus, e.g the dispersion of the refraction index of the extraordinary wave is determined by the relation

$$\frac{\cos^2\theta}{\epsilon_\perp(\omega)} + \frac{\sin^2\theta}{\epsilon_\parallel(\omega)} = \frac{1}{n^2}, \tag{9}$$

where θ is the wave vector angle relative to the optical axis. From (9) it follows that the resonances of the value $n(\omega)$ correspond to the frequencies which satisfy the equation

$$\epsilon_\parallel(\omega)\cos^2\theta + \epsilon_\perp(\omega)\sin^2\theta = 0 \tag{10}$$

This equation determines the frequencies $\omega(k)$ of bulk waves with no regard for retardation. One can easily see that $\omega(k)$ depends on the ratio k_x^2/k_z^2 and has no definite limit at $k^2 = k_x^2 + k_z^2 \to 0$. The dispersion law obtained from equation (10) coincides (at $\mu = \nu = 0$; $l_{1,2}k \ll 1$) with the frequencies of the Coulomb problem found in [2, 3] upon the neglect of the effects of transition regions. In these works surface waves were discussed which may propagate along the boundary of the superlattice perpendicular to the optical axis. With the values of $\epsilon_\parallel$ and $\epsilon_\perp$ in hand one may easily find the spectrum of these surface waves also with due regard to retardation [4]

$$k^2 = \frac{\omega^2}{c^2}\frac{(\epsilon_\perp(\omega)-1)\epsilon_\parallel(\omega)}{\epsilon_\parallel(\omega)\epsilon_\perp(\omega)-1} \tag{11}$$

Within the limit $k \gg \omega/c$ the frequency of a surface wave is determined by the equation

$$\epsilon_\parallel(\omega)\epsilon_\perp(\omega) = 1 \tag{12}$$

which at $k(l_1+l_2) \ll 1$ coincides with that obtained in [2, 3] It is not difficult to obtain the dispersion laws for long-wave surface polaritons for any orientation of the superlattice boundary with respect to its optical axis [4] In particular, if this boundary is parallel to the optical axis, then, instead of (11), we have a relation

$$k^2 = \frac{\omega^2}{c^2}\frac{(\epsilon_\parallel(\omega)-1)\epsilon_\perp(\omega)}{\epsilon_\parallel(\omega)\epsilon_\perp(\omega)-1} \tag{13}$$

for $\mathbf{k}_\parallel$ to the optical axis and

$$k^2 = \frac{\omega^2}{c^2}\frac{\epsilon_\perp(\omega)}{\epsilon_\perp(\omega)+1} \tag{14}$$

for $\mathbf{k}_\perp$ to the optical axis

2. Up to now we assumed that for each layer of a superlattice the approximation of the local macroscopic electrodynamics is valid. This approximation is justified far from excitonic resonances, if the layer thicknesses l_1 and l_2 are far in excess of the characteristic lengths where the system response is formed

In the vicinity of excitonic resonances even at $l_{1,2} \gg a_0$ (a_0 is the Bohr radius of an exciton) the nonlocality of the dielectric permeability must be taken into account Consider now the frequency region near the excitonic resonance in the layer 1 We assume that in this frequency region the dielectric constant of the layer 2 is local $\epsilon_2 = \epsilon_2(\omega)$, but we shall make allowance of the nonlocality of the response in the layer 1

Then, instead of (1) and (3) one should write, respectively

$$\bar{D}_x = \frac{1}{l_1+l_2}\left\{(\epsilon_2 l_2 + \epsilon_\infty l_1)E_x + 4\pi\int_0^{l_1} P_x(z)\mathrm{d}z\right\} \tag{15}$$

$$\bar{E}_x = \frac{1}{l_1+l_2}\left\{\left(\frac{l_2}{\epsilon_2}+\frac{l_1}{\epsilon_\infty}\right)D_z - \frac{4\pi}{\epsilon_\infty}\int_0^{l_1} P_z(z)\mathrm{d}z\right\}, \tag{16}$$

where $\mathbf{P}(z)$ is the vector of the excitonic polarization in the layer 1, ϵ_∞ is the dielectric constant of the layer 1 far from the excitonic resonance In the effective mass approximation "the equation of motion" for the excitonic polarization $\mathbf{P}(z)\exp(ikx - i\omega t)$ may be written as [4]

$$\frac{\mathrm{d}^2P_x}{\mathrm{d}z^2} - Q_\perp^2 P_x = -\frac{q_\perp^2}{4\pi}\epsilon_\infty E_x \tag{17}$$

$$\frac{\mathrm{d}^2P_z}{\mathrm{d}z^2} - Q_\parallel^2 P_z = -\frac{q_\perp^2}{4\pi}D_z \tag{18}$$

where

$$Q_{\parallel(\perp)}^2 = \frac{m^*}{\hbar\omega_\perp}(\omega_{\parallel(\perp)}^2 - \omega^2) + k^2$$

$$q_\perp^2 = \frac{m^*}{\hbar\omega_\perp}(\omega_\parallel^2 - \omega_\perp^2)$$

m^* is the efficient mass of an exciton, $\omega_{\parallel(\perp)}$ are the frequencies of longitudinal and transverse excitons

As it has already been noted, at $l_1 \ll \lambda$ one may assume that E_x and D_z are independent of z inside of the layer 1. With regard to this assumption the solutions of the equations (17) and (18) have the form

$$P_x = A_+ e^{Q_\perp z} + A_- e^{-Q_\perp z} + \frac{q_\perp^2 \epsilon_\infty}{4\pi Q_\perp^2} E_x \tag{19}$$

$$P_z = B_+ e^{Q_\parallel z} + B_- e^{-Q_\parallel z} + \frac{q_\perp^2}{4\pi Q_\parallel^2} D_z \tag{20}$$

The constants $A_\pm$ and $B_\pm$ must be found from the boundary conditions at $z = 0$ and $z = l_1$ These so-called additional boundary conditions are determined by the value of the surface energy connected with the excitonic polarization. If we write this energy in the form

$$W_s = \alpha g \int P^2 \mathrm{d}S \tag{21}$$

and include also the term which leads to nonlocality of the equations of motion

$$W = g \int |\nabla P|^2 \mathrm{d}V \tag{22}$$

then the variation of (21) and (22) gives, in addition to the nonlocal terms in the equation of motion, the following boundary conditions·

$$\frac{\partial P}{\partial n} + \alpha P = 0. \tag{23}$$

Determining the constants $A_\pm$, $B_\pm$ with the use of (23) and substituting (19) and (20) into (15) and (16) we obtain

$$\epsilon_\perp = \frac{1}{l_1 + l_2}\left\{\epsilon_2 l_2 + \epsilon_1(\omega, k) l_1 - \frac{2\alpha q_\perp^2 \epsilon_\infty}{Q_\perp^3 (Q_\perp + \alpha \coth(\frac{1}{2} Q_\perp l_1))}\right\} \tag{24}$$

$$\epsilon_\parallel^{-1} = \frac{1}{l_1 + l_2}\left\{\frac{l_2}{\epsilon_2} + \frac{l_1}{\epsilon_1(\omega, k)} + \frac{2\alpha q_\perp^2}{\epsilon_\infty Q_\parallel^3 (Q_\parallel + \alpha \coth(\frac{1}{2} Q_\parallel l_1))}\right\}, \tag{25}$$

where the nonlocal dielectric constant in the layer 1 $\epsilon_1(\omega, k)$ is determined by the expression

$$\epsilon_1(\omega, k) = \epsilon_\infty \frac{\omega_\parallel^2 - \omega^2 + \hbar\omega_\perp \dfrac{k^2}{m^*}}{\omega_\perp^2 - \omega^2 + \hbar\omega_\perp \dfrac{k^2}{m^*}} = \epsilon_\infty \frac{Q_\parallel^2}{Q_\perp^2}$$

First of all it should be noted that at $\alpha \neq 0$ the resonances in $\epsilon_\perp$ and $\epsilon_\parallel^{-1}$ which correspond to $Q_\perp = 0$ and $Q_\parallel = 0$ disappear since the pole terms $\sim Q_{\perp(\parallel)}^{-2}$ in (24) and (25) are cancelled. Thus, the response nonlocality in the layer 1 leads to a qualitative change in the behaviour of $\epsilon_\perp(\omega)$ and $\epsilon_\parallel(\omega)$. In the case of local dielectric constants $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ the poles of $\epsilon_\perp(\omega)$ and $\epsilon_\parallel^{-1}(\omega)$ coincided with the poles of $\epsilon_{1,2}(\omega)$ and $\epsilon_{1,2}^{-1}(\omega)$ [see (2) and (4)] When the spatial dispersion in the layer 1 is taken into account, the poles of $\epsilon_\perp(\omega)$ and $\epsilon_\parallel^{-1}(\omega)$ appear at the frequencies which satisfy the equations

$$Q_{\perp(\parallel)}(\omega, k) + \alpha \coth(\tfrac{1}{2} Q_{\perp(\parallel)}(\omega, k) l_1) = 0 \tag{26}$$

and not at the frequencies $\omega(k) = \omega_{\perp(\parallel)} + \hbar k^2/2m^*$ at which there is a pole in ϵ_1 and ϵ_1^{-1}.

It may easily be shown that these frequencies correspond to even eigenmodes of polarization in the layer 1 for which $P(l_1/2 + z) = P(l_1/2 - z)$ Odd modes whose frequencies satisfy the equation

$$Q_{\perp(\parallel)}(\omega, k) + \alpha \tanh(\tfrac{1}{2} Q_{\perp(\parallel)}(\omega, k) l_1) = 0 \tag{26'}$$

are not excited by the field homogeneous in the direction z since the average dipole moment for these modes equals to zero. However, with regard to variation of the fields E_x and D_z along the axis z odd modes are also excited. This case will be briefly exemplified below by calculation of $\epsilon_\perp$ Strictly speaking the fields $E_x(z)$ and $D_x(z)$ have the form of Bloch waves:

$$E_x = e^{iqz}\mathscr{E}(z); \qquad D_x(z) = e^{iqz}\mathscr{D}(z)$$

where $\mathscr{E}(z)$ is the continuous and $\mathscr{D}(z)$ is the discontinuous periodic function. At $q(l_1 + l_2) \ll 1$ the notion of the average (over the superlattice period) field may be introduced and one may assume $\mathscr{E}(z) = \bar{\mathscr{E}}$.

$$\bar{E}_x(z) = e^{iqz}\bar{\mathscr{E}}$$
$$\bar{D}_x(z) = e^{iqz}\bar{\mathscr{D}}.$$

Then the nonlocal dielectric constant $\epsilon_\perp(\omega, k, q)$ is determined as·

$$\bar{\mathscr{D}} = \epsilon_\perp(\omega, k, q)\bar{\mathscr{E}}.$$

Solving the equation (17) with $E_x = e^{iqz}\bar{\mathscr{E}}$ in the right-hand side and calculating the value $\bar{\mathscr{D}}$ one may obtain·

$$\epsilon_\perp(\omega, k, q) = \frac{1}{l_1 + l_2}\left\{\epsilon_2 l_2 + \epsilon_1(\omega, \sqrt{k^2 + q^2})\, l_1 - \frac{4\epsilon_\infty q_\perp^2 [Q_\perp(q^2 - \alpha^2)(\cos(q l_1) - \cosh(Q_\perp l_1)) + \alpha(Q_\perp^2 - q^2)\sinh(Q_\perp l_1) + 2\alpha q Q_\perp \sin(q l_1)]}{(Q_\perp^2 + q^2)^2[(Q_\perp + \alpha)^2 e^{Q_\perp l_1} - (Q_\perp - \alpha)^2 e^{-Q_\perp l_1}]}\right\}$$

The same as at $q = 0$ it may easily be verified that the poles $\epsilon_\perp$ which correspond to $Q_\perp^2 + q^2 = 0$ are cancelled out. There remain only the poles which satisfy the equation (26) and (26′) (see also [6, 7]). The solutions of these equations at purely imaginary Q describe the effects of dimensional quantization and exist at any sign of α. They may be represented in the form

$$\omega_n(k) = \omega_{\parallel(\perp)} + \frac{\hbar k^2}{2m^*} + 2y_n^2 \frac{\hbar}{m^* l_1^2} \tag{27}$$

where the value y_n satisfies the equations

$$y = \frac{\alpha l_1}{2} \cot y; \qquad y = -\frac{\alpha l_1}{2} \tan y. \tag{28}$$

At $\alpha < 0$, which corresponds to the negative surface energy (21), there exists a solution of (26′), (26′) at which Q is real It describes the local surface state with the frequency

$$\omega_{\mathrm{LOC}}(k) = \omega_{\parallel(\perp)} + \frac{\hbar k^2}{2m^*} - x^2 \frac{\hbar\alpha^2}{2m^*} \tag{29}$$

where the value x satisfies the equations

$$x = -\coth(\tfrac{1}{2}\alpha l_1 x); \qquad x = -\tanh(\tfrac{1}{2}\alpha l_1 x) \tag{30}$$

One can easily see that at $|\alpha l_1| \gg 1$ the frequency shift $\Delta\omega = -x^2\,\hbar\alpha^2/2m^* = -\hbar\alpha^2/2m^*$ is independent of the thickness of the layer l_1, as it must be for the local state. On the contrary, the solutions of the equations (28) in this limiting case have the form

$$y_n = \frac{\pi}{2} n, \qquad n = 1, 2, \ldots \tag{31}$$

As it has already been noted, the poles which correspond to even n (the odd modes) appear only at $q \neq 0$, so that the peculiarities at these frequencies must be appreciably weaker than at the frequencies corresponding to odd n (the even modes)

Note that the resonance frequencies ω_n (27) are independent on the parallel to the axis z component q of the wave vector **K** This leads to the fact that in the region of the excitonic resonance the dispersion law of ordinary polaritons which is found from the equation

$$K^2 = \frac{\omega^2}{c^2}\,\epsilon_\perp(\omega, K\sin\theta, K\cos\theta) \tag{32}$$

depends on the angle θ between the wave vector **K** and the axis z and differs strongly from the dispersion law in the medium with the dielectric constant $\epsilon_1(\omega, K)$ even at $l_1 \gg l_2$ This difference is connected with the dimensional quantization in the layer 1 which is governed by the boundary conditions (23) Thus, we see again the qualitative role played by the interface.

3. Consider now the inverse limiting case $l_1 \ll a_0 = \epsilon_\infty \hbar^2/m^* e^2$ In this case the excitons in the layer 1 are two-dimensional and the excitonic polarization corresponding to the lowest level of the dimensional quantization has the form [5]

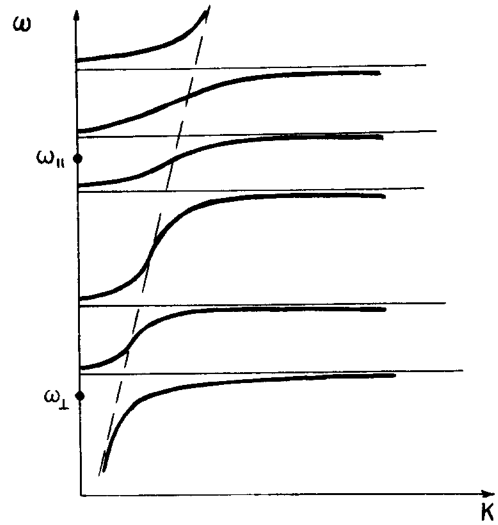

Fig 1. Dispersion law for ordinary polaritons at $\theta = 0$. Horizontal lines correspond to the frequencies of dimensional quantization. There must also exist the Bragg gaps at $\omega = \omega(\mathbf{G}/2)$, where $\mathbf{G}$ – reciprocal lattice vectors (not depicted in the figure).

$$P_i = l_1 \chi_{ij}(\omega, k)\varphi(z) \int_0^{l_1} E_j^{(1)}(z)\varphi(z)\,\mathrm{d}z, \tag{33}$$

where the function $\varphi(z)$ describes the dimensionally quantized transverse – with respect to the film – motion of electrons and holes·

$$\varphi(z) = \frac{2}{l_1} \sin^2\left(\frac{\pi z}{l_1}\right) \tag{34}$$

The susceptibility χ_{ij} for optically isotropic layers includes only two different values $\chi_{xx} = \chi_{yy} = \chi_\perp$ and $\chi_{zz} = \chi_\parallel$ which in the simplest models differ by the factor ~ 1 and are proportional to the value [5]

$$\frac{\epsilon_\infty f}{\omega^2(k) - \omega^2}, \qquad f = \left(\frac{e^2}{\hbar l_1 \epsilon_\infty}\right)^2 \tag{35}$$

where $\hbar\omega(k)$ is the resonance energy of the excitonic transition. Since by definition (34)

$$\int_0^{l_1} \varphi(z)\,\mathrm{d}z = 1 \tag{36}$$

and E_x at $ql_1 \ll 1$ is weakly dependent on z, it follows from (33) and (36)·

$$\int_0^{l_1} P_x(z)\,\mathrm{d}z = l_1 \chi_\perp E_x. \tag{37}$$

Substitution of (37) into (15) gives

$$\epsilon_\perp = \frac{1}{l_1 + l_2}\{\epsilon_2 l_2 + (\epsilon_\infty + 4\pi\chi_\perp(\omega, k))l_1\}. \tag{38}$$

This expression coincides formally with (2), if instead of $\epsilon_1(\omega)$ we assume

$$\epsilon_\infty\left(1 + \frac{4\pi f_\perp}{\omega^2(k) - \omega^2}\right) \tag{39}$$

However, at $l_1 \ll a_0$ the strength of the oscillator $f_\perp$ is determined by the formula (35) and increases with decreasing l_1.

Now we calculate $\epsilon_\parallel$ at $l_1 \ll a_0$. From (33) it follows that

$$D_z = \epsilon_\infty E_z^{(1)} + 4\pi l_1 \chi_\parallel \varphi(z) \int_0^{l_1} E_z^{(1)}\varphi(z)\mathrm{d}z. \tag{40}$$

Multiplying (40) by $\varphi(z)$ and integrating it from 0 to l_1 by z with regard to (36) and also $D_z = \bar{D}_z$ we have.

$$\int_0^{l_1} E_z^{(1)}\varphi(z)\mathrm{d}z = \frac{D_z}{\epsilon_\infty + 4\pi l_1 \chi_\parallel \bar{\varphi}^2} \tag{41}$$

where

$$\bar{\varphi}^2 = \int_0^{l_1} \varphi^2(z)\mathrm{d}z = \frac{3}{2l_1}.$$

Substituting (41) into (33) and (16) with regard to (35) one may easily obtain:

$$\epsilon_\parallel^{-1} = \frac{1}{l_1 + l_2}\left\{\frac{l_2}{\epsilon_2} + \frac{l_1}{\epsilon_\infty}\,\frac{\omega_T^2 - \omega^2}{\omega_L^2 - \omega^2}\right\} \tag{42}$$

where

$$\omega_T^2 = \omega^2(k) + 2\pi f_\parallel$$

$$\omega_L^2 = \omega^2(k) + 6\pi f_\parallel \tag{43}$$

The expression (42) formally coincides with (4) if instead of we take

$$\epsilon_\infty \frac{\omega_L^2 - \omega^2}{\omega_T^2 - \omega^2}. \tag{44}$$

In this case the value of the "longitudinal-transverse" splitting is equal to

$$\omega_L^2 - \omega_T^2 = 4\pi f_\parallel \sim \frac{4\pi}{\hbar^2}\left(\frac{e^2}{l_1\epsilon_\infty}\right)^2$$

and increases with decreasing l_1.

In conclusion it should be noted that though the expressions (38) and (42) may formally be reduced to the form (2) and (4), it can be achieved with the use of two different functions $\epsilon_1(\omega)$ determined by the expressions (39) and (44), whereas in (2) and (4) only one function was used. This should be kept in mind when processing the experiments under conditions when $l_1 \ll a_0$.

4. Consider now one of the examples which shows how a tensor of nonlinear polarizability χ_{ijl} is formed in a superlattice. Suppose this tensor, the same as in cubic crystals of the group Td (GaP a.o.), takes the form

$$\chi_{ijl} = \chi|e_{ijl}|,$$

where e_{ijl} is the totally antisymmetric tensor of the third rank. Consequently,

$$D_x^{(1,2)} = \epsilon_{1,2}E_x^{(1,2)} + 4\pi\chi^{(1,2)}E_y^{(1,2)}E_z^{(1,2)} \tag{45}$$

$$D_z^{(1,2)} = \epsilon_{1,2}E_z^{(1,2)} + 4\pi\chi^{(1,2)}E_x^{(1,2)}E_y^{(1,2)} \tag{46}$$

so that the average (over the superlattice period) value is

$$\bar{D}_x = \epsilon_\perp \bar{E}_x + 4\pi\chi_{123}\bar{E}_y\bar{E}_z$$

where

$$\chi_{123} = \left(\frac{l_1\chi^{(1)}}{\epsilon_1} + \frac{l_2\chi^{(2)}}{\epsilon_2}\right)\frac{\epsilon_\parallel}{l_1 + l_2}. \tag{47}$$

An analogous relation may easily be obtained also for $\bar{D}_y$ which leads to $\chi_{213} = \chi_{123}$. However, the expression for χ_{312} is quite different. One can easily show that

$$\bar{D}_z = \epsilon_\parallel \bar{E}_z + 4\pi\chi_{312}\bar{E}_x\bar{E}_y$$

where

$$\chi_{312} = \frac{1}{l_1 + l_2}\{l_1\chi^{(1)} + l_2\chi^{(2)}\} \neq \chi_{123}. \tag{48}$$

The difference between χ_{312} and χ_{123} is especially noticable near the frequencies when $\epsilon_\parallel^{-1} \approx 0$, i.e. when

$$\frac{l_1}{\epsilon_1} \approx -\frac{l_2}{\epsilon_2}.$$

In this case χ_{123} has the pole whereas χ_{321} has no peculiarity.

5. In the last section of this paper we shall consider briefly the calculation of the superlattice linear response with regard to gyrotropy. The same as before we assume that the optical properties inside each of the superlattice layers are isotropic. Therefore, for each of the layers with regard to gyrotropy we have [4].

$$\mathbf{D}^{(1,2)} = \epsilon_{1,2}E^{(1,2)} + \gamma^{(1,2)}\operatorname{rot}E^{(1,2)} + \tfrac{1}{2}[\nabla\gamma^{(1,2)} \times \mathbf{E}^{(1,2)}], \tag{49}$$

where $\gamma^{(1,2)}$ is the gyrotropy constant. The terms with $\nabla\gamma^{(1,2)}$ differ from zero only near the interfaces of the layers. The multiplier $\frac{1}{2}$ in these terms provides the fulfilment of the symmetry principle for kinetic coefficients. Since $\operatorname{rot} E \propto H$ is continuous at the boundaries of the layers, and $\nabla\gamma$ is directed along the axis z and, therefore, E_z does not contribute to the last term in (49) one obtains

$$D = \hat{\epsilon}E + \gamma \operatorname{rot} E + \tfrac{1}{2}[\nabla\gamma \times E]$$

where

$$\gamma = \frac{1}{l_1 + l_2}(l_1\gamma^{(1)} + l_2\gamma^{(2)}) \qquad (50)$$

The last term differs from zero only at the superlattice boundary.

In conclusion one may say that the linear crystal optics of superlattices bears rich information on the dynamics of interfaces. In particular, by studying propagation of polaritons in superlattices it is possible to determine the frequencies of surface excitations, which appear at interfaces of layers, and their dependence on outer parameters (temperature, strength of the applied electric and magnetic fields, etc.). Moreover, such investigations may give an idea of the nature of interaction of bulk excitations (excitons) with interfaces which manifests itself in additional boundary conditions (ABC) and determines the value of the constant α. Finally, the study of dispersion laws for polaritons in superlattices with small thicknesses of layers permits one to follow variations in the properties of excitons at their two-dimensionalization.

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