# Angle-suppressed scattering and optical forces on submicrometer dielectric particles

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We show that submicrometer silicon spheres, whose polarizabilities are completely given by their two first Mie coefficients, are an excellent laboratory to test effects of both angle-suppressed and resonant differential scattering cross sections. Specifically, outstanding scattering angular distributions, with zero forward- or backward-scattered intensity, (i.e., the so-called Kerker conditions), previously discussed for hypothetical magnetodielectric particles, are now observed for those Si objects in the near infrared. Interesting new consequences for the corresponding optical forces are derived from the interplay, both in and out of resonance, between the electric- and magnetic-induced dipoles. 

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## 1. INTRODUCTION

The scattering properties of small particles having special electromagnetic properties has long been a topic of theoretical interest [1,2]. Even in the simplest case of small or of dipolar scatterers, remarkable scattering effects of magnetodielectric particles were theoretically established by Kerker *et al.* [3] concerning suppression or minimization of either forward or backward scattering. Nevertheless, no concrete example of such particles that might present those interesting scattering properties in the visible or infrared regions has been proposed yet. Intriguing applications in scattering cancellation and cloaking [4,5] have renewed interest in the field [6].

The interplay between electric and magnetic properties is a key ingredient of determining the scattering characteristics of small objects. It also has an important role in the study of magneto-optical systems [7–10] or in the quest for magnetic plasmons [11]. The unavoidable problems of losses and saturation effects inherent to metamaterials in the optical and near infrared regimes have stimulated the study of high-permittivity particles as their constitutive elements [12–18] with unique electromagnetic properties and antennas based on dielectric resonators [19–21]. Regarding radiation pressure, Ashkin and Dziedzic [22] were the first to observe the effect of both their electric and magnetic resonances, which was theoretically analyzed by Chylek *et al.* [23] also in connection with higher-order Mie coefficients. The first-order resonances were subsequently theoretically studied [24].

In this work, we first address small dielectric particles, described by the first-order Mie coefficients, regarding scattering properties similar to those reported for magnetodielectric spheres [3,25]. Second, we analyze how those scattering effects affect the radiation pressure exerted by the electromagnetic field on such particles [26–34]. This is relevant in the study of light-induced interactions [35–38] and dynamics [39–43] of particles trapped or moved by light, topics with an increasing number of applications.

Only recently, a theory of optical forces on small magnetodielectric particles has been developed [44,45]. This includes pure dielectric particles, which can be well described by their two first electric and magnetic Mie coefficients [45], but again, no concrete particles were addressed. However, in a later work, we have shown that silicon spheres present dipolar magnetic and electric responses, characterized by their respective first-order Mie coefficient, in the near infrared [46], in such a way that either of them can be selected by choosing the illumination wavelength. In the present work, we show that they constitute such a previously quested real example of dipolar particle with either electric and/or magnetic response, of consequences both for their emitted intensity and behavior under electromagnetic forces.

This paper is organized as follows. In Section 2, we discuss the scattering cross-section properties of a magnetodielectric particle, and we propose a generalization of the so-called second Kerker condition [3]. Then, we introduce the real instance of a small Si sphere that illustrates these characteristics. It should be stressed that, as far as we know, this is the first concrete example of such a kind of dipolar magnetodielectric particle, from whose resonances one can observe consequences on both the scattering cross section and the optical forces at different wavelengths in the near infrared. In Section 3, we address the optical force on such particles from an incident plane wave. We obtain an expression for this force in terms of the differential scattering cross sections and discuss the consequences, depending on the polarizabilities. In particular, we study the conditions for a minimum force, as well as the resulting force when the first and generalized second Kerker conditions hold. Then, we illustrate these forces with a small Si sphere. The results indicate that this particle may suffer enhanced radiation pressure, which is mainly due to the resonant induction of its magnetic dipole.

## 2. SCATTERING CROSS SECTIONS: KERKER CONDITIONS

Let us consider a small sphere of radius $a$ immersed in an arbitrary lossless medium with relative dielectric permittivity $\epsilon$ and magnetic permeability $\mu$. Under illumination by an external field of frequency $\omega$, $\mathbf{E} = \mathbf{E}^{(i)}(\mathbf{r})e^{-i\omega t}$, $\mathbf{B} = \mathbf{B}^{(i)}(\mathbf{r})e^{-i\omega t}$, the induced electric and magnetic dipoles $\mathbf{p}$ and $\mathbf{m}$ are written in terms of the electric and magnetic complex polarizabilities $\alpha_e$ and $\alpha_m$ as $\mathbf{p} = \alpha_e \mathbf{E}^{(i)}$ and $\mathbf{m} = \alpha_m \mathbf{B}^{(i)}$. For a small sphere, with constitutive parameters $\epsilon_p$ and $\mu_p$, the dynamic polarizabilities are expressed in terms of the Mie coefficients $a_1$ and $b_1$ as [1] $\alpha_e = 3i\epsilon a_1/(2k^3)$ and $\alpha_m = 3ib_1/(2\mu k^3)$ ($k$ is the wavenumber: $k = \sqrt{\epsilon\mu}\omega/c$), which may be written in the form [45]

$$\alpha_e = \frac{\alpha_e^{(0)}}{1 - i\frac{2}{3\epsilon}k^3\alpha_e^{(0)}}, \qquad \alpha_m = \frac{\alpha_m^{(0)}}{1 - i\frac{2}{3}\mu k^3\alpha_m^{(0)}}. \tag{1}$$

In Eq. (1) $\alpha_e^{(0)}$ and $\alpha_m^{(0)}$ are static polarizabilities. The particle extinction, $\sigma^{(\mathrm{ext})}$, absorption, $\sigma^{(a)}$ and scattering, $\sigma^{(s)}$, cross sections are written in terms of the polarizabilities as

$$\sigma^{(\mathrm{ext})} = 4\pi k \Im\{\epsilon^{-1}\alpha_e + \mu\alpha_m\}, \tag{2}$$

$$\sigma^{(s)} = \frac{8\pi}{3}k^4\{|\epsilon^{-1}\alpha_e|^2 + |\mu\alpha_m|^2\}. \tag{3}$$

The symbol $\Im$ means imaginary part. Energy conservation, i.e., the so-called "optical theorem," Eq. (2), imposes $\sigma^{(\mathrm{ext})} = \sigma^{(s)} + \sigma^{(a)}$.

In terms of the static polarizabilities, the absorption cross section is written as

$$\sigma^{(a)} = 4\pi k[(\epsilon A)^{-1}\Im\alpha_e^{(0)} + \mu B^{-1}\Im\alpha_m^{(0)}],$$
$$A = \left|1 - i\frac{2}{3\epsilon}k^3\alpha_e^{(0)}\right|^2, \qquad B = \left|1 - i\frac{2}{3}\mu k^3\alpha_m^{(0)}\right|^2. \tag{4}$$

In absence of magnetic response, i.e., for an induced pure electric dipole (PED), the far-field radiation pattern is given by the differential scattering cross section, which, averaged over incident polarizations, is [47]

$$\frac{d\sigma_{\mathrm{PED}}^{(s)}}{d\Omega}(\theta) = \frac{k^4}{2}|\epsilon^{-1}\alpha_e|^2(1 + \cos^2\theta), \tag{5}$$

being symmetrically distributed between forward and backward scattering. However, when we consider the contribution of both the electric and magnetic induced dipoles, we obtain [45,47]

$$\frac{d\sigma^{(s)}}{d\Omega}(\theta) = \frac{k^4}{2}(|\epsilon^{-1}\alpha_e|^2 + |\mu\alpha_m|^2)(1 + \cos^2\theta) + 2k^4\frac{\mu}{\epsilon}\Re(\alpha_e\alpha_m^*)\cos\theta, \tag{6}$$

which is mainly distributed in the forward or backward region according to whether $\Re(\alpha_e\alpha_m^*)$ is positive or negative, respectively. The symbol $\Re$ means real part. In particular, the forward ($\theta = 0°$; "+") and backward ($\theta = 180°$; "–") directions, the intensities are simply given by

$$\frac{d\sigma^{(s)}}{d\Omega}(\pm) = k^4|\epsilon^{-1}\alpha_e \pm \mu\alpha_m|^2. \tag{7}$$

This asymmetry, arising from the interference between the electric and magnetic dipolar fields, leads to a number of interesting effects.

i. The intensity in the backscattering direction can be exactly zero whenever

$$\epsilon^{-1}\alpha_e = \mu\alpha_m \Rightarrow \frac{d\sigma^{(s)}}{d\Omega}(180°) = 0. \tag{8}$$

This anomaly corresponds to the so-called *first Kerker condition* [3], theoretically predicted for magnetodielectric particles having $\epsilon_p = \mu_p$.

ii. Although the intensity cannot be zero in the forward direction, (causality imposes $\Im\{\alpha_e\}, \Im\{\alpha_m\} > 0$), in absence of particle absorption, the forward-scattered intensity is near a minimum at

$$\begin{aligned}
&\Re\{\epsilon^{-1}\alpha_e\} = -\Re\{\mu\alpha_m\},\\
&\Im\{\epsilon^{-1}\alpha_e\} = \Im\{\mu\alpha_m\} \Rightarrow \frac{d\sigma^{(s)}}{d\Omega}(0°)\\
&\quad = k^4|2\Im\{\epsilon^{-1}\alpha_e\}|^2\\
&\quad = \frac{16}{9}k^{10}|\epsilon^{-1}\alpha_e|^4 = \left|\frac{2}{3}k^3\epsilon^{-1}\alpha_e\right|^2\frac{d\sigma^{(s)}}{d\Omega}(180°).
\end{aligned} \tag{9}$$

[Notice that the first line of Eq. (9) leads to a minimum of the intensity if in addition $\mathrm{Im}\{\epsilon^{-1}\alpha_e\} = \Im\{\mu\alpha_m\} =$ minimum]. For lossless magnetodielectric particles, Eq. (9) is known as the *second Kerker condition* and leads exactly to a zero minimum of $d\sigma^{(s)}(0°)/d\Omega$ [3,25] when $\epsilon_p = -(\mu_p - 4)/(2\mu_p + 1)$ and the particle scattering is well characterized by the quasi-static approximation $\Re\alpha \approx \Re\alpha^{(0)}$, $\Im\alpha \approx \Im\alpha^{(0)} \approx 0$, of the Rayleigh limit $ka \ll 1$, $k|n_p|a \ll 1$, in which [2,45]$\alpha_e^{(0)} = \epsilon a^3\frac{\epsilon_p-\epsilon}{\epsilon_p+2\epsilon}$, $\alpha_m^{(0)} = \mu^{-1}a^3\frac{\mu_p-\mu}{\mu_p+2\mu}$. As a matter of fact, this condition was derived in [3] under these approximations. It should be remarked that the actual intensity for a very small particle goes as $\sim(ka)^{10}$, which, only when $ka$ is well below 1, would be negligible [6]. Otherwise, as is the case of the small particles here addressed, this intensity is near a nonzero minimum value of $d\sigma^{(s)}(0°)/d\Omega$, as seen in Section 3. Although being of fundamental interest, no concrete example of dipolar magnetodielectric particles that might present, such anomalous scattering in the visible or infrared regions, has been proposed.

Our derivation of the special scattering conditions (8) and (9) was obtained with the unique assumption that the radiation fields are well described by dipolar electric and magnetic fields, including their generalization in terms of the coefficients $a_1$ and $b_1$. This goes well beyond the Rayleigh limit and should apply to any small particle described by Eq. (1) in terms of these two Mie coefficients. The first line of Eq. (9) can then be considered as a *generalized second Kerker condition* and is the first result of this work. Specifically, the second Kerker condition [Eq. (9)] also applies to purely dielectric spheres ($\mu_p = 1$) providing that their scattering properties may be fully described by the two first terms in the Mie expansion.

## 3. INSTANCE OF MAGNETODIELECTRIC PARTICLE: A SILICON SPHERE

A recent work [46] reports that dielectric spheres whose refractive index is around 3.5 and that have size parameter $ka$ between 0.75 and 1.5 produce a plane wave scattering, which is with great accuracy given by only the two first Mie coefficients $a_1$ and $b_1$ [see Eq. (1)]. Here we next show that they are very convenient, real, and unexpected objects for testing Kerker conditions, as well as new scattering effects and their consequences on optical forces.

An example is a silicon sphere of radius $a = 230$ nm, whose refractive index may well be approximated by $\epsilon_p = 3.5$ in the range of near-infrared wavelengths ($\lambda \approx 1.2$–$2\ \mu$m) of this study [46].

Figure 1(a) shows the real and imaginary parts of the polarizabilities [Eq. (1)], whereas Fig. 1(b) contains the differential scattering cross sections in the forward-scattering and backscattering directions. The maxima in $\alpha_e$ and $\alpha_m$ [see Fig. 1(a)] occur around 1300 nm and 1700 nm, respectively, and are well separated from each other. The sharp peaks of the differential scattering cross sections [Fig. 1(b)] are mainly due to the corresponding dominant magnetic dipole contribution $\alpha_m$ near the first Mie resonance. One sees the values of $\lambda \simeq 1825$ nm and 1530 nm at which $\mathfrak{I}\{\alpha_e\} = \mathfrak{I}\{\alpha_m\}$, which are where the first and second Kerker conditions hold for these polarizabilities, respectively.

While the backward intensity drops to zero at the first Kerker condition wavelength, at the frequency of the second condition the radiated intensity is near a nonzero minimum in the forward direction. Dielectric spheres and, in particular, lossless Si particles in the near infrared then constitute a realizable laboratory to observe such a special scattering. This is another main result of the present work.

It should be observed that, for a lossless particle as the one under study, the two Kerker conditions are a consequence of the optical theorem, Eq. (2), written for the electric and for the magnetic dipole, separately. This, in turn, obeys the zero contribution of the self-interaction term between both dipoles [i.e., the last term of Eq. (6)] to the total scattering cross section. Then, if one imposes the equality of imaginary parts $\mathfrak{I}\alpha_e = \mathfrak{I}\alpha_m$ and subtracts from each other the optical theorem equations of each dipole, one immediately derives that $\mathfrak{R}\alpha_e = \pm\mathfrak{R}\alpha_m$.

## 4. EFFECTS ON OPTICAL FORCES

It is of interest to analyze the consequences of anomalous scattering properties in radiation pressure forces. For an incident plane wave, $\mathbf{E}^{(i)} = \mathbf{e}^{(i)} e^{ik\mathbf{s}_0\cdot\mathbf{r}}$ and $\mathbf{B}^{(i)} = \mathbf{b}^{(i)} e^{ik\mathbf{s}_0\cdot\mathbf{r}}$, the time-averaged force on a dipolar particle is written as the sum of three terms [45]:

$$\langle \mathbf{F} \rangle = \langle \mathbf{F}_e \rangle + \langle \mathbf{F}_m \rangle + \langle \mathbf{F}_{e\text{-}m} \rangle = \mathbf{s}_0 F_0 \left[ \frac{1}{a^3} \mathfrak{I}\{\epsilon^{-1}\alpha_e + \mu\alpha_m\} - \frac{2k^3}{3a^3}\frac{\mu}{\epsilon} \mathfrak{R}(\alpha_e \alpha_m^*) \right], \quad (10)$$

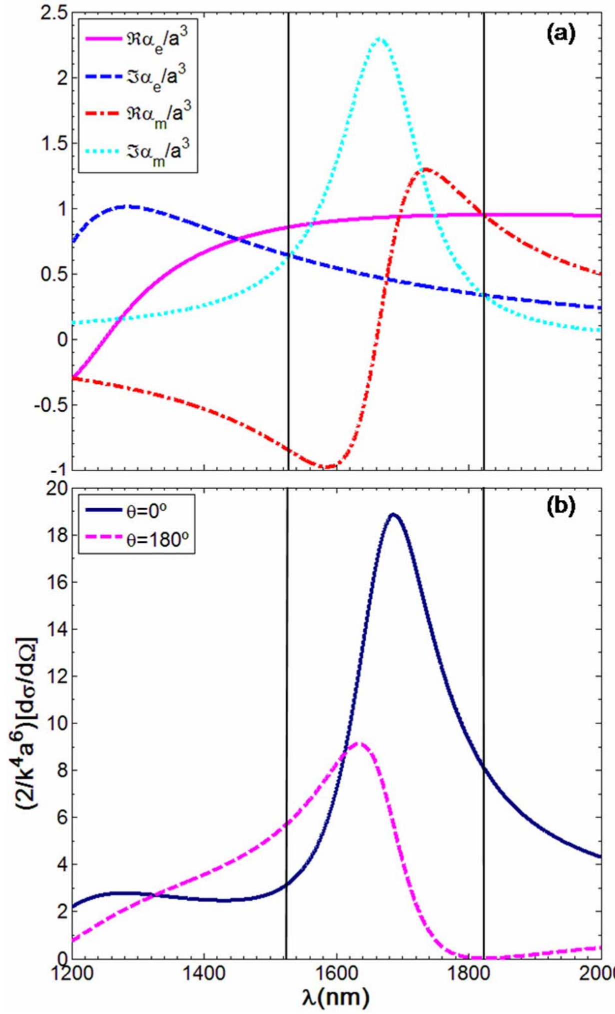


Fig. 1. (Color online) Results for an Si sphere of radius $a = 230$ nm,$\epsilon_p = 12$ and $\mu_p = 1$. The host medium has $\epsilon = \mu = 1$. (a) Normalized real and imaginary parts of both the electric and magnetic polarizabilities. (b) Normalized differential scattering cross section in the forward and backscattering direction. The first and second Kerker conditions are marked by the right and left vertical lines, respectively.

where $F_0 \equiv \epsilon k a^3 |\mathbf{e}^{(i)}|^2/2$. The first two terms, $\langle \mathbf{F}_e \rangle$ and $\langle \mathbf{F}_m \rangle$, correspond to the forces on the induced pure electric and magnetic dipoles, respectively. $\langle \mathbf{F}_{e\text{-}m} \rangle$, due to the interaction between both dipoles [44,45], is related to the asymmetry in the scattered intensity distribution, [cf. the last term in Eq. (6)] [45]. From Eqs. (6) and (10), one derives for the radiation pressure force

$$\langle \mathbf{F} \rangle = \mathbf{s}_0 F_0 \frac{1}{6ka^3} \left[ \frac{d\sigma^{(s)}}{d\Omega}(0°) + 3\frac{d\sigma^{(s)}}{d\Omega}(180°) + \frac{3}{2\pi}\sigma^{(a)} \right]. \quad (11)$$

Equation (11), which is a main result of this work, emphasizes the dominant role of the backward scattering on radiation pressure forces. In turn, this is connected to the asymmetry parameter $\langle \cos(\theta) \rangle$ of the radiation pressure [1,2]. Notice that Eq. (11), which is also valid for a pure dipole, either electric or magnetic, shows that the force due to a plane wave, which is all radiation pressure [45], cannot be negative for ordinary host media with $\epsilon$ and $\mu$ real and positive. This expression also manifests that the weight of the intensity in the backscattering direction is three times that of the forward-scattered power.

Equations (10) and (11) provide an appropriate framework to discuss the interplay between special scattering properties and radiation pressure forces. Let us consider as a reference the standard PED case in absence of absorption, on which the force from the plane wave is

$$\langle \mathbf{F} \rangle_{\mathrm{PED}} = \langle \mathbf{F}_e \rangle = F_0 \frac{2k^3}{3a^3} \mathbf{s}_0 |\epsilon^{-1} \alpha_e|^2. \quad (12)$$

(The following arguments would equally apply with a pure magnetic dipole.) At a fixed electric polarizability, the addition of an extra magnetic dipole always leads to an increase of the total cross section. However, it does not necessarily imply an increase of the total force.

### A. Minimum Force

As shown by Eq. (10), $\langle \mathbf{F} \rangle$ cannot be zero, even if $\sigma^{(a)} = 0$; however, if the particle is lossless, by expressing the bracket of Eq. (10) as a hypersurface of the four variables $\mathfrak{R}\alpha_e$, $\mathfrak{I}\alpha_e$, $\mathfrak{R}\alpha_m$, and $\mathfrak{I}\alpha_m$ ($\mathfrak{I}\alpha_e > 0$, $\mathfrak{I}\alpha_m > 0$), it has the absolute minimum when $\epsilon^{-1}\mathfrak{R}\alpha_e = \mu\mathfrak{R}\alpha_m = 0$, which is trivial, of course.

Nevertheless, the section of the surface Eq. (10) at the planes $\mathfrak{R}\alpha_e = \text{constant}$ and $\mathfrak{I}\alpha_e = \text{constant}$ has minima at $\mu\mathfrak{R}\alpha_m = (1/2)\epsilon^{-1}\mathfrak{R}\alpha_e$ and $\mu\mathfrak{I}\alpha_m = (1/2)\epsilon^{-1}\mathfrak{I}\alpha_e$. Then, Eq. (10) shows that this minimum force is

$$\langle \mathbf{F} \rangle = F_0 \frac{2k^3}{3a^3} \mathbf{s}_0 \frac{3}{4} \left[ \frac{3\sigma^{(a)}}{2\pi k^4} + |\epsilon^{-1}\alpha_e|^2 \right], \quad (13)$$

which for a lossless particle is 3/4 that on a PED, Eq. (12); namely, $\langle \mathbf{F} \rangle = \frac{3}{4} \langle \mathbf{F}_e \rangle$.

{A reciprocal consequence occurs by choosing similar plane cuts for the magnetic polarizability; then an analogous result is obtained with respect to a pure magnetic dipole with the minimum force: $F_0(2k^3)/(3a^3)\mathbf{s}_0(3/4)[3\sigma^{(a)}/(2\pi k^4) + \mu^2|\alpha_m|^2]$ when $\epsilon^{-1}\alpha_e = (1/2)\mu\alpha_m$.}

Also, Eq. (6) shows that now the differential scattering cross section of this magnetodielectric particle is

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{\epsilon^2} |\alpha_e|^2 \left[ \frac{5}{8}(1 + \cos^2\theta) + \cos\theta \right]. \quad (14)$$

### B. Generalization of the Case of a Perfectly Conducting Sphere

On the other hand, let us consider the case in which $\mu\alpha_m = (-1/2)\epsilon^{-1}\alpha_e^*$. Then, from Eqs. (10) and (6), one has for the force on the particle

$$\langle \mathbf{F} \rangle = F_0 \frac{2k^3}{3a^3} \mathbf{s}_0 \epsilon^{-2} \left[ \frac{3}{4} |\alpha_e|^2 + (\mathfrak{R}\alpha_e)^2 \right], \quad (15)$$

and for the corresponding scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{\epsilon^2} \left[ \frac{5}{8} |\alpha_e|^2 (1 + \cos^2\theta) - [(\mathfrak{R}\alpha_e)^2 - (\mathfrak{I}\alpha_e)^2] \cos\theta \right]. \quad (16)$$

Equations (15) and (16) become, for a nonabsorbing Rayleigh particle, for which $\mathfrak{R}\alpha_e \simeq \mathfrak{R}\alpha_e^{(0)}$ and $\mathfrak{I}\alpha_e \simeq 2/(3\epsilon)k^3|\alpha_e^{(0)}|^2$

$$\langle \mathbf{F} \rangle = F_0 \frac{2k^3}{3a^3} \mathbf{s}_0 |\epsilon^{-1}\alpha_e^{(0)}|^2, \quad (17)$$

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{\epsilon^2} |\alpha_e^{(0)}|^2 \left[ \frac{5}{8}(1 + \cos^2\theta) - \cos\theta \right]. \quad (18)$$

Equations (17) and (18) represent a generalization of the force and differential scattering cross section, respectively, that apply to a perfectly conducting sphere at large wavelengths [45], for which $\mu\alpha_m^{(0)} = (-1/2)\epsilon^{-1}\alpha_e^{(0)} \simeq (-1/2)a^3$, which is a particular case of the aforementioned condition $\mu\alpha_m = (-1/2)\epsilon^{-1}\alpha_e^*$.

In addition, the Rayleigh limit, Eq. (17), of Eq. (15) turns out to be 7/4 the force on a lossless PED, Eq. (12), when in Eq. (12) one also takes this Rayleigh limit. [In Eq. (12), the term $F_0\mathbf{s}_0\sigma^{(a)}/(8\pi k a^3)$ should be added if the particle is absorbing.] An analogous result is obtained for a magnetic dipole if the electric polarizability is eliminated instead.

Notice, however, that, since the contribution of the term $\mathfrak{R}(\alpha_e\alpha_m^*)$ integrated over $\Omega$ is zero, both differential cross sections, Eqs. (14) and (16), yield the same total scattering cross section and, hence, the same radiation pressure excluding the component of the self-interaction force $\langle \mathbf{F}_{e-m} \rangle$. (Similar arguments hold for a magnetic dipole by choosing the force hypersurface cut $\mathfrak{R}\alpha_m = \text{constant}$ and $\mathfrak{I}\alpha_m = \text{constant}$.) Thus, we have the interesting result on two particles with the same total scattering cross section, but quite different differential scattering cross sections, in particular in the forward-scattering and backscattering directions, and suffering completely different forces: the former experiences a force, Eq. (15), which in the Rayleigh limit becomes 7/4 that of a pure nonabsorbing dipole, while the latter experiences a minimum force Eq. (13), which becomes 3/4 that of a pure lossless dipole.

### C. Other Relative Minimum Forces: Kerker Conditions

Another minimum force is obtained from Eq. (10) under the condition that $|\epsilon^{-1}\alpha_e|^2$ and $|\mu\alpha_m|^2$ be kept constant. This obviously happens when $\mathfrak{R}(\epsilon^{-1}\alpha_e\mu\alpha_m^*) = |\epsilon^{-1}\alpha_e||\mu\alpha_m|$; then if for instance one keeps the condition $|\mu\alpha_m| = (1/2)|\epsilon^{-1}\alpha_e|$, this force becomes again 3/4 that of a pure dipole, whereas the differential scattering cross section of such particle is 9/2 and 1/2 that of a pure dipole in the forward-scattering and backscattering directions, respectively. This is perfectly explained by Eq. (11).

On the other hand, when $|\epsilon^{-1}\alpha_e|^2 = |\mu\alpha_m|^2$, then Eqs. (10) and (11) show that this minimum force is equal to that of a PED [Eq. (12)]. The differential scattering cross section [Eq. (6)] then is zero in the backscattering direction, but is maximum and equal to four times that of a pure dipole, in the forward direction. An analogous argument can be made as before, with respect to a pure magnetic dipole, if the magnetic parameters are chosen instead.

An important case when $|\epsilon^{-1}\alpha_e|^2 = |\mu\alpha_m|^2$ is that in which $\epsilon_p/\epsilon = \mu\mu_p$, which implies that $\epsilon^{-1}\alpha_e = \mu\alpha_m$, namely, at the first Kerker condition, Eq. (8); then the corresponding force $\langle \mathbf{F} \rangle_{\mathrm{FK}}$ that one obtains from Eqs. (10) and (11) is, eliminating the magnetic constants, exactly equal to the force on a PED [Eq. (12)]. Then $\langle \mathbf{F} \rangle_{\mathrm{FK}} = \langle \mathbf{F}_e \rangle$. [It should be remarked, however, that, in this expression for $\langle \mathbf{F} \rangle_{\mathrm{FK}}$, the term $F_0\mathbf{s}_0\sigma^{(a)}/(4\pi k a^3)$ should now be added to Eq. (12) if the particle is absorbing.] A reciprocal expression for $\langle \mathbf{F} \rangle_{\mathrm{FK}}$ may be written in terms of the magnetic polarizability if one substitutes $\epsilon^{-1}\alpha_e$ by $\mu\alpha_m$.

Thus, the only difference between the two forces, $\langle \mathbf{F} \rangle_{FK}$ on a particle holding the first Kerker condition and that on a PED $\langle \mathbf{F} \rangle_{PED}$, occurs when the particle is absorbing, then being $F_0 \mathbf{s}_0 \sigma^{(a)}/(8\pi k a^3)$. An equivalent result appears for a magnetic dipole. Also, Eq. (6) shows that this pure dipole cross section is nonzero in the backscattering direction, but in the forward direction it is 1/4 of the cross section from a magnetodielectric particle satisfying the first Kerker condition.

At the second Kerker condition, Eq. (9): $\epsilon^{-1}\alpha_e^{(0)} = -\mu\alpha_m^{(0)}$ for Rayleigh lossless particles in the quasi-static approximation, $|\alpha_e|^2 \simeq |\alpha_e^{(0)}|^2$, $\Im\alpha_e \simeq \Im\alpha_e^{(0)} = 0$, so that $d\sigma(0°)/d\Omega = 0$ and Eq. (10) should lead to a force

$$\langle \mathbf{F} \rangle_{SK} = F_0 \frac{2k^3}{a^3} \mathbf{s}_0 |\epsilon^{-1}\alpha_e^{(0)}|^2, \tag{19}$$

which would be three times that on a lossless pure (electric) dipole, Eq. (12), in that quasi-static approximation [as before, we reciprocally argument in terms of a magnetic dipole if $\alpha_m^{(0)}$ is chosen instead]. Hence, the larger weight of the backscattering cross section in the force [which in this case is $4k^4(\alpha_e^{(0)}/\epsilon)^2$] manifested by Eq. (11) would contribute in this situation to such an increase of the averaged force on this particle in comparison with that on a pure dipole.

At the generalized second Kerker condition, Eq. (9), beyond the Rayleigh limit, the corresponding force on the lossless particle then is

$$\langle \mathbf{F} \rangle_{SK} = F_0 \frac{2k^3}{3a^3} \mathbf{s}_0 \epsilon^{-2}[|\alpha_e|^2 + 2(\Re\alpha_e)^2], \tag{20}$$

which, in the Rayleigh limit, $\Re\alpha_e \simeq \Re\alpha_e^{(0)}$, $\Im\alpha_e \simeq 2/(3\epsilon)k^3|\alpha_e^{(0)}|^2$, would become smaller and approximately equal to the quasi-static value [Eq. (19)]. More generally, when $|\alpha_e|^2 \simeq (\Re\alpha_e)^2$, Eq. (20) would again be three times the force on a PED [Eq. (12)].

### D. Summary of the Relationships between Forces on Different Small Spheres and That on a Pure Dipole

To summarize this, we conclude that, at the first generalized Kerker condition, Eq. (8), the interference term of Eq. (10) cancels out the magnetic contribution, and we obtain $\langle \mathbf{F} \rangle = \langle \mathbf{F}_e \rangle$. At the second Kerker condition, Eq. (9), where the backscattering is enhanced, $\langle \mathbf{F} \rangle = 3\langle \mathbf{F}_e \rangle$. Notice that, at both Kerker conditions, the total scattering cross section is exactly the same, although the radiation pressures differ by a factor of 3. These properties are illustrated in Fig. 2, where we show the different contributions to the total time-averaged force on a submicrometer Si particle.

One can conclude from the above discussion derived from Eqs. (10) and (11) that the force on the magnetodielectric particle is near (and equal to for a Rayleigh particle) $R$ times that on a PED ($R$ being a real number equal or larger than 3/4) whenever

$$\mu\Re\alpha_m = (1/2)(1 \pm \sqrt{4R-3})\epsilon^{-1}\Re\alpha_e, \qquad \mu\Im\alpha_m = (1/2)|1 \pm \sqrt{4R-3}|\epsilon^{-1}\Im\alpha_e. \tag{21}$$

An equation analogous to Eq. (21) occurs with a pure magnetic dipole whenever $\epsilon^{-1}\Re\alpha_e$ and $\epsilon^{-1}\Im\alpha_e$ are reciprocally replaced by $\mu\Re\alpha_m$ and $\mu\Im\alpha_m$, respectively. Equation (21) summarizes the cases discussed before and shows that $R$ cannot be smaller than $R = 3/4$, which would correspond to the minimum force [Eq. (13).] The case of the PED corresponds to $R = 1$ and the square root in Eq. (21) with negative sign, whereas $R = 1$ and the plus sign in that square root leads to the first Kerker condition. On the other hand, the case of the generalized second Kerker condition corresponds to $R = 3$ and the minus sign in front of the square root of Eq. (21).

## 5. SILICON AND OTHER HIGH REFRACTIVE INDEX DIELECTRIC SPHERES: A LABORATORY TO TEST OPTICAL FORCES

Figure 2 shows the different contributions to the total time-averaged force on the Si particle studied in Fig. 1, presenting their peaks in the region of wavelengths where the magnetic dipole dominates. Hence, we have the two additional remarkable results of this work as follows.

First, there are regions of the spectrum, near the corresponding electric and magnetic Mie resonances, where $\Im\alpha_e \gg \Re\alpha_e$ and $\Im\alpha_m \gg \Re\alpha_m$. This should be observed in future experiments in contrast with previous observations indicating the opposite result out of resonance [48,49]. [Notice that Eq. (1) shows that, at the resonant values of the static polarizabilities $\alpha_e^{(0)}$ and $\alpha_m^{(0)}$, one has $\Re\alpha_e = \Re\alpha_m = 0$ and $\Im\alpha_e = 3\epsilon/(2k^3)$, $\Im\alpha_m = 3/(\mu 2k^3)$.]

Second, the strong peak in the radiation pressure force is mainly dominated by the first "magnetic" Mie resonance, concretely of $\Im\alpha_m$. This constitutes an illustration of a dipolar dielectric particle on which the optical force is not solely described by the electric polarizability. Also, in such a case the imaginary part of the polarizability is much larger than its real part. As a matter of fact, this is the opposite situation to the usual experiments with optical tweezers out of resonance, in which gradient forces (that are proportional to

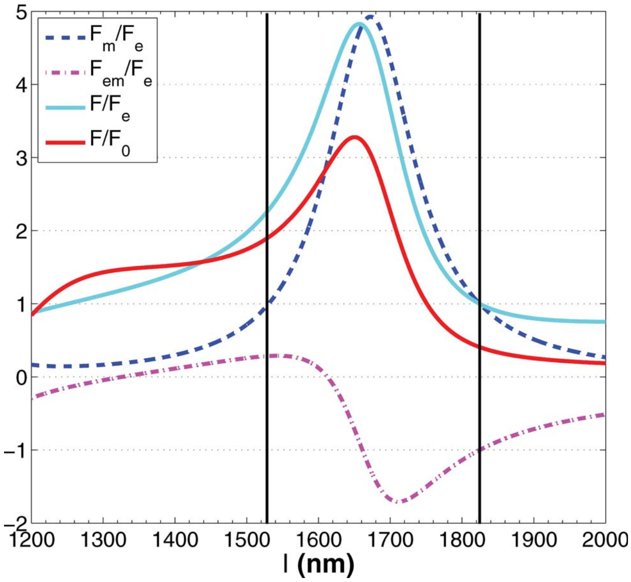


Fig. 2. (Color online) Different contributions to the total radiation pressure, versus the wavelength, for the Si particle of Fig. 1. Normalization is done by either the electric force magnitude $\langle F_e \rangle$ or $F_0 = ka^3|\mathbf{e}^{(i)}|^2/2$. Again, the vertical lines mark, from right to left, the first and second Kerker conditions. Notice that, when the first Kerker condition is fulfilled, i.e., $\Im\alpha_e = \Im\alpha_m$ and $\Re\alpha_e = \Re\alpha_m$, $\langle F \rangle = \langle F_e \rangle = \langle F_m \rangle = -\langle F_{e-m} \rangle$.

$\Re\{\alpha_e\}$) dominate over the radiation pressure or scattering force contribution (which is proportional to $\Im\{\alpha_e\}$) [48,49].

Nonetheless, as the size of the particle increases, and for any dielectric particle, there is a crossover from electric to magnetic response as we approach the first Mie resonance, the point at which there is dominance of the magnetic dipole.

Moreover, just at the resonance and in absence of absorption, $\Re\{\alpha_m\} = 0$ and $\Im\{\alpha_m\} = 3/(\mu 2k^3)$. Then, the radiation pressure contribution of the magnetic term dominates the total force $\langle \mathbf{F} \rangle \simeq \langle \mathbf{F}_m \rangle \approx (3F_0\mathbf{s}_0)/(2k^3a^3)$. Namely, in resonance, the radiation pressure force presents a strong peak, the maximum force being independent of both material parameters and the particle radius. On the other hand, the relationship between polarizabilities leading to Eq. (15) approximately appears in Figs. 1(a) and 1(b) in the zone about $\lambda \simeq 1450$ nm.

In addition, we observe in Fig. 2 that, at the wavelength where the first Kerker condition holds, as expected from Eq. (10), the three components of the force are of equal magnitude, but the electric–magnetic dipole interaction force $\langle \mathbf{F}_{e-m} \rangle$ contributes with negative sign, and hence the total force equals either the electric or magnetic contribution, confirming the previous remarks. On the other hand, at the wavelength where the generalized second Kerker condition is fulfilled, the electric and magnetic force components are equal, and the total force, in agreement with Eq. (20), is almost three times either of them.

## 6. CONCLUSIONS

We have analyzed the scattering properties of magnetodielectric small particles, proposing a generalization of the second Kerker condition, and discussed the consequences for the optical forces. We have shown that real, small dielectric particles made of nonmagnetic materials present scattering properties similar to those previously reported for somewhat hypothetical magnetodielectric particles [3], resulting from an interplay between real and imaginary parts of both electric and magnetic polarizabilities. Then, we have discussed how these scattering effects also affect the radiation pressure on these small particles. Specifically, submicrometer Si (as well as Ge and $TiO_2$) particles constitute an excellent laboratory to observe such remarkable scattering phenomena and force effects in the near infrared region. This kind of scattering will strongly affect the dynamics of particle confinement in optical traps, which is also governed by both the gradient and curl forces [45] and which should be observable as soon as one introduces a spatial distribution of intensity in the incident wavefield, and plays with its polarization. We believe, therefore, that our results should stimulate further experimental and theoretical work in this direction, since they suggest intriguing possibilities in rapid developing fields, ranging from optical trapping and particle manipulation to cloaking and the design of optical metamaterials based on lossless dielectric particles.

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