STRINGY CONFINEMENT OF LIGHT QUARKS

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Linear confining potentials, independent of spin, and conserving chirality are found from the non-perturbative extension of the light front QCD. The V_{qq} potential in baryons is obtained from $V_{q\bar{q}}$ in mesons by multiplying $V_{q\bar{q}}$ by $\frac{1}{2}(\frac{8}{9})^2 \approx 0.4$. In any light meson $V_{q\bar{q}}(r) = \lambda r$, $\lambda = (0.13 \pm 0.03)\mathrm{GeV}^2$, if the gluon and quark condensates are $\langle | (\alpha_s/\pi):G^a_{\mu\nu}G^{\mu\nu}_a: | \rangle = (360 \pm 20\ \mathrm{MeV})^4$, $\langle | \sqrt{\alpha_s}:\bar{\psi}\psi: | \rangle = -(240\ \mathrm{MeV})^3$, and $\Lambda = 150$ MeV. For $p^2 \leqslant m_p^2$ the running quark mass is frozen at the pole value $m_p = m(p^2 = m_p^2) = 331$ MeV. Inside mesons, q and $\bar{q}$, with an average momentum equal to m_p, move in the meson rest frame with velocity $0.7c$, and are "connected" by a string with tension equal to (0.17 ± 0.03) GeV^2. The slope α' of Regge trajectories, the quark and gluon condensates, and the frozen coupling constant $\alpha_s = \alpha_s(p^2 = m_p^2)$ obey the relation $\alpha' \langle | (\alpha_s/\pi):G^a_{\mu\nu}G^{\mu\nu}_a: | \rangle = (16/9\pi)(3\pi\alpha_s^2)^{-1/3}(\langle | \sqrt{\alpha_s}:\bar{\psi}\psi: | \rangle)_{2/3}$ giving $\alpha' = (1.0 \pm 0.2)$ GeV^{-2}.

Stringy confinement is equivalent to constant energy density per unit length, and is found in all linear confining potentials. It is different in character from the confinement caused by a logarithmic potential, in which the energy density is not constant. It is also qualitatively different from two other classes of confinement, which we call "chain", and "heavy". In the "chain" class quarks cannot escape because the coupling constant blows up to infinity. Sometimes, the chain is in an abstract form, being replaced by walls, wich, like the sides in bag models, prevent quarks from escaping, but allow for a "free" motion inside the hadron, as the chain does. Our stringy confinement differs qualitatively from the "chain" class, since the coupling constant α_s in our case is frozen for $p^2 \leqslant m_p^2$, at the value $\alpha_s(p^2 = m_p^2) = 0.88$. In the "heavy" class the mass of constituents becomes infinite, and that prevents quarks from escaping. Again, our stringy confinement differs qualitatively from the "heavy" class, since for $p^2 \leqslant m_p^2$ the quark (antiquark) mass is finite, and it is frozen at the pole value $m_p = m(p^2 = m_p^2) = 331$ MeV.

To derive the linear $q\bar{q}$ and qq potentials we need several results which were obtained in the non-perturbative extension of the light front QCD [1]. Two of these results concern a *single* hard gluon, and a *single* hard quark, moving in external soft fields, and treated in the Feynman calculus, with the virtuality of each particle $p^2 = p_\mu p^\mu$ varying (not fixed even in the numerator), and bounded from below. Two other results, in contrast, concern a few-body system, described within the light front dynamics [2]. There, we have a common "time" $x^+ = x^0 + z$, and diagrams are x^+-ordered. The off-shell continuation is in the "$-$" component of momentum, which plays the rôle of "energy". Each particle is on its mass shell, and a varying virtuality follows from the off-shell continuation in the "$-$" component of momentum. These two sets of results [1] are denoted as (i), (ii), and (iii), (iv), respectively. The current quark mass is set equal to zero for simplicity.

(i) A single hard gluon, with virtuality $p^2 = p_\mu p^\mu$ bounded from below, moving in the external soft gluon field $A^a_{\mathrm{ext.}\,\mu}(x) = \frac{1}{2}x^\sigma G^a_{\mu\sigma}(0)$ acquires a running mass [#1]

$$m^2_{\mathrm{gluon}}(p) = (\pi^2/4p^2)\langle | (\alpha_s/\pi):G^a_{\mu\nu}G^{\mu\nu}_a: | \rangle\ , \tag{1}$$

which is bounded from above by

$$m_{\mathrm{gluon}}(p) < (451 \pm 25)\ \mathrm{MeV}\ , \tag{2}$$

[#1] In eqs. (1) and (3) there is an extra statistical factor 1/2!, which should be included in front of the gluon bubble in the gluon propagator. It is taken into account by Mandelstam [3], and we keep it too if either of the gluons which form the bubble gets broken, and is replaced by the external soft gluon field.

if the gluon condensate is [4] $\langle | (\alpha_s/\pi):G^a_{\mu\nu}G^{\mu\nu}_a: | \rangle = (360 \pm 20\ \mathrm{MeV})^4$. The external, soft gluon field, characterized by the gluon condensate, in addition to forming the gluon running mass in eq. (1), produces an extra term in the hard gluon propagator (see footnote 1)

$$\frac{\pi^2}{16}\frac{\eta_\mu\eta_\nu}{p^4(p\eta)^2}\langle | (\alpha_s/\pi):G^a_{\mu\nu}G^{\mu\nu}_a: | \rangle , \tag{3}$$

where η_μ is a null four-vector, $\eta^2=0$, and the original Feynman hard gluon (massless) propagator $D_{\mu\nu}(p) = (p_\mu\eta_\nu + p_\nu\eta_\mu - p\eta g_{\mu\nu})/p^2 p\eta$ is in the null gauge $A^\mu\eta_\mu = 0$. The crucial result is the appearence of p^{-4} in eq. (3), in contrast to the massive gluon propagator having $(p^2-m^2)^{-1}$, with m given by eq. (1). The term in eq. (3) neither gets changed into $(p^2-m^2)^{-2}$ nor should be confused with the gluon seagull term in the light front formulation [2], which also is proportional to $\eta_\mu\eta_\nu$. There is no contribution, neither to eq. (1) nor to (3) from the soft quark (antiquark) external field, because the trace of the odd number of the γ matrices vanishes.

(ii) A single hard quark, with virtuality p^2 bounded from below, moving in the external soft gluon field is unchanged, if we restrict ourselves to the lowest-dimension gluon operator $(\alpha_s/\pi):G^a_{\mu\nu}G^{\mu\nu}_a:$. However, the same hard quark moving in the external soft quark (antiquark) field acquires the running mass

$$m_{\mathrm{quark}}(p) = -\frac{8\pi}{9}\frac{\sqrt{\alpha_s(p^2)}}{p^2}\langle | \sqrt{\alpha_s}:\bar\psi\psi: | \rangle , \tag{4}$$

which for $\langle | \sqrt{\alpha_s}:\bar\psi\psi: | \rangle = -(240\ \mathrm{MeV})^3$ is bounded from above by

$$m_{\mathrm{quark(p)}} < 331\ \mathrm{MeV} . \tag{5}$$

(iii) In the hadron state with n constituents, and total four-momentum $P^\mu = \sum_{i=1}^n p_i^\mu$, the virtuality of that state is defined as the product of $p^+ \equiv p^0 + p^z$ with the difference of the "$-$" component of the initial (hadron) momentum and $P^- \equiv P^0 - P^z$. Since in the light front dynamics [2] the "$+$" and the "$\perp$" components of the total momentum are conserved, we have the following result for the state virtuality:

$$D_0 = P^+(P^-_{\mathrm{hadron}} - P^-) = M^2 - P_\mu P^\mu , \tag{6}$$

where M is the hadron mass, $M^2 = P^\mu_{\mathrm{hadron}}P_{\mathrm{hadron}\,\mu}$. The virtuality of the ith constituent of hadron is

$$\tilde p_i^2 = (p_i^+/P^+)D_0, \quad i=1,\dots,n, \tag{7}$$

and the sum of individual constituent's virtualities is easily obtained from eq. (7),

$$\sum_{i=1}^n \tilde p_i^2 = D_0 . \tag{8}$$

Because of eqs. (7) and (8) we conjectured in ref. [1], that an approximate account of the interaction of hadron's constituents with soft fields, present in the hadron, can be made if in the running mass formulas eqs. (1) and (4), and in the extra term in the gluon propagator eq. (3), the virtuality of the single particle p^2 is replaced by

$$\operatorname*{average}_{i=1,\dots,n} \tilde p_i^2 = D_0/n . \tag{9}$$

(iv) The internal dynamics of a few-body system is described by the Weinberg [2,5] equation for the hadron's wave function ψ

$$G_0^{-1}\psi = V\psi , \tag{10}$$

where $G_0^{-1} \equiv D_0$, and V is an irreducible kernel, which is an integral operator in momentum space. It is useful to define the variables

$$x_i = p_i^+/P^+ , \quad i=1,\dots,n ,$$

$$\sum_{i=1}^n x_i = 1 , \tag{11}$$

and the relativistic relative momenta defined in the manner of Jacobi,

$$q_i = p_i\Big(\sum_{j\neq i}{}' x_j\Big) - \Big(\sum_{j\neq i}{}' p_j\Big)x_i ,$$

$$i=1,\dots,n ,$$

$$\sum_{i=1}^n q_i = 0 . \tag{12}$$

In terms of these variables the state virtuality D_0 is

$$D_0 = M^2 - \sum_{i=1}^n \frac{-q_i^2 + m_i^2}{x_i} . \tag{13}$$

The irreducible kernel V in eq. (10) contains three sets of terms. In the first set there are two x^+-ordered diagrams of the massive gluon exchanges, with the running gluon mass given by eq. (1). The second set is the gluon seagull diagram. These two sets are neatly combined [2,6], with the result which formally in the *numerator* obeys the conservation of all components, as in a Feynman diagram [6]. The third set is the focus of our interest, and it is given by eq. (3) with the substitution $p^2 \to D_0/n$. There are two x^+-ordered diagrams in this third set. Below we write the expression for one of them, denoting by $-C$ the colour factor ($C=c_F=\frac{4}{3}$ for $q\bar{q}$ in mesons, and $C=C_B=\frac{2}{3}$ for qq in baryons). The square of the coupling constant is $4\pi\alpha_s$ and by w we denote either the v spinor of $\bar{q}$ in mesons, or the u spinor of the other quark in qq in baryons. With this notation we get [#2] from eq. (3)

$$-C\tfrac{1}{4}\pi^3\alpha_s \frac{\bar{u}_{\lambda_1'}\gamma^\mu\eta_\mu u_{\lambda_1}\bar{w}_{\lambda_2'}\gamma^\nu\eta_\nu w_{\lambda_2}}{2(D_0/n)^2(p\eta/P^+)^2}$$

$$\times\langle|(\alpha_s/\pi):G^a_{\mu\nu}G^{\mu\nu}_a:|\rangle . \tag{14}$$

That ends the summary of the results of ref. [1], and now we show their consequences. The γ-matrix structure of either $q\bar{q}$ interaction in mesons, or the qq interaction in baryons, exposed in eq. (14) as $\gamma^0\gamma^\mu\otimes\gamma^0\gamma^\nu$ conserves chirality [7], because the γ_5 matrix commutes with $\gamma^0\gamma^\mu$. The spin structure of these interactions follows from the diagonality of the spinor elements

$$\bar{u}_{\lambda_1'}(p_1')\gamma^\mu\eta_\mu u_{\lambda_1}(p_1)=\bar{u}_{\lambda_1'}(p_1')\gamma^+u_{\lambda_1}(p_1)$$

$$\bar{v}_{\lambda_1'}(p_1')\gamma^+v_{\lambda_1}(p_1)=m_p^{-1}\sqrt{p_1^+p_1'^+}\,\delta_{\lambda_1\lambda_1'} . \tag{15}$$

Note, that the light front spinors in eq. (15) are normalized in the standard way, $\bar{u}u=1$, in contrast to $\bar{u}u=2m$ in refs. [2,8]. The mass m_p in eq. (15) is the pole mass $m_p=m(p^2=m_p^2)=331$ MeV. From the diagonality in helicities in eq. (15) we infer that eq. (14) describes spin independent interactions both in the $q\bar{q}$ case in mesons and in the qq case in baryons.

The $q\bar{q}$ and qq interactions written in eq. (14) depend on the number n of constituents in two factors. The first factor is $(D_0/n)^{-2}$ exposed in eq. (14), and the second factor is hidden in eq. (15) in values of p_1^+, $p_1'^+$, and p_2^+, $p_2'^+$. For mesons the factor $(D_0/n)^{-2}$ brings in 3^2, and for baryons 4^2, since in counting the number of constituents we include as 1 the exchanged object [2], and in baryons we must also include the third spectator quark. To get the n dependence from eq. (15) we restrict ourselves to a domain of phase space in which the momentum transfer between either q and $\bar{q}$, or q and q is mall. This corresponds to a large distance between q and $\bar{q}$, or q and q. In momentum space the values of momenta of constituents are comparable, and denoting them as p_1^+, p_2^+, and $p_1'^+$, $p_2'^+$ before, and after interaction, respectively, we have

$$p_1^+\approx p_2^+\approx P^+(n-1)^{-1}\approx p_1'\approx p_2'^+ . \tag{16}$$

Here P^+ is the "+" component of the total momentum, which is a conserved quantity [2] equal to $\sqrt{s}$, where s is the Lorentz invariant for the hadron, $s=M^2=P^\mu_{\text{hadron}}P_{\text{hadron}\,\mu}$ (in the hadron's rest frame $P^+_{\text{hadron}}=\sqrt{s}$). The factor $(n-1)$ in eq. (16) comes from n, denoting the number of constituents, plus 1 for the exchanged object. Thus, from eqs. (14), (15) and (16) we get for the second factor $(\frac{1}{2})^2$ in mesons, and $(\frac{1}{3})^2$ in baryons. Including yet the colour factors c_B, and c_F, we get the following result for the ratio V_{qq} to $V_{q\bar{q}}$:

$$[-\tfrac{2}{3}\cdot4^2(\tfrac{1}{3})^2]/[-\tfrac{4}{3}\cdot3^2(\tfrac{1}{2})^2]=\tfrac{1}{2}(\tfrac{8}{9})^2\approx0.4 . \tag{17}$$

To get the *linear* character of V we show that in the domain of the phase space corresponding to a large distance between constituents, V has the form $(\boldsymbol{q}-\boldsymbol{q}')^{-4}$, where $\boldsymbol{q}$ is a new relative momentum with only *three* degrees of freedom. It is crucial that we do not get $(q-q')^{-4}$ with q having four degrees of freedom, since then we would get a logarithmic, instead of the linear confining potential. The relative momentum q is different from the one defined in eq. (12), and in fact we have to make two changes of definition of the relative momenta. The first change leads to the manifestly invariant notion of the relative momentum in terms of the Wightman–Gårding [9,10] four-vector obeying an invariant constraint. The second change of the definition of the relative momentum exposes explicitly its three degrees of freedom. The last relative momentum is such, that the Dirac–Poisson bracket of it with all four components of the total momen-

[#2] In the $q\bar{q}$ case in mesons there are two extra "$-$" signs, which result in $+1$. One "$-$" sign comes from the outgoing $\bar{q}$, while the other "$-$" sign results from the fact, that in the numerator of the $\bar{q}$ propagator its argument is the minus quark momentum. Therefore, we have $-\not{p}_2+m=-\Sigma_\beta v_\beta\bar{v}_\beta$, and we find the second "$-$" sign in front of the sum.

tum is zero [11]. The relative momentum $\boldsymbol{q}$ is not a three-vector, but it is a collection of three Lorentz invariants [12], which we only symbolically denote as $\boldsymbol{q}$ to emphasize its three degrees of freedom.

Now we restrict ourselves to the two-body system. If p_1 and p_2 are on-mass-shell four-momenta $p_i^2 = m_i^2$, $i=1,2$, then the Wightman–Gårding [9,10] relative momentum is also defined in the Jacobi manner [2], but $p_i\eta$ from eq. (12) is replaced by p_iP. For simplicity we use the same letter q to denote the Wightman–Gårding relative momentum

$$q=(\tfrac{1}{2}-\nu)p_1-(\tfrac{1}{2}+\nu)p_2,$$

$$\nu\equiv\tfrac{1}{2}(m_1^2-m_2^2)P^{-2}\ . \tag{18}$$

The crucial property of q is the constraint

$$qP=0\ . \tag{19}$$

Also P^2 is the following function of q^2:

$$P^2=[(-q^2+m_1^2)^{1/2}+(-q^2+m_2^2)^{1/2}]^2\ . \tag{20}$$

Eqs. (19) and (20) are consequences of the mass shell conditions $p_i^2=m_i^2$, $i=1,2$, and eq. (18). In the frame of reference in which $\boldsymbol{P}=\boldsymbol{0}$ we have from eq. (19) that $q^0=0$, and therefore q is a space-like relative momentum, most appropriate for defining angles, and partial wave decomposition [10]. The fact that q describes only three degrees of freedom follows from eqs. (19) and (20), and is stated there in manifestly invariant way.

We consider the case $m_1=m_2=m$, and denote $P\equiv M_0^2$,

$$M_0^2=4(-q^2+m^2),$$

$$D_0=s-M_0^2,\quad q=\tfrac{1}{2}(p_1-p_2). \tag{21}$$

In terms of the components of the Wightman–Gårding relative momenta q and q', the contribution from two x^+-ordered diagrams corresponding to the exchange of a massless particle, in the spinless case [11–13], is

$$g^2[\theta(q'^+-q^+)/D_1+\theta(q^+-q'^+)/D_2]$$

$$=g^2\{(q-q')^2$$

$$+|q^+-q'^+|[\sqrt{s}-(M_0^2+M_0'^2)/\sqrt{s}]\}^{-1}\ . \tag{22}$$

In obtaining eq. (22) we used

$$D_1\equiv(p_1'^+-p_1^+)$$

$$\times[\sqrt{s}-p_1^--(p'_{1\perp}-p_{1\perp})^2(p_1'^+-p_1^+)^{-1}-p_2'^-]$$

$$=(q-q')^2$$

$$+(q'^+-q^+)[\sqrt{s}-(M_0^2+M_0'^2)(2\sqrt{s})^{-1}]\ ,$$

and a similar expression for D_2.

The components of the relative momentum q, in which eq. (14) has the factor $(\boldsymbol{q}-\boldsymbol{q}')^{-4}$, are expressed by the following four-dimensional Lorentz scalar products of the Wightman–Gårding relative momentum q with three space-like four-vectors: k, l, m, which together with PM_0^{-1} form a tetrad [12] [#3]:

$$q=(q_k,q_l,q_m)\ ,$$

$$q_k=-qk,\quad q_l=-ql,\quad q_m=-qm\ . \tag{23}$$

In terms of q_k, q_l, and q_m we have

$$M_0^2=4(q_k^2+q_l^2+q_m^2+m^2)\ ,$$

$$(q-q')^2=-(q_k'-q_k)^2-(q_l'-q_l)^2-(q_m'-q_m)^2$$

$$+q_kq_k'(M_0'/M_0+M_0/M_0'-2)\ ,$$

$$|q^+-q'^+|=|q_k/M_0-q_k'/M_0'|\sqrt{s}\ , \tag{24}$$

since the following relations hold: $q_k=q^+M_0s^{-1/2}$, and $q_{l,m}=q_\perp$.

In the domain of phase space in which the relative motion of constituents is small (corresponding to large distances), and in which $(q_k-q_k')^2+(q_l-q_l')^2+(q_m-q_m')^2\ll s$, or equivalently, in terms of the Wightman–Gårding four-vectors q and q', $q^2\approx q'^2$, we get

$$M_0\approx M_0'\ . \tag{25}$$

[#3] In terms of the four components of $P^\mu=(P^0,P^1,P^2,P^3)\equiv(P_0,P_x,P_y,P_z)$, and $N\equiv P^0+M_0$, the components of the tetrad PM_0^{-1}, l, m, k are [14]

$$PM_0^{-1}=(P_0M_0^{-1},P_xM_0^{-1},P_yM_0^{-1},P_zM_0^{-1})\ ,$$

$$l=(P_xM_0^{-1},1+P_x^2M_0^{-1}N^{-1},P_xP_yM_0^{-1}N^{-1},P_xP_zM_0^{-1}N^{-1})\ ,$$

$$m=(P_yM_0^{-1},P_xP_yM_0^{-1}N^{-1},1+P_y^2M_0^{-1}N^{-1},P_yP_zM_0^{-1}N^{-1})$$

$$k=(P_zM_0^{-1},P_xP_zM_0^{-1}N^{-1},P_yP_zM_0^{-1}N^{-1},1+P_z^2M_0^{-1}N^{-1})$$

If in addition to eq. (25) we have

$$M_0 \approx M_0' \approx \sqrt{s}\,, \tag{26}$$

then one of the factors in eq. (22), namely $[\sqrt{s}-(M_0^2+M_0'^2)/2\sqrt{s}]$, and another factor in eq. (24), $(M_0'/M_0+M_0/M_0'-2)$, vanish approximately, and the right-hand side of eq. (22) simplifies to $-g^2[(q_k-q_k')^2+(q_l-q_l')^2+(q_m-q_m')^2]$. Note that eqs. (25) and (26) hold either in the hadron's internal dynamics, which at large distances is dominated by small relative momenta giving $M_0 \approx M_0' \approx 2m_p \approx \sqrt{s}$, or in the case of Regge trajectories in which s is large and the dominating contribution for the Regge behaviour comes from the region $M_0 \approx M_0' \approx \sqrt{s}$.

From the above algebra it follows, that two x^+-ordered contributions from the factor $(D_0 p\eta/P^+)^{-2}$ in eq. (14) give

$$\frac{\theta(q'^+-q^+)}{[D_0(q'-q)\eta/P^+]^2}+\frac{\theta(q^+-q'^+)}{[D_0(q-q')\eta/P^+]^2}$$

$$\approx[(q_k-q_k')^2+(q_l-q_l')^2+(q_m-q_m')^2]^{-2}\,. \tag{27}$$

The remaining momentum dependent factors in eq. (14) are spinor elements, which according to eqs. (15), (16), (25), and (26) give only constant factors in the domain corresponding to large distances between constituents. Therefore, the crucial momentum dependence of eq. (14) in this domain is given by eq. (27), and in fact it is the leading behaviour of the kernel V in eq. (10) at large distances, since the massive gluon exchange is less important than eq. (27), due to the finite gluon running mass. Taking the finite part of the Fourier transform [#4] of eq. (27) we get

$$\frac{1}{(2\pi)^3}\int\frac{\mathrm{d}^3(\boldsymbol{q}-\boldsymbol{q}')\exp[\mathrm{i}(\boldsymbol{q}-\boldsymbol{q}')\boldsymbol{r}]}{(\boldsymbol{q}-\boldsymbol{q}')^4}=-\frac{r}{8\pi}\,, \tag{28}$$

where $r=(r_k^2+r_l^2+r_m^2)^{1/2}$ is the appropriate relative distance, defined in the Lorentz invariant way. Due to the dependence of the right-hand side of eq. (27) on the difference $(q_k-q_k')^2+(q_l-q_l')^2+(q_m-q_m')^2$, the potential corresponding to eq. (14) is diagonal in the position space, i.e., proportional to $\delta^{(3)}(r_{klm}-r_{klm}')$.

Considering the hadron's constituents at large distances, i.e., in the domain corresponding to eqs. (25) and (26), the *average* momentum of constituents in the hadron's rest frame is of the order of m_p. Their energies in this frame are $E_i \approx \sqrt{2m_p^2}$, $i=1$, 2, and the velocities are $v_i \approx 2^{-1/2}c \approx 0.7c$. Thus, within the approximation $0.7 \approx 1$, the hadron's constituents, interacting at large distances with the linear potential in eq. (28), can be thought of as an open string, which has its ends moving with the light velocity, and has a constant energy per unit length.

If the virtuality of a single quark constituent p^2 is smaller than m_p^2, then the running quark mass is frozen [16] at the pole [17] value $m_p=m(p^2=m_p^2)$. This freezing is the consequence of treating m_p as the mass which appears in the equation of motion for the *non-perturbative* soft quark (antiquark) field [17]

$$\not{D}\psi^{\mathrm{NP}}(x)=-\mathrm{i}m_p\psi^{\mathrm{NP}}(x)\,, \tag{29}$$

where ψ^{NP} denotes the non-perturbative soft field. The pole mass m_p is the solution of eq. (4) for $p^2=m_p^2$, and it is determined by the quark condensate, which characterizes the soft quark (antiquark) field. Denoting by k the momentum associated with the soft quark (antiquark) field, and allowing themselves to consider p^2 both above and below m_p^2, Reinders and Stam [16] showed that by averaging over the directions of k in four dimensions, and then going with k to zero, results in the following replacement:

$$\langle\, |(p-k)^-|\,\rangle$$
$$\to p^{-2}\theta(p^2-m_p^2)+m_p^{-2}\theta(m_p^2-p^2)\,. \tag{30}$$

Thus, for $p^2<m_p^2$ instead of the p^{-2} factor in eq. (4) we have m_p^{-2}, and the running mass remains fixed at the value m_p. We note, that in eq. (4) for the running quark mass there appears explicitly the factor $\sqrt{\alpha_s(p^2)}$. Therefore, if $m(p)$ is frozen for $p^2<m_p^2$, then also $\alpha_s(p^2)$ is frozen at the value $\alpha_s(p^2=m_p^2)=0.88$, $[\alpha_s(m_p^2)]^{1/2}=0.94$.

To get the coefficient in front of r in the linear potential we must consider the two-body propagator

[#4] The non-singular part of eq. (28) is taken from the formula in ref. [15] eq. (D. 4), for $D=3$ dimensions:

$$\int\frac{\mathrm{d}^3x\exp(\mathrm{i}px)}{(x^2)^n}=2^{3-2n}\pi^{3/2}|p|^{2n-3}\frac{\Gamma(3/2-n)}{\Gamma(n)}\,,$$

which for $n=2$ gives $-\pi^2|p|$.

G_0, and the volume of integration in the Weinberg equation (10). $G_0^{-1}=D_0$ can be rewritten in such a way, that the internal energy of the hadron $E\equiv\sqrt{s}$ is explicitly exposed:

$$G_0^{-1}=D_0=s-M_0^2=\sqrt{s}\,(\sqrt{s}-M_0^2/\sqrt{s})\ . \tag{31}$$

The integration volume [2,11] is

$$\frac{dx\,d^2q_\perp}{16\pi^3x(1-x)}=2\frac{d^3q_{klm}}{(2\pi)^3M_0}\,, \tag{32}$$

and taking the meson case with $C=c_F=\frac{4}{3}$, we collect all coefficients in front of r using eqs. (14), (15), (16), (28), (31), (32), and (26) to approximate M_0 by $\sqrt{s}$:

$$\lambda=-\tfrac{4}{3}\cdot\tfrac{1}{4}\pi^3\alpha_s(\tfrac{3}{2})^2(P^+/2m)^2$$

$$\times\langle\,|(\alpha_s/\pi){:}G^a_{\mu\nu}G^{\mu\nu}_a{:}|\,\rangle(-1/8\pi)2/\sqrt{s}M_0$$

$$=(3\pi^2\alpha_s/32m_p^2)\langle\,|(\alpha_s/\pi){:}G^a_{\mu\nu}G^{\mu\nu}_a{:}|\,\rangle\ . \tag{33}$$

Using the universal value [4] of the gluon condensate

$$\langle\,|(\alpha_s/\pi){:}G^a_{\mu\nu}G^{\mu\nu}_a{:}|\,\rangle=(360\pm20\ \text{MeV})^4\ , \tag{34}$$

we get from eq. (33)

$$\lambda=(0.13\pm0.03)\ \text{GeV}^2\ . \tag{35}$$

The final task is to get the Regge behaviour, and the slope α' of the Regge trajectory. For that it is useful to denote

$$q_s^2\equiv\tfrac{1}{4}s-m^2\ , \tag{36}$$

and rewrite the propagator in the Weinberg equation (10) in the form

$$G_0=D_0^{-1}=(s-M_0^2)^{-1}=4(q_s^2-q^2)^{-1}$$

$$=(2/q_s)(q_s-q)^{-1}+(2/q_s)(q_s+q)^{-1}\ , \tag{37}$$

where $q^2\equiv q_k^2+q_l^2+q_m^2$. Now we follow Mandelstam [18], and note that for large s and q, which are relevant for the Regge behaviour, the first term on the right-hand side of eq. (37) plays the dominant rôle, so we only retain that term. Then, repeating Mandelstam's arguments [18] we go from the momentum to the position representation [#5], writing

$$|q|=[l(l+1)/r^2+q_r^2]^{1/2}\ ,$$

and note [18], that in the interesting domain $l(l+1)/r^2\gg q_r^2$, and for $|r-R|<R$, where R is some average value, we get

$$|q|\approx l/r+(R/2l)q_r^2\ . \tag{38}$$

For large s we have $q_s\approx\frac{1}{2}\sqrt{s}$, and from eq. (37) we get $G_0\approx4s^{-1/2}(\frac{1}{2}\sqrt{s}-q)^{-1}$. Including that, together with eq. (32), in which for large s and q we take $M_0\approx\sqrt{s}$, we get from eq. (38) and the Weinberg equation (10) the following partial wave (see footnote 5) equation

$$[l/r+(R/2l)q_r^2-\tfrac{1}{2}\sqrt{s}+\tfrac{1}{2}\lambda r]\phi_l(jr)=0\ . \tag{39}$$

Continuing with Mandelstam's arguments [18], we note that the important values of r are near the minimum R of the sum of two r-dependent terms in eq. (39), giving $R^2=2l/\lambda$. Next we expand l/r in powers of $r'=R-r$, and, denoting $\mu\equiv l/R$, we get from eq. (39) the following harmonic oscillator equation:

$$[q_r^2/2\mu+(\mu/R^2)r'^2]\phi_l=(\tfrac{1}{2}\sqrt{s}-2\mu)\phi_l\ . \tag{40}$$

The frequency is $\omega=\sqrt{2}/R$, and the eigenvalues of eq. (40) are

$$\sqrt{s}=4l/R+(2\sqrt{2}/R)(n+\tfrac{1}{2})\ , \tag{41}$$

where n is the quantum number. For large $l=\frac{1}{2}R^2\lambda$, and fixed n, the first term in eq. (41) dominates, giving

$$s\approx16l^2/R^2=16l^2(2l)^{-1}\lambda=8\lambda l\ . \tag{42}$$

Thus, we find the linear Regge trajectory, with the slope α' determined by λ, and we get

$$\alpha'=(8\lambda)^{-1}=(1.0\pm0.2)\ \text{GeV}^{-2}\ ,$$

taking λ from eq. (35). Using the explicit form for λ in eq. (33), and m_p^2 obtained [1] from eq. (4) as

$$m_P^2=(\tfrac{8}{9}\pi\langle\,|\sqrt{\alpha_s}{:}\bar\psi\psi{:}|\,\rangle)^{2/3}\alpha_s^{\ 1/3}\ , \tag{43}$$

we get

[#5] The angular momentum operator in general presents some problems in the light front framework. However, in the case of the linear potential, which has the Fourier transform $\sim[(q_k-q_k')^2+(q_l-q_l')^2+(q_m-q_m')^2]^{-2}$ the rotational symmetry is manifest.

$$\alpha' \langle | (\alpha_s/\pi) : G^a_{\mu\nu} G^{\mu\nu}_a : | \rangle$$
$$= (16/9\pi)(3\pi\alpha_s^2)^{-1/3} (\langle | \sqrt{\alpha_s} : \bar{\psi}\psi : | \rangle)^{2/3} , \quad (44)$$

where $\alpha_s = \alpha_s(p^2 = m_p^2)$. This is an interesting relation, allowing us to correlate the quark and the gluon condensates. A similar relation between these condensates was recently shown by Nussinov [19,5].

The appearence of the linear Regge trajectory for a linear potential, treated with relativistic kinematics, was first obtained in the WKB approximation by Kang and Schnitzer [20], and was also discussed more recently in ref. [21].

Having Regge behaviour with the definite slope α' we can infer the value of the string tension k from the Nambu [22] model

$$k = 1/2\pi\alpha' = (4/\pi)\lambda = (0.17 \pm 0.03) \text{ GeV}^2 ,$$

with λ taken from eq. (35).

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References

[1] J.M. Namysłowski, Nonperturbative extension of the light front QCD, ORO5126-307,U of Md.PP#87-064, submitted to Phys. Rev. D.
[2] J.M. Namysłowski, Progr. Part. Nucl. Phys. Vol. 14, ed. A. Faessler (Pergamon Press, New York, 1984) p. 49.
[3] S. Mandelstam, Phys. Rev. D 20 (1979) 3223.
[4] L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep. 127 (1985) 1.
[5] S. Weinberg, Phys. Rev. 150 (1966) 1313.
[6] S.J. Brodsky, R. Roskies and R. Suaya, Phys. Rev. D 8 (1973) 4574.
[7] A. Casher, Phys. Lett. B 83 (1979) 395.
[8] G.P. Lepage and S.J. Brodsky, Phys. Rev. D 22 (1980) 2157.
[9] A.S. Wightman, in: Lectures on invariance in relativistic quantum mechanics, in dispersion relations and elementary particles, eds. C. De Witt and R. Omnes (Herman, Paris) p. 198.
[10] A.J. McFarlane, Rev. Mod. Phys. 34 (1962) 41.
[11] J.M. Namysłowski, Phys. Rev. D 18 (1978) 3676.
[12] P.M. Fishbane and J.M. Namysłowski, Phys. Rev. D 21 (1980) 2406.
[13] J.M. Namysłowski, Phys. Rev. C 20 (1979) 2356.
[14] R.N. Faustov, Ann. Phys. (NY) 78 (1973) 176.
[15] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Fortschr. Phys. 32 (1984) 585.
[16] L.J. Reinders and K. Stam, Bonn Univ. preprint, Bonn-HE-86-21.
[17] V. Elias and M. Scadron, Phys. Rev. D 30 (1984) 647; V. Elias, M. Scadron and R. Tarrach, Phys. Lett. B 162 (1985) 176.
[18] S. Mandelstam, Phys. Rev. D 20 (1979) 3223.
[19] S. Nussinov, private communication; A. Casher, H. Neuberger and S. Nussinov, Phys. Rev. D 20 (1979) 179.
[20] J.S. Kang and H.J. Schnitzer, Phys. Rev. D 12 (1975) 841.
[21] J. Dias de Deus and J. Pulido, Z. Phys. C 9 (1981) 255; J.L. Basdevant and S. Boukraa, Z. Phys. C 28 (1985) 413; A. Martin, Regge trajectories in the quark model, CERN TH.4259/85.
[22] Y. Nambu, Phys. Rev. D 10 (1974) 4262.