Midterm

COMS E6998 Formal Verification of System Software

Name ____________ **UNI** ____________

Honor Code Statement
I certify that I have not discussed the contents of the exam with any other student. I have not cheated on this exam in any way, nor do I know of anyone else who has cheated.
Signaure: _______________

Directions/Notes

- Write your name and UNI on this cover page.
- No notes, books, or calculators are permitted.
- Be sure to sign the honor code statement when you are finished.
- The points add up to slightly more than 100, but the exam is worth 100 points.

Score Breakdown

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Score															
Value	4	6	8	8	8	8	8	8	8	8	5	5	8	10	102

Problem 1 (4 pt)

Construct a truth table for “$p \to \neg q \wedge (q \vee p)$”

p	q	$\neg q$	$q \vee p$	$\neg q \wedge (q \vee p)$	$p \to \neg q \wedge (q \vee p)$

Problem 2 (6 pt)

Show "$\neg p \lor q \vdash p \to q$" using syntactic reasoning.

Proof.

1. $\neg p \lor q$ (Pre)

Problem 3 (8 pt)

Show "$p \to q \vdash \neg p \lor q$" using syntactic reasoning. *Hint:* you can use rule $\neg\neg e$.

Proof.

1. $p \to q$ (Pre)

Problem 4 (8 pt)

Prove **LEM** (Law of Excluded Middle) " $\vdash p \vee \neg p$" via syntactic reasoning. *Hint:* you can use rule $\neg\neg e$.

Proof.

Problem 5 (8 pt)

Prove $\vdash ((p \to q) \to p) \to p$" using **LEM** via syntactic reasoning.

Proof.

Problem 6 (8 pt)

Use the following premises to deduce the conclusion “r” via syntactic reasoning:

Proof.

1. $s \to p$ **(Pre)**
2. $w \land \neg z$ **(Pre)**
3. $\neg p$ **(Pre)**
4. $\neg z \to (s \lor q) \lor r$ **(Pre)**
5. $w \lor y \to \neg q$ **(Pre)**

Problem 7 (8 pt)

Convert $((p \wedge q) \vee (r \wedge s)) \vee (\neg q \wedge (p \rightarrow t))$ into CNF.

$((p \wedge q) \vee (r \wedge s)) \vee (\neg q \wedge (p \rightarrow t))$

$\equiv$

Problem 8 (8 pt)

Prove the following statement via syntactic reasoning:

$$[\forall x.\, Q(x) \to R(x)],\ [\exists y.\, P(y) \land Q(y)] \ \vdash\ \exists x.\, P(x) \land R(x)$$

Proof.

1. $\forall x.\, Q(x) \to R(x)$ **(Pre)**
2. $\exists y.\, P(y) \land Q(y)$ **(Pre)**

Problem 9 (8 pt)

Prove the following lemma via syntactic reasoning.

Lemma 4.2 $\neg\exists x.\, A \vdash \forall x.\, \neg A$

Proof.

1. $\neg\exists x.\, A$ **(Pre)**

Problem 10 (8 pt)

Show “$\neg\forall x.\, A \ \vdash\ \exists x.\, \neg A$” via syntactic reasoning. *Hint:* you can use **Lemma 4.2**.

Proof.

1. $\neg\forall x.\, A$ **(Pre)**

Problem 11 (10 pt)

Given the following program C:

```
if (x < 5) {
  x := x * x
} else {
  y := 2;
  x := y + x;
}
```

(a) (5pt) Show $\{x \leq -3 \lor x > 9\}\ C\ \{x \geq 9\}$ using inference rules.

Proof.

$\{x \leq -3 \lor x > 9\}$ **(Pre)**

```
if (x < 5) {



  x := x * x


} else {



  y := 2;



  x := y + x;


}
```

$\{x \geq 9\}$

(b) (5pt) Calculate the *weakest precondition* of program C:

$wp(C, x \geq 9)$

$\equiv$

Problem 12 (8 pt)

Prove that

$$\{x = x_0 \wedge y = y_0\}\ x := x + y;\ y := x - y;\ x := x - y\ \{y = x_0 \wedge x = y_0\}$$

Proof.

$\{x = x_0 \wedge y = y_0\}$ **(Pre)**

```
x:=x+y;
```

```
y:=x-y;
```

```
x:=x-y
```

$\{y = x_0 \wedge x = y_0\}$

Problem 13 (10 pt)

Prove that $\{x = x_0\}\ FACT\ \{y = x_0!\}$ for the following program $FACT$:

```
// Program FACT
y := 1;
while (x > 0) {
  y := y * x;
  x := x - 1;
}
```

where we define $x!$ as below:

$$x! \stackrel{\text{def}}{=} \begin{cases} x \times (x-1)! & (\text{if } x > 0) \\ 1 & (\text{otherwise}) \end{cases}$$

Proof.

$\{x = x_0\}$ **(Pre)**

```
y := 1;
```

```
while (x > 0) {
```

```
  y := y * x;
```

```
  x := x - 1;
```

```
}
```

$\{y = x_0!\}$