Supplementary Material of Video Stabilization Using IMU-Based Upright Adjustment Prior

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In the supplementary material, a detailed derivation of the homography matrix decomposition algorithm will be provided.

To independently process the different types of motion within a homography transformation, we develop a new homography decomposition algorithm. This algorithm can decompose a 2D homography transformation between two frames into a combination of different fundamental 2D motions. Specifically, the homography matrix is decomposed into translational motion, rotational motion, scaling motion, shear motion, and perspective motion.

I. Motion Components

For a given homography matrix:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}, \tag{1}$$

the proposed algorithm decomposes the homography matrix $\mathbf{H}$ to obtain five types of motion components. The sum of the degrees of freedom of these components is 8, which matches the 8 degrees of freedom of the homography matrix. In theory, the decomposition can be performed as follows:

$$\mathbf{H} = \mathbf{H}_{trans} \cdot \mathbf{H}_{rot} \cdot \mathbf{H}_{shear} \cdot \mathbf{H}_{sca} \cdot \mathbf{H}_{perp}, \tag{2}$$

where $\mathbf{H}_{trans}$ is translation component, $\mathbf{H}_{rot}$ is rotation component, $\mathbf{H}_{shear}$ is shearing component, $\mathbf{H}_{sca}$ is scaling component, and $\mathbf{H}_{perp}$ is perpective component. Their detailed forms are presented in the paper. It should be noted that the homography matrix has homogeneity. The result on the right side of Eq.(2) needs to be normalized by dividing by the element in the third row and third column, making this element equal to 1.

The Eq.(2) can be transformed into a system of equations regarding the different motion components of the homography transformation. The center of motion (c_x, c_y) is usually set at the center of the frame, as this configuration aligns better with human visual perception. Given the eight unknown elements of the homography matrix $\{h_{11}, h_{12}, h_{13}, h_{21}, h_{22}, h_{23}, h_{31}, h_{32}\}$ and the center of motion (c_x, c_y), the process of solving for motion components $\{t_x, t_y, \theta, \lambda_x, \lambda_y, s_h, \phi_x, \phi_y\}$ can be carried out. The solved motion components can be further converted into their corresponding motion matrices.

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II. Computational Process

In this section, we will provide the detailed computational process for homography matrix decomposition. Considering that the computations for shear in the x-axis direction and shear in the y-axis direction are slightly different, we will present the computational processes for both cases.

A. In the case of shear in the x-axis direction

In Eq.(2), we first compute the right side of the equation, then normalize it so that the element in the third row and third column becomes 1. Setting the normalized right side equal to the left side yields the system of equations:

$$\begin{cases} h_{11} = \dfrac{\lambda_{xc} + \phi_x x_{ct}}{n_\phi}, & \text{(3a)} \\ h_{12} = \dfrac{-\lambda_{ysmc} + \phi_y x_{ct}}{n_\phi}, & \text{(3b)} \\ h_{13} = \dfrac{x_{ct} - c_x h_{11} n_\phi - c_y h_{12} n_\phi}{n_\phi}, & \text{(3c)} \\ h_{21} = \dfrac{\lambda_{xs} + \phi_x y_{ct}}{n_\phi}, & \text{(3d)} \\ h_{22} = \dfrac{-\lambda_{ycps} + \phi_y y_{ct}}{n_\phi}, & \text{(3e)} \\ h_{23} = \dfrac{y_{ct} - c_x h_{21} n_\phi - c_y h_{22} n_\phi}{n_\phi}, & \text{(3f)} \\ h_{31} = \dfrac{\phi_x}{n_\phi}, & \text{(3g)} \\ h_{32} = \dfrac{\phi_y}{n_\phi}. & \text{(3h)} \end{cases}$$

For convenience, we define the following intermediate variables:

$$\begin{cases} n_\phi = 1 - c_x\phi_x - c_y\phi_y, & \text{(4a)} \\ \lambda_{xc} = \lambda_x \cos\theta, & \text{(4b)} \\ \lambda_{xs} = \lambda_x \sin\theta, & \text{(4c)} \\ \lambda_{ysmc} = \lambda_y(\sin\theta - s_h\cos\theta), & \text{(4d)} \\ \lambda_{ycps} = \lambda_y(\cos\theta + s_h\sin\theta), & \text{(4e)} \\ x_{ct} = c_x + t_x, & \text{(4f)} \\ y_{ct} = c_y + t_y. & \text{(4g)} \end{cases}$$

First, from Eq.(3g) and Eq.(3h), we can derive:

$$\phi_x = \frac{h_{31}}{1 + c_x h_{31} + c_y h_{32}}, \tag{5}$$

$$\phi_y = \frac{h_{32}}{1 + c_x h_{31} + c_y h_{32}}. \tag{6}$$

In this way, n_ϕ has become a known element. For the remaining unknowns, we first solve for the intermediate variables. From Eq.(3a), Eq.(3b), Eq.(3c), Eq.(3d), Eq.(3e), and Eq.(3f), we can obtain:

$$x_{ct} = n_\phi(h_{13} + c_x h_{11} + c_y h_{12}), \tag{7}$$

$$y_{ct} = n_\phi(h_{23} + c_x h_{21} + c_y h_{22}), \tag{8}$$

$$\lambda_{xc} = n_\phi(h_{11} - \phi_x(h_{13} + c_x h_{11} + c_y h_{12})), \tag{9}$$

$$\lambda_{xs} = n_\phi(h_{21} - \phi_x(h_{23} + c_x h_{21} + c_y h_{22})), \tag{10}$$

$$\lambda_{ysmc} = -n_\phi(h_{12} - \phi_y(h_{13} + c_x h_{11} + c_y h_{12})), \tag{11}$$

$$\lambda_{ycps} = n_\phi(h_{22} - \phi_y(h_{23} + c_x h_{21} + c_y h_{22})). \tag{12}$$

After solving for the above intermediate variables, we can directly obtain from Eq.(4f) and Eq.(4g):

$$t_x = x_{ct} - c_x, \tag{13}$$

$$t_y = y_{ct} - c_y. \tag{14}$$

Then, according to Eq.(4b), Eq.(4c), Eq.(4d), and Eq.(4e), two sets of solutions can be obtained:

$$\begin{cases} s_h = \dfrac{\lambda_{xs}\lambda_{ycps} - \lambda_{xc}\lambda_{ysmc}}{\lambda_{xc}\lambda_{ycps} + \lambda_{xs}\lambda_{ysmc}}, & (15a) \\ \theta = -2\arctan\dfrac{\lambda_{xc} + \sqrt{z}}{\lambda_{xs}}, & (15b) \\ \lambda_x = -\sqrt{z}, & (15c) \\ \lambda_y = -\dfrac{\lambda_{xc}\lambda_{ycps} + \lambda_{xs}\lambda_{ysmc}}{\sqrt{z}}, & (15d) \end{cases}$$

$$\begin{cases} s_h = \dfrac{\lambda_{xs}\lambda_{ycps} - \lambda_{xc}\lambda_{ysmc}}{\lambda_{xc}\lambda_{ycps} + \lambda_{xs}\lambda_{ysmc}}, & (16a) \\ \theta = -2\arctan\dfrac{\lambda_{xc} - \sqrt{z}}{\lambda_{xs}}, & (16b) \\ \lambda_x = \sqrt{z}, & (16c) \\ \lambda_y = \dfrac{\lambda_{xc}\lambda_{ycps} + \lambda_{xs}\lambda_{ysmc}}{\sqrt{z}}, & (16d) \end{cases}$$

where $z = \lambda_{xc}^2 + \lambda_{xs}^2$. Both sets of solutions satisfy the system of equations and the homography matrix decomposition formula. In practice, the scaling factors between video frames are usually positive numbers; therefore, we adopt the set of solutions from Eq.(16a), Eq.(16b), Eq.(16c), and Eq.(16d) as the result. After calculating all the necessary parameters for the matrix decomposition, we substitute them into each component of the decomposition, thereby completing the homography matrix decomposition.

B. In the case of shear in the y-axis direction

On the other hand, we present the computation process based on shear in the y-axis direction for decomposition in specific cases. We can obtain a new system of equations by Eq.(2):

$$\begin{cases} h_{11} = \dfrac{\lambda_{xcms} + \phi_x x_{ct}}{n_\phi}, & (17a) \\ h_{12} = \dfrac{-\lambda_{ys} + \phi_y x_{ct}}{n_\phi}, & (17b) \\ h_{13} = \dfrac{x_{ct} - c_x h_{11} n_\phi - c_y h_{12} n_\phi}{n_\phi}, & (17c) \\ h_{21} = \dfrac{\lambda_{xspc} + \phi_x y_{ct}}{n_\phi}, & (17d) \\ h_{22} = \dfrac{-\lambda_{yc} + \phi_y y_{ct}}{n_\phi}, & (17e) \\ h_{23} = \dfrac{y_{ct} - c_x h_{21} n_\phi - c_y h_{22} n_\phi}{n_\phi}, & (17f) \\ h_{31} = \dfrac{\phi_x}{n_\phi}, & (17g) \\ h_{32} = \dfrac{\phi_y}{n_\phi}. & (17h) \end{cases}$$

For convenience, we define the following intermediate variables:

$$\begin{cases} n_\phi = 1 - c_x\phi_x - c_y\phi_y, & (18a) \\ \lambda_{xcms} = \lambda_x(\cos\theta - s_h\sin\theta), & (18b) \\ \lambda_{xspc} = \lambda_x(\sin\theta + s_h\cos\theta), & (18c) \\ \lambda_{ys} = \lambda_y\sin\theta, & (18d) \\ \lambda_{yc} = \lambda_y\cos\theta, & (18e) \\ x_{ct} = c_x + t_x, & (18f) \\ y_{ct} = c_y + t_y. & (18g) \end{cases}$$

The solutions for ϕ_x and ϕ_y are the same as before and will not be reiterated. From Eq.(17a), Eq.(17b), Eq.(17c), Eq.(17d), Eq.(17e), and Eq.(17f), we can obtain:

$$x_{ct} = n_\phi(h_{13} + c_x h_{11} + c_y h_{12}), \tag{19}$$

$$y_{ct} = n_\phi(h_{23} + c_x h_{21} + c_y h_{22}), \tag{20}$$

$$\lambda_{xcms} = n_\phi(h_{11} - \phi_x(h_{13} + c_x h_{11} + c_y h_{12})), \tag{21}$$

$$\lambda_{xspc} = n_\phi(h_{21} - \phi_x(h_{23} + c_x h_{21} + c_y h_{22})), \tag{22}$$

$$\lambda_{ys} = -n_\phi(h_{12} - \phi_y(h_{13} + c_x h_{11} + c_y h_{12})), \tag{23}$$

$$\lambda_{yc} = n_\phi(h_{22} - \phi_y(h_{23} + c_x h_{21} + c_y h_{22})). \tag{24}$$

The solutions for t_x and t_y are the same as before and will not be reiterated. According to Eq.(4b), Eq.(4c), Eq.(4d), and Eq.(4e), two sets of solutions can be obtained:

$$\begin{cases} s_h = \dfrac{\lambda_{xspc}\lambda_{yc} - \lambda_{xcms}\lambda_{ys}}{\lambda_{xspc}\lambda_{ys} + \lambda_{xcms}\lambda_{yc}}, & (25a) \\ \theta = -2\arctan\dfrac{\lambda_{yc} + \sqrt{z}}{\lambda_{ys}}, & (25b) \\ \lambda_x = -\dfrac{\lambda_{xcms}\lambda_{yc} + \lambda_{xspc}\lambda_{ys}}{\sqrt{z}}, & (25c) \\ \lambda_y = -\sqrt{z}. & (25d) \end{cases}$$

$$\begin{cases} s_h = \dfrac{\lambda_{xspc}\lambda_{yc} - \lambda_{xcms}\lambda_{ys}}{\lambda_{xspc}\lambda_{ys} + \lambda_{xcms}\lambda_{yc}}, & \text{(26a)} \\ \theta = -2\arctan\dfrac{\lambda_{yc} - \sqrt{z}}{\lambda_{ys}}, & \text{(26b)} \\ \lambda_x = \dfrac{\lambda_{xcms}\lambda_{yc} + \lambda_{xspc}\lambda_{ys}}{\sqrt{z}}, & \text{(26c)} \\ \lambda_y = \sqrt{z}. & \text{(26d)} \end{cases}$$

For the same reasons mentioned earlier, we select the set of solutions with a positive scaling factor (Eq.(26a), Eq.(26b), Eq.(26c), and Eq.(26d)) as the final result.