

2011

# Mathematical Contest In Modeling

## Certificate of Achievement

Be It Known That The Team Of

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Of

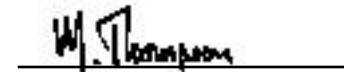
China Agricultural University

Was Designated As  
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Frank R. Giordano, Contest Director

Administered by The Consortium for Mathematics and Its Applications



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## 2011 MCM

### Problem B

## Repeater Coordination

The VHF radio spectrum involves line-of-sight transmission and reception. This limitation can be overcome by “repeaters,” which pick up weak signals, amplify them, and retransmit them on a different frequency. Thus, using a repeater, low-power users (such as mobile stations) can communicate with one another in situations where direct user-to-user contact would not be possible. However, repeaters can interfere with one another unless they are far enough apart or transmit on sufficiently separated frequencies.

In addition to geographical separation, the “continuous tone-coded squelch system” (CTCSS), sometimes nicknamed “private line” (PL), technology can be used to mitigate interference problems. This system associates to each repeater a separate subaudible tone that is transmitted by all users who wish to communicate through that repeater. The repeater responds only to received signals with its specific PL tone. With this system, two nearby repeaters can share the same frequency pair (for receive and transmit); so more repeaters (and hence more users) can be accommodated in a particular area.

For a circular flat area of radius 40 miles radius, determine the minimum number of repeaters necessary to accommodate 1,000 simultaneous users. Assume that the spectrum available is 145 to 148 MHz, the transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency, and there are 54 different PL tones available.

How does your solution change if there are 10,000 users?

Discuss the case where there might be defects in line-of-sight propagation caused by mountainous areas.

# The optimization of repeater coordination

Team 9592  
February 10, 2011

## Summary

Our goal is a model that can determine the minimum number of repeaters necessary to users. We assume that users in the region are uniformly distributed and that repeaters' radiation radius can be regulated.

We analyze how best to optimize the model, in terms of:

- Location of repeaters.
- Reasonable use of PL tones.
- Effective processing methods against complex.

In detail, the first aspect involves user's communication radius and covering methods. The second one needs to consider the number of frequency pairs in the spectrum available and channel multiplex. What's more, the limitation of PL tones provided calls for combinatorial problem. Additionally, how to decompose irregular topography to be regular is the key to the solution.

With the help of genetic algorithm, we use the honeycomb cellular structure to determine repeater's site, which can meet minimum seamless coverage. Based on the statistic, we estimate the effective radius of user and spatial channel number (or the number of frequency pairs). We use C++ program to calculate the minimum number of repeaters necessary to accommodate 1,000 and 10,000 simultaneous users. Based on our model, we find that completely simultaneity needs quite a large number of repeaters, while proper time-sharing multiplexing can sharply decrease the number. By Queuing theory, we simulate that users only need to wait for a few seconds to send their message. Therefore, we treat simultaneity as extremely short waiting time in order to reduce the number of the repeaters.

Through sensitivity analysis, we can see that user radius and the channels number affect the number of repeaters greatly.

As to the mountain areas, we divide obstacles into several segments. We innovatively deal with the mountains as lines. In this way, fade zones (or shadows) become apart. Consequently, we solve the problem of complete coverage and estimates the number of added repeater sites.

As we give the priority to professional background, our model is simple with reasonable assumptions and our solutions are feasible.

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## 1. Introduction

Using the “continuous tone-coded squelch system” (CTCSS), also called “private line” (PL), the transmission and reception of signal will be efficient and safe (Tian). For the same area, the same bandwidth can be used to accommodate more users with the technology. Then, we propose cellular theory to determine the number of necessary repeaters.

## 2. Assumptions and Variables

### 2.1 Assumptions

Based on communication technique given, and the absence of the density and distribution of users, we need to make some assumptions:

- The receiver gets the sender’s message without knowing the sender’s information, but he can get it when they’re communicating.
- The repeaters send the message immediately when get it.
- Each repeater has one PL tone only, then receives and transmits signals with the same PL tone; each user has 54 PL tones and can choose a PL tone to transmit or receive messages. All users use the same equipment.
- Each two repeaters can transmit or receive messages freely and can meet multiple frequency pairs simultaneously. The effective radius of each repeater is further than 80miles and the height of theirs is sufficient to interconnect.

### 2.2 Variables

---

$n$	The number of laps of regular hexagon
$r_n$	Radius of maximum inscribed circle in N laps of regular hexagon
$r_k$	Radius of $k$ th inscribed circle in N laps of regular hexagon
$a_n$	The number of regular hexagons in each lap
$R_u$	The effective radius of users’ radiation
$\rho_r$	The user density per regular hexagon
$m$	The number of PL tones per regular hexagon
$r_c$	Radius of a normal circle
$h$	The number of regular polygon edges
$X$	The number of regular polygons

---

$p$	The channel number
$M$	The total number of users

---

### 3. Models

Our analytic model consists of a honeycomb cellular (Yuan 2010) structure (Figure.1). We use the structure to determine every repeater's site. We give concise proof, that the structure meets the requirements of minimum seamless coverage. According to the known condition, we estimate the effective radius of user and spatial channel number (or the number of frequency pairs). Based on the established model, we try to determine the minimum number of repeaters necessary to accommodate 1,000 and 10,000 simultaneous users. However, in our model, completely simultaneity needs quite a large number of repeaters. We found that proper time-sharing multiplexing sharply decreases the number of repeaters and users just need to wait for a few seconds. Therefore, we attempt to consider simultaneity as extremely short waiting time. The model also discusses the influence of user radius and channel number to the repeater number.

#### 3.1 The effective radius of each user

The effective radius of each user's equipment varies in a large scope. To be specific, user's equipment can be interphone, radio and so forth. After consulting many kinds of products of different companies, we realize that 4miles to 10miles is reasonable effective radius of each user in barrier-free region. MOTO GP2000, for instance, can reach 4km as maximum distance. MOTO CLS1418 can reach 8km. American triple-a interphone can reach up to 18km (Beijing sanxin communication 2007).  $R_u$  can be the value from 4miles to 18miles.

#### 3.2 Service Area of a repeater

Based on our assumption, once a sender's signal reaches a repeater, its information can be transmitted to the entire scope. In a plane area, there are no problems with receiving messages. We define the area where one sender's (or user's) signal can reach as the sender's effective circle (surely it is a circle with radius of  $R_u$ ). Thus wise, we should give the priority to the problem, how to make sure that, for any user, there are at least one repeater inside the user's effective circle. The problem can be transformed into how to use least effective circles to seamlessly cover a circular flat area of 40 miles radius.

### 3.3 The Justification of Cells

#### 3.3.1 Analyzing

To determine the minimum number of repeaters necessary to accommodate 1,000 simultaneous users means using the Minimum number of small of circles in order to meet seamless coverage. However, we cannot meet seamless coverage without overlap. So we use an inscribed regular polygon to replace the circle. Consequently, the question is to use the minimum number of regular polygons to meet seamless coverage without overlap.

#### 3.3.2 Solution

The angle of each internal angle in regular polygon is  $\frac{(h-2)180^\circ}{h}$ .

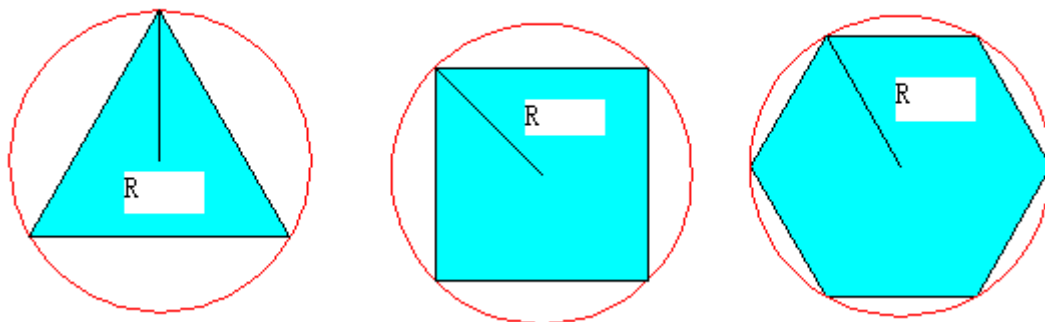
What is more, in the linking node area,

$$X \frac{(h-2)180}{h} = 360^\circ \quad (\text{Zhao\&Zhang 2010}) \quad \text{Eq 1}$$

So,

$$X = 2 + \frac{4}{h-2} \quad \text{Eq 2}$$

In order to let X to be positive integer, we can choose  $h=3, X_3=3, h=4, X_4=4, h=6, X_6=3$ . We can use congruent equilateral triangles, square, hexagon to meet seamless coverage.



**Figure.1 Equilateral triangles, Square, Regular hexagon**

We can see clearly from the three graphs that coverage of the circle is much higher if we use hexagon coverage model. In summary, hexagon coverage model can be proper choice to meet seamless coverage which uses minimum number of circles.

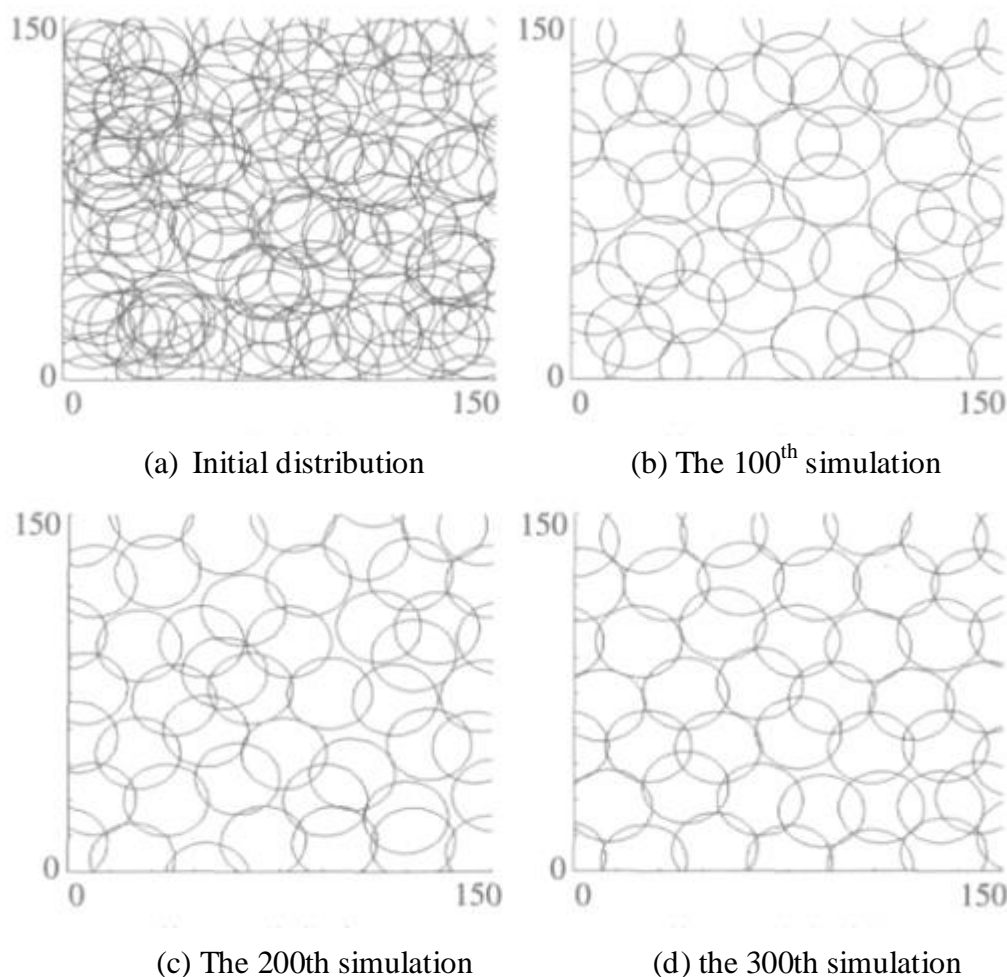
### 3.3.3 Simulation

The simulation is quoted from Lv's essay(2010).

This experiment based on MATLAB 7.0 which use genetic algorithm to simulate optimal method of seamless coverage.

- Monitoring plane is  $150 \times 150$ ,  $R=15$ .
- Initial population random deployment nodes  $n=150$ .
- Based on genetic algorithm, Crossover probability is 0.8.
- Mutation probability is 0.05. This experiment made data records while ran for 100 generations.

We can see that every circle in graph (d) can be replaced by regular hexagon.



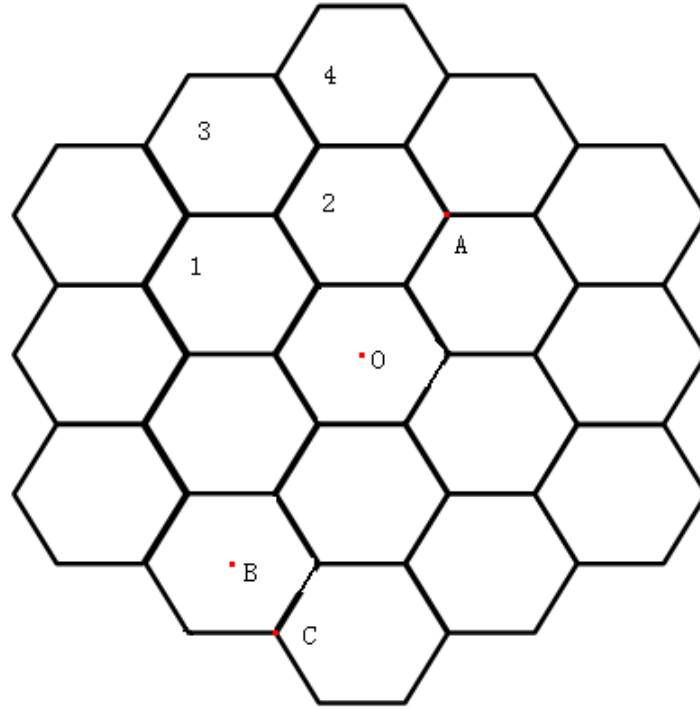
**Figure.2 Simulation Figures (Lv, Cui&Hou 2010)**

### 3.4 The Number of Cells

For a circular flat area of radius 40 miles radius, determine the minimum number of repeaters necessary to accommodate 1,000 simultaneous users. Which means using minimum number of hexagons to meet seamless coverage? The most important thing we must pay attention is the coverage area is circle.



Consider about the maximum inscribed circle in a regular hexagon including  $n$  laps:



**Figure.3 honeycomb cellular structure**

Like figure.3,  $O$  is the center,  $C$  is a vertex of the outermost odd number lap of regular hexagons, while  $A$  is a vertex of adjacent even number lap of regular hexagons, and  $B$  is the center of the outermost regular hexagon. The maximum inscribed circle is defined as a circle with a center  $O$  and crosses  $C$ .

When  $n$  is even,  $r_n(OA)$  is an arithmetic sequence, and  $r_2 = 2$ ,  $r_k - r_{k-2} = 3 (k \geq 2)$ .

$$r_n = \frac{3}{2}n - 1. \quad \text{Eq 3}$$

When  $n$  is odd, the process to solve the distance of  $r_n(OC)$  is difficult. But  $b_n(OB)$  is also an arithmetic sequence, and  $b_3 = 2$ ,  $b_1 = 1$ ,  $b_k - b_{k-2} = 3 (k \geq 2)$ .

$$b_n = \frac{3}{2}(n-1) \quad \text{Eq 4}$$

By the geometry,

$$OC = \sqrt{\left(OB + \frac{1}{2}\right)^2 + \frac{3}{4}}, \quad \text{Eq 5}$$

$$\therefore r_n = \sqrt{\left(b_n + \frac{1}{2}\right)^2 + \frac{3}{4}}. \quad \text{Eq 6}$$

So,

$$\begin{cases} r_n = \frac{3}{2}n - 1 & n \geq 2 \quad \& \quad n \text{ is even;} \\ r_n = \sqrt{\left(\frac{3}{2}(n-1) + \frac{1}{2}\right)^2 + \frac{3}{4}} & n \geq 3 \quad \& \quad n \text{ is odd.} \end{cases} \quad \text{Eq 7}$$

When the number of laps is known, we should calculate number of regular hexagons.  $a_n$  is the number of regular hexagons in each lap.  $a_1 = 1, a_2 = 6$ . As figure.1 shows, when  $k \geq 2$ , in the  $k$ th lap, each cell must be adjacent to a cell in the  $k+1$ th lap, such as cell 2 and 4. There is a cell in the  $k+1$ th lap, between every two cells in the  $k$ th lap, e.g. cell 3 is between cell 1 and 2. Then,

$$a_{k+1} = a_k + a_k = 2a_k, k \geq 2. \quad \text{Eq 8}$$

So,

$$N = 1 + 6 + 6 \times 2^1 + 6 \times 2^2 + \dots + 6 \times 2^{n-2} = 1 + 6 \times \frac{1 - 2^{n-2}}{1 - 2} = 3 \times 2^{n-1} - 5. \quad \text{Eq 9}$$

**Table.1 The numerical relationship of  $R_u$  and  $N$**

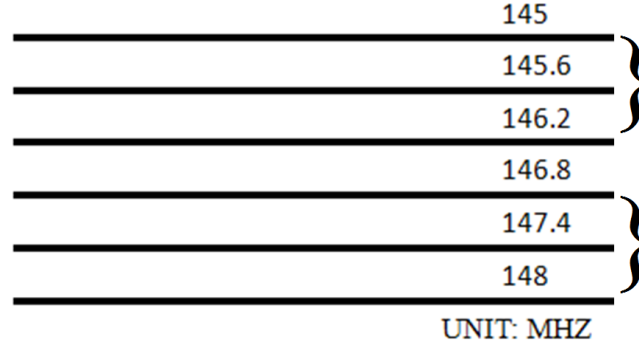
$R_u$	1	3	5	8
N	402,653,179	1,531	187	43

### 3.5 The Number of Frequency pairs (channels)

Channel spacing - the frequency spacing between adjacent frequency allocations - may be 50, 30, 25, 15 or 12.5 kHz, depending upon the convention in use in the area of the repeater. Adjacent channel interference – the interference resulting when a strong signal from another repeater occurs close to the repeater frequency you are monitoring. Your transceiver receiver is unable to reject this signal which may be only 15 or 20 KHz off frequency (Duncan 2007).

According to the given data and the information above, we stipulate that the standard channel width is 15 kHz and channel spacing is 15 kHz. In latter discussion, it can change from 15 to 50 kHz (thus audio signal may be satisfied). Then, we need to calculate the channel number.

In the entire range, the available spectrum is 145 to 148MHz and frequency pairs have 600 kHz separation. So the spectrum can be divided in 5 part, each has 600 kHz bandwidth (145.0, 145.6, 146.2, 146.8, 147.4, 148MHz). One of the bandwidth utilization methods is shown in figure.4. We empty out part 3 for, so that channel adjustment can be more flexible.



**Figure.4 one bandwidth utilization method**

The channel number  $p$ :

$$p = \frac{\text{bandwidth of a part}}{\text{channel width} + \text{channel spacing}} \times 2 = \frac{600\text{KHz}}{15\text{KHz} + 15\text{KHz}} \times 2 = 40 \quad \text{Eq 10}$$

If lower the channel width or optimize the channel adjustment,  $p$  can be more than 60.

### 3.6 Channel capacity of space

This part is of the most importance in our paper. We have got to the point that the spectrum accommodates  $p$  channels. According to the assumption, each repeater can meet multiple frequency pairs simultaneously. With only one PL, a repeater can simultaneously service  $p$  senders, while all other receivers can select those  $p$  channels to listen (because each user has 54 PL tones). Please note that  $p$  only restricts the number of senders, because it is the channel capacity of space that limited senders. Take PLs that help achieve channel multiplexing in to account; the channel capacity can increase several times over. The relational expression is:

$$\begin{aligned} \text{capacity of space} &= \text{the channel number} \times \text{the number of PLs} \\ &= p \times m \end{aligned} \quad \text{Eq 11}$$

So our model indicates that, in a moment, the system can service  $p \times m$  senders at most.

### 3.7 Analysis of the model

In this part, we will determine the function of the number of repeaters and other factors (the channel number  $p$ , user's radius  $R_u$ ). To analyze the model, we ought to add some other assumption for convenience.

#### 3.7.1 Local Assumption

- Population density is uniform. More specifically, the number of people in a cell is constant, once the population is set.
- Nearby repeaters are close enough so that all repeaters in the same cell can be geographically concentrated in one point.

#### 3.7.2 Derivation

Since a repeater can assemble only one PL, the number of PL is equal to the number of repeaters in the same cell. In order to simplify the calculation and avoid combinatorial problem, we stipulate the PL kinds of a cell are consistent with one another. Thus, the number of repeaters is:

$$\begin{aligned} \text{the number of repeaters} &= \text{the number of cells} \times \text{the number of PLs} \\ &= N \times m \end{aligned} \quad \text{Eq 12}$$

As discussed in 3.4, the lap number  $n$  and  $R_u$  constitute function relation. We symbolize the relation as follow:

$$n = \varphi(R_u) \quad \text{Eq 13}$$

Then

$$N = 3 \times 2^{\varphi(R_u)-1} - 5 \quad \text{Eq 14}$$

Next, we'd like to deduce  $m$ . Assume that the number of users is  $M$ . The system is required to accommodate  $M$  simultaneous users. We firstly strictly comply with simultaneity conditions as discussed in Project1. That is to say, one cell includes  $M$  repeaters! Then we apply appropriate time-sharing multiplexing to optimize the model as discussed in Project2.

## Project1

$$m = \left\lceil \frac{M}{p} \right\rceil \quad \text{Eq 15}$$

$\left\lceil \frac{M}{p} \right\rceil$  means taking the smallest integer greater than  $\frac{M}{p}$ .

$$\text{the number of repeaters} = (3 \times 2^{\varphi(R_u)-1} - 5) \times \left\lceil \frac{M}{p} \right\rceil \quad \text{Eq 16}$$

Constraints:

$$54 \times p \geq M \quad \text{Eq 17}$$

The outcome above ought to be unsatisfactory,

## Project2

The destiny of population  $\rho$  is:

$$\rho = \left\lceil \frac{M}{N} \right\rceil \quad \text{Eq 18}$$

We try to lower m as follow

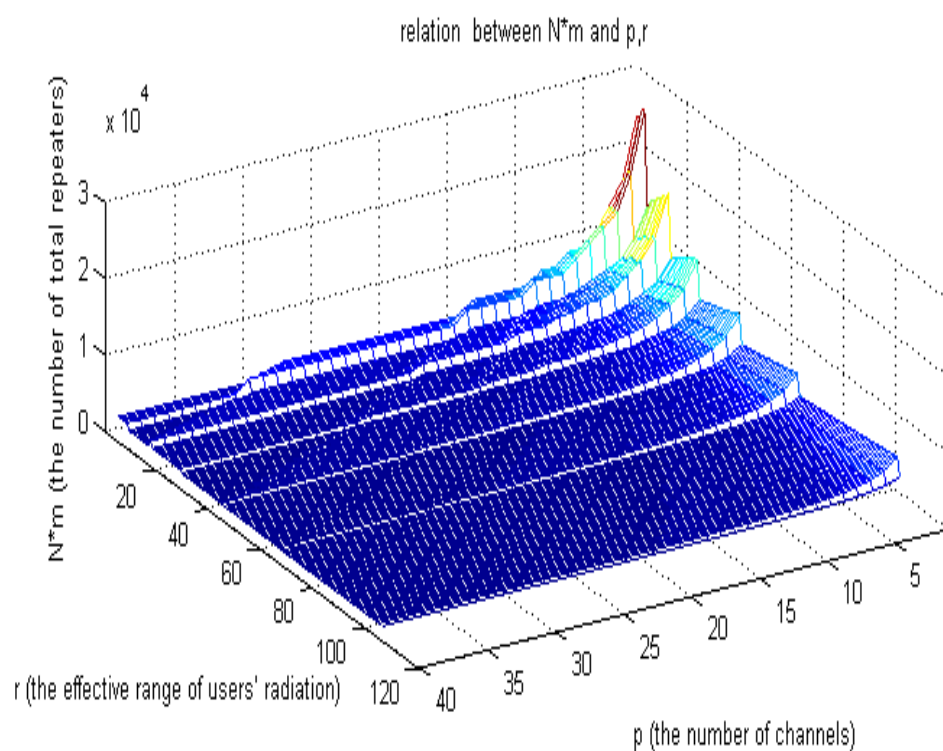
$$m = \left\lceil \frac{\rho}{p} \right\rceil \quad \text{Eq 19}$$

Then we get the result.

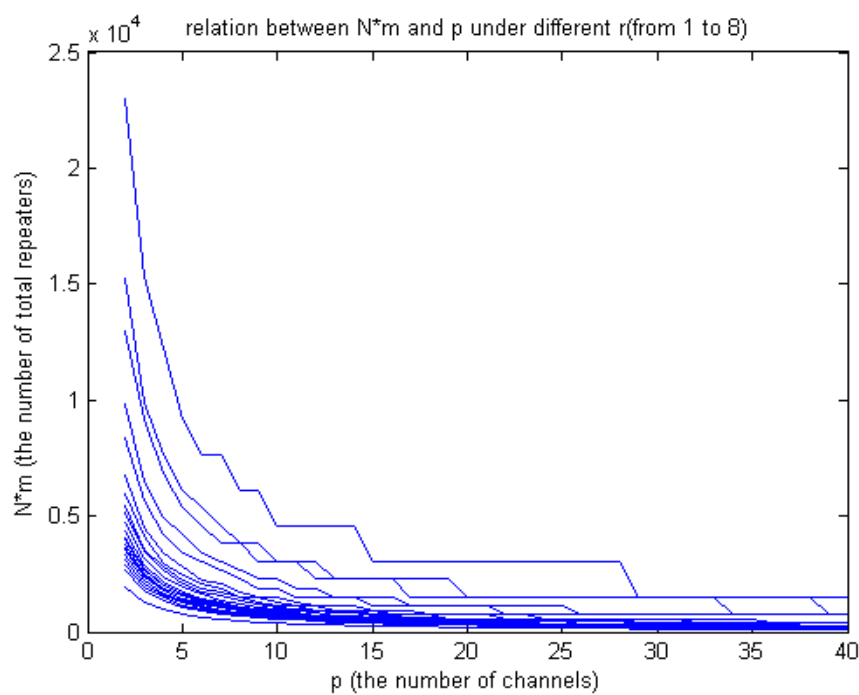
$$\text{the number of repeaters} = (3 \times 2^{\varphi(R_u)-1} - 5) \times \left\lceil \frac{\left\lceil \frac{M}{(3 \times 2^{\varphi(R_u)-1} - 5)} \right\rceil}{p} \right\rceil \quad \text{Eq 20}$$

To reflect the superiority of this project, we will reveal that Project2 can sharply reduce the repeater number. In addition, the user's waiting time is acceptable.

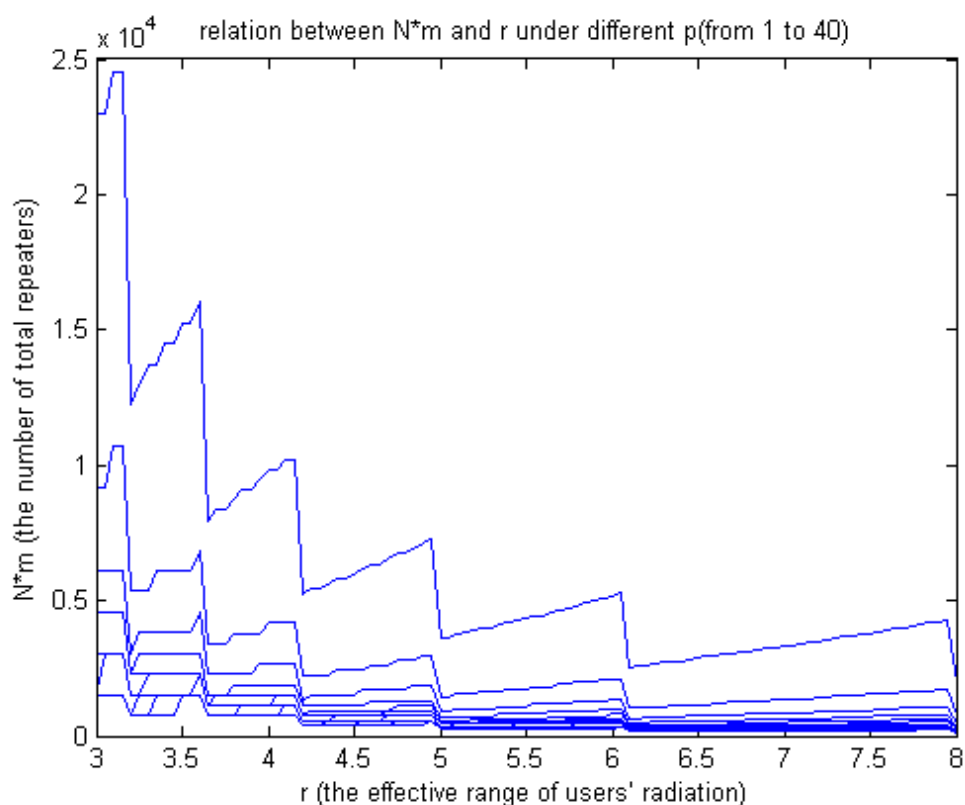
## 4. Sensitivity analysis



**Figure.5 relationship between  $N^*m$  and  $p, r$**



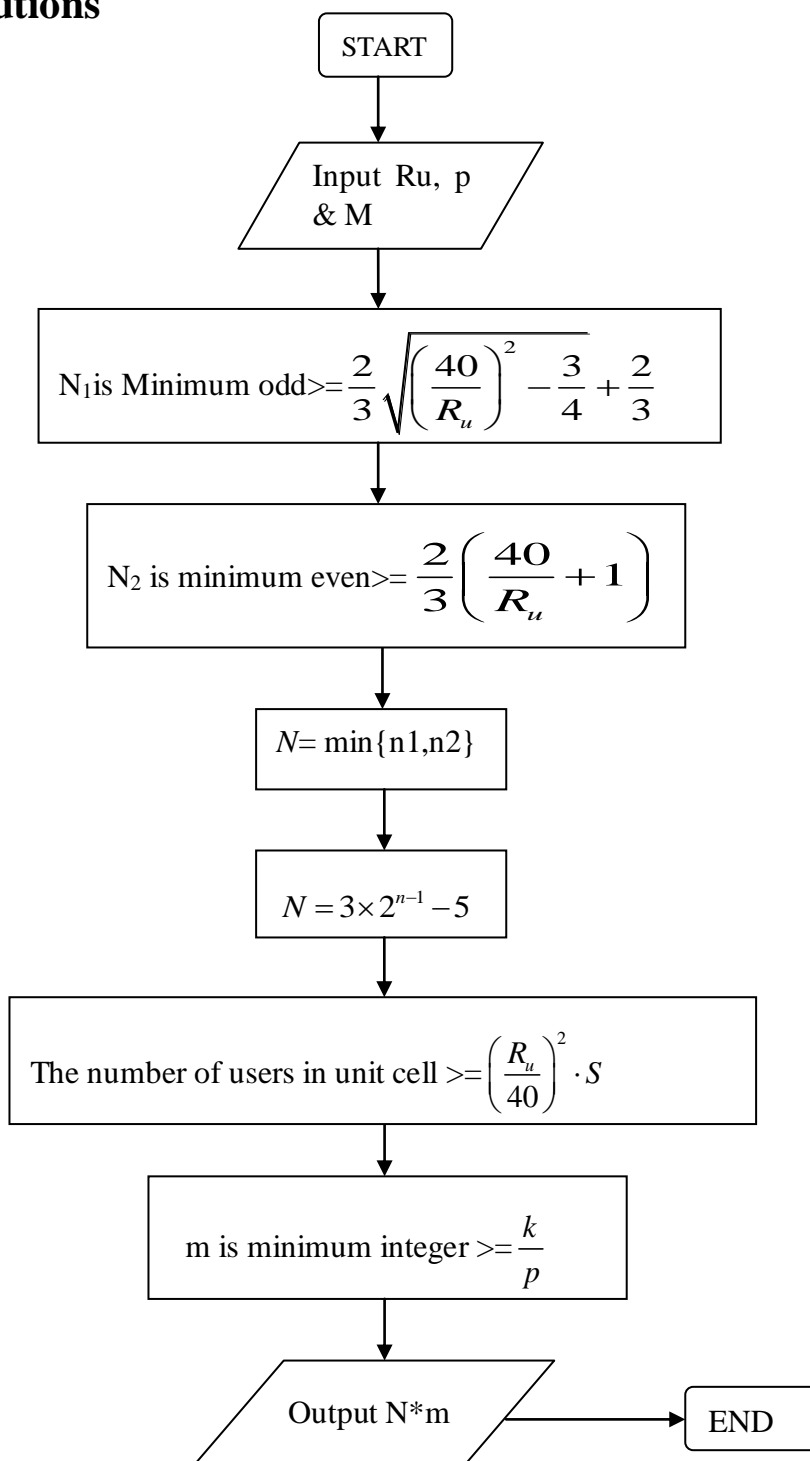
**Figure.6 relationship between  $N^*m$  and  $p$**



**Figure.7 relationship between  $N*m$  and  $r$**

We can see the fact that  $p$  and  $r$  will affect the value of  $N*m$  greatly from figure.5. At the points close to the point  $p=0$  and  $r=0$ , the value  $N*m$  is very great (over 100000). And it drops rapidly as  $P$  or  $R$  increases. At point  $p=40$ ,  $r=8$ , the value has decreased to 209. Figure.6 shows the different curve of the relationship between  $N*m$  and  $r$  under the different values of  $p$ . The curve is serrated. Generally, the value of  $N*m$  and  $R$  have a negative correlation, but  $N*m$  performs a zigzagged decrease as the value of  $P$  increases. The reason is obvious: The increase of  $N$  and  $r$  is discontinuous. There are certain 'gaps' of the adjacent values of  $N*m$ . The curve of relationship between  $N*m$  and  $p$  under different  $r$  is drawn in figure.7. And we can see the very rapid decrease of  $N*m$  with the increase of  $p$ . Generally speaking, the model is sensitive especially at the points close to the point  $p=0$  and  $r=0$ , but the value of  $N*m$  is pretty steady when  $p$  and  $r$  become much far from that point. And in fact,  $r$  is usually between 3 and 7, and  $p$  is usually between 15 and 40 according to the data. So we can see the curve is gentler in this area. There are some tables showing the value of  $p$ ,  $r$  and  $N*m$ .

## 5. Solutions



**Flow chart.1**

**Captions:** M is the total number of the users. We drew this flow chart is to calculate N\*m- the number of the cells.

### 5.1 Problem One

The goal is to determine the minimum number of repeaters necessary to



accommodate 1,000 simultaneous users.

Based on our model, we use C++ program to compute an approximate number of the Repeaters, which equals to  $N \cdot m$ . Then we can obtain Table.4.

**Table.2 There's 1000 users**

$\begin{matrix} m \\ p \\ R_u \end{matrix}$	10	20	30	40
1	1	1	1	1
3	1	1	1	1
5	2	1	1	1
8	4	2	2	1

**Table.3 There's 1000 users**

$\begin{matrix} N \cdot m \\ p \\ R_u \end{matrix}$	10	20	30	40
1	402,653,179	402,653,179	402,653,179	402,653,179
3	1,531	1,531	1,531	1,531
5	374	187	187	187
8	172	86	86	43

From the table, we can see that if the channel number reached 40 and the user's radius is 8 miles, the number of the repeater is only 43.

However, if we use Project1 to calculate the number, the bottom line is:

**Table.4 There's 1000 users (simultaneous)**

$\begin{matrix} N \cdot m \\ p \\ R_u \end{matrix}$	10	20	30	40
8	100,000	50,000	33,000	25,000

In this project, we can ensure the synchronism, while the expense is fancy.

## 5.2 Problem Two

### 5.2.1 Result

If there are 10,000 users, we use the same model to solve this problem. From

table.5 we can see that 430 repeaters can solve this problem.

**Table.5 There's 10000 users**

$\begin{matrix} m \\ p \\ R_u \end{matrix}$	10	20	30	40
1	1	1	1	1
3	6	3	2	2
5	16	8	6	4
8	40	20	14	10

**Table.6 10000 users**

$\begin{matrix} N^*m \\ p \\ R_u \end{matrix}$	10	20	30	40
1	402,653,179	402,653,179	402,653,179	402,653,179
3	9,186	4,593	3,062	3,062
5	2,992	1,496	1,122	748
8	1,720	860	602	430

### 5.2.2 Queuing theory

Queuing theory is a research method to study on services system process. In this kind of system, the time when the client comes and the service time every client spend is at random (Zhao 2000). By studying on every individual random service system, we can find common characteristics. As a result, we can provide scientific basis for improving existing system. Now we use queuing theory to test the model. For simplicity, we simplify Multi server queuing theory into Single server queuing theory.

### 5.2.3 Assumptions

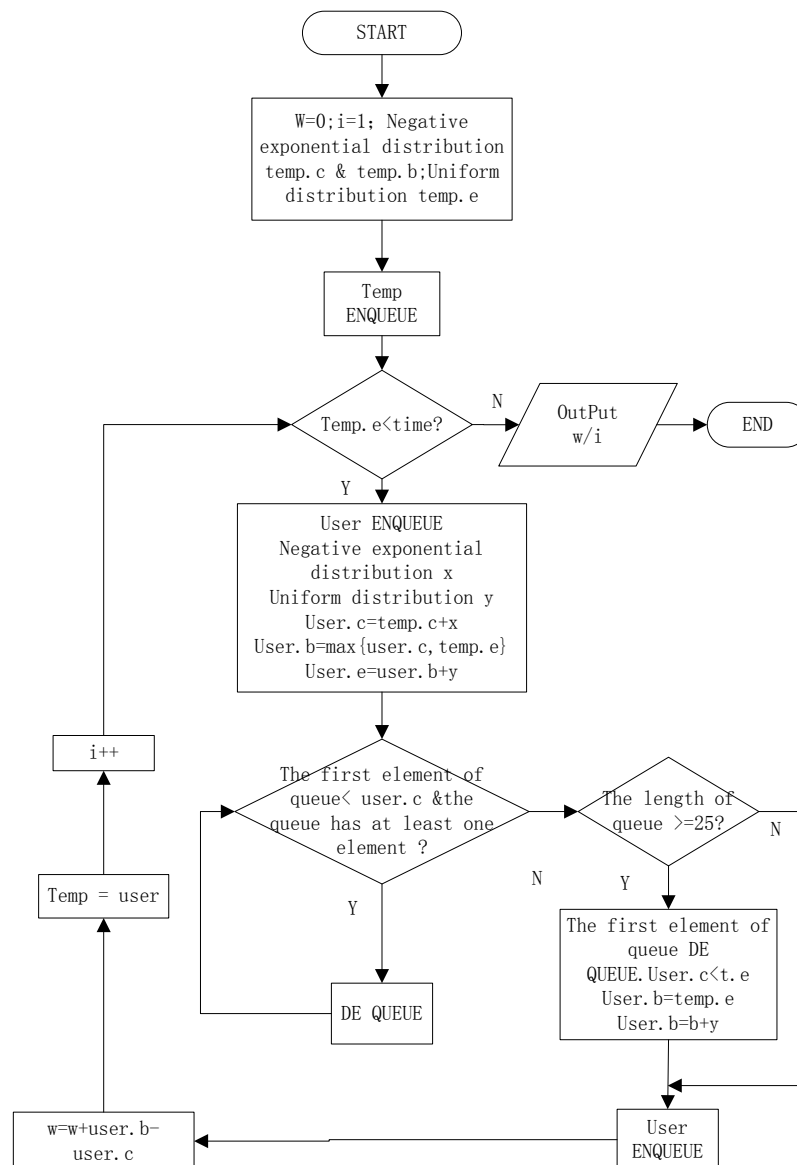
- The number of channel is 40.
- Service time obeys uniform distribution.
- The extent of time between two users obeys Negative exponential distribution. (Du 2002)
- We use only one repeater in one honey comb cellular.

### 5.2.4 Simulation results

40 channels means 40 servers, 10000 people means 10000 users. We simplify 40 channels into one channel. Which means 10000 users now should be considered as 250 users. We use C++ to test our model. We assume Uniform distribution function to be **【1, 30】**.

**Table.7 the relationship between  $\lambda$  and t**

$\lambda$ (parameter of negative exponential distribution function)	1/45	1/40
t[s] (average waiting time)	5.74	6.7



**Flow chart.2**

**Captions:**  $w$ =waiting time;  $c$ =coming time;  $e$ =ending time;  $temp$  is a variable represented a user;  $user$  is a variable represented another user.  $temp.c$ =the

**coming time of temp, and so on.**

So far, we can draw a rational understanding that proper time-sharing multiplexing sharply decreases the number of repeaters and users just need to wait for a few seconds. In reality, the case often happens and will not affect normal communication. Thus, the rationality of our model is proved.

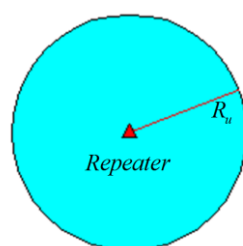
### 5.3 Problem Three

The range of line-of-sight propagation is limited by geographical obstacles. We took this defect into consideration and proposed solutions to avoid signal blind areas. The users enjoy barrier free communications with others when entering the effective range of repeaters.

In the circular flat area of radius 40 miles radius, we place repeaters as the problem A&B. The shadow often occurs in the mountainous areas where the signal emitted from repeaters cannot reach. Users in the shadow cannot get any service from the repeater.

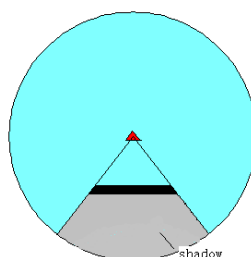
#### 5.3.1 Caption

Figure.8 shows that, in barrier free circumstances, we treat a repeater as a light source whose radiation radius is  $R_u$ . Users in the area that is illuminated can have their information reached the repeater, thus through repeaters the information will be able to reach any other user in the entire spectrum.



**Figure.8 No obstacles**

In figure.9, an obstruction inside perfectly blocks the light and produce a shadow. Users in the shadow cannot have their information reached the repeater due to the VHF frequency characteristic. We call it blocking effect.



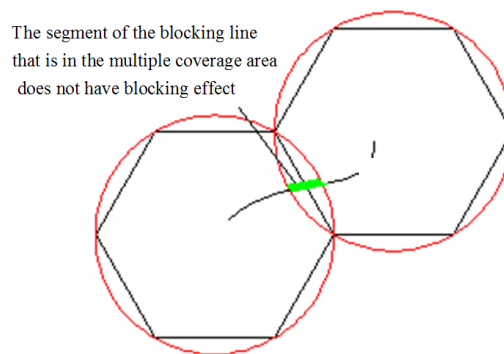
### Figure.9 With obstacles

Now, we place obstacles (mountains) into model1 at random. Meanwhile, shadows emerge. Then our goal is to eliminate all shadows.

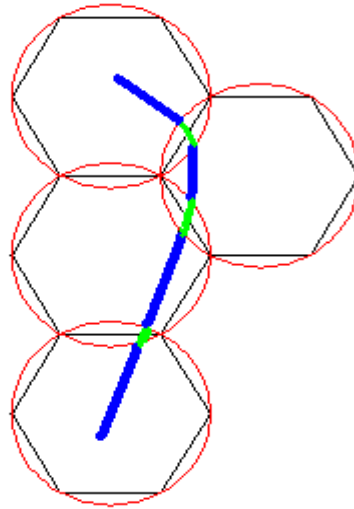
#### 5.3.2 Local assumptions

- In a unit cell, the side of the mountain facing the repeater located in the center of the cell can be covered. Thus, when the opposite side is lightened, the mountain can be simulated as a line from the overlooking perspective. We treat all mountains as lines called 'blocking lines' to simplify the model.
- The shortest distance between any two blocking lines is greater than  $2R_u$  and the minimum curvature radius of any blocking line is greater than  $R_u$ .
- The blocking effect to the line-of-sight propagation caused by mountainous areas is perfect.
- There isn't any blocking line passing through any existing repeater located according to figure.3.

Based on the upper assumption, if a blocking line passes through a multiple coverage area and the two corresponding repeaters are on both sides of the line, the certain segment of the blocking line that is in the multiple coverage area does not have blocking effect (i.e., this part cannot give birth to fade zone or shadow). Figure.10 reveals this situation. Other parts of the blocking line are called effective segments which can produce shadows, as figure.11 shows. Then we get the 5<sup>th</sup> assumption.



**Figure.10**



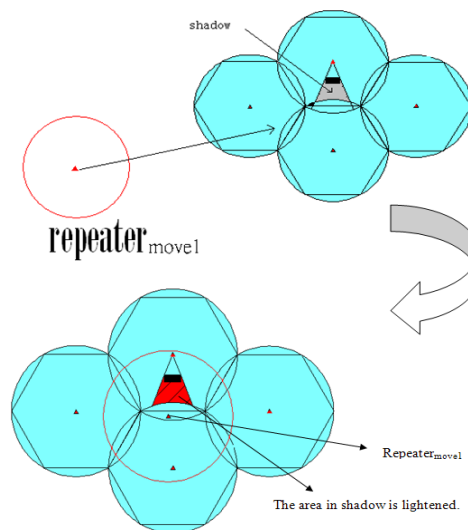
**Figure.11**

- There is only one effective segment inside one single coverage area.

### 5.3.3 The methods

Let's imagine moving a repeater  $Repeater_{move1}$  (regarded as a light source) in the plane in order to illuminate the shadow. The site where  $Repeater_{move1}$  can perfectly illuminate the shadow is the location of the repeater. Thus, any user in the shadow can reach  $Repeater_{move1}$ , so that the entire spectrum is bestowed. This process is shown in figure.12.

Given that only one "light source" does not always suffice, we have to increase the number of  $Repeater_{move}$ . To be specific, move  $Repeater_{move2}$  to cover some other shadow after the positioning of  $Repeater_{move1}$ . Then we turn to place  $Repeater_{move3}$ . The operation continues until the entire spectrum is bestowed. In this way, we ensure the completion of the task.

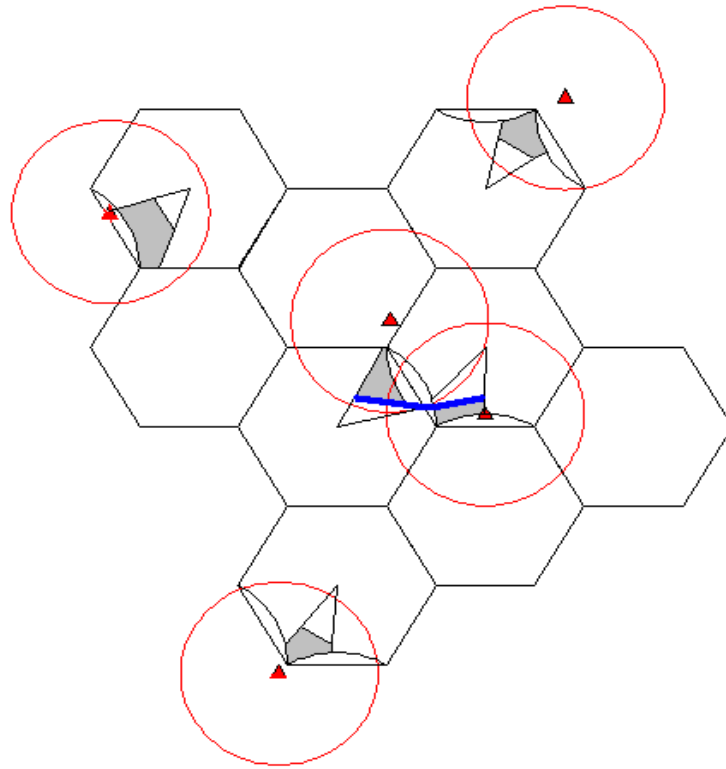


**Figure.12**

### 5.3.5 Evaluation

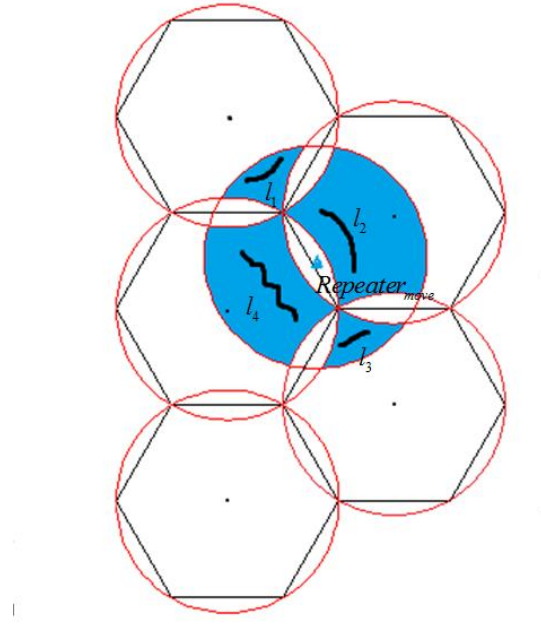
With the help of honeycomb structure, blocking lines could be cut into several effective segments, as can be seen in figure.11. Any shadow caused by one of the effective segments will be surely inside one unit cell. Now, we would like to estimate the number of  $Repeater_{move}$ .

In some simple situations where blocking lines are far from each other (the shortest distance between any two lines is greater than  $2R_u$ ) and smooth in curvature (the minimum curvature radius is greater than  $R_u$ ), the number of  $Repeater_{move}$  is generally equal to the number of effective segments. Figure.13 presents an example.



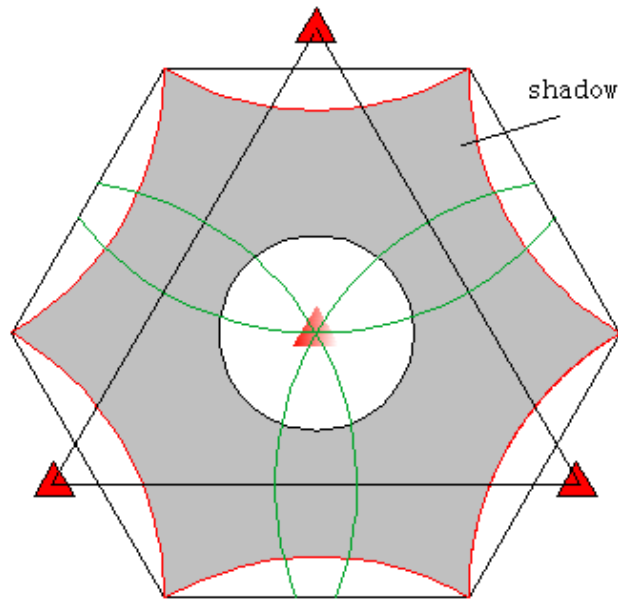
**Figure.13**

Nevertheless, more complex situations exist. Figure indicates that 4 effective segments share the same  $Repeater_{move}$ . As there is no more than one effective segment inside a single coverage area, a  $Repeater_{move}$  can deal with 4 effective segments at most. So, each effective segment appropriates  $1/4Repeater_{move}$ .



**Figure .14 One  $Repeater_{move}$  deals with 4 effective segments ( $l_1, l_2, l_3, l_4$ ) and each effective segment appropriates to  $\frac{1}{4} Repeater_{move}$**

On the other hand, if the effective segment comes to be a circle (such as basins), more  $Repeater_{move}$  have to be employed. We notice from figure.13 that one effective segment may appropriates  $3Repeater_{move}$ . Up to now, we haven't met with the situation of 4.



**Figure.15 the obstacle is a ring**

Thus, the number of  $Repeater_{move}$   $N_{Repeater_{move}}$  and the number of effective segments



$N_{effective\_segment}$  imply such a relationship:

$$\frac{1}{4}N_{effective\_segment} \leq N_{Repeater\_move} \leq 3N_{effective\_segment} \quad \text{Eq 21}$$

The method solves the problem of complete coverage and estimates  $N_{Repeater\_move}$ , the number of added repeater sites. As determining the number of repeaters in each site, we can refer to the previously mentioned algorithm.

## 6. Conclusion

To determine the minimum number of repeaters necessary to users, we develop one simulation model. The model can be applied to solves

- Our strengths:
  1. Simple: The significant advantage of our model is that the solutions are simple with reasonable assumption. As we give the priority to professional background, our analysis ought to be closer to the actual situation.
  2. Lower cost: we use the minimum number of repeaters, and the average cost of the repeater is rather high (e.g. \$1,911), so we can save rather amount of money.
  3. Feasible: Based on the professional background, our model considers many additional facts. As a result, it is feasible.
  4. Flexible: This model fits perfectly in the plains. It is also applicable to mountain areas.
- Our weaknesses:
  1. We fail to test our model because of the absence of actual data.
  2. In order to simplify the model, without considering the case population density which is not uniform.
- Recommendations:
  1. Considering the density of the population, each region should do repeaters coordination according to actual circumstances.
  2. Considering energy loss in the process of transmission, such as thunderstorm weather.

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