Supplementary Materials for Physics-Informed Deep Learning: A Hybrid Paradigm for Traffic State Estimation Informed By Second-Order Traffic Models

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The LWR-PIDL TSE method using the three-parameter LWR model is described as follows:

$$\begin{aligned}
&\rho_t + (Q(\rho))_x = 0,\\
&Q(\rho) = \sigma\Big(a + (b-a)\frac{\rho}{\rho_{max}} - \sqrt{1+y^2}\Big),\\
&u = Q(\rho)/\rho,\\
&a = \sqrt{1+\left(\delta p\right)^2},\\
&b = \sqrt{1+\left(\delta(1-p)\right)^2},\\
&y = \delta\Big(\frac{\rho}{\rho_{max}} - p\Big),
\end{aligned} \tag{1}$$

where δ, p and σ are the three free parameters as the function is named. The parameters σ and p control the maximum flow rate and critical density (where the flow is maximized), respectively. δ controls the roundness level of $Q(\rho)$. In addition to the above-mentioned three parameters, the maximum density ρ_{max} is also part of the model parameters. Thus, the model parameters are $\boldsymbol{\lambda} = (\delta, p, \sigma, \rho_{max})$. The residual is redesigned as follows:

$$\begin{aligned}
\hat{f}(t,x;\theta,\boldsymbol{\lambda}) := &\ \hat{\rho}_t(t,x;\theta)\\
&+ (Q(\hat{\rho}(t,x;\theta);\delta,p,\sigma,\rho_{max}))_x,
\end{aligned} \tag{2}$$

The PIDL structure for this baseline method is given in Fig.1. In this structure, the PUNN estimates the density $\hat{\rho}$ directly, but the averaged velocity $\hat{u}$ is inferred from the model, which is part of the PINN.

The learning loss is

$$\begin{aligned}
&Loss_{\theta,\boldsymbol{\lambda}} = MSE_o + MSE_a\\
&= \tfrac{1}{N_o}\sum_{i=1}^{N_o} \alpha_1 |\hat{\rho}(t_o^{(i)}, x_o^{(i)};\theta) - \rho^{(i)}|^2 + \alpha_2 |\hat{u}(t_o^{(i)}, x_o^{(i)};\theta) - u^{(i)}|^2\\
&+ \tfrac{1}{N_a}\sum_{j=1}^{N_a} \beta |\hat{f}(t_a^{(j)}, x_a^{(j)};\theta,\boldsymbol{\lambda})|^2,
\end{aligned} \tag{3}$$

where both $\hat{u}$ and $\hat{f}$ are calculated by PINN, and only $\hat{\rho}$ is from PUNN.

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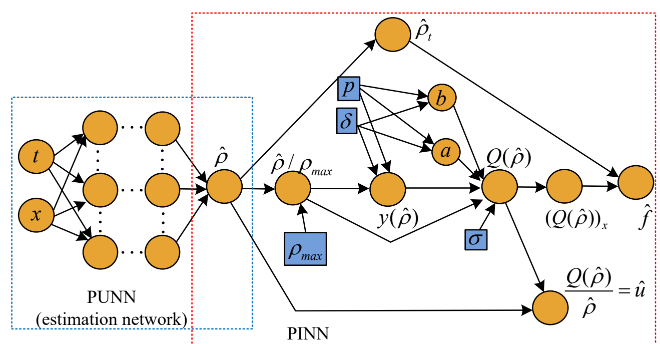

Figure 1: The structure of LWR-PIDL.