

Supplementary Materials for Physics-Informed Deep Learning: A Hybrid Paradigm for Traffic State Estimation Informed By Second-Order Traffic Models

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The LWR-PIDL TSE method using the three-parameter LWR model is described as follows:

$$\begin{aligned}
 \rho_t + (Q(\rho))_x &= 0, \\
 Q(\rho) &= \sigma \left(a + (b - a) \frac{\rho}{\rho_{max}} - \sqrt{1 + y^2} \right), \\
 u &= Q(\rho) / \rho, \\
 a &= \sqrt{1 + (\delta p)^2}, \\
 b &= \sqrt{1 + (\delta(1 - p))^2}, \\
 y &= \delta \left(\frac{\rho}{\rho_{max}} - p \right),
 \end{aligned} \tag{1}$$

where δ , p and σ are the three free parameters as the function is named. The parameters σ and p control the maximum flow rate and critical density (where the flow is maximized), respectively. δ controls the roundness level of $Q(\rho)$. In addition to the above-mentioned three parameters, the maximum density ρ_{max} is also part of the model parameters. Thus, the model parameters are $\lambda = (\delta, p, \sigma, \rho_{max})$. The residual is redesigned as follows:

$$\begin{aligned}
 \hat{f}(t, x; \theta, \lambda) &:= \hat{\rho}_t(t, x; \theta) \\
 &\quad + (Q(\hat{\rho}(t, x; \theta); \delta, p, \sigma, \rho_{max}))_x,
 \end{aligned} \tag{2}$$

The PIDL structure for this baseline method is given in Fig.1. In this structure, the PUNN estimates the density $\hat{\rho}$ directly, but the averaged velocity \hat{u} is inferred from the model, which is part of the PINN.

The learning loss is

$$\begin{aligned}
 Loss_{\theta, \lambda} &= MSE_o + MSE_a \\
 &= \frac{1}{N_o} \sum_{i=1}^{N_o} \alpha_1 |\hat{\rho}(t_o^{(i)}, x_o^{(i)}; \theta) - \rho^{(i)}|^2 + \alpha_2 |\hat{u}(t_o^{(i)}, x_o^{(i)}; \theta) - u^{(i)}|^2 \\
 &\quad + \frac{1}{N_a} \sum_{j=1}^{N_a} \beta |\hat{f}(t_a^{(j)}, x_a^{(j)}; \theta, \lambda)|^2,
 \end{aligned} \tag{3}$$

where both \hat{u} and \hat{f} are calculated by PINN, and only $\hat{\rho}$ is from PUNN.

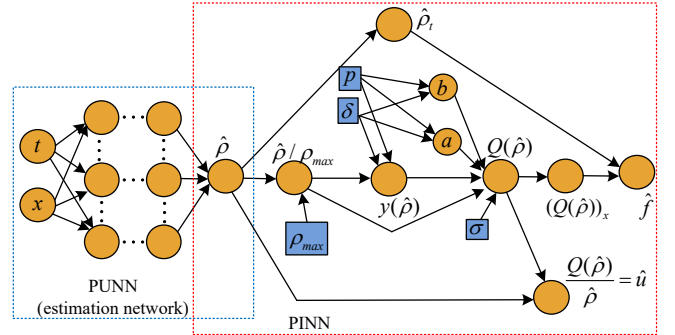


Figure 1: The structure of LWR-PIDL.