SYDE 532 Assignment 4

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Problem 8.10: Numerical/Computational—Spatial Models

We would like to implement the forest-fire model of Table 8.3. At each point in time, each cell can be one of three states: { tree, burning, ash }.

The following GIFs simulate the forest fire rate starting with all ash.

Blue = Tree

Red = Ash

White = Fire

Figure 1: Forest Fire Simulation, f = 0.0001

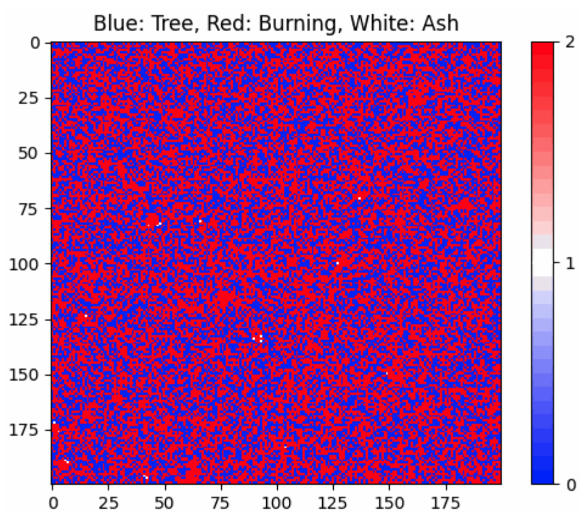

Figure 2: Forest Fire Simulation, f = 0.01

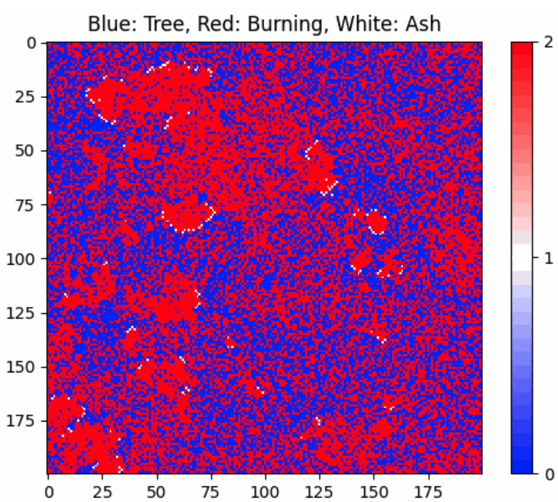

FIgure 3: Forest Fire Simulation, f = 0.01

GIF too large, see .py submission (or code in Appendix).

So p is a measure of forest growth rate, and f is a measure of the likelihood of lightning strikes. The key parameter controlling the scale of the resulting burns is the ratio f/p.

Set p = 0.01. Run simulations for f = 0.0001, f = 0.001, and f = 0.01. Discuss your observations.

Observing the forest growth rate, results show how the simulation is affected by varying *f*, the likelihood of lightning strikes. When the ratio of the probability of lightning strikes (f) to forest growth is less than one, the forest seems to be fairly resilient to fire. The forest appears to grow into a dense forest until a fire begins to spread until the forest regrows. However, when the ratio is equal to (or greater than) one, there seems to be an ongoing spread of fire and the forest does not achieve a large density of trees.

Problem 8.13: Reading Question—Waves

Most of the PDE related discussion in this chapter focused on phenomena related to advection–diffusion, however most definitely one other phenomenon worth understanding is that of waves:

(a) Look up the derivation of the wave equation and offer a succinct summary of the derivation.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

The wave equation is a differential equation that describes how waves propagate in various environments. In the equation, **u** represents the wave function, **t** is time, and **c** is the speed of the wave. A common approach to summarise the derivation starts with considering a small portion of a string under tension [1]. In this case, **u** (the wave) appears through the physical displacement of the string. Using Hooke's law, we know that the force that returns the string to equilibrium (tension of the string) is proportional to its displacement from equilibrium. Using Newton's second law, we derive a relationship between the resulting acceleration (seen in the wave equation as the second time derivative of u) and the spatial derivative (seen as the Laplacian of u). The result summarises how the speed and behaviour of a wave is affected by spatial properties such as tension and density of the medium (i.e. string in this case).

(b)There are many wave–like phenomena present on the earth, from ripples on the water surface of a pond, shock waves in automobile traffic, to large–scale inertial waves, gravity7 waves, and Rossby waves, to name a few. Look up one or more large–scale wave phenomena and offer a qualitative summary of their role in or impact on climate.

An example of an important large-scale wave phenomena is Madden-Julian Oscillation (MJO). MJO is an eastward moving disturbance of clouds, rain, wind, and pressure that travels around the earth near tropical areas [2]. This wave returns to its initial starting point every 30 to 60 days and significantly impacts the weather and climate patterns around the world.

The MJO travels as a system that consists of two phases. The first phase consists of enhanced rainfall. This causes the winds at the surface of the earth to converge and thrust upwards into the atmosphere where the winds diverge, causing significant rainfall. The second is suppressed rainfall. In this stage, the winds at the top of the atmosphere converge and force air downwards. As air sinks it dries, bringing warm and dry weather. These two phases cause the dipole system to propagate from the west to the east, bringing clouds and rain to the enhanced phase and sunshine in the suppressed phase.

This wave phenomenon has a significant impact that reaches far from the topics as well. For example, it has an impact on the timing and strength of monsoons in northern Australia. The increase in MJO convective phases over northern Australia explains most of the increase in rainfall since 1974 in northwest Australia [3]. The MJO also influences other wave phenomena such as El Nino and La Nina. Finally, MJO is in fact influenced by other climate phenomena. ENSO is another phenomena that is *stationary* (not wave-like), but impacts the strength of MJO as well as how fast it propagates.

Overall, MJO is a climate wave phenomena that influences global weather patterns and is an excellent example of how waves come in various forms.

Citations

[1] Admin, “Wave equation - definition, formula, derivation of wave equation,” BYJUS, https://byjus.com/maths/wave-equation/ (accessed Mar. 18, 2024).

[2] J. Gottschalck, RE: still don’t understand. Permalink There is no way to predict with much confidence when people will understand why and how the MJO iitiates, and RE: MJO and the Hot Blob west of Baja Permalink The MJO itself is fairly well constrained to the tropics, “What is the MJO, and why do we care?,” NOAA Climate.gov, https://www.climate.gov/news-features/blogs/enso/what-mjo-and-why-do-we-care (accessed Mar. 16, 2024).

[3] The link between the madden-julian oscillation and rainfall ..., https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2022GL101799 (accessed Mar. 16, 2024).

Code

```
# Problem 8.10: Numerical/Computational—Spatial Models

# We would like to implement a the forest-fire model. At each point in time, each cell
can be one of three states: { tree, burning, ash }.


# define states for the cells

tree = 0

burning = 1

ash = 2


# define the size of the grid

n = 200


p = 0.01

f = 0.0001


import numpy as np

import matplotlib.pyplot as plt

import random


# Function to update the grid

def update_grid(Z, p, f):

# Create a new grid to store the updated values

Z_new = np.zeros((n, n))
```

```
# Loop through each cell in the grid

for i in range(n):

for j in range(n):

# If the cell is a tree and is next to a burning cell, it becomes burning

if Z[i, j] == tree:

if i > 0 and Z[i - 1, j] == burning:

Z_new[i, j] = burning

elif i < n - 1 and Z[i + 1, j] == burning:

Z_new[i, j] = burning

elif j > 0 and Z[i, j - 1] == burning:

Z_new[i, j] = burning

elif j < n - 1 and Z[i, j + 1] == burning:

Z_new[i, j] = burning

else:

if random.random() < f:

Z_new[i, j] = burning

# If the cell is burning, it becomes ash

elif Z[i, j] == burning:

Z_new[i, j] = ash

# If the cell is ash, it becomes a tree with probability p

elif Z[i, j] == ash:

if random.random() < p:

Z_new[i, j] = tree

else:
```

```
Z_new[i, j] = ash

return Z_new
```

```
# Setting up the simulation

Z = np.zeros((n, n))

Z.fill(ash)



# Let's save a GIF of the simulation

images = []



f_values = [0.0001, 0.001, 0.01]

# Run the simulation

for f_val in f_values:

# Run the simulation

for i in range(400):

Z = update_grid(Z, p, f_val)

# Save grid as image for GIF

if i%1 == 0:

plt.imshow(Z, cmap='bwr', interpolation='nearest')

plt.title('Blue: Tree, Red: Burning, White: Ash')

# Label the color bar with state names

plt.colorbar( ticks=[0, 1, 2])

plt.savefig(f'imgs/forest_fire_{f_val}_{i}.png')
```

```python
images.append(f'imgs/forest_fire_{f_val}_{i}.png')

plt.close()



# Save a .gif using the images we saved (forest_fire_{}.png)

with imageio.get_writer(f'forest_fire{f_val}.gif', mode='I') as writer:

for image in images:

writer.append_data(imageio.imread(image))



# Reset the grid to ash

Z.fill(ash)
```