Algebraic geometry

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1. Schemes

1.1 Spectrum

Definition 1.1: Zariski topology

Let A be a commutative ring with unity and let Spec A be the set of all prime ideals of A For any $I \subseteq A$ let us define sets

$$V(I) := \{\mathfrak{p} \in \operatorname{Spec} A \ : \ I \subseteq \mathfrak{p}\}$$

$$D(f) := \operatorname{Spec} A - V(fA)$$

A topology on Spec A such that all sets $V(I)$ are closed and $D(f)$ are open is called the **Zariski topology** on Spec A.

A prime ideal is an ideal P such that

- if $ab \in P$ then $a \in P$ or $b \in P$
- $P \subsetneq A$.

For $\mathfrak{p} \in \operatorname{Spec} A$ we have $\{\mathfrak{p}\}$ is a closed set $\iff$ $\mathfrak{p}$ is a maximal ideal in A.

Proposition 1.2

For a ring A the following hold true:

a) for any $I, J \trianglelefteq A$ we have $V(I) \cup V(J) = V(I \cap J)$
b) for a family of ideals $(I_\alpha)_\alpha$ we have $\bigcap_\alpha V(I_\alpha) = V(\sum_\alpha I_\alpha)$
c) $V(A) = \emptyset$ and $V(0) = \operatorname{Spec} A$.