Linear algebra stuff

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Abstract

We do cool stuff in linear algebra

1 Matrices and linear equations

1.1 Elementary definitions and results

Let $A = [a_{i,j}]$ where $a_{i,j}$ are the entries of A. The matrix A can be explicitly written as:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{bmatrix}$$

and consider the following system of linear equations:

$$\begin{cases} a_{1,1}x_1 & + & a_{1,2}x_2 & + & \dots & + & a_{1,n}x_n & = & 0 \\ a_{2,1}x_1 & + & a_{2,2}x_2 & + & \dots & + & a_{2,n}x_n & = & 0 \\ \vdots \\ a_{n,1}x_1 & + & a_{n,2}x_2 & + & \dots & + & a_{n,n}x_n & = & 0 \end{cases} \tag{1}$$

Proposition 1.1. *The equations in Equation* (1) *have a unique solution* $\iff$ *A is row equivalent to the identity matrix I_n.*

2 Invertible matrices

Definition 2.1. *An $n \times n$ matrix B is invertible if there is some matrix C such that $B \cdot C = I$, where I is the identity matrix.*

Fact 2.2. *Let A be an $n \times n$ matrix. Then the following statements are equivalent:*

1. *A is invertible*

2. *A is row equivalent to the identity matrix I_n*

3 Conclusion

Theorem 3.1. *A is row equivalent to the identity matrix* $I_n \iff$ *The equations in 1 have a unique solution.*

Proof. The theorem follows from Proposition 1.1 and fact 2.2. $\square$

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That's

all,

folks!