Exercise . Prove that in the definition of field extension by radicals (before Thm. 9.6) we ay assume that $L_k \subseteq L_0$ is Galois.

The definition of field extension by radicals given during the lecture goes as follow:

$K \subseteq L$ *is e.b.r. if there exists* k *and*

$$[L \subseteq] L_0 \supseteq L_1 \supseteq \dots \supseteq L_k = K$$

such that for every $i < k$ L_i *is the splitting field of one of the following polynomials:*

- $x^{n_i} - b_i, b_i \in L_{i+1}$ *(*$p \nmid n_i$ *when* $\mathrm{char}(K) = p$*)*
- $x^p - x - b_i, b_i \in L_{i+1}$

Theorem 9.6 states that:

Assume that $K \subseteq L$ *is a finite field extension. Then* $K \subseteq L$ *is an extension by radicals* $\iff$ *there exists* $L' \supseteq L$ *such that* $K \subseteq L'$ *is solvable.*

In the proof of this theorem we assumed that $K \subseteq L_0$ is Galois so now I have to prove that we did not lie.

Firstly, I am going to say that any splitting field is algebraic. Then, $L_k \subseteq L_0$ is a splitting field and so is algebraic. Hence, to show that $L_k \subseteq L_0$ is Galois I only need to show that it is normal and separable.

NORMAL:

I know that $K \subseteq L$ is normal $\iff$ every $f \in K[X]$ splits into linear polynomials in $L[X]$

Take any $f \in L_k[X](= K[X])$.

SEPARABLE:

TO BE CONTINUED