

ZAD 1. $(\forall g \in G) g^2 = e \implies G \text{ is commutative}$

ZAD 3.

First of all, we showed during lecture that every permutation of n elements can be written as a finite product of cycles of form $(1a)$. To recap this, we just need to be able to achieve any transposition (kl) , cus then we can do anything

$$(kl) = (1k)(1l)(1k).$$

Then we want to show that any permutation can be written as a finite product of $(k \ k+1)$. For that we can use previous observation to show that $(1 \ k)$ can be written using $(k \ k+1)$. Using induction:

$$(1 \ k+1) = (1k)(k \ k+1)(1k).$$

Finally, we claim that every $(k \ k+1)$ can be written using (12) and $(12 \dots n)$:

$$(k \ k+1) = (12 \dots n)(k-1 \ k)(12 \dots n)^{-1}$$

which actually boils down to

$$(k \ k+1) = (12 \dots n)^{k-1}(12)(12 \dots n)^{-k+1}.$$