ZAD 1.

Let ≥ 1, *and let* $x_1, \ldots, x_{3n} \in \mathbb{R}^2$ *be points such that* $\|x_i - x_j\| \leq 1$ *for all* i *and* j. *Prove that* $\|x_i - x_j\| > \frac{1}{\sqrt{2}}$ *for at most* $3n^2$ *pairs* (i, j) *with* $i < j$.
Hint: can four of these points have all pairwise distances greater than $\frac{1}{\sqrt{2}}$?

Let us construct a graph G with vertices representing those points and being connected if and only if the distance between them is at least $\frac{1}{\sqrt{2}}$.
Graph G cannot contain a K_4 because if it did contain it, we would have a rhombus with side of length larger than $\frac{1}{\sqrt{2}}$ and so one of the diagonals would be longer than 1. So we have a K_4 free graph with I guess $e(G) \geq t_3(n)$ (cuz like it does not need to be partite?) and so

$$G \simeq T_3(n)$$

which has 3 vertex classes, each of size n. So we have 3n vertices, each of degree 2n and so $e(G) = \frac{1}{2}\sum d(v) = \frac{1}{2}3n \cdot 2n = 3n^2$. So we have at most $3n^2$ edges. This is an awful proof.

ZAD 2.

Let G *be a graph with* $n \geq r + 2 \geq 4$ *vertices and* $t_r(n) + 1$ *edges.*
(a) Show that for every p *with* $r + 1 \leq p \leq n$, G *has a subgraph* H *with* $|H| = p$ *and* $e(H) = t_r(p) + 1$.

ZAD 11.

(a) Let G *be a connected graph of order* $n \geq 1$ *and let* $k < n$ *be such that for any* $v, w \in G$ *with* $v \neq w$ *and* $v \nsim w$ *we have* $d(v) + d(w) \geq k$. *Show that* $P_k \leq G$.

Let us take any v and w such that $vw \notin G$. We know that $d(v) + d(w) \geq k$ and that there exists a path v...w.
If all $u \in N(v) \cup N(w)$ are connected, then we can take k of them and have a P_k.

ZAD 12.

NOPE.

ZAD 13.

Show that a graph G *has an Euler trail* $\iff$ *it has at most 2 vertices of odd degree. (we kinda assume that it is connected)*

$\Longrightarrow$

We have a graph that contains a path that goes through each edge once. Let us suppose that there are at lest 3 vertices of odd degree. But then we would have to start our path at one of then, go through the other odd degree vertex and end and the third odd degree vertex. But the middle vertex has to have the same number of edges coming in and out of it, making one of them redundant. Hence there is no Eulerian path.

$\Longleftarrow$

If we have a graph with 2 vertices of odd degree that are not connected with each other, we can temporarily connect them to obtain a cycle. Now we have a cycle through all the vertices that will become an Eulerian path once the artificial edge is removed.
And we cannot have only one vertex of odd degree because of hand shaking lemma. Elo.

ZAD 14.

Ain't no problem walking around crossing each bridge only one, the problem begins when you want to end where you started.