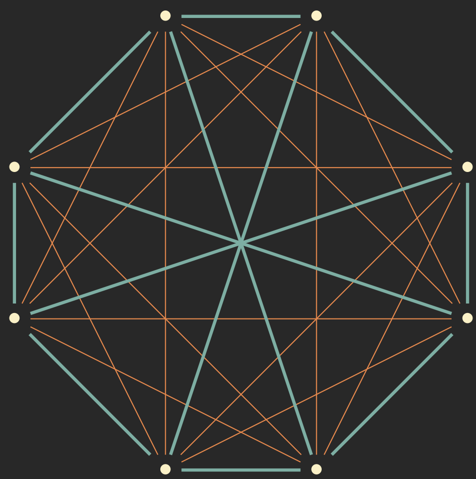
## ZAD. 1.

*(a) Konstruując przykład kolorowania* $K_8$*, pokaż, że* $R(3,4) \geq 9$.



*(b) Pokaż, że jeśli* $R(s-1,t)$ *i* $R(s,t-1)$ *są parzyste, to wtedy tak naprawdę mamy* $R(s,t) \leq R(s-1,t) + R(s,t-1) - 1$.

Let $x = R(s-1,t)$ and $y = R(s,t-1)$. We want to consider graph $K_{x+y-1}$. Let $v \in K_{x+y-1}$ be any vertex then we have $x+y-2$ edges going from it. For my convenience, all blue edges will be removed and only red edges will remain. If we have x edges going from it, then we good as the neighborhood of v will form a $K_x$. If we deleted y edges, then we also good because then we have y neighbors that are blue so they can form a $K_y$ which will have a blue $K_{t-1}$. Now what if we could only find vertices with $(x-1)$ red edges (and $(y-1)$ blue ones)? Well then we have a graph with $(x+y-1)$, an odd number of vertices.

## ZAD. 4.

*Mając dane dwa grafy* G *i* H*, piszemy* $R(G,H)$ *dla najmniejszej liczby* $n \geq 2$ *takiej, że dowolne czerwono-niebieskie kolorowanie* $K_n$ *ma albo czerwony podgraf izomorficzny do* G *albo niebieski izomorficzny do* H.
*(a) Dlaczego* $R(G,H)$ *istnieje?*

Every finite graph is a subset of some $K_n$ so we know that in the worst possible scenario we can just get the higher bound by checking what is the smallest clique for which G is a subgraph and the same for H.

*(b) Pokaż, że* $R(K_{1,t}, K_{r+1}) = rt + 1$ *dla wszystkich* $r, t \geq 1$. *[Wskazówka: Użyj twierdzenia Turána.]*