ZAD 1. $H < G$, G - cyclic $\implies$ H - cyclic

We know that

$$G = \{\ldots, g^{-}k, \ldots, g^{0} = 3, \ldots, g^{k}, \ldots\}$$

Now, because $H < G$, we know that for every $h \in H$, we have $h \in G$ so there exists $k \in \mathbb{N}^{+}$

$$h = g^{k}$$

and because H is a group, also

$$h^{-1} = g^{-k} \in H.$$

Let m be the least positive integer such that

$$g^{m} \in H.$$

We want to show that then $H = < g^{m} >$. Suppose that H is not generated by g^{m}, meaning that there exists an $h \in H$ such that $h = g^{l}$ for $l = m + n$, $0 < n < m$. Then

$$g^{l}g^{-m} = g^{m+n}g^{-m} = g^{m}g^{n}g^{-m} = eg^{n} = g^{n} \in H$$

But then n is smaller than m and we have a contradiction of our supposition.

ZAD 2. $H_1, H_2 < G$; $H_1 \cup H_2 < G$ $\iff$ $H_1 \subseteq H_2 \vee H_2 \subseteq H_1$

$\Leftarrow$

Without loss of generality, let us say that $H_1 \subseteq H_2$. Then

$$H_1 \cup H_2 = H_2$$

and it is trivial.

$\implies$

We know that $H_1 \cup H_2 < G$. Let us suppose that neither H_1 is a subset of H_2 nor H_2 is a subset of H_1. Then let us take any

$$h \in H_1 \ \wedge h \notin H_2$$

$$\eta \in H_2 \ \wedge \ \eta \notin H_1.$$

They are both in $H_1 \cup H_2$ so they are elements of a group. Therefore, their product, $h\eta$ should also be contained within this group. But because $h \notin H_1$, we cannot have $h\eta \in H_2$. Similarly, $\eta \notin H_1$ so $h\eta \notin H_1$. So we cannot have $h\eta \in H_1 \cup H_2$. So we have a contradiction. Therefore, either $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$.