Kombinatoryka & teoria grafów

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SYLABUS - teoria grafów:

1. Basic concepts: graphs, paths and cycles, complete andbipartite graphs
2. Matchings: Hall's Marriage theorem and its variations
3. Forbidden subgraphs: complete bipartite and r-partite subgraphs, chromatic numbers, Tur"an's thorem, asymptotic behaviour og edge density, Erd"os-Stone theorem
4. Hamiltonian cycles (Dirac's Theorem), Eulerian circuits
5. Connectivity: connected and k-connected graphs, Menger's theorem
6. Ramsey theory: edge colourings of graphs, Ramsey's theorem and its variations, asymptotic bounds on Ramsey numbers
7. Planar graphs and colourings: statements of Kuratowski's and Four Colour theorems, proof of Five Colour theorem, graphs on other surfaces and Euler chracteristics, chromatic polynomial, edge colourings and Vizing's theorem
8. Random graphs: further asymptotic bounds on Ramsey numbers, Zarankiewicz numbers and their bounds, graphs of large firth and high chromatic number, cmplete subgraphs in random graphs.
9. Algebraic methods: adjavenvy matrix and its eigenvalues, strongly regular graphs, Moore graphs and their existence.

Contents

Definicja – to jest bardzo wyedukowany tekst w ramce

Troszke mniej wyedukowany tekst, bo jest poza fajna ramka ktora wyglada jak guwno i w sumie to nie wiem czemu ja ja w ten sposob ranie

1 Motivation

Graph – collection of vertices joined by edges.

Seven bridges of Konigsberg – can one walk around the city crossing each bridge only once? NO

Simultaneous representation of cosets: let G be a finite group and $H \subseteq G$ be a subgroup of G, then there exists elements $a_1, \ldots, a_k \in G$ such that $G = a_1H \cup \ldots \cup a_kH$ and there exists $b_1, \ldots, b_k \in G$ such that $G = Hb_1 \cup \ldots \cup Hb_k$.

Can we take $a_i = b_i$? YES (Hall's Marriage Theorem)

Map coloring problem
Suppose we have a political map and we want to color its regions so that given any two regions sharing a border they have different colors.
How many colors do we need? 4 will alwas be enough.

Fremat's Last Theorem modulo p (where p is prime) – FLT stated that if $x^n + y^n = z^n$, $x, y, z, n \in \mathbb{Z}$ and $n \geq 3$ then $x = 0 \vee y = 0 \vee z = 0$. Given a prime p do there exist $x, y, z \in \mathbb{Z}$ such that $x^n + y^n = z^n (\mod p)$ but $x, y, z \not\equiv 0 (\mod p)$? YEP for p large enough.

PROOF
Let $G = (\mathbb{Z}/p\mathbb{Z})^X$ and $H = \{g^n : g \in G\} \leq G$. Given $h \in H$, the polynomial $x^n - h \in (\mathbb{R}/p\mathbb{Z})[X]$ (ring of remainder modulo p????) has degree n so it has at most n roots. Therefore, $\frac{|G|}{|H|} \leq n$, so there are at most n left cosets of H in G. Suppose there exists $a, b, c \in gH$ such that $a + b = c$. Then $g^{-1}a + g^{-1}b = g^{-1}c$ and $g^{-1}a \ldots \in H$, which implies that $g^{-1}a = x^n \ldots$. It is enough to show that for any integer k that is large enough if the set $\{1, 2, \ldots, k - 1\}$ is partitioned into n parts, then some part contains a, b, c such that $a + b = c$. This will be shown using graph theory and its such a fucking mess.

Graph is an ordered pair $G = (V, E)$ where
$\hookrightarrow$ V is a set of vertices $\hookrightarrow$ E is a set of edges (unordered pairs $\{v, w\}$ where $v, w \in V$ and $v \neq w$)

We write $v \in G$ to write $v \in V(G)$ and denote $\{v, w\} \in E(G)$ as vw:
$\hookrightarrow$ v, w are endpoints of $vw \in E(G)$
$\hookrightarrow$ vw is incident to v

Unless specified otherwise, graph G is finite. If $|V| = \infty$ we say that G is an infinite graph.

Order of G is $|G| := |V(G)|$
Size of G is $e(G) := |E(G)|$