

# Kombinatoryka & teoria grafów

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## SYLABUS – teoria grafów:

1. Basic concepts: graphs, paths and cycles, complete and bipartite graphs
2. Matchings: Hall's Marriage theorem and its variations
3. Forbidden subgraphs: complete bipartite and  $r$ -partite subgraphs, chromatic numbers, Turán's theorem, asymptotic behaviour of edge density, Erdős-Stone theorem
4. Hamiltonian cycles (Dirac's Theorem), Eulerian circuits
5. Connectivity: connected and  $k$ -connected graphs, Menger's theorem
6. Ramsey theory: edge colourings of graphs, Ramsey's theorem and its variations, asymptotic bounds on Ramsey numbers
7. Planar graphs and colourings: statements of Kuratowski's and Four Colour theorems, proof of Five Colour theorem, graphs on other surfaces and Euler characteristics, chromatic polynomial, edge colourings and Vizing's theorem
8. Random graphs: further asymptotic bounds on Ramsey numbers, Zarankiewicz numbers and their bounds, graphs of large first and high chromatic number, complete subgraphs in random graphs.
9. Algebraic methods: adjacency matrix and its eigenvalues, strongly regular graphs, Moore graphs and their existence.

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Definicja – to jest bardzo wy-  
dukowany tekst w ramce

Troszke mniej wyedukowany tekst, bo jest poza  
fajna ramka ktora wyglada jak guwno i w sumie  
to nie wiem czemu ja ja w ten sposob ranie

# 1 Structural properties

## 1.1 The basics

**Graph** – an ordered pair  $G = (V, E)$ :

↪ **vertices**  $:= V$  [singular *vertex*]

↪ **edges**  $:= E$ ,  $\{v, w\} := vw$

For an edge  $vw$ ,  $v \neq w$  we say that  $v, w$  are its **endpoints** and that it is **incident** to  $v$  (or  $w$ ).

Dla krawedzi  $vw$ ,  $v \neq w$  mówimy, że  $v, w$  są jej **koncami** i że jest krawedzią **padającą** na  $v$  (lub  $w$ ).

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Graphs  $G$  and  $H$  are **isomorphic** ( $G \simeq H$ ) if there exists  $f: V(G) \xrightarrow[1-1]{\text{on}} V(H)$  such that

$(\forall v, w \in V(G)) \quad vw \in E(G) \iff f(v)f(w) \in E(H)$

$G$  is a **subgraph** of  $H$  [ $G \leq H$ ] if  $V(G) \subseteq V(H)$  and  $E(G) \subseteq E(H)$ .

If  $G$  is **H-free** if it has no subgraphs isomorphic to  $H$ .