ZAD. 1.

Describe the orbits of the natural action of $GL_n(\mathbb{R})$ *on* $\mathbb{R}^n$.

The natural action of $GL_n(\mathbb{R})$ on $\mathbb{R}^n$ would be for $A \in GL_n(\mathbb{R})$

$$\phi_A(x) = Ax.$$

An orbit for $\mathbb{R}^n$ is a set

$$O(x) = \{\phi_A(x) \ : \ A \in GL_n(\mathbb{R})\}.$$

In this case an orbit is a set of vectors y such that there exists an invertible matrix A for which $Ax = y$. So these are all isometries of x. And some more?

ZAD. 2.

Let $(A, +)$ *be a commutative group.*
(a) Show that the following formula:

$$(\forall\ a \in A)\ 0 \cdot a = a,\ \ 1 \cdot a = -a$$

(gives an action of $\mathbb{Z}_2$ *on A by automorphism.*

I guess this means that for all $x \in \mathbb{Z}_2$ we have that $\phi_x(a)$ defined as above is an automorphism. So let us take any two $a, b \in A$. Then we have

$$\phi_0(a) + \phi_0(b) = a + b = \phi_0(a + b)$$

$$\phi_1(a + b) = -(a + b) = (-b) + (-a) = (-a) + (-b) = \phi_1(a) + \phi_1(b)$$

(b) Describe the homomorphism

$$\psi : \mathbb{Z}_2 \to \mathrm{Aut}(A)$$

which corresponds to the action from (a) above.

So we send 0 to identity and 1 to function such that $a \mapsto a^{-1}$, let us call it f. This is a homomorphism, because

$$\psi(0) = id$$

$$\psi(1) = f$$

$$\psi(0 + 1) = \psi(0) \circ \psi(1) = id \circ f = f = \psi(1) = \psi(1 + 0)$$
$$\psi(1 + 1) = \psi(1) \circ \psi(1) = f \circ f = id = \psi(0) = \psi(1 + 1)$$

(c) For which groups A, the homomorphism ψ *from (b) above is a monomorphism?*

For groups that have at least three elements.

ZAD. 3.

Assume that there is $g \in G \setminus \{e\}$ *such that* $\mathrm{ord}(g) \neq 2$. *Show that:*

$$\mathrm{Aut}(G) \neq \{id_G\}.$$

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ZAD. 4.

Show that

$$\mathrm{Aut}(\mathbb{Z}_2 \times \mathbb{Z}_2) \simeq S_3.$$

Ok. So. Big brain time. Cuz Dominik had a proof that is waaay tooooo loooooooooong.

The group on the right has 4 elements: (0, 0), (0, 1), (1, 0), (1,1). For a homomorphism we only need to determine where does (0, 1) and (1, 0) go. Neither of them can go onto (0, 0) cus its the neutral element.