ZAD 1.

Let ≥ 1*, and let* $x_1, \ldots, x_{3n} \in \mathbb{R}^2$ *be points such that* $\|x_i - x_j\| \leq 1$ *for all* i *and* j*. Prove that* $\|x_i - x_j\| > \frac{1}{\sqrt{2}}$ *for at most* $3n^2$ *pairs* (i, j) *with* $i < j$.
Hint: can four of these points have all pairwise distances greater than $\frac{1}{\text{sqrt2}}$?

Let us create a graph G with vertices $1, \ldots, 3n$ such that i corresponds to x_i. Let us connect x_i and x_j iff $\|x_i - x_j\| > \frac{1}{\text{sqrt2}}$. If a vertex is connected with 3 other vertices, then one pair of those cannot be connected with each other. Otherwise, we would have a square with side of length $> \frac{1}{\text{sqrt2}}$ and so the diagonal would be greater than $\frac{1}{\text{sqrt2}}\sqrt{2} = 1$ giving us a contradiction. I dunno, I dont wanna think right now.

ZAD 2.

Let G *be a graph with* $n \geq r + 2 \geq 4$ *vertices and* $t_r(n) + 1$ *edges.*
(a) Show that for every p *with* $r + 1 \leq p \leq n$*,* G *has a subgraph* H *with* $|H| = p$ *and* $e(H) = t_r(p) + 1$.