Kombinatoryka & teoria grafów

by a fish

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SYLABUS – teoria grafów:

1. Basic concepts: graphs, paths and cycles, complete andbipartite graphs
2. Matchings: Hall’s Marriage theorem and its variations
3. Forbidden subgraphs: complete bipartite and r-partite subgraphs, chromatic numbers, Tur"an’s thorem, asymptotic behaviour og edge density, Erd"os-Stone theorem
4. Hamiltonian cycles (Dirac’s Theorem), Eulerian circuits
5. Connectivity: connected and k-connected graphs, Menger’s theorem
6. Ramsey theory: edge colourings of graphs, Ramsey’s theorem and its variations, asymptotic bounds on Ramsey numbers
7. Planar graphs and colourings: statements of Kuratowski’s and Four Colour theorems, proof of Five Colour theorem, graphs on other surfaces and Euler chracteristics, chromatic polynomial, edge colourings and Vizing’s theorem
8. Random graphs: further asymptotic bounds on Ramsey numbers, Zarankiewicz numbers and their bounds, graphs of large firth and high chromatic number, cmplete subgraphs in random graphs.
9. Algebraic methods: adjavenvy matrix and its eigenvalues, strongly regular graphs, Moore graphs and their existence.

Contents

1 Structural properties

1.1 Basic definitions

Graph – an ordered pair $G = (V, E)$:
$\hookrightarrow$ vertices $:= V$ [singular: *vertex*]
$\hookrightarrow$ edges $:= E$, $\{v, w\} := vw$

For an edge vw, $v \neq w$ we say that v, w are its endpoints and that it is incident to v (or w).

Dla krawedzi vw, $v \neq w$ mowimy, ze v, w sa jej koncami i ze jest krawedzia padajaca na v (lub w).

Graphs G and H are isomorfic ($G \simeq H$) if there exists $f : V(G) \xrightarrow[1-1]{on} V(H)$ such that
$(\forall\, v, w \in V(G))\; vw \in E(G) \iff f(v)f(w) \in E(H)$

Meaning that edges are like an operation on a group of vertices

G is a subgraph of H [$G \leq H$] if $V(G) \subseteq V(H)$ and $E(G) \subseteq E(H)$.

If G is H-free if it is has no subgraphs isomorfphic to H.

Grafy G i G sa izomorficzne, jezeli istnieje $f : V(G) \xrightarrow{1} -1]naV(H)$ takie, ze
$(\forall\, v, w \in V(G))\; vw \in E(G) \iff f(v)f(w) \in E(H)$

G jest podgrafem H [$G \leq H$] jezeli $V(G) \subseteq V(H)$ oraz $E(G) \subseteq E(H)$.

G jest H-free (wolny od H?), jezeli nie ma podgrafow izomorficznych z H.

A cycle of length $n \geq 3$ [C_n] is a graph with vertices

$$V(C_n) = [n]$$

and edges:

$$E(C_n) = \{i(i+1) \ : \ i \leq i \leq n-1\} \cup \{1n\}.$$

A path of length $n-1$ [P_{n-1}] is a graph with vertices

$$V(P_{n-1}) = [n]$$

and edges

$$E(P_{n-1}) = \{i(i+1) \ : \ 1 \leq i \leq n-1\}.$$

Cykl dlugosci $n \geq 3$ [C_n] to graf z wierzcholkami

$$V(C_n) = [n]$$

i krawiedziami:

$$E(C_n) = \{i(i+1) \ : \ i \leq i \leq n-1\} \cup \{1n\}.$$

Sciezka dlugosci $n-1$ [P_{n-1}] to graf z wierzcholkami

$$V(P_{n-1}) = [n]$$

i krawedziami

$$E(P_{n-1}) = \{i(i+1) \ : \ 1 \leq i \leq n-1\}.$$

An induced by $A \subseteq V(G)$ subgraph of G is $G[A] = (A, E_A)$

A connected component of G is a subgraph $G[W] \leq G$ where $W \subseteq V$ is an equivalence class under $\approx$ given by

$$v \approx w \iff \text{exists a path } v \ldots w \text{ in } G$$

A graph is connected if $v \approx w$ for every $v, w \in V$ (G has at most one connected component).