

CaseStudy2

June 24, 2018

1 Case Study #2: Santa Claus Route Optimization Problem

2 Executive Summary:

In this MVP, I'm trying to find the optimal route for Santa such that the weight-miles (weight * miles) is as low as possible in his night of sleigh flights. I chose to use the optimization method I mentioned on our group phone call: the Simulated Annealing (SA) algorithm (https://en.wikipedia.org/wiki/Simulated_annealing). It's great for problems like this with a massive search space (Traveling Salesperson and combinatorial optimization-type problems). It won't find the global, best solution, but it'll find a pretty good solution (and in a fraction of the time too). Optimizing this route is difficult. There are 100,000 destinations around the world, and if you assume that the sleigh can hold all gifts without the need to return periodically to the North Pole, there are $(100,000 - 1)!$ possible routes to consider. In this problem, Santa has to periodically return to the North Pole, which means there are even more routes to consider. At a high level, when implementing the SA algorithm, you take an initial route, evaluate it, then randomly perturb the route, and see if it performs any better. In this exercise, I assumed that for every trip out of the North Pole, Santa is carrying as close to 1000 lbs on the sleigh as possible, and for each perturbation in each iteration of the algorithm, I randomly swapped a small amount of gifts around. This way, his night's worth of routes change a bit with each iteration. Because this is just intended as a POC and given the limited time, I only ran the algorithm shown below for 100 iterations. This POC shows that the route improves by 0.55% even in a small number of iterations. As this web app demonstrates, simulated annealing can require 100,000s of iterations before dramatic improvements are made: <http://toddwschneider.com/posts/traveling-salesman-with-simulated-annealing-r-and-shiny/> Given additional time to work on the next version of this MVP, I would have tried:

- * Instead of perturbing the list of gifts and locations between each trip of 1000 pounds, I could have perturbed the entire route.
- * If I had access to a computer cluster, it would be effortless to speed up the computation here.

2.0.1 Library Imports and defining functions

```
In [1]: import time
import pandas as pd
import numpy as np
from multiprocessing import Pool, cpu_count
from geopy.distance import distance # Function to calculate the distance between two cities
from matplotlib import pyplot as plt
```

```

import matplotlib.gridspec as gs
%matplotlib inline

pd.set_option('display.max_rows', 500)
pd.options.mode.chained_assignment = None

# I have 4 registered cores to work with on my laptop. I'll use that to speed the calc
core_count = cpu_count()
print core_count

# Set the random seed
np.random.seed(512)

```

4

```

In [2]: # Function to swap a small number of rows (this is the small perturbation in the overa
def shuffle_a_few_rows(df, n_swaps):
    """
    df: the dataframe you're working with
    n_swaps: the number swaps you want to do
    """
    for i in range(0, n_swaps):
        loc1 = np.random.randint(0, len(df))
        loc2 = np.random.randint(0, len(df))
        b, c = df.iloc[loc1].copy(), df.iloc[loc2].copy()
        df.iloc[loc1], df.iloc[loc2] = c, b
    return df

# Function that to evaluate the weight-miles (weight * miles) associated with each nor
# *This can be parallelized easily*
def evaluate_individual_trip_route(trip_number):
    """
    trip_number: the integer trip number from the route dataframe
    """
    sub_df = new_df.loc[new_df['trip_number'] == trip_number]
    sub_df['shift_Latitude'] = sub_df['Latitude'].shift(-1)
    sub_df['shift_Longitude'] = sub_df['Longitude'].shift(-1)
    #sub_df.loc[sub_df.index[-1], 'shift_Latitude'] = 90
    #sub_df.loc[sub_df.index[-1], 'shift_Longitude'] = 0
    weight_miles = 0
    current_sleigh_weight = sub_df['Weight'].sum() + 10
    weight_miles += (distance(North_Pole, (sub_df['Latitude'].values[0], sub_df['Longi
    for row in sub_df.itertuples():
        if np.isnan(row[7]):
            weight_miles += (distance((row[2], row[3]), North_Pole).miles) * (current_

```

```

        else:
            current_sleigh_weight -= row[4]
            weight_miles += (distance((row[2], row[3]), (row[7], row[8])).miles) * (current_sleigh_weight)
    return weight_miles

```

2.0.2 Create an initial random route, and evaluate it

```

In [3]: # Create a list to store the results (total weight-miles the whole night) from each iteration
final_results = []

# Setting the north pole latitude and longitude
North_Pole = (90, 0)

### Read in the gifts dataset
gifts = pd.read_csv("~/Desktop/gifts.csv")

# Do an initial shuffling of the dataset
new_df = gifts.sample(frac = 1, random_state = 512).reset_index()[['GiftId', 'Latitude', 'Longitude', 'Weight']]

# Calculate cumulative sum for weights
new_df['cumul_weight'] = new_df['Weight'].cumsum()

# Break up the gifts into trips (trips can't exceed 1000 lbs)
new_df['trip_number'] = new_df['cumul_weight'] / 1000
new_df['trip_number'] = new_df['trip_number'].astype(int)

# Evaluate the overall route, using parallel processing
pool = Pool(processes = core_count)
results = pool.map(evaluate_individual_trip_route, range(0,1410))
pool.close()

# Get the total weight-miles for that initial, random route. It's terrible, not surprising
old_result = sum(results)
final_results.append(old_result)

# Set up the old_df variable for the SA algorithm
old_df = new_df.copy()

```

2.0.3 Setup the SA hyperparameters (temperature, number of iterations to try, etc.)

```

In [4]: temperature = 1.0 # Initial temperature
count = 0 # Start the count at 0
alpha = 0.9995 # The rate at which the temperature will drop over each iteration
iteration_num = 100 # Total number of iterations to stop at

```

2.0.4 Run the SA algorithm. A while loop begins here...

```
In [5]: while count < iteration_num:
        temperature *= alpha
        count += 1

        ##### Swap 100 (0.1% of all routes) individual routes randomly (https://stackoverflow.com/questions/1220807/shuffle-a-few-rows-in-pandas)
        new_df = shuffle_a_few_rows(old_df, 10)

        ##### Evaluate the new_dataset

        # Calculate cumulative sum for weights
        new_df['cumul_weight'] = new_df['Weight'].cumsum()

        # Break up the gifts into trips (trips can't exceed 1000 lbs)
        new_df['trip_number'] = new_df['cumul_weight'] / 1000
        new_df['trip_number'] = new_df['trip_number'].astype(int)

        pool = Pool(processes = core_count)
        results = pool.map(evaluate_individual_trip_route, range(0,1410))
        pool.close()

        # Evaluate the new route list
        new_result = sum(results)

        #### Compare new result with old result, decide whether to accept the challenger route

        if new_result < old_result:
            old_df = new_df.copy()
            old_result = new_result
            p = None
            final_results.append(old_result)

            print "Iteration: {0} \t Temperature: {1} \t P: {2} \t Score: {3}".format(count, temperature, p, new_result)

        else:
            uniform_rand_num = np.random.uniform(0,1)
            p = 1e8 * np.exp(-((np.log(new_result - old_result))/temperature))

            if p > uniform_rand_num:
                old_df = new_df.copy()
                old_result = new_result
                final_results.append(old_result)
                print "Iteration: {0} \t Temperature: {1} \t P: {2} \t Score: {3}".format(count, temperature, p, new_result)
            else:
                print "Iteration: {0} \t Temperature: {1} \t P: {2} \t Score: {3}".format(count, temperature, p, new_result)
                final_results.append(old_result)
```

continue

Iteration: 1	Temperature: 0.9995	P: None	Score: 2.89249626279e+11
Iteration: 2	Temperature: 0.99900025	P: None	Score: 2.89239171925e+
Iteration: 3	Temperature: 0.998500749875	P: None	Score: 2.889027924
Iteration: 4	Temperature: 0.9980014995	P: 0.517253958905	Score: 2.8
Iteration: 5	Temperature: 0.99750249875	P: 0.384969918726	Score: 2.8
Iteration: 6	Temperature: 0.997003747501	P: 0.192422144307	Score: 2
Iteration: 7	Temperature: 0.996505245627	P: 2.63093349851	Score: 2.8
Iteration: 8	Temperature: 0.996006993004	P: None	Score: 2.890725230
Iteration: 9	Temperature: 0.995508989508	P: 0.24802243159	Score: 2.8
Iteration: 10	Temperature: 0.995011235013	P: 0.215666371708	Score: 2
Iteration: 11	Temperature: 0.994513729396	P: 0.134463953551	Score: 2
Iteration: 12	Temperature: 0.994016472531	P: 0.148656708144	Score: 2
Iteration: 13	Temperature: 0.993519464295	P: None	Score: 2.88819412
Iteration: 14	Temperature: 0.993022704563	P: 0.128300611373	Score: 2
Iteration: 15	Temperature: 0.99252619321	P: 0.102557436692	Score: 2
Iteration: 16	Temperature: 0.992029930114	P: 0.0877554325677	Score:
Iteration: 17	Temperature: 0.991533915149	P: None	Score: 2.89512904
Iteration: 18	Temperature: 0.991038148191	P: 0.622032776672	Score: 2
Iteration: 19	Temperature: 0.990542629117	P: None	Score: 2.89141548
Iteration: 20	Temperature: 0.990047357802	P: 1.25346848712	Score: 2
Iteration: 21	Temperature: 0.989552334123	P: 4.76983419232	Score: 2
Iteration: 22	Temperature: 0.989057557956	P: 0.174898399152	Score: 2
Iteration: 23	Temperature: 0.988563029177	P: None	Score: 2.89179873
Iteration: 24	Temperature: 0.988068747663	P: 0.216753122695	Score: 2
Iteration: 25	Temperature: 0.987574713289	P: 0.189213045067	Score: 2
Iteration: 26	Temperature: 0.987080925932	P: 0.180407978065	Score: 2
Iteration: 27	Temperature: 0.986587385469	P: 1.28586520044	Score: 2
Iteration: 28	Temperature: 0.986094091777	P: 0.341627962218	Score: 2
Iteration: 29	Temperature: 0.985601044731	P: 0.375987666516	Score: 2
Iteration: 30	Temperature: 0.985108244208	P: None	Score: 2.88630166
Iteration: 31	Temperature: 0.984615690086	P: 0.0954032625088	Score:
Iteration: 32	Temperature: 0.984123382241	P: 0.0525683032876	Score:
Iteration: 33	Temperature: 0.98363132055	P: 0.0628071676595	Score: 2
Iteration: 34	Temperature: 0.98313950489	P: 0.0850294079813	Score: 2
Iteration: 35	Temperature: 0.982647935137	P: 0.103948877329	Score: 2
Iteration: 36	Temperature: 0.98215661117	P: 0.112592270821	Score: 2
Iteration: 37	Temperature: 0.981665532864	P: 0.140502377735	Score: 2
Iteration: 38	Temperature: 0.981174700098	P: 0.619562566309	Score: 2
Iteration: 39	Temperature: 0.980684112748	P: 0.121706922638	Score: 2
Iteration: 40	Temperature: 0.980193770691	P: 0.524456864111	Score: 2
Iteration: 41	Temperature: 0.979703673806	P: None	Score: 2.88976590
Iteration: 42	Temperature: 0.979213821969	P: None	Score: 2.88533611
Iteration: 43	Temperature: 0.978724215058	P: 0.980364885017	Score: 2
Iteration: 44	Temperature: 0.978234852951	P: 7.61482776364	Score: 2
Iteration: 45	Temperature: 0.977745735524	P: 0.686543039713	Score: 2
Iteration: 46	Temperature: 0.977256862656	P: None	Score: 2.88465673

Iteration: 47	Temperature: 0.976768234225	P: 0.515636180028	Score: 2.884505902
Iteration: 48	Temperature: 0.976279850108	P: None	Score: 2.884505902
Iteration: 49	Temperature: 0.975791710183	P: 0.128453139363	Score: 2.884505902
Iteration: 50	Temperature: 0.975303814328	P: None	Score: 2.888073332
Iteration: 51	Temperature: 0.974816162421	P: 0.140618396511	Score: 2.888073332
Iteration: 52	Temperature: 0.974328754339	P: 0.114720182613	Score: 2.888073332
Iteration: 53	Temperature: 0.973841589962	P: 0.309780042022	Score: 2.888073332
Iteration: 54	Temperature: 0.973354669167	P: 6.53437233328	Score: 2.888073332
Iteration: 55	Temperature: 0.972867991833	P: 0.619509370388	Score: 2.888073332
Iteration: 56	Temperature: 0.972381557837	P: 0.230758867983	Score: 2.888073332
Iteration: 57	Temperature: 0.971895367058	P: 0.810366761929	Score: 2.888073332
Iteration: 58	Temperature: 0.971409419374	P: 0.0939262086143	Score: 2.888073332
Iteration: 59	Temperature: 0.970923714665	P: 0.0971091677924	Score: 2.888073332
Iteration: 60	Temperature: 0.970438252807	P: None	Score: 2.894463242
Iteration: 61	Temperature: 0.969953033681	P: None	Score: 2.890526282
Iteration: 62	Temperature: 0.969468057164	P: 1.03096478602	Score: 2.890526282
Iteration: 63	Temperature: 0.968983323135	P: 0.189131232554	Score: 2.890526282
Iteration: 64	Temperature: 0.968498831474	P: 0.43403943251	Score: 2.890526282
Iteration: 65	Temperature: 0.968014582058	P: None	Score: 2.879772842
Iteration: 66	Temperature: 0.967530574767	P: 0.0748004233869	Score: 2.879772842
Iteration: 67	Temperature: 0.96704680948	P: 0.0357864966124	Score: 2.879772842
Iteration: 68	Temperature: 0.966563286075	P: 0.11533752878	Score: 2.879772842
Iteration: 69	Temperature: 0.966080004432	P: 0.107562946482	Score: 2.879772842
Iteration: 70	Temperature: 0.96559696443	P: 0.057865039715	Score: 2.879772842
Iteration: 71	Temperature: 0.965114165948	P: 0.0441425712607	Score: 2.879772842
Iteration: 72	Temperature: 0.964631608865	P: 0.0570606339528	Score: 2.879772842
Iteration: 73	Temperature: 0.96414929306	P: 0.0432372909521	Score: 2.879772842
Iteration: 74	Temperature: 0.963667218414	P: 0.047242195215	Score: 2.879772842
Iteration: 75	Temperature: 0.963185384804	P: 0.0446192663043	Score: 2.879772842
Iteration: 76	Temperature: 0.962703792112	P: 0.046660821786	Score: 2.879772842
Iteration: 77	Temperature: 0.96222440216	P: 0.0562486770757	Score: 2.879772842
Iteration: 78	Temperature: 0.961741328996	P: 0.0317037080542	Score: 2.879772842
Iteration: 79	Temperature: 0.961260458331	P: 0.0473427510934	Score: 2.879772842
Iteration: 80	Temperature: 0.960779828102	P: 0.0228080230809	Score: 2.879772842
Iteration: 81	Temperature: 0.960299438188	P: 0.0296348937234	Score: 2.879772842
Iteration: 82	Temperature: 0.959819288469	P: 0.0259625604013	Score: 2.879772842
Iteration: 83	Temperature: 0.959339378825	P: 0.0213239390589	Score: 2.879772842
Iteration: 84	Temperature: 0.958859709135	P: 0.025943401562	Score: 2.879772842
Iteration: 85	Temperature: 0.958380279281	P: 0.0259542517775	Score: 2.879772842
Iteration: 86	Temperature: 0.957901089141	P: 0.0290801421499	Score: 2.879772842
Iteration: 87	Temperature: 0.957422138597	P: 0.0401776785774	Score: 2.879772842
Iteration: 88	Temperature: 0.956943427527	P: 0.0295293316544	Score: 2.879772842
Iteration: 89	Temperature: 0.956464955814	P: 0.0253106231089	Score: 2.879772842
Iteration: 90	Temperature: 0.955986723336	P: 0.0247420176356	Score: 2.879772842
Iteration: 91	Temperature: 0.955508729974	P: 0.0243557679352	Score: 2.879772842
Iteration: 92	Temperature: 0.955030975609	P: 0.0324639036763	Score: 2.879772842
Iteration: 93	Temperature: 0.954553460121	P: 0.0251154936626	Score: 2.879772842
Iteration: 94	Temperature: 0.954076183391	P: 0.0216268468474	Score: 2.879772842

Iteration: 95	Temperature: 0.953599145299	P: 0.0190749211822	Score:
Iteration: 96	Temperature: 0.953122345727	P: 0.0185969516035	Score:
Iteration: 97	Temperature: 0.952645784554	P: 0.0238962960655	Score:
Iteration: 98	Temperature: 0.952169461662	P: 0.0216572392355	Score:
Iteration: 99	Temperature: 0.951693376931	P: 0.0219966463459	Score:
Iteration: 100	Temperature: 0.951217530242	P: 0.0284560235701	Score:

```
In [6]: fig=plt.figure(figsize=(10, 6), dpi= 80, facecolor='w', edgecolor='k')
```

```
g1 = gs.GridSpec(1,1)
```

```
ax1 = plt.subplot(g1[0,0])
```

```
plt.plot((final_results))
```

```
plt.xlabel('Iteration Number', {'fontsize': 12})
```

```
plt.ylabel('Total Score For The Night (100s of billions of weight-miles)', {'fontsize': 12})
```

```
plt.title('')
```

```
Out[6]: Text(0.5,1,'')
```

