BOOLEAN ALGEBRA

Boolean Algebra

- **A Boolean algebra is a closed algebraic system containing a set *K* of two or more elements and the two operators · and + which refer to logical AND and logical OR.**

Basic Identities

(1) $x + 0 = x$	(2) $x \cdot 0 = 0$
(3) $x + 1 = 1$	(4) $x \cdot 1 = x$
(5) $x + x = x$	(6) $x \cdot x = x$
(7) $x + x' = 1$	(8) $x \cdot x' = 0$

Basic Identities

- **Commutative Law**

  9. $x \cdot y = y \cdot x$

  10. $x + y = y + x$

- **Associative Law**

  11. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

  12. $x + (y + z) = (x + y) + z$

- ◇ **Distributive**

  13. $x \bullet (y+z) = xy + xz$

  14. $x + (y \bullet z) = (x + y)(x + z)$

- ◇ **Involution**

  15. $(x')' = x$

Duality

- **A true Boolean Algebra expression can be converted into another true Boolean algebra expression by making the following substitutions:**

  1 ↔ 0 **• ↔ +**

DeMorgan's Theorem

- **DeMorgan's Theorem provides the final link in taking any expression, including an expression with NAND or NOR, and converting it to S-o-P or P-o-S form.**

  16. $(x+y)' = x' \bullet y'$

  17. $(x \bullet y)' = x' + y'$

Function Minimization using Boolean Algebra

Examples:

- $a + ab$

  $= a(1+b)$ *(Identity – 3)*

  $=a$

- $a(a + b)$

  $= a.a + ab$ *(Distributive Law)*

  $= a+ab$

  $=a(1+b)$ *(Identity – 3)*

  $=a$

- $a + a'b$

  $= (a + a')(a + b)$ (Distributive Law)

  $=1(a + b)$

  $=a+b$

- $a(a' + b)$

  $= a.\ a' + ab$ (Distributive Law)

  $=0+ab$

  $=ab$

Show That

1. $ab + ab' = a$
2. $(a + b)(a + b') = a$

- **ab + ab′ = a**

L.H.S = ab + ab′

= a(b+b′)

= a(1)

= a

= R.H.S

- $(a + b)(a + b') = a$

L.H.S. $= (a+b)(a+b')$

$= a + (b*b')$ (Distributive Law)

$= a + (0)$ (Identity – 8)

$= a + 0$

$= a$ (Identity – 1)

= R.H.S.