#Solution-1:

a) $\int_{4}^{9} 3\sqrt{x}\, dx$

Here, We know $\sqrt{x} = x^{1/2}$, so the integrand becomes $3x^{1/2}$.

The antiderivative of $3x^{1/2}$ is:

$\int 3x^{1/2}dx \Rightarrow 3.\frac{1}{3}.x^{3/2}$

Now, evaluating from 4 to 9, we get,

$\left[x^{3/2}\right]_{4}^{9} = (9^{3/2} - 4^{3/2})$

= 2(27) - 2(8)
= 54 - 16
= 38

b)$\int ln(x)dx$

Using integration by parts.

Let u= $ln(x)$, $dv=dx$.

Then du = $\frac{1}{x}dx$ and v = x.

Using Integration by part formula:

$\int u\, dv = uv - \int u\, du$

$\int ln(x)\, dx = xln(x) - \int x\, .\frac{1}{x}dx$

$\Rightarrow xln(x) - x$

Evaluating from 1 to e:

$[x\, ln(x) \;-\; x\,]_{1}^{e} \;=\; [eln(e) - e] - [1\, .\, ln(1) - 1]$

$= [e - e] - [0 - 1]$

$= 0 + 1$

$=1$

c) $\int_0^1 cos^{-1}(x)\, dx$

Here, the indefinite integral of the given equation becomes,

$$\int_0^1 cos^{-1}(x)\, dx = xcos^{-1}(x) - \sqrt{1 - x^2} + C$$

Applying the definite integral limits:

$$= xcos^{-1}(x) - \sqrt{1 - x^2}\,]_0^1$$

Evaluating at X = 0:

$xcos^{-1}(x)$ = 0.cos^{-1}(0) - $\sqrt{1 - 0^2}$] = -1

Evalauting at x=1:

$xcos^{-1}(x)$ = 1.cos^{-1}(1) - $\sqrt{1 - 1^2}$ = 0

Substituting , we get,
=[0]-[-1]
= 1

Thus, value of $\int_0^1 cos^{-1}(x)\, dx$ = 1.

(d) $\int_{-1}^{1} \pi \cos\left(\frac{\pi x}{2}\right)\, dx$
The constant π can be factored out:

$$\pi \int_{-1}^{1} \cos\left(\frac{\pi x}{2}\right)\, dx$$

The antiderivative of $\cos(kx)$ is:

$$\int \cos(kx)\, dx = \frac{\sin(kx)}{k}$$

Here, $k = \frac{\pi}{2}$, so:

$$\int \cos\left(\frac{\pi x}{2}\right)\, dx = \frac{2}{\pi} \sin\left(\frac{\pi x}{2}\right)$$

Now evaluate the definite integral:

$$\pi \cdot \left[\frac{2}{\pi} \sin\left(\frac{\pi x}{2}\right)\right]_{-1}^{1} = 2\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

Simplify:

$$\sin\left(\frac{\pi}{2}\right) = 1, \quad \sin\left(-\frac{\pi}{2}\right) = -1$$

$$2(1 - (-1)) = 2(2) = 4$$

Result: $\int_{-1}^{1} \pi \cos\left(\frac{\pi x}{2}\right)\, dx = 4$

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2.

(a) $\int x^2 \cos(x^3)\, dx$
Let $u = x^3$, so $du = 3x^2\, dx$, or $\frac{1}{3}\, du = x^2\, dx$.
The integral becomes:

$$\int x^2 \cos(x^3)\, dx = \frac{1}{3} \int \cos(u)\, du$$

The antiderivative of $\cos(u)$ is $\sin(u)$, so:

$$\frac{1}{3} \sin(u) + C$$

Substitute back $u = x^3$:

$$\frac{\sin(x^3)}{3} + C$$

Result: $\int x^2 \cos(x^3)\, dx = \frac{\sin(x^3)}{3} + C$

(b) $\int \frac{\cos(3t)}{1+\sin(3t)}\,dt$
Let $u = 1 + \sin(3t)$, so $du = 3\cos(3t)\,dt$, or $\frac{1}{3}\,du = \cos(3t)\,dt$.
The integral becomes:

$$\frac{1}{3}\int \frac{1}{u}\,du$$

The antiderivative of $\frac{1}{u}$ is $\ln|u|$, so:

$$\frac{1}{3}\ln|u| + C$$

Substitute back $u = 1 + \sin(3t)$:

$$\frac{\ln(1+\sin(3t))}{3} + C$$

Result: $\int \frac{\cos(3t)}{1+\sin(3t)}\,dt = \frac{\ln(1+\sin(3t))}{3} + C$

3.

The function is $f(x) = x^3 - 5x^2 + 30$, and we will evaluate over $x\varepsilon[0, 4]$.

The average value of a function is:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$

Substituting f(x) = $x^3 - 5x^2 + 30$ and a=0, b=4:

$$f_{avg} = \frac{1}{4-0}\int_0^4 (x^3 \; - \; 5x^2 \; + \; 30)dx$$

Evaluating the integral,

$$= \frac{x^4}{4} - \frac{5x^3}{3} + 30\text{x}$$

Evaluating from 0 to 4,

$$\left[\frac{x^4}{4} \; - \; \frac{5x^3}{3} \; + \; 30x\right]_0^4 \qquad = \left[\frac{4^4}{4} - \frac{5(4)^3}{3} + 30(4)\right]$$

$$= 19.33$$

x	f(x)
0	30
0.8	22.24
1.6	-60.79
2.4	-136.09
3.2	-204.02
4	-245.13