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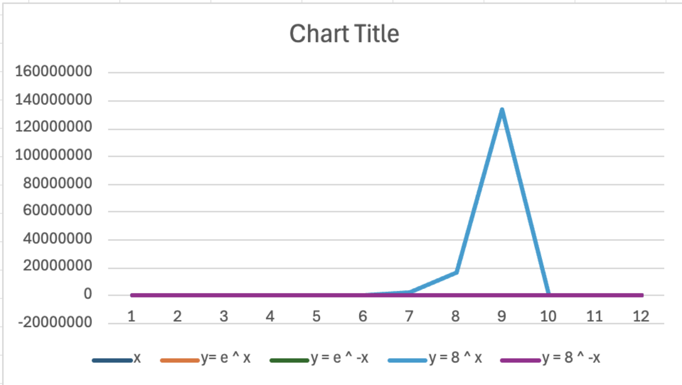
19707
Calculus-I

Q.no.1) 1. Plot each following group of functions in one graph respectively by Excel, covering the appropriate domain of x and y.
a. $y = ex$, $y = e —x$, $y = 8x$, $y = 8 —x$
b. $y = 0.9x$, $y = 0.6x$, $y = 0.3x$, $y = 0.1x$

Solution:

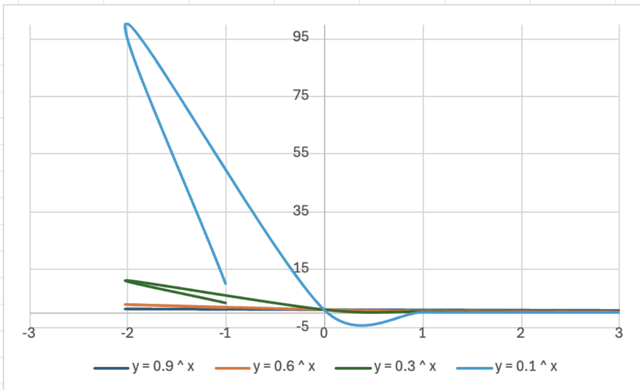
Q.No.1a)

x	y= e ^ x	y = e ^ -x	y = 8 ^ x	y = 8 ^ -x
1	2.718282	0.367879441	8	0.125
2	7.389056	0.135335283	64	0.015625
3	20.08554	0.049787068	512	0.001953
4	54.59815	0.018315639	4096	0.000244
5	148.4132	0.006737947	32768	3.05E-05
6	403.4288	0.002478752	262144	3.81E-06
7	1096.633	0.000911882	2097152	4.77E-07
8	2980.958	0.000335463	16777216	5.96E-08
9	8103.084	0.00012341	134217728	7.45E-09
-1	0.367879	2.718281828	0.125	8
-2	0.135335	7.389056099	0.015625	64
-4	0.018316	54.59815003	0.00024414	4096



1b)

1b				
x	y = 0.9 ^ x	y = 0.6 ^ x	y = 0.3 ^ x	y = 0.1 ^ x
-1	1.111111	1.666666667	3.33333333	10
-2	1.234568	2.777777778	11.1111111	100
0	1	1	1	1
1	0.9	0.6	0.3	0.1
2	0.81	0.36	0.09	0.01
2.7	0.75241	0.251772549	0.03874605	0.001995
3.6	0.684342	0.158980735	0.01311102	0.000251
4.5	0.622431	0.100387728	0.00443655	3.16E-05
5.4	0.566121	0.063389416	0.00150126	3.98E-06
6.3	0.514906	0.040026985	0.000508	5.01E-07
7.2	0.468324	0.025274874	0.0001719	6.31E-08
8.1	0.425956	0.015959715	5.8168E-05	7.94E-09



Q.No.2) Given $f(x) = 10x$, prove that $\frac{f(x+h)-f(x)}{h} = 10^x(\frac{10^h-1}{h})$
and verify it by the plot in Excel.

ANSWER:

Given we have, $f(x) = 10^x$

Then, $f(x\ +\ h) = \ 10^{x+h} = 10^{x}.\ 10^{h}$

Now, $f(x\ +\ h)$ - $f(x) = 10^{x}.\ 10^{h}$ - 10^{x}

Or, $f(x\ +\ h)$ - $f(x) = \ 10^{x}(\ 10^{h} - 1)$

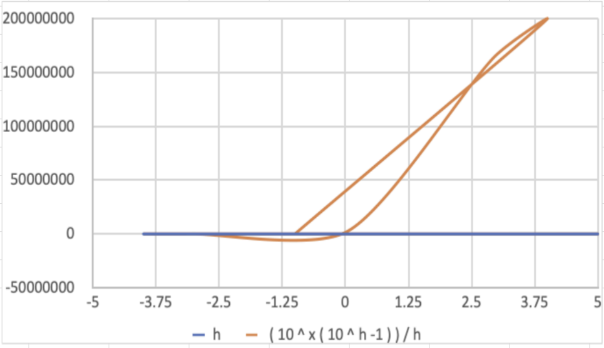
Dividing the equation by h on both sides, we get,

$\frac{f(x+h) - f(x)}{h} = \ 10^{x}(\frac{10^{h} - 1}{h})$

Hence, we have proved the equation.

Q.no.2)

x	h	(10 ^ x (10 ^ h -1)	10 ^ h -1	10 ^ x
-1	4	249.975	9999	0.1
4	5	199998000	99999	10000
3	6	166666500	999999	1000
0	7	1428571.286	9999999	1
-3	8	12499.99988	99999999	0.001
-4	-1	0.00009	-0.9	0.0001
-2	4	24.9975	9999	0.01
3	-4	249.975	-0.9999	1000
5	-3	33300	-0.999	100000

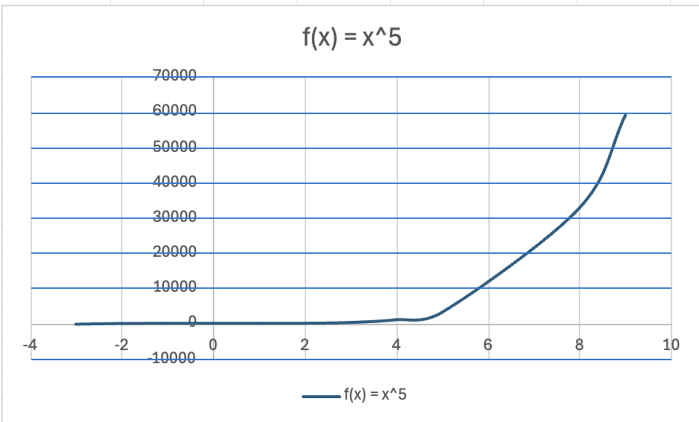


Q.No.3) . Compare the functions *f*(*x*) = *x*5 and *g*(*x*) = 5*x* by plotting curve in Excel and which function grows more rapidly when x is large? And prove it mathematically.

Graph of f(x)

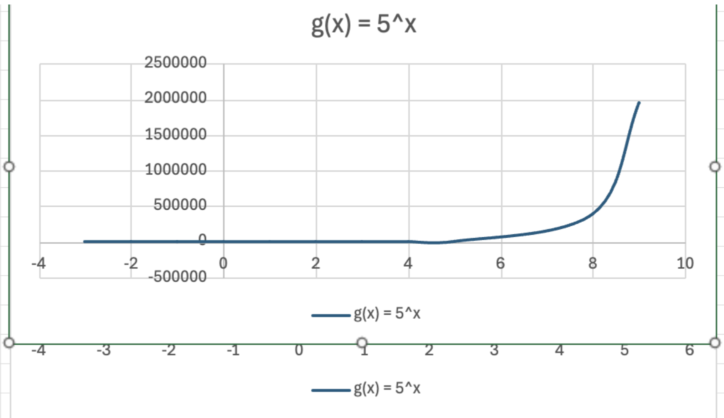
Q.No.3)

x	f(x) = x^5	g(x) = 5^x
-3	-243	0.008
-2	-32	0.04
-1	-1	0.2
0	0	1
1	1	5
2	32	25
3	243	125
4	1024	625
5	3125	3125
8	32768	390625
9	59049	1953125



Graph of g(x)

x	g(x) = 5^x
-3	0.008
-2	0.04
-1	0.2
0	1
1	5
2	25
3	125
4	625
5	3125
8	390625
9	1953125



From the graph we can observe that, $g(x) = 5^x$ grows faster than $f(x) = x^5$ when x is large.

For small values of xxx, $f(x)=x^5$ grows more gradually.

As x increases, $g(x)=5^x$ overtakes and grows significantly faster than f(x)f(x)f(x), especially when x is large.

Mathematical calculation:

$f(x) = x^5$:
Here, when x value , the value of f(x) function increases rapidly.
• Limit of $f(x) = x^5$ as x approaches infinity:
$\lim_{x\to\infty} x^5 = \infty$
• Limit of $g(x) = 5^x$ as x approaches infinity:
$\lim_{x\to\infty} 5^x = \infty$

However, we can further compare the growth rates by calculating the ratio of the functions for large values of x. For example, let's consider x = 100: $f(100) = 100^5 = 10{,}000{,}000{,}000$ $g(100) = 5^{100} = 7.8886091 \times 10^{69}$

Both limits are infinite, indicating that both functions grow without bound as x approaches infinity. However, we can observe that the exponential function $g(x) = 5^x$ grows much more rapidly compared to the polynomial function $f(x) = x^5$. Thus, mathematically, $g(x) = 5^x$ grows more rapidly than $f(x) = x^5$ when x is large.

Step 1: Consider the ratio of $g(x) = 5^x$ and $f(x) = x^5$:

$$\lim_{x\to\infty} \frac{g(x)}{f(x)} = \lim_{x\to\infty} \frac{5^x}{x^5}$$

Step 2: Apply L'Hopital's Rule:

This is an indeterminate form of $\frac{\infty}{\infty}$, so we can apply L'Hopital's Rule multiple times by differentiating the numerator and denominator:

- Differentiate the numerator 5^x repeatedly:

$$\frac{d}{dx}5^x = 5^x \ln(5)$$

- Differentiate the denominator x^5 repeatedly:

$$\frac{d}{dx}x^5 = 5x^4, \quad \frac{d}{dx}5x^4 = 20x^3, \quad \ldots, \frac{d}{dx}120x = 120$$

After several applications of L'Hopital's Rule, the limit will approach:

$$\lim_{x\to\infty} \frac{5^x}{x^5} = \infty$$

Conclusion, Since the limit tends to infinity, this means that g(x) = 5^x grows much faster than f(x) = x^5 for large x

Q.No.4)

Plot the function $f(x) = \frac{1-e^{1/x}}{1+e^{1/x}}$ in **Excel**. And then prove that $f(x)$ is an odd function.

A function is odd if:

f(-x) = -f(x)

The given function is:

$f(x) = \frac{1-e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$

We know,

$f(-x) = \frac{1-e^{-\frac{1}{x}}}{1+e^{-\frac{1}{x}}}$

Multiplying numerator and denominator by $e^{\frac{1}{x}}$,we get,

$$f(-x) = \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

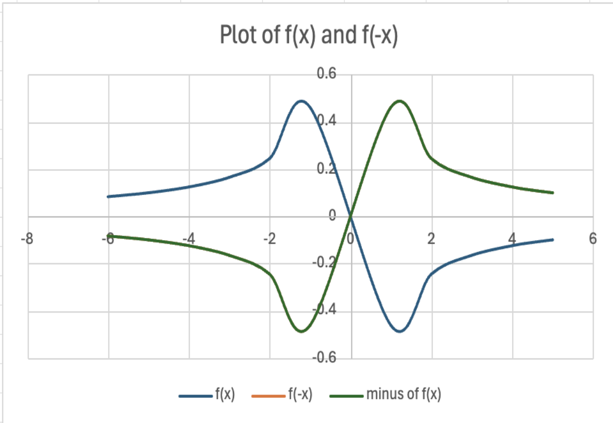
$$f(-x) = -\frac{1-e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} = -f(x)$$

Hence, f(-x) = -f(x) proved.

Plot:

Q.No.4)

x	f(x)	f(-x)	minus of f(x)	f(-x)= -f(x)
-6	0.083141	-0.083140966	-0.08314097	0
-5	0.099668	-0.099667995	-0.09966799	0
-4	0.124353	-0.124353002	-0.124353	0
-3	0.16514	-0.165140413	-0.16514041	0
-2	0.244919	-0.244918662	-0.24491866	0
-1	0.462117	-0.462117157	-0.46211716	0
1	-0.46212	0.462117157	0.46211716	0
2	-0.24492	0.244918662	0.24491866	0
3	-0.16514	0.165140413	0.16514041	0
4	-0.12435	0.124353002	0.124353	0
4.5	-0.11066	0.110656111	0.11065611	0
5	-0.09967	0.099667995	0.09966799	0



Here, we can see that f(-x) = -f(x) for the given x-values that supports the odd symmetry of the function. Therefore, f(x) is an od function.

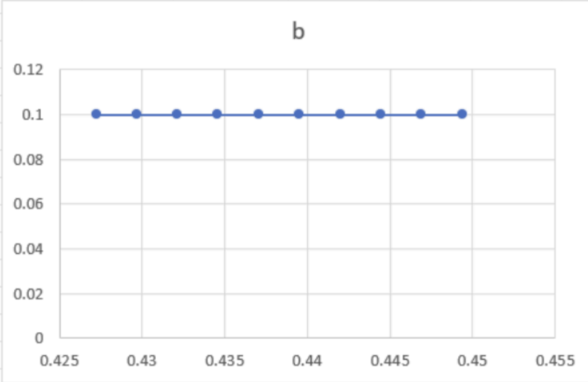
Q.No.5) For the parametrized function $f(x) = 1 / (1 + a * e \wedge bx)$
a. where a > 0. How does the graph change when b changes by showing a group of curves by Excel?
b. How does it change when changes in Excel?

Solution:

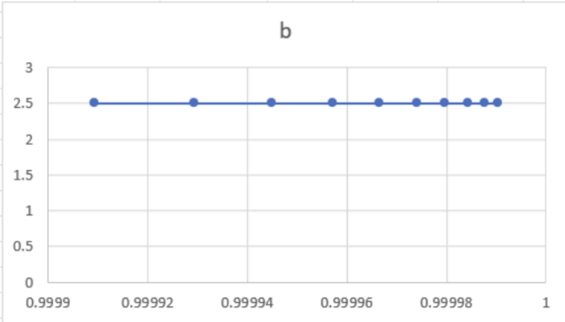
a) in this the value of b is changing.

NUMBER 5 a)

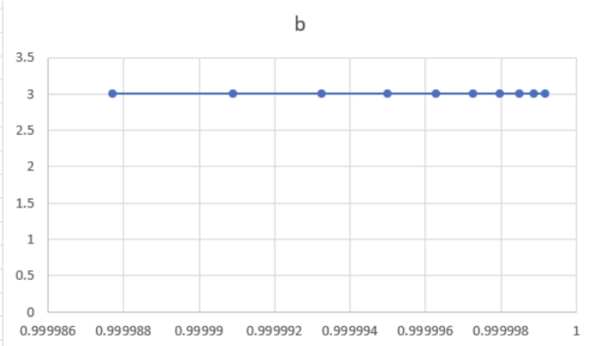
x	a	1/(1+ae^bx)	b
-4.9	2	0.449387146	0.1
-4.8	2	0.446914035	0.1
-4.7	2	0.444443548	0.1
-4.6	2	0.441975805	0.1
-4.5	2	0.439510924	0.1
-4.4	2	0.437049023	0.1
-4.3	2	0.434590219	0.1
-4.2	2	0.432134631	0.1
-4.1	2	0.429682372	0.1
-4	2	0.42723356	0.1



x	a	1/(1+ae^bx)	b
-4.9	2	0.99999043	2.5
-4.8	2	0.999987712	2.5
-4.7	2	0.999984222	2.5
-4.6	2	0.99997974	2.5
-4.5	2	0.999973986	2.5
-4.4	2	0.999966598	2.5
-4.3	2	0.999957111	2.5
-4.2	2	0.99994493	2.5
-4.1	2	0.99992929	2.5
-4	2	0.999909208	2.5

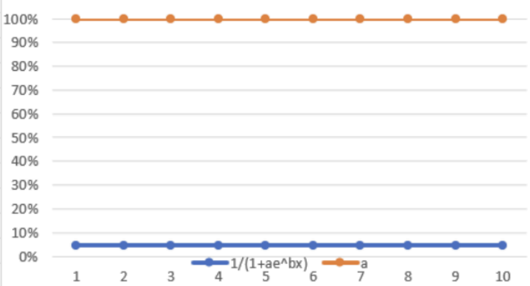


x	a	1/(1+ae^bx)	b
-4.9	2	0.999999174	3
-4.8	2	0.999998885	3
-4.7	2	0.999998495	3
-4.6	2	0.999997969	3
-4.5	2	0.999997258	3
-4.4	2	0.999996299	3
-4.3	2	0.999995004	3
-4.2	2	0.999993256	3
-4.1	2	0.999990897	3
-4	2	0.999987712	3

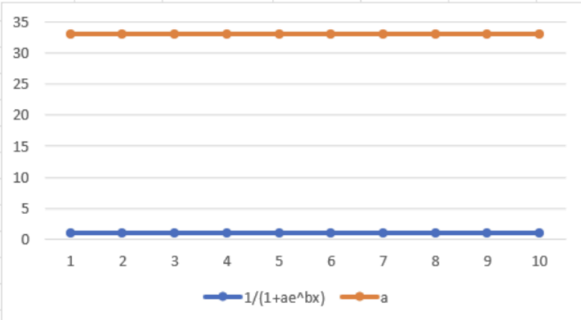


b) b is fixed but a value is changing.

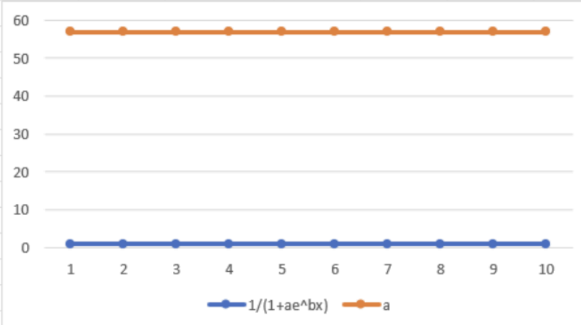
Number 5	b)		
x	b	1/(1+ae^bx)	a
-4.9	1.5	1	21
-4.8	1.5	1	21
-4.7	1.5	1	21
-4.6	1.5	1	21
-4.5	1.5	1	21
-4.4	1.5	1	21
-4.3	1.5	1	21
-4.2	1.5	1	21
-4.1	1.5	1	21
-4	1.5	1	21



x	b	1/(1+ae^bx)	a
-4.9	1.5	1	32
-4.8	1.5	1	32
-4.7	1.5	1	32
-4.6	1.5	1	32
-4.5	1.5	1	32
-4.4	1.5	1	32
-4.3	1.5	1	32
-4.2	1.5	1	32
-4.1	1.5	1	32
-4	1.5	1	32



x	b	1/(1+ae^bx)	a
-4.9	1.5	1	57
-4.8	1.5	1	57
-4.7	1.5	1	57
-4.6	1.5	1	57
-4.5	1.5	1	57
-4.4	1.5	1	57
-4.3	1.5	1	57
-4.2	1.5	1	57
-4.1	1.5	1	57
-4	1.5	1	57



Q.no.6)

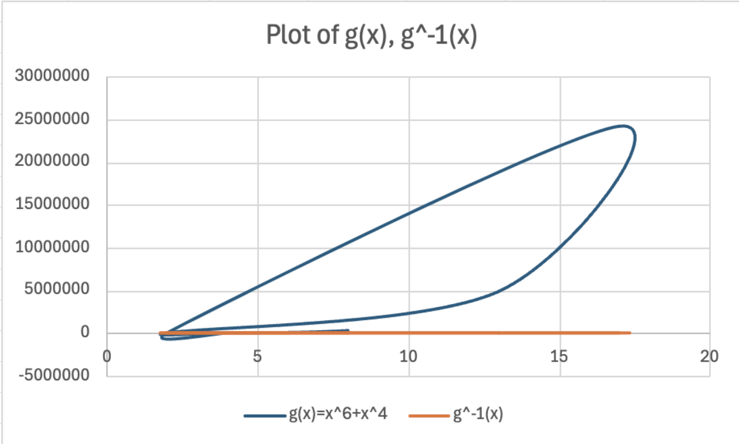
If $g(x) = x^6 + x^4, x \geq 0$, find $g^{-1}(x)$ expression. And that, plot $y = g(x)$, $y = x$, and $y = g^{-1}(x)$ in one graph by **Excel**

Solution:

Unfortunately, finding an explicit algebraic expression for g ^ -1 is not possible. y^6+y^4-x=0

Q.No.6

x	g(x)=x^6+x	g^-1(x)
6	47952	2.08542E-05
4	4352	0.000229779
2	80	0.0125
13	4855370	2.05958E-07
17	24221090	4.12863E-08
2	80	0.0125
4	4352	0.000229779
6	47952	2.08542E-05
7	120050	8.32986E-06
8	266240	3.75601E-06



Q.no.7)

When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge given by $Q(t) = Q0(1 — e —t/a)$ (The maximum charge capacity is $Q0$ and t is measured in seconds.)

a. Find the inverse of this function and explain its meaning.
b. How long does it take to recharge the capacitor to 90% of capacity if a = 2 showing in the plot by Excel?

Solution:

The given function is:

$$Q(t) = Q_0\left(1 - e^{-\frac{t}{\alpha}}\right)$$

$$\text{or, } \frac{Q(t)}{Q_0} = \left(1 - e^{-\frac{t}{\alpha}}\right)$$

$$\text{or, } e^{-\frac{t}{\alpha}} = 1 - \frac{Q(t)}{Q_0}$$

Now, take natural logarithm on both sides,

$$\ln\left(e^{-\frac{t}{\alpha}}\right) = \ln\left(1 - \frac{Q(t)}{Q_0}\right)$$

$$-\frac{t}{\alpha} = \ln\left(1 - \frac{Q(t)}{Q_0}\right)$$

$t = -\alpha . \ln(1 - \frac{Q(t)}{Q_0})$

The inverse function $t = -\alpha . \ln(1 - \frac{Q(t)}{Q_0})$ gives us the time it takes for the capacitor to reach a certain charge Q, given the constant α and the maximum charge Q0.

b) Time to reach 90% capacity when α=2.

At 90% capacity, Q(t) = $0.9*Q_0$

$t = -2 * \ln(1 - \frac{0.9*Q_0}{Q_0})$

t= -2*ln(1-0.9)

t= -2.ln(0.1)

ln(0.1)~ -2.3026

So, t= -2.(-2.3026)

t= 4.6052 seconds

So, it takes 4,61 seconds to recharge the capacitor to 90% of its capacity.

Q.no.7)

a	q	t=- a ln (1 – (Q / Q0))
2	0.9	4.6