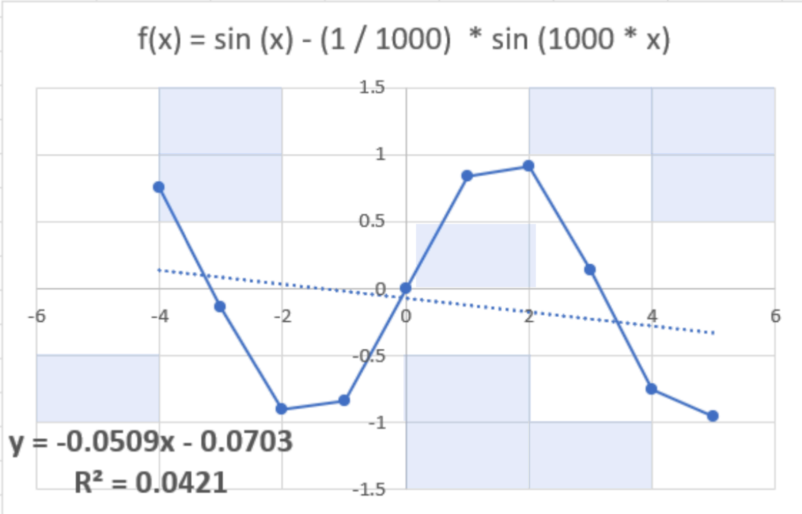
1. a) Graph in Excel the function $f(x) = sin(x) - \frac{1}{1000} * sin(1000x)$ in the viewing rectangle [-2π, 2π] by [-4, 4]. What slope does the graph appear to have at the origin?

ANSWER:
From the graph, we can see that, the slope of the graph appears to have at the origin is: -0.0509 x – 0.0703

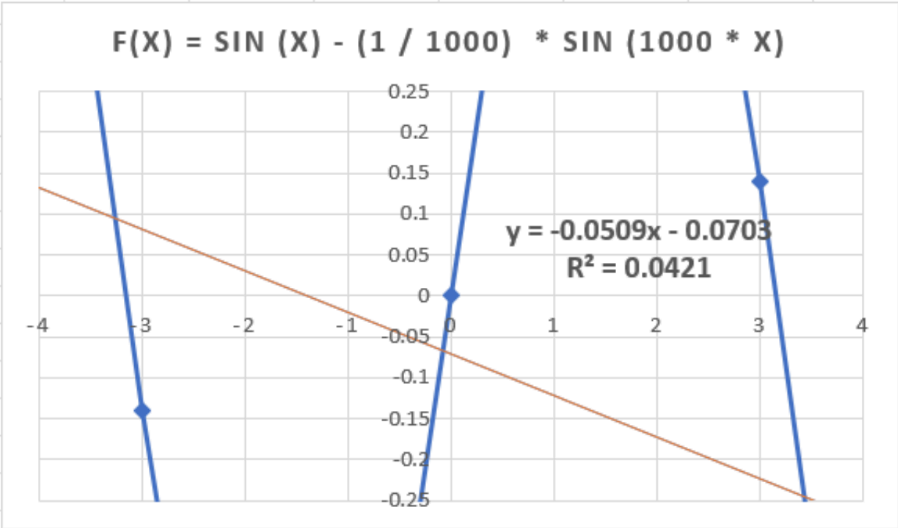
x	f(x) = sin (x) - (1 / 1000) * sin (1000 * x)
-4	0.756118992
-3	-0.140900818
-2	-0.908367387
-1	-0.840644105
0	0
1	0.840644105
2	0.908367387
3	0.140900818
4	-0.756118992
5	-0.957936308



(b) Zoom in to the viewing window [-0.4, 0.4] by [-0.25, 0.25] in Excel and estimate the value of $f'(0)$. Does this agree with your answer from part (a)?
ANSWER:

I have adjusted the values of x-axis and y-axis on the graph to match the viewing window [-0.4, 0.4] by [-0.25, 0.25]. We can observe that, the slope is still the same agreeing with part a.

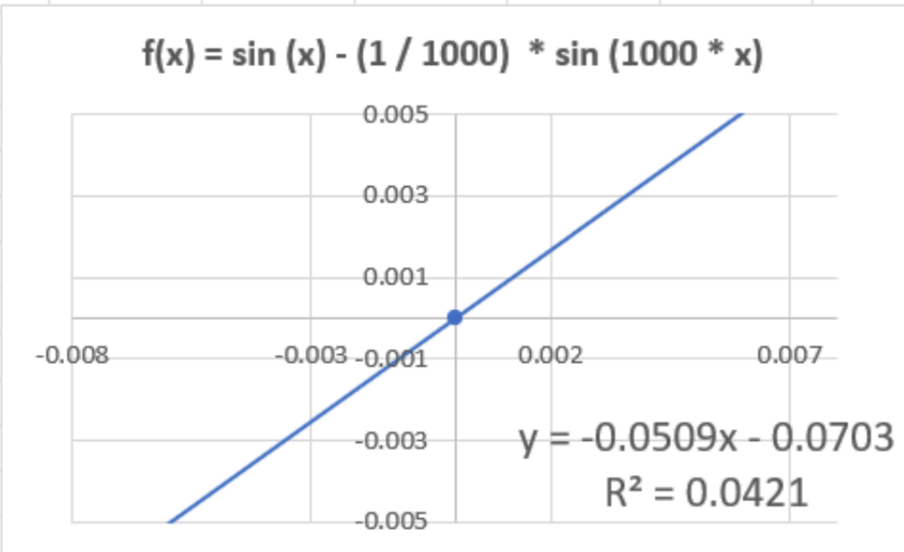
b)



(c) Now zoom in to the viewing window [-0.008, 0.008] by [-0.005, 0.005] in Excel. Do you wish to revise your estimate for $f'(0)$?

ANSWER: We can strongly say that the slope of the curve at 0 which is f'(0) is -0.0509. The slope remains the same.

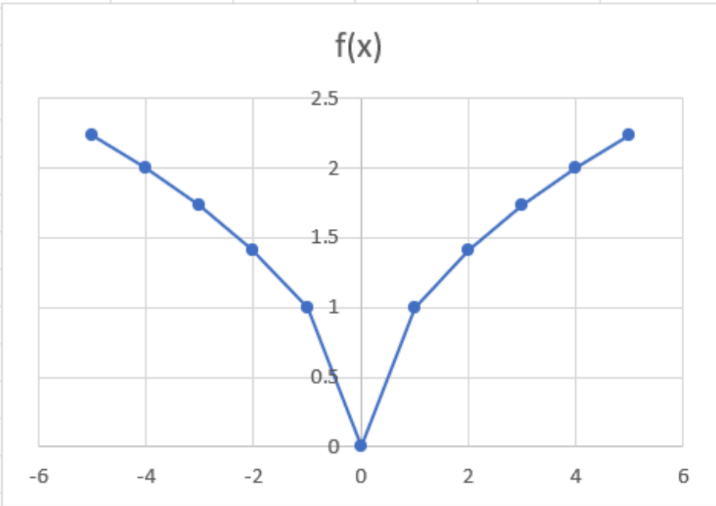
c)



2.. Graph in Excel the function $f(x) = x + \sqrt{|x|}$. Zoom in repeatedly, first toward the

point (-1, 0) and then toward the origin. What is different about the behavior of f in the vicinity of these two points? What do you conclude about the differentiability of f ?

ANS 02	
x	f(x)
-5	2.236067977
-4	2
-3	1.732050808
-2	1.414213562
-1	1
0	0
1	1
2	1.414213562
3	1.732050808
4	2
5	2.236067977



From the graph. When we zoom in to the point (-1, 0), the graph becomes smooth and continuous. Because there is no sharp comer in the graph, the function f(x) is differentiable at x = -1. At this point, the derivative exists and is continuous.

Zooming in closer to the origin (0,0) reveals a sharp 'V' shape. The graph has a sharp comer, and there is no derivative at this point. As a result, the function f(x) is not differentiable at the origin.

3. The left-hand and right-hand derivatives of f at a are defined by

$$f'_-(a) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$

and

$$f'_+(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

if these limits exist. Then $f'(a)$ exists if and only if these one-sided derivatives exist and are equal.

(a) Find $f'_-(4)$ and $f'_+(4)$ for the function

$$f(x) = \begin{cases} 0 & if\ x \le 0 \\ 5 - x & if\ 0 < x < 4 \\ \dfrac{1}{5-x} & if\ x \ge 4 \end{cases}$$

(b) Sketch the graph of *f*.
(c) Where is *f* discontinuous?
(d) Where is *f* not differentiable?

Finding the left derivative at x=4,
For x<4, then f(x)=5−x.

$$f_-'(4) = lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0^-} \frac{f(4+h)-f(4)}{h}$$

Substituting f(x) = 5-x, we get,

$$f_-'(4) = \lim_{h\to 0^-} \frac{(5-(4+h)-(5-4)}{h}$$

$$= \lim_{h\to 0^-} \frac{-h}{h}$$

$$= -1$$

Finding the right derivative at x=4,
For x≥4, then f(x)=$\frac{1}{5-x}$

$$f_+'(4) = \lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0^-} \frac{f(4+h)-f(4)}{h}$$

Substituting f(x)=$\frac{1}{5-x}$ *, we get,*

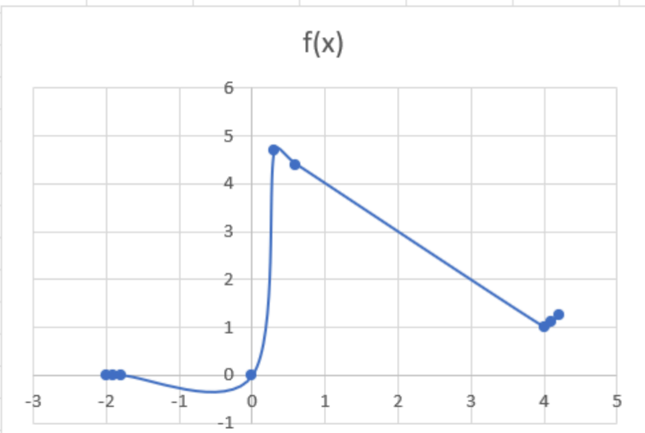
$$f+'(4) = \lim_{h \to 0^+} \frac{\left(\frac{1}{5-(4+h)}\right) - \left(\frac{1}{5-h}\right)}{h}$$

$$= \lim_{h \to 0^+} \frac{\left(\frac{1}{1-h}\right) - 1}{h}$$

$$f+'(4) = \lim_{h \to 0^+} \frac{1}{1-h} = \mathbf{1.}$$

b)Here, f(x) = =IF(x<=0, 0, IF(x<4, 5-x, 1/(5-x)))

ANS 3 b	
x	f(x)
-2	0
-1.9	0
-1.8	0
0	0
0.3	4.7
0.6	4.4
4	1
4.1	1.111111
4.2	1.25



c) Where is f discontinuous?

ANSWER:
Only at x=0, f(x) is discontinuous. At x = 0, f(x) has left-hand limit 0and right hand limit 5 which both are not equal.

d) Where is f not differentiable?

ANSWER:
At x=0 and 4, f(x) is differentiable. At x = 0, f(x) is discontinuous, so, it must be not differentiable and at x = 4, f' (4)≠ f'+(4), so, it is not differentiable

4. If f is a differentiable function and $g(x) = xf(x)$, use the definition of a derivative to show that $g'(x) = xf'(x) + f(x)$

$$g'(x) = \lim_{x \to a} \frac{g(x)-g(a)}{x-a}$$

Substituting g(x) = x f(x) in the following expression, we get,

$$g'(x) = \lim_{x \to a} \frac{xf(x)-af(a)}{x-a}$$

$$g'(x) = \lim_{x \to a} \frac{xf(x)-af(x)+af(x)-af(a)}{x-a}$$

$$g'(x) = \lim_{x \to a} \frac{(x-a)f(x)+a(f(x)-f(a))}{x-a}$$

$$g'(x) = \lim_{x \to a} \frac{(x-a)f(x)}{x-a} + \frac{a(f(x)-f(a))}{x-a}$$

In this expression, the first term is used to simplify the f(x) and the second term is used to simplify to af’(a).

Therefore we can say that, $g'(x) = f'(x)$ + af’(a)
Here, a is a constant, replacing a with x, we get the following,

$g'(x) = xf'(x) + f(x)$ [PROVED]

5.

a)
Given, V = 0.106 m^3 and P = 50 kPa.
Where:
P is the pressure of the gas
V is the volume of the gas
and k is a constant.
Substitute in PV = k
k = (0.106 * 50) kPa · m^3
= 5.3 kPa · m^3
Here, PV = 5.3

V = $\frac{5.3}{P}$

b)Calculate dV / dP when P = 50 kPa. What is the meaning of the derivative?
What are its units?
ANSWER:
From the output of number, a, we can say that,

V = $\frac{5.3}{P}$

Using the differential laws on both sides with respect to P, we get the following results,

$\frac{dV}{dP} = \frac{5.3}{P^2}$

$= \frac{5.3}{50^2}$ {As, we know, P = 50 kPa}

= -0.00212 m^3 / kPa

So, the unit of $\frac{dV}{dP}$ = m^3/ kPa which can also be written as m^3/kPa. This unit represents the change in volume per unit change in pressure and reflects how the volume of the gas responds to changes in pressure.

6. Car tires need to be inflated properly because overinflation or underinflation can cause premature tread wear. The data in the table show tire life L (in thousands of miles) for a certain type of tire at various pressures P (in lb/in^2).

P	26	28	31	35	38	42	45
L	50	66	78	81	74	70	59

a. Use a calculator to model tire life with a quadratic function of the pressure.

b. Use the model to estimate $\frac{dL}{dP}$ when $P = 30$ and when $p = 40$. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

ANSWER:

To model tire life with a quadratic function of pressure, we'll use the quadratic Equation:

The quadratic model for L(P)L(P)L(P) is given as:

L(P)=aP^2+bP+cL

P=[26,28,31,35,38,42,45] and L=[50,66,78,81,74,70,59]

L(P) = a * P ^2 + b * P + c
L = 0.275 * P^2 + 19.75 * P – 273.55

b) Use the model to estimate *dL* / *dP* when P = 30 and when p = 40. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

ANSWER:
If we differentiate the function with respect to 'P', we get the following,

$$\frac{dL}{dP} = -0.55P + 19.75$$

$$\frac{dL}{dP}(P{=}30) = -0.55\ (30) + 19.75$$
$$= -16.5 + 19.75$$
$$= 3.25\ \text{lb/in}^2$$

$$\frac{dL}{dP}(P{=}40) = -0.55\ (40) + 19.75$$
$$= -22 + 19.75$$
$$= -2.25\ \text{lb/in}^2$$

Units of the derivative:
The unit of tire life L is thousands of miles.
The unit of pressure P is lb/in^2.
So, the unit of $\frac{dL}{dP}$ is thousands of miles per lb/in^2, which means it represents the change in tire life per unit change in pressure