Ronish Shrestha(19707)
Assignment 3

1. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t — 1.86t2$
(a) Find the average velocity over the given time intervals:

(i) [1, 2]
ANSWER:
When t=1, then y1 = 10t – 1.86(t)^2
= 10(1) – 1.86(1) = 10 – 1.86 = 8.14m

When t=2, then y2 = 10t – 1.86(t)^2
= 10 x 2 – 1.86 x 4 = 20 – 7.2 = 12.56m

Average velocity, Δv = Δy – Δt = (12.56 – 8.14) / (2 – 1) = 4.42 m/s

(ii) [1, 1.5]
ANSWER:
When t=1, then y1 = 10t – 1.86(t)^2
= 10 x 1 – 1.86 x 1 = 10 – 1.86 = 8.14m

When t=1.5, then y2 = 10t – 1.86 x (t)^2
= 10 x 1.5 – 1.86 x 2.25 = 10.815m

Average velocity, Δv = Δy – Δt = (10.815 – 8.14) / (1.5-1) = 5.35 m/s

(iii) [1, 1.1]
ANSWER:

When t=1, then y1 = 10t – 1.86(t)^2
= 10 x 1 – 1.86 x 1 = 10 – 1.86 = 8.14m

When t=1.1, then y2 = 10t – 1.86 x (t)^2
= 10 x 1.1 – 1.86 x 1.21 = 8.7494m

Average velocity, Δv = Δy – Δt = (8.7494 – 8.14) / (1.1-1) = 6.094 m/s

(iv) [1, 1.01]
ANSWER:

When t=1, then y1 = 10t – 1.86(t)^2
= 10 x 1 – 1.86 x 1 = 10 – 1.86 = 8.14m

When t=1.01, then y2 = 10t – 1.86 x (t)^2
= 10 x 1.01 – 1.86 x 1.0201 = 8.2026m

Average velocity, Δv = Δy – Δt = (8.2026 – 8.14) / (1.01-1) = 6.2614 m/s

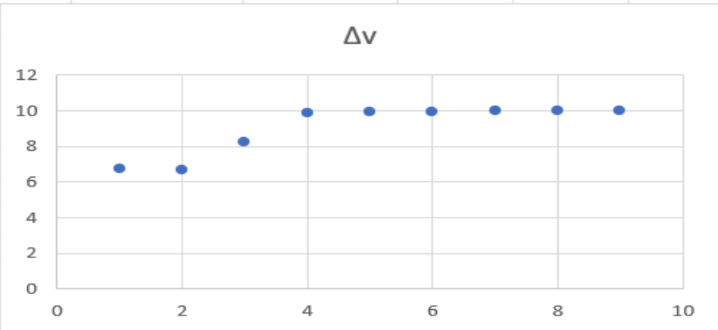
(v) [1, 1.001]
ANSWER:
When t=1, then y1 = 10t – 1.86(t)^2
= 10 x 1 – 1.86 x 1 = 10 – 1.86 = 8.14m

When t=1.001, then y2 = 10t – 1.86 x (t)^2
= 10 x 1.001 – 1.86 x 1.002001 = 8.1462m

Average velocity, Δv = Δy – Δt = (8.1462 – 8.14) / (1.001-1) = 6.278 m/s

(b) Estimate the instantaneous velocity in Excel when t = 1

T	Y= 10t - 1.8t^2	ΔY	Δt	Δv
0.9	7.542	0.13448	0.02	6.724
0.92	7.67648	0.0667	0.01	6.67
0.93	7.74318	-7.34606	-0.89	8.254
0.04	0.39712	-0.09874	-0.01	9.874
0.03	0.29838	-0.0991	-0.01	9.91
0.02	0.19928	-0.19828002	-0.0199	9.96382
0.0001	0.000999982	0.008998218	0.0009	9.99802
0.001	0.0099982	0.0898218	0.009	9.9802
0.01	0.09982	-0.09982	-0.01	9.982



2. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion s = 2sin (πt) +3cos (πt), where t is measured in seconds.
(a) Find the average velocity during each time period:

(i) [1, 2]
ANSWER:
When t=1, then s1 = 2sin (πt) +3cos (πt)
= 2sin (π x 1) +3cos (π x 1)
= 2sin (π) +3cos (π) = 2 x 0 + 3 x (-1) = -3 cm

When t=2, then s2 = 2sin (πt) +3cos (πt)
= 2sin (π x 2) +3cos (π x 2)

$= 2\sin(2\pi) + 3\cos(2\pi) = 3$ cm

Average velocity, $\Delta v = \Delta s – \Delta t = (3 – (-3)) / (2-1) = 6$ cm/s

(ii) [1, 1.1]
ANSWER:
When t=1, then s1 = $2\sin(\pi t) + 3\cos(\pi t)$
$= 2\sin(\pi \times 1) + 3\cos(\pi \times 1)$
$= 2\sin(\pi) + 3\cos(\pi) = 2 \times 0 + 3 \times (-1) = -3$ cm

When t=1.1, then s2 = $2\sin(\pi t) + 3\cos(\pi t)$
$= 2\sin(\pi \times 1.1) + 3\cos(\pi \times 1.1)$
$= 2\sin(1.1\pi) + 3\cos(1.1\pi) = -3.47$ cm

Average velocity, $\Delta v = \Delta s – \Delta t = (-3.47 – (-3)) / (1.1-1) = -4.7$ cm/s

(iii) [1, 1.01]
ANSWER:
When t=1, then s1 = $2\sin(\pi t) + 3\cos(\pi t)$
$= 2\sin(\pi \times 1) + 3\cos(\pi \times 1)$
$= 2\sin(\pi) + 3\cos(\pi) = 2 \times 0 + 3 \times (-1) = -3$ cm

When t=1.01, then s2 = $2\sin(\pi t) + 3\cos(\pi t)$
$= 2\sin(\pi \times 1.01) + 3\cos(\pi \times 1.01)$
$= 2\sin(1.01\pi) + 3\cos(1.01\pi) = -3.061$ cm

Average velocity, $\Delta v = \Delta s – \Delta t = (-3.061 – (-3)) / (1.01-1) = -6.1$cm/s

(iv) [1, 1.001]
ANSWER:
When t=1, then s1 = $2\sin(\pi t) + 3\cos(\pi t)$
$= 2\sin(\pi \times 1) + 3\cos(\pi \times 1)$
$= 2\sin(\pi) + 3\cos(\pi) = 2 \times 0 + 3 \times (-1) = -3$ cm

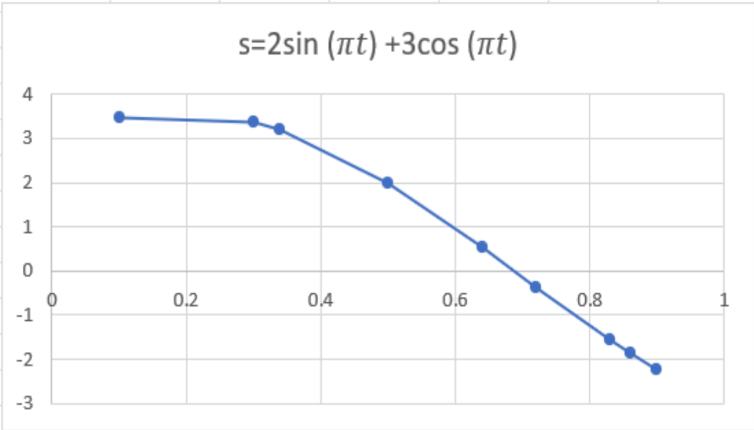
When t=1.001, then s2 = $2\sin(\pi t) + 3\cos(\pi t)$
$= 2\sin(\pi \times 1.001) + 3\cos(\pi \times 1.001)$
$= 2\sin(1.001\pi) + 3\cos(1.001\pi) = -3.006$ cm

Average velocity, $\Delta v = \Delta s – \Delta t = (-3.006 – (-3)) / (1.001-1) = -6$ cm/s

(b) Estimate the instantaneous velocity of the particle in Excel when $t = 1$

number 2

t	$s=2\sin(\pi t)+3\cos(\pi t)$
0.1	3.471203538
0.3	3.381389746
0.34	3.197874382
0.5	2
0.64	0.53231623
0.72	-0.371245484
0.83	-1.56414325
0.86	-1.862922574
0.9	-2.23513556



3. (a) Estimate the value of

lim

$$\lim_{x \to 0} \frac{\sin(x)}{\sin(\pi x)}$$

by graphing the function $f(x) = (\sin x) / (\sin \pi x)$ in Excel. State your answer correct to two decimal places.

ANSWER:

$$\lim_{x \to 0} \frac{\sin(x)}{\sin(\pi x)}$$

$$\Rightarrow \lim_{x \to 0} \frac{\sin(x).\pi x.x}{\sin(\pi x).\pi x.x}$$

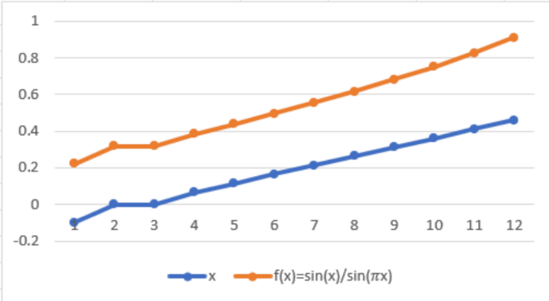
$$\Rightarrow \lim_{x \to 0} \frac{\sin(x)}{x} \, . \lim_{x \to 0} \frac{\pi x}{\sin(\pi x)} . \lim_{x \to 0} \frac{x}{\pi x}$$

$\Rightarrow 1.1.\frac{1}{\pi}$

$\Rightarrow$ 0.32

number 3

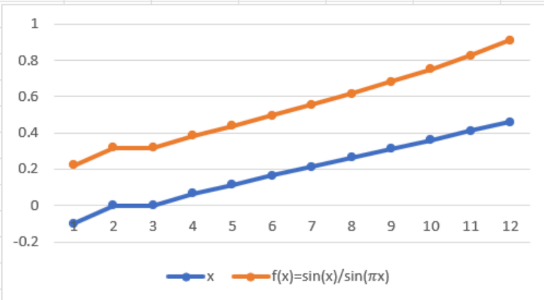
sin (pi() * x	sin (x)	x	f(x)=sin(x)/sin(πx)
-0.30902	-0.099833417	-0.1	0.323067723
-0.00314	-0.001	-0.001	0.318310357
-0.00314	-0.001	-0.001	0.318310357
0.202787	0.064954239	0.065	0.32030724
0.352005	0.114249977	0.1145	0.324569172
0.492727	0.163265831	0.164	0.331351271
0.621558	0.211881724	0.2135	0.340888078
0.735388	0.259978561	0.263	0.353525771
0.83147	0.307438515	0.3125	0.369753158
0.907484	0.354145321	0.362	0.390249476
0.961598	0.39998456	0.4115	0.415958302
0.992504	0.444843937	0.461	0.448203875



b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approaches 0 in Excel.

number 3

sin (pi() * x	sin (x)	x	f(x)=sin(x)/sin(πx)
-0.30902	-0.099833417	-0.1	0.323067723
-0.00314	-0.001	-0.001	0.318310357
-0.00314	-0.001	-0.001	0.318310357
0.202787	0.064954239	0.065	0.32030724
0.352005	0.114249977	0.1145	0.324569172
0.492727	0.163265831	0.164	0.331351271
0.621558	0.211881724	0.2135	0.340888078
0.735388	0.259978561	0.263	0.353525771
0.83147	0.307438515	0.3125	0.369753158
0.907484	0.354145321	0.362	0.390249476
0.961598	0.39998456	0.4115	0.415958302
0.992504	0.444843937	0.461	0.448203875



4.(a) Estimate the value of the limit

$\lim_{x \to 0} (1 + x)^{\frac{1}{x}}$ to five decimal places. Does this number look familiar?

Solution:

L = $\lim_{x \to 0} (1 + x)^{\frac{1}{x}}$

Taking natural logarithm on both sides,

ln(l) = $\lim_{x \to 0} ln\left((1 + x)^{\frac{1}{x}}\right)$

= $\lim_{x \to 0} \frac{1}{x} . ln(1 + x)$ [; $ln(a^{b}) = b.ln$(a)]

$$= \lim_{x \to 0} \frac{ln(1+x)}{x}$$

Now, applying L-Hospital's rule, we get,

$$\lim_{x \to 0} \frac{\frac{d}{dx} ln(1+x)}{\frac{d}{dx}[x]}$$

$$\Rightarrow \lim_{x \to 0} \frac{1}{1+x} = 1$$

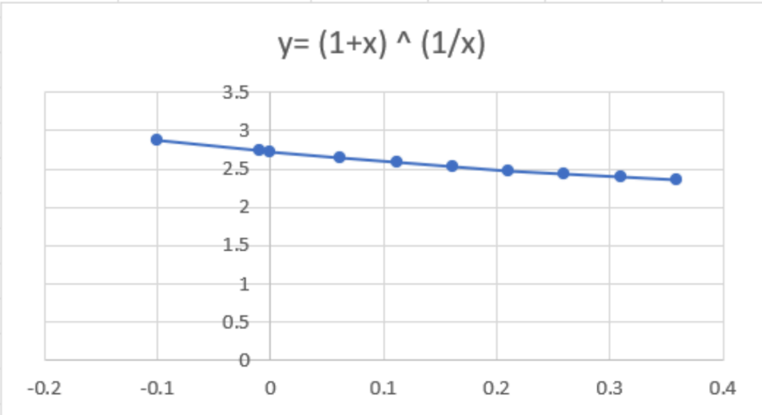
We found that ln(L) = 1, therefore, L= e^1 = e

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e \sim 2.71828$$

b)

number 4

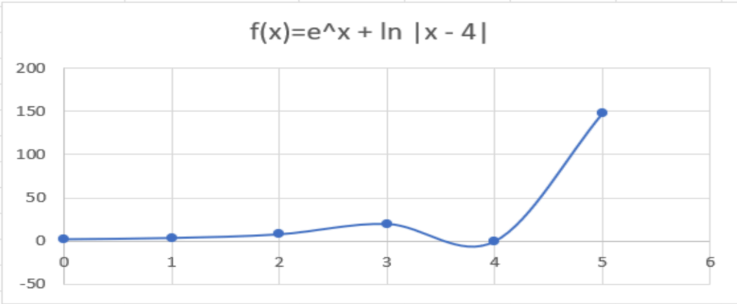
x	y= (1+x) ^ (1/x)
-0.1	2.867971991
-0.01	2.731999026
-0.001	2.719642216
0.062	2.638536907
0.1115	2.580739084
0.161	2.527461767
0.2105	2.478175374
0.26	2.43243116
0.3095	2.389846243
0.359	2.350091857



5.

a)

number 5	
x	f(x)=e^x + ln |x - 4|
0	2.386294361
1	3.816894117
2	8.082203279
3	20.08553692
4	#NUM!
5	148.4131591

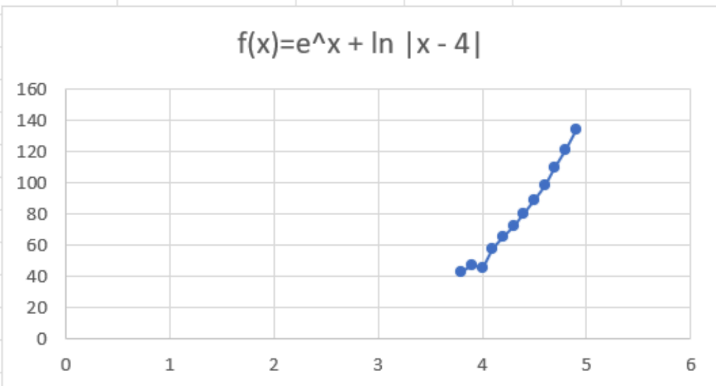


No, I don’t think the graph is an accurate representation of f because from Excel, we can easily observe
that if x =4 then f(x) is undefined number. However, if x is in a domain of 0 ≤ x ≤ 5 means the value so f x
are these 0,1,2,3,4,5 then the range of f(x) = y values should be 0 ≤ y ≤ 150

5b)

e

B	
x	f(x)=e^x + ln |x - 4|
3.8	43.09174658
3.9	47.09986401
4.0001	45.39326975
4.1	58.0377025
4.2	65.07689313
4.3	72.4958209
4.4	80.53457793
4.5	89.32398412
4.6	98.97349002
4.7	109.5904975
4.8	121.287274
4.9	134.1844192



This graph represents better than the previous one and here, the domain of x is 3.8 ≤ x ≥ 4.9 and
suppose, y = f(x) and the domain of y is 43 ≤ y ≥ 135.

6. Use numerical to find the value of the limit and verify it in Excel

$$\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$$

Solution:

Let y = f(x) = $\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$

$\Rightarrow \lim_{x \to 1} \frac{(x-1)(x^2 + x + 1)}{\sqrt{x} - 1}$

$\Rightarrow \lim_{x \to 1} \frac{((\sqrt{x})^2 - 1)(x^2 + x + 1)}{\sqrt{x} - 1}$

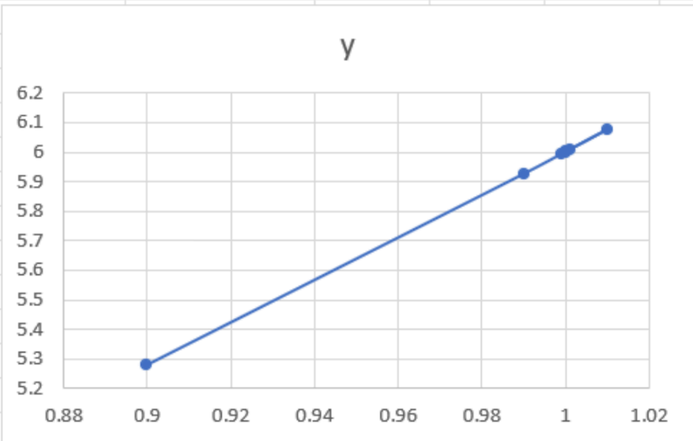
$\Rightarrow \lim_{x \to 1} \frac{(\sqrt{x} + 1)(\sqrt{x} - 1)(x^2 + x + 1)}{\sqrt{x} - 1}$

$\Rightarrow \lim_{x \to 1} (\sqrt{x} + 1)(x^2 + x + 1)$

⇒ (1+1) (1+1+1) = 2(3) = 6

Therefore, y= f(x) = 6

number 6	
x	y
0.9	5.280931738
0.99	5.925312187
0.999	5.992503125
1.01	6.075312812
1.001	6.007503125
1.0001	6.000750031
1.00001	6.000075



(b) How close to 1 does x have to be to ensure that the function in part (a) is within a distance 0.5 of its
limit?

y = f(x) = $\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$

| f (x) – 6 | < 0.5
= -0.5 < f(x) – 6 < 0.5
=6 - 0.5 < f(x) – 6 +6< 0.5 + 6
= 5.5 < f(x) < 6.5
= 5.5 < $\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$ < 6.5

So, using the calculator, the function of $f(x)$ within 0.5 of its limit when $0.93 < x < 1.06$.