

Recursion - 2

```
void solve (int N) {  
    if (N == 0) return  
    solve (N-1)  
    print(N)  
}
```

}

output: 1 2 3

```
void solve (int N) {  
    if (N == 0) return  
    print(N)  
    solve (N-1)  
}
```

}

output: 3 2 1

```
solve (N=3) {  
    solve (N=2) {  
        solve (N=1) {  
            solve (N=0) {  
                return  
            }
```

}

print(1)

}

print(2)

}

print(3)

}

```
solve (N=3) {  
    print(3)  
    solve (N=2) {  
        print(2)  
        solve (N=1) {  
            print(1)  
            solve (N=0) {  
                return  
            }
```

}

}

}

}

```

void solve (int N) {
    if (N == 0) return
    print(N)
    solve (N-1)
}

```

Stack overflow error

```

solve (N = -3) {
    print (-3)
    solve (N = -4) {
        print (-4)
        solve (N = -5) {
            ...
            infinity
        }
    }
}

```

never been $N=0$

Tower of Hanoi

There are N disks placed on tower A of different size.
 Move all the disk from A to C. (using tower B)

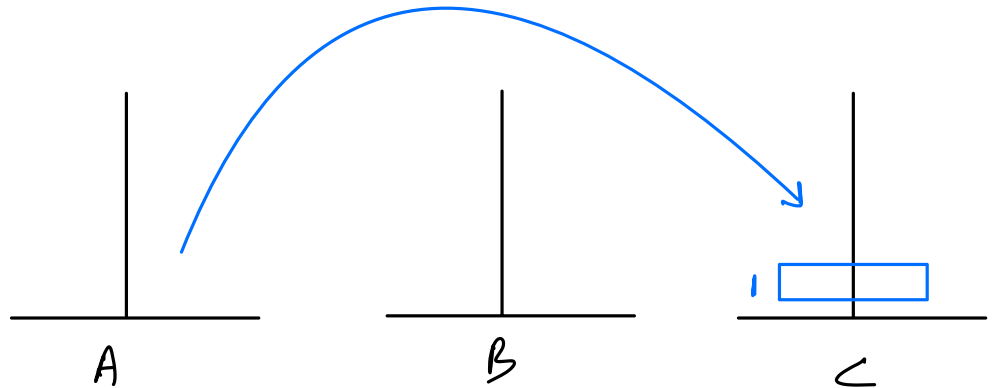
Constraint :

1. Only 1 disk can be moved in 1 step
2. Large disk can't be placed on a small disk at any step.

Print the movement of disk from A to C in minimum steps.

$N=1$

o/p: 1, $A \rightarrow C$

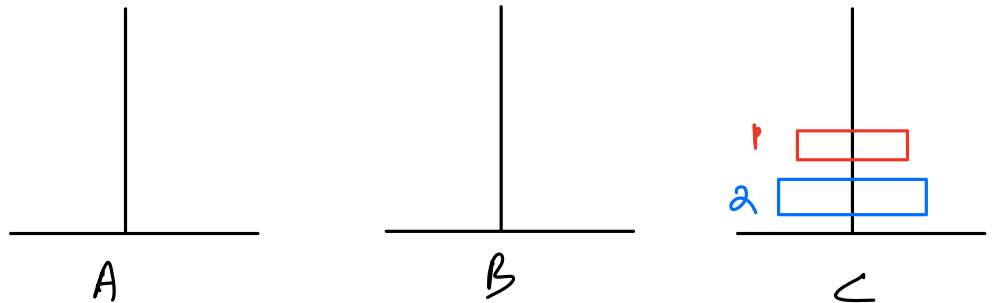


$N=2$

o/p: 1, $A \rightarrow B$

2, $A \rightarrow C$

1, $B \rightarrow C$



$N=3$

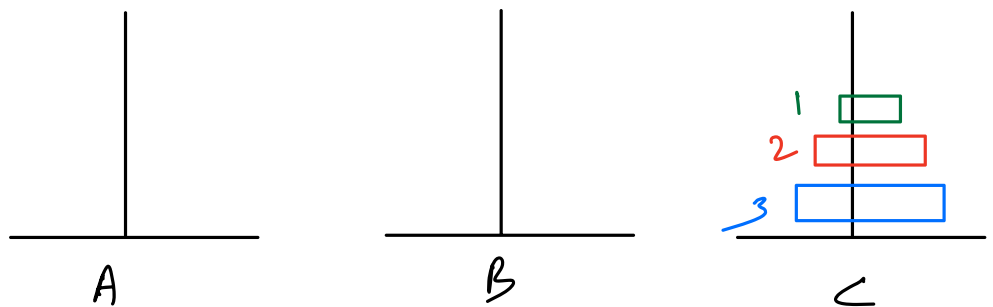
o/p: 1, $A \rightarrow C$
2, $A \rightarrow B$
1, $C \rightarrow B$

→ move 2 disks from A to B

3, $A \rightarrow C$ → 3rd disk, $A \rightarrow C$

1, $B \rightarrow A$
2, $B \rightarrow C$
1, $A \rightarrow C$

→ move 2 disks from B to C



if we have N disks to move

1. Move $N-1$ disks from $A \rightarrow B$
2. N^{th} disk, $A \rightarrow C$
3. Move $N-1$ disks from $B \rightarrow C$

void $Ton(N, A, B, C)$ {

if ($N == 0$) { return }

$Ton(N-1, A, C, B)$

print ($N, "A \rightarrow C"$)

$Ton(N-1, B, A, C)$

}

// move N^{th} disk from $A \rightarrow C$

N	# steps		
1	$1 = 1$	$2-1$	2^1-1
2	$1+1+1 = 3$	$4-1$	2^2-1
3	$3+1+3 = 7$	$8-1$	2^3-1
4	$7+1+7 = 15$	$16-1$	2^4-1
\vdots			
N			2^n-1

TC of recursive code = $O(2^n)$

TOH (N=3, 'A', 'B', 'C') {

TOH (N=2, A, C, B) {

TOH (N=1, A, B, C) {

TOH (N=0, A, C, B) { return }

print (1, A → C)

TOH (N=0, B, A, C) { return }

}

print (2, A → B)

TOH (N=1, C, A, B) {

TOH (N=0, C, B, A) { return }

print (1, C → B)

TOH (N=0, A, C, B) { return }

}

}

print (3, A → C)

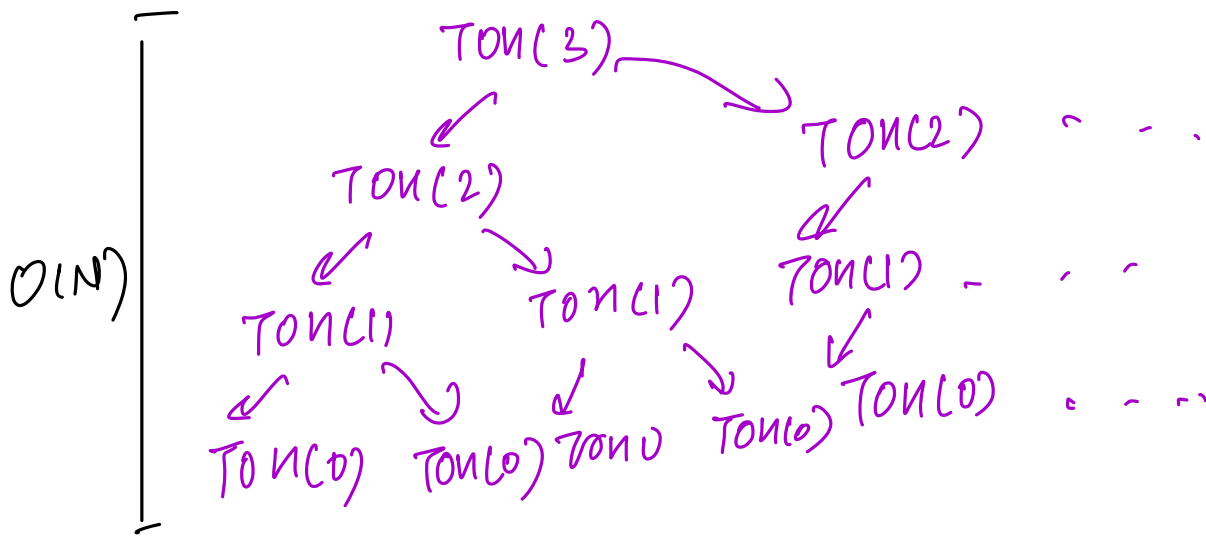
TOH (N=2, B, A, C) {

⋮

}

~~toh(0...)~~
~~toh(1...)~~
toh(2...)
toh(3...)

SC = $O(N)$



Question

Print all valid parenthesis of length $2N$ for a given value of N .

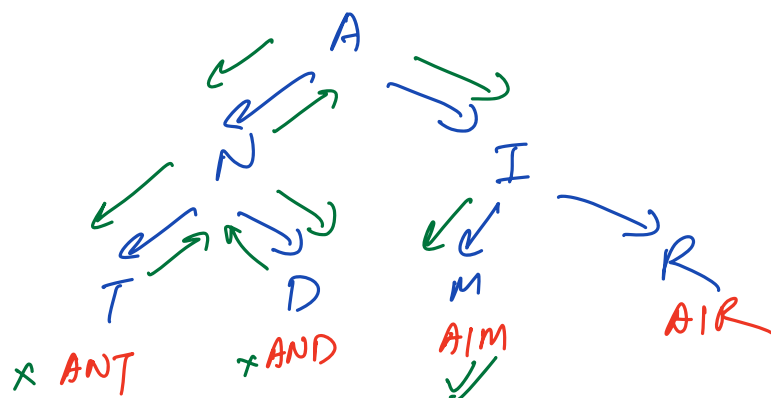
1. travel from left to right
open \geq # close
2. total # open = total # close

$N=1$ $()$ ~~$()$~~

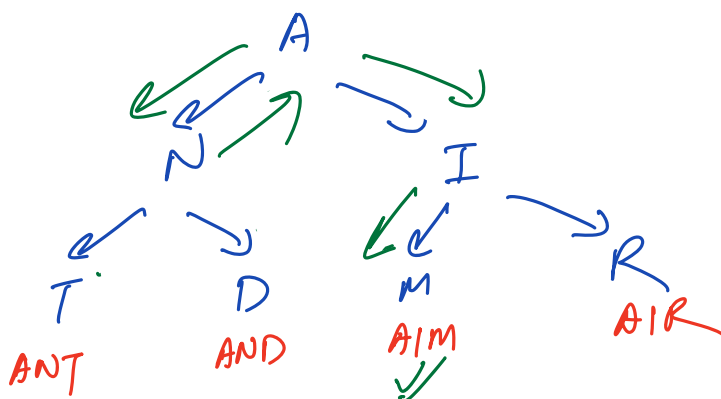
$N=2$ $(())$, ~~$((())$~~ , $() ()$

$N=3$ $((()))$, $(() ())$, $(()) ()$, $() (())$, $() (())$

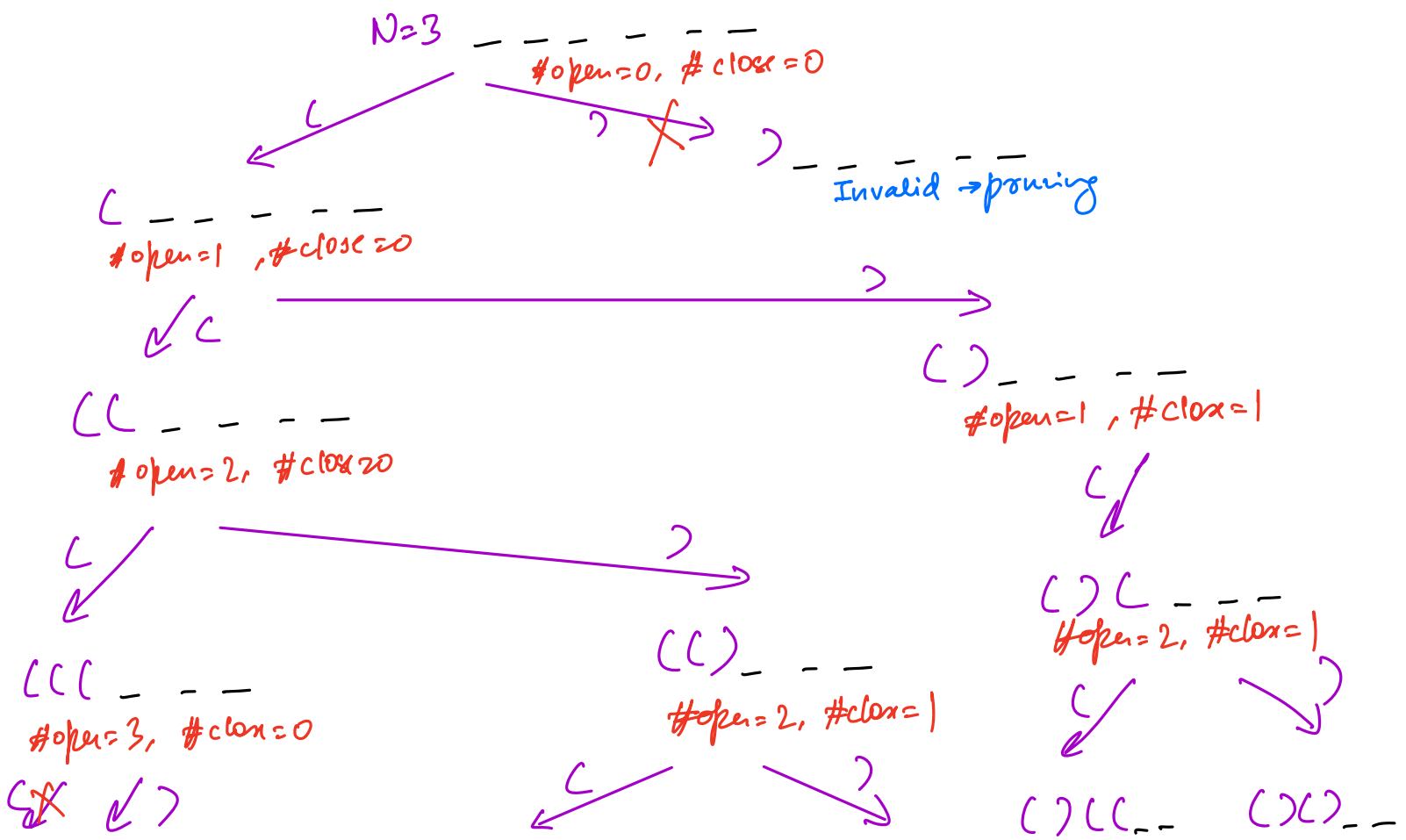
Backtracking ?



AIM = ?



better approach



((() _ _
open = 3
close = 1

↓

((()) _
open = 3
close = 2

↓

((()))
open = 3
close = 3

(() (_ _
open = 3
close = 1

↓

(() () _
open = 3
close = 2

↓

(() ())
open = 3
close = 3

(()) _ _
open = 2
close = 2

↓

(()) (_
open = 3
close = 2

↓

(()) ()
open = 3
close = 3

open = 3
close = 1

↓

(()) (() _
open = 3
close = 3

open = 3
close = 3

↓

(()) () ()
open = 3
close = 3

```
void solve(N, open, close, str) {
    if (open == N && close == N) {
        print(str)
        return
    }
    if (open < N) {
        solve(N, open+1, close, str + '(')
    }
    if (close < open) {
        solve(N, open, close+1, str + ')')
    }
}
```


TC : ?

at every point there are 2 choices

total calls : $\approx 2 \times 2 \times 2 \dots \times 2$
2n times

$$\approx 2^{2n} \approx 4^n$$

TC: $O(2^n)$

eg - a
-- 00
01 y
10
11
--- b

SC: $O(N)$

Tip not Rule :

if constraint on N is very small like

$$N \leq 20, \quad N \leq 15 \text{ etc.}$$

then probably expected TC = $O(2^n)$, $O(n!)$...
which could lead to recursion based soln.