

Stacks 2 : Nearest Smaller / Greater Element

Ques 1 → Given an integer array, find the index of nearest smaller element of left of index in array A.

If it is not present → mark -1

A = [8 2 4 7 7 5 3 10]

ans → [-1 -1 1 2 2 2 1 6]

Brute force → for i, iterate to left & find nearest smaller element.

TC = $O(N^2)$ SC = $O(1)$

A = [8 - - - - 5 - - - -]

for any element X can index 0 be the answer?

if $X > 8 \Rightarrow X > 5$ ~~index = 0~~

for any index > 5 closer index is 5 wrt. 0

$A = \begin{bmatrix} 8 & 2 & 4 & 7 & 7 & 15 & 3 & 10 \end{bmatrix}$
 $\begin{bmatrix} -1 & -1 & 1 & 2 & 2 & 4 & 1 & 6 \end{bmatrix}$

possible answer indices \rightarrow
~~0~~ ~~1~~ ~~2~~
~~3~~ ~~4~~ ~~5~~
 6 7

 last index is checked first (LIFO)
stack

7
 6
~~5~~
~~4~~
 3
~~2~~
 1
~~0~~

```

for ( i = 0 to n-1 ) {
    while ( !st.isEmpty() && A[st.peek()] >= A[i] ) {
        st.pop()
    }
    if ( st.isEmpty() ) ans[i] = -1
    else ans[i] = st.peek()
    st.push(i)
}
return ans
  
```

TC = O(N)
 SC = O(N)

Q2 \rightarrow if, find the nearest smaller or equal element on left

Q3 \rightarrow if, find the nearest greater element on left

Q.4 \rightarrow i , find the nearest greater or equal element on left

Q.5 \rightarrow i , find the nearest smaller element on right

for ($i = n-1$ to 0) {

while ($!st.isEmpty()$ & $A[st.peek()] \geq A[i]$) {

st.pop()

}

if ($st.isEmpty()$) $ans[i] = -1$

else $ans[i] = st.peek()$

st.push(i)

}

return ans



Q.6 \rightarrow i , find the nearest smaller or equal element on right

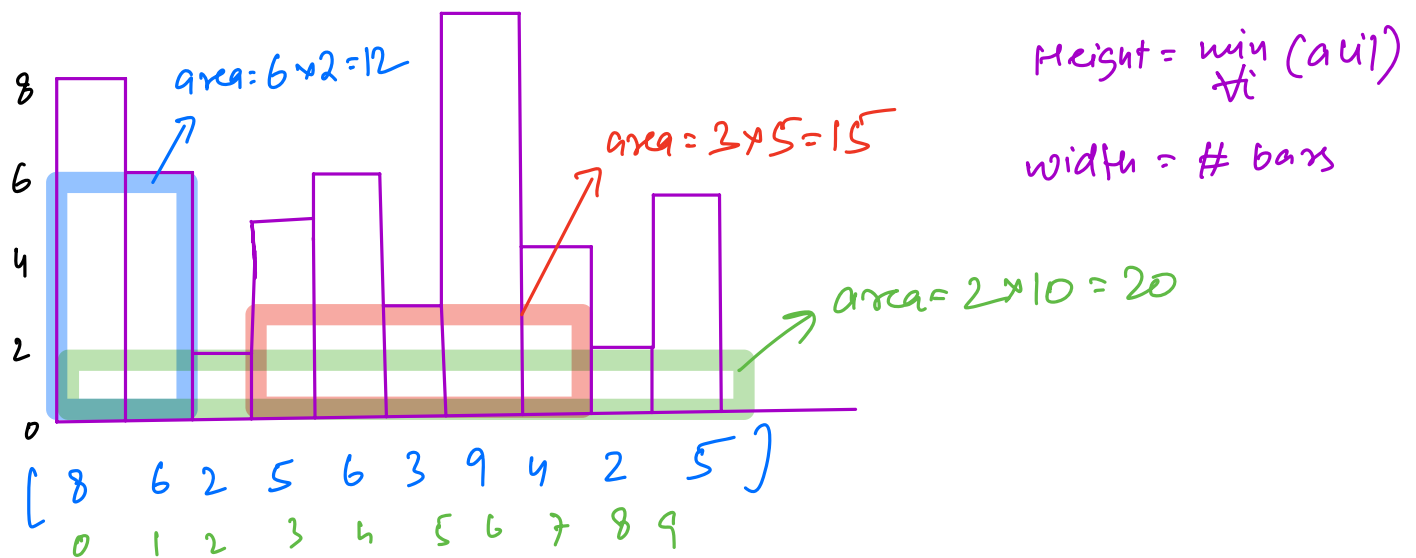
Q.7 \rightarrow i , find the nearest greater element on right

Q.8 \rightarrow i , find the nearest greater or equal element on right

Ques → Given an integer array A,
 $a_i, A[i]$ = height of i^{th} bar

width of each bar = 1

find the area of largest rectangle formed by
continuous bars.



Bruteforce : \forall subarray i, j find H & w & calculate
area. Max area is ans.

$$TC = O(N^2)$$

ans = 0

for ($i = 0$ to $N-1$) {

$H = a[i]$

 for ($j = i$ to $N-1$) {

$H = \min(H, a[j])$

$w = j - i + 1$

$ans = \max(ans, H * w)$

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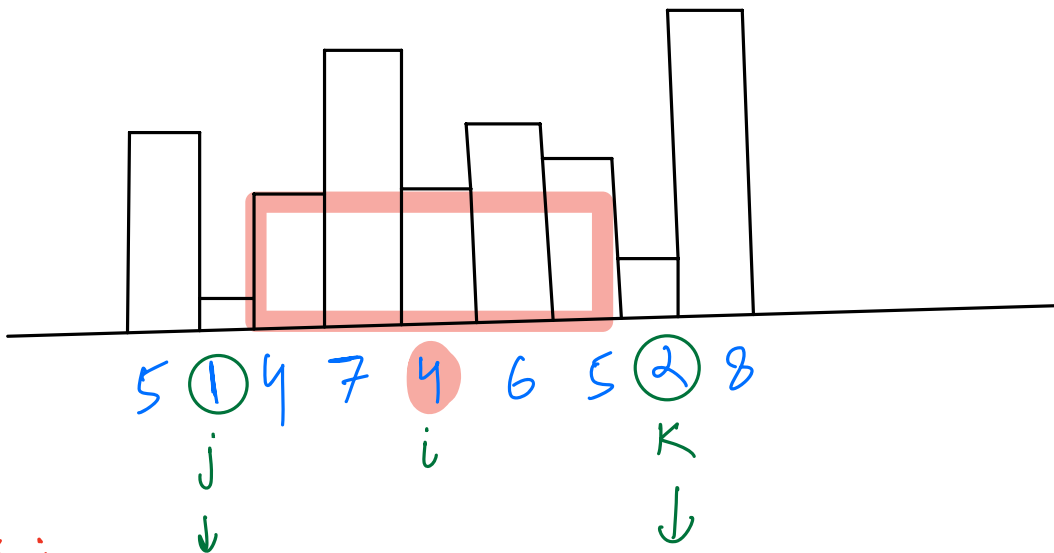
}

return ans

if subarray \longrightarrow min element

if width \longrightarrow height

if height \longrightarrow width



steps :

1. nearest smaller element on left
2. nearest smaller element is right

$$H = a[i]$$

$$W = K - 1 - (j + 1) + 1 = K - j - 1$$

$$\text{subarray} \rightarrow [j+1, K-1]$$

3. ans = 0
for $i = 0$ to $n-1$ {
 $j = \text{left}(i)$
 $K = \text{right}(i)$

$$j = -1 \quad K = n$$

$$W = n - (-1) - 1 \\ = n + 1 - 1 = n$$

if (K == -1) k = n
 $am = \max(am, a[i] * (k - j - 1))$

}

return am

TC = $O(N + N + N)$
 ↑ ↑ ↑
 break break main logic
 left array right array

= $O(N)$

SC = $O(N)$

Ques Given an integer array with distinct elements, find (max - min) of subarrays & return their sum as answer.

$A = [2, 5, 3]$

		(max - min)		
2		$\rightarrow 2 - 2 = 0$	$2 \times (1 - 3)$ $+$ $5 \times (4 - 1)$ $+$ $3 \times (1 - 2)$ $= -4 + 15 - 3$ $= 8$	
2	5	$\rightarrow 5 - 2 = 3$		
2	5	3		$\rightarrow 5 - 2 = 3$
5		$\rightarrow 5 - 5 = 0$		
5	3	$\rightarrow 5 - 3 = 2$		
3		$\rightarrow 3 - 3 = 0$		

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contribution technique

Breakdown:

TC = $O(N^2)$

SC = $O(1)$

ans = 0

for (i = 0 to n-1) { [i, ...]

mx = a[i], mn = a[i]

for (j = i to n-1) { [i, j]

mx = max(mx, a[j])

mn = min(mn, a[j])

ans += (mx - mn)

}

}

return ans

TC = $O(N^2)$

SC = $O(1)$

Contribution technique

$$\text{ans} = \sum_i \text{contribution of } a[i]$$

$a[i] * \left(\begin{array}{l} \# \text{ subarrays} \\ \text{where } a[i] \text{ is max} \end{array} \right)$

$- \left(\begin{array}{l} \# \text{ subarrays} \\ \text{where } a[i] \text{ is min} \end{array} \right)$

A = [2 10 3 5 6 4 7 1 3]
 0 1 2 3 4 5 6 7 8

subarrays where a[4] is max?
= 6

3 5 6
5 6
6

nearest greater < start <= i } [j+1, i) → i-j
on left (j)

3 5 6 4
5 6 4
6 4

i <= end < nearest greater } [i, k-1]
on right (K) → k-1-i+1
→ k-i

$$\# \text{ subarrays} = (i-j) \times (k-i)$$

$$\text{ans} = \sum_i \text{contribution of } a(i)$$

$a(i) \times \left(\begin{array}{l} \# \text{ subarrays} \\ \text{where } a(i) \text{ is max} \end{array} - \begin{array}{l} \# \text{ subarrays} \\ \text{where } a(i) \text{ is min} \end{array} \right)$

$$(i - \text{greater left}(i)) \times (\text{greater right}(i) - i)$$

$$(i - \text{smaller left}(i)) \times (\text{smaller right}(i) - i)$$

$$TC = O(N + N + N + N + N)$$

$$= O(N)$$

$$SC = O(N)$$