Bit Manipulation Basi

Decimal Number System $\rightarrow \{0,1,2,3,4,5,6,7,8,9\} \Rightarrow$ base 10

$342 \rightarrow 300 + 40 + 2$

$= 3\times10^2 + 4\times10^1 + 2\times10^0$

$\underset{3\,2\,1\,0}{2536} \rightarrow 2\times10^3 + 5\times10^2 + 3\times10^1 + 6\times10^0$

Binary Number System $\rightarrow \{0,1\} \Rightarrow$ base 2

$\underset{2\,1\,0}{110} \Rightarrow 1\times2^2 + 1\times2^1 + 0\times2^0 = 6$ ← decimal no. equivalent of $(110)_2$

$1011 \Rightarrow 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 = 8+2+1 = 11$

$(1011)_2 = (11)_{10}$

Binary to Decimal conversion

$$\underset{4}{1}\ \underset{3}{0}\ \underset{2}{1}\ \underset{1}{0}\ \underset{0}{1} \Rightarrow 1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$$

$$= 16 + 0 + 4 + 0 + 1 = 21$$

$$\overset{6}{1}\ \overset{5}{0}\ \overset{4}{1}\ \overset{3}{1}\ \overset{2}{0}\ \overset{1}{1}\ \overset{0}{0}$$

$0\times2^0 = 0$

$1\times2^1 = 2$

$0\times2^2 = 0$

$1\times2^3 = 8$

$1\times2^4 = 16$

$0\times2^5 = 0$

$1\times2^6 = 64$

$$\overline{\underline{90}}$$

$$1\ 0\ 1\ 1\ 0\ 1\ 0$$

$2^6 \quad 2^4 \quad 2^3 \quad 2^1$

$= 64+16+8+2$

$= 90$

$$(1011010)_2 = (90)_{10}$$

Decimal to Binary

$(20)_{10}$

		remainder
2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$\Rightarrow (10100)_2$

$2^4 + 2^2 = 16+4 = 20$

Binary representation of $(45)_{10}$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$\Rightarrow (101101)_2$

$2^5 \quad 2^3 \quad 2^2 \quad 2^0$

$32+8+4+1 = 45$

Addition in Decimal

$$
\begin{array}{rccc}
 & 1 & 1 & \\
 & 3 & 6 & 8 \\
+ & 4 & 3 & 5 \\
\hline
 & 8\,1 & 0 & 1\,3
\end{array}
= 8\ 0\ 3
$$

$9+9 = 18$

$1+9+9 = 19$

Addition of Binary no.

$$\begin{array}{r} 1\ 0\ 1\ 0\ 1 \\ +\quad 1\ 1\ 0\ 1 \\ \hline 1\ 0\ 0\ 0\ 1\ 0 \end{array}$$

$\Rightarrow 1\ 0\ 0\ 0\ 1\ 0$

$$\begin{array}{r} 1\ 0\ 1\ 1\ 0 \\ +\ 0\ 0\ 1\ 1\ 1 \\ \hline 1\ 1\ 1\ 0\ 1 \end{array}$$

$\Rightarrow 1\ 1\ 1\ 0\ 1$

$(2)_{10} = (10)_2$

$(3)_{10} = (11)_2$

$$\begin{array}{r|l} 2 & 3 \\ \hline 2 & 1 \quad 1 \\ \hline & 0 \quad 1 \end{array} \uparrow \ (11)_2$$

Bitwise Operators $\rightarrow$ AND, OR, XOR, NOT

& | ^ !/~

AND : $x \,\&\, y$ = 1 iff both x and y are 1.

= 0 if any is 0.

OR : $x \,|\, y$ = 1 if any one is 1

= 0 iff both are 0

XOR: $x \wedge y$ = 1 if x and y are different

= 0 if they are same

NOT: $\sim x$ = 1 if x is 0

= 0 if x is 1

$\sim 0 = 1$

$\sim 1 = 0$

A	B	A&B	A|B	A^B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

A^B → addition without carry

(0+0=0)

(0+1=1)

(1+0=1)

(1+1 = ~~1~~0)

5 & 6 ⇒

$$\begin{array}{r} 5 = 101 \\ 6 = 110 \\ \hline 100 \end{array} = (4)_{10}$$

$\boxed{5\&6 = 4}$

20 & 45

$20 = 010100$ AND

$45 = 101101$

$(000100)_2 = (4)_{10}$

92 | 154

$92 = 01011100$ OR

$154 = 10011010$

$11011110 = (222)_{10}$

92 ^ 154

$92 = 01011100$ XOR

$154 = 10011010$

$(11000110)_2 = (198)_{10}$

20 ^ 45

$20 = 010100$

$45 = 101101$

$(111001)_2 = (57)_{10}$

Negative numbers

$(-45)_{10} = (?)_2$

integer → 32 bits

31	30	29	.	.	.	.	.	2	1	0

Most Significant Bit (MSB)

$$2^{31} > \underbrace{2^{30} + 2^{29} + \cdots + 2^2 + 2^1 + 2^0}_{31 \text{ terms}}$$

$$\text{sum} = a\left(\frac{r^n - 1}{r-1}\right)$$

$$= 1\left(\frac{2^{31}-1}{2-1}\right) = 2^{31}-1$$

2's complement

→ assume all no. are 8 bits

$5 \rightarrow 00000101$

1's complement of 5 → 11111010 (flip all bits)

$+1$ → $+\;1$

2's complement of 5 → 11111011

$2^7\; 2^6\; 2^5\; 2^4\; 2^3\; 2^1\; 2^0$

$= 128+64+32+16+8+2+1$

$= 251 \Rightarrow -5$

Range of 8 bit no. : -128 to 127

-2^7 to 2^7-1

$= -128+64+32+16+8+2+1$

$= -5$

$11111111 \Rightarrow -2^7+(2^6+2^5+\dots 2^0)$

$= -2^7+(2^7-1) = -1$

$10000000 \Rightarrow -2^7$

$01111111 \Rightarrow \cancel{-2^7\times 0}\,(2^6+2^5+\dots 2^0)$

$\Rightarrow 2^7-1$

$00000000 \Rightarrow 0$

$-3 = (?)_2$

$3 = 00000011$

1's comp $= 11111100$

2's $= 11111101$

$(-3)_{10} = (11111101)_2$

$(-10)_{10} = (?)_2$

$10 \Rightarrow 00001010$

1's complement $\Rightarrow$ 11110101

$+1$

2's complement $\Rightarrow$ $(11110110)_2$

for n bit no. range $= -2^{n-1}$ to $2^{n-1}-1$

32 bit integer

$[2^{10} = 1024 \approx 10^3]$

$\Rightarrow -2^{31}$ to $2^{31}-1$

$2^{31} = 2^{30} \times 2$
$= (2^{10})^3 \times 2$
$= (10^3)^3 \times 2 = 2\times 10^9$

$\Rightarrow -2\times 10^9$ to 2×10^9

$\Rightarrow -2147483648$ to 2147483647

64 bit no.

$\Rightarrow -2^{63}$ to $2^{63}-1$

$\Rightarrow -8\times 10^{18}$ to 8×10^{18}

Importance of constraints

int a = 10^5

int b = 10^6

int c = a*b ✗ [$a*b = 10^5 \times 10^6 = 10^{11}$]

↳ overflow, wrong value

long c = a*b ✗

↳ overflow during multiplication

MUL a, b, temp (int)

CPY temp, c

long c = long (a*b) ✗

↳ overflow during multiplication

long c = (long) a * (long) b ✓

↳ will work

MUL a, b, temp (long)

CPY temp, c

long c = (long) a * b ✓

↳ will work

Question

Given array of size N, calc. sum of all elements.

$1 <= N <= 10^5$

$1 <= a[i] <= 10^6$

~~int~~ long ans = 0

for (i = 0 to n)

  ans += a[i]

print(ans)

if all $a[i] = 10^6$ &

$n = 10^5$

$sum = 10^6 \times 10^5 = 10^{11}$