# Maths 2 : Combinatorics Basics

1. Addition & Multiplication Rule
2. Permutations Basics
3. Combinatorics Basics & Properties
4. Pascal Triangle
5. $N^{th}$ column title

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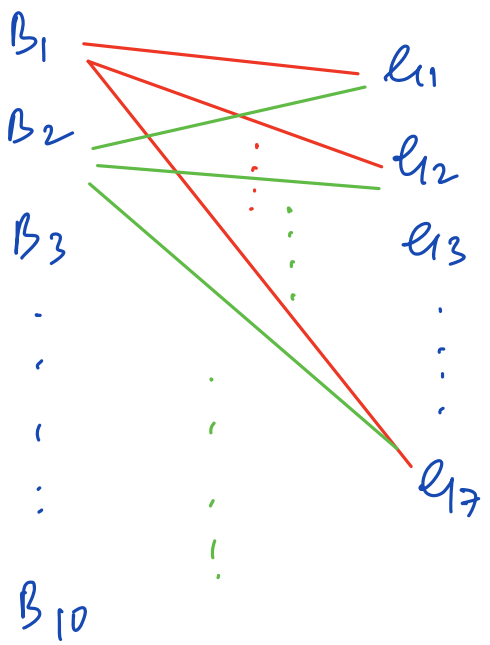
Q → 10 Boys 7 Girls

How many pairs of 1:1 mapping possible?

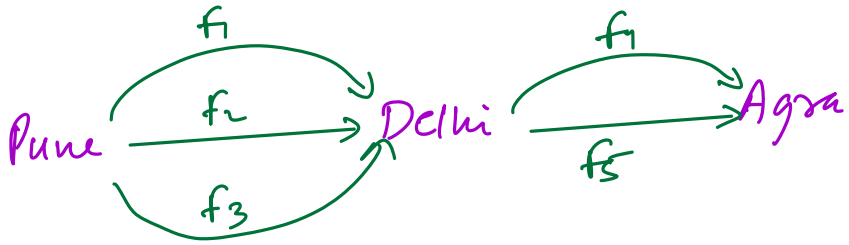



$B_1$ → 7 pairs ($G_1$ to $G_7$)

$B_2$ → 7 pairs ($G_1$ to $G_7$)

total pairs = $10 \times 7 = 70$

Q →




# ways to travel from Pune to Agra via Delhi?

Pune → Delhi **AND** Delhi → Agra

ways (Pune → Delhi) = 3

ways (Delhi → Agra) = 2

# ways → $f_1 f_4$ $f_2 f_4$ $f_3 f_4$

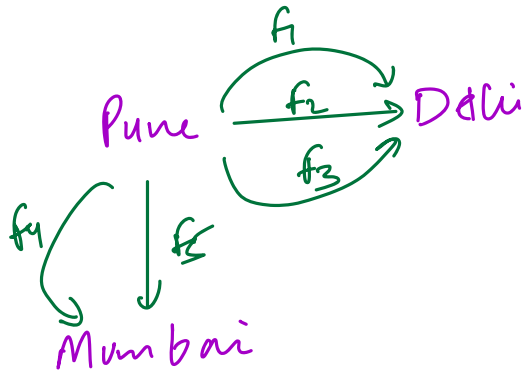
$f_1 f_5$ $f_2 f_5$ $f_3 f_5$

ways (Pune → Delhi) **X** ways (Delhi → Agra)

$3 \times 2 = 6$

AND → (*)

Q →



# ways to travel from Pune to Delhi or Mumbai.

Pune → Delhi **OR** Pune → Mumbai

ways (Pune → Delhi) = 3

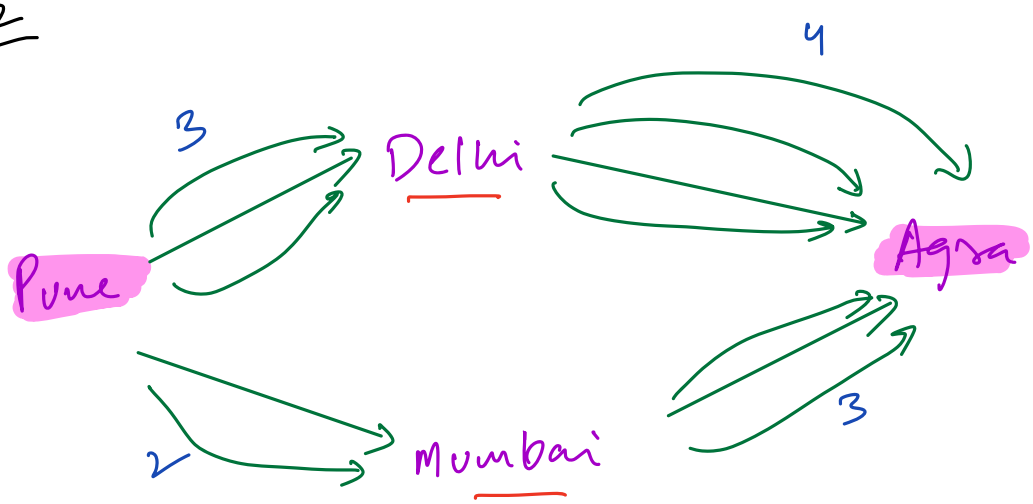
ways (Pune → Mumbai) = 2

ways (Pune → Delhi) **+** ways (Pune → Mumbai)

3 + 2 = 5

OR → ⊕

Quiz




ways (Pune → Delhi) × ways (Delhi → Agra)

= 3 × 4 = 12

**OR**

ways (Pune → Mumbai) × ways (Mumbai → Agra)

$2 \times 3 = 6$

total ways $= 12 + 6 = 18$

Permutations → Arrangement of objects

order matters ⇒ $(i,j) \neq (j,i)$

# ways to arrange 3 distinct characters

a, b, c ⇒ abc, acb, bac, bca, cab, cba  ans = 6

$3 \times 2 \times 1 = 6$ (3!)

4 distinct characters → $4 \times 3 \times 2 \times 1 = 24$ (4!)

# ways to arrange N distinct characters =

$N \times (N-1) \times (N-2) \ldots \times 1 = N!$

Q → find # ways to arrange R out of N characters.

d, a, t, e

N=4, R=2

4 × 3 = 12

da, at, te, dt - - - ...

| | 1 | 2 | 3 | . . . . . | R |
|---|---|---|---|---|---|
| # choices → | N | N-1 | N-2 | | N-(R-1) = N-R+1 |

$$\frac{N \times (N-1) \times (N-2) \cdots \times (N-R+1) \times (N-R) \times (N-R-1) \cdots \times 1}{(N-R) \times (N-R-1) \cdots \times 1}$$

$$= \frac{N!}{(N-R)!}$$

$$\boxed{{}^{N}P_{R} = \frac{N!}{(N-R)!}} \qquad R <= N$$

↳ # ways to arrange R elements out of total N elements

Combinations → selection of objects

order doesn't matter → (i,j) = (j,i)

select 3 out of 4 distinct characters

d, a, t, e

| | | | |
|---|---|---|---|
| d,a,t | d,a,e | a,t,e | d,t,e |
| dta | dea | aet | det |
| adt | ade | tae | tde |
| atd | aed | tea | ted |
| tad | eda | eta | edt |
| tda | ead | eat | etd |

selection AND arrangement = Permutation

$^4C_3 \times 3! = {}^4P_3$

$$^4C_3 = \frac{^4P_3}{3!} = \frac{4!}{(4-3)! \times 3!} = \frac{24}{1 \times 6} = 4$$

$$^NC_R = \frac{^NP_R}{R!} = \frac{N!}{(N-R)! \times R!}$$

# Properties of ${}^{N}C_{R}$

1. # ways to **not select anything** $= {}^{N}C_{0} = 1$

$${}^{N}C_{0} = \frac{N!}{0!\,(N-0)!} = \frac{N!}{1 \times N!} = 1$$

2. # ways to **select everything** $= {}^{N}C_{N} = 1$

3. # ways to select $(N-R)$ items $= {}^{N}C_{N-R}$

$$= \frac{N!}{(N-R)!\,(N-(N-R))!}$$

$$= \frac{N!}{(N-R)!\,R!} = {}^{N}C_{R}$$

$$\boxed{{}^{N}C_{R} = {}^{N}C_{N-R}} \qquad 0 \le R \le N$$

4. — — — — . . . . . $N^{th}$ item — $N$ items — select $R$ items

${}^{N}C_{R} =$

select — remaining to select $= R-1$, total remaining $= N-1$ → ${}^{N-1}C_{R-1}$

OR

not select — remaining to select $= R$, total remaining $= N-1$ → ${}^{N-1}C_{R}$

$+$

$$\boxed{{}^{N}C_{R} = {}^{N-1}C_{R-1} + {}^{N-1}C_{R}}$$

d, a, t, e  $n=4$, $R=2$

$${}^4C_2 = \frac{4\times 3}{2} = 6$$

e, d
e, a
e, t
3

d, a
d, t
a, t
3

$3 + 3 = 6$

$${}^{N-1}C_{R-1} + {}^{N-1}C_R = \frac{(N-1)!}{(R-1)!\times(N-1-R-1)!} + \frac{(N-1)!}{R!\times(N-1-R)!}$$

$$(N-R)! = (N-R)\times(N-R-1)!$$

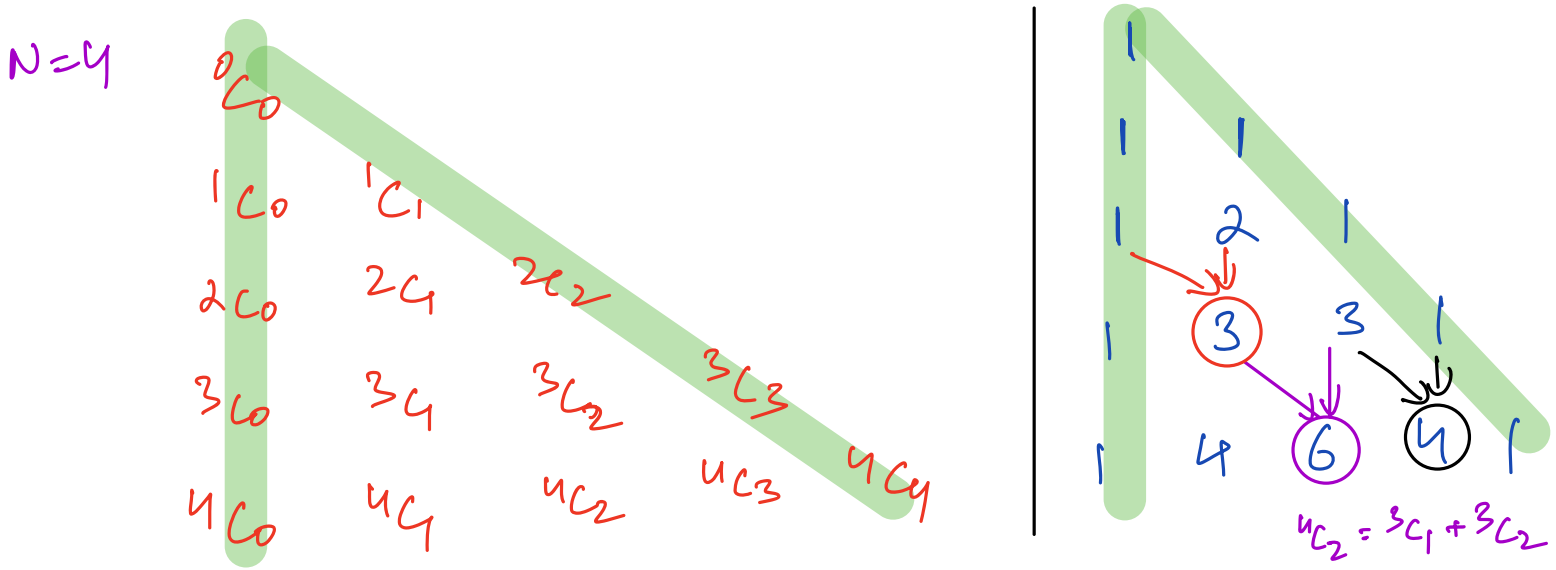
$$R\times(R-1)!$$

$$= \frac{(N-1)!}{(R-1)!\,(N-R-1)!}\left[\frac{1}{(N-R)} + \frac{1}{R}\right]$$

$$\frac{R+N-R}{R(N-R)} = \frac{N}{(N-R)\times R}$$

$$= \frac{(N-1)!\times N}{(R-1)!\times R\times(N-R-1)!\times(N-R)} = \frac{N!}{R!\times(N-R)!} = {}^NC_R$$

Q → Generate Pascal Triangle for given input N.




TC to calculate N! = O(N)

Brute force : to find $^NC_R$, calculate the value

TC: $O(N^2 \times N) = O(N^3)$

$^NC_0 = 1$ | $^NC_N = 1$ | $^NC_R = {}^{N-1}C_{R-1} + {}^{N-1}C_R$

nCr

```
for (i=0 to N) {
    nCr[i][0] = 1
    nCr[i][i] = 1
    for (j=1 to (i-1)) {
        nCr[i][j] = (nCr[i-1][j-1] + nCr[i-1][j]) %M
```

3 3

3

return nCr

TC : $O(N^2)$ SC : $O(1)$ / $O(N^2)$

## Question

Given a +ive integer N, find $N^{th}$ column title.

A B C . . . . . Z AA AB . . . . AZ BA BB . . . . ZZ AAA . . . .

1 2 3 26 27 28 52 53 54 . . . .

N = 4 ans = "D"

N = 28 ans = "AB"

N = 50 ans = "AX"

observation → base 26 number system

0 1 2 . . . . 25

A B Z

0 1 2 . . . . 9 10 11 . . . . 19 20 21 . . . 29 . . . .

| | | |
|---|---|---|
| 26 | (50-1) | 23 → X |
| 26 | (1-1) | 0 → A |
| | 0 | |

AX

N = 10,000

| | | |
|---|---|---|
| 26 | 10000 - 1 | 15 → P |
| 26 | 384 - 1 | 19 → T |
| 26 | 14 - 1 | 13 → N |
| | 0 | |

"NTP"

```
ans = ""
while (N > 0) {
    N = N - 1
    ans = (char)('A' + (N % 26)) + ans;
    N = N / 26
}
return ans
```

TC = O(log26(N))

SC = O(1)

# Sorted permutation Rank

What is the rank of given string → distinct char. w.r.t sorted order of its permutation.

eg "acb"

| | |
|---|---|
| abc | 1 |
| acb | 2 |
| bac | 3 |
| bca | 4 |
| cab | 5 |
| cba | 6 |

ans = 2

"312"

123, 132, 213, 231, 312, 321

"date"

a ___ ⇒ 3! = 6

dae_ ⇒ 1! = 1

date ⇒ 1 + 6 + 1 = 8

0 1 2 3 4 n=5
t r u n K

# char < 't'

K ____ ⇒ 4! = 24
r ____ ⇒ 4! = 24
u ____ ⇒ 4! = 24
} 4! ↓ 24 × 3 ↓ = 72

t K ___ ⇒ 3! = 6
t r ___ ⇒ 3! = 6
} 6 × 2 = 12

t r K __ ⇒ 2! = 2 ]

⋮

ans = 1

→ create factorial array ⇒ fact[N+1]

0 → 0!
1 → 1!
2 → 2!
⋮

```
for (i = 0 to n-1) {
    cnt = 0
    for (j = i+1 to n-1) {
        if (s[j] < s[i])
            cnt++
    }
    ans = (ans + (cnt x fact[N-i-1]) % M) % M
}
return ans
```

factorial array

TC : O(N²)  SC : O(N) factorial array