Maths 3 : Prime Numbers

Prime Number : Positive Numbers that have exactly 2 factors are prime number.

smallest → 2

-1 ⇒ factors = {-1, 1}

Check if the given integer is prime ?

```
bool checkPrime(N) {
    // prime → factors 1 & N
    for(i=2; i*i<=N; ++i) {   // 2 → √N
        if(N%i==0)
            return false
    }
    return true
}
```

i*i <= 2

TC : O(√N)

SC : O(1)

Given an integer N, print all prime numbers from 1 to N.

N = 10 o/p ⇒ 2 3 5 7

N = 15 o/p ⇒ 2 3 5 7 11 13

Bruteforce →

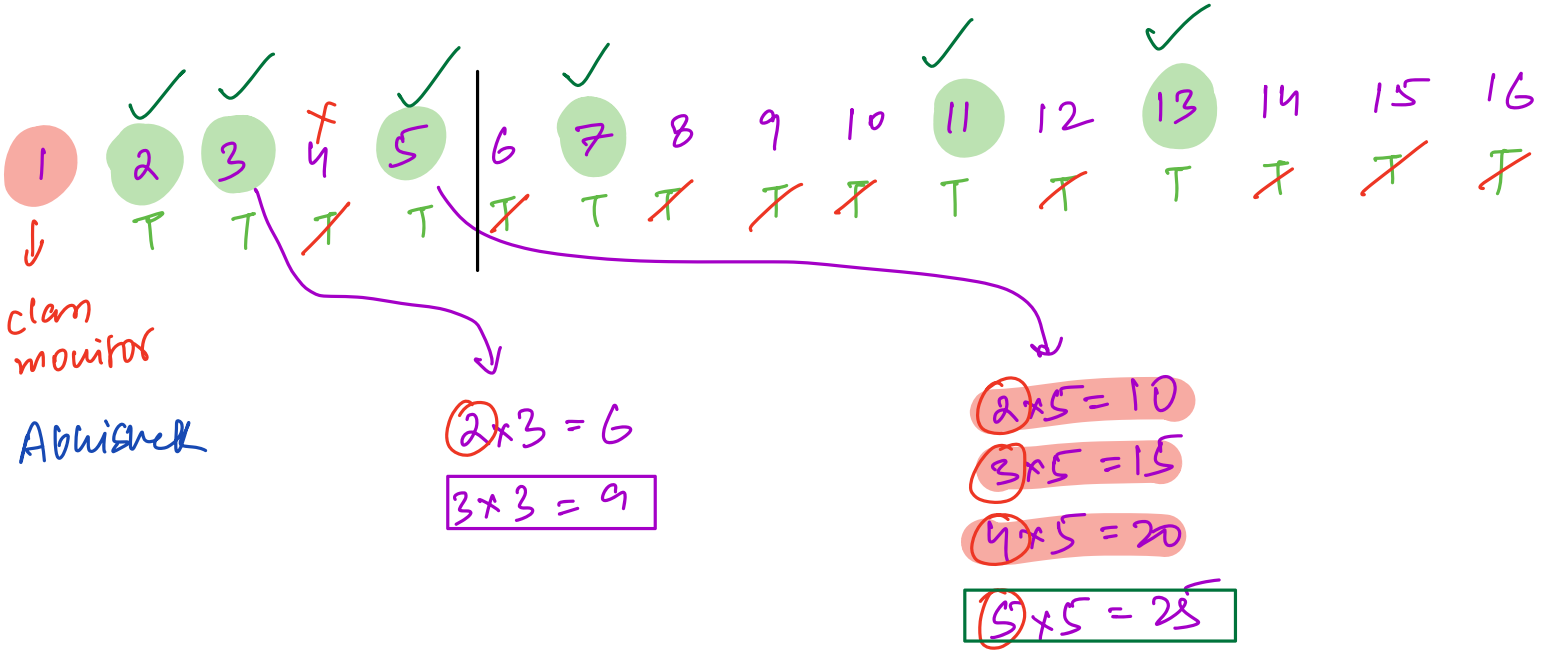
```
for (i = 2 to N) {
    if (checkPrime(i))
        print(i)
}
```

TC: $O(N\sqrt{N})$

SC: $O(1)$

Sieve of Eratosthenes

class teacher → Manoj → chocolates to prime roll no.



```
fi isprime[i] = true
isprime[0] = isprime[1] = false
for(i=2; i*i <= N; ++i){
    if(isprime[i]){
        for(j = i*i; j<=N; j+=i){
            isprime[j] = false
        }
    }
}
```

SC: O(N)
isprime array

```
for(i=2 to N){
    if(isprime[i])
        print(i)
}
```

Time Complexity

i	j	# iterations
2	4, 6, 8, ...	~N/2
3	9, 12, 15, ...	~N/3
4	———	0
5	25, 30, 35, ...	~N/5
		⋮

$$\frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \cdots < \frac{N}{2} + \frac{N}{3} + \frac{N}{4} + \frac{N}{5} \cdots$$

$$= \sum_{i=2}^{N} \frac{N}{i} = N \sum_{i=2}^{N} \frac{1}{i}$$

$$\int \frac{1}{x} dx = \log x$$

$$= N \log N$$

$TC < O(N \log N)$

$\downarrow$

exact $\Rightarrow$ $O(N \log(\log N))$

Q → Given a positive integer N, count the # factors for all numbers from 1 to N.

N = 6

1	2	3	4	5	6
1	2	2	3	2	4

#factors →

N = 10

1 ✓	2 ✓	3 ✓	4 ✓	5 ✓	6 ✓	7 ✓	8 ✓	9 ✓	10 ✓
~~0~~ 1	~~0~~ × 2	~~0~~ × 2	~~0~~ × × 3	~~0~~ × 2	~~0~~ × × 3 4	~~0~~ × 2	~~0~~ × × 3 4	~~0~~ × × 3	~~0~~ × × 3 4

#factors

1 2 2 3 2 4 2 4 3 4

```
if i, factors(i) = 1
for(i=2; i<=N; i++){
    for(j=i; j<=N; j+=i){
        factors(j)++   // i is a factor of j
    }
}
return factors
```

i = 3
j = 3, 6, 9, 12 ....

iterations = $\frac{N}{2} + \frac{N}{3} + \frac{N}{4} + \dots + 1$

TC: $O(N \log N)$

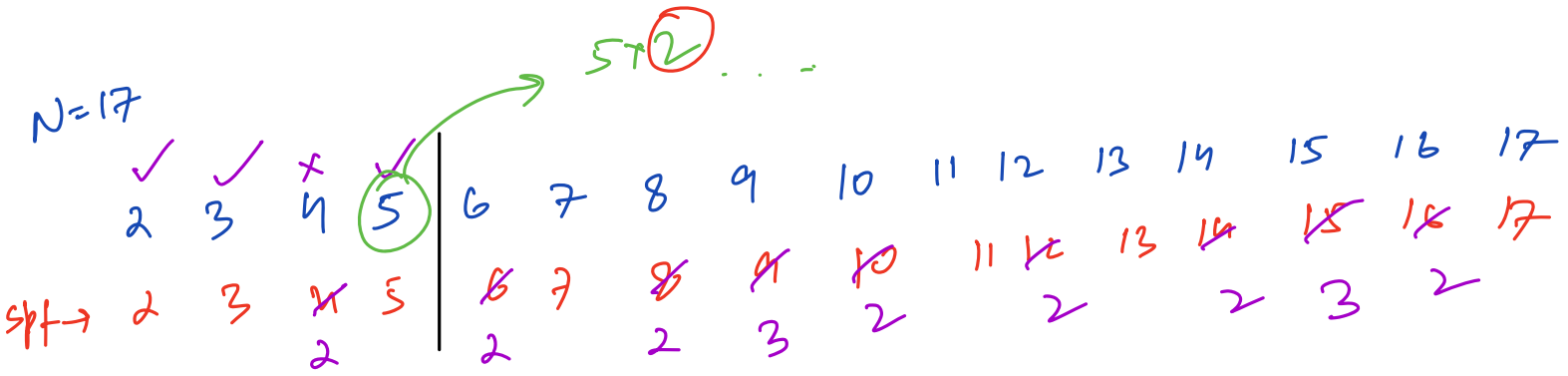
SC: $O(1)$

Q → Given a positive integer N, find the smallest prime factor for all numbers from 2 to N.

N = 10

	2	3	4	5	6	7	8	9	10
spf →	2	3	2	5	2	7	2	3	2

spf[prime no.] = same prime no.



$\forall i$, $spf[i] = i$

```
for(i=2; i*i<=N; i++){
    if(spf[i]==i){
        for(j=i*i; j<=N; j+=i){
            if(i<spf[j])  spf[j]=i
```

3

3

3

return spf

TC: $O(N \log(\log N))$

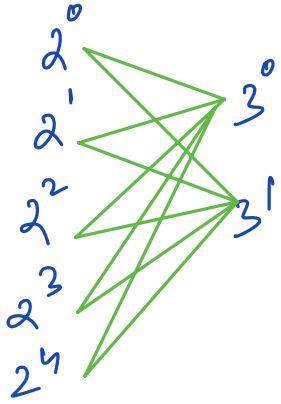
SC: $O(1)$

Prime factorization

$N = 48 \rightarrow 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1$

2	48
2	24
2	12
2	6
3	3
	1

#factors →



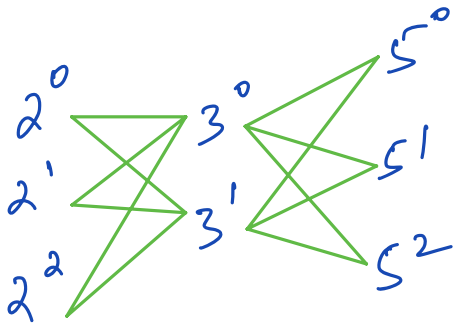
1, 3, 2, 6, 4, 12, 8, 24, 16, 48

count = 10

$5 \times 2 = 10$

$N = 300 \rightarrow 2 \times 2 \times 3 \times 5 \times 5 = 2^2 \times 3^1 \times 5^2$

2	300
2	150
3	75
5	25
5	5
	1



$3 \times 2 \times 3 = 18$

1	5	25
3	15	75
2	10	50
6	30	150
4	20	100
12	60	300

factors of 300

$2^{(2)} \times 3^{(1)} \times 5^{(2)}$

$(2+1) \times (1+1) \times (2+1) = 18$

$$N = p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \cdots \times p_k^{a_k}$$

$$\#\text{factors} = (a_1+1) \times (a_2+1) \times (a_3+1) \times \cdots \times (a_k+1)$$

$$= \prod_{i} (a_i+1)$$

use spf to find # factors

$N = 702$

spf[702] = 2

702/2 = 351

spf[351] = 3

351/3 = 117

117/3 = 39

39/3 = 13

spf[13] = 13

13/13 = 1

pow

2^{1}

ans = 1

* (1+1)

3^{2}

* (2+1)

13^{1}

* (1+1)

$= 2 \times 4 \times 2 = 16$

// calculate spf() → TC = O(N log log N)
SC = O(N)

```
ans=1
while (N>1) {
    pow=0
    s = spf[N]  // smallest prim factor
    while (N%s == 0) {
        pow++
        N/=s
    }
    ans *= (pow+1)
}
return ans
```

N = 404 ans = 1

pow=0 N=404

s = spf[404] = 2

404/2 = 202

202/2 = 101 ans = 1 x (2+1) = 3

s = spf[101] = 101

101/101 = 1 ans = 3 x 2 = 6

$2^2 \times 101^1$

1 2 4

101 202 404

min(s) = 2

$N \to \frac{N}{2} \to \frac{N}{4} \to \dots$

TC = O(log N)

SC = O(1)

6 Dec (Wed) → 2 Dec (Sat)

8 Dec (Fri) → 5 Dec (Tues)

11 Dec (Mon) → 12 Dec (Tues)

2^{70} 10^9

1024

$2^{10} = 10^3$

$2^{70} = 10^{3 \times 7} = 10^{21}$

String S = 

0 n-1

length $n <= 10^6$