

Searching I : Binary Search on Array

Search Space → eg. Library

Target → eg. Book

Condition → helps in finding target by reducing search space.

Binary Search → divide search space into 2 parts & keep neglecting one part based on condition

└→ organised data
⇒ use binary search

Question 1

Given a sorted array of distinct elements. find index of a given element K , if not present return -1.

$A = [3, 6, 9, 12, 14, 19, 20, 22, 25, 27]$

Indices: 0 1 2 3 4 5 6 7 8 9

Annotations: $r = mid - 1$, mid (circled around 14), $12 < 14$, $K = 12$

Bruteforce → for (i=0 to n-1) {
 if (A[i] == K)
 return i
}
return -1

// linear search

TC = $O(N)$

SC = $O(1)$

Binary Search → 3 steps

// 1. Define search space [0, n-1] // index l to r

l=0, r=n-1

while (l <= r) {

// 2. Check if middle element is the answer

mid = (l+r)/2

// $l + (r-l)/2$

if (A[mid] == K)
 return mid

// 3. Decide whether to go left or right

if (K < A[mid])

 r = mid - 1

else

 l = mid + 1

}

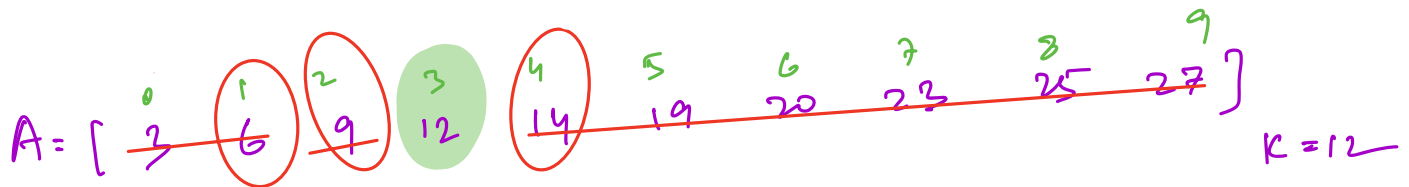
return -1

TC: $N \rightarrow N/2 \rightarrow N/4 \rightarrow \dots \rightarrow 1$

iterations = $\log_2 N$

$TC = O(\log_2 n)$

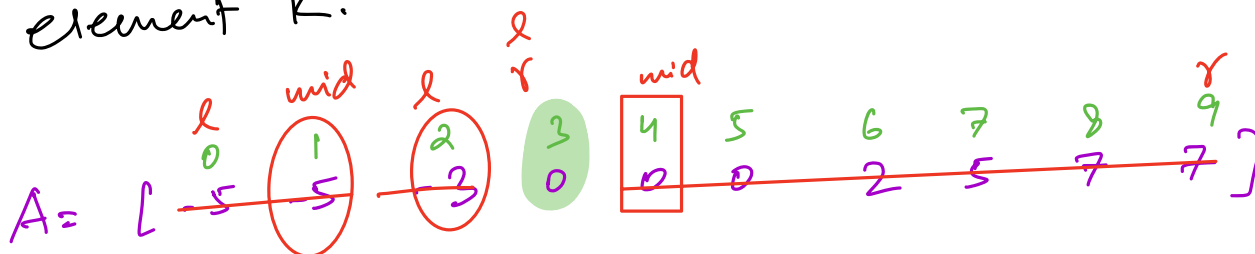
$SC = O(1)$



l	r	mid	
0	9	$(0+9)/2 = 4$	$K < 14 \rightarrow \text{go left}$
0	3	$(0+3)/2 = 1$	$K > 6 \rightarrow \text{go right}$
2	3	$(2+3)/2 = 2$	$K > 9 \rightarrow \text{go right}$
3	3	$(3+3)/2 = 3$	$K = 12$

Question 2

Given a sorted array. find first index of given element K .



$K = 0$ $ans = 3$

$K = 1$ $ans = -1$

// 1. Define search space $[0, n-1]$ // index l to r

$l = 0, r = n - 1$

while ($l \leq r$) {

// 2. check if middle element is the answer

$mid = (l + r) / 2$ // $l + (r - l) / 2$

if ($A[mid] == K$ && ($mid == 0$ || $A[mid - 1] != K$))

return mid

// 3. Decide whether to go left or right

if ($K \leq A[mid]$)

$r = mid - 1$

else

$l = mid + 1$

}

return -1

$K < A[mid] \rightarrow$ go left

$K = A[mid] \rightarrow$ go left

$K > A[mid] \rightarrow$ go right

Question 3

Given an array where every element occurs twice except for 1 element that appears once. Find the unique element. All equal pair of elements are together. (unsorted but organised)

$A = [\overset{0}{9} \overset{1}{8} \overset{2}{5} \overset{3}{5} \overset{4}{6} \overset{5}{2} \overset{6}{2}]$
ans = 6

Idea 1 : $ans = \bigoplus_{i=0}^{n-1} A[i]$ // xor of all elements

TC = $O(N)$ SC = $O(1)$

// 1. Define search space $[0, n-1]$ // index l to r

$l = 0, \quad r = n-1$

while ($l \leq r$) {

// 2. check if middle element is the answer

$mid = (l+r)/2$ // $l + (r-l)/2$

if ($(mid == 0 \parallel A[mid-1] \neq A[mid]) \&\&$
 $(mid == N-1 \parallel A[mid+1] \neq A[mid])$)

return $A[mid]$

// 3. Decide whether to go left or right

$A = [\overset{0}{9} \overset{1}{8} \overset{2}{5} \overset{3}{5} \overset{4}{6} \overset{5}{2} \overset{6}{2}]$

even-odd → go right odd-even ← go left

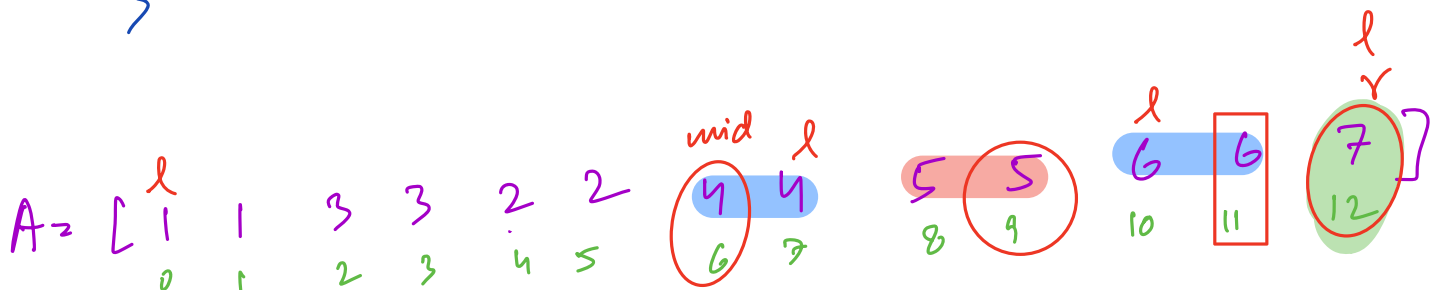
2 2 5 5 6 6 9 1 1
 0 1 2 3 4 5 6 7 8

```

if (mid == 0 || A[mid-1] != A[mid]) { // (mid, mid+1) pair
    if (mid % 2 == 0) l = mid + 1 // (even-odd)
    // or +2
    else r = mid - 1 // (odd-even)
}
else { // (mid-1, mid)
    if (mid % 2 == 0) r = mid - 1 // (odd-even)
    // or -2
    else l = mid + 1 // (even-odd)
}
}
}

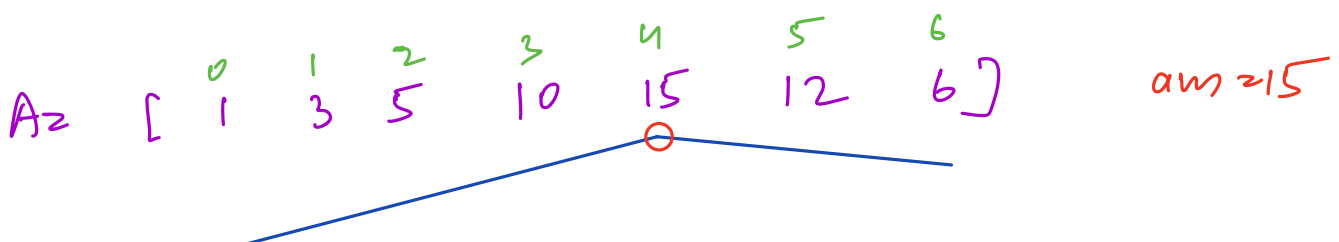
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TC = $O(\log N)$
 SL = $O(1)$



Question 4

Given an increasing-decreasing array. Find
 max element. (Peak element)



// 1. Define search space $[0, n-1]$ // index l to r

$l=0$, $r=n-1$

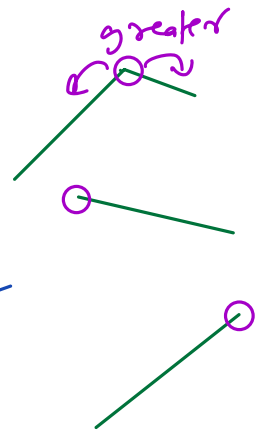
while ($l \leq r$) {

// 2. Check if middle element is the answer

$mid = (l+r)/2$ // $l + (r-l)/2$

if ($(mid == 0 \parallel A[mid-1] < A[mid]) \&\&$
 $(mid == N-1 \parallel A[mid+1] < A[mid])$))

return $A[mid]$



// 3. Decide whether to go left or right

if ($mid == 0 \parallel A[mid-1] < A[mid]$) {

$l = mid + 1$

else

$r = mid - 1$



}

$TC = O(\log N)$

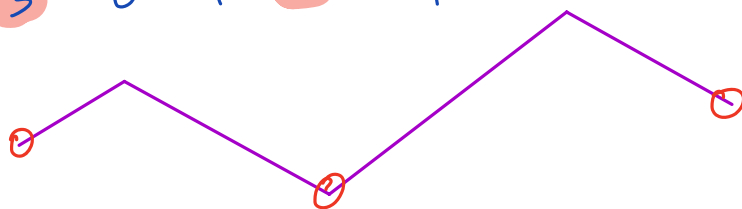
$SC = O(1)$

Question 5

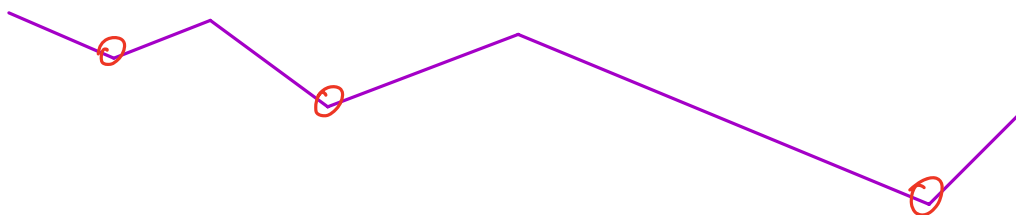
Given an array of distinct elements, find any one local minima i.e.

$$A[i-1] > A[i] < A[i+1]$$

$A = [\overset{0}{1} \overset{1}{3} \overset{2}{6} \overset{3}{1} \overset{4}{0} \overset{5}{9} \overset{6}{8}]$



$A = [\overset{0}{9} \overset{1}{7} \overset{2}{8} \overset{3}{3} \overset{4}{5} \overset{5}{6} \overset{6}{2} \overset{7}{1} \overset{8}{0} \overset{9}{4}]$



Bruteforce : $\forall i$, check if $A[i]$ is local minima

$$TC = O(N) \quad . \quad SC = O(1)$$

Search in $TC < O(N)$?

Binary Search

// 1. Define search space $[0, n-1]$ // index l to r

$l=0$, $r=n-1$

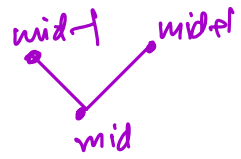
while ($l \leq r$) {

// 2. Check if middle element is the answer

$mid = (l+r)/2$ // $l + (r-l)/2$

if ($(mid == 0 \parallel A[mid-1] > A[mid]) \&\&$
 $(mid == N-1 \parallel A[mid+1] > A[mid])$))

return $A[mid]$



// 3. Decide whether to go left or right

if ($mid == 0 \parallel A[mid-1] > A[mid]$) {

$l = mid + 1$

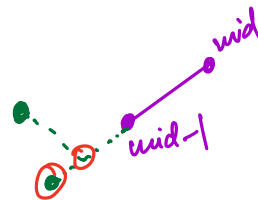
else

$r = mid - 1$



\Rightarrow go right

3

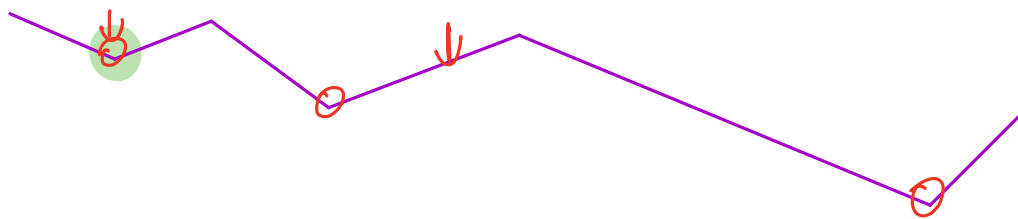


$TC = O(\log N)$

$SC = O(1)$

$A = \begin{bmatrix} 9 & 7 & 8 & 3 & 5 & 6 & 2 & 1 & 0 & 4 \end{bmatrix}$

Indices: 0 1 2 3 4 5 6 7 8 9
 Values: 9 7 8 3 5 6 2 1 0 4
 Red annotations: \downarrow at index 0, \downarrow at index 4, and a red line connecting index 4 to index 9.



$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \end{bmatrix}$

Red annotations: \downarrow at index 0, \downarrow at index 2, and a red line connecting index 0 to index 5.