

Arrays 2 : Two Dimensional

Question 1

Given a row-wise & column-wise sorted matrix, find out whether element K is present or not.

A =

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	12

$$K = 13$$

ans = true

$$K = 0$$

ans = true

$$K = -1$$

ans = false

Bruteforce \rightarrow $\forall i, j$ check $a[i][j]$

$$TC: O(N \times M)$$

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	12

$$K = 0$$

optional

If binary search

$$TC: O(N \log M)$$

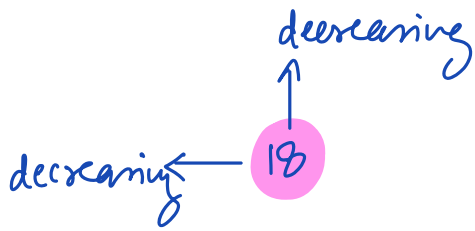
or

$$O(M \log N)$$

$$-5 < 0$$

\Rightarrow go to large values

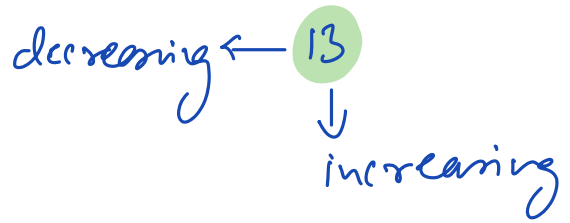
-5 \rightarrow increasing
 \downarrow
increasing



$18 > 0$
 \Rightarrow go to smaller element

	0	1	2	3
0	-5	-2	1 ← 13	14
1	-4	0	3	11
2	-3	2	6	12

$k=0$



	0	1	
0	-5	-2 ← 1	14
1	-4	0	3
2	-3	2	6

$13 > 0$
 \Rightarrow go to left

$1 > 0$
 \Rightarrow go to left

	0	1	
0	-5	-2 ←	
1	-4	0 ↓	
2	-3	2	

$-2 < 0$
 \Rightarrow go to down

	1	
1	-4	0 ↓
2	-3	2

FOUND !!
 return true

Code

$i=0, j=m-1$ // top right (OR bottom left)

while($i < n$ & $j >= 0$) {

if ($a[i][j] == K$) return true

else if ($a[i][j] > K$) $j--$ // left

else $i++$ // down

}

return false

$i \rightarrow 0$ to $n-1$

N times

$j \rightarrow m-1$ to 0

M times

TC: $O(N+M)$

SC: $O(1)$

Question 2

Given a binary ^{$0/1$} matrix sorted row-wisely. $000...111...$

find the smallest row index with max # 1's.

A =

	0	1	2
0	0	1	1
1	0	0	1
2	1	1	1

ans = 2

[answer is index]

Brute force - count #1's in each row

TC: $O(N \times M)$

SC: $O(1)$

A =

	0	1	2	3
0	0	0	1	1
1	0	1	1	1
2	0	0	0	0
3	1	1	1	1
4	0	0	1	1

count = 2
 $am = 0 \neq 3$

3
 4

Code

$i=0, j=m-1, am=0$

while ($i < n$ & $j \geq 0$) {

while ($j \geq 0$ & $a[i][j] == 1$) {

$j--$ // left

$am = i$

}

$i++$ // down

}

return am

} nested loop
 $\Rightarrow O(N \times M)$

TC: $O(N \times M)$

SC: $O(1)$

Question-3

Given the matrix, print boundary elements
clockwise.

N rows

M cols

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

$0,0$ $0,m-1$ $n-1,0$ $n-1,m-1$

output: 1 2 3 4 5 10 15 20

25 24 23 22 21 16 11 6

1	2	3
4	5	6
7	8	9

output: 1 2 3 6

9 8 7 4

Code

$i=0, j=0$

for ($k = 0$ to $m-2$) { $\rightarrow m-1$ times

 print ($a[i][j]$)

$j++$ // right

$i=0, j=0$
first row $L \rightarrow R$

}
for ($k = 0$ to $n-2$) { $\rightarrow n-1$ times

 print ($a[i][j]$)

$i++$ // down

$i=0, j=m-1$
last column top
↓
down

```

for (k = 0 to m-2) { → m-1 times
    print (a[i][j])
    j-- // left
}
for (k = 0 to n-2) { → N-1 times
    print (a[i][j])
    i-- // up
}

```

$i = n-1, j = m-1$
 last row $L \leftarrow R$
 $i = n-1, j = 0$
 first column \uparrow top
 bottom

$$= m-1 + N-1 + m-1 + N-1$$

$$TC: O(N+m)$$

$$SC: O(1)$$

Question 4

Print elements in spiral order in clock wise direction. (square matrix) $\rightarrow N \times N$

A =

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

output: 1 2 3 4 5 10 15 20 25
 24 23 22 21 16 11 6
 7 8 9 14 19 18 17
 12 13

$N=5$

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

r c
 0 0
 1 1
 2 2

N
 $5 \rightarrow -2$
 $3 \rightarrow -2$
 1

$r=0, c=0$

while ($n > 1$) {

$i=r, j=c$

for ($k=0$ to $n-2$) {

print ($a[i][j]$)

$j++$ // right

}

for ($k=0$ to $n-2$) {

print ($a[i][j]$)

$i++$ // down

}

for ($k=0$ to $n-2$) {

print ($a[i][j]$)

$j--$ // left

}

because square
matrix, no M

$TC: O(N^2)$

$SC: O(1)$

```
for (k = 0 to n-2) {
```

```
    print (a[i][j])
```

```
    i-- // up
```

```
}
```

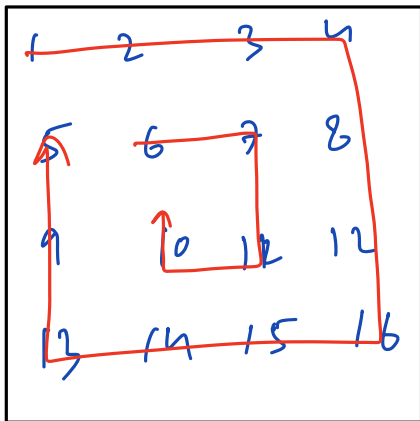
```
    r++, c++, n-- = 2
```

```
}
```

```
if (n == 1)
```

```
    print (a[r][c])
```

} → executed if n is odd

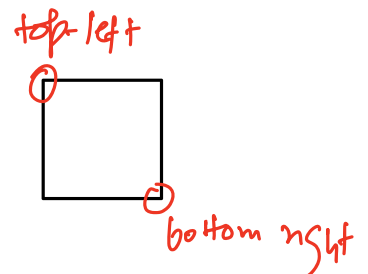


Sub matrix

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

A →

sub matrix



Question

find the # submatrix in which $a[i][j]$ is present for given i, j .

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

$(i, j) = (1, 2)$

ans = 12

$$2 \times 3 \times (3-1) \times (3-2)$$

$$2 \times 3 \times 2 \times 1$$

$$= 12$$

1 2 3 2 3 3
4 5 6 5 6 6

4 5 6 5 6 6

4 5 6 5 6 6
7 8 9 8 9 9

1 2 3 2 3 3
4 5 6 5 6 6
7 8 9 8 9 9

	0	1	2	3	4
0	1	2	3	4	5
1	6	7	8	9	10
2	11	12	13	14	15
3	16	17	18	19	20
4	21	22	23	24	25

possible
top-left cells = $(i+1) \times (j+1)$

bottom-right
cells = $(n-i) \times (m-j)$

submatrix containing $a[i][j] =$
 $= (i+1) \times (j+1) \times (n-i) \times (m-j)$

Question 5

find sum of all submatrix sum.

$ans = 0$

for $(i=0$ to $N-1)$ {

for $(j=0$ to $M-1)$ {

$ans += (i+1) \times (j+1) \times (n-i) \times (m-j) \times a[i][j]$

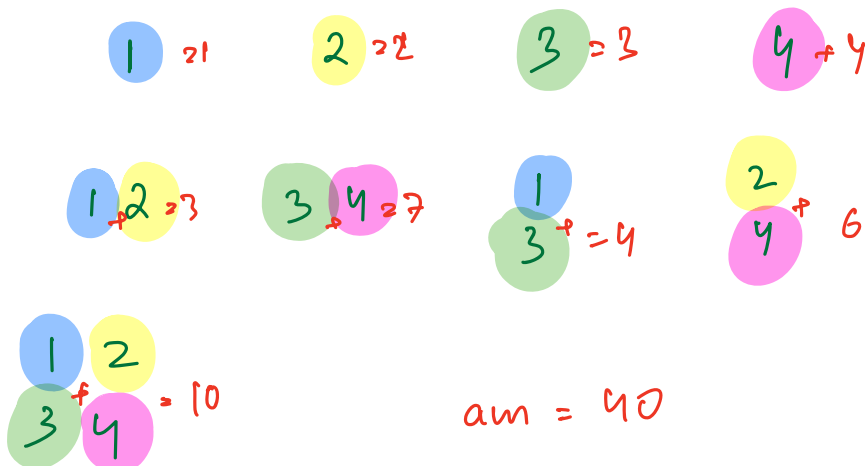
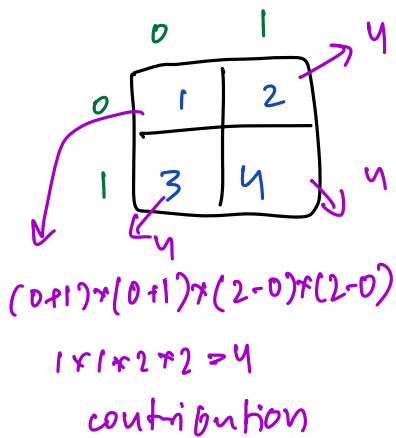
}

}

return ans

TC: $O(N \times M)$

SC: $O(1)$



ans = 40

$$= 1 \times 4 + 2 \times 4 + 3 \times 4 + n \times 4 = 40$$