Fault-Tolerant Quantum Computation by Hybrid Qubits with Bosonic Cat Code and Single Photons

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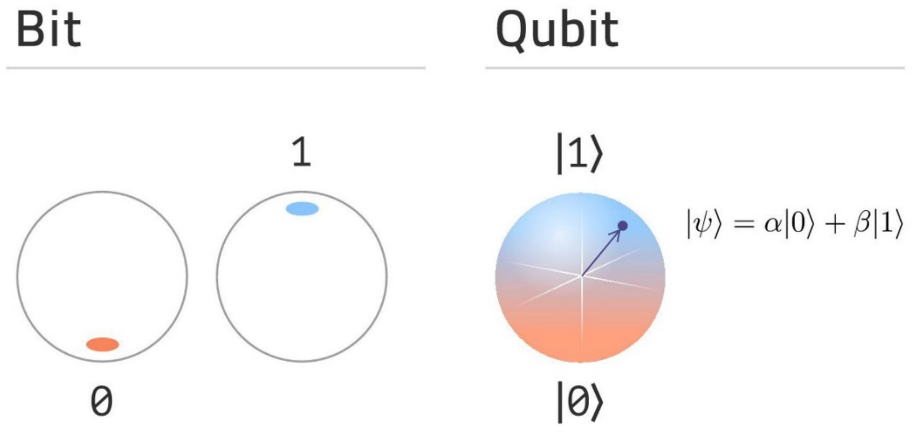
24.10.25 QEC Seminar

Outline

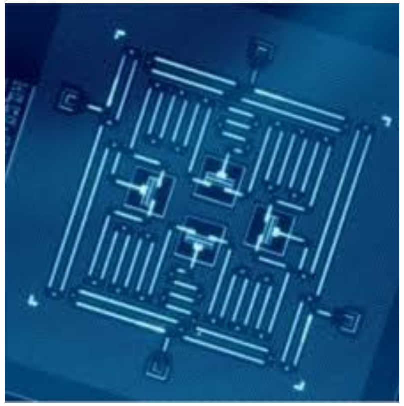
- Encoding qubits in bosonic modes
  - Discrete variables / Continuous variables
  - Bosonic error correction code
- Hybrid quantum computation
  - Hybrid qubits
  - Hybrid fusion gate
  - Fault-tolerance simulation
- Physical realization

Quantum computing

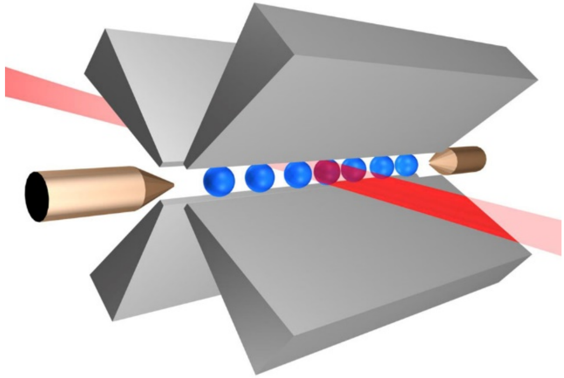
- Qubit: Quantum + Bit



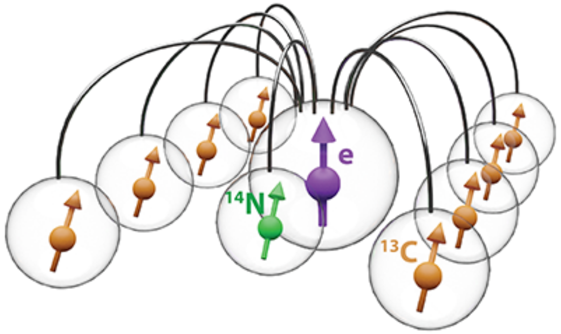
Superconductor



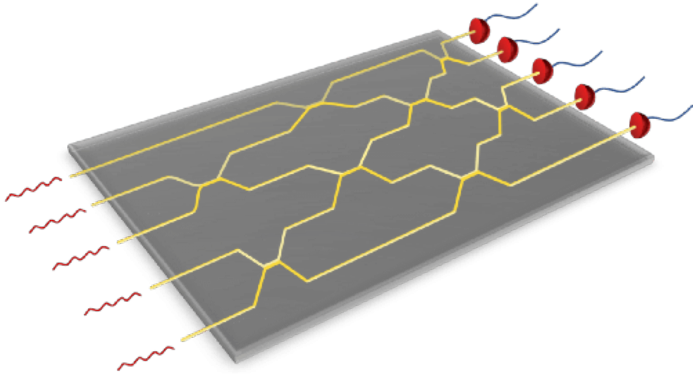
Trapped ion



NV center



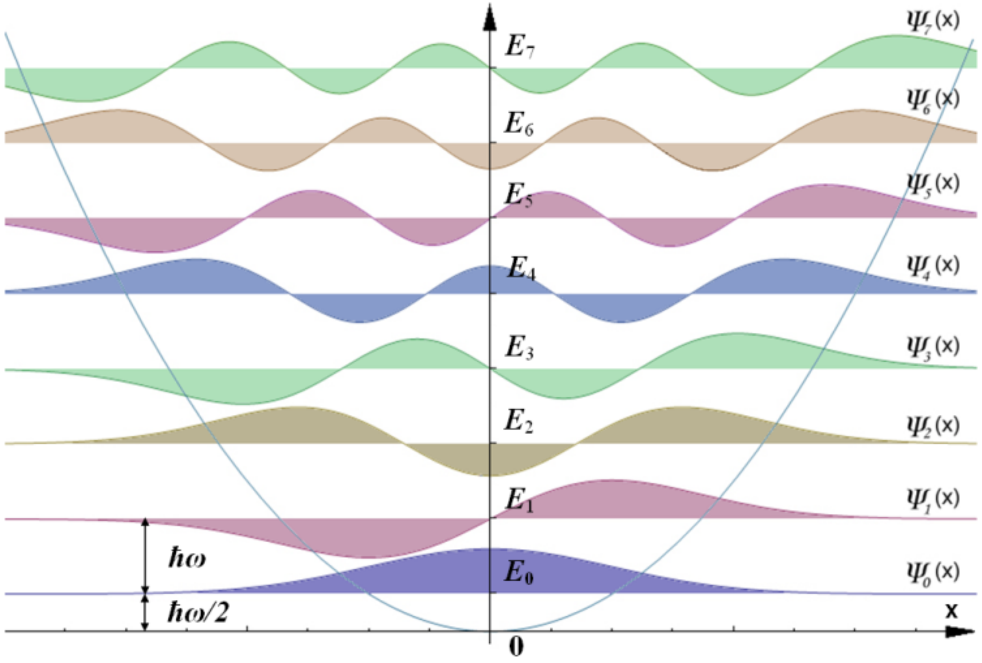
Optical system



Quantum computing

- Bosonic system

$$\hat{H} = \frac{1}{2}m\omega\hat{x}^2 + \frac{1}{2m}\hat{p}^2 = \hbar\omega\left(\hat{n} + \frac{1}{2}\right)$$

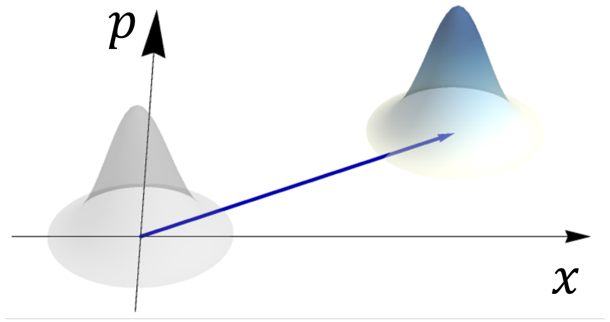


Discrete variable: $\hat{n}$

$|0\rangle$: upper path

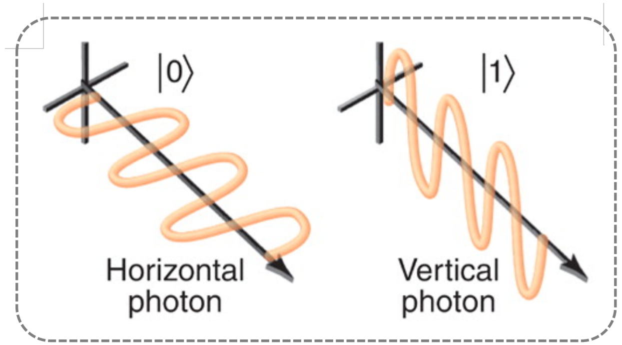
$|1\rangle$: lower path

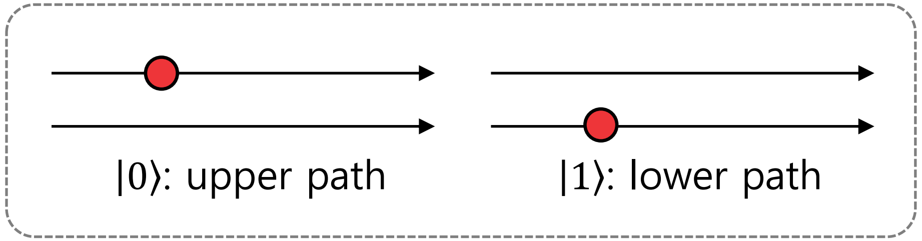
Continuous variable: $\hat{x}, \hat{p}$



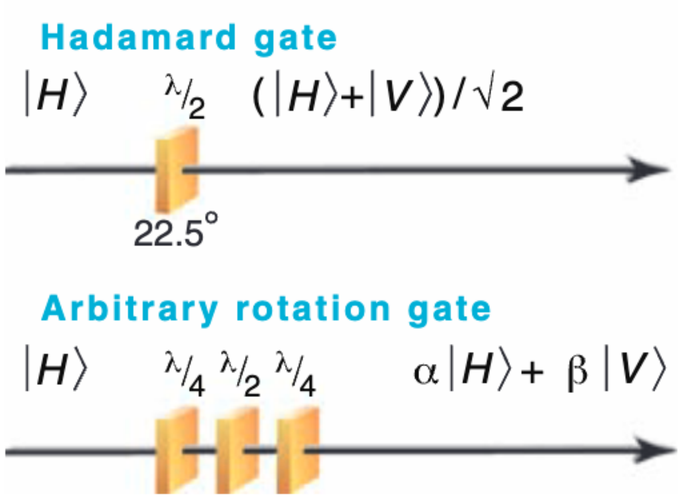
Quantum computing

- Single-photon encoding (dual-rail encoding)

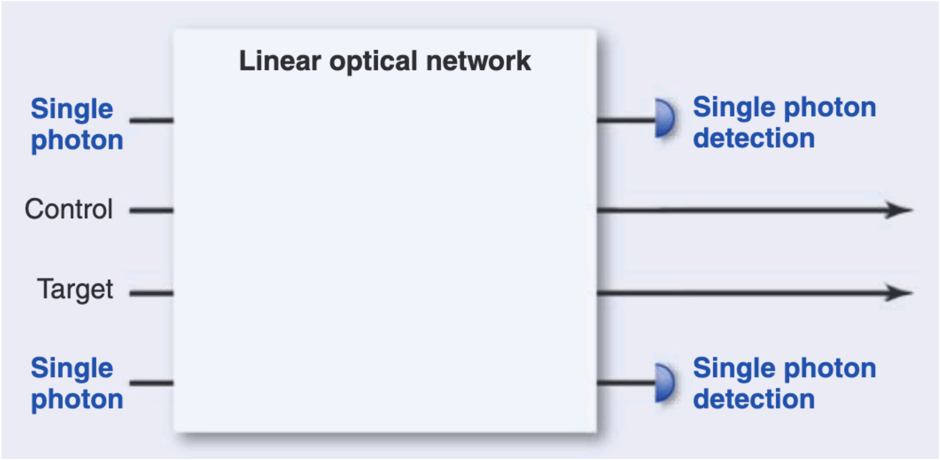




- Single-qubit gates: linear optical implementation



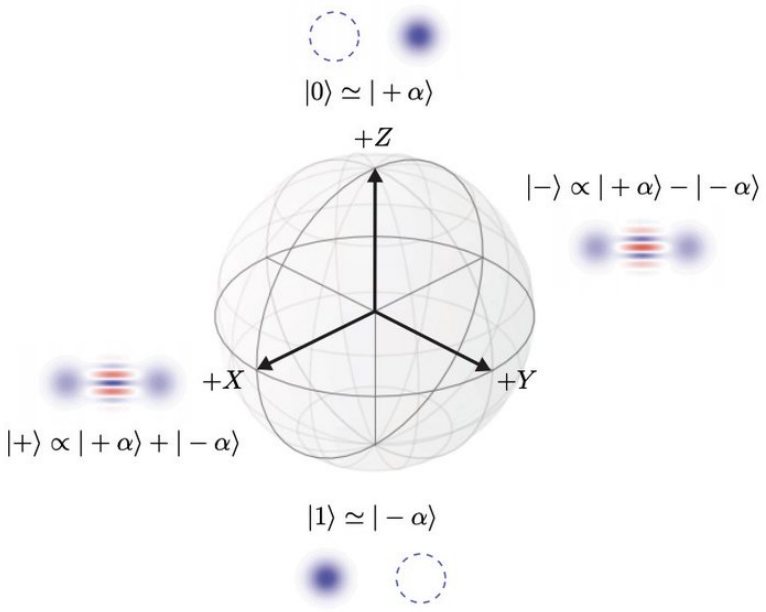
- Two-qubit gates: nondeterministic

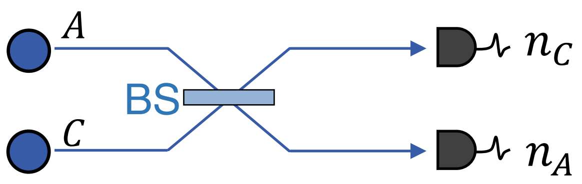


✻ J. O'Brien, Science **318**, 5856 (2007).

Quantum computing

- Coherent-state encoding





n_A	n_C	Result
Even	0	$\vert\Psi_+\rangle$
Odd	0	$\vert\Psi_-\rangle$
0	Even	$\vert\Phi_+\rangle$
0	Odd	$\vert\Phi_-\rangle$
0	0	Fail

$p_{\text{fail}} \to 0$

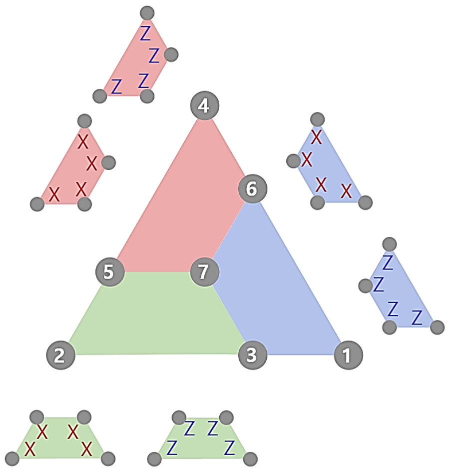
- Near-deterministic Bell measurement

✻ H. Jeong *et al.*, Phys. Rev. A **64**, 052308 (2001).

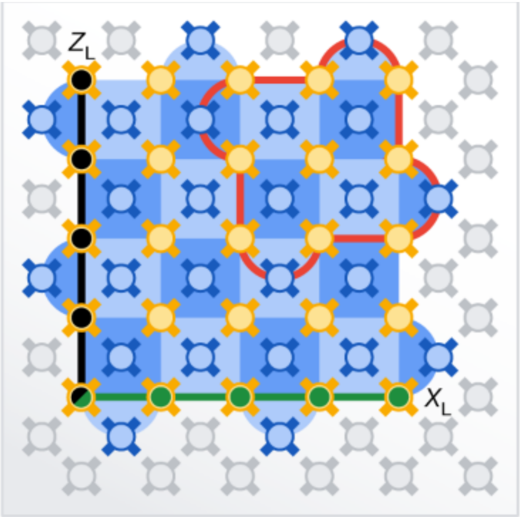
✻ Ralph *et al.*, Phys. Rev. A **68**, 042319 (2003).

Quantum error correction

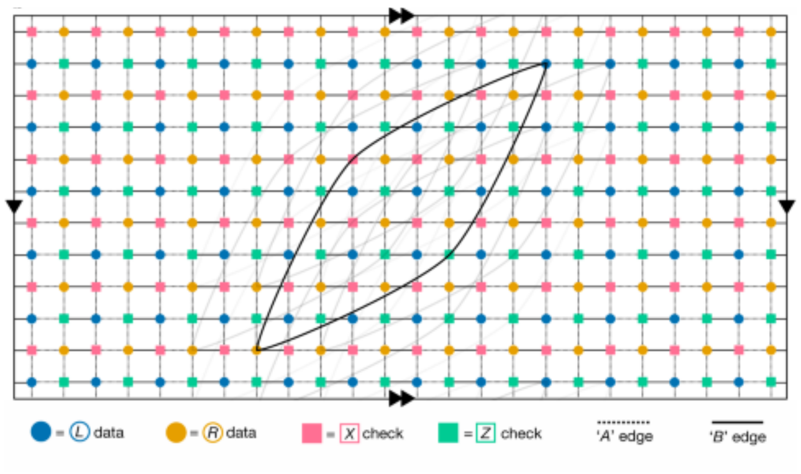
- Protect qubits from noise
- Typically encoding a logical qubit into a number of physical qubits



Steane code



Surface code

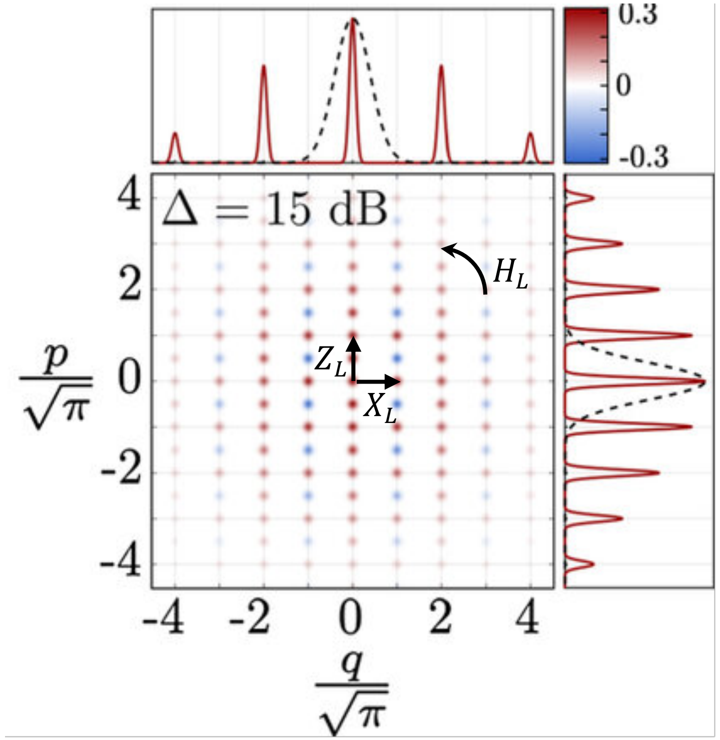


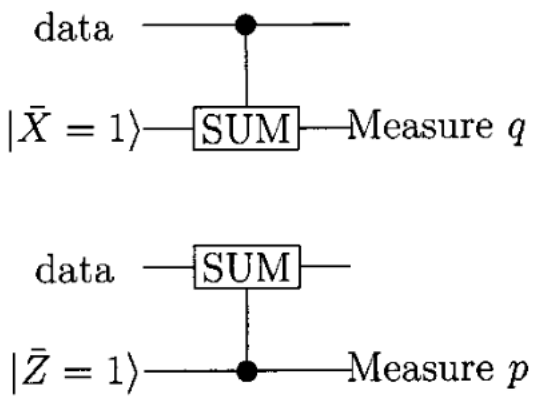
qLDPC code

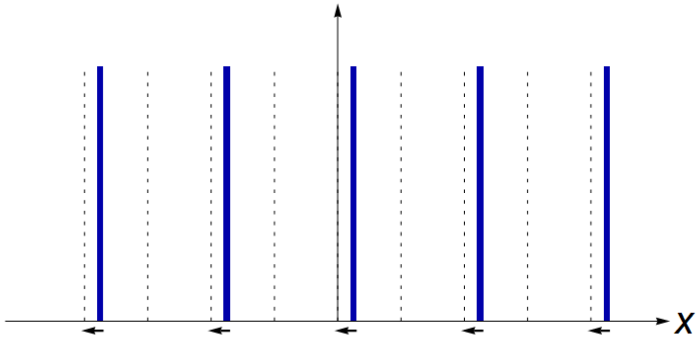
Bosonic error correction code

- Error correction in single bosonic mode

- GKP code



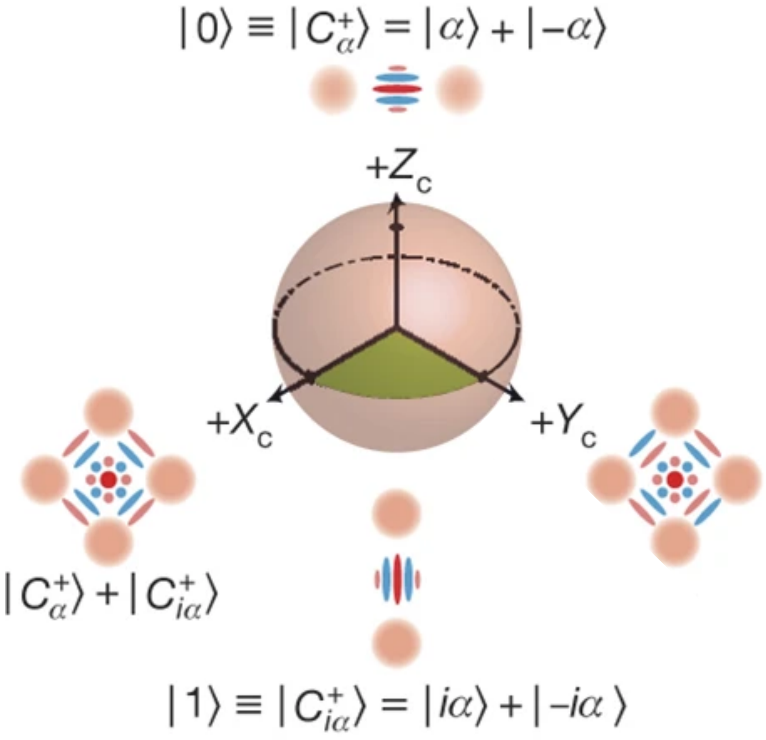




✻ D. Gottesman, A. Kitaev, and J. Preskill, PRA **64**, 012310 (2001).

Bosonic error correction code

- Cat code
  - Encoding qubit into even cat states
    $$|\mathcal{C}_\alpha^+\rangle = \mathcal{N}(|\alpha\rangle + |-\alpha\rangle)$$
    $$|\mathcal{C}_{i\alpha}^+\rangle = \mathcal{N}(|i\alpha\rangle + |-i\alpha\rangle)$$
  - Photon loss induces a parity change
    $$\hat{a}(|\alpha\rangle + |-\alpha\rangle) \;\rightarrow\; |\alpha\rangle - |-\alpha\rangle$$
  - Syndrome measurement:
    parity measurement

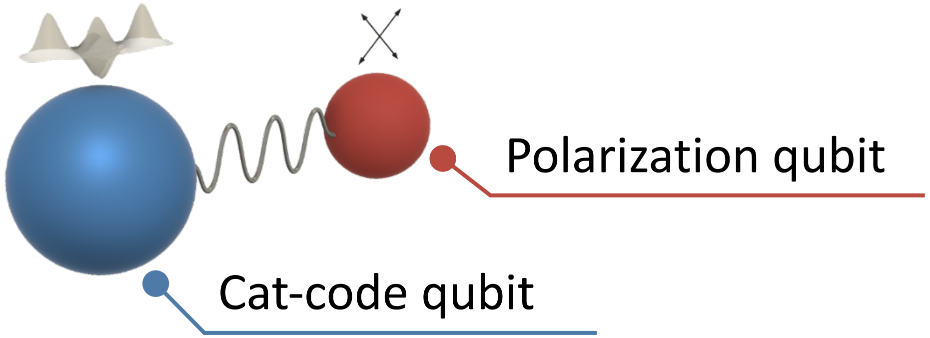


✻ M. Bergmannn *et al.*, PRA **94**, 042332 (2016).
✻ A. L. Grimsmo *et al.*, PRX **10**, 011058 (2020).
✻ N. Ofek *et al.*, Nature **536**, 441 (2016).

Hybrid quantum computation

- Hybrid qubit

$$|0_L\rangle = |+\rangle|\mathcal{C}_\alpha^+\rangle$$
$$|1_L\rangle = |-\rangle|\mathcal{C}_{i\alpha}^+\rangle$$



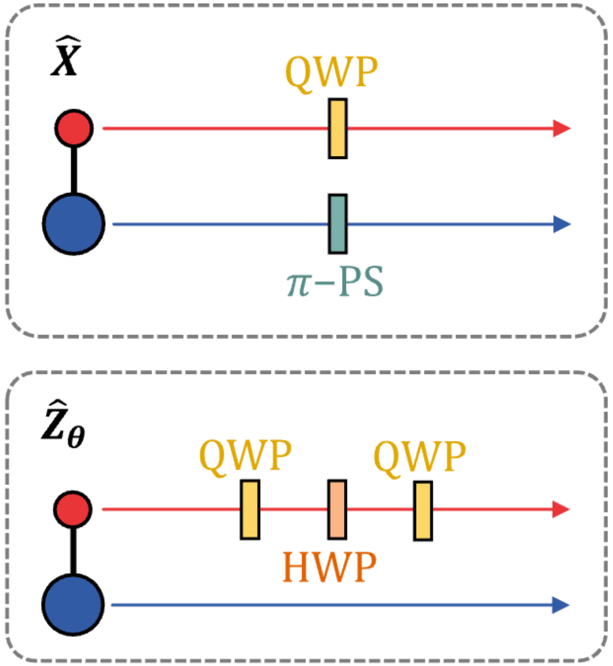
- Key features
  1) Photon loss correction by cat code
  2) Linear optical implementation of single-qubit gates
  3) Efficient hybrid fusion operation

✻ J. Lee *et al.*, PRX Quantum **5**, 030322 (2024).

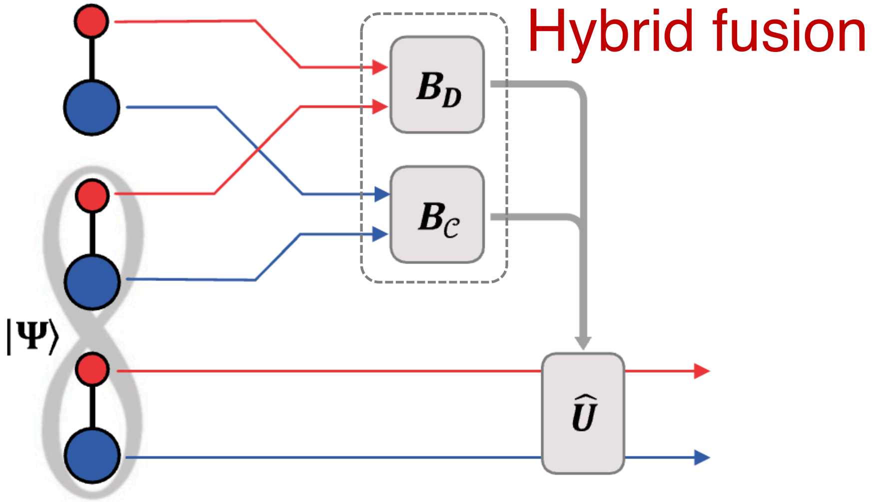
Hybrid quantum computation

- Universal gate set: $\{X, Z_\theta, H, CZ\}$

- Single-qubit rotation



- Gate teleportation: H, CZ



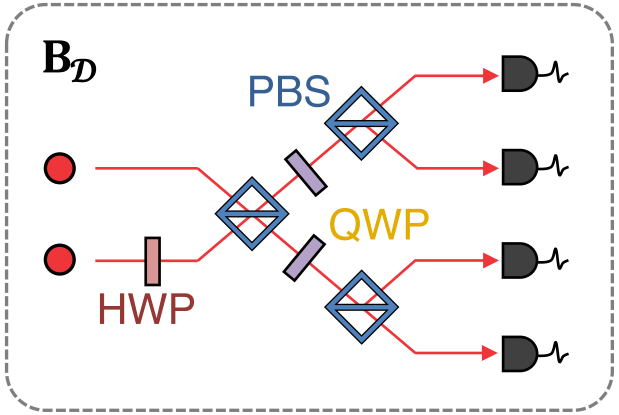
Hybrid quantum computation

- Hybrid fusion gate: Near-deterministic Bell measurement

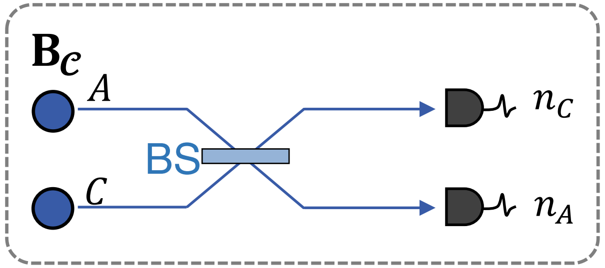
+

Syndrome measurement by parity check

- DV Bell measurement



- CV Bell measurement

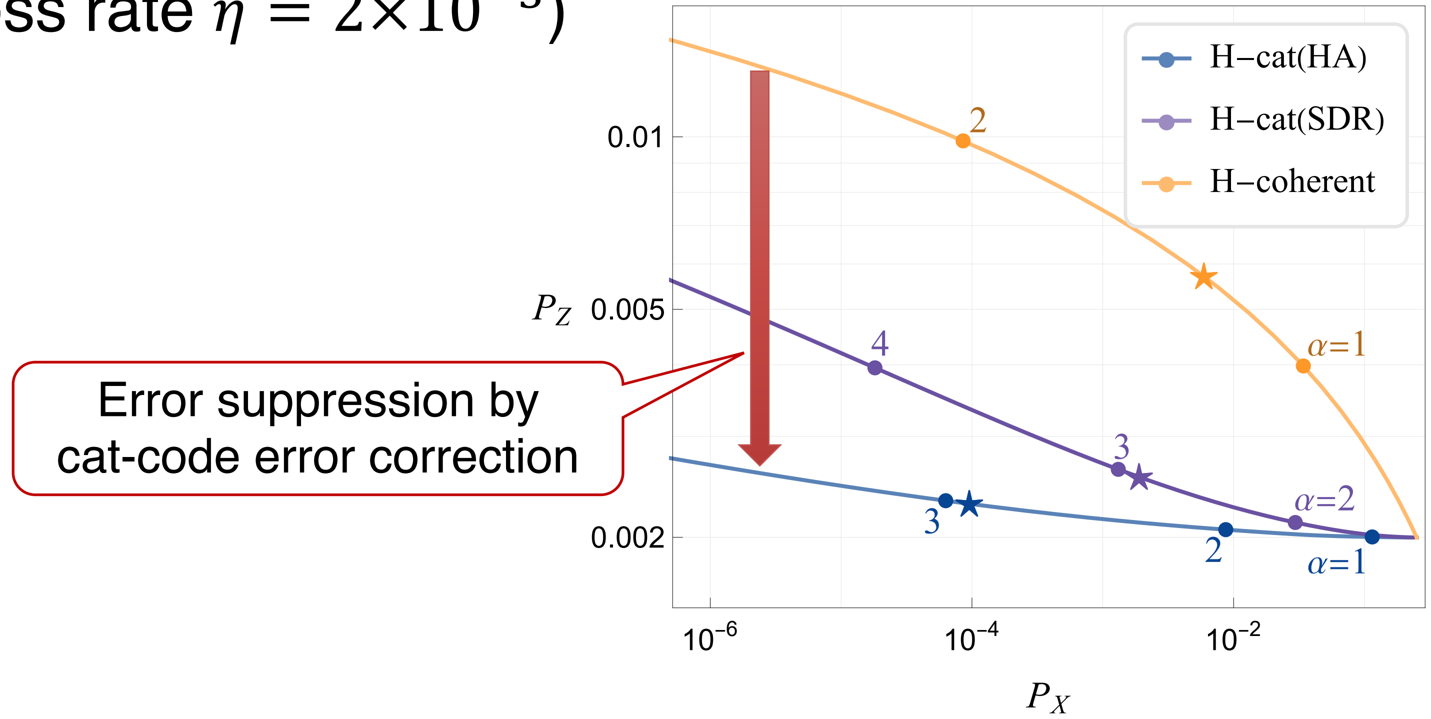


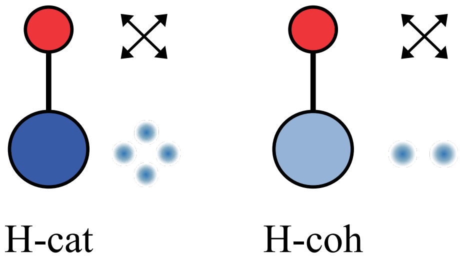
Measurement Pattern	No photon loss			Single photon loss		
	N	Decision	X error	N	Decision	X error
$n_A = 0$ or $n_C = 0$	$4k$	$\vert\Phi_{\mathcal{C}}^{+}\rangle$	p_X	$4k+3$	$\vert\Phi_{\mathcal{C}}^{+}\rangle$	p_X
	$4k+2$	$\vert\Phi_{\mathcal{C}}^{-}\rangle$	None	$4k+1$	$\vert\Phi_{\mathcal{C}}^{-}\rangle$	None
$n_A \neq 0$ and $n_C \neq 0$	$4k$	$\vert\Psi_{\mathcal{C}}^{+}\rangle$		$4k+3$	$\vert\Psi_{\mathcal{C}}^{+}\rangle$	
	$4k+2$	$\vert\Psi_{\mathcal{C}}^{-}\rangle$		$4k+1$	$\vert\Psi_{\mathcal{C}}^{-}\rangle$	

✻ J. Hastrup and U. Andersen, PRR **4**, 043065 (2022).
✻ D. Su *et al.*, PRA **106**, 042614 (2022).

Hybrid quantum computation

- Error rates P_X and P_Z after hybrid fusion (loss rate $\eta = 2\times10^{-3}$)

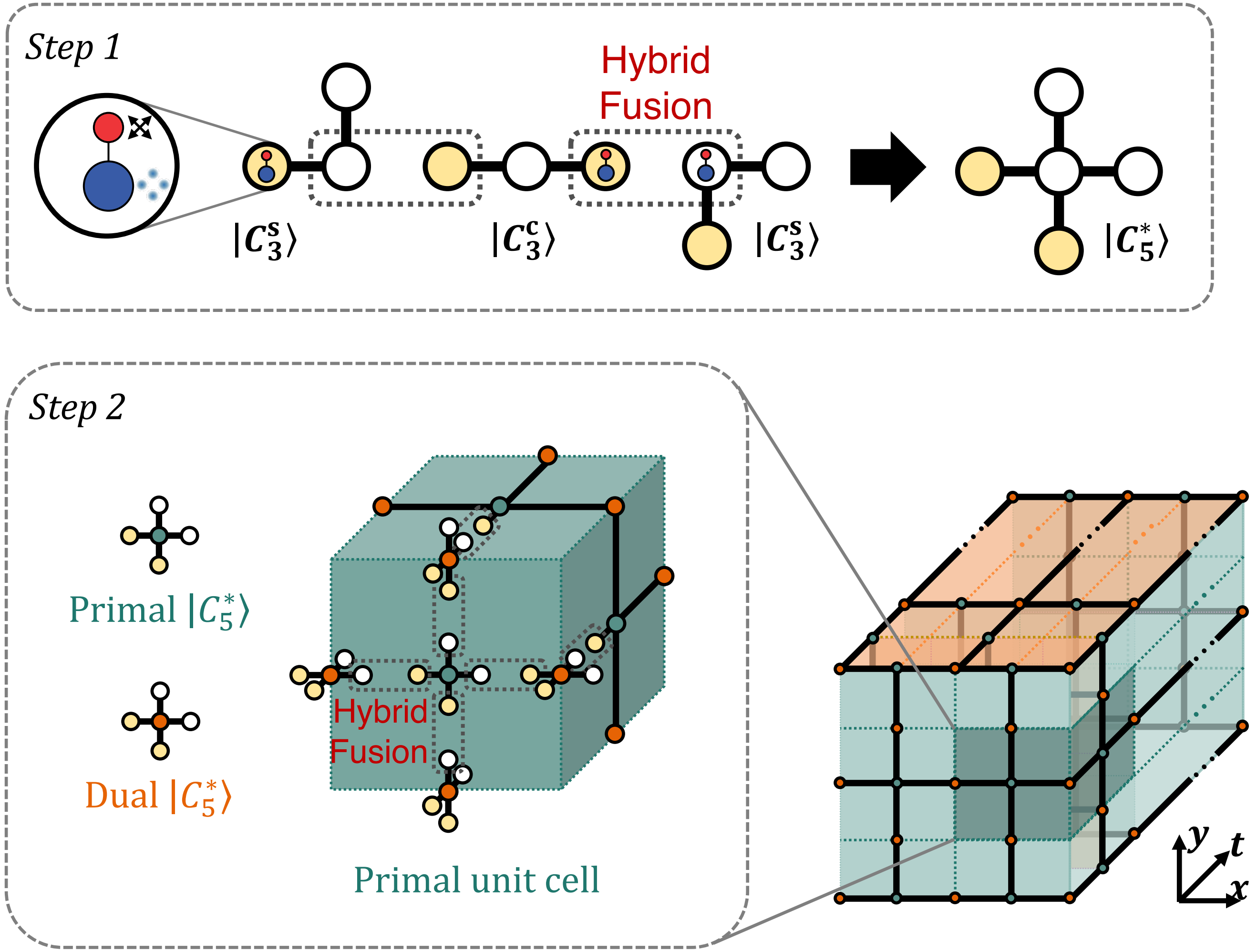




- $P_X \uparrow$ as $\alpha \downarrow$: Nonorthogonality of coherent states
- $P_Z \uparrow$ as $\alpha \uparrow$: Fragile against photon loss

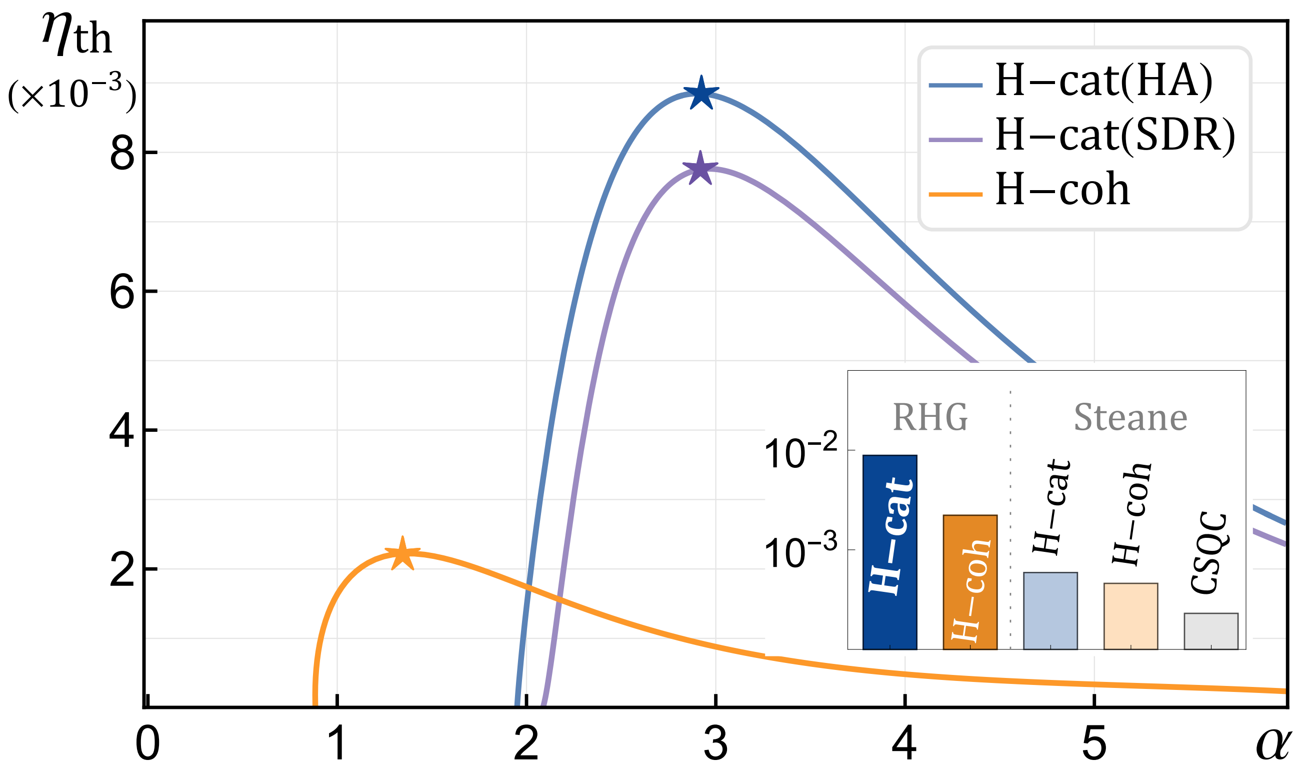
Fault-tolerance simulation

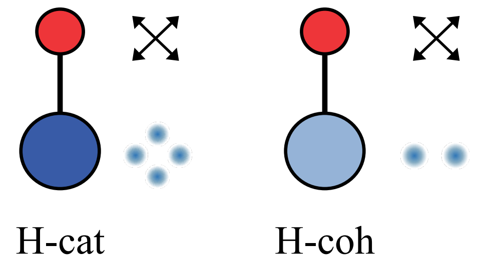
- RHG lattice simulation
  1) Build RHG lattice using hybrid fusion
  2) Assign error rates (error propagation)
  3) Find the error pattern using MWPM
  4) Count uncorrectable error chains and identify a logical error



Fault-tolerance simulation

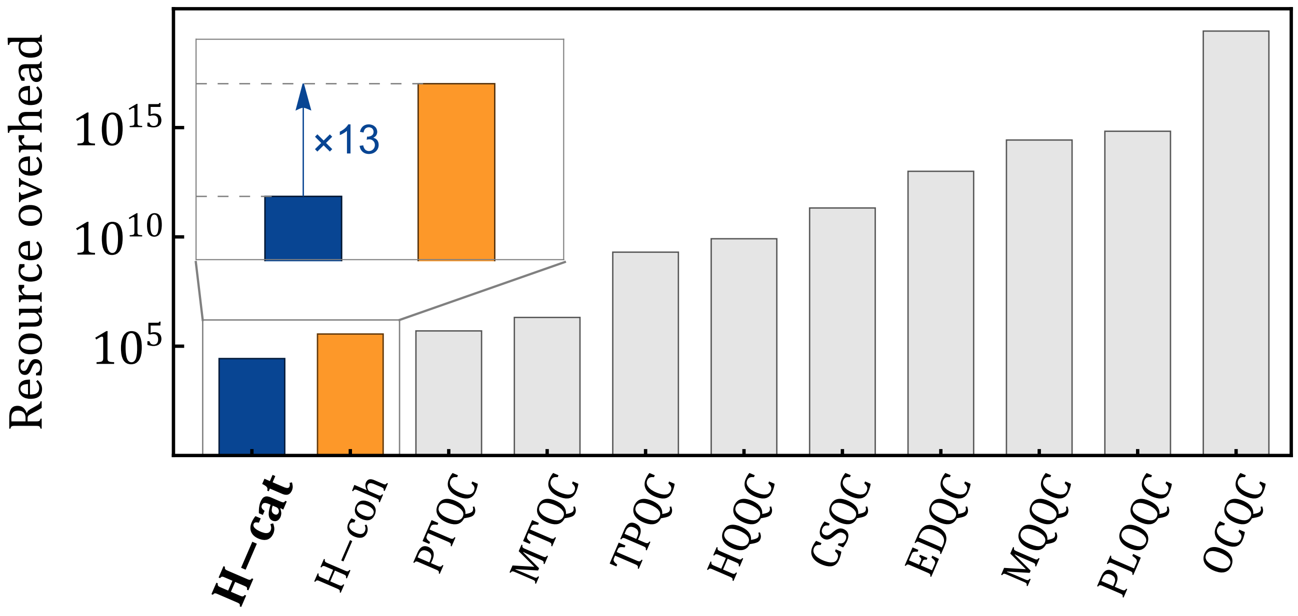
- Loss threshold: 0.89% (improved by a factor of 4)
- Best threshold in CV or hybrid optical quantum computing





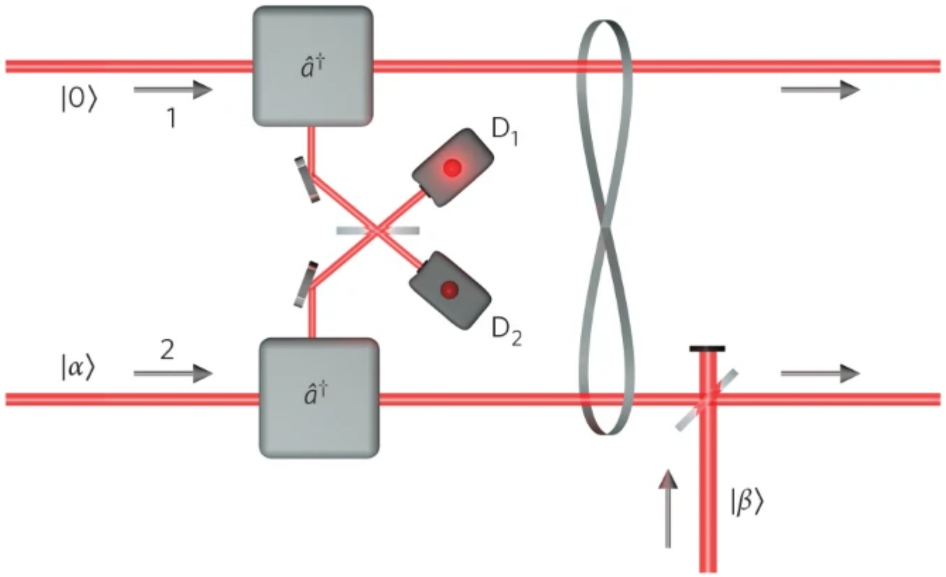
Fault-tolerance simulation

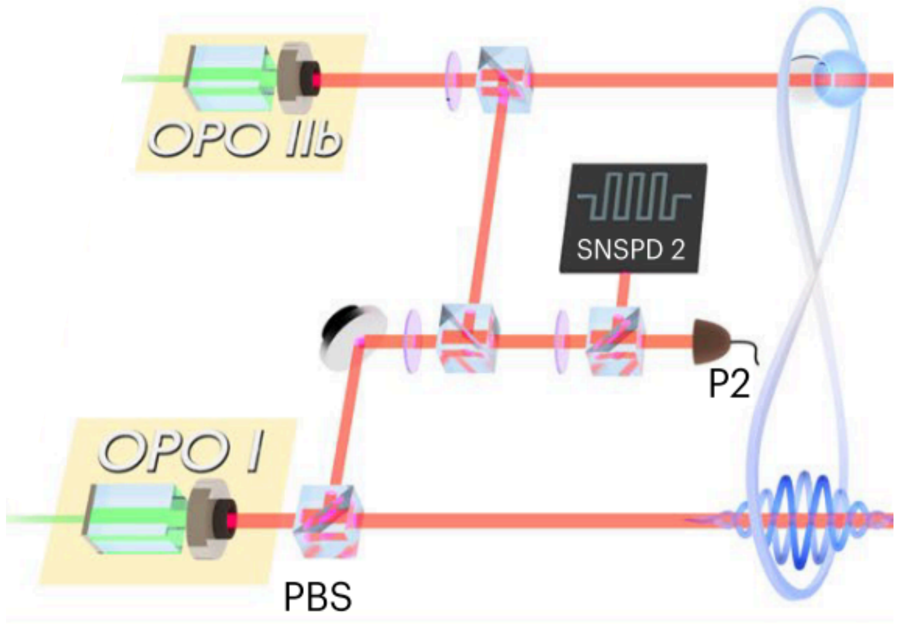
- Count the number of H-coh pairs as unit resources.
- Resource efficiency improved at least by an order of magnitude!



Physical implementation

- Generation of hybrid pairs (H-coh)

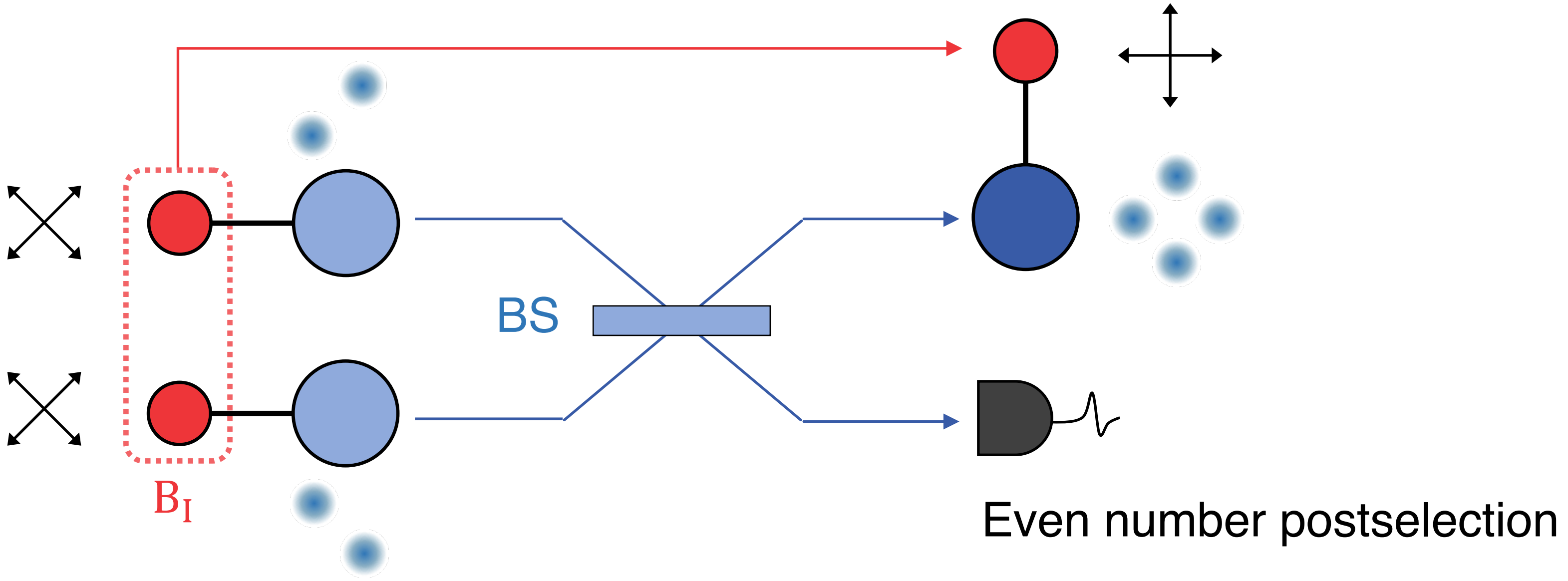




✻ H. Jeong *et al.*, Nat. Photonics 8, 564 (2014).
✻ O. Morin *et al.*, Nat. Photonics 8, 570 (2014).

Physical implementation

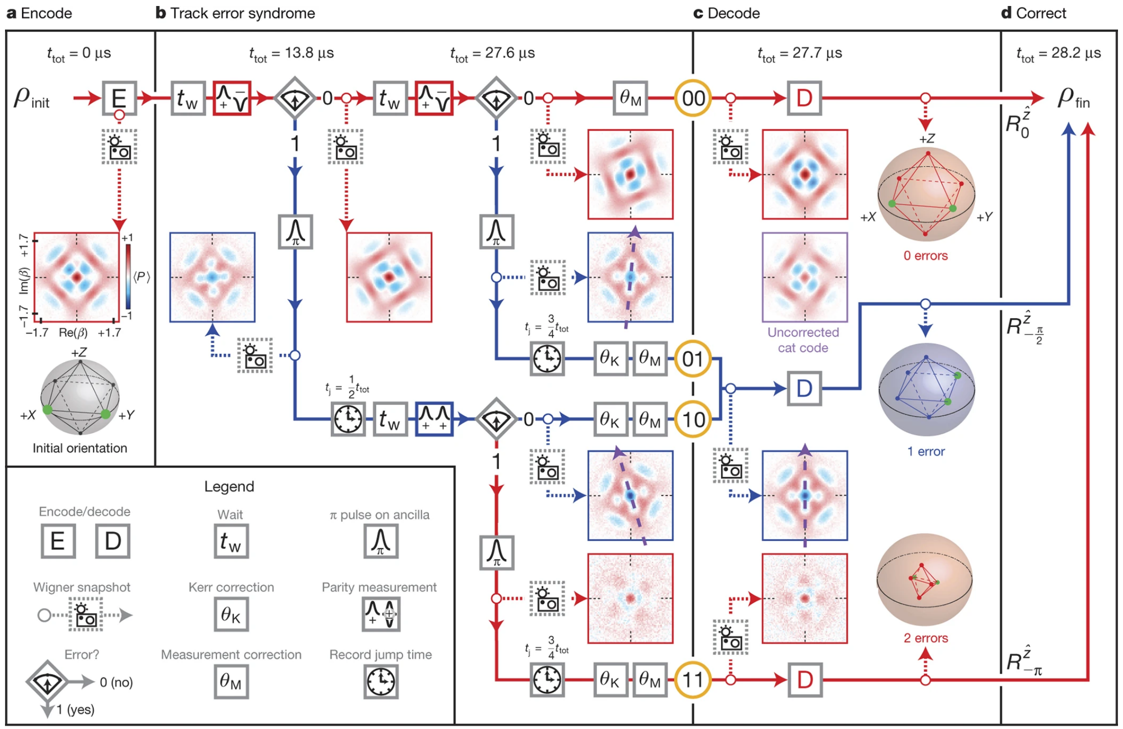
- H-cat pair generation from H-coh pairs
  - 1 H-cat ≈ 8 H-coh
    : 2(# of H-cat pair) x 2(DV success rate) x 2(CV success rate)

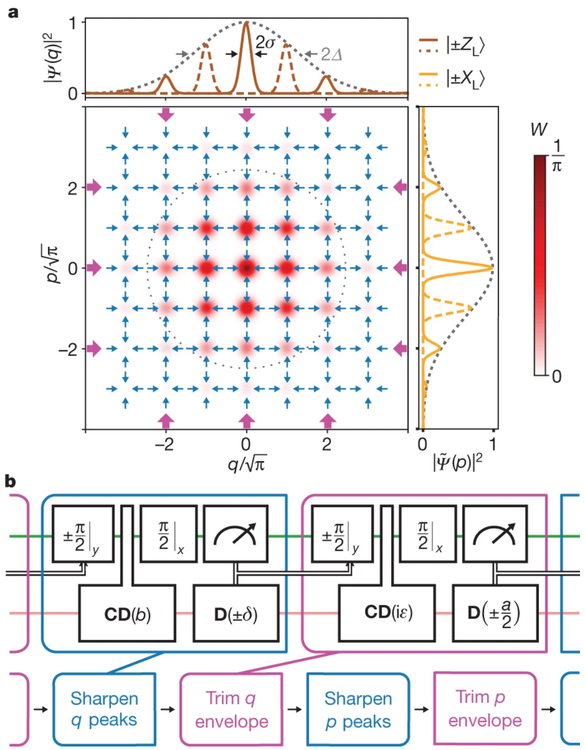


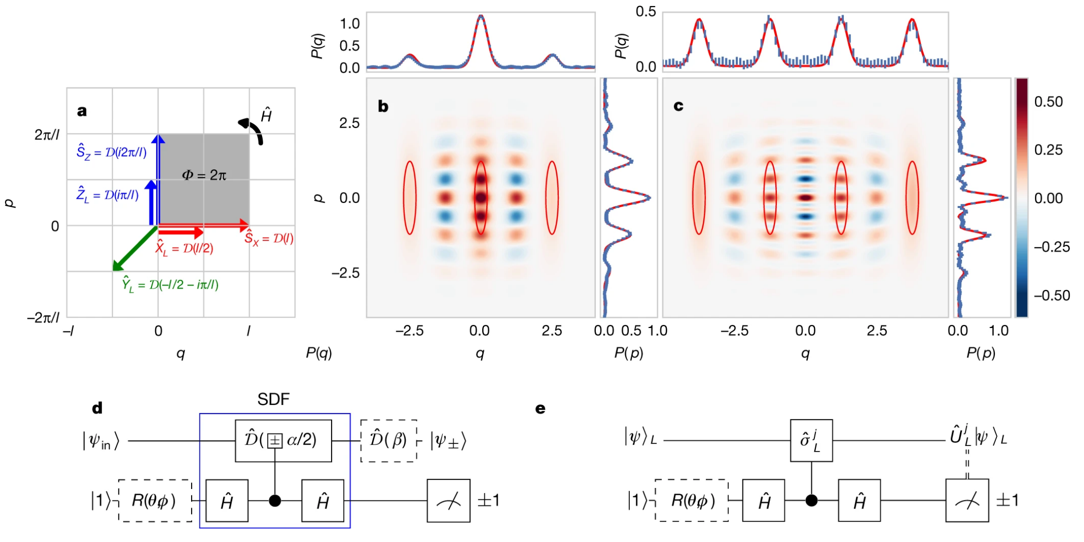
✻ J. Hastrup *et al.*, Opt. Lett. **45**, 640 (2020).

Physical implementation

- Bosonic code in superconductor / ion trap







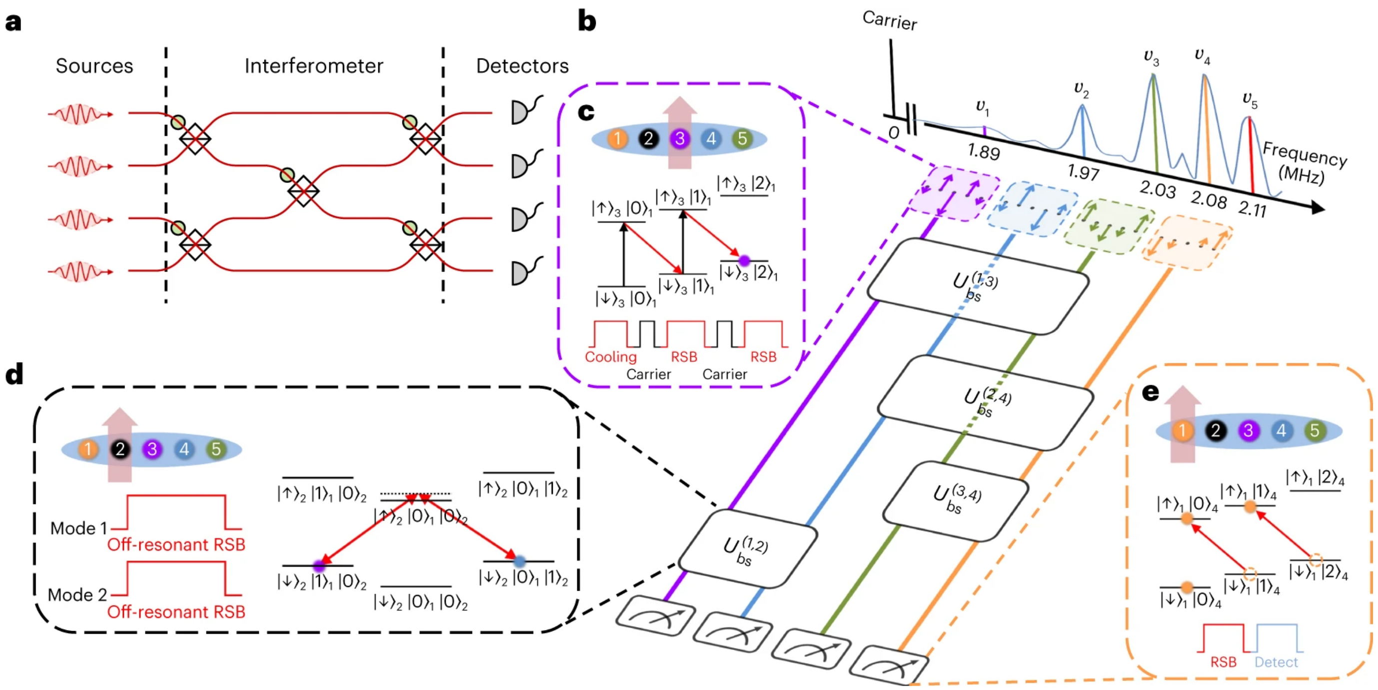
✻ N. Ofek *et al.*, Nature **536**, 441 (2016).

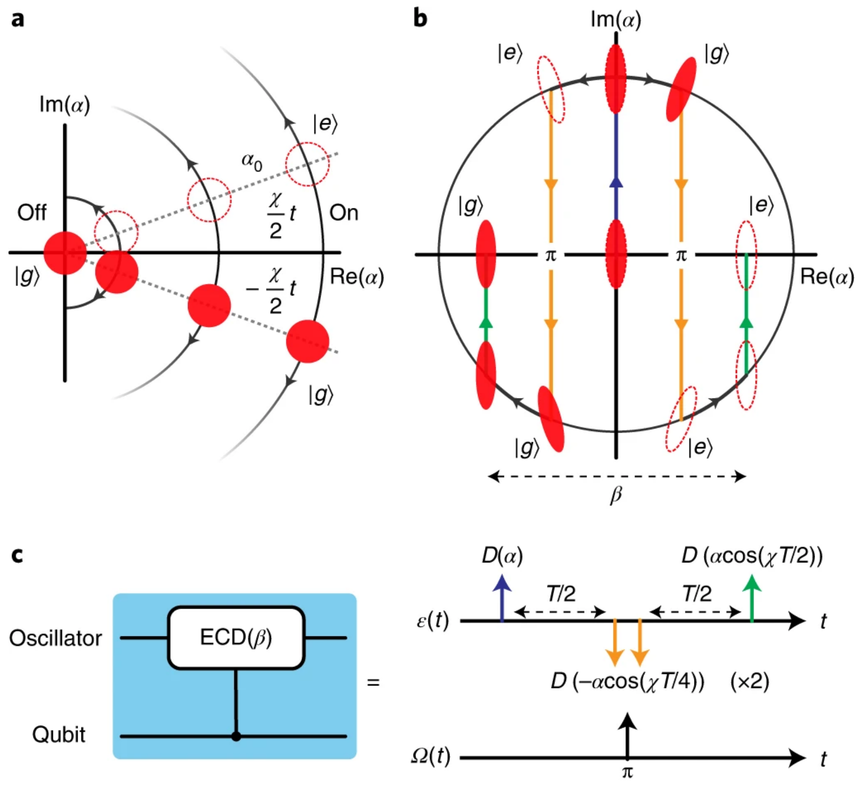
✻ P. Campagne-Ibarcq *et al.*, Nature **584**, 368 (2020).

✻ C. Flühmann *et al.*, Nature **566**, 513 (2019).

Physical implementation

- DV-CV nonlinear interaction





- DV-CV hybrid approach available in various platforms

✻ A. Eickbusch *et al.*, Nat. Phys. **18**, 1464 (2022).

✻ W. Chen *et al.*, Nat. Phys. **19**, 877 (2023).

Conclusion

- Summary
  - We suggest hybrid quantum computation using single photon and cat code.
  - Hybrid fusion operation conducts the Bell measurement as well as photon loss detection.
  - Improvement in loss threshold and resource efficiency is shown.
- Future works
  - Outer codes exhibiting error-bias property
  - Hybrid approach with various bosonic codes