Dynamical Quantum Error-Correction

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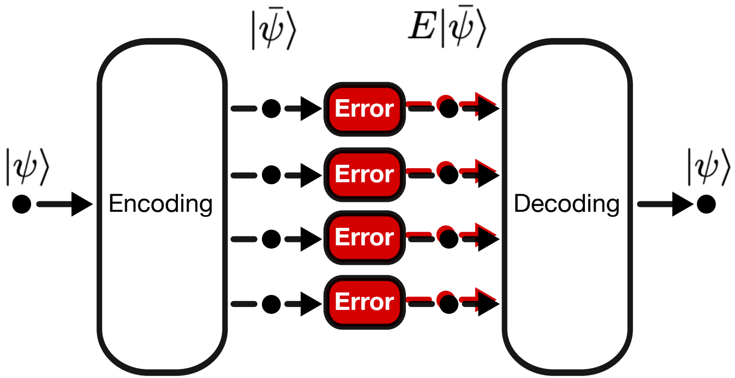
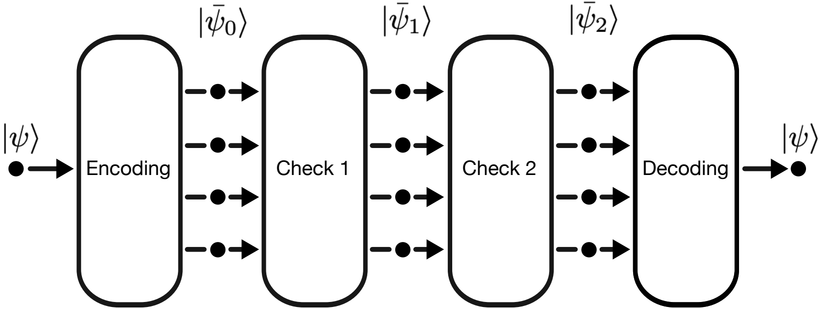
September 13, 2024

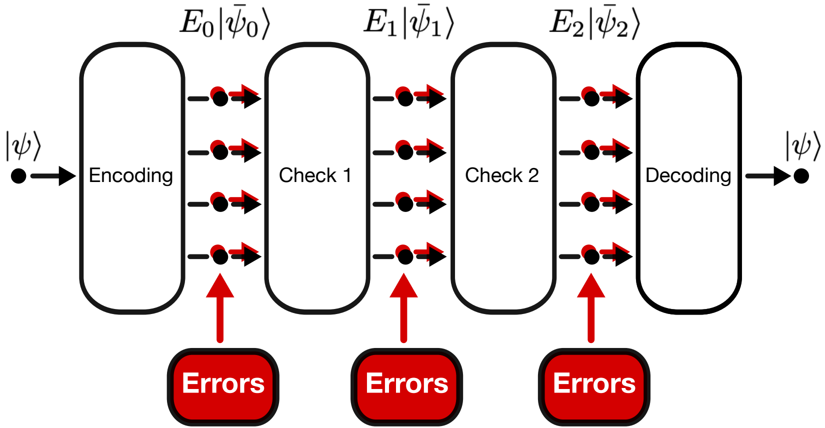
Quantum Error-Correction



Dynamical QEC: We can do more, temporally

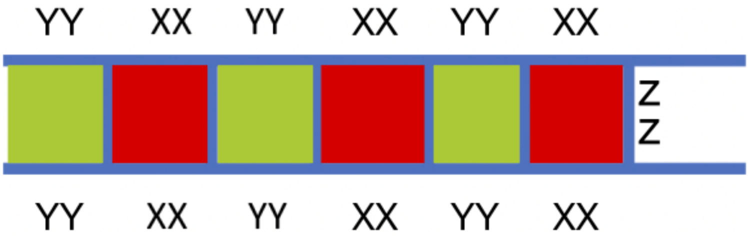


More errors



$E_0|\bar{\psi}_0\rangle$
$E_1|\bar{\psi}_1\rangle$
$E_2|\bar{\psi}_2\rangle$
$|\psi\rangle$
Encoding
Check 1
Check 2
Decoding
$|\psi\rangle$
Errors
Errors
Errors

Dynamical Codes - Error-correction over time



Periodic measurement sequence – *Floquet code*:

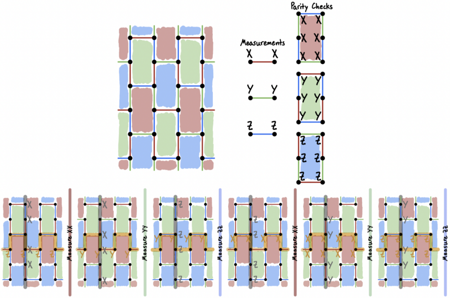
$$ZZ \to XX \to ZZ \to YY \to ZZ \to XX \to ZZ \to YY \to \dots$$

round (mod 4)	0	1	2	3
check	ZZ	XX	ZZ	YY
stabilizer	$\langle G, R, C_z\rangle$	$\langle G, R, C_x\rangle$	$\langle G, R, C_z\rangle$	$\langle G, R, C_y\rangle$
syndrome	G	R	R	G

Content

Content

Floquet Codes I



Dynamically Generated Logical Qubits

Matthew B. Hastings[1,2] and Jeongwan Haah[2]

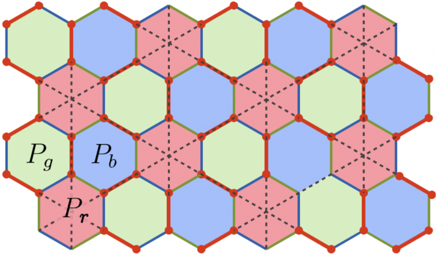
[1]Station Q, Microsoft Quantum, Santa Barbara, CA 93106-6105, USA
[2]Microsoft Quantum and Microsoft Research, Redmond, WA 98052, USA

A Fault-Tolerant Honeycomb Memory

Craig Gidney, Michael Newman, Austin Fowler, and Michael Broughton

Google Quantum AI, Santa Barbara, California 93117, USA

December 15, 2021



Floquet Codes without Parent Subsystem Codes

Margarita Davydova[1,2] Nathanan Tantivasadakarn[3,4] and Shankar Balasubramanian[5,*]

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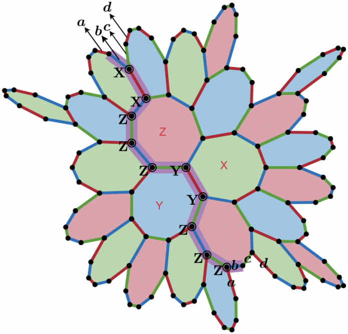
Floquet Codes II



Fault-tolerant hyperbolic Floquet quantum error correcting codes

Ali Fahimniya,[1,2,*] Hossein Dehghani,[1,2] Kishor Bharti,[1,2,3] Sheryl Mathew,[1]
Alicia J. Kollár,[2] Alexey V. Gorshkov,[1,2] and Michael J. Gullans[1]

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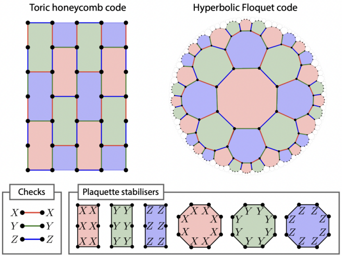
(Dated: September 25, 2023)



Constructions and performance of hyperbolic and semi-hyperbolic Floquet codes

Oscar Higgott[1] and Nikolas P. Breuckmann[2]

Stabilizer codes review

- *Pauli group*: $\mathcal{P} = \{\pm 1, \pm i\} \cdot \{I, X, Y, Z\}$.
- n-qubit Pauli group: $\mathcal{P}_n := \mathcal{P}^{\otimes n}$.
- n-qubit *stabilizer group* $\mathcal{S} \subseteq \mathcal{P}_n$ generated by mutually commuting $g_1, \ldots, g_m \in \mathcal{P}_n$:

$$\begin{gathered} \mathcal{S} = \langle g_1, \ldots, g_m \rangle = \{s_w : w \in \{0,1\}^m\} \subseteq \mathcal{P}^{\otimes n} \\ s_w = g_1^{w_1} \ldots g_m^{w_m} \end{gathered} \tag{1}$$

- *Stabilizer code* of $\mathcal{S}$:

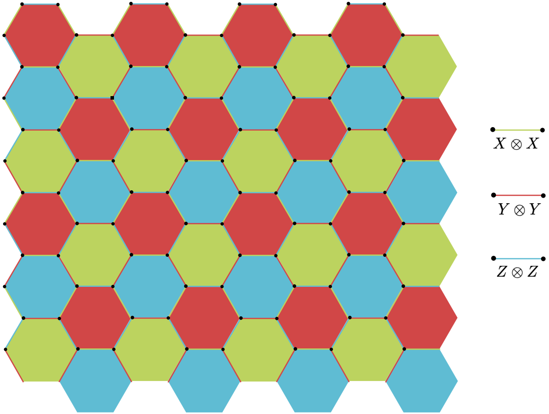
$$\mathcal{C}(\mathcal{S}) = \{|\psi\rangle : \forall s \in \mathcal{S}, s|\psi\rangle = |\psi\rangle\} \tag{2}$$

Stabilizer update

Update $\mathcal{S} \to \mathcal{S}'$ after measuring Pauli $M \in \mathcal{P}_n$:

1. If $+M$ or $-M$ in $\mathcal{S}$:
   $\mathcal{S}' = \mathcal{S}$ (measurement outcome is deterministic).
2. If $+M, -M \notin \mathcal{S}$ and M commutes with all $s \in \mathcal{S}$:
   $\mathcal{S}' = \langle \mathcal{S}, +M \rangle$ if outcome is $+1$ and $\mathcal{S}' = \langle \mathcal{S}, -M \rangle$ if outcome is -1.
3. If $+M, -M \notin \mathcal{S}$ and $\exists s \in \mathcal{S}$ that does not commute with M:
   $\mathcal{S}' = \langle \mathcal{S}^*, +M \rangle$ if outcome is $+1$ and $\mathcal{S}' = \langle \mathcal{S}^*, -M \rangle$ if outcome is -1, for $\mathcal{S}^* = \{s \in \mathcal{S} : [s, M] = 0\}$.

Initializing the Floquet code

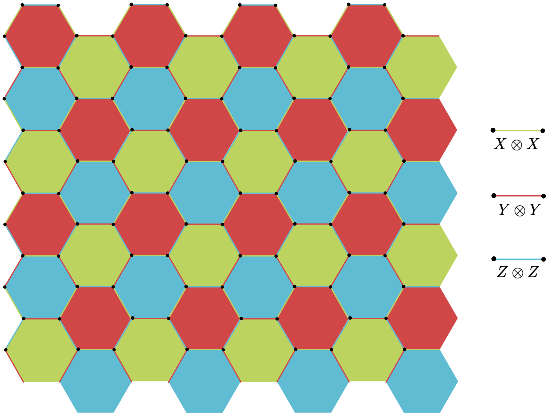


Measurements:

$C_g \to C_r \to C_b \to C_g \to \dots$

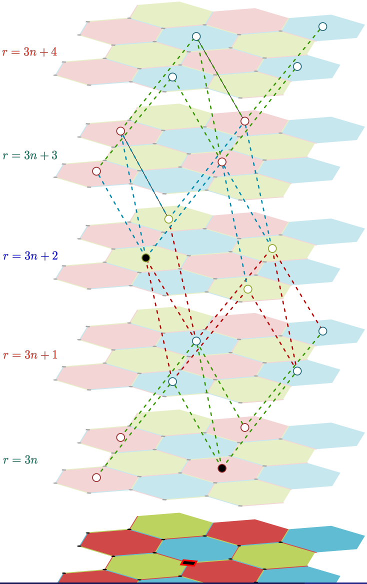
- $\mathcal{S}_{-1} = \{I\}$
- $\mathcal{S}_0 = \langle C_g \rangle$
- $\mathcal{S}_1 = \langle A_b, C_r \rangle$
- $\mathcal{S}_2 = \langle A_g, A_b, C_b \rangle$
- $\mathcal{S}_3 = \langle A_r, A_g, A_b, C_g \rangle$
- $\mathcal{S}_4 = \langle A_r, A_g, A_b, C_r \rangle$
- $\vdots$

Syndrome Inference



round (mod 3)	0	1	2
check	green	red	blue
stabilizer	$\langle A_r, A_g, A_b, C_g \rangle$	$\langle A_r, A_g, A_b, C_r \rangle$	$\langle A_r, A_g, A_b, C_b \rangle$
syndrome	A_r	A_b	A_g

Decoding

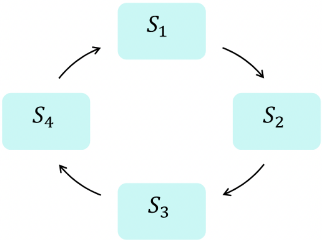


Identify flipped plaquettes A_r, A_g, A_b.
Single-qubit Z error before $r = 3n$:

- Flipped green check $\Rightarrow$ flipped red plaquette
- Flipped red check $\Rightarrow$ no flipped plaquette
- Flipped blue check $\Rightarrow$ flipped green plaquette

Multiple errors: minimum weight perfect matching on space-time lattice

Floquet Code generalizations and constructions I



Error Correction in Dynamical Codes

Xiaozhen Fu[1] and Daniel Gottesman[1,2]

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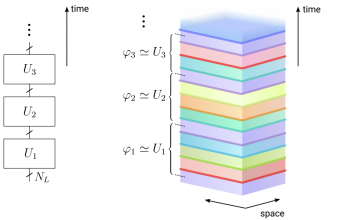
- Sequence of Check measurements: $C_0, C_1, \ldots$
- Sequence of ISGs: $\mathcal{S}_0, \mathcal{S}_1, \ldots$
- Not necessarily periodic

Floquet Code generalizations and constructions II



Quantum computation from dynamic automorphism codes

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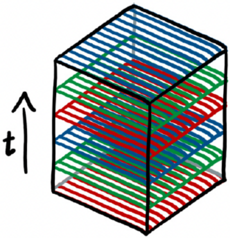
Ideas:

- Dynamic automorphism: Evolution of logical operators over time (as weight 2 or 3 check measurements are performed)
- Apply transversal unitary U on logical state $|\bar{\psi}\rangle$ using check measurements
- "Native" quantum computation on dynamical codes
- Full Clifford group transversal gates
- Transversal T-gate using adaptive check measurements

Floquet Code generalizations and constructions III



Anyon Condensation and the Color Code

Markus S. Kesselring,[1,*] Julio C. Magdalena de la Fuente,[1] Felix Thomsen,[2] Jens Eisert,[1,3] Stephen D. Bartlett,[2] and Benjamin J. Brown[2]

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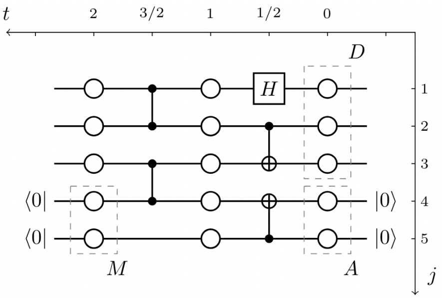
[3] *Helmholtz-Zentrum Berlin für Materialien und Energie, Berlin 14109, Germany*

Content

Spacetime Codes I



Sparse Quantum Codes from Quantum Circuits

Dave Bacon,[1] Steven T. Flammia,[2] Aram W. Harrow,[3] and Jonathan Shi[4]

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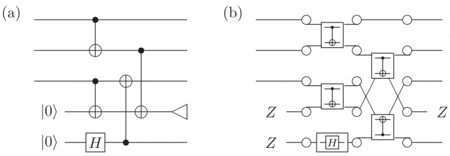
[3] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA, USA*

[4] *Department of Computer Science, Cornell University, Ithaca, NY, USA*

(Dated: February 20, 2017)



Opportunities and Challenges in Fault-Tolerant Quantum Computation

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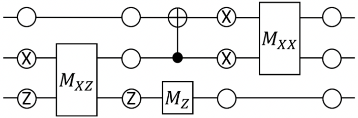
Spacetime Codes II

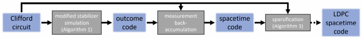


Spacetime codes of Clifford circuits

Nicolas Delfosse, Adam Paetznick

Microsoft Quantum, Redmond, Washington 98052, USA

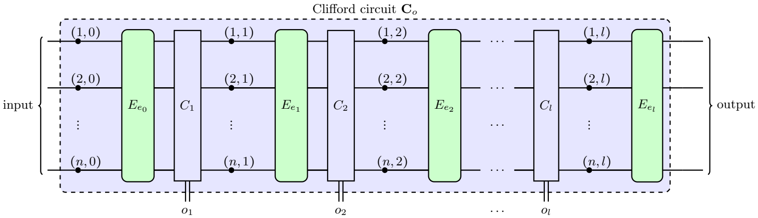
May 29, 2023



Ideas:

- Input: n-qubits with code $\mathcal{C}$
  Output: n'-qubits with code $\mathcal{C}'$
- Arbitrary circuit C with Clifford gates + Pauli measurements
- For depth-l circuit we get spacetime stabilizer group $\mathcal{S}_{st} \subseteq \mathcal{P}_{(l+1)n}$
- Noisy circuit $C(F)$ for "circuit fault" $F \in \mathcal{P}_{(l+1)n}$
- Can we correct $C(F)\rho C(F)$?

Error-correction with spacetime code I



1. Fault $F = \bigotimes_{r=0}^{l} E_{e_r}$ gives noisy outcome $o_{\text{noisy}} := o + f$ and output state ρ_{o+f} ($o = o_1, \ldots, o_l$ is the noiseless outcome)
2. $\mathcal{O}$ = all possible output $o = o_1, \ldots, o_l$ in the absence of error
3. $\mathcal{O}$ is a linear code with parity checks $\mathcal{O}^{\perp}$

Error-correction with spacetime code II

4. For measurement M_r in layer C_r, we get stabilizer group

$$\mathcal{S}_{st} = \left\langle \left\{ \prod_{r=1}^{l} [M_r^{u_r}]_r : u \in \mathcal{O}^{\perp} \right\} \right\rangle \tag{3}$$

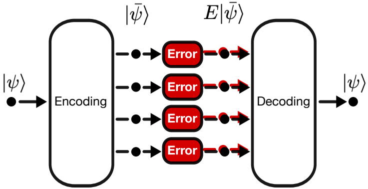
5. Obtain syndromes $\{y = o_{\text{noisy}} \cdot u : u \in \mathcal{O}^{\perp}\}$ of stabilizer $\mathcal{S}_{st}$ from measurements $M_1, \ldots, M_l$.
6. Compute the most likely fault $\hat{F} = \hat{E}_{e_l} \otimes \cdots \otimes \hat{E}_{e_0}$ given syndromes and most likely outcome flip $\hat{f} \in \{0,1\}^l$. i.e. decode the spacetime code.
7. Propagate each errors $\hat{E}_{e_1}, \ldots, \hat{E}_{e_l}$ to the last layer.

   i.e. $U_{r+1} E_{e_r} = U_{r+1} E_{e_r} U_{r+1}^{\dagger} U_{r+1}$

   $\hat{E}_{\text{out}} := \hat{E}_{e_l} U_l \hat{E}_{e_{l-1}} U_{l-1} \hat{E}_{e_{l-2}} \ldots U_1 \hat{E}_{e_0} U_1^{\dagger} \ldots U_{l-1}^{\dagger} U_l^{\dagger} \in \mathcal{P}_n$
8. Obtain circuit decoder: $o_{\text{noisy}} \mapsto (\hat{f}, \hat{E}_{\text{out}})$

Content

Beyond current QEC paradigm



Knill-Laflamme condition[1] for error $\mathcal{E}(\rho) = \sum_e E_e \rho E_e$: For all i, j, e, e',

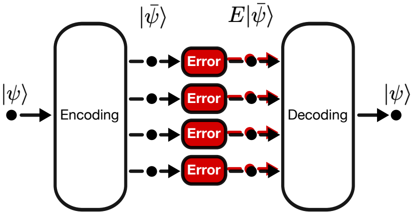
$$\langle i|_{Q_0} E_e^\dagger E_{e'} |j\rangle_{Q_0} = \lambda_{e,e'} \delta_{i,j} \ . \tag{4}$$

Questions:

Analogue of Knill-Laflamme for dynamical codes?
What is the most general thing we can do to correct errors?

[1] Knill and Laflamme, "Theory of quantum error-correcting codes".

We can do even more: Adaptivity



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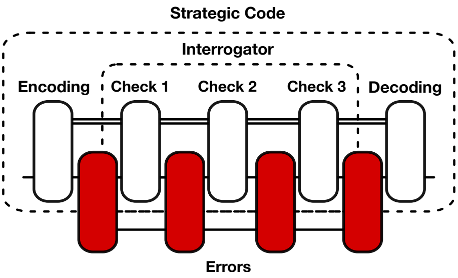

Strategic Code: A Unified Spatio-Temporal Framework for Quantum Error-Correction

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(Dated: May 29, 2024)

Error correction

A strategic code $\mathbf{C}$ can be defined by $(Q_0, \mathbf{T}, \mathcal{D})$ for initial codespace Q_0, interrogator $\mathbf{T} := \{\mathbf{T}_m\}_m$, and decoder $\mathcal{D} := \{\mathcal{D}_m\}_m$ for m being the final memory.

General error-correction

Strategic code $\mathbf{C} = (Q_0, \mathbf{T}, \mathcal{D})$ corrects error $\mathbf{E}$ if for all $|\psi\rangle \in Q_0$,

$$\mathcal{D}_m(\mathbf{E} * \mathbf{T}_m * |\psi\rangle\langle\psi|) \propto |\psi\rangle\langle\psi| \,. \tag{5}$$

But we can characterize error-correction only by initial codespace Q_0 and interrogator $\mathbf{T}$.

Linear decomposition of interrogator and error

Interrogator with final memory $m := m_1, \ldots, m_l$ is of the form

$$\mathbf{T}_m = \bigotimes_{r=1}^{l} |C_{m_r|m_{r-1}}\rangle\!\rangle\langle\!\langle C_{m_r|m_{r-1}}| \,. \tag{6}$$

Error $\mathbf{E}$ can be decomposed as

$$\mathbf{E} = \sum_e \mathbf{E}_e = \sum_e |E_e\rangle\!\rangle\langle\!\langle E_e| \,. \tag{7}$$

Universal Necessary and Sufficient Condition (Algebraic)

Suppose $\{|i\rangle_{Q_0}\}_i$ spans Q_0 and $|\psi(i,m,e)\rangle := \mathbf{E}_e * \mathbf{T}_m(|i\rangle_{Q_0})$ is final state given input $|i\rangle_{Q_0}$, final memory m and error sequence $\mathbf{E}_e$ (by decomposition $\mathbf{E} = \sum_e \mathbf{E}_e$).

Theorem 1

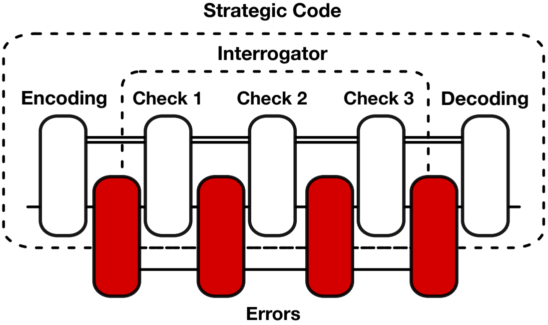
$(\mathcal{C}^{(0)}, \mathbf{T})$ corrects error $\mathbf{E} = \sum_e \mathbf{E}_e$ if and only if $\forall m, e, e', i.j$

$$\langle \psi(i,m,e) | \psi(j,m,e') \rangle = \lambda_{m,e,e'} \delta_{i,j} \ . \tag{8}$$

Static QEC condition

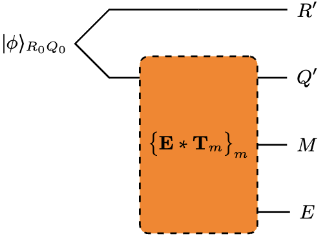
Knill-Laflamme condition[2] for error $\mathcal{E}(\rho) = \sum_e E_e \rho E_e$ as a special case of Theorem 1:

$$\langle i|_{Q_0} E_e^\dagger E_{e'} |j\rangle_{Q_0} = \lambda_{e,e'} \delta_{i,j} \ . \tag{9}$$



[2] Knill and Laflamme, "Theory of quantum error-correcting codes".

Universal Necessary and Sufficient Condition (Information-Theoretic) I



Output state:

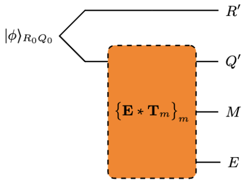
$$|\psi_m\rangle_{R'Q'ME} := \mathbf{E} * \mathbf{T}_m(|\phi\rangle_{R_0Q_0}) \tag{10}$$

Universal Necessary and Sufficient Condition (Information-Theoretic) II

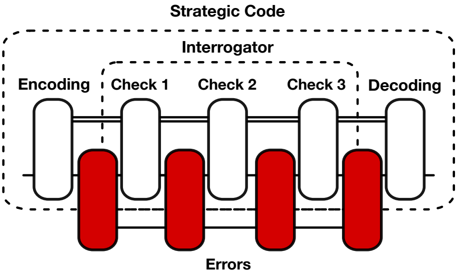
Theorem 2 (informal)

The following are equivalent:

1. A strategic code $(Q_0, \mathbf{T})$ corrects error $\mathbf{E}$.
2. $S(\rho_m^{R'ME}) = S(\rho_m^{R'}) + S(\rho_m^{ME})$ for all final memory m.
3. $I_{\rho_m^{R'ME}}(R' : ME) = 0$ for all final memory m.
4. $I_c(\mathbf{E} * \mathbf{T}_m) = S(\rho^{Q_0})$ for all final memory m.



Info-theoretic static QEC condition



Static QEC condition[3] for codespace Q_0 and error map $\mathcal{E}$

$$\begin{aligned} S(\rho^{R'E}) &= S(\rho^{R'}) + S(\rho^{E}) \\ I_{\rho^{R'E}}(R':E) &= 0 \\ I_c(\mathcal{E}) &= S(\rho^{Q_0}) \end{aligned} \tag{11}$$

[3] Nielsen et al., "Information-theoretic approach to quantum error correction and reversible measurement"; Cerf and Cleve, "Information-theoretic interpretation of quantum error-correcting codes".

Content

Outlook

1. What other advantages can we obtain from dynamical QEC?
2. Are there errors $\mathbf{E}$ which cannot be corrected without adaptivity? How to characterize them?
3. Universal fault-tolerant quantum computation using dynamical QEC?

Dynamical Quantum Error-Correction

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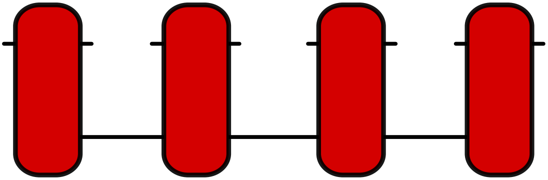
September 13, 2024

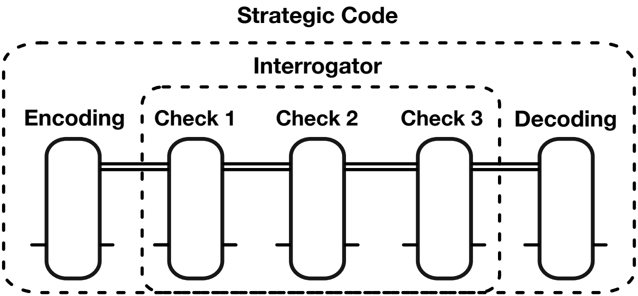
Objects in a general QEC scenario I

$\mathbf{E}$ =

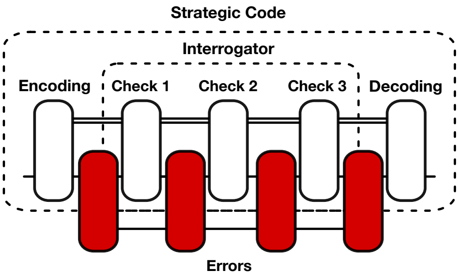


Errors

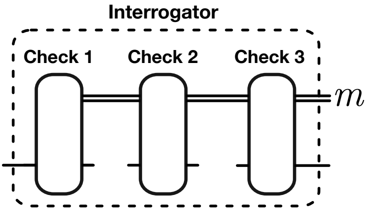
$\mathbf{C}$ =

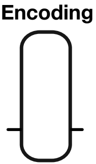


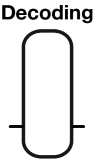
Objects in a general QEC scenario II

$\mathbf{E} * \mathbf{C} =$ 

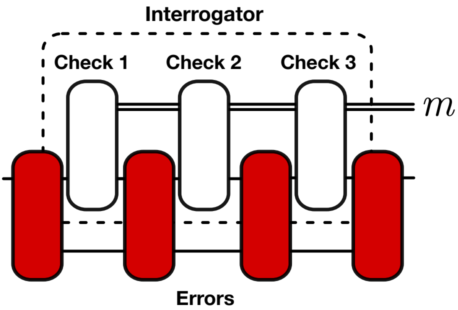
$\rightarrow$ a CP map

$\mathbf{T}_m =$ 

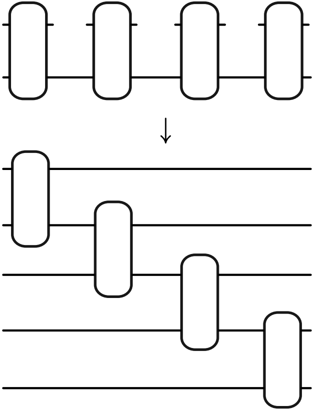
$\mathcal{C} =$ 

$\mathcal{D} =$ 

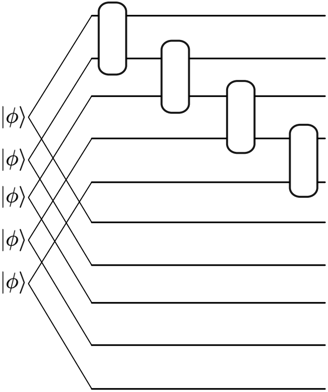
Objects in a general QEC scenario III

$\mathbf{E} * \mathbf{T}_m =$  $\to$ a CP map

Operator rerpesentation



→



$\mathbf{C}$

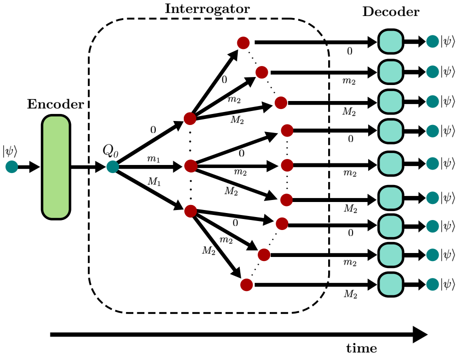
$\mathbf{C}$

positive semidefinite

“Quantum Comb” / “Quantum Strategy” / “Process Tensor”

Code trajectory and adaptivity

Adaptive code



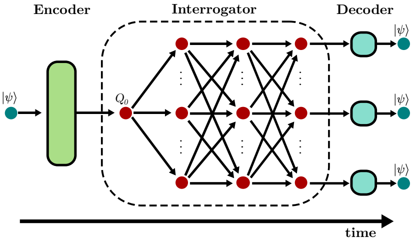
Interrogator:

$\mathbf{T}_m = \bigotimes_{r=1}^{l} \mathbf{C}_{m_r|m_{1:r-1}}$

for $\sum_{m_r} \mathbf{C}_{m_r|m_{1:r-1}} \leftrightarrow$ CPTP map

Decoder: $\{\mathcal{D}_m\}_m$

Non-adaptive code



Interrogator:

$\mathbf{T}_m = \bigotimes_{r=1}^{l} \mathbf{C}_{m_r}$

for $\sum_{m_r} \mathbf{C}_{m_r} \leftrightarrow$ CPTP map

Decoder: $\{\mathcal{D}_m\}_m$

$$m := m_{1:l} \quad , \quad m_{1:r} := m_1, \ldots, m_r$$

Quantum Combs and Link Product

Link product "$*$"

For linear operators $F \in \mathscr{H}_A \otimes \mathscr{H}_C$ and $G \in \mathscr{H}_C \otimes \mathscr{H}_B$,

$$F * G := \mathrm{Tr}_C((F^{\top_C} \otimes I_B)(I_A \otimes G)) \ . \tag{12}$$

- (Tensor product) If the operator space $\mathscr{H}_C$ has trivial dimension then $F * G = F \otimes G$.
- (Hilbert-Schmidt inner-product) If $\mathscr{H}_A, \mathscr{H}_B$ have a trivial dimension then $F * G = \mathrm{Tr}(F^\top G)$.

Vectorized Kraus representation

We can represent the channel resulting from the interaction between interrogator $\mathbf{T}_m$ and error $\mathbf{E}$ as

$$\begin{aligned} \mathbf{E} * \mathbf{T}_m &= \sum_e \sum_{o \in O_m} |E_e\rangle\rangle\langle\langle E_e| * |C_{m,o}\rangle\rangle\langle\langle C_{m,o}| \\ &= \sum_e \sum_{o \in O_m} |K_{e,m,o}\rangle\rangle\langle\langle K_{e,m,o}| \,, \end{aligned} \tag{13}$$

for

$$|C_{m,o}\rangle\rangle = |C_{o_1}\rangle\rangle \otimes \bigotimes_{r=2}^{l} |C_{m_r|m_{r-1}}\rangle\rangle \,. \tag{14}$$

General decomposition of interrogator and error

Interrogator with final memory $m := m_l$ and memory encoding function $f_r(o_1, \ldots, o_r) =: m_r$ for each r can be decomposed as

$$\mathbf{T}_m = \sum_{o \in O_m} \mathbf{T}_{m,o} = \sum_{o \in O_m} \bigotimes_{r=1}^{l} |C_{o_r|m_{r-1}}\rangle\!\rangle\langle\!\langle C_{o_r|m_{r-1}}| \tag{15}$$

where $O_m = \{o_1, \ldots, o_l : f_l(o_1, \ldots, o_l) = m\}$ is the set of trajectories ending in final memory m.
Error $\mathbf{E}$ can be decomposed as

$$\mathbf{E} = \sum_e \mathbf{E}_e = \sum_e |E_e\rangle\!\rangle\langle\!\langle E_e| \,, \tag{16}$$

where $\{\mathbf{E}_e\}_e$ is the set of error sequences. (e.g. Pauli error channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3} X\rho X + \frac{p}{3} Y\rho Y + \frac{p}{3} Z\rho Z)$.

Necessary and sufficient condition

Theorem

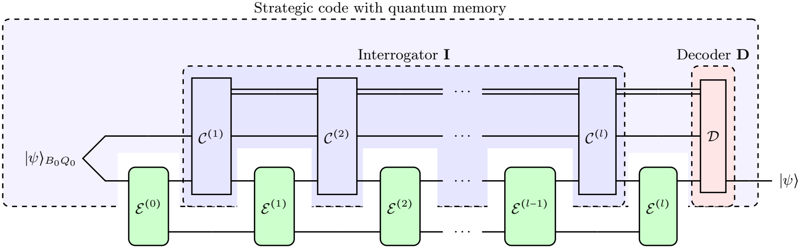
A strategic code $(Q_0, \mathbf{T})$ corrects $\mathbf{E}$ if and only if

$$\langle\!\langle E_{e'}|\Big(|C_m\rangle\!\rangle\langle\!\langle C_{m,o}| \otimes |j\rangle\langle i|\Big)|E_e\rangle\!\rangle = \lambda_{e',e,m,o}\delta_{j,i} \tag{17}$$

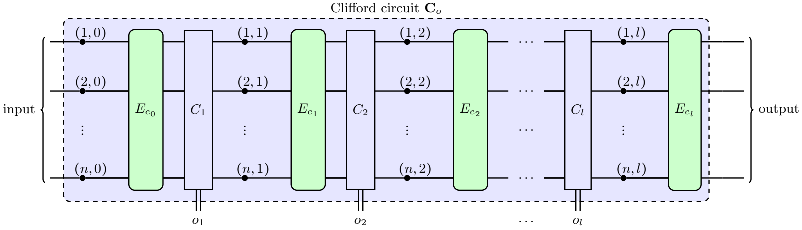
for a constant $\lambda_{e',e,m,o} \in \mathbb{C}$, and for all m, all check measurement outcome sequence $o \in O_m$, all pairs of error sequences e, e', and all i, j.

Here $|C_m\rangle\!\rangle := \sum_{o \in O_m} |C_{m,o}\rangle\!\rangle$ and $|C_{m,o}\rangle\!\rangle$ is an eigenvector of interrogator operator $\mathbf{T}_m = \sum_{o \in O_m} |C_{m,o}\rangle\!\rangle\langle\!\langle C_{m,o}|$ and $|E_e\rangle\!\rangle$ is eigenvector of the error $\mathbf{E} = \sum_e |E_e\rangle\!\rangle\langle\!\langle E_e|$. Vectors $|i\rangle, |j\rangle$ are orthonormal basis vectors of initial codespace $\mathscr{S}_{Q_0}$.

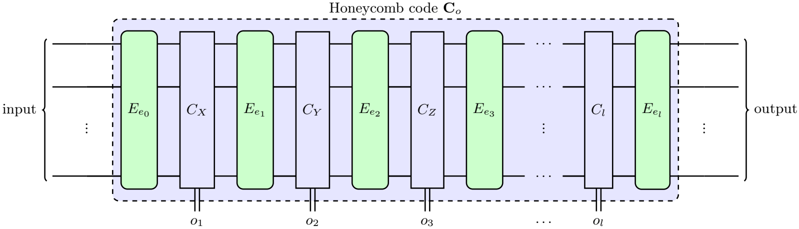
Strategic Code with Quantum Memory



Spacetime Code



Hastings-Haah Floquet Code



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[3] Michael A Nielsen et al. "Information-theoretic approach to quantum error correction and reversible measurement". In: *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 454.1969 (1998), pp. 277–304 (cit. on p. 32).