50.050/50.550 – Advanced Algorithms

January–April Term, 2026

Homework Set 1

Due by: Week 2 Friday (6 February 2026) 1pm.
Please submit your homework online via eDimension.

Question 1. [Countable sets] In Chapter 2, Example 20 (page 159) of the course textbook [Rosen], it was shown that the set $\mathbb{Q}^+$ of positive rational numbers is countable. Using a similar argument, please justify **in your own words** why the set $\mathbb{Q}$ of all rational numbers is countable. Your justification should be direct and should involve describing an explicit and direct bijection from $\mathbb{N}$ to $\mathbb{Q}$. (Hints: Use a similar idea of "following a path" of rational numbers. A diagram would be useful.) No credit would be given if you show the countability of $\mathbb{Q}$ indirectly via two or more bijections (e.g. from $\mathbb{N}$ to $\mathbb{Q}^+$ and then from $\mathbb{Q}^+$ to $\mathbb{Q}$). [5 marks]

Solution. Similarly to the argument that $\mathbb{Q}^+$ is countable, we wish to construct a sequence of all rational numbers. Observe that every rational number can be expressed as $\frac{p}{q}$, where p is an integer, and q is a positive integer. [2 marks]

We can construct a sequence of rational numbers, starting first with those rational numbers $\frac{p}{q}$ satisfying $|p| + q = 1$, then those rational numbers $\frac{p}{q}$ satisfying $|p| + q = 2$, and so on, skipping any rational number we have previously already listed in our sequence. [2 marks]

For each $k \in \mathbb{Z}^+$, when we consider those rational numbers $\frac{p}{q}$ satisfying $|p| + q = k$ (whether already listed or not), we could begin by considering the smallest possible value for p, and incrementally consider larger values for p, so that the rational numbers to be considered are: $\frac{-(k-1)}{1}, \frac{-(k-2)}{2}, \ldots, \frac{0}{k}, \ldots, \frac{k-2}{2}, \frac{k-1}{1}$.

Omitting those rational numbers already previously listed, this gives us the sequence with initial terms $\frac{0}{1}, \frac{-1}{1}, \frac{1}{1}, \frac{-2}{1}, \frac{-1}{2}, \frac{1}{2}, \frac{2}{1}, \frac{-3}{1}, \frac{-1}{3}, \ldots$ Since we have defined a sequence containing all possible rational numbers, where the terms of the sequence can be indexed by $\mathbb{N}$ (e.g. 0th term is $\frac{0}{1}$, 4th term is $\frac{-1}{2}$), we have shown that the set of rational numbers is countable. [1 mark]

Question 2. [Logic puzzle] Several new food stalls are scheduled to open at the SUTD canteen this term. Each stall is using a different advertising method to increase sales. Match each stall to its owner and advertising method, and determine its grand opening date.

- The set of opening dates is:
  {11th February, 14th February, 17th February, 20th February, 23rd February}.
- The set of stalls is:
  {vegetarian stall, chicken rice stall, Muslim food stall, burger stall, Asian Delights stall}.
- The set of stall owners is:
  {Kumar, Amanda, Thomas, Claire, Bee Bee See}.
- The set of advertising methods is:
  {posters, email ads, brochures, Instagram posts, YouTube ad videos}.

The following eight propositions are true:

1. The vegetarian stall will not be opening on 20th February, and isn't using brochures.
2. The stall owned by Kumar will open 9 days after the stall owned by Claire.
3. Of the two stalls using Instagram posts and YouTube ad videos, one is the Muslim food stall, and the other will open on 11th February.
4. The stall using email ads will open 3 days after the stall owned by Thomas.
5. The stall using brochures will open 6 days after the burger stall.
6. The stall owned by Amanda is either the stall using Instagram posts or the stall opening on 14th February, but not both.
7. The chicken rice stall will not be opening on 20th February.
8. The stall owned by Claire will open 6 days before the burger stall.

Please solve this logic puzzle, and fill in the following table with the correct entries. Please justify your answer with **as much details as possible**, and please **show all intermediate logical reasoning steps**. Please define all notation/terminology that you introduce (if any). You may explain your solution in your own words. [8 marks]

Opening Date	Stall	Stall Owner	Advertising Method
11th February			
14st February			
17th February			
20th February			
23rd February			

Solution. For this question, 7 marks shall be allocated to the intermediate reasoning steps, while 1 mark shall be allocated to the correct final answer (in table format). For the last mark, no partial 0.5 mark will be awarded to a final table that is not completely correct. For the first 7 marks, we are looking out for correctness (in your logical reasoning) and clarity.
There could be multiple possible ways to solve this logic puzzle. Here is one possible way: Consider the following sequence of intermediate consequences.

(i) By statement (5) and statement (8), the burger stall will open on 17th February, which implies the stall using brochures will open on 23 February, and the stall owned by Claire will open on 11th February. [1 mark]

(ii) Combined with statement (2), the stall owned by Kumar will open on 20th February. [0.5 mark]

(iii) By statement (6), the stall owned by Amanda either will open on 14th February, or is using Instagram posts. In the latter case, that stall must be the burger stall that will open on 17 February. However, this would contradict statement (3). [1 mark]

(iv) So the stall owned by Amanda will open on 14th February, and isn't using Instagram posts. [0.5 mark]

(v) Combined with statement (4), the stall owned by Thomas will open on 17th February, and the stall using email ads will open on 20th February, which then implies the stall owned by Bee Bee See will open on 23rd February. [1 mark]

(vi) By statement (6) and statement (3), the stall using Instagram posts will open on 11th February, and is not the Muslim food stall. [1 mark]

(vii) Hence, we infer that the Muslim food stall is using YouTube ad videos, and will open on 14th February, which in turn implies the burger stall is using posters and will open on 17th February. [1 mark]

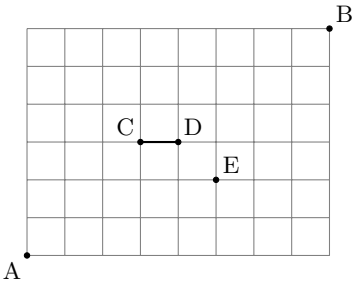
(viii) By statement (1), the vegetarian stall must be opening on 11th February. [0.5 mark]

(ix) Combined with statement (7), the chicken rice stall will open on 23rd February, and the Asian Delights stall will open on 20th February. [0.5 mark]

The correct details for each stall are thus given in the table below. [1 mark]

Opening Date	Stall	Stall Owner	Advertising Method
11th February	Vegetarian	Claire	Instagram posts
14st February	Muslim food	Amanda	YouTube ad videos
17th February	Burger	Thomas	Posters
20th February	Asian Delights	Kumar	Email ads
23rd February	Chicken rice	Bee Bee See	Brochures

Question 3. [Counting] This is an extension of Example 5 from Week 1's cohort class. There is a larger rectangular grid given below, with five points A, B, C, D, E marked out. An ant wants to crawl from point A to point B via the line segments of this rectangular grid given below.



(i) How many possible shortest paths are there from point A to point B that go through point E? Please justify your answer. [1.5 marks]

(ii) How many possible shortest paths are there from point A to point B that go along line segment CD (i.e. this line segment CD is included as part of the shortest path)? Please justify your answer. [1.5 marks]

Solution. (i) We represent each shortest path from A to B that passes through E as an ordered pair of a shortest path from A to E and a shortest path from E to B. By similar logic to Example 5, there are $\binom{7}{2}$ shortest paths from A to E and $\binom{7}{4}$ shortest paths from E to B, for a total of $\binom{7}{2}\binom{7}{4} = 735$ shortest paths. [1 mark reasoning, 0.5 mark answer]

(ii) We represent each shortest path from A to B that passes along line segment CD as an ordered triple of a shortest path from A to C, the path C to D (along line segment CD), and a shortest path from D to B. By similar logic to Example 5, there are $\binom{6}{3}$ shortest paths from A to C, one path CD, and $\binom{7}{3}$ shortest paths from D to B, for a total of $\binom{6}{3}\binom{7}{3} = 700$ shortest paths. [1 mark reasoning, 0.5 mark answer]

Question 4. [Counting] There are 14 chairs arranged sequentially in a row from left to right. There are a total of 14 students, comprising 10 undergraduate students and 4 post-graduate students, who have to be seated at these 14 chairs. Assume that no two students are allowed to be seated in the same chair. The names of the 4 post-graduate students are Amy, Bob, Claire, and Dan. Suppose that the 4 post-graduate students are required to be seated in the order of Amy, Bob, Claire, and Dan, from left to right. (There may be undergraduate students seated between any two post-graduate students, or to the left of the left-most post-graduate student Amy, or to the right of the right-most post-graduate student Dan.) Also, we are required to have at least 4 undergraduate students between Amy and Bob, and at most 1 undergraduate student between Claire and Dan. What is the total possible number of seating arrangements satisfying all specified requirements? Please justify your answer with details, and please be as clear as possible. [4 marks]

Solution. First, let us ignore the chairs, and think about the 5 spaces, which we denote by $S_1, \ldots, S_5$, and defined as follows: to the left of Amy, in between Amy and Bob, in between Bob and Claire, in between Claire and Dan, to the right of Dan. These spaces can be visualized as follows:

$$\underline{\quad S_1 \quad} \text{ Amy } \underline{\quad S_2 \quad} \text{ Bob } \underline{\quad S_3 \quad} \text{ Claire } \underline{\quad S_4 \quad} \text{ Dan } \underline{\quad S_5 \quad}$$

Next, for each $1 \leq i \leq 5$, let n_i denote the number of undergraduate students in space S_i. We are given that there must be at least 4 undergraduate students in S_2, and at most 1 undergraduate student in S_4. Hence, $n_2 \geq 4$, and $n_4 \leq 1$.
Consider the task of determining the total number of possible 5-tuples $(n_1, n_2, n_3, n_4, n_5)$.
[1 mark for task set-up in terms of 5 spaces]

Note that $\sum_{i=1}^{5} n_i = 10$, which equals the total number of undergraduate students. Since n_4 equals either 0 or 1, we can split this task into two subtasks:

- Subtask 1: Determine total number of 5-tuples of the form $(n_1, n_2, n_3, 0, n_5)$, where $n_2 \geq 4$, $n_1 + n_2 + n_3 + n_5 = 10$, and $n_1, n_3, n_5 \in \mathbb{N}$.
- Subtask 2: Determine total number of 5-tuples of the form $(n_1, n_2, n_3, 1, n_5)$, where $n_2 \geq 4$, $n_1 + n_2 + n_3 + n_5 = 9$, and $n_1, n_3, n_5 \in \mathbb{N}$.

If we introduce the new variable $n_2' := n_2 - 4$, then Subtask 1 is equivalent to determining the total number of 4-tuples of the form (n_1, n_2', n_3, n_5), whose entries n_1, n_2', n_3, n_5 take on values in $\mathbb{N}$, and whose sum equals 6. Using the stars and bars method (from Week 1's cohort class), where we have a total of 6 stars and 3 bars (to split a row of 6 stars into four segments), we infer that the total number of such 4-tuples equals $\binom{6+3}{3} = \binom{9}{3} = 84$. [0.5 mark]

Analogously, using the same new variable $n_2' := n_2 - 4$, Subtask 2 is equivalent to determining the total number of 4-tuples of the form (n_1, n_2', n_3, n_5), whose entries n_1, n_2', n_3, n_5 take on values in $\mathbb{N}$, and whose sum equals 5. Again, using the stars and bars method (from Week 1's cohort class), where we have a total of 5 stars and 3 bars (to split a row of 5 stars into four segments), we infer that the total number of such 4-tuples equals $\binom{5+3}{3} = \binom{8}{3} = 56$. [0.5 mark]

Hence, the total number of possible 5-tuples $(n_1, n_2, n_3, n_4, n_5)$ equals $84 + 56 = 140$. [0.5 mark]

Now, from left to right, if we ignore the post-graduate students, then there will be 10 undergraduate students, arranged in some order. There are 10! possible permutations of these 10 undergraduate students. [0.5 mark]

Therefore, the total possible number of seating arrangements equals $(10!) \times 140 = 508032000$.
[1 mark for correct final answer. The expression $(10!) \times 140$ gets full credit.]

Question 5. (**For post-graduate students only**) Mathematical notation is very important when presenting a new idea (approach/algorithm/framework etc.) in a research paper. The notation has to be clear, precise, and most importantly, correct. Unfortunately, there are numerous examples of publications where the mathematical notation is not completely correct. There are "comparatively less serious" inaccuracies, such as incorrectly defining a dataset to be "$\mathcal{D} = \{x_i, y_i\}|_{i=1}^{n}$", when it should be defined as "$\mathcal{D} = \{(x_i, y_i)\}_{i=1}^{n}$". It is not rare to see a dataset being defined as a set, but subsequently used as a probability distribution. (A set is not a probability distribution!) It is sadly very very common to see publications where some of the notation is undefined, or where the notation is inconsistent (e.g. a function f having different

number of inputs in different parts of the paper). Occasionally, there are "even more serious" inaccuracies, such as expressions of the form $\frac{\mathbf{v}}{\mathbf{w}}$, where $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^d$ (which does not make mathematical sense), or expressions of the form $\mathbf{a} + \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are vectors of different lengths, or "illegal" expressions such as "$x \in K$", but K is a number. (A number K is not a set, and so does not contain any elements.)

Find one published journal/conference/workshop paper of your choice, and identify **three** instances of incorrect use of mathematical notation in that single paper. The three instances could be the same kind of error on different lines of the paper. Please clearly indicate the name of the publication as well as the name of the journal/conference/workshop where that paper has been published in. You may indicate the required information about the paper as a reference (e.g. in MLA or APA format). Please also provide screenshots of the relevant portion of the paper (e.g. using a snipping tool[1]), with the incorrect use of mathematical notation clearly indicated. For each incorrect instance, either explain why it is incorrect, or explain what the corrected notation should be. For the convenience of the grader, please include a pdf copy of the paper when you submit HW1 via eDimension. [5 marks]

Solution. Grading rubric:

- 0.5 mark for indication of paper name and journal/conference/workshop name.
- For each of the three instances of incorrect use of mathematical notation:
  - 0.5 mark for valid identification of incorrect use of mathematical notation.
  - 1 mark for either a valid explanation of why the identified mathematical notation is incorrect, or a valid proposed correction of the mathematical notation.

[1] For Windows users, the Windows snipping tool is useful.