

50.050/50.550 – Advanced Algorithms

January–April Term, 2026

Homework Set 1

Due by: Week 2 Friday (6 February 2026) 1pm.

Please submit your homework online via eDimension.

Question 1. [Countable sets] In Chapter 2, Example 20 (page 159) of the course textbook [Rosen], it was shown that the set \mathbb{Q}^+ of positive rational numbers is countable. Using a similar argument, please justify **in your own words** why the set \mathbb{Q} of all rational numbers is countable. Your justification should be direct and should involve describing an explicit and direct bijection from \mathbb{N} to \mathbb{Q} . (Hints: Use a similar idea of “following a path” of rational numbers. A diagram would be useful.) No credit would be given if you show the countability of \mathbb{Q} indirectly via two or more bijections (e.g. from \mathbb{N} to \mathbb{Q}^+ and then from \mathbb{Q}^+ to \mathbb{Q}). [5 marks]

Question 2. [Logic puzzle] Several new food stalls are scheduled to open at the SUTD canteen this term. Each stall is using a different advertising method to increase sales. Match each stall to its owner and advertising method, and determine its grand opening date.

- The set of opening dates is:
 {11th February, 14th February, 17th February, 20th February, 23rd February}.
- The set of stalls is:
 {vegetarian stall, chicken rice stall, Muslim food stall, burger stall, Asian Delights stall}.
- The set of stall owners is:
 {Kumar, Amanda, Thomas, Claire, Bee Bee See}.
- The set of advertising methods is:
 {posters, email ads, brochures, Instagram posts, YouTube ad videos}.

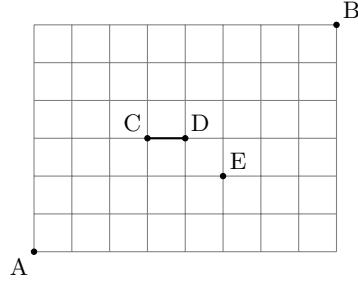
The following eight propositions are true:

1. The vegetarian stall will not be opening on 20th February, and isn't using brochures.
2. The stall owned by Kumar will open 9 days after the stall owned by Claire.
3. Of the two stalls using Instagram posts and YouTube ad videos, one is the Muslim food stall, and the other will open on 11th February.
4. The stall using email ads will open 3 days after the stall owned by Thomas.
5. The stall using brochures will open 6 days after the burger stall.
6. The stall owned by Amanda is either the stall using Instagram posts or the stall opening on 14th February.
7. The chicken rice stall will not be opening on 20th February.
8. The stall owned by Claire will open 6 days before the burger stall.

Please solve this logic puzzle, and fill in the following table with the correct entries. Please justify your answer with **as much details as possible**, and please **show all intermediate logical reasoning steps**. Please define all notation/terminology that you introduce (if any). You may explain your solution in your own words. [8 marks]

Opening Date	Stall	Stall Owner	Advertising Method
11th February			
14st February			
17th February			
20th February			
23rd February			

Question 3. [Counting] This is an extension of Example 5 from Week 1's cohort class. There is a larger rectangular grid given below, with five points A, B, C, D, E marked out. An ant wants to crawl from point A to point B via the line segments of this rectangular grid given below.



- (i) How many possible shortest paths are there from point A to point B that go through point E? Please justify your answer. [1.5 marks]
- (ii) How many possible shortest paths are there from point A to point B that go along line segment CD (i.e. this line segment CD is included as part of the shortest path)? Please justify your answer. [1.5 marks]

Question 4. [Counting] There are 14 chairs arranged sequentially in a row from left to right. There are a total of 14 students, comprising 10 undergraduate students and 4 post-graduate students, who have to be seated at these 14 chairs. Assume that no two students are allowed to be seated in the same chair. The names of the 4 post-graduate students are Amy, Bob, Claire, and Dan. Suppose that the 4 post-graduate students are required to be seated in the order of Amy, Bob, Claire, and Dan, from left to right. (There may be undergraduate students seated between any two post-graduate students, or to the left of the left-most post-graduate student Amy, or to the right of the right-most post-graduate student Dan.) Also, we are required to have at least 4 undergraduate students between Amy and Bob, and at most 1 undergraduate student between Claire and Dan. What is the total possible number of seating arrangements satisfying all specified requirements? Please justify your answer with details, and please be as clear as possible. [4 marks]

Question 5. (For post-graduate students only) Mathematical notation is very important when presenting a new idea (approach/algorithm/framework etc.) in a research paper. The notation has to be clear, precise, and most importantly, correct. Unfortunately, there are numerous examples of publications where the mathematical notation is not completely correct. There are “comparatively less serious” inaccuracies, such as incorrectly defining a dataset to be “ $\mathcal{D} = \{x_i, y_i\}_{i=1}^n$ ”, when it should be defined as “ $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^n$ ”. It is not rare to see a dataset being defined as a set, but subsequently used as a probability distribution. (A set is not a probability distribution!) It is sadly very common to see publications where some of the notation is undefined, or where the notation is inconsistent (e.g. a function f having different number of inputs in different parts of the paper). Occasionally, there are “even more serious” inaccuracies, such as expressions of the form $\frac{\mathbf{v}}{\mathbf{w}}$, where \mathbf{v} and \mathbf{w} are vectors in \mathbb{R}^d (which does not make mathematical sense), or expressions of the form $\mathbf{a} + \mathbf{b}$, where \mathbf{a} and \mathbf{b} are vectors of different lengths, or “illegal” expressions such as “ $x \in K$ ”, but K is a number. (A number K is not a set, and so does not contain any elements.)

Find one published journal/conference/workshop paper of your choice, and identify **three** instances of incorrect use of mathematical notation in that single paper. The three instances could be the same kind of error on different lines of the paper. Please clearly indicate the name

of the publication as well as the name of the journal/conference/workshop where that paper has been published in. You may indicate the required information about the paper as a reference (e.g. in MLA or APA format). Please also provide screenshots of the relevant portion of the paper (e.g. using a snipping tool¹), with the incorrect use of mathematical notation clearly indicated. For each incorrect instance, either explain why it is incorrect, or explain what the corrected notation should be. For the convenience of the grader, please include a pdf copy of the paper when you submit HW1 via eDimension. [5 marks]

¹For Windows users, the Windows snipping tool is useful.