

# Physics of Semiconductor Devices

## Lecture 14-16

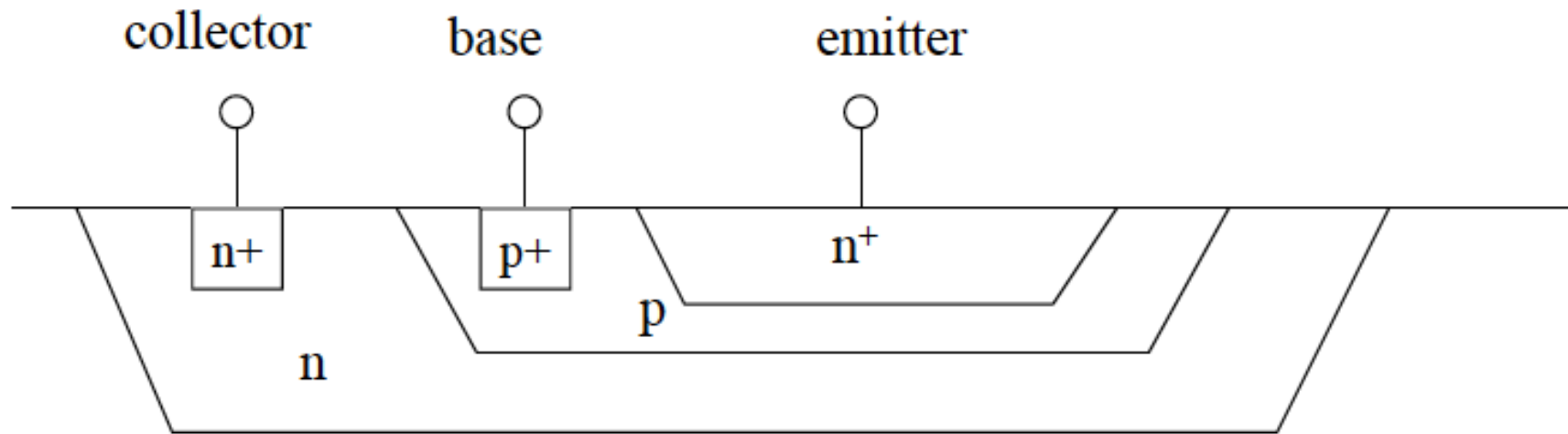
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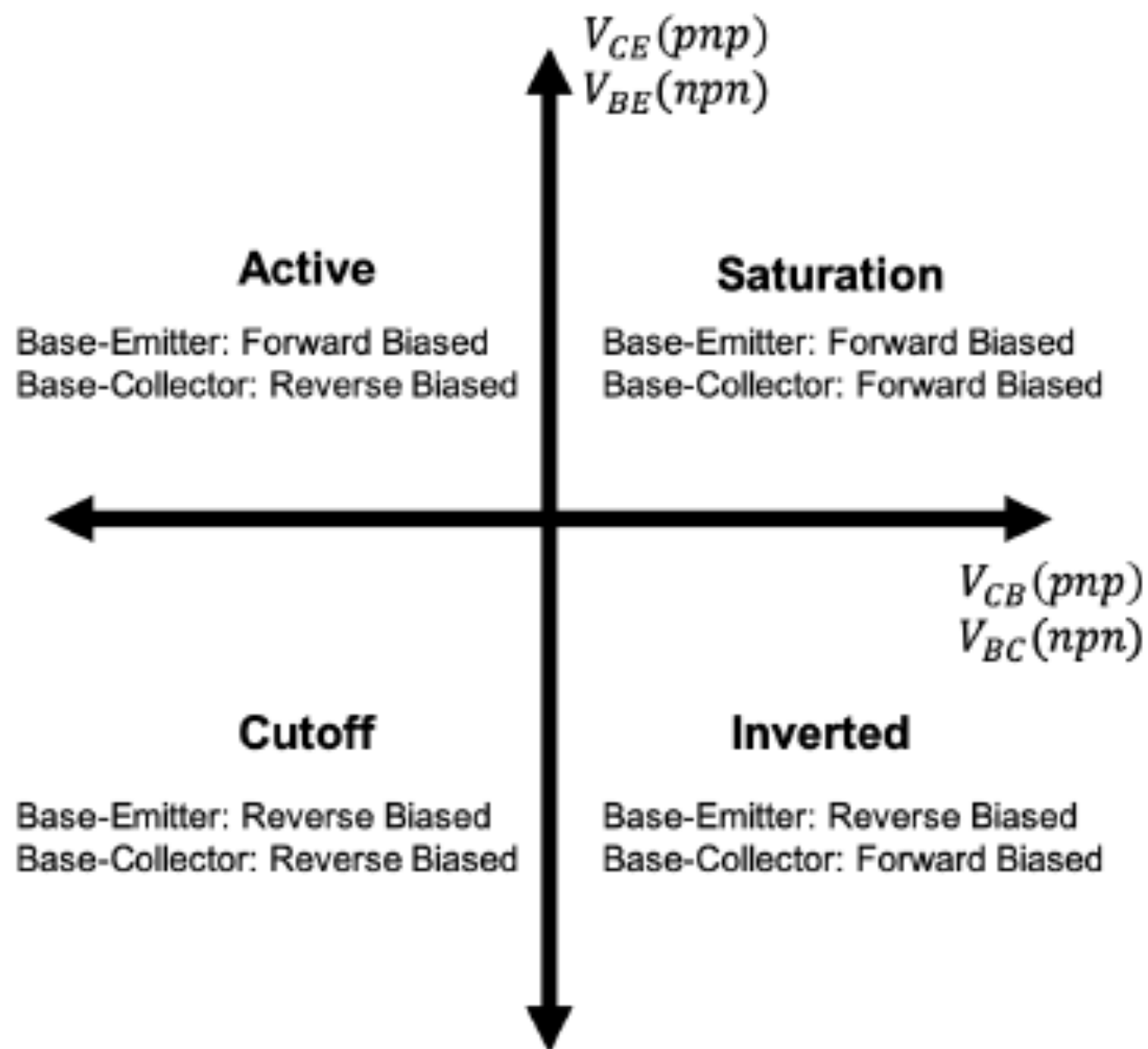
# bipolar transistors

nnp transistor

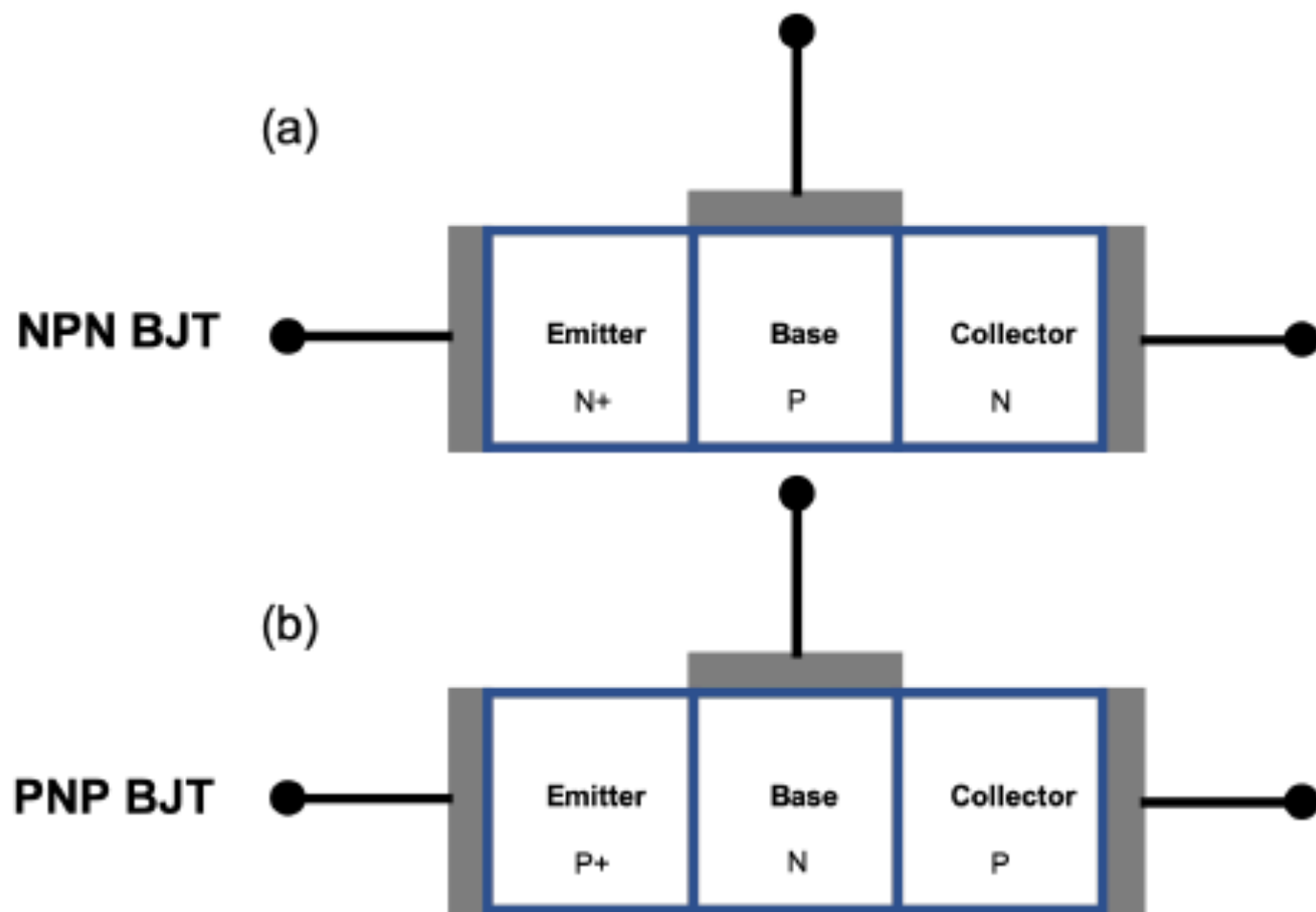


lightly doped p substrate

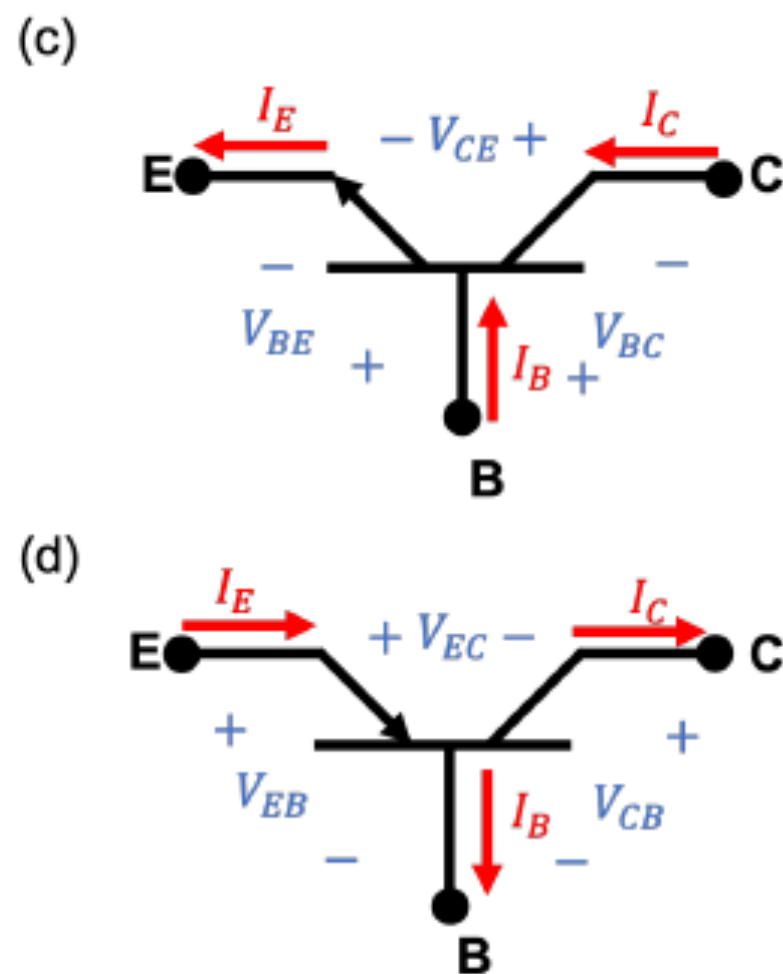
Used in front-end high-frequency receivers (mobile telephones).

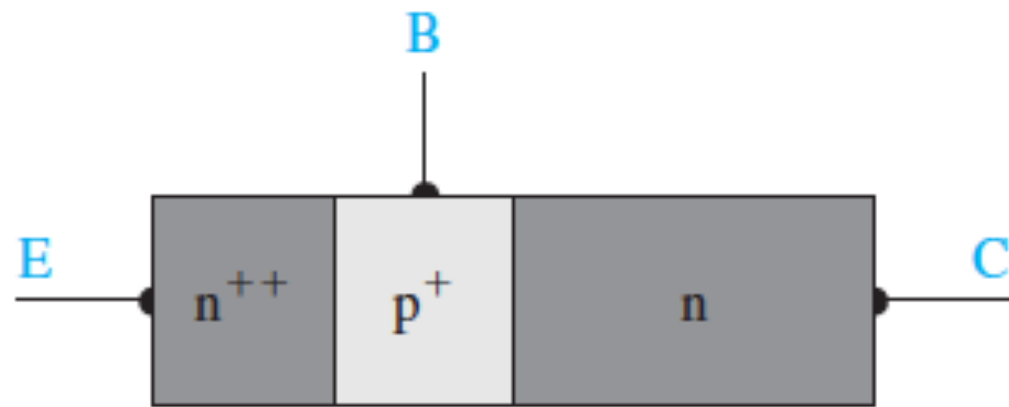


## Schematic Representation

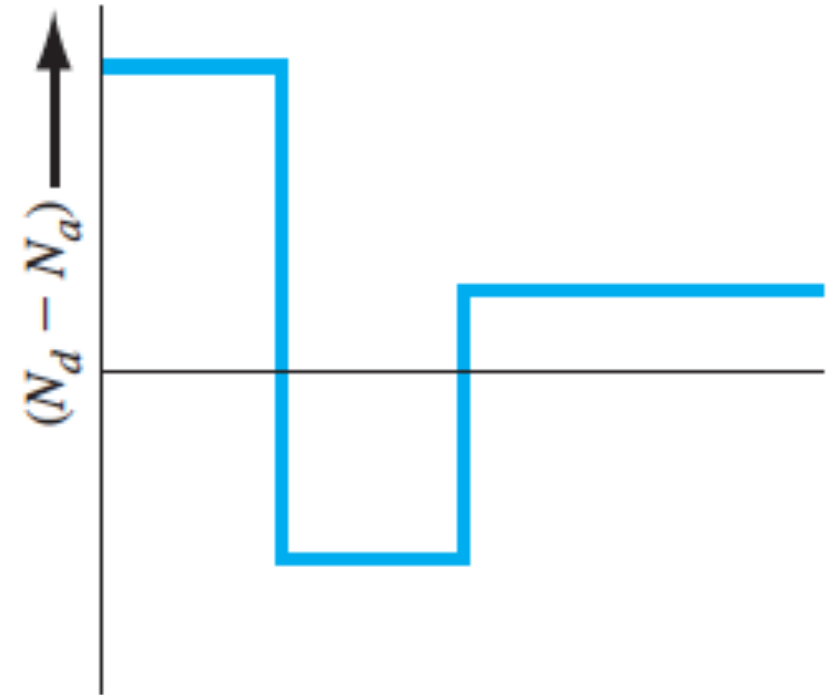


## Circuit Symbol





(a)



(b)

**Figure 12.3** | Idealized doping profile of a uniformly doped npn bipolar transistor.

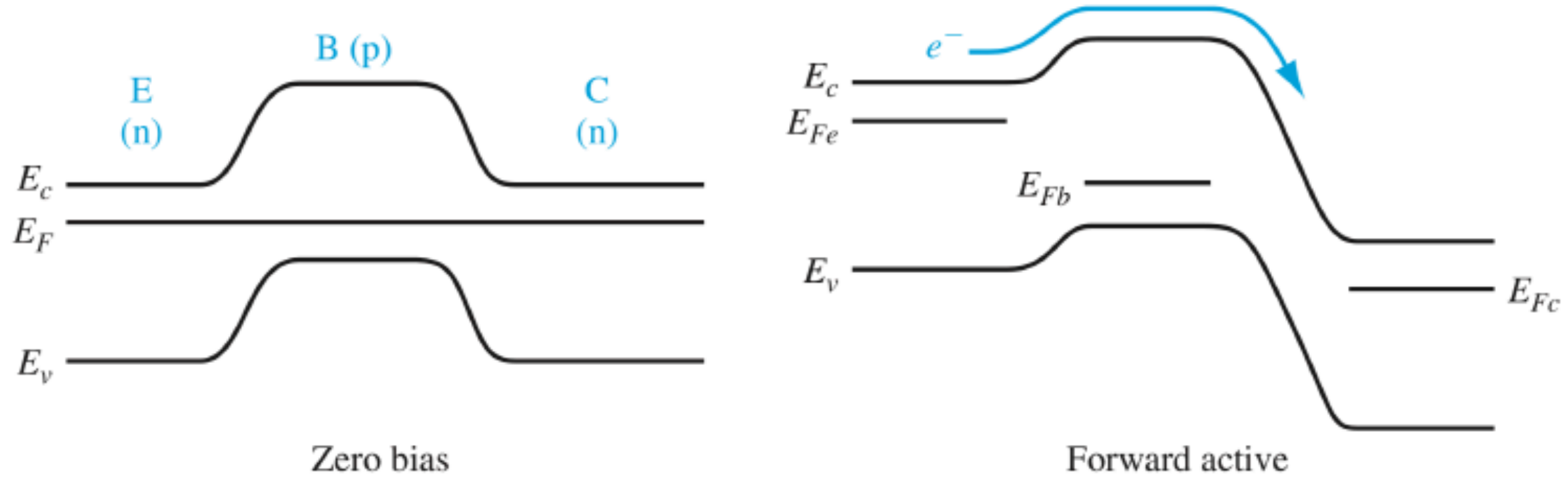
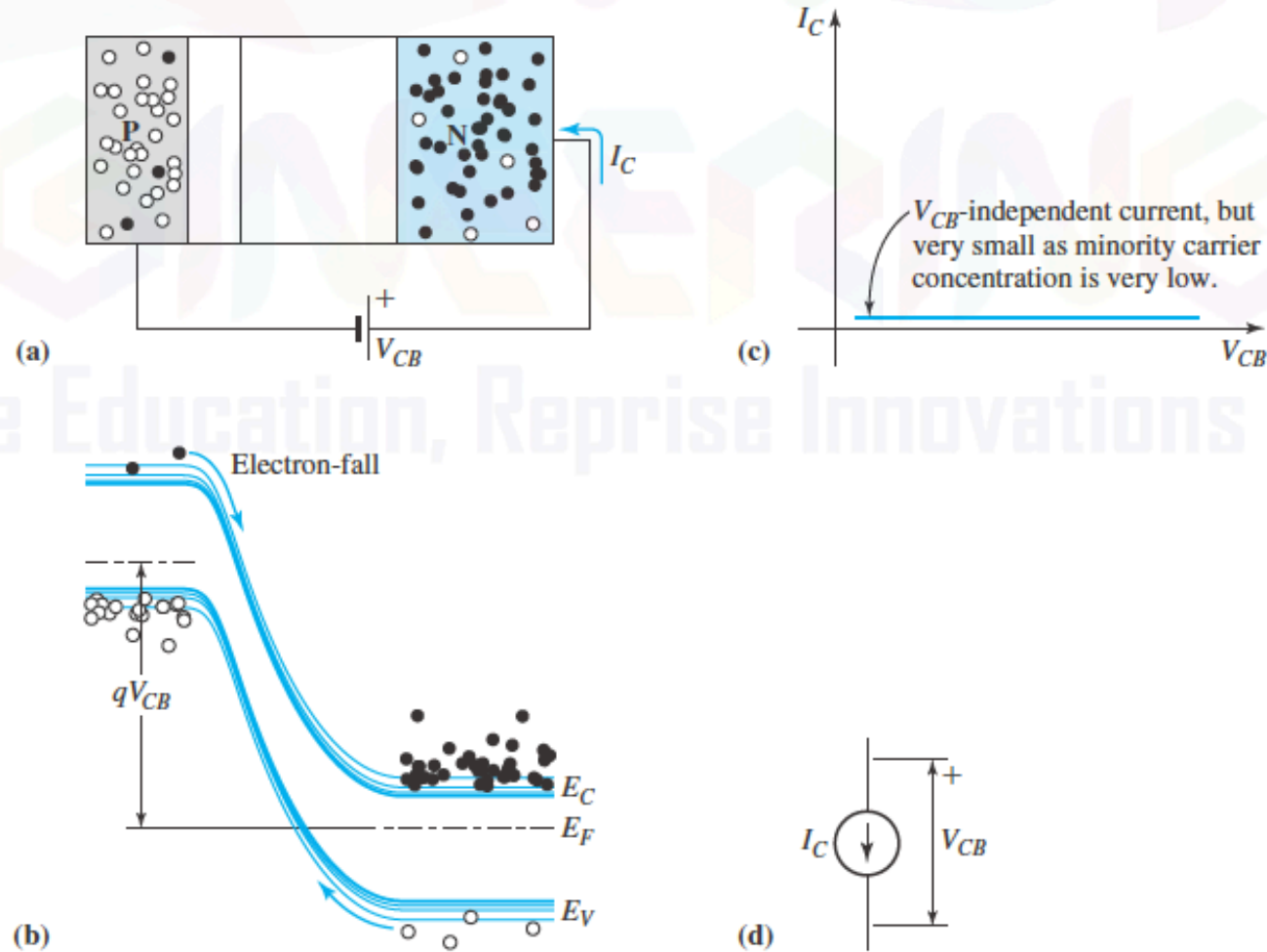
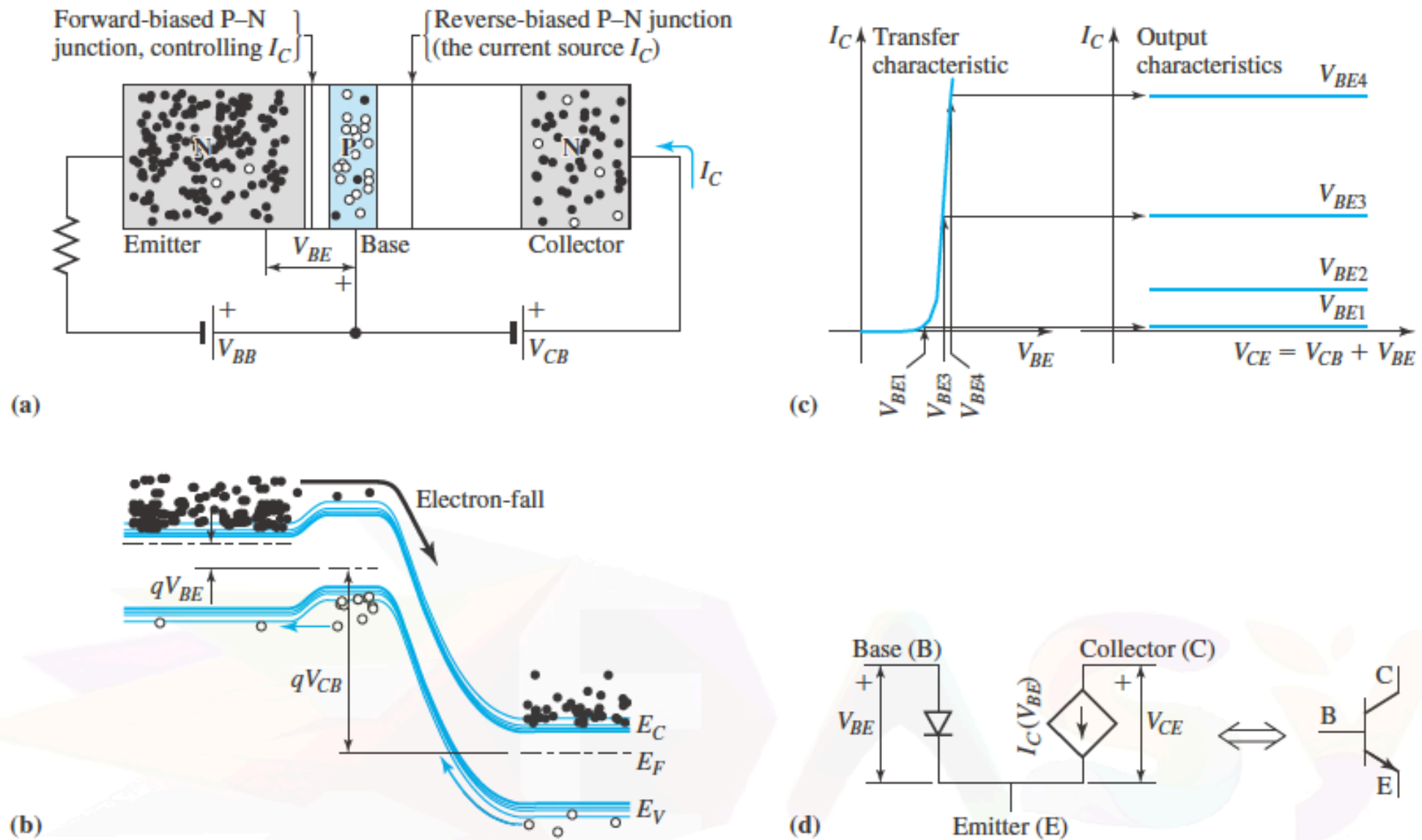


Figure 5: (left) Equilibrium and (right) forward active band diagrams of an npn BJT

# BJT as a Voltage-Controlled Current Source

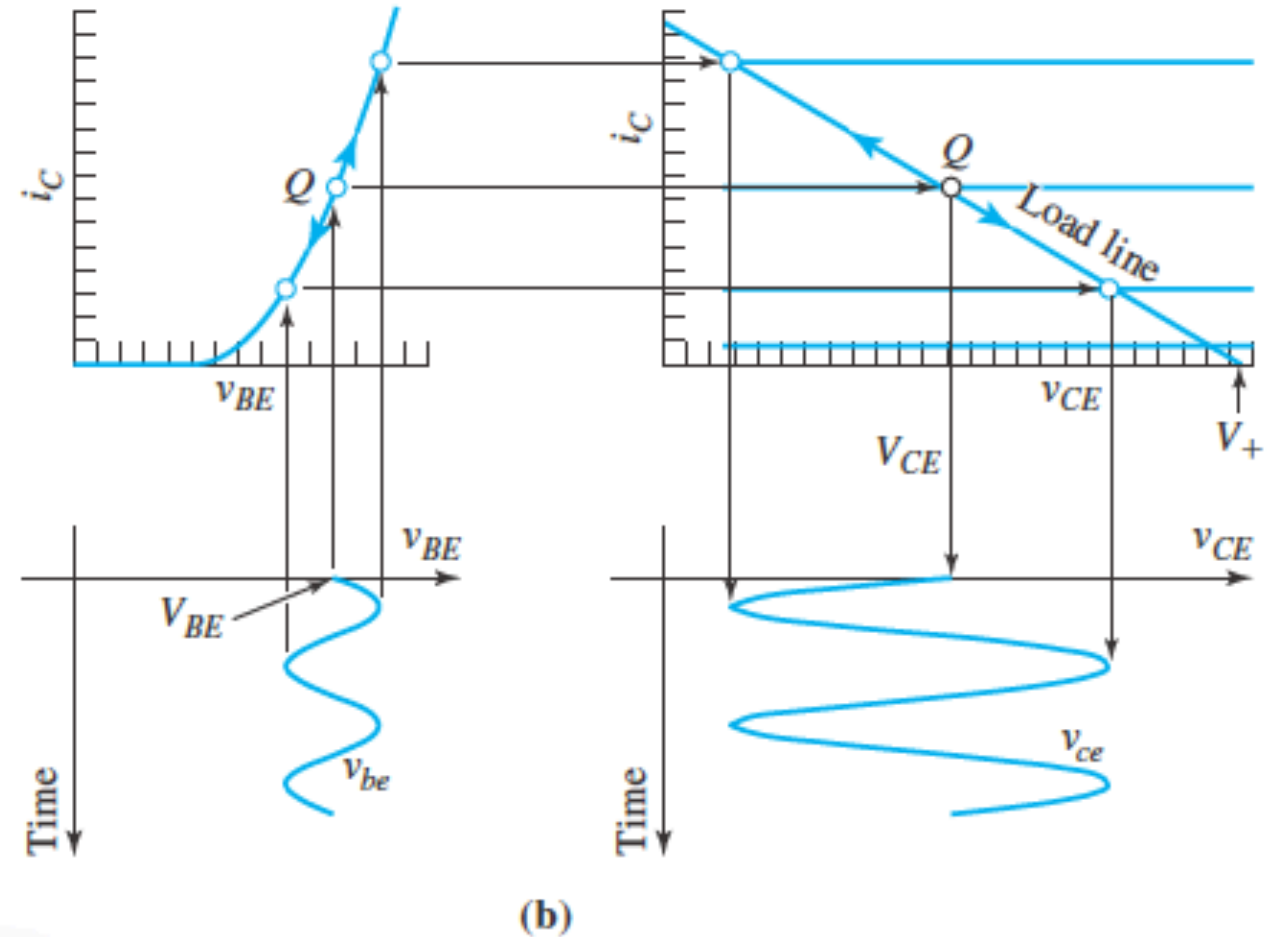
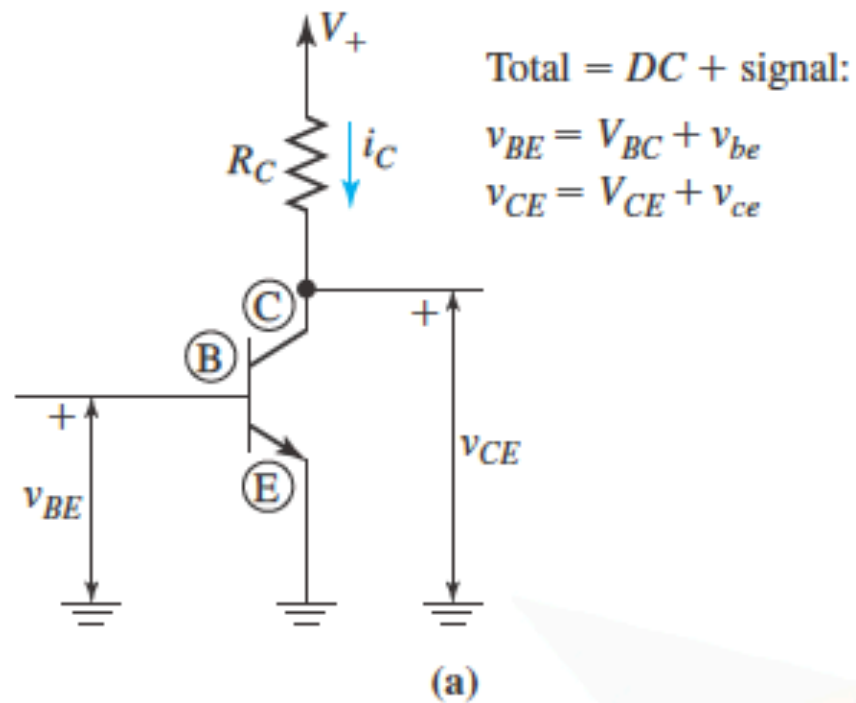


**Figure 9.1** Reverse-biased P-N junction as a current source. (a) Cross section. (b) Energy-band diagram. (c)  $I$ - $V$  characteristic. (d) Current-source symbol.

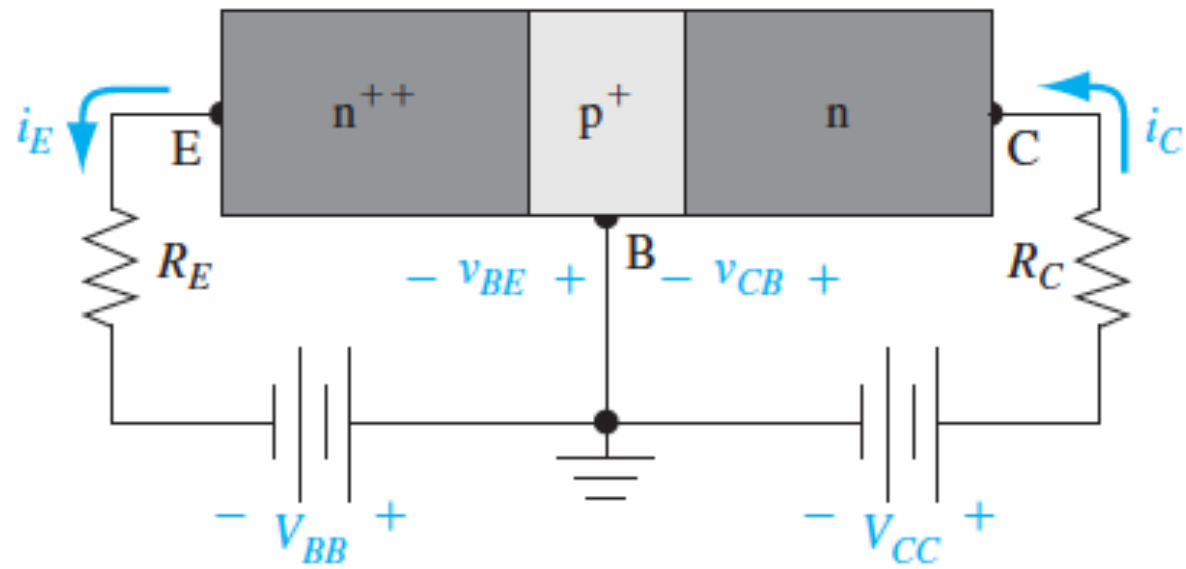


**Figure 9.2** Summary of NPN BJT operation as a voltage-controlled current source. (a) Cross section showing the three regions, their names, the two junctions, and the biasing arrangement. (b) Energy-band model. (c) Main current-voltage characteristics. (d) An equivalent circuit (left) and the symbol (right).

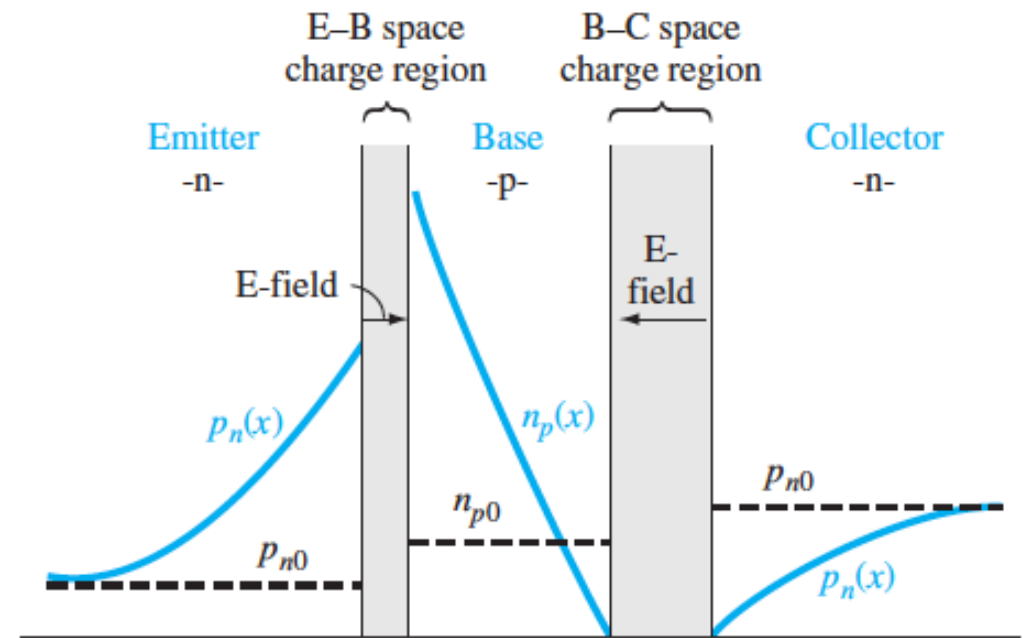




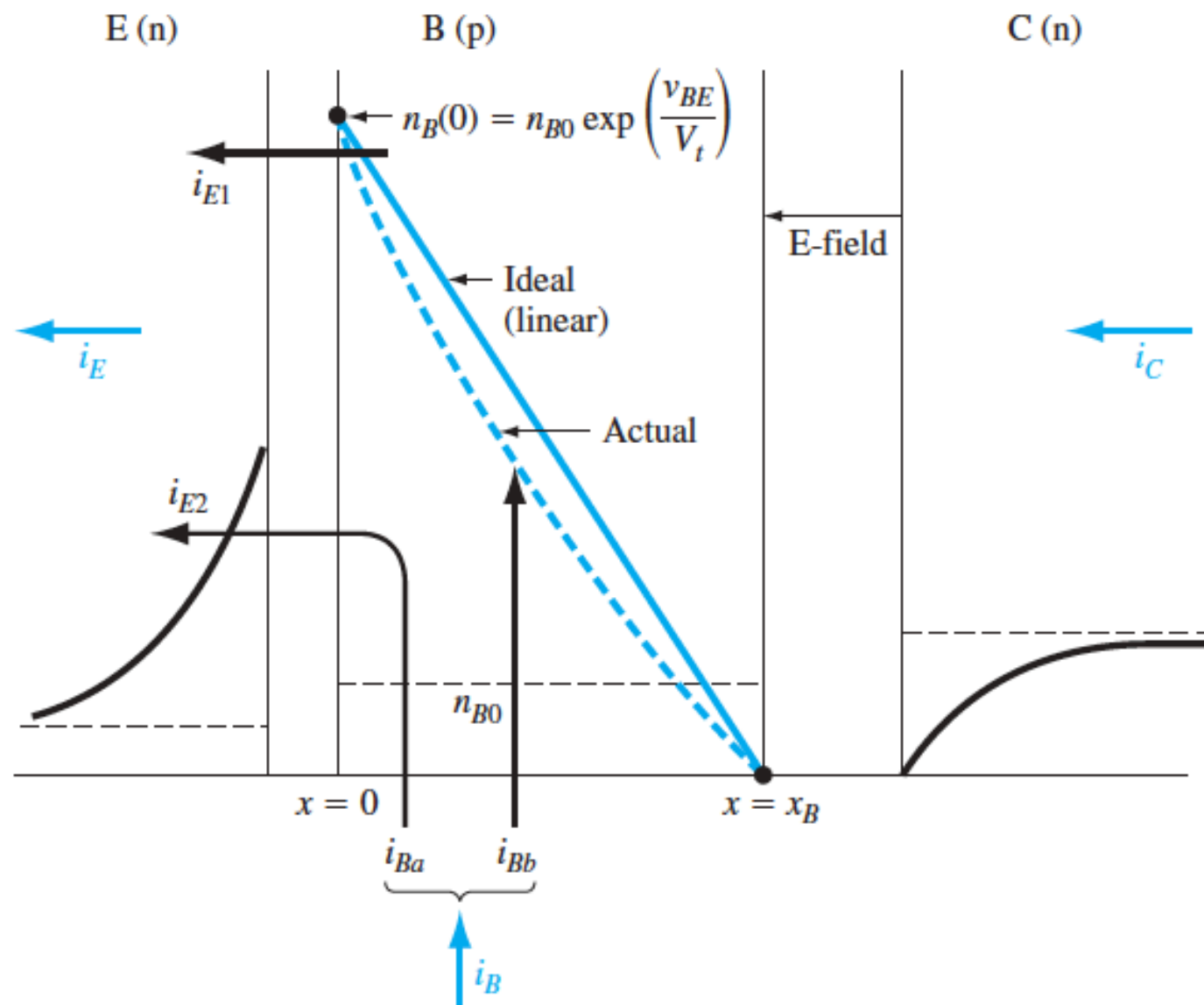
**Figure 9.3** Principles of voltage amplification by a voltage-controlled current source. (a) A BJT is connected to a DC power supply through a loading resistor to create the principal amplifier circuit. (b) Graphic analysis of the amplifier circuit.



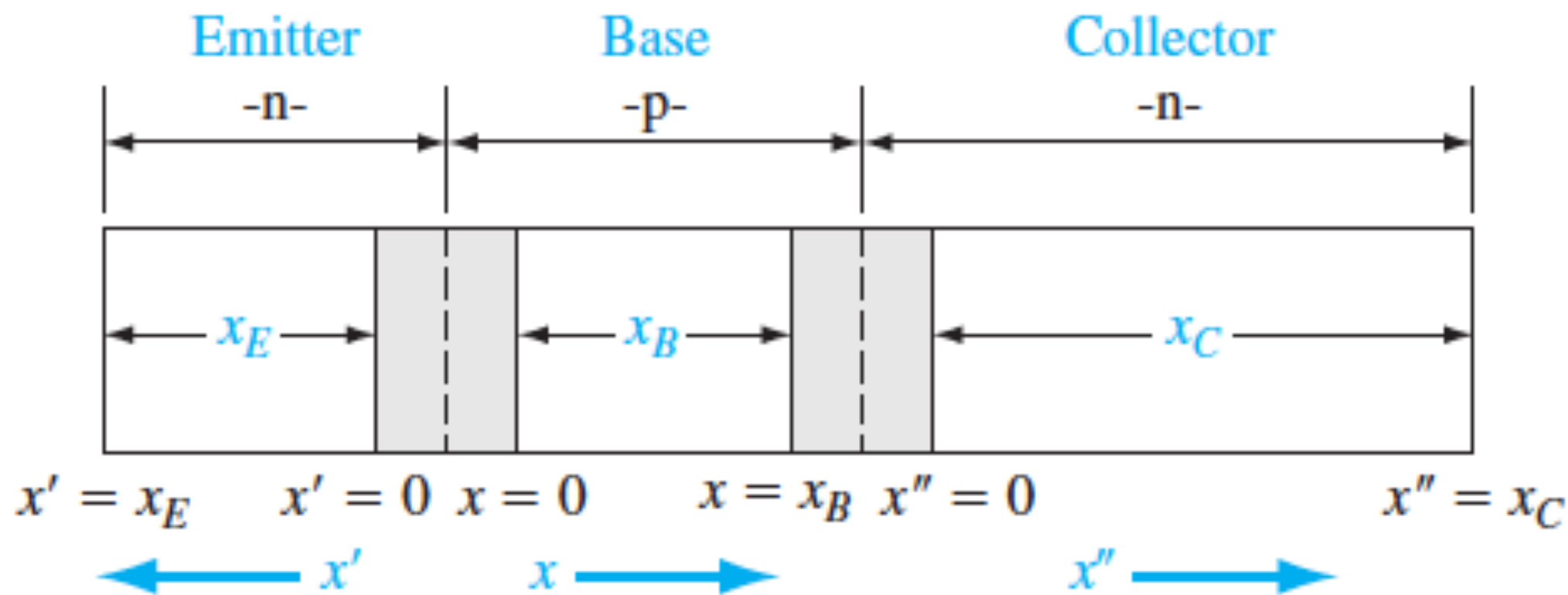
(a)



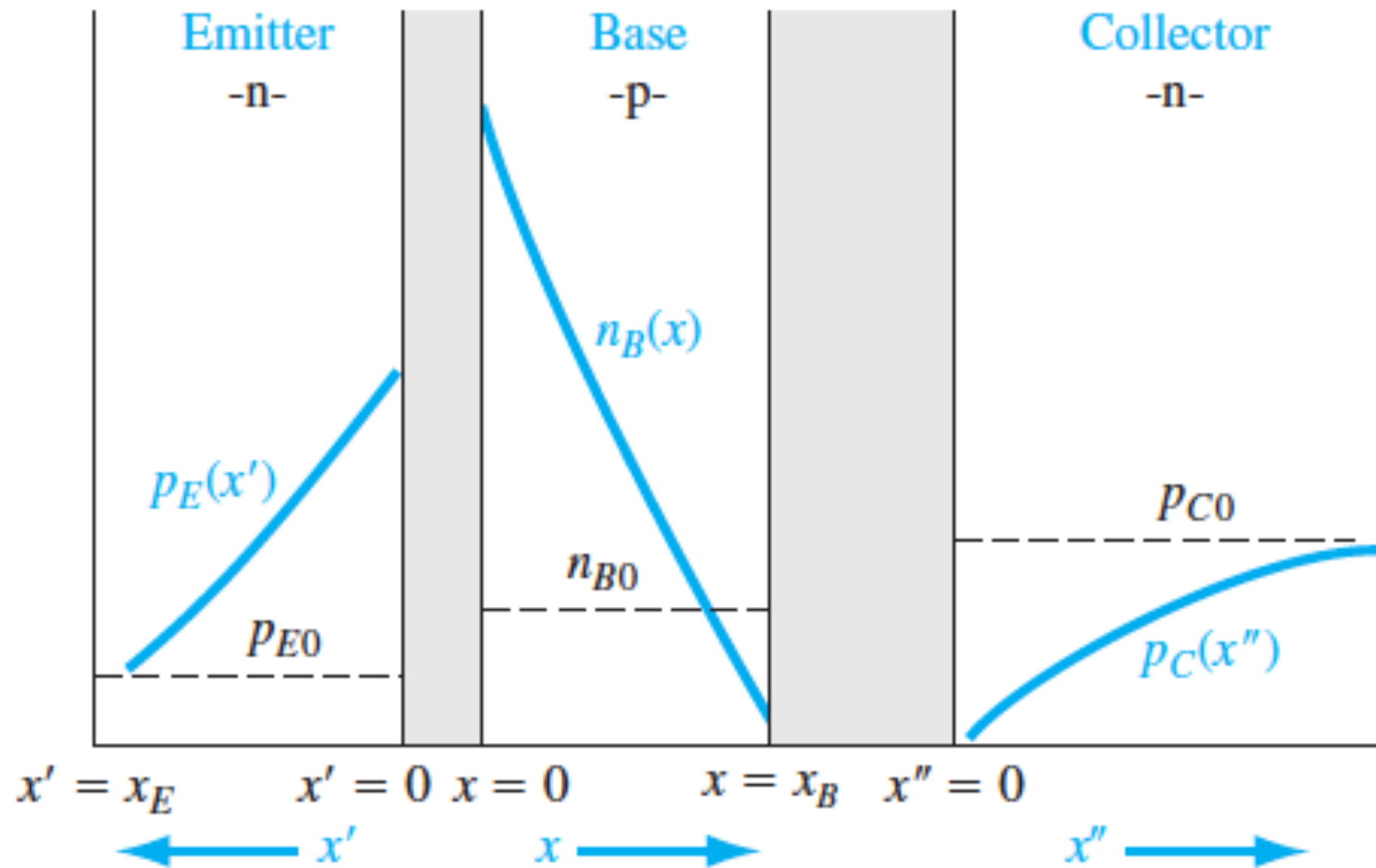
(b)



**Figure 12.6** | Minority carrier distributions and basic currents in a forward-biased npn bipolar transistor.



**Figure 12.13** | Geometry of the npn bipolar transistor used to calculate the minority carrier distribution.



**Figure 12.14** | Minority carrier distribution in an npn bipolar transistor operating in the forward-active mode.

**Base Region** The steady-state excess minority carrier electron concentration is found from the ambipolar transport equation, which we discussed in detail in Chapter 6. For a zero electric field in the neutral base region, the ambipolar transport equation in steady state reduces to

$$D_B \frac{\partial^2(\delta n_B(x))}{\partial x^2} - \frac{\delta n_B(x)}{\tau_{B0}} = 0 \quad (12.9)$$

where  $\delta n_B$  is the excess minority carrier electron concentration, and  $D_B$  and  $\tau_{B0}$  are the minority carrier diffusion coefficient and lifetime in the base region, respectively. The excess electron concentration is defined as

$$\delta n_B(x) = n_B(x) - n_{B0} \quad (12.10)$$

The general solution to Equation (12.9) can be written as

$$\delta n_B(x) = A \exp\left(\frac{+x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right) \quad (12.11)$$

The excess minority carrier electron concentrations at the two boundaries become

$$\delta n_B(x = 0) \equiv \delta n_B(0) = A + B \quad (12.12a)$$

and

$$\delta n_B(x = x_B) \equiv \delta n_B(x_B) = A \exp\left(\frac{+x_B}{L_B}\right) + B \exp\left(\frac{-x_B}{L_B}\right) \quad (12.12b)$$

The B–E junction is forward biased, so the boundary condition at  $x = 0$  is

$$\delta n_B(0) = n_B(x = 0) - n_{B0} = n_{B0} \left[ \exp\left(\frac{eV_{BE}}{kT}\right) - 1 \right] \quad (12.13a)$$

The B–C junction is reverse biased, so the second boundary condition at  $x = x_B$  is

$$\delta n_B(x_B) = n_B(x = x_B) - n_{B0} = 0 - n_{B0} = -n_{B0} \quad (12.13b)$$

From the boundary conditions given by Equations (12.13a) and (12.13b), the coefficients  $A$  and  $B$  from Equations (12.12a) and (12.12b) can be determined. The results are

$$A = \frac{-n_{B0} - n_{B0} \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] \exp \left( \frac{-x_B}{L_B} \right)}{2 \sinh \left( \frac{x_B}{L_B} \right)} \quad (12.14a)$$

and

$$B = \frac{n_{B0} \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] \exp \left( \frac{x_B}{L_B} \right) + n_{B0}}{2 \sinh \left( \frac{x_B}{L_B} \right)} \quad (12.14b)$$



Then, substituting Equations (12.14a) and (12.14b) into Equation (12.9), we can write the excess minority carrier electron concentration in the base region as

$$\delta n_B(x) = \frac{n_{B0} \left\{ \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] \sinh \left( \frac{x_B - x}{L_B} \right) - \sinh \left( \frac{x}{L_B} \right) \right\}}{\sinh \left( \frac{x_B}{L_B} \right)} \quad (12.15a)$$

Using the approximation that  $\sinh(x) \approx x$  for  $x \ll 1$ , the excess electron concentration in the base is given by

$$\delta n_B(x) \approx \frac{n_{B0}}{x_B} \left\{ \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] (x_B - x) - x \right\} \quad (12.15b)$$

**Emitter Region** Consider, now, the minority carrier hole concentration in the emitter. The steady-state excess hole concentration is determined from the equation

$$D_E \frac{\partial^2 [\delta p_E(x')]}{\partial x'^2} - \frac{\delta p_E(x')}{\tau_{E0}} = 0 \quad (12.16)$$

where  $D_E$  and  $\tau_{E0}$  are the minority carrier diffusion coefficient and minority carrier lifetime, respectively, in the emitter. The excess hole concentration is given by

$$\delta p_E(x') = p_E(x') - p_{E0} \quad (12.17)$$

The general solution to Equation (12.16) can be written as

$$\delta p_E(x') = C \exp\left(\frac{+x'}{L_E}\right) + D \exp\left(\frac{-x'}{L_E}\right) \quad (12.18)$$

where  $L_E = \sqrt{D_E \tau_{E0}}$ . If we assume the neutral emitter length  $x_E$  is not necessarily long compared to  $L_E$ , then both exponential terms in Equation (12.18) must be retained.

The excess minority carrier hole concentrations at the two boundaries are

$$\delta p_E(x' = 0) \equiv \delta p_E(0) = C + D \quad (12.19a)$$

and

$$\delta p_E(x' = x_E) \equiv \delta p_E(x_E) = C \exp\left(\frac{x_E}{L_E}\right) + D \exp\left(\frac{-x_E}{L_E}\right) \quad (12.19b)$$

Again, the B–E junction is forward biased, so

$$\delta p_E(0) = p_E(x' = 0) - p_{E0} = p_{E0} \left[ \exp\left(\frac{eV_{BE}}{kT}\right) - 1 \right] \quad (12.20a)$$

An infinite surface recombination velocity at  $x' = x_E$  implies that

$$\delta p_E(x_E) = 0 \quad (12.20b)$$

Solving for  $C$  and  $D$  using Equations (12.19) and (12.20) yields the excess minority carrier hole concentration in Equation (12.18):

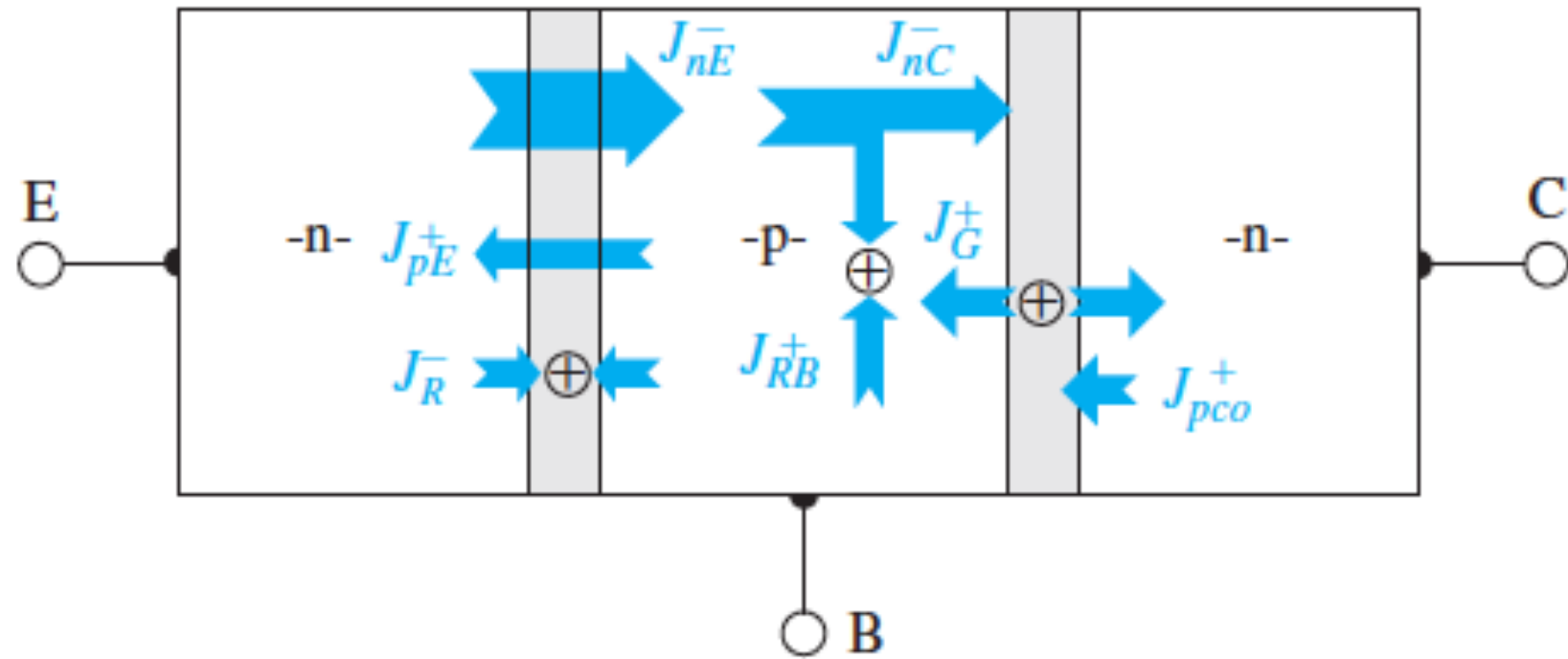
$$\delta p_E(x') = \frac{p_{E0} \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] \sinh \left( \frac{x_E - x'}{L_E} \right)}{\sinh \left( \frac{x_E}{L_E} \right)} \quad (12.21a)$$

*This excess concentration will also vary approximately linearly with distance if  $x_E$  is small. We find*

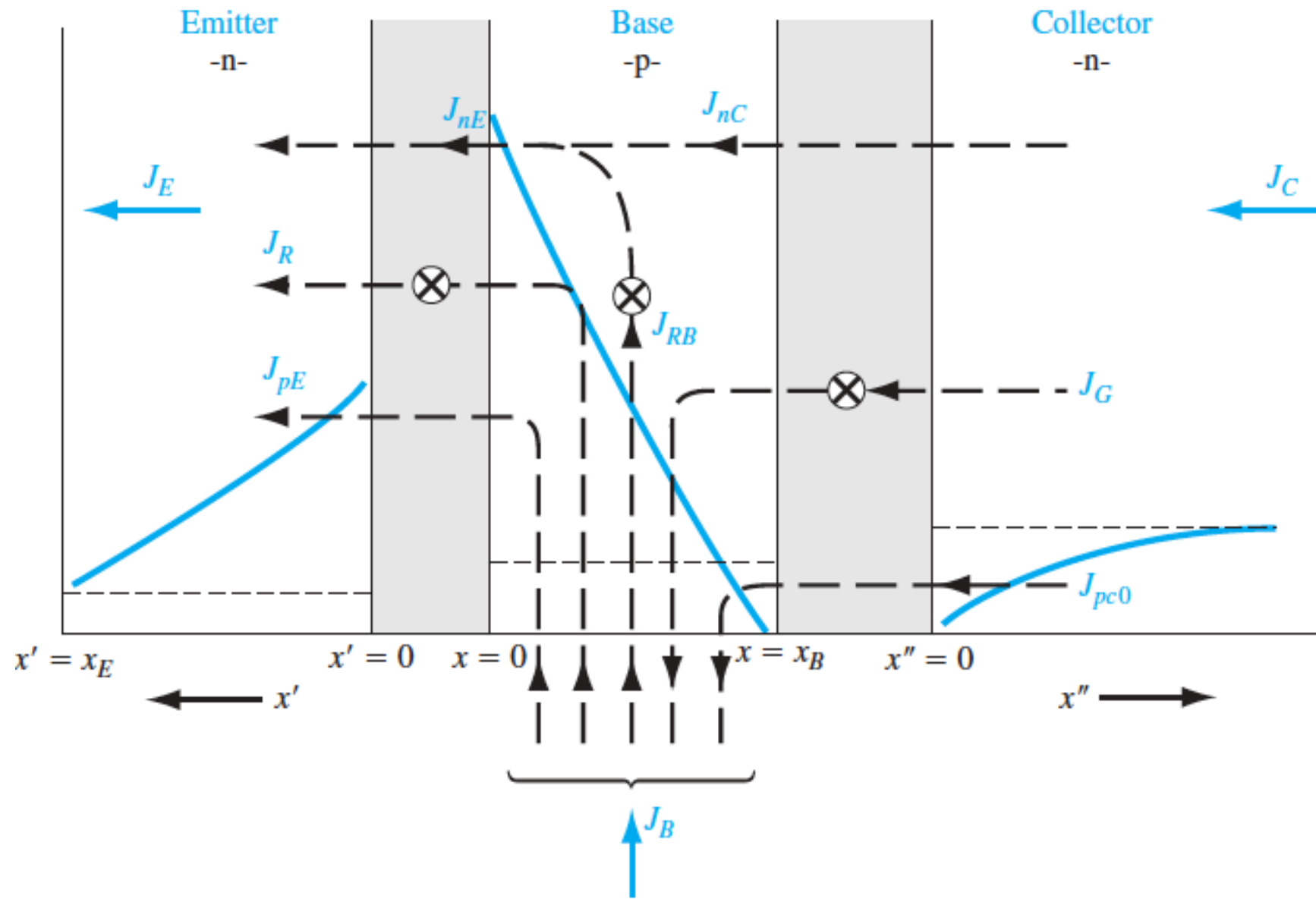
$$\delta p_E(x') \approx \frac{p_{E0}}{x_E} \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] (x_E - x') \quad (12.21b)$$

If  $x_E$  is comparable to  $L_E$ , then  $\delta p_E(x')$  shows an exponential dependence on  $x_E$ .

## 12.3.1 Current Gain—Contributing Factors



**Figure 12.18** | Particle current density or flux components in an npn bipolar transistor operating in the forward-active mode.



**Figure 12.19** | Current density components in an npn bipolar transistor operating in the forward-active mode.

The dc common-base current gain is defined as

$$\alpha_0 = \frac{I_C}{I_E} \quad (12.27)$$

If we assume that the active cross-sectional area is the same for the collector and emitter, then we can write the current gain in terms of the current densities, or

$$\alpha_0 = \frac{J_C}{J_E} = \frac{J_{nC} + J_G + J_{pc0}}{J_{nE} + J_R + J_{pE}} \quad (12.28)$$

We are primarily interested in determining how the collector current will change with a change in emitter current. The small-signal, or sinusoidal, common-base current gain is defined as

$$\alpha = \frac{\partial J_C}{\partial J_E} = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}} \quad (12.29)$$



We can rewrite Equation (12.29) in the form

$$\alpha = \left( \frac{J_{nE}}{J_{nE} + J_{pE}} \right) \left( \frac{J_{nC}}{J_{nE}} \right) \left( \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \right) \quad (12.30a)$$

or

$$\alpha = \gamma \alpha_T \delta \quad (12.30b)$$

The factors in Equation (12.30b) are defined as:

$$\gamma = \left( \frac{J_{nE}}{J_{nE} + J_{pE}} \right) \equiv \text{emitter injection efficiency factor} \quad (12.31a)$$

$$\alpha_T = \left( \frac{J_{nC}}{J_{nE}} \right) \equiv \text{base transport factor} \quad (12.31b)$$

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \equiv \text{recombination factor} \quad (12.31c)$$



**Emitter Injection Efficiency Factor** Consider, initially, the emitter injection efficiency factor. We have from Equation (12.31a)

$$\gamma = \left( \frac{J_{nE}}{J_{nE} + J_{pE}} \right) = \frac{1}{\left( 1 + \frac{J_{pE}}{J_{nE}} \right)} \quad (12.32)$$

We derived the minority carrier distribution functions for the forward-active mode in Section 12.2.1. Noting that  $J_{nE}$ , as defined in Figure 12.19, is in the negative  $x$  direction, we can write the current densities as

$$J_{pE} = -eD_E \left. \frac{d[\delta p_E(x')]}{dx'} \right|_{x'=0} \quad (12.33a)$$

and

$$J_{nE} = (-)eD_B \left. \frac{d[\delta n_B(x)]}{dx} \right|_{x=0} \quad (12.33b)$$

where  $\delta p_E(x')$  and  $\delta n_B(x)$  are given by Equations (12.21) and (12.15), respectively.

Taking the appropriate derivatives of  $\delta p_E(x')$  and  $\delta n_B(x)$ , we obtain

$$J_{pE} = \frac{eD_E p_{E0}}{L_E} \left[ \exp\left(\frac{eV_{BE}}{kT}\right) - 1 \right] \cdot \frac{1}{\tanh(x_E/L_E)} \quad (12.34a)$$

and

$$J_{nE} = \frac{eD_B n_{B0}}{L_B} \left\{ \frac{1}{\sinh(x_B/L_B)} + \frac{[\exp(eV_{BE}/kT) - 1]}{\tanh(x_B/L_B)} \right\} \quad (12.34b)$$

Positive  $J_{pE}$  and  $J_{nE}$  values imply that the currents are in the directions shown in Figure 12.19. If we assume that the B–E junction is biased sufficiently far in the forward bias so that  $V_{BE} \gg kT/e$ , then

$$\exp\left(\frac{eV_{BE}}{kT}\right) \gg 1$$

and also

$$\frac{\exp(eV_{BE}/kT)}{\tanh(x_B/L_B)} \gg \frac{1}{\sinh(x_B/L_B)}$$

The emitter injection efficiency, from Equation (12.32), then becomes

$$\gamma = \frac{1}{1 + \frac{p_{E0} D_E L_B}{n_{B0} D_B L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}} \quad (12.35a)$$

If we assume that all the parameters in Equation (12.35a) except  $p_{E0}$  and  $n_{B0}$  are fixed, then in order for  $\gamma \approx 1$ , we must have  $p_{E0} \ll n_{B0}$ . We can write

$$p_{E0} = \frac{n_i^2}{N_E} \quad \text{and} \quad n_{B0} = \frac{n_i^2}{N_B}$$

where  $N_E$  and  $N_B$  are the impurity doping concentrations in the emitter and base, respectively. Then the condition that  $p_{E0} \ll n_{B0}$  implies that  $N_E \gg N_B$ . For the emitter injection efficiency to be close to unity, the emitter doping must be large compared to the base doping. This condition means that many more electrons from the n-type emitter than holes from the p-type base will be injected across the B–E space charge region. If both  $x_B \ll L_B$  and  $x_E \ll L_E$ , then the emitter injection efficiency can be written as

$$\gamma \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} \quad (12.35b)$$

**Base Transport Factor** The next term to consider is the base transport factor, given by Equation (12.31b) as  $\alpha_T = J_{nC}/J_{nE}$ . From the definitions of the current directions shown in Figure 12.19, we can write

$$J_{nC} = (-)eD_B \left. \frac{d[\delta n_B(x)]}{dx} \right|_{x=x_B} \quad (12.36a)$$

and

$$J_{nE} = (-)eD_B \left. \frac{d[\delta n_B(x)]}{dx} \right|_{x=0} \quad (12.36b)$$

Using the expression for  $\delta n_B(x)$  given in Equation (12.15), we find that

$$J_{nC} = \frac{eD_B n_{B0}}{L_B} \left\{ \frac{[\exp(eV_{BE}/kT) - 1]}{\sinh(x_B/L_B)} + \frac{1}{\tanh(x_B/L_B)} \right\} \quad (12.37)$$

The expression for  $J_{nE}$  is given in Equation (12.34a).

If we again assume that the B–E junction is biased sufficiently far in the forward bias so that  $V_{BE} \gg kT/e$ , then  $\exp(eV_{BE}/kT) \gg 1$ . Substituting Equations (12.37) and (12.34b) into Equation (12.31b), we have

$$\alpha_T = \frac{J_{nC}}{J_{nE}} \approx \frac{\exp(eV_{BE}/kT) + \cosh(x_B/L_B)}{1 + \exp(eV_{BE}/kT) \cosh(x_B/L_B)} \quad (12.38)$$

In order for  $\alpha_T$  to be close to unity, the neutral base width  $x_B$  must be much smaller than the minority carrier diffusion length in the base  $L_B$ . If  $x_B \ll L_B$ , then  $\cosh(x_B/L_B)$  will be just slightly greater than unity. In addition, if  $\exp(eV_{BE}/kT) \gg 1$ , then the base transport factor is approximately

$$\alpha_T \approx \frac{1}{\cosh(x_B/L_B)} \quad (12.39a)$$



For  $x_B \ll L_B$ , we may expand the cosh function in a Taylor series, so that

$$\alpha_T \approx \frac{1}{\cosh (x_B/L_B)} \approx \frac{1}{1 + \frac{1}{2}(x_B/L_B)^2} \approx 1 - \frac{1}{2}(x_B/L_B)^2 \quad (12.39b)$$

The base transport factor  $\alpha_T$  will be close to one if  $x_B \ll L_B$ . We can now see why we indicated earlier that the neutral base width  $x_B$  would be less than  $L_B$ .

**Recombination Factor** The recombination factor is given by Equation (12.31c). We can write

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \approx \frac{J_{nE}}{J_{nE} + J_R} = \frac{1}{1 + J_R/J_{nE}} \quad (12.40)$$

We have assumed in Equation (12.40) that  $J_{pE} \ll J_{nE}$ . The recombination current density, due to the recombination in a forward-biased pn junction, was discussed in Chapter 8 and can be written as

$$J_R = \frac{ex_{BE}n_i}{2\tau_0} \exp\left(\frac{eV_{BE}}{2kT}\right) = J_{r0} \exp\left(\frac{eV_{BE}}{2kT}\right) \quad (12.41)$$

where  $x_{BE}$  is the B–E space charge width.

The current  $J_{nE}$  from Equation (12.34b) can be approximated as

$$J_{nE} = J_{s0} \exp\left(\frac{eV_{BE}}{kT}\right) \quad (12.42)$$

where

$$J_{s0} = \frac{eD_B n_{B0}}{L_B \tanh(x_B/L_B)} \quad (12.43)$$

The recombination factor, from Equation (12.40), can then be written as

$$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)} \quad (12.44)$$

The recombination factor is a function of the B–E voltage. As  $V_{BE}$  increases, the recombination current becomes less dominant and the recombination factor approaches unity.